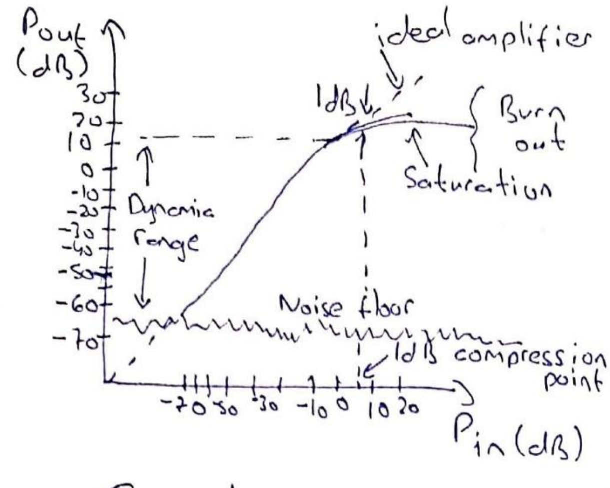
Noise in Microwave Systems

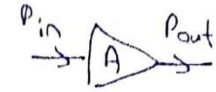
Noise can be passed into a microwave system from external sources or generated within the system itself. The noise level of a system sets the lower limit on the strength of a signal that can be detected. It is generally desired to minimize the noise level of a receiver to achieve the best performance.

Dynamic range and sources of noise

For a component, there is a range of signal levels over which deterministic and linear assumptions are valid; this range is called the dynamic range of the component.

(Linear: the output is directly proportional to the input
Deterministic: the output is predictable from the input)





Consider a microwave transistor amplifier having a gain of K dB, as shown in Figure. At very low input power levels, the output will be dominated by the noise of the amplifier. This level is often called the noise floor of the component or system. Above the noise floor, the amplifier has a range of input powers for which $P_{out} = K P_{in}$ is closely approximated. This is the usable dynamic range of the component.

At the upper end of the dynamic range, the output begins to saturate, meaning that the output power no longer increases linearly as the input power increases. 1 dB compression point is defined as the input power level for which the output is 1 dB below that of the ideal amplifier. If the input power is excessive, the amplifier can be destroyed.

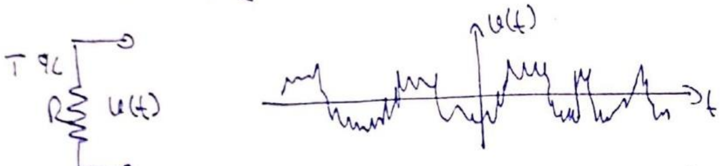
Here, the dynamic range is defined as the difference of the output powers which are the output power corresponding to the 1 dB compression point and the output power corresponding to the noise floor.

Noise is usually generated by the random motions of charges or charge carriers in devices and materials. There are several sources of noise:

- Thermal noise: (Johnson or Nyquist noise) Caused by thermal vibration of bound charges.
- Shot noise: Caused by the random fluctuations of charge carriers in an electron tube o solid-state device.
- Flicker noise: ($1/f$ noise) Occurs in solid-state components and vacuum tubes and varies inversely with frequency.
- Plasma noise: Caused by random motion of charges in an ionized gas.
- Quantum noise: Results from the quantized nature of charge carriers and photons.

Noise power and equivalent noise temperature

Consider a resistor at a temperature of T (°K) as depicted in the figure. The electrons in this resistor are in random



motions that produce small, random voltage fluctuations that is proportional to the temperature at the resistor terminals. This voltage has a value given by Planck's black body radiation law,

$$v_n = \sqrt{\frac{4hfBR}{e^{hf/kT}-1}}$$

$h = 6{,}546 \cdot 10^{-34}$ J-sec : Planck's constant
$k = 1{,}380 \cdot 10^{-23}$ J/°K : Boltzmann's "
T is the temperature (°K)
B is the bandwidth of the system (Hz)
f is the center frequency of the bandwidth (Hz)
R is the resistance (Ω)

At microwave frequencies this result can be simplified by making use of the fact that $hf \ll kT$. Using the first two terms of a Taylor series for the exponential gives,

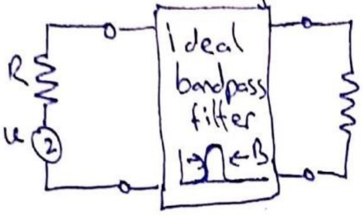
$$e^{hf/kT} - 1 \simeq \frac{hf}{kT}$$

and the above result reduces to

$$v_n = \sqrt{4kTBR}$$

This is the Rayleigh-Jeans approximation and valid in microwave region. Such a noise has a power spectral density which is independent of frequency is referred to as a white noise.

The noisy resistor can be replaced with a Thevenin equivalent circuit consisting of a noiseless resistor and a generator with a voltage v. Connecting a load resistor R results in



maximum power transfer from the noisy resistor. The power delivered to the load is then,

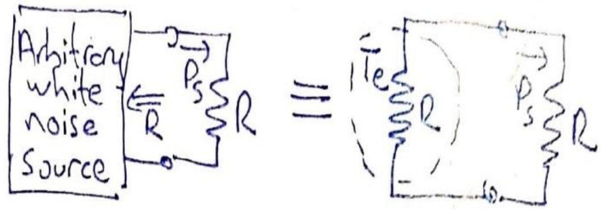
$$P_n = \left(\frac{v_n}{2R}\right)^2 R = \frac{v_n^2}{4R} = \underline{kTB}$$

- As $B \xrightarrow{\text{goes}} 0$, $P_n \to 0$ (systems with smaller B collect less noise power)
- As $T \to 0$, $P_n \to 0$ (cooler devices generate less noise power)
- As $B \to \infty$, $P_n \to \infty$ does not occur in reality because the above app. is not valid as $f \to \infty$. The first form for v_n must be used in this case.

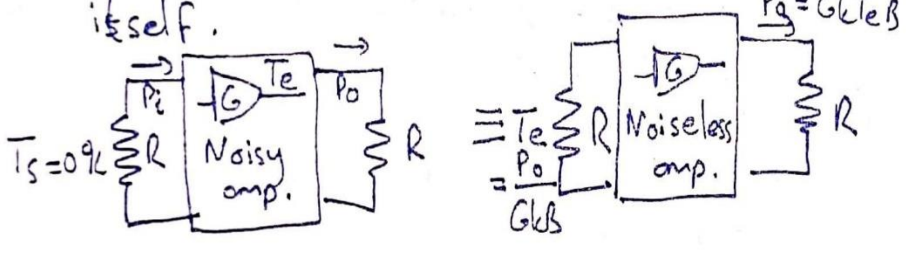
If an arbitrary source of noise is "white", it can be modeled as an equivalent thermal noise source, such as a noisy resistor R, and characterized with temperature T_e where T_e is an equivalent noise temperature.

T_e is selected so that the same noise power is delivered to the load, that is,

$$T_e = \frac{P_s}{kB}$$



Consider a noisy amplifier with a bandwidth B and gain G. Let the amplifier be matched to noiseless source with temperature of $T_s = 0°K$, then the P_i to the amplifier will be zero and the P_o will be due only to the noise generated by the amplifier itself.



We can obtain the same P_o by driving an ideal noiseless amplifier with a resistor of temperature T_e,

$$T_e = \frac{P_o}{GkB}$$

so that the P_o in both cases is GkT_eB. Here, T_e is the equivalent noise temperature of the amplifier.

In practice, the 0 °K source temperature cannot be achieved then the Y-factor method can be applied to determine the T_e of a component. Let the amplifier be matched to two loads (sources) of temperatures T_1 and T_2 ($T_1 > T_2$). Output powers will be,

$$P_1 = GkT_1B + GkT_eB$$
$$P_2 = GkT_2B + GkT_eB$$

Define the Y-factor as

$$Y = \frac{P_1}{P_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1$$

which is determined via the power measurements. Then T_e will be,

$$T_e = \frac{T_1 - YT_2}{Y - 1}$$

Example:

For an X-band amplifier, $G = 20$ dB and $B = 1$ GHz. The following data is obtained: $T_1 = 290$ °K $\rightarrow P_1 = -62.0$ dBm
$T_2 = 77$ K $\rightarrow P_2 = -64.7$ dBm

Determine the T_e of the amplifier. If the source has $T_s = 450$°K, $P_o = ?$

$$Y = (P_1 - P_2)\,dB = (-62) - (-64.7) = 2,7 \text{ dB} = 1,86 \text{ (numeric value)}$$

$$T_e = \frac{T_1 - YT_2}{Y-1} = \frac{290 - (1,86)(77)}{1,86 - 1} = 170 \text{ °K}$$

$$P_o = GkT_sB + GkT_eB = 100\,(1,38 \cdot 10^{-23})(10^9)(450 + 170)$$
$$= 8,56 \cdot 10^{-10} \text{ W} = -60,7 \text{ dBm}.$$

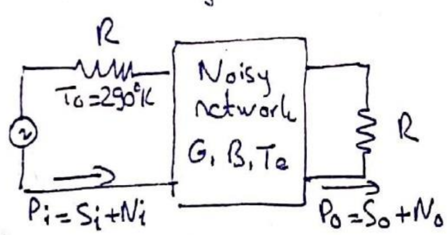
Noise figure

When noise and a desired signal are applied to the input of a noisy network, the output noise power will be increased more than the output signal power, so that the output signal-to-noise ratio will be reduced. The noise figure, F, is a measure of this reduction in S/N and is defined as

$$F = \frac{S_i/N_i}{S_o/N_o} \geq 1$$

By definition, N_i is assumed to be the noise power resulting from a matched resistor at $T_o = 290\,°K$; that is, $N_i = kT_oB$.

Consider the figure below. The input noise power is $N_i = kT_oB$ and the output noise power is a sum of the amplified input noise with the gain G and the noise generated by the noisy network: $N_o = kGB(T_o + T_e)$. The output signal power is $S_o = GS_i$. The noise figure F will be,



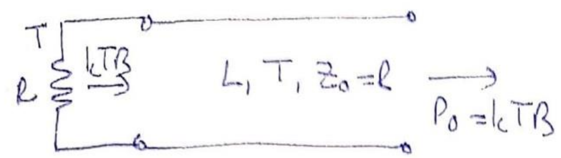
$$F = \frac{S_i / kT_oB}{GS_i / kGB(T_o + T_e)} = 1 + \frac{T_e}{T_o} \geq 1$$

In dB, $F = 10\log(1 + T_e/T_o)\ dB \geq 0$

If the network were noiseless, T_e would be zero, giving $F = 1$ (0 dB). Using above equation, T_e will be,

$$T_e = (F-1)T_o$$

An important special case occurs in practice when the two-port network is a passive, lossy component held at a temperature T. The loss factor, L, can be defined as $L = 1/G > 1$.



Because the entire system is in thermal equilibrium at T and has driving point impedance of R, the output noise power must be $P_o = kTB$. This power comes from the source resistor and from the noise generated by the line itself. Thus,

$$P_o = kTB = GkTB + GN_{added}$$

Solving this equation for N_{added},

$$N_{added} = \frac{1-G}{G} kTB = (L-1)kTB.$$

This result shows that the lossy line has an T_e given by

$$\underline{T_e = (L-1)T}$$

and using $F = 1 + T_e/T_o$, the noise figure F is,

$$\underline{F = 1 + (L-1)\frac{T}{T_o}}$$

- If the line is at temperature T_o, then $F = L$

Exomple

A 10-12 GHz amplifier has a gain of 20 dB, a noise figure of 3.5 dB and an output power of 10 dBm at its 1 dB compression point. What is the dynamic range of this amplifier?

The upper end of the dynamic range is 10 dBm corresponding to 1 dB comp. point. The lower end is set by the output noise power N_o, due to the amplifier itself.

The equivalent noise temperature of the amplifier is

$$T_e = (F-1)T_0 = (10^{3,5/10} - 1)290 = 359\ °K$$

The output noise power is,

$$N_o = G k T_e B = 20 + 10\log \frac{(1{,}38 \cdot 10^{-23})(359)(2 \cdot 10^{9})\ W}{10^{-3}\ W}$$

$$= -60.0\ dBm$$

So the dynamic rang is

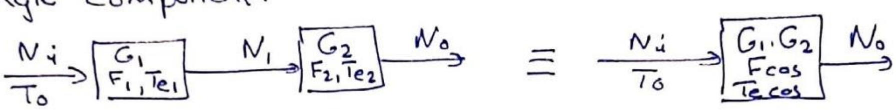
$$10\ dBm - (-60.0\ dBm)$$

$$= \underline{\underline{70.0\ dB}}$$

Noise figure of a cascaded system

If we know the F (or T_e) of the individual stages, we can determine the F (or T_e) of the cascade connection of stages.

Consider the two cascaded networks as shown the figure below, we wish to find the overall F_{cas} and T_{ecas} as if it were a single component.



The noise power at the output of the first stage is

$$N_1 = G_1 k T_0 B + G_1 k T_{e1} B$$

and the second stage is

$$N_o = G_2 N_1 + G_2 k T_{e2} B = G_1 G_2 k B \left(T_0 + T_{e1} + \frac{1}{G_1} T_{e2}\right)$$

$$= G_1 G_2 k B \left(T_{cas} + T_0\right)$$

where the noise temperature of the cascade system is,

$$T_{cas} = T_{e1} + \frac{1}{G_1} T_{e2}$$

and using $F = 1 + T_e / T_0$, the noise figure of the cascade system is,

$$F_{cas} = F_1 + \frac{1}{G_1} (F_2 - 1)$$

These equations show that the noise characteristics of a cascaded system are dominated by the characteristics of the first stage. For the best system noise performance, the first stage should have a low noise figure and at least moderate gain.

These equations can be generalized to an arbitrary number of stages, as follows:

$$T_{cas} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots$$

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

Example:

An antenna is connected to a low-noise amplifier with a piece of coaxial transmission line. The amplifier parameters are; $G = 15$ dB, $B = 100$ MHz, $T_e = 150$ °K. The coaxial line has an attenuation of 2 dB. Find the F of the cascoded network. What would be the F if the amplifier were placed at the antenna, eliminating the transmission line? Assume all components are at an ambient temperature of $T = 300$°K

The loss factor of the line is $L = 10^{2/10} = 1{,}58$, and the F of the line is

$$F_\ell = 1 + (L-1)\frac{T}{T_0} = 1 + (1.58-1)\frac{300}{290} = 1.60 = 2{,}04 \text{ dB}$$

The F of the amplifier is,

$$F_a = 1 + \frac{T_e}{T_0} = 1 + \frac{150}{290} = 1.52 = 1{,}81 \text{ dB}$$

The F of the cascode is,

$$F_{cas} = F_\ell + \frac{1}{G_\ell}(F_a - 1) = 1{,}60 + 1{,}58\,(1{,}52 - 1) = 2{,}42$$

$$= 3.84 \text{ dB}$$

Without the transmission line, F would be that of the amplifier itself, or 1.81 dB. We see that the effect of the lossy line reduces the F of the system by about 2 dB.