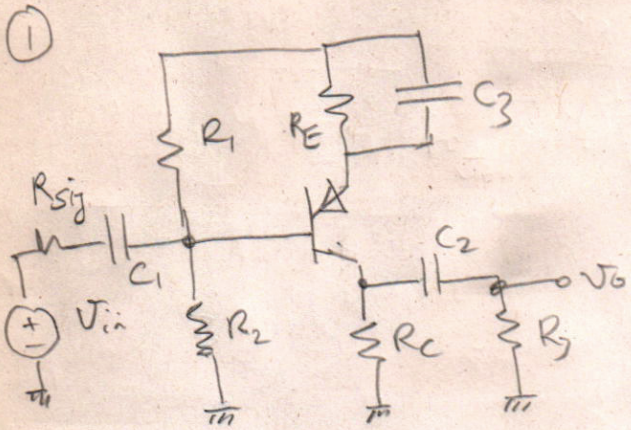
EHB 335E HW#2 Solutions

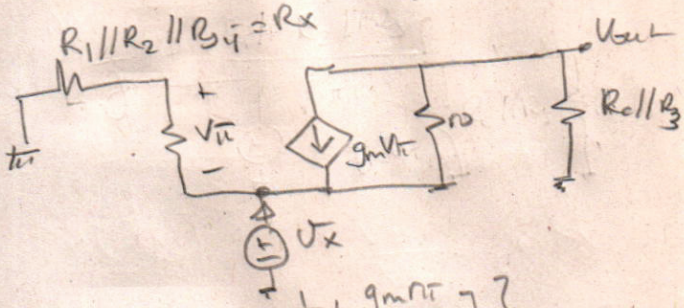
(1)



$$R_{C_1} = R_{sig} + R_1 \parallel R_2 \parallel r_\pi$$

$$R_{C_2} = (R_C \parallel r_o) + R_3$$

$R_{C_3} = ?$



$$i_x = \frac{V_x}{r_\pi + R_x} - g_m V_\pi + \frac{V_x - V_{out}}{r_o} \qquad V_\pi = -\frac{V_x r_\pi}{r_\pi + R_x}$$

$$i_x = \frac{V_x}{r_\pi + R_x} + \frac{g_m r_\pi V_x}{r_\pi + R_x} + \frac{V_x}{r_o} - \frac{V_{out}}{r_o} \Rightarrow i_x = V_x \left\{ \frac{g_m r_\pi + 1}{r_\pi + R_x} + \frac{1}{r_o}\left[1 - \frac{\frac{1}{r_o} + \frac{g_m r_\pi}{r_\pi + R_x}}{\frac{1}{R_C \parallel R_3} + \frac{1}{r_o}}\right]\right\}$$

$$R_x = \frac{V_x}{i_x} \Rightarrow R_{C_3} = R_E \parallel R_x$$

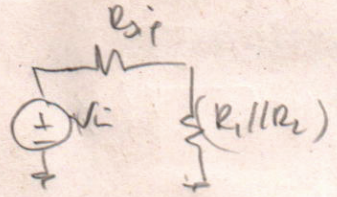
$$\frac{V_{out}}{R_C \parallel R_3} + \frac{V_{out} - V_x}{r_o} - \frac{g_m r_\pi V_x}{r_\pi + R_x} = 0$$

$$f_1 = \frac{1}{2\pi R_{C_1} C_1}; \quad f_2 = \frac{1}{2\pi R_{C_2} C_2}; \quad f_3 = \frac{1}{2\pi R_{C_3} C_3}$$

$$V_{out} = \frac{\left(\frac{1}{r_o} + \frac{g_m r_\pi}{r_\pi + R_x}\right) V_x}{\left(\frac{1}{R_C \parallel R_3} + \frac{1}{r_o}\right)}$$

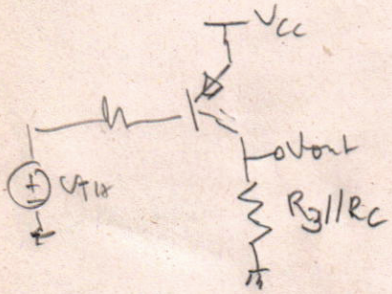
$$f_{-3dB} = f_1 + f_2 + f_3 \quad \text{(Short-Circuit Time Constants Method)}$$

Midband gain $\Rightarrow$



$$R_{TH} = R_{sig} \parallel R_1 \parallel R_2$$

$$V_{TH} = \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{sig}} \cdot V_i$$



$$\frac{V_{out}}{V_{TH}} = \frac{r_\pi}{r_\pi + R_{TH}} \times (-g_m)(R_3 \parallel R_C)$$

$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_{TH}} \cdot \frac{V_{TH}}{V_{in}} = \frac{r_\pi (-g_m)(R_3 \parallel R_C)}{r_\pi + R_{TH}} \times \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{sig}}$$

$A_V = -g_m (R_C \parallel R_3)$ Miller approximation: $C_{\mu 1} = C_\mu [1 + g_m (R_C \parallel R_3)]$ High frequency poles

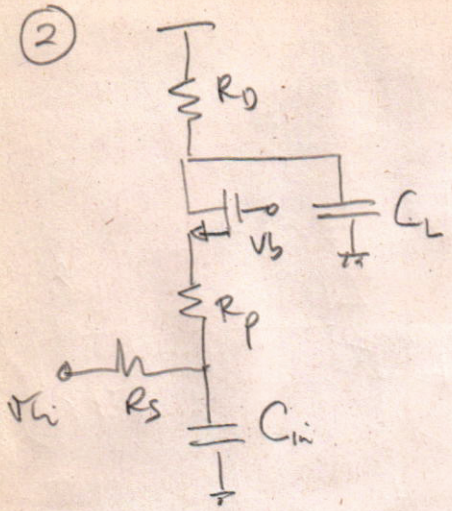
$$C_{\mu 2} = C_\mu \left[1 + \frac{1}{g_m (R_C \parallel R_3)}\right]$$

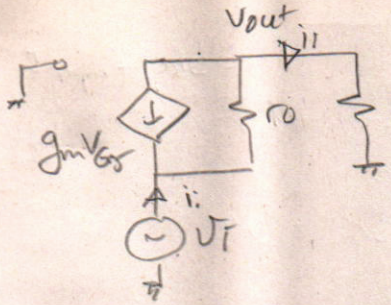
$R_{C_\pi} = r_\pi \parallel R_{TH}$ $R_{C_{\mu 2}} = R_C \parallel R_3 \parallel r_o$ $f_1 = \dfrac{1}{2\pi (C_\mu + C_{\mu 1})(r_\pi \parallel R_{TH})}$

$R_{C_{\mu 1}} = r_\pi \parallel R_{TH}$

$$f_2 = \frac{1}{2\pi C_{\mu 2} (R_C \parallel R_3 \parallel r_o)}$$

(2)





$V_{out} = i_i R_D$

$i_i = -g_m V_{gs} + \frac{V_{in} - V_{out}}{r_o}$

$i_{in} + i_{in}\frac{R_D}{r_o} = +g_m V_{in}$

$\frac{V_{in}}{i_{in}} = \frac{1 + \frac{R_D}{r_o}}{g_m} = \frac{r_o + R_D}{g_m r_o} = R_x$

$R_{Cin} = R_s \parallel \left(R_p + \frac{r_o + R_D}{g_m r_o}\right) \Rightarrow \tau_{Cin} = R_{Cin} \cdot C_{in} \Rightarrow f_{in} = \frac{1}{2\pi \tau_{Cin}}$

$R_{CL} = R_D \parallel \left[r_o + (R_p + R_s) + g_m r_o (R_p + R_s)\right]$

$\Rightarrow \tau_{CL} = R_{CL} \cdot C_{CL} \Rightarrow f_2 = \frac{1}{2\pi \tau_{CL}}$

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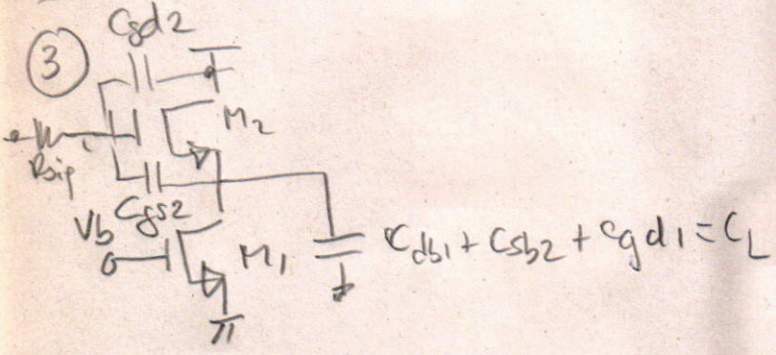
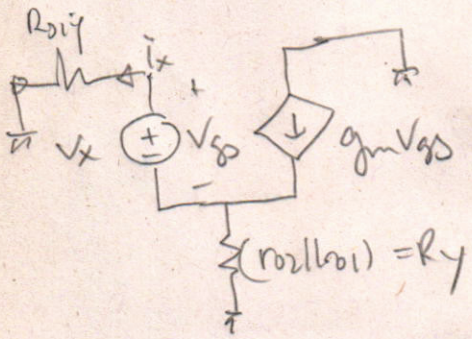
(3)



$R_{CL} = r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{m2}}$ $C_L = C_{db1} + C_{sb2} + C_{gd1}$

$R_{gd2} = R_{sig}$

$R_{gs2} = ?$



$V_g = i_x R_{sig}$ $V_s = (g_m V_{gs} - i_x) R_y$

$V_x = V_g >$

$\frac{V_s}{R_y} + i_x = g_m (i_x R_{sig} - V_s)$

$V_s\left(\frac{1}{R_y} + g_m\right) = i_x (g_m R_{sig} - 1)$

$V_s = i_x \left(\frac{g_m R_{sig} - 1}{\frac{1}{R_y} + g_m}\right)$

$V_x = i_x R_{sig} - i_x \left(\frac{g_m R_{sig} - 1}{\frac{1}{R_y} + g_m}\right)$

$\frac{V_x}{i_x} = R_x = \frac{\left(\frac{R_{sig}}{R_y} + \cancel{g_m R_{sig}} - \cancel{g_m R_{sig}} + 1\right)}{\frac{1}{R_y} + g_m} = \frac{1 + \frac{R_{sig}}{R_y}}{\frac{1}{R_y} + g_m} = R_{gs2}$

$\tau_1 = R_{CL} \cdot C_L$ $\tau_2 = R_{gs2} \cdot C_{gs2}$ $\tau_3 = R_{gd2} \cdot C_{gd2}$

$f_{-3dB} \approx \frac{1}{2\pi \sum_{i=1}^{3} \tau_i}$

(4) check solutions with your TA!