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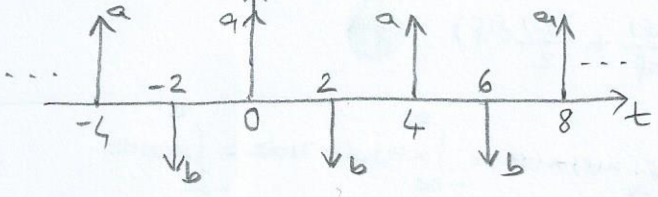
EHB 351

Analog Haberleşme

(Arasınav1 Çözümleri)

36P 1 a)

$$x(t)=\sum_n c_n e^{jn\omega_0 t}\ , \quad \omega_0=\frac{2\pi}{4}\quad (x(t)\ ,\ T_0=4 \text{ periyotlu})$$

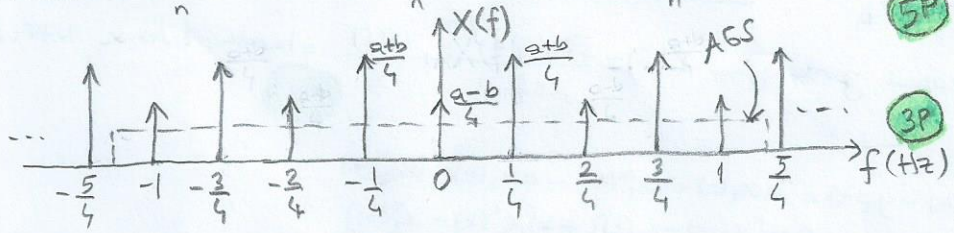


$$c_n=\frac{1}{4}\int_{T_0} x(t)e^{-jn\omega_0 t}dt=\frac{1}{4}\left[\int_{-1}^{1} a\delta(t)e^{-jn\omega_0 t}dt-\int_{1}^{3} b\delta(t-2)e^{-jn\omega_0 t}dt\right]$$

$$=\frac{1}{4}\left[a\int_{-1}^{1}\delta(t)dt-be^{-jn\frac{2\pi}{4}.2}\int_{1}^{3}\delta(t-2)dt\right]=\frac{1}{4}\left[a-be^{-jn\pi}\right]=\frac{1}{4}\left[a-b(-1)^n\right] \quad 10P$$

b) $$X(f)=\mathcal{F}\left\{\sum_n c_n e^{jn\omega_0 t}\right\}=\sum_n c_n\delta(f-nf_0)=\sum_n\frac{1}{4}\left[a-b(-1)^n\right]\delta\left(f-\frac{n}{4}\right)\ ,\quad f_0=\frac{1}{T_0}=\frac{1}{4}$$

5P



3P

c) $H(f)=\Pi(f/2.4)$

$$Y(f)=X(f)H(f)=\frac{1}{4}\Big[(a-b)\left[\delta(f)+\delta\left(f-\tfrac{1}{2}\right)+\delta\left(f+\tfrac{1}{2}\right)+\delta(f-1)+\delta(f+1)\right]$$
$$+(a+b)\left[\delta\left(f-\tfrac{1}{4}\right)+\delta\left(f+\tfrac{1}{4}\right)+\delta\left(f-\tfrac{3}{4}\right)+\delta\left(f+\tfrac{3}{4}\right)\right]\Big] \quad 5P \quad \left(=\sum_{n=-4}^{4}\frac{a-b(-1)^n}{4}\delta\left(f-\frac{n}{4}\right)\right)$$

$$y(t)=\mathcal{F}^{-1}\{Y(f)\}=\frac{1}{4}\Big[(a-b)\left[1+e^{j2\pi\frac{1}{2}t}+e^{-j2\pi\frac{1}{2}t}+e^{j2\pi t}+e^{-j2\pi t}\right]$$
$$+(a+b)\left[e^{j2\pi\frac{1}{4}t}+e^{-j2\pi\frac{1}{4}t}+e^{j2\pi\frac{3}{4}t}+e^{-j2\pi\frac{3}{4}t}\right]\Big]$$

$$=\frac{a-b}{4}+\frac{(a-b)}{2}\cos\pi t+\frac{(a-b)}{2}\cos 2\pi t+\frac{(a+b)}{2}\cos\frac{\pi}{2}t+\frac{(a+b)}{2}\cos\frac{3\pi}{2}t$$

5P

$$y(t)=\sum_n y_n e^{jn\omega_0 t}\ \Rightarrow\ y_n=\begin{cases}\frac{a-b}{4}\ , & |n|=0,2,4\\ \frac{a+b}{4}\ , & |n|=1,3\\ 0\ , & \text{dışında}\end{cases}$$

d) $$R_x(\tau)=\sum_n|c_n|^2e^{jn\omega_0\tau}=\frac{1}{16}\sum_n\left[a-b(-1)^n\right]^2e^{jn\frac{2\pi}{4}\tau} \quad 2P$$

$$G_x(f)=\sum_n|c_n|^2\delta(f-nf_0)=\frac{1}{16}\sum_n\left[a-b(-1)^n\right]^2\delta\left(f-\frac{n}{4}\right) \quad 2P$$

$$R_y(\tau)=\sum_n|y_n|^2e^{jn\omega_0\tau}=\left(\frac{a-b}{4}\right)^2+\frac{(a-b)^2}{8}\cos\pi\tau+\frac{(a-b)^2}{8}\cos 2\pi\tau+\frac{(a+b)^2}{8}\cos\frac{\pi}{2}\tau+\frac{(a+b)^2}{8}\cos\frac{3\pi}{2}\tau$$

2P

$$G_y(f)=\sum_n|y_n|^2\delta(f-nf_0)=\left(\frac{a-b}{4}\right)^2\left[\delta(f)+\delta\left(f-\tfrac{1}{2}\right)+\delta\left(f+\tfrac{1}{2}\right)+\delta(f-1)+\delta(f+1)\right]$$
$$+\left(\frac{a+b}{4}\right)^2\left[\delta\left(f-\tfrac{1}{4}\right)+\delta\left(f+\tfrac{1}{4}\right)+\delta\left(f-\tfrac{3}{4}\right)+\delta\left(f+\tfrac{3}{4}\right)\right]$$

2P

30P ② a) $\mathcal{F}\{sgn(t)\} = \frac{1}{j\pi f}$

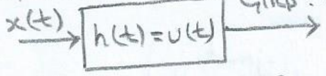
$$u(t) = \frac{1+sgn(t)}{2} \Rightarrow \mathcal{F}\{u(t)\} = \mathcal{F}\left\{\frac{1+sgn(t)}{2}\right\} = \frac{\delta(f)}{2} + \frac{1}{j2\pi f}$$ 5P

b) <u>İntegral Teoremi:</u>

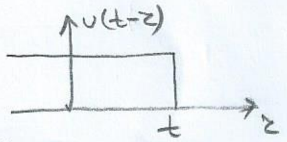
$$x(t) \longleftrightarrow X(f)$$

$$\int_{-\infty}^{t} x(\tau)d\tau \longleftrightarrow \frac{X(f)}{j2\pi f} + \frac{X(0)}{2}\delta(f)$$ 5P

<u>İspat:</u>

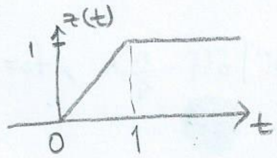


$$\text{Çıkış}: x(t)*u(t) = \int_{-\infty}^{\infty} x(\tau)u(t-\tau)d\tau = \int_{-\infty}^{t} x(\tau)d\tau$$



$$\underbrace{\mathcal{F}\{x(t)*u(t)\}}_{\mathcal{F}\left\{\int_{-\infty}^{t} x(\tau)d\tau\right\}} = \mathcal{F}\{x(t)\}\mathcal{F}\{u(t)\} = X(f)\left[\frac{1}{j2\pi f} + \frac{\delta(f)}{2}\right] = \frac{X(f)}{j2\pi f} + \frac{X(f)\delta(f)}{2} = \frac{X(f)}{j2\pi f} + \frac{X(0)}{2}\delta(f)$$ 10P //

c)



$x(t) = \Pi(t-1/2)$ olmak üzere, $z(t) = \int_{-\infty}^{t} x(\tau)d\tau$ yazılabilir.

Zamanda öteleme teoremi de kullanılarak $X(f) = sinc(f)e^{-j\pi f}$ bulunur.

İntegral teoremi yardımıyla, $Z(f) = \frac{sinc(f)e^{-j\pi f}}{j2\pi f} + \frac{\delta(f)}{2}$ olarak bulunur. NOT: $X(0)=1$. 10P

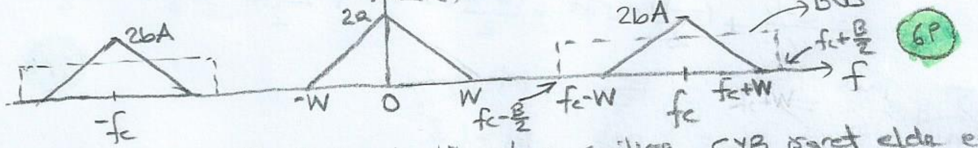
34P ③ a)

$$\begin{aligned} z(t) = y_1(t) - y_2(t) &= ax_1(t) + bx_1^2(t) - ax_2(t) - bx_2^2(t) \\ &= a[x_1(t) - x_2(t)] + b[x_1^2(t) - x_2^2(t)] \\ &= a[x_1(t) - x_2(t)] + b[x_1(t) - x_2(t)][x_1(t) + x_2(t)] \\ &= [x_1(t) - x_2(t)][a + b(x_1(t) + x_2(t))] \end{aligned}$$

$$\left.\begin{aligned} x_1(t) &= x(t) + A\cos w_c t \\ x_2(t) &= A\cos w_c t - x(t) \end{aligned}\right\} \Rightarrow \begin{aligned} x_1(t) - x_2(t) &= 2x(t) \\ x_1(t) + x_2(t) &= 2A\cos w_c t \end{aligned}$$

$$\Rightarrow z(t) = 2x(t)[a + 2bA\cos w_c t] = 2ax(t) + 4bAx(t)\cos w_c t$$ 9P

$$Z(f) = 2aX(f) + 2bA[X(f-f_c) + X(f+f_c)]$$



6P

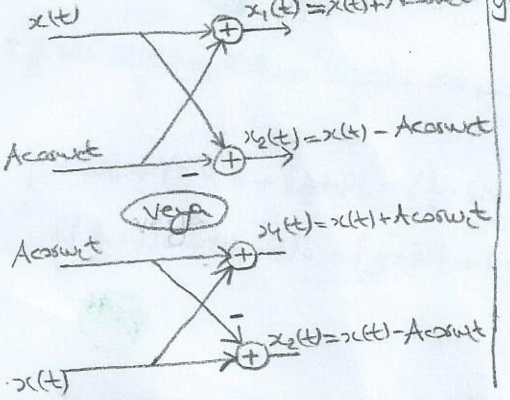
b) BGS yardımıyla $\pm f_c$ frekanslı bileşenler seçilirse GYB işaret elde edilir.

$f_c - \frac{B}{2} \geq W$ ve $B \geq 2W$ olmalı. Buradan $2(f_c - W) \geq B \geq 2W$ koşulu elde edilir.

Bir diğer koşul, $f_c - W \geq W$ ve $f_c \geq 2W$ olmasıdır. 6P

BGS'in merkez frekansı f_c, kazancı 1 seçilebilir. Bu durumda $x_c(t) = \underbrace{4bA}_{A_c}x(t)\cos w_c t$ olur. GYB işaret

c)



Sistemin giriş kısmında solda gösterilen değişiklik yapılırsa,

$$\begin{aligned} z(t) &= [x_1(t) - x_2(t)][a + b(x_1(t) + x_2(t))] \\ &= 2A\cos w_c t\,[a + 2bx(t)] \\ &= \underbrace{2aA}_{A_c}[1 + \underbrace{\tfrac{2b}{a}}_{m}x(t)]\cos w_c t \end{aligned}$$

biçiminde doğrudan GM işaret elde edilir. BGS'e de gerek kalmaz. Bu yolla karmaşıklık da azalır. 13P