EHB 315E Digital Signal Processing
Fall 2020
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# HOMEWORK 1 - SOLUTIONS

**1 [20 pts]** Make an accurate sketch of each of the discrete-time signals

(a) $x(n) = u(n+3) + 0.5u(n-1)$
(b) $x(n) = \delta(n+3) + 0.5\delta(n-1)$
(c) $x(n) = 2^n \cdot \delta(n-4)$
(d) $x(n) = 2^n \cdot u(-n-2)$
(e) $x(n) = (-1)^n u(-n-4)$

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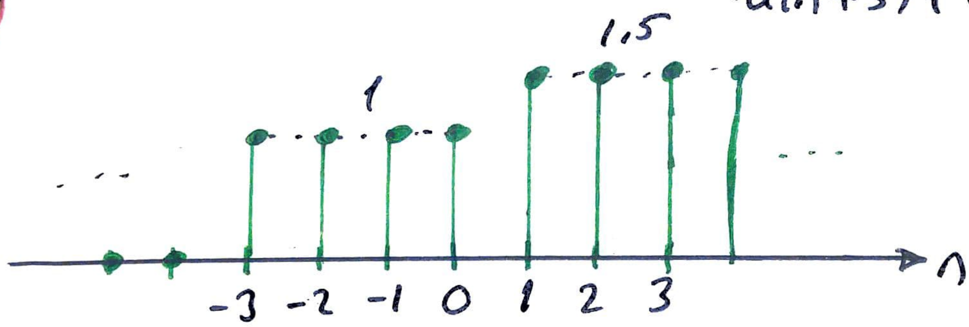
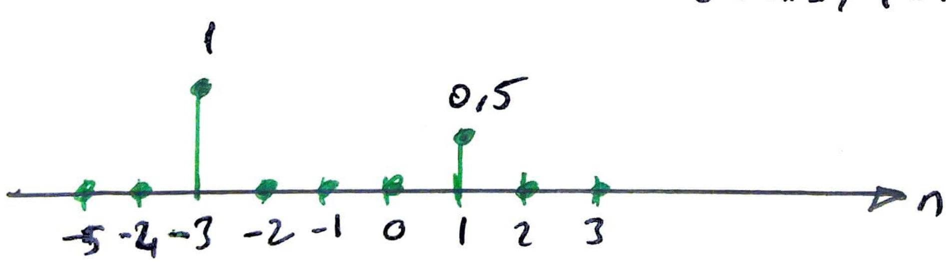
(a) $u(n+3) + 0{,}5\,u(n-1)$



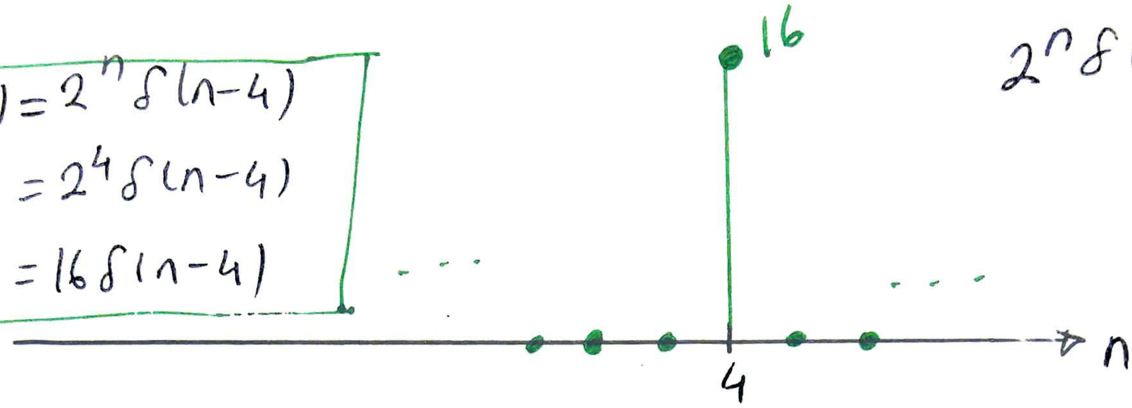

(b) $\delta(n+3) + 0{,}5\,\delta(n-1)$



c

$$x(n) = 2^n \delta(n-4)$$
$$= 2^4 \delta(n-4)$$
$$= 16\delta(n-4)$$

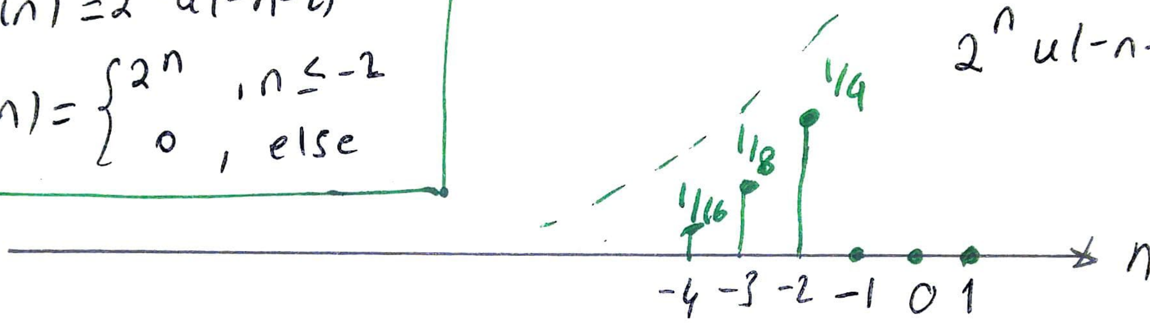
$2^n \delta(n-4)$




d

$$x(n) = 2^n u(-n-2)$$
$$x(n) = \begin{cases} 2^n & , n \leq -2 \\ 0 & , \text{else} \end{cases}$$

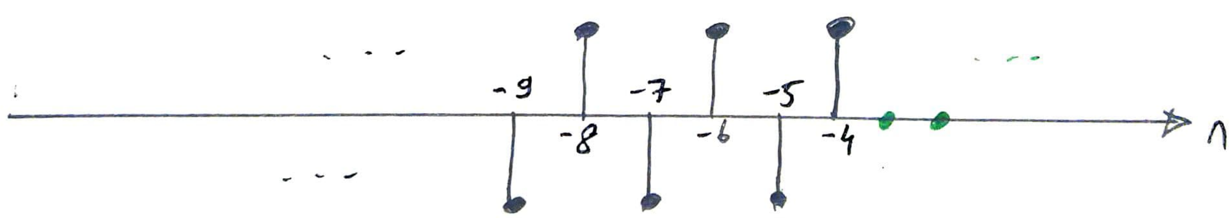
$2^n u(-n-2)$




e

$$x(n) = (-1)^n u(-n-4)$$
$$x(n) = \begin{cases} (-1)^n & , n \leq -4 \\ 0 & , \text{else} \end{cases}$$

$(-1)^n u(-n-4)$



**2 [20 pts]** Determine which of the following signals is periodic. If a signal is periodic, determine its period

(a) $x(n) = e^{j(2\pi n/5)}$

(b) $x(n) = \sin(\pi n/9) \cdot \cos(\pi n/12) + \cos(2\pi n/15)$

(c) $x(n) = ne^{j\pi n}$

(d) $x(n) = e^{jn}$

(e) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-3k] + \delta[n-k^2]$

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Periodicity condition is as follows:

$$\boxed{x(n) = x(n+N)}$$

(a) $e^{j(2\pi n/5)} = e^{j(2\pi(n+N)/5)}$

$$\boxed{\omega_0 N = 2\pi k}$$

$\frac{2\pi}{5} \cdot N = 2\pi k \longrightarrow N = 5 \ (k=1)$

$\boxed{x(n) \text{ is PERIODIC w/ } N=5}$

(b) $x(n) = \underbrace{\sin(\pi n/9)}_{x_1(n)} \cdot \underbrace{\cos(\pi n/12)}_{x_2(n)} + \underbrace{\cos(2\pi n/15)}_{x_3(n)}$

$x_1(n) \longrightarrow N_1 = 18$

$x_2(n) \longrightarrow N_2 = 24$

$N_{12} = \frac{18 \cdot 24}{\gcd(18,24)} = 72$

$x_3(n) \longrightarrow N_3 = 15$

$N = \frac{15 \cdot 72}{\gcd(15,72)} = 360$

$x(n)$ is PERIODIC with $N=360$

(c) $x(n) = \underbrace{n}_{x_1(n)}\underbrace{e^{j\pi n}}_{x_2(n)}$

$x_2(n)$ is periodic $\rightarrow \pi N = 2\pi k$, $N=2$

$x_1(n)$ is not periodic $\rightarrow$

$x_1(n)$

Multiplication of periodic and non-periodic signals creates non-periodic signals $\rightarrow$ $\boxed{\text{Hence, } x(n) \text{ is NOT PERIODIC}}$

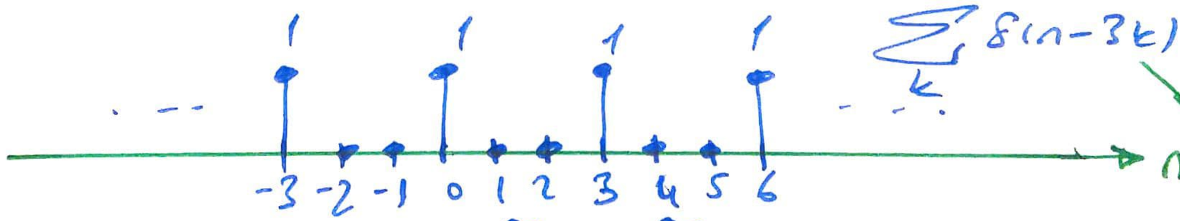
(d) $e^{jn} = e^{j(n+N)} \rightarrow e^{jn} = e^{jn}\underbrace{e^{jN}}$ If $e^{jN}$ is 1, the signal is periodic.

$e^{jN} = 1$

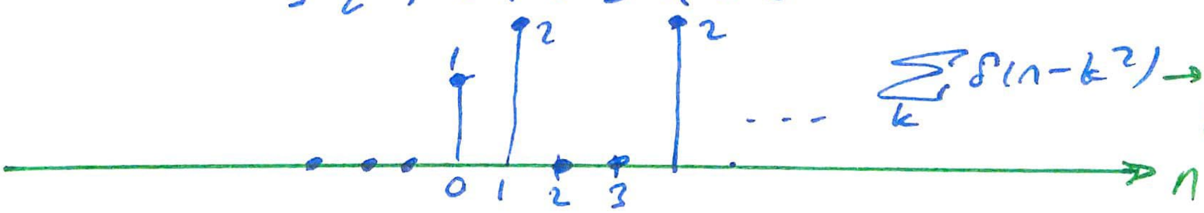
$N = 2\pi k$, for $k \in \mathbb{Z}$

Since $2\pi k$ is an irrational number, $\boxed{e^{jN} \text{ IS NOT PERIODIC}}$

(e) $x(n) = \sum_{k=-\infty}^{\infty} \delta(n-3k) + \delta(n-k^2) = \sum_k \delta(n-3k) + \sum_k \delta(n-k^2)$



$\sum_k \delta(n-3k) \rightarrow$ PERIODIC



$\sum_k \delta(n-k^2) \rightarrow$ NOT PERIODIC

Summation of periodic and non-periodic signals produces non-periodic signals. Therefore, $x(n)$ is $\boxed{\text{NOT PERIODIC.}}$

**3 [20 pts]** Derive and sketch the convolution $x(n) = (f * g)(n)$ where

(a) $f(n) = 2\delta(n+10) + 2\delta(n-10)$

$g(n) = 3\delta(n+5) + 3\delta(n-5)$

(b) $f(n) = (-1)^n$

$g(n) = \delta(n) + \delta(n-1)$

(c) $f(n) = u(n) - u(n-5)$

$$g(n) = \sum_{k=0}^{\infty} \delta(n-5k)$$

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(a)

$x(n) = f(n) * g(n)$

$= f(n) * (3\delta(n+5) + 3\delta(n-5))$

$= 3f(n+5) + 3f(n-5)$

$= 3[2\delta(n+15) + 2\delta(n-5)] + 3[2\delta(n+5) + 2\delta(n-15)]$

$= 6\delta(n+15) + 6\delta(n-5) + 6\delta(n+5) + 6\delta(n-15)$

(b)

$x(n) = f(n) * g(n)$

$= f(n) * (\delta(n) + \delta(n-1))$

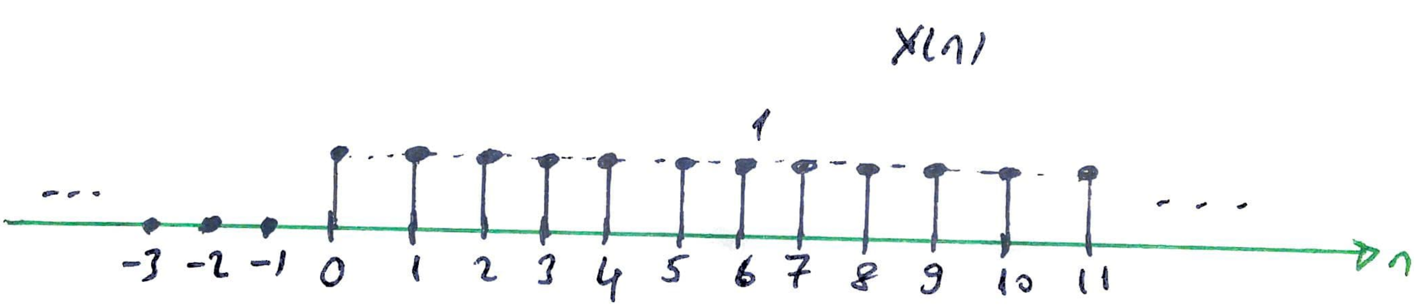
$= f(n) + f(n-1)$

$= (-1)^n + (-1)^{n-1}$

$= 0$

(c)

$$
\begin{aligned}
x(n) &= f(n) * g(n) \\
&= g(n) * f(n) \\
&= g(n) * \left(\delta(n)+\delta(n-1)+\delta(n-2)+\delta(n-3)+\delta(n-4)\right) \\
&= g(n)+g(n-1)+g(n-2)+g(n-3)+g(n-4) \\
&= \sum_{k=0}^{\infty}\Big[\delta(n-5k)+\delta(n-1-5k)+\delta(n-2-5k)+ \\
&\qquad \delta(n-3-5k)+\delta(n-4-5k)\Big]
\end{aligned}
$$



4 **[20 pts]** A discrete-time system is described by the following rule

$$y(n) = 0.5x(2n) + 0.5x(2n-1)$$

where $x$ is the input signal, and $y$ the output signal.

(a) Sketch the output signal, $y(n)$, produced by the 4 -point input signal, $x(n)$ is defined below:

$$x(n) = 2\delta(n) + 3\delta(n-1) + 2\delta(n-2) + \delta(n-3)$$

(b) Classify the system as:

i. causal/non-casual

ii. linear/nonlinear

iii. time-invariant/ time varying

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a

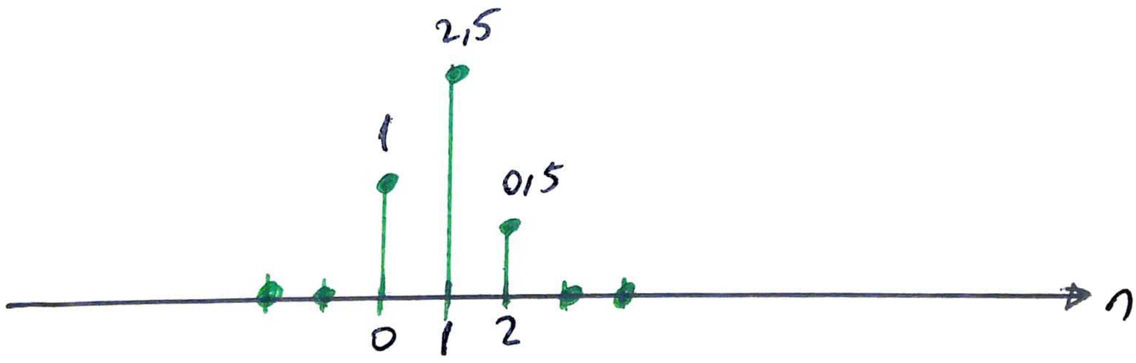
$$x(2n) = 2\delta(n) + 2\delta(n-1)$$

$$x(2n-1) = 3\delta(n-1) + \delta(n-2)$$

$$y(n) = 0{,}5\, x(2n) + 0{,}5\, x(2n-1)$$

$$= \delta(n) + \delta(n-1) + 1{,}5\,\delta(n-1) + 0{,}5\,\delta(n-2)$$

$$= \delta(n) + 2{,}5\,\delta(n-1) + 0{,}5\,\delta(n-2)$$



(b) i. CAUSALITY

The output depends only on present and past values of the input $\longrightarrow$ system is $\boxed{\text{CAUSAL}}$.

ii. LINEARITY

$$T[x_1(n)] = 0{,}5\,x_1(2n) + 0{,}5\,x_1(2n-1)$$

$$T[x_2(n)] = 0{,}5\,x_2(2n) + 0{,}5\,x_2(2n-1)$$

$$\begin{aligned}T[a x_1(n) + b x_2(n)] &= 0{,}5\,(a x_1(2n) + b x_2(2n)) + \\ &\quad 0{,}5\,(a x_1(2n-1) + b x_2(2n-1)) \\ &= a(0{,}5\,x_1(2n) + 0{,}5\,x_1(2n-1)) + \\ &\quad b(0{,}5\,x_2(2n) + 0{,}5\,x_2(2n-1)) \\ &= a\,T[x_1(n)] + b\,T[x_2(n)]\end{aligned}$$

the system is $\boxed{\text{LINEAR}}$

iii. TIME-INVARIANCE

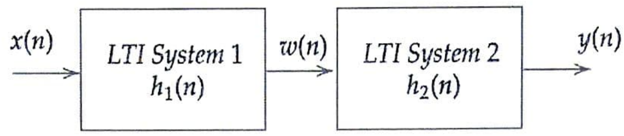
$$T[x_1(n)] = T[x(n-n_0)] = 0{,}5\,x(2n-n_0) + 0{,}5\,x(2n-n_0-$$

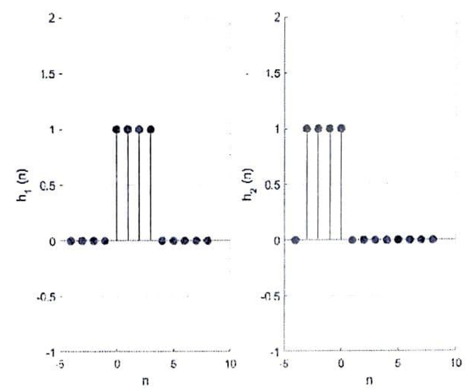
$$y(n-n_0) = 0{,}5\,x(2(n-n_0)) + 0{,}5\,x(2(n-n_0)+1)$$

$$T[x(n-n_0)] \neq y(n-n_0)$$

the system is $\boxed{\text{time-varying}}$

5 **[20 pts]** Consider cascade connection of two LTI systems in the figure below:




(a) Determine and sketch the overall impulse response of the cascade system

(b) Determine and sketch $w(n)$ if $x(n) = (-1)^n \cdot u(n)$. Also determine the overall output $y(n)$

(c) Determine the overall system is (i) causal, (ii) stable, (iii) memoryless, (iv) linear, (v) time-invariant

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a

$$h_1(n) = u(n) - u(n-4) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)$$

$$h_2(n) = u(n+3) - u(n-1) = \delta(n+3) + \delta(n+2) + \delta(n+1) + \delta(n)$$

$\boxed{\text{Soln a}}$

Let $h(n)$ denote the overall impulse response

$\boxed{\text{ANALYTICAL SOLUTION}}$

$$h(n) = h_1(n) * h_2(n)$$

$$= h_1(n) * [\delta(n+3) + \delta(n+2) + \delta(n+1) + \delta(n)]$$

$$= h_1(n+3) + h_1(n+2) + h_1(n+1) + h_1(n)$$

$$= \delta(n+3) + 2\delta(n+2) + 3\delta(n+1) + 4\delta(n) + 3\delta(n-1) + 2\delta(n-2) + \delta(n-3)$$

(b) $w(n) = x(n) * h_1(n)$

$$= x(n) * (\delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3))$$

$$= x(n) + x(n-1) + x(n-2) + x(n-3)$$

$$= (-1)^n u(n) + (-1)^{n-1} u(n-1) + (-1)^{n-2} u(n-2) + (-1)^{n-3} u(n-3)$$

$$= (-1)^n u(n) - (-1)^n u(n-1) + (-1)^n u(n-2) - (-1)^n u(n-3)$$

$$= (-1)^n [u(n) - u(n-1) + u(n-2) - u(n-3)]$$

$$w(n) = (-1)^n [\delta(n) + \delta(n-2)]$$

$y(n) = w(n) * h_2(n)$

$$= w(n) * (\delta(n+3) + \delta(n+2) + \delta(n+1) + \delta(n))$$

$$= w(n+3) + w(n+2) + w(n+1) + w(n)$$

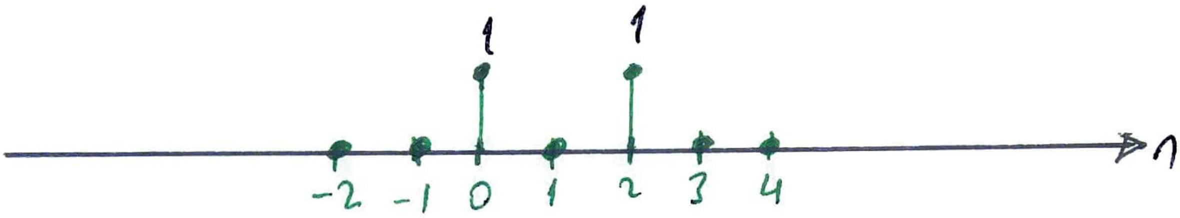
$$= (-1)^{n+3}[\delta(n+3) + \delta(n+1)] + (-1)^{n+2}[\delta(n+2) + \delta(n)] + (-1)^{n+1}[\delta(n+1) + \delta(n-1)] + (-1)^n[\delta(n) + \delta(n-2)]$$

$$= -(-1)^n[\delta(n+3) + \delta(n+1)] + (-1)^n[\delta(n+2) + \delta(n)] - (-1)^n[\delta(n+1) + \delta(n-1)] + (-1)^n[\delta(n) + \delta(n-2)]$$

$$= (-1)^n[-\delta(n+3) - \delta(n+1) + \delta(n+2) + \delta(n) - \delta(n+1) - \delta(n-1) + \delta(n) + \delta(n-2)]$$

$$= (-1)^n[-\delta(n+3) + \delta(n+2) - 2\delta(n+1) + 2\delta(n) - \delta(n-1) + \delta(n-2)]$$

$$w(n) = (-1)^n [\delta(n) + \delta(n-2)]$$




(c) $h(n) = \delta(n+3) + 2\delta(n+2) + 3\delta(n+1) + 4\delta(n) + 3\delta(n-1) + 2\delta(n-2) + \delta(n-3)$

i. NOT CAUSAL

ii. STABLE

iii. NOT MEMORYLESS

iv. LINEAR

v. TIME-INVARIANT