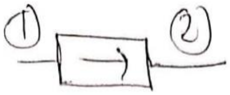
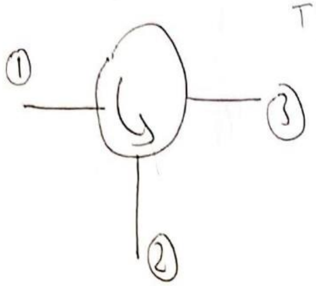
Isolater :

$$S = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$



we obtain $b_2 = a_1$ and when a_2 wave applies the port 2 there is no wave at port 1.

Circulator

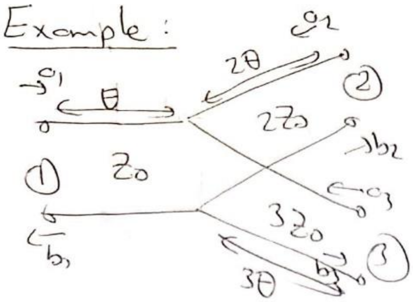


An interesting non-reciprocal 3-port is the circulator. The S-matrix corresponding to the ideal form of a circulator is,

$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad S_{ii} = 0$$

The ideal circulator is lossless $(S^* S = 1)$

Example:



Find the S matrix of this three-port circuit when $\theta = \pi/2$

$$l_1 = l_2 = l_3 = 0$$

$$S_{11} = \Gamma_1 = \left.\frac{b_1}{a_1}\right|_{a_2=a_3=0}$$

$$S_{11} = \frac{(2Z_0 // 3Z_0) - Z_0}{(2Z_0 // 3Z_0) + Z_0} = \frac{1}{11}, \quad S_{22} = \frac{(Z_0 // 3Z_0) - 2Z_0}{(Z_0 // 3Z_0) + 2Z_0} = -5/11$$

$$S_{33} = \Gamma_3 = \frac{(Z_0 // 2Z_0) - 3Z_0}{(Z_0 // 2Z_0) + 3Z_0} = -7/11$$

$$V_i = \sqrt{Z_{0i}}\,(a_i + b_i)$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=a_3=0} \qquad V_{b_1} = V_{a_2} + V_{b_2} \Rightarrow \sqrt{Z_0}\, b_1 = \sqrt{2Z_0}\,(a_2 + b_2)$$

$$S_{12} = \sqrt{2}\,(1 + S_{22}) = \frac{6\sqrt{2}}{11}$$

$$S_{13} = \left.\frac{b_1}{a_3}\right|_{a_1=a_2=0} = \sqrt{3}\,(1 + S_{33}) = \frac{4\sqrt{3}}{11}$$

$$S_{21} = \frac{1}{\sqrt{2}}(1 + S_{11}) = \frac{6\sqrt{2}}{11} = S_{12}, \quad S_{23} = \sqrt{\frac{3}{2}}(1 + S_{33}) = \frac{2\sqrt{6}}{11}$$

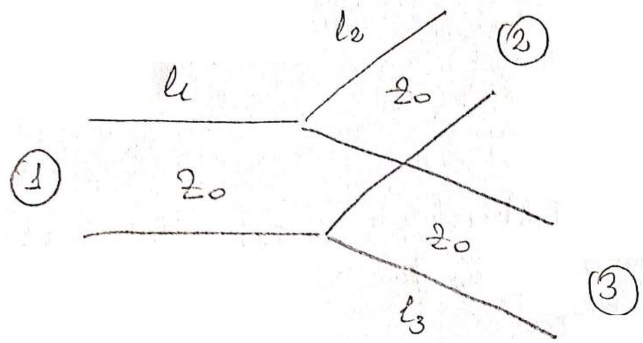
$$S_{31} = \frac{1}{\sqrt{3}}(1 + S_{11}) = \frac{4\sqrt{3}}{11} = S_{13}, \quad S_{32} = \sqrt{\frac{2}{3}}(1 + S_{22}) = \frac{2\sqrt{6}}{11} = S_{23}$$

$$[S] = \frac{1}{11}\begin{bmatrix} 1 & 6\sqrt{2} & 4\sqrt{3} \\ 6\sqrt{2} & -5 & 2\sqrt{6} \\ 4\sqrt{3} & 2\sqrt{6} & -7 \end{bmatrix} \Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

$$l_1 \neq 0, \; l_2 \neq 0, \; l_3 \neq 0, \quad \beta l_1 = \frac{\pi}{2}, \quad \beta l_2 = \pi, \quad \beta l_3 = \frac{3\pi}{2}$$

$$S_{ij}' = S_{ij}\, e^{-j\beta(l_i + l_j)}$$

$$\Rightarrow S' = \frac{1}{11}\begin{bmatrix} -1 & j6\sqrt{2} & 4\sqrt{3} \\ j6\sqrt{2} & -5 & -j2\sqrt{6} \\ 4\sqrt{3} & -j2\sqrt{6} & 7 \end{bmatrix}$$



$l_1 = \frac{\lambda}{2}$

$l_2 = \frac{\lambda}{6}$

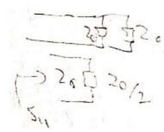
$l_3 = \frac{\lambda}{4}$

$l_1 = l_2 = l_3 = 0$ tam simetrik dır

diğer ikisi kapı sonlandırılırsa

$$\begin{bmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{11} & S_{12} \\ S_{12} & S_{12} & S_{11} \end{bmatrix}$$

$$S_{11} = \rho_1 = -\frac{1}{3} = 0.33 \angle \pi$$



$$2|S_{12}|^2 + |S_{11}|^2 = 1$$

$$|S_{12}| = \frac{2}{3}$$

$$S_{12}^*(S_{11} + S_{12}) = -S_{12}S_{11}^* \qquad S_{12} = 0.66 \angle 0$$

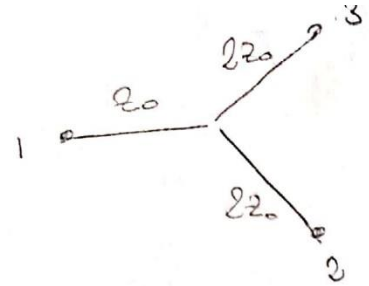
$$S'_{ij} = S_{ij}\, e^{-j(\beta l_i + \beta l_j)} \qquad \beta l_1 = \pi \qquad \beta l_2 = \frac{\pi}{3} \qquad \beta l_3 = \frac{\pi}{2}$$

$$S'_{11} = 0.33\, e^{-j(\pi+\pi+\pi)} = 0.33\, e^{-j\pi}$$

$$S'_{12} = 0.66\, e^{-j(0+\pi+\pi/3)} = 0.66\, e^{-j\frac{4\pi}{3}}$$

$$S'_{13} = 0.66\, e^{-j(0+\pi+\frac{\pi}{2})} = 0.66\, e^{-j\frac{3\pi}{2}}$$

$$\frac{1}{3}\begin{bmatrix} (1 \angle -\pi) & (2 \angle -4\pi/3) & (2 \angle -3\pi/2) \\ (2 \angle -4\pi/3) & (1 \angle -2\pi/3) & (2 \angle -5\pi/6) \\ (2 \angle -3\pi/2) & (2 \angle -5\pi/6) & (1 \angle 0) \end{bmatrix}$$



$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{22} & S_{23} \\ S_{12} & S_{23} & S_{22} \end{bmatrix}$$

$S_{11} = 0 \qquad 2|S_{12}|^2 = 1 \qquad |S_{12}| = \frac{1}{\sqrt{2}}$

S_{22} için $\qquad 2Z_0 \,//\, Z_0 \rightarrow \frac{2Z_0}{3}$

$$\rho_2 = \rho_3 = \frac{\frac{2Z_0}{3} - 2Z_0}{\frac{2Z_0}{3} + 2Z_0} = -\frac{1}{2}$$

$|S_{12}|^2 + |S_{23}|^2 + |S_{22}|^2 = 1$

$\frac{1}{2} + |S_{23}|^2 + \frac{1}{4} = 1 \qquad |S_{23}| = \frac{1}{2}$

$[S^*]^T[S] = [I]$ kayıpsız

$S_{11} S_{21}^* + S_{12} S_{22}^* + S_{13} S_{23}^* = 0$

$S_{22}^* + S_{23}^* = 0 \qquad S_{23} = \frac{1}{2} \qquad S_{12} = \frac{1}{\sqrt{2}}$

$$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/2 & 1/2 \\ 1/\sqrt{2} & 1/2 & -1/2 \end{bmatrix}$$