

13. Consider the set of all linear combinations of the form  $x_1n_1 + x_2n_2 + \dots + x_sn_s$ . Clearly, the integers  $abs(n_1), abs(n_2), \dots, abs(n_s)$  themselves belong to this set. Now consider the smallest positive integer  $c$  that belongs to this set. Clearly,  $c \leq abs(n_1)$ ,  $c \leq abs(n_2)$ ,  $\dots$  and  $c \leq abs(n_s)$ . Consider the remainder  $r$  obtained by dividing  $abs(n_1)$  by  $c$ . We can write  $r$  as  $abs(n_1) - qc$  for some quotient  $q$ . Since  $abs(n_1)$  and  $c$  can both be written in the linear combination form above, so too can  $r$ . However, since  $c$  is the smallest positive such linear combination,  $r$  cannot be positive. In other words, we must have  $r = 0$ , or equivalently,  $c|abs(n_1)$ . We can similarly derive that  $c|abs(n_2)$ ,  $\dots$  and  $c|abs(n_s)$ . This tells us that  $c$  is a common divisor of  $n_1, n_2, \dots$  and  $n_s$ . In particular  $c|d$ , where  $d = GCD(n_1, n_2, \dots, n_s)$ . Since  $c$  can be expressed as a linear combination of the above form, so too can  $d$ . This concludes the proof. In fact, since  $d$  must divide all integers of the linear combination form above, we must have  $d|c$ , which leads us to the conclusion that  $c = d$ .