Recitation hour (27.05.2020)

1) One generates a number x from a uniform distribution on the interval $[0, \theta]$.

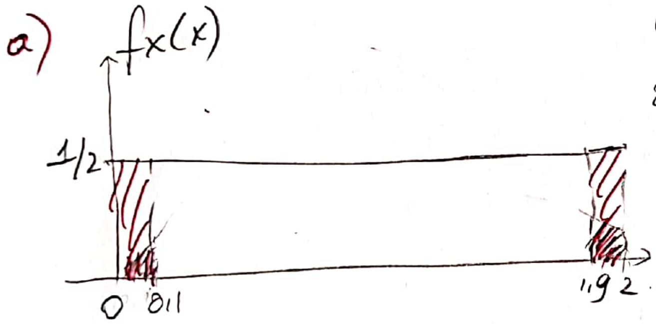
One decides $H_0: \theta = 2$ against $H_A: \theta \neq 2$ by rejecting H_0 if $x \leq 0{,}1$ or $x \geq 1{,}9$

a) Compute the probability of a type I error.

b) Compute the probability of a type II error if the true value of θ is $2{,}5$

soln.

a)



α : level of significance or type I error

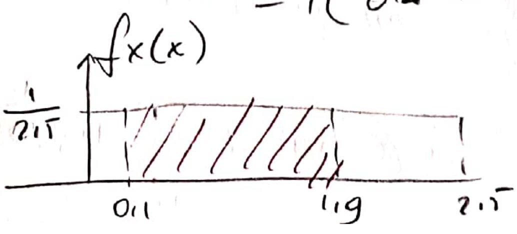
$$f_X(x) = \frac{1}{b-a} = \frac{1}{2-0} = \frac{1}{2}$$

the probability critical region $= 0{,}1 \cdot \frac{1}{2} + 0{,}1 \cdot \frac{1}{2} = 0{,}1$

$\alpha = 0{,}1$.

b) $P(\text{type II}) = P(\text{don't reject } H_0 \mid \theta = 2{,}5)$

$= P(0{,}1 < x < 1{,}9 \mid \theta = 2{,}5) = \frac{1}{2{,}5}(1{,}9 - 0{,}1) = \frac{1{,}8}{2{,}5} = 0{,}72$



2) An industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 19 water samples and measure the pH level of each. The sample mean and sample std. are 6.7 and 0.24 respectively.

Is there enough evidence to reject the company's claim

i) at $\alpha = 0.05$

ii) at $\alpha = 0.1$.

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soln.

the standard deviation of data is unknown, therefore we consider t-test.

$\mu_0 = 6.8$

$\bar{x} = 6.7$

$\sigma_x = 0.24$

$n = 19$ samples. $DF = n - 1 = 19 - 1 = 18$ //

$H_0: \mu = 6.8$ $\quad H_a: \mu \neq 6.8$

$$t = \frac{\bar{x} - \mu_0}{\frac{\sigma_x}{\sqrt{n}}} = -1.82$$

2.1

1.82

1.73

0.05   0.056   0.1

from the t-table.

P-value $= 0.056$

i) for $\alpha = 0.05$

since $p > \alpha \Rightarrow$ we do not have enough evidence to reject H_0: Therefore, we accept H_0.

ii) for $\alpha = 0.1$

since $p < \alpha$, the H_0 in the critical region.
Therefore we reject H_0.

3) Let X be a Gaussian rv with unknown mean and unknown variance. A set of 10 independent measurements of X yields

$$\sum_{j=1}^{10} X_j = 350 \quad \text{and} \quad \sum_{j=1}^{10} X_j^2 = 12{,}645$$

a) Find a 90% confidence interval for the mean of X.
b) Find a 90% confidence interval for the variance of X.

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soln.

$n = 10$ samples.

$$\bar{\mu}_x = \frac{1}{n} \sum_{j=1}^{n} X_j = \frac{1}{10} \cdot \sum_{j=1}^{n} X_j = \frac{350}{10} = 35$$

$$\bar{\sigma}_x^2 = \frac{1}{n-1} \sum_{j=1}^{n} (X_j - \bar{\mu}_x)^2$$

$$= \frac{1}{n-1} \left[\sum_{j=1}^{n} X_j^2 - 2\bar{\mu}_x \underbrace{\sum_{j=1}^{n} X_j}_{n \cdot \bar{\mu}_x} + \sum_{j=1}^{n} \bar{\mu}_x^2 \right]$$

$$= \frac{1}{n-1} \left[\sum_{j=1}^{n} X_j^2 - 2\bar{\mu}_x (n\bar{\mu}_x) + n\bar{\mu}_x^2 \right] =$$

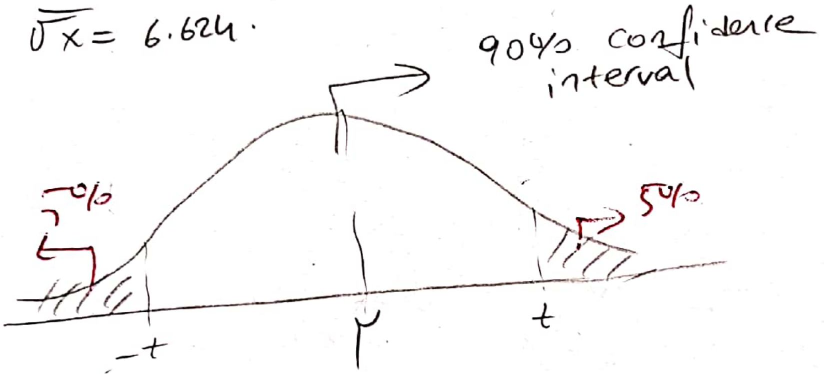
3

$$\bar{\sigma}_x^2 = \frac{1}{n-1}\left[\sum_{j=1}^{n} x_j^2 - 2n\bar{\mu}_x^2 + n\bar{\mu}_x^2\right]$$

$$= \frac{1}{n-1}\left[\sum_{j=1}^{n} x_j^2 - n\bar{\mu}_x^2\right]$$

$$= \frac{1}{9}\left[12645 - 10 \cdot 35^2\right] = 43.88$$

$\bar{\sigma}_x = 6.624.$



$$P(X > t) = 1 - P(X < t) = 0{,}95$$

$$P(X < t) = 0{,}95$$

since, $n = 10$, $DF = n - 1 = 9$ //

from t-table.

$t = 1{,}83$ //

$$t = \frac{\bar{\mu}_x - \mu_0}{\frac{\bar{\sigma}_x}{\sqrt{n}}} \Rightarrow \frac{35 - \mu}{\frac{6.624}{\sqrt{10}}} =$$

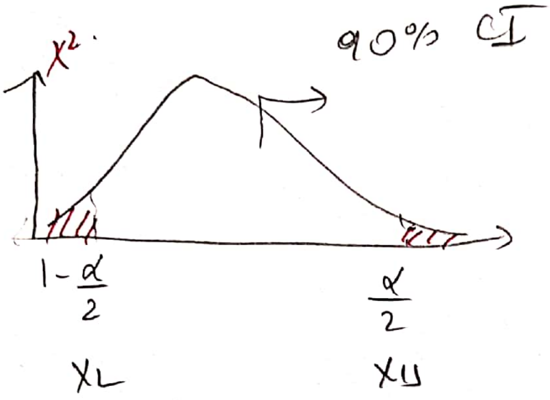
$$\left[\bar{\mu}_x - \frac{6.624}{\sqrt{10}} \cdot t < \mu < \bar{\mu}_x + \frac{6.624}{\sqrt{10}} \cdot t\right]$$

$$\left[35 - \frac{6.624}{\sqrt{10}} \cdot 1{,}83 < \mu < 35 + \frac{6.624}{\sqrt{10}} \cdot 1{,}83\right] = \left[31.17 , 38.83\right]$$

gives 90% C.I.

b) 90% CI

$\alpha = 0.1$.



$$\chi^2 = \frac{(n-1)\,\sigma_{\bar{x}}^2}{\sigma^2}.$$

$\sigma_{\bar{x}} = 6.624$

$\sigma_{\bar{x}}^2 = 43.88$

$$\left[\frac{(n-1)\,\sigma_{\bar{x}}^2}{\chi^2_{\chi U}}, \frac{(n-1)\,\sigma_{\bar{x}}^2}{\chi^2_{\chi L}}\right]$$

$$\chi^2_{\chi L} = \chi^2_{1-\frac{\alpha}{2}} = \chi^2_{0.95} = 3.325$$

$$\chi^2_{\chi U} = \chi^2_{\frac{\alpha}{2}} = \chi^2_{0.05} = 16.92.$$

$$\left[\frac{9\cdot(43.88)}{16.92}, \frac{9\cdot(43.88)}{3.325}\right] = \left[23.34, 118.77\right] = 90\%\ CI$$

	cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
	one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
	two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
	df											
	1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
	2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
	3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
	4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
	5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
	6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
	7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
	8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
Q3a)	9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
	10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
	11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
	12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
	13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
	14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
	15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
	16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
	17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
Q2	18	0.000	0.688	0.862	1.067	1.330	1.734 1.82	2.101	2.552	2.878	3.610	3.922
	19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
	20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
	21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
	22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
	23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
	24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
	25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
	26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
	27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
	28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
	29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
	30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
	40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
	60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
	80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
	100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
	1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
	z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
		0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
							Confidence Level					

Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of Critical Value							
	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01
1	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
Q3b) 9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892