EHB 335E ~~HW~~ #5 Solutions

(1)

$$H(j) = \frac{3\times10^4}{\left(1+\frac{jf}{2\times10^5}\right)\left(1+\frac{jf}{3\times10^5}\right)\left(1+\frac{jf}{2\times10^6}\right)}$$

$$\phi = -\left[\arctan\left(\frac{f}{2\times10^5}\right) + \arctan\left(\frac{f}{3\times10^5}\right) + \arctan\left(\frac{f}{2\times10^6}\right)\right]$$

ϕ should be $-135°$ for a $45°$ phase margin

Trial & error: $f = 4\times10^5 \Rightarrow \phi = -63.4° - 53.1° - 11.3° = -127.8°$

$f = 4.2\times10^5 \Rightarrow \phi = -64.5° - 54.5° - 11.9° = -131°$

$f = 4.4\times10^5 \Rightarrow \phi = -65.56° - 55.7° - 12.4° = -133.7°$

$f = 4.5\times10^5 \Rightarrow \phi = -66.1° - 56.3° - 12.7° = -135.1°$

So, $f = 4.5\times10^5$ Hz $= 450$ kHz

$$|H(j450\,\text{Hz})| = \left|\frac{3\times10^4}{\left|1+\frac{j\,4.5\times10^5}{2\times10^5}\right|\left|1+\frac{j\,4.5\times10^5}{3\times10^5}\right|\left|1+\frac{j\,4.5\times10^5}{2\times10^6}\right|}\right| = \frac{3\times10^4}{2.46\times1.8\times1.025}$$

$= 6610 = 6.61\times10^3$

$\Rightarrow$ open loop gain

$|1+A\beta| = 0$ $\quad \beta = \left|\frac{1}{H(j450\,\text{Hz})}\right| = 1.51\times10^{-4}$

At the phase margin:

$1 = |A(j\omega)\beta|$

Closed loop gain $\Rightarrow \dfrac{A_o}{1+KA_o} = \dfrac{3\times10^4}{1+1.51\times10^{-4}\cdot3\times10^4} = 5425 = 74.7$ dB

(set $f=0$)

Q2

$$H(s) = \frac{A_0}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)} \qquad \omega_1 = 2\pi \times 10^4 \quad A_0 = 10^4$$
$$\omega_2 = 2\pi \times 10^5$$

Closed loop
$$H_{CL}(s) = \frac{H(s)}{1+KH(s)} = \frac{A_0}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right) + KA_0}$$

Denominator:
$$1 + s\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right) + \frac{s^2}{\omega_1\omega_2} + KA_0 = 0$$

$$\Delta = b^2 - 4ac = 0 \qquad \left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)^2 - \frac{4(1+KA_0)}{\omega_1\omega_2} = 0 \Rightarrow \left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)^2 \frac{\omega_1\omega_2}{4} - 1 = KA_0$$

for coincident poles

$$K = \frac{1}{A_0}\left[\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)^2 \frac{\omega_1\omega_2}{4} - 1\right] = 2.025 \times 10^{-4}$$

$$f_1 = \left|-\frac{b}{2a}\right| = \left|\frac{\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)}{2\left(\frac{1}{\omega_1}\cdot\frac{1}{\omega_2}\right)}\right| = 5.5 \times 10^4 \text{ Hz}$$

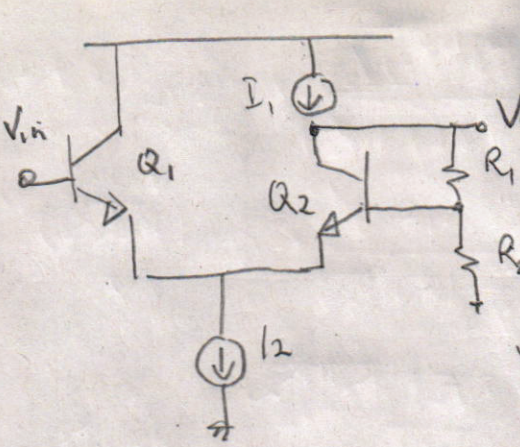
b) Set $s=0$ $\quad H_{CL}(s) = \frac{A_0}{1+KA_0} = \frac{10^4}{1+10^4 \cdot 2.025\times10^{-4}} = 3305.8$
$= 70.4\text{ dB}$

Set $s = 5.5\times10^4 \cdot 2\pi$

$$|H_{CL}(s)| = \frac{10^4}{\left(1+j\frac{5.5\times10^4}{10^4}\right)\left(1+j\frac{5.5\times10^4}{10^5}\right)+KA_0} = \frac{10^4}{\sqrt{\left(1+KA_0-\frac{(5.5\times10^4)^2}{10^4\cdot10^5}\right)^2 + \left(\frac{5.5\times10^4}{10^4}+\frac{5.5\times10^4}{10^5}\right)^2}}$$

$H_{CL}(s) = 1652.9 = 64.4\text{ dB}$

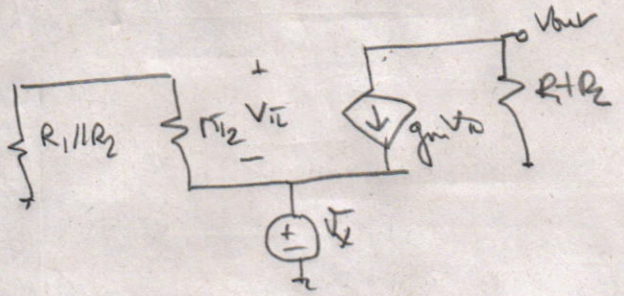
Q3)

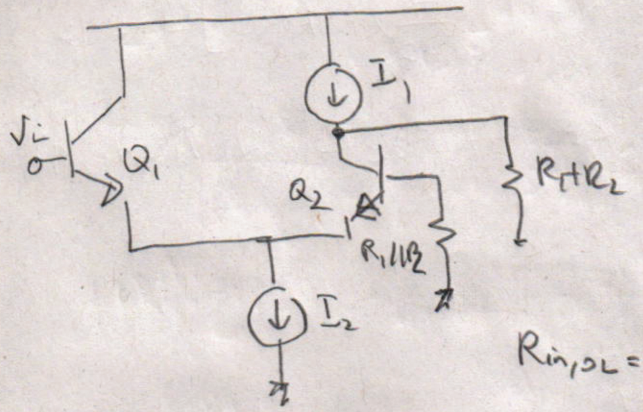


$$A_{v,OL} = \frac{V_x}{V_{in}} \cdot \frac{V_{out}}{V_x}$$

$$\frac{V_x}{V_{in}} = A_1 = \frac{r_{\pi2} + (R_1 \| R_2)}{r_{\pi1} + r_{\pi2} + (R_1 \| R_2)}$$

$$\frac{V_{out}}{V_x} \Rightarrow A_{v2}$$





$$\frac{V_{out}}{V_x} = \frac{r_{\pi2}}{r_{\pi2} + R_1 \| R_2} \times g_{m2}(R_1+R_2) = \frac{g_{m2}(R_1+R_2)}{1 + \frac{R_1 \| R_2}{r_{\pi2}}} = A_{v2}$$

$R_{in,OL} = r_{\pi1} + r_{\pi2} + R_1 \| R_2$ $\quad R_{out,OL} = R_1 + R_2$ $\quad K = \frac{R_2}{R_1+R_2}$

$A_{v,CL} \Rightarrow$ note that the feedback is solely applied on Q_2 not including Q_1

$$A_{v,CL2} = \frac{A_{v2}}{1 + K A_{v2}} = \frac{g_{m2}(R_1+R_2)}{1 + \frac{R_1 \| R_2}{r_{\pi2}} + \frac{R_2}{R_1+R_2} g_{m2}(R_1+R_2)}$$

$$R_{in,CL2} = (1 + K A_{v2}) \left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta+1} \right) \qquad R_{out,CL2} = R_{out,CL} = \frac{R_1+R_2}{1 + K A_{v2}}$$

$$R_{in,CL} = r_{\pi1} + (\beta+1) R_{in,CL2} = r_{\pi1} + (1 + K A_{v2})(r_{\pi2} + R_1 \| R_2)$$

$$A_{v,CL} = \left. \frac{V_x}{V_{in}} \right|_{CL} \times A_{v,CL2} = \frac{(r_{\pi2} + R_1 \| R_2)(1 + K A_{v2})}{\left[r_{\pi1} + (1 + K A_{v2})(r_{\pi2} + R_1 \| R_2) \right]} \times \frac{g_{m2}(R_1+R_2)}{\left[1 + \frac{R_1 \| R_2}{r_{\pi2}} + \frac{R_2 \, g_{m2}(R_1+R_2)}{R_1+R_2} \right]}$$