– EHB 335E – ANALOG ELECTRONICS – 11219 –

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EHB 335E HW#7

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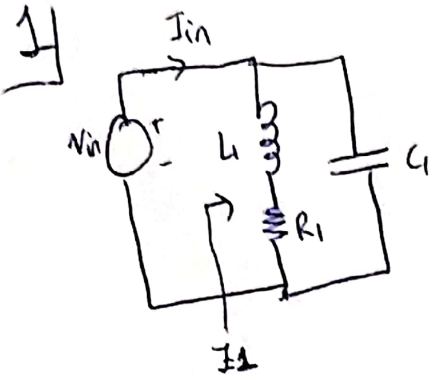
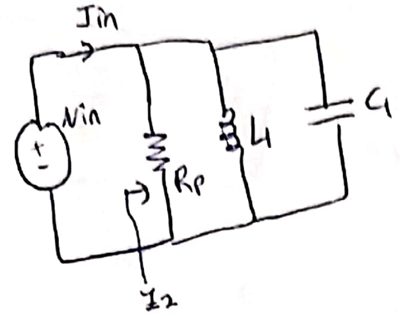
1)





Let us calculate Z_1 first; $(sL_1+R_1) \parallel \frac{1}{sC_1}$

$$Z_1(s) = (sL_1+R_1) \parallel \frac{1}{sC_1} = \frac{\frac{sL_1+R_1}{sC_1}}{sL_1+R_1+\frac{1}{sC_1}} = \frac{sL_1+R_1}{s^2L_1C_1+sC_1R_1+1}$$

$$Z_1(j\omega) = \frac{j\omega L_1+R_1}{(1-\omega^2L_1C_1)+j\omega C_1R_1} = \frac{(R_1-\omega^2R_1L_1C_1+\omega^2C_1R_1L_1)}{(1-\omega^2L_1C_1)^2+\omega^2C_1^2R_1^2} + j\frac{\omega L_1(1-\omega^2L_1C_1)-\omega C_1R_1^2}{(1-\omega^2L_1C_1)^2+\omega^2C_1^2R_1^2}$$

$$((1-\omega^2L_1C_1)-j\omega C_1R_1)$$

Also, calculate $Z_2(s) = R_p \parallel sL_1 \parallel \frac{1}{sC_1} = R_p \parallel \left(\frac{L_1/C_1}{sL_1+\frac{1}{sC_1}}\right) = R_p \parallel \left(\frac{sL_1}{s^2L_1C_1+1}\right)$

$$Z_2(s) = \frac{sR_pL_1}{s^2L_1C_1R_p+R_p+sL_1} \Rightarrow Z_2(j\omega) = \frac{j\omega L_1R_p}{R_p(1-\omega^2L_1C_1)+j\omega L_1} \times \frac{R_p(1-\omega^2L_1C_1)-j\omega L_1}{R_p(1-\omega^2L_1C_1)-j\omega L_1}$$

$$Z_2(j\omega) = \frac{\omega^2L_1^2R_p}{R_p^2(1-\omega^2L_1C_1)^2+\omega^2L_1^2} + j\frac{R_p^2\omega L_1(1-\omega^2L_1C_1)}{R_p^2(1-\omega^2L_1C_1)^2+\omega^2L_1^2}$$

→ By equating real and imaginary parts we can find a solution or by simply

$$\frac{1}{\frac{1}{R_1+j\omega L_1}+j\omega C_1} = \frac{1}{\frac{1}{R_p}+\frac{1}{j\omega L_1}+j\omega C_1} \Rightarrow \frac{1}{R_1+j\omega L_1} = \frac{1}{R_p}+\frac{1}{j\omega L_1}$$

$$\frac{1}{R_1+j\omega L_1} = \frac{j\omega L_1+R_p}{j\omega L_1R_p}$$

$$Z_1(j\omega) = Z_2(j\omega)$$

↳ By equating these impedances

$$\cancel{j\omega L_1R_p} = j\omega L_1R_1+R_pR_1-\omega^2L_1^2+\cancel{j\omega L_1R_p}$$

$$R_pR_1 = -j\omega L_1R_1+\omega^2L_1^2$$

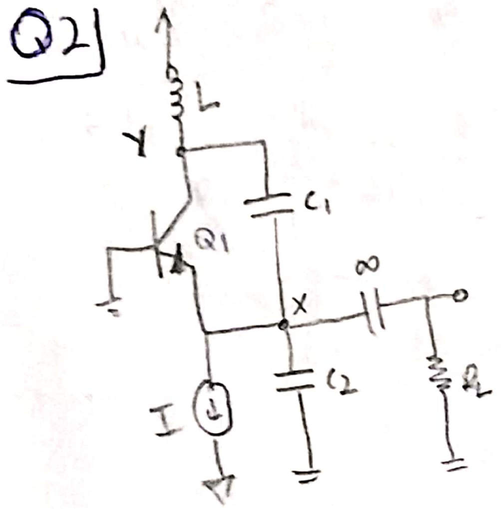
$$R_p = -j\omega L_1+\frac{\omega^2L_1^2}{R_1}$$

$$R_P = -j\omega L_1 + \frac{\omega^2 L_1^2}{R_1}$$

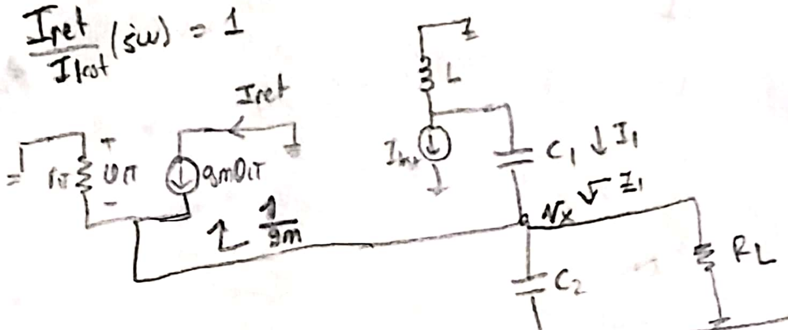
$$R_P = -j\omega L_1 \left(1 - j\frac{\omega L_1}{R_1}\right)$$ we know that $\frac{j\omega L_1}{R_1} \gg 1$

$$R_P \approx -j\omega L_1\left(-j\frac{\omega L_1}{R_1}\right) \Rightarrow \boxed{R_P \approx \frac{\omega^2 L_1^2}{R_1}}$$

Q2)



- Break the loop at node Y.
- $\frac{I_{ret}}{I_{test}}(j\omega) = 1$



$I_{ret} = g_m v_\pi = -g_m v_x = -g_m I_1 Z_1$; $\boxed{Z_1 = \frac{1}{Y_{Q_1}+Y_{C_2}+Y_{R_L}} = \frac{1}{g_m+sC_2+1/R_L}}$

$I_{ret} = -g_m \frac{1}{g_m+sC_2+1/R_L} I_1$ $\qquad I_1 = -I_{test}\cdot\frac{sL}{\frac{1}{sC_1}+sL+Z_1}$

$$I_{ret} = -g_m\frac{1}{g_m+sC_2+1/R_L}\left(-I_{test}\frac{sL}{\frac{1}{sC_1}+sL+\frac{1}{g_m+sC_2+1/R_L}}\right)$$

$$\frac{I_{ret}}{I_{test}} = \frac{g_m L s}{\left(g_m+sC_2+(1/R_L)\right)\left(\frac{1}{sC_1}+sL\right)+1} = \frac{g_m L s}{\frac{g_m}{sC_1}+g_m sL+\frac{C_2}{C_1}+s^2C_2L+\frac{1}{sR_LC_1}+\frac{sL}{R_L}+1}$$

$$\frac{I_{ret}}{I_{test}}(j\omega) = \frac{j\omega g_m L}{\frac{g_m}{j\omega C_1}+j\omega g_m L+\frac{C_2}{C_1}-\omega^2C_2L+\frac{1}{j\omega R_LC_1}+\frac{j\omega L}{R_L}+1} = 1$$

↳ we only want this term to be left so that $\frac{j\omega g_m L}{j\omega g_m L} = 1$

Therefore, imaginary and real parts in the denominator should be zero.

$$\Rightarrow \left(\frac{\omega L}{R_L}-\frac{g_m}{\omega C_1}-\frac{1}{\omega R_LC_1}=0\right) \quad ; \quad \left(\frac{C_2}{C_1}+1-\omega^2C_2L=0\right)$$

↳ Imaginary Part $\qquad$ ↳ Real Part

$$\Rightarrow \frac{C_2}{C_1} + 1 = w^2 C_2 L$$

$$\frac{C_1 + C_2}{C_1} = w^2 C_2 L \Rightarrow w^2 = \frac{C_1 + C_2}{C_1 C_2 L} \Rightarrow W = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}}$$

$\hookrightarrow$ oscillation frequency.

$$\Rightarrow \frac{wL}{R_L} - \frac{g_m}{wC_1} - \frac{1}{wR_L C_1} = 0$$

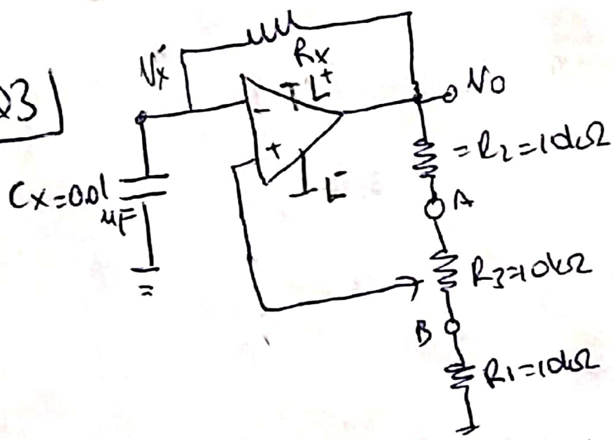
$$\frac{wL}{R_L} = \frac{1}{wC_1}\left(g_m + \frac{1}{R_L}\right) \Rightarrow w^2 L C_1 = g_m R_L + 1$$

$$\frac{C_1 + C_2}{\cancel{C_1} C_2 \cancel{L}} \cancel{L} \cancel{C_1} = g_m R_L + 1$$

$$\frac{C_1}{C_2} + \cancel{1} = g_m R_L + \cancel{1} \Rightarrow \boxed{g_m R_L = \frac{C_1}{C_2}}$$

$\Rightarrow$ To ensure the oscillation $\boxed{g_m R_L \geq \frac{C_1}{C_2}}$ $\rightarrow$ oscillation condition.

Q3)



We know saturation voltages $L^+ = -L^- = 10V$ and since this is an astable multivibrator, the switching voltage is $N_x = L^+ \cdot \frac{20k\Omega}{30k\Omega} = \frac{2}{3} L^+$

$$N_x = L^+ - \left(L^+ + \frac{2}{3} L^+\right) e^{-t/RC}$$

$$\frac{2}{3} L^+ = L^+ - \left(\frac{5}{3} L^+\right) e^{-t/RC}$$

$$\frac{1}{3} \cancel{L^+} = \frac{5}{3} \cancel{L^+} e^{-t/RC} \Rightarrow e^{-t/RC} = \frac{1}{5}$$

let's take the natural logarithm of both sides $\ln\left(e^{-t/RC}\right) = -\ln 5 \Rightarrow t/RC = \ln 5 \Rightarrow t = RC \ln 5$

$$t = \frac{T}{2} = \frac{1}{2 f_{osc}} = \frac{1}{2 \times 500} = 10^{-3} s \Rightarrow R_x C_x = \frac{t}{\ln 5} \Rightarrow \frac{10^{-3}}{\ln 5} = R_x C_x = 0.621 \text{ ms}$$

a) $$R_x = \frac{0.621 \times 10^{-3}}{C_x} = \frac{0.621 \times 10^{-3}}{0.01 \times 10^{-6}} = 62.1 \times 10^3 \Omega = 62.1 k\Omega \; ; \; \boxed{R_x = 62.1 k\Omega}$$

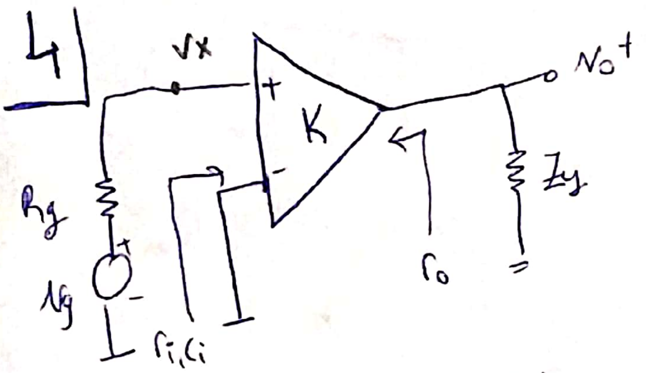
b) Now, potentiometer is connected to node B.

$$N_x = L^+ \cdot \frac{10}{30} = \frac{1}{3} L^+ \Rightarrow N_x = L^+ - \left(L^+ + \frac{1}{3} L^+\right) e^{-t/R_x C_x} \Rightarrow \frac{1}{3} L^+ = L^+ - \frac{4}{3} L^+ e^{-t/R_x C_x} \Rightarrow e^{-t/R_x C_x} = \frac{1}{2}$$

$$\Rightarrow \frac{t}{R_x C_x} = -\ln 2 \Rightarrow t = -\ln 2 (R_x C_x) = -\ln 2 (62.1 \times 10^3)(10^{-8}) = 4.3 \times 10^{-4} s \; ; \; t = \frac{T}{2} = \frac{1}{2 f_{osc}}$$

$$\Rightarrow f_{osc} = \frac{1}{2t} = \frac{1}{2(4.3 \times 10^{-4})} = 1.1628 kHz \; ; \; \boxed{f_{oscillation} = 1.1628 kHz}$$

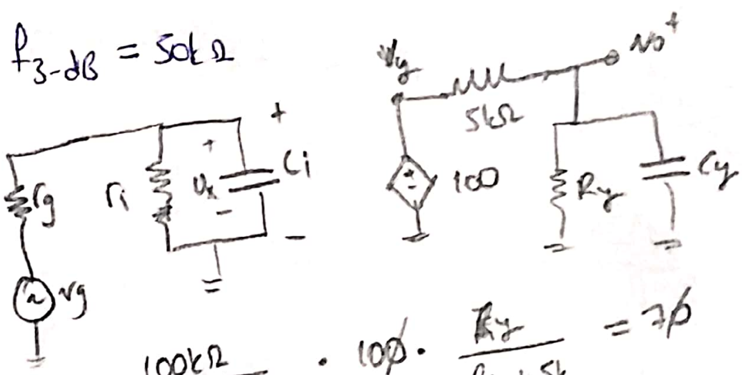
4)



$A_{v,ul} = 100$; $r_i = 100k\Omega$; $C_i = 50pF$

$r_o = 5k\Omega$, if Z_y connected $\Rightarrow A_{v,l} = 70$

$f_{3-dB} = 50k\Omega$



When Z_y is connected

$$A_{v,L} = \frac{V_o^+}{V_g} = \frac{V_x}{V_g} \cdot \frac{V_y}{V_x} \cdot \frac{V_o^+}{V_y} = \frac{100k\Omega}{100k\Omega + 5k\Omega} \cdot 100 \cdot \frac{R_y}{R_y + 5k} = 70$$

$$\Rightarrow \frac{R_y}{R_y + 5k} = \frac{7}{10} \cdot \frac{105}{100} = 0.735 \Rightarrow 0.735 R_y + 3675 = R_y$$

$$3675 = 0.265 R_y$$

$$R_y = \frac{3675}{0.265} = 13.868 \ k\Omega$$

- $\tau_{in} = R_{in} C_{in} = 50pF\ (100k \parallel 5k\Omega)$
  $= 5 \times 10^{-11} \left(\frac{5 \times 10^8}{105 \times 10^3} \right) = 0.238\ \mu s$

- $f_{in} = \frac{1}{2\pi \tau_{in}} = \frac{1}{2\pi (0.238 \mu s)} = 668.45\ kHz$

- $\tau_{out} = R_{out} C_{out} = (13.868k\Omega \parallel 5k\Omega)\ C_{out} \Rightarrow C_{out} = \frac{1}{\tau_{out}\,(13.868k\Omega \parallel 5k\Omega)}$

$\frac{1}{2\pi \tau_{out}} = 50k\Omega \Rightarrow \tau_{out} = 3.183\ \mu s$

$C_{out} = 866.11\ pF$

We should compensate the output pole. By using the inductor formula given as;

$L = \frac{R_A R_B^2 C_{out}}{2 r_i}$; $R_A = R_g + r_i = 5k\Omega + 13.868k\Omega = 18.868k\Omega$

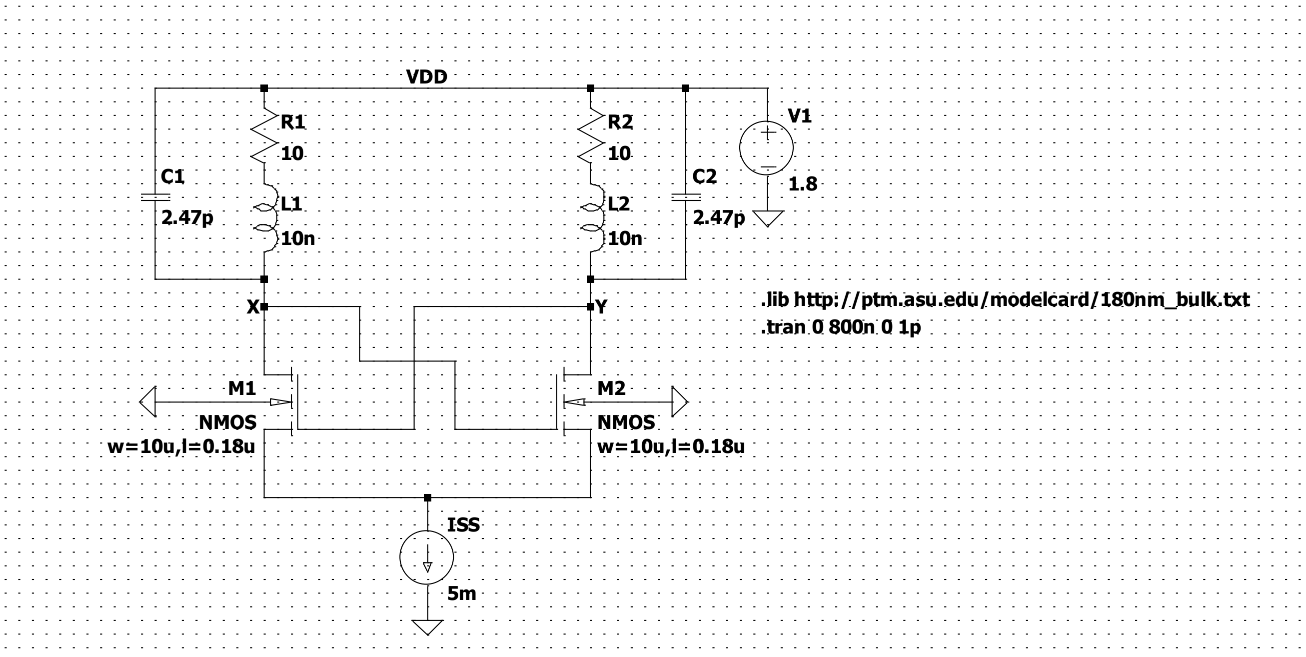
$R_B = R_g \parallel r_i = 5k\Omega \parallel 13.868k\Omega = 3.675k\Omega$

$$L = \frac{(18.868k\Omega)(3.675k\Omega)^2 (866.11 \times 10^{-12})}{2(13.868k\Omega)} = 7.957 \times 10^{-3} H$$

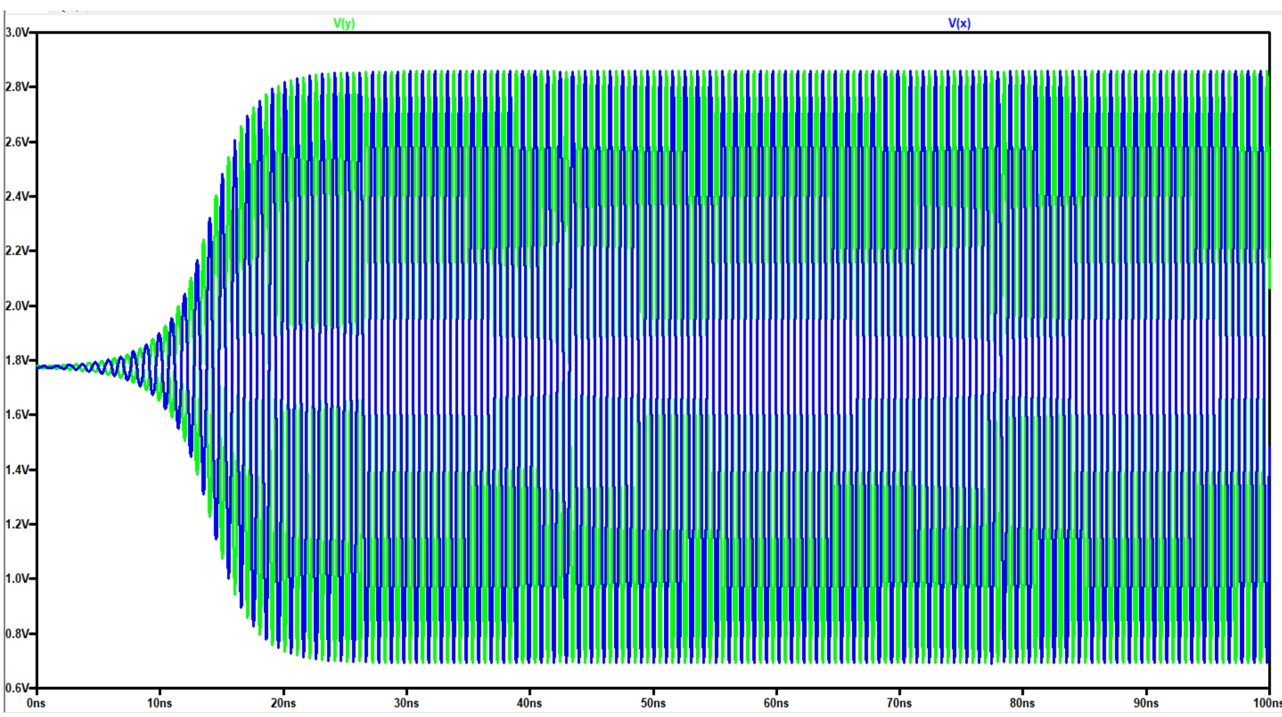
$\approx 7.96mH$

4TH QUESTION IS SIMULATED USING LTSPICE SIMULATION PROGRAM

First, we draw the circuit, and import the models as requested in the question.

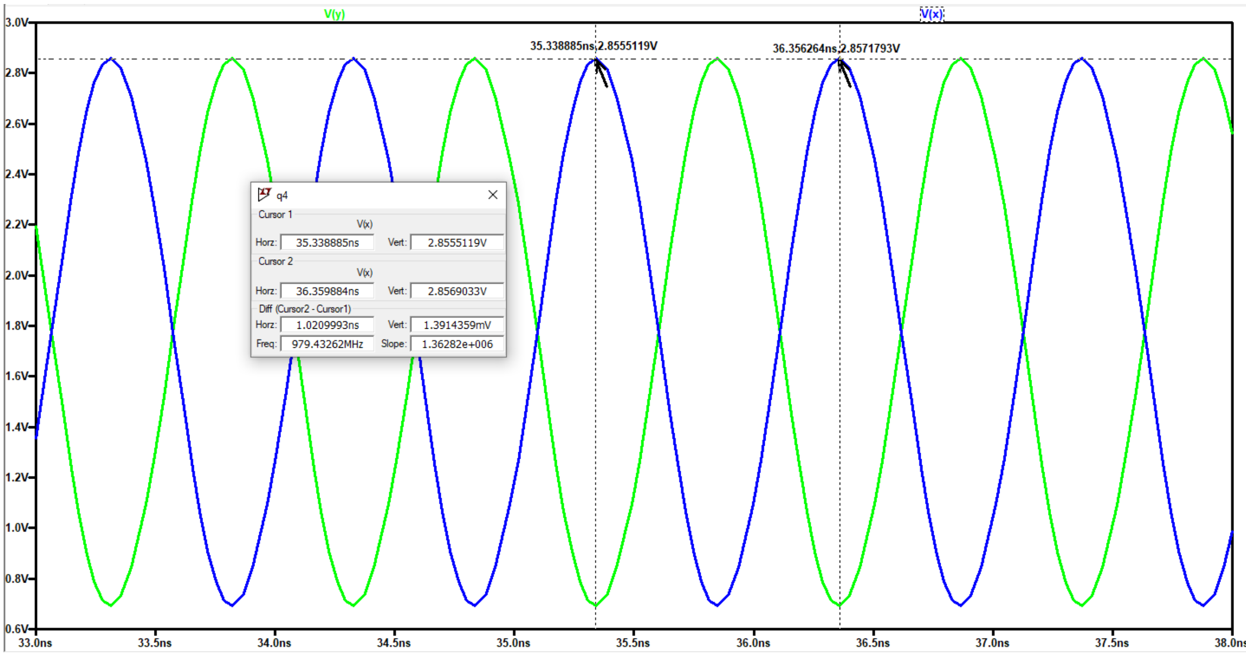


Then, since we are looking for the 1GHZ of oscillation frequency, we should find a period of 1ns. Therefore, we are starting to record the data points by the .tran 0 200n 100n transient analysis command because it requires some time for the circuit to reach the steady-state. I thought 100ns will be adequate.



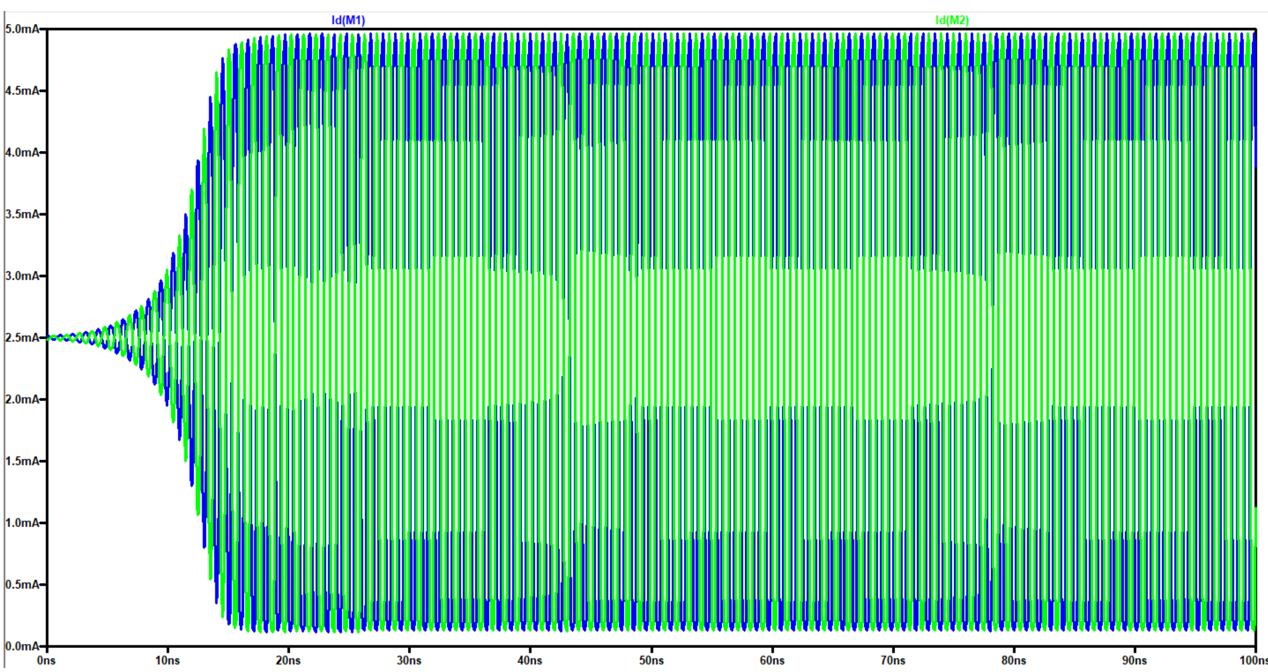
As it can be seen above that, 100ns of time is required for the circuit to start to oscillate.

Since I want to measure the time between two peak points of the sinusoidal waveform, I have to zoom in an area that I can easily show the time between two peaks.

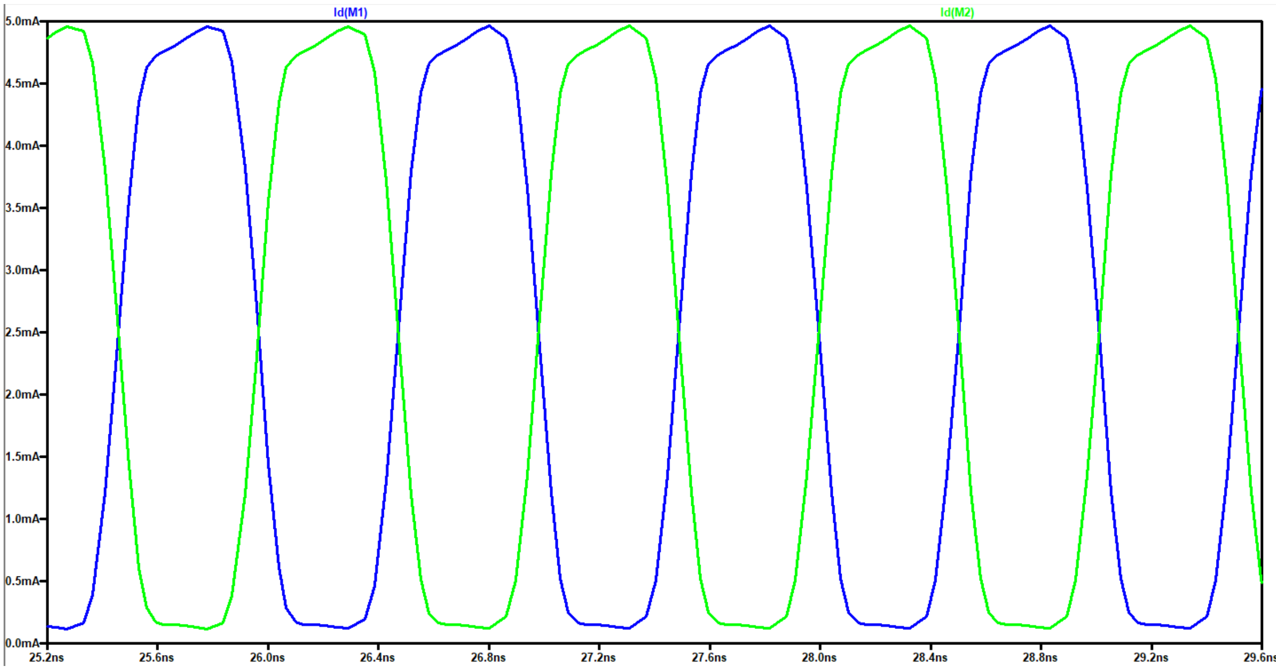


As it can be seen from the cursors, the time between two peaks of the sinusoid is approximately 1ns, which is corresponding to 1GHZ of oscillation frequency. Namely, for a capacitor value of 2.47 pF, we get a 1 GHZ of oscillation frequency.

Drain currents of M1 and M2 can be seen from the figure:



If we zoom in an area to see the steady state sinusoidal, we see these waveforms as shown



Now, let’s investigate the tail current value that ceases the oscillation. In order to find this value, we decrease the value of Iss by small steps until we don’t get an oscillation at the output.

For Iss = 2.12 mA, the output still oscillates but for the values lower than 2.12 mA, oscillation ceases.