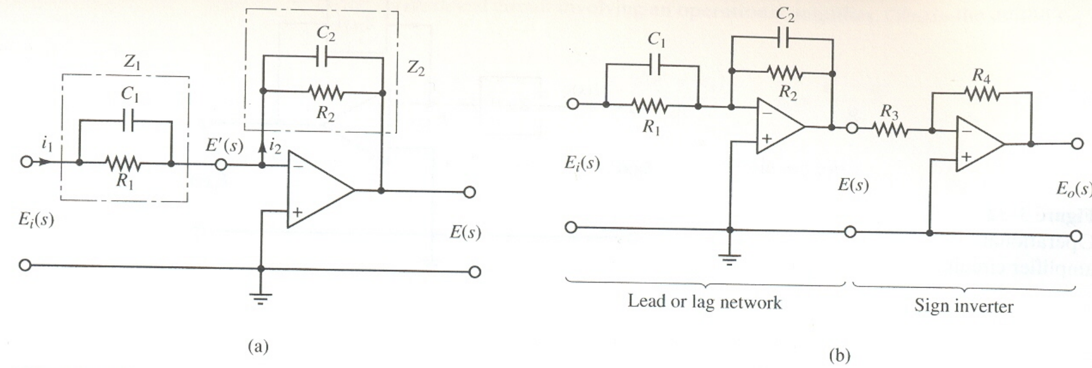


Figure 3–33
(a) Operational-amplifier circuit; (b) operational-amplifier circuit used as a lead or lag compensator.

Lead or Lag Networks Using Operational Amplifiers. Figure 3–33(a) shows an electronic circuit using an operational amplifier. The transfer function for this circuit can be obtained as follows: Define the input impedance and feedback impedance as Z_1 and Z_2, respectively. Then

$$Z_1 = \frac{R_1}{R_1 C_1 s + 1}, \qquad Z_2 = \frac{R_2}{R_2 C_2 s + 1}$$

Hence, referring to Equation (3–73), we have

$$\frac{E(s)}{E_i(s)} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1}\frac{R_1 C_1 s + 1}{R_2 C_2 s + 1} = -\frac{C_1}{C_2}\frac{s + \dfrac{1}{R_1 C_1}}{s + \dfrac{1}{R_2 C_2}} \tag{3–74}$$

Notice that the transfer function in Equation (3–74) contains a minus sign. Thus, this circuit is sign inverting. If such a sign inversion is not convenient in the actual application, a sign inverter may be connected to either the input or the output of the circuit of Figure 3–33(a). An example is shown in Figure 3–33(b). The sign inverter has the transfer function of

$$\frac{E_o(s)}{E(s)} = -\frac{R_4}{R_3}$$

The sign inverter has the gain of $-R_4/R_3$. Hence the network shown in Figure 3–33(b) has the following transfer function:

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 R_4}{R_1 R_3}\frac{R_1 C_1 s + 1}{R_2 C_2 s + 1} = \frac{R_4 C_1}{R_3 C_2}\frac{s + \dfrac{1}{R_1 C_1}}{s + \dfrac{1}{R_2 C_2}}$$

$$= K_c \alpha \frac{Ts + 1}{\alpha T s + 1} = K_c \frac{s + \dfrac{1}{T}}{s + \dfrac{1}{\alpha T}} \tag{3–75}$$

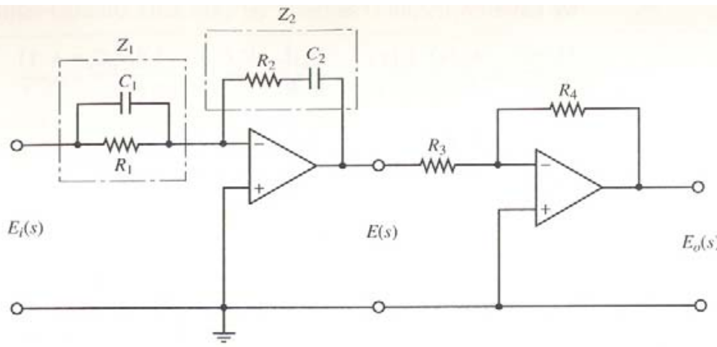


Figure 3–34
Electronic PID controller.

where

$$T = R_1C_1, \qquad \alpha T = R_2C_2, \qquad K_c = \frac{R_4C_1}{R_3C_2}$$

Notice that

$$K_c\alpha = \frac{R_4C_1}{R_3C_2}\frac{R_2C_2}{R_1C_1} = \frac{R_2R_4}{R_1R_3}, \qquad \alpha = \frac{R_2C_2}{R_1C_1}$$

This network has a dc gain of $K_c\alpha = R_2R_4/(R_1R_3)$.

Note that this network is a lead network if $R_1C_1 > R_2C_2$, or $\alpha < 1$. It is a lag network if $R_1C_1 < R_2C_2$.

PID Controller Using Operational Amplifiers. Figure 3–34 shows an electronic proportional-plus-integral-plus-derivative controller (a PID controller) using operational amplifiers. The transfer function $E(s)/E_i(s)$ is given by

$$\frac{E(s)}{E_i(s)} = -\frac{Z_2}{Z_1}$$

where

$$Z_1 = \frac{R_1}{R_1C_1s + 1}, \qquad Z_2 = \frac{R_2C_2s + 1}{C_2s}$$

Thus

$$\frac{E(s)}{E_i(s)} = -\left(\frac{R_2C_2s + 1}{C_2s}\right)\left(\frac{R_1C_1s + 1}{R_1}\right)$$

Noting that

$$\frac{E_o(s)}{E(s)} = -\frac{R_4}{R_3}$$

we have

$$\frac{E_o(s)}{E_i(s)} = \frac{E_o(s)}{E(s)}\frac{E(s)}{E_i(s)} = \frac{R_4R_2}{R_3R_1}\frac{(R_1C_1s + 1)(R_2C_2s + 1)}{R_2C_2s}$$

$$= \frac{R_4R_2}{R_3R_1}\left(\frac{R_1C_1 + R_2C_2}{R_2C_2} + \frac{1}{R_2C_2s} + R_1C_1s\right)$$

$$= \frac{R_4(R_1C_1 + R_2C_2)}{R_3R_1C_2}\left[1 + \frac{1}{(R_1C_1 + R_2C_2)s} + \frac{R_1C_1R_2C_2}{R_1C_1 + R_2C_2}s\right] \tag{3–76}$$

Notice that the second operational-amplifier circuit acts as a sign inverter as well as a gain adjuster.

When a PID controller is expressed as

$$\frac{E_o(s)}{E_i(s)} = K_p\left(1 + \frac{T_i}{s} + T_ds\right)$$

K_p is called the proportional gain, T_i is called the integral time, and T_d is called the derivative time. From Equation (3–76) we obtain the proportional gain K_p, integral time T_i, and derivative time T_d to be

$$K_p = \frac{R_4(R_1C_1 + R_2C_2)}{R_3R_1C_2}$$

$$T_i = \frac{1}{R_1C_1 + R_2C_2}$$

$$T_d = \frac{R_1C_1R_2C_2}{R_1C_1 + R_2C_2}$$

When a PID controller is expressed as

$$\frac{E_o(s)}{E_i(s)} = K_p + \frac{K_i}{s} + K_ds$$

K_p is called the proportional gain, K_i is called the integral gain, and K_d is called the derivative gain. For this controller

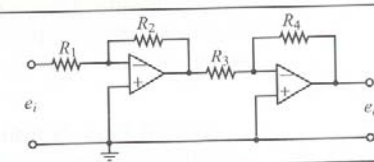
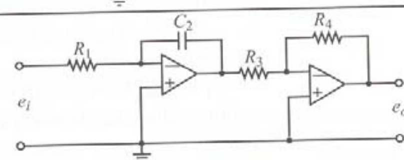
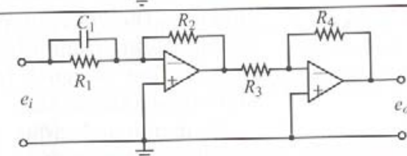
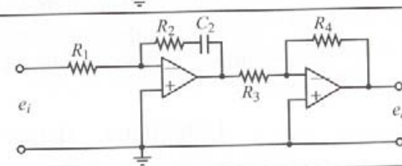
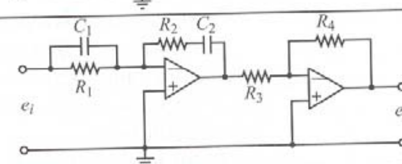
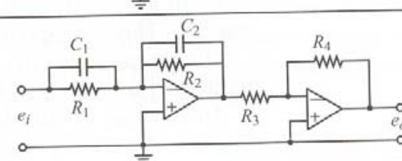
$$K_p = \frac{R_4(R_1C_1 + R_2C_2)}{R_3R_1C_2}$$

$$K_i = \frac{R_4}{R_3R_1C_2}$$

$$K_d = \frac{R_4R_2C_1}{R_3}$$

Table 3-1 shows a list of operational-amplifier circuits that may be used as controllers or compensators.

Table 3–1 Operational-Amplifier Circuits That May Be Used as Compensators

	Control Action	$G(s) = \frac{E_o(s)}{E_i(s)}$	Operational Amplifier Circuits
1	P	$\frac{R_4}{R_3}\frac{R_2}{R_1}$	
2	I	$\frac{R_4}{R_3}\frac{1}{R_1 C_2 s}$	
3	PD	$\frac{R_4}{R_3}\frac{R_2}{R_1}(R_1 C_1 s + 1)$	
4	PI	$\frac{R_4}{R_3}\frac{R_2}{R_1}\frac{R_2 C_2 s + 1}{R_2 C_2 s}$	
5	PID	$\frac{R_4}{R_3}\frac{R_2}{R_1}\frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{R_2 C_2 s}$	
6	Lead or lag	$\frac{R_4}{R_3}\frac{R_2}{R_1}\frac{R_1 C_1 s + 1}{R_2 C_2 s + 1}$	
7	Lag–lead	$\frac{R_6}{R_5}\frac{R_4}{R_3}\frac{[(R_1 + R_3) C_1 s + 1](R_2 C_2 s + 1)}{(R_1 C_1 s + 1)[(R_2 + R_4) C_2 s + 1]}$