Facultatea de Automatica si Calculatoare

Automatica si Informatica Aplicata

UNIVERSITATEA TEHNICĂ DIN CLUJ-NAPOCA

Proiect Teoria Sistemelor I

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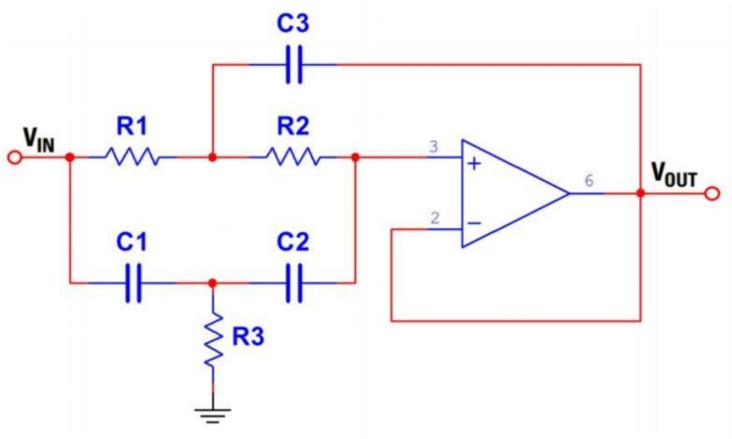
An universitar: 2020-2021

Filtru Sallen-Key de tip Notch



Condiţia efectului *notch*:

- R1 = R2 = R;
- R3 = R/2;
- *C1 = C2 = C;*
- *C3 = 2C.*

R1=8.2k, R2=700, R3 = 480k, C1=62*10^{-6}, C2 =6.8*10^{-6}F, C3 = 66*10^{-6}F

1. Modelul matematic al circuitului

$$x_1 = u_{C_1}$$

$$x_2 = u_{C_2}$$

$$x_3 = u_{C_3}$$

$$i_{C_1} = i_{C_2} + i_{R_3}$$

$$i_{R_3} = \frac{u_{R_3}}{R_3} = \frac{u - u_{C_1}}{R_3} = \frac{u - x_1}{R_3}$$

$$i_C = C \cdot \frac{d}{\mathrm{d}t} u_C \Rightarrow C_1 \cdot \dot{x_1} = C_2 \dot{x_2} + \frac{u - x_1}{R_3}$$

$$i_{C_2} + i_{R_2} = 0 \Rightarrow C_2 \dot{x_2} + \frac{u_{R_2}}{R_2} = 0$$

$$\dot{x_2} = -\frac{u_{R_2}}{R_2 C_2}$$

$$u_{R_2} = u_{C_3} \Rightarrow \dot{x_2} = -\frac{x_3}{R_2 C_2}$$

$$C_1 \dot{x_1} = C_2 \dot{x_2} + \frac{u - x_1}{R_3}$$

$$\Rightarrow C_1 \dot{x_1} = -\frac{x_3}{R_2} + \frac{u - x_2}{R_3}$$

$$\Rightarrow \boldsymbol{\dot{x_1}} = \frac{\mathbf{1}}{\boldsymbol{C_2}} \left(\frac{\boldsymbol{u - x_1}}{\boldsymbol{R_3}} - \frac{\boldsymbol{x_3}}{\boldsymbol{R_2}} \right)$$

$$i_{R_1} = i_{C_3} + i_{R_2}$$

$$\frac{u_{R_1}}{R_1} = C_3 \dot{x_3} + \frac{u_{R_2}}{R_2}$$

$$\Rightarrow \frac{x_1 + x_2 - x_3}{R_1} = C_3 \dot{x_3} + \frac{x_3}{R_2}$$

$$\Rightarrow \boldsymbol{\dot{x_3}} = \frac{\mathbf{1}}{\boldsymbol{C_3}} \left(\frac{\boldsymbol{x_1 + x_2 - x_3}}{\boldsymbol{R_1}} - \frac{\boldsymbol{x_3}}{\boldsymbol{R_2}} \right)$$

$$u = x_1 + u_{R_2} = x_1 + x_2 + y$$

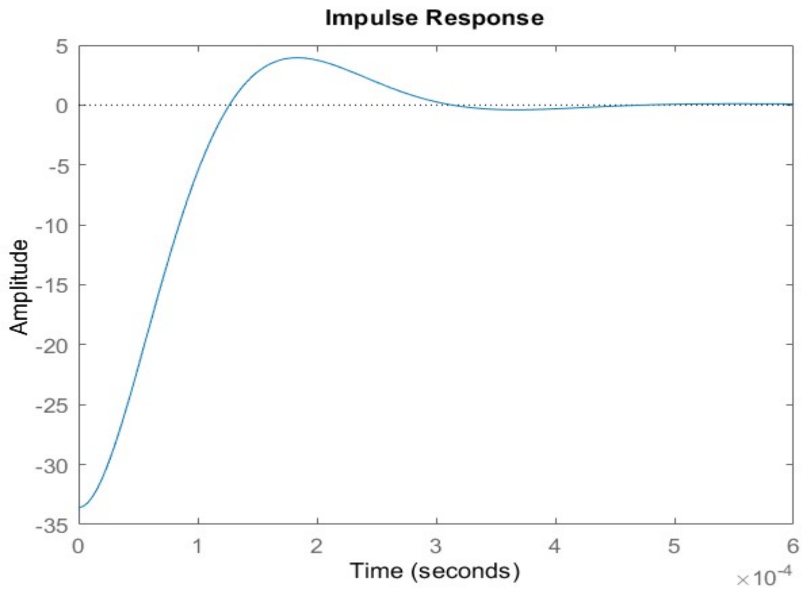
$$\Rightarrow \boldsymbol{y = u - x_1 - x_2}$$

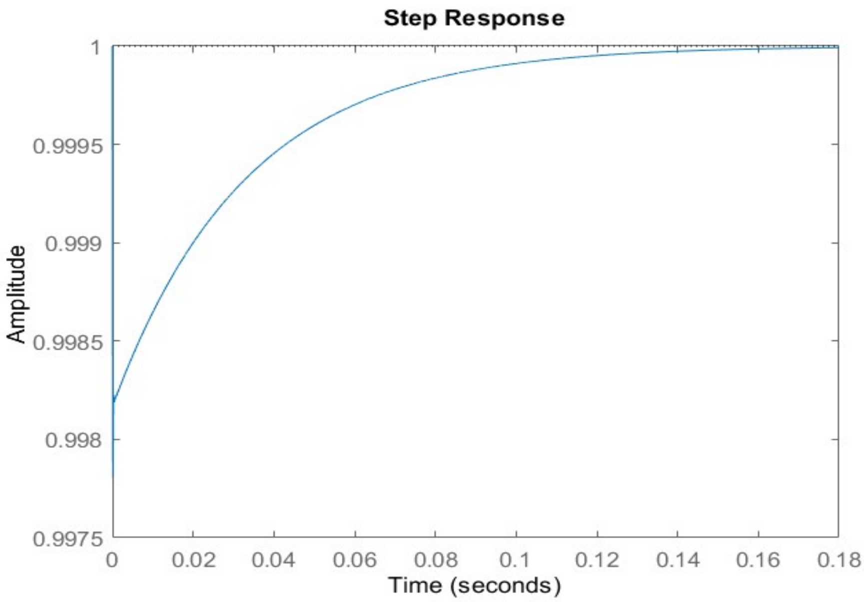
$$\dot{X} = \begin{pmatrix} -\frac{1}{R3C1} & 0 & -\frac{1}{R2C1} \\ 0 & 0 & -\frac{1}{R2C2} \\ \frac{1}{R1C3} & \frac{1}{R1C3} & -\frac{1}{C3}\left(\frac{1}{R1} + \frac{1}{R2}\right) \end{pmatrix} \cdot \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} + \begin{pmatrix} \frac{1}{R3C1} \\ 0 \\ 0 \end{pmatrix} \cdot u$$

$$y = \begin{pmatrix} -1 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} + (1) \cdot u$$

Pentru valorile date obtinem:

```
ans =

  A =
                 x1           x2           x3
   x1         -33.6            0   -2.304e+04
   x2             0            0   -2.101e+05
   x3          1848         1848   -2.349e+04

  B =
         u1
   x1  33.6
   x2     0
   x3     0
```





2. Functia de transfer

$$H(s) = C(s \cdot I - A)^{-1} \cdot B + D$$

$$\Rightarrow H(s) = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} s + \frac{1}{C_1R_3} & 0 & \frac{1}{C_1R_2} \\ 0 & s & \frac{1}{C_2R_2} \\ -\frac{1}{C_3R_1} & -\frac{1}{C_3R_1} & s + \frac{1}{C_3R_1} + \frac{1}{C_3R_2} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \frac{1}{C_1R_3} \\ 0 \\ 0 \end{bmatrix} + 1$$

$$\det(M) = \left(s + \frac{1}{C_1R_3}\right) \cdot s \cdot \left(s + \frac{1}{C_3R_1} + \frac{1}{C_3R_2}\right) + s \cdot \frac{1}{C_3C_1R_2R_1} + \left(s + \frac{1}{C_1R_3}\right) \cdot \frac{1}{C_3R_1} \cdot \frac{1}{C_2R_2} =$$

$$= s^3 + \frac{s^2}{C_3R_1} + \frac{s^2}{C_3R_2} + \frac{s^2}{C_1R_3} + \frac{s}{C_3C_1R_3R_1} + \frac{s}{C_3C_1R_3R_2} + \frac{s}{C_3C_1R_2R_1} + \frac{s}{C_3C_2R_2R_1} + \frac{1}{C_3C_2C_1R_3R_2R_1}$$

$$M^* = \begin{bmatrix} s + \frac{1}{C_1R_3} & 0 & -\frac{1}{C_3R_1} \\ 0 & s & -\frac{1}{C_3R_1} \\ \frac{1}{C_1R_2} & \frac{1}{C_2R_2} & s + \frac{1}{C_3R_1} + \frac{1}{C_3R_2} \end{bmatrix}$$

$$a_{11} = (-1)^{1+1} \cdot \det \begin{bmatrix} s & -\frac{1}{C_3R_1} \\ \frac{1}{C_2R_2} & s + \frac{1}{C_3R_1} + \frac{1}{C_3R_2} \end{bmatrix} = s^2 + \frac{s}{C_3R_1} + \frac{s}{C_3R_2} + \frac{1}{C_3C_2R_2R_1}$$

$$a_{12} = (-1)^{1+2} \cdot \det \begin{bmatrix} 0 & s \\ \frac{1}{C_1R_2} & s + \frac{1}{C_3R_1} + \frac{1}{C_3R_2} \end{bmatrix} = -\frac{1}{C_3C_1R_2R_1}$$

$$a_{13} = (-1)^{1+3} \cdot \det \begin{bmatrix} 0 & s \\ \frac{1}{C_1R_2} & \frac{1}{C_2R_2} \end{bmatrix} = -\frac{s}{C_1R_2}$$

$$a_{21} = (-1)^{2+1} \cdot \det \begin{bmatrix} 0 & -\frac{1}{C_3R_1} \\ \frac{1}{C_2R_2} & s + \frac{1}{C_3R_1} + \frac{1}{C_3R_2} \end{bmatrix} = -\frac{1}{C_3C_2R_2R_1}$$

$$a_{22} = (-1)^{2+2} \cdot \det \begin{bmatrix} s + \frac{1}{C_1R_3} & -\frac{1}{C_3R_1} \\ \frac{1}{C_1R_2} & s + \frac{1}{C_3R_1} + \frac{1}{C_3R_2} \end{bmatrix} = s^2 + \frac{s}{C_3R_1} + \frac{s}{C_2R_2} + \frac{s}{C_1R_3} + \frac{1}{C_3C_1R_3R_1} + \frac{1}{C_3C_1R_3R_1} + \frac{1}{C_3C_1R_3R_2} + \frac{1}{C_3C_1R_2R_1}$$

$$a_{23} = (-1)^{2+3} \cdot \begin{bmatrix} s + \frac{1}{C_1R_3} & 0 \\ \frac{1}{C_1R_2} & \frac{1}{C_2R_2} \end{bmatrix} = \frac{s}{C_2R_2} + \frac{1}{C_2C_1R_3R_2}$$

$$a_{31} = (-1)^{3+1} \cdot \det \begin{bmatrix} 0 & -\frac{1}{C_3R_1} \\ s & -\frac{1}{C_3R_1} \end{bmatrix} = \frac{s}{C_3R_1}$$

$$a_{32} = (-1)^{3+2} \cdot \det \begin{bmatrix} s + \frac{1}{C_1R_3} & -\frac{1}{C_3R_1} \\ 0 & -\frac{1}{C_3R_1} \end{bmatrix} = \frac{s}{C_3R_1} + \frac{1}{C_3C_1R_3R_1}$$

$$a_{33} = (-1)^{3+3} \cdot \det \begin{bmatrix} s + \frac{1}{C_1R_3} & 0 \\ 0 & s \end{bmatrix} = s^2 + \frac{s}{C_1R_3}$$

$$\Rightarrow M^* = \begin{bmatrix} s^2 + \frac{s}{C_3R_1} + \frac{s}{C_3R_2} + \frac{1}{C_3C_2R_2R_1} & -\frac{1}{C_3C_1R_2R_1} & -\frac{s}{C_1R_2} \\ -\frac{1}{C_3C_2R_2R_1} & s^2 + \frac{s}{C_3R_1} + \frac{s}{C_2R_2} + \frac{s}{C_1R_3} + \frac{1}{C_3C_1R_3R_1} + \frac{1}{C_3C_1R_3R_1} + \frac{1}{C_3C_1R_3R_2} + \frac{1}{C_3C_1R_2R_1} & \frac{s}{C_2R_2} + \frac{1}{C_2C_1R_3R_2} \\ \frac{s}{C_3R_1} & \frac{s}{C_3R_1} + \frac{1}{C_3C_1R_3R_1} & s^2 + \frac{s}{C_1R_3} \end{bmatrix}$$

$$\Rightarrow H(s) = \frac{-s^2 - \frac{s}{C_3C_1R_3R_1} - \frac{s}{C_3C_1R_3R_2} - \frac{1}{C_3C_2C_1R_3R_2R_1} + \frac{1}{C_3C_2C_1R_3R_2R_1}}{s^3 + \frac{s^2}{C_3R_1} + \frac{s^2}{C_3R_2} + \frac{s^2}{C_1R_3} + \frac{s}{C_3C_1R_3R_1} + \frac{s}{C_3C_1R_3R_2} + \frac{s}{C_3C_1R_2R_1} + \frac{s}{C_3C_2R_2R_1} + \frac{1}{C_3C_2C_1R_3R_2R_1}} + 1$$

$$H(s) = \frac{s^3 + s^2\left(\frac{1}{C_3R_1} + \frac{1}{C_3R_2}\right) + s\left(\frac{1}{C_3C_1R_2R_1} + \frac{1}{C_3C_2R_2R_1}\right) + \frac{1}{C_3C_2C_1R_3R_2R_1}}{s^3 + s^2\left(\frac{1}{C_1R_3} + \frac{1}{C_3R_1} + \frac{1}{C_3R_2}\right) + s\left(\frac{1}{C_3C_1R_3R_1} + \frac{1}{C_3C_1R_3R_2} + \frac{1}{C_3C_1R_2R_1} + \frac{1}{C_3C_2R_2R_1}\right) + \frac{1}{C_3C_2C_1R_3R_2R_1}}$$

Pentru valorile date obtinem:

$$H(s) = \frac{s^3 + 23492s^2 + 4307.5 \cdot 10^5 \cdot s + 130.4 \cdot 10^8}{s^3 + 23525s^2 + 4315.4 \cdot 10^5 \cdot s + 130.4 \cdot 10^8}$$

In Matlab am obtinut:

```
[num,den]=ss2tf(A,B,C,D)
H=tf(num,den)
```

```
H =

  s^3 + 2.349e04 s^2 + 4.308e08 s + 1.304e10
  ------------------------------------------
  s^3 + 2.353e04 s^2 + 4.315e08 s + 1.304e10
```

3. Singularitarile sistemului

Singularitatile sistemului reprezinta zerourile si polii sitemului. Zerourile sunt radacinile ecuatiei de la numarator, iar polii sunt radacinile ecuatiei de la numitor.

Zerourile:

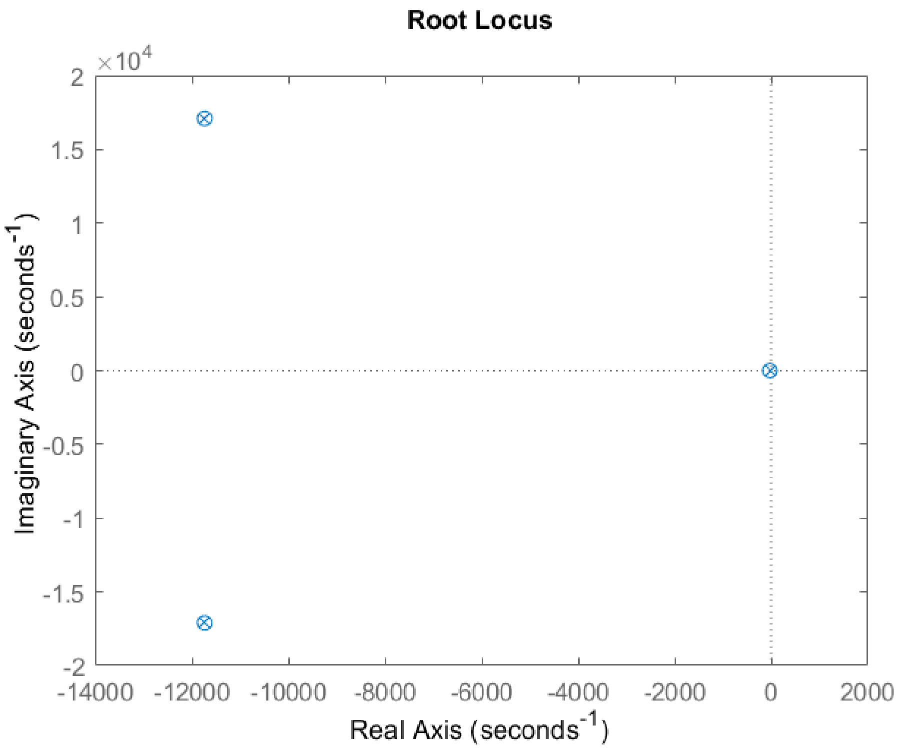
$s1 = -0.003 \cdot 10^4$

$s2,3 = -1.1731 \pm 1.71i \cdot 10^4$

Polii:

$s1 = -0.003 \cdot 10^4$

$s2,3 = -1.1748 \pm 1.7112i \cdot 10^4$



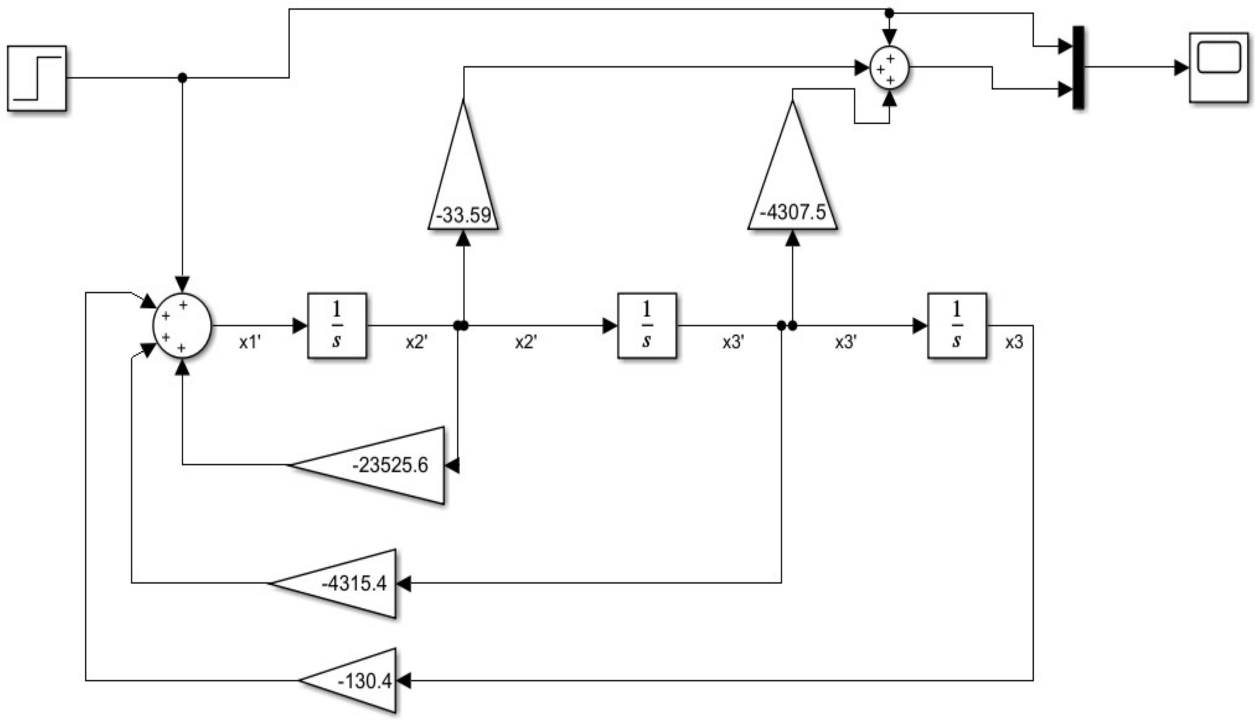
4. <u>Realizari de stare FCC si FCO</u>

<u>FCC</u>

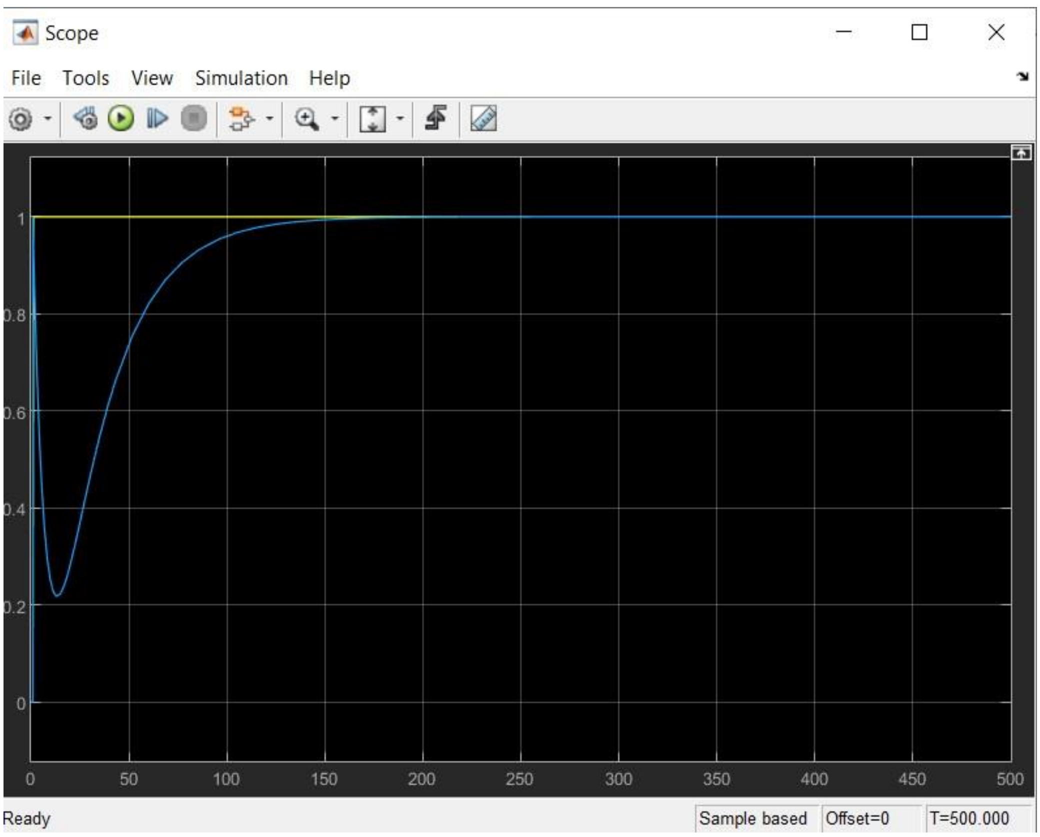
$$\begin{bmatrix} A_{FCC} & B_{FCC} \\ C_{FCC} & D_{FCC} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -(\frac{1}{R1C3}+\frac{1}{R2C3}+\frac{1}{R3C1}) & -\frac{1}{R1R3C1C3}+\frac{1}{R2R3C1C3}+\frac{1}{R1R2C1C3}+\frac{1}{R1R2C2C3} & -\frac{1}{R1R2R3C1C2C3} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} -\frac{1}{R3C1} & -(\frac{1}{R1R3C1C3}+\frac{1}{R2R3C1C3}) & 0 \end{bmatrix} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_{FCC} & B_{FCC} \\ C_{FCC} & D_{FCC} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -23525.6 & -4315.4 & -130.4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} -33.59 & -4307.5 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

Modelul Simulink al circuitului FCC:



-33.59
-4307.5
x1'
x2'
x2'
x3'
x3'
x3
-23525.6
-4315.4
-130.4



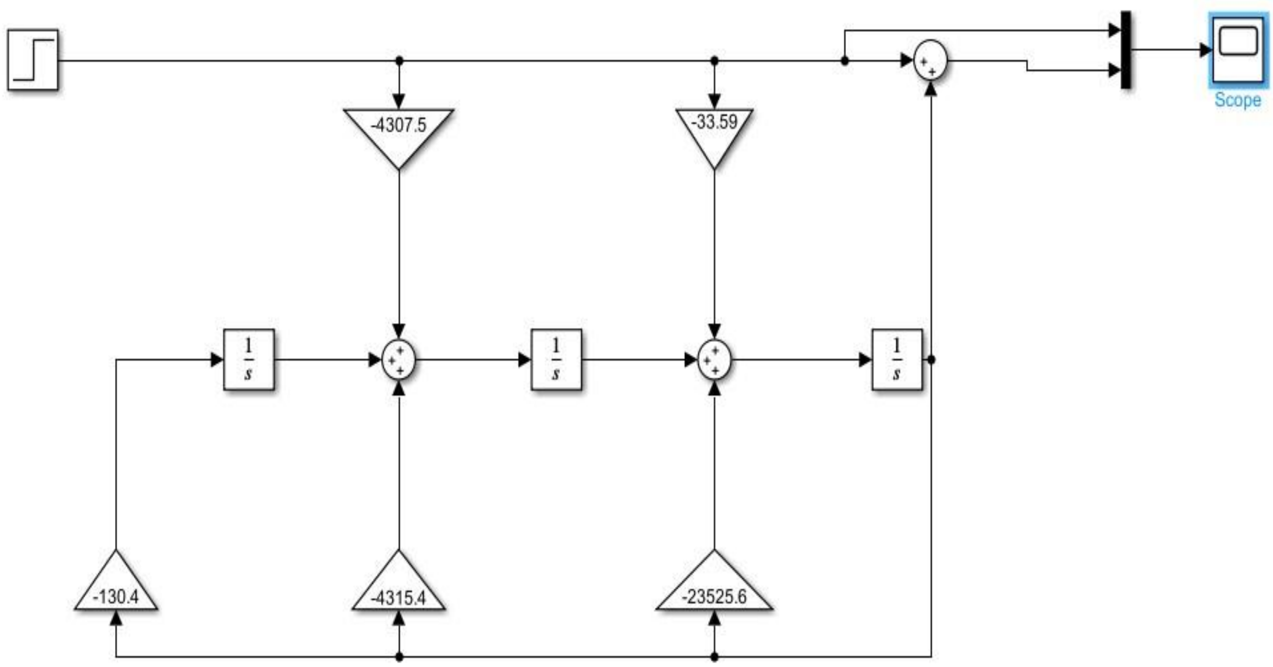
Scope
File Tools View Simulation Help
1
0.8
0.6
0.4
0.2
0
0 50 100 150 200 250 300 350 400 450 500
Ready
Sample based
Offset=0
T=500.000

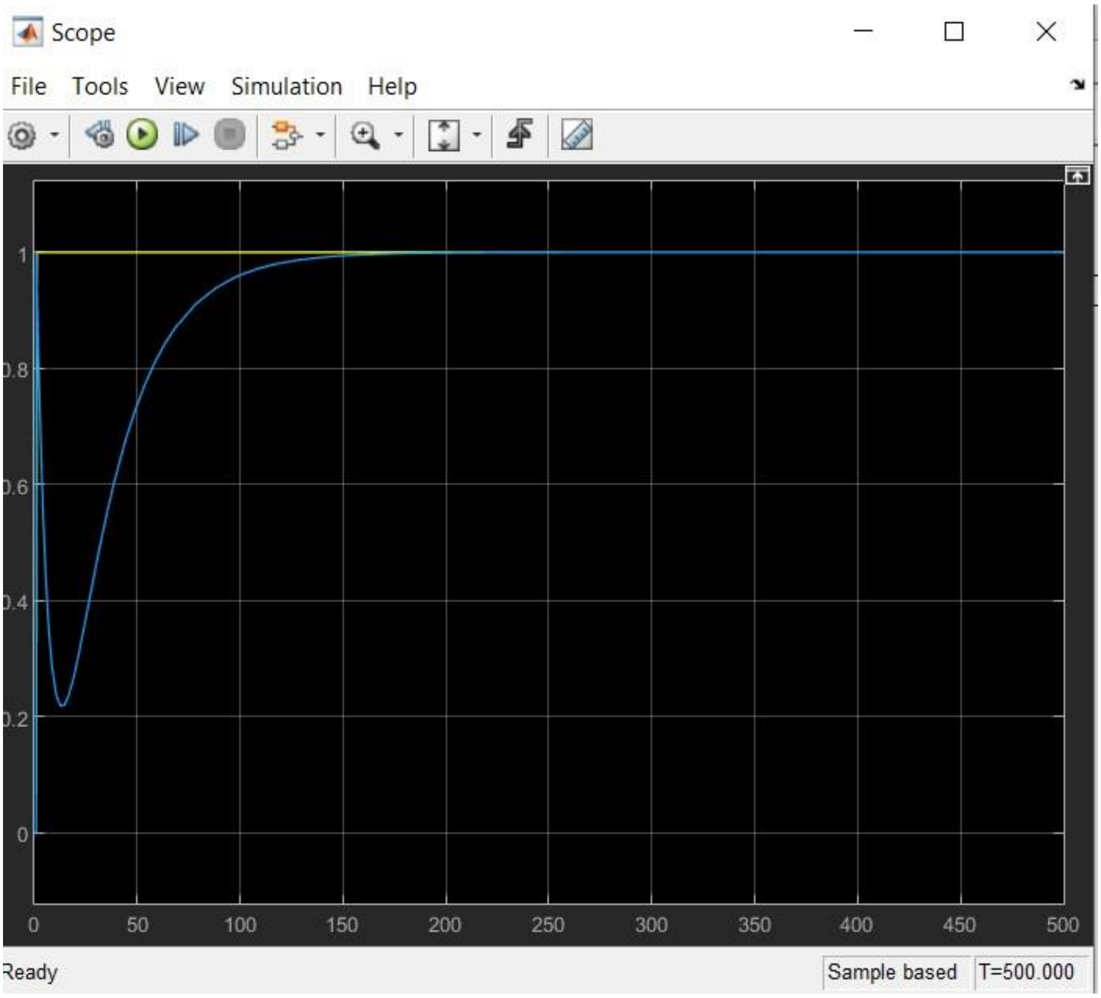
FCO

$$\begin{bmatrix} A_{\mathrm{FCO}} & B_{\mathrm{FCO}} \\ C_{\mathrm{FCO}} & D \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -\left(\frac{1}{C_3R_1}+\frac{1}{C_3R_2}+\frac{1}{C_1R_3}\right) & 1 & 0 \\ -\left(\frac{1}{C_3C_1R_3R_1}+\frac{1}{C_3C_1R_3R_2}+\frac{1}{C_3C_1R_2R_1}+\frac{1}{C_3C_2R_2R_1}\right) & 0 & 1 \\ -\frac{1}{R_3R_2R_1C_3C_2C_1} & 0 & 0 \end{bmatrix} & \begin{bmatrix} -\frac{1}{C_1R_3} \\ -\left(\frac{1}{C_3C_1R_3R_1}+\frac{1}{C_3C_1R_3R_2}\right) \\ 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_{FCO} & B_{FCO} \\ C_{FCO} & D_{FCO} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -23525.6 & 1 & 0 \\ -4315.4 & 0 & 1 \\ -130.4 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -33.59 \\ -4307.5 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

Modelul Simulink al circuitului FCO:





5. <u>Determinarea functiei de transfer in forma minimala</u>

Pentru determinarea formei minimale trebuie sa calculam parametrii Markov prin impartirea polinoamelor din functia de transfer.

Obținem parametrii Markov:

$$\gamma_0 = 1$$
$$\gamma_1 = 33.6$$
$$\gamma_2 = -0.01 \cdot 10^5$$
$$\gamma_3 = 238.8 \cdot 10^5$$
$$\gamma_4 = -4.3809 \cdot 10^{11}$$

$\gamma_5 = 1.0319 \cdot 10^{16}$

Cu care vom construi matricea Henkel

$$H_{3x3} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 \\ \gamma_2 & \gamma_3 & \gamma_4 \\ \gamma_3 & \gamma_4 & \gamma_5 \end{bmatrix}$$

$$H_{3X3} = \begin{bmatrix} 33.6 & -0.01 \cdot 10^5 & 238.8 \cdot 10^5 \\ -0.01 \cdot 10^5 & 238.8 \cdot 10^5 & -4.3809 \cdot 10^{11} \\ 238.8 \cdot 10^5 & -4.3809 \cdot 10^{11} & 1.0319 \cdot 10^{16} \end{bmatrix}$$

Calculăm determinantul matricii pentru a verifica dacă funcția este în formă minimală.

$\det(H3\ 3\square) \neq 0$=> rang=3

$\mathrm{rang}(H_{3x3}) = 3 =$ ordinul sistemului

$\Rightarrow H(s)$ este în formă minimală.

6. Stabilitatea sistemului cu ajutorul tabelului Ruth-Hurwitz

$$\det(s \cdot I - A) = s^3 + s^2\left(\frac{1}{C_1R_1} + \frac{1}{C_3R_2} + \frac{1}{C_3R_1}\right) + s\left(\frac{1}{C_3C_1R_3R_1} + \frac{1}{C_3C_1R_3R_2} + \frac{1}{C_3C_1R_2R_1} + \frac{1}{C_3C_2R_2R_1}\right) + \frac{1}{C_3C_2C_1R_3R_2R_1}$$

$\square\det = s^3 + 23525.6s^2 + 4315.5s + 130.4$

Rezolvam ecuatia $\det(s\,I - A) = 0$

$s^3 + 23525.6s^2 + 4315.5s + 130.4 = 0$

```
ans =

   1.0e+04 *

   -2.3525
   -0.0000
   -0.0000
```

$Re\{\gamma_{1,2,3}\} < 0 \Rightarrow Sistem\ stabil\ extern(1)$

$$\det(\gamma I - A) = det\begin{bmatrix} \gamma + \frac{1}{C1R3} & 0 & \frac{1}{C1R3} \\ 0 & \gamma & \frac{1}{C2R2} \\ -\frac{1}{C3R1} & -\frac{1}{C3R1} & \gamma + \frac{1}{C3R1} + \frac{1}{C3R2} \end{bmatrix}$$

$$= det\begin{bmatrix} \gamma + 33.6 & 0 & 33.6 \\ 0 & \gamma & 2.1008 \cdot 10^5 \\ -1848 \cdot 10^3 & -1848 \cdot 10^3 & \gamma + 2.3493 \cdot 10^4 \end{bmatrix}$$

$$= \gamma^3 + 2.3527 \cdot 10^4 \cdot \gamma^2 + 3.8823 \cdot 10^{11} \cdot \gamma + 1.3045 \cdot 10^{13}$$

Tabelul Ruth-Hurtwitz

γ^3	1	$3.8823 \cdot 10^{11}$
γ^2	$2.3527 \cdot 10^4$	$1.3045 \cdot 10^{13}$
γ^1	$3.8768 \cdot 10^{11}$	0
γ^0	$1.3045 \cdot 10^{13}$	0

$$c_1 = \frac{det\begin{bmatrix} 1 & 3.8823 \cdot 10^{11} \\ 2.3525 \cdot 10^4 & 1.3045 \cdot 10^{13} \end{bmatrix}}{2.3527 \cdot 10^4} = 3.8768 \cdot 10^{11}$$

$$c_2 = \frac{det\begin{bmatrix} 2.3525 \cdot 10^4 & 1.3045 \cdot 10^{13} \\ 3.8768 \cdot 10^{11} & 0 \end{bmatrix}}{3.8768 \cdot 10^{11}} = 1.3045 \cdot 10^{13}$$

Nu exista schimbari de semn pe prima coloana=> sistem stabil intern(2)

(1)(2)=>sistem stabil intern si extern

7. Determinarea stabilitatii interne prin ecuatia Lyapunov

```
Q = eye(length(A));
P= lyap(A',Q);
P
eig(P)
```

```
P =

    0.0268   -0.0029    0.0002
   -0.0029    0.0004   -0.0003
    0.0002   -0.0003    0.0022


ans =

    0.0000
    0.0023
    0.0271
```

$$P = \begin{pmatrix} 0.0268 & -0.0029 & 0.0002 \\ -0.0029 & 0.0004 & -0.0003 \\ 0.0002 & -0.0003 & 0.0022 \end{pmatrix}$$

Avem sistemul descris prin ecuatia: x(t)=Ax(t)⇔x=Ax

$V(x) = x^t P x,\ \ P = P^t$

=>$\dot{V(x)} = \dot{x}^t P x + x^t P \dot{x} = (x^t A^t) P x + x^t P (A x) = x^t (A^t P + P A) x$

=>$A^t P + P A < 0$

$A^t P + P A = -Q \qquad Q = Q^t > 0 \qquad Q = I_3$

$$A^t P + P A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$V(x) = x^t P x = [x1 \quad x2 \quad x3] \cdot P \cdot \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$$

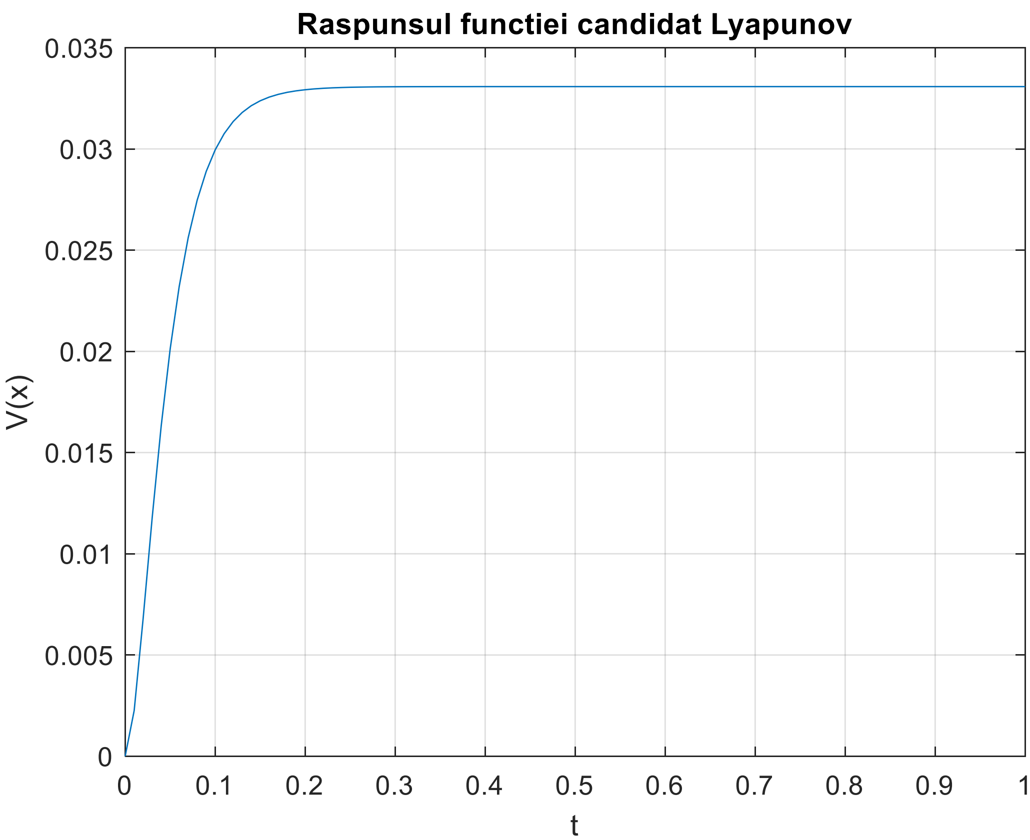
$$V(x) = [x1 \quad x2 \quad x3]\begin{bmatrix} 0.0268 & -0.0029 & 0.002 \\ -0.0029 & 0.0004 & -0.0003 \\ 0.0002 & -0.0003 & 0.0022 \end{bmatrix}\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$$
$$= \begin{bmatrix} 0.0268x1 - 0.0029x2 + 0.0002x3 \\ -0.0029x1 + 0.0004x2 - 0.0003x3 \\ 0.0002x1 - 0.0003x2 + 0.0022x3 \end{bmatrix}\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} =$$

$$= 0.0268x1^2 - 0.0029x2x1 + 0.0002x3x1 - 0.0029x1x2 + 0.0004x2^2 - 0.0003x3x2 + 0.0002x1x3 - 0.0003x2x3 + 0.0022x3^2 =$$

$= 0.0268x1^2 + 0.0004x2^2 + 0.0022x3^2 - 0.0058x1x2 + 0.0004x1x3 - 0.0006x2x3$ –functia de energie

$eig(P) = \{0\,;0.0023\,;0.0271\}$

Valorile propii ale matricei P sunt pozitive => sistemul este intern asimptotic stabil



8. Expresiile analitice ale celor trei raspunsuri

$$H(s)=\frac{s^3+23492s^2+4307.5\cdot 10^5\cdot s+130.4\cdot 10^8}{s^3+23525.6s^2+4315.4\cdot 10^5\cdot s+130.4\cdot 10^8}$$

Zerourile:
$s_1=-0.003\cdot 10^4$
$s_{2,3}=-1.173\pm 1.71i\cdot 10^4$

Polii:
$s_1=-0.003\cdot 10^4$
$s_{2,3}=-1.1748\pm 1.7112i\cdot 10^4$

$$H(s)=\frac{(s+30.33)(s^2+2.346\cdot 10^4s+4.3\cdot 10^8)}{(s+30.28)(s^2+2.35\cdot 10^4s+4.308\cdot 10^{8)}}$$

$$\frac{(s+30.33)(s^2+2.346\cdot 10^4s+4.3\cdot 10^8)}{(s+30.28)(s^2+2.35\cdot 10^4s+4.308\cdot 10^{8)}}=\frac{A}{s+30.28}+\frac{Bs+C}{s^2+2.35\cdot 10^4s+4.308\cdot 10^8}$$

$$=\frac{s^2A+2.35\cdot 10^4s\cdot A+4.308\cdot 10^8\cdot A+Bs^2+30.28Bs+Cs+30.28C}{(s+30.28)(s^2+2.35\cdot 10^4s+4.308\cdot 10^{8)}}$$

……

$$\Rightarrow H(s)=1-\frac{30.28}{s+30.28}+\frac{4.308\cdot 10^8}{s^2+2.35\cdot 10^4s+4.308\cdot 10^8}$$

$$h(t)=L^{-1}\{H(s)\}$$

$$y(t)=L^{-1}\left\{H(s)\cdot\frac{1}{s}\right\}$$

$$y_r(t)=L^{-1}\left\{H(s)\cdot\frac{1}{s^2}\right\}$$

Raspunsul pondere

$$h(t)=L^{-1}\{H(s)\}=\delta(t)-30.28\cdot e^{-30.28t}+25178.88619\cdot e^{-11750t}\cdot\sin(\sqrt{292737500}\,t)$$

Componenta tranzitorie: $-30.20\cdot e^{-30.28t}+25178.88619\cdot e^{-11750t}\cdot\sin\left(\sqrt{292737500}t\right)$

Componenta permanenta: 0

Raspunsul indicial

$$y(t)=L^{-1}\left\{H(s)\cdot\frac{1}{s}\right\}=$$

$$=-0.68675\cdot e^{-11750t}\cdot\sin\left(\sqrt{292737500}t\right)+1-\frac{30.28}{t+30.28}+\frac{4.308\cdot 10^4}{t^2+2.35\cdot 10^4\cdot t+4.308\cdot 10^8}+e^{-30.28t}-e^{-11750t}\cdot\cos(\sqrt{292737500}t)$$

Componenta tranzitorie: $-0.68675 \cdot e^{-11750t} \cdot \sin\left(\sqrt{292737500}t\right) + e^{-30.28t} - e^{-11750t} \cdot \cos(\sqrt{292737500}t)$

Componenta permanenta : $1 - \frac{30.28}{t+30.28} + \frac{4.308 \cdot 10^4}{t^2+2.35 \cdot 10^4 \cdot t+4.308 \cdot 10^8}$

Raspunsul rampa

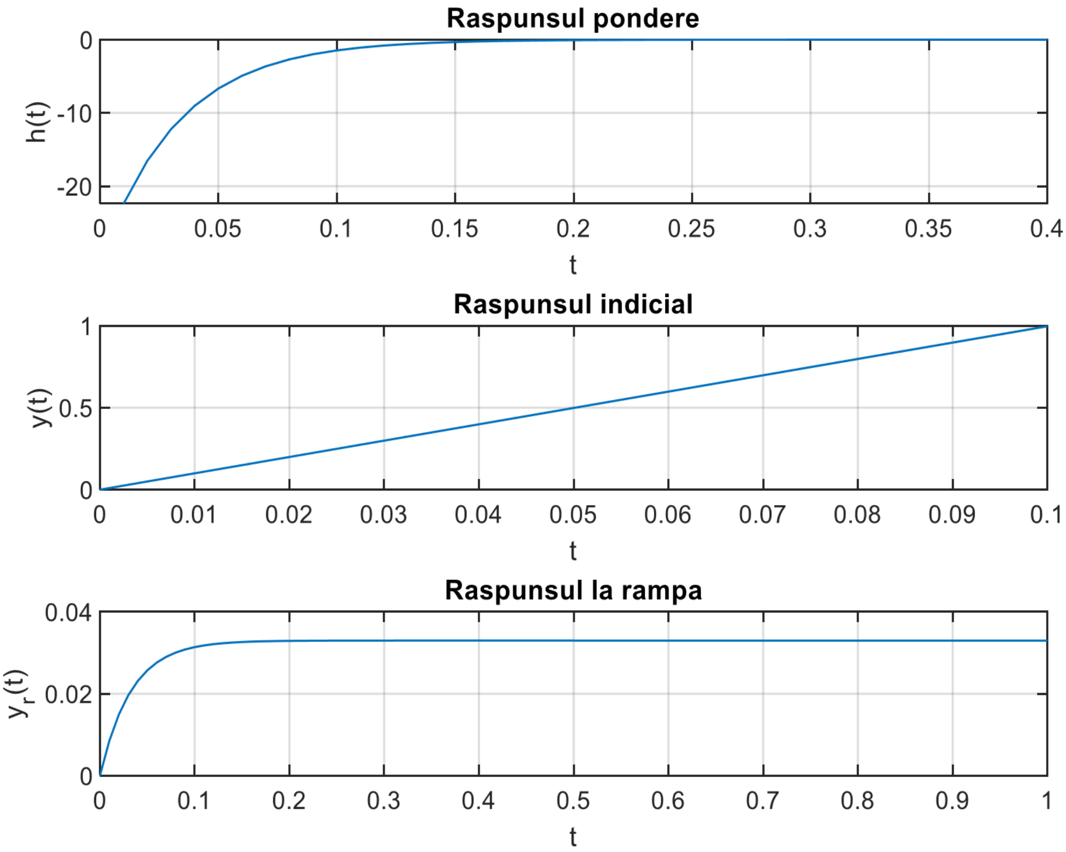
$$y_r(t) = L^{-1}\left\{H(s) \cdot \frac{1}{s^2}\right\} =$$

$$= t + 0.03297 - \frac{0.9983316}{t + 30.28} + \frac{0.14203476 \cdot 10^8}{t^2 + 2.35 \cdot 10^4 \cdot t + 4.308 \cdot 10^8} + 0.00005 \cdot e^{-11750t} \cdot \cos(17109.57334t) - 0.00002 \cdot e^{-11750t} \cdot \sin(17109.57334t) - 0.03302 \cdot e{-}30.28t$$

Componenta tranzitorie: $0.00005 \cdot e^{-11750t} \cdot \cos(17109.57334t) - 0.00002 \cdot e^{-11750t} \cdot \sin(17109.57334t) - 0.03302 \cdot e^{-30.28t}$

Componenta permanenta: $t + 0.03297 - \frac{0.9983316}{t+30.28} + \frac{0.14203476 \cdot 10^8}{t^2+2.35 \cdot 10^4 \cdot t+4.308 \cdot 10^8}$

```
t = 0:0.01:0.4; h = dirac(t) - 30.28.*exp(-
30.28.*t)+25178.88619.*exp(-
11750.*t).*sin(sqrt(292737500).*t);
figure; subplot(311);
plot(t, h); grid; xlabel('t'); ylabel('h(t)'); title("Raspunsul
pondere"); t = 0:0.1:0.1; y = -0.68675.*exp(-
11750*t).*sin(sqrt(292737500).*t) + 1-
[30.28\(t+30.28)]+
[4.308e+8\(t.*t+2.35e+4.*t+4.308e+8)]+exp(30.28.*t).*-exp(-
11750.*t).*cos(sqrt(292737500).*t); subplot(312);
plot(t, y); grid;xlabel('t'); ylabel('y(t)'); title("Raspunsul
indicial"); t = 0:0.01:1; yr = t+ 0.03297-
[0.9983316\(t+30.28)]+[0.14203476e+8\(t.*t+2.35e+4.*t+4.308e+8
)]+0.00005.*exp(-11750.*t).*cos(17109.57334.*t)-
0.00002.*exp(11750.*t).*sin(17109.57334.*t)-0.03302.*exp(-
30.28.*t); subplot(313);
plot(t, yr); grid; xlabel('t'); ylabel('y_{r}(t)');
title("Raspunsul la rampa");
```



9. Performantele sistemului

Utilizam functia zpk(H) si obtinem:

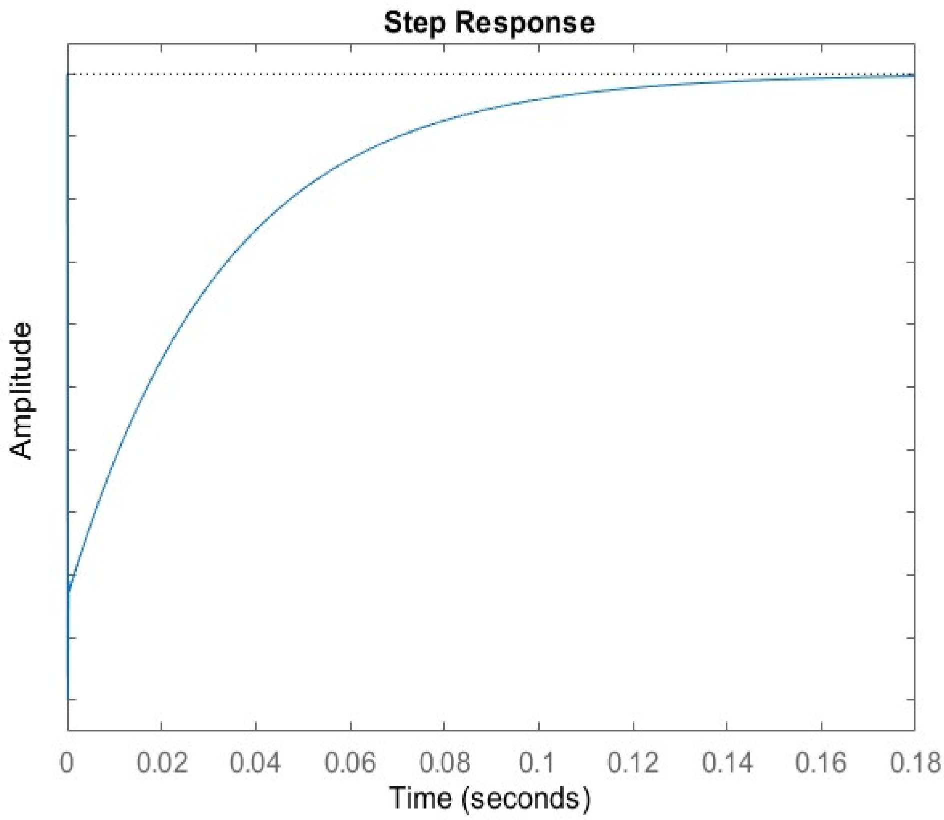
```
ans =

  (s+30.33) (s^2 + 2.346e04s + 4.3e08)
  -------------------------------------
  (s+30.28) (s^2 + 2.35e04s + 4.308e08)
```

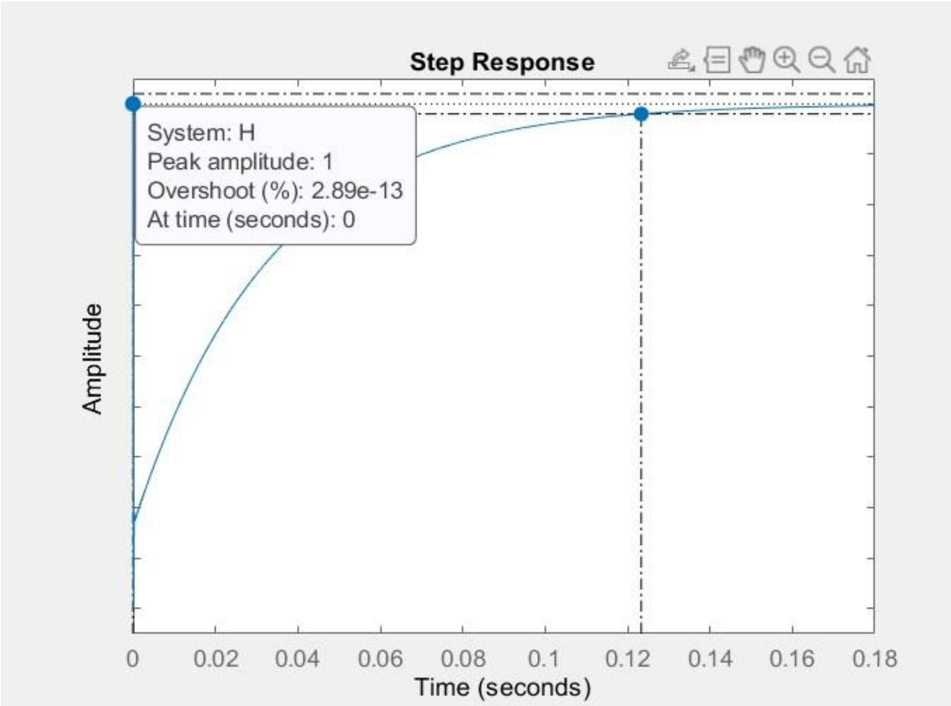
Deducem:

- Constanta de timp $k = H(0) = \frac{30.33 \cdot 4.3 \cdot 10^8}{30.28 \cdot 4.308 \cdot 10^8} = 0.999 \cong 1$
- $T_1 = \frac{1}{30.28} = 0.0331$
- $T_2 = \frac{1}{30.33} = 0.0329$
- Pulsatia naturala 1: $\omega_{n1} = \sqrt{4.308 \cdot 10^8}$
- Factorul de amortizare 1: $\zeta_1 = \frac{2.35 \cdot 10^4}{2 \cdot \sqrt{4.308 \cdot 10^8}} = 0.5661$

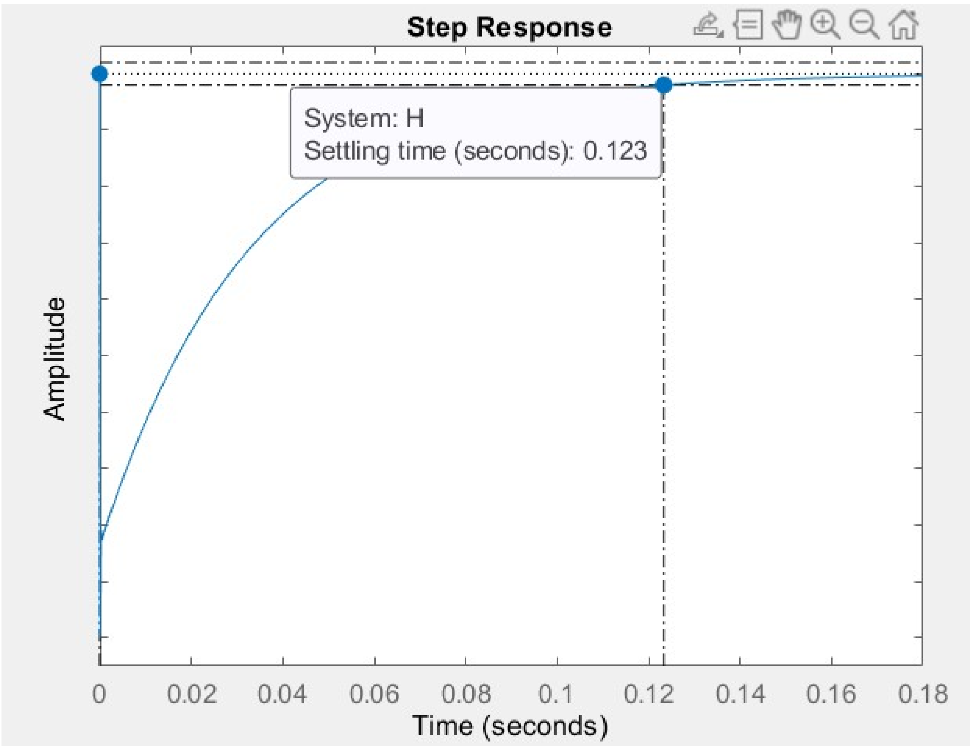
- Pulsatia naturala 2: $\omega_{n2} = \sqrt{4.3 \cdot 10^8}$
- Factorul de amortizare 2: $\text{ב}_2 = \frac{2.346 \cdot 10^4}{2 \cdot \sqrt{4.3 \cdot 10^8}} = 0.5656$
- Pulsatia de oscilatie: $\omega_{osc} = \omega_n \sqrt{1 - \text{ב}^2} = 1.7109 \cdot 10^4$
- Eroarea stationara la pozitei: $_{ssp} = \lim_{s \to 0}(1 - H(s)) = 1 - H(0) = 0$
- Eroarea stationara la viteza: $_{ssv} = \lim \frac{(1 - H(s))}{} = 2.376 \cdot 10^8 s \to 0 \qquad s$

Raspunsul la treapta:





- Suprareglajul $\sigma = 2.89 \cdot 10^{-13}\%$



- Timpul de raspuns: $t_r = 0.123\ secunde$ -este dat de polii dominanti

10. Sistem de reglare cu regulator proportional

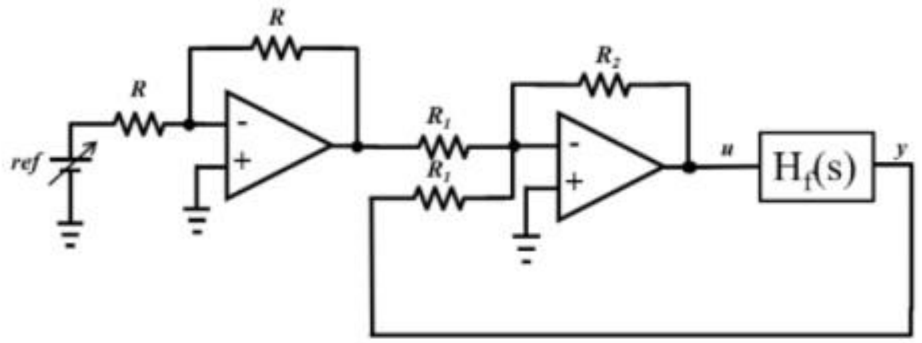


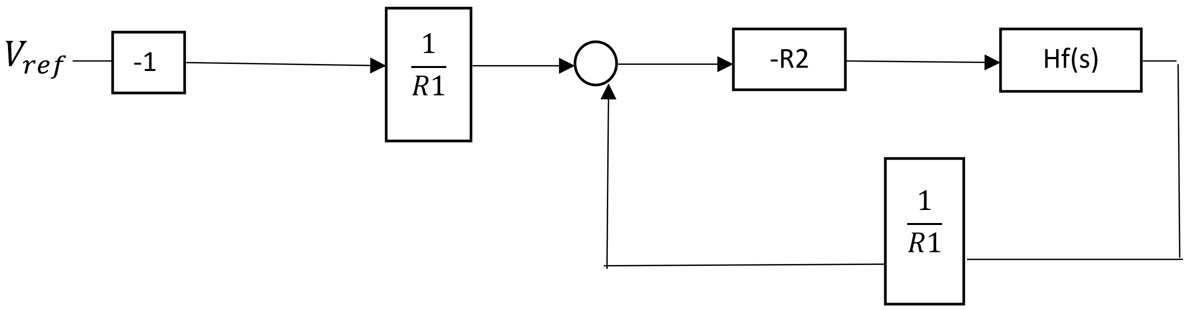
Figure 1: Structura unui sistem de reglare cu regulator proporţional

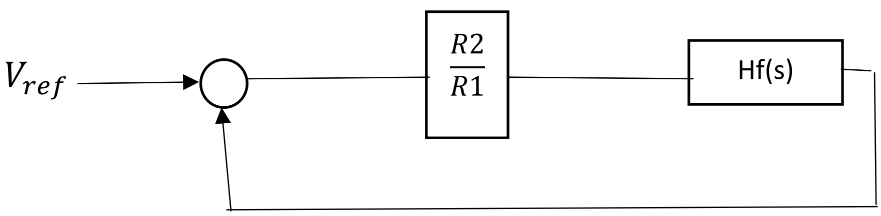
a) Functia de transfer

$$H_{des}(s) = H_f(s) =$$

$$= \frac{s^3 + \frac{2}{R_1 \cdot C_3} \cdot s^2 + \frac{2}{R_2^2 \cdot C_2 \cdot C_3} \cdot s + \frac{1}{R_2^2 \cdot C_2^2 \cdot R_3 \cdot C_3}}{s^3 + \left(\frac{2}{R_1 \cdot C_3} + \frac{1}{R_3 \cdot C_1}\right) \cdot s^2 + \left(\frac{2}{R_1 \cdot R_3 \cdot C_1 \cdot C_3} + \frac{2}{R_1^2 \cdot C_1 \cdot C_3}\right) \cdot s + \frac{1}{R_2^2 \cdot C_2^2 \cdot R_3 \cdot C_3}}$$

$$H_{des}(s) = H_f(s) = \frac{s^3 + 0.00369 \cdot 10^6 \cdot s^2 + 9.0945 \cdot 10^3 \cdot s + 1.393 \cdot 10^9}{s^3 + 3723.6 \cdot s^2 + 19.686 \cdot 10^6 \cdot s + 1.393 \cdot 10^9}$$





$V_+ = V_- = 0$

$\frac{V_{ref}-V_-}{Z_1} = \frac{V_- - V_{ref}}{Z_2}$ $\qquad H = \frac{V_{out}}{V_{ref}} = -\frac{Z_2}{Z_1}$

Functia de transfer pe cale de reactie este $H_r(s) = -85.3$

Functia de transfer pe cale de reactie este $H_d(s) = \frac{R2}{R1} \cdot H_f(s)$

Functia de transfer in bucla inchisa este $H_o(s) = \frac{\frac{R2}{R1} \cdot H_f(s)}{1+\frac{R2}{R1} \cdot H_f(s)}$

b) Trasarea si interpretarea LR ($k = \frac{R2}{R1}$)

Numar de poli: n=3

1.0e+03 *

$\hat{s}_1$ =-1.8259 + 4.0112i

$\hat{s}_2$ = -1.8259 - 4.0112i

$\hat{s}_3$ = -0.0717

Numar de zerouri: m=3

1.0e+03 *

$s_1{}^o$ = -1.7383 + 1.8720i

$s_2{}^o$ = -1.7383 - 1.8720i

$s_3{}^o$ = -0.2135

Putem afla numarul de asimptote => n-m= 0 asimptote

Unghiurile de plecare din poli:

$\Phi_{\widehat{sj}} = \sum L\widehat{s_j} - s_i^o - \sum L\widehat{s_j} - \widehat{s_l} - (2 \cdot l + 1) \cdot_{\pi}$

$\Phi_{\widehat{s1}} = L - 2.8259 + L - 0.8259 + 1.8720i + L - 0.8259 - 1.8720i - L - 2.8259$

$$- 4.0112i - L - 2.8259 + 4.0112i - (2 \cdot l + 1) \cdot \pi$$
$$= 180^o - 144.463^o - 35.537^o - 163.0725^o + 163.0725_o$$
$$- (2 \cdot l + 1) \cdot \pi = \pi$$

$$\Phi_{\widehat{s2}} = L - 1.8259 + 4.0112i + 0.2153 + L - 0.8259 + 1.8720i + L - 0.8259$$
$$- 1.8720i - L - 1.8259 + 4.0112i \pm 1.8259 + 4.0112i - L - 1.8259$$
$$+ 4.0112i + 0.0717 - (2 \cdot l + 1) \cdot \pi$$
$$= 45^o + 121.7595^o + 90^o + 16.9275^o - (2 \cdot l + 1) \cdot \pi = 331.9275$$

$$\Phi_{\widehat{s3}} = 2 \cdot \pi - \Phi_{\widehat{s2}} = 28.0725_o$$

Unghiurile de sosire in zerouri:

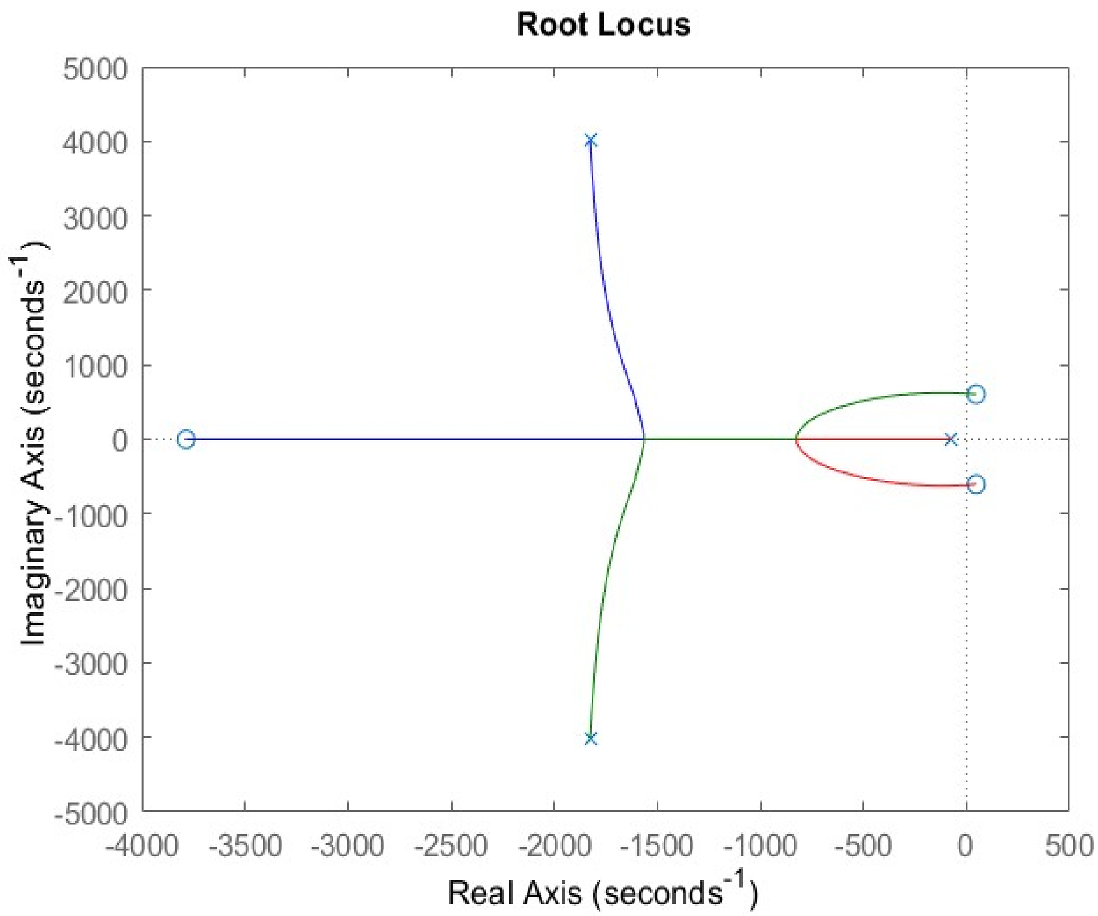
$$\Phi_{\widehat{sj}} = -\sum Ls_j^o - s_i^o - \sum Ls_j^o - \widehat{s_l} - (2 \cdot l + 1) \cdot \pi$$

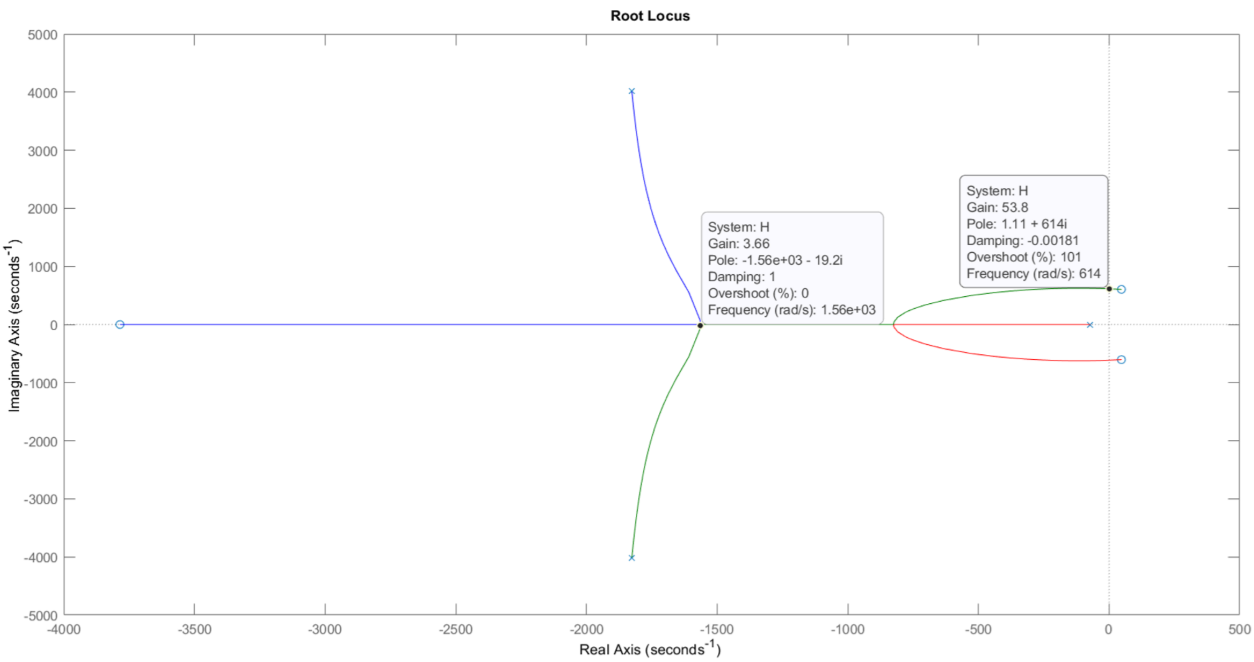
Puncte de apropiere/desprindere:

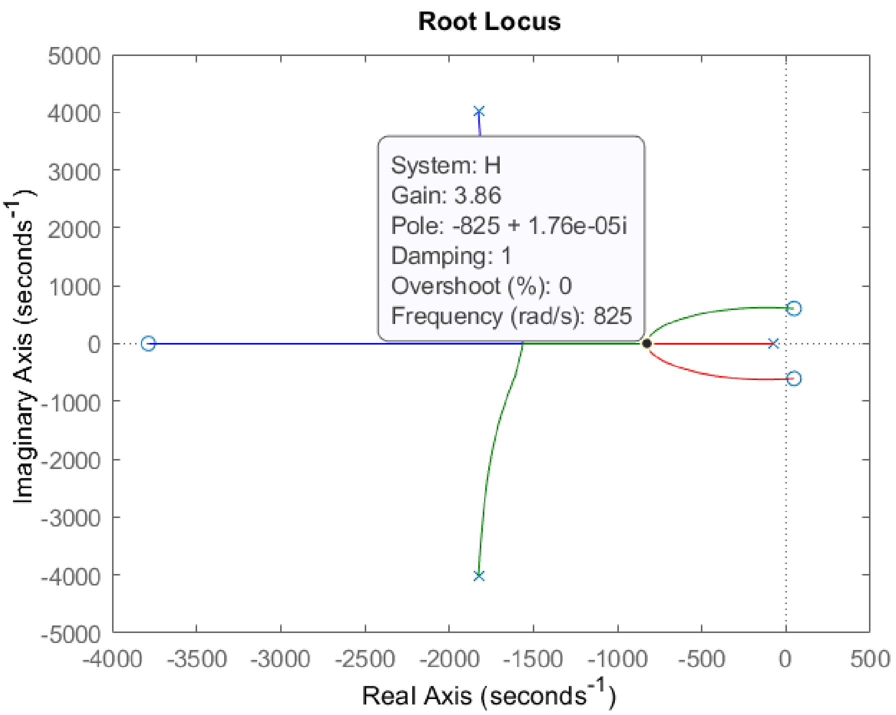
$$\begin{cases} 1 + \frac{R2}{R1} \cdot H_{des(s)} = 0 \\ \frac{d}{ds} \cdot H_{des}(s) = 0 \end{cases}$$

Intersectie cu axa imagianra:

$$s^3 + 3723.6 \cdot s^2 + 19.686 \cdot 10^6 \cdot s + 1.393 \cdot 10^9 + k$$
$$\cdot (s^3 + 0.00369 \cdot 10^6 \cdot s^2 + 9.0945 \cdot 10^3 \cdot s + 1.393 \cdot 10^9) = 0$$


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
5000
4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
-5000
-4000
-3500
-3000
-2500
-2000
-1500
-1000
-500
0
500


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
System: H
Gain: 3.66
Pole: -1.56e+03 - 19.2i
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 1.56e+03
System: H
Gain: 53.8
Pole: 1.11 + 614i
Damping: -0.00181
Overshoot (%): 101
Frequency (rad/s): 614



$k_{cr} \approx 53.8$

$k_{apr} \approx 3.86$

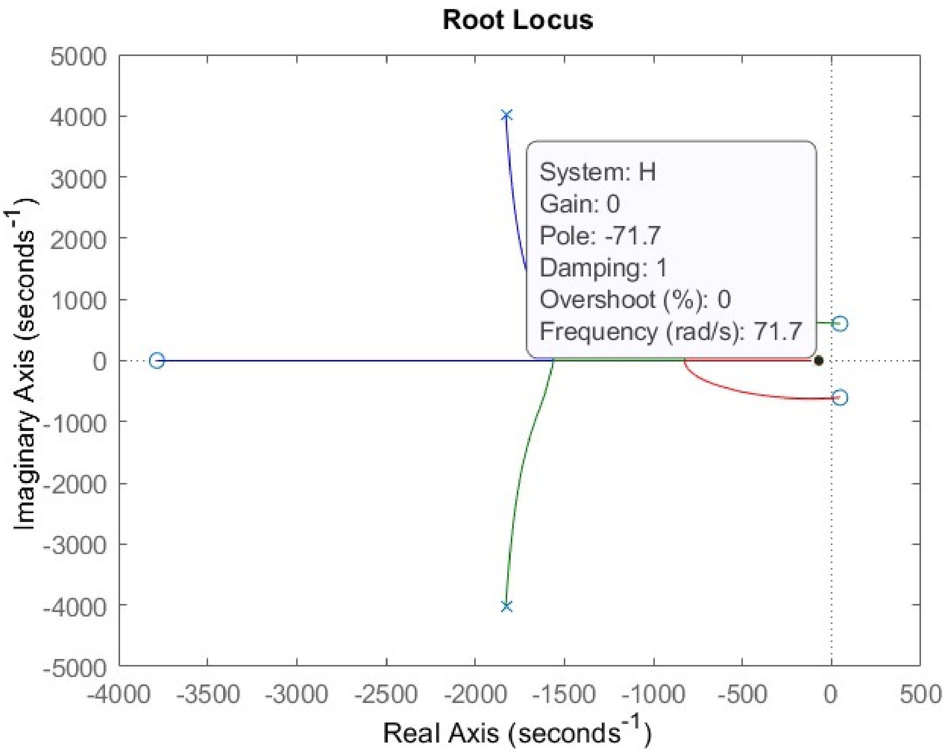
Stabilitate: Sistemul este stabil pentru $k \in (0, k_{cr})$

Moduri si regimuri de functionare:

- $k \in (0, 3.66)$: regim aperiodic amortizat cu modul $e^{\hat{s}_1 t}, e^{\hat{s}_2 t}, e^{\hat{s}_3 t}$
- $k = 3.66$: regim aperiodic critic amortizat cu modurile $e^{-3.66t}, te^{-3.66t}$
- $k \in (3.66, 53.8)$: regim oscilant amortizat cu modurile $e^{Re(\hat{s}_{1,}s_2)t}\sin(Im(s_{1,}s_2)t), e^{\hat{s}_3 t}$
- $k = 53.8$: regim oscilant intretinut cu modurile $\sin(Im(s_1)t), \sin(Im(s_2)t), e^{\hat{s}_3 t}$
- $k \in (53.8, \infty)$: regim oscilant neamortizat cu modurile $e^{Re(\hat{s}_{1,}s_2)t}\sin(Im(s_{1,}s_2)t), e^{\hat{s}_3 t}$

Sensibilitatea este mare: se schimba stabilitatea dupa k, se schimba regimurile dupa k si polii dominati parcurg un drum infinit.

c) c1) Pentru ca suprareglaj sa fie minim, factorul de amortizare trebuie sa fie maxim – k=0



c2)Pentru a avea cel mai mic timp de raspuns, $\xi\omega$ trebuie sa fie maxim si valoarea polului sa fie cat mai mare sau partea reala a polilor complexi conjugati sa fie maxima.

11. Sistem de reglare cu regulator de tip Lead/Lag

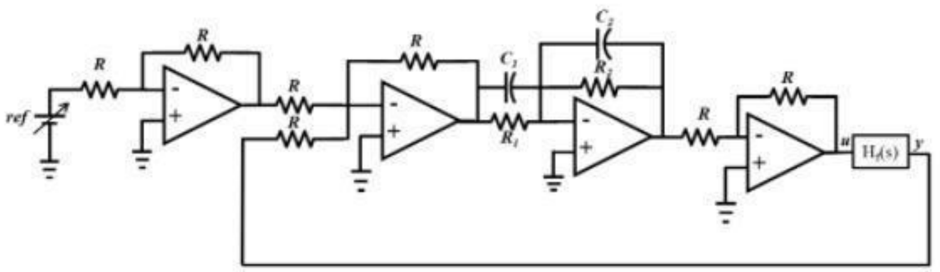


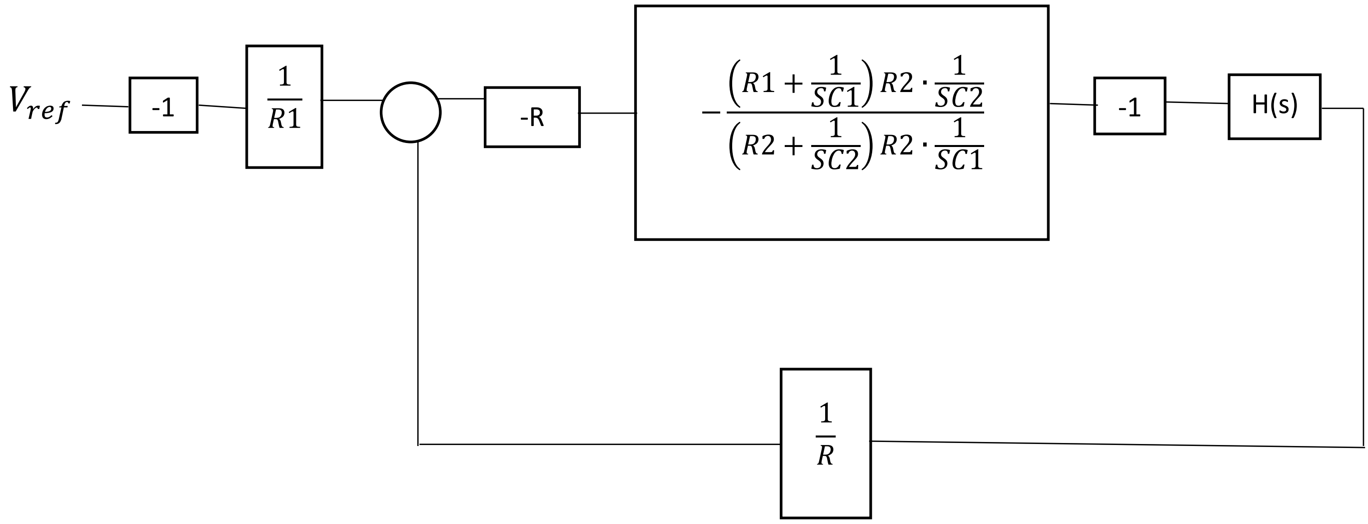
Figure 2: Structura unui sistem de reglare cu regulator de tip *Lead/Lag* (cu avans/întârziere de fază)

a)Functia de transfer:

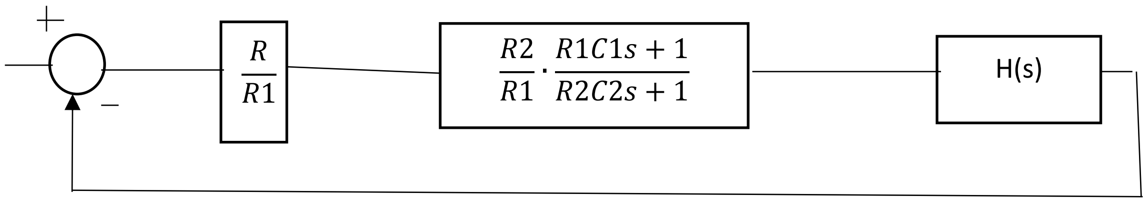
$$H_{des}(s) = H_f(s)$$

$$= \frac{s^3 + \frac{2}{R1C3}\cdot s^2 + \frac{2}{R2^2C2C3}\cdot s + \frac{1}{R2^2C2^2R3C3}}{s^3 + \left(\frac{2}{R1C3} + \frac{1}{R3C1}\right)\cdot s^2 + \left(\frac{2}{R1R3C1C3} + \frac{2}{R1^2C1C3}\right)\cdot s + \frac{1}{R2^2C2^2R3C3}}$$

$$H_{des}(s) = H_f(s) = \frac{s^3 + 0.00369 \cdot 10^6 \cdot s^2 + 9.0945 \cdot 10^3 \cdot s + 1.393 \cdot 10^9}{s^3 + 3723.6 \cdot s^2 + 19.686 \cdot 10^6 \cdot s + 1.393 \cdot 10^9}$$



$$-H = \frac{Z2}{Z1} = -\frac{\frac{R2\cdot\frac{1}{sC2}}{R2+\frac{1}{sC2}}}{\frac{R1\cdot\frac{1}{sC1}}{R1+\frac{1}{sC1}}} = \frac{\left(R1+\frac{1}{sC1}\right)R2\cdot\frac{1}{sC2}}{\left(R2+\frac{1}{sC2}\right)R1\cdot\frac{1}{sC1}}$$



Functia de transfer pe cale de reactie: $H_r(s) = 1$

Functia de transfer pe cale directa: $H_d(s) = \frac{R}{R2}\cdot\frac{R2C2s+1}{R1C1s+1}\cdot H_f(s)$

Functia de transfer in bucla inchisa: $H_o(s) = \frac{\frac{R}{R2}\cdot\frac{R2C2s+1}{R1C1s+1}\cdot H_f(s)}{1+\frac{R}{R2}\cdot\frac{R2C2s+1}{R1C1s+1}\cdot H_f(s)}$

b) Functia de transfer

$$H_o(s) =$$

$$= \frac{\frac{R}{R_2}\cdot\frac{R_2\cdot C_2\cdot s+1}{R_1\cdot C_1\cdot s+1}\cdot\frac{s^3+\frac{2}{R_1\cdot C_3}\cdot s^2+\frac{2}{R_2^2\cdot C_2\cdot C_3}\cdot s+\frac{1}{R_2^2\cdot C_2^2\cdot R_3\cdot C_3}}{s^3+\left(\frac{2}{R_1\cdot C_3}+\frac{1}{R_3\cdot C_1}\right)\cdot s^2+\left(\frac{2}{R_1\cdot R_3\cdot C_1\cdot C_3}+\frac{2}{R_1^2\cdot C_1\cdot C_3}\right)\cdot s+\frac{1}{R_2^2\cdot C_2^2\cdot R_3\cdot C_3}}}{1+\frac{R}{R_2}\cdot\frac{R_2\cdot C_2\cdot s+1}{R_1\cdot C_1\cdot s+1}\cdot\frac{s^3+\frac{2}{R_1\cdot C_3}\cdot s^2+\frac{2}{R_2^2\cdot C_2\cdot C_3}\cdot s+\frac{1}{R_2^2\cdot C_2^2\cdot R_3\cdot C_3}}{s^3+\left(\frac{2}{R_1\cdot C_3}+\frac{1}{R_3\cdot C_1}\right)\cdot s^2+\left(\frac{2}{R_1\cdot R_3\cdot C_1\cdot C_3}+\frac{2}{R_1^2\cdot C_1\cdot C_3}\right)\cdot s+\frac{1}{R_2^2\cdot C_2^2\cdot R_3\cdot C_3}}}$$

$$= \frac{\frac{R}{R_2}\cdot\frac{R_2\cdot C_2\cdot s+1}{R_1\cdot C_1\cdot s+1}\cdot\frac{s^3+0.00369\cdot 10^6\cdot s^2+9.0945\cdot 10^3\cdot s+1.393\cdot 10^9}{s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9}}{1+\frac{R}{R_2}\cdot\frac{R_2\cdot C_2\cdot s+1}{R_1\cdot C_1\cdot s+1}\cdot\frac{s^3+0.00369\cdot 10^6\cdot s^2+9.0945\cdot 10^3\cdot s+1.393\cdot 10^9}{s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9}}$$

$$= \frac{R\cdot(R_2\cdot C_2\cdot s+1)\cdot(s^3+0.00369\cdot 10^6\cdot s^2+9.0945\cdot 10^3\cdot s+1.393\cdot 10^9)}{R_2\cdot(R_1\cdot C_1\cdot s+1)\cdot(s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9)+s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9}$$

c)Trasarea si interpretarea locului radacinilor

Fie $R1 = 10k\Omega = 100\Omega$, R=200Ω, R2=2Ω, C2=1mF=10^-3F, C1=1mF=10^-3F

$$H = \frac{\frac{200}{R_2}\cdot\frac{R_2\cdot C_2\cdot s+1}{100\cdot s+1}\cdot\frac{s^3+0.00369\cdot 10^6\cdot s^2+9.0945\cdot 10^3\cdot s+1.393\cdot 10^9}{s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9}}{1+\frac{200}{R_2}\cdot\frac{R_2\cdot C_2\cdot s+1}{100\cdot s+1}\cdot\frac{s^3+0.00369\cdot 10^6\cdot s^2+9.0945\cdot 10^3\cdot s+1.393\cdot 10^9}{s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9}}$$

$$= \frac{\frac{2}{R_2}\cdot\frac{R_2\cdot C_2\cdot s+1}{s+0.01}\cdot\frac{s^3+0.00369\cdot 10^6\cdot s^2+9.0945\cdot 10^3\cdot s+1.393\cdot 10^9}{s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9}}{1+\frac{2}{R_2}\cdot\frac{R_2\cdot C_2\cdot s+1}{s+0.01}\cdot\frac{s^3+0.00369\cdot 10^6\cdot s^2+9.0945\cdot 10^3\cdot s+1.393\cdot 10^9}{s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9}}$$

$$H = \frac{T_1\cdot\frac{(s+\frac{1}{2\cdot 10^{-3}})\cdot(2\cdot s^3+2\cdot 0.00369\cdot 10^6\cdot s^2+2\cdot 9.0945\cdot 10^3\cdot s+2\cdot 1.393\cdot 10^9)}{2\cdot(s+0.01)(s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9)}}{1+T_1\cdot\frac{(s+\frac{1}{2\cdot 10^{-3}})\cdot(2\cdot s^3+2\cdot 0.00369\cdot 10^6\cdot s^2+2\cdot 9.0945\cdot 10^3\cdot s+2\cdot 1.393\cdot 10^9)}{(s+0.01)(s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9)}}$$

$$H = \frac{T_1\cdot\frac{(s+500)\cdot(2\cdot s^3+2\cdot 0.00369\cdot 10^6\cdot s^2+2\cdot 9.0945\cdot 10^3\cdot s+2\cdot 1.393\cdot 10^9)}{2\cdot(s+0.01)(s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9)}}{1+T_1\cdot\frac{(s+500)\cdot(2\cdot s^3+2\cdot 0.00369\cdot 10^6\cdot s^2+2\cdot 9.0945\cdot 10^3\cdot s+2\cdot 1.393\cdot 10^9)}{(s+0.01)(s^3+3723.6\cdot s^2+19.686\cdot 10^6\cdot s+1.393\cdot 10^9)}}$$

$$H = \frac{T_1\cdot\frac{2s^4+8380s^3+3708189s^2+2.7951\cdot 10^9\cdot s+1393\cdot 10^9}{2s^4+7447.22s^3+3.9372\cdot 10^7\cdot s^2+6.7230\cdot 10^9\cdot s+0.0278\cdot 10^9}}{1+T_1\cdot\frac{2s^4+8380s^3+3708189s^2+2.7951\cdot 10^9\cdot s+1393\cdot 10^9}{s^4+3723.61s^3+1.9686\cdot 10^7\cdot s^2+1.3932\cdot 10^9\cdot s+0.01393\cdot 10^9}}$$

```
H =

  s^4 + 500 s^3 + 0.9674 s^2 + 0.000568 s + 4.542e-06
  ----------------------------------------------------
   s^4 - 1e-06 s^3 + 2.906e06 s^2 - 48480 s + 136900
```

Polii:

1.0e+03 *

-0.0000 + 1.7047i

-0.0000 - 1.7047i

0.0000 + 0.0002i

0.0000 - 0.0002i

Zerouri:

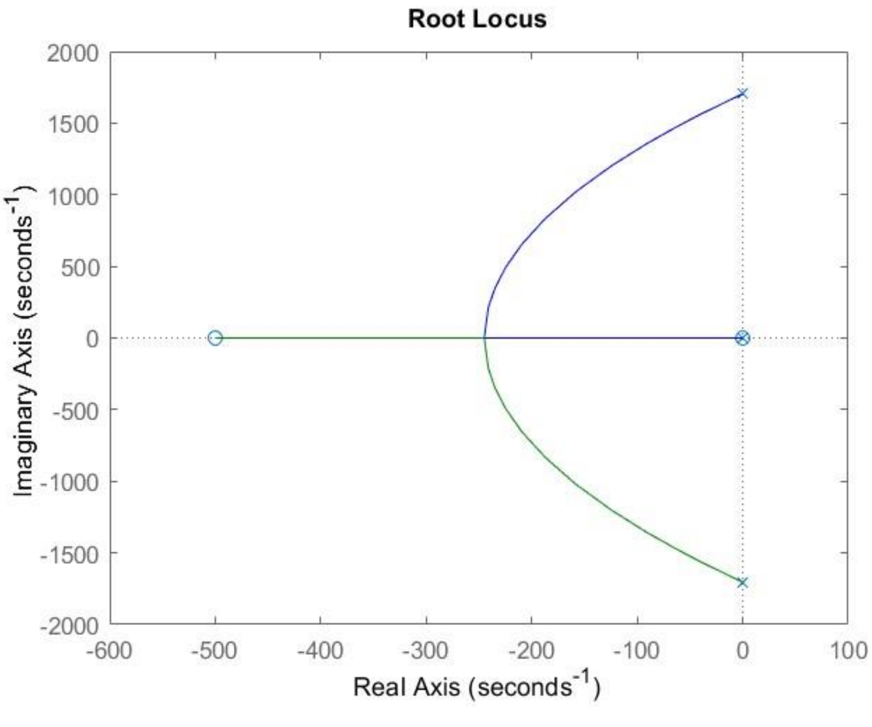
1.0e+02 *

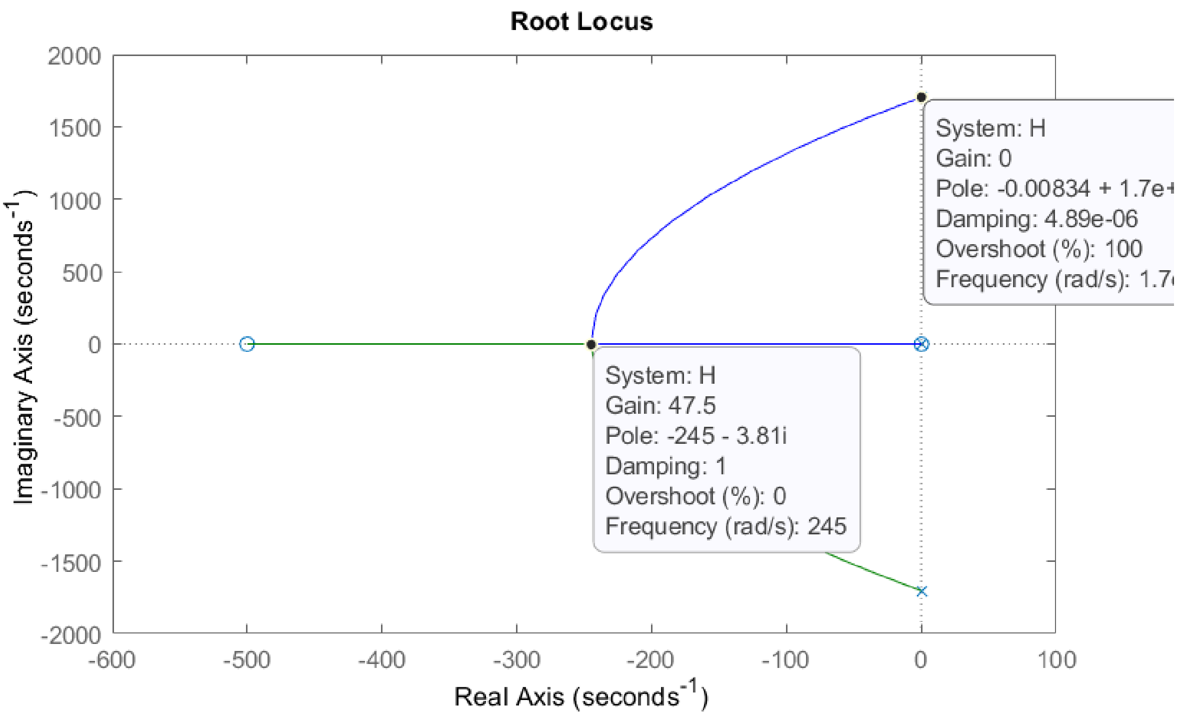
-5.0000 + 0.0000i

-0.0000 + 0.0000i

0.0000 + 0.0000i

0.0000 - 0.0000i





$k_{apr} \approx 47.5$

$k_{cr} = 0$

Stabilitate: Sistemul este stabil pentru $k \in (0, k_{cr})$

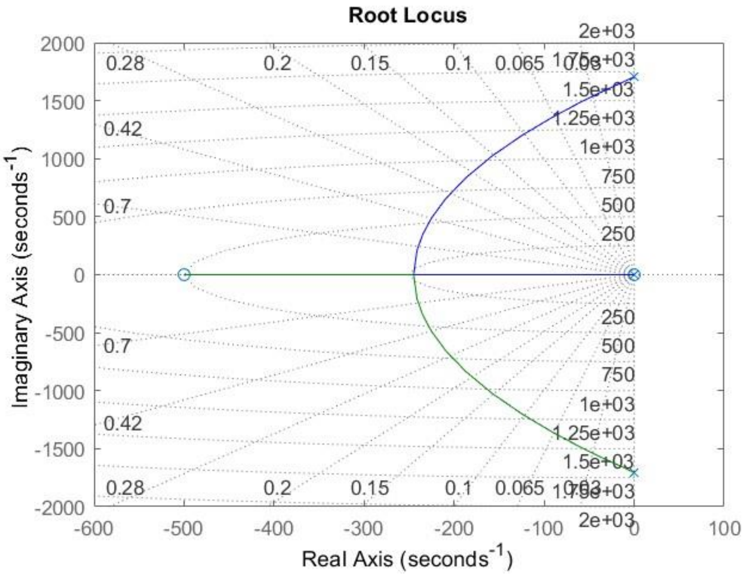
Moduri si regimuri de functionare:

- k=0 : regim oscilant intretinut cu modurile $\sin(Im(\hat{s}_1))$, $\sin(Im(\hat{s}_2))$
- $k \in (0, 47.5)$: regim oscilant amortizat cu modurile $e^{Re(\hat{s}1,s\hat{2})t}\sin(Im(\hat{s}_1,\hat{s}_2)t)$, $e^{\hat{s}3t}$
- $k = 47.5$: regim aperiodic critic amortizat cu modurile $e^{47.5t}$, $te^{47.5t}$
- $k \in (47.5\ ,\infty)$: regim aperiodic amortizat cu modurile $e^{\hat{s}1t}$, $e^{\hat{s}2t}$, $e^{\hat{s}3t}$, $e^{\hat{s}4t}$

Sensibilitatea este mare: se schimba stabilitatea dupa k, se schimba regimurile dupa k si polii dominati parcurg un drum infinit.

d) d1)Pentru a avea pulsatia de oscilatie maxima partea imaginara a polilor complecsi trebuie sa fie maxima.

d2)Pentru a avea pulsatia naturala maxima, modului trebuie sa fie maxim pe cercul cel mai indepartat de LR.



d3)Pentru a fi la limita de stabilitate regimul trebuie sa fie oscilant intretinut (polii pe axa).