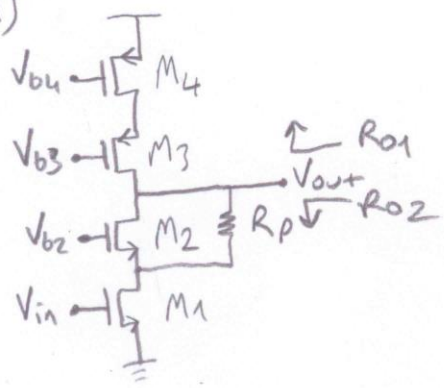
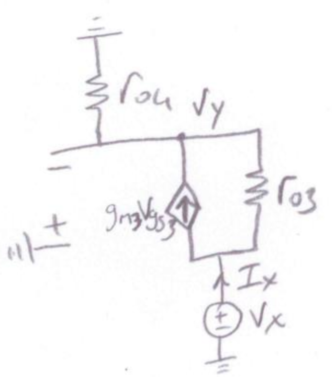
# EHB 335E - HOMEWORK I - SOLUTIONS - ALICAN ÇAĞLAR

① a)




=>



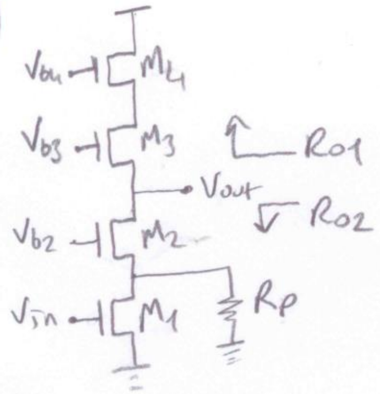

$V_{gs3} = -I_x \cdot r_{o4}$

$V_x - (I_x + I_x \cdot r_{o4} \cdot g_{m3}) r_{o3} = I_x \cdot r_{o4}$

$R_{o2} = g_{m2} \cdot r_{o1} \cdot r_{o2} \| R_p + r_{o2} \| R_p + r_{o1}$ <= $R_{o1} = \dfrac{V_x}{I_x} = g_{m3} \cdot r_{o4} \cdot r_{o3} + r_{o3} + r_{o4}$

$A_v = -g_{m1} \cdot R_{o1} \| R_{o2}$

b)

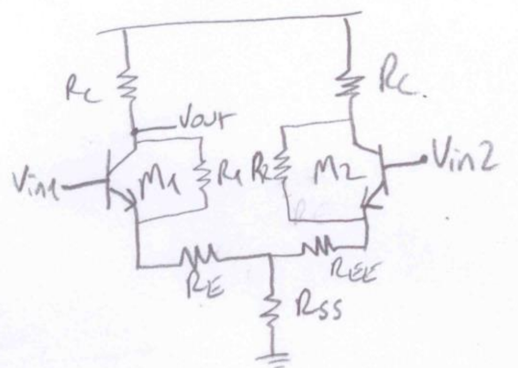



$R_{o1} = g_{m3} \cdot r_{o4} \cdot r_{o3} + r_{o3} + r_{o4}$

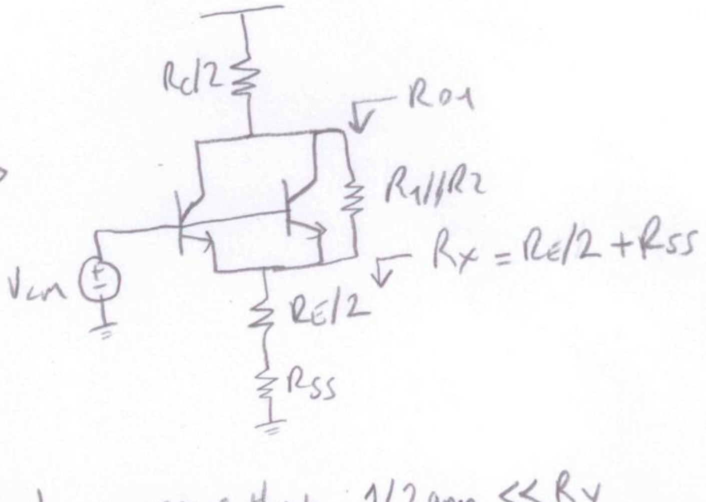
$R_{o2} = g_{m2} \cdot r_{o2} \cdot r_{o1} \| R_p + r_{o1} \| R_p + r_{o2}$

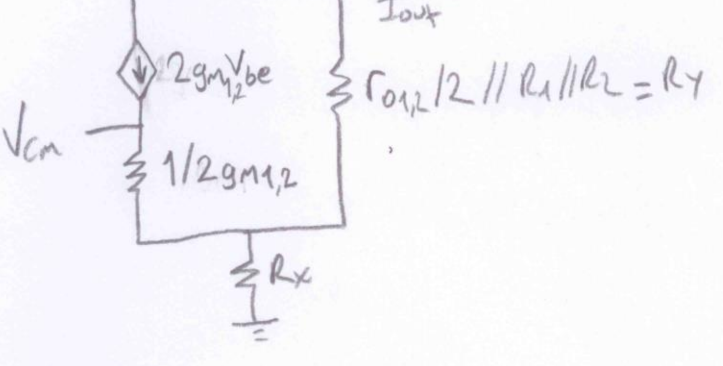
$A_v = -g_{m1} \cdot R_{o1} \| R_{o2}$

②




=>



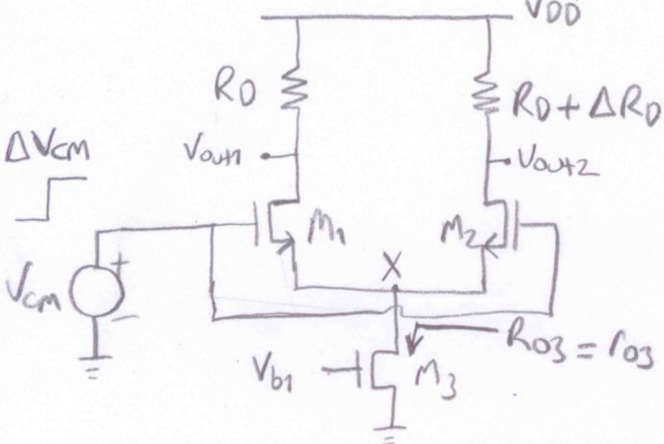

If we assume that $1/2g_{m1,2} \ll R_Y$

$G_m = \dfrac{I_{out}}{V_{cm}} = \dfrac{1}{1/2g_{m1,2} + R_X}$

$R_{o1} = g_{m1,2} \cdot R_Y \cdot R_X + R_Y + R_X$

$A_{cm} = -G_m \cdot R_{o1} \| R_C/2$

③ a)




$$\Delta V_{GS} = \Delta V_{GS1} = \Delta V_{GS2}$$
$$\Delta I_D = \Delta I_{D1} = \Delta I_{D2}$$

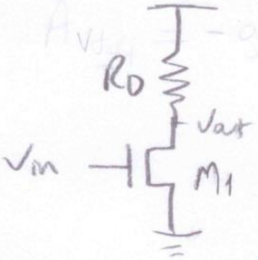
$$\Delta V_{cm} = \Delta V_{GS} + 2\Delta I_D . R_{o3}$$
$$= \frac{\Delta I_D}{g_{m1}} + 2\Delta I_D . r_{o3}$$

$$\Delta V_{out} = \Delta V_{out1} - \Delta V_{out2} = -\Delta R_D . \Delta I_D$$

$$A_{vcm} = \left|\frac{\Delta V_{out}}{\Delta V_{cm}}\right| = \frac{\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}}$$

X node appears as virtual grand when differential input signals are applied.

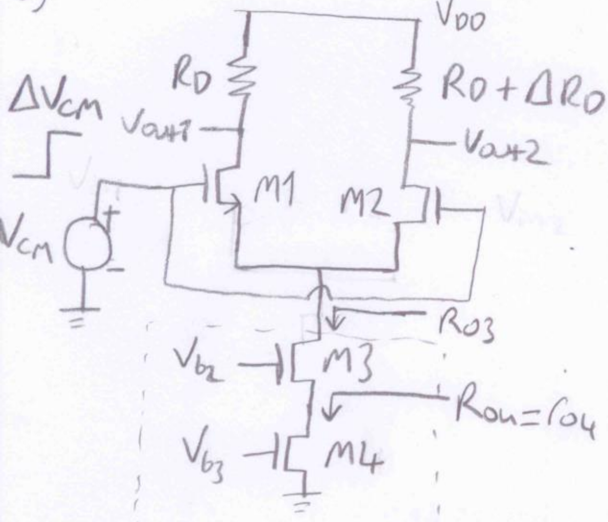
So,




$$\Rightarrow A_{vdiff} = -g_{m1}R_D$$

$$CMRR = \frac{|A_{vdiff}|}{|A_{vcm}|} = \frac{R_D}{\Delta R_D} . (1 + 2g_{m1}.r_{o3})$$

b)

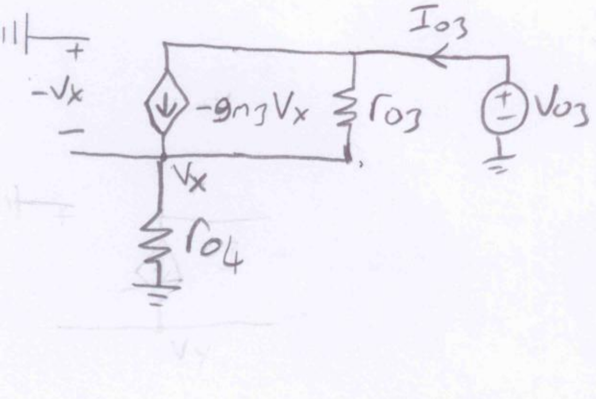



$$\Delta V_{cm} = \Delta V_{GS} + 2\Delta I_D R_{o3} = \Delta I_D[1/g_{m1} + 2.R_{o3}]$$
$$\Delta V_{out} = -\Delta R_D . \Delta I_D$$
$$A_{vcm} = \left|\frac{\Delta V_{out}}{\Delta V_{cm}}\right| = \frac{\Delta R_D}{1/g_{m1} + 2R_{o3}}$$

$$A_{vdiff} = -g_{m1}.R_D$$

$$\Rightarrow CMRR = \left|\frac{A_{vdiff}}{A_{vcm}}\right| = \frac{R_D}{\Delta R_D} . (1 + 2g_{m1}g_{m3}r_{o3}r_{o4})$$

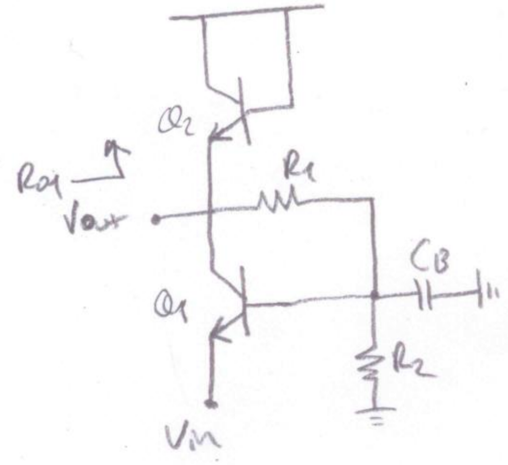
⇓

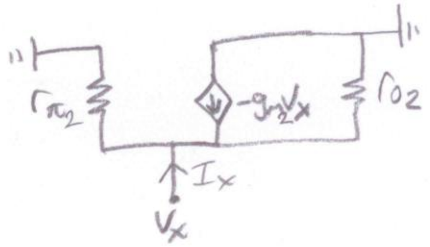



$$V_{o3} = (I_{o3} + g_{m3}V_x)r_{o3} + I_{o3}.r_{o4}$$
$$V_x = I_{o3}.r_{o4}$$

$$R_{o3} = \frac{V_{o3}}{I_{o3}} = g_{m3}.r_{o3}.r_{o4} + r_{o4} + r_{o3}$$
$$\cong g_{m3}.r_{o3}.r_{o4}$$

4

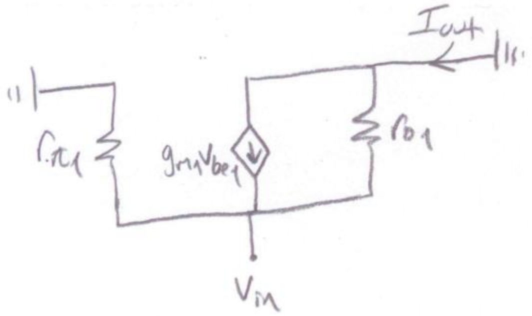



$$(I_x - g_{m2} V_x)\, r_{\pi 2} \| r_{o2} = V_x$$

$$R_{o1} = \frac{V_x}{I_x} = \frac{r_{\pi 2} \| r_{o2}}{1 + g_{m2} r_{\pi 2} \| r_{o2}}$$

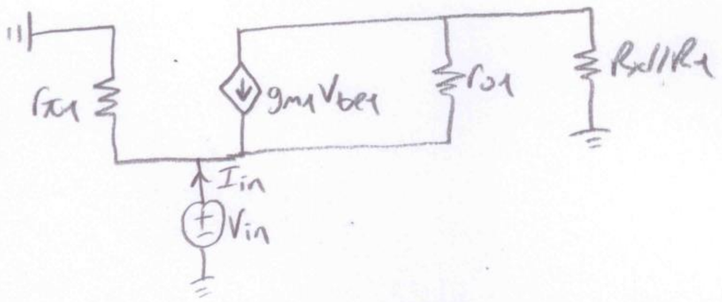
$$R_{out} = R_{o1} \| R_1 \| r_{o1}$$




$$0 - (I_{out} - g_{m1} V_{in})\, r_{o1} = V_{in}$$

$$G_m = \frac{I_{out}}{V_{in}} = \frac{r_{o1}}{1 + g_{m1} r_{o1}}$$

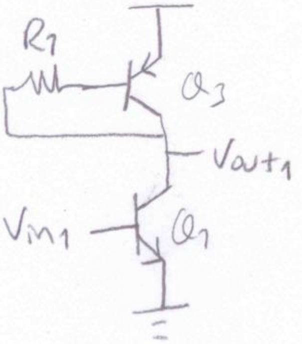
$$A_v = G_m . R_{out}$$



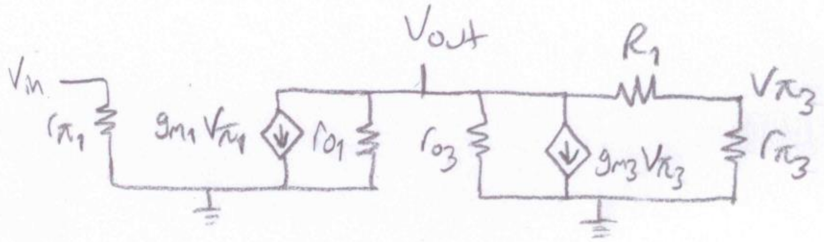

$$\left(I_{in} - \frac{V_{in}}{r_{\pi 1}} - g_{m1} V_{in}\right) r_{o1} + R_x \| R_1 \left(I_{in} - \frac{V_{in}}{r_{\pi 1}}\right) = V_{in}$$

$$R_{in} = \frac{V_{in}}{I_{in}} = \frac{r_{\pi 1} r_{o1} + R_x \| R_1 . r_{\pi 1}}{r_{\pi 1} + R_x \| R_1 + r_{o1} + g_{m1} . r_{\pi 1} . r_{o1}}$$

⑤ Half of the amplifier

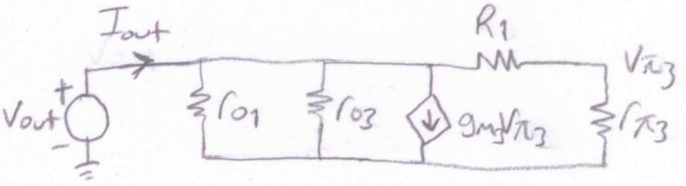



Small signal equivalent circuit




$$G_m = \frac{I_{out}}{V_{in}} = g_{m1}$$

Calculation of Rout



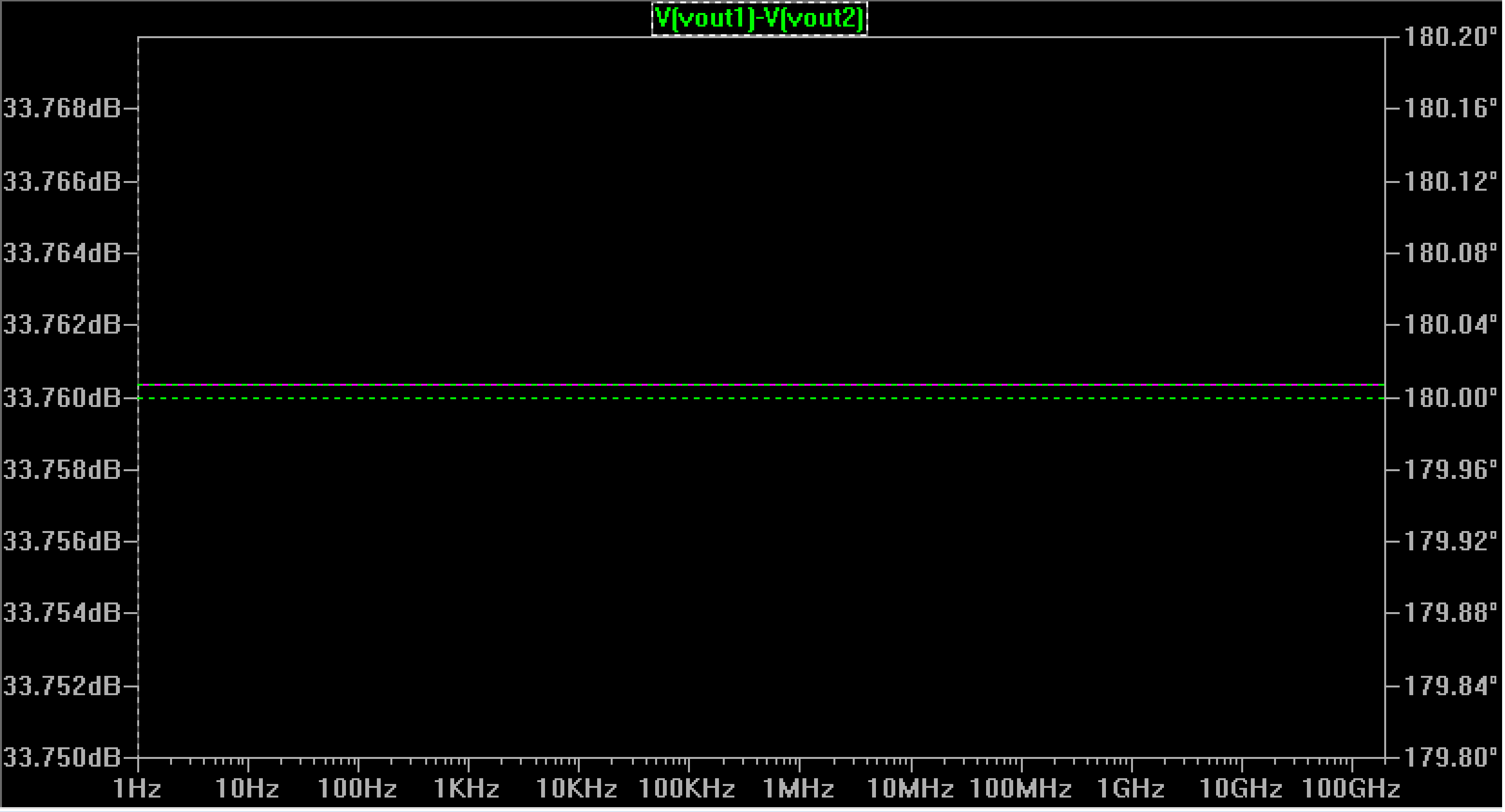

$$\frac{V_{out}}{V_{\pi 3}} = \frac{R_1 + r_{\pi 3}}{r_{\pi 3}} \Rightarrow V_{\pi 3} = \frac{V_{out}\, r_{\pi 3}}{R_1 + r_{\pi 3}}$$

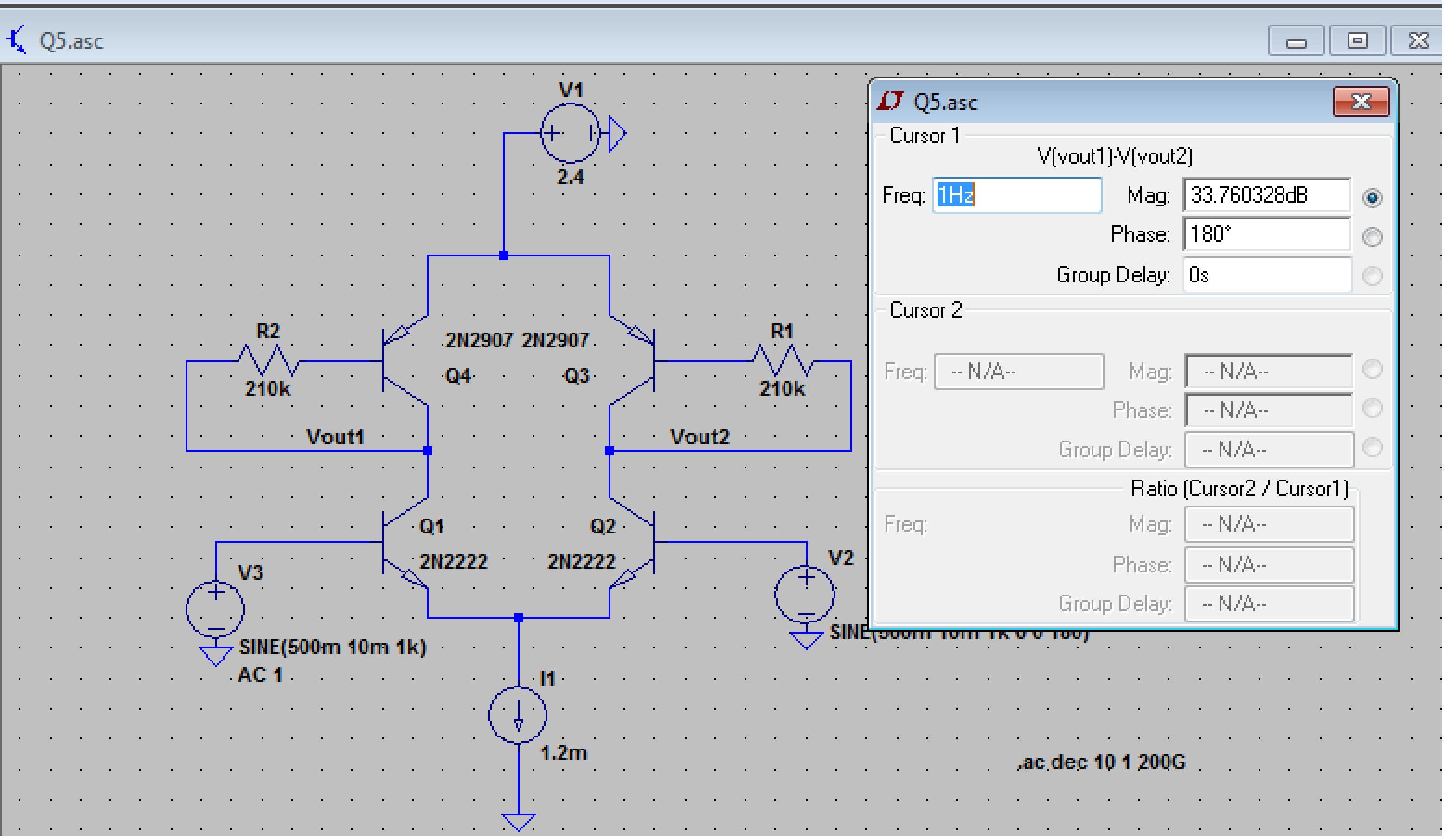
$$(I_{out} - g_{m3} V_{\pi 3})\underbrace{(r_{o1} \| r_{o3} \| (R_1 + r_{\pi 3}))}_{R_{eq}} = V_{out}$$

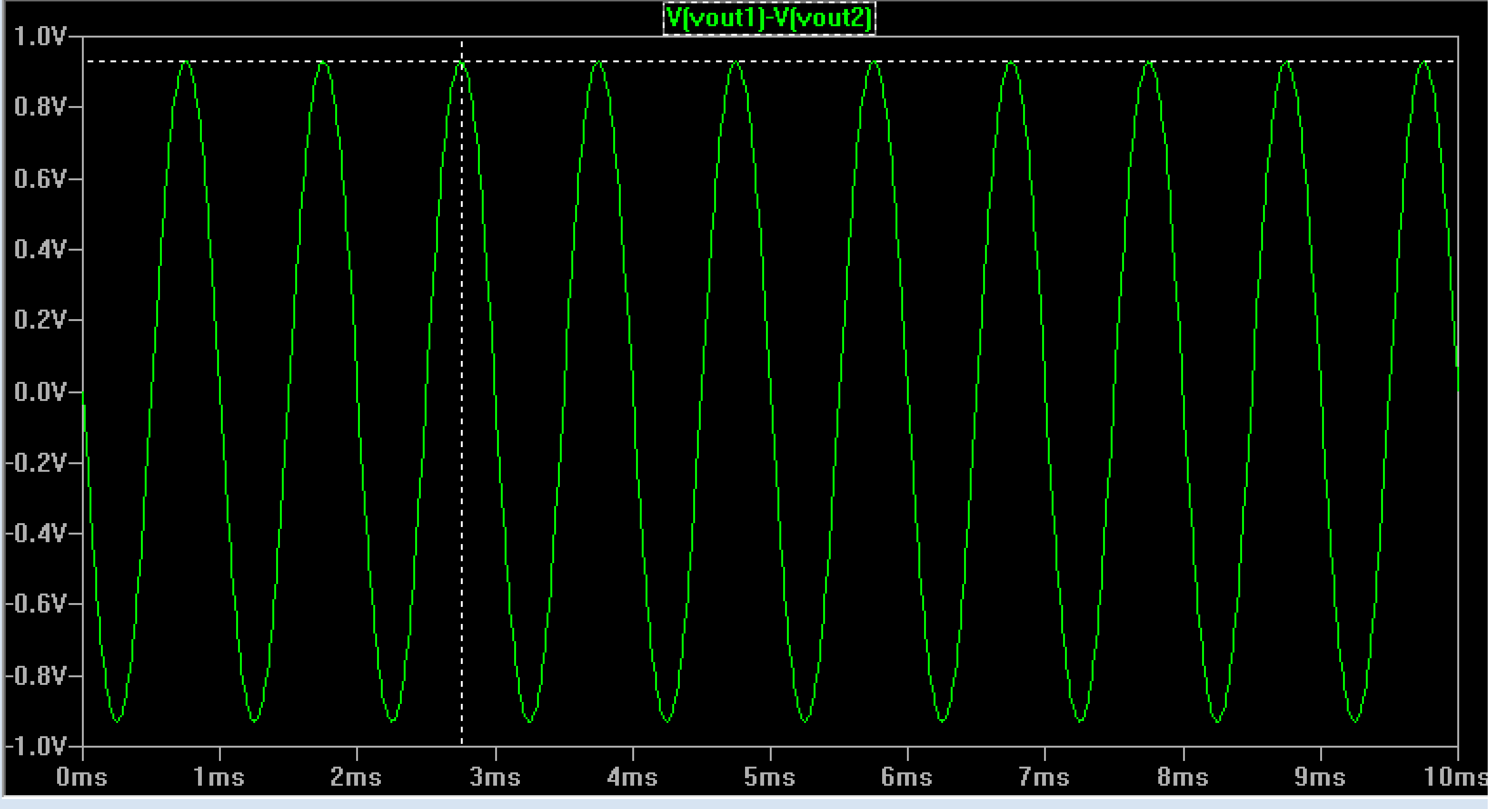
$$\frac{V_{out}}{I_{out}} = R_{out} = \frac{R_{eq}(R_1 + r_{\pi 3})}{g_{m3} r_{\pi 3} R_{eq} + R_1 + r_{\pi 3}} \cong \frac{R_1 + r_{\pi 3}}{g_{m3} r_{\pi 3}}$$

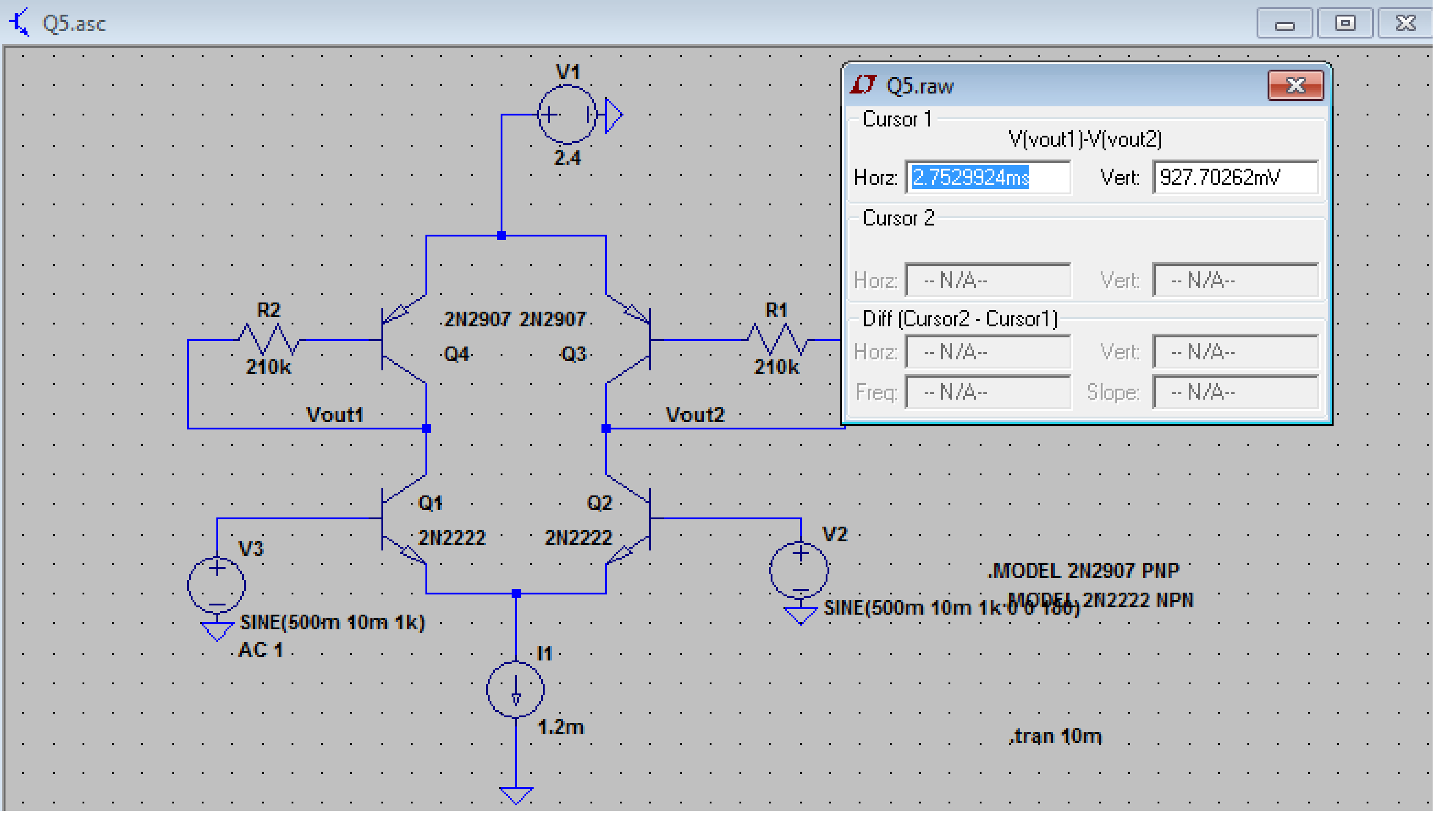
$$A_v = \frac{V_{out}}{V_{in}} = -g_{m1} \cdot \frac{R_1 + r_{\pi 3}}{g_{m3} r_{\pi 3}}$$

Due to constant current, we can't change value of $g_{m1}$, $g_{m3}$ and $r_{\pi 3}$. However, we can increase $R_1$ to obtain higher gain. Note that after a certain $R_1$ value, input transistors will go into saturation region leading to gain drop.


V(vout1)-V(vout2)
33.768dB
33.766dB
33.764dB
33.762dB
33.760dB
33.758dB
33.756dB
33.754dB
33.752dB
33.750dB
180.20°
180.16°
180.12°
180.08°
180.04°
180.00°
179.96°
179.92°
179.88°
179.84°
179.80°
1Hz
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz
10MHz
100MHz
1GHz
10GHz
100GHz


Q5.asc
V1
2.4
2N2907 2N2907
Q4
Q3
R2
210k
R1
210k
Vout1
Vout2
Q1
Q2
2N2222
2N2222
V3
V2
SINE(500m 10m 1k)
AC 1
I1
1.2m
.ac dec 10 1 200G
Q5.asc
Cursor 1
V(vout1)-V(vout2)
Freq: 1Hz
Mag: 33.760328dB
Phase: 180°
Group Delay: 0s
Cursor 2
Freq: -- N/A--
Mag: -- N/A--
Phase: -- N/A--
Group Delay: -- N/A--
Ratio (Cursor2 / Cursor1)
Freq:
Mag: -- N/A--
Phase: -- N/A--
Group Delay: -- N/A--


V(vout1)-V(vout2)
1.0V
0.8V
0.6V
0.4V
0.2V
0.0V
-0.2V
-0.4V
-0.6V
-0.8V
-1.0V
0ms
1ms
2ms
3ms
4ms
5ms
6ms
7ms
8ms
9ms
10ms


Q5.asc
V1
2.4
Q5.raw
Cursor 1
V(vout1)-V(vout2)
Horz: 2.7529924ms
Vert: 927.70262mV
Cursor 2
Horz: -- N/A--
Vert: -- N/A--
Diff (Cursor2 - Cursor1)
Horz: -- N/A--
Vert: -- N/A--
Freq: -- N/A--
Slope: -- N/A--
R2
210k
2N2907 2N2907
Q4
Q3
R1
210k
Vout1
Vout2
Q1
Q2
2N2222
2N2222
V3
V2
SINE(500m 10m 1k)
AC 1
SINE(500m 10m 1k 0 0 180)
.MODEL 2N2907 PNP
.MODEL 2N2222 NPN
I1
1.2m
.tran 10m