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TEL313E-Electromagnetic Waves

Midterm1

1-) (20 Points)

a) Write the Maxwell Equations .

b) Write the name and unit of all terms (E,H...) in Maxwell Equations .

c) Write the Maxwell Equation in phasor form (time dependence of EM wave I assumed to be e^{jwt})

d) Write the i) Poynting vector ii) complex Poynting vector and then describe the relation between them.

a) $\nabla \times \vec{H} = \vec{J}_v + \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$ $\qquad \nabla \cdot \vec{B} = 0.$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\qquad \nabla \cdot \vec{D} = \rho.$

b) $\vec{E}$: Electric field (V/m)

$\vec{H}$: Magnetic field (A/m)

ρ : Charge density (C/m³)

J_v : Current density (A/m²)

$\vec{D}$: Electric Displacement field (C/m²)

$\vec{B}$: Magnetic Flux Density (Tesla)

σ : Conductivity (S/m)

c) $e^{jwt} \Rightarrow \frac{\partial}{\partial t} \rightarrow j\omega$

$\nabla \times \vec{H} = \vec{J}_v + \sigma \vec{E} + j\omega \vec{D}.$ $\qquad \nabla \cdot \vec{B} = 0.$

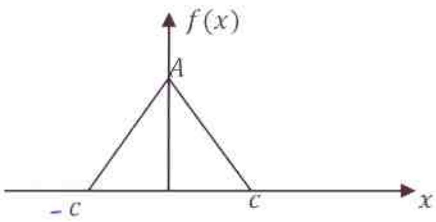
$\nabla \times \vec{E} = -j\omega \vec{B}$ $\qquad \nabla \cdot \vec{D} = \rho.$

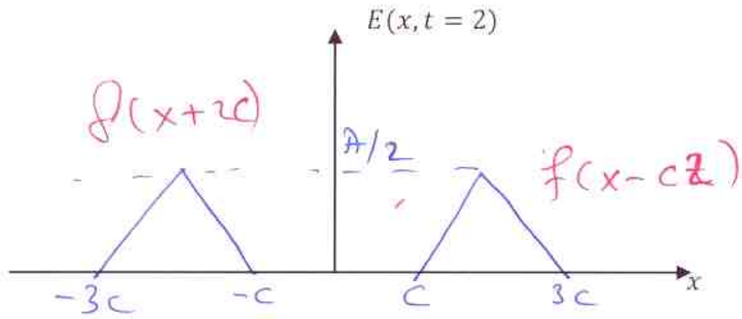
d) i) $\vec{P} = \vec{E}(r;t) \times \vec{H}(r;t).$

ii) $\vec{P}_c = \frac{1}{2} \vec{E}(r) \times \vec{H}^*(r)$

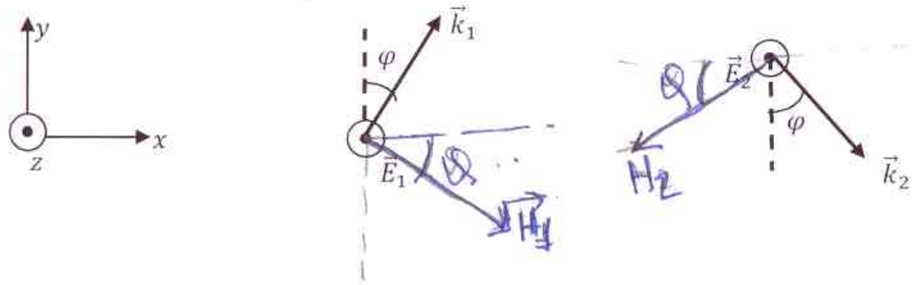
$Re\{\vec{P}_c\} = \frac{1}{T} \int_0^T \vec{P} \, dt$

2-) (15 Points) Assume that E (x,t) is the solution of wave equation. And the initial conditions are $E(x,0) = f(x)$ and $\frac{\partial E}{\partial t}(x,0) = 0$. Under these initial conditions, $E(x,t)$ becomes $E(x,t) = (f(x-ct)+f(x+ct))/2$. If f(x) is a function given in figure below, plot the $E(x,t)$ at t=2sec





3-) (45 Points) An electromagnetic wave is represented by the superposition of two plane waves of equal frequency $(\omega_1 = \omega_2 = \omega)$ which are propagating within a free space $(\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon_0}, \boldsymbol{\mu} = \boldsymbol{\mu_0})$ see figures below. The electric field of the first and second wave is given by $\vec{E}_1 = E_0 e^{-j\vec{k}_1.\vec{r}}\vec{e}_z$ and $\vec{E}_2 = E_0 e^{-j\vec{k}_2.\vec{r}}\vec{e}_z$, respectively.



a) Complete the figure with the magnetic fields $\vec{H}_1$ and $\vec{H}_2$. Determine the magnitudes (absolute value) of $\vec{H}_1$ and $\vec{H}_2$

b) What are the magnitude s of $\vec{k}_1$ and $\vec{k}_2$ in terms of ω. Write down the $\vec{k}_1.\vec{r}$ and $\vec{k}_2.\vec{r}$ as functions of x,y,z

c) Calculate the electric and magnetic field of the total wave as functions of x,y,z. Simplify the result using the given hints.

d) What is direction of propagation of the total wave?

e) Describe the polarization of the total wave.

Hints: $Cosx = \frac{e^{jx}+e^{-jx}}{2}, Sinx = \frac{e^{jx}-e^{-jx}}{2j}$

a) $\vec{n}_1 = \vec{e}_x \sin\theta + \vec{e}_y \cos\theta$

$\vec{H}_1 = \frac{1}{Z_0}\vec{n}_1 \times \vec{E}_1 = \frac{E_0}{Z_0}\cdot\left\{\cos\theta\,\vec{e}_z - \sin\theta\,\vec{e}_y\right\} e^{-j\vec{k}_1\cdot\vec{r}}$

$\vec{n}_2 = \vec{e}_x \sin\theta - \vec{e}_y \cos\theta.$

$\vec{H}_2 = \frac{1}{Z_0}\vec{n}_2 \times \vec{E}_2 = \frac{E_0}{Z_0}\cdot\left\{-\cos\theta\,\vec{e}_x - \sin\theta\,\vec{e}_y\right\} e^{-j\vec{k}_2\cdot\vec{r}}$

$|\vec{H}_1| = |\vec{H}_2| = \frac{E_0}{Z_0}$ $\qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi.$

b) $|\vec{k}_1| = |\vec{k}_2| = \frac{\omega}{c} = k$ $\qquad$ in free space $\Rightarrow v = c$. speed of light
and same frequency $\Rightarrow \omega$.

$\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$

$\vec{k}_1\vec{r} = k\,\vec{n}_1\cdot\vec{r}$
$= k\cdot(\sin\theta\, x + \cos\theta\, y)$

$\vec{k}_2\vec{r} = k\,\vec{n}_2\cdot\vec{r}$
$= k(\sin\theta\, x - \cos\theta\, y).$

3-)

c) $\vec{E} = \vec{E_1} + \vec{E_2}$

$= E_0 \left\{ e^{-jk\sin\theta x - jk\cos\theta y} + e^{-jk\sin\theta x + jk\cos\theta y} \right\} \vec{e_z}$

$= E_0 e^{-jk\sin\theta x} \cdot \left\{ e^{-jk\cos\theta y} + e^{jk\cos\theta y} \right\} \vec{e_z}$

$= E_0 \cdot e^{-jk\sin\theta x} \cdot 2\cos(k\cos\theta y)\, \vec{e_z}$

$\vec{H} = \vec{H_1} + \vec{H_2}$

$= H_x \vec{e_x} + H_y \vec{e_y}$

$H_x = \frac{E_0}{Z_0} \cos\theta \cdot \left\{ e^{-jk\sin\theta x - jk\cos\theta y} - e^{-jk\sin\theta x + jk\cos\theta y} \right\} \vec{e_x}$

$= \frac{E_0}{Z_0} \cdot \cos\theta\, e^{-jk\sin\theta x} \cdot -2j \sin(k\cos\theta y)\, \vec{e_x}$

$H_y = -\frac{E_0}{Z_0} \sin\theta \left\{ e^{-jk\sin\theta x} e^{-jk\cos\theta y} + e^{-jk\sin\theta x} \cdot e^{jk\cos\theta y} \right\} \vec{e_y}$

$= -\frac{E_0}{Z_0} \sin\theta\, e^{-jk\sin\theta x} \cdot 2\cos(k\cos\theta y)\, \vec{e_y}$

d) $e^{-jk\sin\theta x} \Rightarrow$ Direction of propagation $\vec{e_x}$

e) $\vec{E} = (E_0 e^{-jk\sin\theta x} \cdot 2\cos(k\cos\theta y))\, \underline{\vec{e_z}}$

$\vec{E}(x,y;t) = Re\left\{ \vec{E} \cdot e^{j\omega t} \right\}$

$= 2\cos(k\cos\theta y) \cdot E_0 \cdot \cos(\omega t - k\sin\theta x)\, \vec{e_z}$

$\vec{E}$ only has a "z" component $\Rightarrow$ Linear polarization.

4-) (20 Points) To shield a room from radio interference, the room must be enclosed in a layer of copper five skin-depths thick. If the frequency to be shielded against in **10 kHz** to **1 GHz**, what should be the thickness of the copper (in millimeters)? For copper, $\boldsymbol{\varepsilon = \varepsilon_0}, \boldsymbol{\mu = \mu_0}$ and $\sigma = 5.8 \times 10^7$ S/m.

$10\ kHz \Rightarrow$ Loss tangent $= \dfrac{\sigma}{\omega \varepsilon_0} = \dfrac{5.8 \cdot 10^7}{2\pi 10^4 \cdot \frac{1}{36\pi} \cdot 10^{-9}} = 1.04\ 10^{14} \gg 1.$ Good conductor.

$1\ GHz \Rightarrow$ Loss tangent $= \cancel{1.04\ 10^{8}}$ $= 1.04 \cdot 10^8 \gg 1$ Good conductor

$\Rightarrow 10\ kHz \Rightarrow \alpha_1 = \sqrt{\pi f \mu \sigma} = 15.13 \cdot 10^2$

$\delta_1 = \dfrac{1}{\alpha_1} = 0.66$ mm.

Thickness $= 5\delta_1 = 3.3$ mm.

$\Rightarrow 1\ GHz \Rightarrow \alpha_2 = 15.13\ 10^5$

$\delta_2 = \dfrac{1}{\alpha_2} = 0.66\ 10^{-3}$ mm.

$5\delta_2 = 3.3\ 10^{-3}$ mm.

For whole frequency range
Thickness $= 5\delta_1$
$= 3.3$ mm.

Complex permittivity, $\varepsilon_c = \varepsilon' - j\varepsilon'', \varepsilon' = \varepsilon, \varepsilon'' = \frac{\sigma}{\omega}$, *Loss tangent* $= \frac{\varepsilon''}{\varepsilon'}$.

Low − loss Dielectrics: *Loss tangent* $\ll 1 \Rightarrow \alpha \cong \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}}$ *and* $\beta \cong \omega\sqrt{\mu\varepsilon'}[1 + \frac{1}{8}(\frac{\varepsilon''}{\varepsilon'})^2]$

Good Conductors: *Loss tangent* $\gg 1 \Rightarrow \alpha = \beta \cong \sqrt{\pi f \mu \sigma}$

$(\varepsilon_0 = \frac{1}{36\pi}10^{-9}\frac{F}{m}, \mu_0 = 4\pi 10^{-7}\ H/m)$

Good Luck...