12. Given that L is Turing recognizable, some enumerator E enumerates L. Consider another TM D that operates as follows. For a given input i, D runs E until it prints $< M_i >$, the ith item. Then D runs M_i on input i. Finally, if M_i accepts, D rejects, and if M_i rejects, D accepts. Clearly, D always terminates on all inputs, and is thus a decider itself. Let's call the (decidable) language of D as L_D.

Now, let's assume that there is some $< M > \in L$ such that M decides L_D. Then $< M >$ must appear at some index k in the enumeration done by E. Now, since M and D are both deciders for L_D, M must accept input k iff D accepts input k. However, by definition, D rejects k if M accepts k. Therefore, our supposition that such a TM M exists is wrong, which implies that the language L_D has no decider in L. This concludes the proof.