10. Given that the polynomial has a root at x_0, we can say the following.

$$\begin{aligned}
c_1{x_0}^n + c_2{x_0}^{n-1} + \cdots + c_n x_0 + c_{n+1} &= 0 \\
c_1{x_0}^n &= -(c_2{x_0}^{n-1} + \cdots + c_n x_0 + c_{n+1}) \\
|c_1{x_0}^n| &= |c_2{x_0}^{n-1} + \cdots + c_n x_0 + c_{n+1}| \\
|c_1{x_0}^n| &<= |c_2{x_0}^{n-1}| + \cdots + |c_n x_0| + |c_{n+1}| \\
|c_1{x_0}^n| &<= |c_2||{x_0}^{n-1}| + \cdots + |c_n||x_0| + |c_{n+1}| \\
|c_1{x_0}^n| &<= c_{max}(|{x_0}^{n-1}| + \cdots + |x_0| + 1)
\end{aligned}$$

Now, if $c_{n+1} = 0$, then $x = 0$ is also a root of the polynomial. In that case, the given inequality holds true.

Else, we must have $|x_0| >= 1$. Therefore, we get

$$\begin{aligned}
|c_1{x_0}^n| &<= c_{max} n |{x_0}^{n-1}| \\
|x_0| &<= n c_{max}/|c_1| \\
|x_0| &< (n c_{max}/|c_1|) + 1 \\
&<= (n c_{max}/|c_1|) + (c_{max}/|c_1|) \\
&<= (n+1) c_{max}/|c_1|
\end{aligned}$$

This concludes the proof.