1) Aşağıdaki fark denklemi verilen kareli ve LZD sistemin impuls cevabını bulunuz.

$$y[n] - \frac{1}{2}y[n-1] = x[n] - \frac{1}{4}x[n-1]$$

$$x[n-n_0] \longleftrightarrow e^{-j\omega n_0} \cdot X(e^{j\omega})$$

$$Y(e^{j\omega}) - \frac{1}{2}e^{-j\omega}Y(e^{j\omega}) = X(e^{j\omega}) - \frac{1}{4}e^{-j\omega}X(e^{j\omega})$$

$$Y(e^{j\omega})\left[1 - \frac{1}{2}e^{-j\omega}\right] = X(e^{j\omega})\left[1 - \frac{1}{4}e^{-j\omega}\right]$$

$$Y(e^{j\omega}) = X(e^{j\omega})\,H(e^{j\omega}) \Rightarrow H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

Böylece,

$$H(e^{j\omega}) = \frac{1 - \frac{1}{4}e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}}$$

$$= \frac{1}{1 - \frac{1}{2}e^{-j\omega}} + \frac{-1/4\,e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}}$$

$$\alpha^n u[n] \overset{F}{\longleftrightarrow} \frac{1}{1 - \alpha e^{-j\omega}} \qquad |\alpha| < 1$$

$$h[n] = \mathcal{F}^{-1}\left\{H(e^{j\omega})\right\} = \mathcal{F}^{-1}\left\{\frac{1}{1 - \frac{1}{2}e^{-j\omega}}\right\} + \mathcal{F}^{-1}\left\{\frac{-1/4\,e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}}\right\}$$

$$\therefore h[n] = \left(\frac{1}{2}\right)^n u[n] - \frac{1}{4}\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

2) Ayrık-zamanlı, nedensel ve LTI bir sistem için giriş-çıkış fark denklemi

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$$

olarak verilmektedir.

a) Bu sistemin frekans cevabı $H(e^{j\omega})$'yı bulunuz.

b) Bu sistemin girişine $x[n] = \left(\frac{1}{3}\right)^{n-1} u[n-1]$ uygulandığında çıkış işareti $y[n]$ ne olur?

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$$x[n-n_0] \xleftrightarrow{\mathcal{F}} e^{-j\omega n_0} X(e^{j\omega})$$

a) $$Y(e^{j\omega}) - \frac{3}{4}e^{-j\omega}Y(e^{j\omega}) + \frac{1}{8}e^{-j2\omega}Y(e^{j\omega}) = 2X(e^{j\omega})$$

$$Y(e^{j\omega})\left[1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}\right] = 2X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}$$

$-\frac{1}{4}e^{-j\omega}$ $\quad$ $-\frac{1}{2}e^{-j\omega}$

b)

$$H(e^{j\omega}) = \frac{A}{1 - \frac{1}{4}e^{-j\omega}} + \frac{B}{1 - \frac{1}{2}e^{-j\omega}} = \frac{2}{\left(1 - \frac{1}{4}e^{-j\omega}\right)\left(1 - \frac{1}{2}e^{-j\omega}\right)}$$

$\left(1 - \frac{1}{2}e^{-j\omega}\right)$ $\quad$ $\left(1 - \frac{1}{4}e^{-j\omega}\right)$

$$\frac{A}{2} + \frac{B}{4} = 0 \qquad A + B = 2$$

$$\Rightarrow B = 4, \quad A = -2$$

$$H(e^{j\omega}) = \frac{-2}{1-\frac{1}{4}e^{-j\omega}} + \frac{4}{1-\frac{1}{2}e^{-j\omega}}$$

$$h[n] = \mathcal{F}^{-1}\{H(e^{j\omega})\} = \mathcal{F}^{-1}\left\{\frac{-2}{1-\frac{1}{4}e^{-j\omega}}\right\} + \mathcal{F}^{-1}\left\{\frac{4}{1-\frac{1}{2}e^{-j\omega}}\right\}$$

$$\alpha^n u[n] \xleftrightarrow{\mathcal{F}} \frac{1}{1-\alpha e^{-j\omega}} \quad |\alpha|<1 \text{ ise}$$

$$h[n] = -2\left(\frac{1}{4}\right)^n u[n] + 4\left(\frac{1}{2}\right)^n u[n]$$

c) $x[n] = \left(\frac{1}{3}\right)^{n-1} u[n-1]$

$$X(e^{j\omega}) = \frac{e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}}$$

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$$

$$= \frac{2e^{-j\omega}}{\left(1-\frac{1}{3}e^{-j\omega}\right)\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

$$= \frac{A}{1-\frac{1}{3}e^{-j\omega}} + \frac{B}{1-\frac{1}{2}e^{-j\omega}} + \frac{C}{1-\frac{1}{4}e^{-j\omega}}$$

$$Y(e^{j\omega})\left(1-\frac{1}{3}e^{-j\omega}\right)\Big|_{e^{-j\omega}=3} = A + \frac{B\left(1-\frac{1}{3}e^{-j\omega}\right)}{1-\frac{1}{2}e^{-j\omega}} + \frac{C\left(1-\frac{1}{3}e^{-j\omega}\right)}{1-\frac{1}{4}e^{-j\omega}}\Bigg|_{e^{-j\omega}=3}$$

$$\frac{2e^{-j\omega}}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)} = A + \frac{B\left(1-\frac{1}{3}e^{-j\omega}\right)}{1-\frac{1}{2}e^{-j\omega}} + \frac{C\left(1-\frac{1}{3}e^{-j\omega}\right)}{1-\frac{1}{4}e^{-j\omega}}$$

$\Rightarrow$ $e^{-j\omega}=3$ için $\rightarrow$ A'yı hesaplayalım

$$\frac{2\times 3}{\left(1-\frac{3}{2}\right)\left(1-\frac{3}{4}\right)} = A = -48$$

$\Rightarrow$ B'yi hesaplamak için

$$Y(e^{j\omega})\left(1-\frac{1}{2}e^{-j\omega}\right)\Big|_{e^{-j\omega}=2} \Rightarrow \frac{2\times 2}{\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)} = B = 24$$

$\Rightarrow$ C'yi hesaplamak için

$$Y(e^{j\omega})\left[1-\frac{1}{4}e^{-j\omega}\right]\Big|_{e^{-j\omega}=4} \Rightarrow \frac{2\times 4}{\left(1-\frac{4}{2}\right)\left(1-\frac{4}{3}\right)} = C = 24.$$

$$Y(e^{j\omega}) = \frac{-48}{1-\frac{1}{3}e^{-j\omega}} + \frac{24}{1-\frac{1}{2}e^{-j\omega}} + \frac{24}{1-\frac{1}{4}e^{-j\omega}}$$

$$y[n] = \mathcal{F}^{-1}\left\{Y(e^{j\omega})\right\} = -48\left(\frac{1}{3}\right)^n u[n] + 24\left(\frac{1}{2}\right)^n u[n] + 24\left(\frac{1}{4}\right)^n u[n]$$

3) LZD bir sistemin impuls cevabı

$h[n] = (0.9)^n u[n-1]$

olarak verilmektedir.

a) Sistemin frekans cevabı $H(e^{jw})$'yi bulunuz.

b) Bu sistemin girişine $x[n] = (0.5)^n u[n]$ işareti uygulanırsa çıkış işareti ne olur?

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$$h[n] = (0.9)^n u[n-1]$$

$$= (0.9)(0.9)^{n-1} u[n-1]$$

$$x[n-n_0] \xleftrightarrow{F} e^{-jwn_0} X(e^{jw})$$

$$\alpha^n u[n] \xleftrightarrow{F} \frac{1}{1-\alpha e^{-jw}}$$

$$\Rightarrow H(e^{jw}) = \frac{0.9\, e^{-jw}}{1-0.9e^{-jw}}$$

b) $$x[n] = \left(\frac{1}{2}\right)^n u[n] \xleftrightarrow{F} \frac{1}{1-\frac{1}{2}e^{-jw}}$$

$$Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw})$$

$$= \frac{0.9e^{-jw}}{\left(1-\frac{1}{2}e^{-jw}\right)\left(1-0.9e^{-jw}\right)}$$

$$= \frac{A}{1-\frac{1}{2}e^{-jw}} + \frac{B}{1-0.9e^{-jw}}$$

$$\left(1-0.9e^{-jw}\right) \qquad \left(1-0.5e^{-jw}\right)$$

Buradan,

$$A+B=0 \quad \Big\} \quad A = -9/4$$

$$0.9A+0.5B=0.9 \qquad B = 9/4$$

$$Y(e^{jw}) = \frac{-9/4}{1-\frac{1}{2}e^{-jw}} + \frac{9/4}{1-0.9e^{-jw}}$$

$$y[n] = \mathcal{F}^{-1}\left\{Y(e^{jw})\right\} = -9/4\left(\frac{1}{2}\right)^n u[n] + 9/4\left(0.9\right)^n u[n]$$