En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting)

$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ data kümesi için bu veriye en az hata (toplam hata) ile uyan bir $f(x)$ fonksiyonu arıyoruz.

$$i'nci\ data\ için\ hata:\ \epsilon_i = y_i - f(x_i) \Longrightarrow hataların\ toplamını\ minimum\ yapmak\ istiyoruz$$

$\epsilon_i = y_i - f(x_i)$ pozitif / negatif / sıfır her türlü değeri alabilir bu nedenle

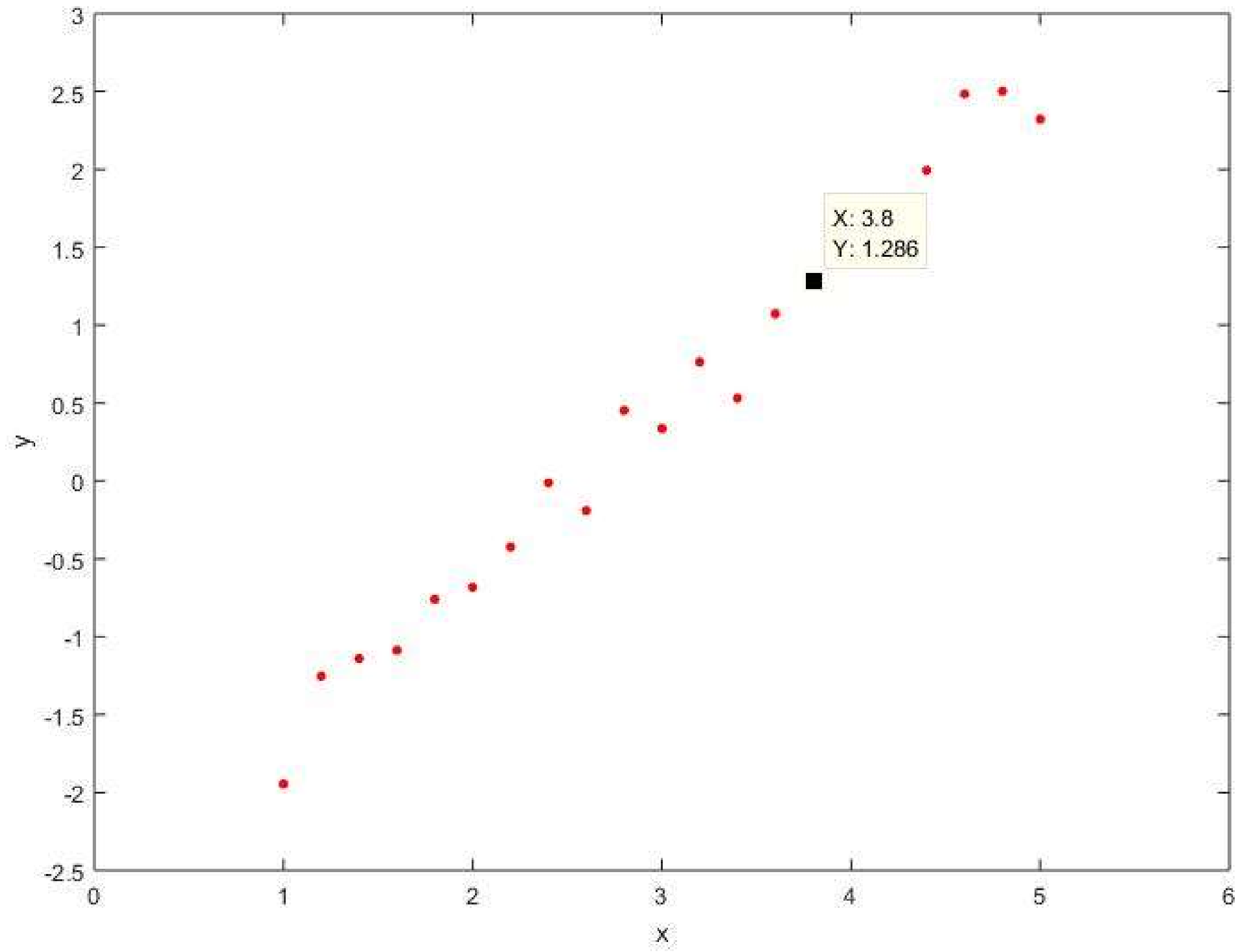
$$\sum_{i=1}^{n} \epsilon_i = \sum_{i=1}^{n} y_i - f(x_i) \longrightarrow \min\ çok\ anlamlı\ değil!!$$

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{f}(x_i)]^2}\ : \text{Ortalama karesel hatanın karekökü (Root Mean Square Error: RMSE)}$$

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{f}(x_i)]^2} \longrightarrow \min\ yapan\ \hat{f}\ fonksiyonu\ data\ için\ en\ küçük\ kareler\ yaklaşımı\ ile\ en\ iyi\ model$$

En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting)

x_i	y_i	x_i	y_i
1.0	−1.945	4.2	1.582
1.2	−1.253	4.4	1.993
1.4	−1.140	4.6	2.473
1.6	−1.087	4.8	2.503
1.8	−0.760	5.0	2.322
2.0	−0.682		
2.2	−0.424		
2.4	−0.012		
2.6	−0.190		
2.8	0.452		
3.0	0.337		
3.2	0.764		
3.4	0.532		
3.6	1.073		
3.8	1.286		
4.0	1.502		



En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting)

x_i	y_i	x_i	y_i
1.0	−1.945	4.2	1.582
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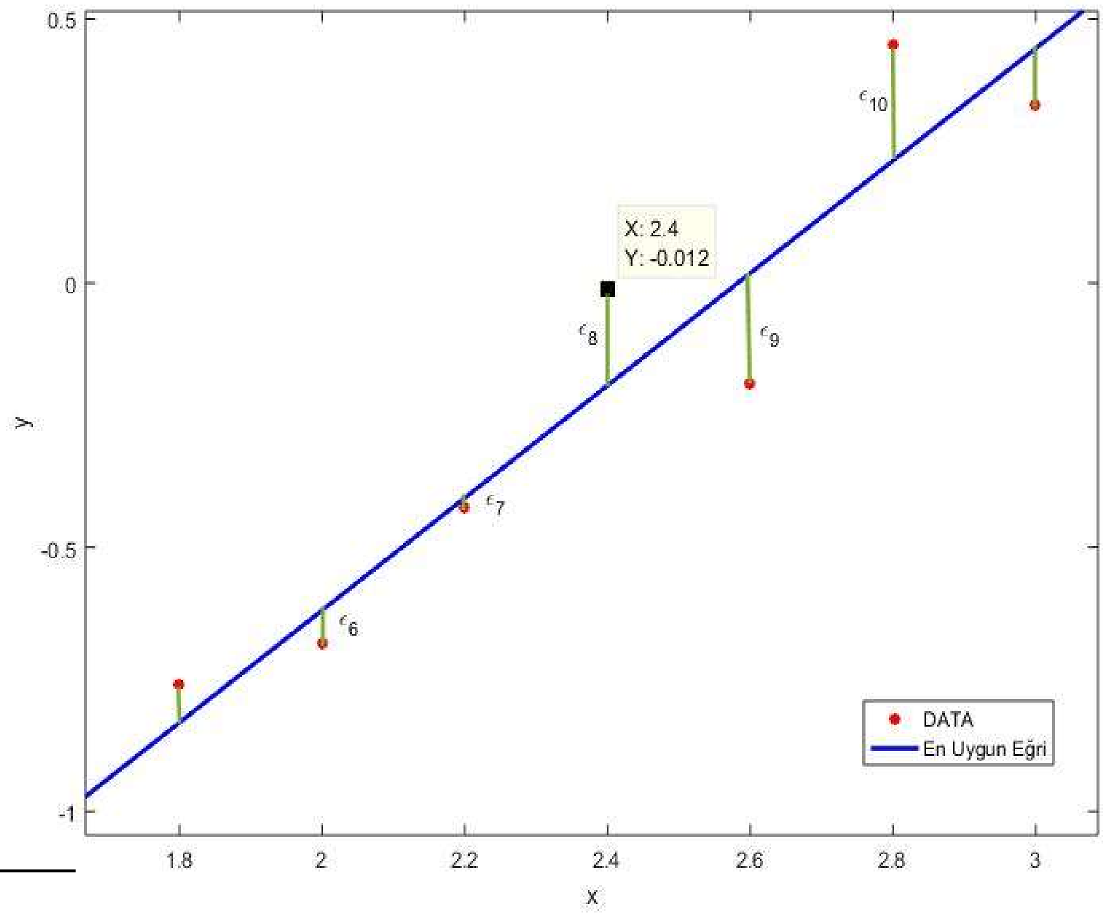
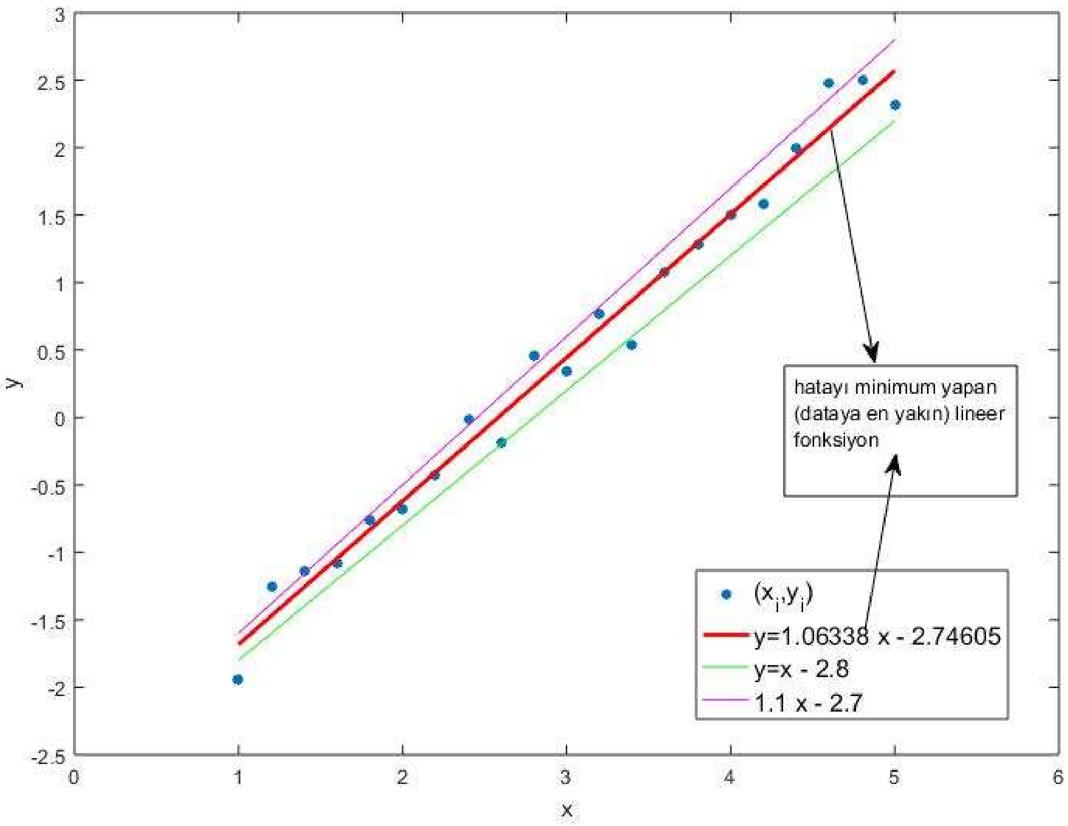


$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {\epsilon_i}^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{f}\,(x_i)]^2} \rightarrow min$$

En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting)



x_i	y_i	x_i	y_i
1.0	−1.945	4.2	1.582
1.2	−1.253	4.4	1.993
1.4	−1.140	4.6	2.473
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3.6	1.073		
3.8	1.286		
4.0	1.502		

En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting)

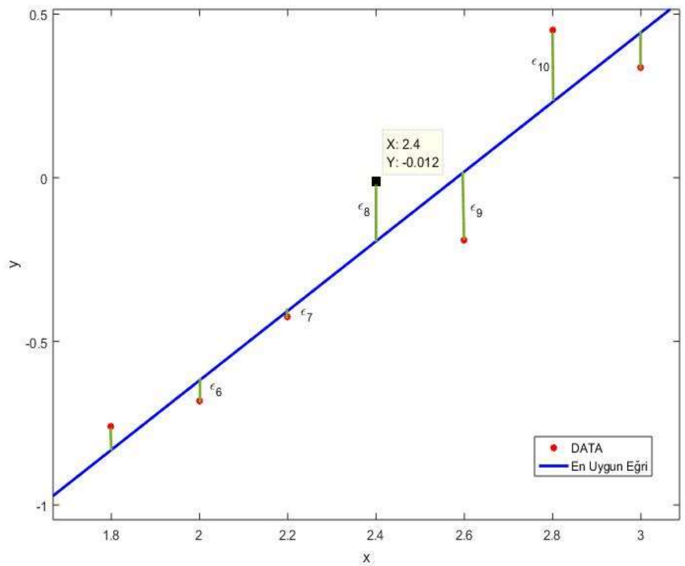
Lineer Yaklaşım: (Lineer Regresyon)

$f(x) = mx + b$ *olarak önerelim* $\Longrightarrow$ *En iyi m ve b yi bulmaya çalışacağız.*

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {\epsilon_i}^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} [y_i - f\,(x_i)]^2} \rightarrow minimum\ yapacak\ m\ ve\ b\ ?$$

$$G(b,m) = \sum_{i=1}^{n} [f\,(x_i) - y_i]^2 = \sum_{i=1}^{n} [mx_i + b - y_i]^2 \rightarrow min$$

$$\frac{\partial G}{\partial b} = 0 \;\; ve \;\; \frac{\partial G}{\partial m} = 0$$



En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting)

$$\frac{\partial G}{\partial b} = \sum_{i=1}^{n} 2[mx_i + b - y_i] = 0 \;\Rightarrow\; \sum_{i=1}^{n} 2b + \sum_{i=1}^{n} 2x_i m = \sum_{i=1}^{n} 2y_i \;\Rightarrow\; n\,b + \left(\sum_{i=1}^{n} x_i\right) m = \sum_{i=1}^{n} y_i \quad (1)$$

$$\frac{\partial G}{\partial m} = \sum_{i=1}^{n} 2[mx_i + b - y_i]x_i = 0 \;\Rightarrow\; \sum_{i=1}^{n} 2x_i b + \sum_{i=1}^{n} 2x_i^2 m = \sum_{i=1}^{n} 2y_i\, x_i \;\Rightarrow\; \left(\sum_{i=1}^{n} x_i\right) b + \left(\sum_{i=1}^{n} x_i^2\right) m = \sum_{i=1}^{n} y_i\, x_i \quad (2)$$

(1) ve (2) den b ve m çözülerek $f(x) = mx + b$ bulunur.

$$\begin{bmatrix} n & \left(\sum_{i=1}^{n} x_i\right) \\ \left(\sum_{i=1}^{n} x_i\right) & \left(\sum_{i=1}^{n} x_i^2\right) \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} y_i\, x_i \end{bmatrix}$$

Çözümün varlığı için sistemin determinantı sıfırdan farklı olmalı$\Rightarrow$

$$n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2 \neq 0$$

En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting)

x_i	y_i	x_i	y_i
1.0	-1.945	4.2	1.582
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3.0	0.337		
3.2	0.764		
3.4	0.532		
3.6	1.073		
3.8	1.286		
4.0	1.502		

$$Data\ Sayısı{:}\ n = 21$$

$$\sum_{i=1}^{n} x_i = 0{,}2 \sum_{j=5}^{25} j = 0{,}2 \times \left(\frac{25 \times 26}{2} - \frac{4 \times 5}{2}\right) = 63$$

$$\sum_{i=1}^{n} x_i^2 = 219{,}8$$

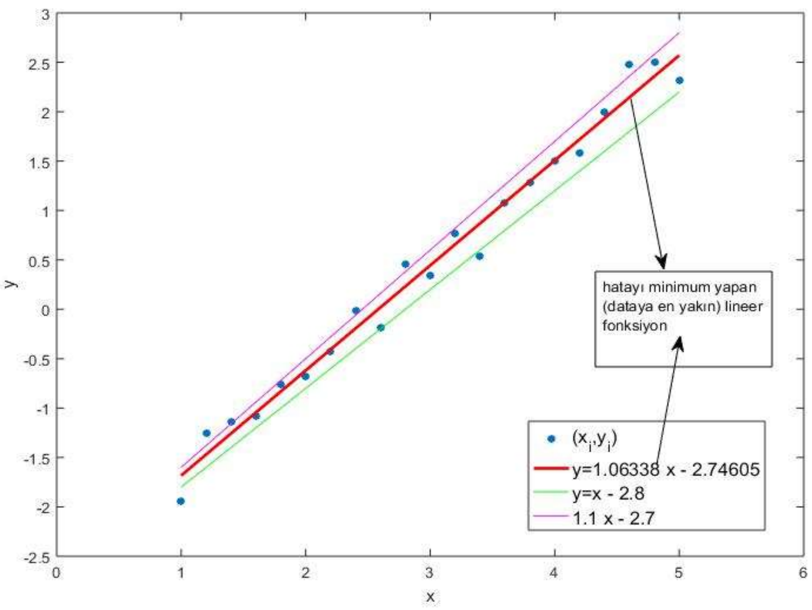
$$\sum_{i=1}^{n} y_i = 9{,}326$$

$$\sum_{i=1}^{n} x_i y_i = 60{,}73$$

$$\begin{bmatrix} 21 & 63 \\ 63 & 219{,}8 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} 9{,}326 \\ 60{,}73 \end{bmatrix} \Longrightarrow \begin{array}{c} b = -2.74605 \\ m = 1.06338 \end{array}$$

$$f(x) = mx + b = 1.06338x - 2.74605$$

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {\epsilon_i}^2} = 0.171$$



En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting)

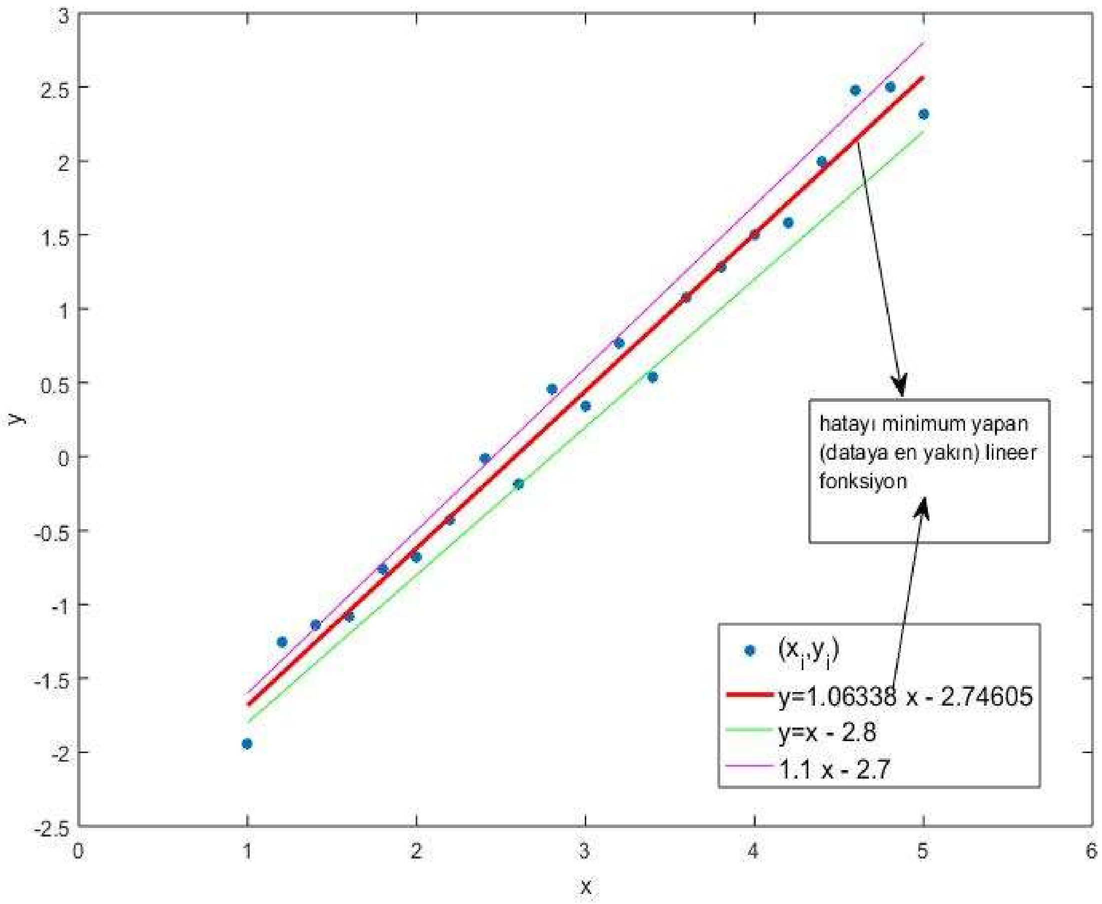
$$En\ iyi\ eğri:\ f(x) = mx + b = 1.06338x - 2.74605$$

$$f(x) = x - 2.8 \Longrightarrow E = 0.309$$

$$f(x) = 1.1x - 2.7 \Longrightarrow E = 0.236$$

$$\hat{f}(x) = mx + b = 1.06338x - 2.74605 \Longrightarrow E = 0.171$$

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {\epsilon_i}^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} [y_i - \hat{f}(x_i)]^2} \longrightarrow min$$



En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting)

$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ data kümesi için bu veriye en az hata (toplam hata) ile uyan bir $f(x)$ fonksiyonu arıyoruz.

En iyi eğri : $f(x) = mx + b$

İsterlerimiz:

$f(x_1) = y_1 \Longrightarrow b + mx_1 = y_1$

$f(x_2) = y_2 \Longrightarrow b + mx_2 = y_2$

$\vdots$

$f(x_n) = y_n \Longrightarrow b + mx_n = y_n$

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \Longrightarrow$$

$$[\boldsymbol{L}]_{n\times 2} \begin{bmatrix} b \\ m \end{bmatrix}_{2\times 1} = [\boldsymbol{y}]_{n\times 1} \Longrightarrow \boldsymbol{L}^T \boldsymbol{L} \begin{bmatrix} b \\ m \end{bmatrix} = \boldsymbol{L}^T \boldsymbol{y}$$

$\begin{bmatrix} b \\ m \end{bmatrix} = (\boldsymbol{L}^T \boldsymbol{L})^{-1} \boldsymbol{L}^T \boldsymbol{y}$: Least Squares Solution

$$\boldsymbol{L}^T \boldsymbol{L} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} = \begin{bmatrix} n & \left(\sum_{i=1}^{n} x_i\right) \\ \left(\sum_{i=1}^{n} x_i\right) & \left(\sum_{i=1}^{n} x_i^2\right) \end{bmatrix}$$

$$\boldsymbol{L}^T \boldsymbol{y} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} y_i\, x_i \end{bmatrix}$$

$$\begin{bmatrix} n & \left(\sum_{i=1}^{n} x_i\right) \\ \left(\sum_{i=1}^{n} x_i\right) & \left(\sum_{i=1}^{n} x_i^2\right) \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} y_i\, x_i \end{bmatrix}$$

En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting):

Yüksek Mertebe Eğri Uydurma

$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ data kümesi için bu veriye en az hata (toplam hata) ile uyan bir $f(x)$ fonksiyonu arıyoruz.

$i'nci\ data\ için\ hata:\ \epsilon_i = y_i - f(x_i) \Longrightarrow hataların\ toplamını\ minimum\ yapmak\ istiyoruz$

$\epsilon_i = y_i - f(x_i)$ pozitif / negatif / sıfır her türlü değeri alabilir bu nedenle

$$\sum_{i=1}^{n} \epsilon_i = \sum_{i=1}^{n} y_i - f(x_i) \longrightarrow \min\ çok\ anlamlı\ değil!!$$

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{f}(x_i)]^2} \ : \text{Ortalama karesel hatanın karekökü (Root Mean Square Error: RMSE)}$$

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{f}(x_i)]^2} \longrightarrow \min\ yapan\ \hat{f}\ fonksiyonu\ data\ için\ en\ küçük\ kareler\ yaklaşımı\ ile\ en\ iyi\ model$$

Yüksek Mertebe Eğri Uydurma

Lineer fonksiyon yerine çeşitli Özel Fonksiyonlar seçip bunların kombinasyonu yardımıyla bir yaklaşım yapabiliriz:

$$\hat{f}(x) = a_1\varphi_1(x) + a_2\varphi_2(x) + \cdots + a_m\varphi_m(x)$$

$$\varphi_j(x); j = 1,2,\ldots,m \ : Seçilmiş\ (Baz)\ Fonksiyonları$$

$$a_j\ ; j = 1,2,\ldots,m \ : Bilinmeyen\ Katsayılar$$

Örneğin 2. derece bir polinomla veri uydurma yapmak istersek ;

$$\varphi_1(x) = 1\ ;\ \ \varphi_2(x) = x\ ;\ \ \varphi_3(x) = x^2 \Longrightarrow$$

$$\hat{f}(x) = a_1\varphi_1(x) + a_2\varphi_2(x) + a_3\varphi_3(x) = a_1 + a_2 x + a_3 x^2$$

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{f}\,(x_i)]^2} \longrightarrow \min\ yapan\ \hat{f}\ fonksiyonu\ ? \Longrightarrow a_j\ ; j = 1,2,\ldots,m\ \ katsayılarının\ belirlenmesi$$

En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting):

Yüksek Mertebe Eğri Uydurma

$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ veri kümesi

$\varphi_1(x) = 1\,;\ \ \varphi_2(x) = x\,;\ \ \varphi_3(x) = x^2 \Longrightarrow$

$\hat{f}(x) = a_1\varphi_1(x) + a_2\varphi_2(x) + a_3\varphi_3(x) = a_1 + a_2 x + a_3 x^2$

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{f}\,(x_i)]^2} \longrightarrow \min \Longrightarrow$$

$$G(a_1, a_2, a_3) = \sum_{j=1}^{n}[f\,(x_j) - y_j]^2 = \sum_{j=1}^{n}[a_1\varphi_1(x_j) + a_2\varphi_2(x_j) + a_3\varphi_3(x_j) - y_j]^2 \rightarrow min \Longrightarrow$$

$$\frac{\partial G}{\partial a_i} = 0\,;\, i = 1,2,3$$

$$\frac{\partial G}{\partial a_1} = 0, \qquad \frac{\partial G}{\partial a_2} = 0, \qquad \frac{\partial G}{\partial a_3} = 0$$

$$\frac{\partial G}{\partial a_i} = \sum_{j=1}^{n} 2[a_1\varphi_1(x_j) + a_2\varphi_2(x_j) + a_3\varphi_3(x_j) - y_j]\varphi_i(x_j)\,;\, i = 1,2,3$$

Yüksek Mertebe Eğri Uydurma

$(x_1,y_1),(x_2,y_2),\dots,(x_n,y_n)$ veri kümesi

$$\hat{f}(x)=a_1\varphi_1(x)+a_2\varphi_2(x)+a_3\varphi_3(x)=a_1+a_2x+a_3x^2$$

$$\frac{\partial G}{\partial a_i}=\sum_{j=1}^{n}2\left[a_1\varphi_1(x_j)+a_2\varphi_2(x_j)+a_3\varphi_3(x_j)-y_j\right]\varphi_i(x_j)=0\ \ ;\ i=1,2,3\Longrightarrow$$

$$\left[\sum_{j=1}^{n}\varphi_1(x_j)\varphi_i(x_j)\right]a_1+\left[\sum_{j=1}^{n}\varphi_2(x_j)\varphi_i(x_j)\right]a_2+\left[\sum_{j=1}^{n}\varphi_3(x_j)\varphi_i(x_j)\right]a_3=\sum_{j=1}^{n}y_j\varphi_i(x_j)\ \ ;\ \ i=1,2,3$$

3 denklem
3 bilinmeyen

$$\varphi_1(x)=1\ ;\ \ \varphi_2(x)=x\ ;\ \ \varphi_3(x)=x^2\Longrightarrow$$

$$na_1+\left(\sum_{j=1}^{n}x_j\right)a_2+\left(\sum_{j=1}^{n}x_j^2\right)a_3=\sum_{j=1}^{n}y_j\qquad(\mathbf{1})$$

$$\left(\sum_{j=1}^{n}x_j\right)a_1+\left(\sum_{j=1}^{n}x_j^2\right)a_2+\left(\sum_{j=1}^{n}x_j^3\right)a_3=\sum_{j=1}^{n}y_jx_j\qquad(\mathbf{2})$$

$$\left(\sum_{j=1}^{n}x_j^2\right)a_1+\left(\sum_{j=1}^{n}x_j^3\right)a_2+\left(\sum_{j=1}^{n}x_j^4\right)a_3=\sum_{j=1}^{n}y_jx_j^2\qquad(\mathbf{3})$$

En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting):

Yüksek Mertebe Eğri Uydurma

$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ veri kümesi

$$\hat{f}(x) = a_1\varphi_1(x) + a_2\varphi_2(x) + a_3\varphi_3(x) = a_1 + a_2 x + a_3 x^2$$

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{f}\,(x_i)]^2} \longrightarrow \min \Longrightarrow$$

$$\frac{\partial G}{\partial a_i} = \sum_{j=1}^{n} 2[a_1\varphi_1(x_j) + a_2\varphi_2(x_j) + a_3\varphi_3(x_j) - y_j]\varphi_i(x_j) = 0 \quad ; \; i = 1,2,3 \Longrightarrow$$

$$\begin{bmatrix} n & \left(\sum_{j=1}^{n} x_j\right) & \left(\sum_{j=1}^{n} x_j^2\right) \\ \left(\sum_{j=1}^{n} x_j\right) & \left(\sum_{j=1}^{n} x_j^2\right) & \left(\sum_{j=1}^{n} x_j^3\right) \\ \left(\sum_{j=1}^{n} x_j^2\right) & \left(\sum_{j=1}^{n} x_j^3\right) & \left(\sum_{j=1}^{n} x_j^4\right) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} y_j \\ \sum_{j=1}^{n} y_j x_j \\ \sum_{j=1}^{n} y_j x_j^2 \end{bmatrix}$$

En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting):

Yüksek Mertebe Eğri Uydurma: Genel HAL

$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ veri kümesi

$\varphi_j(x); j = 1,2, \dots, m$: *Seçilmiş (Baz) Fonksiyonları*

$\hat{f}(x) = a_1\varphi_1(x) + a_2\varphi_2(x) + \cdots + a_m\varphi_m(x)$

$a_j\,; j = 1,2, \dots, m$: *Bilinmeyen Katsayılar*

$$G(a_1, a_2, \dots, a_m) = \sum_{j=1}^{n} [f(x_j) - y_j]^2 = \sum_{j=1}^{n} [a_1\varphi_1(x_j) + a_2\varphi_2(x_j) + \cdots + a_m\varphi_m(x_j) - y_j]^2 \rightarrow min \Longrightarrow$$

$$\frac{\partial G}{\partial a_i} = \sum_{j=1}^{n} 2[a_1\varphi_1(x_j) + a_2\varphi_2(x_j) + \cdots + a_m\,\varphi_m(x_j) - y_j]\varphi_i(x_j) = 0 \;\; ; \; i = 1,2, \dots, m \Longrightarrow$$

$$\sum_{k=1}^{m} a_k \left[\sum_{j=1}^{n} \varphi_k(x_j)\varphi_i(x_j) \right] = \sum_{j=1}^{n} y_j\varphi_i(x_j) \;\; ; \;\; i = 1,2, \dots, m$$

$$[\boldsymbol{K}]_{m\times m}[\boldsymbol{a}]_{m\times 1} = [\tilde{\boldsymbol{y}}]_{m\times 1} \; ; \quad m \times m \; lik \; bir \; lineer \; sistem$$

Yüksek Mertebe Eğri Uydurma: Örnek

$$\hat{f}(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$\begin{bmatrix} n & \left(\sum_{j=1}^{n} x_j\right) & \left(\sum_{j=1}^{n} x_j^2\right) & \left(\sum_{j=1}^{n} x_j^3\right) \\ \left(\sum_{j=1}^{n} x_j\right) & \left(\sum_{j=1}^{n} x_j^2\right) & \left(\sum_{j=1}^{n} x_j^3\right) & \left(\sum_{j=1}^{n} x_j^4\right) \\ \left(\sum_{j=1}^{n} x_j^2\right) & \left(\sum_{j=1}^{n} x_j^3\right) & \left(\sum_{j=1}^{n} x_j^4\right) & \left(\sum_{j=1}^{n} x_j^5\right) \\ \left(\sum_{j=1}^{n} x_j^3\right) & \left(\sum_{j=1}^{n} x_j^4\right) & \left(\sum_{j=1}^{n} x_j^5\right) & \left(\sum_{j=1}^{n} x_j^6\right) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} y_j \\ \sum_{j=1}^{n} y_j x_j \\ \sum_{j=1}^{n} y_j x_j^2 \\ \sum_{j=1}^{n} y_j x_j^3 \end{bmatrix}$$

$$\begin{bmatrix} 21 & 10.5 & 7.175 & 5.5125 \\ 10.5 & 7.175 & 5.5125 & 4.5166 \\ 7.175 & 5.5125 & 4.5166 & 3.8541 \\ 5.5125 & 4.5166 & 3.8541 & 3.3821 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 24.118 \\ 13.234 \\ 9.4683 \\ 7.5594 \end{bmatrix}$$

$$[\boldsymbol{K}]_{4\times4}[\boldsymbol{a}]_{4\times1} = [\hat{\boldsymbol{y}}]_{4\times1} \Longrightarrow \quad [\boldsymbol{a}] = [\boldsymbol{K}]^{-1}[\hat{\boldsymbol{y}}] \Longrightarrow$$

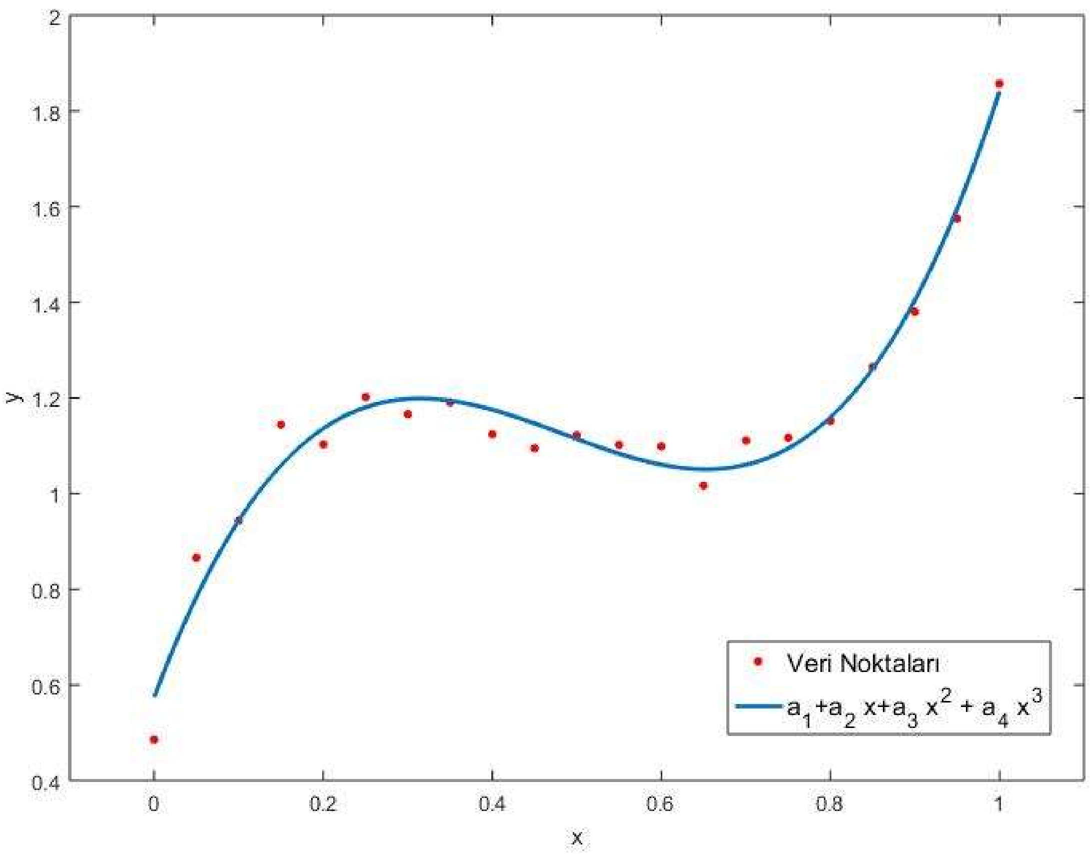
$$[\boldsymbol{a}] = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 0.5747 \\ 4.7259 \\ -11.1282 \\ 7.6687 \end{bmatrix}$$

x_i	y_i	x_i	y_i
0.00	0.486	0.80	1.152
0.05	0.866	0.85	1.265
0.10	0.944	0.90	1.380
0.15	1.144	0.95	1.575
0.20	1.103	1.00	1.857
0.25	1.202		
0.30	1.166		
0.35	1.191		
0.40	1.124		
0.45	1.095		
0.50	1.122		
0.55	1.102		
0.60	1.099		
0.65	1.017		
0.70	1.111		
0.75	1.117		

Yüksek Mertebe Eğri Uydurma: Örnek

x_i	y_i	x_i	y_i
0.00	0.486	0.80	1.152
0.05	0.866	0.85	1.265
0.10	0.944	0.90	1.380
0.15	1.144	0.95	1.575
0.20	1.103	1.00	1.857
0.25	1.202		
0.30	1.166		
0.35	1.191		
0.40	1.124		
0.45	1.095		
0.50	1.122		
0.55	1.102		
0.60	1.099		
0.65	1.017		
0.70	1.111		
0.75	1.117		

$$\hat{f}(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$



Yüksek Mertebe Eğri Uydurma: Örnek

$$\hat{f}(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$[\boldsymbol{K}]_{4\times4}[\boldsymbol{a}]_{4\times1} = [\hat{\boldsymbol{y}}]_{4\times1} \Longrightarrow \quad [\boldsymbol{a}] = [\boldsymbol{K}]^{-1}[\hat{\boldsymbol{y}}] \Longrightarrow$$

$$\begin{bmatrix} 21 & 10.5 & 7.175 & 5.5125 \\ 10.5 & 7.175 & 5.5125 & 4.5166 \\ 7.175 & 5.5125 & 4.5166 & 3.8541 \\ 5.5125 & 4.5166 & 3.8541 & 3.3821 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 24.118 \\ 13.234 \\ 9.4683 \\ 7.5594 \end{bmatrix}$$

$$[\boldsymbol{a}] = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 0.5747 \\ 4.7259 \\ -11.1282 \\ 7.6687 \end{bmatrix}$$

$cond(K) = \|K\|\|K^{-1}\| = 220 \gg 1$ $Kötü\ Koşullu\ Problem$ (stabilite sıkıntılı!!!)

$$\tilde{\hat{\boldsymbol{y}}} = \hat{\boldsymbol{y}} + \widehat{\boldsymbol{\delta y}} = \begin{bmatrix} 24.118 \\ 13.234 \\ 9.4683 \\ 7.5594 \end{bmatrix} + \begin{bmatrix} 0.01 \\ -0.01 \\ 0.01 \\ -0.01 \end{bmatrix} \Longrightarrow$$

$$[\boldsymbol{a}'] = \begin{bmatrix} a_1' \\ a_2' \\ a_3' \\ a_4' \end{bmatrix} = \begin{bmatrix} 0.7408 \\ 2.6825 \\ -6.1538 \\ 4.455 \end{bmatrix}$$

$$\|\widehat{\boldsymbol{\delta y}}\| = 0.01 \qquad max \left\| \frac{\widehat{\boldsymbol{\delta y}}}{\hat{\boldsymbol{y}}} \right\| = \frac{0.01}{7.5594} = 0.001322$$

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0.55	1.102		
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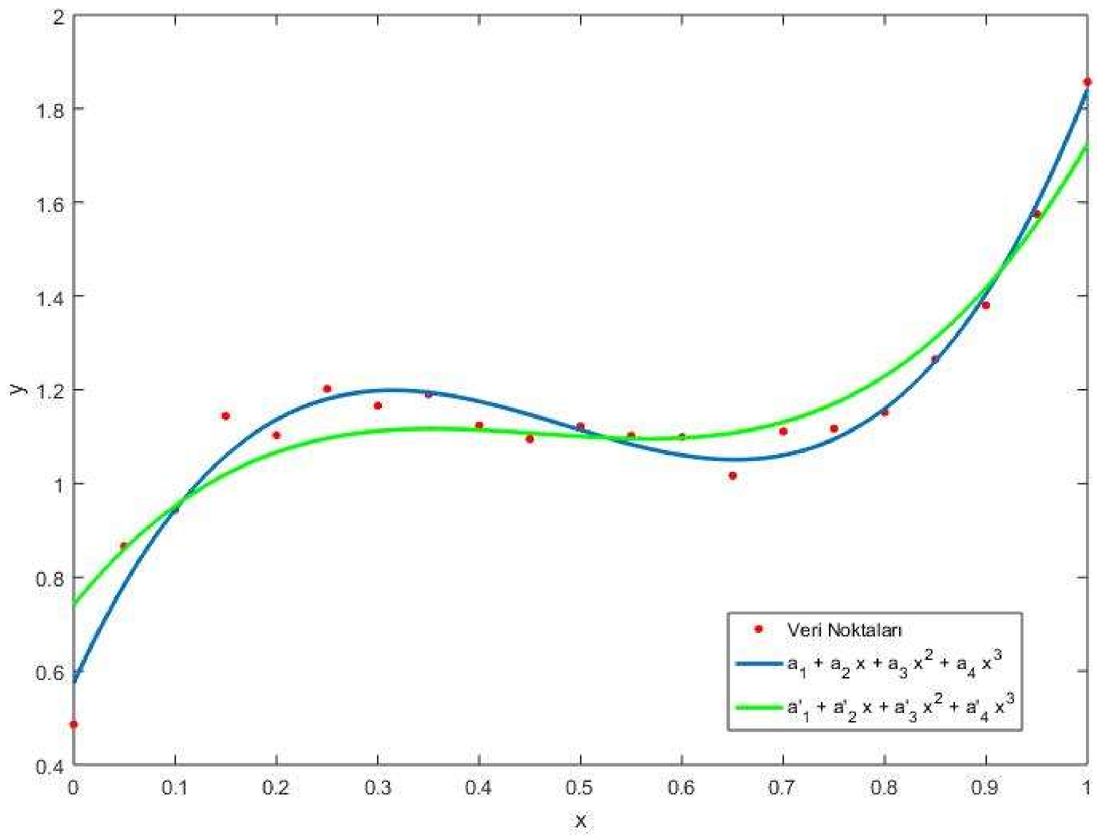
0.65	1.017		
0.70	1.111		
0.75	1.117		

$$\hat{f}(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$



En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting):

Yüksek Mertebe Eğri Uydurma: Genel HAL

$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ veri kümesi

$x_1, x_2, \dots x_n \in [\alpha, \beta]$

$\hat{f}(x) = a_1\varphi_1(x) + a_2\varphi_2(x) + \cdots + a_m\varphi_m(x)$

$\varphi_j(x); j = 1,2, \dots, m$: *Seçilmiş (Baz) Fonksiyonları*

$$\sum_{k=1}^{m} a_k \left[\sum_{j=1}^{n} \varphi_k(x_j)\varphi_i(x_j) \right] = \sum_{j=1}^{n} y_j \varphi_i(x_j) \;;\; i = 1,2, \dots, m$$

$a_j \,; j = 1,2, \dots, m$: *Bilinmeyen Katsayılar*

$$\varphi_j(x) = T_{j-1}\left(\frac{2x-\alpha-\beta}{\beta-\alpha}\right);\ \alpha \le x \le \beta; j = 1,2, \dots m; \quad \textit{Modifiye Chebyshev Polinomları} \Longrightarrow$$

$$\varphi_1(x) = T_0(2x-1) = 1$$

$$\varphi_2(x) = T_1(2x-1) = 2x-1$$

$$\varphi_3(x) = T_2(2x-1) = 2(2x-1)^2 - 1$$

$$\varphi_4(x) = T_3(2x-1) = 4(2x-1)^3 - 3(2x-1)$$

Yüksek Mertebe Eğri Uydurma: *Chebyshev Polinomları*

$$\hat{f}(x) = a_1\varphi_1(x) + a_2\varphi_2(x) + a_3\varphi_3(x) + a_4\varphi_4(x)$$

$$\varphi_1(x) = T_0(2x-1) = 1$$

$$\varphi_2(x) = T_1(2x-1) = 2x-1$$

$$\varphi_3(x) = T_2(2x-1) = 2(2x-1)^2-1$$

$$\varphi_4(x) = T_3(2x-1) = 4(2x-1)^3-3(2x-1)$$

$$[\boldsymbol{K}]_{4\times4}[\boldsymbol{a}]_{4\times1} = [\hat{\boldsymbol{y}}]_{4\times1} \Longrightarrow$$

$$[\boldsymbol{a}] = [\boldsymbol{K}]^{-1}[\hat{\boldsymbol{y}}] \Longrightarrow$$

$$\begin{bmatrix} 21 & 0 & -5.6 & 0 \\ 0 & 7.7 & 0 & -2.8336 \\ -5.6 & 0 & 10.4664 & 0 \\ 0 & -2.8336 & 0 & 11.0105 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 24.118 \\ 2.351 \\ -6.011 \\ 1.5235 \end{bmatrix} \Longrightarrow$$

$$[\boldsymbol{a}] = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1.1609 \\ 0.3935 \\ 0.0468 \\ 0.2396 \end{bmatrix}$$

$$cond(K) = \|K\|\|K^{-1}\| = 4.8 \approx 1 \ (well\ conditioned)$$

En Küçük Kareler/Eğri Uydurma (Least Squares Data Fitting):

Yüksek Mertebe Eğri Uydurma: *Chebyshev Polinomları*

$$\hat{f}(x) = a_1\varphi_1(x) + a_2\varphi_2(x) + a_3\varphi_3(x) + a_4\varphi_4(x)$$

$$\varphi_j(x) = T_{j-1}\left(\frac{2x-\alpha-\beta}{\beta-\alpha}\right);\ \alpha \le x \le \beta; j = 1,2,\dots m;$$

$$Modifiye\ Chebyshev\ Polinomları \Longrightarrow$$

$$\varphi_1(x) = T_0(2x-1) = 1$$

$$\varphi_2(x) = T_1(2x-1) = 2x-1$$

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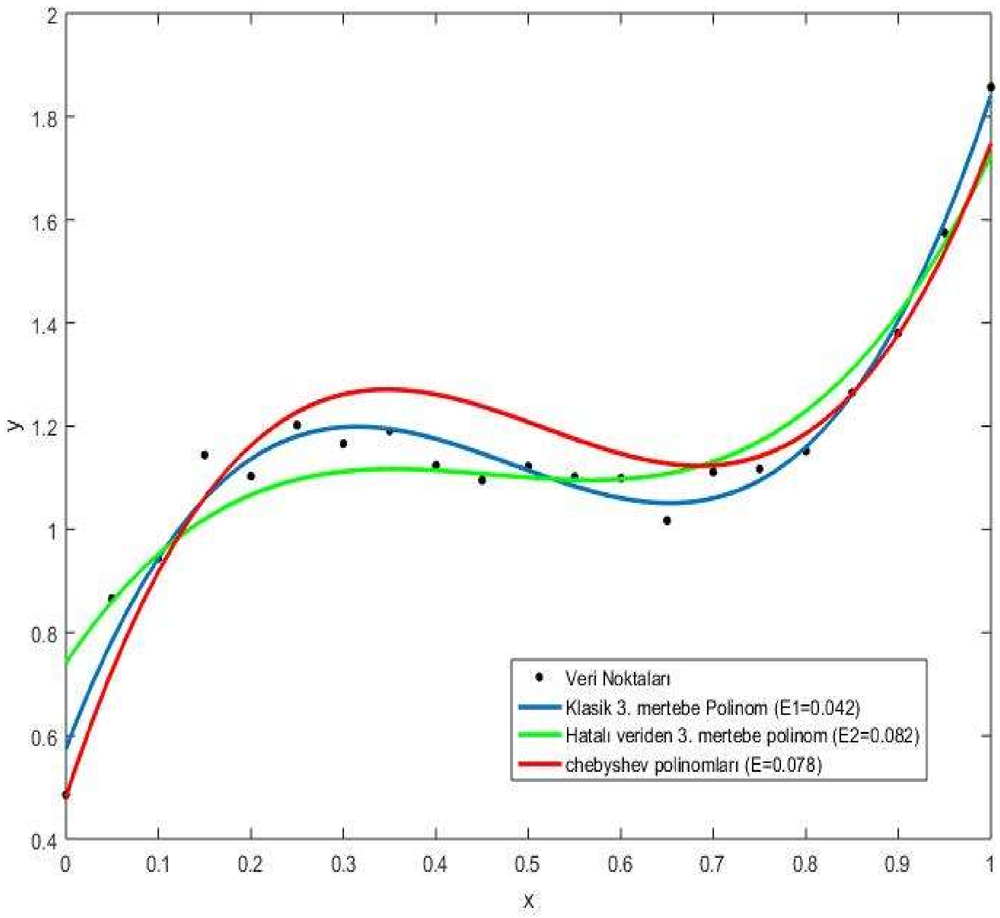
$$\varphi_4(x) = T_3(2x-1) = 4(2x-1)^3-3(2x-1)$$



$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[y_i - \hat{f}\,(x_i)]^2}$$