TEL351 - UYGULAMA I

1. $x(t) = \cos(2\pi f_0 t) + \cos(4\pi f_0 t) + \sin^2(2\pi f_0 t)$

şeklinde verilen işaretin Fourier serisi katsayılarını ve ortalama gücünü bulunuz.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \cdot e^{j2\pi kt/T}$$

$$P = \sum_{k=-\infty}^{\infty} |a_k|^2$$, Parseval eşitliği

$$x(t) = \cos(2\pi f_0 t) + \cos(4\pi f_0 t) + \frac{1}{2} - \frac{1}{2}\cos(4\pi f_0 t)$$

$$= \frac{1}{2} + \cos(2\pi f_0 t) + \frac{1}{2}\cos(4\pi f_0 t)$$

$$= \frac{1}{2} + \frac{1}{2}e^{j2\pi f_0 t} + \frac{1}{2}e^{-j2\pi f_0 t} + \frac{1}{4}e^{j4\pi f_0 t} + \frac{1}{4}e^{-j4\pi f_0 t}$$

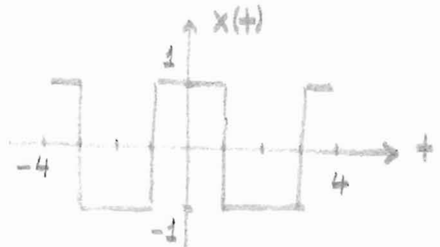
$T = \frac{1}{f_0}$ olduğundan

$a_0 = \frac{1}{2}$, $a_1 = a_{-1} = \frac{1}{2}$, $a_2 = a_{-2} = \frac{1}{4}$, diğer a_k'lar sıfır. //

$$P = \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{4}\right)^2 = \frac{7}{8} \quad //$$

2. Aşağıdaki periyodik işaretlerin Fourier serisi spektrumlarını bulunuz

a.



$$a_k = \frac{1}{T}\int_T x(t) \cdot e^{-j2\pi kt/T} dt \quad , \quad k = -\infty, \ldots, \infty$$

$$= \frac{1}{4}\int_{-1}^{1} e^{-j2\pi kt/4} \cdot dt - \frac{1}{4}\int_{1}^{3} e^{-j2\pi kt/4} dt$$

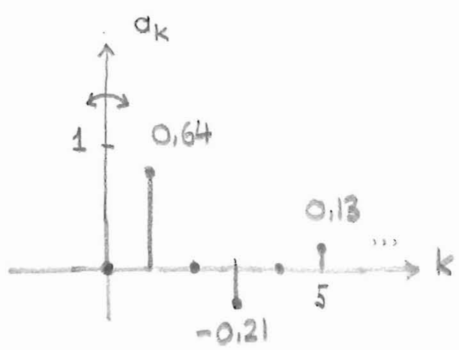
$$a_k = -\frac{1}{j2\pi k}\left[e^{-j\pi k t/2}\right]\Bigg|_{-1}^{1} + \frac{1}{j2\pi k}\left[e^{-j\pi k t/2}\right]\Bigg|_{1}^{3}$$

$$= \frac{1}{j2\pi k}\left(-e^{-j\pi k/2} + e^{j\pi k/2} + e^{-j3\pi k/2} - e^{-j\pi k/2}\right)$$

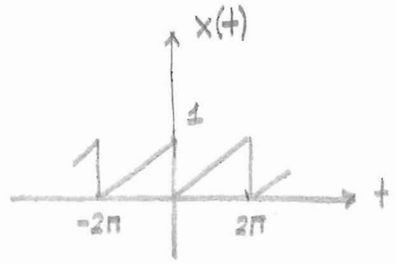
$$e^{-j3\pi k/2} = e^{-j3\pi k/2} \cdot \underbrace{e^{j2\pi k}}_{1} = e^{j\pi k/2}$$

$$a_k = \frac{2}{\pi k}\left(\frac{1}{2j}\right)\left(e^{j\pi k/2} - e^{-j\pi k/2}\right) = \frac{2}{\pi k}\sin\left(\frac{\pi k}{2}\right) \quad //$$

$$a_0 = \frac{1}{4}\int_{-1}^{3} x(t).dt = \frac{0}{4} = 0$$



b.



$$a_k = \frac{1}{2\pi}\int_{0}^{2\pi} \frac{t}{2\pi} e^{-j2\pi k t/2\pi} dt$$

$$= \frac{1}{(2\pi)^2}\int_{0}^{2\pi} t\, e^{-jkt}.dt$$

Parçalı integral : $\int u.dv = u.v - \int v.du$

$u = t$, $v = -\frac{1}{jk} e^{-jkt}$

$$a_k = \frac{1}{(2\pi)^2} \frac{-t}{jk} e^{-jkt} \Big|_0^{2\pi} + \frac{1}{(2\pi)^2} \frac{1}{jk} \int_0^{2\pi} e^{-jkt} . dt$$

$$= \frac{1}{(2\pi)^2} \frac{-1}{jk} \left[2\pi . e^{-j2\pi k} - 0 . e^{-j0} \right] - \frac{1}{(2\pi)^2} \frac{1}{k^2} e^{-jkt} \Big|_0^{2\pi}$$

$$= \frac{j}{2\pi k} - \frac{1}{(2\pi k)^2} \left(e^{-j2\pi k} - 1 \right)$$

$$= \frac{1}{(2\pi k)^2} \left[j2\pi k - e^{-j2\pi k} + 1 \right]$$

$$a_k = \frac{j}{2\pi k} \quad , \quad k \neq 0 \quad //$$

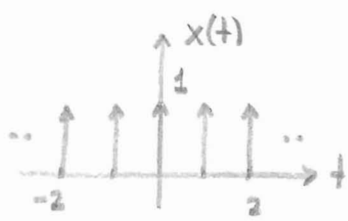
$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} x(t) . dt$$

$$= \frac{\pi}{2\pi} = \frac{1}{2} \quad //$$

$$|a_k| = \begin{cases} \frac{1}{2} \, , & k=0 \\ \frac{1}{2\pi k} \, , & k \neq 0 \end{cases}$$

$$\angle a_k = \begin{cases} 0 \, , & k=0 \\ \pi/2 \, , & k>0 \\ -\pi/2 \, , & k<0 \end{cases}$$

3. $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$ işaretinin Fourier serisi katsayılarını ve Fourier spektrumunu bulunuz



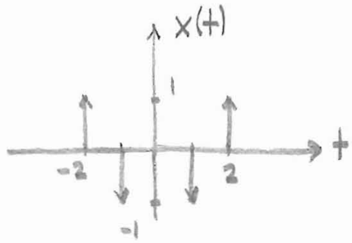
$$a_k = \frac{1}{1}\int_0^1 \delta(t) . e^{-j2\pi k t} . dt$$

$$= e^{-j2\pi k.0} = 1 \;//$$

$$X(f) = \sum_{n=-\infty}^{\infty} a_n . \delta\left(f - \frac{n}{T}\right)$$, Ayrık spektrum

$$= \sum_{n=-\infty}^{\infty} \delta(f-n) \;//$$

4 Aşağıdaki isaretin Fourier dönüsümünü bulunuz



Periyodik isaretlerin Fourier dönüsümü iki asamada bulunur.

Aperiyodik isaretlerin Fourier donusumu tek asamada bulunabilir

$$X(f) = \int_{-\infty}^{\infty} x(t) . e^{-j2\pi f t} . dt$$

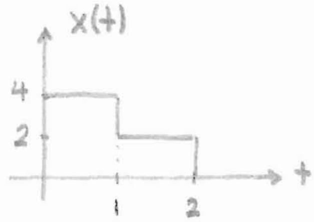
$$= \int_{-\infty}^{\infty} \left[\delta(t-2) - \delta(t-1) - \delta(t+1) + \delta(t+2) \right] . e^{-j2\pi f t} . dt$$

$$= e^{-j4\pi f} - e^{-j2\pi f} - e^{j2\pi f} + e^{j4\pi f}$$

$$= 2.\cos(4\pi f) - 2.\cos(2\pi f) \;//$$

TEL351 UYGULAMA II

1. Aşağıdaki işaretin Fourier dönüşümünü bulunuz.



x(t) aperiyodik olduğundan .

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t} \cdot dt$$

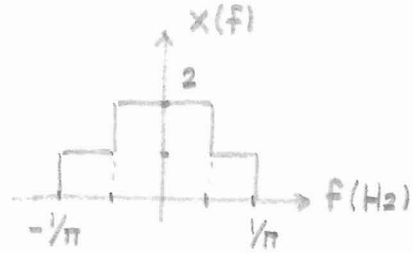
$$= \int_{0}^{1} 4 \cdot e^{-j2\pi f t} \cdot dt + \int_{1}^{2} 2 \cdot e^{-j2\pi f t} \cdot dt$$

$$= \frac{4}{-j2\pi f} e^{-j2\pi f t} \Big|_{0}^{1} + \frac{2}{-j2\pi f} e^{-j2\pi f t} \Big|_{1}^{2}$$

$$= \frac{4}{-j2\pi f} \left[e^{-j2\pi f} - 1 \right] + \frac{2}{-j2\pi f} \left[e^{-j4\pi f} - e^{-j2\pi f} \right]$$

$$= \frac{1}{j2\pi f} \left[4 - 2 \cdot e^{-j2\pi f} - 2 \cdot e^{-j4\pi f} \right] \quad //$$

2. Aşağıda Fourier dönüşümü verilen x(t)'yı bulunuz.



X(f) sürekli olduğundan x(t) aperiyodiktir.

$$x(t) = \int_{-\infty}^{\infty} X(f) \, e^{j2\pi f t} \, df$$

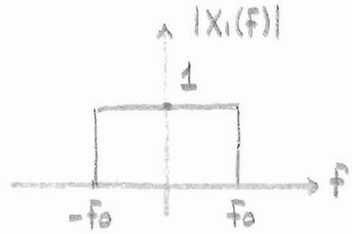
$$= \int_{-1/\pi}^{1/\pi} e^{j2\pi f t} \cdot df + \int_{-1/2\pi}^{1/2\pi} e^{j2\pi f t} \cdot df$$

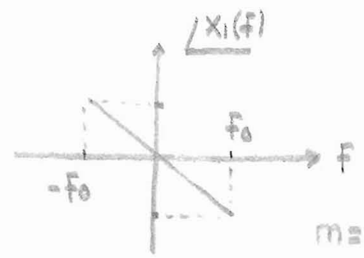
$$x(t) = \frac{1}{j2\pi t}\left[e^{j2t} - e^{-j2t}\right] + \frac{1}{j2\pi t}\left[e^{jt} - e^{-jt}\right]$$

$$= \frac{1}{\pi t}\left[\sin 2t + \sin t\right] \quad //$$

3. Aşağıda Fourier spektrumları verilen işaretleri bulunuz

a.





$m = -2\pi f t_0$

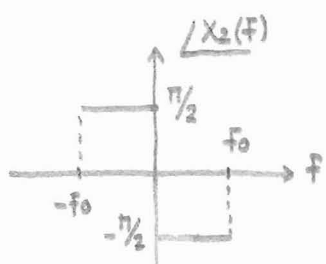
$$X_1(f) = 1 . e^{-j2\pi f t_0} \quad , \quad |f| < f_0$$

$$x_1(t) = \int_{-f_0}^{f_0} e^{-j2\pi f t_0} . e^{j2\pi f t} . df = \int_{-f_0}^{f_0} e^{j2\pi f (t-t_0)} . df$$

$$= \frac{1}{j2\pi(t-t_0)}\left[e^{j2\pi f_0 (t-t_0)} - e^{-j2\pi f_0 (t-t_0)}\right]$$

$$= \frac{\sin\left[2\pi f_0 (t-t_0)\right]}{\pi(t-t_0)} = 2f_0 . \operatorname{sinc}\left[2f_0(t-t_0)\right] \quad //$$

b. $|X_2(f)| = |X_1(f)|$



$$X_2(f) = \begin{cases} e^{-j\pi/2} = -j \ , & 0 < f < f_0 \\ e^{j\pi/2} = j \ , & -f_0 < f < 0 \\ 0 \ , & \text{diğer} \end{cases}$$

$$x_2(t) = \int_{-f_0}^{0} J \cdot e^{j2\pi ft} . df - \int_{0}^{f_0} J \cdot e^{j2\pi ft} . df$$

$$= \frac{1}{2\pi t} \left[1 - e^{-j2\pi f_0 t} - e^{j2\pi f_0 t} + 1 \right]$$

$$= \frac{1}{\pi t} \left[1 - \cos(2\pi f_0 t) \right] \quad //$$

c. $|X_3(f)| = |X_1(f)|$

$\angle X_3(f) = 0$

$$X_3(f) = \Pi\left(\frac{f}{2f_0}\right)$$

$$\Pi(f) \xleftrightarrow{\mathbb{F}^{-1}} \operatorname{sinc}(-t)$$

$$h(at) \xleftrightarrow{\mathbb{F}} \frac{1}{|a|} H\left(\frac{f}{2a}\right)$$

$$x_3(t) = |2f_0| . \operatorname{sinc}(-2f_0 t)$$

$$= 2f_0 . \operatorname{sinc}(2f_0 t) \quad //$$

4. $x(t) = e^{-\alpha t} . u(t)$, $\alpha > 0$, işaretinin

a. Enerji spektral yoğunluğunu

b. Otokorelasyon fonksiyonunu

c. Toplam enerjisini bulunuz

$$X(f) = \int_{0}^{\infty} e^{-\alpha t} . e^{-j2\pi ft} . dt$$

$$= -\frac{1}{\alpha + j2\pi f} \left[e^{-\infty} - 1 \right] = \frac{1}{\alpha + j2\pi f}$$

$$S_x(f) = |X(f)|^2 = X(f)\, X^*(f)$$

$$= \frac{1}{\alpha + j2\pi f} \; \frac{1}{\alpha - j2\pi f} = \frac{1}{\alpha^2 + 4\pi^2 f^2} \quad //$$

$$R_x(\tau) = \int_{-\infty}^{\infty} S_x(f)\, e^{j2\pi f\tau} . df$$

Hatırlatma :

$$e^{-\alpha|t|} \xleftrightarrow{\mathbb{F}} \frac{1}{\alpha - j2\pi f} + \frac{1}{\alpha + j2\pi f} = \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$$

$$R_x(\tau) = \frac{1}{2\alpha} \cdot e^{-\alpha|\tau|} \quad //$$

Parseval teoreminden

$$\mathcal{E} = \int_{-\infty}^{\infty} |X(f)|^2 . df = \int_{-\infty}^{\infty} S_x(f) . df = R_x(0)$$

$$= \frac{1}{2\alpha} \quad //$$

5. $X(f) = \Pi(f-5)$ 'in ters Fourier dönüşümünü bulunuz.

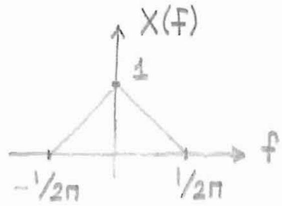
$$\Pi(t-5) \xleftrightarrow{\mathbb{F}} e^{-j10\pi f} . \operatorname{sinc}(f)$$

$$\Pi(f-5) \xleftrightarrow{\mathbb{F}^{-1}} e^{-j10\pi(-t)} . \operatorname{sinc}(-t)$$

$$x(t) = e^{j10\pi t} . \operatorname{sinc}(t) \quad //$$

TEL351 - UYGULAMA III

1. Aşağıda verilen $X(f)$ için



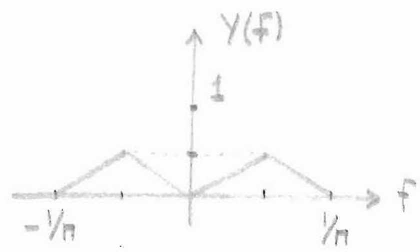
$y(t) = x(t) . p(t)$ 'nin spektrumunu çiziniz

a. $p(t) = \cos t$

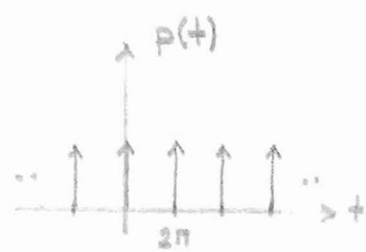
$Y(f) = X(f) * P(f)$

$P(f) = \frac{1}{2} \delta(f - 1/2\pi) + \frac{1}{2} \delta(f + 1/2\pi)$

$Y(f) = \frac{1}{2} X(f - 1/2\pi) + \frac{1}{2} X(f + 1/2\pi)$



b. $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - 2\pi n)$



$p(t)$ periyodik olduğundan

$P(f) = \sum_{n=-\infty}^{\infty} a_n \delta\left(f - \frac{n}{T}\right)$

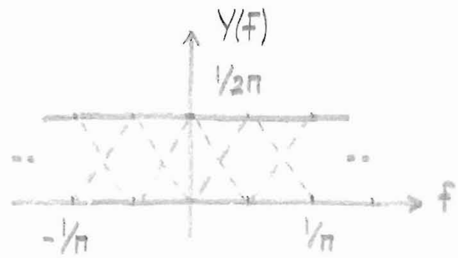
$a_k = \frac{1}{T} \int_T p(t) . e^{-j2\pi kt/T} . dt$

$= \frac{1}{2\pi} \int_0^{2\pi} \delta(t) . e^{-jkt} . dt = \frac{1}{2\pi}$

$$P(f) = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi} \delta\left(f - \frac{n}{2\pi}\right)$$

$$Y(f) = X(f) * P(f)$$

$$= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} X\left(f - \frac{n}{2\pi}\right)$$



$$Y(f) = \frac{1}{2\pi}$$

2. İmpuls cevabı

$$h(t) = \frac{\sin(4t-4)}{\pi(t-1)}$$

olan DZD bir sisteme $x(t)$ uygulandığında çıkış ne olur?

a. $x(t) = \cos\left(6t + \pi/2\right)$

$$\text{sinc}(t) \xleftrightarrow{\mathbb{F}} \Pi(f)$$

$$\text{sinc}(t) = \frac{\sin \pi t}{\pi t}$$

$$\text{sinc}\left(\frac{4t}{\pi}\right) = \frac{\sin 4t}{4t}$$

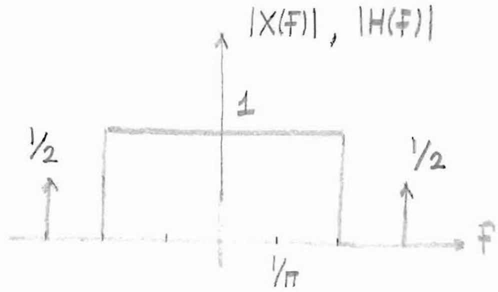
$$\frac{4}{\pi} \text{sinc}\left(\frac{4t}{\pi}\right) = \frac{\sin 4t}{\pi t} \xleftrightarrow{\mathbb{F}} \Pi\left(\frac{\pi f}{4}\right)$$

$$\frac{\sin(4t-4)}{\pi(t-1)} \xrightarrow{\mathbb{F}} e^{-j2\pi f} \cdot \Pi\left(\frac{\pi f}{4}\right) = H(f)$$

$$x(t) = \frac{1}{2} e^{j6t} \cdot \underbrace{e^{j\pi/2}}_{j} + \frac{1}{2} e^{-j6t} \cdot \underbrace{e^{-j\pi/2}}_{-j}$$

$$x(t) = \frac{j}{2} e^{j6t} - \frac{j}{2} e^{-j6t}$$

$$X(f) = \frac{j}{2} \delta(f - 6/2\pi) - \frac{j}{2} \delta(f + 6/2\pi)$$



$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f)\, H(f) = 0 \longrightarrow y(t) = 0$$

b. $$x(t) = \frac{\sin(4t+4)}{\pi(t+1)}$$

$$X(f) = e^{j2\pi f} \cdot \Pi\left(\frac{\pi f}{4}\right)$$

$$Y(f) = X(f)\, H(f) = \Pi\left(\frac{\pi f}{4}\right) \longrightarrow y(t) = \frac{\sin 4t}{\pi t}$$

3. $g(t) = \dfrac{2a}{t^2+a^2}$ işareti için % 99 bant genişliğini bulunuz.

$$e^{-a|t|} \xleftrightarrow{\mathcal{F}} \frac{2a}{a^2 + (2\pi f)^2}$$

$$e^{-a|t|} \xleftrightarrow{\mathcal{F}^{-1}} \frac{2a}{a^2 + (2\pi t)^2}$$

$$e^{-a|2\pi f|} \xleftrightarrow{\mathcal{F}^{-1}} \frac{1}{2\pi} \frac{2a}{a^2+t^2}$$

$$G(f) = 2\pi \cdot e^{-a|2\pi f|}$$

$$\mathcal{E} = \int_{-\infty}^{\infty} |G(f)|^2 \cdot df$$

$$\mathcal{E} = 2\int_0^{\infty} 4\pi^2 . e^{-a4\pi f} . df$$

$$= \frac{8\pi^2}{-a4\pi}\left[0-1\right] = \frac{2\pi}{a} \quad , \text{ Toplam Enerji}$$

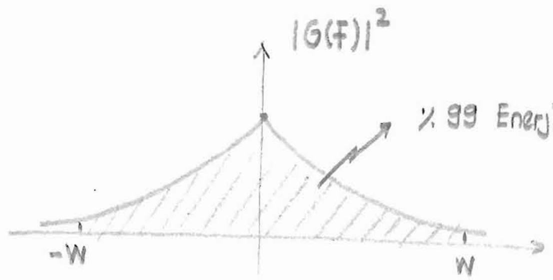
$$\mathcal{E}_W = 2\int_0^{W} 4\pi^2 . e^{-a4\pi f} . df$$

$$= \frac{2\pi}{a}\left[1 - e^{-a4\pi W}\right] = 0{,}99\ \frac{2\pi}{a}$$

$$e^{-a4\pi W} = 0{,}01$$

$$a.4\pi W = 4{,}6052$$

$$W = \frac{0{,}3665}{a}\ Hz$$



g(t)'nin bant genişliği sonsuzdur.

4. Herhangi bir $x(t)$ işareti için

$$x_1(t) = \sum_{n=-\infty}^{\infty} x(t - nT_0)$$

işaretinin

a. $X_1(f)$'i $X(f)$ cinsinden bulunuz

b. $x_1(t)$ periyodik midir, neden?

$$h(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

$$x_1(t) = x(t) * h(t)$$

$$X_1(f) = X(f) . H(f)$$

$$H(f) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$$

Böylelikle

$$X_1(f) = \frac{X(f)}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$$

$X_1(f)$ ayrık olduğundan $x_1(t)$ periyodiktir.