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TEL313E-Electromagnetic Waves

Midterm Exam 2

1- (10) a- Write the Maxwell Equations in differential form

b- Write the name and unit of all terms (E,H...) in Maxwell Equations

c- Write the SWR (voltage standing wave ratio)

a) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{H} = \vec{J}_v + \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$ $\nabla \cdot \vec{D} = \rho$.

b) $\vec{E}$: Electric field (V/m)

$\vec{H}$: Magnetic field (A/m)

$\vec{D}$: (Electric) Displacement Field C/m^2

$\vec{B}$: Magnetic Flux Density (Tesla)

ρ: charge density C/m^3

$\vec{J}$: Current density A/m^2

σ: Conductivity (S/m)

c) $SWR = \frac{|E_{max}|}{|E_{min}|} = \frac{1-|\Gamma|}{1+|\Gamma|}$

2- (10 Points) Consider the uniform electromagnetic plane wave with frequency $f = \frac{600}{2\pi} MHz$ in a lossless dielectric medium ($\varepsilon = \varepsilon_r \varepsilon_0, \mu = \mu_0$). If the electric field can be written in the following form $\vec{E} = E_0 e^{-j(2x+2\sqrt{3}z)}\vec{e_y}$ (E_0: constant)

a) Determine the wavenumber k, propagation direction, $\vec{n}$ and dielectric constant ε_r.

b) Find the corresponding phasor expression of magnetic field $\vec{H}$?

a) $\vec{E} = E_0 e^{-j4\left(\frac{x}{2} + \frac{\sqrt{3}}{2}z\right)} \vec{e}_y = E_0 \cdot e^{-jk \cdot \vec{n} \cdot \vec{r}}$ $\boxed{k = 4}$

$\boxed{\hat{n} = \frac{1}{2}\vec{e}_x + \frac{\sqrt{3}}{2}\vec{e}_z}$

$k = \omega\sqrt{\varepsilon_r \varepsilon_0 \mu_0} = 4$

$\cancel{2}\pi \frac{6.10^8}{\cancel{2\pi}} \cdot \frac{1}{3.10^8} \cdot \sqrt{\varepsilon_r} = 4 \Rightarrow \boxed{\varepsilon_r = 4}$

b) $\vec{H} = \frac{1}{\eta}\hat{n} \times \vec{E} = \frac{1}{60\pi} \cdot \left(\frac{1}{2}\vec{e}_x + \frac{\sqrt{3}}{2}\vec{e}_z\right) \times E_0 e^{-j4\vec{n}\cdot\vec{r}} \vec{e}_y$

$= \frac{E_0}{60\pi} \cdot \left(\frac{1}{2}\vec{e}_z - \frac{\sqrt{3}}{2}\vec{e}_x\right) e^{-jk\vec{n}\cdot\vec{r}}$

$\eta = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{120\pi}{2} = 60\pi$

3- (25 Points) The electric field intensity of uniform plane wave propagate in the +z direction in seawater is $\vec{E} = 100\cos(10^7\pi t)\vec{e}_x$ V/m at $z = 0$. The constitutive parameters for the seawater $\varepsilon_r = 72, \mu_r = 1, \sigma = 4S/m$

a- Determine the attenuation constant, phase constant, complex characteristic impedance, phase velocity, wavelength and the skin depth.

b- Determine the amplitude of electric field intensity at $z = 0.8$

c- Determine the instantaneous expression of magnetic field ?

a) $k = \sqrt{\omega^2 \varepsilon\mu - j\sigma\omega\mu} = \beta - j\alpha.$

Loss tagent: $\dfrac{\sigma}{\omega\varepsilon} = 200 \gg 1 \Rightarrow$ Good conductor

Attenuation Constant $\alpha = \sqrt{\pi f \mu \sigma} = \sqrt{\pi \cdot \frac{10^7}{2} \cdot 4\pi \cdot 10^{-7} \cdot 4} \cong 8.8858$ (Np/m)

Phase Constant $\alpha = \beta$

Charactristic impedance.

$$Z = \frac{\omega\mu}{k} = \frac{10^7\pi \cdot 10^{-7} \cdot 4\pi}{\frac{4\pi}{\sqrt{2}} \cdot (1-j)} = \pi \frac{\sqrt{2}}{2}(1+j)$$

phase velocity

$$v_p = \frac{\omega}{\beta} = 3.53 \, 10^6 \text{ m/sn.}$$

Wavelength $\lambda = \dfrac{2\pi}{\beta} = 0.707$ m.

Skin depth $\delta = \dfrac{1}{\alpha} = 0.112$ m.

b) $\vec{E} = 100 \cdot e^{-jkz}\vec{e}_x = 100\, e^{-\alpha z} \cdot e^{-j\beta z}\vec{e}_x$

$z = 0.8 \qquad \vec{E} = \underbrace{0.082}_{\text{Amplitude}}\, e^{-j\beta z}\vec{e}_x$

c) $\vec{H} = \dfrac{1}{Z}\vec{n} \times \vec{E} = \dfrac{1}{Z}\vec{e}_z \times 100\, e^{-jkz}\vec{e}_x = \dfrac{100}{\pi \cdot e^{j\frac{\pi}{4}}} \cdot e^{-\alpha z} \cdot e^{-j\beta z}\vec{e}_x$

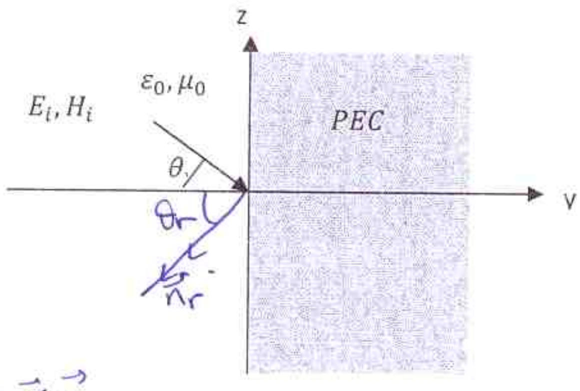
$= \dfrac{100}{\pi} \cdot \left(\dfrac{1}{\sqrt{2}} - \dfrac{j}{\sqrt{2}}\right) e^{-\alpha z} \cdot e^{-j\beta z}\vec{e}_y$

$H(z,t) = Re\{H e^{j\omega t}\}$

$= \dfrac{100}{\pi\sqrt{2}} \cdot e^{-\alpha z} \cdot \cos(\omega t - \beta z)\vec{e}_y$

4-) (30 Points) A parallel polarized plane wave ($\vec{H}_i // \vec{e}_x$), with incident angle θ_i is incident on a free space-PEC medium ($y = 0$) as shown in Figure below. . Assume that the frequency of the wave is 300MHz and the amplitude of the incident magnetic field is 1.

a) Write the instantaneous expressions of incident magnetic field $\vec{H}_i$ and electric field $\vec{E}_i$ in free space.
b) Determine the phasor expressions of total electric and magnetic fields.
c) Write the propagation direction of total wave.
d) What is the time-averaged power in free-space?



a) $\vec{H}_i = 1\, e^{-jk\vec{n}_i \cdot \vec{r}}\, \vec{e}_x$ $\qquad \vec{n} = \cos\theta_i \vec{e}_y - \sin\theta_i \vec{e}_z$ $\qquad Z_1 = Z_0 = 120\pi$

$K = \omega\sqrt{\varepsilon_0\mu_0} = \dfrac{3.10^8 . 2\pi}{3 \cdot 10^8} = 2\pi$

$\vec{E}_i = -Z_0\, \vec{n}_i \times \vec{H}_i$

$= Z_0 . (\sin\theta_i \vec{e}_y + \cos\theta_i \vec{e}_z)\, e^{-jk\vec{n}_i \cdot \vec{r}}$

$\vec{H}_i(y,z;t) = \mathrm{Re}\left\{ e^{-jk\vec{n}_i \cdot \vec{r}}\, e^{j\omega t}\, \vec{e}_x \right\} = \cos(\omega t - (k\cos\theta_i y - k\sin\theta_i z))\vec{e}_x$

$\vec{E}_i(y,z;t) = Z_0 (\sin\theta_i \vec{e}_y + \cos\theta_i \vec{e}_z) \cos(\omega t - (k\cos\theta_i y - k\sin\theta_i z))$

b) Reflected Wave : $\vec{n}_r = -\cos\theta_i \vec{e}_y - \sin\theta_i \vec{e}_z$

$\vec{H}_r = \Gamma\, e^{-jk\vec{n}_r \cdot \vec{r}}\, \vec{e}_x$

$\vec{E}_r = -Z_0\, \vec{n}_r \times \vec{H}_r = \Gamma Z_0 . (\sin\theta_r \vec{e}_y - \cos\theta_r \vec{e}_z)\, e^{-jk\vec{n}_r \cdot \vec{r}}$

$\theta_i = \theta_r$

$\vec{n}_r \cdot \vec{r} = -\cos\theta_i y - \sin\theta_i z$

4-) (Continued)

Boundary Condition.

$$\left(\vec{E}_i + \vec{E}_r\right)_{tan}\Big|_{y=0} = 0.$$

$$Z_0 \cos\theta_i\, e^{+jk\sin\theta_i z} - \Gamma Z_0 \cos\theta_i\, e^{jk\sin\theta_i z} = 0.$$

$$\boxed{\Gamma = 1}$$

$$\vec{E}_{tot} = \vec{E}_i + \vec{E}_r$$
$$= \vec{e}_y \cdot \left\{ Z_0 \sin\theta_i\, e^{-jk(\cos\theta_i y - \sin\theta_i z)} + Z_0 \sin\theta_i\, e^{-jk(-\cos\theta_i y - \sin\theta_i z)} \right\}$$
$$+ \vec{e}_z \cdot \left\{ Z_0 \cos\theta_i\, e^{-jk(\cos\theta_i y - \sin\theta_i z)} - Z_0 \cos\theta_i\, e^{-jk(-\cos\theta_i y - \sin\theta_i z)} \right\}$$
$$= \vec{e}_y\, 2Z_0 \sin\theta_i \cdot \cos(k\cos\theta_i y)\, e^{jk\sin\theta_i z}$$
$$+ \vec{e}_z \cdot -2jZ_0 \cos\theta_i \sin(k\cos\theta_i y)\, e^{jk\sin\theta_i z}.$$

$$\vec{H}_{tot} = \vec{H}_i + \vec{H}_r$$
$$= 2\cos(k\cos\theta_i y)\, e^{jk\sin\theta_i z}\, \vec{e}_x$$

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c) Propagation direction $\vec{n} = \vec{e}_z$

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d) $\vec{P}_c = \frac{1}{2} \vec{E}_{tot} \times \vec{H}^{*}_{tot}$ $\qquad Re\{\vec{P}_c\} = ?$

$$\vec{P}_c = \frac{1}{2} \cdot \left\{ \cancel{4}\, Z_0 \sin\theta_i \cos^2(k\cos\theta_i y)(-\vec{e}_z) + 4jZ_0 \cos\theta_i \sin(k\cos\theta_i y) \cdot \cos(k\cos\theta_i y) \cdot \vec{e}_y \right\}$$

$Re\{\vec{P}_c\}$ = Time-averaged power density $= 2Z_0 \sin\theta_i \cos^2(k\cos\theta_i y)\,(-\vec{e}_z)$.

4- (25 Points) Consider a plane wave incident on a planar boundary at $x = 0$ from a dielectric medium as shown in the Figure. The right-hand circularly polarized incident electric field is

$$\vec{E}_i = E_0\left[2\vec{e}_z \sin(\omega t-(k_x x + k_y y)) + (\sqrt{3}\vec{e}_y - \vec{e}_x)\cos(\omega t - (k_x x + k_y y))\right]$$

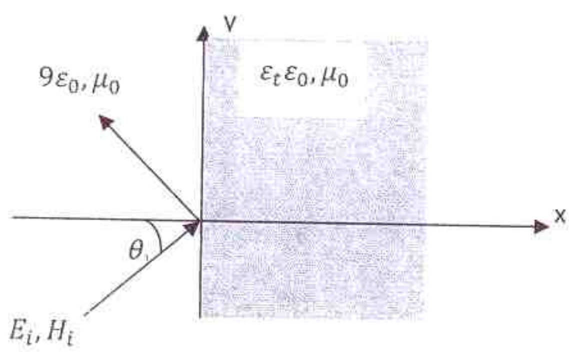
Where E_0 is a real constant. Reflected wave is

$$\vec{E}_r = E_0\left[\Gamma^{\text{perp}} 2\vec{e}_z \sin(\omega t - (-k_x x + k_y y)) + \Gamma^{\text{par}}(-\sqrt{3}\vec{e}_y - \vec{e}_x)\cos(\omega t - (-k_x x + k_y y))\right]$$

a- What is the incident angle, θ_i?

b- For $k_y = 1$, determine the frequency and the wavelength

c- In the case of $\varepsilon_t = 3\varepsilon_0$, find the polarization of the reflected wave.



a) $\vec{n}_i = \cos\theta_i \vec{e}_x + \sin\theta_i \vec{e}_y$ $\qquad k\vec{n}_i \cdot \vec{r} = k_x x + k_y y$

$= k\cos\theta_i x + k\sin\theta_i y$.

$\vec{E}^{perp}: \vec{e}_z$

$\vec{E}^{par}: \cos\theta_i \vec{e}_y - \sin\theta_i \vec{e}_x$

$$\left.\begin{array}{l}\frac{\sqrt{3}}{2} = \cos\theta_i \\ \frac{1}{2} = \sin\theta_i\end{array}\right\} \theta_i = 30^\circ.$$

b) $k_y = 1$ $\quad k\sin\theta_i = 1 \Rightarrow k\frac{1}{2} = 1 \quad \Rightarrow \boxed{k = 2}$.

$\omega\sqrt{9\varepsilon_0\mu_0} = 2 \Rightarrow \frac{2\pi f \cdot 3}{3\cdot 10^8} = 2 \quad \Rightarrow \boxed{f = 10^8/\pi}$ $\qquad \lambda = v/f = \frac{10^8}{10^8/\pi}$ $\boxed{\lambda = \pi}$

Complex permittivity, $\varepsilon_c = \varepsilon' - j\varepsilon'', \varepsilon' = \varepsilon, \varepsilon'' = \frac{\sigma}{\omega}$, *Loss tangent* $= \frac{\varepsilon''}{\varepsilon'}, k = \beta - j\alpha$

Low − loss Dielectrics: *Loss tangent* $\ll 1 \Rightarrow \alpha \cong \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}}$ *and* $\beta \cong \omega\sqrt{\mu\varepsilon'}[1 + \frac{1}{8}(\frac{\varepsilon''}{\varepsilon'})^2]$

Good Conductors: *Loss tangent* $\gg 1 \Rightarrow \alpha = \beta \cong \sqrt{\pi f\mu\sigma}$, $(\varepsilon_0 = \frac{1}{36\pi}10^{-9}\frac{F}{m}, \mu_0 = 4\pi 10^{-7}\,H/m)$

Brewster angle: $\sin\theta_b = \sqrt{\frac{1}{1+\frac{\varepsilon_1}{\varepsilon_2}}}$ *Critical Angle*: $\sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

Good Luck...

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c) $\varepsilon_t = 3\varepsilon_0$.

Brewster angle: $\sin\theta_B = \sqrt{\frac{1}{1+\frac{9\varepsilon_0}{3\varepsilon_0}}} = \frac{1}{2}$

$\theta_B = 30^\circ = \theta_i \Rightarrow$ No reflection for parallel-polarized wave.

$\vec{E}_r = \vec{e}_z\Gamma(\ \ )$ Linear polarization. ONLY Perpendicular polarization reflected