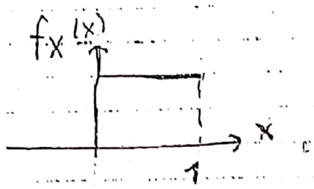
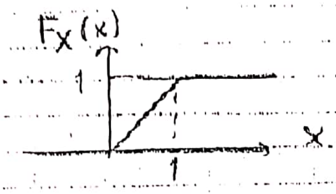
$X \rightarrow \boxed{g(?)} \rightarrow Y$, uniform distribution $[0,1]$ arasında uniform dağılsın istiyoruz.





Hangi $g(?)$ için $F_Y(y) = y$ olur?

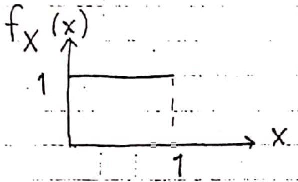
Given this for checking

$$F_Y(y) = P[Y \leq y]$$
$$= P[g(X) \leq y] \qquad g(X) = F_X(x)$$
$$= P[F_X(x) \leq y] \qquad \text{CDF of } X \rightarrow \text{invertible}$$
$$= P[F_X^{-1}(F_X(x)) \leq F_X^{-1}(y)]$$

CDF'i y olan dağılım

uniform dağılım sanırım

$$= P[X \leq F_X^{-1}(y)]$$
$$= F_X(F_X^{-1}(y)) = y$$



random var 0 ve 1 arasında değerler alabilir ve aynı değeri alırlar.

A Function of Random Var Bitti

Sınavda Kesin Var

Bir şeyin randomnessliğini varyans arttırır.

Mean, $E[x]$ (expected value), μ_X, $\mu(x)$, Mean of X
(beklenen değer)
ortalama

σ_X : Standard Deviation $\qquad \sigma_X^2$: variance

$$E[X] = \int_{-\infty}^{\infty} x \, f_X(x)\, dx$$

linear operator RV

$$\sigma_X^2 = E[\underbrace{(X-\mu_X)}_{Y}{}^2] \qquad Y = X - \mu_X$$

$$= \int_{-\infty}^{\infty} (X-\mu_X)^2 f_X(x)\, dx$$

$$E[Y] = E[X - \mu_X]$$
$$= E[X] - E[\mu_X]$$
$$= \mu_X - \mu_X = 0$$

deterministik şeyin beklenen değeri kendisidir.

centralized around origin

CamScanner ile tarand

$$E[g(X)] = \int_{-\infty}^{\infty} g(X) f_X(x)\, dx$$

21/10/16

$E[\overset{RV}{X}]$, μ_X, $\mu(X)$

expectation operator

For a cont. random var. X, μ_X is the mean

$P[X = \mu_X] = 0$

Var $[X]$, σ_X^2 $\quad \sigma_X^2 = E[(X-\mu_X)^2] = \int_{-\infty}^{\infty} (x-\mu_x)^2 f_X\, x\, dx$

$E[g(x)] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$ $\quad$ centeralized around origin

$E[(X-\mu_X)] = E[X] - E[\mu_X]$

$= \mu_X - \mu_X = 0$

$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$

$\sigma_X^2 = E[(X-\mu_X)^2]$

$= E[X^2 - 2X\mu_X + \mu_X^2]$

$= E[X^2] - 2\mu_X E[X] + \mu_X^2$

$E[X^2] = \sigma_X^2 + 2\mu_X E[X] - \mu_X^2$

$= \sigma_X^2 + \mu_X^2$

X is a RV with mean μ_X, variance σ_X^2

$Y = 2X + 2$; $E[Y]$, σ_Y^2

$E[2X+2] = E[2X] + E[2] = 2E[X] + E[2]$

$= 2\mu_X + 2$

$Y = aX + b$ $\quad \mu_Y = a\mu_X + b$

$\sigma_Y^2 = E[(Y-\mu_Y)^2] = E[(2X+2-2\mu_X-2)^2]$

$= E[(2X - 2\mu_X)^2] = E[4(X-\mu_X)^2]$

$= 4E[(X-\mu_X)^2]$

$= 4\sigma_X^2$

Variance $2^2 = 4\sigma_X^2$ oldu.

Variance has been affected only a (By sorunda)

2/11/16

Common Pdfs

$E[x] = \int x\, f(x)\, dx$

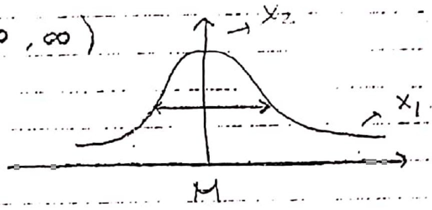
1) Gaussian: $X \sim N(\mu, \sigma^2)$ → 2^{nd} order statistics

1^{st} order statistics

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\int_{-\infty}^{\infty} f_X \, dx = 1$$

$R_X: (-\infty, \infty)$



Given $\mu = 0$, $\sigma^2 = 1$; then Gaussian Random Var. is called as Normal RV.

$Y = g(x)$; Y is $N(0,1)$

$$Y = \frac{X-\mu}{\sigma} = \frac{x}{\sigma} - \frac{\mu}{\sigma}$$

$$g(u) = \frac{u}{\mu} - \frac{\mu}{\sigma}$$

$\mu_1 = \mu_2$

$\sigma_1^2 > \sigma_2^2$

wider ↑ σ^2 ↑

very big variance ≈ uniformlu dist. benziyor

2) Exponential $\left(\text{mean} = \frac{1}{\lambda}\right)$ $\left(\sigma^2 = \frac{1}{\lambda^2}\right)$

$f_X = \lambda e^{-\lambda x} u(x)$; $R_X: [0, \infty)$ $\lambda > 0$ always

pdf için
→ hep pozitif
→ $\int_{-\infty}^{\infty} = 1$

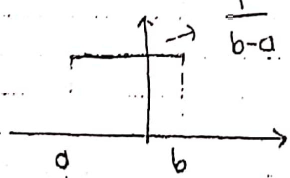
3) Gamma:

$$f_X = \frac{x^{\alpha-1}}{\sqrt{\alpha}\, \beta^{\alpha}} e^{-\frac{x}{\beta}} u(x) \quad ; \quad R_X: [0, \infty), \quad \alpha, \beta > 0$$

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx \qquad n! = \int_0^{\infty} x^n e^{-x} dx$$

$$\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$$

4) Uniform:



$E[x] = \mu_X = \frac{b+a}{2}$ $\sigma_X^2 = \frac{b^2 - a^2}{12}$ → $\frac{(a-b)^2}{12}$ olacak sanırım tekrar bul!

$$f_X(x) = \frac{1}{b-a}\left[u(x-a) - u(x-b)\right]$$

* { Gaussian
Uniformly Dist

5) Rayleigh: $\frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}} u(x)$; $R_X : [0, \infty)$

Discrete Random Variable

$X = x$, $P_X(x)$, $P_X(k)$ — dummy var.

probability mass function. RV

1) Bernoulli: $P(X=1) = p$, $P(X=0) = q = 1-p$

2) Binomial (# of successes in n independent trials).
(n is constant)

$P(X=k) = \binom{n}{k} p^k q^{n-k}$ $k = 0, 1, 2, \ldots, n$

# of succes.

3) Geometric

$P(X=k) = p \cdot q^{k-1}$; $k = 1, 2, 3 \cdots, \infty$ $p + q = 1$ ilk successte bitiriyoruz

prob. of success

4) Pascal

$P(X=k) = \binom{k-1}{r-1} p^r q^{k-r}$ r success görene kadar devam in k trials

rth successte bitiriyoruz

k nın küçük değeri = r
r nin en büyük değeri = k

5) Poisson (random events occur at a certain rate over a period of time)

mostly depends for a particular product

$P[X=x] = e^{-\mu} \frac{\mu^x}{x!}$; $x = 0, 1, 2, \ldots$ *

$E[X] = \sigma_X^2 = \mu$

Expectation for Discrete RVs

CRV $X \rightarrow f_X \rightarrow$ probability density func.

DRV $X \rightarrow P[X=k] \rightarrow$ prob. mass func.

sınırlar depends on range of the RV

$E[X] = \int_{-\infty}^{\infty} x f_X \, dx$

$E[X=x] = \sum_{x \in R_X} x \cdot P[X=x]$
E[X]

fair coin toss
H → 0
T → 1

$E[g(x)] = \int_{-\infty}^{\infty} g(x) f_X \, dx$

$E[X] = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{2}$

$E[g(x)] = \sum g(x) P[X=x]$

ex: $X \sim$ binomial (n, p). Find $E[X=x]$, $\bar{x}$, μ_x cevap: np

$\sigma^2 = np.(1-p)$

$$\boxed{E[x^2] = (1^{st} \text{moment})^2 + (2^{nd} \text{central moment})}$$

Moment

1) m-th moment (about the origin) $m \geq 1$; $m = 1, 2, 3, \ldots$

$$E[X^m] = \int x^m f_X \, dx \; / \; \sum x^m P[X=x]$$

$E[(X-0)^m]$

1st order moment: mean   2nd order moment: variance

If $m=1$; μ_x 1st moment gives us the mean

2) m-th central moment (about the mean)

$$E[(x-\mu)^m] = \int (x-\mu)^m f_X \, dx \; / \; \sum (X-\mu)^m P[X=x]$$

If $m=2$, it gives the variance

Moment Generating Function of RV X

$$\underbrace{\Phi_X(s)}_{a+bj} = E[e^{sx}] = \int_{-\infty}^{\infty} e^{sx} f_X \, dx \; / \; \sum e^{sx} P[X=x]$$

$$\frac{d\,\Phi(s)}{ds} = \frac{d}{ds} E[e^{sx}] = E\left[\frac{d}{ds} e^{sx}\right] = E[x e^{sx}]\Big|_{s=0} = E[x] = \mu_x$$

Inequalities

Markov Inequality: For non negative RV X

$a > 0$, $P[x \geq a] \leq \dfrac{E[x]}{a}$

$$E[x] \geq a \int_a^{\infty} f_X \, dx$$

$P[X \geq a]$

$$E[x] = \int_0^{\infty} x f_X \, dx \geq \int_a^{\infty} x f_X \, dx \geq a \int_a^{\infty} f_X \, dx$$

for nonnegative old için
m dan değil 0 dan

X ve f_X nonnegative old için

CamScanner ile tarand

Chebyshev Inequality:

$$P[|X-\mu| \geq a] \leq \frac{\sigma_X^2}{a^2}, \quad a>0$$

→ proof

Quiz 1) X : Head → −1 $\quad P[H] = P[T] = \frac{1}{2}$
Tail → 1

$$E[X] = \sum_{x \in R_X} P[X=x] \cdot x$$

$$= \frac{1}{2} \cdot -1 + \frac{1}{2} \cdot 1 = 0$$

unfair coin: $P[H]=p \quad P[T]=1-p$

Quiz 2)

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$Y = g(x) = \frac{X-(-2)}{\sqrt{2}} \qquad E[Y] = \frac{E[X]-(-2)}{\sqrt{2}} = 0$$

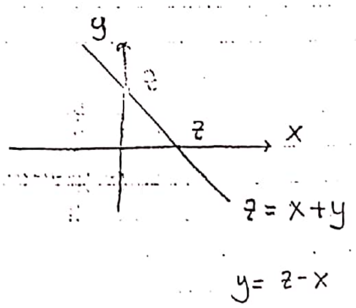
$Y \rightarrow (0, 1)$, μ, σ^2

$g(x) = Y$ → CDF of X

Function of Two Random Variables

$Z = g(X, Y)$, $\quad X+Y$, $\frac{X}{Y}$, $\min(X,Y)$, $\max(X,Y)$, $\frac{\max(X,Y)}{\min(X,Y)}$

Ex. $Z = X+Y$



$$F_Z(z) = P[Z \leq z] = P[X+Y \leq z]$$

$$F_Z(z) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{z-x} f_{XY}(x,y)\, dy\, dx$$

joint pdf of x and y

$$= \int_{y=-\infty}^{\infty} \int_{-\infty}^{z-y} f_{XY}(x,y)\, dx\, dy$$