Otomatik Kontrol Sistemleri

Hafta 3

Doç. Dr. Volkan Sezer

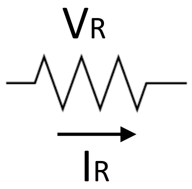
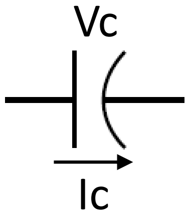
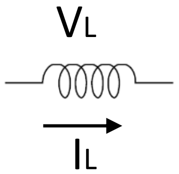
Elektrik Devrelerinin Modellenmesi

Kaynaklar ve Devre Elemanları

- Kaynaklar:

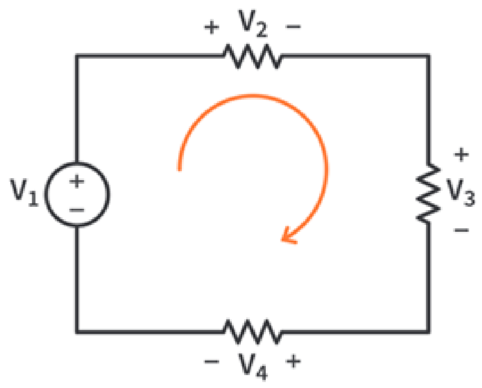
  Gerilim Kaynağı: V(t) veya e(t)

  Akım Kaynağı: i(t)

		Gerilim-Akım	**Akım-Gerilim**	**Empedans**
Direnç		$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$\frac{V(s)}{I(s)} = R$
Kapasitör		$v(t) = \frac{1}{C}\int_0^1 i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$\frac{V(s)}{I(s)} = \frac{1}{sc}$
Endüktans		$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^1 v(\tau)d\tau$	$\frac{V(s)}{I(s)} = Ls$

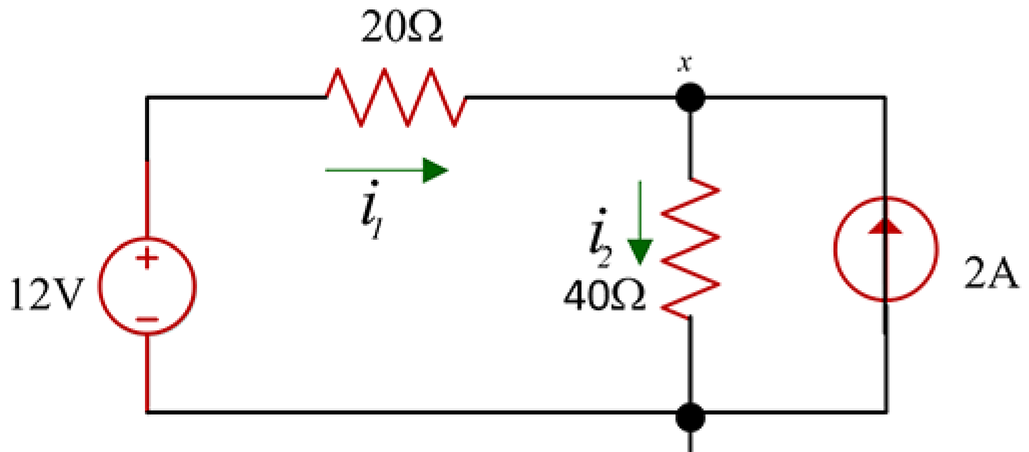
Temel Kanunlar

- Kirchhoff gerilimler kanununa göre, bir çevre boyunca karşılaşılan gerilimlerin toplamı sıfırdır.



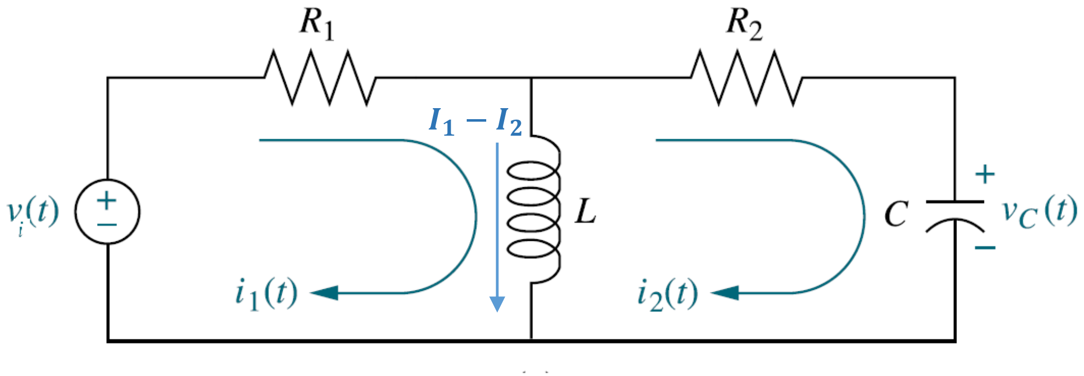
$$-V_1 + V_2 + V_3 + V_4 = 0$$

- Kirchhoff akımlar kanununa göre, bir düğüme giren ve o düğümden çıkan akımların toplamı sıfırdır.



$$I_1 + 2 - I_2 = 0$$

Örnek



Yukarıdaki devrede, $I_2(s)/V_i(s)$ transfer fonksiyonunu elde ediniz.

1. Çevreye ait Kirchhoff Gerilimler Yasası:

$$V_i(t) = V_{R_1} + V_L$$

$$V_i(t) = R_1 \cdot i_1 + L \cdot \frac{d(i_1 - i_2)}{dt}$$

$$V_i(s) = R_1 \cdot I_1(s) + L \cdot s \cdot (I_1(s) - I_2(s))$$

$$\boxed{V_i(s) = I_1(s)(R_1 + sL) - I_2(s) \cdot sL} \quad (1)$$

2. Çevreye ait Kirchhoff Gerilimler Yasası:

$$V_L = V_{R_2} + V_C$$

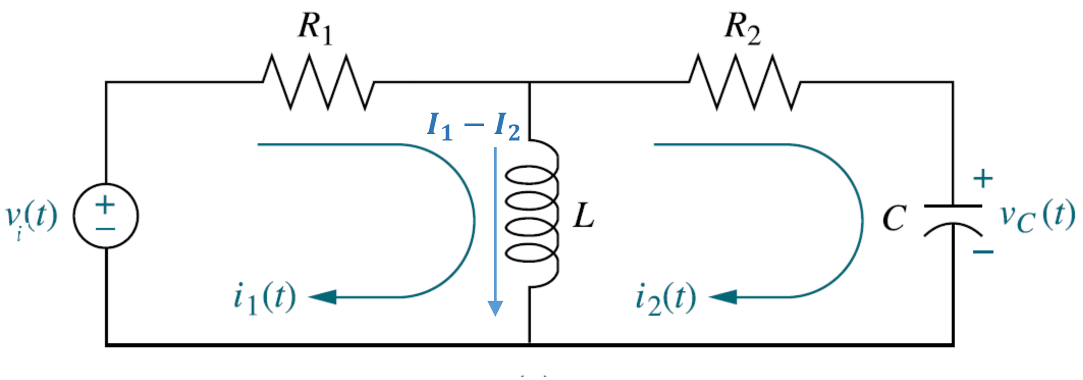
$$L \cdot \frac{d(I_1 - I_2)}{dt} = R_2 \cdot i_2 + \frac{\int i_2 \, dt}{C}$$

$$L \cdot s(I_1(s) - I_2(s)) = R_2 \cdot I_2(s) + \frac{1}{sC} \cdot I_2(s)$$

$$I_1(s) \cdot Ls = I_2(s)\left(R_2 + \frac{1}{sC} + Ls\right)$$

$$\boxed{I_1(s) = I_2(s) \cdot \frac{\left(R_2 + \frac{1}{sC} + Ls\right)}{Ls}} \quad (2)$$

Örnek



Yukarıdaki devrede, $I_2(s)/V_i(s)$ transfer fonksiyonunu elde ediniz.

$$\boxed{V_i(s) = I_1(s)(R_1 + sL) - I_2(s)\cdot sL} \quad (1)$$

$$\boxed{I_1(s) = I_2(s)\frac{\left(R_2 + \frac{1}{sC} + Ls\right)}{Ls}} \quad (2)$$

(2) denklemini (1)'de yerine yazarsak, V_i ile I_2 arasındaki ilişkiyi buluruz.

$$V_i(s) = I_2(s)\frac{\left(R_2 + \frac{1}{sC} + Ls\right)}{Ls}(R_1 + sL) - I_2(s)\cdot sL$$

$$V_i(s) = I_2(s)\left(\frac{R_2 + \frac{1}{sC} + Ls}{Ls}(R_1 + sL) - sL\right)$$

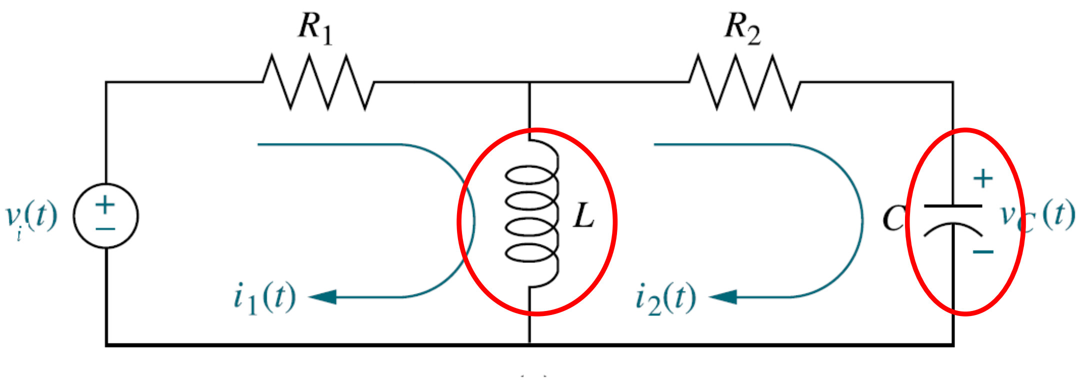
$$\frac{I_2(s)}{V_i(s)} = \frac{Ls}{\left(R_2 + \frac{1}{sC} + Ls\right)(R_1 + sL) - s^2L^2}$$

$$\frac{I_2(s)}{V_i(s)} = \frac{Ls}{R_1R_2 + R_1\frac{1}{sC} + R_1Ls + sLR_2 + \frac{L}{C} + s^2L^2 - s^2L^2}$$

$$\frac{I_2(s)}{V_i(s)} = \frac{s^2LC}{R_1R_2sC + R_1 + R_1Ls^2C + s^2LCR_2 + sL}$$

$$\frac{I_2(s)}{V_i(s)} = \frac{s^2LC}{s^2(LCR_2 + LCR_1) + s(R_1R_2C + L) + R_1}$$

Örnek



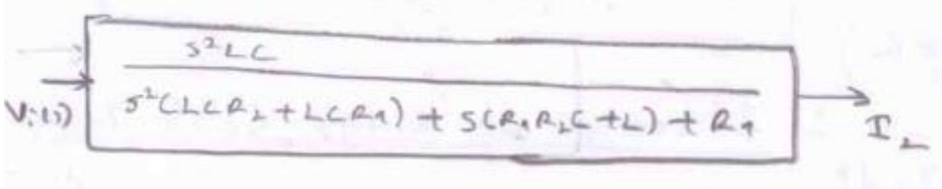
Görüldüğü gibi, sistemde 2 tane aktif eleman yer aldığı için, transfer fonksiyonu da 2. dereceden çıkmıştır.

Yukarıdaki devrede, $I_2(s)/V_i(s)$ transfer fonksiyonunu elde ediniz.

$$\boxed{V_i(s) = I_1(s)(R_1 + sL) - I_2(s)\cdot sL} \quad (1)$$

$$\boxed{I_1(s) = I_2(s)\frac{\left(R_2 + \frac{1}{sC} + Ls\right)}{Ls}} \quad (2)$$

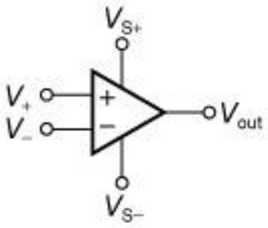
Blok diyagram olarak şöyle de gösterilebilir:



$$\frac{I_2(s)}{V_i(s)} = \frac{s^2LC}{s^2(LCR_2+LCR_1)+s(R_1R_2C+L)+R_1}$$

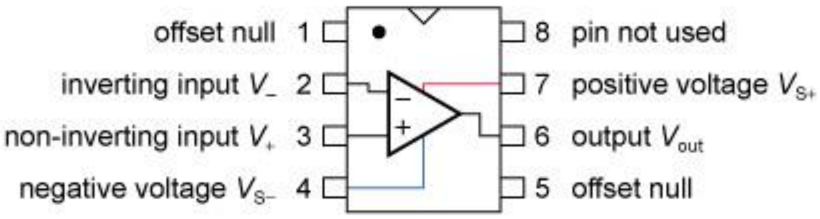
İşlemsel Kuvvetlendiriciler (Op-Amp)

İşlemsel Kuvvetlendiriciler



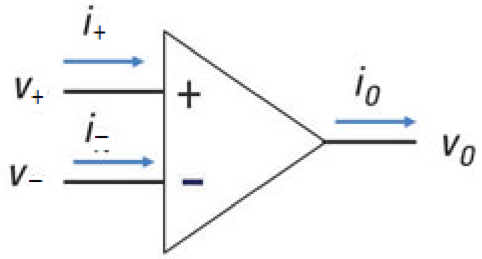
(a)

(b)



(c)

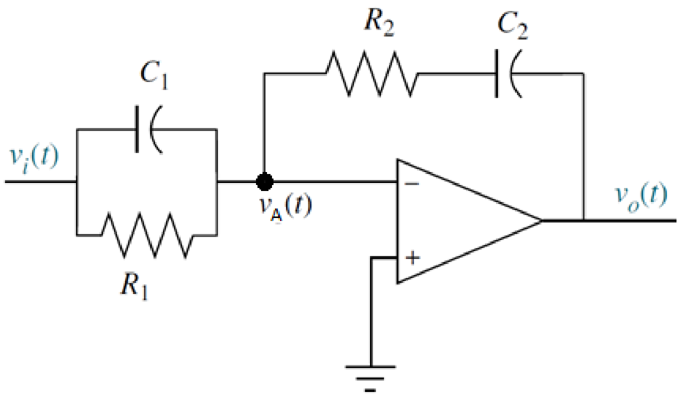
https://www.open.edu/openlearn/science-maths-technology/introduction-electronics/content-section-3.3



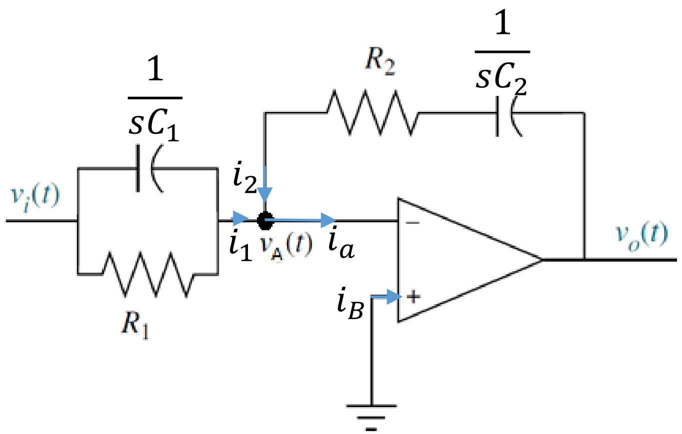
İstenilen transfer fonksiyonlarını gerçeklemek için kullanılabilirler.
Geri besleme bağlantısına sahip Opamplar için 2 temel kural vardır:

- Yüksek (idealde sonsuz) giriş empedansına sahip oluğu her iki girişi de akım çekmez (I+ = I- =0).
- Giriş gerilim seviyeleri birbirine eşittir (V+ = V-)

İşlemsel Kuvvetlendiriciler-Örnek



Empedans eşdeğer devresini çizelim.



$$V_A = 0, I_A = 0, I_B = 0$$

$$I_1 = \frac{V_I - \cancel{V_A}}{\frac{1}{\frac{1}{1/sC_1} + \frac{1}{R_1}}} = \frac{V_I}{\frac{1}{sC_1 + \frac{1}{R_1}}} = \frac{V_I}{\frac{1}{\frac{sC_1+1}{R_1}}} = \frac{V_I(sC_1+1)}{R_1}$$

$$I_2 = \frac{V_0 - \cancel{V_A}}{R_2 + \frac{1}{sC_2}} = \frac{V_o}{\frac{sC_2R_2 + 1}{sC_2}} = \frac{V_o sC_2}{sC_2R_2 + 1}$$

$I_A = 0$ olduğu için, $I_1 + I_2 = 0 \rightarrow \boxed{I_1 = -I_2}$

$$\frac{V_I(sC_1+1)}{R_1} = \frac{V_o sC_2}{sC_2R_2+1}$$

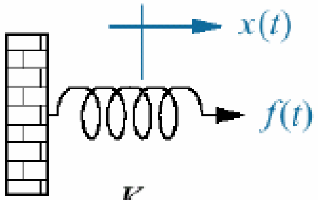
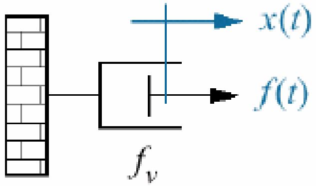
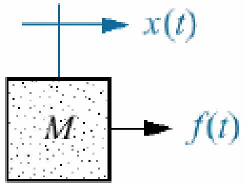
$$\frac{V_0}{V_i} = \frac{-(sC_2R_2 + 1)(sC_1R_1+1)}{sC_2R_1}$$

$$\boxed{\frac{V_0}{V_i} = \frac{-(s^2C_1C_2R_1R_2 + s(C_2R_2 + C_1R_1) + 1)}{sC_2R_1}}$$

PID Kontrolörün Transfer Fonksiyonu!

Mekanik Sistemlerin Modellenmesi: Ötelemeli Sistem Modelleme

Sistem Elemanları

Component	Force-velocity	Force-displacement	
Spring  K	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	Yay Katsayısı
Viscous damper  f_v	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	Viskoz Sürtünme Katsayısı
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Kütle

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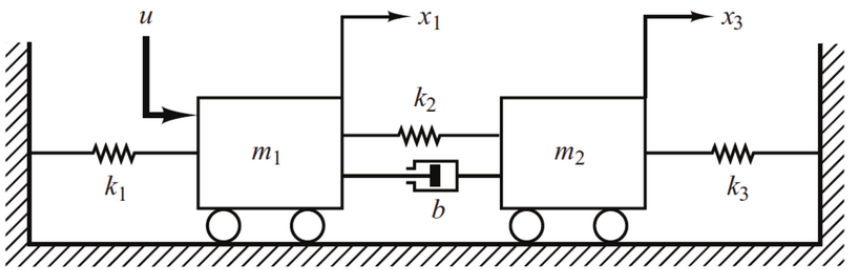
Ötelemeli Sistemler İçin Serbest Cisim Diyagramı Yöntemi

For each mass, we calculate force values by its own motion while other mass motions are ignored.

Then we ignore its own motion and calculate the force values coming from other masses' motion.

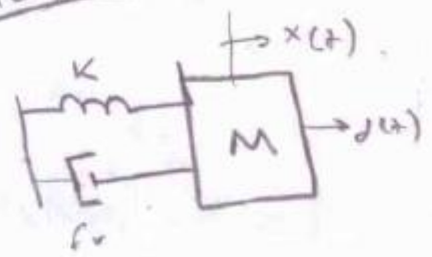
Finally we apply Newton's law which says 'sum of the forces on each mass is zero' $\sum F = 0$.

We obtain differential equations and using Laplace Transform, we obtain transfer functions.



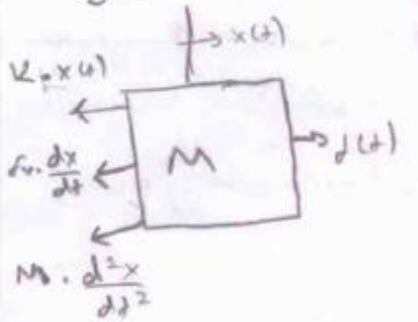
Serbest Cisim Diyagramı Örnek-1

Örnek 1)



$\frac{X(s)}{F(s)}$ transfer fonk-siyonunu bulunuz.

serbest cisim diyagramı



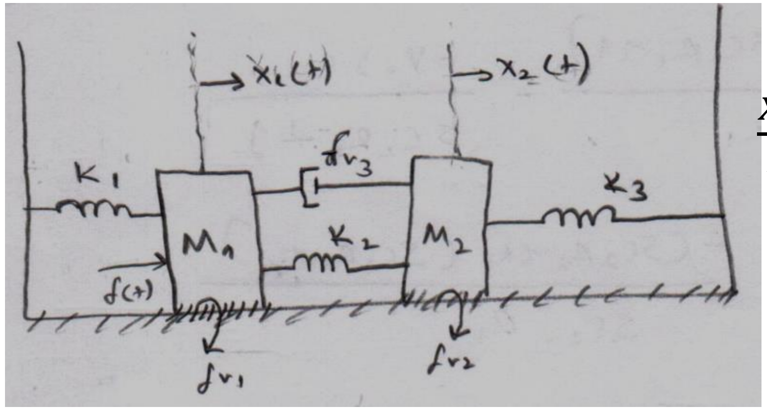
$$M.\frac{d^2x}{dt^2} + f_v.\frac{dx}{dt} + K.x(t) = f(t)$$

$$M.s^2.X(s) + f_v.s.X(s) + K.X(s) = F(s)$$

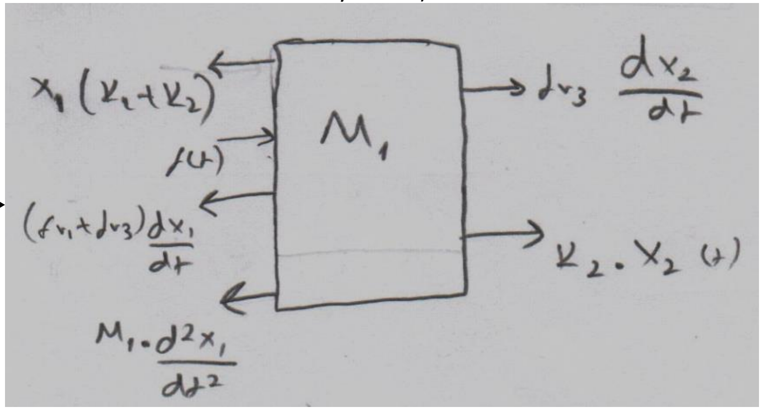
$$X(s)\left(Ms^2 + f_v \cdot s + K\right) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

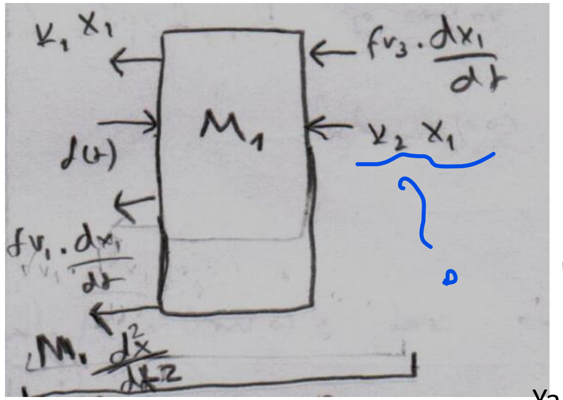
Serbest Cisim Diyagramı Örnek-2



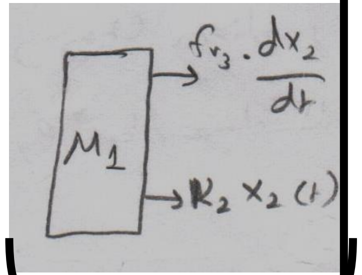
$\frac{X_2(s)}{F(s)}$=? (f(t) giriş, fv1, fv2 and fv3 viskoz sürtünme katsayılarıdır)



M1 için Serbest Cisim Diyagramı



Yalnızca M1'in hareketinden gelen kuvvetler



Yalnızca M2'nin hareketinden gelen kuvvetler

Süperpozisyon ile toplanabilir!

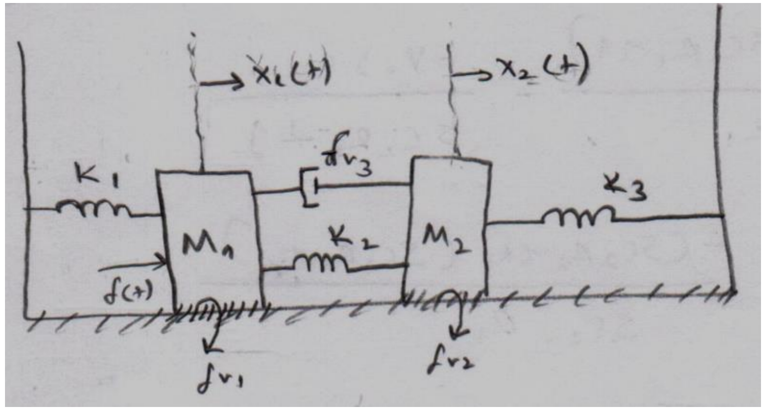
$$x_1(t)(K_1+K_2)+(f_{v1}+f_{v3})\frac{dx_1}{dt}+M_1\frac{d^2x_1}{dt^2} = f(t)+f_{v3}\frac{dx_2}{dt}+K_2X_2(t)$$

Laplace

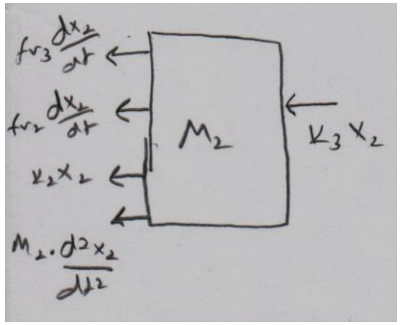
$$X_1(s)(K_1+K_2)+(f_{v1}+f_{v3})s.X_1(s)+M_1s^2X_1(s) = F(s)+f_{v3}sX_2(s)+K_2X_2(s)$$

$$\boxed{F(s)=X_1(s)(M_1s^2+(f_{v1}+f_{v3})s+(K_1+K_2)) - X_2(s)(f_{v3}s+K_2)} \quad (1)$$

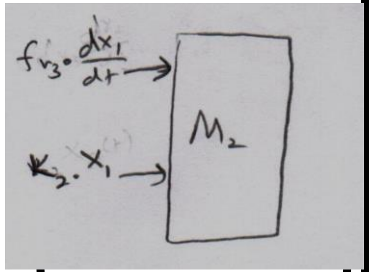
Serbest Cisim Diyagramı Örnek-2



M2 İçin Serbest Cisim Diyagramı

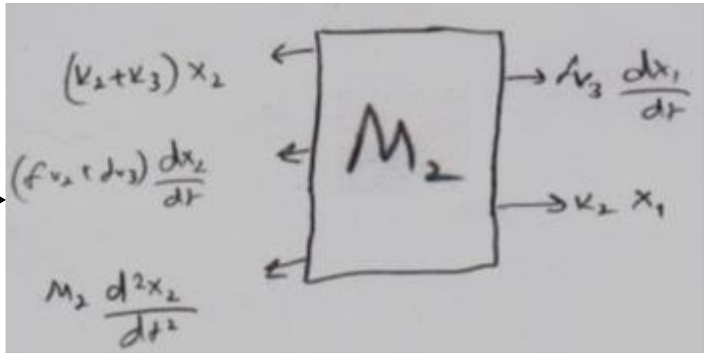


Yalnızca M2'nin hareketinden gelen kuvvetler



Yalnızca M1'in hareketinden gelen kuvvetler

Süperpozisyon ile toplanabilir!



$$(K_2+K_3)x_2 + (f_{v_2}+f_{v_3})\frac{dx_2}{dt} + M_2\frac{d^2x_2}{dt^2} = f_{v_3}\frac{dx_1}{dt} + K_2.x_1(t)$$

Laplace

$$(K_2+K_3)X_2(s) + (f_{v_2}+f_{v_3}).s.X_2(s) + M_2s^2X_2(s) = f_{v_3}.s.X_1(s) + K_2.X_1(s)$$

$$X_2(s)\left((K_2+K_3)+s(f_{v_2}+f_{v_3})+M_2s^2\right) = X_1(s)\left(f_{v_3}s+K_2\right)$$

$$\boxed{X_1(s) = \frac{X_2(s)\left((K_2+K_3)+s(f_{v_2}+f_{v_3})+M_2s^2\right)}{f_{v_3}.s+K_2}} \quad (2)$$

$\frac{X_2(s)}{F(s)}$ ifadesi; (2) denklemi , (1)'de yerine koyarak elde edilebilir.

Pratik Yöntem

$$\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_1 \end{bmatrix} X_1(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_2 \end{bmatrix} X_2(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_3 \end{bmatrix} X_3(s) - \cdots\cdots = \begin{bmatrix} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_1 \end{bmatrix}$$

$$\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_2 \end{bmatrix} X_2(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_2 \text{ and } x_1 \end{bmatrix} X_1(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_2 \text{ and } x_3 \end{bmatrix} X_3(s) - \cdots\cdots = \begin{bmatrix} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_2 \end{bmatrix}$$

$$\vdots$$

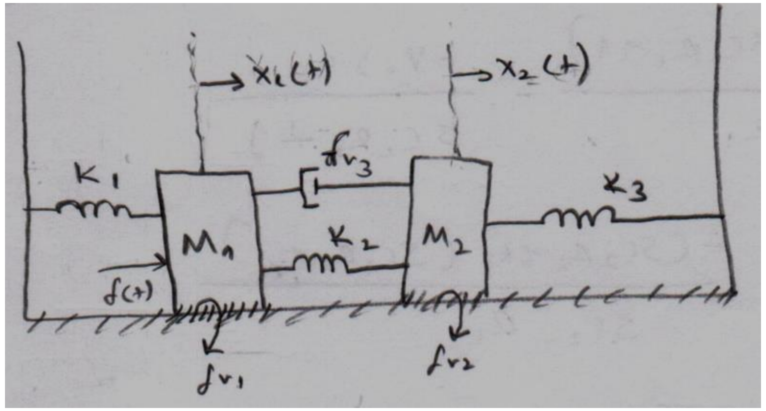
$$\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_n \end{bmatrix} X_n(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_n \text{ and } x_1 \end{bmatrix} X_1(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_n \text{ and } x_2 \end{bmatrix} X_2(s) \cdots\cdots = \begin{bmatrix} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_n \end{bmatrix}$$

Kütlenin empedansı: $M*S^2$

Viskoz sürtünme elemanı empedansı: fv*S

Yay empedansı: K

Pratik Yöntem



$$\begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{connected}\\\text{to the motion}\\\text{at } x_1\end{bmatrix} X_1(s) - \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\x_1 \text{ and } x_2\end{bmatrix} X_2(s) - \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\x_1 \text{ and } x_3\end{bmatrix} X_3(s) - \ldots\ldots = \begin{bmatrix}\text{Sum of}\\\text{applied forces}\\\text{at } x_1\end{bmatrix}$$

$$\begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{connected}\\\text{to the motion}\\\text{at } x_2\end{bmatrix} X_2(s) - \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\x_2 \text{ and } x_1\end{bmatrix} X_1(s) - \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\x_2 \text{ and } x_3\end{bmatrix} X_3(s) - \ldots\ldots = \begin{bmatrix}\text{Sum of}\\\text{applied forces}\\\text{at } x_2\end{bmatrix}$$

$$\vdots$$

$$\begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{connected}\\\text{to the motion}\\\text{at } x_n\end{bmatrix} X_n(s) - \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\x_n \text{ and } x_1\end{bmatrix} X_1(s) - \begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\x_n \text{ and } x_2\end{bmatrix} X_2(s) \ldots\ldots = \begin{bmatrix}\text{Sum of}\\\text{applied forces}\\\text{at } x_n\end{bmatrix}$$

Kütlenin empedansı: M*S^2

Viskoz sürtünme elemanı empedansı: fv*S

Yay empedansı: K

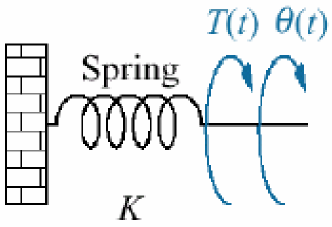
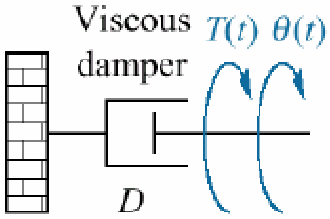
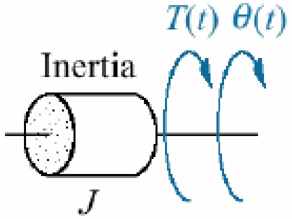
$$[M_1s^2+(f_{v_1}+f_{v_3})s+(K_1+K_2)]X_1(s)-(f_{v_3}s+K_2)X_2(s)=F(s) \qquad (1)$$

$$[M_2s^2+(f_{v_2}+f_{v_3})s+(K_2+K_3)]X_2(s)-(f_{v_3}s+K_2)X_1(s)=0 \qquad (2)$$

$\frac{X_2(s)}{F(s)}$ ifadesi; (2) denklemi , (1)'de yerine koyarak elde edilebilir.

Mekanik Sistemlerin Modellenmesi: Dönen Sistem Modelleme

Sistem Elemanları

Component	Torque-angular velocity	Torque-angular displacement
	$T(t) = K\int_0^t \omega(\tau)\,d\tau$	$T(t) = K\theta(t)$
	$T(t) = D\omega(t)$	$T(t) = D\dfrac{d\theta(t)}{dt}$
	$T(t) = J\dfrac{d\omega(t)}{dt}$	$T(t) = J\dfrac{d^2\theta(t)}{dt^2}$

Yay Katsayısı

Viskoz Sürtünme Katsayısı

Eylemsizlik Momenti

«Norman S. Nise-Control Systems Engineering»

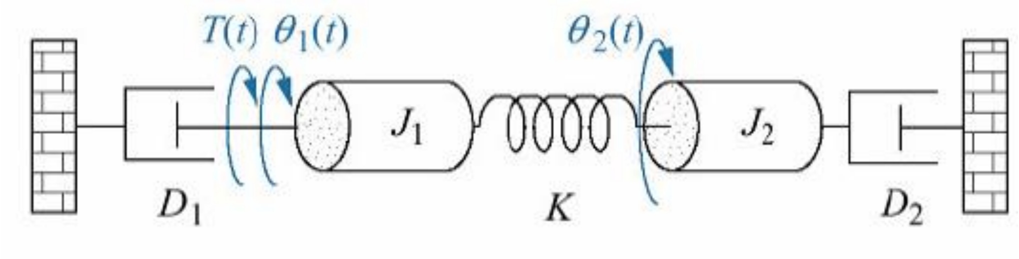
Dönen Sistemler İçin Serbest Cisim Diyagramı Yöntemi

For each inertia, we calculate torque values by its own motion while other inertia motions are ignored.

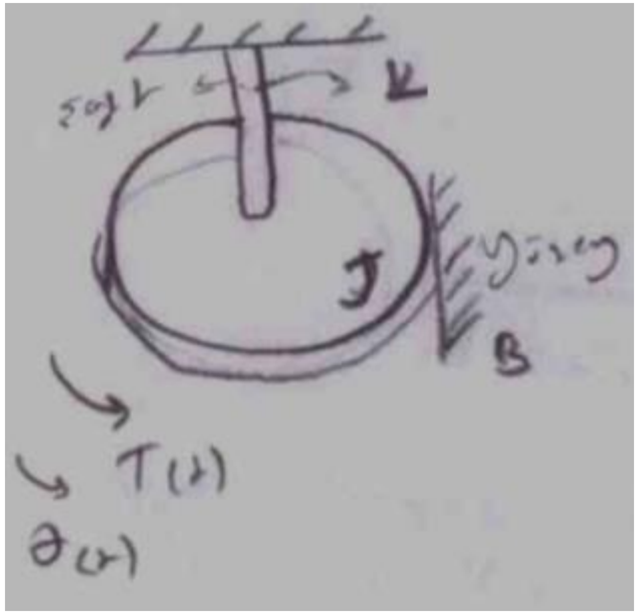
Then we ignore its own motion and calculate the torque values coming from other inertias motion.

Finally we apply Newton's law which says 'sum of the torques on each inertia is zero' $\sum T = 0$.

We obtain differential equations and using Laplace Transform, we obtain transfer functions.

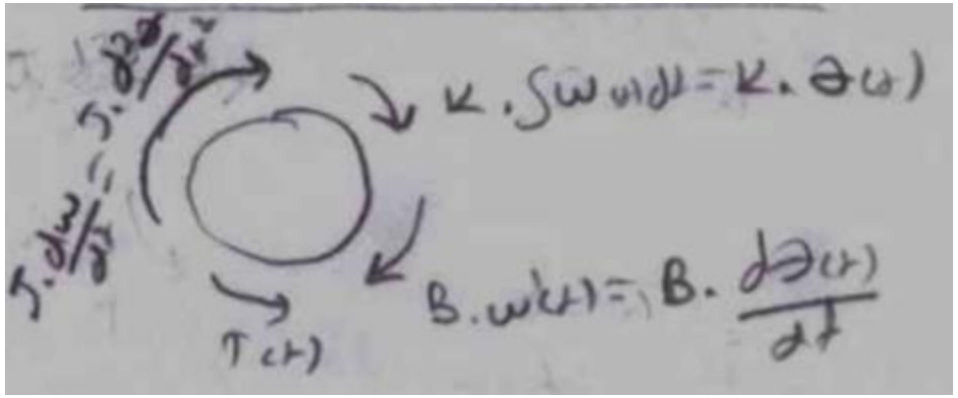


Serbest Cisim Diyagramı Örnek-1



Yüke bağlı şaftın, esnek bir yapısı vardır. Bunu modelleyen yay katsayısı K'dır. Yük ve yüzey arasındaki viskoz sürtünme katsayısı B'dir. Eylemsizlik momenti ise J ve sisteme uygulanan giriş torku T'dir. $\frac{\theta(s)}{T(s)}$ transfer fonksiyonunu bulunuz.

Serbest Cisim Diyagramı

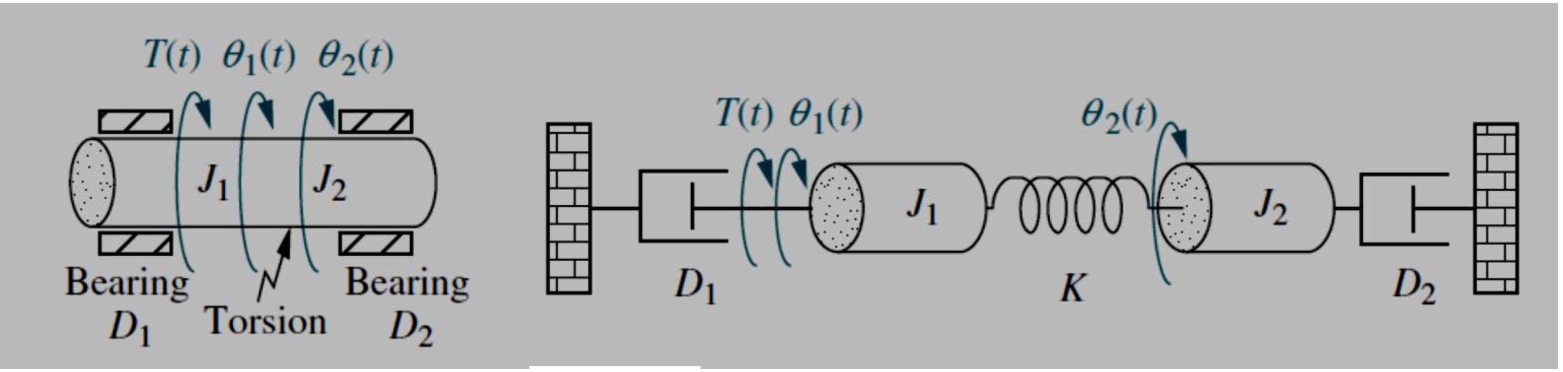


$$J.\frac{d^2\theta}{dt^2} + B.\frac{d\theta}{dt} + K.\theta_{(t)} = T_{(t)}$$

$$J.s^2.\theta_{(s)} + B.s.\theta_{(s)} + K.\theta_{(s)} = T_{(s)}$$

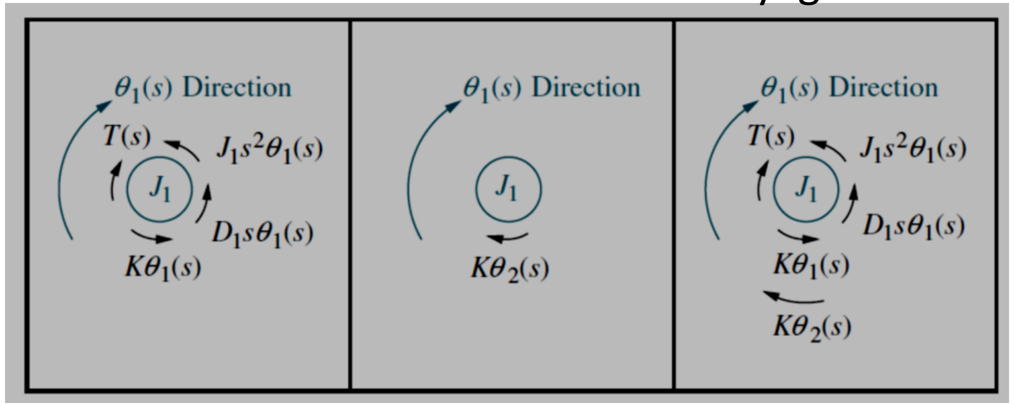
$$\frac{\theta_{(s)}}{T_{(s)}} = \frac{1}{Js^2 + Bs + K}$$

Serbest Cisim Diyagramı Örnek-2



$\theta_2(s)/T(s)$ =? «Control Systems Engineering-Norman Nise»

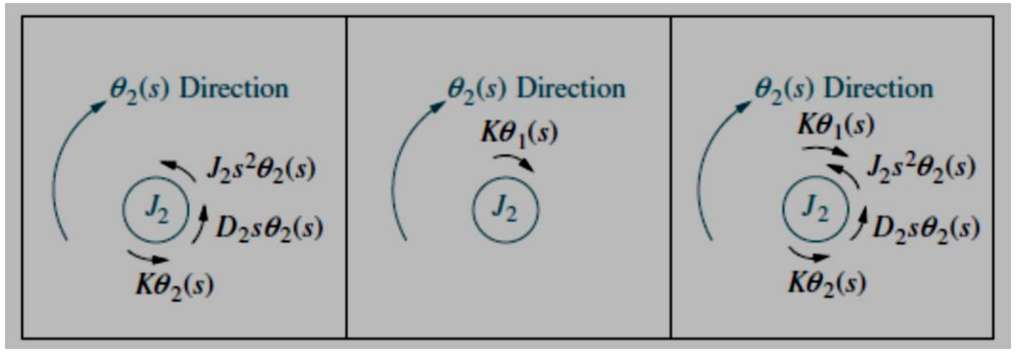
Silindir 1'e ait serbest cisim diyagramı



$$T(s) + K\theta_2(s) = J_1s^2\theta_1(s) + D_1s\theta_1(s) + K\theta_1(s)$$

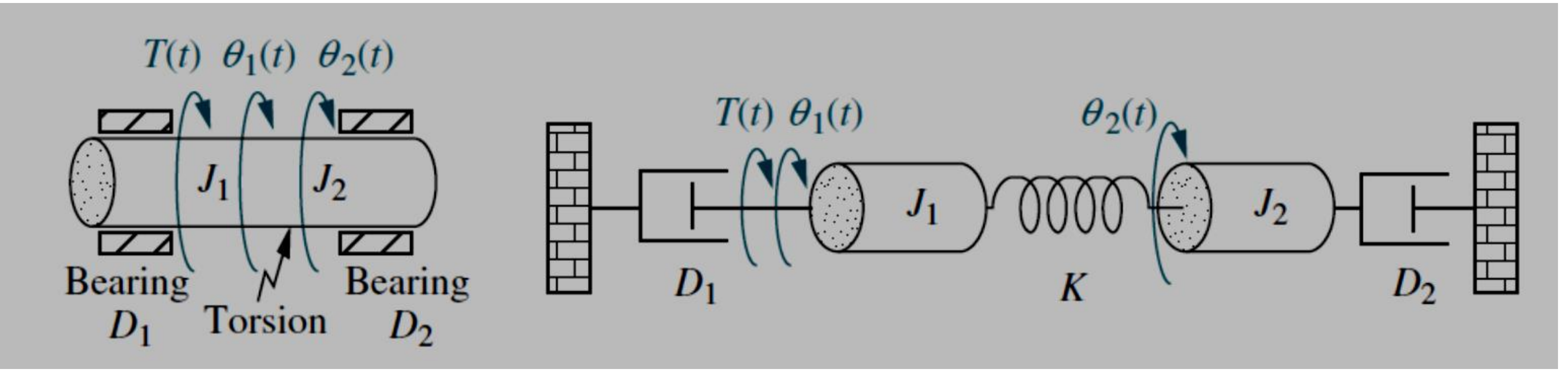
$\theta_2(s)/T(s)$ Elde edilen 2 denklem yardımıyla bulunabilir.

Silindir 2'ye ait serbest cisim diyagramı



$$K\theta_1(s) = J_2s^2\theta_2(s) + D_2s\theta_2(s) + K\theta_2(s)$$

Ex2-Practical Method



$\theta_2(s)/T(s)$ =?

$$\begin{bmatrix}\text{Sum of impedances connected to the motion at } x_1\end{bmatrix} X_1(s) - \begin{bmatrix}\text{Sum of impedances between } x_1 \text{ and } x_2\end{bmatrix} X_2(s) - \begin{bmatrix}\text{Sum of impedances between } x_1 \text{ and } x_3\end{bmatrix} X_3(s) - \ldots\ldots = \begin{bmatrix}\text{Sum of applied forces at } x_1\end{bmatrix}$$

$$-\begin{bmatrix}\text{Sum of impedances between } x_2 \text{ and } x_1\end{bmatrix} X_1(s) + \begin{bmatrix}\text{Sum of impedances connected to the motion at } x_2\end{bmatrix} X_2(s) - \begin{bmatrix}\text{Sum of impedances between } x_2 \text{ and } x_3\end{bmatrix} X_3(s) - \ldots\ldots = \begin{bmatrix}\text{Sum of applied forces at } x_2\end{bmatrix}$$

$$\vdots$$

$$\begin{bmatrix}\text{Sum of impedances connected to the motion at } x_2\end{bmatrix} X_n(s) - \begin{bmatrix}\text{Sum of impedances between } X_n \text{ and } X_1\end{bmatrix} X_1(s) - \begin{bmatrix}\text{Sum of impedances between } X_n \text{ and } x_2\end{bmatrix} X_2(s) \ldots\ldots = \begin{bmatrix}\text{Sum of applied forces at } X_n\end{bmatrix}$$

X1, X2, … Xn yerine, $\theta_1, \theta_2 .. \theta_n$ koyabilirsiniz.

Eylemsizlik Empedansı: J*S^2

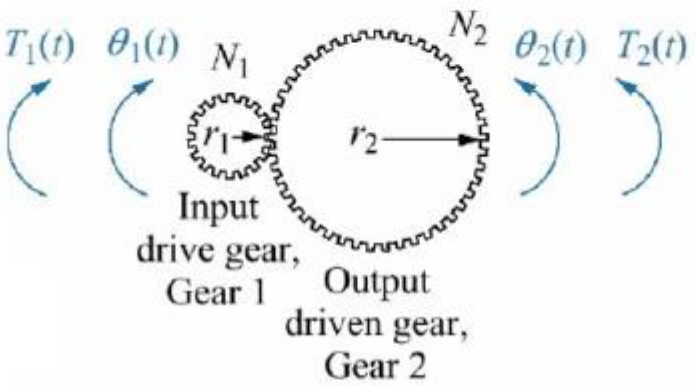
Dönen viskoz sürtünme empedansı: D*S

Dönen yay empedansı: K

$$(J_1 s^2 + D_1 s + K)\theta_1(s) - K\theta_2(s) = T(s)$$

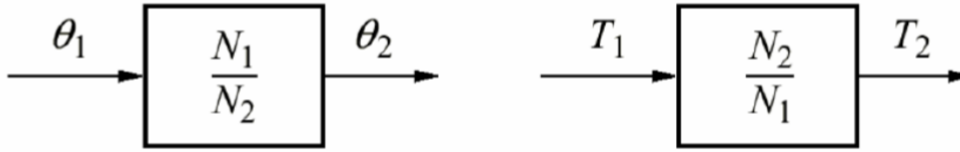
$$(J_2 s^2 + D_2 s + K)\theta_2(s) - K\theta_1(s) = 0$$

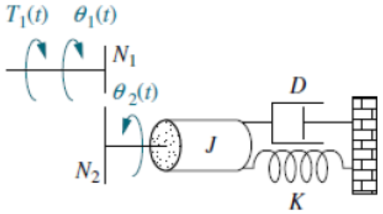
Dişli Sistemleri

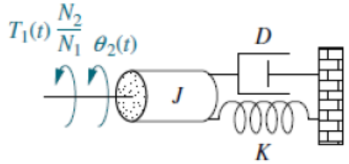


$$T_1\theta_1 = T_2\theta_2$$

$$\frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}$$

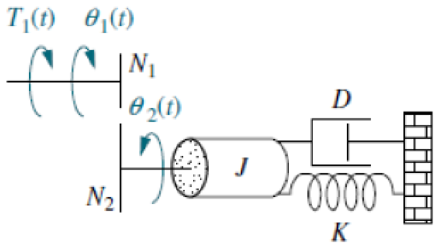


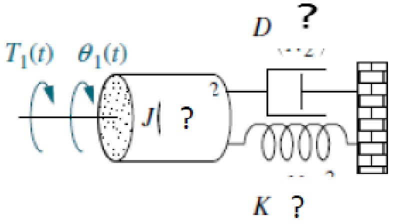




$$(Js^2 + Ds + K)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

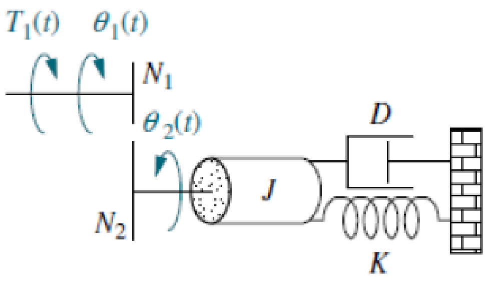
Çıkıştaki eşdeğer sistem

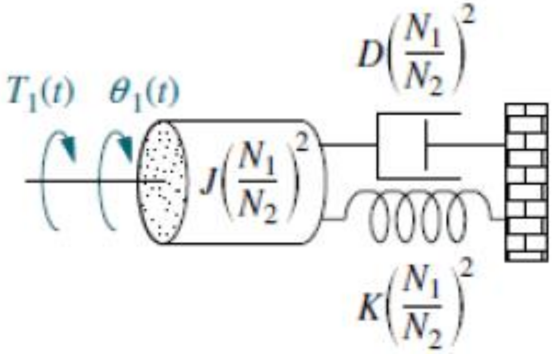




Girişteki eşdeğer sistem?

Dişli Sistemleri





$$(Js^2 + Ds + K)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

$$\theta_2 = \theta_1\frac{N_1}{N_2}$$

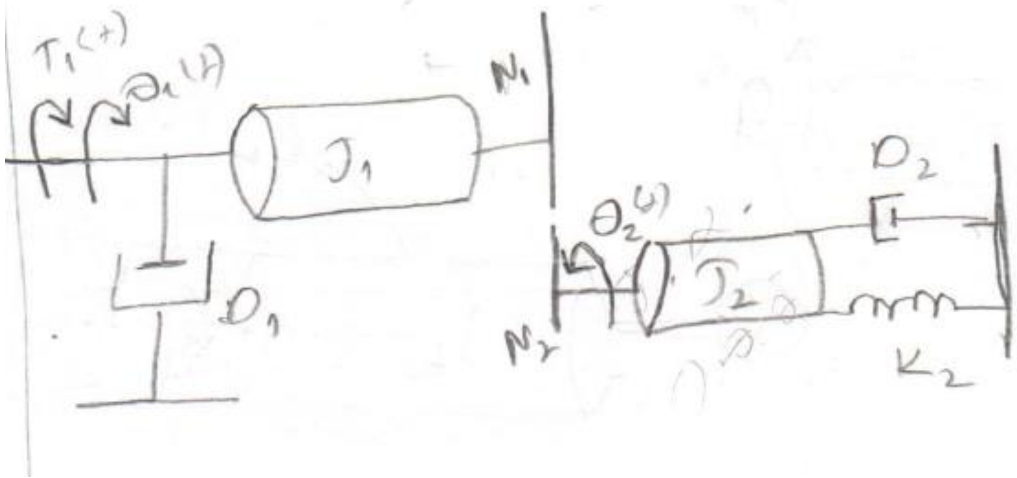
$$(Js^2 + Ds + K)\frac{N_1}{N_2}\theta_1(s) = T_1(s)\frac{N_2}{N_1}$$

$$\left[J\left(\frac{N_1}{N_2}\right)^2 s^2 + D\left(\frac{N_1}{N_2}\right)^2 s + K\left(\frac{N_1}{N_2}\right)^2\right]\theta_1(s) = T_1(s)$$

Sonuç: Dönen sistemlerde dişli kullanılması durumunda, Mekanik empedanslar (eylemsizlik, sürtünme, yay), aşağıdaki ifadeyle çarpılarak hesaplanır!

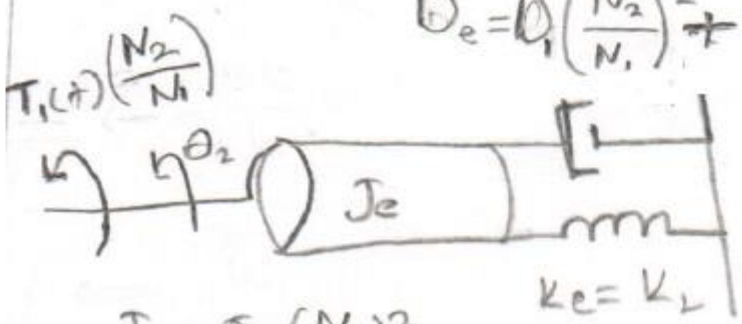
$$\left(\frac{Hedef\ şafttaki\ diş\ sayısı}{Kaynak\ şafttaki\ diş\ sayısı}\right)^2$$

Dişli Sistemleri-Örnek



Dişlinin sol tarafındaki komponentleri sağ tarafına yansıtarak eşdeğer sistemi yazalım:

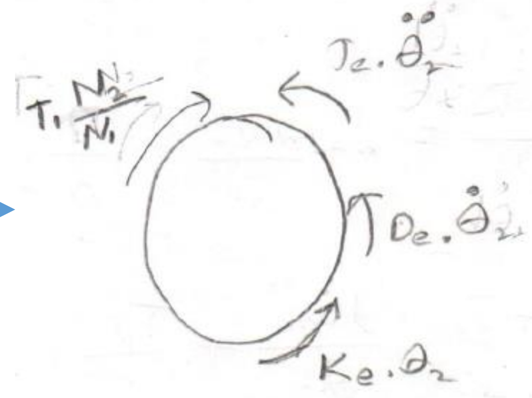
$$D_e = D_1\left(\frac{N_2}{N_1}\right)^2 + D_2$$



$$J_e = J_1\left(\frac{N_2}{N_1}\right)^2 + J_2$$

Burayi bir daha incele

$$\frac{\theta_2(s)}{T_1(s)}$$ 'i elde ediniz.



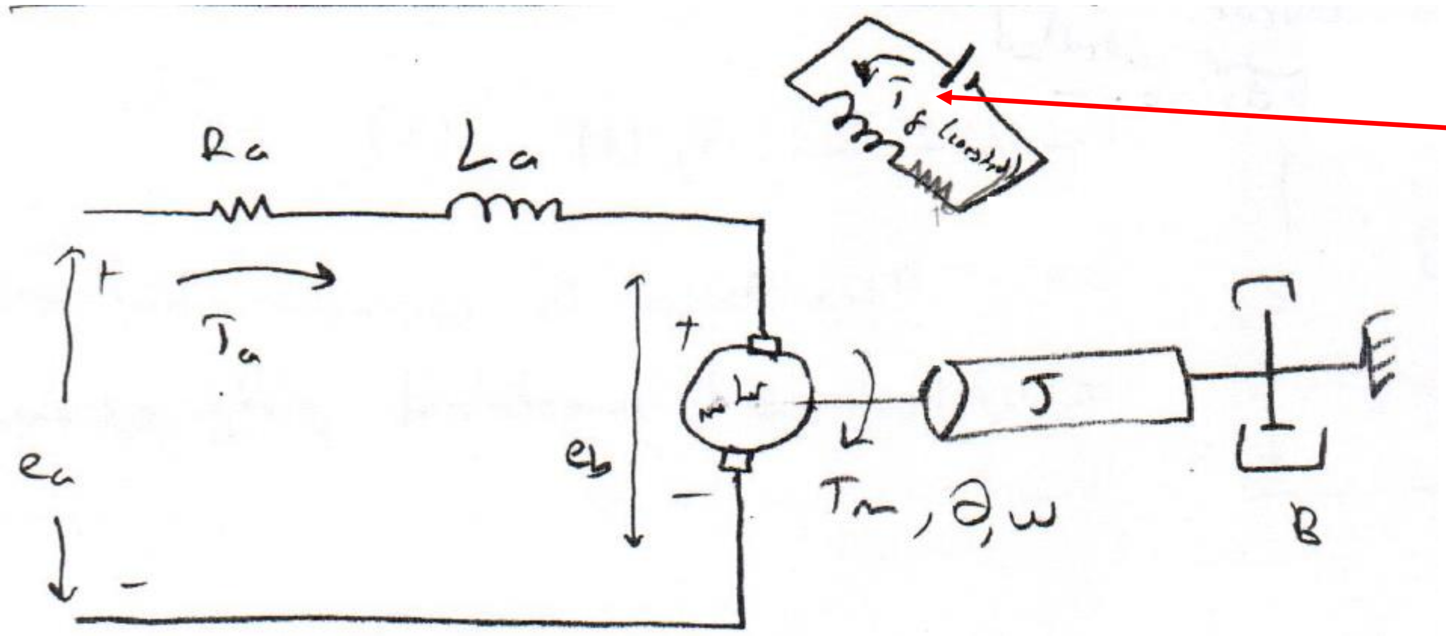
$$T_1 . \frac{N_2}{N_1} = J_2 . \ddot{\theta}_2 + D_e . \dot{\theta}_2 + K_e . \theta_2$$

$$T_1 \frac{N_2}{N_1} = \left(s^2 . \left(J_1\left(\frac{N_2}{N_1}\right)^2 + J_2\right) + s . \left(D_1\left(\frac{N_2}{N_1}\right)^2 + D_2\right) + K_2\right)\theta_2$$

$$G(s) = \frac{\theta_2(s)}{T_1(s)} = \frac{N_2/N_1}{s^2\left(J_1\left(\frac{N_2}{N_1}\right)^2 + J_2\right) + s\left(D_1\left(\frac{N_2}{N_1}\right)^2 + D_2\right) + K_2}$$

Elektromekanik Sistemler

Armatur Kontrollü DC Motor

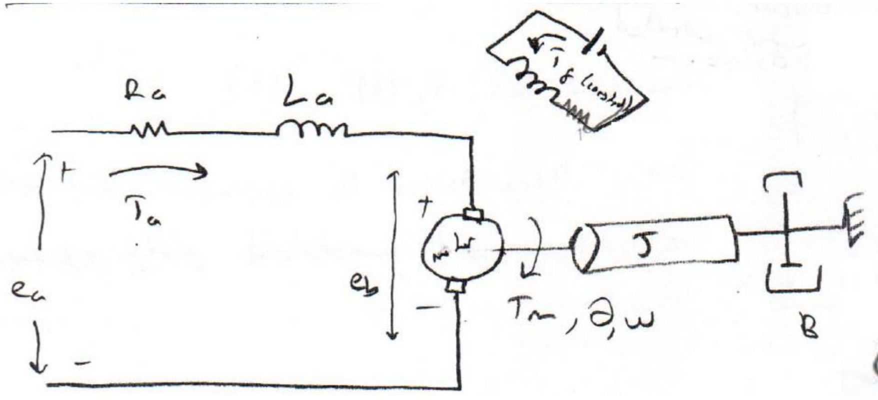


i_f $(alan\ akımı)$
sabit

Tork kontrolü, i_a ‘yı değiştirerek gerçekleştirilir. i_a akımı, e_a gerilimini değiştirerek ayarlanır ve ters emk geriliminden (e_b) doğrudan etkilenir .

$$\frac{W(s)}{E_a(s)}$$ transfer fonksiyonunu bulalım. .

Armature Controlled DC Motor



1) Akım Denklemi

$$e_a(t) = L_a \cdot \frac{di_a}{dt} + R_a \cdot i_a(t) + e_b$$

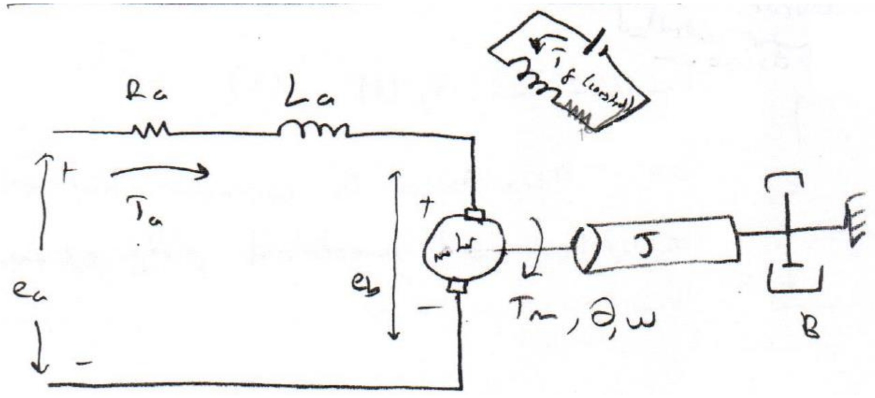
$$e_b = k_b \cdot w$$

k_b constant

$$E_a(s) - K_b w(s) = I_a(s)(Ls + R_a)$$

$$\Rightarrow I_a(s) = \frac{E_a - K_b \cdot W(s)}{L \cdot s + R_a} \quad (1)$$

Armature Controlled DC Motor



1) Akım Denklemi

$$e_a(t) = L_a \cdot \frac{di_a}{dt} + R_a \cdot i_a(t) + e_b$$

$$e_b = K_b \cdot w$$

$\hookrightarrow$ constant

$$E_a(s) - K_b\, w(s) = I_a(s)\,(Ls + R_a)$$

$$\boxed{\Rightarrow I_a(s) = \frac{E_a - K_b \cdot W(s)}{L \cdot s + R_a} \qquad (1)}$$

2)Motor Tork Üretimi

$$T_m = K_m \cdot i_a(t) \cdot \phi(t)$$

→ magnetic flux

$$\phi(t) = K_f \cdot i_f$$

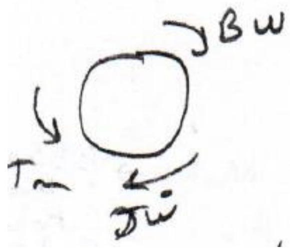
$$T_m = K_m \cdot i_a(t) \cdot K_f \cdot i_f$$

constant

$$\boxed{T_m = K_a \cdot i_a(t)} \quad (\text{where } K_a = K_m \cdot K_f \cdot i_f)$$

Elektriksel ve mekanik sistem arasındaki bağlantıyı sağlayan ifadedir!

3)Moment Denklemi



$$T_m = Bw + J\dot{w}$$

$$K_a \cdot i_a = B \cdot w + J \cdot s \cdot w$$

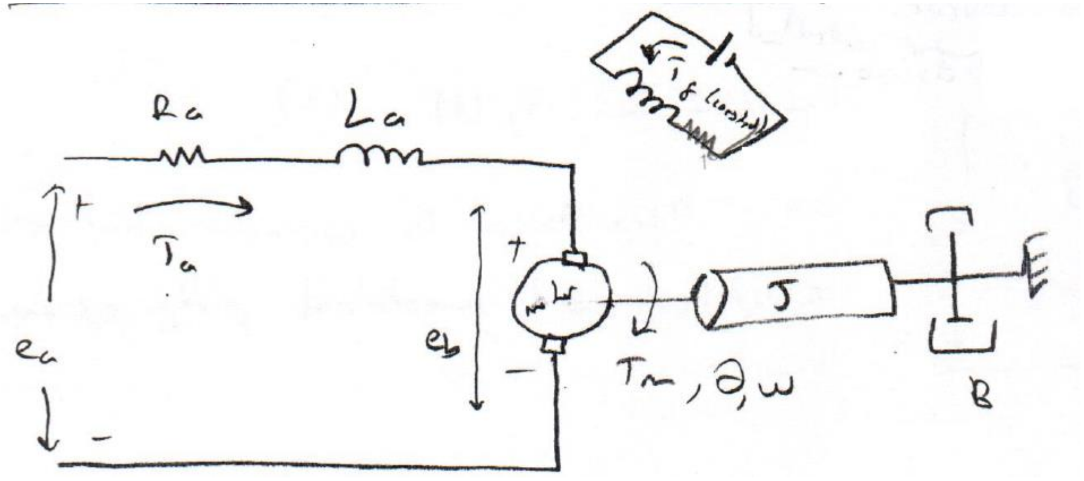
Bu ifadede, ia yerine (1)’deki karşılığını koyalım.

$$K_a \frac{E_a - K_b w}{Ls + R_a} = w(B + Js)$$

$$\frac{(Ls + R_a)\, w\, (B + Js)}{K_a} = E_a - K_b w$$

$$\frac{w\left[(Ls + R_a)(B + Js) + K_b K_a\right]}{K_a} = E_a \Rightarrow \boxed{\frac{W(s)}{E_a(s)} = \frac{K_a}{(Ls + R_a)(B + Js) + K_b K_a}}$$

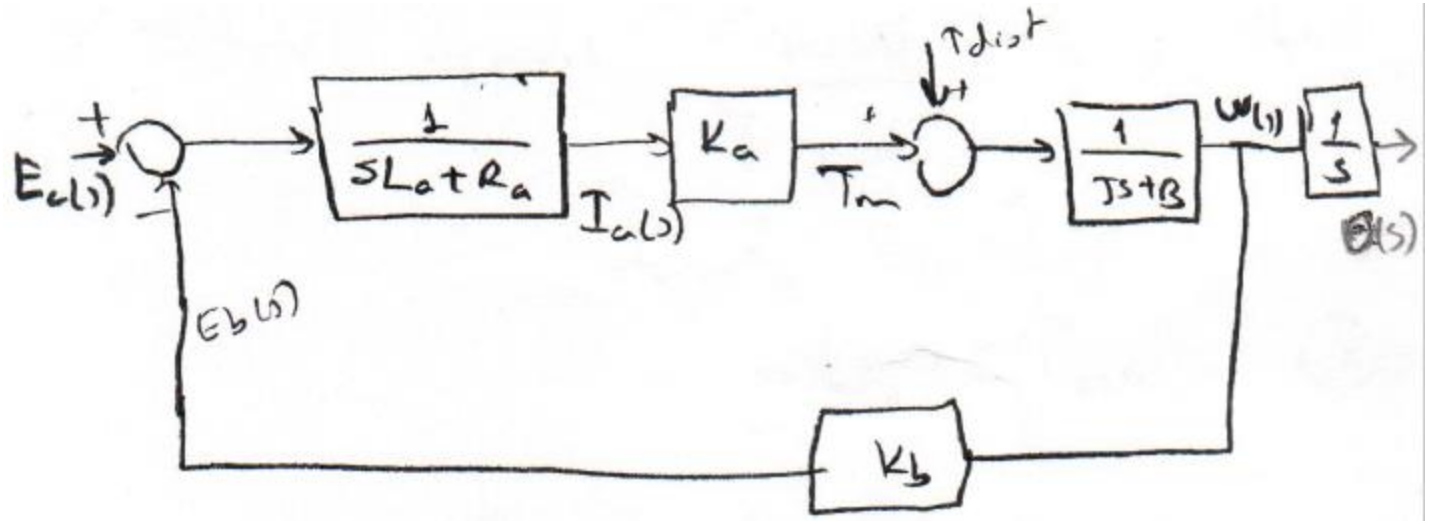
Armature Controlled DC Motor

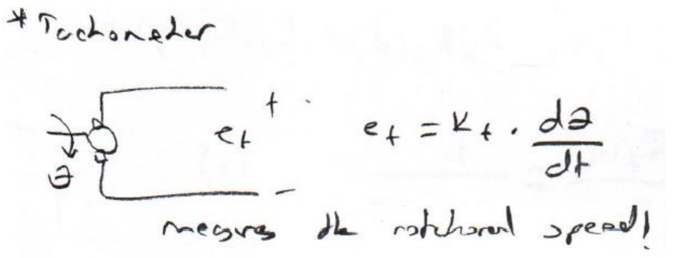


$$\boxed{\frac{W(s)}{E_a(s)} = \frac{K_a}{(Ls+R_a)(B+Js)+K_bK_a}}$$

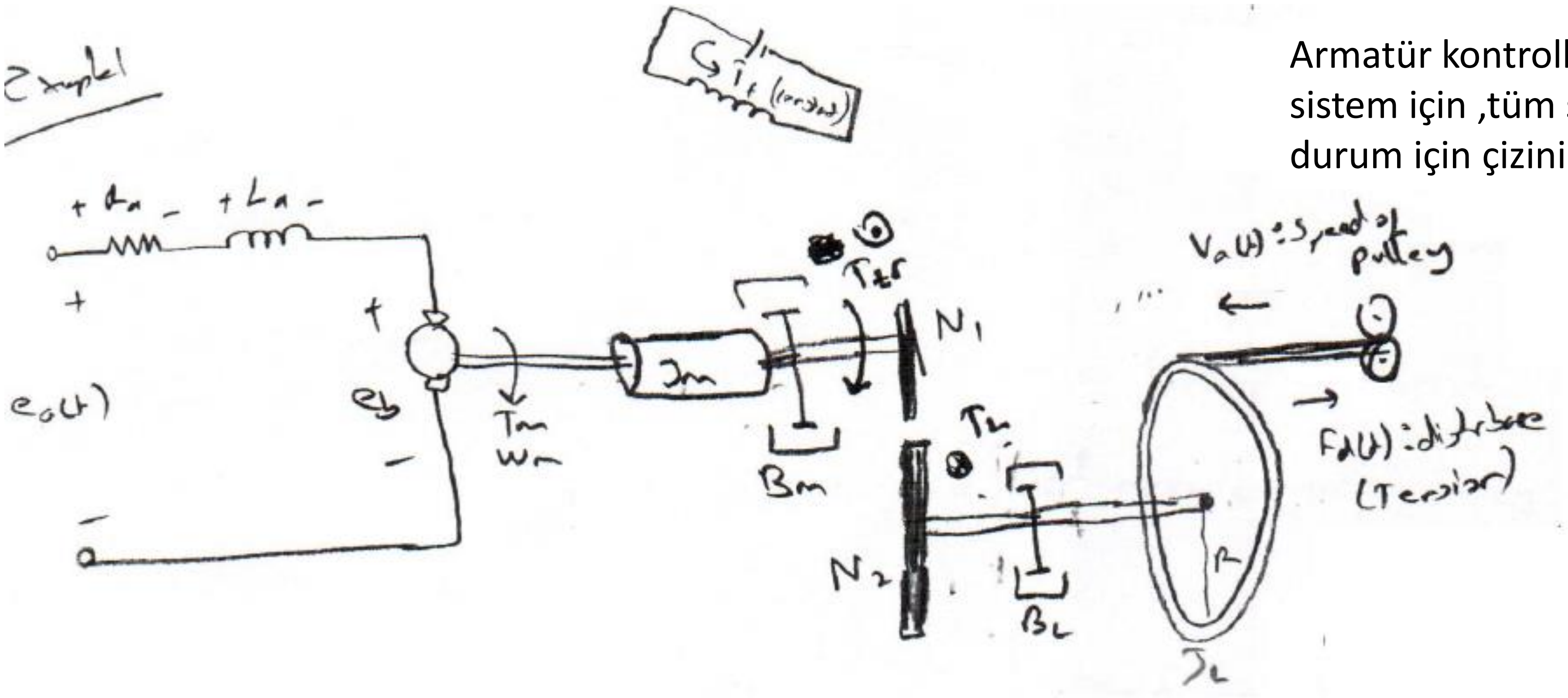
J ve B toplam eylemsizlik ve sürtünme olarak düşünülebilir..

Aşağıdaki blok diyagram, sistemi özetlemektedir.





Example



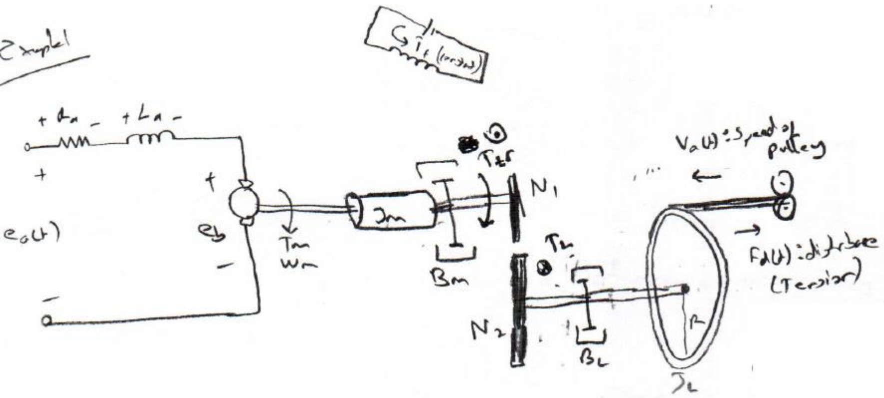
Armatür kontrollü doğru akım motoru içeren şekildeki sistem için ,tüm sisteme ait blok diyagramı aşağıdaki 2 durum için çiziniz.

- Bozucu olması durumunda
- Bozucu olmaması durumunda

$$\frac{N_1}{N_2} = \mu$$

R:Kasnak yarıçapı

Example



$$e_a(t) = R_a \cdot i_a + L_a \frac{di_a}{dt} + \underbrace{e_b(t)}_{K_b \cdot W_m(t)}$$

$$T_m = K_a \cdot i_a(t)$$

$$T_m = J_m \cdot \dot{W}_m + B_m \cdot W_m + J_L \cdot \left(\frac{N_1}{N_2}\right)^2 \dot{W}_m + B_L \left(\frac{N_1}{N_2}\right)^2 \cdot W_m$$

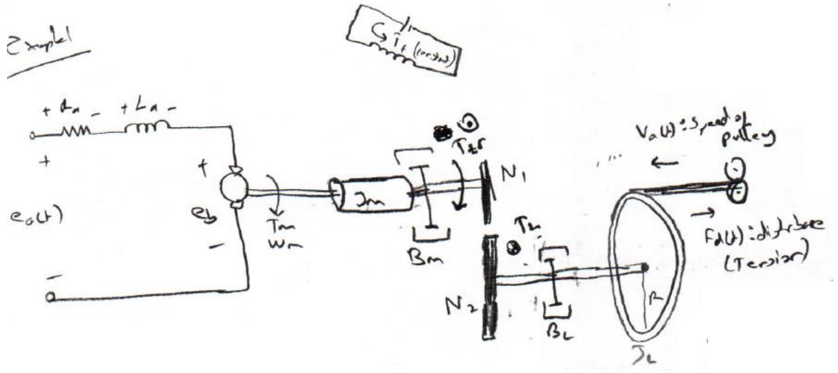
$$T_m = \underbrace{\left[J_m + J_L \left(\frac{N_1}{N_2}\right)^2\right]}_{J_{eq}} \dot{W}_m + \underbrace{\left[B_m + B_L \left(\frac{N_1}{N_2}\right)^2\right]}_{B_{eq}} W_m$$

$$T_m(s) = J_{eq} s W_m + B_{eq} W_m$$

Jeq: Motor tarafındaki eşdeğer eylemsizlik.
Beq: Motor tarafındaki eşdeğer sürtünme.

Örnek

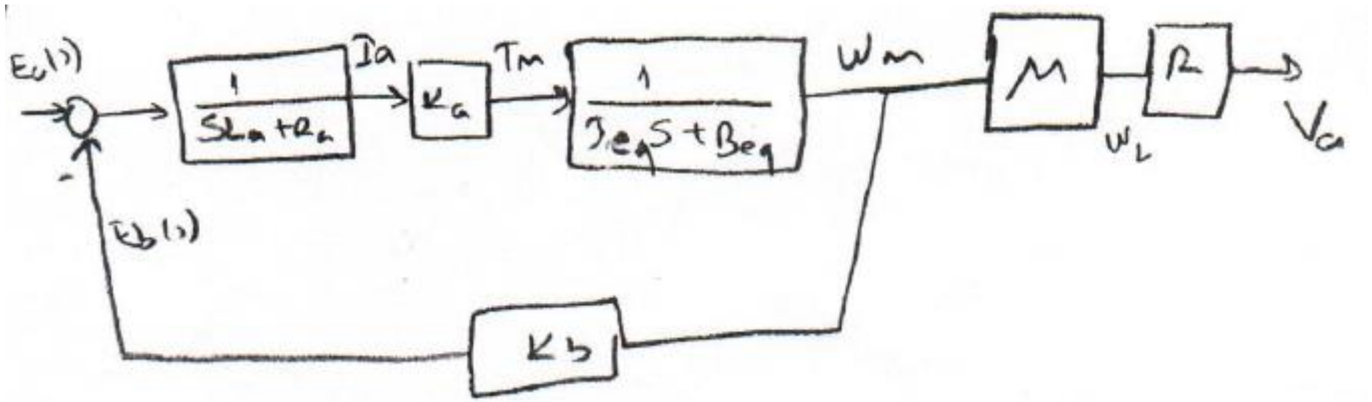
$V_a = w_L \cdot R$
m/s, rad/s, m



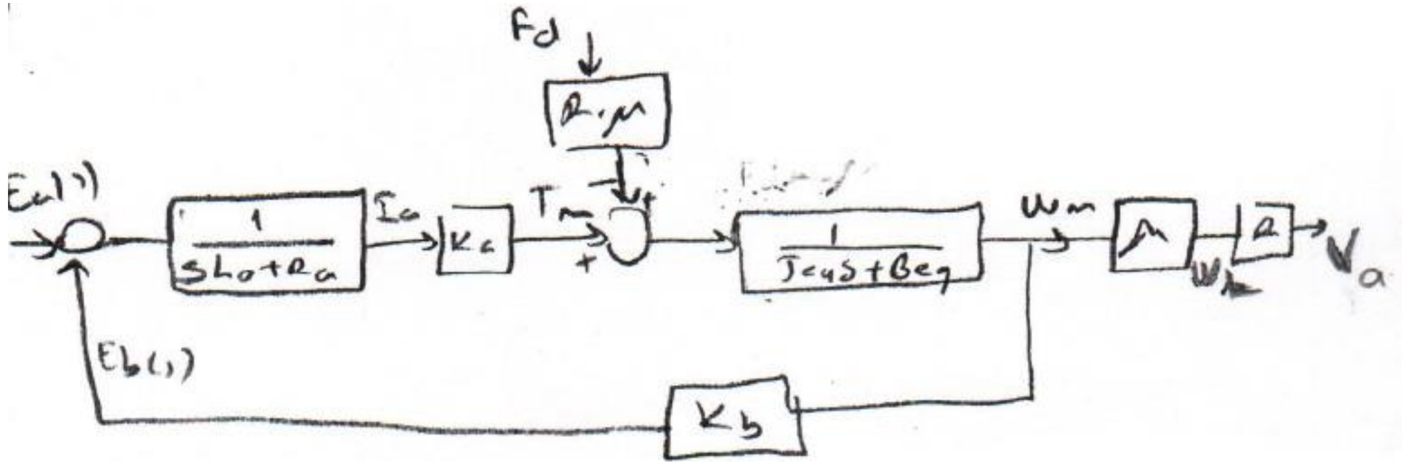
$$T_m = K_a \cdot I_a(t)$$

$$T_m(s) = J_{eq} s\, W_m + B_{eq} W_m$$

Bozucu Olmadığında



Bozucu Olduğunda



Bu blok diyagramlara bakarak şunlar elde edilebilir:

- Bozucu olmadığı durumda Va ve Ea arasındaki transfer fonksiyonu
- Bozucu olduğu durumda, Va çıkışının Ea ve Fd cinsinden ifadesi (Süperpozisyon özelliğini hatırla!)