EHB 335E HW#7

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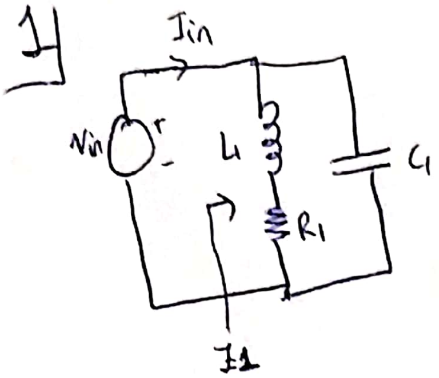
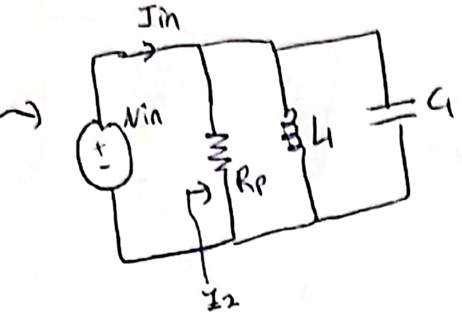
1)





Let us calculate Z_1 first; $(sL_1+R_1) \parallel \frac{1}{sC_1}$

$$Z_1(s) = (sL_1+R_1) \parallel \frac{1}{sC_1} = \frac{\frac{sL_1+R_1}{sC_1}}{sL_1+R_1+\frac{1}{sC_1}} = \frac{sL_1+R_1}{s^2L_1C_1+sC_1R_1+1}$$

$$Z_1(j\omega) = \frac{j\omega L_1+R_1}{(1-\omega^2L_1C_1)+j\omega C_1R_1} = \frac{(R_1-\omega^2R_1L_1C_1+\omega^2C_1R_1L_1)}{(1-\omega^2L_1C_1)^2+\omega^2C_1^2R_1^2} + j\frac{\omega L_1(1-\omega^2L_1C_1)-\omega C_1R_1^2}{(1-\omega^2L_1C_1)^2+\omega^2C_1^2R_1^2}$$

$$((1-\omega^2L_1C_1)-j\omega C_1R_1)$$

Also, calculate $Z_2(s) = R_p \parallel sL_1 \parallel \frac{1}{sC_1} = R_p \parallel \left(\frac{L_1/C_1}{sL_1+\frac{1}{sC_1}}\right) = R_p \parallel \left(\frac{sL_1}{s^2L_1C_1+1}\right)$

$$Z_2(s) = \frac{sR_pL_1}{s^2L_1C_1R_p+R_p+sL_1} \Rightarrow Z_2(j\omega) = \frac{j\omega L_1R_p}{R_p(1-\omega^2L_1C_1)+j\omega L_1} \times \frac{R_p(1-\omega^2L_1C_1)-j\omega L_1}{R_p(1-\omega^2L_1C_1)-j\omega L_1}$$

$$Z_2(j\omega) = \frac{\omega^2L_1^2R_p}{R_p^2(1-\omega^2L_1C_1)^2+\omega^2L_1^2} + j\frac{R_p^2\omega L_1(1-\omega^2L_1C_1)}{R_p^2(1-\omega^2L_1C_1)^2+\omega^2L_1^2}$$

→ By equating real and imaginary parts we can find a solution or by simply

$$\frac{1}{\frac{1}{R_1+j\omega L_1}+j\omega C_1} = \frac{1}{\frac{1}{R_p}+\frac{1}{j\omega L_1}+j\omega C_1} \Rightarrow \frac{1}{R_1+j\omega L_1} = \frac{1}{R_p}+\frac{1}{j\omega L_1}$$

$$\frac{1}{R_1+j\omega L_1} = \frac{j\omega L_1+R_p}{j\omega L_1R_p}$$

$$Z_1(j\omega) = Z_2(j\omega)$$

↳ By equating these impedances

$$\cancel{j\omega L_1R_p} = j\omega L_1R_1 + R_pR_1 - \omega^2L_1^2 + \cancel{j\omega L_1R_p}$$

$$R_pR_1 = -j\omega L_1R_1 + \omega^2L_1^2$$

$$R_p = -j\omega L_1 + \frac{\omega^2L_1^2}{R_1}$$

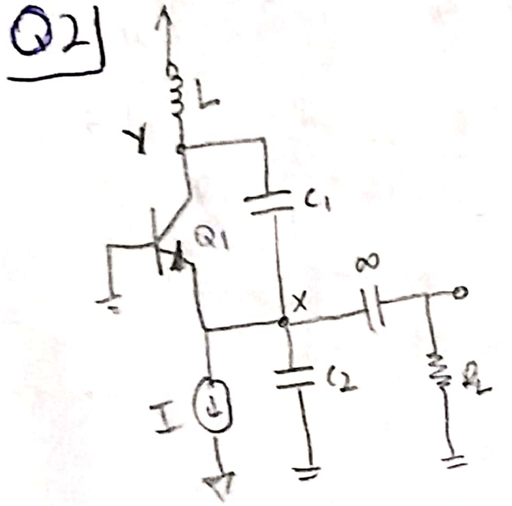
$$R_p = -j\omega L_1 + \frac{\omega^2 L_1^2}{R_1}$$

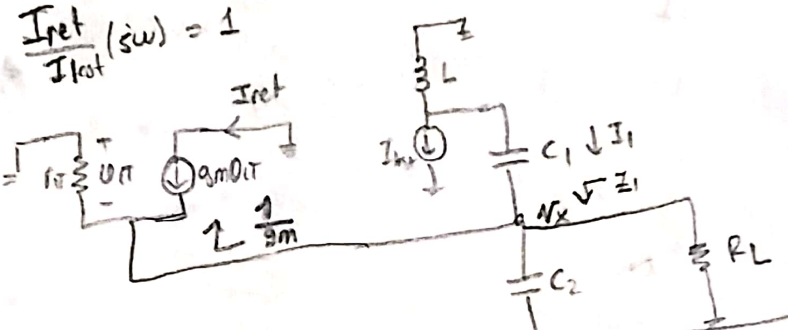
$$R_p = -j\omega L_1 \left(1 - j\frac{\omega L_1}{R_1}\right)$$ we know that $\frac{j\omega L_1}{R_1} \gg 1$

$$R_p \approx -j\omega L_1\left(-j\frac{\omega L_1}{R_1}\right) \Rightarrow \boxed{R_p \approx \frac{\omega^2 L_1^2}{R_1}}$$

Q2)

- Break the loop at node Y.
- $\frac{I_{ret}}{I_{test}}(j\omega) = 1$





$I_{ret} = g_m v_\pi = -g_m v_x = -g_m I_1 Z_1$; $\boxed{Z_1 = \frac{1}{Y_{Q_1} + Y_{C_2} + Y_{R_L}} = \frac{1}{g_m + sC_2 + 1/R_L}}$

$$I_{ret} = -g_m \frac{1}{g_m + sC_2 + 1/R_L} I_1 \qquad I_1 = -I_{test} \cdot \frac{sL}{\frac{1}{sC_1} + sL + Z_1}$$

$$I_{ret} = -g_m \frac{1}{g_m + sC_2 + 1/R_L}\left(-I_{test} \frac{sL}{\frac{1}{sC_1} + sL + \frac{1}{g_m + sC_2 + 1/R_L}}\right)$$

$$\frac{I_{ret}}{I_{test}} = \frac{g_m L s}{\left(g_m + sC_2 + (1/R_L)\right)\left(\frac{1}{sC_1} + sL\right) + 1} = \frac{g_m L s}{\frac{g_m}{sC_1} + g_m sL + \frac{C_2}{C_1} + s^2 C_2 L + \frac{1}{sR_L C_1} + \frac{sL}{R_L} + 1}$$

$$\frac{I_{ret}}{I_{test}}(j\omega) = \frac{j\omega g_m L}{\frac{g_m}{j\omega C_1} + j\omega g_m L + \frac{C_2}{C_1} - \omega^2 C_2 L + \frac{1}{j\omega R_L C_1} + \frac{j\omega L}{R_L} + 1} = 1$$

↳ we only want this term to be left so that $\frac{j\omega g_m L}{j\omega g_m L} = 1$

Therefore, imaginary and real parts in the denominator should be zero.

$$\Rightarrow \left(\frac{\omega L}{R_L} - \frac{g_m}{\omega C_1} - \frac{1}{\omega R_L C_1} = 0\right) ; \left(\frac{C_2}{C_1} + 1 - \omega^2 C_2 L = 0\right)$$

↳ Imaginary Part ↳ Real Part

$$\Rightarrow \frac{C_2}{C_1} + 1 = w^2 C_2 L$$

$$\frac{C_1 + C_2}{C_1} = w^2 C_2 L \Rightarrow w^2 = \frac{C_1 + C_2}{C_1 C_2 L} \Rightarrow W = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}}$$

$\hookrightarrow$ oscillation frequency.

$$\Rightarrow \frac{wL}{R_L} - \frac{gm}{wC_1} - \frac{1}{wR_L C_1} = 0$$

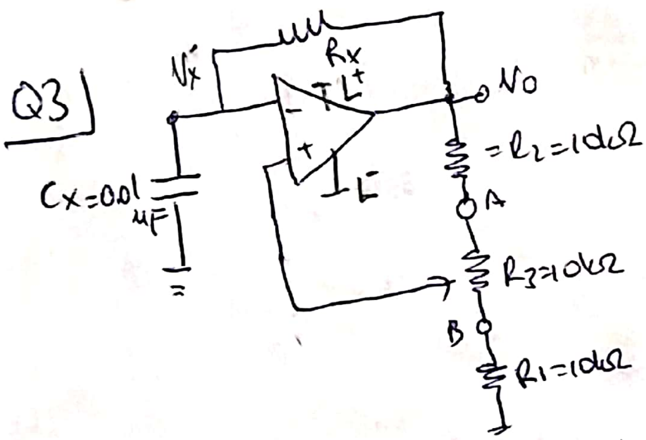
$$\frac{wL}{R_L} = \frac{1}{wC_1}\left(gm + \frac{1}{R_L}\right) \Rightarrow w^2 L C_1 = gmR_L + 1$$

$$\frac{C_1 + C_2}{\cancel{C_1} C_2 \cancel{L}} \cancel{L} \cancel{C_1} = gmR_L + 1$$

$$\frac{C_1}{C_2} + \cancel{1} = gmR_L + \cancel{1} \Rightarrow \boxed{gmR_L = \frac{C_1}{C_2}}$$

$\Rightarrow$ To ensure the oscillation $\boxed{gmR_L \geq \frac{C_1}{C_2}}$ $\rightarrow$ oscillation condition.

Q3)



We know saturation voltages $L^+ = -L^- = 10V$ and since this is an astable multivibrator, the switching voltage is $V_x = L^+ \cdot \frac{20k\Omega}{30k\Omega} = \frac{2}{3}L^+$

$$V_x = L^+ - \left(L^+ + \frac{2}{3}L^+\right) e^{-t/RC}$$

$$\frac{2}{3}L^+ = L^+ - \left(\frac{5}{3}L^+\right) e^{-t/RC}$$

$$\frac{1}{3}\cancel{L^+} = \frac{5}{3}\cancel{L^+} e^{-t/RC} \Rightarrow e^{-t/RC} = \frac{1}{5}$$

let's take the natural logarithm of both sides $\ln\left(e^{-t/RC}\right) = -\ln 5 \Rightarrow t/RC = \ln 5 \Rightarrow t = RC \ln 5$

$$t = \frac{T}{2} = \frac{1}{2 f_{osc}} = \frac{1}{2 \times 500} = 10^{-3} s \Rightarrow R_x C_x = \frac{t}{\ln 5} \Rightarrow \frac{10^{-3}}{\ln 5} = R_x C_x = 0.621 \text{ ms}$$

a) $$R_x = \frac{0.621 \times 10^{-3}}{C_x} = \frac{0.621 \times 10^{-3}}{0.01 \times 10^{-6}} = 62.1 \times 10^3\ \Omega = 62.1\ k\Omega ; \boxed{R_x = 62.1\ k\Omega}$$

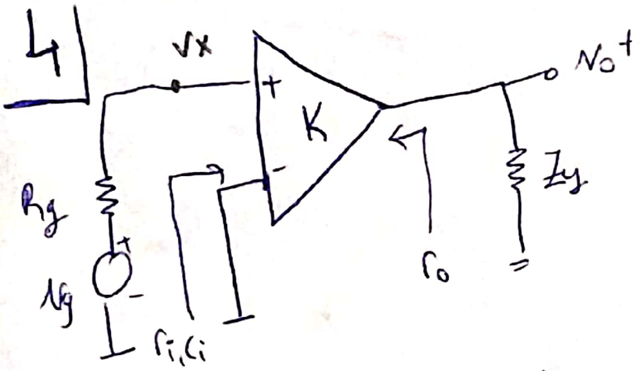
b) Now, potentiometer is connected to node B.

$$V_x = L^+ \cdot \frac{10}{30} = \frac{1}{3}L^+ \Rightarrow V_x = L^+ - \left(L^+ + \frac{1}{3}L^+\right) e^{-t/R_x C_x} \Rightarrow \frac{1}{3}L^+ = L^+ - \frac{4}{3}L^+ e^{-t/R_x C_x} \Rightarrow e^{-t/R_x C_x} = \frac{1}{2}$$

$$\Rightarrow \frac{t}{R_x C_x} = -\ln 2 \Rightarrow t = -\ln 2 (R_x C_x) = -\ln 2 (62.1 \times 10^3)(10^{-8}) = 4.3 \times 10^{-4} s \ ; \ t = \frac{T}{2} = \frac{1}{2 f_{osc}}$$

$$\Rightarrow f_{osc} = \frac{1}{2t} = \frac{1}{2(4.3 \times 10^{-4})} = 1.1628\ kHz ; \boxed{f_{oscillation} = 1.1628\ kHz}$$

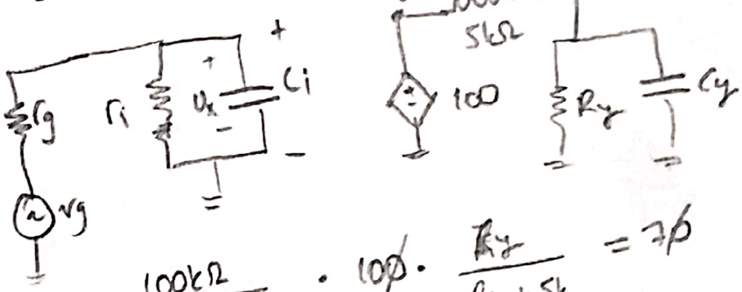
4)



$A_{v,ul} = 100$; $r_i = 100k\Omega$; $C_i = 50pF$

$r_o = 5k\Omega$, if Z_y connected $\rightarrow A_{v,l} = 70$

$f_{3\text{-}dB} = 50k\Omega$



When Z_y is connected

$$A_{v,L} = \frac{V_o^+}{v_g} = \frac{V_x}{v_g}\cdot\frac{V_y}{V_x}\cdot\frac{V_o^+}{V_y} = \frac{100k\Omega}{100k\Omega + 5k\Omega}\cdot 100\cdot\frac{R_y}{R_y + 5k} = 70$$

$$\Rightarrow \frac{R_y}{R_y + 5k} = \frac{7}{10}\cdot\frac{105}{100} = 0.735 \Rightarrow 0.735R_y + 3675 = R_y$$

$$3675 = 0.265R_y$$

$$R_y = \frac{3675}{0.265} = 13.868\ k\Omega$$

- $\tau_{in} = R_{in}C_{in} = 50pF\,(100k \parallel 5k\Omega)$

$$= 5\times10^{-11}\left(\frac{5\times10^{8}}{105\times10^{3}}\right) = 0.238\ \mu s$$

- $f_{in} = \dfrac{1}{2\pi\tau_{in}} = \dfrac{1}{2\pi(0.238\mu s)} = 668.45\ kHz$

- $\tau_{out} = R_{out}C_{out} = (13.868k\Omega \parallel 5k\Omega)\,C_{out} \Rightarrow C_{out} = \dfrac{\tau_{out}}{(13.868k\Omega \parallel 5k\Omega)}$

$\dfrac{1}{2\pi\tau_{out}} = 50k\Omega \Rightarrow \tau_{out} = 3.183\ \mu s$

$C_{out} = 866.11\ pF$

We should compensate the output pole. By using the inductor formula given as;

$L = \dfrac{R_A R_B^2 C_{out}}{2r_i}$; $R_A = R_g + r_i = 5k\Omega + 13.868k\Omega = 18.868k\Omega$

$R_B = R_g \parallel r_i = 5k\Omega \parallel 13.868k\Omega = 3.675k\Omega$

$$L = \frac{(18.868k\Omega)(3.675k\Omega)^2(866.11\times10^{-12})}{2(13.868k\Omega)} = 7.957\times10^{-3}\ H$$

$$\approx 7.96mH$$