$$= \frac{1}{n^2} n \left(\mu^2 + \sigma^2 + (n-1)\mu^2 \right) - 2\mu^2 + \mu^2$$

$$= \frac{1}{n} \left(n\mu^2 + \sigma^2 \right) - 2\mu^2 + \mu^2$$

$$= \mu^2 + \frac{1}{n} \sigma^2 - \mu^2$$

$$= \frac{1}{n} \sigma^2 \quad // \qquad n \uparrow \quad \sigma_x^2 \downarrow$$

sample sayısı ↑ variance ↓

$$\sum_{i=1}^{n} (\ldots)^2_i = \sum_{i=1}^{n} \sum_{j=1}^{n} ()_i ()_j$$

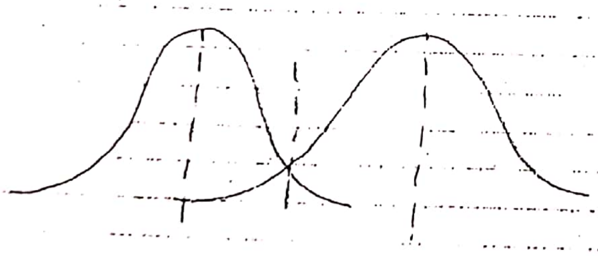
Truth / Reality — Detection; Decision

Decision \ Reality	1	0
1	TP	FP
0	FN	TN

$$\frac{TP + TN}{TP + FP + FN + TN} = \text{probability of correct detection}$$

$$\text{Sensitivity} = \frac{TP}{TP + FN}$$ gerçekten hata varken hangi oranda yakalanmış

$$\text{Specificity} = \frac{TN}{TN + FP}$$ hata yokken yok olduğunu bulma



Hypothesis Testing

H_0 / H_1

Type I error
II error

Critical value

α, β

Sample size

Power of test $(1-\beta)$

Hypothesis testing concerns on how to use a random sample to judge if it is evidence that supports or not the hypothesis.

H_0 : null hypothesis

H_1 : alternate hypothesis

ex RAM Chips , defective rate of chips → 5% 10s of 1000s

Let p denote the true defective probabity.

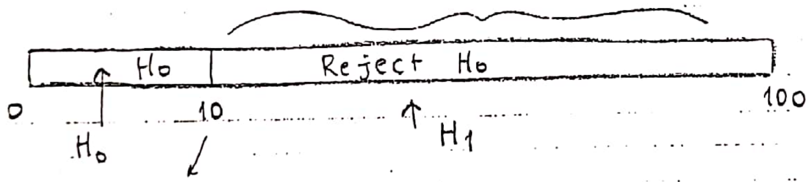
H_0 : $p = 0.05$ (firmanın iddiası doğru)

H_1 : $p > 0.05$ (firmanın iddiasından fazla)

Select 100 chips.

$n \cdot p = 5$ critical value $\geq n \cdot p$
100, 0.05 (threshold)

critical range $\equiv 10$



critical value

	H_0	H_1
H_0	✓	Type II error
H_1	Type I error	✓

(firma haksız iken onlar doğru demek)

α, β nın küçük olmasını isteriz

α ve β, critical value ile değişebilir

CV ↑ β ↑ α ↓
↓ ↓ ↑

Given Ho is true, you REJECT Ho.

α : the level of significance

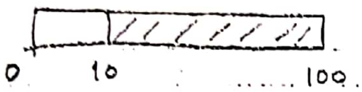
$n \uparrow$ $\alpha \downarrow$ $\beta \downarrow$

critical value değiştirildikçe α ve β yı aynı anda düzeltemeyiz.

$\alpha = P[\text{Type I error}] = P[H_1 / H_0]$

$= P[X \geq 10 \text{ when } p = 0.05]$

binomial distribution $= \sum_{x=10}^{100} b(x;\ n=100,\ p=0.05)$



$= \sum_{x=10}^{100} \binom{100}{x} \underbrace{(0.05)^x}_{p} \underbrace{(0.95)^{100-x}}_{(1-p)}$

$= 0.0282 = \alpha$: the level of significance
$=$ Type I error

$\beta = P[\text{Type II error}] = P[H_0 / H_1]$

sample sayısı asıl sayıya eşit olduğunda
$\alpha = \beta = 0$
bütün ürünlerin kontrol edildiği

We cannot compute β since the true p is not known.

(p, 0.05'ten büyük ve kesin belli değil
$0.05 < p < 1$ arasında herşey olabilir ama kesin değil.)

However, we can compute it for testing $H_0 : p = 0.05$ against
$H_1 : p = 0.1$

$\beta = P[\text{Type II error}] = P[H_0 / H_1]$

$= P[X < 10 \text{ when } p = 0.1]$

$= \sum_{x=0}^{9} \binom{100}{x} (0.1)^x (0.9)^{100-x}$

$= 0.45$

bu p'ye bağlı

$\cancel{P[X < 9 \text{ when } p = 0.09]}$

$n \times p =$ ~~β hesaplamak için atanan değer~~

~~dizilime yapılır~~

$\cancel{= P[X < 6 \text{ when } p = 0.06]}$

$\cancel{= \sum_{x=0}^{5} \binom{100}{x} (0.06)^x (0.94)^{100-x}} \ll 0.45$

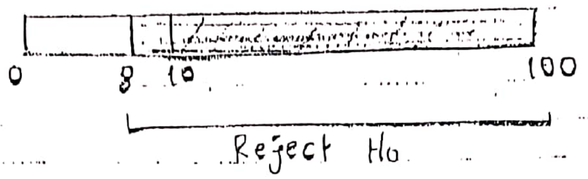
α için 10 değil de 6 falan seçip bak.

effect of critical value → check?

Effect of the Critical Value.

$n = 100$ $\quad$ $CV = 10 \downarrow 8$

$p = 0.05$



$\alpha = P[\,X \geq 8 \text{ when } p = 0.05\,]$

$$= \sum_{x=8}^{100} \binom{100}{x} (0.05)^x (0.95)^{100-x}$$

$= 0.128$ (10 iken 0.028) ✓

Design sorusu gelebilir.

Hypothesis Testing

Critical value, α, β, n, p $\qquad$ Th ↑ β ↑ α ↓

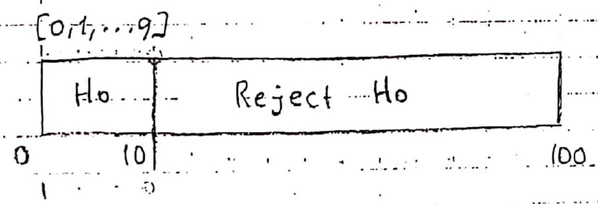
Threshold $\qquad$ n ↑ β ↓ α ↓

Reality / decision	Ho	H1 problemli
Ho problemsiz	✓	β
H1	α	✓

$n = 100$
$p = 0.05$
$n \cdot p = 5$

Critical value > n·p

bu tablonun bütün hücreleri için Th değeri aynı



$$\alpha = b(n, p, Th) = \sum_{x=10}^{100} \binom{100}{x} (0.05)^x (0.95)^{100-x}$$

$\beta = b(n = 100,\ p^{0.1}$, Th için ilk kabul edilen value geçerli) ~~önce anlatılan gibi değil~~

$$\beta = \sum_{x=0}^{9} \binom{100}{x} (0.1)^x (0.9)^{100-x}$$

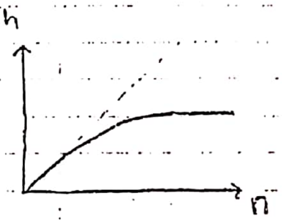
CHECK!
(sample mean'in variance n↑ azalır, ∞'da variance = 0) ✓
(sample mean'in beklenen değeri, mean'e eşit oluyor) ✓

Th = 8 için ... $\beta = \sum_{x=0}^{7} \binom{100}{x} (0.1)^x (0.9)^{100-x}$

$= \quad < 0.45$

n artarsa hem α hem β azalır. n toplam sayıya eşit olursa $n = N$, $\alpha = \beta = 0$.

yeni $n = 150$ $\quad p = 0.05$



$\alpha = \sum_{x=Th}^{150} \binom{150}{x} (0.05)^x (0.95)^{150-x}$

Th was 10 when $n = 100$.

$n = 150 \rightarrow Th = 15$ (n=100 ikenki oranı korumak mantıklı ama ondan büyük seçemeyiz. 100 → 150 artış çok fazla olmadığı için o oranla Th hesaplanabilir.)

$= 0.0085 < \alpha_{n=100}$

$\beta = \sum_{x=0}^{14} \binom{150}{x} (0.1)^x (0.9)^{150-x} = \ldots < \beta_{earlier}$ olmalı

? check

Power of Test : $1 - \beta$ $\quad \beta \downarrow$ test kuvvetlenir.

$\beta = 0$ test kuvveti $= 1$

n sayısı toplam sayıya eşit old zaman

* Approximation

$n = 100$ $\quad p = 0.05$ $\quad Th = 10$

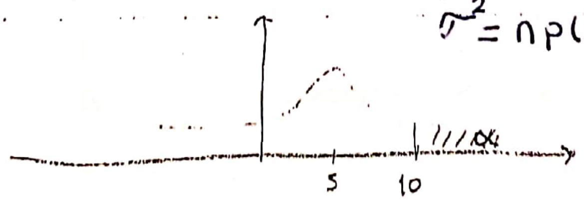
$\alpha = \sum_{x=10}^{100} \binom{100}{x} p^x (1-p)^{100-x}$

normal dist. geçerken

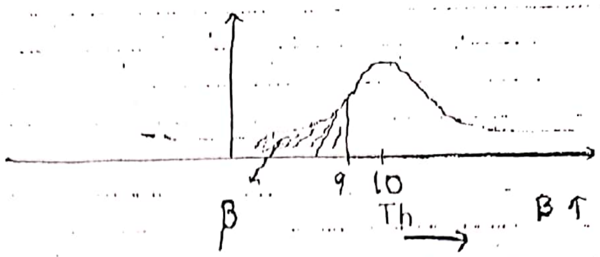
$\mu = n \cdot p = 5$

$\sigma^2 = np(1-p)$

Th ↑ α ↓



$\mu = n \cdot p_\beta = 100 \cdot 0.1 = 10$



Th ↑ β ↑

Approximating Binomial Distribution
Using Normal Distribution

Theorem: If X is a binomial random var. with n trials and probability of success is p, then limiting form of the distribution.

$$Z = \frac{X - np}{\sqrt{np(1-p)}}$$

p: number of success

Gaussian dist with mean = 0 variance = 1 ↳ normal dist.

$X \geq 12$ örneğin → binomial

$Z \geq \ldots$ tablodan

CDF of X

$F_X(x) = P[X \leq x]$

0.01 . . . .

App. error decreases as n increases and
# of samples

p does NOT set too close to zero.

ex: Let's recompute α with normal appx.

final
n = 150
Th = 12 (critical value)

$$\alpha = P[\text{Type I error}] = P[X \geq 12 \text{ when } p = 0.05]$$

$$= \sum_{x=12}^{150} \binom{150}{x} (0.05)^x (0.95)^{150-x}$$

$$\approx P\left[Z \geq \frac{12 - 150 \cdot 0.05}{\sqrt{150 \cdot 0.05 (1-0.05)}}\right] = P[Z \geq 1.69]$$

$$= 1 - \underbrace{P[Z \leq 1.69]}_{F_Z(1.69)}$$

$0.0455 = 1 - 0.9545 =$

originally 0.074 . not too bad

Let's increase n to 500. Th = 40 (oransal, daha küçük de alına bilir.)

Let's set critical value to 40.

$$\alpha \approx P\left[Z \geq \frac{40 - 500 \cdot 0.05}{\sqrt{500 \cdot 0.05\,(1-0.05)}}\right]$$

$= P[Z \geq 3.08]$

$= 1 - P[Z \leq 3.08]$

$= 1 - 0.999 = 0.001 \rightarrow$ Ho'ı reject etme olasılığı 1000'de 1.

Th = 35 old. zaman ne olacak

$$\alpha \approx P\left[Z \geq \frac{35 - 500 \cdot 0.05}{\sqrt{500 \cdot 0.05 \cdot 0.95}}\right]$$

$= P[Z \geq 2.05]$

n = 150,

Ho	→ Reject Ho

12

n = 500,

	→ Reject Ho

32

$$\alpha \approx P\left[Z \geq \frac{32 - 25}{\sqrt{500 \cdot 0.05 \cdot 0.95}}\right]$$

$= P[Z \geq 1.44]$

$= 1 - P[Z \leq 1.44]$

$= 1 - 0.9251$

$= 0.0749$

$= 0.075$ ↳ %7.5

1000'de 1'de %7.5'a geldi

artan n'in α'yı azaltmasını bekliyoruz ama 1000'de bir'e kadar da azalmasını değil. Burda çok azalmış.

The same example continue with different parameters :

Testing against the alternate hypothesis :

$H_1 : p = 0.1$

$n = 500$, $Th = 40$ (critical value) (sağ tarafa dahil)

$$\beta = \sum_{x=0}^{39} \binom{500}{x} (0.1)^x (0.9)^{500-x}$$

hypothesis testing design finalde

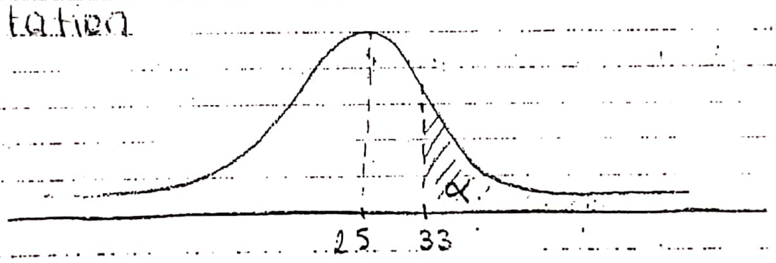
$$\approx P\left[Z \leq \frac{39 - 500 \cdot 0.1}{\sqrt{500 \cdot 0.1 \cdot 0.9}} \right]$$

$$= P[Z \leq -1.69] = 0.068$$

%7 oranında onları yanlış iken bizim onları doğru almamız.

Visual Interpretation

$n = 500$
$p = 0.05$



$Z \sim (np, np(1-p))$

$$Z \geq \frac{X - np}{\sqrt{np(1-p)}}$$

$Y = aX + b$

$E[Y] = aE[X] + b$

$Var[Y] = a^2 Var[X]$

$Th \uparrow \quad \alpha \downarrow$

(0,1)

$$P[Y \geq 33] = P\left[Z \geq \frac{33 - 25}{\sqrt{23.75}} \right]$$

mean = 25
$Var[Y] = 23.75$

$N(25, 23.75)$
$\mu \quad \sigma^2$
$np \quad np(1-p)$

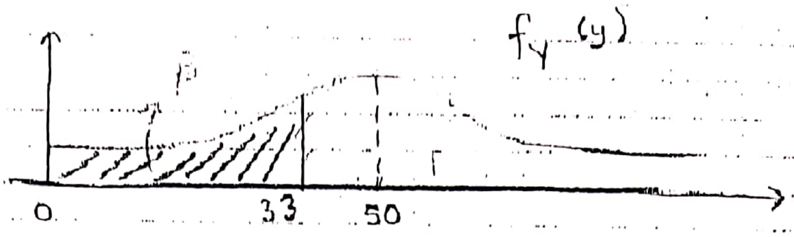
$= P[Z \geq 1.64]$

$= 1 - P[Z \leq 1.64]$

$= 1 - 0.9495$

$= 0.0505$ //

$n = 500$
$p = 0.1$ olsun
$Y = N(50, 45)$



$\beta \lesssim 1/2$ den büyük

$\beta \approx P[Y \leq 33]$

$= P\left[Z \leq \dfrac{33-50}{\sqrt{45}}\right]$

$= P[Z \leq -2.5]$

$= 0.062$

Th $\uparrow$ $\beta \uparrow$

Central Limit Theorem

Dağılımlar iid ise toplamları Gauss'a convergence eder.
Bu yaklaşıklık $n \uparrow$ artar.

The central limit theorem says that the mean and variance of some of independent identically dist. random samples can be computed as sample mean $n \mu$ and $n\sigma^2$ with Gaussianly distributed.

Samples $s_1, s_2, \ldots, s_n$ are i.i.d with mean μ and variance σ^2.

If $Y = \sum_{i=1}^{n} s_i$, then Y is Gaussianly distr. with $n \cdot \mu$ and $n\sigma^2$.

As n increases, appx. success increases.

$s_1 + s_2 + \cdots + s_n$
$\sigma^2 \quad \sigma^2$

$Y = X_1 + X_2$

$\sigma_Y^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2$

if X_1 and X_2 are independent