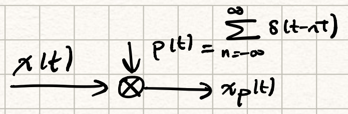
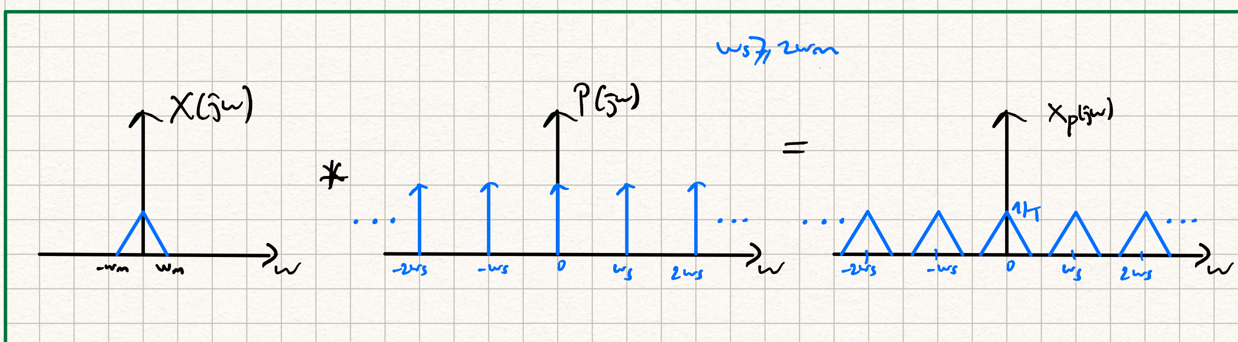


$$x_p(t) = x(t)p(t)$$
$$= x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

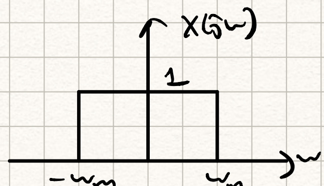
$$x_p(t) = x(t)p(t) \xleftrightarrow{\mathcal{F}} X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$P(j\omega) = \mathcal{F}\{p(t)\} = \frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi n}{T}\right)$$

$$X_p(j\omega) = \frac{1}{\cancel{2\pi}} X(j\omega) * \frac{\cancel{2\pi}}{T}\sum_{n=-\infty}\delta\left(\omega - n\frac{2\pi}{T}\right)$$
$$= \frac{1}{T}\sum_{n=-\infty}^{\infty} X(j\omega) * \delta\left(\omega - n\frac{2\pi}{T}\right) \rightarrow \omega_s = \frac{2\pi}{T}$$
$$= \frac{1}{T}\sum_{n=-\infty}^{\infty} X(j(\omega - n\omega_s))$$



① x(t) işaretinin en düşük örnekleme frekansı ws olarak verilsin. Aşağıdaki işaretler için en küçük örnekleme frekansını hesaplayınız.

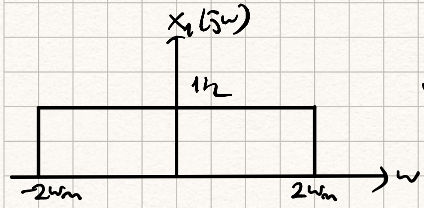


$\omega_s = 2\omega_m$ olarak verilsin.

a) $x_1(t) = x(2t)$

$$x(at) \xleftrightarrow{\mathcal{F}} \frac{1}{|a|} X\left(\frac{j\omega}{|a|}\right)$$

$x_1(t) = x(2t) \Rightarrow X_1(j\omega) = \frac{1}{2} X\left(\frac{j\omega}{2}\right)$

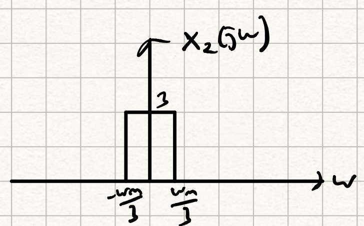


$\omega_{s_1} = 2\cdot \underbrace{2\omega_m}_{\omega_s} = 2\omega_s$

$= 4\omega_m$

b) $x_2(t) = x\left(\frac{t}{3}\right)$

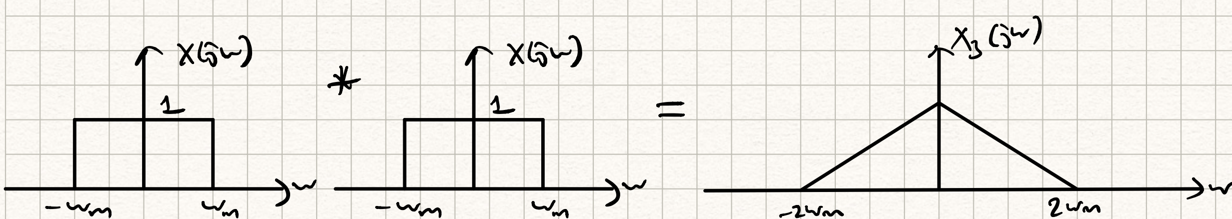
$x_2(t) = x\left(\frac{t}{3}\right) \xleftrightarrow{\mathcal{F}} X_2(j\omega) = 3X(j3\omega)$



$\omega_{s_2} = 2\frac{\omega_m}{3} = \frac{\omega_s}{3}$

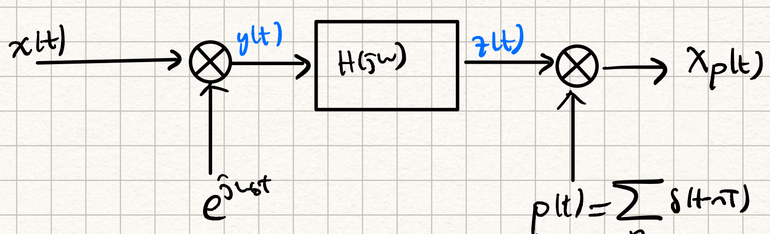
c) $x_3(t) = x^2(t)$

$x_3(t) = x(t)x(t) \xleftrightarrow{\mathcal{F}} X_3(j\omega) = \frac{1}{2\pi} X(j\omega) * X(j\omega)$



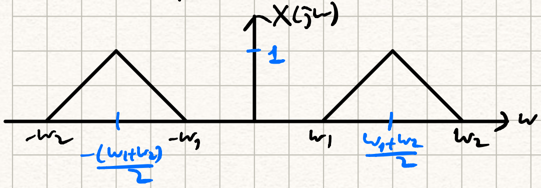
$\omega_{s_3} = 2\cdot \underbrace{2\omega_m}_{\omega_s} = 2\omega_s$

(2)



$x(t)$ işareti yukarıdaki gibi örneklenmektedir. $\omega_0 = \frac{1}{2}(\omega_1 + \omega_2)$ ve transfer fonksiyonu $H(j\omega)$ olarak verilen alçak geçiren süzgecin kesim frekansı $\omega_c = \frac{1}{2}(\omega_2 - \omega_1)$ olarak verilmektedir.

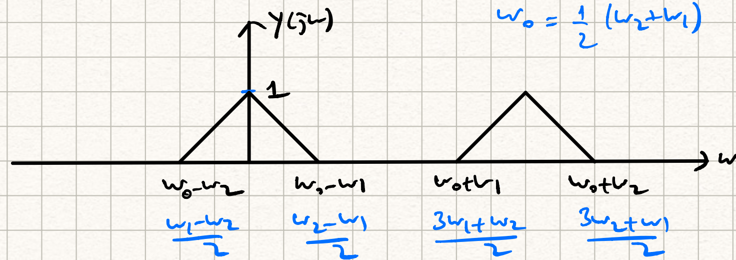
a) $x(t)$ işaretinin spektrumu aşağıdaki gibi verildiğine göre örneklenmiş $x_p(t)$ işaretinin spektrumunu çiziniz.

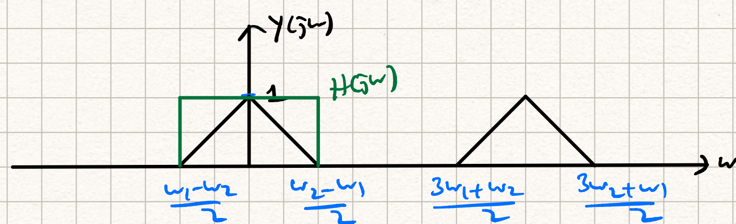


$$y(t) = x(t)\, e^{-jw_0 t} \Rightarrow \mathcal{F}\{e^{jw_0 t}\} = 2\pi\, \delta(w-w_0)$$

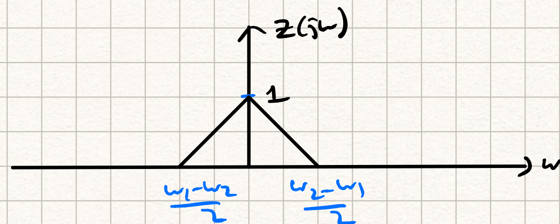
$$Y(jw) = \frac{1}{2\pi}\, X(jw) * 2\pi\, \delta(w-w_0) = X(j(w-w_0))$$

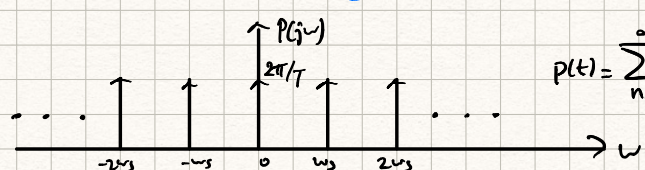
$w_0 = \frac{1}{2}(w_2+w_1)$





$$Z(jw) = Y(jw)\, H(jw)$$

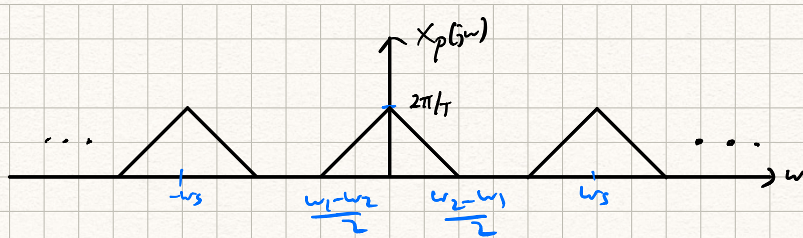




$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$\boxed{w_s = \frac{2\pi}{T}}$$

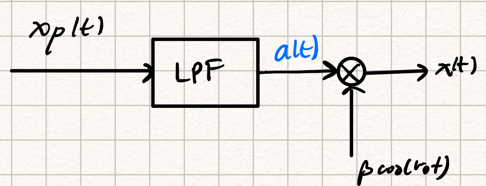
$$x_p(t) = z(t)\,p(t)\ , \quad X_p(jw) = \sum_{n=-\infty}^{\infty} X\left(j\left(w - n\frac{2\pi}{T}\right)\right) = \sum_{n=-\infty}^{\infty} X(j(w-nw_s))$$

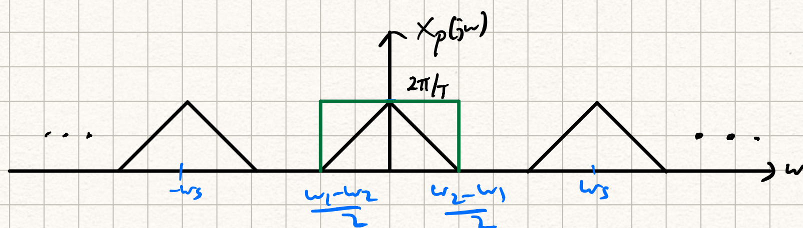


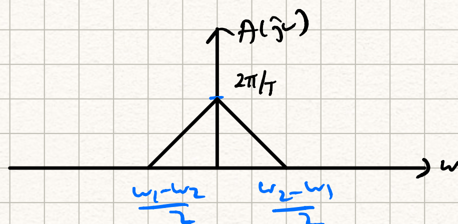
b) $x(t)$ işaretinin $x_p(t)$ örneklerinden geri elde edilebilmesi için gereken en büyük örnekleme periyodu T ne olmalıdır?

$\omega_s \geq 2\frac{(\omega_2-\omega_1)}{2} = \omega_2-\omega_1$ olmalıdır.

c) $x(t)$ işareti $x_p(t)$ örneklerinden geri elde edilebilen bir sistem tasarlayınız.







$$\cos(\omega_0 t) \xleftrightarrow{\mathcal{F}} \pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$$

$$a(t)\cos(\omega_0 t) \xleftrightarrow{\mathcal{F}} \pi A(j(\omega-\omega_0)) + \pi A(j(\omega+\omega_0))$$