EHB 351 — 24.12.2020

Analog Haberleşme

(Arasınav Çözümleri)

33P ① $x(t) = 1 + \cos 2\pi t + \cos^3 2\pi t$

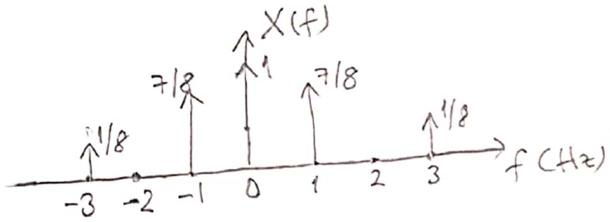
a) $\cos^3 2\pi t = \dfrac{\cos 6\pi t + 3\cos 2\pi t}{4}$ olduğu için,

$$x(t) = 1 + \frac{7}{4}\cos 2\pi t + \frac{1}{4}\cos 6\pi t = 1 + \frac{7}{8}e^{j2\pi t} + \frac{7}{8}e^{-j2\pi t} + \frac{1}{8}e^{j6\pi t} + \frac{1}{8}e^{-j6\pi t}$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \Rightarrow \omega_0 = 2\pi, \quad T = \frac{1}{f_0} = 1 \text{ sn.}$$

$c_0 = 1$, $c_1 = c_{-1} = \frac{7}{8}$, $c_3 = c_{-3} = \frac{1}{8}$. $c_n = 0$, $|n| = 2$ ve $|n| \geq 4$ için

b) $x(t) = \sum_n c_n e^{jn\omega_0 t} \Rightarrow X(f) = \sum_n c_n \delta(f - nf_0) = \delta(f) + \frac{7}{8}\delta(f-1) + \frac{7}{8}\delta(f+1) + \frac{1}{8}\delta(f-3) + \frac{1}{8}\delta(f+3)$



c)

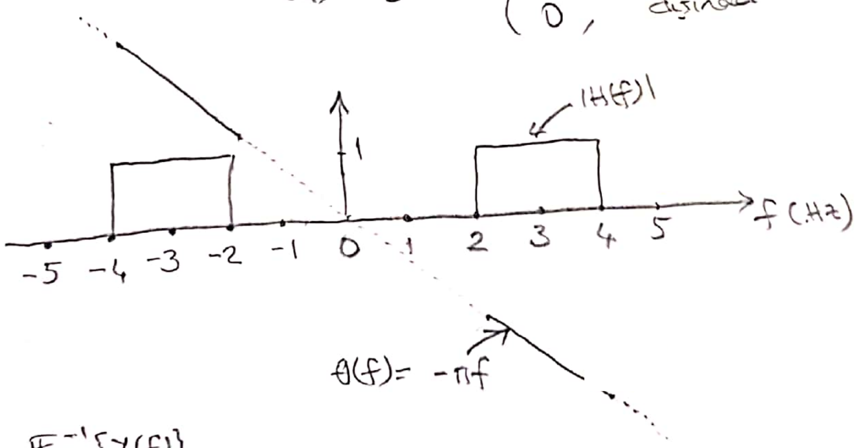
$x(t) \rightarrow \boxed{H(f)} \rightarrow y(t)$

Kanal

$$H(f) = \begin{cases} e^{-j\pi f}, & 2 \leq |f| \leq 4 \text{ Hz} \\ 0, & \text{dışında} \end{cases}$$

$$\Rightarrow H(f) = |H(f)| e^{j\overbrace{\arg H(f)}^{\theta(f)}} \Rightarrow |H(f)| = \begin{cases} 1, & 2 \leq |f| \leq 4 \text{ Hz} \\ 0, & \text{dışında} \end{cases}$$

$$\theta(f) = \arg H(f) = \begin{cases} -\pi f, & 2 \leq |f| \leq 4 \text{ Hz} \\ 0, & \text{dışında} \end{cases}$$



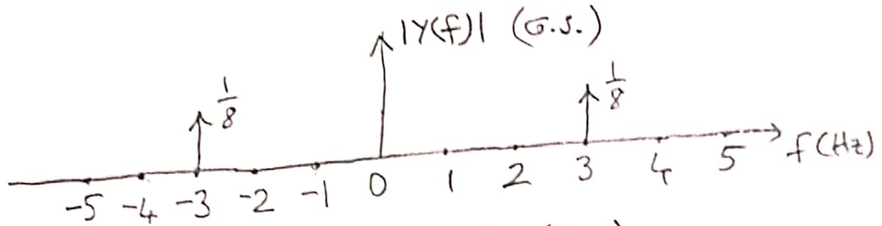
d) $y(t) = \mathcal{F}^{-1}\{Y(f)\}$

$Y(f) = X(f)H(f)$

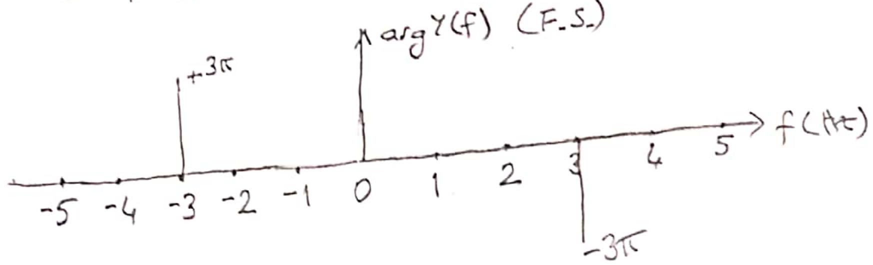
$$= \frac{e^{-j\pi f}}{8}\delta(f-3) + \frac{e^{-j\pi f}}{8}\delta(f+3) = \frac{e^{-j3\pi}}{8}\delta(f-3) + \frac{e^{j3\pi}}{8}\delta(f+3)$$

$$= -\frac{1}{8}\left[\delta(f-3) + \delta(f+3)\right]$$

$$y(t) = -\frac{1}{4}\cos 6\pi t$$



|Y(f)| (G.S.)
1/8
1/8
f (Hz)
-5 -4 -3 -2 -1 0 1 2 3 4 5



arg Y(f) (F.S.)
+3π
-3π
f (Hz)
-5 -4 -3 -2 -1 0 1 2 3 4 5

34P (2) a) $s(t) = \sum_n c_n e^{jnw_c t}$, $w_c = 2\pi f_c = \frac{2\pi}{T}$

$$c_n = \frac{1}{T}\int_T s(t) e^{-jnw_c t} = \frac{1}{T}\int_T s(t)\cos(nw_c t)dt - j\frac{1}{T}\int_T s(t)\sin(nw_c t)dt$$

$$= \underbrace{\langle s(t)\cos(nw_c t)\rangle}_{a_n} - j\underbrace{\langle s(t)\sin(nw_c t)\rangle}_{b_n} = a_n - jb_n$$

Sorulmuyor

$s(t)$ reel ve çift ise $a_n = a_{-n}$, $b_n = 0 \Rightarrow c_n = a_n = a_{-n} = c_{-n}$

c_n reel $\quad c_n$ çift

$$\Rightarrow s(t) = \sum_n c_n e^{jnw_c t} = c_0 + \sum_{n=-\infty}^{-1} c_n e^{jnw_c t} + \sum_{n=1}^{\infty} c_n e^{jnw_c t}$$

$$= c_0 + \sum_{k=1}^{\infty} c_{-k} e^{-jkw_c t} + \sum_{n=1}^{\infty} c_n e^{jnw_c t}$$

$$= c_0 + \sum_{n=1}^{\infty} (c_n e^{jnw_c t} + c_{-n} e^{-jnw_c t})$$

$c_n = c_{-n}$ olduğu için,

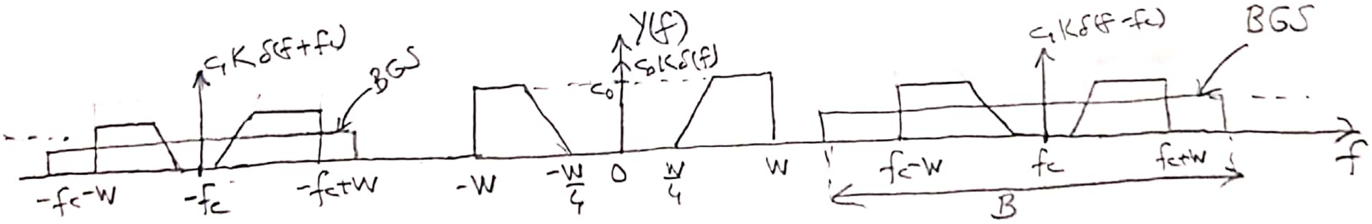
$$s(t) = c_0 + \sum_{n=1}^{\infty} c_n (e^{jnw_c t} + e^{-jnw_c t}) = c_0 + \sum_{n=1}^{\infty} 2c_n \cos(nw_c t)$$

b) $y(t) = x_1(t)s(t) + Ks(t) = (x_1(t)+K)s(t)$

$$= c_0(x_1(t)+K) + 2c_1(x_1(t)+K)\cos w_c t + 2c_2(x_1(t)+K)\cos 2w_c t + \dots$$

c) $Y(f) = c_0(X_1(f) + K\delta(f)) + c_1[X_1(f-f_c) + K\delta(f-f_c)] + c_1[X_1(f+f_c) + K\delta(f+f_c)]$

$$+ c_2[X_1(f-2f_c) + K\delta(f-2f_c)] + c_2[X_1(f+2f_c) + K\delta(f+2f_c)] + \dots$$



d) BGS sadece $2c_1(x_1(t)+K)\cos w_c t$ terimini geçirirse çıkışta klasik GM işaret elde ederiz, BGS'in merkez frekansı f_c olmalı.
Süzgecin kazancı 1 varsayılırsa,

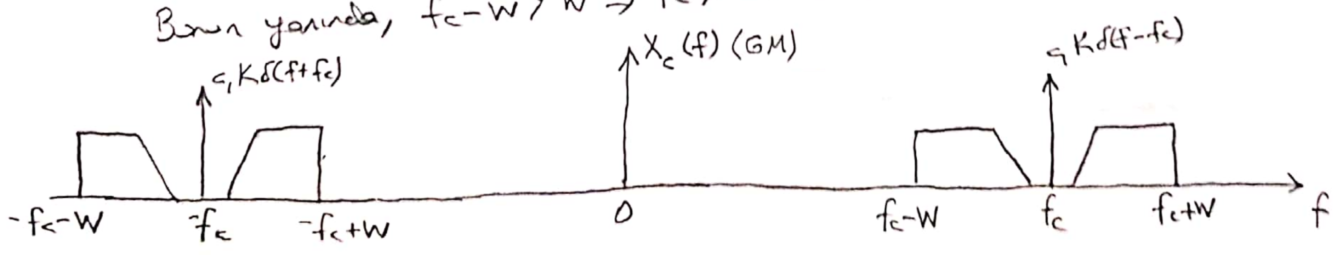
$$x_c(t) = 2c_1(x_1(t)+K)\cos w_c t$$

$$= \underbrace{2c_1 K}_{A_c}\left(1 + \frac{x_1(t)}{K}\right)\cos w_c t$$

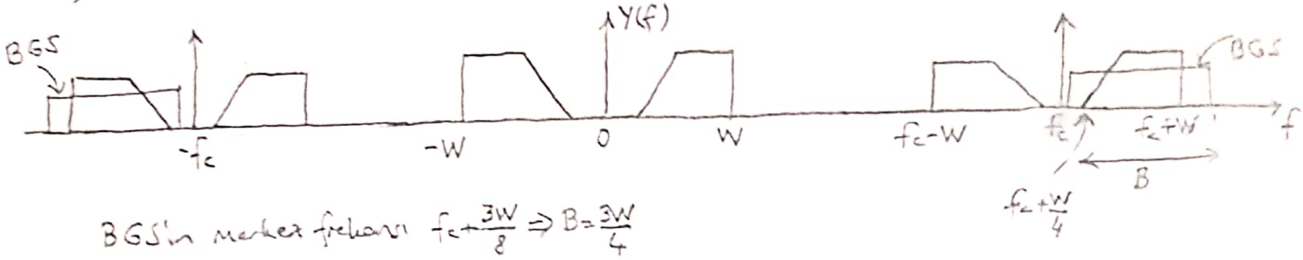
Bu işaret de $x_c(t) = A_c(1 + m x(t))\cos w_c t$ formunda yazılabilir ($\langle x(t)\rangle = 0$, $\max_t |x(t)| = 1$)

BGS'in bant genişliği $B \geq 2W$ olmalı.
Ayrıca, $f_c - \frac{B}{2} \geq W$ olmalı. $\Rightarrow B \leq 2(f_c - W)$ olmalı $\Big\} \Rightarrow 2W \leq B \leq 2(f_c - W)$

Bunun yanında, $f_c - W > W \Rightarrow f_c > 2W$ olmalı.



e)



BGS'in merkez frekansı $f_c+\frac{3W}{8} \Rightarrow B=\frac{3W}{4}$

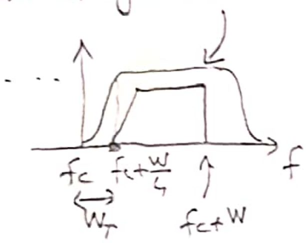
" " " $f_c+\frac{W}{2} \Rightarrow B=2W$

Başka çözümler de var.

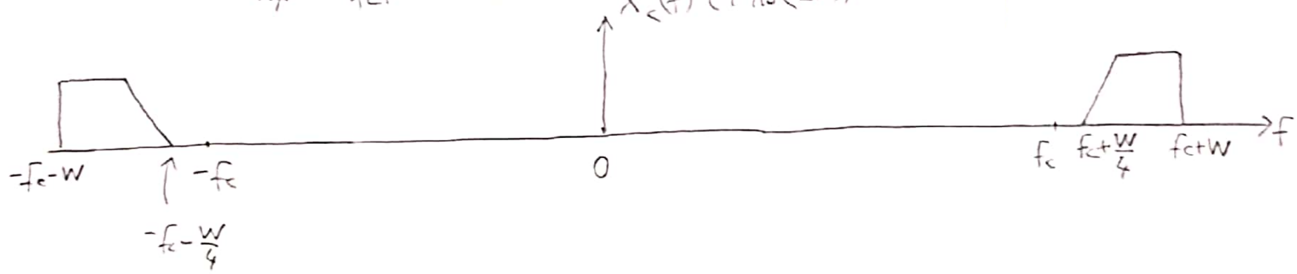
$f_c-W>W \Rightarrow f_c>2W$ olmalı.

$x_c(t) = \frac{A_c}{2}\left[x_1(t)\cos w_c t - \hat{x}_1(t)\sin w_c t\right]$, $\hat{x}_1(t) = x_1(t) * \frac{1}{\pi t}$

İdeal olmayan BGS kullanılırsa,



$\frac{f_c}{100} \leq W_T = \frac{W}{4} \Rightarrow f_c \leq 25W$



5) a) $x_1(t)$: TYB+C (Üst) , $f_{c1} = 90$ kHz

$x_2(t)$: ÇYB , $f_{c2} = 100$ kHz

$x_3(t)$: TYB+C (Alt) , $f_{c3} = 110$ kHz

b) $x_{FDM}(t) = 8\cos 2\pi f_{c1}t + 2x_1(t)\cos 2\pi f_{c1}t - 2\hat{x}_1(t)\sin 2\pi f_{c1}t$

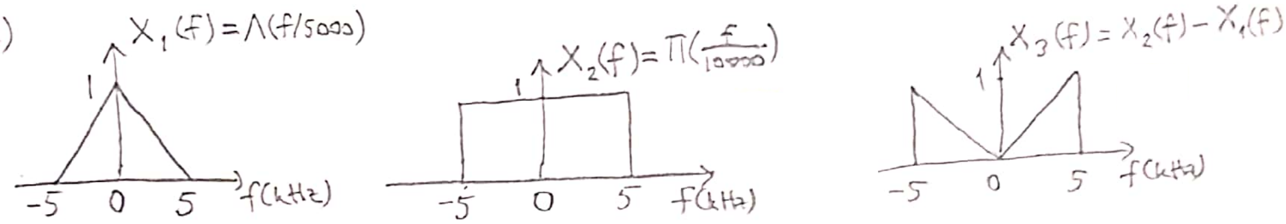
$+ 8x_2(t)\cos 2\pi f_{c2}t$

$+ 16\cos 2\pi f_{c3}t + 6x_3(t)\cos 2\pi f_{c3}t + 6\hat{x}_3(t)\sin 2\pi f_{c3}t$

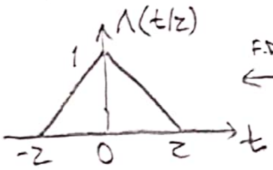
$\hat{x}_1(t) = x_1(t) * \frac{1}{\pi t}$

$\hat{x}_3(t) = x_3(t) * \frac{1}{\pi t}$

c)

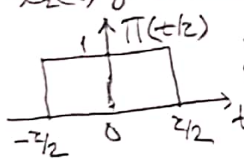


✓ $x_1(t)$'yi bulmak için,



$\xleftrightarrow{F.D.}$ $\tau\,\mathrm{sinc}^2(f\tau)$ dönüşümünü kullanalım (derste gösterilmişti). Dualite teoreminden $\tau\,\mathrm{sinc}^2(t\tau) \xleftrightarrow{F.D.} \Lambda(-f/\tau) = \Lambda(f/\tau)$. Buradan $x_1(t) = 5000\,\mathrm{sinc}^2(5000t)$ bulunur.

✓ $x_2(t)$'yi bulmak için,



$\xleftrightarrow{F.D.}$ $\tau\,\mathrm{sinc}(f\tau)$ dönüşümünü kullanalım (derste gösterilmişti). Dualite teoreminden $\tau\,\mathrm{sinc}(t\tau) \xleftrightarrow{F.D.} \Pi(-f/\tau) = \Pi(f/\tau)$. Buradan $x_2(t) = 10000\,\mathrm{sinc}(10000t)$ bulunur.

✓ $x_3(t)$'yi bulmak için, $X_3(f) = X_2(f) - X_1(f)$ eşitliğini kullanalım. $\Rightarrow x_3(t) = x_2(t) - x_1(t) = 10000\,\mathrm{sinc}(10000t) - 5000\,\mathrm{sinc}^2(5000t)$

d)