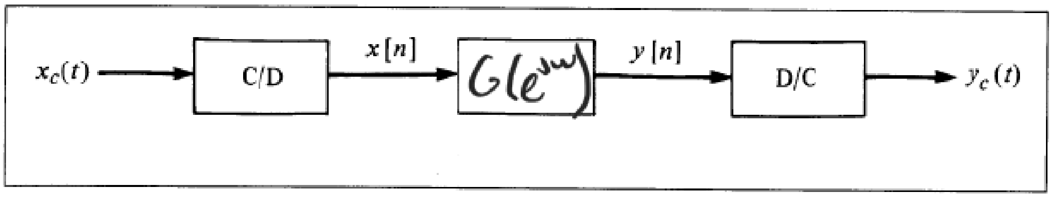
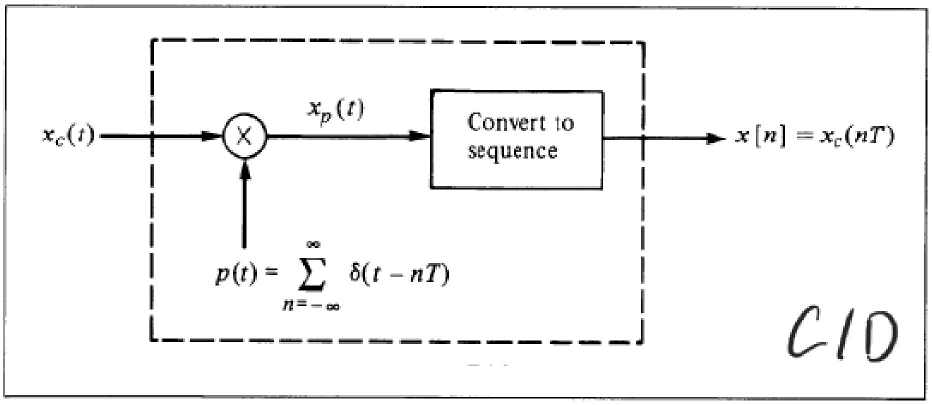
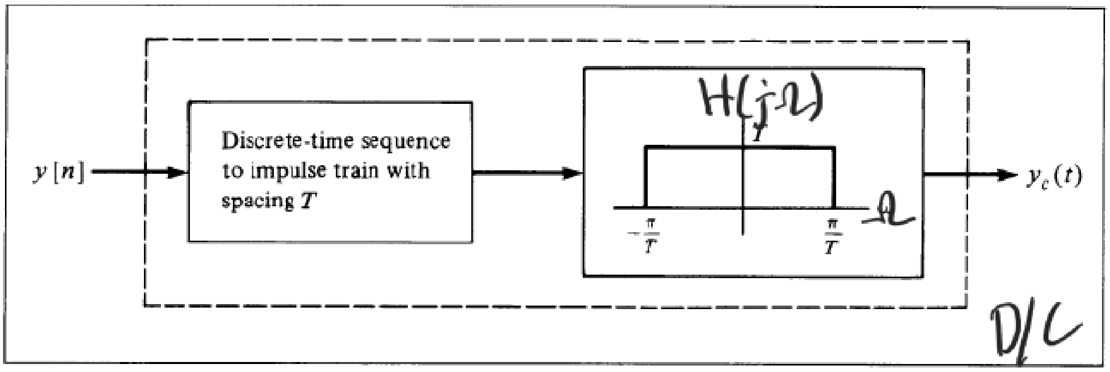
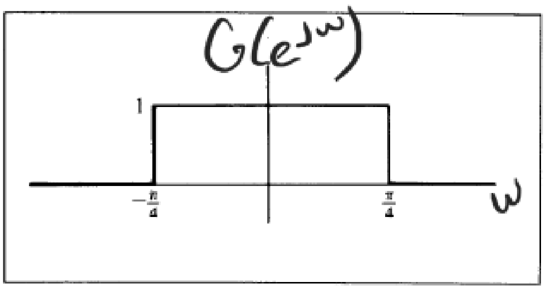
EHB 315E – Digital Signal Processing

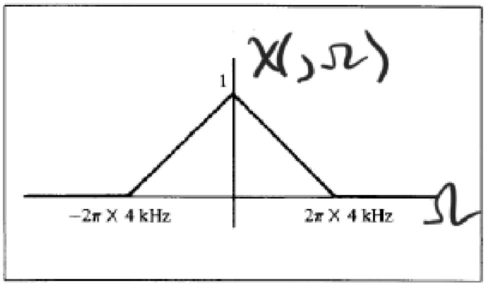
1. **Consider the system in the figure for discrete-time processing of a continuous-time signal using sampling period T.**



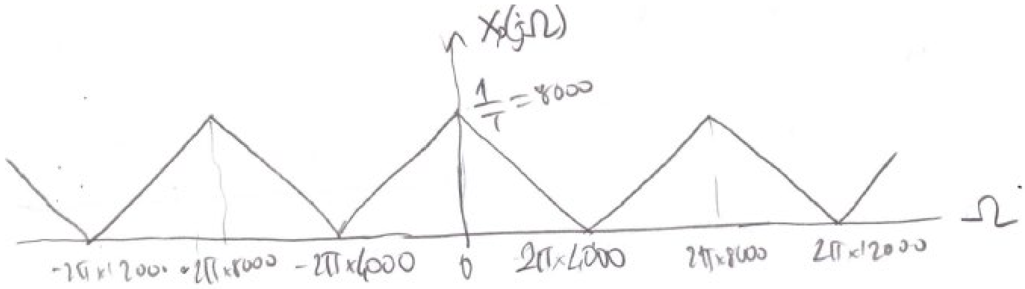




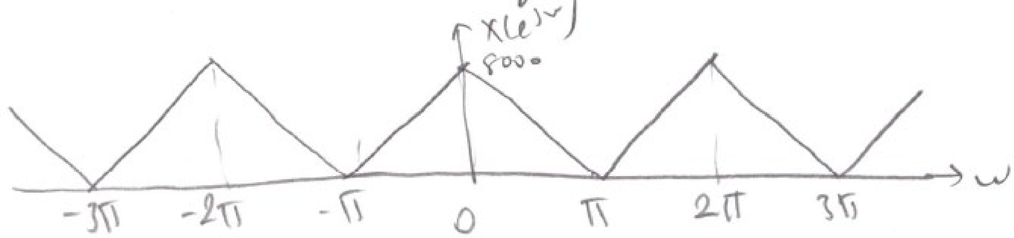


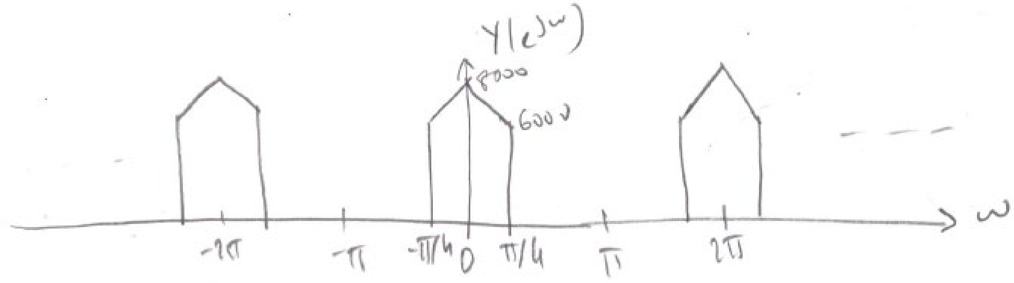


The sampling frequency is 8 kHz. Sketch the frequency response of each output of the blocks.

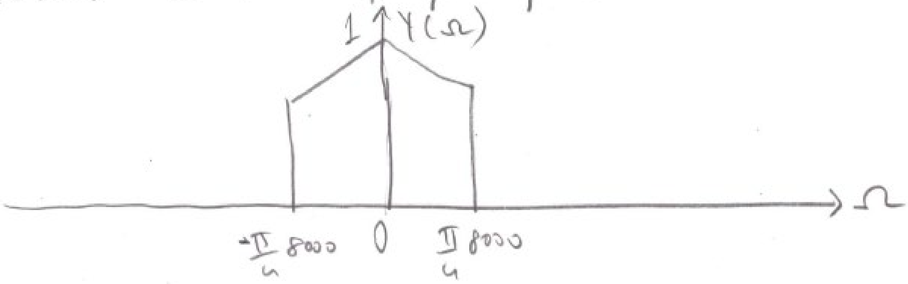


$X(e^{j\omega})$ is the rescaling of the frequency axis.





$Y_c(\Omega)$ is the frequency scaled version of $Y(e^{j\omega})$

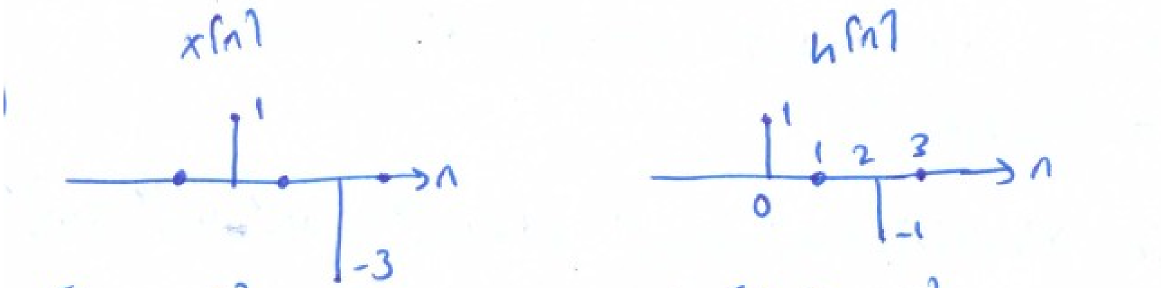


2. Suppose we have two sequences $x[n]$ and $h[n]$ as follows:

$x[n]=\delta[n-1]-3\delta[n-3]$

$h[n]=\delta[n]-\delta[n-2]$

a) Calculate $y[n]=x[n]\circledast h[n]$ by doing four-point circular convolution directly.
b) Calculate four-point DFTs $X[k]$ and $H[k]$.
c) Calculate $y[n]$ by multiplying the DFTs $X[k]$ and $H[k]$, and performing inverse DFT.



$x=\{\underset{\uparrow}{0},1,0,-3\}$ $\qquad$ $h=\{\underset{\uparrow}{1},0,-1,0\}$

a)

$$y[n]=x[n]\,④\,h[n]=\sum_{m=0}^{N-1}x[m]\,h[\langle n-m\rangle_4]$$

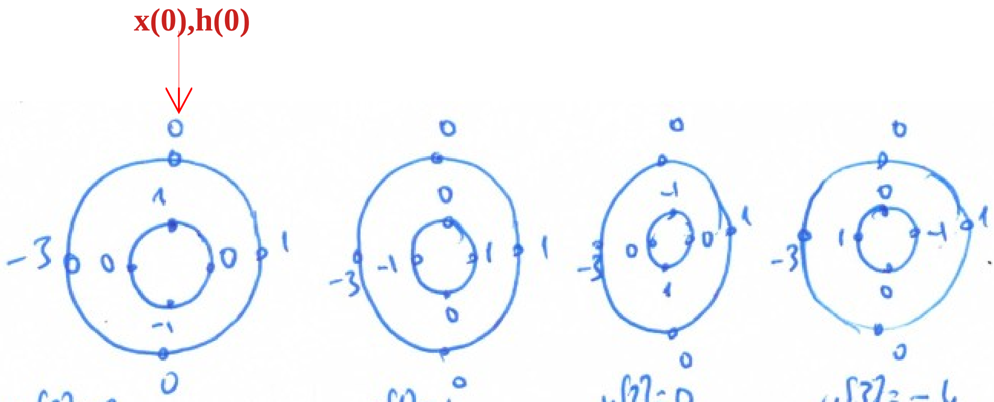
$$y[0]=x[0]h[0]+x[1]h[3]+x[2]h[2]+x[3]h[1]=0$$

$$y[1]=x[0]h[1]+x[1]h[0]+x[2]h[3]+x[3]h[2]=4$$

$$y[2]=x[0]h[2]+x[1]h[1]+x[2]h[0]+x[3]h[3]=0$$

$$y[3]=x[0]h[3]+x[1]h[2]+x[2]h[1]+x[3]h[0]=-4$$

$$y=\{\underset{\uparrow}{0},4,0,-4\}$$



$y[0]=0$ $\quad y[1]=4$ $\quad y[2]=0$ $\quad y[3]=-4$

$$y=\{\underset{\uparrow}{0},4,0,-4\}$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 0 \\ -4 \end{bmatrix} \qquad y=\{0,4,0,-4\}$$

b)

$$X[k]=\sum_{n=0}^{3} x[n]\,W_4^{nk} \qquad W_4=e^{-j\frac{2\pi}{4}} \qquad X[k]=W_4^{k}+(-3)\,W_4^{3k}$$

$$H[k]=\sum_{n=0}^{3} h[n]\,W_4^{nk} \qquad H[k]=1+(-1)\,W_4^{2k}$$

$W_4^{k}=(-j)^k$
$W_4^{2k}=(-1)^k$
$W_4^{3k}=j^k$

$X[0]=-2 \qquad X[1]=-j-3j=-4j \qquad X[2]=-1+3=2 \qquad X[3]=j+3j=4j$

$$X=\{-2,-4j,2,4j\}$$

$H[0]=0 \qquad H[1]=1+1=2 \qquad H[2]=0 \qquad H[3]=1+1=2$

$$H=\{0,2,0,2\}$$

c)

$$Y(k)=X(k)H(k)=(W_4^{k}-3W_4^{3k})(1-W_4^{2k})=W_4^{k}-W_4^{3k}-3W_4^{3k}+3\underbrace{W_4^{5k}}_{W_4^{k}}$$

$$=4W_4^{k}-4W_4^{3k}$$

$y[n]=4\delta[n-1]-4\delta[n-3] \qquad y[n]=\{0,4,0,-4\}$