Recitation Hour (13.05.2020)

1) You independently draw 100 data points from a normal distribution.

a) Suppose you know the distribution is $N(\mu, 4)$ $(4=\sigma^2)$ and you want to test the null hypothesis $H_0: \mu_0 = 3$ against the alternative hypothesis $H_A: \mu_0 \neq 3$

If you want a significance level of. $\alpha = 0.05$.

What's your rejection region?

b) Suppose the 100 data points have sample mean 5.

What's the p-value for this data?

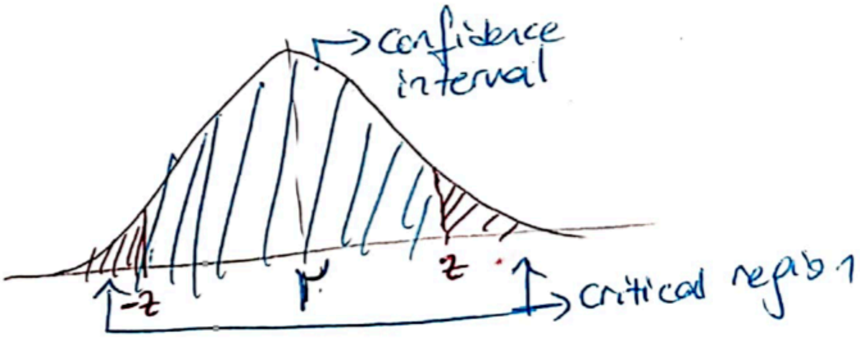
Soln.

a). $H_0: \mu_0 = 3$

$H_A: \mu_0 \neq 3$

$$z = \frac{X - \mu_0}{\sigma/\sqrt{n}} = \frac{X-3}{2/10} = 5(X-3).$$

$\alpha = P[\text{Type I error}]$: the probability of rejection H_0 / H_0 is true.



$$\alpha = 2P(Z < z) = 2(1 - P(Z<z)).$$

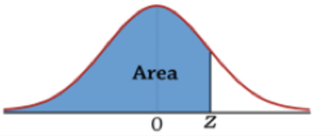
Therefore

$$P(Z<z) = P(Z < 5(X-3)) = \frac{\alpha}{2} = 0.025 \quad (1)$$

$$P\left(Z < -5(x-3)\right) = 1 - P\left(Z < 5(x-3)\right) = 0.025$$

$$P\left(Z < 5(x-3)\right) = 0.975$$

from the z-table



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

$$z = 5(x-3) = 1.96.$$

At $\alpha = 0.05$ we reject Ho if

$$z < -1.96 \quad \text{or} \quad z > 1.96.$$

b) $\mu = 5$ for 100 samples

$$z = \frac{5-3}{2/10} = 10.$$

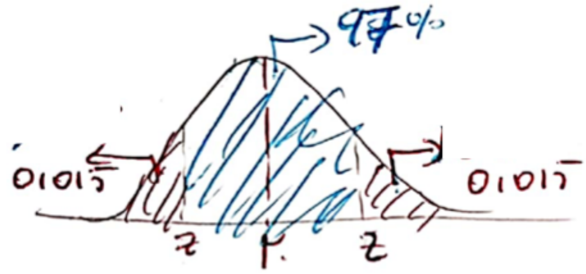
$$p = P(Z > z) = P(Z > 10) = 0 //$$

since $p < \alpha$, we should reject Ho.

2) Suppose that $x_1, x_2, \dots, x_n$ are identically independent distributed rvs. and drawn from $N(\mu, \sigma^2)$.

Suppose that a data set is taken and we have $n=49$ samples with $\bar{\mu} = 92$: sample mean, and $\bar{\sigma} = 0{,}75$ sample standard deviation.

Find a 97% confidence interval for μ.



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0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517
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2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951

$$P(Z < -z) = 1 - P(Z < z) = 0{,}015$$

$$z = \frac{X - 92}{0{,}75/\sqrt{n}} = \frac{X - 92}{0{,}75/7}$$

from the z-table

$$z = \frac{7(X - 92)}{0{,}75} = 2{,}17$$

$$P\left(-2{,}17 < \frac{(X-92)}{0{,}75/7} < 2{,}17\right) = 0{,}97$$

$$\left(92 - \frac{0{,}75}{7} \cdot 2{,}17 < X < 92 + \frac{0{,}75}{7} \cdot 2{,}17\right)$$

$$[\,91{,}7675 ; 92{,}2325\,]$$

3) The manufacturer of the patent medicine claimed that it was 90% effective in relieving an allergy for a period of 8 hours. In a sample of 200 people who had the allergy, the medicine provided relief for 160 people.

a) Determine whether the manufacturer's claim is legitimate by using 0.01 as the level of significance.

b) Find the P-value of the test.

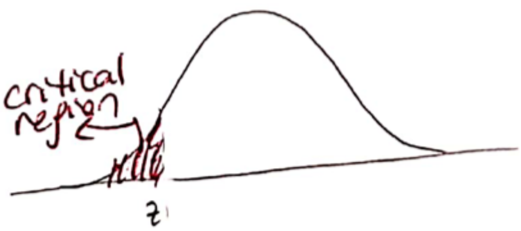
soln

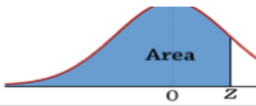
Let p denote the probability of obtaining relief by using medicine.

$H_0: p = 0.9$ and the claim is correct.

$H_1: p < 0.9$ and the claim is false

one-tailed test





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2.5	0.9938	0.9940	0.9941	0.9943	0.9945
2.6	0.9953	0.9955	0.9956	0.9957	0.9959
2.7	0.9965	0.9966	0.9967	0.9968	0.9969

$P(Z < z) = 1 - P(Z < z) = \alpha = 0.01$

$P(Z < -z) = 0.99$

from the z table

$z = 2.33$

Therefore,

If $Z < -2.33$, we reject H_0.

Since

$z = \dfrac{X - \mu}{\sigma}$

$z = \dfrac{160 - 180}{4.23} = -4.73.$

and

$-4.73 < -2.33$

we reject H_0.

$\mu = np = 200 \cdot 0.9 = 180$

$\sigma = \sqrt{nqp} = \sqrt{200 \cdot 0.9 \cdot 0.1} = 4.23$

3 b) P-value of the test is

$P(Z \leq -4.73) \approx 0$ ↳ from the Z-table.

which shows that the claim is almost certainly false.

If Ho were true, it is almost certain that a random sample of 200 patients who used the medicine would include more than 160 people who found relief.