Name:

No:

16.10.2007

GUIDED WAVES AND FIELDS

Quiz 1

1. Write the Maxwell's equations

$$1.\ rot\vec{E} + \frac{\partial}{\partial t}\vec{B} = 0 \qquad 3.\ div\vec{D} = \rho$$

$$2.\ rot\vec{H} - \frac{\partial}{\partial t}\vec{D} = \vec{J}_v + \sigma\vec{E} \qquad 4.\ div\vec{B} = 0$$

2. Write brief expressions for the following quantities:

a) Wave equation $\longrightarrow \Delta u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$

b) Complex wave number $\rightarrow k^2 = \omega^2 \varepsilon\mu + i\omega\sigma\mu$

c) Phase velocity $\rightarrow v = \frac{\omega}{|grad\, \alpha(\vec{r})|}$

d) Wavelength $\Rightarrow \lambda = \frac{2\pi}{k}$

3. Write the following expression in time domain.

$u(x) = -i.e^{i(2y+3x)}$

4. Write the following expression in complex domain.

$u(x,y,t) = \sin(2x-3y+t)$

5. The electric field of an electromagnetic wave is given by:

$E_x(\vec{r},t) = \sqrt{2}E_0 \sin 3z \cos\omega t$, $\qquad E_y = E_z = 0$, $\qquad (\rho \equiv 0,\ J_v \equiv 0,\ \sigma \equiv 0)$

a) Find $\vec{H}(\vec{r},t)$.

b) Find the propagation direction $\vec{n}$.

c) Find the relative dielectric permittivity ε_r.

$$3.\ u(x,t) = Re\left\{\underbrace{-i.e^{i(2y+3x)}}_{u(x)}.e^{-i\omega t}\right\} = Re\left\{e^{-i\frac{\pi}{2}}.e^{i(2y+3x)}.e^{-i\omega t}\right\}$$

$$= Re\left\{e^{i(2y+3x-\frac{\pi}{2}-\omega t)}\right\} = \cos\left(2y+3x-\frac{\pi}{2}-\omega t\right)$$

$$= \boxed{\sin(2y+3x-\omega t)}$$

4. $u(x,y,t) = \sin(2x-3y+t) = \cos\left(2x-3y+t-\frac{\pi}{2}\right) = \cos\left(-2x+3y-t+\frac{\pi}{2}\right)$

$$= \mathrm{Re}\left\{ e^{i\left(-2x+3y+\frac{\pi}{2}-t\right)} \right\} = \mathrm{Re}\left\{ \underbrace{e^{i\frac{\pi}{2}} \cdot e^{i(-2x+3y)}}_{u(x,y)} \cdot e^{-it} \right\}$$

$$\Rightarrow \boxed{u(x,y) = i \cdot e^{i(-2x+3y)}}$$

5. $\vec{E}_x(\vec{r},t) = \sqrt{2}\, E_0 \sin 3z \cos\omega t \, \vec{e}_x = \sqrt{2}\, E_0 \sin 3z \cos(-\omega t) \cdot \vec{e}_x$

$$= \mathrm{Re}\left\{ \sqrt{2}\, E_0 \sin 3z \cdot e^{-i\omega t} \cdot \vec{e}_x \right\}$$

$$\Rightarrow \boxed{\vec{E}_x(\vec{r}) = \sqrt{2}\, E_0 \sin 3z \cdot \vec{e}_x}$$

a) $\mathrm{rot}\,\vec{E}(\vec{r}) - i\omega\mu \vec{H}(\vec{r}) = 0 \quad \Rightarrow \quad \vec{H}(\vec{r}) = \frac{1}{i\omega\mu}\, \mathrm{rot}\,\vec{E}(\vec{r})$

$$\vec{H}(\vec{r}) = \frac{1}{i\omega\mu} \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \frac{1}{i\omega\mu}\left(\frac{\partial}{\partial z} E_x \vec{e}_y - \cancelto{0}{\frac{\partial}{\partial y} E_x \vec{e}_z} \right)$$

$$\begin{matrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{matrix}$$

$$= \frac{1}{i\omega\mu} \cdot \frac{\partial}{\partial z} \sqrt{2}\, E_0 \sin 3z \cdot \vec{e}_y = \boxed{\frac{3\sqrt{2}\, E_0}{i\omega\mu} \cos 3z \cdot \vec{e}_y}$$

$$\Rightarrow \vec{H}(\vec{r},t) = \mathrm{Re}\left\{ \frac{3\sqrt{2}\, E_0}{i\omega\mu} \cos 3z \cdot e^{-i\omega t} \cdot \vec{e}_y \right\} = \mathrm{Re}\left\{ \frac{3\sqrt{2}\, E_0}{i\omega\mu} \cos 3z \cdot e^{i\left(-\omega t - \frac{\pi}{2}\right)} \cdot \vec{e}_y \right\}$$

$$= \frac{3\sqrt{2}\, E_0}{\omega\mu} \cos 3z \cdot \cos\left(-\omega t - \frac{\pi}{2}\right) \cdot \vec{e}_y = \boxed{\frac{3\sqrt{2}\, E_0}{\omega\mu} \cos 3z \cdot \sin\omega t \cdot \vec{e}_y}$$

b) Dalganın ilerleme yönü, enerjinin taşındığı yönle aynı olduğundan,

$$\vec{n} \parallel \vec{P} = \frac{1}{2} \vec{E} \times \vec{H}^*$$

$$\vec{n} = \vec{e_x} \times \vec{e_y} = \boxed{\vec{e_z}}$$

c) $\Delta E(\vec{r}) + k^2 E(\vec{r}) = 0$

$$\frac{dE_x}{dz} = 3\sqrt{2} E_0 \cos 3z$$

$$\frac{d^2E_x}{dz^2} = -9\sqrt{2} E_0 \sin 3z$$

$$-9\sqrt{2} E_0 \sin z + k^2 . \sqrt{2} E_0 \sin z = 0$$

$$\Rightarrow k^2 = 9 = \omega^2 \varepsilon \mu = \omega^2 \varepsilon_0 \varepsilon_r \mu_0 \cancel{\mu_r}^{1}$$

$$\Rightarrow \boxed{\varepsilon_r = \frac{9}{\omega^2 \varepsilon_0 \mu_0}}$$