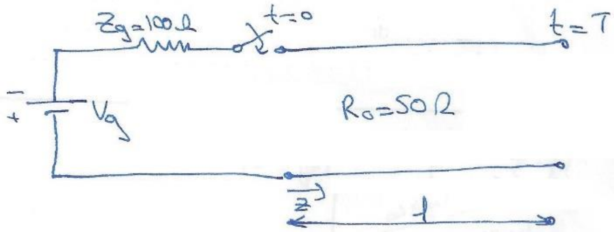
Wave Responses

Step-Function Response



The figure above shows a step function of V_g voltz from a d-c generator with an internal impedance of $100\ \Omega$, which is connected to an initially uncharged, open circuited lossless line of $R_o = 50\ \Omega$ at $t=0$. The voltage and current responses along the line can be described with the following steps:

1. Let T be the time required for the wave to travel once from the sending end to the receiving end. When the switch is closed at $t=0$, the first incident voltage to the sending end of the line is,

$$V_+ = \frac{V_g R_0}{R_0 + Z_g} = \frac{V_g}{3} \text{ volts}$$

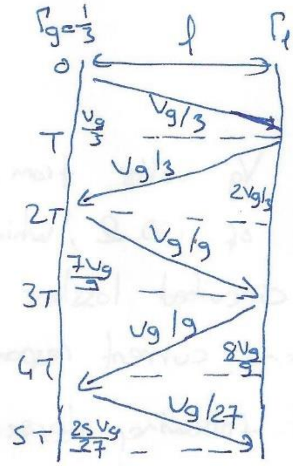
2. The first reflected voltage from the receiving end is

$$V_- = V_+ \Gamma_\ell = \frac{V_g}{3}\left(\frac{Z_\ell - R_0}{Z_\ell + R_0}\right) = \frac{V_g}{3} \text{ volts.}$$

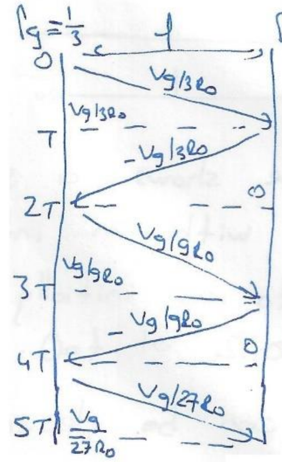
3. The first reflected voltage from the sending end is

$$V_{+r} = V_- \Gamma_g = \frac{V_g}{3}\left(\frac{Z_g - R_0}{Z_g + R_0}\right) = \frac{V_g}{9} \text{ volts}$$

4. The second reflected voltage from the receiving end is $V_g/9$ and so on.
5. The current along the line can be obtaind by dividing the voltage by R_o and changing the sign of Γ_ℓ and Γ_g from positive to negative.
6. The figure below shows the zig zag diagrams for voltage and current on a lossless line



voltage



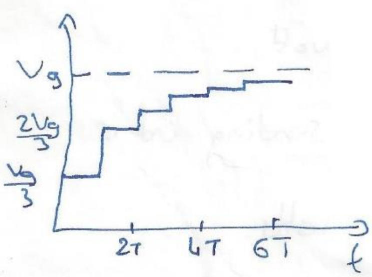
current

7. The line voltage and current, respectively, are the sum of the two travelling waves as shown the figure below. Thus, as $t \to \infty$,

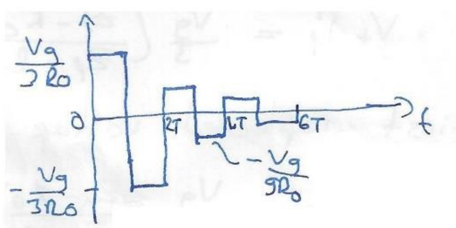
$$V_{line} = \frac{2V_g}{3}\left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \cdots\right] = \frac{2V_g}{3}\left[\frac{1}{1-\frac{1}{3}}\right] = V_g$$

and

$$I_{line} = \frac{V_g}{3R_0}\left[1 - 1 + \frac{1}{3} - \frac{1}{3} + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 + \cdots\right] = 0$$



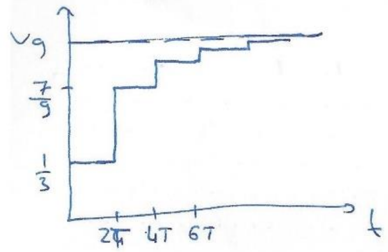
line voltage



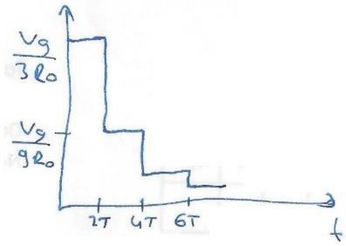
line current

8. The sending end voltage and current are the sum of the two travelling waves of the sending end as illustrated in fig. below

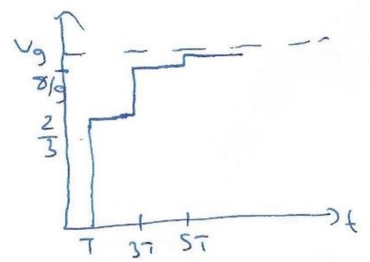
9. The receiving-end voltage and current are also sum of the two travelling waves. Since the receiving end is an open circuit, the receiving end voltage should be equal to the source voltage, and its current should be zero.



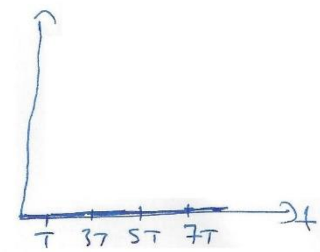
Sending-end voltage



Sending-end current

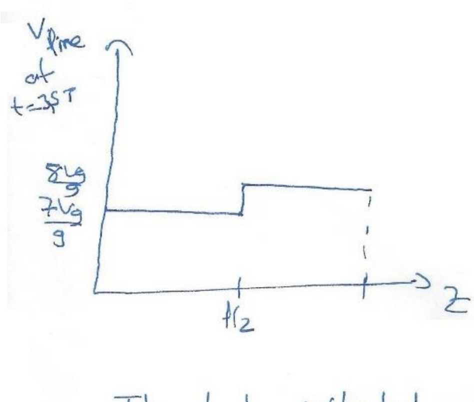


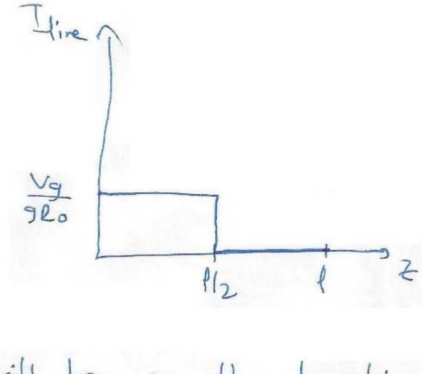
Receiving-end voltage



Receiving-end current

10. The voltage and current at any time t_i on the line can be calculated from zigzag diagrams or voltage and current series. The voltage and current at any time t_i will be the sum of all sending and reflected wave values at a location on the line. These values which are the function of location z are shown in figures below:





The last reflected wave will be on the location of $z = l/2$ in the (−) z direction. Thus the values of voltage and current are changed after $z = l/2$ location.