1. A second-order recursive system is describerd by the LCCDE

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) - x(n-1)$$

a) Find the unit sample response h(n) of this system.

b) Find the system's response to the input $x(n) = \left(\frac{1}{2}\right)^n u(n)$.

Term in $x(n)$	Particular Solution
C	C_1
Cn	$C_1 n + C_2$
Ca^n	$C_1 a^n$
$C\cos(n\omega_0)$	$C_1\cos(n\omega_0) + C_2\sin(n\omega_0)$
$C\sin(n\omega_0)$	$C_1\cos(n\omega_0) + C_2\sin(n\omega_0)$
$Ca^n\cos(n\omega_0)$	$C_1 a^n\cos(n\omega_0) + C_2 a^n\sin(n\omega_0)$
$C\delta(n)$	None

a) $r^2 - \frac{3}{4}r + \frac{1}{8} = \left(r - \frac{1}{4}\right)\left(r - \frac{1}{2}\right)$

$y_h(n) = A_1\left(\frac{1}{2}\right)^n + A_2\left(\frac{1}{4}\right)^n \quad n \geq 0$

$y(0) = \frac{3}{4}y(-1) - \frac{1}{8}y(-2) + x(0) - x(1) = 1$ $\qquad 1 = A_1 + A_2$

$y(1) = \frac{3}{4}y(0) - \frac{1}{8}y(-1) + x(1) - x(0) = \frac{3}{4} - 1 = -\frac{1}{4}$ $\qquad -\frac{1}{4} = \frac{1}{2}A_1 + \frac{1}{4}A_2$ $\qquad A_1 = -2$, $A_2 = 3$

$y(n) = -2\left(\frac{1}{2}\right)^n + 3\left(\frac{1}{4}\right)^n \quad n \geq 0$, $\quad h(n) = \left[-2\left(\frac{1}{2}\right)^n + 3\left(\frac{1}{4}\right)^n\right]u(n)$

b) $y_p(n) = Kn\left(\frac{1}{2}\right)^n$

$Kn\left(\frac{1}{2}\right)^n = \frac{3}{4}K(n-1)\left(\frac{1}{2}\right)^{n-1} - \frac{1}{8}K(n-2)\left(\frac{1}{2}\right)^{n-2} + \left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^{n-1}$

dividing through by $\left(\frac{1}{2}\right)^n$

$Kn = \frac{3}{4}K(n-1)\,2 - \frac{1}{8}K(n-2)\,4 + 1 - 2$

$Kn = \frac{3}{2}K(n-1) - \frac{1}{2}K(n-2) - 1$, $\quad K = -2$

$y(n) = -2n\left(\frac{1}{2}\right)^n + A_1\left(\frac{1}{2}\right)^n + A_2\left(\frac{1}{4}\right)^n \quad n \geq 0$

$y(0) = 1 \quad y(1) = \frac{1}{4}$ $\qquad 1 = A_1 + A_2$, $\quad \frac{1}{4} = -1 + \frac{1}{2}A_1 + \frac{1}{4}A_2$ $\qquad A_1 = 4$, $A_2 = 3$

$y(n) = \left[-2n\left(\frac{1}{2}\right)^n + 4\left(\frac{1}{2}\right)^n - 3\left(\frac{1}{4}\right)^n\right]u(n)$