Problem Section 7

Monday Feb 26 2024

Learning Outcomes

The problems are designed to build conceptual understanding and problem-solving skills. The emphasis is on learning to find, evaluate and build confidence. The specific tasks include:

- Calculate one and two-sided P-values
- Find the Type I and Type II errors for a given decision rule
- Find an empirical P-value
- Back up and support work with relevant explanations

Exercises

1. A children's game uses a six sided die with a picture of a ghost named Hugo on one side and numbers on the other sides. If the die is fair, the ghost should be rolled 1/6 of the time. You test the die by rolling it $n = 10$ times and and the ghost is rolled $x = 3$ times. Calculate the P-value for an exact binomial test of the hypothesis

$$H_0 : \pi = \frac{1}{6} \quad H_1 : \pi \neq \frac{1}{6}$$

2. As input for a new inflation model, economists predicted that the average cost of a hypothetical "food basket" in western WA in July would be \$145.75. The standard deviation (σ_0) of basket prices was assumed to be \$9.50, a figure that has held fairly constant over the years. To check their prediction, a sample of twenty-five baskets representing different parts of the region were checked in late July, and the average cost was \$149.75.

a. Let μ_0 denote the true mean price of the food basket in July in Western WA. Write the null and alternative hypothesis.

b. Suppose the test will be based on $\bar{X}$ the sample mean. What is its sampling distribution? (You may assume the CLT applies)

c. Calculate the P-value associated with $\bar{x} = \$149.75$.

3. An experimenter takes a sample of size 4 - X_1, X_2, X_3, X_4 - from the Poisson probability model,

$$f(x) = e^{-\lambda_0} \frac{\lambda_0^x}{x!} \quad x = 0, 1, 2, \ldots$$

and wishes to test $H0 : \lambda_0 = 6$ versus $H_1 : \lambda_0 < 6$. The test will be based on the statistic $S = X_1 + X_2 + X_3 + X_4$.

   a. Find the P-value associated with observing $s_{obs} = 15$.

   b. Suppose we decide to conduct the test at level $\alpha = 0.1$. What values of s_{obs} will you reject H_0 for?

   c. Find the Type I error rate for your test in part b.

d. Calculate the Type II error rate for your test in part b. when $\lambda_0 = 4$.

4. Suppose the following sample - denoted by $X_1, X_2, \ldots, X_n$ - are drawn from a $Norm(0, \sigma_0^2)$ distribution:

```
sample_df <- tibble(
        x= c(-0.58319935, -1.36090219,  0.38663763, -1.54365592,
              0.87083945, -0.69187830, 0.45898841, -2.82556635,
              0.01777137, -0.62753863 , 0.54611381, -1.39731591,
              -1.72584231,  0.91371529,  0.18096064, -0.53063107,
              -0.76604739, -1.97107704, 0.56394712,  1.13707563))
```

We want to test $H_0 : \sigma_0^2 = 1$ versus $\sigma_0^2 > 1$ using the sample variance:

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

as our test statistic.

a. Calculate s^2_{obs}, the observed value of the estimator S^2. Save it in a variable called `obs_s2`.

```
# calculate obs_s2, the observed value for S^2
```

b. Simulate the sampling distribution of S^2 under the null hypothesis. This means generate a new x assuming the null hypothesis is true and calculate s^2 for each such dataset. Fill in blanks in the code below and then remove the `eval=F` chunk option when knitting.

```
set.seed(2626)                          #random number seed
B <- 1000                               # number of replications
nsamp <- ___                            # sample size n
sigma2_null <- ___                      #null value

#repeatedly sample from a Norm(0,sigma2null) and calculate the
#sample variance s2star from each sample
null_sim_df <- tibble(
                s2star = replicate(n = ___, expr = ___)
)
```

c. Make a histogram of the values of `s2star` you have simulated under the null hypothesis, and mark the observed value with a vertical line.

d. Calculate the empirical P-value. (Hint: S^2 is an unbiased estimator of the true value of σ_0^2. Based on this fact, do large values, small values or both large and small values support H_1?)