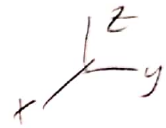
1) **Q:** Açısal frekansı ω, genliği E_0, x ve y bileşenleri sıfırdan farklı olan, +z yönünde ilerlemekte olan, sol el dairesel polarize dalganın elektrik alan vektörünün genel ifadesini yazınız. Karşılık düşen manyetik alan vektörünü yazınız. Poynting vektörünün zaman bölgesi ifadesini yazınız.

A:

a)
$$\vec{E}(z,t) = \frac{E_0}{\sqrt{2}}\cdot\cos(\omega t - kz + \phi)\,\vec{e_x} + \frac{E_0}{\sqrt{2}}\cdot\cos(\omega t - kz + \phi + \pi/2)\cdot\vec{e_y}$$



$$\left(\text{Genlik} \checkmark ;\ \sqrt{\frac{E_0^2}{2} + \frac{E_0^2}{2}} = E_0\right)$$

$$\bar{E}(z) = \frac{E_0}{\sqrt{2}}\cdot e^{+j(kz-\phi)}\,\vec{e_x} + \frac{E_0}{\sqrt{2}}\cdot e^{+j(kz-\phi-\pi/2)}\,\vec{e_y}$$

b)
$$\vec{H}(z) = \frac{1}{\eta}\,\vec{n}\times\vec{E} = \frac{1}{\eta}(\vec{e_z})\times\vec{E}(z)$$

$\vec{H}(z,t) = Re\{\vec{H}(z)\,e^{j\omega t}\}$

Ortam kayıpsızsa, η reel.

$$= \frac{1}{\eta}\left(\frac{E_0}{\sqrt{2}}\cdot e^{+j(kz-\phi)}(\vec{e_y}) + \frac{E_0}{\sqrt{2}}\cdot e^{+j(kz-\phi-\pi/2)}(-\vec{e_x})\right)$$

$$= \frac{E_0}{\sqrt{2}\eta}\,e^{+j(kz-\phi)}\,\vec{e_y} - \frac{E_0}{\sqrt{2}\eta}\cdot e^{+j(kz-\phi-\pi/2)}\,\vec{e_x}$$

$$\Rightarrow \vec{H}(z,t) = -\frac{E_0}{\sqrt{2}\eta}\cdot\cos(\omega t - kz + \phi + \pi/2)\,\vec{e_x} + \frac{E_0}{\sqrt{2}\eta}\cos(\omega t - kz + \phi)\,\vec{e_y}$$

$$= \frac{E_0}{\sqrt{2}\eta}\cdot\sin(\omega t - kz + \phi)\,\vec{e_x} + \frac{E_0}{\sqrt{2}\eta}\cdot\cos(\omega t - kz + \phi)\,\vec{e_y}$$

$$\vec{E}(z,t) = \frac{E_0}{\sqrt{2}}\cdot\cos(\omega t - kz + \phi)\,\vec{e_x} - \frac{E_0}{\sqrt{2}}\cdot\sin(\omega t - kz + \phi)\cdot\vec{e_y}$$

c)
$$\vec{P} = \vec{E}(z,t)\times\vec{H}(z,t) = \begin{vmatrix} i & j & k \\ E_x & E_y & 0 \\ H_x & H_y & 0 \end{vmatrix} = \hat{k}\left(E_x H_y - E_y H_x\right) \quad (\hat{k} = \vec{e_z})$$

$$= \vec{e_z}\left(\frac{E_0^2}{2\eta}\cdot\left[\cos^2(\omega t - kz + \phi) + \sin^2(\omega t - kz + \phi)\right]\right) = \vec{e_z}\cdot\frac{E_0^2}{2\eta} \,//$$

2)

Ortalama güç yoğunluğu,

$$\boldsymbol{P_{av}} = Re\{\boldsymbol{P_c}\} = Re\left\{\frac{1}{2}\boldsymbol{E}\times\boldsymbol{H}^*\right\} = Re\left\{\frac{1}{2}\boldsymbol{E}\times\left(\frac{1}{\eta}\boldsymbol{n}\times\boldsymbol{E}\right)^*\right\} = Re\left\{\frac{1}{2\eta^*}|\boldsymbol{E}|^2\boldsymbol{e}_x\right\}$$

Dalga empedansı

$$\eta = Z = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0 + j\dfrac{\sigma}{\omega}}} = \sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0 + j\dfrac{\sigma}{\omega}}} = \sqrt{\frac{4\pi\times 10^{-7}}{3\dfrac{1}{36\pi}10^{-9} + j\dfrac{10^{-2}}{2\pi\times 10^8}}}$$

$$= 194.24 - j53.801 \quad \Rightarrow \quad \eta^* = 194.24 + j53.801$$

Zayıflama sabiti (α),

$$k = \beta + j\alpha = \sqrt{\omega^2\epsilon\mu + j\omega\sigma\mu}$$

$$= \sqrt{(2\pi\times 10^8)^2\cdot 3\frac{1}{36\pi}10^{-9}\cdot 4\pi\times 10^{-7} + j(2\pi\times 10^8)\cdot 10^{-2}\cdot 4\pi\times 10^{-7}}$$

$$= 3.7753 + j1.0457 \quad \Rightarrow \quad \alpha = 1.0457$$

$x = 0$ ‘daki yüzeyden geçen güç,

$$P_1 = \int_S \boldsymbol{P}\cdot d\boldsymbol{s} = \int_{z=0}^{z=3}\int_{y=0}^{y=1} Re\left\{\frac{1}{2\eta^*}|\boldsymbol{E}_1|^2\boldsymbol{e}_x\right\}\cdot\boldsymbol{e}_x ds = \int_{z=0}^{z=3}\int_{y=0}^{y=1} Re\left\{\frac{1}{2\eta^*}|10|^2\right\}dydz$$

$x = 2$ ‘deki yüzeyden geçen güç

$$P_2 = \int_S \boldsymbol{P}\cdot d\boldsymbol{s} = \int_{z=0}^{z=3}\int_{y=0}^{y=1} Re\left\{\frac{1}{2\eta^*}|\boldsymbol{E}_2|^2\boldsymbol{e}_x\right\}\cdot\boldsymbol{e}_x ds = \int_{z=0}^{z=3}\int_{y=0}^{y=1} Re\left\{\frac{1}{2\eta^*}|10e^{-\alpha x}|^2\right\}dydz$$

Fark,

$$P_1 - P_2 = Re\left\{\frac{1}{2\eta^*}|10|^2\right\}\cdot 3 - Re\left\{\frac{1}{2\eta^*}|10e^{-\alpha x}|^2\right\}\cdot 3 = Re\left\{\frac{3}{2\eta^*}(|10|^2 - |10e^{-\alpha x}|^2)\right\}$$

$$= Re\left\{\frac{3}{2(194.24 - j53.801)}(10^2 - (10\cdot e^{-1.0457\cdot 2})^2)\right\} = \mathbf{0.7062}$$

Kutu içinde ısıya dönüşen toplam ortalama güç,

$$P = \frac{1}{2}\int_V \sigma|\boldsymbol{E}|^2 dv = \frac{1}{2}\int_{z=0}^{3}\int_{y=0}^{1}\int_{x=0}^{2} 10^{-2}\cdot(10\cdot e^{-1.0457\cdot x})^2$$

$$= \frac{1}{2}\int_{z=0}^{3}\int_{y=0}^{1}\int_{x=0}^{2} e^{-2.0914\cdot x}dxdydz = \mathbf{0.7063}$$

3)

a)

$$\nabla \times \boldsymbol{H}(r) = -j\omega\epsilon \boldsymbol{E}(r) = \boldsymbol{e}_x\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \boldsymbol{e}_y\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \boldsymbol{e}_z\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$= -\boldsymbol{e}_x\frac{\partial H_y}{\partial z} + \boldsymbol{e}_y\frac{\partial H_x}{\partial z} = -\boldsymbol{e}_x(-2)(-2j)e^{-j\left(2z-\frac{\pi}{2}\right)} + \boldsymbol{e}_y(-2j)e^{-j\left(2z-\frac{\pi}{2}\right)}$$

$$= -4je^{-j\left(2z-\frac{\pi}{2}\right)}\boldsymbol{e}_x - 2je^{-j\left(2z-\frac{\pi}{2}\right)}\boldsymbol{e}_y$$

Then,

$$-j\omega 4\epsilon_0 \boldsymbol{E}(z) = -4je^{-j\left(2z-\frac{\pi}{2}\right)}\boldsymbol{e}_x - 2je^{-j\left(2z-\frac{\pi}{2}\right)}\boldsymbol{e}_y \quad \Rightarrow \quad \boldsymbol{E}(z) = \frac{e^{-j\left(2z-\frac{\pi}{2}\right)}}{\epsilon_0\omega}\boldsymbol{e}_x + \frac{e^{-j\left(2z-\frac{\pi}{2}\right)}}{2\epsilon_0\omega}\boldsymbol{e}_y$$

$$\boldsymbol{E}(z,t) = Re\left\{\boldsymbol{E}(z)e^{-j\omega t}\right\} = \frac{\cos\left(\omega t + 2z - \frac{\pi}{2}\right)}{\epsilon_0\omega}\boldsymbol{e}_x + \frac{\cos\left(\omega t + 2z - \frac{\pi}{2}\right)}{2\epsilon_0\omega}\boldsymbol{e}_y$$

b)

$$k = \beta + j\alpha = \sqrt{\omega^2\epsilon\mu + j\omega\sigma\mu}$$

$$\sigma = 0$$

$$k = \beta = \frac{\omega}{v} \quad \Rightarrow \quad \omega = \beta v = 2\frac{1}{\sqrt{4\epsilon_0\mu_0}} = 3 \times 10^8\ rad/s$$

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{4\epsilon_0\mu_0}} = 1.5 \times 10^8\ m/s$$

4)

$$\boldsymbol{H}(z) = 3e^{j(\omega t-\beta z)}\boldsymbol{e}_x + 4e^{j\left(\omega t-\beta z+\frac{\pi}{2}\right)}\boldsymbol{e}_y \quad mA/m$$

$$\boldsymbol{E} = -\eta\boldsymbol{n}\times\boldsymbol{H} = -\frac{\eta_0}{2}(\boldsymbol{e}_z)\times\left(3e^{j(\omega t-\beta z)}\boldsymbol{e}_x + 4e^{j\left(\omega t-\beta z+\frac{\pi}{2}\right)}\boldsymbol{e}_y\right)$$

$$= \boldsymbol{e}_x 754 e^{j\left(\omega t-\beta z+\frac{\pi}{2}\right)} - \boldsymbol{e}_y 565.5 e^{j(\omega t-\beta z)} \quad mV/m$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{\eta_0}{2} = 60\pi$$

W

$$\boldsymbol{P}_{av} = Re\left\{\frac{1}{2}E\times H^*\right\} = \frac{1}{2\eta}|E|^2\boldsymbol{n} = \frac{1}{\eta_0}\left|\sqrt{0.5655^2+0.754^2}\right|^2\boldsymbol{e_z} = 0.0024\boldsymbol{e}_z \quad \cancel{m}W/m^2$$

yz düzleminin normal vektörü e_x dir. O halde,

$$\int_S \boldsymbol{P}d\boldsymbol{s} = \int_S 0.0024\,\boldsymbol{e}_z ds\boldsymbol{e}_z = 0.0024\,[mW/m^2\,]\times(\pi 5^2\,[m^2]) = 0.188\ \cancel{m}W \quad W$$