6. Since $gcd(a_i, n) = 1$ for any i, and $gcd(a, n) = 1$, it is obvious that $gcd(aa_i, n) = 1$ for any i.

We next prove that aa_i is not $\equiv aa_j \mod (n)$ if $i \neq j$. If not, $aa_i \equiv aa_j \mod (n)$, and $n|a(a_i - a_j)$. However, since $gcd(a, n) = 1$, this implies that $n|(a_i - a_j)$ or in other words $a_i \equiv a_j \mod (n)$, which is a contradiction.

This implies that if $a_1, a_2, \cdots a_{\phi(n)}$ is a reduced residue modulo n, so is $aa_1, aa_2, \cdots aa_{\phi(n)}$. This concludes the proof.

7. Consider the set of integers $< n$ that are coprime to n. Call these $a_1, a_2, \cdots a_{\phi(n)}$. Clearly these form a reduced residue system modulo n. Now for any a such that $gcd(a, n) = 1$, we know from exercise 6 above that $aa_1, aa_2, \cdots aa_{\phi(n)}$ is also a reduced residue system modulo n.

Now, for any i, $aa_i = qn + r_i$, where $0 < r_i < n$ and $gcd(r_i, n) = 1$. In other words, $r_i = a_j$ for some j. Thus, for each i, there is a unique j such that $aa_i \equiv a_j \mod (n)$.

Therefore,

$$\begin{aligned} aa_1 aa_2 \cdots aa_{\phi(n)} &\equiv a_1 a_2 \cdots a_{\phi(n)} \mod (n) \\ a^{\phi(n)} a_1 a_2 \cdots a_{\phi(n)} &\equiv a_1 a_2 \cdots a_{\phi(n)} \mod (n) \\ \therefore a^{\phi(n)} &\equiv 1 \mod (n) \end{aligned}$$

where the last line is due to the fact that we can cancel out the term $a_1 a_2 \cdots a_{\phi(n)}$ on both sides due to it being relatively coprime to n. This concludes the proof.