

1a. Let's call the number of solutions for  $n$  seating positions arranged in a straight line as  $S(n)$ . In any such solution, either the  $n$ th seat is chosen or not. The number of solutions for the latter case is  $S(n - 1)$ , and that for the former case is  $S(n - 2)$  (because when the  $n$ th seat is chosen, the  $(n - 1)$ th seat cannot be). Thus,  $S(n) = S(n - 1) + S(n - 2)$ . For the base case, we can easily see that  $S(0) = 1$  (because the act of not choosing anything is itself a solution) and  $S(1) = 2$  (because we can either choose the only seat or not). Thus, we see that  $S(n)$  has the same recurrence as  $F(n)$ , the Fibonacci sequence, but  $S(0) = F(1)$  and  $S(1) = F(2)$ . Therefore, by induction,  $S(n) = F(n + 1)$ . This concludes the proof.

1b. Let's call the number of solutions for  $n$  seating positions arranged in a circle as  $T(n)$ . We can see that almost any solution to the problem where seats are arranged in a straight line can be a solution to the case where the seats are arranged in a circle. The only cases we must exclude are those where seat 1 and seat  $n$  are both chosen. For  $n \geq 4$ , the number of such cases is  $S(n - 4)$ , since both 1 and  $n$  are chosen, neither seat 2 nor seat  $n - 1$  can be. So, the number of solutions for  $n$  seats arranged in a circle is simply  $S(n) - S(n - 4) = F(n + 1) - F(n - 3) = F(n) + F(n - 1) - F(n - 3) = F(n) + F(n - 2)$ . For the cases  $n = 3$  and  $n = 2$ , we can manually verify that this equation holds. This concludes the proof.