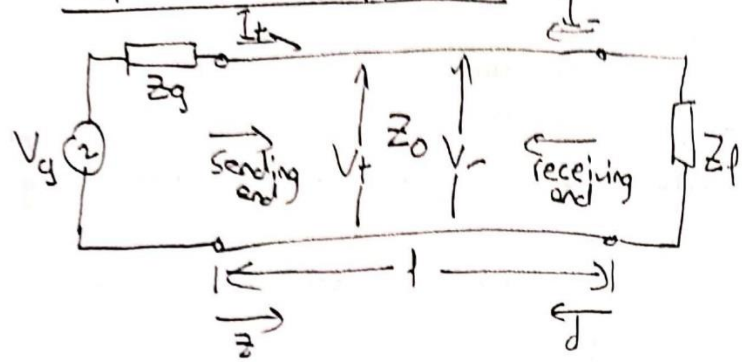
Reflection and Transmission Coefficients:

Reflection Coefficient:



If the load impedance equals the line characteristic impedance, there is no reflected traveling wave. ($V_- = \frac{I_\ell}{2}(Z_\ell - Z_0)e^{-\gamma\ell}$, $Z_\ell = Z_0 \Rightarrow V_- = 0$)

Because of the voltage-current relationship of the load point is fixed by the load impedance, we usually prefer to write down the expessions for the receiving end. Remember,

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z} \quad \text{and} \quad I = I_+ e^{-\gamma z} + I_- e^{\gamma z} = Y_0 (V_+ e^{-\gamma z} - V_- e^{\gamma z})$$

If the line is of length ℓ, the voltage and current at the receiving end become:

$$V = V_+ e^{-\gamma\ell} + V_- e^{\gamma\ell} \quad \text{and} \quad I = Y_0 (V_+ e^{-\gamma\ell} - V_- e^{\gamma\ell})$$

and Z_ℓ is,

$$Z_\ell = \frac{V_\ell}{I_\ell} = Z_0 \frac{V_+ e^{-\gamma\ell} + V_- e^{\gamma\ell}}{V_+ e^{-\gamma\ell} - V_- e^{\gamma\ell}}$$

We define the reflection coefficient, designated by Γ, as

Reflection coefficient = $\dfrac{\text{Reflected voltage or negative reflected current}}{\text{Incident voltage or current}}$

or $\boxed{\Gamma \equiv \dfrac{V_{ref}}{V_{inc}} = \dfrac{-I_{ref}}{I_{inc}}} = |\Gamma| e^{j\theta}$ (generally a complex quantity)

$V_- e^{\gamma l}$: ref. vol. at $z=l$
$V_+ e^{\gamma l}$: inc. vol at $z=l$

at any point on the line.

$|\Gamma|$ is never greater than 1 for a positive load.

The reflection coefficient at the receiving end is $\boxed{\Gamma_l = \dfrac{V_- e^{\gamma l}}{V_+ e^{\gamma l}} = \dfrac{Z_l - Z_0}{Z_l + Z_0}}$

The generalized reflection coefficient is (at any z point on the line)

$$\Gamma \equiv \frac{V_- e^{\gamma z}}{V_+ e^{-\gamma z}} = \frac{Z - Z_0}{Z + Z_0}$$

where Z is the line impedance at point z.

If we let $z = l - d$, Γ at a point located distance d from the receiving end is,

$$\Gamma_d = \frac{V_- e^{\gamma(l-d)}}{V_+ e^{-\gamma(l-d)}} = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} e^{-2\gamma d} = \Gamma_l e^{-2\gamma d}$$

$$\Gamma_d = \Gamma_l e^{-2\gamma d} = |\Gamma_l| e^{j\theta_l} e^{-2\alpha d} e^{-j2\beta d} = |\Gamma_l| e^{-2\alpha d} e^{j(\theta_l - 2\beta d)}$$

For a lossless line, $\overset{\text{the magnitude of } \Gamma}{|\Gamma|}$ remains constant, $\overset{\text{the phase of } \Gamma}{\angle\Gamma}$ changes with $-2\beta d$.

When $Z_l = Z_0$, there will be no reflection from the receiving end, then $\Gamma_l = 0$, so Γ_d also equals to zero. Thus, any $Z_l \neq Z_0$ impedance will create a reflected traveling wave towards to $-z$ direction. If $Z_s \neq Z_0$, this sending end impedance will also create a reflected wave towards to $+z$ direction.

Example:

A transmission line has a characteristic impedance of $50 - j0{,}01\,\Omega$ and terminates in an impedance of $73 + j42{,}50\,\Omega$. Determine Γ of the load.

$$\Gamma = \frac{Z_l - Z_0}{Z_l + Z_0} = 0{,}377\angle 42{,}7 = 0{,}2735 + j0{,}2525$$

Transmission Coefficient:

A reflection coefficient Γ exists at any point along an improperly terminated line.

The transmission coefficient T is defined as,

$$T \equiv \frac{\text{Transmitted voltage}}{\text{incident voltage}} = \frac{V_{tr}}{V_{inc}}$$

Let we define P_{inc} as incident power to the load, P_{ref} as reflected power from the load and P_{tr} as the transmitted power on the load.

Let the traveling waves at the receiving end be,

$$V_+ e^{-\gamma z} + V_- e^{\gamma z} = V_{tr} e^{-\gamma z} \quad \text{and} \quad Y_0 (V_+ e^{-\gamma z} - V_- e^{\gamma z}) = \frac{V_{tr}}{Z_\ell} e^{-\gamma z}$$

when we multiply the second eq. by Z_ℓ and substitute the result into the first eq. we get

$$\Gamma_\ell = \frac{V_- e^{\gamma z}}{V_+ e^{-\gamma z}} = \frac{Z_\ell - Z_0}{Z_\ell + Z_0}$$

and we use this result in the first eq. again

$$\underline{T = \frac{V_{tr}}{V_+} = \frac{2Z_\ell}{Z_\ell + Z_0}}$$

The net average power carried by the inc. and ref. waves is,

$$\langle P_{it} \rangle = \frac{|V_+ e^{-\alpha z}|^2}{2Z_0} - \frac{|V_- e^{\alpha z}|^2}{2Z_0}$$

and the average power carried to the load by the transmitted wave is,

$$\langle P_{tr} \rangle = \frac{|V_{tr} e^{-\alpha z}|^2}{2Z_\ell}$$

Setting $\langle P_{it} \rangle = \langle P_{tr} \rangle$ and using $\Gamma_\ell = \frac{Z_\ell - Z_0}{Z_\ell + Z_0}$ and $T = \frac{2Z_\ell}{Z_\ell + Z_0}$, we obtain

$$\underline{T^2 = \frac{Z_\ell}{Z_0}(1 - \Gamma^2)}$$

Example

A transmission line has $Z_0 = 75 + j0{,}01\,\Omega$ and $Z_\ell = 70 + j50\,\Omega$. a) $\Gamma = ?$ b) $T = ?$
c) verify the relationship between Γ and T d) verify that $T = 1 + \Gamma$

a) $$\Gamma = \frac{Z_\ell - Z_0}{Z_\ell + Z_0} = 0{,}33\angle 76{,}68 = 0{,}08 + j0{,}32$$

b) $$T = \frac{2Z_\ell}{Z_\ell + Z_0} = 1{,}12\angle 16{,}51^\circ = 1{,}08 + j0{,}32$$

c) $$T^2 = 1{,}25\angle 33{,}02 \quad , \quad \frac{Z_\ell}{Z_0}(1-\Gamma^2) = 1{,}25\angle 33^\circ$$

Thus we have verified the equation.

d) $$T = 1{,}08 + j0{,}32 = 1 + 0{,}08 + j0{,}32 = 1 + \Gamma$$

Thus we have verified that

$$\boxed{T = 1 + \Gamma}$$