EHB 351

Analog Haberleşme

(Ara Sınav 1 Çözümleri)

40P (1) a) Zamanda öteleme teoremi:

$$x(t) \longleftrightarrow X(f)$$

$$x(t-t_0) \longleftrightarrow e^{-j2\pi f t_0} X(f) \quad (4)$$

İspat: $y(t) = x(t-t_0) \Rightarrow Y(f) = \int_{-\infty}^{\infty} y(t) e^{-j2\pi f t} dt = \int_{-\infty}^{\infty} x(t-t_0) e^{-j2\pi f t} dt$

$u = t - t_0$
$du = dt$

$$\Rightarrow Y(f) = \int_{u=-\infty}^{\infty} x(u) e^{-j2\pi f(u+t_0)} du = e^{-j2\pi f t_0} \int_{-\infty}^{\infty} x(u) e^{-j2\pi f u} du = e^{-j2\pi f t_0} X(f) \quad (4)$$

b) $m(t) = \pi e^{-2\pi|t-5|}$

$x(t) = \pi e^{-2\pi|t|}$ olarak tanımlansın. $\Rightarrow m(t) = x(t-5)$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt = \pi \int_{-\infty}^{0} e^{2\pi t} . e^{-j2\pi f t} dt + \pi \int_{0}^{\infty} e^{-2\pi t} e^{-j2\pi f t} dt$$

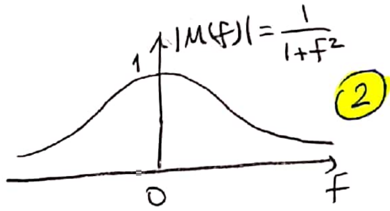
$$= \pi \int_{-\infty}^{0} e^{2\pi(1-jf)t} dt + \pi \int_{0}^{\infty} e^{-2\pi(1+jf)t} dt$$

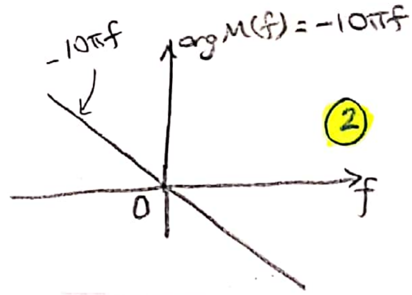
$$= \frac{\pi e^{2\pi(1-jf)t}}{2\pi(1-jf)} \Bigg|_{-\infty}^{0} + \frac{\pi e^{-2\pi(1-jf)t}}{-2\pi(1+jf)} \Bigg|_{0}^{\infty} = \frac{1}{2}\left[\frac{1}{1-jf} + \frac{1}{1+jf}\right] = \frac{1}{2}\left[\frac{1+jf+1-jf}{(1-jf)(1+jf)}\right] = \frac{1}{1+f^2}$$

Zamanda öteleme teoreminden, (10)

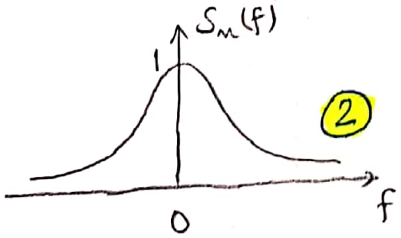
$m(t) = x(t-5)$

$$M(f) = e^{-j2\pi f 5} X(f) = \frac{e^{-j2\pi f 5}}{1+f^2} = |M(f)| e^{j \arg M(f)}$$

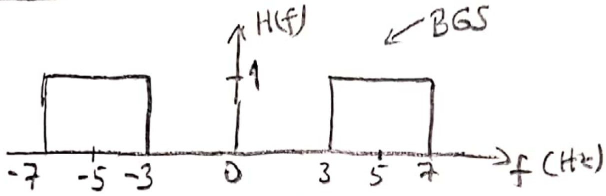




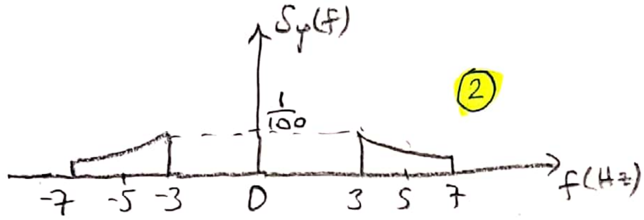
c) $S_m(f) = |M(f)|^2 = \frac{1}{(1+f^2)^2}$ (4)



d)



$$\underbrace{|Y(f)|^2}_{S_y(f)} = \underbrace{|M(f)|^2}_{S_m(f)} |H(f)|^2 = \begin{cases} \dfrac{1}{(1+f^2)^2}, & 3 \leq |f| \leq 7 \\ 0, & \text{dışında} \end{cases}$$ (4)



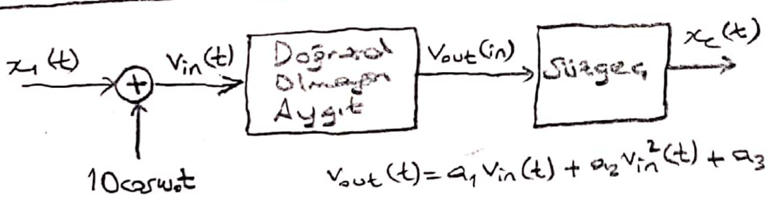
e) $$E_m = \int_{-\infty}^{\infty} S_m(f)\,df = 2 \times \int_{0}^{\infty} \frac{df}{(1+f^2)^2} = 2 \times \frac{\pi}{4} = \frac{\pi}{2}\ [J]$$

Veriliyor.

$$E_y = \int_{-\infty}^{\infty} S_y(f)\,df = 2 \times \int_{3}^{7} \frac{df}{(1+f^2)^2} = 2 \times \left[\frac{f}{2(1+f^2)} \Big|_3^7 + \frac{1}{2} \operatorname{arctg} f \Big|_3^7 \right] = 0.0239$$ (6)

$$\frac{E_y}{E_m} = 0.015$$ (Yaklaşık %1,5'i çıkışa aktarılmış).

SOP (2)



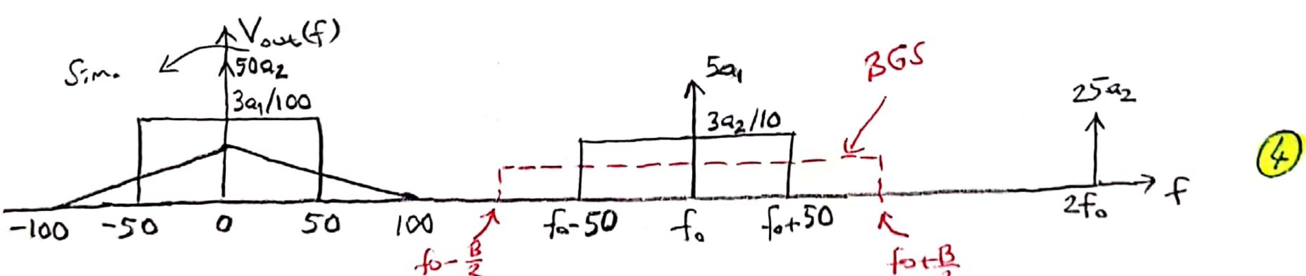
$$V_{out}(t) = a_1 V_{in}(t) + a_2 V_{in}^2(t) + a_3 V_{in}^3(t)$$

a) $a_1 \neq 0,\ a_2 \neq 0,\ a_3 = 0 \Rightarrow$

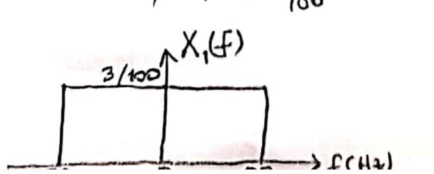
i) $V_{out}(t) = a_1 V_{in}(t) + a_2 V_{in}^2(t)$

$$= a_1 (x_1(t) + 10\cos w_0 t) + a_2 (x_1(t) + 10\cos w_0 t)^2$$

$$= a_1 x_1(t) + \underline{10 a_1 \cos w_0 t} + a_2 x_1^2(t) + \underline{20 a_2 x_1(t) \cos w_0 t} + 100 a_2 \underbrace{\cos^2 w_0 t}_{\frac{1+\cos 2w_0 t}{2}}$$ (4)



Not: $x_1(t) = 3\,\text{sinc}(100t) \Rightarrow X_1(f) = \frac{3}{100} \Pi\left(\frac{f}{100}\right)$



ii) Çıkışta GM olması için,

$f_0 = f_c = 1000$ Hz seçilmeli.

BGS kullanılmalı, merkez frekansı $f_0 = 1000$ Hz olmalı

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Ayrıca,

$100 \le f_0 - \frac{B}{2} \le f_0 - 50 \Rightarrow 100 \le 1000 - \frac{B}{2} \le 950 \Rightarrow 100 \le B \le 1800$

✓ Not1: $f_0 + \frac{B}{2} \ge f_0 + 50$ koşulu buna denktir.

✓ Not2: Yukarıdaki koşullar sağlandığında $f_0 + \frac{B}{2} < 2f_0$ koşulu da sağlanır.

iii) $x_c(t) = 10a_1 \cos w_c t + 20 a_2 x_1(t) \cos w_c t$ $(f_0 = f_c)$

$= 10a_1\left(1 + \frac{2a_2}{a_1} x_1(t)\right)\cos w_c t$

$x_1(t)$ normalize formda değil.

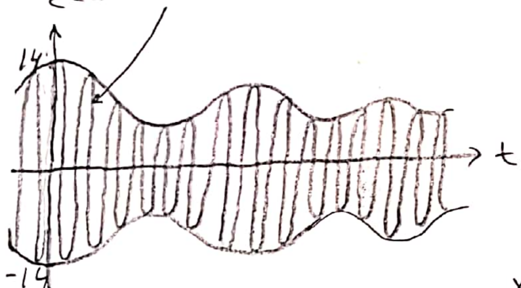
$x_1(t) = a + b x(t) \Rightarrow a = \langle x_1(t) \rangle = \lim_{T \to \infty} \frac{3}{T} \int_{-T/2}^{T/2} \mathrm{sinc}(100t)\,dt = 0$ $\Rightarrow x(t) = \mathrm{sinc}(100t)$, $x_1(t) = 3x(t)$

$b = \max_t |x_1(t) - a| = 3$

$x_c(t) = \underbrace{10a_1}_{A_c}\left(1 + \underbrace{\frac{6a_2}{a_1}}_{m} x(t)\right)\cos w_c t \Rightarrow m = \frac{6a_2}{a_1} = \frac{6 \times 0.1}{0.8} = 0.75$ 5

$A_c = 10a_1 = 8$ V $\left(\begin{matrix} a_1 = 0.8 \\ a_2 = 0.1 \end{matrix}\right)$

$x_c(t) = 8(1 + 0.75 x(t)) \cos w_c t$



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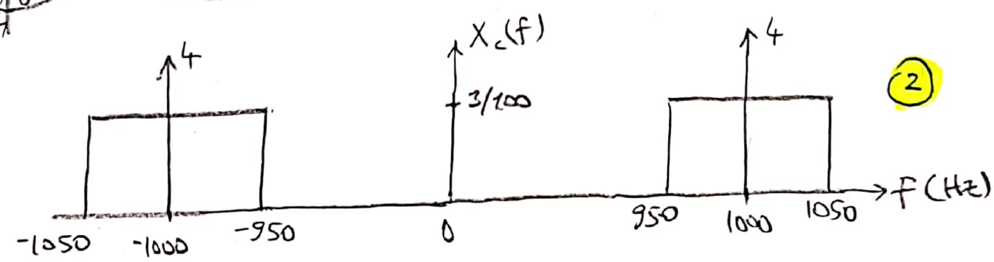

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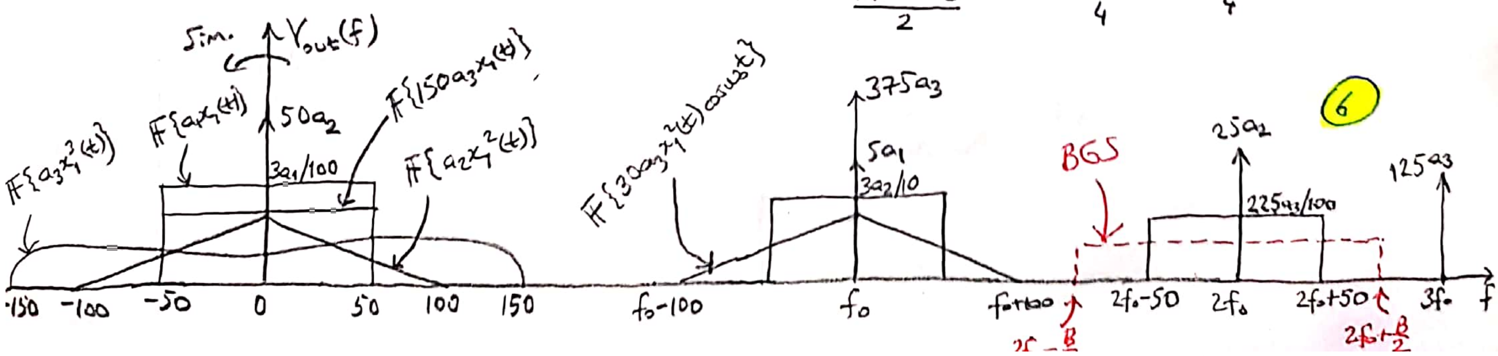
b) $a_1 \ne 0,\ a_2 \ne 0,\ a_3 \ne 0 \Rightarrow$

i) $V_{out}(t) = a_1 v_{in}(t) + a_2 v_{in}^2(t) + a_3 v_{in}^3(t)$

$= a_1(x_1(t) + 10\cos w_0 t) + a_2(x_1(t) + 10\cos w_0 t)^2 + a_3(x_1(t) + 10\cos w_0 t)^3$

$= a_1 x_1(t) + 10a_1 \cos w_0 t + a_2 x_1^2(t) + 20a_2 x_1(t)\cos w_0 t + 100a_2 \cos^2 w_0 t \rightarrow \frac{1 + 2\cos w_0 t}{2}$ 6

$+ a_3 x_1^3(t) + 30a_3 x_1^2(t)\cos w_0 t + 300a_3 x_1(t)\underbrace{\cos^2 w_0 t}_{\frac{1+\cos 2w_0 t}{2}} + 1000a_3 \underbrace{\cos^3 w_0 t}_{\frac{3}{4}\cos w_0 t + \frac{1}{4}\cos(3w_0 t)}$



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ii) Çıkışta GM almak için,

$2f_0 = f_c = 1000$ Hz $\Rightarrow f_0 = 500$ Hz seçilmeli.

BGS kullanılmalı, merkez frekansı $2f_0 = 1000$ Hz olmalı. 5

Ayrıca,

$$f_0 + 100 \leq 2f_0 - \frac{B}{2} \leq 2f_0 - 50 \Rightarrow 600 \leq 1000 - \frac{B}{2} \leq 950 \Rightarrow 100 \leq B \leq 800$$

✓ Not 1: $2f_0 + \frac{B}{2} \geq 2f_0 + 50$ koşulu buna denktir.

✓ Not 2: Yukarıdaki koşullar sağlandığında $2f_0 + \frac{B}{2} < 3f_0$ koşulu da sağlanır.

iii) $x_c(t) = 50\, a_2 \cos \underbrace{2\omega_0}_{\omega_c} t + 150\, a_3 x_1(t) \cos \underbrace{2\omega_0}_{\omega_c} t$

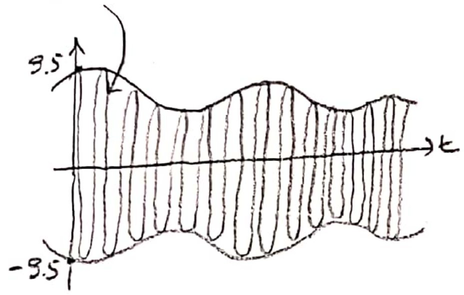
$a_1 = 0.8$, $a_2 = 0.1$, $a_3 = 0.01$

$= 50 a_2 \left(1 + \frac{3a_3}{a_2} x_1(t)\right) \cos\omega_c t$ 5

$= \underbrace{50 a_2}_{A_c} \left(1 + \underbrace{\frac{9a_3}{a_2}}_{m} x(t)\right) \cos\omega_c t \Rightarrow m = \frac{9a_3}{a_2} = \frac{0.09}{0.1} = 0.9$

$A_c = 50 a_2 = 5$ V

$x_c(t) = 5\,(1 + 0.9\, x(t)) \cos\omega_c t$



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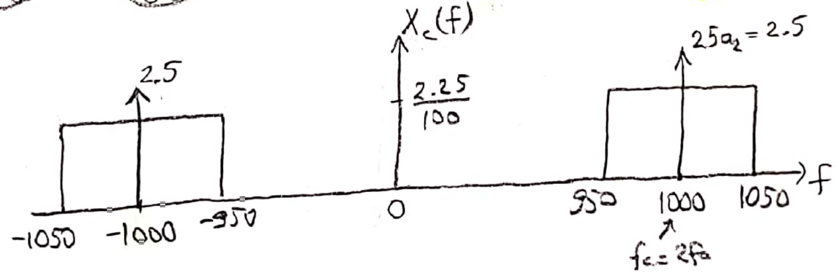

c)

a_1	a_2	a_3	f_c (f_0 cinsinden)	Çıkışta Elde Edilen Modülasyon Türü
0	≠0	≠0	$2f_0$	GM
≠0	0	≠0	$2f_0$	ÇYB
≠0	≠0	0	f_0	GM
0	0	≠0	$2f_0$	ÇYB
≠0	0	0	—	—
0	≠0	0	f_0	ÇYB
≠0	≠0	≠0	$2f_0$	GM

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