3a. The base case is for $n = 1$, where we see the inequality obviously holds true because $1! > (1/e)^1$. If we assume the inequality to hold true for n, then we are left to prove the inequality true for $n + 1$.

$$(n+1)! = (n+1)n!$$
$$(n+1)! > (n+1)(n/e)^n$$

Multiplying numerator and denominotor on the RHS by $(n + 1)^n$, we get

$$(n+1)! > (n+1)^{n+1}(n/(n+1))^n/e^n$$
$$(n+1)! > (n+1)^{n+1}(1/(1+1/n)^n)/e^n$$

Now, since $e > (1 + 1/n)^n$ for all n, we get

$$(n+1)! > (n+1)^{n+1}(1/e)/e^n$$
$$(n+1)! > (n+1)^{n+1}/e^{n+1}$$
$$(n+1)! > ((n+1)/e)^{n+1}$$

This concludes the proof by induction. Let's now prove by construction (although not asked for in the exercise).

Consider the following expansion of $n!e^n$.

$$n!e^n = \prod_{k=1}^{n} ke$$

We are now going to replace each of the e's with an inequality based on the fact that $e > (1 + 1/r)^r$ for all positive integers r.

$$n!e^n > \prod_{k=1}^{n} k(1+1/k)^k$$
$$n!e^n > \prod_{k=1}^{n} (k+1)^k/k^{k-1}$$

We can now cancel each of the denominators on the RHS with the preceding numerator, and obtain

$$n!e^n > (n+1)^n$$
$$n!e^n > n^n$$
$$\therefore n! > (n/e)^n$$

This concludes the proof.

3b. We'll apply the arithmetico-geometric inequality on the set of integers $1, 2, 3, ..., n$. This gives us

$$\sqrt[n]{\prod_{k=1}^{n} k} < (\sum_{k=1}^{n} k)/n$$
$$\sqrt[n]{n!} < (n(n+1)/2)/n$$
$$n! < ((n+1)/2)^n$$

Multiplying numerator and denominator on the RHS by n^n, we get

$$n! < ((n+1)/n)^n (n/2)^n$$
$$n! < (1+1/n)^n (n/2)^n$$

Once again, since $e > (1+1/n)^n$ for all n, we get the desired result $n! < e(n/2)^n$.