İSTANBUL TEKNİK ÜNİVERSİTESİ

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HOMEWORK II

Electromagnetic Waves

EHB 313E

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$$E_x = -i.w.\mu.\frac{\pi}{b}.\sin\left(\frac{\pi}{b}y\right).\sin\left(\frac{\pi}{h}.z\right) \quad , \quad E_y = E_z = 0$$

For The Magnetic Field

By using the Time Harmonic Maxwell's Equation;

$$\rightarrow \nabla \times \vec{E} = -i.w.\mu.\vec{H}$$

$$E = E_x.\vec{e}_x + 0.\vec{e}_y + 0.\vec{e}_z = E_x.\vec{e}_x$$

$$\nabla \times E = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & 0 & 0 \end{vmatrix} = \vec{e}_y.\frac{\partial}{\partial z}.E_x - \vec{e}_z\frac{\partial}{\partial y}E_x$$

$$\frac{\partial}{\partial z}E_x = -i.w.\mu.\frac{\pi^2}{h.b}.\sin\left(\frac{\pi}{b}y\right)\cos\left(\frac{\pi}{h}z\right)$$

$$\frac{\partial}{\partial y}E_x = -i.w.\mu.\frac{\pi^2}{b^2}\cos\left(\frac{\pi}{b}y\right).\sin\left(\frac{\pi}{h}z\right)$$

$$-i.w.\mu.\vec{H} = -iw\mu\left[\frac{\pi^2}{hb}.\sin\left(\frac{\pi}{b}y\right).\cos\left(\frac{\pi}{h}.z\right).\vec{e}_y - \frac{\pi^2}{b^2}.\cos\left(\frac{\pi}{b}y\right).\sin\left(\frac{\pi}{h}z\right)\vec{e}_z\right]$$

$$\vec{H}(y,z) = \frac{\pi^2}{hb}.\sin\left(\frac{\pi}{b}y\right).\cos\left(\frac{\pi}{h}.z\right)\vec{e}_y - \frac{\pi^2}{b^2}.\cos\left(\frac{\pi}{b}y\right).\sin\left(\frac{\pi}{h}z\right)\vec{e}_z \quad //$$

The magnetic field vector shown above is in the phasor domain. The time domain representation is given in the next page.

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$$\vec{H}(y,z)=\frac{\pi^2}{hb}\left[-\frac{i}{2}\left(e^{i\frac{\pi}{b}y}-e^{-i\frac{\pi}{b}y}\right)\frac{1}{2}\left(e^{i\frac{\pi}{h}z}+e^{-i\frac{\pi}{h}\cdot z}\right)\right]\vec{e}_y$$

$$-\frac{\pi^2}{b^2}\left[\frac{1}{2}\left(e^{i\frac{\pi}{b}y}+e^{-i\frac{\pi}{b}y}\right)\left(\frac{-i}{2}\right)\left(e^{i\frac{\pi}{h}z}-e^{-i\frac{\pi}{h}z}\right)\right]\vec{e}_z$$

$$\vec{H}(y,z)=\frac{\pi^2}{hb}\left(\frac{-i}{4}\right)\left[e^{i\pi\left(\frac{y}{b}+\frac{z}{h}\right)}+e^{i\pi\left(\frac{y}{b}-\frac{z}{h}\right)}-e^{i\pi\left(\frac{z}{h}-\frac{y}{b}\right)}-e^{i\pi\left(-\frac{y}{b}-\frac{z}{h}\right)}\right]\vec{e}_y$$

$$-\frac{\pi^2}{b^2}\left(\frac{-i}{4}\right)\left[e^{i\pi\left(\frac{y}{b}+\frac{z}{h}\right)}-e^{i\pi\left(\frac{y}{b}-\frac{z}{h}\right)}+e^{i\pi\left(\frac{z}{h}-\frac{y}{b}\right)}-e^{i\pi\left(-\frac{y}{b}-\frac{z}{h}\right)}\right]\vec{e}_z$$

$$\vec{H}(y,z,t)=Re\left\{H(y,z)\cdot e^{iwt}\right\}$$

$$\vec{H}(y,z,t)=\frac{\pi^2}{4hb}\left[\sin\left(\frac{y\pi}{b}+\frac{z\pi}{h}+wt\right)+\sin\left(\frac{y\pi}{b}-\frac{z\pi}{h}+wt\right)-\sin\left(\frac{z\pi}{h}-\frac{y\pi}{b}+wt\right)-\sin\left(-\frac{y\pi}{b}-\frac{z\pi}{h}+wt\right)\right]\vec{e}_y$$

$$-\frac{\pi^2}{4b^2}\left[\sin\left(\frac{y\pi}{b}+\frac{z\pi}{h}+wt\right)-\sin\left(\frac{y\pi}{b}-\frac{z\pi}{h}+wt\right)+\sin\left(\frac{z\pi}{h}-\frac{y\pi}{b}+wt\right)-\sin\left(-\frac{y\pi}{b}-\frac{z\pi}{h}+wt\right)\right]\vec{e}_z$$

At the last step, the magnetic field vector is found in the time domain.

The Relation Between 'w', 'b' and 'h'

The Helmholtz equation for electric field vector;

$$\nabla^2\vec{E}+k^2\vec{E}=0 \Rightarrow (\nabla^2+k^2)\vec{E}=0$$

Where

$k^2=w^2\varepsilon\mu-iw\mu\sigma$ (In free space $\sigma=0$ and $k^2=w^2\varepsilon\mu$)

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$$\nabla^2 \vec{E} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}$$

$$= 0 + i w \mu \frac{\pi^3}{b^3} \sin\left(\frac{\pi}{b} y\right) \sin\left(\frac{\pi}{h} z\right) + i w \mu \frac{\pi^3}{h^2 b} \sin\left(\frac{\pi}{b} y\right) \sin\left(\frac{\pi}{h} z\right)$$

$$= \left(-\frac{\pi^2}{b^2} - \frac{\pi^2}{h^2}\right) \underbrace{\left[-i w \mu \frac{\pi}{b} \sin\left(\frac{\pi}{b} y\right) \sin\left(\frac{\pi}{h} z\right)\right]}_{\vec{E}}$$

By Using Helmholtz Equation;

$$\left(\nabla^2 + k^2\right) \vec{E} = 0$$

$$\underbrace{\left(-\frac{\pi^2}{b^2} - \frac{\pi^2}{h^2} + k^2\right)}_{=0} \vec{E} = 0$$

$$k^2 = \frac{\pi^2}{b^2} + \frac{\pi^2}{h^2}$$

} "k^2" has only the real part, therefore, "σ" equals to the "0" due to the free space.

$$w^2 \cdot \varepsilon \cdot \mu = \frac{\pi^2}{b^2} + \frac{\pi^2}{h^2}$$

$$\boxed{w = \frac{1}{\sqrt{\varepsilon \cdot \mu}} \cdot \left(\frac{\pi^2}{b^2} + \frac{\pi^2}{h^2}\right)^{1/2}}$$