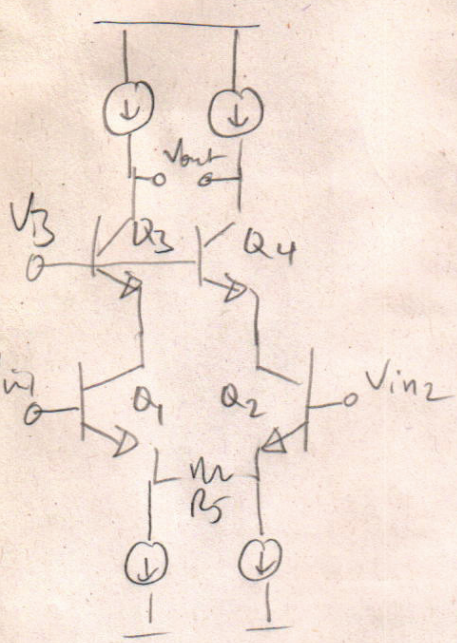
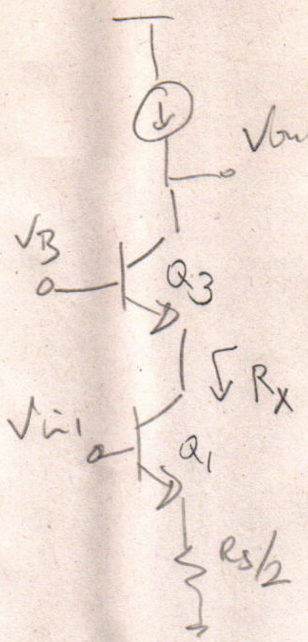
EHB335E HW#1 Solutions

(1)



Half Circuit



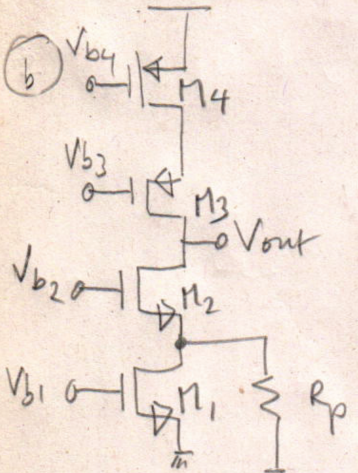
$$G_m = \frac{g_{m1}}{1 + g_{m1}\frac{R_S}{2}}$$

$$R_X = r_{o1} + \left(\frac{R_S}{2} \| r_{\pi 1}\right) + g_{m1} r_{o1} \left(\frac{R_S}{2} \| r_{\pi 1}\right)$$

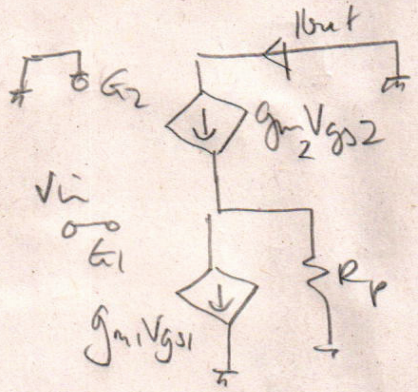
$$R_{out} = r_{o3} + (R_X \| r_{\pi 3}) + g_{m3}(R_X \| r_{\pi 3}) r_{o3}$$

$$A_v = -G_m R_{out}$$

(b)



Find Gm:



$$i_{out} = g_{m2}(0 - V_{S2}) = -g_{m2}V_{S2}$$

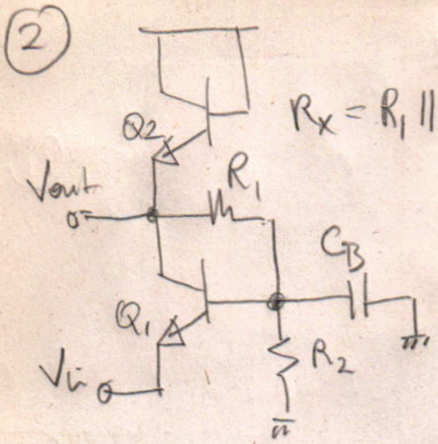
$$i_{out} = \frac{V_{S2}}{R_P} + g_{m1}(V_{in} - \cancel{V_{S1}})$$

$$i_{out} + \frac{1}{g_{m2}R_P} i_{out} = g_{m1}V_{in}$$

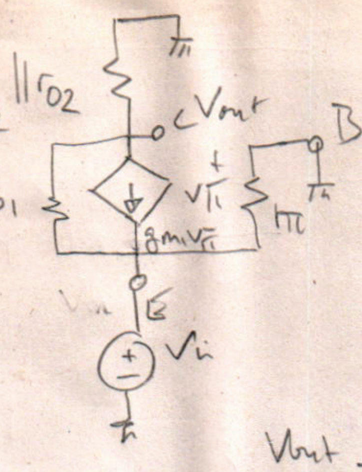
$$G_m = \frac{i_{out}}{V_{in}} = \frac{g_{m1}}{1 + \frac{1}{g_{m2}R_P}} = \frac{g_{m1}g_{m2}R_P}{1 + g_{m2}R_P}$$

$$R_{out} = (r_{o3} + r_{o4} + g_{m3}r_{o3}r_{o4}) \| \left[r_{o2} + (R_P \| r_{o1}) + g_{m2}r_{o2}(R_P \| r_{o1})\right]$$

②



$R_x = R_1 \| \frac{1}{g_{m2}} \| r_{o2}$



$$\frac{V_{out}}{R_x} + \frac{V_{out}-V_{in}}{r_{o1}} + g_m V_\pi = 0$$

$$V_{in} = -V_\pi$$

$$\frac{V_{out}}{R_x} + \frac{V_{out}}{r_{o1}} = \left(g_m + \frac{1}{r_{o1}}\right) V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{\left(g_m + \frac{1}{r_{o1}}\right)}{\left(\frac{1}{R_x} + \frac{1}{r_{o1}}\right)}$$

To check the answer let $r_{o1} \to \infty$ $\frac{V_{out}}{V_{in}} = g_m R_x$, which is correct.

$$i_{in} = \frac{V_{in} - V_{out}}{r_{o1}} - g_m V_\pi + \frac{V_{in}}{r_{\pi 1}}$$

$$i_{in} = \frac{V_{in} - V_{out}}{r_{o1}} + V_{in}\left(g_{m1} + \frac{1}{r_{\pi 1}}\right),$$ use the gain expression.

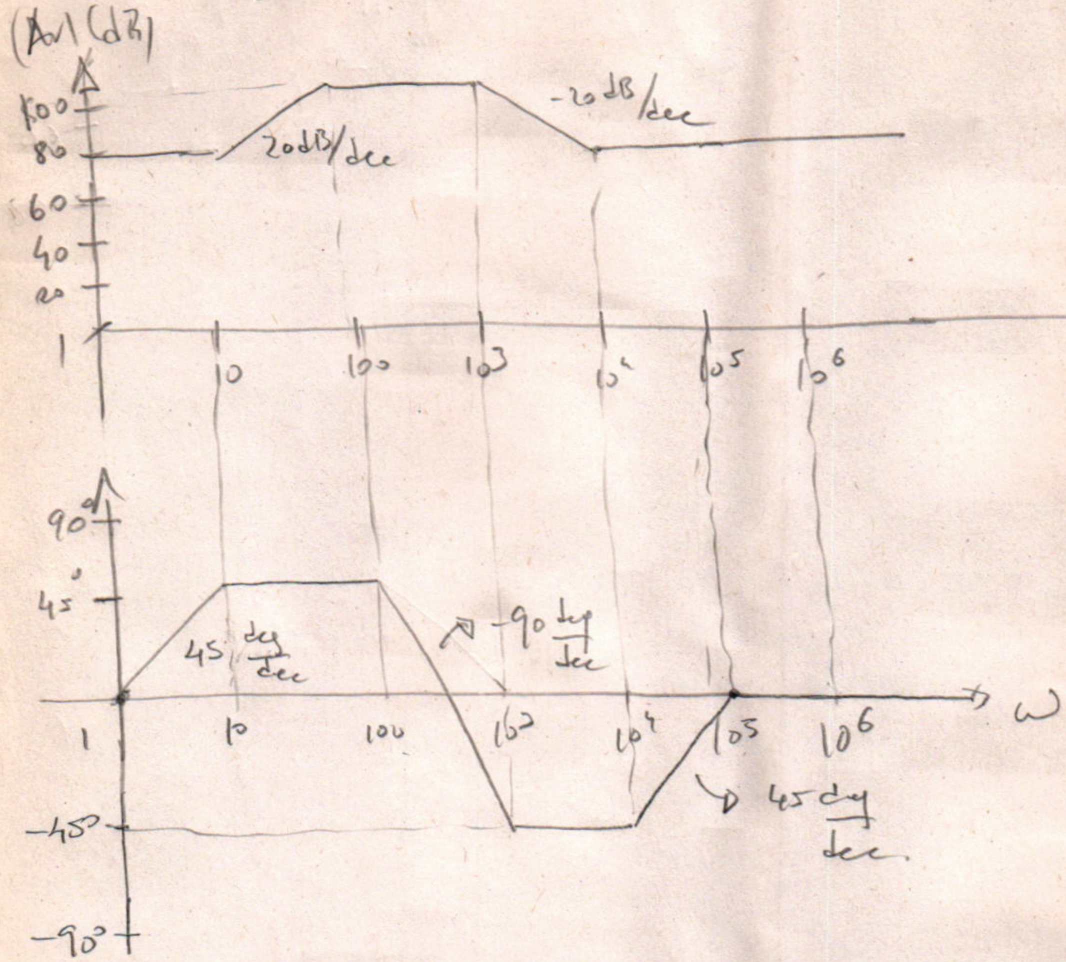
$$i_{in} = \frac{V_{in}}{r_{o1}} - \frac{V_{in}}{r_{o1}} \frac{\left(g_m + \frac{1}{r_{o1}}\right)}{\frac{1}{R_x} + \frac{1}{r_{o1}}} + V_{in} g_m = \frac{\frac{1}{R_x} + \cancel{\frac{1}{r_{o1}}} - g_m - \cancel{\frac{1}{r_{o1}}} + g_m r_{o1}\left(\frac{1}{R_x} + \frac{1}{r_{o1}}\right)}{\left(\frac{1}{R_x} + \frac{1}{r_{o1}}\right) r_{o1}} V_{in}$$

$$i_{in} = \frac{\frac{1}{R_x} - \cancel{g_m} + \frac{g_m r_{o1}}{R_x} + \cancel{g_m}}{r_{o1}\left(\frac{1}{R_x} + \frac{1}{r_{o1}}\right)} V_{in} \Rightarrow \frac{V_{in}}{i_{in}} = R_{in} = \frac{r_{o1}\left(\frac{1}{R_x} + \frac{1}{r_{o1}}\right)}{\frac{(1 + g_m r_{o1})}{R_x}} = \frac{(r_{o1} + R_x)\cancel{r_{o1}}}{\cancel{r_{o1}}(1 + g_m r_{o1})} = \frac{r_{o1} + R_x}{1 + g_m r_{o1}}$$

To check the answer let $r_{o1} \to \infty$ $\frac{V_{in}}{i_{in}} = R_{IN} = \frac{1}{g_m}$, as expected.

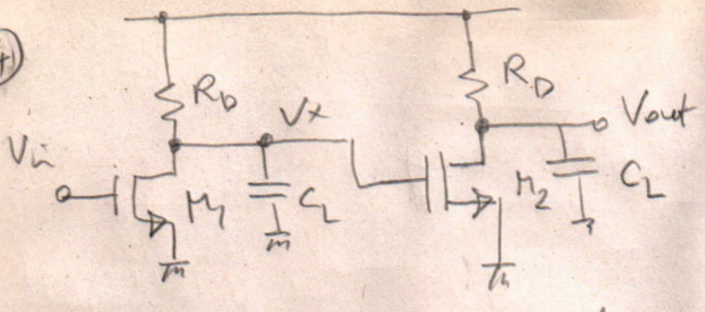
(3) $$H(s) = \frac{10^4 (10+s)(10000+s)}{(100+s)(1000+s)} = \frac{10^4 \cancel{10} \cdot \cancel{10^5} \left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^5}\right)}{\cancel{10^5} \left(1+\frac{s}{10^2}\right)\left(1+\frac{s}{10^3}\right)}$$

$A_v = 20\log_{10} 10^4 = 80\,\mathrm{dB}$



Check MATLAB solutions with your TA!

(4)



$$\frac{V_x}{V_{in}} = -g_{m1}\left(R_D \| \frac{1}{sC_L}\right) \qquad \frac{V_{out}}{V_x} = -g_{m2}\cdot\left(R_D \| \frac{1}{sC_L}\right)$$

$$\frac{V_{out}}{V_{in}} = g_{m1}g_{m2}\left(R_D \| \frac{1}{sC_L}\right)^2 = g_{m1}g_{m2}\left(\frac{R_D\cdot\frac{1}{sC_L}}{R_D+\frac{1}{sC_L}}\right)^2$$

$$\frac{V_{out}}{V_{in}} = g_{m1}g_{m2}\frac{R_D^2}{(1+sC_LR_D)^2} = \frac{g_{m1}g_{m2}\cancel{R_D^2}}{C_L^2\cancel{R_D^2}}\frac{1}{\left(s+\frac{1}{C_LR_D}\right)^2}$$

$$\frac{V_{out}}{V_{in}}(j\omega) = \frac{g_{m1}g_{m2}}{C_L^2}\frac{1}{\left(\omega^2+\frac{1}{C_L^2R_D^2}\right)} = \frac{A}{\omega^2+\frac{1}{C_L^2R_D^2}}$$

-3dB BW:

$$\frac{A}{\omega^2+\frac{1}{C_L^2R_D^2}} = \frac{A}{\sqrt{2}}\frac{1}{\frac{1}{C_L^2R_D^2}} \qquad \omega_p = \frac{1}{C_LR_D}$$

$$\frac{\sqrt{2}-1}{C_L^2R_D^2} = \omega^2 \Rightarrow \omega = \frac{\sqrt{\sqrt{2}-1}}{C_LR_D} = \sqrt{\sqrt{2}-1}\,\omega_p$$

(5) Check solutions with your TA!