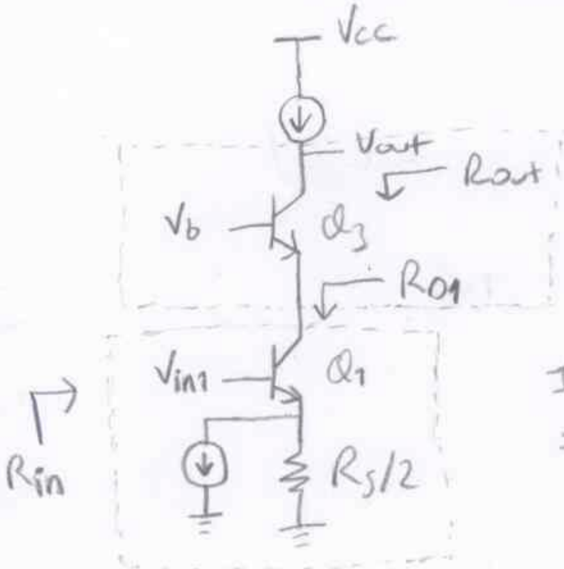
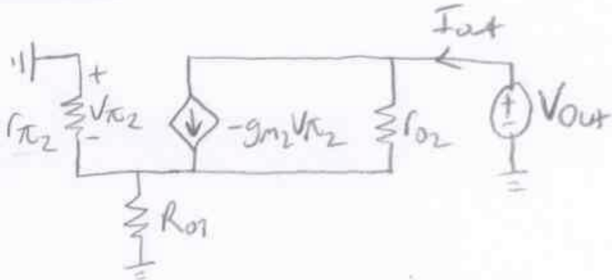
EHB 335E - HOMEWORK I - SOLUTIONS - ALICAN ÇAĞLAR

(1)

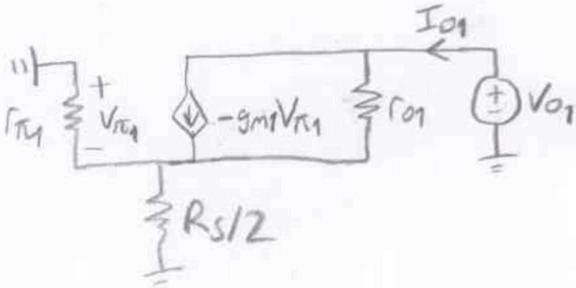
Half circuit of the differential amplifier



I ⟹



II ⟹



Calculating impedance seen looking into Q1's collector, R_{o1}:

$$V_{o1} = (I_{o1} + g_{m1}V_{\pi 1})r_{o1} + I_{o1}(r_{\pi 1} \parallel R_s/2) \qquad V_{\pi 1} = I_{o1}\cdot(R_s/2 \parallel r_{\pi 1})$$

$$\frac{V_{o1}}{I_{o1}} = R_{o1} = g_{m1}(R_s/2 \parallel r_{\pi 1})\cdot r_{o1} + r_{o1} + (r_{\pi 1} \parallel R_s/2)$$

Calculating output impedance, R_{out}:

Small signal equivalent circuit I and II are identical, so their output impedance will be similar:

$$\frac{V_{out}}{I_{out}} = R_{out} = g_{m2}(R_{o1} \parallel r_{\pi 2})\cdot r_{o2} + r_{o2} + (r_{\pi 2} \parallel R_{o1})$$

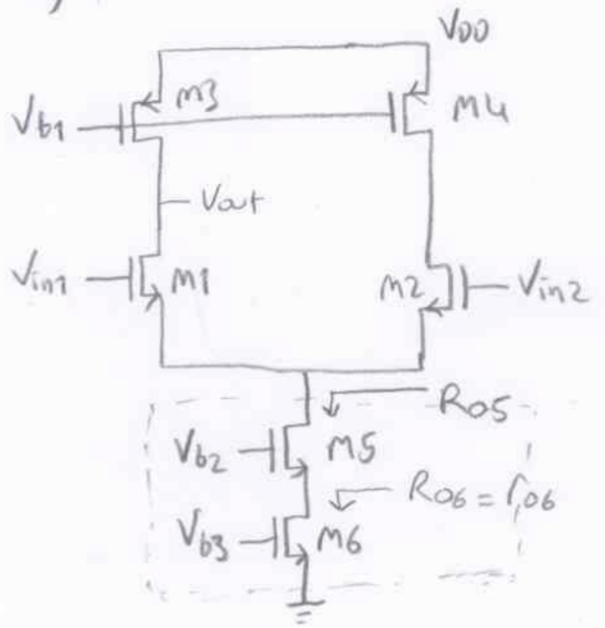
Transconductance of the amplifier, G_m:

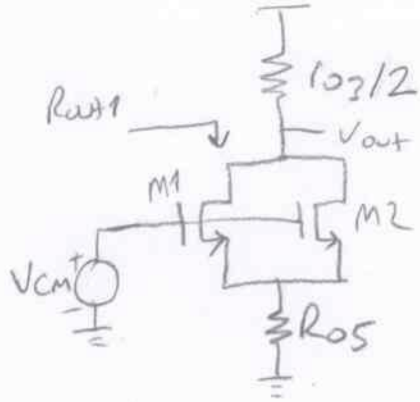
$$G_m = \frac{I_{out}}{V_{in}} = \frac{i_C}{R_{in}\cdot i_B} = \frac{\beta}{R_{in}} \qquad R_{in} = r_{\pi 1} + (\beta+1)R_s/2$$

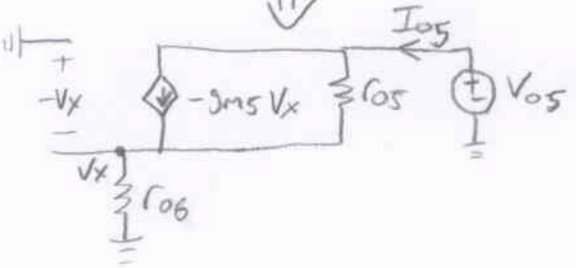
$$\cong \frac{g_{m1}}{1+g_{m1}R_s/2}$$

$$\Rightarrow \boxed{A_v = G_m \cdot R_{out}}$$

② a)





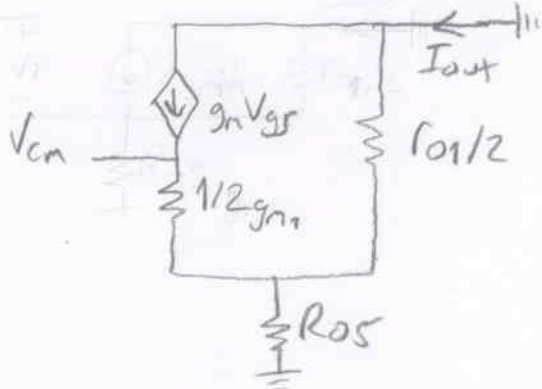


$V_{o5} = (I_{o5} + g_{m5}V_x)r_{o5} + I_{o5}.r_{o6}$

$V_x = I_{o5}.r_{o6}$

$R_{o5} = \frac{V_{o5}}{I_{o5}} = g_{m5}r_{o5}r_{o6} + r_{o5} + r_{o6}$

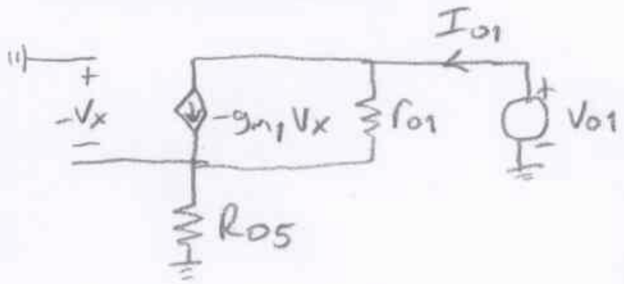
Because of two parallel transistor, transconductance will be $2g_{m1,2}$.



If we assure that $1/2g_{m1} \ll r_{o1}/2$.

$$\Rightarrow G_m = \frac{I_{out}}{V_{cm}} = \frac{1}{1/2g_{m1} + R_{o5}}$$

<u>Calculation of R_{out1}</u>



$V_{o1} = (I_{o1} + g_{m1}V_x)r_{o1} + I_{o1}R_{o5}$

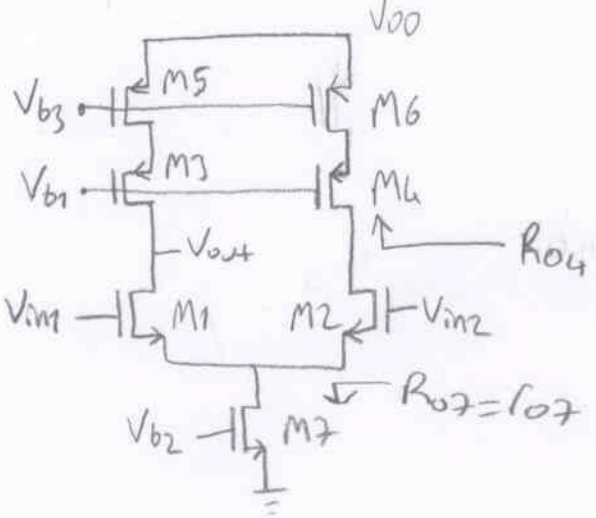
$V_x = I_{o1}.R_{o5}$

$R_{out1} = g_{m1}r_{o1}.R_{o5} + R_{o5} + r_{o1}$

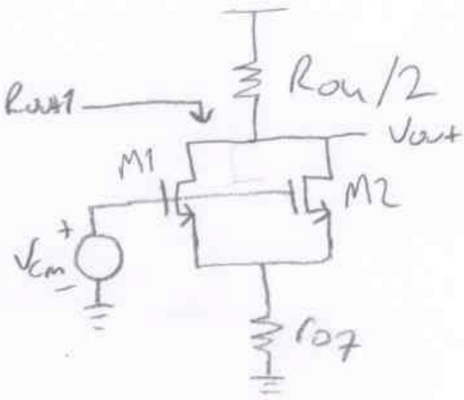
$R_{out} = R_{out1} // r_{o3}/2$

$\boxed{A_{cm} = -G_m.R_{out}}$

② b)



$\Rightarrow$



For R_{o4}, we can use the expression of R_{o5} in Part a).

$$R_{o4} = g_{m4} r_{o4} r_{o6} + r_{o4} + r_{o6}$$

This circuit has similar structure to the differential amplifier in Part a).

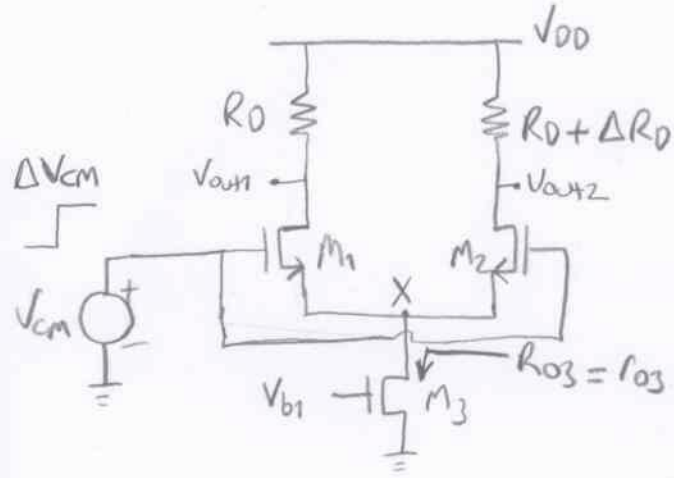
So, $$G_m = \frac{I_{out}}{V_{cm}} = \frac{1}{1/2g_{m1} + r_{o7}}$$

$$R_{out1} = g_{m1} r_{o1} r_{o7} + r_{o1} + r_{o7}$$

$$R_{out} = R_{out1} \parallel R_{o4}/2$$

$$A_{cm} = -G_m \cdot R_{out} = \frac{-R_{out}}{1/2g_{m1} + r_{o7}}$$

③ a)



$$\Delta V_{GS}=\Delta V_{GS1}=\Delta V_{GS2}$$
$$\Delta I_D=\Delta I_{D1}=\Delta I_{D2}$$

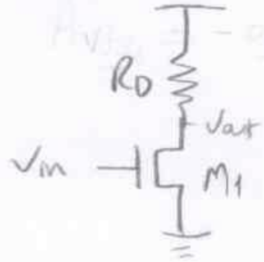
$$\Delta V_{cm}=\Delta V_{GS}+2\Delta I_D.R_{o3}$$
$$=\frac{\Delta I_D}{g_{m1}}+2\Delta I_D.r_{o3}$$

$$\Delta V_{out}=\Delta V_{out1}-\Delta V_{out2}=-\Delta R_D.\Delta I_D$$

$$A_{vcm}=\left|\frac{\Delta V_{out}}{\Delta V_{cm}}\right|=\frac{\Delta R_D}{\frac{1}{g_{m1}}+2r_{o3}}$$

X node appears as virtual grand when differential input signals are applied.

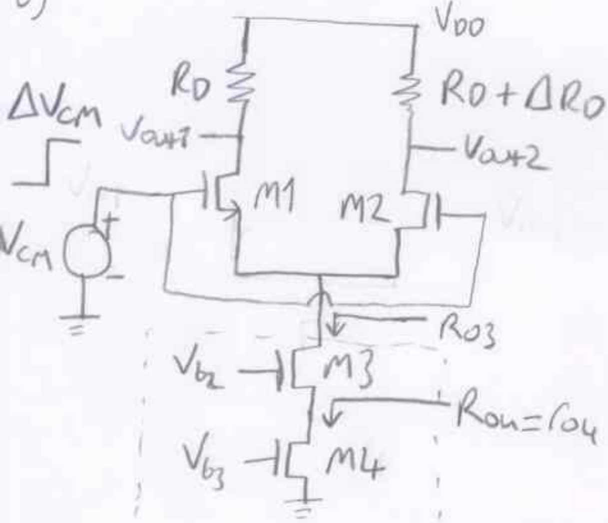
So,



$$\Rightarrow A_{Vdiff}=-g_{m1}R_D$$

$$CMRR=\frac{|A_{vdiff}|}{|A_{vcm}|}=\frac{R_D}{\Delta R_D}.(1+2g_{m1}.r_{o3})$$

b)

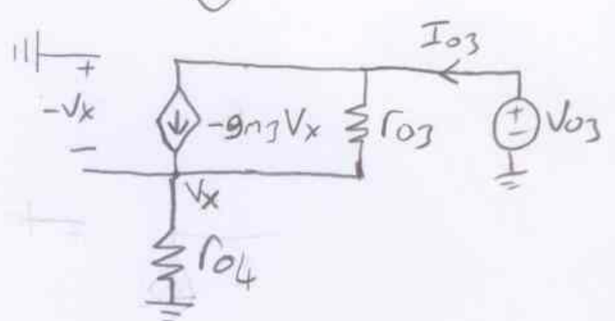


$$\Delta V_{cm}=\Delta V_{GS}+2\Delta I_D R_{o3}=\Delta I_D[1/g_{m1}+2.R_{o3}]$$
$$\Delta V_{out}=-\Delta R_D.\Delta I_D$$
$$A_{vcm}=\left|\frac{\Delta V_{out}}{\Delta V_{cm}}\right|=\frac{\Delta R_D}{1/g_{m1}+2R_{o3}}$$

$$A_{vdiff}=-g_{m1}.R_D$$

$$\Rightarrow CMRR=\left|\frac{A_{vdiff}}{A_{vcm}}\right|=\frac{R_D}{\Delta R_D}.(1+2g_{m1}g_{m3}r_{o3}r_{o4})$$

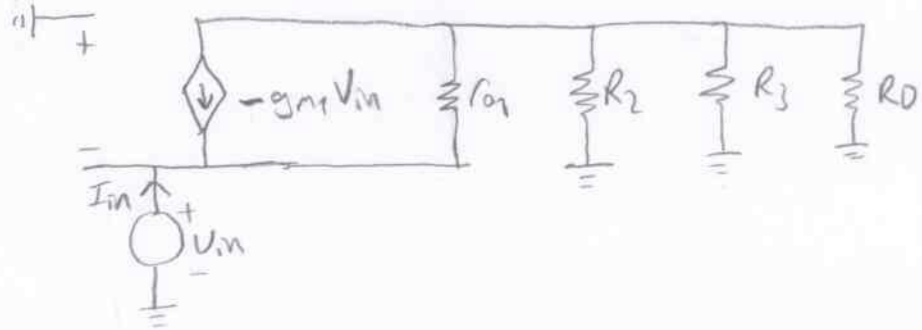
$\Downarrow$



$$V_{o3}=(I_{o3}+g_{m3}V_x)r_{o3}+I_{o3}.r_{o4}$$
$$V_x=I_{o3}.r_{o4}$$

$$R_{o3}=\frac{V_{o3}}{I_{o3}}=g_{m3}.r_{o3}.r_{o4}+r_{o4}+r_{o3}$$
$$\cong g_{m3}.r_{o3}.r_{o4}$$

(4) Calculating input resistance R_{in}

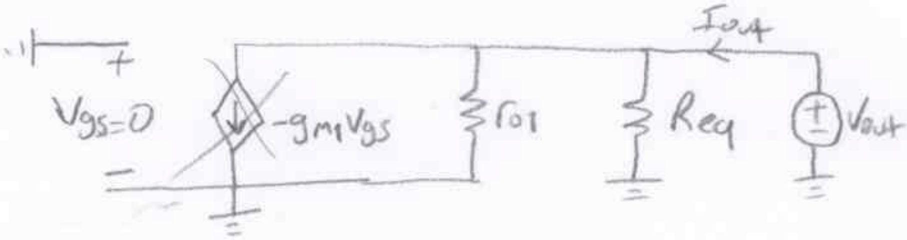


$R_{eq} = R_2 // R_3 // R_D$

$$V_{in} = (I_{in} - g_{m1}V_{in})\, r_{o1} + I_{in}.R_{eq}$$

$$R_{in} = \frac{V_{in}}{I_{in}} = \frac{r_{o1} + R_{eq}}{1 + g_{m1} r_{o1}}$$

Calculation of output resistance, R_{out}

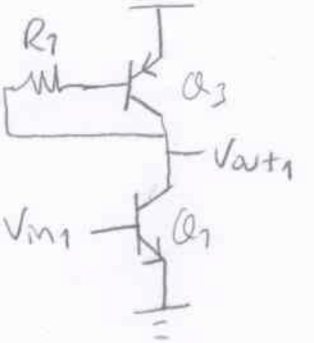


$$R_{out} = \frac{V_{out}}{I_{out}} = r_{o1} // R_{eq} \simeq R_{eq}$$

Equal amount of current flows through input and output.

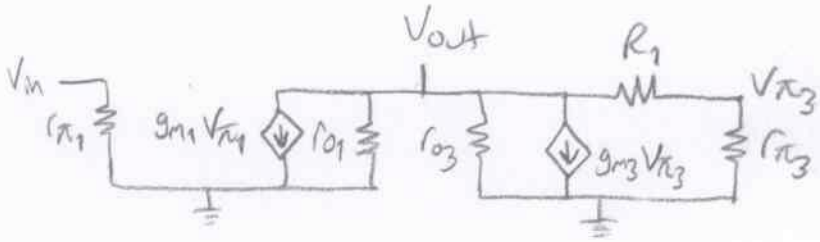
So, $$A_V = \frac{R_{out}}{R_{in}} = \frac{(1 + g_{m1} r_{o1}) R_{eq}}{r_{o1} + R_{eq}}$$

⑤ Half of the amplifier



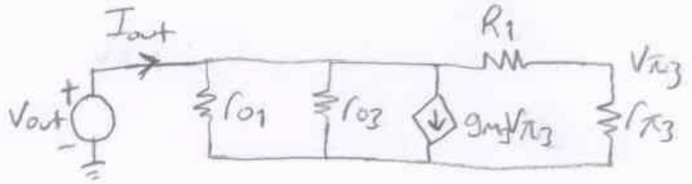
$\Rightarrow$

Small signal equivalent circuit



$\Downarrow$

Calculation of Rout



$$G_m = \frac{I_{out}}{V_{in}} = g_{m1}$$

$$\frac{V_{out}}{V_{\pi 3}} = \frac{R_1 + r_{\pi 3}}{r_{\pi 3}} \Rightarrow V_{\pi 3} = \frac{V_{out}\, r_{\pi 3}}{R_1 + r_{\pi 3}}$$

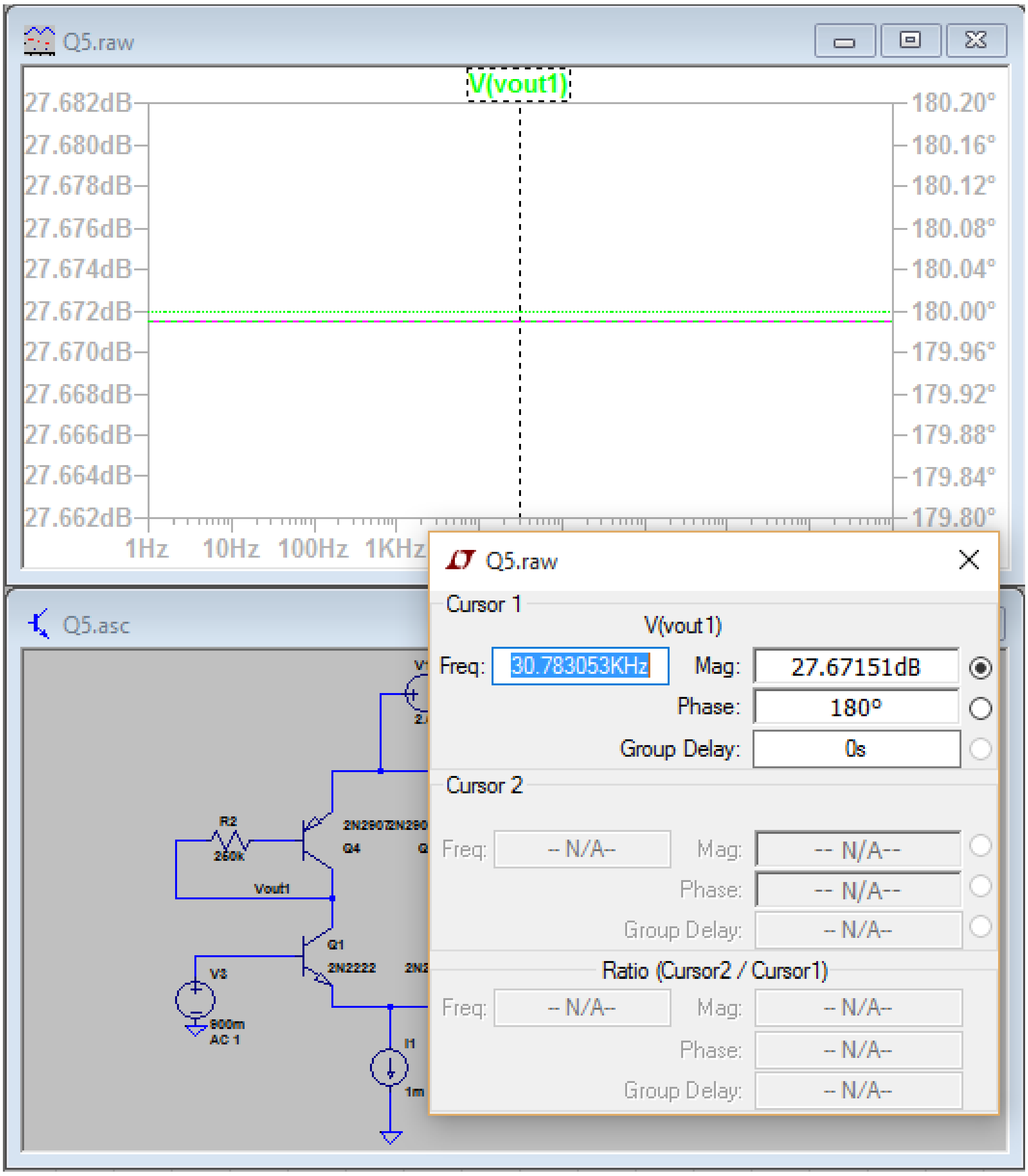
$$(I_{out} - g_{m3} V_{\pi 3})\underbrace{\left(r_{o1} // r_{o3} // (R_1 + r_{\pi 3})\right)}_{R_{eq}} = V_{out}$$

$$\frac{V_{out}}{I_{out}} = R_{out} = \frac{R_{eq}(R_1 + r_{\pi 3})}{g_{m3} r_{\pi 3} R_{eq} + R_1 + r_{\pi 3}} \cong \frac{R_1 + r_{\pi 3}}{g_{m3} r_{\pi 3}}$$

$$A_V = \frac{V_{out}}{V_{in}} = -g_{m1} \cdot \frac{R_1 + r_{\pi 3}}{g_{m3} r_{\pi 3}}$$

Due to constant current, we can't change value of g_{m1}, g_{m3} and $r_{\pi 3}$. However, we can increase R_1 to obtain higher gain. Note that after a certain R_1 value, input transistors will go into saturation region leading to gain drop.

AC simulation of the amplifier



<u>**Transient response of the amplifier**</u>