Electro Magnetic Waves

- $\nabla f = \vec{e}_x \frac{\partial f}{\partial x} + \vec{e}_y \frac{\partial f}{\partial y} + \vec{e}_z \frac{\partial f}{\partial z} = \text{grad } f$
- $\nabla \cdot \vec{f} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = \text{div } \vec{f}$
- $\nabla \times \vec{F} = \text{curl } \vec{F} = \text{rot } \vec{E} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}$

$$\boxed{\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 E}$$

- $\nabla^2 \vec{F} = \Delta \vec{F} = \text{lap } \vec{F} = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2}$

Integral Theorem

- $\int_V (\nabla \cdot \vec{F})\, d\vartheta = \oint_S \vec{F} \cdot dS$
- $\int_S (\nabla \times \vec{F}) \cdot dS = \oint_C F \cdot dl$

Maxwel Equations

- $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$
- $\nabla \cdot \vec{H} = \frac{\partial D}{\partial t} + J_v$
- $\nabla \cdot \vec{D} = \rho_v$
- $\nabla \cdot \vec{B} = 0$

- $\vec{D} = \varepsilon \vec{E}$
- $\vec{B} = \mu \vec{H}$
- $\vec{J} = \sigma \cdot \vec{E}$

- $F = q(E + \vec{V} \times B)$ Lorentz Force

ε: dielectric permittivity [F/m]
μ: magnetic permeability [H/m]
σ: electric conductivity [s/m]

Boundry Conditions

- $E_{1t} = E_{2t}$
- $B_{1n} = B_{2n}$
- $D_{1n} - D_{2n} = \rho_s$
- $H_{1t} - H_{2t} = J_s$

$$V = \frac{1}{4\pi\varepsilon_0} \int_{\vartheta'} \frac{\rho}{R}\, d\vartheta'$$

$$A = \frac{\mu}{4\pi} \int_{\vartheta'} \frac{J}{R}\, d\vartheta'$$

$$E = \frac{1}{4\pi\varepsilon_0} \int_{\vartheta'} \rho \frac{\bar{R}}{R^3}\, d\vartheta'$$

$$B = \frac{\mu}{4\pi} \int_{\vartheta'} J \frac{\bar{R}}{R^3}\, d\vartheta'$$

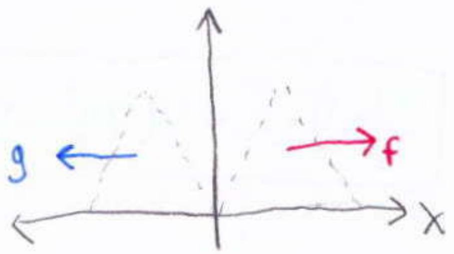
Wave Equations

$$\left(\nabla^2 - \frac{1}{C^2}\frac{\partial}{\partial t^2}\right) u(r;t) = 0$$

$$C_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

- $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu\frac{\partial J_v}{\partial t} + \nabla\frac{\rho_v}{\varepsilon}$
- $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{H} = -\nabla \times J_v$

For free space $\rho_v = 0$, $J_v = 0$



$$u(x,t) = \underbrace{f(x-ct)}_{+x\ \text{direction}} + \underbrace{g(x+ct)}_{-x\ \text{direction}}$$

travelling wave

- $\oint E.dl = -\frac{\partial}{\partial t}\int_s B.ds = \frac{\partial \Phi}{\partial t}$

$\oint_s D.ds = \int \rho_v dv = Q$

Φ : Magnetic Flux

Q : Total Electric Charge

Simple Medium $(\varepsilon, \mu, \sigma)$

$\vec{D} = \varepsilon\vec{E}$, $\vec{B} = \mu\vec{H}$, $\vec{J} = \sigma\vec{E}$ } Linear

$\varepsilon \neq \varepsilon(r)$, $\mu \neq \mu(r)$, $\sigma \neq \sigma(r)$ } Homogenous

$\vec{D} \parallel \vec{E}$, $\vec{B} \parallel \vec{H}$ } Isotropic

$\varepsilon \neq \varepsilon(t)$, $\mu \neq \mu(t)$ } Stationary

$\varepsilon = \varepsilon_0\varepsilon_r$ $\quad \varepsilon_r \geq 1$

$\mu = \mu_0\mu_r$ $\quad \mu_r \approx 1$ (most case)

Time Harmonic Waves (Monochromatic waves – Single Frequency)

$$u(\vec{r};t) = u(\vec{r})\cos(wt - \alpha(\vec{r}))$$

- $wt - kx = \text{constant}$
- $w\,dt - k\,dx = 0 \Rightarrow \frac{dx}{dt} = \frac{w}{k} = v_p$

$\boxed{v_p = \frac{w}{k}} \rightarrow f.\lambda = v_p = \frac{2\pi f}{k}$

$\boxed{\lambda = \frac{2\pi}{k}} \leftarrow \lambda . f = \frac{2\pi f}{k}$

k: Wave number

Phasor Representation

$E(r;t) = \mathrm{Re}\{E(r)e^{iwt}\}$

E(r): Phasor

$$\boxed{\frac{\partial^n}{\partial t^n}E(r;t) = \mathrm{Re}\{(iw)^n E(\vec{r})e^{iwt}\}}$$

EHB 313E

Time Harmonic Wave Eq.

- $\nabla \times \vec{H} = \dfrac{\partial \vec{D}}{\partial t} + \vec{J} \quad \Rightarrow \quad \nabla \times \vec{H} = i w \varepsilon \vec{E} + \vec{J}$
- $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \quad \Rightarrow \quad \nabla \times \vec{E} = -i w \mu \vec{H}$
- $\nabla \cdot \vec{D} = \rho_v \quad \Rightarrow \quad \nabla \cdot \vec{E} = \dfrac{\rho_v}{\varepsilon}$
- $\nabla \cdot \vec{B} = 0 \quad \Rightarrow \quad \nabla \cdot \vec{B} = 0$

Helmholtz Equation (Reduced Wave Equation)

- $\nabla^2 \vec{E} + k^2 \vec{E} = \dfrac{1}{\varepsilon} \nabla \rho_v$

$k^2 = w^2 \mu \varepsilon - i w \mu \sigma$

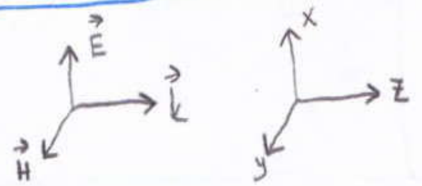
k : wave number spatial frequency

$$k^2 = \underbrace{w^2 \mu \varepsilon_0}_{k_0^2} \underbrace{\left(\varepsilon_r - i \frac{\sigma}{w \varepsilon_0} \right)}_{\varepsilon_{cr}}$$

ε_{cr} : Complex Relative Permittivity

→ Wave frequency and the length of antenna are inversly propotional

Plane wave Solution



$$\boxed{E_x = \underbrace{E_0^+ e^{-ikz}}_{+z \text{ direction}} + \underbrace{E_0^- e^{ikz}}_{-z \text{ direction}}}$$

wave propogating

Loss Tangent

$$\boxed{\tan \delta = \frac{\sigma}{w \varepsilon}}$$

δ : Loss Angle

$\tan \delta \gg 1$; good conductor

$\tan \delta \ll 1$; good insulator (dielectric)

- $E_x(z;t) = \mathrm{Re}\{E_x(z) e^{iwt}\}$

$= \mathrm{Re}\{E_0^+(z) . e^{-ikz} . e^{iwt}\}$ | $E_x(z) = |E_0^+| e^{i\phi}$

- $E_x(z;t) = |E_0^+| \cos(wt - kz + \phi)$

$\Downarrow$

- $w\,dt - k\,dz = 0 \Rightarrow$ phase velocity $\boxed{v_p = \dfrac{dz}{dt} = \dfrac{w}{k}}$

- "λ" and "v_p" depends on $(\varepsilon, \mu, \sigma)$ of medium

Intrinsic Impedence

using $(\nabla \times \vec{E} = -i w \mu \vec{H})$

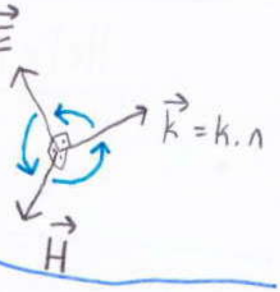
$$\eta = \frac{E_x}{H_y} = \frac{w \mu}{k} \Rightarrow \boxed{\eta = \sqrt{\frac{\mu}{\varepsilon}} \; [\Omega]} = 120\pi$$

Transverse ElectroMagnetic Waves (TEM)

- $E = E_0 \, e^{-i(k_x x + k_y y + k_z z)} = E_0 \, e^{-i\bar{k}\bar{r}}$
- $\vec{k} = k_x . \vec{e}_x + k_y \vec{e}_y + k_z \vec{e}_z \Rightarrow$ • $k^2 = k_x^2 + k_y^2 + k_z^2 = w^2 . \varepsilon . \mu$
- $\vec{k} = k . \vec{n} \Rightarrow$ • $\vec{n} = \dfrac{k_x \vec{e}_x + k_y \vec{e}_y + k_z \vec{e}_z}{\sqrt{k_x^2 + k_y^2 + k_z^2}}$ $\hat{n}$: direction of propagation
- $r = x \vec{e}_x + y \vec{e}_y + z \vec{e}_z$ r: radius vector
- $\boxed{E = E_0 \, e^{-i\vec{k}.\vec{r}} = E_0 \, e^{-i.k.\vec{n}.\vec{r}}}$ Plane Wave

Example

$\vec{k} = k \, \vec{e}_z$ $\vec{E} = E_x . \vec{e}_x$ $\vec{H} = H_y . \vec{e}_y$



For TEM Waves

$\vec{E}(\vec{r}) = \eta . \vec{H}(\vec{r}) \times \vec{n}$

$\vec{H}(\vec{r}) = \dfrac{1}{\eta} . \vec{n} \times \vec{E}(\vec{r})$

Properties

i) $\vec{E} . \vec{k} = 0$

ii) $\vec{E} \perp \vec{H} = 0$

iii) $(\vec{E} \times \vec{H}) \, // \, \vec{k}$

iv) $\eta = \dfrac{\vec{E}(r)}{\vec{H}(r)}$

Conducting Media $\approx$ Lossy Media

Plane Waves In Conducting Media ($\sigma \neq 0$)

- $k^2 = w^2 \varepsilon \mu - i w \sigma \mu$
- $\boxed{k = w \sqrt{\varepsilon_c \mu}}$
- $\boxed{\varepsilon_c = \varepsilon - i \dfrac{\sigma}{w}}$

$\boxed{\gamma = ik} \Rightarrow \boxed{\gamma = \alpha + i\beta}$

$(\nabla^2 + k^2)\vec{E} = 0 \Rightarrow (\nabla^2 - \gamma^2)\vec{E} = 0$

$e^{-ikz} = e^{-\gamma z} = e^{-\alpha z} e^{-i\beta z}$

$E = E_0 e^{-ikz} = E_0 \underbrace{e^{-\alpha z}}_{\text{causes exponentially decaying}} \underbrace{e^{-i\beta z}}_{\text{phase factor}}$

α: attenuation constant $\left[\frac{Np}{m}\right]$

β: phase constant $\left[\frac{rad}{m}\right]$

γ: propagation constant

k: wave number

For free space $\alpha = 0$
$k = \beta = \dfrac{2\pi}{\lambda}$

* Low Loss Dielectrics ($\frac{\sigma}{w\varepsilon} \ll 1$)

$$k^2 = w^2 \mu \varepsilon - i w \mu \sigma = w^2 \mu \varepsilon \left(1 + \frac{\sigma}{i w \varepsilon}\right)$$

$$k = \underbrace{w\sqrt{\mu \varepsilon}}_{\text{constant}} \left(1 + \underbrace{\frac{\sigma}{i w \varepsilon}}_{\ll 1}\right)^{1/2}$$

$$\left(1 + \frac{\sigma}{i w \varepsilon}\right)^{1/2} = 1 + \frac{1}{2}\frac{\sigma}{i w \varepsilon} + \frac{1}{8}\left(\frac{\sigma}{w \varepsilon}\right)^2$$

$$(\gamma = ik = \alpha + i\beta)$$

- $\boxed{\alpha = w\sqrt{\mu\varepsilon} \cdot \frac{\sigma}{2 w \varepsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}$
- $\boxed{\beta = w\sqrt{\varepsilon \mu}\left[1 + \frac{1}{8}\left(\frac{\sigma}{w\varepsilon}\right)^2\right]}$
- $\boxed{\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon}}\left(1 + i\frac{\sigma}{2 w \varepsilon}\right)}$

wave impedence

- $v_p = \frac{w}{\beta} = \frac{1}{\sqrt{\varepsilon\mu}}\left[1 - \frac{1}{8}\left(\frac{\sigma}{w\varepsilon}\right)^2\right]$

* Good Conductors ($\frac{\sigma}{w\varepsilon} \gg 1$)

- $\left(1 + \frac{\sigma}{i w \varepsilon}\right)^{1/2} \approx \sqrt{\frac{\sigma}{w\varepsilon}}$
- $\gamma = i w \sqrt{\varepsilon\mu} \cdot \sqrt{\frac{\sigma}{w\varepsilon}}$

$$\boxed{\gamma = \alpha + i\beta = (1+i)\sqrt{\pi f w \mu \sigma}}$$

$$\boxed{\alpha = \beta = \sqrt{\pi f w \mu \sigma}}$$

- $\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{i w \mu}{\sigma}}$ $\quad \left(\varepsilon_c = \frac{\sigma}{i w}\right)$
- $\eta_c = (1+i)\sqrt{\frac{\pi f \mu}{\sigma}} \Rightarrow \boxed{\eta_c = (1+i)\left(\frac{\alpha}{\sigma}\right)}$
- $\boxed{v_p = \frac{w}{\beta} = \sqrt{\frac{2w}{\mu\sigma}}}$

Skin Depht (δ_s)

- $\boxed{\delta_s = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}}$

- Frekans arttıkça skin depth azalır, ölçülen alan azdır
- Frekans azaldıkça ölçülen alan artar fakat resolution azalır.

Polarization of EM waves

$$\boxed{\vec{E}(z) = \vec{e}_x E_x(z) + \vec{e}_y E_y(z)}$$

For RHCP $\quad E_y(z) = -i E_{y0} e^{-ikz}$

For LHCP $\quad E_y(z) = i E_{y0} e^{-ikz}$

For Linearly P. $\quad E_y(z) = E_{y0} e^{-ikz}$

For RHCP

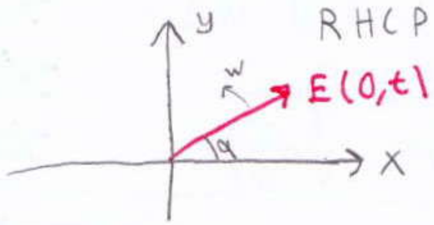
$$E(z;t) = Re\left\{\left[\vec{e}_x E_{x0} e^{-ikz} + \vec{e}_y e^{-i\frac{\pi}{2}} E_{y0} e^{-ikz}\right] e^{iwt}\right\}$$

$$E(z;t) = \vec{e}_x E_{x0} \cos(wt - kz) + \vec{e}_y E_{y0} \cos\left(wt - kz - \frac{\pi}{2}\right)$$

$$E(0;t) = \vec{e}_x E_{x0} \cos(wt) + \vec{e}_y E_{y0} \sin(wt)$$

$$\left[\frac{E_x(0;t)}{E_{x0}}\right]^2 + \left[\frac{E_y(0,t)}{E_{y0}}\right]^2 = 1$$

- $\vec{E}$ is elliptically polarized if $E_{x0} \neq E_{y0}$
- $\vec{E}$ is circullarly " if $E_{x0} = E_{y0}$



$$\boxed{\alpha = \tan^{-1}\left(\frac{E_y(0,t)}{E_x(0,t)}\right) = wt}$$

$$\boxed{\text{LHCP} \Rightarrow \vec{E}(z) = \vec{e}_x E_{x0} e^{-ikz} + i\vec{e}_y E_{y0} e^{-ikz}}$$

$$\boxed{\text{Linearly Polarized} > \vec{E}(z) = \left[\vec{e}_x E_{x0} + \vec{e}_y E_{y0}\right] \cos wt}$$

$$E(z;k) = E_0 \cos\left[(w_0 + \Delta w)t - (\beta_0 + \Delta\beta) z\right] + E_0 \cos\left[(w_0 - \Delta w)t - (\beta - \Delta\beta) z\right]$$

$$= 2E_0 \cos\left[(\Delta wt - \Delta\beta z)\right] \cos\left[(wt - \beta z)\right]$$

$\boxed{\vartheta_p = \frac{dz}{dt} = \frac{w}{\beta}}$ ⇒ Phase Velocity $\boxed{\vartheta_g = \frac{dz}{dt} = \frac{\Delta w}{\Delta\beta}}$ ⇒ Group Velocity

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Poynting Vector

$$\boxed{\vec{P} = \vec{E} \times \vec{H}}$$

Poynting Vector

$$\boxed{-\int P.dS = \frac{\partial}{\partial t}\int_{\vartheta}(w_e + w_m)\, d\vartheta + \int_{\vartheta} P_\sigma \, d\vartheta}$$

$\boxed{W_e = \frac{\varepsilon}{2} E^2 = \frac{\varepsilon}{2} \bar{E}.\bar{E}^*}$ Electric Energy Density

$\boxed{w_m = \frac{\mu}{2} H^2 = \frac{\mu}{2} \bar{H}.\bar{H}^*}$ Magnetic Energy Density

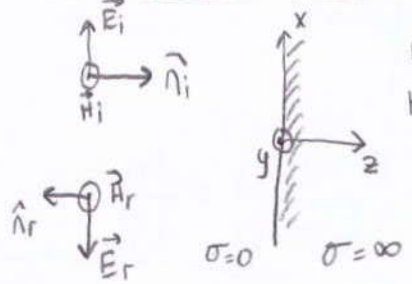
$\boxed{P_\sigma = \sigma E^2 = \sigma.\bar{E}.\bar{E}^*}$ Ohmic Power Density

Instanteneous Power Density: $\boxed{\vec{P}(r;t) = \vec{E}(r;t) \times \vec{H}(r;t)}$

Avarage Power Density: $\boxed{P_{ave}(\bar{r}) = \frac{1}{2} \text{Re}\left[E(\bar{r}) \times H^*(\bar{r})\right] \left[\frac{W}{m^2}\right]}$

Midterm 2 EMB 3I3E

Normal Incidence at a Plane Conducting Media



$E_2(z)=0$

$H_2(z)=0$

$\vec{E}_i(z)=\vec{e}_x E_{oi}\, e^{-i\beta_1 z}$ $\qquad$ $\vec{E}_r(z)=\vec{e}_x E_{or}\, e^{i\beta_1 z}$

$\vec{H}_i(z)=\vec{e}_y \dfrac{E_{oi}}{\eta_1} e^{-i\beta_1 z}$ $\qquad$ $\vec{H}_r(z)=\vec{e}_y \dfrac{E_{or}}{\eta_1} e^{i\beta_1 z}$

$E_1(0)=E_2(0) \Rightarrow E_i(z)+E_r(z)=0$

$\boxed{E_{oi}=-E_{or}}$

$\boxed{E_1(z)=\vec{e}_x E_{oi}\left(e^{-i\beta_1 z}-e^{i\beta_1 z}\right)=-\vec{e}_x E_{oi}.2i.\sin\beta_1 z}$

$H_r(z)=\underbrace{\hat{n}_r}_{-\vec{e}_z}\times\dfrac{E_r(z)}{\eta_1}=+\vec{e}_y\dfrac{E_{oi}}{\eta_1}.e^{i\beta_1 z}$

$\boxed{H_1(z)=\vec{e}_y.2.\dfrac{E_{oi}}{\eta_1}.\cos\beta_1 z}$

Using Phasor Representation

$\boxed{E_1(z;t)=\vec{e}_x\, 2.E_{io}.\sin\beta_1 z.\sin wt}$

$\boxed{H_1(z;t)=\vec{e}_y\, 2.\dfrac{E_{io}}{\eta_1}.\cos\beta_1 z.\cos wt}$

$E_1=0 \quad H_1=\max$

$\beta_1 z=-n.\pi \quad n=0,1,2\ldots$

$z=-n.\dfrac{\lambda}{2}$

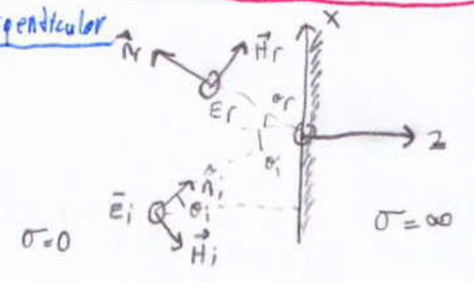
$E_1=\max \quad H_1=0$

$\beta_1 z=-(2n+1).\dfrac{\pi}{2} \quad n=0,1,2$

$z=-(2.n+1)\dfrac{\lambda}{4}$

Oblique Incidence at a Plane Conducting Media

Perpendicular



θ_i = Oblique Incident angle

$E_r=?$

$H_r=?$

$\theta_r=?$

$\hat{n}_i=\vec{e}_x\sin\theta_i+\vec{e}_z\cos\theta_i$

$\vec{r}=x\vec{e}_x+y\vec{e}_y+z\vec{e}_z \qquad k.\vec{n}_i.\vec{r}=k(z\cos\theta_i+x\sin\theta_i)$

$E_i(z,x)=\vec{e}_y.E_{io}.e^{-i\beta_1(z\cos\theta_i+x\sin\theta_i)}$

$H_i(z,x)=\dfrac{E_i}{\eta}\left(z.\sin\theta_i-x\cos\theta_i\right).e^{-i\beta_1(z\cos\theta_i+x\sin\theta_i)}$

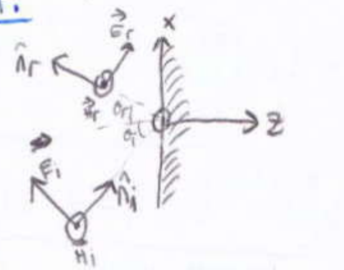
$\hat{n}_r=\vec{e}_x.\sin\theta_r-\vec{e}_z\cos\theta_r$

$\boxed{E_{io}=-E_{ro}} \quad \boxed{\theta_i=\theta_r} \rightarrow$ Snell's Law

$\vec{E}_r(x,z)=-\vec{e}_y E_{io}.e^{-i\beta_1(x.\sin\theta_i-z\cos\theta_i)}$

$\vec{H}_r(x,z)=\dfrac{1}{\eta_1}\left[\hat{n}_r\times\vec{E}_r(x,z)\right]$

Parallel P.



$\boxed{v_p=\dfrac{w}{\beta_{1x}}}$

$\boxed{\beta_{1x}=\beta_1\sin\theta_1}$

$H_1(x,z)=\vec{e}_y 2.\dfrac{E_{io}}{\eta_1}\cos(\beta_1 z\cos\theta_i)e^{-j\beta_1\sin\theta_i}$

$E_1(x,z)=-2E_{io}\left[\vec{e}_x j.\cos\theta_i.\sin(\beta_1 z\cos\theta_i)\right.$

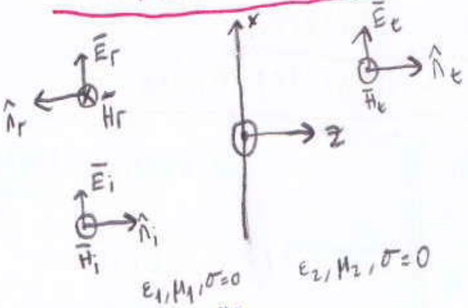
$\left.+\vec{e}_z\sin\theta_i.\cos(\beta_1 z\cos\theta_i)\right]e^{-j\beta_1 x\sin\theta_i}$

$E_1(x,z)=-\vec{e}_y.2.E_{io}.\sin(\beta_1.z.\cos\theta_i)e^{-i\beta_1 x\sin\theta_i}$

$H_1(x,z)=-2.\dfrac{E_{io}}{\eta_1}\left[\vec{e}_x\cos\theta_1.\cos(\beta_1 z\cos\theta_1)\right.$

$\left.+\vec{e}_z j\sin\theta_i.\sin(\beta_1 z.\cos\theta_i)\right]e^{-i\beta_1 x\sin\theta_i}$

Normal Incidence at a Plane Dielectric



Incident Wave

$E_i(z) = \vec{e}_x E_{i0} e^{-i\beta_1 z}$ $\quad H_i(z) = \vec{e}_y \frac{E_{i0}}{\eta_1} e^{-i\beta_1 z}$

Reflected Wave

$E_r(z) = \vec{e}_x . E_{r0} . e^{+i\beta_1 z}$ $\quad H_r(z) = -\vec{e}_y \frac{E_{r0}}{\eta_1} e^{+i\beta_1 z}$

Transmitted Wave

$E_t(z) = \vec{e}_x . E_{t0} . e^{-i\beta_2 z}$ $\quad H_t(z) = e_y \frac{E_{t0}}{\eta_2} e^{-i\beta_2 z}$

(i) $E_i(0) + E_r(0) = E_t(0)$

(ii) $H_i(0) + H_r(0) = H_t(0)$

$$\boxed{E_{r0} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} E_{i0}} \qquad \boxed{E_{t0} = \frac{2\eta_2}{\eta_1 + \eta_2} E_{i0}}$$

Γ: Reflection coefficient $\qquad \tau$: Transmission Coefficient

$$\boxed{1 + \Gamma = \tau}$$

• If $\sigma_2 = \infty$ (PEC) $\rightarrow \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2(\sigma)}} = 0 \quad \tau = 0 \quad \Gamma = -1$

• If $\sigma_2 \neq \infty$ (NOT PEC)

$$\boxed{E_1(z) = \vec{e}_x E_{i0}\Big[\underbrace{e^{-i\beta_1 z}(1+\Gamma)}_{\text{travelling wave}} + \underbrace{2i\sin\beta_1 z}_{\text{standing wave}}\Big]}$$

$E_1(z) = \vec{e}_x . E_{i0} . e^{-i\beta_1 z}\left[1 + \Gamma . e^{i2\beta_1 z}\right]$

Standing Wave Ratio

$$\boxed{S = SWR = \frac{|E_{max}|}{|E_{min}|} = \frac{1+|\Gamma|}{1-|\Gamma|}} \qquad \boxed{\Gamma = \frac{S-1}{S+1}}$$

$\Gamma \in (-1,1)$	$S \in (1,\infty)$	In dB = $20 \log S$

$\Gamma = 1$ OPEN Circuit

$\Gamma = 0$ Perfectly Matched

$\Gamma = -1$ Short Circuit

$H_t(z) = \vec{e}_y . \frac{\tau}{\eta_2} . E_{i0} . e^{-i\beta_2 z}$

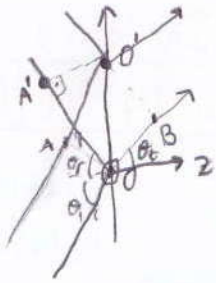
$E_t(z) = \vec{e}_x . \tau . E_{i0} . e^{-i\beta_2 z}$

Problem 8-29 'u 4öz

Oblique Incidence at a Plane Dielectric

(i) $\overline{OO'} \sin\theta_r = \overline{O'O} \sin\theta_i$

$$\boxed{\theta_r = \theta_i}$$



(ii) $\frac{\overline{OB}}{v_{p_2}} = \frac{\overline{AO'}}{v_{p_1}} \Rightarrow \frac{\overline{OO'}\sin\theta_t}{\overline{OO'}\sin\theta_i} = \frac{v_{p_2}}{v_{p_1}}$

$$\boxed{\frac{\sin\theta_t}{\sin\theta_i} = \frac{v_{p_2}}{v_{p_1}} = \frac{\beta_1}{\beta_2} = \frac{n_1}{n_2} = \frac{\eta_2}{\eta_1} = \sqrt{\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}}}$$

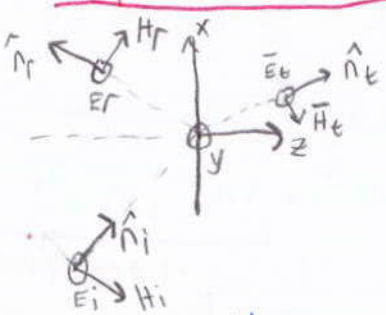
FOR TOTAL REFLECTION

$\theta_t = \frac{\pi}{2} \Rightarrow \frac{1}{\sin\theta_i} = \sqrt{\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}} \Rightarrow \sin\theta_i = \sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}}$

$$\boxed{\theta_c = \arcsin\left(\sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}}\right)}$$

Critical Angle

Perpendicular Polarization (Dielectric)



Incident Field

$\hat{n}_i = \vec{e}_x \sin\theta_i + \vec{e}_z \cos\theta_i$ $\qquad \beta_1 \cdot \hat{n}_i \cdot \bar{r} = \beta_1 (x \sin\theta_i + z\cos\theta_i)$

$E_i = \vec{e}_y \cdot E_{i0} \cdot e^{-i\beta_1 \hat{n}_i \cdot \bar{r}}$

$H_i = \frac{E_{i0}}{\eta_1} \cdot (-e_z \sin\theta_i + e_x \cdot \cos\theta_i) \cdot e^{-i\beta_1 \hat{n}_i \cdot \bar{r}}$

Reflected Field

$\hat{n}_r = +\vec{e}_x \sin\theta_r - \vec{e}_z \cos\theta_r$

Transmitted Field

$\hat{n}_t = \vec{e}_x \sin\theta_t + \vec{e}_z \cos\theta_t$

Boundary Conditions

(i) $E_i(x,0) + E_r(x,0) = E_t(x,0)$

(ii) $H_i(x,0) + H_r(x,0) = H_t(x,0)$

Extra Equation (Phase Matching)

$\beta_1 \cdot \sin\theta_i = \beta_1 \sin\theta_r = \beta_2 \sin\theta_t$

$\boxed{\theta_i = \theta_r}$ $\qquad \boxed{\frac{\beta_1}{\beta_2} = \frac{\sin\theta_t}{\sin\theta_i}}$

$$\boxed{\Gamma_\perp = \frac{(\eta_2/\cos\theta_t) - (\eta_1/\cos\theta_i)}{(\eta_2/\cos\theta_t) + (\eta_1/\cos\theta_i)}}$$

$$\boxed{\tau_\perp = \frac{2 \cdot (\eta_2/\cos\theta_t)}{(\eta_2/\cos\theta_t) + (\eta_1/\cos\theta_i)}}$$

$\boxed{1 + \Gamma_\perp = \tau_\perp}$

using: $E_i + E_{r0} = E_{t0}$, $\frac{1}{\eta_1}(E_{i0} - E_{r0}) \cdot \cos\theta_i = \frac{1}{\eta_2} E_{t0} \cos\theta_t$

Brewster Angle θ_B

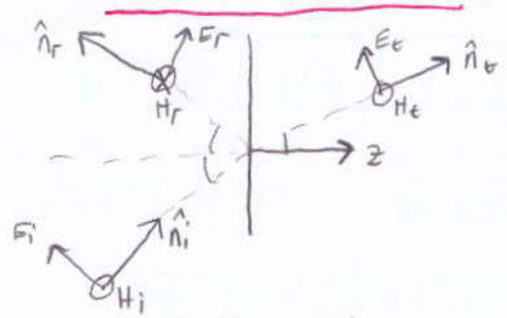
$\Gamma_\perp = 0 \Rightarrow \eta_2 \cos\theta_i = \eta_1 \cos\theta_t \Rightarrow \cos\theta_B = \frac{\eta_2}{\eta_1}\cos\theta_t$

$$\boxed{\theta_B = \arccos\left(\frac{\eta_2}{\eta_1}\cos\theta_t\right)}$$

if $\varepsilon_1 = \varepsilon_2$ $\quad \mu_1 \neq \mu_2$

$$\boxed{\sin\theta_B = \frac{1}{\sqrt{1 + (\mu_1/\mu_2)}}}$$

Parallel Polarization



Incident Field

$\hat{n}_i = \vec{e}_x \sin\theta_i + \vec{e}_z \cos\theta_i$

$E_i(x,z) = E_{i0} \cdot (\vec{e}_x \cos\theta_i - \vec{e}_z \sin\theta_i) \cdot e^{-i\beta_1 \hat{n}_i \cdot \vec{r}}$

$H_i(x,z) = \frac{E_{i0}}{\eta_1} \cdot \vec{e}_y \, e^{-i\beta_1 \hat{n}_i \cdot \vec{r}}$

Boundary Condition (@ z=0)

(i) $(E_{i0} + E_{r0}) \cos\theta_i = E_{t0} \cos\theta_t$

(ii) $\frac{1}{\eta_1}(E_{i0} - E_{r0}) = \frac{1}{\eta_2} E_{t0}$

$$\boxed{\Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}}$$

$$\boxed{\tau_\parallel = \frac{2 \cdot \eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}}$$

$$\boxed{1 + \Gamma_\parallel = \tau_\parallel \left(\frac{\cos\theta_t}{\cos\theta_i}\right)}$$

Brewster Angle (θ_B)

if $\Gamma_\parallel = 0$ $\qquad \eta_2 \cos\theta_t = \eta_1 \cos\theta_i$

$$\sin^2\theta_{B_\parallel} = \frac{1 - \mu_2\varepsilon_1/\mu_1\varepsilon_2}{1 - (\varepsilon_1/\varepsilon_2)^2}$$

- if $(\mu_1 = \mu_2)$

$$\sin\theta_B = \sqrt{\frac{1}{1 + (\varepsilon_1/\varepsilon_2)}}$$

Waveguides

$$E(x,y,z,t) = \mathrm{Re}\left\{ E(x,y) \cdot e^{i\omega t} \cdot e^{-\gamma z} \right\}$$

- $\nabla^2 E + k^2 E = 0$ • $\nabla^2 H + k^2 H = 0$
- $\nabla_{xy}^2 E + \nabla_z^2 E = \nabla_{xy}^2 E + \gamma^2 E$
- $\boxed{\left[\nabla_{xy}^2 + (\gamma^2 + k^2)\right] E = 0}$ • $\boxed{h^2 = \gamma^2 + k^2}$
- $\boxed{\left[\nabla_{xy}^2 + (\gamma^2 + k^2)\right] H = 0}$

4 Eq.

$$H_x^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial H_z^0}{\partial x} - i\omega\varepsilon \frac{\partial E_z^0}{\partial y}\right)$$

$$H_y^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial H_z}{\partial y} + i\omega\varepsilon \frac{\partial E_z^0}{\partial x}\right)$$

$$E_x^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial E_z^0}{\partial x} + i\omega\mu \frac{\partial H_z^0}{\partial y}\right)$$

$$E_y^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial E_z^0}{\partial y} - i\omega\mu \frac{\partial H_z^0}{\partial x}\right)$$

TEM_z Waves ($E_z = 0$, $H_z = 0$)

$\gamma_{TEM}^2 + k^2 = 0$ $\gamma_{TEM} = ik = i\omega\sqrt{\mu\varepsilon}$

TEM waves cannot propogates in the Wg.

$$\boxed{Z_{TEM} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \sqrt{\frac{\mu}{\varepsilon}} = \eta}$$

TM_z Waves ($E_z \neq 0$, $H_z = 0$)

$$\nabla_{xy}^2 E_z + (\gamma^2 + k^2) E_z = 0$$

$$H_x^0 = \frac{i\omega\varepsilon}{h^2} \frac{\partial E_z^0}{\partial y}$$

$$H_y^0 = -\frac{i\omega\varepsilon}{h^2} \frac{\partial E_z^0}{\partial x}$$

$$E_x^0 = -\frac{\gamma}{h^2} \frac{\partial}{\partial x} E_z^0$$

$$E_y^0 = -\frac{\gamma}{h^2} \frac{\partial}{\partial y} E_z^0$$

$$(E_T^0)_{TM} = \vec{e}_x E_x^0 + \vec{e}_y E_y^0$$

$$(E_T^0)_{TM} = -\frac{\gamma}{h^2} \nabla_T E_z^0$$

$$\boxed{Z_{TM} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{\gamma}{i\omega\varepsilon}}$$

$$\gamma^2 = h^2 - k^2 \rightarrow \gamma = \sqrt{h^2 - \omega^2\mu\varepsilon}$$

where $\gamma = 0$

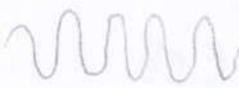
$$h^2 = \omega_c^2 \mu\varepsilon \Rightarrow \boxed{f_c = \frac{h}{2\pi\sqrt{\mu\varepsilon}} \ [Hz]}$$

cut off frequency

- $\boxed{\gamma = h\sqrt{1 - \left(\frac{f}{f_c}\right)^2}} \rightarrow$ $f > f_c$ or $f < f_c$

if $f > f_c$, $\gamma = i\beta$

$e^{-i\beta z}$



→ oscillations
propagating mode

if $f < f_c$, $\gamma = \alpha$

$e^{-\alpha z}$

evanescent mode
→ exponentially decay

if $f > f_c$, $\gamma = i\beta$

$$\beta = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\boxed{\lambda = \frac{2\pi}{k} = \frac{1}{f\sqrt{\varepsilon\mu}} = \frac{v_T}{f}}$$

$$\boxed{\lambda_g = \frac{2\pi}{\beta} = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}} > \lambda}$$

$v_p = \frac{\lambda_g}{\lambda} v > v$ $v_g = \frac{\lambda}{\lambda_g} \cdot v < v$

$\boxed{v_p \cdot v_g = v^2}$ $\boxed{v = \frac{1}{\sqrt{\varepsilon\mu}}}$

$$\boxed{Z_{TM} = \eta\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} < \eta$$

$$\vec{H} = \frac{1}{Z_{TM}}\left(e_z \times \vec{E}\right)$$

TE_z Waves ($E_z = 0$, $H_z \neq 0$)

$$\nabla_{xy}^2 H + (\gamma^2 + k^2) H = 0$$

4 Eq.

$$H_x^0 = -\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial x}$$

$$H_y^0 = -\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial y}$$

$$E_x^0 = -\frac{i w \mu}{h^2}\frac{\partial H_z^0}{\partial y}$$

$$E_y^0 = +\frac{i w \mu}{h^2}\frac{\partial H_z^0}{\partial x}$$

$$(H_T^0)_{TE} = -\frac{\gamma}{h^2}\nabla_T H_z^0$$

$$\boxed{Z_{TE} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{i w \mu}{\gamma}}$$

$$\vec{E} = Z_{TE} \cdot (\vec{H} \times \vec{e}_z)$$

$$\gamma = h\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

if $f > f_c$ $\gamma = i\beta$ Propagating Mode

$\gamma = i\beta$ $\beta = k\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$$\boxed{Z_{TE} = \frac{i w \mu}{i w\sqrt{\epsilon\mu}\cdot\sqrt{1-(f_c/f)^2}} = \frac{\eta}{\sqrt{1-(f_c/f)^2}}}$$

$> \eta$

if $f < f_c$ $\gamma = \alpha$ Evanescent Mode

	Wave Impedence	W_g Wavelength
TEM	$\eta = \sqrt{\frac{\mu}{\epsilon}}$	$\lambda = \frac{2\pi}{k} = \frac{1}{f\sqrt{\epsilon\mu}}$
TM	$\eta\sqrt{1-(f_c/f)^2}$	$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$
TE	$\frac{\eta}{\sqrt{1-(f_c/f)^2}}$	$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$

Rectangular Waveguides

TM Waves

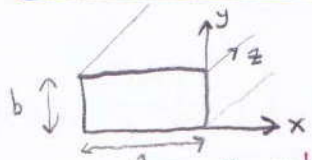
$H_z = 0$ $E_z(x,y,z) = E_z^0(x,y)e^{-\gamma z}$

$$\boxed{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right) E_z^0(x,y) = 0}$$

$$E_z^0(x,y) = X(x) \cdot Y(y)$$

- $X''Y + XY'' + h^2 XY = 0$
- $\underbrace{\frac{X''}{X}}_{-k_x^2} + \underbrace{\frac{Y''}{Y}}_{-k_y^2} + h^2 = 0$
- $\boxed{h^2 = k_x^2 + k_y^2}$

$X(x) = A\sin(k_x x) + B\cos(k_x x)$
$Y(y) = C\sin(k_y y) + D\cos(k_y y)$



Boundary Condition

x-Direction
$E_z^0(0,y) = 0$
$E_z^0(a,y) = 0$

y-direction
$E_z^0(x,0) = 0$
$E_z^0(x,b) = 0$

$X(0) \cdot Y(y) = 0 \Rightarrow B = 0$ $X(x) \cdot Y(0) = 0 \Rightarrow D = 0$

$$\boxed{E_z^0(x,y) = E_0 \sin(k_x \cdot x) \cdot \sin(k_y \cdot y)}$$

- $E_z^0(a,y) = 0$ $\boxed{k_x = \frac{m \cdot \pi}{a}}$
- $E_z^0(x,b) = 0$ $\boxed{k_y = \frac{n\pi}{b}}$

$$\boxed{E_z(x,y) = E_0 \sin\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right)}$$

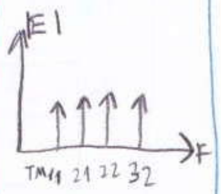
$$\boxed{h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\gamma^2 = h^2 - k^2$$

$$\boxed{\gamma = i\beta = i\sqrt{w^2 \varepsilon \mu - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

$$\boxed{f_c = \frac{1}{2\sqrt{\varepsilon\mu}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$\boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}}$$



TE Waves

$$H_z \neq 0 \qquad E_z = 0$$

$$H_z(x,y) = X(x)\, Y(y)$$

$$\boxed{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right) H_z^{\circ}(x,y) = 0}$$

$$H_z(x,y) = H_0 \cos\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right)$$

$$\boxed{(f_c)_{TE_{10}} = \frac{1}{2a\sqrt{\mu\varepsilon}} = \frac{v}{2a}}$$

$$\boxed{(f_c)_{mn} = \frac{1}{2\sqrt{\varepsilon\mu}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \qquad \lambda_c = \frac{c}{f_c}$$

$$\boxed{\beta = \sqrt{w^2 \mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

Circular Wave Guides

$$TE_{nm}: \; f_{c_{nm}} = \frac{q'_{nm}}{2\pi a\sqrt{\varepsilon\mu}} \qquad n = 0,1,2,\ldots \quad m = 1,2,3,\ldots$$

$$TM_{nm}: \; f_{c_{nm}} = \frac{q_{nm}}{2\pi a\sqrt{\varepsilon\mu}} \qquad n = 0,1,2,\ldots \quad m = 1,2,3,\ldots$$