3a. Consider any processing operation that has two types A and B. Now, let's count the number of ways we can pick k objects from n and apply processing operation A on l of the k objects and operation B on the remaining $k-l$ objects. We can count this in two equivalent ways.

- $\binom{n}{k}\binom{k}{l}$, which involves picking k objects from n, and then picking the l from amongst those k.
- $\binom{n}{l}\binom{n-l}{k-l}$, which involves picking l objects from n, and then picking the $k-l$ from amongst the remaining $n-l$.

This concludes the proof.

3b. Let's say we have m boys and n girls to pick a total of k people from. We can count this in two equivalent ways.

- $\binom{m+n}{k}$, which involves picking k objects from amongst the total of $m+n$ people.
- $\sum_{i=0}^{k}\binom{m}{i}\binom{n}{k-i}$, which involves counting $k+1$ exclusive possibilities, wherein in each possibility, we pick a certain number i of boys from amongst the m boys, and the remaining $k-i$ required people from the n girls.

This concludes the proof.

3d. The algebraic proof is based on differentiating the two sides of the binomial expression for $(1+t)^n$ and then replacing t with 1. But this is a bit mundane. So let's give a combinatorial proof instead.

Consider n objects, and say we need to do two things - first pick at least one of them, and then designate a leader from amongst those picked. We can count this in two equivalent ways.

- $\sum_{i=1}^{k}\binom{n}{i}i$, which involves counting k exclusive possibilities, wherein in each possibility, we first pick a certain number i of objects from among the n objects, and then designate one of those i objects as leader.
- $n2^{n-1}$, which involves first picking a leader, which can be done in n ways, and then picking 0 or more objects from the remaining $n-1$ objects, which can be done in 2^{n-1} ways.

This concludes the proof.