13. Consider the set of all linear combinations of the form $x_1n_1+x_2n_2+...+x_sn_s$. Clearly, the integers $abs(n_1)$, $abs(n_2)$, ... $abs(n_s)$ themselves belong to this set. Now consider the smallest positive integer c that belongs to this set. Clearly, $c <= abs(n_1)$, $c <= abs(n_2)$, ... and $c <= abs(n_s)$. Consider the remainder r obtained by dividing $abs(n_1)$ by c. We can write r as $abs(n_1) - qc$ for some quotient q. Since $abs(n_1)$ and c can both be written in the linear combination form above, so too can r. However, since c is the smallest positive such linear combination, r cannot be positive. In other words, we must have $r = 0$, or equivalently, $c|abs(n_1)$. We can similarly derive that $c|abs(n_2)$, ... and $c|abs(n_s)$. This tells us that c is a common divisor of n_1, n_2, ... and n_s. In particular $c|d$, where $d = GCD(n_1, n_2, ...n_s)$. Since c can be expressed as a linear combination of the above form, so too can d. This concludes the proof. In fact, since d must divide all integers of the linear combination form above, we must have $d|c$, which leads us to the conclusion that $c = d$.

17. The proof is a special case of proof for exercise 18. Replace m and n both with 2.

18. Assume that $\sqrt[n]{m}$ is rational i.e. can be expressed as a ratio of integers a/b. We'll prove that this leads to a contradiction. Raise both sides of the equation to the nth power. This yields $m = a^n/b^n$. The prime factorization of a^n and b^n must contain primes raised to an integer that is a multiple of n. Even as the primes in the numerator and denominator cancel out to yield m (on the LHS), the resulting prime factorization must still contain primes raised to an integer that is a multiple of n. This implies that m is the nth power of some integer, which contradicts what we have been told about m. This concludes the proof.