DFT

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-jk(2\pi/N)n} = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

Inverse DFT

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{jk(2\pi/N)n} = \frac{1}{N} \sum_{k=0}^{N-1} X_k W_N^{-nk}$$

N sıfır sayı olmak üzere, aşağıdaki özellikleri kullanarak AFD'lerini bulunuz.

a) $x_0[n] = \delta[n]$

$$X(k) = \sum_{n=0}^{N-1} \delta[n] W_N^{kn}, \quad 0 \le k \le N-1$$

$$= \delta(0) W_N^0 = \delta(0) = 1$$

b) $x[n] = \delta[n-n_0], \quad 0 \le n_0 \le N-1$

$$X[k] = \sum_{n=0}^{N-1} \delta[n-n_0] W_N^{kn}, \quad 0 \le k \le$$

TABLE 6.3
Properties of the DFT

Property	Time Domain	Frequency Domain
1. Linearity	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
2. Time-shifting	$x[n-m]$	$e^{-j2\pi km/N} X(k)$
3. Frequency-shifting (modulation)	$e^{-j2\pi k_0 n/N} x[n]$	$X(k-k_0)$
4. Time reversal	$x[-n]$	$X(-k)$
5. Conjugation	$x^*[n]$	$X^*(-k)$
6. Time-convolution	$x_1[n] \otimes x_2[n]$	$X_1[k]X_2[k]$
7. Frequency-convolution	$x_1[n]x_2[n]$	$\frac{1}{N} X_1[k] \otimes X_2[k]$
8. Parseval's relation	$E_x = \sum_{n=0}^{N-1} \lvert x[n] \rvert^2$	$E_x = \frac{1}{N} \sum_{k=0}^{N-1} \lvert X[k] \rvert^2$

1) $x[n] = \cos\left(\pi \frac{n}{2}\right)$, $n = 0,1,2,3$ $\quad x[0] = 1$, $x[1] = 0$, $x[2] = -1$, $x[3] = 0$

$h[n] = 2^n$, $\quad n = 0,1,2,3$

a) $x[n]$ işaretinin AFD $X(k)$'yı bulunuz.

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot \underbrace{e^{j\frac{2\pi}{N}nk}}_{W_4^{nk}} = \sum_{n=0}^{3} x[n] e^{j\frac{2\pi}{4}nk}$$

$$X[k] = \cos(0) + \cos\left(\frac{\pi}{2}\right) \cdot e^{j\frac{\pi}{2}k} + \cos(\pi) \cdot e^{j\pi k} + \cos\left(\frac{3\pi}{2}\right) \cdot e^{j\frac{3\pi}{2}k}$$

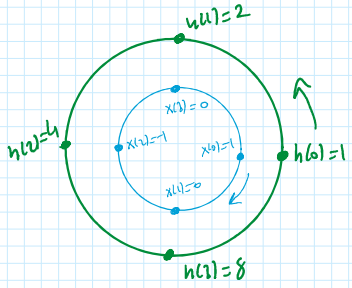
$$= 1 - W_4^{2k} = 1 - e^{j\pi k}$$

b) $h[n]$ işaretinin AFD $H(k)$'yı bulunuz.

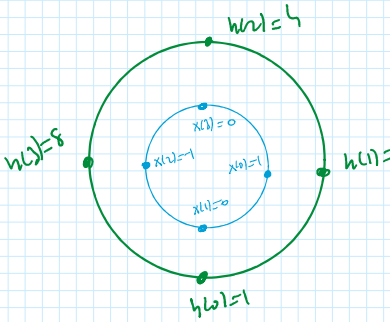
$$H(k) = \sum_{n=0}^{3} 2^n \cdot e^{j\frac{2\pi}{4} \cdot nk}$$

$$= 1 + 2 \cdot W_4^k + 4 W_4^{2k} + 8 W_4^{3k}$$

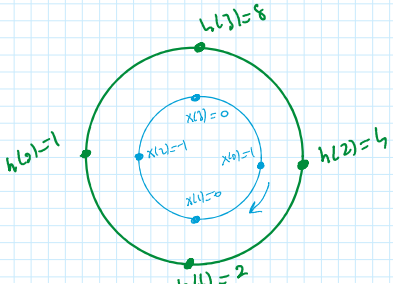
c) $y[n] = x[n] \textcircled{N} h[n]$ dairesel konvolüsyon ile bulunuz.



$y(0) = 1 \cdot 1 - 4 \cdot 1 = -3$



$y(1) = 2 - 8 = -6$



$y(2) = 4 - 1 = 3$

$h(1) = 2$

$x[n] \textcircled{N} h[n] = y[n]$

$$\begin{bmatrix} x[0] & x[N-1] & x[N-2] & \cdots & x[1] \\ x[1] & x[0] & x[N-1] & \cdots & x[2] \\ x[2] & x[1] & x[N] & \cdots & x[3] \\ \vdots & \vdots & \vdots & & \vdots \\ x(N-1) & x[N-2] & x(N-3) & \cdots & x[0] \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \\ h[2] \\ \vdots \\ h[N-1] \end{bmatrix} = \begin{bmatrix} y[0] \\ y[1] \\ \vdots \\ y[N-1] \end{bmatrix}$$

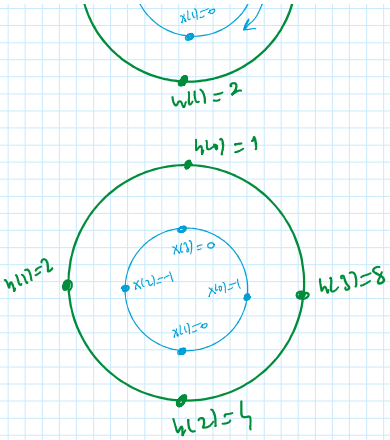
$x[0] = 1 \quad x[1] = 0 \quad x[2] = -1 \quad x[3] = 0$

$h[0] = 1 \quad h[1] = 2 \quad h[2] = 4 \quad h[3] = 8$

$$\begin{bmatrix} x(0) & x(3) & x(2) & x(1) \\ x(1) & x(0) & x(3) & x(2) \\ x(2) & x(1) & x(0) & x(3) \\ x(3) & x(2) & x(1) & x(0) \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ h(2) \\ h(3) \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 1-4 \\ 2-8 \\ -1+4 \\ -2+8 \end{bmatrix} = \begin{bmatrix} -3 \\ -6 \\ 3 \\ 6 \end{bmatrix}$$

1 0 -1 0



$y[1] = 1 \cdot 8 - 1 \cdot 2 = 6$

(a) $y[n]$'i AFD'den yararlanarak bulunuz.

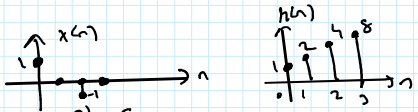
$$Y[k] = X[k]H[k] = (1 - W_4^{2k})(1 + 2W_4^{k} + 4W_4^{2k} + 8W_4^{3k})$$

$$= 1 + 2W_4^{k} + 4W_4^{2k} + 8W_4^{3k} - W_4^{2k} - 2W_4^{3k} - 4W_4^{4k} - 8W_4^{5k}$$

$$= -3 - 6W_4^{k} + 3W_4^{2k} + 6W_4^{3k}$$

$$y[n] = -3\delta[n] - 6\delta[n-1] + 3\delta[n-2] + 6\delta[n-3]$$

(b) $y[n] = x[n] \circledast h[n]$ dairesel konvolüsyon sonucunu bulunuz.



$y[0] = 1 \quad y[1] = 2 \quad y[2] = 3 \quad y[3] = -24 + 8 = 6 \quad y[4] = -4 \quad y[5] = -8 \quad y[6] = 0, \; y[7] = 0$

$3 + 4 - 1 = 6$ noktalı

n	0	1	2	3	4	5	6	7
$y[n]$	1	2	3	6	-4	-8	0	0
$y[n+4]$	-4	-8	6	0				
n için ④ $h[n]$	-3	-6	3	6				

(3) $x[n] = \delta[n] + 2\delta[n-2] + \delta[n-3]$

a) 4 noktalı AFD'sini bulunuz.

$$X[k] = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi kn}{N}} = \sum_{n=0}^{3} x[n] W_4^{kn}, \quad k = 0, 1, \ldots, 3$$

$$X[k] = \underbrace{x[0]}_{1} + \underbrace{x[1]}_{0} W_4^{k} + \underbrace{x[2]}_{1} W_4^{2k} + \underbrace{x[3]}_{1} W_4^{3k}$$

$$= 1 + 2e^{-j\frac{2\pi 2k}{4}} + e^{-j\frac{2\pi 3k}{4}}, \quad k = 0, 1, \ldots, 3$$

$X[0] = 4$

$X[1] = -1 - j$

$X[2] = 2$

$X[3] = -1 + j$

b) $y[n] = x[n] \circledast x[n] \iff Y[k] = X[k]X[k]$

$$Y[k] = (1 + 2W_4^{2k} + W_4^{3k})(1 + 2W_4^{2k} + W_4^{3k})$$

b) $y[n] = x[n] \circledast x[n] \Leftrightarrow Y[k] = X[k]X[k]$

$Y[k] = (1+2W_4^{2k}+W_4^{3k})(1+2W_4^{2k}+W_4^{3k})$

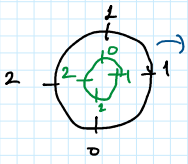
$= 1+2W_4^{2k}+W_4^{3k}+2W_4^{2k}+4W_4^{4k}+2W_4^{5k}+W_4^{3k}+2W_4^{5k}+W_4^{6k}$

$= 1+4W_4^{2k}+2W_4^{3k}+4W_4^{4k}+4W_4^{5k}+W_4^{6k}$

$W_4^0 \quad W_4^k \quad W_4^{2k}$

$Y(k) = 5+4W_4^k+5W_4^{2k}+2W_4^{3k}$

$y[n] = 5\delta[n]+4\delta[n-1]+5\delta[n-2]+2\delta[n-3]$



$y[0] = 5$

$y[1] = 2+2 = 4$

$y[2] = 2+1+2 = 5$

$y[3] = 1+1 = 2$

4) $x[n] = \{1, 2, 2, 1\}$

$h[n] = \{1, 2, 3\}$

$x[n] \rightarrow [h(n)] \rightarrow y(n)$

Sistem çıkışını DFT'de 8 nokta ve 4 nokta kullanarak bulunuz.

Giriş işareti → $L=4$

İmpuls yanıtı → $M=3$ örneğe sahip

$L+M-1 = 4+3-1 = 6$

a) $X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{3} x(n)W_8^{kn}$

$X[k] = 1+2W_8^k+2W_8^{2k}+W_8^{3k}$

$H[k] = 1+2W_8^k+3W_8^{2k}$

$Y[k] = X(k)H(k)$

$= (1+2W_8^k+2W_8^{2k}+W_8^{3k})(1+2W_8^k+3W_8^{2k})$

$= 1+2W_8^k+3W_8^{2k}+2W_8^k+4W_8^{2k}+6W_8^{3k}+2W_8^{2k}$

$+4W_8^{3k}+6W_8^{4k}+W_8^{3k}+2W_8^{4k}+3W_8^{5k}$, $k = \{0, 1, \ldots, 7\}$

$y(n) = \{1, 4, 9, 11, 8, 3, 0, 0\}$ → 8 uzunluklu

b) 4 noktalık:

$X[k] = 1+2W_4^k+2W_4^{2k}+W_4^{3k}$

$H(k) = 1+2W_4^k+3W_4^{2k}$

$Y(k) = X[k]H(k) = (1+2W_4^k+3W_4^{2k})(1+2W_4^k+2W_4^{2k}+W_4^{3k})$

$Y(k) = 1+2W_4^k+2W_4^{2k}+W_4^{3k}+2W_4^k+4W_4^{2k}+2W_4^{4k}$

$+3W_4^{2k}+6W_4^{3k}+6W_4^{4k}+3W_4^{5k}$

$Y(k) = 9+7W_4^k+9W_4^{2k}+11W_4^{3k}$

$y[n] = \{9, 7, 9, 11\}$ → örtüşme

$y[0] = y'(0)+y'(4)$ → önceki parça

$y(1) = y'(1)+y'(5)$

$y(2) = y'(2)$

$y(3) = y'(3)$