EHB 315E Digital Signal Processing
Fall 2020
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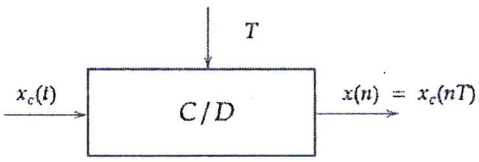
HOMEWORK 4 - SOLUTIONS

1 [20 pts] Each of the following continuous-time signals is used as the input $x_c(t)$ for an ideal C/D converter as shown in the following figure with the sampling period T specified. In each case, find the resulting discrete-time signal $x(n)$



(a) $x_c(t) = \cos(2\pi(1000)t)$, $T = (1/3000)$sec

(b) $x_c(t) = \sin(2\pi(1000)t)/(\pi t)$, $T = (1/5000)$sec

S1 (a) $x(n) = x_c(nT)$

$$x(n) = \cos(2\pi n(1000)/3000)$$

$$= \cos\left(\frac{2\pi n}{3}\right)$$

(b) $$x(n) = \frac{\sin(2\pi 1000 n/5000)}{\pi n/5000}$$

$$= \frac{\sin(2\pi n/5)}{\pi n/5000}$$

2 **[20 pts]** Determine the group delay for $0 < \omega < \pi$ for each of the following sequences:

(a) $x_1[n] = \begin{cases} n-1, & 1 \le n \le 5 \\ 9-n, & 5 < n \le 9 \\ 0, & \text{otherwise} \end{cases}$

(b) $x_2[n] = \left(\frac{1}{2}\right)^{|n-1|} + \left(\frac{1}{2}\right)^{|n|}$

S2 (a) $x_1(n) = \delta(n-2) + 2\delta(n-3) + 3\delta(n-4) + 4\delta(n-5)$
$+ 3\delta(n-6) + 2\delta(n-7) + \delta(n-8)$

Notice that $x_1(n)$ is a symmetric sequence centered at $n=5$. Hence the phase of $X_1(e^{j\omega})$ is

$$\arg[X_1(e^{j\omega})] = -5\omega$$

Thus,

$$\mathrm{grd}[X_1(e^{j\omega})] = \frac{-d}{d\omega}\left[\arg(X_1(e^{j\omega}))\right]$$
$$= \frac{-d}{d\omega}(-5\omega) = \boxed{+5}$$

(b) $$x_2(n) = \left(\frac{1}{2}\right)^{|n|}\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{2}\right)^{|n|}$$
$$= \left(\frac{1}{2}\right)^{|n|}\left(\left(\frac{1}{2}\right)^{-1} + 1\right)$$
$$= \left(\frac{1}{2}\right)^{|n|}(2+1) = \left(\frac{1}{2}\right)^{|n|} \cdot 3 = 3\left(\frac{1}{2}\right)^{|n|}$$

$x_2(n)$ sequence is also symmetric around $n = \frac{1}{2}$.
Also, this sequence has linear phase.

$$\arg[X_2(e^{j\omega})] = -\frac{\omega}{2} \rightarrow \mathrm{grd}[X_2(e^{j\omega})] = \boxed{\frac{1}{2}}$$

3 [20 pts] For each of the following system functions, state whether or not it is a minimum-phase system. Justify your answers:

(a) $H_1(z) = \frac{(1-2z^{-1})(1+\frac{1}{2}z^{-1})}{(1-\frac{1}{3}z^{-1})(1+\frac{1}{3}z^{-1})}$

(b) $H_2(z) = \frac{(1+\frac{1}{4}z^{-1})(1-\frac{1}{4}z^{-1})}{(1-\frac{2}{3}z^{-1})(1+\frac{2}{3}z^{-1})}$

(c) $H_3(z) = \frac{1-\frac{1}{3}z^{-1}}{(1-\frac{j}{2}z^{-1})(1+\frac{j}{2}z^{-1})}$

(d) $H_4(z) = \frac{z^{-1}(1-\frac{1}{3}z^{-1})}{(1-\frac{j}{2}z^{-1})(1+\frac{j}{2}z^{-1})}$

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(a) A system is called minimum-phase when all its poles and zeros inside the unit circle.

$P^1_{H1} = \frac{1}{3}$ $\quad z^1_{H1} = 2$

$P^2_{H1} = \frac{1}{3}$ $\quad z^2_{H1} = -\frac{1}{2}$

z^1_{H1} is outside the unit circle. Hence, it is not minimum phase

(b) $P^1_{H2} = \frac{2}{3}$ $\quad z^1_{H2} = -\frac{1}{4}$

$P^2_{H2} = -\frac{2}{3}$ $\quad z^2_{H2} = \frac{1}{4}$

All poles and zeros are inside the unit circle. So, the system is minimum-phase

(c) $P^1_{H3} = j/2$ $\quad z^1_{H3} = \frac{1}{3}$

$P^2_{H3} = -j/2$ $\quad z^1_{H3} = 0$

$H_3(z)$ is minimum-phase system since its poles and zeros are inside the unit circle.

(d) $P^1_{H4} = -j/2$ $\quad z^1_{H4} = \frac{1}{3}$

$P^2_{H4} = j/2$ $\quad z^2_{H4} = \infty$

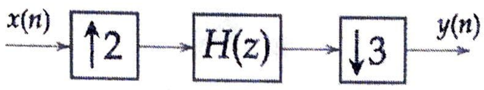
$z^2_{H4} = \infty$ is outside the unit circle, so it is not minimum phase.

4 **[20 pts]** The signal $x(n)$ and impulse response $h(n)$ of an LTI system are given as follows:

$$x(n) = \delta(n) + 2\delta(n-1) + 3\delta(n-2) + 2\delta(n-3) + \delta(n-4)$$

$$h(n) = \delta(n) + 2\delta(n-1)$$

If the input signal $x(n)$ is applied to the following system, what is the output signal?



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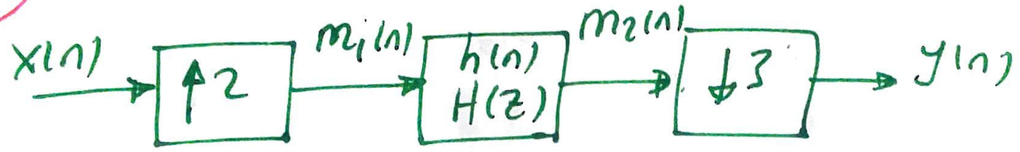

$m_1(n) = x(n/2)$

$m_1(n) = \delta(n) + 2\delta(n-2) + 3\delta(n-4) + 2\delta(n-6) + \delta(n-8)$

$m_2(n) = m_1(n) * h(n)$

$= m_1(n) * [\delta(n) + 2\delta(n-1)]$

$= m_1(n) + 2m_1(n-1)$

$= \delta(n) + 2\delta(n-1) + 2\delta(n-2) + 4\delta(n-3) + 3\delta(n-4) + 6\delta(n-5) + 2\delta(n-6) + 4\delta(n-7) + \delta(n-8) + 2\delta(n-9)$

$y(n) = m_2(3n)$

$= \delta(n) + 4\delta(n-1) + 2\delta(n-2) + 2\delta(n-3)$

5 [20 pts] This problem deals with linear-phase FIR filters with zeros at $z = 1$ and $z = -1$ of multiplicity two. For each of the following linear-phase filters: sketch the pole-zero diagram, impulse response, and classify each filter as type I, II, III, IV. Comment on your observations, especially with regard to what you know about the zero locations of linear-phase filters:

(a) $H(z) = \left(1 - z^{-1}\right)\left(1 + z^{-1}\right)$

(b) $H(z) = \left(1 - z^{-1}\right)^2\left(1 + z^{-1}\right)$

(c) $H(z) = \left(1 - z^{-1}\right)\left(1 + z^{-1}\right)^2$

(d) $H(z) = \left(1 - z^{-1}\right)^2\left(1 + z^{-1}\right)^2$

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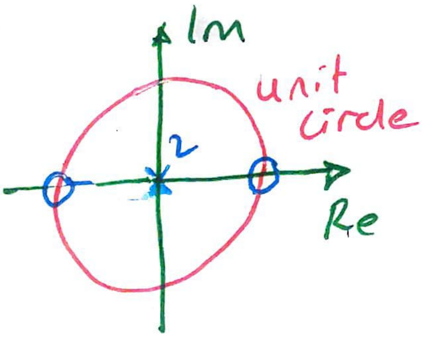
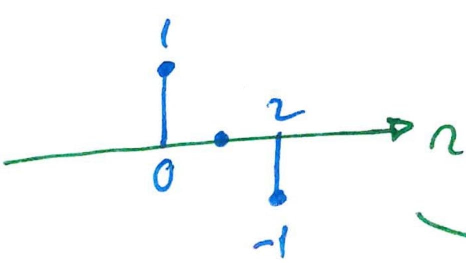
(a)



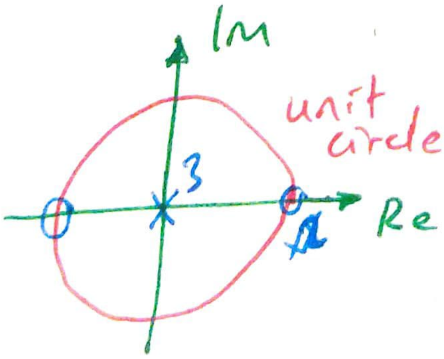
$$H(z) = \underbrace{(1-z^{-1})}_{X_1(z)}\underbrace{(1+z^{-1})}_{X_2(z)}$$

$$h(n) = x_1(n) * x_2(n)$$
$$= (\delta(n) - \delta(n-1)) * (\delta(n) + \delta(n-1))$$
$$= \delta(n) + \delta(n-1) - \delta(n-1) - \delta(n-2)$$
$$= \delta(n) - \delta(n-2)$$



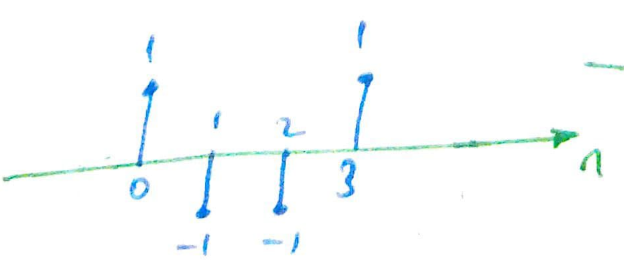
TYPE III Linear Phase FIR Filter

(b)



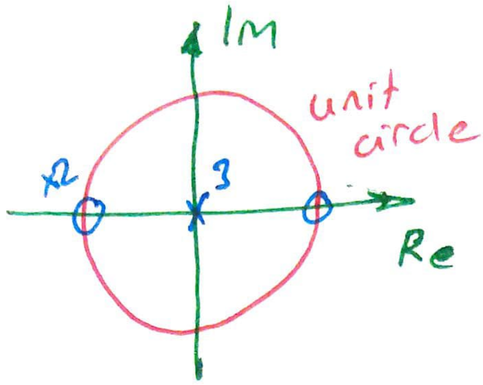
$$h(n) = [\delta(n) - \delta(n-1)] * [\delta(n) - \delta(n-1)] * [\delta(n) + \delta(n-1)]$$

$$h(n) = \delta(n) - \delta(n-1) - \delta(n-2) + \delta(n-3)$$



TYPE II Linear Phase FIR Filter

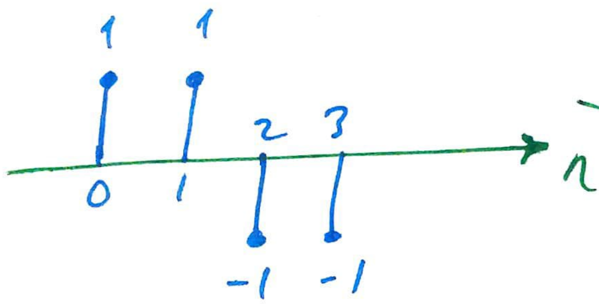
(c)



$$h(n) = [\delta(n)-\delta(n-1)] * [\delta(n)+\delta(n-1)] * [\delta(n)+\delta(n-1)]$$

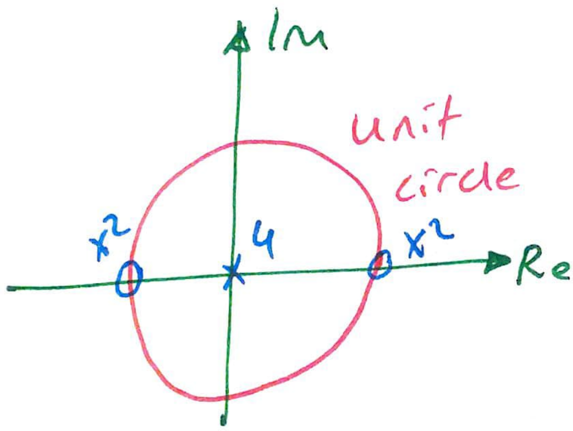
$$h(n) = [\delta(n)-\delta(n-1)] * [\delta(n)+2\delta(n-1)+\delta(n-2)]$$

$$h(n) = \delta(n)+\delta(n-1)-\delta(n-2)-\delta(n-3)$$



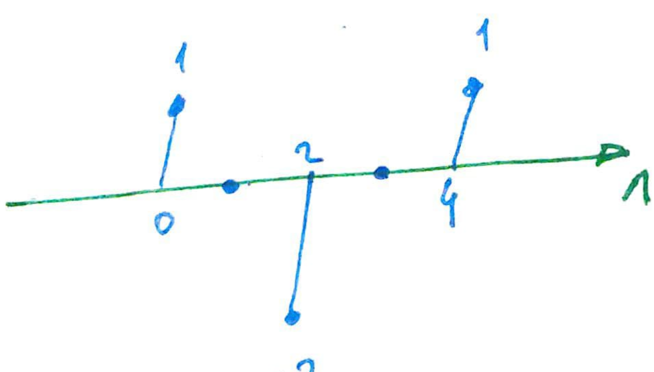
TYPE IV Linear Phase FIR Filter

(d)



$$h(n) = [\delta(n)-\delta(n-1)] * [\delta(n)-\delta(n-1)] * [\delta(n)+\delta(n-1)] * [\delta(n)+\delta(n-1)]$$

$$h(n) = \delta(n)-2\delta(n-2)+\delta(n-4)$$



TYPE I Linear Phase FIR Filter