

3a. Let's consider the expression  $n!e^n$ .

$$n!e^n = \prod_{k=1}^n ke$$

We are now going to replace each of the  $e$ 's with an inequality based on the fact that  $e > (1 + 1/r)^r$  for all positive integers  $r$ .

$$\begin{aligned} n!e^n &> \prod_{k=1}^n k(1 + 1/k)^k \\ n!e^n &> \prod_{k=1}^n (k+1)^k / k^{k-1} \end{aligned}$$

We can now cancel each of the denominators on the RHS with the preceding numerator, and obtain

$$\begin{aligned} n!e^n &> (n+1)^n \\ n!e^n &> n^n \\ \therefore n! &> (n/e)^n \end{aligned}$$

This concludes the proof.