İSTANBUL TEKNİK ÜNİVERSİTESİ

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HOMEWORK III

Electromagnetic Waves

EHB 313E

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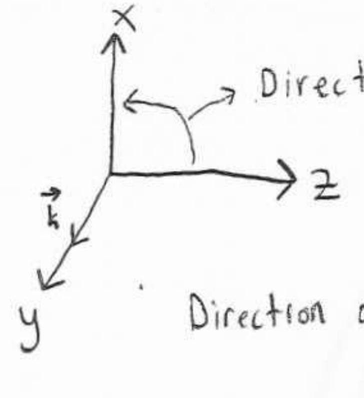
1)



Direction of ratation of RHCP waves

Direction of propogation : $\vec{e}_y$

- $f = 10^8$ [Hz]
- $v_p = \dfrac{1}{\sqrt{\varepsilon_r \varepsilon \mu_0}} = \dfrac{3}{2}\, 10^8$ [m/
- $w = 2\pi . 10^8$ [rad/s]
- $k = \dfrac{w}{v_p} = \dfrac{4\pi}{3}$ [$\dfrac{rad}{m}$

For The Phusor Representations;

$$E(y) = E_{z0}\, e^{-iky}\, \vec{e}_z - i\, E_{x0}\, e^{-iky}\, \vec{e}_x$$

For The Time Domain Representations;

$$E(y;t) = E_{z0} \cos(wt - ky)\, \vec{e}_z + E_{x0} \sin(wt - ky)\, \vec{e}_x$$

For RHCP : $E_{z0} = E_{x0}$

$$|E(y;t)| = \sqrt{E_{z0}^2 \underbrace{\left[\sin^2(wt-ky) + \cos^2(wt-ky)\right]}_{1}} = E_{z0} = 3 \text{ mV/m}$$

$$\boxed{E(y;t) = 3.10^{-3} \cos(wt - ky)\, \vec{e}_z + 3.10^{-3} \sin(wt - ky)\, \vec{e}_x \text{ [V/m]}}$$

For Magnetic Field Vector $H(y;t)$;

$$\vec{H}(y) = \frac{1}{\eta}\, \vec{n} \times \vec{E}(y) \qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} = 188{,}5\ \Omega$$

$$\vec{n} = \vec{e}_y$$

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$$H(y) = \frac{1}{188,5}\left[\vec{e}_y \times \left(3.10^{-3} e^{-iky}\vec{e}_z - i\,3.10^{-3} e^{-iky}\vec{e}_x\right)\right]$$

$$H(y) = \frac{1}{188,5}\left[3.10^{-3} e^{-iky}\vec{e}_x + i.3.10^{-3} e^{-iky}\vec{e}_z\right]$$

$$\boxed{H(y;t) = 1,59\times10^{-5}\cos(wt-ky)\vec{e}_x - 1,59\times10^{-5}\sin(wt-ky)\vec{e}_z \quad [A/m]}$$

The exact expressions of the magnetic and electric field vectors are written below;

$$E(y;t) = 3.\cos\left(2\pi 10^8 t - \frac{4\pi}{3}y\right)\vec{e}_z + 3\sin\left(2\pi 10^8 t - \frac{4\pi}{3}y\right)\vec{e}_x \quad [mV/m]$$

$$H(y;t) = 15,9\cos\left(2\pi 10^8 t - \frac{4\pi}{3}y\right)\vec{e}_x - 15,9\sin\left(2\pi 10^8 t - \frac{4\pi}{3}y\right)\vec{e}_z \quad [\mu A/m]$$

2) $$\vec{E}(z;t) = \vec{e}_x E_0 \cos\left[3.10^8\pi t - \frac{3.10^8\pi}{v} z + \theta\right]$$

$$\vec{E}(z;t) = \vec{e}_x E_0 \cos\left[wt - \underbrace{\frac{w}{v}}_{k} z + \theta\right]$$

$$\vec{E}(z;t) = \vec{e}_x E_0 \cos\left[wt - kz + \theta\right]$$

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a) $\varepsilon_r = 4 \quad \mu_r = 1 \quad \sigma = 0$ (Lossless)

$w = 3 . \pi . 10^8$ [rad/s]

$f = \frac{w}{2\pi} \Rightarrow \boxed{f = \frac{3}{2} \cdot 10^8 \ [Hz]}$ → frequency

$$\boxed{v = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_0}} = \frac{3 . 10^8}{\sqrt{4}} = \frac{3}{2} \cdot 10^8 \ [m/s]}$$ → phase velocity

$$\boxed{k = \frac{w}{v} = \frac{3\pi . 10^8}{\frac{3}{2} 10^8} = 2\pi \left[\frac{rad}{m}\right]}$$ → wave number

$$\boxed{\lambda = \frac{2\pi}{k} = 1 \ [m]}$$ → wavelength

b) $\vec{E}_1(z;t) = \vec{e}_x \, 8 . \sin\left[3 \times 10^8 \pi \left(t - \frac{z}{v}\right)\right]$

$\vec{E}_1(z;t) = \vec{e}_x . 8 . \sin[wt - kz]$

$$\boxed{\vec{E}_1(z) = -i . 8 \, e^{-ikz} \, \vec{e}_x}$$

$\vec{E}_2(z;t) = \vec{e}_x \, 6 . \cos\left[3 . 10^8 \pi \left(t - \frac{z}{v}\right) - \frac{5\pi}{6}\right]$

$\vec{E}_2(z;t) = \vec{e}_x \, 6 . \cos\left[wt - kz - \frac{5\pi}{6}\right]$

$\vec{E}_2(z) = 6 . e^{-ikz} e^{-i5\pi/6} \, \vec{e}_x$

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$$\vec{E}(z) = \vec{E}_1(z) + \vec{E}_2(z)$$

$$= -i8\, e^{-ikz}\, \vec{e}_x + 6 e^{-ikz}\, e^{-i\frac{5\pi}{6}}\, \vec{e}_x$$

$$= e^{-ikz}\, \vec{e}_x \left[-8i + 6e^{-i\frac{5\pi}{6}}\right]$$

$$= e^{-ikz}\, \vec{e}_x \left[-8i + 6\cos\frac{5\pi}{6} - 6\sin\frac{5\pi}{6}\right]$$

$$= e^{-ikz}\, \vec{e}_x \left[-5{,}196 - 11i\right]$$

$$= e^{-ikz}\, \vec{e}_x \left[12{,}165 \times e^{i1{,}36\pi}\right]$$

$$\vec{E}(z) = 12{,}165\, e^{i1{,}36\pi}\, e^{-ikz}\, \vec{e}_x \quad [V/m]$$

$$\vec{E}(z;t) = Re\left\{\vec{E}(z)\, e^{iwt}\right\}$$

$$\boxed{\vec{E}(z;t) = 12{,}165 \cos\left(wt - kz + 1{,}36\pi\right)\vec{e}_x\ [V/m]}$$

$\boxed{E_0 = 12{,}165}$ // $\boxed{\theta = 1{,}36\pi\ [rad]}$ // or $\boxed{\theta = 244{,}7^\circ}$ //

c) $\vec{E}(z;t) = Re\left\{E(z) . e^{iwt}\right\}$

$$\vec{E}(z) = 12{,}165 . e^{i \times 1{,}36\pi}\, e^{-ikz}\, \vec{e}_x$$

$$\boxed{\vec{E}(z) = 12{,}165 . e^{i1{,}36\pi}\, e^{-i2\pi z}\, \vec{e}_x \quad [V/m]}$$

↓
This result is also found in part (b)

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d) $\eta = \sqrt{\dfrac{\mu_0}{\varepsilon_r \varepsilon_0}} = \dfrac{377}{\sqrt{4}} = 188,5 [\Omega]$

$\vec{n} = \vec{e}_z$ $\Rightarrow$ Direction of Propagation

$$\vec{H}(z) = \frac{1}{\eta} \, \vec{n} \times E(z)$$

$$\vec{H}(z) = \frac{1}{188,5} \left[\vec{e}_z \times \left(12,165 \, e^{i\,1,36\pi} \, e^{-i2\pi z} \, \vec{e}_x \right) \right]$$

$$\boxed{\vec{H}(z) = 64,5 \, e^{i\,1,36\pi} \, e^{-i2\pi z} \, \vec{e}_y \quad [mA/m]}$$

$$\vec{H}(z;t) = Re\left\{ H(z) \cdot e^{iwt} \right\}$$

$$\boxed{H(z;t) = 64,5 \, \cos\left(3\times10^8 \pi t - 2\pi z + 1,36\pi\right) \vec{e}_y \quad [mA/m]}$$

3) $\varepsilon_r = 9 \quad \mu_r = 1 \quad \sigma = 0$

a) $f = 10^9 [Hz]$, $w = 2\pi \, 10^9 \; [rad/s]$

$k = w\sqrt{\varepsilon_r \varepsilon_0 \mu_0} \Rightarrow \boxed{k = 20\pi \; \left[\frac{rad}{m}\right]}$ $\rightarrow$ wave number

$\lambda = \frac{2\pi}{k} \Rightarrow \boxed{\lambda = 0,1 \; [m]}$ $\longrightarrow$ wave length

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$$v_p = \frac{w}{k} \Rightarrow \boxed{v_p = 10^8 \, [m/s]} \rightarrow \text{phase velocity}$$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} \Rightarrow \boxed{\eta = 125{,}67 \, [\Omega]} \rightarrow \text{impadence of medium}$$

b) $f = 10 \text{ kHz}$, $w = 2\pi \, 10^4 \, [rad/s]$

$$k = w\sqrt{\varepsilon_r \varepsilon_0 \mu_0} \Rightarrow \boxed{k = 2\pi \, 10^{-4} \left[\frac{rad}{m}\right]}$$

$$\lambda = \frac{2\pi}{k} \Rightarrow \boxed{\lambda = 10^4 \, [m]}$$

$$v_p = \frac{w}{k} \Rightarrow \boxed{v_p = 10^8 \, [m/s]} \qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} \Rightarrow \boxed{\eta = 125{,}67 \, [\Omega]}$$

c) $f = 6 \, 10^6 \, [Hz]$ $\quad w = 12\pi \, 10^6 \left[\frac{rad}{s}\right]$

$$k = w\sqrt{\varepsilon_r \varepsilon_0 \mu_0} \Rightarrow \boxed{k = 0{,}12 \, \pi \left[\frac{rad}{m}\right]}$$

$$\lambda = \frac{2\pi}{k} \Rightarrow \boxed{\lambda = 16{,}67 \, [m]}$$

$$v_p = \frac{w}{k} \Rightarrow \boxed{v_p = 10^8 \, m/s} \qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} \Rightarrow \boxed{\eta = 125{,}67 \, [\Omega]}$$

Consequently, all three parts show that the phase **v**elocity (v_p) and the impedence of the medium (η) are independent from the frequency. They depend on "ε_r" and "μ_r" of the medium.