

9. Any integer  $n$  can be written as a product of prime powers. Say  $n = \prod_{i=1}^l p_i^{a_i}$ . Then we can say that  $f(n) = \prod_{i=1}^l f(p_i^{a_i})$ , for any multiplicative function  $f$ , because the prime powers are relatively co-prime. Therefore, such a function is purely determined by its values on prime powers. This concludes the proof.

15a. Imagine a function  $f(n) = 1$  for all values of  $n$ . Then, we can say that  $\nu(n) = \sum_{d|n} 1 = \sum_{d|n} f(d)$ . Thus, by the Mobius inversion theorem,  $\sum_{d|n} \mu(n/d)\nu(d) = f(n) = 1$ . This concludes the proof.

15b. Imagine a function  $f(n) = n$  for all values of  $n$ . Then, we can say that  $\sigma(n) = \sum_{d|n} d = \sum_{d|n} f(d)$ . Thus, by the Mobius inversion theorem,  $\sum_{d|n} \mu(n/d)\sigma(d) = f(n) = n$ . This concludes the proof.