Quiz 2 - Solutions

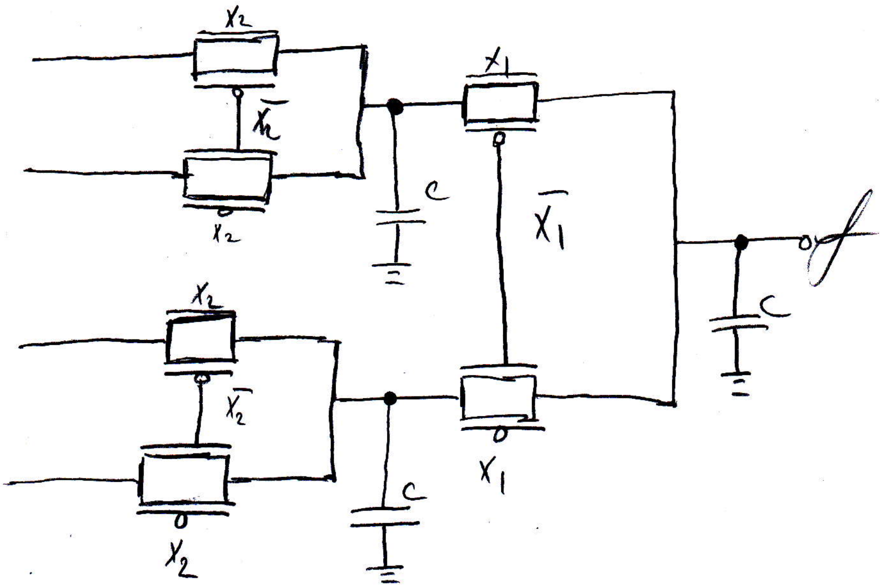
1) $f = x_1\bar{x}_3 + \bar{x}_1\bar{x}_2x_3 + x_2\bar{x}_3$ → missing variables added for shannon expansion.

$$= x_1\bar{x}_3(x_2+\bar{x}_2) + \bar{x}_1\bar{x}_2x_3 + x_2\bar{x}_3(x_1+\bar{x}_1)$$

$$= x_1x_2\bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + \bar{x}_1\bar{x}_2x_3 + \cancel{x_1x_2\bar{x}_3} + \bar{x}_1x_2\bar{x}_3$$

$$= x_1(x_2\bar{x}_3 + \bar{x}_2\bar{x}_3) + \bar{x}_1(\bar{x}_2x_3 + x_2\bar{x}_3)$$

$$= x_1(x_2(\bar{x}_3) + \bar{x}_2(\bar{x}_3)) + \bar{x}_1(x_2(\bar{x}_3) + \bar{x}_2(x_3))$$



worst case → 2 pass transistors → each dual section must have either both (4 transistors total) open or both closed transistors, so we can use $R_n // R_p$ for each dualics.

$$t_{PLH} = t_{PHL} = 0.69((R_n//R_p).C + 2(R_n//R_p).C)$$

$$= 3(R_n//R_p) = 40.10^3$$

$$\frac{1}{R_n} + \frac{1}{R_p} = \frac{1}{R_n//R_p} \rightarrow \frac{(W/L)_n}{12k} + \frac{(W/L)_p}{24k} = \frac{1}{4k} \rightarrow 2(W/L)_n + (W/L)_p = 6$$

→ now we can check possible values for $(W/L)_n$ and $(W/L)_p$ and find an approximate value or interval.

$\rightarrow (W/L)_n' \quad (W/L)_p$

$(W/L)_n$	$(W/L)_p$	
0	6	$\longrightarrow$ X
0,5	5	$\longrightarrow$ 3,5
1	4	$\longrightarrow$ 5
1,5	3	$\longrightarrow$ 4,5
2	2	$\longrightarrow$ 4
2,5	1	$\longrightarrow$ 3,5 $\longrightarrow$ min (W/L) = 1 ✓✓
2,75	0,5	$\longrightarrow$ 3,25
2,85	0,1	$\longrightarrow$ 3,05

$\rightarrow$ as $(W/L)_n$ converges to 3 and $(W/L)_p$ to '0' we can have minimum as 3 but this not a practical solution for real devices.

2-)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

$\rightarrow$ X: output of first part on circuit.

$X = AB + \bar{A}\bar{B}$, $Sum = X.Cin + \bar{X}.0 = X.Cin$

$Cout = \bar{X}.Cin + X.A$

- First we inspect delay situations on node 'X'.

$\rightarrow$ Delays occured on 'sum' and 'Cout' are sum of delay of 'X' and transistors before them.

$\rightarrow$ Different cases for X should be inspected

$AB \rightarrow A'B'$		X	
X X $\longrightarrow$	1 1	1	$\rightarrow t_{PLH(x)} = 0.69.(R_n/2).C = 41.4\,ns$
1 0 $\longrightarrow$	0 0	1	$\rightarrow t_{PLH(x)} = 0.69.(R_p.C + 2R_p.C) = 496{,}8\,ns$ ✓✓
0 1 $\longrightarrow$	0 0	1	$\rightarrow t_{PLH(x)} = 0.69.(2.R_p.C) = 331{,}2\,ns$
X X $\longrightarrow$	0 1	0	$\rightarrow t_{PHL(x)} = 0.69.(R_n.C) = 82{,}8\,ns$ ✓✓
X X $\longrightarrow$	1 0		

$\rightarrow$ Highest delay values for HL and LH.

worst cases: after calculating each delay value on X, we can consider 'sum' and 'cout' values and their t_{PHL} and t_{PLH} values.

Cout $t_{PHL}(Cout) = \underset{AB \rightarrow (10 \rightarrow 00)}{t_{PLH}(x)} + 0{,}69.R_n.C = 579\,ns \rightarrow$ $t_{PLH}(x)$ and A=0 but for t_{PLH}, A must be 1, so that situation wouldn't occur.

$t_{PLH}(Cout) = t_{PHL}(x) + 0.69.R_p.C = 248{,}4\,ns$ ✓✓

$\underset{AB \rightarrow (XX \rightarrow 11)}{t_{PLH}(x)} + 0{,}69.R_n.C = 123{,}6\,ns$ XX

Sum $t_{PHL}(sum) = \underset{AB \rightarrow (10 \rightarrow 00)}{t_{PLH}(x)} + 0{,}69.R_n.C = 579\,ns$ $\rightarrow$ $t_{PHL}(x) + 0{,}69 R_p C$ could be used but it would have lower value.

$t_{PLH}(sum) = \underset{AB \rightarrow (10 \rightarrow 00)}{t_{PLH}(x)} + 0.69.R_n.C = 579\,ns$