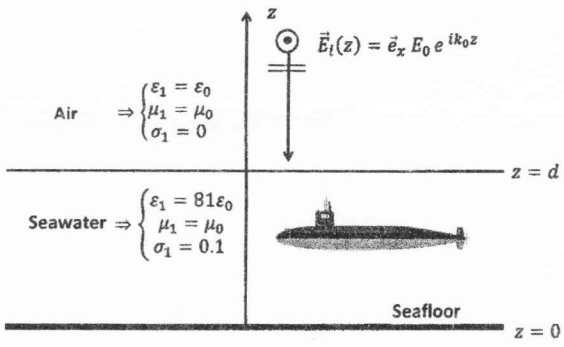
Electromagnetic Waves Midterm #2
(Dec. 24th, 2013)

1) In the illustration, the $z = 0$ plane represents the seafloor which is assumed to be perfect electric conductor (PEC) and the $z = d$ plane is the sea surface. Constitutive parameters of air and seawater are given $\varepsilon_1 = \varepsilon_0,\ \mu_1 = \mu_0,\ \sigma_1 = 0$ and $\varepsilon_2 = 81\varepsilon_0,\ \mu_2 = \mu_0,\ \sigma_2 = 0.1$ respectively. An incident plane wave in air propagating along $-z$ direction is given with the electric field $\vec{E}_i(z,t) = \vec{e}_x E_0 \cos(\omega t + 0.01z)$.



a) Find the frequency of the incident wave.
b) Express the electric and magnetic fields in the regions $z \in (0, d)$ and $z > d$.
c) Define the boundary conditions on the planes $z = 0$ and $z = d$ respectively.
d) Using the field expressions of (b) together with boundary conditions, derive all expressions required to calculate the reflected wave in region $z > d$ and upgoing and downgoing waves inside the region $z \in (0, d)$ (You do not need to solve the equations.)
e) Decide if the seawater is a good conductor at this frequency?
f) Calculate the skin depth.
g) An antenna mounted on the submarine depicted in the figure can detect signals with electric field as low as $1\mu V/m$. Given the submarine is $h = 2m$ below the sea surface, find the minimum of required amplitude E_0 for the incident plane wave to be able to communicate with the submarine? (Ignore the reflected wave coming from the seafloor assuming $d \gg h$).

Hint: For good conductors: $\alpha = \beta = \sqrt{\pi f \mu \sigma},\ \eta_c = (1+i)\frac{\alpha}{\sigma}$, for low loss dielectrics: $\alpha \cong \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}},\ \beta \cong \omega\sqrt{\mu\varepsilon'}\left[1 + \frac{1}{8}\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right],\ \eta_c \cong \sqrt{\frac{\mu}{\varepsilon'}}\left(1 + i\frac{\varepsilon''}{2\varepsilon'}\right),\ \varepsilon' = \varepsilon_0\varepsilon_r,\ \varepsilon'' = \frac{\sigma}{\omega}$

2) An antenna which is connected to a rectangular waveguide is fed with a signal at $f = 3GHz$. The operating frequency f is known to be at the center of two cutoff frequencies within the dominant mode region. It is required that the cutoff frequencies of the first three mode satisfy $(f_{c10} < f_{c20} \leq f_{c01})$
a) Determine the dimensions (a, b) of the waveguide.
b) Calculate the phase constant, the phase velocity and the wave impedance at $f = 3GHz$.
c) What is the average power of transmitted wave at $f = 1GHz$
d) Determine the first 5 mode with lowest cutoff frequencies.
e) Determine the operating frequency range in order the first 3 modes to propagate.

Good Luck...
Assist. Prof. Mehmet Çayören