

Mathematik für Informatiker 2

Übungsblatt 7

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Übung 5:Dienstag 12:00

Aufgabe 1

a)

$$\begin{aligned}z^5 &= 16(1 - \sqrt{3}i) \\z &= \sqrt[5]{16(1 - \sqrt{3}i)} \\z &= \sqrt[5]{16} \sqrt[5]{1 - \sqrt{3}i}\end{aligned}$$

b)

$$iz^2 + (1 - i)z - 3 = 0$$

Substitution: $z = x$

$$ix^2 + (1 - i)x - 3 = 0$$

$$\begin{aligned}x_{1,2} &= \frac{-1 + i \pm \sqrt{(1 - i)^2 - 4 \cdot i \cdot (-3)}}{2i} \\&= \frac{-1 + i \pm \sqrt{1 - 2i - 1 + 12i}}{2i} \\&= \frac{-1 + i \pm \sqrt{10i}}{2i} \\z_{1,2} &= \frac{-1 + i \pm \sqrt{10i}}{2i}\end{aligned}$$

c)

$$z^4 + z^2 + iz^2 + i = 0$$

Substitution: $z^2 = x$

$$x^2 + \underbrace{x + ix}_{(i+1)x} + i = 0$$

$$\begin{aligned}
 x_{1,2} &= \frac{-i-1 \pm \sqrt{-1+2i+1-4 \cdot 1 \cdot i}}{2} \\
 &= \frac{-i-1 \pm \sqrt{-2i}}{2} \\
 &= \frac{-i-1 \pm \sqrt{-1 \cdot 2i}}{2} \\
 &= \frac{-i-1 \pm i \cdot \sqrt{2i}}{2} \\
 &= \frac{-i-1 \pm (i+2i)}{2} \\
 &= \frac{2i-1}{2}, \frac{-4i-1}{2}
 \end{aligned}$$

$$z_{1,2} = \pm \sqrt{\frac{2i-1}{2}}$$

Aufgabe 2

$$\frac{1}{1-z-z^2-z^3}$$

Nullstellen von Nennerpolynom:

$$z = 1$$

$$(z^3 - z^2 - z + 1) : (z - 1) = z^2 - 1$$

$$\Rightarrow z^2 = 1 \Rightarrow z_2 = 1, z_3 = -1$$

$$\Rightarrow \frac{1}{(z-1)^2 \cdot (z+1)} = \frac{A}{(z+1)} + \frac{B}{(z-1)} + \frac{C}{(z-1)^2}$$

$$\Rightarrow 1 = \frac{A \cdot (z-1)^2 \cdot (z+1)}{(z+1)} + \frac{B \cdot (z-1)^2 \cdot (z+1)}{(z-1)} + \frac{C \cdot (z-1)^2 \cdot (z+1)}{(z-1)^2}$$

$$= A \cdot (z-1)^2 + B \cdot (z-1)(z+1) + C \cdot (z+1)$$

$$\Rightarrow z = 1 \Rightarrow 1 = 2C \Rightarrow C = 0,5$$

$$z = -1 \Rightarrow 1 = 4A \Rightarrow A = 0,25$$

$$\Rightarrow z \text{ beliebig, z.B. } 0$$

$$\Rightarrow 1 = 0,25 \cdot 1 + B \cdot (-1) \cdot 1 + 0,5 \cdot 1$$

$$1 = 0,25 - B + 0,5$$

$$B = 0,75 - 1 + 0,5$$

$$\Rightarrow B = -0,25$$

$$\Rightarrow \frac{1}{1-z-z^2+z^3} = \frac{0,25}{(z+1)} - \frac{0,25}{(z-1)} + \frac{0,5}{(z-1)^2}$$

Aufgabe 3

a) $\frac{1}{z^3 - iz^2 - z + i}$
 $z = 1 : 1^3 - i - 1 + i = 0?$
 $(z^3 - iz^2 - z + i) : (z - 1) = z^2 - iz + z - i$

$$\begin{aligned} z_{1/2} &= \frac{-(-i+1) \pm \sqrt{(-i+1)^2 - 4 - (-i)}}{2} \\ &= \frac{i-1 \pm \sqrt{1-2i-1+4i}}{2} \\ &= \frac{i-1 \pm \sqrt{2i}}{2} \\ &= \frac{i-1 \pm (i+1)}{1} \\ z_1 &= \frac{2i}{2} = i \\ z_2 &= \frac{-2}{2} = -1 \end{aligned}$$

$$\begin{aligned} \frac{a}{z-1} + \frac{b}{z-i} + \frac{c}{z+1} &= \frac{1}{z^3 - iz^2 - z + i} = \frac{1}{(z-1)(z-i)(z+1)} \\ \left(\frac{a}{z-1} + \frac{b}{z-i} + \frac{c}{z+1} \right) \cdot (z^3 - iz^2 - z + i) &= 1 \\ a - (z-i)(z+1) + b(z-1)(z+1) + c(z-1)(z-i) &= 1 \\ \text{Sei } z &= 1 \end{aligned}$$

$$\begin{aligned} a \cdot (1-i)(2) + b(0)(2) + c(0)(1-i) &= 1 \\ 2a(1-i) &= 1 \\ 2a - 2ai &= 1 \\ a &= \frac{1}{2(1-i)} = \frac{1}{2-2i} \end{aligned}$$

Sei $z = i$

$$\begin{aligned} a \cdot (0)(2) + b(i-1)(i+1) + c(2)(0) &= 1 \\ b(i-1)(i+1) &= 1 \\ b &= \frac{1}{(i-1)(i+1)} \end{aligned}$$

Sei $z = -1$

$$a \cdot (z)(0) + b \cdot (z)(0) + c(-z)(-1-i) = 1$$

$$-2c(-1-i) = 1$$

$$c = \frac{1}{(-z)(-1-i)} = \frac{1}{z+zi}$$

$$= \frac{1}{z-1} + \frac{\frac{1}{(i-1)(i+1)}}{z-i} + \frac{1}{z+1}$$

$$= \frac{1}{(z-zi)(z-1) + (i-1)(i+1)(z-i) + (z+zi)(z+1)}$$

$$= \frac{1}{(z-zi)(z-1)} + \frac{1}{(i-1)(i+1)(z-i)} + \frac{1}{(z+zi)(z+1)}$$

b) $\frac{z-1}{z^4+z^2}$

$$z = 0: 0 + 0 = 0$$

$$z^2 = x$$

$$z_1 = 0$$

$$x^2 + x$$

$$z_{2/3} \pm \sqrt{-1}$$

$$x(x+1) = 0 \quad z_2 = i; z_3 = -i$$

$$\frac{a}{z-0} + \frac{b}{(z-0)^2} + \frac{c}{z-i} + \frac{d}{z+i} = \frac{z-1}{z^4+z^2} = \frac{-1}{(z-0)^2(z-i)(z+i)}$$

$$a(z)(z-i)(z+i) + b(z-i)(z+i) + c(z)^2(z+i) + d(z)^2(z-i) = z-1$$

Sei $z = 0$

$$a \cdot 0 + b(-i)(i) + c \cdot 0 + d \cdot 0 = -1$$

$$b = \frac{-1}{(-i)(i)} = -1$$

Sei $z = i$

$$a \cdot 0 + b \cdot 0 + c \cdot i^2 \cdot (2i) + d \cdot 0 = i-1$$

$$c = \frac{i-1}{-(2i)}$$

Sei $z = -i$

$$a \cdot 0 + b \cdot 0 + c \cdot 0 + d(-i)^2(-2i) = -i-1$$

$$d = \frac{-i-1}{2i}$$

Sei $z = 1$

$$\begin{aligned}
 a(1-i)(1+i) + (-1)(1-i)(1+i) + \frac{i-1}{-(2i)}(1+i) + \frac{-i-1}{2i}(1-i) &= 0 \\
 a(1-i)(1+i) + (-1+i)(1+i) + \frac{(i-1)(1+i)}{-(2i)} + \frac{(-i-1)(1-i)}{2i} &= 0 \\
 a \cdot 2 - 2 + 2 \cdot \frac{(i-1)(1+i)}{-2i} &= 0
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow 2a - 2 + 2 \frac{-2}{-2i} &= 0 \\
 \Rightarrow 2a - 2 + \frac{2}{i} &= 0 \\
 \Rightarrow 2a &= 2 - \frac{2}{i} \\
 \Rightarrow a &= \frac{2 - \frac{2}{i}}{2} = \frac{2 - 2 \left(\frac{1}{i}\right)}{2} = 1 - \frac{1}{i}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \frac{z-1}{(z-0)^2(z-i)(z+i)} &= \frac{1-\frac{1}{i}}{z-0} + \frac{-1}{(z-0)^2} + \frac{i-1}{-2i} \frac{1}{z-i} + \frac{-i-1}{2i} \frac{1}{z+i} \\
 \Rightarrow \frac{z-1}{(z-0)^2(z-i)(z+i)} &= \frac{1-\frac{1}{i}}{z} - \frac{1}{z^2} + \frac{i-1}{-2iz-2} + \frac{-i-1}{2iz-2}
 \end{aligned}$$

Aufgabe 4

$$\frac{2x^2+7x+5}{(x^2+x+2)(x^2+1)}$$

Nullstellen:

$$(x^2+1) \Rightarrow \text{keine}$$

$$x^2+x+2 \Rightarrow x_{1,2} = \frac{-1 \pm \sqrt{1^2-4 \cdot 1 \cdot 2}}{2 \cdot 1} \quad \nexists \text{keine Nullstelle}$$

$$\Rightarrow \frac{Ax+B}{4x^2+x+2} + \frac{Cx+D}{x^2+1}$$

$$\Rightarrow \frac{2x^2+7x+5}{(x^2+x+2)(x^2+1)} = \frac{Ax+B}{x^2+x+2} + \frac{Cx+D}{x^2+1}$$

$$\begin{aligned}
 \frac{2x^2+7x+5}{(x^2+x+2)(x^2+1)} &= \frac{(Ax+B)(x^2+1)}{(x^2+x+2)(x^2+1)} + \frac{(Cx+D)(x^2+x+2)}{(x^2+x+2)(x^2+1)} \\
 &= \frac{(Ax+B)(x^2+1) + (Cx+D)(x^2+x+2)}{(x^2+x+2)(x^2+1)} \\
 &= \frac{Ax^3 + Ax + Bx^2 + B + Cx^3 + Cx^2 + 2Cx + Dx^2 + Dx + D}{(x^2+x+2)(x^2+1)} \\
 \frac{2x^2+7x+5}{(x^2+x+2)(x^2+1)} &= \frac{x^3(A+C) + x^2(B+C+D) + x(A+2C+D) + (B+D)}{(x^2+x+2)(x^2+1)}
 \end{aligned}$$

$$\Rightarrow A + C = 0 \qquad \qquad \qquad \Rightarrow A = C \qquad (1)$$

$$B + C + D = 2 \qquad (2)$$

$$A + 2C + D = 7 \qquad (3)$$

$$B + D = 5 \qquad (4)$$

$$\Rightarrow (1) \text{ in } (3) \Rightarrow A + 2A + D = 7 \Rightarrow 3A + D = 7 \Rightarrow D = 7 - 3A \qquad (5)$$

$$\Rightarrow (1) \& (5) \text{ in } (2) \Rightarrow B + A + 7 - 3A = 2 \Rightarrow B - 2A = -5 \Rightarrow B = 2A - 5 \qquad (6)$$

$$\Rightarrow (6) \& (5) \text{ in } (4) \Rightarrow 2A - 5 + 7 - 3A = 5 \Rightarrow 2A - 3A = 3 \Rightarrow -A = 3 \Rightarrow A = -3 \qquad (7)$$

$$B = 2 \cdot (-3) - 5 \qquad \qquad \qquad = 11$$

$$C = A \qquad \qquad \qquad = -3$$

$$D = 7 - 3 \cdot (-3) \qquad \qquad \qquad = 7 + 9 = 16$$

$$\Rightarrow \frac{2x^2+7x+5}{(x^2+x+2)(x^2+1)} = \frac{-3x-11}{x^2+x+2} + \frac{-3x+16}{x^2+1} = \frac{3x+11}{x^2+x+2} - \frac{3x-16}{x^2+1}$$