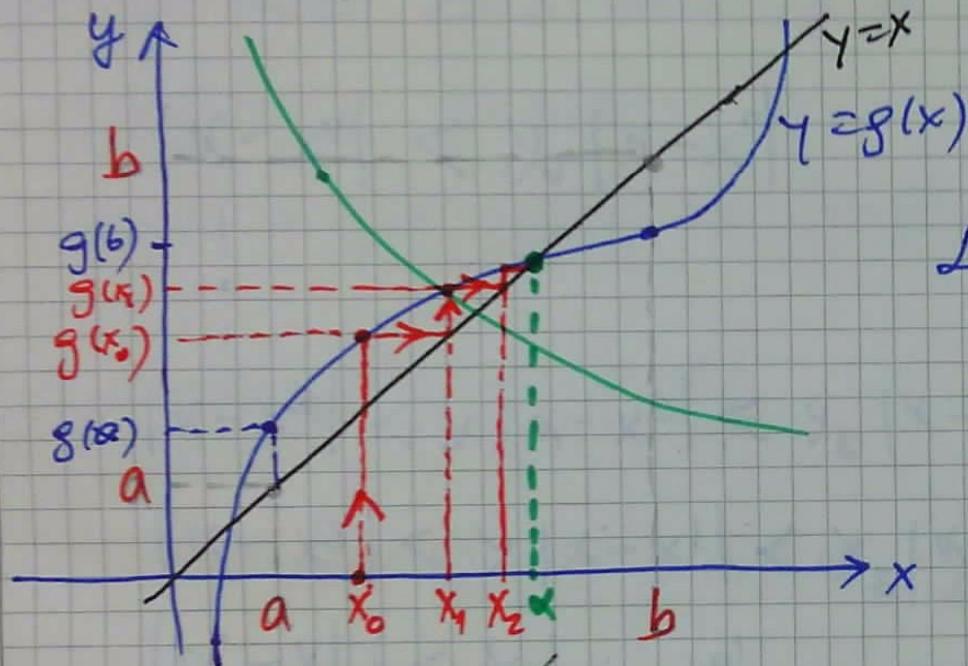


MAT 202 5. Hafta 30.03.2021

KÖK BULMA : SABİT NOKTA İTERASYONU

①



$g(x)$ $[a, b]$ aralığında kalacak ve
 $|g'(x)| < 1 \quad \forall x \in (a, b)$

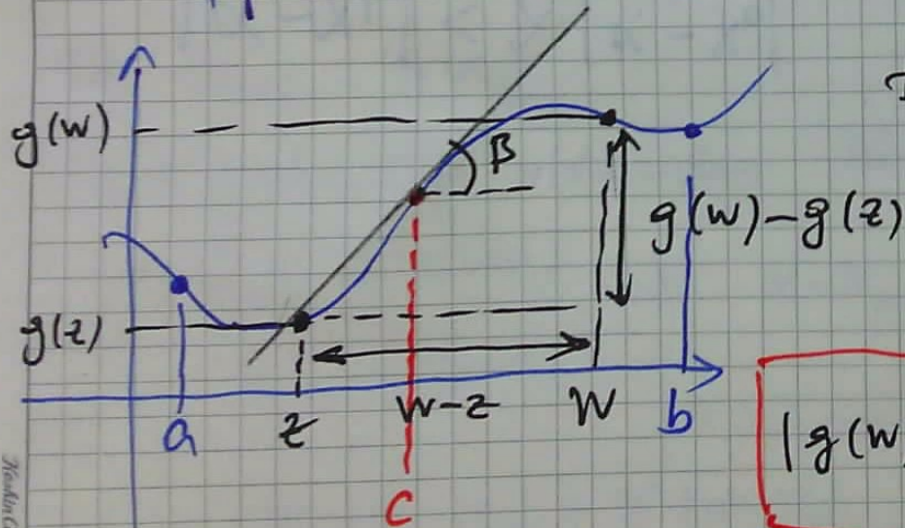
Lemmaya göre en az bir eğim vardır.

Ortalama değeri teoremine
 göre herhangi $w, z \in (a, b)$
 için

$$\tan \beta = g'(c)$$

$$g(w) - g(z) = g'(c)(w - z)$$

• (aşağıda şekilde bir $c \in (z, w)$
 ara noktası mevcuttur



$$|g(w) - g(z)| = |g'(c)| \cdot |w - z| \leq \lambda |w - z|$$

(2)

S1. İki farklı çözümler olan α ve β ile gösterelim ($\alpha \neq \beta$)

$$\alpha = g(\alpha) ; \beta = g(\beta)$$

$$\alpha - \beta = g(\alpha) - g(\beta) \Rightarrow |\alpha - \beta| = |g(\alpha) - g(\beta)| \leq \lambda |\alpha - \beta|$$

$$(g(\alpha) - g(\beta) = g'(c)(\alpha - \beta))$$

$$(1 - \lambda) |\alpha - \beta| \leq 0$$

tek olan durum
 $\alpha = \beta$

$$\lambda < 1 \Rightarrow 1 - \lambda > 0$$

$$|\alpha - \beta| \geq 0$$

meburen $\alpha = \beta \Rightarrow$ tek çözüm vardır

$$S2. x_{n+1} = g(x_n)$$

$a \leq g(x_n) \leq b$ olduğundan tüm x_n 'ler $[a, b]$ aralığında kalacaktır

$$x_{n+1} = g(x_n) \Rightarrow \alpha - x_{n+1} = g(\alpha) - g(x_n) = g'(c_n)(\alpha - x_n)$$

$$\alpha = g(\alpha)$$

$$|\alpha - x_{n+1}| \leq \lambda |\alpha - x_n| ; n \geq 0 \leq \lambda |\alpha - x_n|$$

$$(\lambda < 1)$$

$$\dots \Rightarrow |\alpha - x_n| \leq \lambda^n |\alpha - x_0| ; n \geq 0 \Rightarrow$$

$$|\alpha - x_n| \leq \lambda^n |\alpha - x_0| \quad ; n \geq 1$$

$$\lim_{n \rightarrow \infty} \lambda^n |\alpha - x_0| \rightarrow 0$$

($\lambda < 1$)

$$\rightarrow \lim_{n \rightarrow \infty} x_n = \alpha$$

$$x_{n+1} = g(x_n) \text{ iterasyon}$$

$$S3. \quad |\alpha - x_0| \leq |\alpha - x_1| + |x_1 - x_0|$$

$$= \underbrace{|g(\alpha) - g(x_0)|}_{= g'(\xi) \cdot |\alpha - x_0|} + |x_1 - x_0| = g'(\xi) \cdot |\alpha - x_0| + |x_1 - x_0|$$

$$\leq \lambda |\alpha - x_0| + |x_1 - x_0|$$

$$(1 - \lambda) |\alpha - x_0| \leq |x_1 - x_0|$$

$$|\alpha - x_0| \leq \frac{1}{1 - \lambda} |x_1 - x_0|$$

82 den $|\alpha - x_n| \leq \lambda^n |\alpha - x_0|$ idi

$$|\alpha - x_n| \leq \lambda^n |\alpha - x_0| \leq \frac{\lambda^n}{1 - \lambda} \underbrace{|x_1 - x_0|}_{\text{constant}} \quad n \geq 1$$

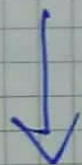
S4. $\lim_{n \rightarrow \infty} \frac{\alpha - x_{n+1}}{\alpha - x_n} = g'(\alpha)$

(4)

$$\alpha - x_{n+1} = g(\alpha) - g(x_n) = g'(c_n)(\alpha - x_n)$$

$$\lim_{n \rightarrow \infty} \frac{\alpha - x_{n+1}}{\alpha - x_n} = \lim_{n \rightarrow \infty} g'(c_n) \quad \text{çünkü } c_n \text{ } \alpha \text{ ile } x_n \text{ arasında}$$

her zaman bir değeri $\lim_{n \rightarrow \infty} x_n = \alpha$



$$\lim_{n \rightarrow \infty} \frac{\alpha - x_{n+1}}{\alpha - x_n} = \underline{\underline{g'(\alpha)}}$$

(5)

$$I_1 \text{ iter} \rightarrow g(x) = 5 + x - x^2 \rightarrow g'(x) = 1 - 2x$$

$$g'(\alpha) = 1 - 2\sqrt{5} < -1$$

$$|g'(\alpha)| > 1 \Rightarrow \text{IRAKSAK}$$

$$I_2 \text{ iter} \quad g(x) = \frac{5}{x} \rightarrow g'(x) = -\frac{5}{x^2} \rightarrow g'(\alpha) = -\frac{5}{5} = -1 \Rightarrow \text{IRAKSAK}$$

$$|g'(\alpha)| = 1 \text{ (Salim)} \nearrow$$

$$I_3 \text{ iter} \quad g(x) = 1 + x - \frac{x^2}{5} \Rightarrow g'(x) = 1 - \frac{2x}{5} \quad g'(\alpha) = 1 - \frac{2\sqrt{5}}{5} \approx 0,106 < 1$$

\Rightarrow YAKINSAK

$$I_4 \text{ iter} \quad g(x) = \frac{1}{2} \left(x + \frac{5}{x} \right) \rightarrow g'(x) = \frac{1}{2} \left(1 - \frac{5}{x^2} \right) \rightarrow g'(\alpha) = 0 < 1$$

YAKINSAK



$$x = \underline{\underline{g(x)}}$$

$$|g'(x)| < 1$$

⑥

$$r_n = \frac{\alpha - x_n}{\alpha - x_{n-1}} \xrightarrow{(n \gg 1)} g'(\alpha) = 0,105773$$

I_3 iterasyonu için

n	x_n	$\alpha - x_n$	r_n
0	2,5	$-2,64 \cdot 10^{-1}$	—
1	2,25	$-1,39 \cdot 10^{-2}$	0,0528
2	2,2375	$-1,43 \cdot 10^{-3}$	0,1028
3	2,23621875	$-1,57 \cdot 10^{-4}$	0,1053
4	2,23608389	$-1,59 \cdot 10^{-5}$	0,1055
5	2,23606966	$-1,68 \cdot 10^{-6}$	0,1056
6	2,23606815	$-1,77 \cdot 10^{-7}$	0,1056
7	2,23606800	$-1,87 \cdot 10^{-8}$	0,1056

$$|g'(\alpha)| < 1 \rightarrow \text{YAK}$$

$$g'(\alpha) > 1 \rightarrow \text{ırak}$$

$$g'(\alpha) < -1 \rightarrow \text{Salak}$$

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Ders
Başlar

$$I_3 \text{ için } g'(\alpha) \approx 0,106 < 1$$

$$\sqrt{5} \approx 2,23606797749979$$