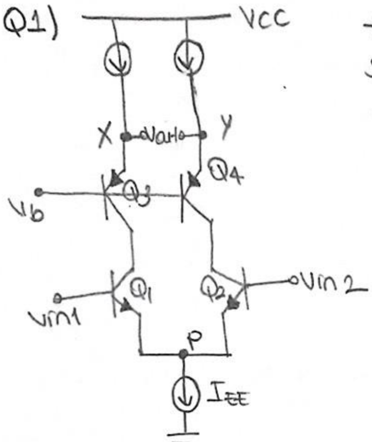
Elif Ding
040170040

EHB 262 HW #6

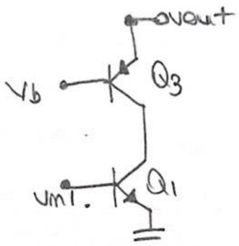
Q1)



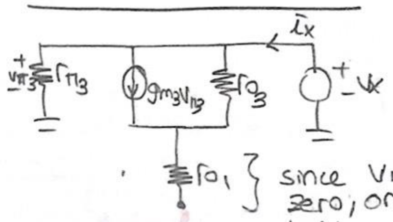
→Node P does not change with time in small signal analysis. Therefore it is AC Ground. So we get two half circuits which are symmetric. Gain is equal to the half-circuits' gains.

$$A_v = -G_m R_{out}$$

(current source become opened)



to calculate Rout ;



} since Vin is zero; only r_{o1} left.

$$\rightarrow V_x = \left(i_x - g_{m3}V_x - \frac{V_x}{r_{\pi 3}}\right) r_{o3} + \left(i_x - \frac{V_x}{r_{\pi 3}}\right) r_{o1}$$

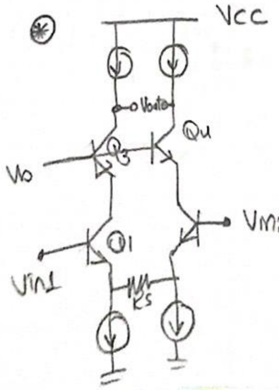
$$\rightarrow \frac{V_x}{i_x} = \frac{r_{o1} + r_{o3}}{1 + g_{m3}r_{o3} + \frac{g_{m3}r_{o3}}{\beta_3} + \frac{g_{m3}r_{o1}}{\beta_3}} = R_{out}$$

[since β is too big denominator is equal to $1 + g_{m3}r_{o3}$ approximetly]

$$\boxed{R_{out} \simeq \frac{r_{o1} + r_{o3}}{1 + g_{m3}r_{o3}}}$$

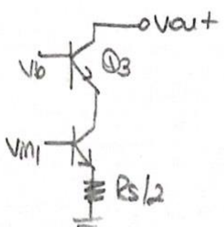
$$G_m = \frac{i_o}{V_{in}} \approx g_{m1} \longrightarrow \boxed{A_v = -g_{m1} \cdot \frac{r_{o1} + r_{o3}}{1 + g_{m3}r_{o3}}} = \frac{V_Y - V_X}{V_{in1} - V_{in2}}$$

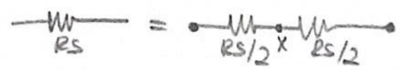
⊛



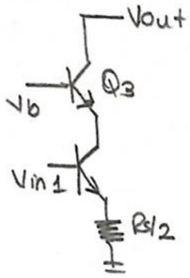
Same logic exists for this diff also.

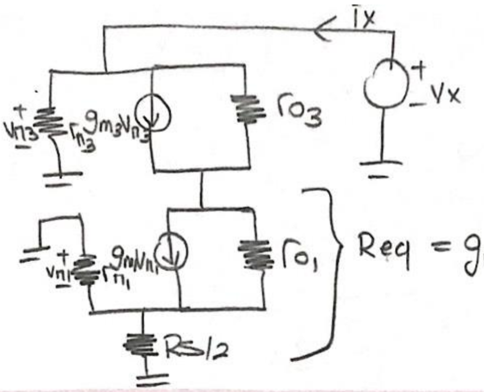
Half circuit ;





can be written in series. X point does not change with time. So it is also AC GND. The point is in symmetry line.





$$\frac{V_x}{i_x} = R_{out}$$

$$R_{eq} = g_{m1} r_{o1} \left(\frac{R_s}{2} // r_{\pi 1} \right) + r_{o1} + \left(\frac{R_s}{2} // r_{\pi 1} \right)$$

$$\boxed{\frac{V_x}{i_x} = g_{m3} r_{o3} \left(R_{eq} // r_{\pi 3} \right) + r_{o3} + \left(R_{eq} // r_{\pi 3} \right) = R_{out}}$$

Gm calculation;

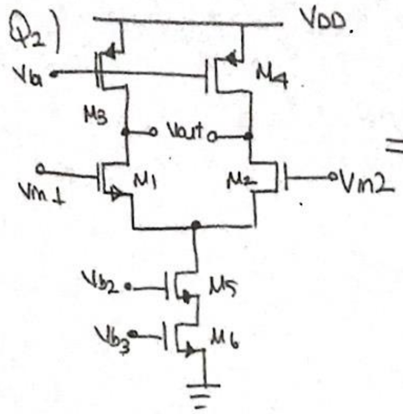
$$V_{\pi 1} \approx \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{R_s}{2}} \cdot v_{in} = \frac{1}{1 + g_{m1}\frac{R_s}{2}} \cdot v_{in}$$

we neglet early effect in Gm calculations for ease

$$G_m = \frac{i_o}{v_{in}} = \frac{g_{m1} \cdot V_\pi}{v_{in}} = \frac{g_{m1}}{1 + g_{m1}\frac{R_s}{2}}$$

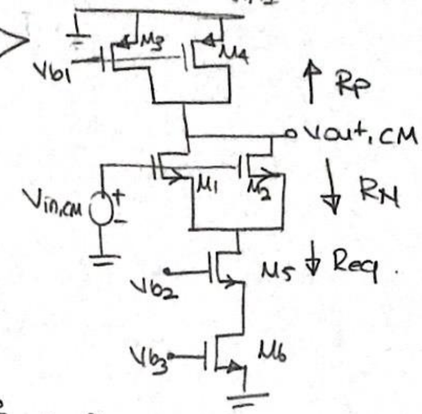
$$\boxed{A_v = \frac{-g_{m1}}{1 + g_{m1}\frac{R_s}{2}} \left[g_{m3} r_{o3} \left(R_{eq} // r_{\pi 3} \right) + r_{o3} + \left(R_{eq} // r_{\pi 3} \right) \right]}$$

$Q_2)$



$A_{CM}? = \frac{V_{out}}{V_{in1}}$

$\Rightarrow$



$R_P = r_{o3} \parallel r_{o4}$ } since they're identical $\Rightarrow \frac{r_{o3}}{2}$

$$R_N = \frac{r_{o1}}{2} + R_{eq} + \cancel{2}g_{m1}\frac{r_{o1}}{\cancel{2}} \cdot R_{eq} \approx g_{m1} r_{o1} R_{eq} \approx g_{m1} r_{o1} g_{m5} r_{o5} r_{o6}$$

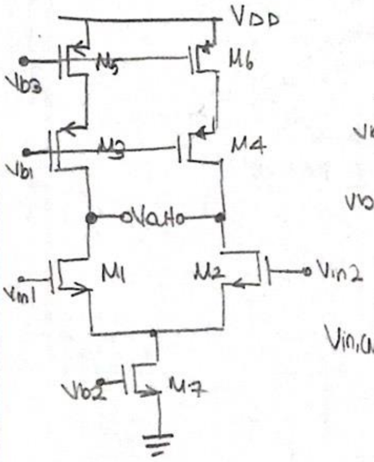
$R_{out} = R_P \parallel R_N = \frac{r_{o3}}{2} \parallel g_{m1} r_{o1} g_{m5} r_{o5} r_{o6}$ } since second term is way too bigger than the first

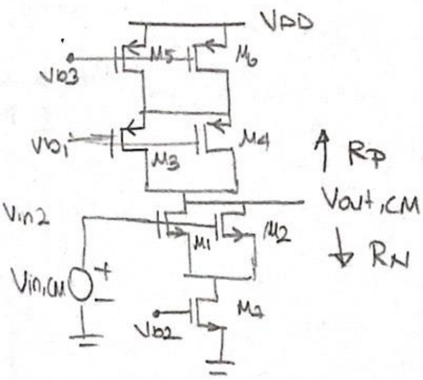
$\boxed{\approx r_{o3}/2}$

Gm calculation

$$G_m = \frac{i_o}{V_{in,CM}} = \frac{2 g_{m1} V_{gs1}}{V_{in,CM}} = \frac{2g_{m1}}{V_{in,CM}} \cdot \frac{\frac{1}{2g_{m1}}}{\frac{1}{2g_{m1}} + R_{eq}} \cdot V_{in,CM} \approx \frac{1}{R_{eq}}$$

$$\rightarrow A_{CM} = -G_m R_{out} = \frac{-r_{o3}}{2R_{eq}} = \boxed{\frac{-r_{o3}}{2 g_{m5} r_{o5} r_{o6}}}$$





$$R_P = 2 g_{m3} \frac{r_{o3}}{2}\frac{r_{o5}}{2} + \frac{r_{o3}}{2} + \frac{r_{o5}}{2} \approx \frac{g_{m3} r_{o3} r_{o5}}{2}$$

$$R_N = 2 g_{m1} \frac{r_{o1}}{2} r_{o7} + \frac{r_{o1}}{2} + r_{o7} \approx g_{m1} r_{o1} r_{o7}$$

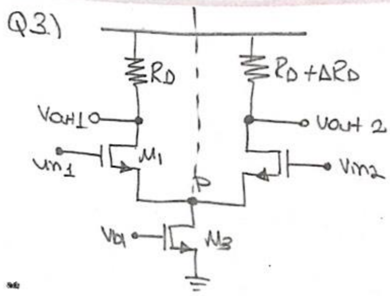
} $R_{out} = R_N \parallel R_P$

Gm calculation

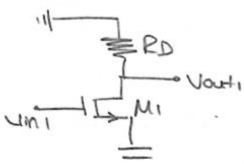
$$G_m = \frac{i_o}{V_{in,CM}} = \frac{2g_{m1}V_{gs1}}{V_{in,CM}} = \frac{\cancel{2g_{m1}}}{\cancel{V_{in,CM}}} \cdot \frac{1}{\frac{\cancel{2g_{m1}}}{\frac{1}{2g_{m1}} + r_{o7}}} \cdot \cancel{V_{in,CM}} \approx \frac{1}{r_{o7}}$$

$$\Rightarrow A_{CM} = -\frac{1}{r_{o7}} \cdot \left(\frac{g_{m3}r_{o3}r_{o5}}{2} \,//\, g_{m1}r_{o1}r_{o7} \right)$$

Q3.)

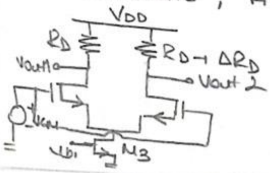


we can use half-circuit to calculate A_{DM}



$$A_{DM} = \frac{V_{out}}{V_{in1}} = -g_{m1} R_D.$$

To calculate, A_{CM-DM} we have;



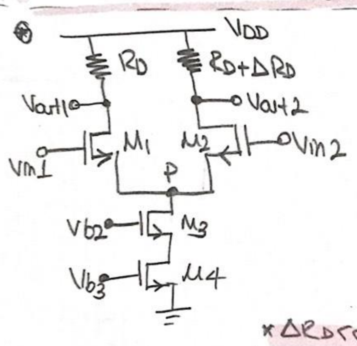
$$A_{CM-DM} = \frac{\Delta V_{out}}{\Delta V_{CM}}$$

$$\Delta V_{CM} = \Delta V_{GS} + 2\Delta I_D r_{o3} \Rightarrow$$

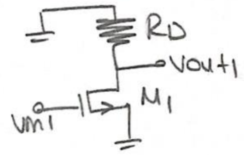
$$\Delta V_{out} = \Delta V_{out1} - \Delta V_{out2} = -\Delta R_D \Delta I_D \Rightarrow$$

$$A_{CM-DM} = \frac{-\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}}$$

$$\boxed{CMMR = \frac{A_{DM}}{A_{CM-DM}} = \frac{g_{m1} R_D}{\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}}} = (1 + 2g_{m1} r_{o3}) \frac{R_D}{\Delta R_D}}$$



$$A_{DM} = \frac{V_{out1}}{V_{in1}} \Rightarrow$$

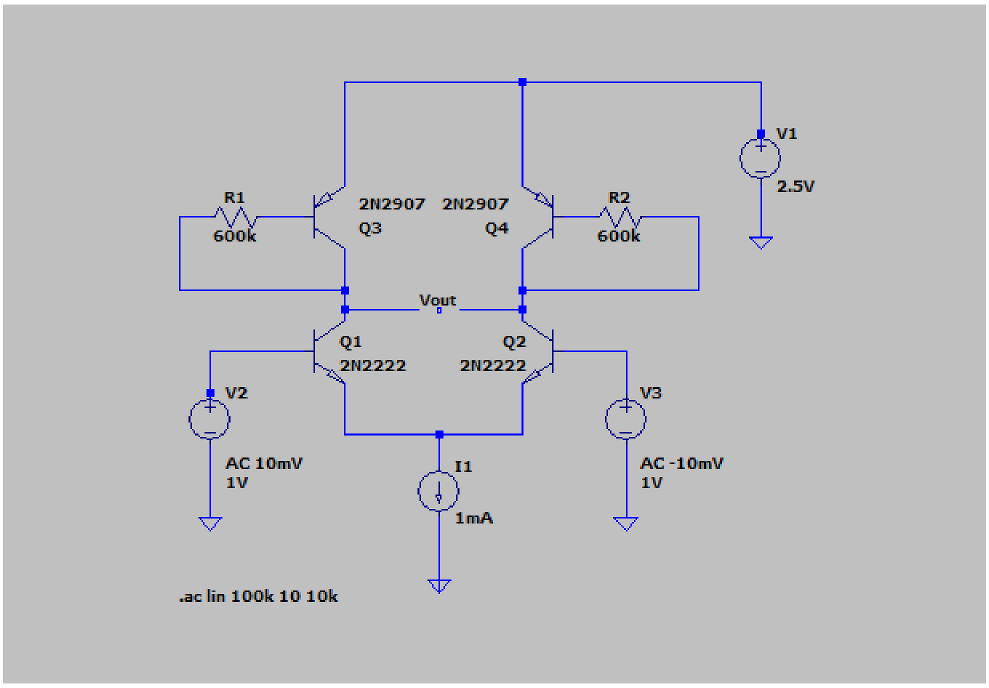


$$A_{DM} = -g_{m1} R_D.$$

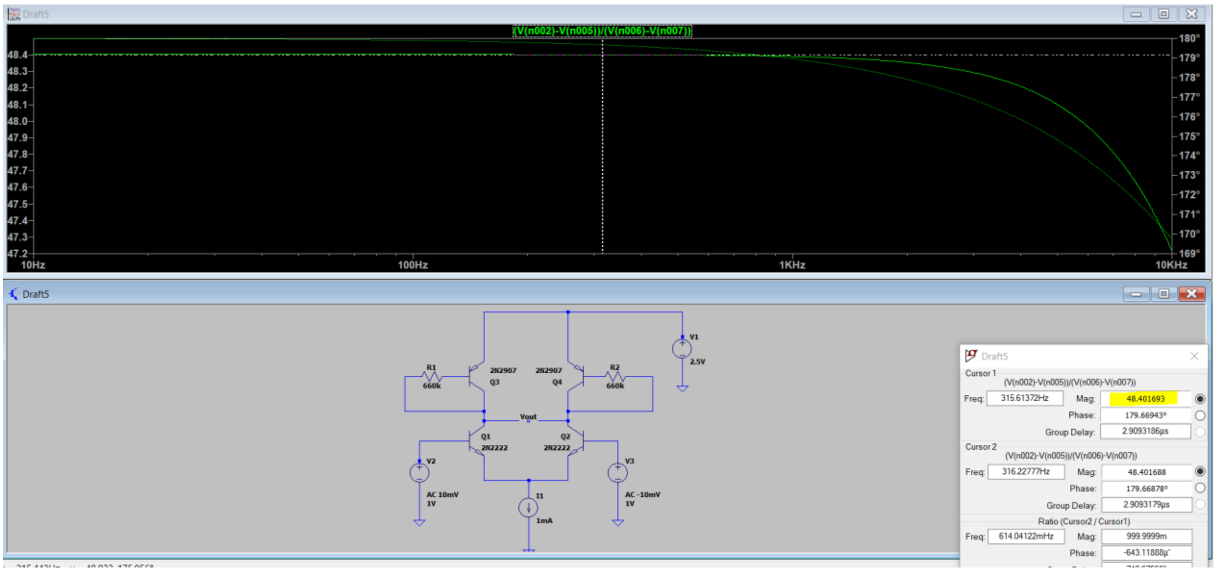
$$A_{CM-DM} = -\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2[g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}]}$$

* $\Delta R_D \ll R_D$ so we could use half circuit since gains are nearly equal.

$$\boxed{CMMR = \frac{A_{DM}}{A_{CM-DM}} = \left(1 + 2g_{m1}[g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}]\right) \frac{R_D}{\Delta R_D}}$$



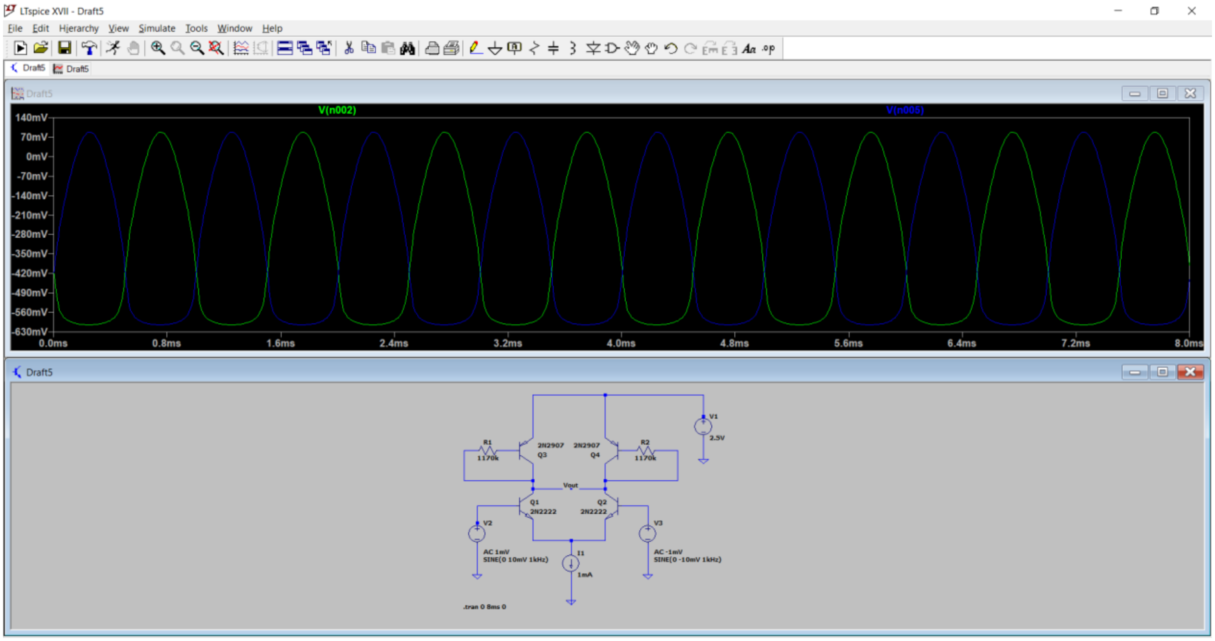
Gain:

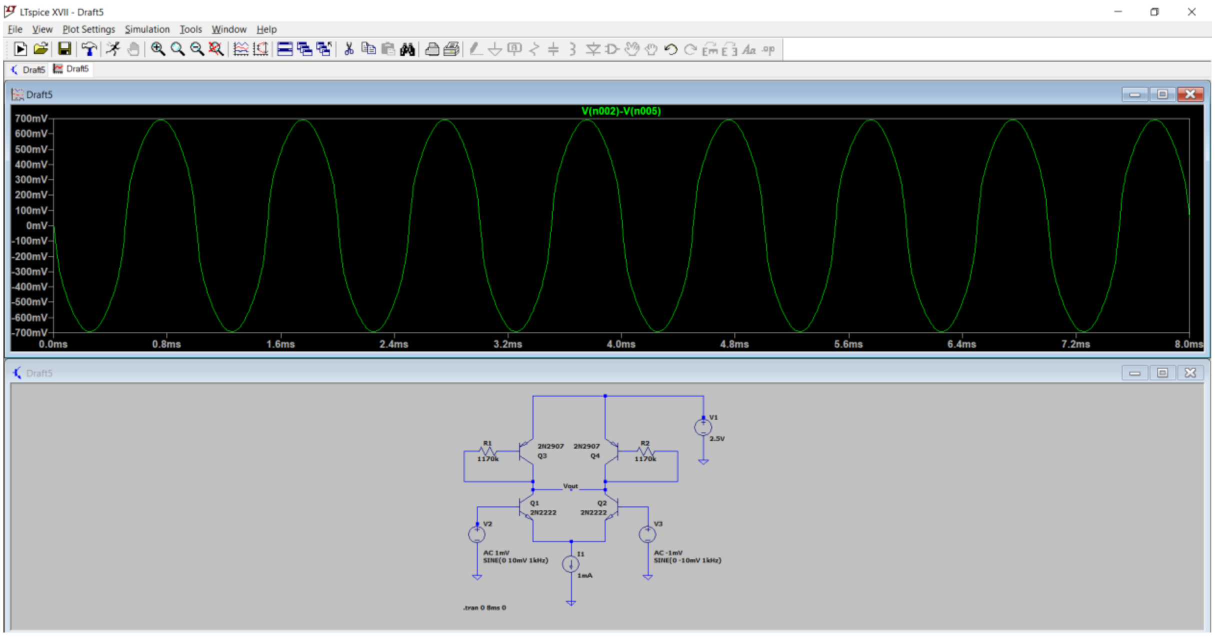


Transient Response:

V(n002)=Vout1

V(n005)=Vout2


LTspice XVII - Draft5
V(n002)
V(n005)
.tran 0 8ms 0


LTspice XVII - Draft5
V(n002)-V(n005)
.tran 0 8ms 0