Aufgabe 1

a)

Aufgabe 2

$\frac{1}{1-z-z^2-z^3}$
Nullstellen von Nennerpolynom:
$z = 1$
$(z^3 - z^2 - z + 1) : (z - 1) = z^2 - 1$
$\Rightarrow z^2 = 1 \Rightarrow z_2 = 1, z_3 = -1$
$\Rightarrow \frac{1}{(z-1)^2 \cdot (z+1)} = \frac{A}{(z+1)} + \frac{B}{(z-1)} + \frac{C}{(z-1)^2}$
$\Rightarrow 1 = \frac{A \cdot (z-1)^2 - (z+1)}{(z+1)} + \frac{B \cdot (z-1)^2 \cdot (z+1)}{(z-1)} + \frac{C \cdot (z-1)^2 \cdot (z+1)}{(z-1)^2}$
$= A \cdot (z-1)^2 + B \cdot (z-1)(z+1) + C \cdot (z+1)$
$\Rightarrow z = 1 \Rightarrow 1 = 2C \Rightarrow C = 0,5$
$z = -1 \Rightarrow 1 = 4A \Rightarrow A = 0,25$
$\Rightarrow$ z beliebig, z.B 0

$$\begin{aligned} \Rightarrow 1 &= 0,25 \cdot 1 + B \cdot (-1) \cdot 1 + 0,5 \cdot 1 \\ 1 &= 0,25 - B + 0,5 \\ B &= 0,75 - 1 + 0,5 && \Rightarrow B = -0,25 \end{aligned}$$

$\Rightarrow \frac{1}{1-z-z^2+z^3} = \frac{0,25}{(z+1)} - \frac{0,25}{(z-1)} + \frac{0,5}{(z-1)^2}$

Aufgabe 3

a) $\frac{1}{z^3-iz^2-z+i}$
$z = 1 : 1^3 - i - 1 + i = 0?$
$\left(z^3 - iz^2 - z + i\right) : (z-1) = z^2 - iz + z - i$

$$\begin{aligned} z_{1/2} = \frac{-(-i+1) \pm \sqrt{(-i+1)^2 - 4 - (-i)}}{2} \\ &= \frac{i - 1 \pm \sqrt{1 - 2i - 1 + 4i}}{2} \\ &= \frac{i - 1 \pm \sqrt{2i}}{2} \\ &= \frac{i - 1 \pm (i+1)}{1} \\ z_1 &= \frac{2i}{2} = i \\ z_2 &= \frac{-2}{2} = -1 \end{aligned}$$

$\frac{a}{z-1} + \frac{b}{z-i} + \frac{c}{z+1} = \frac{1}{z^3-iz^2-z+i} = \frac{1}{(z-1)(z-i)(z+1)}$
$\left(\frac{a}{z-1} + \frac{b}{z-i} + \frac{c}{z+1}\right) \cdot \left(z^3 - iz^2 - z + i\right) = 1$
$a - (z-i)(z+1) + b(z-1)(z+1) + c(z-1)(z-i) = 1$
Sei $z = 1$

$$\begin{aligned} a \cdot (1-i)(2) + b(0)(2) + c(0)(1-i) &= 1 \\ 2a(1-i) &= 1 \\ 2a - 2ai &= 1 \\ a &= \frac{1}{2(1-i)} = \frac{1}{2-2i} \end{aligned}$$

Sei $z = i$

$$\begin{aligned} a \cdot (0)(2) + b(i-1)(i+1) + c(2)(0) &= 1 \\ b(i-1)(i+1) &= 1 \\ b &= \frac{1}{(i-1)(i+1)} \end{aligned}$$

Sei $z = -1$

$$\begin{aligned} a \cdot (2)(0) + b \cdot (2)(0) + c(-2)(-1-i) &= 1 \\ -2c(-1-i) &= 1 \\ c &= \frac{1}{(-2)(-1-i)} = \frac{1}{2+2i} \end{aligned}$$

$$\frac{\frac{1}{2-2i}}{z-1} + \frac{\frac{1}{(i-1)(i+1)}}{z-i} + \frac{\frac{1}{2+2i}}{z+1} = \frac{1}{-}$$

$$= \frac{1}{(z-zi)(z-1)} + \frac{1}{(i-1)(i+1)(z-i)} + \frac{1}{(z+zi)(z+1)}$$

b) $\frac{z-1}{z^4+z^2}$
$z = 0{:}0 + 0 = 0$
$z^2 = x$ $\quad z_1 = 0$
$x^2 + x$ $\quad z_{2/3} \pm \sqrt{-1}$
$x(x+1) = 0$ $\quad z_2 = i; z_3 = -i$

$$\frac{a}{z-0} + \frac{b}{(z-0)^2} + \frac{c}{z-i} + \frac{d}{z+i} = \frac{z-1}{z^4+z^2} = \frac{-1}{(z-0)^2(z-i)(z+i)}$$

$a(z)(z-i)(z+i) + b(z-i)(z+i) + c(z)^2(z+i) + d(z)^2(z-i) = z-1$
Sei $z = 0$

$$\begin{aligned} a \cdot 0 + b(-i)(i) + c \cdot 0 + d \cdot 0 &= -1 \\ b &= \frac{-1}{(-i)(i)} \qquad = -1 \end{aligned}$$

Sei $z = i$

$$a \cdot 0 + b \cdot 0 + c \cdot i^2 \cdot (2i) + d \cdot 0 = i - 1c \qquad = \frac{i-1}{-(2i)}$$

Sei $z = -i$

$$a \cdot 0 + b \cdot 0 + c \cdot 0 + d\,(-i)^2\,(-2i) = -i - 1$$
$$d = \frac{-i-1}{2i}$$

Sei $z = 1$

$$a(1-i)(1+i) + (-1)(1-i)(1+i) + \frac{i-1}{-(2i)}(1+i) + \frac{-i-1}{2i}(1-i) = 0$$
$$a(1-i)(1+i) + (-1+i)(1+i) + \frac{(i-1)(1+i)}{-(2i)} + \frac{(-i-1)(1-i)}{2i} = 0$$
$$a \cdot 2 - 2 + 2 \cdot \frac{(i-1)(1+i)}{-2i} = 0$$

$$\Rightarrow 2a - 2 + 2\frac{-2}{-2i} = 0$$
$$\Rightarrow 2a - 2 + \frac{2}{i} = 0$$
$$\Rightarrow 2a = 2 - \frac{2}{i}$$
$$\Rightarrow a = \frac{2 - \frac{2}{i}}{2} \qquad = \frac{2 - 2\left(\frac{1}{i}\right)}{2} = 1 - \frac{1}{i}$$

$$\Rightarrow \frac{z-1}{(z-0)^2\,(z-i)\,(z+i)} = \frac{1-\frac{1}{i}}{z-0} + \frac{-1}{(z-0)^2} + \frac{i-1}{-2i}\,\frac{1}{z-i} + \frac{-i-1}{2i}\,\frac{1}{z+i}$$
$$\Rightarrow \frac{z-1}{(z-0)^2(z-i)(z+i)} = \frac{1-\frac{1}{i}}{z} - \frac{1}{z^2} + \frac{i-1}{-2iz-2} + \frac{-i-1}{2iz-2}$$

Aufgabe 4

$\frac{2x^2+7x+5}{(x^2+x+2)(x^2+1)}$
Nullstellen:
$\left(x^2+1\right) \Rightarrow$ keine
$x^2 + x + 2 \Rightarrow x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 2}}{2-1}$ $\lightning$keine Nullstelle
$\Rightarrow \frac{Ax+B}{4x^2+x+2} + \frac{Cx+D}{x^2+1}$

$\Rightarrow \frac{2x^2+7x+5}{(x^2+x+2)(x^2+1)} = \frac{Ax+B}{x^2+x+2} + \frac{Cx+D}{x^2+1}$

$$
\begin{aligned}
\frac{2x^2+7x+5}{(x^2+x+2)(x^2+1)} &= \frac{(Ax+B)\left(x^2+1\right)}{(x^2+x+2)(x^2+1)} + \frac{(Cx+D)\left(x^2+x+2\right)}{(x^2+x+2)(x^2+1)} \\
&= \frac{(Ax+B)\left(x^2+1\right)+(Cx+D)\left(x^2+x+2\right)}{(x^2+x+2)(x^2+1)} \\
&= \frac{Ax^3+Ax+Bx^2+B+Cx^3+Cx^2+2Cx+Dx^2+Dx+D}{(x^2+x+2)(x^2+1)} \\
\frac{2x^2+7x+5}{(x^2+x+2)(x^2+1)} &= \frac{x^3(A+C)+x^2(B+C+D)+x(A+2C+D)+(B+D)}{(x^2+x+2)(x^2+1)}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow A+C &= 0 & \Rightarrow A &= C & (1) \\
B+C+D &= 2 & & & (2) \\
A+2C+D &= 7 & & & (3) \\
B+D &= 5 & & & (4)
\end{aligned}
$$

$$\Rightarrow (1) \text{ in } (3) \Rightarrow A+2A+D=7 \Rightarrow 3A+D=7 \Rightarrow D=7-3A \tag{5}$$

$$\Rightarrow (1)\&(5) \text{ in } (2) \Rightarrow B+A+7-3A=2 \Rightarrow B-2A=-5 \Rightarrow B=2A-5 \tag{6}$$

$$\Rightarrow (6)\&(5) \text{ in } (4) \Rightarrow 2A-5+7-3A=5 \Rightarrow 2A-3A=3 \Rightarrow -A=3 \Rightarrow A=-3 \tag{7}$$

$$
\begin{aligned}
B &= 2\cdot(-3)-5 & &= 11 \\
C &= A & &= -3 \\
D &= 7-3\cdot(-3) & &= 7+9=16
\end{aligned}
$$

$\Rightarrow \frac{2x^2+7x+5}{(x^2+x+2)(x^2+1)} = \frac{-3x-11}{x^2+x+2} + \frac{-3x+16}{x^2+1} = \frac{3x+11}{x^2+x+2} - \frac{3x-16}{x^2+1}$