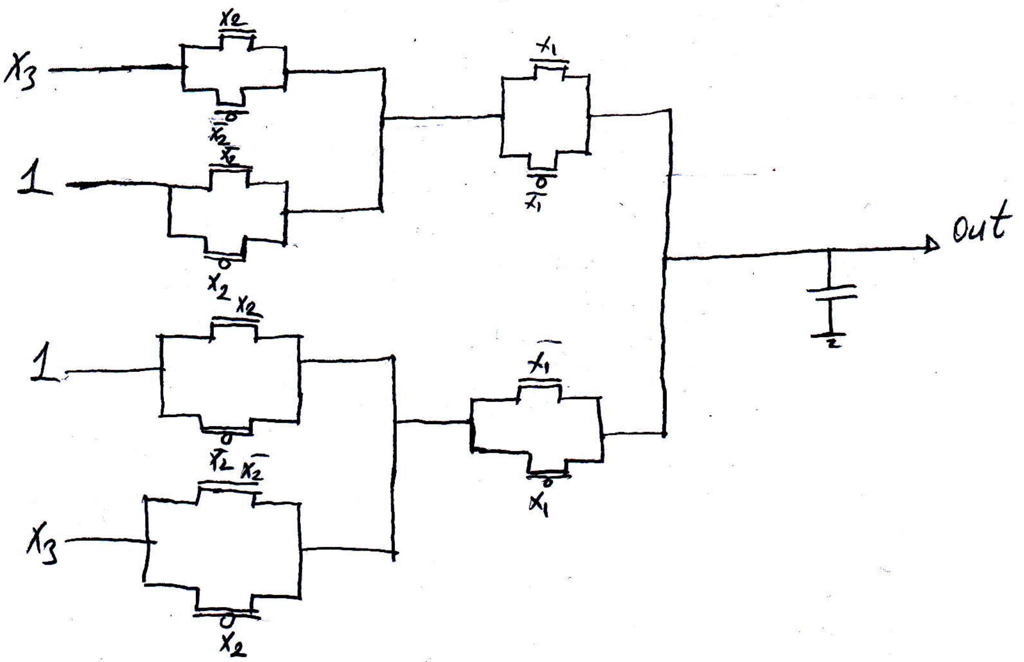
Quiz 2 - Solutions

1.) $f = x_1\bar{x}_2 + \bar{x}_1 x_2 + x_3 \xrightarrow{\text{rearrange}} x_1\bar{x}_2 + \bar{x}_1 x_2 + x_3(x_1+\bar{x}_1)$

$$= x_1(\bar{x}_2 + x_3) + \bar{x}_1(x_2 + x_3) \longrightarrow x_1(\bar{x}_2 + x_3(x_2+\bar{x}_2)) + \bar{x}_1(x_2 + x_3(x_2+\bar{x}_2))$$

$$= x_1[x_2(x_3) + \bar{x}_2(1+x_3)] + \bar{x}_1[x_2(1+x_3) + \bar{x}_2(x_3)]$$

$$= x_1[x_2 x_3 + \bar{x}_2(1)] + \bar{x}_1[x_2(1) + \bar{x}_2 x_3] \checkmark$$



worst case → 2 transistors
or best case ↗ → $2\times(R_n \| R_p).10pF = 80n \Rightarrow 2.(R_n\|R_p).10.10^{-12} = 80.10^{-9}$

$R_n \| R_p = 4k$

$$\frac{(\frac{W}{L})_n}{12k} + \frac{(\frac{W}{L})_p}{24k} = \frac{1}{4k} \rightarrow 2\left(\frac{W}{L}\right)_n + \left(\frac{W}{L}\right)_p = 6$$

then; $\left(\frac{W}{L}\right)_n = 2$, $\left(\frac{W}{L}\right)_p = 2$

$\left(\frac{W}{L}\right)_n = 1$, $\left(\frac{W}{L}\right)_p = 4$

$\left(\frac{W}{L}\right)_n = 1.5$, $\left(\frac{W}{L}\right)_p = 3$

⋮

Possible ratios

→ Appropriate values can be determined due to manufacturing technology.

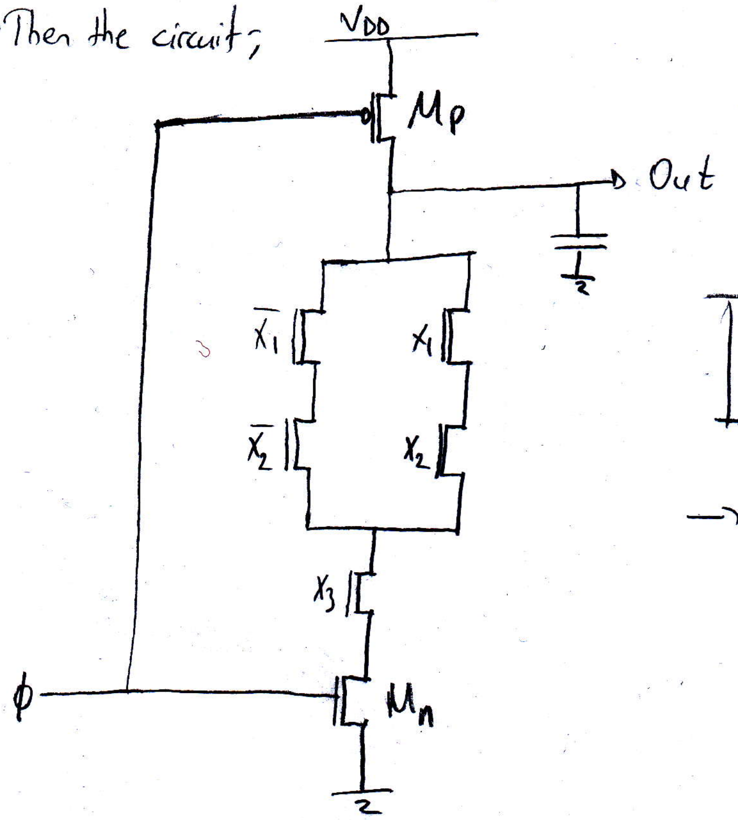
2-) $f = x_1\bar{x}_2 + \bar{x}_1 x_2 + x_3$

→ Converting f to AOI canonical form. (And-or-Invert)

$$f = \overline{\overline{x_1\bar{x}_2 + \bar{x}_1 x_2 + x_3}} = \overline{\overline{x_1\bar{x}_2}\cdot\overline{\bar{x}_1 x_2}\cdot\bar{x}_3} = \overline{(\bar{x}_1+x_2)\cdot(x_1+\bar{x}_2)\cdot\bar{x}_3}$$

$$= \overline{\Big[\underbrace{x_1\cancel{\bar{x}_1}}_{0} + \bar{x}_1\cdot\bar{x}_2 + x_1 x_2 + \underbrace{\cancel{x_2\bar{x}_2}}_{0}\Big]\cdot\bar{x}_3} = \overline{\big[\bar{x}_1\bar{x}_2 + x_1 x_2\big]\cdot\bar{x}_3} \quad ✓$$

→ Then the circuit;



→ Pull-up → $t_{PLH(WC,BC)} = 0{,}69 \cdot R_p \cdot C_{out}$

$$R_p \cdot C_{out} = \frac{24\cdot 10^3}{\left(\frac{W}{L}\right)_p} \cdot 10\cdot 10^{-12} = 80\cdot 10^{-9}$$

$$\boxed{\left(\frac{W}{L}\right)_p = 3}$$

→ Pull-down → worst case

3 series network transistors

1 evaluate transistor

total 4 transistors along the path for Worst Case $1 \to 0$

$$t_{PHL(WC)} = 0{,}69 \cdot 4 \cdot R_n \cdot C_{out} = 0{,}69 \cdot 80\cdot 10^{-9}$$

$$\Rightarrow 4 \cdot R_n \cdot C_{out} = 80 \cdot 10^{-9}$$

$$4 \cdot \frac{12\cdot 10^3}{\left(\frac{W}{L}\right)_n} \cdot 10\cdot 10^{-12} = 80 \cdot 10^{-9}$$

$$\boxed{\left(\frac{W}{L}\right)_n = 6}$$