The same example continue with different parameters :

Testing against the alternate hypothesis :

$H_1 : p = 0.1$

$n = 500$, $Th = 40$ (critical value) (sağ tarafa dahil)

$$\beta = \sum_{x=0}^{39} \binom{500}{x} (0.1)^x (0.9)^{500-x}$$

hypothesis testing design finalde ☆

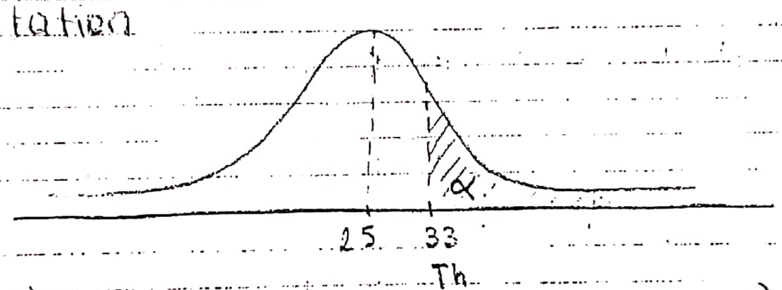
$$\approx P\left[Z \leq \frac{39 - 500 \cdot 0.1}{\sqrt{500 \cdot 0.1 \cdot 0.9}} \right]$$

$$= P[Z \leq -1.69] = 0.068$$

% 7 oranında onları yanlış iken bizim onları doğru almamız.

Visual Interpretation

$n = 500$
$p = 0.05$



$Z \sim (np, \ np(1-p))$

$$Z \geq \frac{X - np}{\sqrt{np(1-p)}}$$

$Y = aX + b$

$E[Y] = a E[X] + b$

$Var[Y] = a^2 Var[X]$

$Th \uparrow \quad \alpha \downarrow$

$$P[Y \geq 33] = P\left[Z \geq \frac{33 - 25}{\sqrt{23.75}} \right]$$

(0, 1)

mean = 25

$Var[Y] = 23.75$

$N(25, 23.75)$

$\mu \quad \sigma^2$

$np \quad np(1-p)$

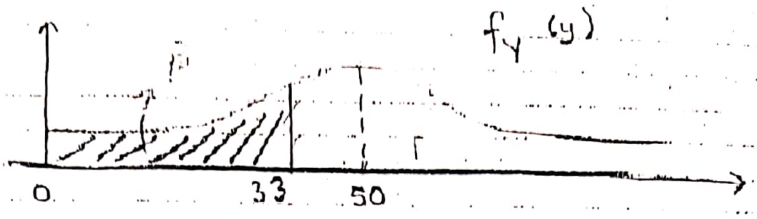
$= P[Z \geq 1.64]$

$= 1 - P[Z \leq 1.64]$

$= 1 - 0.9495$

$= 0.0505$ //

$n = 500$
$p = 0.1$ olsun
$Y = N(50, 45)$



$\beta \leq 1/2$ den büyük

$\beta \approx P[Y \leq 33]$

$= P\left[Z \leq \frac{33-50}{\sqrt{45}}\right]$

$= P[Z \leq -2.5]$

$= 0.062$

Th $\uparrow$ $\beta \uparrow$

Central Limit Theorem

Dağılımlar iid ise toplamları Gauss'a convergence eder.
Bu yaklaşıklık $n \uparrow$ artar.

The central limit theorem says that the mean and variance of some of independent identically dist. random samples can be computed as sample mean $n\mu$ and $n\sigma^2$ with Gaussianly distributed

Samples $s_1, s_2, \ldots, s_n$ are i.i.d with mean μ and variance σ^2.

If $Y = \sum_{i=1}^{n} s_i$, then Y is Gaussianly distr. with $n\mu$ and $n\sigma^2$.

As n increases, appx. success increases.

$s_1 + s_2 + \cdots + s_n$

$\sigma^2 \quad \sigma^2$

$Y = X_1 + X_2$

$\sigma_Y^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2$

if X_1 and X_2 are independent ✓

$(\frac{1}{n})^2 = \frac{1}{n^2}$ [unbiased, check]

Sample mean $= \bar{x}$

$n\sigma^2$

$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ where x_i's are i.i.d. with μ, σ^2.

$\bar{X}$ is Gaussianly distributed with mean μ and variance $\frac{\sigma^2}{n} = \left(\frac{1}{n}\right)^2 n\sigma^2$ from the Central Limit Theorem.

Summary

* α and β are related; decreasing one generally increases the other one.

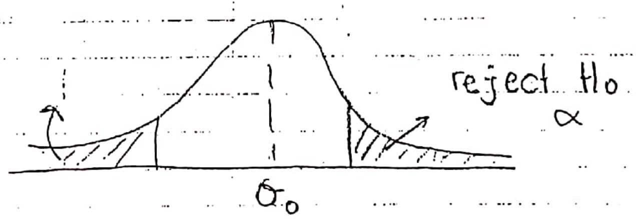
*** α can be set to a desired value by adjusting the critical value.

* Increasing n decreases both α and β.

30/11/16

One Tailed Vs Two Tailed Test

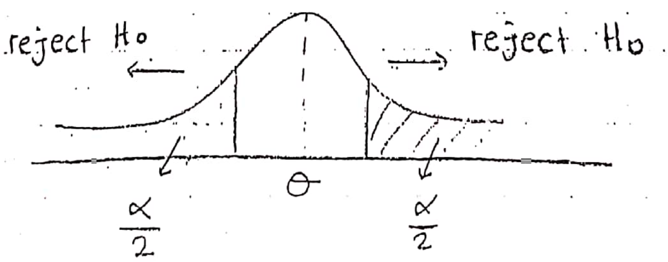
one tail
H_0 : $\theta = \theta_0$
H_1 : $\theta > \theta_0$



Two Tail test (Sample mean örnek verilebilir)

H_0 : $\theta = \theta_0$

H_1 : $\theta \neq \theta_0$



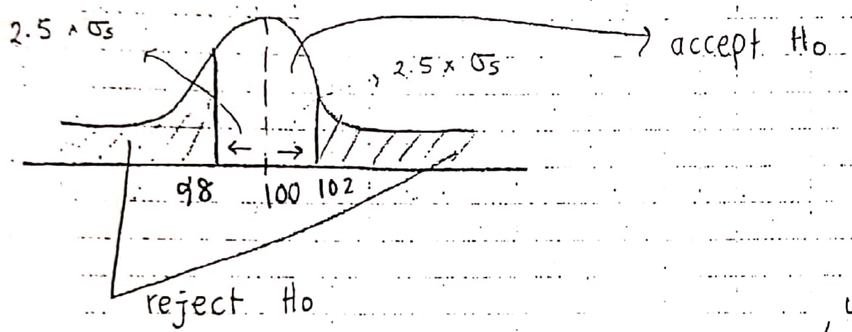
Ex: Consider a production line of resistors that are supposed to be 100 ohms with standard division 8.

$H_0 : \mu = 100\ \Omega$

$H_1 : \mu \neq 100\ \Omega$

Let $\bar{X}$ be the sample mean for a sample size $n = 100$.

$$\bar{X} \sim N\left(100, \left(\frac{8}{\sqrt{100}}\right)^2\right) = \bar{X} \sim N(100, 0.64)$$



union

$$\alpha : \text{Type I error} = P[\bar{X} < 98 \text{ when } \mu = 100] + P[\bar{X} > 102 \text{ when } \mu = 100]$$

normal Gaussian

$$= P\left[Z < \frac{98-100}{0.8}\right] + P\left[Z > \frac{102-100}{0.8}\right]$$

$$= \underbrace{P[Z < -2.5]}_{\frac{\alpha}{2}} + \underbrace{P[Z > 2.5]}_{\frac{\alpha}{2}}$$

$$= 2 \cdot P[Z < -2.5]$$

$$= 2 \cdot F_Z(-2.5) = 2 \cdot (0.0062) = 0.0124$$

% 1.2 olasılıkla doğru olanı reddediyorum.

%98.76 oranında, firmanın iddiası doğru iken, doğruluğunu kabul etmek.

$n = 30$'dan az ise Student's t Test 4/...

* sample sayısı az old zamanlarda normal dist. app. çalışmıyor. Student's t test kullanılıyor bu durumlarda.

confidence interval → significance level → α check!
(z test, normal dist app.)

Hypothesis Testing Using p-value

p-value is the lowest level of significance at which the observed value of a test statistic is significant.

Reject H_0 if $p < \alpha$

Ex: A batch of 100 resistors have an average of 101.5 Ω. Standard deviation of the population is 5.

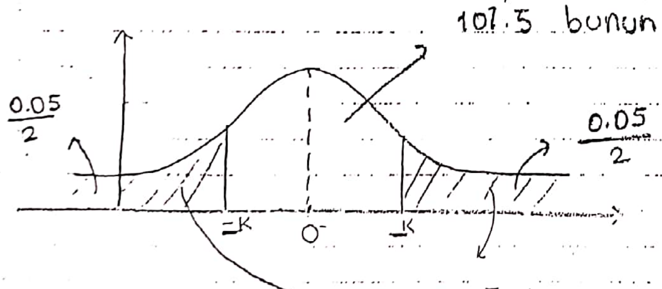
Test whether the population mean is 100 Ω at a level of significance 0.05. $\alpha = 0.05$

$H_0: \mu = 100$
$H_1: \mu \neq 100$ } two-sided.

p → critical value → α
α → critical value →

Test statistic: $\bar{X}$ sample mean

101.5 bunun içindeyse accept Ho.



$-K \to 0.025$ $K \to 1 - \frac{\alpha}{2} = 0.975$

- bu değerleri bulmak istiyoruz

101.5 bunun içindeyse reject Ho...

Reject Ho if   ±1.96

$\bar{X} < 100 - z_{0.025} \frac{\sigma}{\sqrt{n}}$ $\bar{X} < 99.02$

$\bar{X} > 100.98$

$\bar{X} > 100 + z_{0.025} \frac{\sigma}{\sqrt{n}}$ $\bar{X} = 101.5$ → so reject Ho.

$0.025 = \frac{\alpha}{2}$ α'ı daha da küçültebilirim.