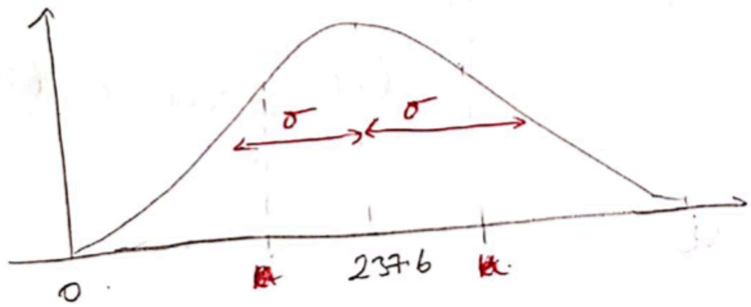
Recitation Hour

1) The number of students using the ATM on campus daily is normally distributed with a mean of 237.6 and a standard deviation of 26.3. You take a random sample of 30 days. Find two values "a" and "b" that are symmetric around the mean such that the probability is 0.95 that the sample mean is greater than "a" and less than "b".

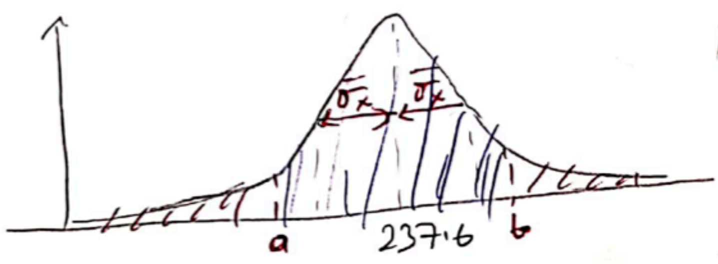
soln



n : # of samples $= 30$

$\mu_{\bar{x}}$: sample mean $= \mu = 237.6$

$\sigma_{\bar{x}}$: sample variance $= \dfrac{\sigma}{\sqrt{n}} = \dfrac{26.3}{\sqrt{30}} = 4.80$

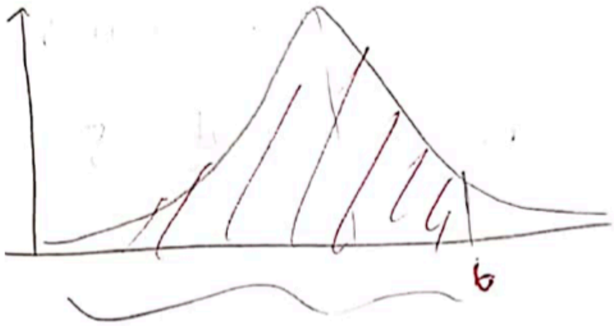


$P(a < x < b)$

$P(a < x < b) = ?$

$P(a < \quad < b) = 1 - 2P(X < a) = 0.95$

$P(X < a) = 0.025$



$P(X < b) = 0.95 + P(X < a) = 0.975$ //

$P(X < b) = P\left(Z < \frac{b - \mu_x}{\sigma_x}\right)$

$Z = \frac{b - \mu_x}{\sigma_x} = \frac{b - 237.6}{4.80} = 1.96$

⇓ from the Z-table

$Z = \frac{b - 237.6}{4.80} = 1.96$

$b = 247.01 \; , \; a = 228.12$

z	0.00	0.01	0.02	0.03	0.04	3	0.06
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406

2) Times spent studying by students in the week before final exams follow a normal distribution with standard deviation 9 hours. A random sample of 5 students was taken in order to estimate the mean study for the population of all students.

a) What's the standard error of the sampling distribution for the mean score?

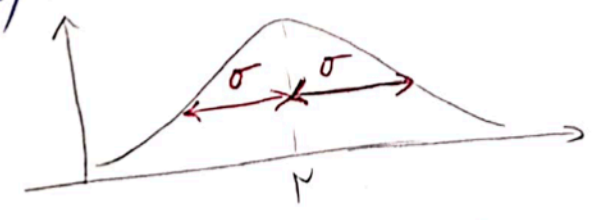
b) What's the probability that the sample mean exceeds the population mean by more than 2.1 hours?

c) What's the probability that the sample mean is more than 3.2 hours below the population mean?

d) What's the prob. that the sample mean differs from the population mean by ∓ 4.1 hours?

~~ans~~
soln.

a)



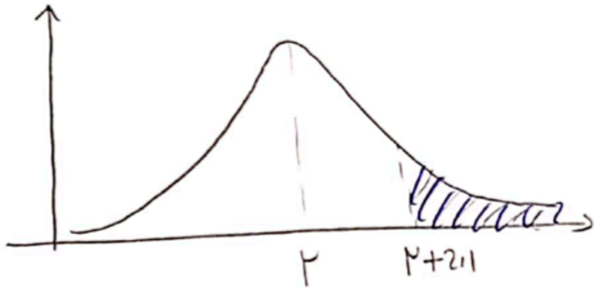
n: # of samples $= 5$

$\sigma = 9$ hours.

the standard error of the sampling dist. = the sample std.

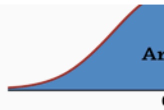
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{9}{\sqrt{5}} = 4.025$$

b)



$P(X > \mu + 2.1) = ?$

$1 - P(X < \mu + 2.1) = ?$



z	0.00	0.01	0.02	0.03	0.
0.0	0.5000	0.5040	0.5080	0.5120	0.5
0.1	0.5398	0.5438	0.5478	0.5517	0.5
0.2	0.5793	0.5832	0.5871	0.5910	0.5
0.3	0.6179	0.6217	0.6255	0.6293	0.6
0.4	0.6554	0.6591	0.6628	0.6664	0.6
0.5	0.6915	0.6950	0.6985	0.7019	0.7
0.6	0.7257	0.7291	0.7324	0.7357	0.7
0.7	0.7580	0.7611	0.7642	0.7673	0.7
0.8	0.7881	0.7910	0.7939	0.7967	0.7
0.9	0.8159	0.8186	0.8212	0.8238	0.8
1.0	0.8413	0.8438	0.8461	0.8485	0.8
1.1	0.8643	0.8665	0.8686	0.8708	0.8
1.2	0.8849	0.8869	0.8888	0.8907	0.8
1.3	0.9032	0.9049	0.9066	0.9082	0.9
1.4	0.9192	0.9207	0.9222	0.9236	0.9
1.5	0.9332	0.9345	0.9357	0.9370	0.9
1.6	0.9452	0.9463	0.9474	0.9484	0.9
1.7	0.9554	0.9564	0.9573	0.9582	0.9
1.8	0.9641	0.9649	0.9656	0.9664	0.9
1.9	0.9713	0.9719	0.9726	0.9732	0.9
2.0	0.9772	0.9778	0.9783	0.9788	0.9
2.1	0.9821	0.9826	0.9830	0.9834	0.9
2.2	0.9861	0.9864	0.9868	0.9871	0.9
2.3	0.9893	0.9896	0.9898	0.9901	0.9
2.4	0.9918	0.9920	0.9922	0.9925	0.9
2.5	0.9938	0.9940	0.9941	0.9943	0.9
2.6	0.9953	0.9955	0.9956	0.9957	0.9
2.7	0.9965	0.9966	0.9967	0.9968	0.9
2.8	0.9974	0.9975	0.9976	0.9977	0.9
2.9	0.9981	0.9982	0.9982	0.9983	0.9
3.0	0.9987	0.9987	0.9987	0.9988	0.9
3.1	0.9990	0.9991	0.9991	0.9991	0.9
3.2	0.9993	0.9993	0.9994	0.9994	0.9
3.3	0.9995	0.9995	0.9995	0.9996	0.9
3.4	0.9997	0.9997	0.9997	0.9997	0.9

$P(X < \mu + 2.1) = P\left(Z < \frac{\mu + 2.1 - \mu_X}{\sigma_X}\right)$.

$Z = \frac{\mu + 2.1 - \mu_X}{\sigma_X} \Rightarrow \mu_X = \mu$

$\sigma_X = 4.025$

$Z = \frac{2.1}{4.025} = 0.5217$ //

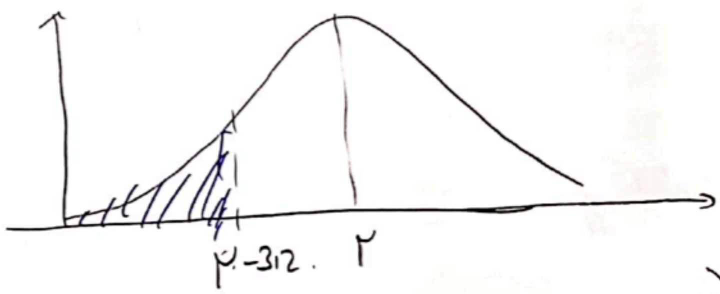
$P\left(Z < \frac{\mu + 2.1 - \mu_X}{\sigma_X}\right) = \boxed{0.6985}$

Since, $P(X > \mu + 2.1) = 1 - P(Z < 0.5217)$

$P(X > \mu + 2.1) = \boxed{0.3025}$

c) the sample mean 3.2 hours below the population mean.

$P(X < \mu - 3.2) = ?$



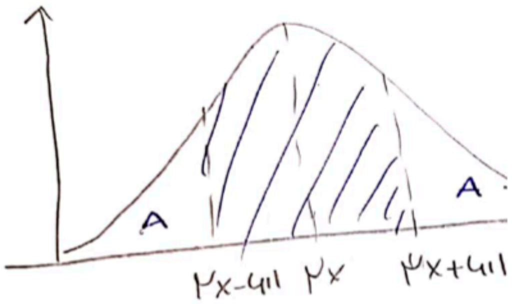
$P(X < \mu - 3.2) = P\left(Z < \frac{\mu - 3.2 - \mu}{4.025}\right) = P\left(Z < -\frac{3.2}{4.025}\right)$

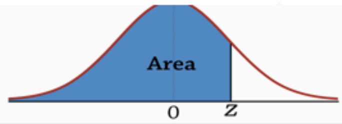
4

$$P(Z<-0{,}7950) = 1 - \underbrace{P(Z<0{,}7950)}_{0{,}7881} = \underline{0{,}2119}$$

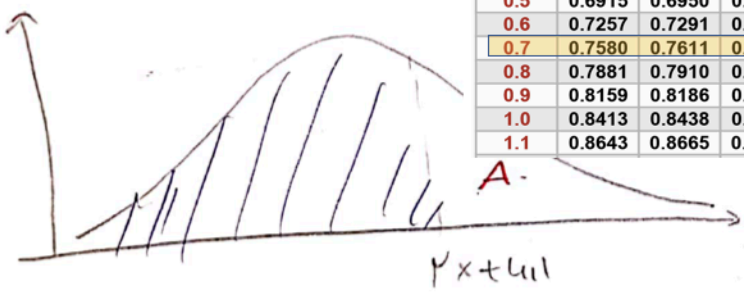
d) $\mu_X \mp 411$ hours.

$$P(\mu_X - 411 < X < \mu_X + 411) = 1 - 2P(X > \mu_X + 411).$$





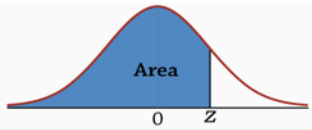
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830



$$P(X<\mu_X+411) = P\left(Z < \frac{\mu_X+411-\mu_X}{\sigma_X}\right)$$

$$z = \frac{411}{4{,}025} = 1{,}0186 \;//$$

$$P(Z<1{,}0186) \sim 0{,}8461$$



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1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279

$$P(X<\mu_X+411) = 0{,}8461$$

$$P(X>\mu_X+411) = 1 - P(X<\mu_X+411)$$
$$= 0{,}1539$$

Therefore, for the $\mu_X \mp 411$

$$P(\mu_X-411<X<\mu_X+411) = 1-2\cdot 0{,}1539 = 0{,}6922 \;//$$