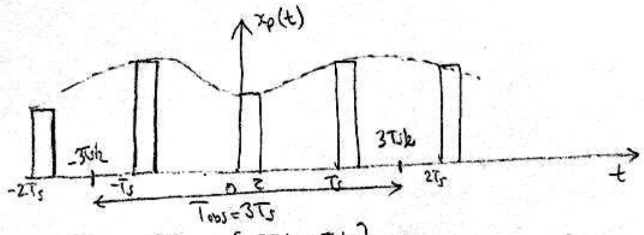
The average power consumption of the signal $x_p(t)$ over resistive load of 1Ω is given by the following relationship:

$$P_T = \langle x_p^2(t) \rangle = \lim_{T_{obs}\to\infty} \frac{1}{T_{obs}} \int_{-T_{obs}/2}^{T_{obs}/2} x_p^2(t)\,dt$$

T_{obs} : Observation interval $= (2M+1)T_s$, $M = 0, 1, 2, \ldots$

$x_p(t)$: PAM signal $= \sum_k x(kT_s)h(t-kT_s)$

$$= \lim_{M\to\infty} \frac{1}{(2M+1)T_s} \int_{-\frac{(2M+1)T_s}{2}}^{(2M+1)T_s/2} x_p^2(t)\,dt \quad , M = 0, 1, 2, \ldots$$



For $M=1$, $T_{obs} = 3T_s$ $[-3T_s/2, 3T_s/2]$

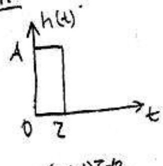
$$P_T = \lim_{M\to\infty} \frac{1}{(2M+1)T_s} \int_{-(2M+1)T_s/2}^{(2M+1)T_s/2} \left[\sum_{k=-M}^{M} x(kT_s)h(t-kT_s)\right]^2 dt$$

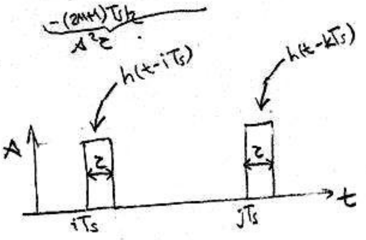
$$= \lim_{M\to\infty} \frac{1}{(2M+1)T_s} \int_t \sum_i \sum_k x(iT_s)x(kT_s)\,h(t-iT_s)h(t-kT_s)\,dt$$

$$= \lim_{M\to\infty} \frac{1}{(2M+1)T_s} \sum_i \sum_k x(iT_s)x(kT_s) \int_{-(2M+1)T_s/2}^{(2M+1)T_s/2} h(t-iT_s)h(t-kT_s)\,dt$$

$$= \begin{cases} 0 , & \text{if } i \neq k \text{ since } h(t-iT_s) \text{ functions are orthogonal} \\ \lim_{M\to\infty} \frac{1}{(2M+1)T_s} \sum_{i=-M}^{M} x^2(iT_s) \underbrace{\int_{-(2M+1)T_s/2}^{(2M+1)T_s/2} h^2(t-iT_s)\,dt}_{A^2\tau} , & i=k \end{cases}$$

Proof:





if $i=k$, then $h(t-iT_s)h(t-kT_s) = h^2(t)$
" $= 0$ otherwise

$$\int_{-(2M+1)T_s/2}^{(2M+1)T_s/2} h^2(t-iT_s)\,dt = \int_{iT_s}^{iT_s+\tau} A^2\,dt = A^2(iT_s+\tau-iT_s) = A^2\tau$$

We obtain

$$P_T = \lim_{M\to\infty} \frac{A^2\tau}{(2M+1)T_s} \sum_{i=-M}^{M} x^2(iT_s)$$

Now, let us write $x(t)$ as, (from the sampling theorem),

$$x(t) = \sum_{i=-\infty}^{\infty} x(iT_s)\,\mathrm{sinc}\left[\frac{1}{T_s}(t-iT_s)\right]$$

In that case, time average of $x^2(t)$, i.e. $\langle x^2(t)\rangle$ can be written as follows:

$$\langle x^2(t)\rangle = \lim_{T_{obs}\to\infty} \frac{1}{T_{obs}} \int_{-T_{obs}/2}^{T_{obs}/2} x^2(t)\,dt$$

$$= \lim_{M\to\infty} \frac{1}{(2M+1)T_s} \int_{-(2M+1)T_s/2}^{(2M+1)T_s/2} \sum_i \sum_k x(iT_s)\,x(kT_s)\,\mathrm{sinc}\left[\frac{t-iT_s}{T_s}\right]\cdot \mathrm{sinc}\left[\frac{t-kT_s}{T_s}\right] dt$$

Sinc(.) functions are also orthogonal, i.e.

$$\lim_{M\to\infty} \int_{-(2M+1)T_s/2}^{(2M+1)T_s/2} \mathrm{sinc}\left[\frac{t-iT_s}{T_s}\right]\mathrm{sinc}\left[\frac{t-kT_s}{T_s}\right] dt = \begin{cases} 0, & i\neq k \\ T_s, & i=k \end{cases}$$

Proof:

It can be shown that

$$\mathcal{F}\left\{\mathrm{sinc}\,\frac{t-iT_s}{T_s}\right\} = T_s\,\Pi(fT_s)\,e^{-j2\pi f iT_s}.$$

Using Parseval theorem, $\left(\int_{-\infty}^{\infty} f(t)g(t)\,dt = \int_{-\infty}^{\infty} F(f)\,G^*(f)\,df\right)$

$$\lim_{M\to\infty} \int_{-(2M+1)T_s/2}^{(2M+1)T_s/2} \mathrm{sinc}\left[\frac{t-iT_s}{T_s}\right]\mathrm{sinc}\left[\frac{t-kT_s}{T_s}\right] dt = \int_{-\infty}^{\infty} \underbrace{\mathrm{sinc}\left[\frac{t-iT_s}{T_s}\right]}_{f(t)} \underbrace{\mathrm{sinc}\left[\frac{t-kT_s}{T_s}\right]}_{g(t)} dt = \int_{-\infty}^{\infty} T_s^2\,\Pi(fT_s)\,e^{-j2\pi f iT_s}\cdot e^{j2\pi f kT_s}\,df$$

$$= T_s^2 \int_{-1/2T_s}^{1/2T_s} e^{j2\pi T_s(k-i)f}\,df = T_s^2 \left.\frac{e^{j2\pi T_s(k-i)f}}{j2\pi T_s(k-i)}\right|_{-1/2T_s}^{1/2T_s}$$

$$= T_s^2\,\frac{e^{j\pi(k-i)} - e^{-j\pi(k-i)}}{j2\pi T_s(k-i)} = T_s\,\mathrm{sinc}(k-i) = \begin{cases} 0, & i\neq k \\ T_s, & i=k \end{cases}$$

$\langle x^2(t)\rangle$ becomes

$$\langle x^2(t)\rangle = \lim_{M\to\infty} \frac{1}{(2M+1)T_s}\,T_s \sum_{i=-M}^{M} x^2(iT_s) = \lim_{M\to\infty} \frac{1}{(2M+1)} \sum_{i=-M}^{M} x^2(iT_s),$$

Hence,

$$P_T = \frac{A^2\tau}{T_s}\langle x^2(t)\rangle.$$

//