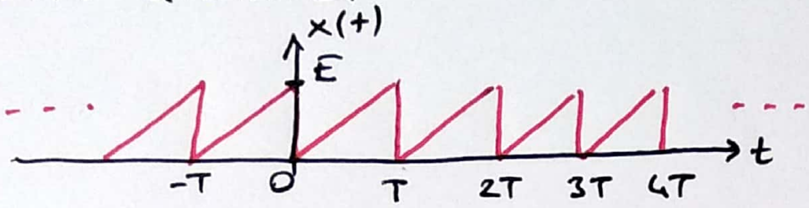
EHB 351
Analog Haberleşme

19/11/2020

Uygulama 1

1) a) Şekildeki periyodik $x(t)$ işaretine ilişkin frekans ve güç spektrumunu bularak değişimini çiziniz.



b) $T = 10^{-3}$ sn ise kesim frekansı $f_c = 1500\,Hz$ olan sıfır faz kaymalı, birim genlikli bir ideal alçak geçiren süzgeç (AGS) girişine $x(t)$ işareti uygulandığında, çıkışta elde edilen $y(t)$ işaretinin ifadesini yazınız. Ayrıca, $y(t)$ ortalama gücünü bulunuz.

Cevaplar

a) $$c_n = \frac{1}{T}\int_0^T \frac{Et}{T} e^{-j\frac{2\pi}{T}nt}\,dt = \frac{E}{T^2}\int_0^T t e^{-j\frac{2\pi}{T}nt}\,dt$$

$$\left[\begin{array}{ll} u = t & e^{-j\frac{2\pi}{T}nt}dt = dv \\ du = dt & \dfrac{e^{-j\frac{2\pi}{T}nt}}{-j\frac{2\pi}{T}n} = v \end{array}\right]$$

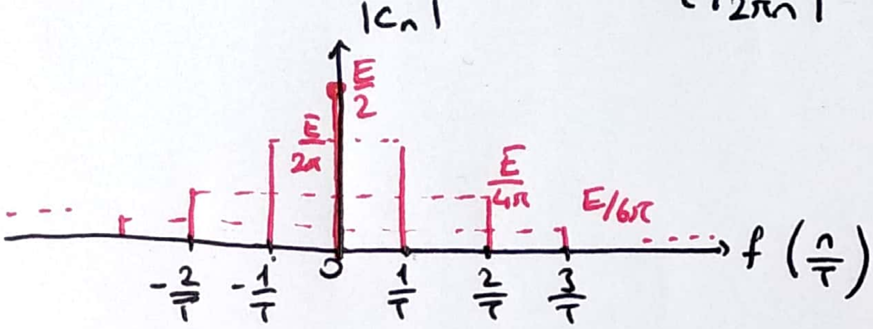
$$c_n = \frac{E}{T^2}\left[-\frac{T}{j2\pi n} t e^{-j\frac{2\pi}{T}nt}\Big|_0^T - \int_0^T \left(-\frac{T}{j2\pi n}\right) e^{-j\frac{2\pi}{T}nt}\,dt \right]$$

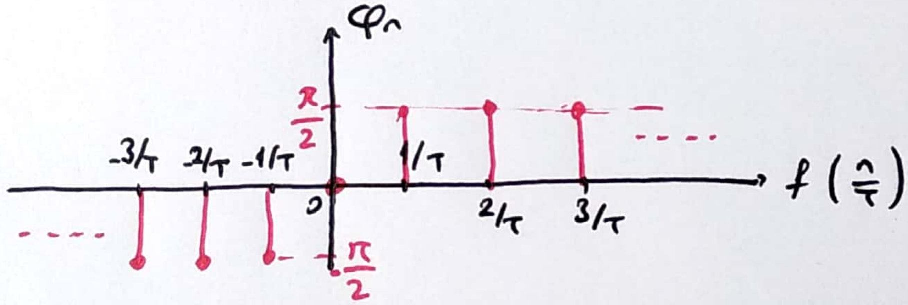
$$c_n = \frac{E}{T^2}\left[-\frac{T^2}{j2\pi n}\underbrace{e^{-j2\pi n}}_{1} + \frac{T}{j2\pi n}\underbrace{\int_0^T e^{-j\frac{2\pi}{T}nt}\,dt}_{0} \right]$$

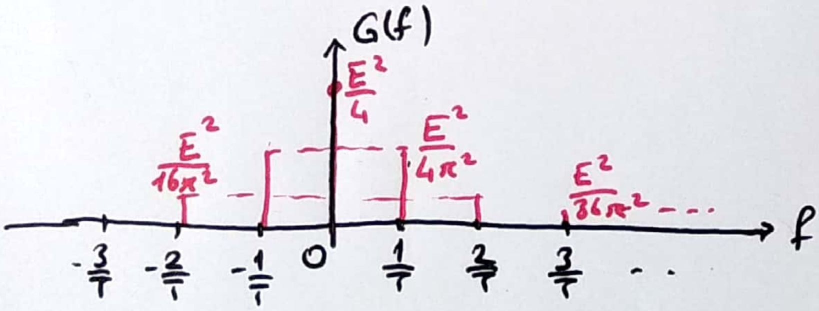
$$c_n = \frac{E}{T^2}\left(-\underset{(j)}{\frac{T^2}{j2\pi n}} + \frac{T}{j2\pi n}\cdot 0 \right) = \frac{jE}{2\pi n} \qquad n \neq 0$$

$$c_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{T}\int_0^T \frac{Et}{T}\,dt = \frac{E}{2}$$

$$c_n = \begin{cases} E/2 & n=0 \\ \frac{jE}{2\pi n} & n \neq 0 \end{cases} \equiv c_n = \begin{cases} E/2 & n=0 \\ \left|\frac{E}{2\pi n}\right| e^{-j\frac{\pi}{2}} & n<0 \\ \left|\frac{E}{2\pi n}\right| e^{j\frac{\pi}{2}} & n>0 \end{cases}$$







b) $$y(t) = \frac{E}{2} + \frac{E}{2\pi} e^{j\frac{\pi}{2}} e^{j\frac{2\pi}{T}t} + \frac{E}{2\pi} e^{-j\frac{\pi}{2}} e^{-j\frac{2\pi}{T}t}$$

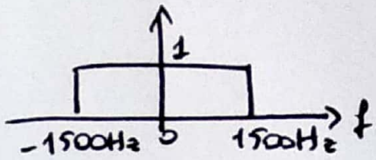
$$y(t) = \frac{E}{2} + \frac{E}{\pi}\cos\left(2\pi 10^3 t + \pi/2\right)$$

$$= \frac{E}{2} - \frac{E}{\pi}\sin\left(2\pi 10^3 t\right)$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi}{T}nt}$$

$$P_y = \frac{E^2}{4} + 2 \cdot \frac{E^2}{4\pi^2},$$

AGS



2) $t\,x(t) \xrightarrow{\mathcal{F}} \frac{j}{2\pi} \frac{dX(f)}{df}$ bunu tanıtlayınız.

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$\frac{dX(f)}{df} = \int_{-\infty}^{\infty} x(t)\, \frac{d e^{-j2\pi f t}}{df}\, dt = \int_{-\infty}^{\infty} \underbrace{x(t)(-j2\pi t)}_{x^*(t)}\, e^{-j2\pi f t}\, dt$$

$$\frac{dX(f)}{df} = \mathcal{F}\{x^*(t)\} = \mathcal{F}\{x(t)(-j2\pi t)\}$$

$$\mathcal{F}\{t\,x(t)\} = \frac{1}{\underset{(j)}{-j2\pi}} \frac{dX(f)}{df} = \frac{j}{2\pi} \frac{dX(f)}{df}.$$