Kaynak: Oppenheim, Willsky, "Signals and Systems"

Laplace Dönüşümü

$$e^{st} \longrightarrow H(s)e^{st},$$

Sistem çıkışı

$$\begin{aligned} y(t) &= \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)\,d\tau \\ &= \int_{-\infty}^{+\infty} h(\tau)e^{s(t-\tau)}\,d\tau. \end{aligned}$$

$$y(t) = e^{st}\int_{-\infty}^{+\infty} h(\tau)e^{-s\tau}\,d\tau.$$

$$y(t) = H(s)e^{st},$$

Sistemin transfer fonksiyonu

$$H(s) = \int_{-\infty}^{+\infty} h(\tau)e^{-s\tau}\,d\tau.$$

Örnek :

Giriş-çıkış ilişkisi

$$y(t) = x(t-3).$$

şeklinde verilen sistemin girişine

$$x(t) = e^{j2t},$$

İşareti uygulanması durumunda , sistem çıkışı

$$y(t) = e^{j2(t-3)} = e^{-j6}e^{j2t}.$$

Sistemin impuls cevabı

$$h(t) = \delta(t-3).$$

Sistemin transfer fonksiyonu

$$H(s) = \int_{-\infty}^{+\infty} \delta(\tau-3)e^{-s\tau}\,d\tau = e^{-3s},$$

ve

$$H(j2) = e^{-j6}.$$

bulunur.

Girişe

$$x(t) = \cos(4t) + \cos(7t).$$

işareti uygulanması durumunda, sistem çıkışı

$$y(t) = \cos(4(t-3)) + \cos(7(t-3)).$$

$$x(t) = \frac{1}{2}e^{j4t} + \frac{1}{2}e^{-j4t} + \frac{1}{2}e^{j7t} + \frac{1}{2}e^{-j7t},$$

$$y(t) = \frac{1}{2}e^{-j12}e^{j4t} + \frac{1}{2}e^{j12}e^{-j4t} + \frac{1}{2}e^{-j21}e^{j7t} + \frac{1}{2}e^{j21}e^{-j7t},$$

$$\begin{aligned} y(t) &= \frac{1}{2}e^{j4(t-3)} + \frac{1}{2}e^{-j4(t-3)} + \frac{1}{2}e^{j7(t-3)} + \frac{1}{2}e^{-j7(t-3)} \\ &= \cos(4(t-3)) + \cos(7(t-3)). \end{aligned}$$

$$H(j4) = e^{-j12}.$$

Şeklindedir.

Sonuç olarak,

$$y(t) = H(s)e^{st},$$

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}\, dt.$$

ifadesi sistemin transfer fonksiyonu olarak adlandırılır ve

$$s = j\omega$$

İçin sistemin frekans cevabına karşı gelir.

x(t) işaretinin Laplace dönüşümü

$$\boxed{X(s) \triangleq \int_{-\infty}^{+\infty} x(t)e^{-st}\, dt,}$$

$$s = \sigma + j\omega.$$

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s).$$

$$s = j\omega,$$

alınarak , Fourier dönüşümü

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\, dt,$$

İşaretin Fourier ve Laplace dönüşümleri arasındaki ilişki

$$X(s)\big|_{s=j\omega} = \mathfrak{F}\{x(t)\}.$$

$$X(\sigma + j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-(\sigma + j\omega)t}\,dt,$$

$$X(\sigma + j\omega) = \int_{-\infty}^{+\infty} [x(t)e^{-\sigma t}]e^{-j\omega t}\,dt.$$

Örnek :

$$x(t) = e^{-at}u(t), \quad a > 0$$

şeklinde verilen işaretin Laplace dönüşümünü hesaplayalım.

İşaretin Fourier dönüşümü

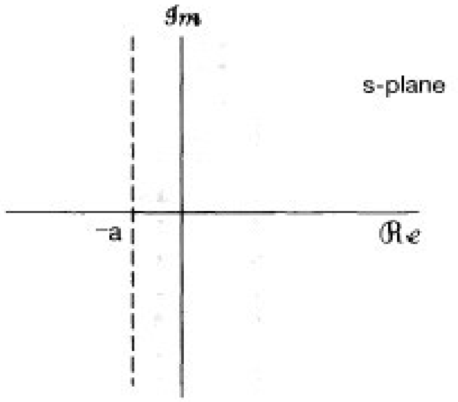
$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-at}u(t)e^{-j\omega t}\,dt = \int_0^{\infty} e^{-at}e^{-j\omega t}\,dt = \frac{1}{j\omega + a}, \quad a > 0.$$

Laplace dönüşümü

$$X(s) = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}dt = \int_0^{\infty} e^{-(s+a)t}dt,$$

$$X(s) = \frac{1}{s+a}, \quad \Re e\{s\} > -a.$$

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \quad \Re e\{s\} > -a.$$



(a)

Yakınsaklık bölgesi

Örnek :

$$x(t) = -e^{-at}u(-t).$$

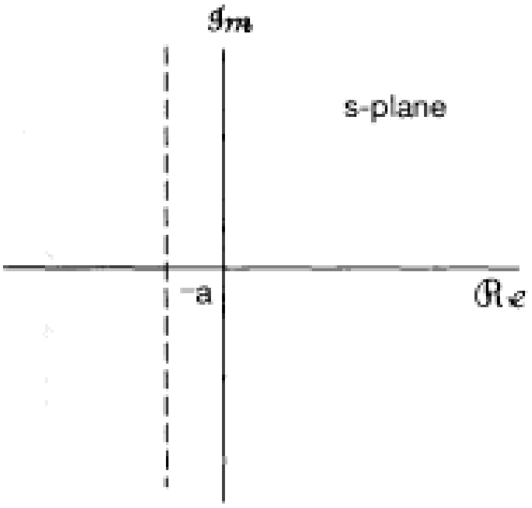
şeklinde verilen işaretin Laplace dönüşümünü hesaplayalım.

$$\begin{aligned} X(s) &= -\int_{-\infty}^{\infty} e^{-at}e^{-st}u(-t)\,dt \\ &= -\int_{-\infty}^{0} e^{-(s+a)t}\,dt, \end{aligned}$$

$$X(s) = \frac{1}{s+a}.$$

$\mathcal{R}e\{s\} < -a;$

$$-e^{-at}u(-t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+a}, \qquad \mathcal{R}e\{s\} < -a.$$



(b)

Yakınsaklık bölgesi

Örnek :

$$x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t).$$ şeklinde verilen işaretin Laplace dönüşümünü bulalım.

$$X(s) = \int_{-\infty}^{\infty}\left[3e^{-2t}u(t) - 2e^{-t}u(t)\right]e^{-st}\,dt$$
$$= 3\int_{-\infty}^{\infty} e^{-2t}e^{-st}u(t)\,dt - 2\int_{-\infty}^{\infty} e^{-t}e^{-st}u(t)\,dt.$$

$$X(s) = \frac{3}{s+2} - \frac{2}{s+1}.$$

$$e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+1}, \qquad \Re e\{s\} > -1,$$
$$e^{-2t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}, \qquad \Re e\{s\} > -2.$$

Terimlerin yakınsaklık bölgeleri dikkate alınarak

$$3e^{-2t}u(t) - 2e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{s-1}{s^2+3s+2}, \quad \Re e\{s\} > -1.$$

elde edilir.

Örnek :

$$x(t) = e^{-2t}u(t) + e^{-t}(\cos 3t)u(t).$$ şeklinde verilen işaret için,

Euler bağıntısından

$$x(t) = \left[e^{-2t} + \frac{1}{2}e^{-(1-3j)t} + \frac{1}{2}e^{-(1+3j)t}\right]u(t),$$

İşaretin Laplace dönüşümü

$$X(s) = \int_{-\infty}^{\infty} e^{-2t}u(t)e^{-st}\,dt$$
$$+ \frac{1}{2}\int_{-\infty}^{\infty} e^{-(1-3j)t}u(t)e^{-st}\,dt$$
$$+ \frac{1}{2}\int_{-\infty}^{\infty} e^{-(1+3j)t}u(t)e^{-st}\,dt.$$

$$e^{-2t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+2}, \quad \Re e\{s\} > -2,$$

$$e^{-(1-3j)t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+(1-3j)}, \quad \Re e\{s\} > -1,$$

$$e^{-(1+3j)t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+(1+3j)}, \quad \Re e\{s\} > -1.$$

$$\frac{1}{s+2} + \frac{1}{2}\left(\frac{1}{s+(1-3j)}\right) + \frac{1}{2}\left(\frac{1}{s+(1+3j)}\right), \quad \Re e\{s\} > -1,$$

$$e^{-2t}u(t) + e^{-t}(\cos 3t)u(t) \xleftrightarrow{\mathcal{L}} \frac{2s^2+5s+12}{(s^2+2s+10)(s+2)}, \quad \Re e\{s\} > -1.$$

elde edilir.

Örnek :

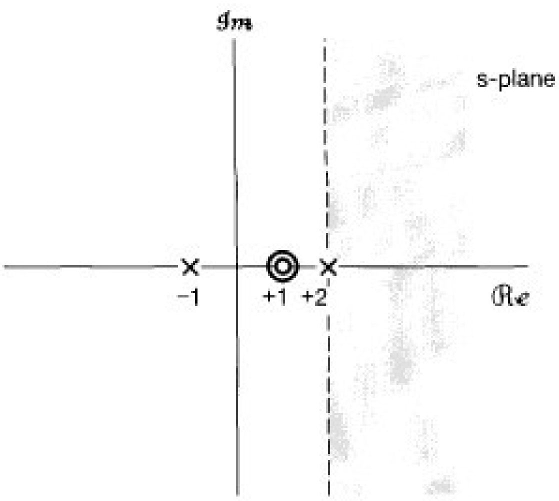
$$x(t) = \delta(t) - \frac{4}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t).$$

şeklinde verilen işaretin Laplace dönüşümü için,

$$\mathcal{L}\{\delta(t)\} = \int_{-\infty}^{+\infty} \delta(t)e^{-st}\,dt = 1,$$

$$X(s) = 1 - \frac{4}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s-2}, \quad \Re e\{s\} > 2,$$

$$X(s) = \frac{(s-1)^2}{(s+1)(s-2)}, \quad \Re e\{s\} > 2,$$



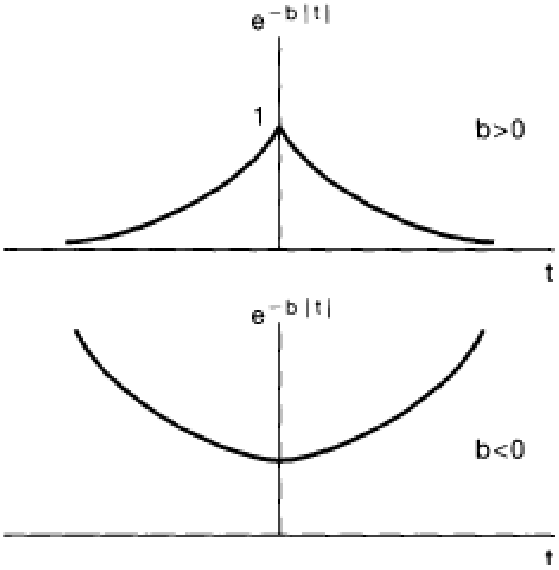
Yakınsaklık Bölgesi

Örnek :

$$x(t) = e^{-b|t|},$$

şeklinde verilen işaretin Laplace dönüşümünü bulalım.

$$x(t) = e^{-bt}u(t) + e^{+bt}u(-t).$$

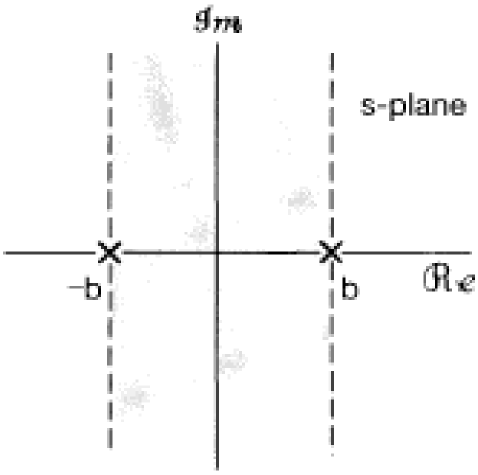


$$e^{-bt}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+b}, \quad \Re e\{s\} > -b,$$

$$e^{+bt}u(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{-1}{s-b}, \quad \Re e\{s\} < +b.$$

Sonuç olarak,

$$e^{-b|t|} \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+b} - \frac{1}{s-b} = \frac{-2b}{s^2-b^2}, \quad -b < \Re e\{s\} < +b.$$



Ters Laplace Dönüşümü

$$X(\sigma + j\omega) = \mathfrak{F}\{x(t)e^{-\sigma t}\} = \int_{-\infty}^{+\infty} x(t)e^{-\sigma t}e^{-j\omega t}\,dt$$

$$x(t)e^{-\sigma t} = \mathfrak{F}^{-1}\{X(\sigma + j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\sigma + j\omega)e^{j\omega t}\,d\omega,$$

Eşitliğin 2 tarafını da $e^{\sigma t}$ ile çarparak,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\sigma + j\omega)e^{(\sigma + j\omega)t}\,d\omega.$$

$$\boxed{x(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} X(s)e^{st}\,ds.}$$

Örnek :

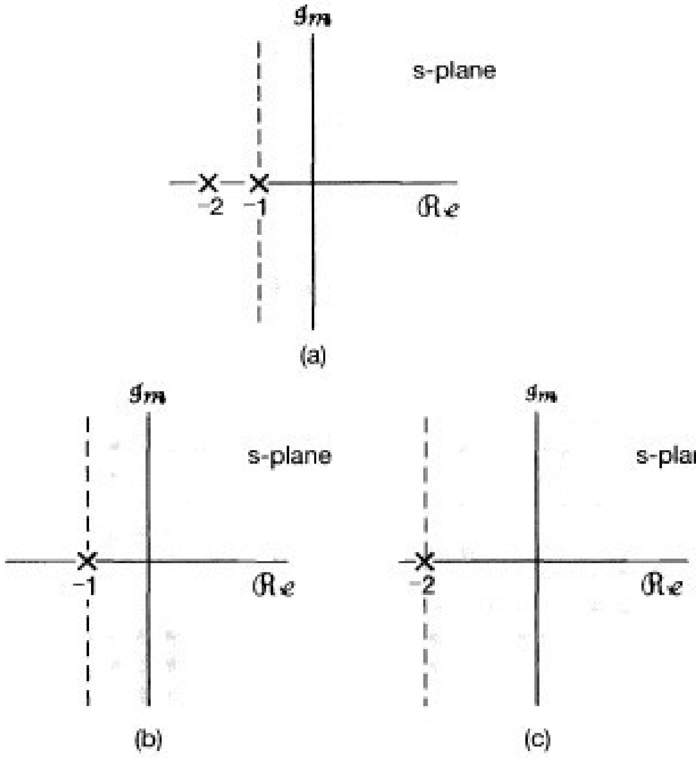
$$X(s) = \frac{1}{(s+1)(s+2)}, \qquad \Re e\{s\} > -1.$$

ifadesini kısmi kesirlere ayrıştırarak,

$$X(s) = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}.$$

$$A = [(s+1)X(s)]|_{s=-1} = 1,$$
$$B = [(s+2)X(s)]|_{s=-2} = -1.$$

$$X(s) = \frac{1}{s+1} - \frac{1}{s+2}.$$



$$e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+1}, \qquad \mathcal{Re}\{s\} > -1,$$

$$e^{-2t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}, \qquad \mathcal{Re}\{s\} > -2.$$

$$[e^{-t} - e^{-2t}]u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{(s+1)(s+2)}, \qquad \mathcal{Re}\{s\} > -1.$$

Yakınsaklık bölgesi

$\mathcal{Re}\{s\} < -2.$ olarak verilmiş olsaydı,

$$-e^{-t}u(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+1}, \qquad \mathcal{Re}\{s\} < -1,$$

$$-e^{-2t}u(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}, \qquad \mathcal{Re}\{s\} < -2,$$

$$x(t) = [-e^{-t} + e^{-2t}]u(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{(s+1)(s+2)}, \qquad \mathcal{Re}\{s\} < -2.$$

Yakınsaklık bölgesi

$-2 < \Re e\{s\} < -1.$ şeklinde verilmesi durumunda

$$x(t) = -e^{-t}u(-t) - e^{-2t}u(t) \overset{\mathfrak{L}}{\longleftrightarrow} \frac{1}{(s+1)(s+2)}, \quad -2 < \Re e\{s\} < -1.$$

elde edilir.

Laplace dönüşümünün Özellikleri

Section	Property	Signal	Laplace Transform	ROC
		$x(t)$	$X(s)$	R
		$x_1(t)$	$X_1(s)$	R_1
		$x_2(t)$	$X_2(s)$	R_2
9.5.1	Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
9.5.2	Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
9.5.3	Shifting in the s-Domain	$e^{s_0t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
9.5.4	Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
9.5.5	Conjugation	$x^*(t)$	$X^*(s^*)$	R
9.5.6	Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
9.5.7	Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
9.5.8	Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
9.5.9	Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\Re e\{s\} > 0\}$

Initial- and Final-Value Theorems

9.5.10 If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s \to \infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \longrightarrow \infty$, then

$$\lim_{t \to \infty} x(t) = \lim_{s \to \infty} sX(s)$$

Lineerlik :

$$x_1(t) \overset{\mathfrak{L}}{\longleftrightarrow} X_1(s)$$

$$x_2(t) \overset{\mathfrak{L}}{\longleftrightarrow} X_2(s)$$

$$\boxed{ax_1(t) + bx_2(t) \overset{\mathfrak{L}}{\longleftrightarrow} aX_1(s) + bX_2(s), \text{ with ROC containing } R_1 \cap R_2.}$$

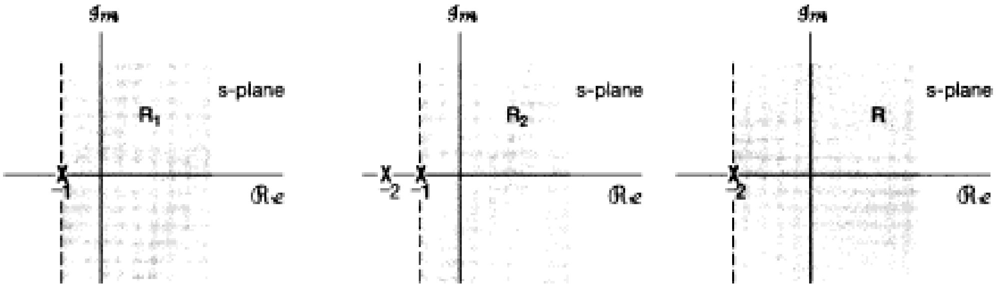
Örnek :

$$x(t) = x_1(t) - x_2(t),$$ Şeklinde verilen işaret için,

$$X_1(s) = \frac{1}{s+1}, \qquad \Re e\{s\} > -1,$$

$$X_2(s) = \frac{1}{(s+1)(s+2)}, \qquad \Re e\{s\} > -1.$$

$$X(s) = \frac{1}{s+1} - \frac{1}{(s+1)(s+2)} = \frac{s+1}{(s+1)(s+2)} = \frac{1}{s+2}.$$



Zamanda Öteleme :

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s), \qquad \text{with ROC} = R,$$

$$\boxed{x(t-t_0) \overset{\mathcal{L}}{\longleftrightarrow} e^{-st_0}X(s), \quad \text{with ROC} = R.}$$

S-domeninde Öteleme :

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s), \qquad \text{with ROC} = R,$$

$$\boxed{e^{s_0 t}x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s-s_0), \quad \text{with ROC} = R + \Re e\{s_0\}.}$$

Zamanda Ölçekleme :

$$x(t) \xleftrightarrow{\mathcal{L}} X(s), \quad \text{with ROC} = R,$$

$$\boxed{x(at) \xleftrightarrow{\mathcal{L}} \frac{1}{|a|}X\left(\frac{s}{a}\right), \quad \text{with ROC } R_1 = aR.}$$

Eşlenik Alma :

$$x(t) \xleftrightarrow{\mathcal{L}} X(s), \quad \text{with ROC} = R,$$

$$\boxed{x^*(t) \xleftrightarrow{\mathcal{L}} X^*(s^*), \quad \text{with ROC} = R.}$$

$X(s) = X^*(s^*)$ reel x(t) için

Konvolüsyon :

$$x_1(t) \xleftrightarrow{\mathcal{L}} X_1(s), \quad \text{with } ROC = R_1,$$

$$x_2(t) \xleftrightarrow{\mathcal{L}} X_2(s), \quad \text{with } ROC = R_2,$$

$$\boxed{x_1(t) * x_2(t) \xleftrightarrow{\mathcal{L}} X_1(s)X_2(s), \quad \text{with ROC containing } R_1 \cap R_2.}$$

Zaman Domeninde Türev :

$$x(t) \xleftrightarrow{\mathcal{L}} X(s), \quad \text{with ROC} = R,$$

$$\boxed{\frac{dx(t)}{dt} \xleftrightarrow{\mathcal{L}} sX(s), \quad \text{with ROC containing } R.}$$

S Domeninde Türev :

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt,$$

$$\frac{dX(s)}{ds} = \int_{-\infty}^{+\infty} (-t)x(t)e^{-st}dt.$$

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s), \qquad \text{with ROC} = R,$$

$$\boxed{-tx(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{dX(s)}{ds}, \qquad \text{with ROC} = R.}$$

Örnek :

$$x(t) = te^{-at}u(t).$$

şeklinde verilen işaretin Laplace dönüşümünü bulalım.

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \qquad \Re e\{s\} > -a,$$

$$te^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} -\frac{d}{ds}\left[\frac{1}{s+a}\right] = \frac{1}{(s+a)^2}, \qquad \Re e\{s\} > -a.$$

Genel olarak,

$$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{(s+a)^n}, \qquad \Re e\{s\} > -a.$$

dönüşüm çifti bulunabilir.

Örnek :

$$X(s) = \frac{2s^2+5s+5}{(s+1)^2(s+2)}, \qquad \Re e\{s\} > -1.$$

Kısmi kesirlere ayrıştırılarak,

$$X(s) = \frac{2}{(s+1)^2} - \frac{1}{(s+1)} + \frac{3}{s+2}, \qquad \Re e\{s\} > -1.$$

$$x(t) = [2te^{-t} - e^{-t} + 3e^{-2t}]u(t).$$

Zaman Domeninde İntegrasyon :

$$x(t) \xleftrightarrow{\mathfrak{L}} X(s), \qquad \text{with ROC} = R,$$

$$\boxed{\int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{\mathfrak{L}} \frac{1}{s}X(s), \quad \begin{array}{l}\text{with ROC containing}\\ R \cap \{\Re e\{s\} > 0\}.\end{array}}$$

Laplace Dönüşüm Çiftleri

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\Re e\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\Re e\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re e\{s\} < 0$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} > -\alpha$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} > -\alpha$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} < -\alpha$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
12	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
13	$[e^{-\alpha t}\cos \omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
14	$[e^{-\alpha t}\sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
15	$u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$

LZD sistemlerin Laplace Domeni Analizi

Nedensellik ve Kararlılık

-Nedensel bir sistemin yakınsaklık bölgesi en sağdaki kutbun sağındadır

-Kararlı bir sistemin yakınsaklık bölgesi jw eksenini içerir.

-Genel olarak kararlı ve nedensel bir sistemin kutupları sol yarı düzlemde bulunmalıdır.

Örnek :

$$h(t) = e^{2t}u(t)$$

şeklinde verilen impuls cevabı integre edilebilir olmadığından sistem kararlı değildir.

$$H(s) = \frac{1}{s-2}, \qquad \Re e\{s\} > 2,$$

Sistemin kutbu sağ yarı düzlemdedir.

Sistemin modelleyen diferansiyel denklemin Laplace dönüşümü alınarak,

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}.$$

$$\left(\sum_{k=0}^{N} a_k s^k\right) Y(s) = \left(\sum_{k=0}^{M} b_k s^k\right) X(s),$$

$$H(s) = \frac{\left\{\sum_{k=0}^{M} b_k s^k\right\}}{\left\{\sum_{k=0}^{N} a_k s^k\right\}}.$$

şeklinde sistemin transfer fonksiyonu bulunabilir.

Örnek :

$$\frac{dy(t)}{dt} + 3y(t) = x(t).$$

$$sY(s) + 3Y(s) = X(s).$$

$$H(s) = \frac{Y(s)}{X(s)},$$

$$H(s) = \frac{1}{s+3}.$$

YB belirtilmediği ya da sistemin hakkında ön bilgi verilmediği için YB 2 farklı şekilde seçilerek,

$\mathcal{R}e\{s\} > -3.$ 17

$$h(t) = e^{-3t}u(t),$$

$$\mathcal{R}e\{s\} < -3.$$

$$h(t) = -e^{-3t}u(-t).$$

Örnek :

Giriş ve çıkış işaretleri aşağıdaki gibi verilen sistemi modelleyen diferansiyel denklemi bulunuz.

$$x(t) = e^{-3t}u(t),$$

$$y(t) = [e^{-t} - e^{-2t}]u(t).$$

şeklinde verilen işaretlerin Laplace dönüşümü alınarak,

$$X(s) = \frac{1}{s+3}, \qquad \Re e\{s\} > -3,$$

$$Y(s) = \frac{1}{(s+1)(s+2)}, \qquad \Re e\{s\} > -1.$$

Sistemin transfer fonksiyonu

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+3}{(s+1)(s+2)} = \frac{s+3}{s^2+3s+2}.$$

Sistemi modelleyen diferansiyel denklem

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + 3x(t).$$

bulunur.