

①

## ~ PART I ~

1. most general form:

$$① \operatorname{rot} \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$② \operatorname{rot} \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J}_v + \sigma \vec{E}$$

$$③ \operatorname{div} \vec{D} = \rho$$

$$④ \operatorname{div} \vec{B} = 0$$

simplest form: for complex domain, in simple medium

$$① \operatorname{rot} \vec{E} - j\omega \mu \vec{H} = 0$$

$$② \operatorname{rot} \vec{H} + j\omega \epsilon \vec{E} = \vec{J}_v + \sigma \vec{E}$$

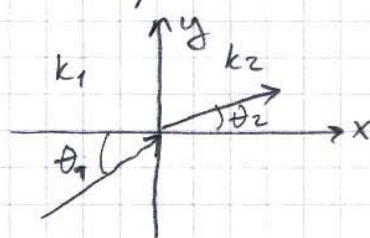
$$③ \operatorname{div} \vec{E} = \rho / \epsilon$$

$$④ \operatorname{div} \vec{H} = 0$$

2. a) Brewster Angle:  $\theta_B = \arctan \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

b) Skin depth:  $\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$

c) Snell's Relation:



$$\Rightarrow k_1 \sin \theta_1 = k_2 \sin \theta_2$$

$$3. a) u(y, t) = e^{-y} \sin(3y + 6 \cdot 10^8 t) = e^{-y} \cos(3y + 6 \cdot 10^8 t - \frac{\pi}{2})$$

$$= e^{-y} \cos(-3y + \frac{\pi}{2} - 6 \cdot 10^8 t) = \operatorname{Re} \left\{ e^{-y} \cdot e^{i(-\frac{3}{2}y + \frac{\pi}{2} - 6 \cdot 10^8 t)} \right\}$$

$$= \operatorname{Re} \left\{ e^{-y} \cdot e^{-i3y} \cdot \underbrace{e^{i\frac{\pi}{2}}}_{i} \cdot \underbrace{e^{-i6 \cdot 10^8 t}}_{e^{-i\omega t}} \right\}$$

$$\Rightarrow \boxed{u(y) = i \cdot e^{-y} \cdot e^{-i3y}} \Rightarrow \boxed{\vec{n} = -\vec{e}_y}$$

$$b) u(x, t) = \cos 2x \cdot \sin 3t = \frac{1}{2} [\sin(2x + 3t) - \sin(2x - 3t)]$$

$$= \frac{1}{2} [\cos(2x + 3t - \frac{\pi}{2}) - \cos(2x - 3t - \frac{\pi}{2})]$$

$$= \frac{1}{2} [\cos(-2x + \frac{\pi}{2} - 3t) - \cos(2x - \frac{\pi}{2} - 3t)]$$

$$\begin{aligned}
 &= \text{Re} \left\{ \frac{1}{2} \left[ e^{-i2x} \cdot e^{i\frac{\pi}{2}} \cdot e^{-i3t} - e^{i2x} \cdot e^{-i\frac{\pi}{2}} \cdot e^{-i3t} \right] \right\} \\
 &= \text{Re} \left\{ \frac{i}{2} \left( e^{-i2x} + e^{i2x} \right) \cdot e^{-i3t} \right\} \\
 &\quad \underbrace{\left( e^{-i2x} + e^{i2x} \right)}_{u(x)} \quad \underbrace{e^{-i3t}}_{e^{-i\omega t}}
 \end{aligned}$$

$$u(x) = \frac{i}{2} (e^{-i2x} + e^{i2x})$$

$$= i \cdot \cos(2x)$$

$$\Rightarrow \vec{n}_- = -\vec{e}_x, \quad \vec{n}_+ = \vec{e}_x$$

(ters yönde ilerleyen iki dalga)

$$c) u(x, y, z, t) = 3e^{-z} \cdot \cos(ax + by - \omega t)$$

$$= \text{Re} \left\{ 3 \cdot e^{-z} \cdot e^{i(ax + by - \omega t)} \right\} = \text{Re} \left\{ \underbrace{3 \cdot e^{-z} \cdot e^{i(ax + by)}}_{u(x, y, z)} \cdot e^{-i\omega t} \right\}$$

$$\Rightarrow u(x, y, z) = 3 \cdot e^{-z} \cdot e^{i(ax + by)}$$

$$ax + by = k \cdot \vec{n} \cdot \vec{r} = \underbrace{(a\vec{e}_x + b\vec{e}_y)}_{k \cdot \vec{n}} \cdot \underbrace{(x\vec{e}_x + y\vec{e}_y)}_{\vec{r}}$$

$$\Rightarrow \vec{n} = \frac{a\vec{e}_x + b\vec{e}_y}{\sqrt{a^2 + b^2}}$$

$$4. \vec{E}(x, y) = e^{i(5x + 12y)} \cdot \vec{e}_z$$

a) yukarıdaki ifade,  $e^{ik\vec{n}\vec{r}}$  formunda yazılabildiğine göre düzlem dalgadır. Bu durumda,

$$k \cdot \vec{n} \cdot \vec{r} = \underbrace{(5\vec{e}_x + 12\vec{e}_y)}_{k \cdot \vec{n}} \cdot \underbrace{(x\vec{e}_x + y\vec{e}_y)}_{\vec{r}}$$

$$\Rightarrow \vec{n} = \frac{5\vec{e}_x + 12\vec{e}_y}{\sqrt{5^2 + 12^2}} \Rightarrow \vec{n} = \frac{5}{13}\vec{e}_x + \frac{12}{13}\vec{e}_y$$

$$b) k = 13$$

$$c) \lambda = \frac{2\pi}{k} \Rightarrow \lambda = \frac{2\pi}{13}$$



$\lambda \rightarrow$  dalga boyu

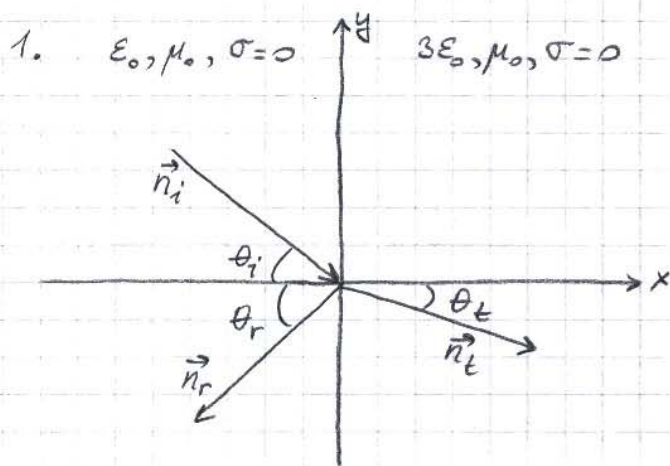
Boş uzayda dalganın faz hızı, ışık hızına eşit olacağından,

$$f = \frac{3 \cdot 10^8}{\frac{2\pi}{13}} = \boxed{\frac{39}{2\pi} \cdot 10^8 \text{ Hz} = f}$$

e) Düzlem dalga için,

$$\begin{aligned}\vec{H} &= \frac{1}{z_0} \cdot \vec{n} \times \vec{E} = \frac{1}{z_0} \cdot \left( \frac{5}{13} \vec{e}_x + \frac{12}{13} \vec{e}_y \right) \times e^{i(5x+12y)} \cdot \vec{e}_z \\ &= \frac{1}{z_0} \cdot e^{i(5x+12y)} \cdot \left( \frac{5}{13} \underbrace{\vec{e}_x \times \vec{e}_z}_{-\vec{e}_y} + \frac{12}{13} \underbrace{\vec{e}_y \times \vec{e}_z}_{\vec{e}_x} \right)\end{aligned}$$

$$\Rightarrow \boxed{\vec{H} = \frac{1}{z_0} \cdot e^{i(5x+12y)} \cdot \left( \frac{12}{13} \vec{e}_x - \frac{5}{13} \vec{e}_y \right)}, \quad z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 120\pi$$



$$\vec{n}_i = \frac{1}{2}\vec{e}_x - \frac{\sqrt{3}}{2}\vec{e}_y$$

$$= \cos\theta_i \vec{e}_x - \sin\theta_i \vec{e}_y$$

$$\left. \begin{aligned} \cos\theta_i &= \frac{1}{2} \\ \sin\theta_i &= \frac{\sqrt{3}}{2} \end{aligned} \right\} \Rightarrow \boxed{\theta_i = 60^\circ}$$

$$\underline{\underline{\theta_r = \theta_i = 60^\circ}}$$

a) Snell's Law:

$$k_1 \cdot \sin\theta_i = k_2 \sin\theta_t$$

$$\omega\sqrt{\epsilon_0\mu_0} \cdot \frac{\sqrt{3}}{2} = \omega\sqrt{3\epsilon_0\mu_0} \cdot \sin\theta_t \Rightarrow \sin\theta_t = \frac{1}{2} \Rightarrow \boxed{\theta_t = 30^\circ}$$

propagation direction of the reflected wave:

$$\vec{n}_r = -\cos\theta_r \vec{e}_x - \sin\theta_r \vec{e}_y$$

$$\boxed{\vec{n}_r = -\frac{1}{2}\vec{e}_x - \frac{\sqrt{3}}{2}\vec{e}_y}$$

propagation direction of the transmitted wave:

$$\vec{n}_t = \cos\theta_t \vec{e}_x - \sin\theta_t \vec{e}_y$$

$$\boxed{\vec{n}_t = \frac{\sqrt{3}}{2}\vec{e}_x - \frac{1}{2}\vec{e}_y}$$

b) 1st Way:

incident field:  $\vec{E} = \underset{\text{amplitude}}{1} \cdot e^{ik_1 \vec{n}_i \vec{r}} \cdot \vec{e}_z$

$$k_1 = \omega\sqrt{\epsilon_0\mu_0} = 6 \cdot 10^8 \sqrt{\frac{10^{-9}}{36\pi} \cdot 4\pi \cdot 10^{-7}} = 6 \cdot 10^8 \cdot \frac{10^{-8}}{3} \Rightarrow \boxed{k_1 = 2}$$



$$\Rightarrow \vec{E}_i = e^{i \cdot 2 \cdot (\frac{1}{2} \vec{e}_x - \frac{\sqrt{3}}{2} \vec{e}_y) \cdot (x \vec{e}_x + y \vec{e}_y)} \cdot \vec{e}_z$$

$$\boxed{\vec{E}_i = e^{i(x - \sqrt{3}y)} \cdot \vec{e}_z}$$

$$z_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = z_0 \quad (\text{free space})$$

$$\vec{H}_i = \frac{1}{z_0} \vec{n}_i \times \vec{E}_i = \frac{1}{z_0} \left( \frac{1}{2} \vec{e}_x - \frac{\sqrt{3}}{2} \vec{e}_y \right) \times e^{i(x - \sqrt{3}y)} \vec{e}_z$$

$$= \frac{1}{z_0} \cdot e^{i(x - \sqrt{3}y)} \left( \frac{1}{2} \vec{e}_x \times \vec{e}_z - \frac{\sqrt{3}}{2} \vec{e}_y \times \vec{e}_z \right)$$

$$\boxed{\vec{H}_i = \frac{1}{z_0} \cdot e^{i(x - \sqrt{3}y)} \cdot \left( -\frac{\sqrt{3}}{2} \vec{e}_x - \frac{1}{2} \vec{e}_y \right)}$$

Components tangential to the interface:

$$\boxed{\begin{aligned} \vec{E}_{it} &= \vec{E}_i = e^{i(x - \sqrt{3}y)} \vec{e}_z \\ \vec{H}_{it} &= -\frac{1}{2z_0} \cdot e^{i(x - \sqrt{3}y)} \cdot \vec{e}_y \end{aligned}}$$

Reflected field:  $\vec{E}_r = A_1 \cdot e^{ik_1 \vec{n}_r \cdot \vec{r}} \cdot \vec{e}_z$

$A_1$ : Reflection Coefficient

$$\vec{E}_r = A_1 \cdot e^{i(-x - \sqrt{3}y)} \cdot \vec{e}_z$$

$$\vec{H}_r = \frac{1}{z_0} \vec{n}_r \times \vec{E}_r = \frac{1}{z_0} \left( -\frac{1}{2} \vec{e}_x - \frac{\sqrt{3}}{2} \vec{e}_y \right) \times A_1 e^{i(-x - \sqrt{3}y)} \cdot \vec{e}_z$$

$$= \frac{A_1}{z_0} e^{i(-x - \sqrt{3}y)} \left( -\frac{1}{2} \vec{e}_x \times \vec{e}_z - \frac{\sqrt{3}}{2} \vec{e}_y \times \vec{e}_z \right)$$

$$\vec{H}_r = \frac{A_1}{z_0} e^{i(-x - \sqrt{3}y)} \left( -\frac{\sqrt{3}}{2} \vec{e}_x + \frac{1}{2} \vec{e}_y \right)$$

tangential Components:

$$\boxed{\begin{aligned} \vec{E}_{rt} &= \vec{E}_r = A_1 \cdot e^{i(-x - \sqrt{3}y)} \cdot \vec{e}_z \\ \vec{H}_{rt} &= \frac{A_1}{2z_0} e^{i(-x - \sqrt{3}y)} \cdot \vec{e}_y \end{aligned}}$$

transmitted field:  $\vec{E}_t = A_2 \cdot e^{ik_2 \vec{n}_t \cdot \vec{r}} \cdot \vec{e}_z$

$$k_2 = \omega \sqrt{3\epsilon_0 \mu_0} = k_1 \cdot \sqrt{3} = \underline{2\sqrt{3}}$$

$$\Rightarrow \vec{E}_t = A_2 \cdot e^{i \cdot 2\sqrt{3} \left( \frac{\sqrt{3}}{2} \vec{e}_x - \frac{1}{2} \vec{e}_y \right) \cdot (x\vec{e}_x + y\vec{e}_y)} \cdot \vec{e}_z$$

$$\Rightarrow \boxed{\vec{E}_t = A_2 \cdot e^{i(3x - \sqrt{3}y)} \cdot \vec{e}_z}$$

$$z_2 = \sqrt{\frac{\mu_0}{3\epsilon_0}} = \frac{z_0}{\sqrt{3}}$$

$$\begin{aligned} \Rightarrow \vec{H}_t &= \frac{1}{z_2} \vec{n}_t \times \vec{E}_t = \frac{\sqrt{3}}{z_0} \left( \frac{\sqrt{3}}{2} \vec{e}_x - \frac{1}{2} \vec{e}_y \right) \times A_2 \cdot e^{i(3x - \sqrt{3}y)} \cdot \vec{e}_z \\ &= \frac{\sqrt{3} A_2}{z_0} \cdot e^{i(3x - \sqrt{3}y)} \cdot \left( \frac{\sqrt{3}}{2} \vec{e}_x \times \vec{e}_z - \frac{1}{2} \vec{e}_y \times \vec{e}_z \right) \end{aligned}$$

$$\boxed{\vec{H}_t = \frac{\sqrt{3} A_2}{z_0} \cdot e^{i(3x - \sqrt{3}y)} \cdot \left( -\frac{1}{2} \vec{e}_x - \frac{\sqrt{3}}{2} \vec{e}_y \right)}$$

Components tangential to the interface:

$$\begin{aligned} \vec{E}_{tt} &= \vec{E}_t = A_2 \cdot e^{i(3x - \sqrt{3}y)} \cdot \vec{e}_z \\ \vec{H}_{tt} &= -\frac{3A_2}{2z_0} \cdot e^{i(3x - \sqrt{3}y)} \cdot \vec{e}_y \end{aligned}$$

Tam yüzey üzerinde yani  $x=0$ 'da, yüzeyin solunda ve sağında kalan alanların teğet bileşenlerinin toplamı eşit olmalı.

Elektrik alanı için:

$$\vec{E}_{it} + \vec{E}_{rt} = \vec{E}_{tt}$$

$$e^{i(x - \sqrt{3}y)} \cdot \vec{e}_z + A_1 \cdot e^{i(-x - \sqrt{3}y)} \cdot \vec{e}_z \Big|_{x=0} = A_2 \cdot e^{i(3x - \sqrt{3}y)} \cdot \vec{e}_z \Big|_{x=0}$$

$$\Rightarrow e^{-i\sqrt{3}y} + A_1 \cdot e^{-i\sqrt{3}y} = A_2 \cdot e^{-i\sqrt{3}y}$$

$$\Rightarrow \boxed{1 + A_1 = A_2} \quad (1)$$



Manyetik alan için:

$$\vec{H}_{it} + \vec{H}_{rt} = \vec{H}_{tt}$$

$$\left. \frac{-1}{2\epsilon_0} \cdot e^{i(x-\sqrt{3}y)} \cdot \vec{e}_y + \frac{A_1}{2\epsilon_0} \cdot e^{i(-x-\sqrt{3}y)} \cdot \vec{e}_y \right|_{x=0} = \left. -\frac{3A_2}{2\epsilon_0} \cdot e^{i(3x-\sqrt{3}y)} \cdot \vec{e}_y \right|_{x=0}$$

$$\Rightarrow \frac{-1}{2\epsilon_0} \cdot e^{i\sqrt{3}y} \cdot \vec{e}_y + \frac{A_1}{2\epsilon_0} \cdot e^{-i\sqrt{3}y} \cdot \vec{e}_y = \frac{-3A_2}{2\epsilon_0} \cdot e^{i\sqrt{3}y} \cdot \vec{e}_y$$

$$\Rightarrow \boxed{-1 + A_1 = -3A_2} \quad (2)$$

① ve ② denklemlerinden,

$$A_2 = A_1 + 1 \Rightarrow -1 + A_1 = -3(A_1 + 1) \Rightarrow -1 + A_1 = -3A_1 - 3$$

$$\Rightarrow 4A_1 = -2$$

$$\Rightarrow \boxed{A_1 = -\frac{1}{2}} \Rightarrow A_2 = 1 - \frac{1}{2} = \boxed{\frac{1}{2} = A_2}$$

2nd Way:

$$A_1 = \frac{z_2 \cos \theta_i - z_0 \cos \theta_t}{z_2 \cos \theta_i + z_0 \cos \theta_t}$$

$$z_2 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{z_0}{\sqrt{3}}, \quad \theta_i = 60^\circ, \theta_t = 30^\circ$$

$$A_1 = \frac{\frac{z_0}{\sqrt{3}} \cdot \frac{1}{2} - z_0 \cdot \frac{\sqrt{3}}{2}}{\frac{z_0}{\sqrt{3}} \cdot \frac{1}{2} + z_0 \cdot \frac{\sqrt{3}}{2}} = \frac{\cancel{z_0} \left( \frac{1-3}{2\sqrt{3}} \right)}{\cancel{z_0} \left( \frac{1+3}{2\sqrt{3}} \right)} = \frac{-2}{4} \Rightarrow \boxed{A_1 = -\frac{1}{2}}$$

$$A_2 = 1 + A_1 = 1 - \frac{1}{2} \Rightarrow \boxed{A_2 = \frac{1}{2}}$$

c) Magnetic field vector of the transmitted wave:

$$\vec{H}_t = \frac{\sqrt{3}}{z_0} \cdot A_2 \cdot e^{i(3x-\sqrt{3}y)} \cdot \left( -\frac{1}{2} \vec{e}_x - \frac{\sqrt{3}}{2} \vec{e}_y \right)$$

$$\Rightarrow \boxed{\vec{H}_t = \frac{\sqrt{3}}{2z_0} \cdot e^{i(3x-\sqrt{3}y)} \left( -\frac{1}{2} \vec{e}_x - \frac{\sqrt{3}}{2} \vec{e}_y \right)}$$

$$2. \vec{H}(x, y) = (1+i) \cdot e^{i\pi[(2+i)x+2y]} \cdot (A\vec{e}_x + \vec{e}_y)$$

First, we need to reorganize the expression of  $\vec{H}$ .

$$\vec{H}(x, y) = (1+i) \cdot e^{i2\pi x} \cdot e^{-\pi x} \cdot e^{i2\pi y} \cdot (A\vec{e}_x + \vec{e}_y)$$

$$\Rightarrow \boxed{\vec{H}(x, y) = (1+i) \cdot e^{-\pi x} \cdot e^{i2\pi(x+y)} \cdot (A\vec{e}_x + \vec{e}_y)}$$

a) It must satisfy:  $\text{div} \vec{H} = 0$

$$\text{div} \vec{H} = \frac{\partial}{\partial x} H_x + \frac{\partial}{\partial y} H_y + \frac{\partial}{\partial z} H_z = 0$$

$$= \frac{\partial}{\partial x} [(1+i) e^{-\pi x} \cdot e^{i2\pi(x+y)} \cdot A] + \frac{\partial}{\partial y} [(1+i) \cdot e^{-\pi x} \cdot e^{i2\pi(x+y)}]$$

$$= (1+i) \cdot A \cdot (-\pi + i2\pi) \cdot e^{-\pi x} \cdot e^{i2\pi(x+y)} + (1+i) \cdot i2\pi \cdot e^{-\pi x} \cdot e^{i2\pi(x+y)}$$

$$= (1+i) \cdot e^{-\pi x} \cdot e^{i2\pi(x+y)} \cdot \underbrace{[A \cdot (-\pi + i2\pi) + i2\pi]}_{=0} = 0$$

$$\Rightarrow A = \frac{-i2\pi}{\pi(-1+2i)} \Rightarrow \boxed{A = \frac{-4+2i}{5}}$$

b) Since it can be written in the form  $H_0 \cdot e^{-\beta x} \cdot e^{i\alpha \vec{n} \vec{r}}$ , where

$k = \alpha + i\beta$ ,  $\alpha$ : phase constant

$\beta$ : attenuation constant

$$e^{i\alpha \vec{n} \vec{r}} = e^{i2\pi(x+y)} = e^{i(2\pi x + 2\pi y)} = e^{i(2\pi \vec{e}_x + 2\pi \vec{e}_y) \cdot (x\vec{e}_x + y\vec{e}_y)}$$

$$\Rightarrow \alpha \vec{n} = 2\pi \vec{e}_x + 2\pi \vec{e}_y$$

$$\Rightarrow \vec{n} = \frac{2\pi \vec{e}_x + 2\pi \vec{e}_y}{\underbrace{\sqrt{(2\pi)^2 + (2\pi)^2}}_{\alpha}} = \frac{2\pi(\vec{e}_x + \vec{e}_y)}{2\pi\sqrt{2}} \Rightarrow \boxed{\vec{n} = \frac{1}{2}\vec{e}_x + \frac{1}{2}\vec{e}_y}$$



$$\Rightarrow \alpha = 2\sqrt{2}\pi, \beta = \pi \Rightarrow k = 2\sqrt{2}\pi + i\pi$$

$$k^2 = (2\sqrt{2}\pi + i\pi)^2 = \omega^2 \epsilon \mu + i\omega \sigma \mu$$

$$8\pi^2 + i4\sqrt{2}\pi^2 - \pi^2 = 7\pi^2 + i4\sqrt{2}\pi^2 = \omega^2 \epsilon \mu + i\omega \sigma \mu \quad (*)$$

$$\Rightarrow \omega^2 \epsilon \mu = 7\pi^2$$

$$\omega^2 \cdot \frac{10^{-9}}{36\pi^9} \cdot 4\pi \cdot 10^{-9} = 7\pi^2 \Rightarrow \omega^2 = 9\pi^2 \cdot 10^{16}$$

$$\Rightarrow \omega = 3\pi \cdot 10^8 = 2\pi f$$

$$\Rightarrow f = 1.5 \cdot 10^8 \text{ Hz} \Rightarrow \boxed{f = 150 \text{ MHz}}$$

For  $\vec{H}(x, y, t)$ , let's reorganize  $\vec{H}(x, y)$  again.

$$\vec{H}(x, y) = (1+i) \cdot e^{-\pi x} \cdot e^{i2\pi(x+y)} \cdot \left[ \left( \frac{-4+2i}{5} \right) \vec{e}_x + \vec{e}_y \right]$$

$$\vec{H}(x, y) = \sqrt{2} \underbrace{\left( \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} \right)}_{e^{i45^\circ}} \cdot e^{-\pi x} \cdot e^{i2\pi(x+y)} \cdot \left[ \underbrace{\frac{2}{\sqrt{5}} \left( \frac{-2}{\sqrt{5}} + i\frac{1}{\sqrt{5}} \right)}_{e^{i\theta}} \vec{e}_x + \vec{e}_y \right]$$

$$\left. \begin{aligned} \cos\theta &= \frac{-2}{\sqrt{5}} \\ \sin\theta &= \frac{1}{\sqrt{5}} \end{aligned} \right\} \Rightarrow \theta \approx 153^\circ$$

$$= \frac{2\sqrt{2}}{\sqrt{5}} \cdot e^{-\pi x} \cdot e^{i45^\circ} \cdot e^{i153^\circ} \cdot e^{i2\pi(x+y)} \cdot \vec{e}_x + \sqrt{2} \cdot e^{-\pi x} \cdot e^{i45^\circ} \cdot e^{i2\pi(x+y)} \cdot \vec{e}_y$$

$$\vec{H}(x, y, t) = \text{Re} \left\{ \left( \frac{2\sqrt{2}}{\sqrt{5}} \cdot e^{-\pi x} \cdot e^{i[2\pi(x+y)+198^\circ]} \cdot \vec{e}_x + \sqrt{2} \cdot e^{-\pi x} \cdot e^{i[2\pi(x+y)+45^\circ]} \cdot \vec{e}_y \right) \cdot e^{-i3\pi \cdot 10^8 t} \right\}$$

$$= \text{Re} \left\{ \frac{2\sqrt{2}}{\sqrt{5}} \cdot e^{-\pi x} \cdot e^{i[2\pi(x+y)+198^\circ]} \cdot e^{-i3\pi \cdot 10^8 t} \cdot \vec{e}_x + \sqrt{2} \cdot e^{-\pi x} \cdot e^{i[2\pi(x+y)+45^\circ]} \cdot e^{-i3\pi \cdot 10^8 t} \cdot \vec{e}_y \right\}$$

$$= \text{Re} \left\{ \sqrt{2} \cdot e^{-\pi x} \left( \frac{2}{\sqrt{5}} e^{i[2\pi(x+y)+198^\circ-3\pi \cdot 10^8 t]} \cdot \vec{e}_x + e^{i[2\pi(x+y)+45^\circ-3\pi \cdot 10^8 t]} \cdot \vec{e}_y \right) \right\}$$

$$\boxed{\vec{H}(x, y, t) = \sqrt{2} \cdot e^{-\pi x} \left( \frac{2}{\sqrt{5}} \cos[2\pi(x+y)-3\pi \cdot 10^8 t + 198^\circ] \vec{e}_x + \cos[2\pi(x+y)-3\pi \cdot 10^8 t + 45^\circ] \vec{e}_y \right)}$$

c) From the equation (\*),

$$\omega \sigma \mu = 4\sqrt{2} \pi^2 \Rightarrow \sigma = \frac{4\sqrt{2} \pi^2}{\omega \mu}$$

$$\sigma = \frac{4\sqrt{2} \pi^2}{3\pi \cdot 10^8 \cdot 4\pi \cdot 10^{-7}} \Rightarrow \boxed{\sigma = \frac{\sqrt{2}}{30}}$$