[illegible]

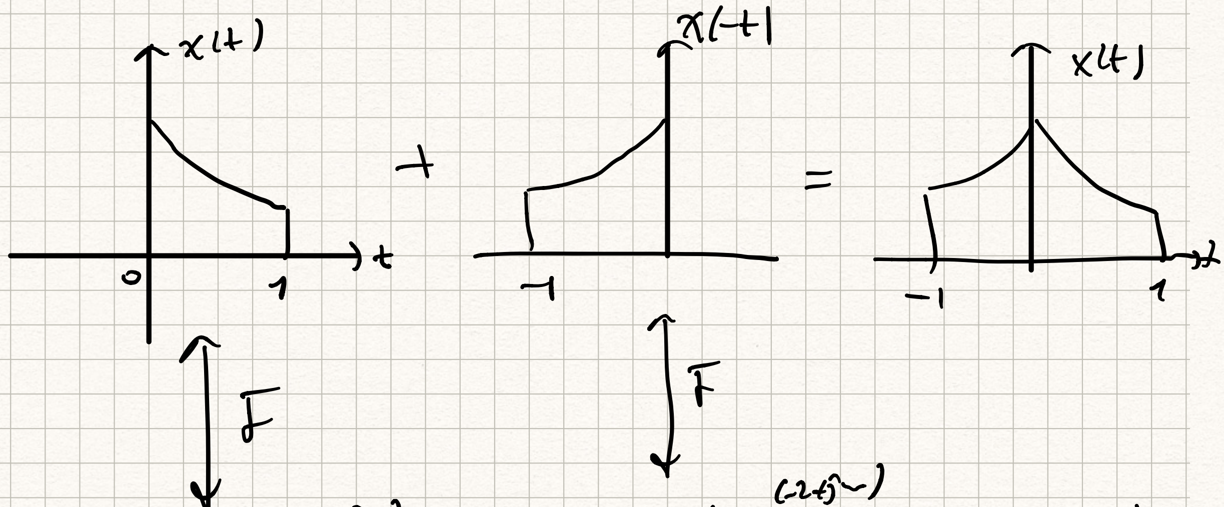
1

$$X(j\omega) = \int_0^1 e^{-2t} \cdot e^{-j\omega t} \cdot dt = \int_0^1 e^{t(-2-j\omega)} dt$$

$$= \frac{e^{t(-2-j\omega)}}{-2-j\omega} \Big|_0^1 = \frac{1-e^{-(2-j\omega)}}{2+j\omega}$$

a

$$x_1(t) = x(-t) + x(t) \xrightarrow{\mathcal{F}} X_1(j\omega) = X(j\omega) + X(-j\omega)$$



$$X(j\omega) = \frac{1-e^{(-2-j\omega)}}{2+j\omega} + X(-j\omega) = \frac{1-e^{(-2+j\omega)}}{2-j\omega} = X_1(j\omega)$$

$$X_1(j\omega) = \frac{1-e^{(-2-j\omega)}}{\underset{(2-j\omega)}{2+j\omega}} + \frac{1-e^{(-2+j\omega)}}{\underset{(2+j\omega)}{2-j\omega}}$$

$$X_1(j\omega) = \frac{2-2e^{(-2-j\omega)} - j\omega + j\omega e^{(-2-j\omega)} + 2 - 2e^{(-2+j\omega)} + j\omega - j\omega e^{(-2+j\omega)}}{4+\omega^2}$$

$$X_1(j\omega) = \frac{4 + (j\omega-2)e^{(-2-j\omega)} + (-j\omega-2)e^{(-2+j\omega)}}{4+\omega^2}$$

(b) $x_2(t) = t \cdot x(t)$

$$X_2(j\omega) = j\frac{dX(j\omega)}{d\omega} = j\frac{d}{d\omega}\left(\frac{1-e^{-2-j\omega}}{2+j\omega}\right)$$

$$X_2(j\omega) = j\left(\frac{j\cdot e^{(-2-j\omega)}(2+j\omega) - (1-e^{(-2-j\omega)})j}{(2+j\omega)^2}\right)$$

$$= j\left(\frac{2j\, e^{(-2-j\omega)} - \omega e^{(-2-j\omega)} - j + j e^{(-2-j\omega)}}{(2+j\omega)^2}\right)$$

$$= j\left(\frac{3j\, e^{(-2-j\omega)} - \omega e^{(-2-j\omega)} - j}{(2+j\omega)^2}\right)$$

$$= \frac{-3e^{-2-j\omega} - j\omega e^{-2-j\omega} + 1}{(2+j\omega)^2} = -\frac{e^{-j\omega-2}\left(j\omega - e^{j\omega+2} + 3\right)}{(j\omega+2)^2}$$

(2)

a) $x(t) \xrightarrow{FS} a_k$

$x(t-2) \xrightarrow{FS} a_k e^{-jk\omega_0 2} = a_k e^{-jk2\pi 2/T}$

$\frac{2\,dx(t)}{dt} \longrightarrow a_k 2jk\omega_0 = a_k 2jk\frac{2\pi}{T} = a_k \frac{4jk\pi}{T}$

$c_k = a_k\left(e^{-j4\pi k/T} + \frac{jk4\pi}{T}\right)$

b) $c_k = \frac{1}{T}\int_0^T z(t)\, e^{-jk2\pi t/T}\, dt$

$= \frac{1}{T}\int_0^T x(t)\, e^{-jk2\pi t/T}\, dt + \frac{1}{T}\int_0^T y(2t)\, e^{-jk2\pi t/T}\, dt$

$= a_k + \frac{1}{T}\int_0^T y(2t)\, e^{-j2\pi t/T}\, dt \qquad (z = 2t)$

$= a_k + \frac{1}{2T}\int_0^{2T} y(z)\, e^{-jk2\pi z/2T}\, dz$

$c_k = a_k + b_k$

3

a) $x(t) = \dfrac{e^{j2t}}{1+t^2}$

$x_1(t) = \dfrac{1}{1+t^2} \Rightarrow x(t) = e^{j2t} x_1(t)$ olsun.

$X(j\omega) = X_1(j(\omega-2))$

$e^{-|t|} \overset{\mathcal{F}}{\longleftrightarrow} \dfrac{2}{1+\omega^2}$ olduğuna göre

Dualite özelliğinden,

$\dfrac{2}{1+t^2} \overset{\mathcal{F}}{\longleftrightarrow} 2\pi e^{-|\omega|}$ olduğu bilinir.

$X_1(j\omega) = \pi e^{-|\omega|}$ olacak.

$X(j\omega) = \pi e^{-|\omega-2|}$ olarak bulunur.

b) $X(j\omega) = \dfrac{4\sin^2(\omega)}{\omega^2} = \underbrace{\dfrac{2\sin(\omega)}{\omega}}_{X_1(j\omega)} \cdot \underbrace{\dfrac{2\sin(\omega)}{\omega}}_{X_1(j\omega)}$

$X_1(j\omega) = \dfrac{2\sin(\omega)}{\omega} \overset{\mathcal{F}^{-1}}{\longleftrightarrow} x_1(t) = \begin{cases} 1, & |x| \leq 1 \\ 0, & \text{diğer} \end{cases}$

$X(j\omega) = X_1(j\omega) X_1(j\omega) \longleftrightarrow x(t) = x_1(t) * x_1(t)$

$x(t) = \int_{-\infty}^{\infty} x_1(\tau) x_1(t-\tau) d\tau = \begin{cases} t+2, & -2<t<0 \\ -t+2, & 0<t<2 \\ 0, & |t|>2 \end{cases}$