19. Consider a TM M that works as follows. On input x, M computes the sequence $f(x), f(f(x)), f(f(f(x))), \cdots$, until the sequence hits 1. If it hits 1, M accepts. Even if M runs forever on some input, we can use H to determine in a finite number of steps whether M accepts or not.

Now, consider another TM N that works as follows. On input w, it neglects w and for each natural number i, starting at 1, runs H on $< M, i >$. If H rejects, then N accepts. If H accepts, then N proceeds onto the next natural number. Clearly, N accepts iff there were a counter-example to the $3x + 1$ problem, else it runs forever.

Now, run H on $< N, w >$. If H accepts, then it indicates that there is a counter-example to the $3x+1$ problem; we can now run N confidently and hope to obtain the actual counter-example. If H rejects, then that indicates that no such counter-example exists.

25. Suppose $T = \{< M >| \ M$ is a TM that accepts w^R whenever it accepts $w\}$ is decidable. Then some TM H decides it. We'll use this to build a decider N for A_{TM}.

N works as follows. On input $< M, w >$, N does the following.

1. constructs a TM M', which works as follows. On input x, M' runs M on w. If M accepts, M' accepts. Else, M' accepts x iff x is lexicographically smaller than x^R.

2. runs H on $< M' >$. If H accepts, then N accepts, else rejects.

Clearly N accepts $< M, w >$ iff H accepts $< M' >$. By definition, H accepts $< M' >$ iff $< M' > \in T$. Now, given the above construction, if M accepts w, then M' accepts all strings and thus $\in T$. And if M does not accept w, then M' only accepts lexicographically smaller counterpart strings (e.g. ab but not ba) and thus is not $\in T$. Thus, $< M' > \in T$ iff M accepts w.

Putting all this together, we see that N accepts $< M, w >$ iff M accepts w, implying that N is a decider for A_{TM}. This contradicts the fact that A_{TM} is undecidable. Therefore, such a decider H (and thus such a decider N) cannot exist. This concludes the proof.