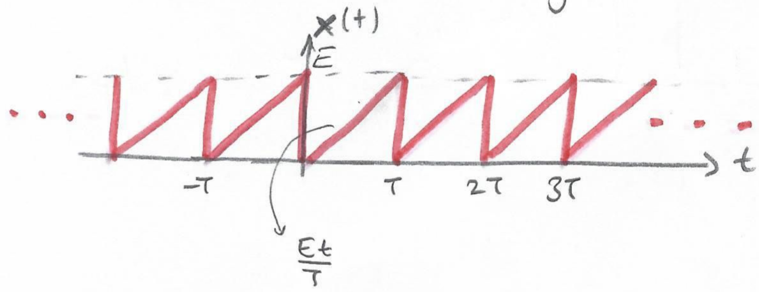
EHB 351

Analog Haberleşme

1) a. Şekildeki periyodik x(t) işaretine ilişkin frekans ve güç spektrumunu bularak değişimini çiziniz.



b. $T = 10^{-3}$ sn ise kesim frekansı $f_c = 1500$ Hz olan sıfır faz kaymalı, birim genlikli bir ideal alçak geçiren süzgeç girişine x(t) işareti uygulandığında, çıkışta elde edilen y(t) işaretinin ifadesini yazınız. Ayrıca, y(t)'nin ortalama gücünü bulunuz.

Cevap

$$C_n = \frac{1}{T}\int_0^T \frac{E}{T} t \cdot e^{-j\frac{2\pi}{T}nt}\,dt = \frac{E}{T^2}\left[\int_0^T t e^{-j\frac{2\pi}{T}nt}\,dt\right]$$

$$\left(\begin{array}{ll} u = t & dv = e^{-j\frac{2\pi}{T}nt}\,dt \\ du = dt & v = \dfrac{e^{-j\frac{2\pi}{T}nt}}{-j\frac{2\pi}{T}n} \end{array}\right)$$

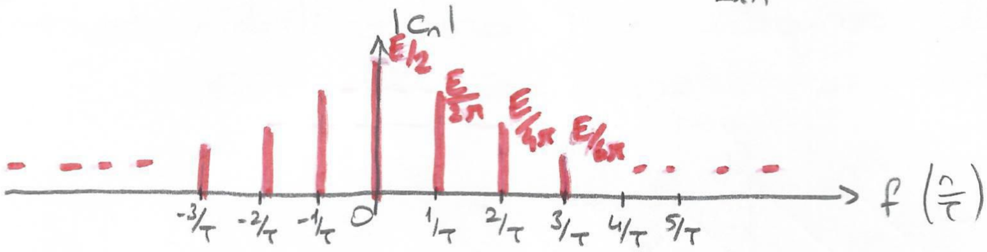
$$C_n = \frac{E}{T^2}\left[-\frac{T}{j2\pi n} t e^{-j\frac{2\pi}{T}nt}\Big|_0^T - \int_0^T \left(-\frac{T}{j2\pi n}\right) e^{-j\frac{2\pi}{T}nt}\,dt\right]$$

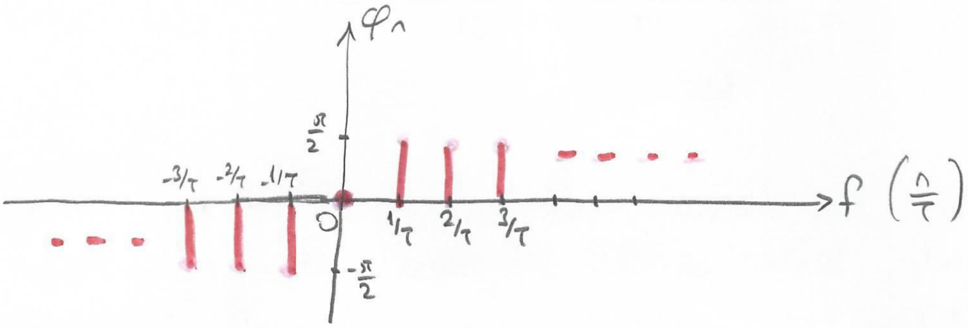
$$= \frac{E}{T^2}\left[-\frac{T^2}{j2\pi n} e^{-j2\pi n} + \frac{T}{j2\pi n}\underbrace{\int_0^T e^{-j\frac{2\pi}{T}nt}\,dt}_{=0}\right]$$

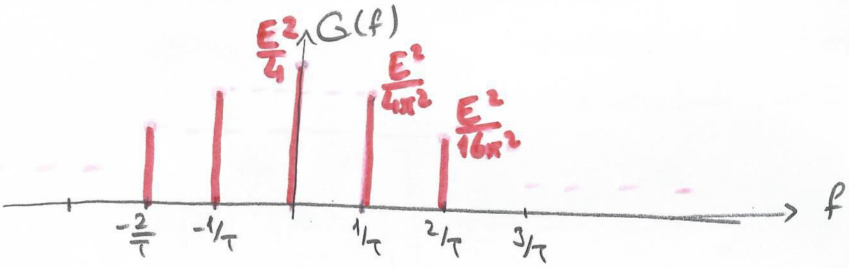
$$C_n = -\frac{E}{j2\pi n}\overbrace{e^{-j2\pi n}}^{1} = \frac{jE}{2\pi n} \qquad n \neq 0$$

$$C_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{T}\int_0^T \frac{E}{T} t\,dt = \frac{E}{2}$$

$$c_n = \begin{cases} E/2, & n=0 \\ j\dfrac{E}{2\pi n}, & n\neq 0 \end{cases} \equiv \quad c_n = \begin{cases} \left|\dfrac{E}{2\pi n}\right| e^{-j\pi/2} & n<0 \\ E/2 & n=0 \\ \dfrac{E}{2\pi n} e^{j\pi/2} & n>0 \end{cases}$$



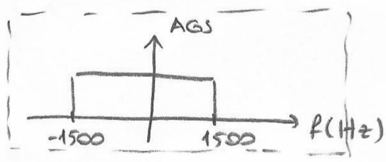




(**NOT**: $R(\tau) = \sum_{\ell} |c_\ell|^2 e^{j\frac{2\pi}{T}\tau \ell}$

$G(f) = \mathcal{F}\{R(\tau)\} = \sum_{\ell} |c_\ell|^2 \delta(f - \frac{\ell}{T})$)

b) $y(t) = \dfrac{E}{2} + \dfrac{E}{2\pi} e^{j\pi/2} e^{j2\pi 10^3 t} + \dfrac{E}{2\pi} e^{-j\pi/2} e^{-j2\pi 10^3 t}$



$= \dfrac{E}{2} + \dfrac{E}{\pi}\cos(2\pi 10^3 t + \frac{\pi}{2})$

$= \dfrac{E}{2} - \dfrac{E}{\pi}\sin(2\pi 10^3 t + \pi/2)$

$P_y = \left(\dfrac{E}{2}\right)^2 + \dfrac{1}{2}\left(\dfrac{E}{\pi}\right)^2$

2) a) $x(t)=e^{-3|t|}$ işaretinin Fourier dönüşümünü bulunuz.

b) a'daki sonuçtan ve Fourier dönüşüm teoremlerinden yararlanarak

i) $s_1(t)=\dfrac{6}{t^2+9}$ ii) $s_2(t)=\dfrac{6}{4t^2+9}$

iii) $s_3(t)=\dfrac{1}{t^2+1}$ $\underline{\text{iv}}$) $s_4(t)=\dfrac{\cos w_0 t}{t^2+1}$ $w_0=2\pi f_0$

işaretlerinin Fourier dönüşümünü bulunuz.

Cevap:

a) $X(f)=\int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}dt=\int_{-\infty}^{\infty} e^{-3|t|}e^{-j2\pi f t}dt$

$$X(f)=\int_{-\infty}^{0} e^{3t}e^{-j2\pi f t}dt+\int_{0}^{\infty} e^{-3t}e^{-j2\pi f t}dt$$

$$=\left.\frac{e^{(3-j2\pi f)t}}{3-j2\pi f}\right|_{-\infty}^{0}+\left.\frac{e^{-(3+j2\pi f)}}{-(3+j2\pi f)}\right|_{0}^{\infty}=\frac{1}{3-j2\pi f}+\frac{1}{3+j2\pi f}$$

$$X(f)=\frac{6}{4\pi^2 f^2+9}$$

Yani $\quad e^{-3|t|} \longleftrightarrow \dfrac{6}{4\pi^2 f^2+9}$

$$\left[\begin{array}{l} \left.\begin{array}{l} x(t) \longleftrightarrow X(f) \\ X(t) \longleftrightarrow x(-f) \end{array}\right\} \text{Dualite} \\ \\ \left.\begin{array}{l} y(t) \longleftrightarrow Y(f) \\ y(at) \longleftrightarrow \frac{1}{|a|} Y(f/a) \end{array}\right\} \text{Ölçekleme} \end{array}\right]$$

b. Dualiteyi kullanırsak

$$y(t) = \frac{6}{4\pi^2 t^2+9} \longleftrightarrow e^{-3|-f|} = e^{-3|f|} = Y(f)$$

i) $s_1(t) = y\left(\frac{t}{2\pi}\right) \longleftrightarrow 2\pi\, Y(2\pi f) = 2\pi e^{-6\pi|f|}$

ii) $s_2(t) = s_1(2t) \Rightarrow S_2(f) = \frac{1}{2} S_1\left(\frac{f}{2}\right) = \pi e^{-3\pi|f|}$

iii) $s_3(t) = \frac{3}{2} s_1(3t) \Rightarrow S_3(f) = \frac{3}{2}\frac{1}{3} S_1\left(\frac{f}{3}\right) = \frac{1}{2} 2\pi e^{-6\pi|f/3|} = \pi e^{-2\pi|f|}$

iv) $s_4(t) = s_3(t)\cos w_0 t \qquad e^{jw_0 t} x(t) \longleftrightarrow X(f-f_0)$

$$\Downarrow$$

$$S_4(f) = \frac{S_3(f-f_0)+S_3(f+f_0)}{2} = \frac{\pi}{2}\left[e^{-2\pi|f-f_0|} + e^{-2\pi|f+f_0|}\right]$$

$$\left(s_1(3t) = \frac{6/9}{\frac{9t^2}{9}+1} = \frac{2/3}{t^2+1} \right)$$