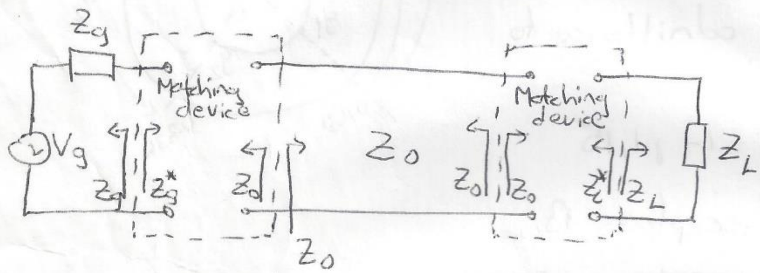
Impedance Matching

A line terminated in its characteristic impedance has a standing wave ratio of one and transmits a given power without reflection. In circuit theory, maximum power transfer requires the load impedance to be equal to the complex conjugate of the generator. We refer to this condition as a conjugate match. In transmission-line problems, matching simply means terminating the line in its characteristic impedance.

Usally, the input impedance to the load itself (for ex. an antenna) is not equal to the characteristic impedance (Z_0) of the connected transmission line. Furthermore, the output impedance of the transmitter may not be equal to the Z_0 of the line. Matching devices are necessary to match the line to the load and transmitter. A complete matched transmission-line system is shown below:



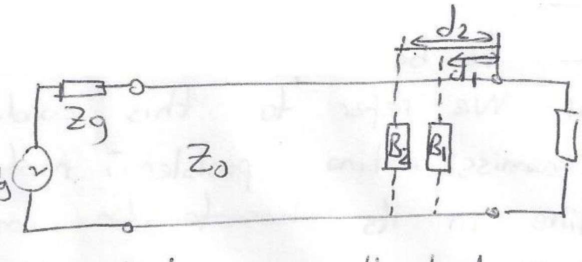
In an actual transmissio-line system, the trosmitter is ordinarily match to the coaxial cable for maximum power transfer. Because of the variable loads, however, an impedance-matching device is often required at the load side.

Matching involves parallel connections on the transmission line, so it is necessary to solve matching problems using admittances rather than impedances.

Double-Susceptance Matching:

Let's explain this technique with a numeric example:
A transmission line has the following parameters;

$Y_\ell = 0{,}004 - j\,0{,}004\ \mho$
$Z_0 = 100\ \Omega$
$s = 2{,}9$ and $f = 1\ GHz$

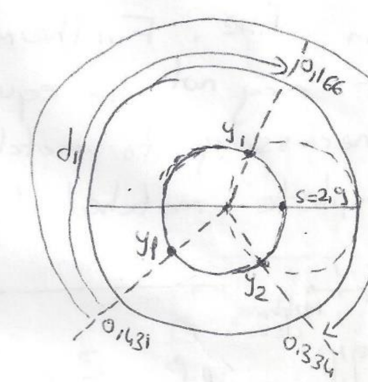


Determine the value and the distance from the load of an inductance or a capacitance that can match the transmission line.

- The normalized load admittance is,

$$y_\ell = Y_\ell Z_0 = 0{,}4 - j\,0{,}4$$

- Plot on SWR s circle on the chart.



- The normalized line admittance to be matched are,

$$y_1 = 1 + j\,1{,}15 \quad . \quad y_2 = 1 - j\,1{,}15$$

- For an inductive susceptance B_1,

$$Y_1 = -j b_1 Y_0 = -j(1{,}15)\frac{1}{100} = -jB_1\ \mho \quad , \quad B_1 = 0{,}0115 = 1/\omega L$$

and $L = \dfrac{1}{2\pi(10^9)(0{,}0115)} = 0{,}0138\ \mu H$

- For a capacitive susceptance B_2,

$$Y_2 = +jB_2 = +j\,b_2 Y_0 = +j(1{,}15)\frac{1}{100} = j\,0{,}0115\ \mho \Rightarrow B_2 = 0{,}0115 = \omega C$$

and $C = \dfrac{B_2}{\omega} = \dfrac{0{,}0115}{2\pi(10^9)} = 1{,}83\ pF$

- The distance to the tuner from the load based on B_1 is

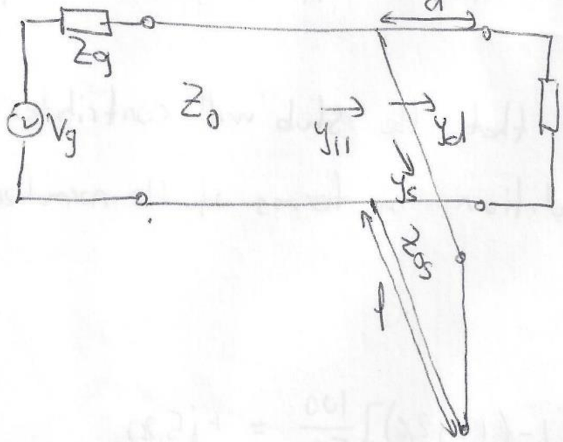
$$d_1 = [0{,}166 + (0{,}50 - 0{,}431)]\,\lambda = 0{,}231 \cdot 30 = 6{,}93\ cm$$

- The distance to the tuner from the load based on B_2 is

$$d_2 = [0{,}334 + (0{,}50 - 0{,}431)]\,\lambda = 0{,}403 \cdot 30 = 12{,}09\ cm\,.$$

Single Stub Matching

Short-circuited transmission lines are more commonly used as matching section becouse of their susceptive properties. Short-circuited sections are prefesoble to open-circuited ones, beccuse a good short circuit is easier to obtain than a good open circuit.



The stub must be located at a point on the line where the real part of the admittance, looking toward the load is Y_0. In the normalized unit, y_{11} must be in the form

$$y_{11} = y_d \pm y_s = 1$$

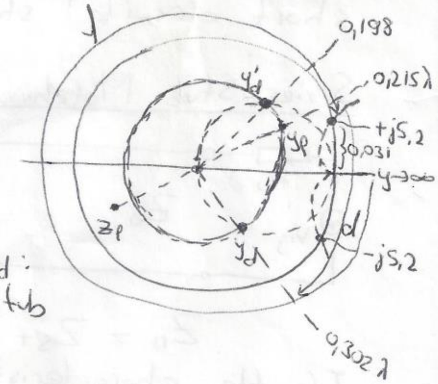
If the stub has the same Z_0 as that of the line. Otherwise, total admittances must bee used:

$$Y_{11} = Y_d \pm Y_s = Y_0$$

We then adjust the stub length so that its susceptence just cancels out the susceptance of the line at the junction.

Example :

A lossless line of characteristic impedance $R_0 = 50\Omega$ is to be matched to a load $Z_\ell = 5{,}58 - j10{,}41\ \Omega$ by means of a lossless short-circuited stub with $Z_{os} = 100\ \Omega$. Find the stub position (closest to the load) and length so that match is obtained.



- normalized load admittance is,

$$y_\ell = \frac{1}{z_\ell} = \frac{Z_0}{Z_\ell} = 2 + j\,3{,}732$$

- Draw an SWR circle through the point y_ℓ; the circle intersects the unit circle at y_d: and $y_d = 1 - j2.6$. This point permits the stub to be attached as closely as possible to the load.

Since the $Z_{os} \neq Z_0$, the condition for impedance matching requires,

$$Y_{11} = Y_d + Y_s$$

where Y_s is the susceptance that the stub will contribute.

We can rewrite this condition in terms of the normalized values :

$$y_{11} Y_0 = y_d Y_0 + y_s Y_{os}$$

and $\quad y_s = (y_{11} - y_d)\dfrac{Y_0}{Y_{os}} = [1 - (1 - j2{,}6)]\dfrac{100}{50} = +j5{,}20$

The distance between the load and the stub position is,

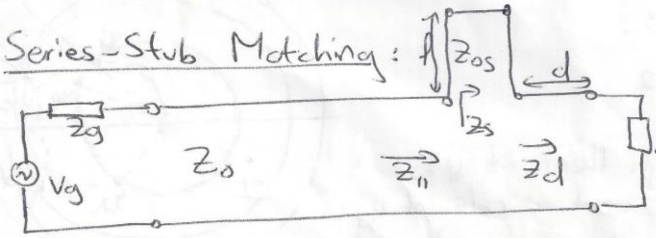
$$d = (0{,}302 - 0{,}215)\lambda = 0{,}087\lambda$$

Since the stub contributes a susceptance of $+j5{,}20$, enter $+j5{,}20$ on the chart and determine the required distance ℓ from the short-circuited end ($y = \infty$), by transversing the chart toward the generator until the point reaching $+j5{,}20$. Then,

$$\ell = (0{,}50 - 0{,}031)\lambda = 0{,}469\ \lambda.$$

If an inductive stub is required, $y_d' = 1 + j2{,}6$ and the susceptance of the stub will be, $y_s' = -j5{,}2$, the position of the stub from the load is $d' = [0{,}50 - (0{,}215 - 0{,}198)]\lambda = 0{,}483\ \lambda$, and the lengt of the short-circuited stub is $\ell' = 0{,}031\ \lambda$.

Series-Stub Matching:



For the series-stub matching problem, the impedance Z_{11} which is the sum of $Z_s + Z_d$ must be Z_o to obtain a matched section on the line:

$$Z_{11} = Z_s + Z_d = Z_o$$

If the characteristic impedance Z_{os} of the stub equals that of the line this equation becomes,

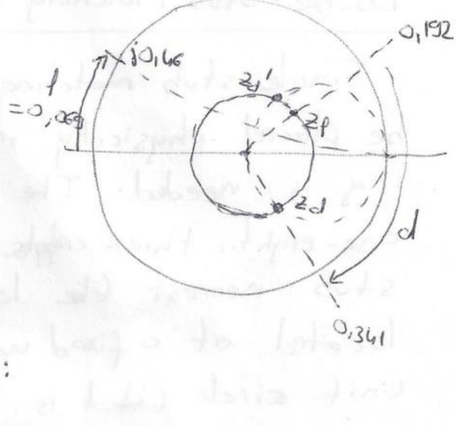
$$z_{11} = z_s + z_d$$

where z_{11}, z_s and z_d are the normalized impedances.

Example:

A single short-circuited stub with $Z_{os} = 200\,\Omega$ is to be placed in series with a line to match the load $Z_L = 150 + j100\,\Omega$ to the line with $R_o = 100\,\Omega$. Determine the length l of the tuner and its distance d from the load required to match the line.

The normalized load impedance z_l is,



$$z_l = 1{,}5 + j1$$

- Draw an SWR circle through z_l toward the generator; the s circle intersects the $r=1$ unit resistance circle at z_d. Read $z_d = 1 - j0{,}92$; $z_d' = 1 + j0{,}92$ is also a solution.
- Measure the distance d between z_l and z_d:

$$d = (0{,}314 - 0{,}192)\lambda = 0{,}14\,\lambda$$

which means that the stub can be placed in series with the line $0{,}14\,\lambda$ from the load.

The impedance Z_{11} can be calculated from, (note that $Z_{os} \neq Z_o$)

$$Z_{11} = Z_s + Z_d \quad \text{then} \quad z_{11} Z_o = z_s Z_{os} + z_d Z_o$$

$$z_s = (z_{11} - z_d)\frac{Z_o}{Z_{os}} = (1 - 1 + j0{,}92)\left(\frac{100}{200}\right) = j0{,}46$$

which indicates that the series stub must contribute $+j0{,}46$

The length of the stub is $l = 0{,}069\,\lambda$. (from the $z=0$ termination impedance for the short-circuited series stub.)

- Quarter Wave Transformer:

For the lossless line, we have $Z_1 \cdot Z_2 = Z_{oq}^2$, where Z_1 and Z_2 is the line impedances at the begining and the ~~end~~ receiving end of the quarter-length line and Z_{oq} is the characteristic impedance of the line. For the lossless line, Z_{oq} has a real number and Z_1 and Z_2 in general must have real number. If the load impedance Z_2 has real value then this transformer converts the Z_2 impedance to Z_1 impedance. In the matching problem Z_1 equals to Z_0 of the transmission line.

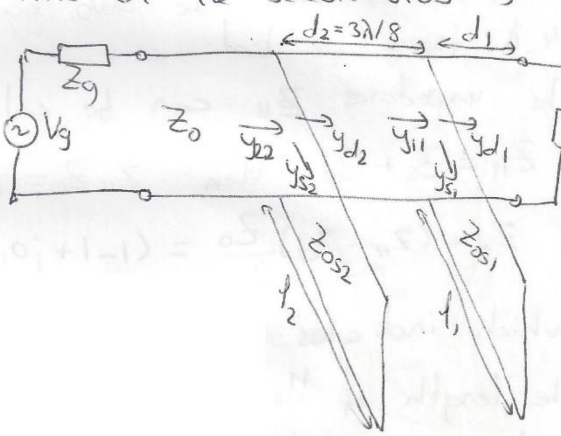
Double-Stub Matching:

Single-stub matching is impractical at times because the stub cannot be placed physically in the ideal location, and so double-stub matching is needed. The fixed distance between two stub is usually one-eight., three eights. or five-eights. of a wavelength. We use the stub nearest the load to adjust the susceptance. The stub is located at a fixed wavelength from the constant conductance unit circle (that is, $g=1$) on an appropriate constant SWR circle. The admittance of the line at the second stub is

$$y_{22} = y_{d2} \pm y_{s2} = 1$$
(if $Z_0 = Z_{os2}$)

or

$$Y_{22} = Y_{d2} \pm Y_{s2} = Y_0$$



Example:

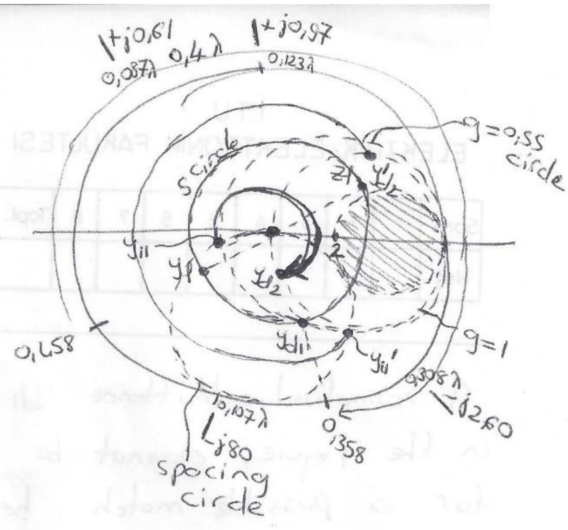
$Z_L = 200 + j200\,\Omega$, $Z_0 = 100\,\Omega = Z_{0s1} = Z_{0s2}$.

The first stub is $0{,}40\lambda$ (d_1) from the load.
The spacing between the two stubs is $3\lambda/8$. Determine the length of the short-circuited stubs when matching is achieved. What terminations are forbidden for matching the line by the double-stub device?

the normalized load impedance z_ℓ is $z_\ell = 2 + j2$

- Plot on SWR (s) circle and read the normalized load admittance 180° out of phase with z_ℓ on the SWR circle:

$$y_\ell = 0{,}25 - j0{,}25$$



- Draw the spacing circle of $\frac{3}{8}\lambda$ by rotating the constant conductance unit circle ($g=1$) through a distance of $2\beta d = 2\beta\frac{3}{8}\lambda = \frac{3}{2}\pi$ toward the load. Point y_{11} must be on this spacing circle, since y_{d2} will be on the $g=1$ circle (y_{11} and y_{d2} are $3/8\ \lambda$ apart).
- Move y_ℓ for a distance of $0{,}40\lambda$ from $0{,}458$ to $0{,}358$ along the SWR circle toward the generator and read y_{d1} from the chart: $y_{d1} = 0{,}55 - j1{,}08$

y_{d1} must be on both SWR circle and real circle of the y_{11} because real parts of y_{11} and y_{d1} are the same.

- There are two possible solutions for y_{11}. They can be found by carrying y_{d1} along the constant conductance $g=0{,}55$ circle, which intersects the spacing circle at two points: (y_{11} must be on both $g=0{,}55$ circle and spacing circle, because the real parts of the y_{d1} and y_{11} are the same (the difference comes from the pure imaginary y_{s1}), and after rotating the distance of $3/8\ \lambda$ toward the generator from y_{11}, the admittance must be reached the appropriate points on the unit conductance circle ($g=1$) since y_{d2} has the real part of $g=1$). Two solutions for y_{11} are:

$$y_{11} = 0{,}55 - j0{,}11 \quad \text{and} \quad y_{11}' = 0{,}55 - j1{,}88$$

- Since $y_{i1} = y_{d1} + y_{s1}$ then $y_{s1} = y_{i1} - y_{d1} = (0{,}55 - j0{,}11) - (0{,}55 - j1{,}08)$
$= +j0{,}97$

Similarly, $y_{s1}' = -j0{,}80$

- The length of stub 1 are $l_1 = (0{,}25 + 0{,}123)\lambda = 0{,}373\,\lambda$ and,
$l_1' = (0{,}25 - 0{,}107)\lambda = 0{,}143\,\lambda$

- The $\frac{3}{8}\lambda$ section of line transform y_{i1} to y_{d2} and y_{i1}' to y_{d2}' along their constant SWR circles, respectively. That is,

$$y_{d2} = 1 - j0{,}61 \quad , \quad y_{d2}' = 1 + j2{,}60$$

- The stub 2 must contribute $y_{s2} = +j0{,}61$ and $y_{s2}' = -j2{,}60$
- The lengths of stub 2 are $l_2 = (0{,}25 + 0{,}087)\lambda = 0{,}337\lambda$, $l_2' = (0{,}308 - 0{,}25)\lambda = 0{,}058\,\lambda$

A normalized admittance y_{d1} located inside the shaded area, as shown in the figure, cannot be brought to lie on the locus of y_{i1} or y_{i1}' for a possible match, because the spacing circle and the $g=2$ circle are mutally tangent. Thus the area of a $g=2$ circle is called the forbidden region of the normalized load admittance.