S- Parameters

Z parameters for a two-port device are defined as,

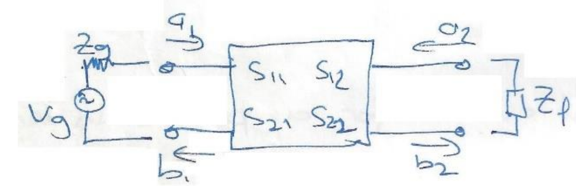
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

and $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$, $Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$

It is not easy to measure H, Y, Z parameters since the implementation of an open or short circuit is difficult in RF and some devices such as, power transistor, tunnel diyod, may be instable when we try to obtain open or short circuit for these devices. In this case some other parameters which is called S-parameters are convenient. These parameters are defined by using travelling waves as,

$$b_1 = S_{11} a_1 + S_{12} a_2$$
$$b_2 = S_{21} a_1 + S_{22} a_2$$

for two-port device given below



where a_i's are the incident waves and b_i's are the outgoing waves.

For a n-port device, the general definition is given as,

$$b_i = \sum_j^n S_{ij} a_j \quad , \quad i = 1, 2, \dots n$$

when $i = j$, (and the other ports are matched with their terminations)

$S_{ij} = S_{ii} = S_{jj} = \Gamma_{ij}$ is the reflection coefficient

and for $i > j$, (and the same condition with first case)

$S_{ij} = T_{ij}$ is the forward transmission coefficient

and for $i < j$ (and the same cond.)

$S_{ij} = T_{ij}$ is the backward transmission coefficient.

For general case,

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + \dots + S_{12}a_n$$
$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + \dots + S_{2n}a_n$$
$$\vdots$$
$$b_n = S_{n1}a_1 + S_{n2}a_2 + S_{n3}a_3 + \dots + S_{nn}a_n$$

and in matrix form,

$$\underline{b} = \underline{S}\,\underline{a}$$

where $\underline{b}$ and $\underline{a}$ are the column matrices and $\underline{S}$ is the n by n scattering matrix defined as,

$$S = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1n} \\ S_{21} & S_{22} & \dots & S_{2n} \\ \vdots & & & \\ S_{n1} & S_{n2} & \dots & S_{nn} \end{bmatrix}$$

We can define voltage and current for a port k in terms of incident and reflected travelling waves as,

$$V_k = V_k^+ + V_k^- \quad \text{and} \quad I_k = \frac{V_k^+}{Z_{0k}} - \frac{V_k^-}{Z_{0k}}$$

or $V_k^+ = \frac{1}{2}(V_k + Z_{0k} I_k)$ and $V_k^- = \frac{1}{2}(V_k - Z_{0k} I_k)$

and the mean incident power for port k, $\frac{1}{2} V_k^+ I_k^{+*} = \dfrac{|V_k^+|^2}{2 Z_{0k}^*}$

The normalized incident and reflected voltages of port k are defined as,

$$a_k = \frac{V_k^+}{\sqrt{Z_{0k}}} \quad \text{and} \quad b_k = \frac{V_k^-}{\sqrt{Z_{0k}}}$$

and using the definitions above, we can write,

$$a_k = \frac{1}{2}\left(\frac{V_k}{\sqrt{Z_{0k}}} + \sqrt{Z_{0k}}\, I_k\right) \quad \text{and} \quad b_k = \frac{V_k^-}{\sqrt{Z_{0k}}} = \frac{1}{2}\left(\frac{V_k}{\sqrt{Z_{0k}}} - \sqrt{Z_{0k}}\, I_k\right)$$

Properties of the S-parameters

1. Symmetry property:

When the network is linear and passive, but not necessarily loss-free, we have by reciprocity $S_{ij} = S_{ji}$. That is, the scattering matrix is symmetrical about the principal diagonal.

When the network is loss-free, which is so for most microwave networks, there are two important properties of the scattering coefficients. Thus,

2. Unitary property:

Let the incident power at port 1 be P_1, (when all other ports are matched) so that $P_1 = a_1 a_1^*$. Since there is no loss in the network, this power will reappear at all the ports including 1 (by reflection). Thus for n-ports,

$$P_1 = a_1 a_1^* = b_1 b_1^* + b_2 b_2^* + \cdots + b_n b_n^*$$

Using the S-matrix relation $\underline{b} = \underline{S}\,\underline{a}$ with $a_2 = a_3 = \cdots = a_n = 0$ we obtain

$$a_1 a_1^* = a_1 S_{11} a_1^* S_{11}^* + a_1 S_{21} \cdot a_1^* S_{21}^* + \cdots + a_1 S_{n1} \cdot a_1^* S_{n1}^*$$

from which

$$\underline{1 = |S_{11}|^2 + |S_{21}|^2 + \cdots + |S_{n1}|^2}$$

By reciprocity, we can also write the above equation as,

$$1 = |S_{11}|^2 + |S_{12}|^2 + \cdots + |S_{1n}|^2$$

These equations imply that the sum of the squares of the amplitudes of any row or column of the scattering matrix, for a lossless network, add to unity.

3. Zero property:

Another property is obtained from the conservation of energy principle: the total power entering, P_{in}, of all ports must equal the total power out, P_{out}, from all ports. For simplicity, we examine a two-port network:

$$P_{in} = a_1 a_1^* + a_2 a_2^* = P_{out} = b_1 b_1^* + b_2 b_2^*$$

$$a_1 a_1^* + a_2 a_2^* = \overbrace{(a_1 S_{11} + a_2 S_{12})}^{b_1}(a_1^* S_{11}^* + a_2^* S_{12}^*) + (a_1 S_{21} + a_2 S_{22})(a_1^* S_{21}^* + a_2^* S_{22}^*)$$

Multiplying the right hand side out and using the unitary conditions yields,

$$0 = a_1 a_2^* (S_{11} S_{12}^* + S_{21} S_{22}^*) + a_2 a_1^* (S_{12} S_{11}^* + S_{22} S_{21}^*)$$

The second term of the equation is the conjugate of the first term. Also $w + w^* = 2\,\mathrm{Re}(w)$ from which the above equation becomes

$$0 = \mathrm{Re}\,(a_1 a_2^* (S_{11} S_{12}^* + S_{21} S_{22}^*))$$

Now a_1 and a_2 are two independent inputs whose reference planes (and hence phase) are also independent, so that $a_1 a_2^* \neq 0$. Thus

$$(S_{11} S_{12}^* + S_{21} S_{22}^*) = 0$$

This condition is for a two port network. Using a similar process to the above, it may be shown for any n-port network, that

$$S_{11} S_{12}^* + S_{21} S_{22}^* + S_{31} S_{32}^* + \cdots + S_{n1} S_{n2}^* = 0$$

This equation implies that the sum of the products of the members of the first column and the conjugate of the corresponding members of the second column are zero. In the same way we can prove a similar condition for any pair of columns of the scattering matrix. Since the scattering matrix is symmetrical about the principal diagonal, any two rows combine similarly to yield zero.

4. Phase shift property

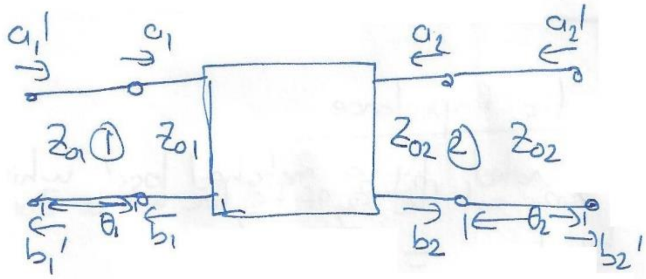
If we insert a line of length $\beta_k \ell_k$ to a port k, we obtain a new n-port network and any S_{ij} parameter which includes

k. port will be multiplied by $e^{-j\beta_k \ell_k}$. Let show the new scattering matrix by S', then,

$$\underline{S'} = \phi \, \underline{S} \, \phi$$

where $$\phi = \begin{bmatrix} \phi_{11} & 0 & 0 & \cdots & 0 \\ 0 & \phi_{22} & 0 & \cdots & 0 \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & \phi_{nn} \end{bmatrix}, \quad \phi_{11} = \phi_{22} = \phi_{kk} = e^{-j\beta_k \ell_k}$$

$k = 1, 2, \cdots n$



$$\theta_i = \beta_i \ell_i$$

$$S_{ij}' = S_{ij} e^{-j(\theta_i + \theta_j)}$$

Each element of the new S' matrix can be calculated by using the relation above in terms of the old matrix element and the electrical length of the inserted lines.