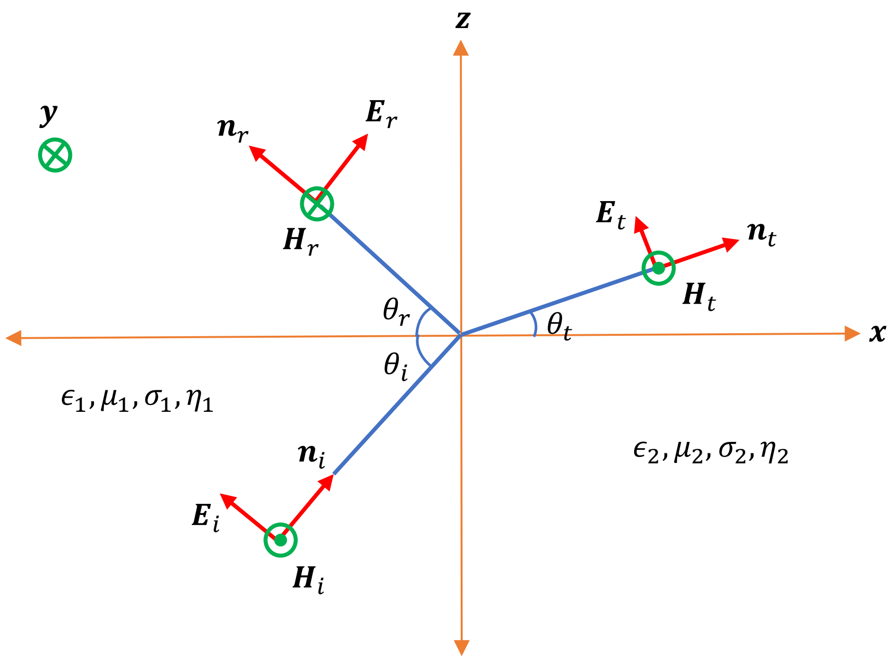
Q1. Wave reflection and transmission at oblique incidence with parallel polarization

Electric field of an EM wave is given as follows:

$$\boldsymbol{E}_i(x,z,t) = -6\cos(\omega t - 2x - 3z)\,\boldsymbol{e}_x + 4\cos\left(\omega t - 2x - 3z + \frac{\pi}{2}\right)\boldsymbol{e}_z$$

The wave are propagating in air toward a medium with $\epsilon_r = 9$ and $x \geq 0$. Both of the media are lossless and non-magnetic. Then, determine all electric and magnetic field components in both region.



A:

Write the incident field in phasor domain,

$$\boldsymbol{E}_i(z) = -6e^{j(2x+3z)}\boldsymbol{e}_x + 4e^{j\left(2x+3z-\frac{\pi}{2}\right)}\boldsymbol{e}_z$$

In order to find θ_i,

$$k_1 = \sqrt{2^2 + 3^2} = \sqrt{13} \approx 3.606$$

$$k_1 cos\theta_i = 2 \quad \Rightarrow \quad \theta_i = \cos^{-1}\left(\frac{2}{\sqrt{13}}\right) = 56.3°$$

And k_2,

$$\frac{k_2}{k_1} = \frac{\sqrt{\omega^2\epsilon_2\mu_2 + i\omega\sigma_2\mu_2}}{\sqrt{\omega^2\epsilon_1\mu_1 + i\omega\sigma_1\mu_1}} \quad \Rightarrow \quad k_2 = k_1\frac{\omega\sqrt{\mu_0 9\epsilon_0}}{\omega\sqrt{\mu_0\epsilon_0}} = 3\sqrt{13} = 10.817$$

By the Snell's law,

$$\theta_t = \sin^{-1}\left(\frac{k_1}{k_2} sin\theta_i\right) = \sin^{-1}\left(\frac{1}{3}\sin(56.3)\right) \cong 16.1°$$

We can see that the configuration has parallel polarization which means that electric field vector lies on the plane of incidence. For this case, the reflection and transmission coefficients for electric field vector can be written as,

$$\Gamma = \frac{\eta_2 cos\theta_t - \eta_1 cos\theta_i}{\eta_2 cos\theta_t + \eta_1 cos\theta_i} = \frac{\sqrt{\frac{\mu_0}{9\epsilon_0}}\cos(16.1°) - \sqrt{\frac{\mu_0}{\epsilon_0}}\cos(56.3°)}{\sqrt{\frac{\mu_0}{9\epsilon_0}}\cos(16.1°) + \sqrt{\frac{\mu_0}{\epsilon_0}}\cos(56.3°)} \cong -0.27$$

$$\mathrm{T} = (1+\Gamma)\frac{cos\theta_i}{cos\theta_t} = 0.42$$

Incident wave,

$$\boldsymbol{n}_i = cos\theta_i \boldsymbol{e}_x + sin\theta_i \boldsymbol{e}_z$$

$$\boldsymbol{E}_i(z) = -6e^{j(2x+3z)}\boldsymbol{e}_x + 4e^{j\left(2x+3z-\frac{\pi}{2}\right)}\boldsymbol{e}_z = (-6\boldsymbol{e}_x - j4\boldsymbol{e}_z)\cdot e^{j(2x+3z)}$$

$$\boldsymbol{H}_i = \frac{1}{\eta_1}\boldsymbol{n}\times\boldsymbol{E}_i = \frac{1}{\eta_0}(cos\theta_i\boldsymbol{e}_x + sin\theta_i\boldsymbol{e}_z)\times(-6\boldsymbol{e}_x - j4\boldsymbol{e}_z)\cdot e^{j(2x+3z)}$$

$$= \frac{1}{\eta_0}\left(4jcos\theta_i\boldsymbol{e}_y - 6sin\theta_i\boldsymbol{e}_y\right)\cdot e^{j(2x+3z)} = (-0.0132 + j0.0059)\cdot e^{j(2x+3z)}\boldsymbol{e}_y$$

Reflected wave,

$$\boldsymbol{n}_r = -cos\theta_r\boldsymbol{e}_x + sin\theta_r\boldsymbol{e}_z$$

$$k_1\boldsymbol{n}_r\boldsymbol{r} = k_1(-cos\theta_r\boldsymbol{e}_x + sin\theta_r\boldsymbol{e}_z)(x\boldsymbol{e}_x + z\boldsymbol{e}_z) = -2x + 3z$$

$$\boldsymbol{E}_r(z) = \Gamma\cdot(-6\boldsymbol{e}_x - j4\boldsymbol{e}_z)\cdot e^{j(-2x+3z)} = (1.62\boldsymbol{e}_x + j1.08\boldsymbol{e}_z)\cdot e^{j(-2x+3z)}$$

$$\boldsymbol{H}_r = \frac{1}{\eta_1}\boldsymbol{n}_r\times\boldsymbol{E}_r = \frac{1}{\eta_0}(-cos\theta_r\boldsymbol{e}_x + sin\theta_r\boldsymbol{e}_z)\times(1.62\boldsymbol{e}_x + j1.08\boldsymbol{e}_z)\cdot e^{j(-2x+3z)}$$

$$= \frac{1}{\eta_0}\left(j1.08\cdot cos\theta_r\boldsymbol{e}_y + 1.62\cdot sin\theta_r\boldsymbol{e}_y\right)\cdot e^{j(-2x+3z)}$$

$$= (0.0036 + j0.0016)\cdot e^{j(-2x+3z)}\boldsymbol{e}_y$$

Transmitted wave,

$$\boldsymbol{n}_t = cos\theta_t\boldsymbol{e}_x + sin\theta_t\boldsymbol{e}_z$$

$$k_2\boldsymbol{n}_t\boldsymbol{r} = 3\sqrt{13}(\cos 16.1°\,\boldsymbol{e}_x + \sin 16.1°\,\boldsymbol{e}_z)(x\boldsymbol{e}_x + z\boldsymbol{e}_z) = 10.392x + 2.999z$$

$$\boldsymbol{E}_t(z) = \mathrm{T}\cdot(-6\boldsymbol{e}_x - j4\boldsymbol{e}_z)\cdot e^{j(10.392x+2.999z)} = (-2.52\boldsymbol{e}_x - j1.68\boldsymbol{e}_z)\cdot e^{j(10.392x+2.999z)}$$

$$\boldsymbol{H}_t = \frac{1}{\eta_2}\boldsymbol{n}_t\times\boldsymbol{E}_t = \frac{3}{\eta_0}(cos\theta_t\boldsymbol{e}_x + sin\theta_r\boldsymbol{e}_z)\times(-2.52\boldsymbol{e}_x - j1.68\boldsymbol{e}_z)\cdot e^{j(10.392x+2.999z)}$$

$$= \frac{3}{\eta_0}\left(j1.68 \cdot \cos 16.1° \, \boldsymbol{e}_y - 2.52 \cdot \sin 16.1° \, \boldsymbol{e}_y\right) \cdot e^{j(10.392x+2.999z)}$$

$$= (-0.0056 + j0.0128)e^{j(10.392x+2.999z)}\boldsymbol{e}_y$$