ÖDEV 4

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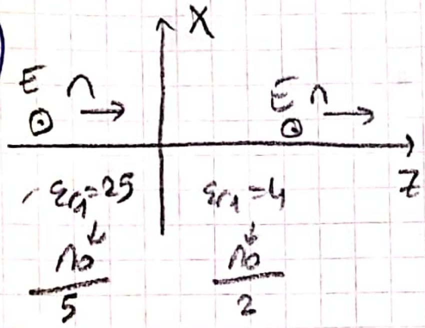
1-) Dik polarize düzlem dalga $E_0 = 10$

a)



$\eta = \sqrt{\frac{\mu}{\varepsilon}}$ $\quad T = \frac{2\eta_2}{\eta_1+\eta_2}$ $\quad \Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$

$$\Gamma = \frac{\eta_0\left(\frac{1}{2}-\frac{1}{5}\right)}{\eta_0\left(\frac{1}{2}+\frac{1}{5}\right)} = 0.42 \qquad T = \frac{\frac{2\eta_0}{2}}{\eta_0\left(\frac{1}{2}+\frac{1}{5}\right)} = 1.42$$

b) $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1.42}{0.58} = 2.5$

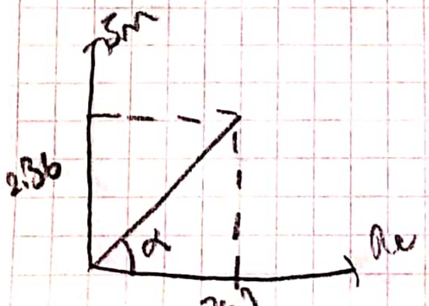
c) $E = \vec{e_y}\, 14.2\, e^{jkz}$ $\quad H_z = \frac{1}{60\pi}\left(-\vec{e_x}\right) e^{jkz}$

$$P_{Avg} = \left\{\frac{1}{2} E \times H^*\right\} = \frac{1}{2}\,\frac{(14.2)^2}{60\pi} = 0.534\, \vec{e_z} \quad W/m^2$$

2-) Kayıp tanjant $\frac{\sigma}{\omega.\varepsilon} = 0.03 \ll 1$ Az kayıplı dielektrik ortam

$$k^2 = \omega^2 \mu \varepsilon + j\sigma\mu\omega = \frac{\omega^2 \varepsilon_r}{c^2} + j\sigma\mu\omega$$

$$k^2 = \frac{36\pi^2 10^{16}.2}{(3\times10^8)^2} + j\,10^{-3}\, 4\pi 10^{-7}\, 6\pi 10^{8} = 78.3 + j2.36$$



Genlik $= \sqrt{2.36^2 + 78.3^2} = 78.33$

$\alpha = \frac{2.36}{78.3} = 1.72$

$k^2 = 78.33 . e^{j0.03}$

$k = 8.85 . e^{j0.015}$

$k = 8.85\,(1 + j0.015) = 8.85 + j0.13$

$\tau = \frac{2\eta_2}{\eta_2+\eta_1}$ $\quad \eta = 120\pi$ $\quad \eta_2 = \frac{6\pi 10^8\, 4\pi 10^{-7}}{8.85 \angle 0.015} = 266.74 \simeq 266.7$ ree

$$\frac{537.41}{266.7+120\pi\pi} = 0.182 \qquad E_T = 82 \cdot e^{-0.13z} \cdot e^{-j\beta_2 z} e_x$$

$$H_T = \frac{82}{266.7} e^{-0.13z} e^{-j\beta_2 z} e_y$$

$$P_{av} = \frac{1}{2} Re\{E \times H^*\}$$

$$= \frac{1}{2} Re\left\{82 \cdot e^{-0.13} \cdot e^{j\beta t} \frac{82}{266.7} e^{-0.13z} \cdot e^{-j\beta_2 z}\right\} e_z = 12.6\, e^{-0.26z}$$

$$\frac{12.6 \times 10^{-12}}{12.6} = e^{-0.26z_t} \qquad 10^{-2} = e^{-0.26z_t} \rightarrow \ln 10^{-2} = 0.26z_t$$

$$z_t = 17.7m$$

3-) a) $\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2}$ $\quad \sin\theta_2 = \sin 30° \frac{1}{2} = 0.41$

$\theta_2 = 24.62°$

$\sin\theta_3 = \frac{1.2}{1.7} \sin 24.62° = 0.29$ $\qquad \sin\theta_4 = \frac{1.7}{1.5} \sin 17.1 = 0.33$

$\theta_3 = 17.1$ $\qquad \theta_4 = 19.47$

b) $x_2 = 2cm$ $\qquad 3 + 0.91 + 0.92 + 1.41 = 6.253cm$

$x_3 = 3cm$

$x_4 = 4cm$

4-) $E_i = 3\cos(\omega t - x - y\sqrt{3})\vec{e_z} \rightarrow E_t = 3e^{j(x+\sqrt{3}y)}\vec{e_z}$

$E_i = 3e^{j2n_i t}$ $\quad \vec{n_i} = \frac{\vec{e_x}}{2} + \frac{\sqrt{3}\vec{e_y}}{2}$ $\quad \vec{n_r} = \frac{-\vec{e_x}}{2} + \frac{\sqrt{3}}{2}\vec{e_y}$

$E_r = \Gamma\, 3e^{j2n_r R}$

a) $k_1 = 2$ $\quad k = \frac{\omega}{v}$ $\quad k_1 = \frac{\omega\sqrt{\varepsilon_{r1}}}{c}$ $\quad \frac{\omega}{c} = 1$ $\quad k_2 = \frac{\omega\sqrt{\varepsilon_{r2}}}{c} = 3$

$\sin\theta_i = \frac{\sqrt{3}}{2}$, $\theta_i = 60°$ $\qquad \frac{1}{\sqrt{3}} = \sin\theta_t$, $\theta_t = 35.26°$

b) $\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}} = \frac{120\pi}{\sqrt{\varepsilon_1}} = 60\pi$ $\quad \eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}} = \frac{120\pi}{\sqrt{\varepsilon_2}} = 40\pi$

$$r_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = -0.42 \qquad 1 + r_\perp = \tau_\perp = 0.58$$

$E_i = 3 e^{j2\left(\frac{x}{2} + \frac{\sqrt{3}}{2}y\right)} \vec{e_z}$ $\quad E_r = -1.26 e^{j2\left(-\frac{x}{2} + \frac{\sqrt{3}y}{2}\right)} \vec{e_z}$

$E_T = 1.74 e^{j3(0.81x + 0.57y)} \vec{e_z}$

$x = 0$ için $\Rightarrow E_i + E_r = E_T$

$3 e^{j\sqrt{3}y} \vec{e_y} - 1.26 e^{j\sqrt{3}y} = 1.74 e^{j0.57}$ $\quad 3 - 1.26 = 1.74$

$H_i = \frac{1}{\eta_1} \vec{n_r} \vec{E} \rightarrow \frac{3 e^{j2\left(\frac{x}{2} + \frac{\sqrt{3}}{2}y\right)}}{60\pi} e_{H_i}$ $\quad e_{H_i} = \left(\frac{\vec{e_x}}{2} + \frac{\sqrt{3}}{2}\vec{e_y}\right) \times \vec{e_z}$

$= \frac{-\vec{e_y}}{2} + \frac{\sqrt{3}}{2}\vec{e_x}$

$H_i = 0.0159 e^{j2\left(\frac{x}{2} + \frac{\sqrt{3}}{2}y\right)} \left(\frac{-\vec{e_y}}{2} + \frac{\sqrt{3}}{2}\vec{e_x}\right)$

$\vec{e_{H_r}} = \left(\frac{-\vec{e_x}}{2} + \frac{\sqrt{3}}{2}\vec{e_y}\right) \times \vec{e_z} = \frac{\vec{e_y}}{2} + \frac{\sqrt{3}\vec{e_x}}{2}$ $\quad H_r = -0.0066 e^{j2\left(-\frac{x}{2} + \frac{\sqrt{3}}{2}\right)} \left(\frac{\vec{e_y}}{2} + \frac{\sqrt{3}\vec{e_x}}{2}\right)$

$\vec{e_{H_T}} = (0.81\vec{e_x} + 0.57\vec{e_y}) \times \vec{e_z} = -0.81\vec{e_y} + 0.57\vec{e_x}$

$H_T = 0.013 e^{j3(0.81x + 0.57y)} (-0.81\vec{e_y} + 0.57\vec{e_x})$

5) $E_i = 2e^{j(4x+2z)} - 4e^{j(4x+2z)} \vec{e_z}$ $\quad E_i = 2e^{j2\sqrt{5}\left(\frac{2x}{\sqrt{5}} + \frac{z}{\sqrt{5}}\right)} - 4e^{j2\sqrt{5}\left(\frac{2x}{\sqrt{5}}+\frac{z}{\sqrt{5}}\right)} \vec{e_z}$

$\sin\theta_i = \frac{1}{\sqrt{5}} \quad \frac{\sin\theta t}{\sin\theta i} = \frac{1}{1{,}7} \rightarrow \sin\theta t = \frac{1}{\sqrt{5}\times 1{,}7} \quad \theta t = 15{,}25°$

$n_i = \left(\frac{2}{\sqrt{5}}\vec{ex} + \frac{1}{\sqrt{5}}\vec{ez}\right) \quad \vec{n_r} = \left(-\frac{2}{\sqrt{5}}\vec{ex} + \frac{1}{\sqrt{5}}\vec{ez}\right)$

$\vec{n_t} = (\cos 15{,}25\vec{ex} + \sin 15{,}25\vec{ez})$

$r_{//} = \frac{n_2\cos\theta t - n_1\cos\theta i}{n_2\cos\theta t + n_1\cos\theta i} \quad n_1 = 120\pi \quad n_2 = \sqrt{\frac{\mu_0}{\varepsilon}} = \frac{120\pi}{\sqrt{\varepsilon_r}} = 70{,}58\pi$

$r_{//} = -0{,}223 \quad T_{//} = (1+r_{//})\left(\frac{\cos\theta i}{\cos\theta t}\right) = 0{,}719$

$E_r = 0{,}447e^{j2\sqrt{5}\left(\frac{-2x}{\sqrt{5}} + \frac{z}{\sqrt{5}}\right)}\vec{ex} + 0{,}894\, e^{j2\sqrt{5}\left(-\frac{2x}{\sqrt{5}} + \frac{z}{\sqrt{5}}\right)}\vec{ez}$

$\frac{k_1}{k_2} = \frac{n_1}{n_2} \Rightarrow \frac{2\sqrt{5}}{k_2} = \frac{1}{1{,}7} = 7{,}6$

$E_t \Rightarrow |E_i| = \sqrt{20} \quad \vec{e_{Et}} = 0{,}96\vec{ex} + 0{,}26\vec{ez}$

$E_t = 3{,}21(0{,}26\vec{ex} - 0{,}96\vec{ez})e^{7{,}6j(0{,}96x+0{,}26z)}$

$H_i = \frac{1}{120\pi}\left(4\frac{2}{\sqrt{5}} + \frac{2}{\sqrt{5}}\right)e^{j\bar{n}R} \Rightarrow H_i = \frac{4{,}47}{120\pi}e^{j(4x+2z)}\vec{ey}$

$H_r = \frac{1}{120\pi}\left(0{,}894\frac{2}{\sqrt{5}} + 0{,}447\frac{1}{\sqrt{5}}\right)e^{jR\bar{n}}\vec{ey} = \frac{1}{120\pi}e^{j(-4x+2z)}\vec{ey}$

$n_t \times E_t = 0{,}92\vec{ey} + 0{,}067\vec{ey} \quad H_t = \frac{3{,}17}{70{,}5\pi}e^{j7{,}6(0{,}96x+0{,}26z)}\vec{ey}$