MICROWAVE ENGINEERING

LECTURE NOTES

Transmission Lines

In ordinary circuit theory, we assume that all circuit elements are constants. But at high frequencies, some parameters of the transmission line (such as inductances and capacitances of short parts of line) are distributed along the line and we must take into account the effects of these parameters. These effects especially important when the wavelength is short in comparison with the physical dimensions of the line.

Basic transmission-line theory is derived from distributed circuit concepts, based on traveling-wave theory, so we can apply this theory to all types of lines.

We can classify the transmission lines according to their physical characteristics or functional properties.

We can classify the transmission lines according to their physical characteristics or functional properties.

* In terms of physical characteristics:

- <u>Two-wire transmission line:</u> consists of two parallel conducting wires such as power lines and telephone lines. This line is not often used at frequency above a few hundred MHz since the radiation losses become excessive at higher frequencies.

- <u>Coaxial transmission line:</u> Consists of an inner conductor and a concentric outer conducting sheath separated by a dielectric medium (such as TV cables). Since the fields are entirely confined within the dielectric region, radiation loss is very low.

- <u>Stripline:</u> Consists of two parallel conducting strips separated by a dielectric slab (such as microtrip line). This line is commonly used in microwave integrated circuits (MICs).

- <u>Digital transmission line:</u> Consists of an inner core of higher refractive index surrounded by an outer cover of lower refractive index (such as fiber). Radiation loss is extremely low and major application includes the long-distance links.

In terms of functional properties:

- **Uniform line**: All parameters of the line are considered to be uniformly distributed.
- **Linear line**: All parameters of the line are assumed to be independent of signal level.
- **Lossless line**: The series resistance and shunt conductance of the line are taken to be zero.
- **Lossy line**: These parameters are not zero.
- **Distortionless line**: The ratio of the series resistance of the line to its series inductance equals the ratio of the shunt conductance of the line to its shunt capacitance.

Transmission-Line Equations

We can analyze a transmission line either by solving Maxwell's field equations, which involves three space variables and time variable, or by using the method of distributed circuit theory which involves only one space and time variable.

We use, here, the second method in terms of the voltage, current and impedance along the line in both the time and frequency domains.

Basic Parameters of Transmission-Line Equations.

In distributed circuit theory, we assume that each incremental length of a transmission line has its own circuit parameters (R, L, G, C). The basic parameters are:

R : series resistance (Ω / ℓ) (Ohm per unit length)
L : " inductance (H/ℓ)
G : shunt conductance ($\mho / \ell$)
C : " capacitance (F/ℓ)

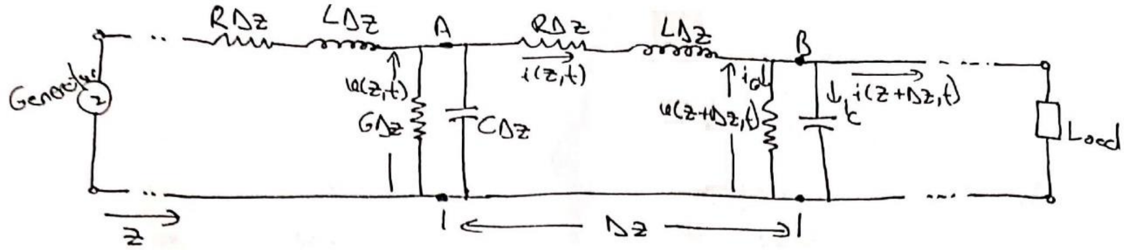
Although these parameters are frequency-dependent (σ, μ, ε are functions of frequency), they are determined basically by the physical configuration of the line:

Parameter	Two-wire line	Coaxial Line	Parallel Stripline	Unit
R	$R_s / \pi a$	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{w}$	Ω/m
L	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu}{2\pi}\ln(b/a)$	$\mu d/w$	H/m
G	$\frac{\pi\sigma}{\cosh^{-1}(D/2a)}$	$\frac{2\pi\sigma}{\ln(b/a)}$	$\sigma w/d$	$\mho/m$
C	$\frac{\pi\varepsilon}{\cosh^{-1}(D/2a)}$	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\varepsilon w/d$	F/m
Z_0	$\sqrt{\frac{R+j\omega L}{G+j\omega C}}$	$\frac{60}{\sqrt{\varepsilon_r}}\ln(b/a)$	$\frac{d}{w}\sqrt{\frac{\mu}{\varepsilon}}$	Ω

$R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$: surface resistance of a conductor
μ_c : permeability of a conductor (H/m)
σ_c : conductivity " " " ($\mho$/m)
D : distance between the two wires (m)
a : radius of the wire or center conductor (m)
b : " " " outer hollow " (m)
w : width of a stripline (m)
d : separation distance of a stripline (m)

Transmission-Line Equations in the Time Domain :

The electrical equivalent of a physical two-wire transmission line is shown below :



$u(z,t)$ and $i(z,t)$ are the instantaneous voltage and current along the line and we can express these functions as,

$$u(z,t) = V(z)\,u(t) \quad \text{and} \quad i(z,t) = I(z)\,i(t)$$

V and I are complex quantities of the sinusoids (called phasors):

$V(z) = V e^{-\gamma z}$, $I(z) = I e^{-\gamma z}$, $\gamma = \alpha + j\beta$: propagation constant

α : attenuation constant (Np/m)

β : phase ,, (rad/m)

$$\frac{\partial u(z,t)}{\partial z} = \lim_{\Delta z \to 0} \frac{u(z+\Delta z,t) - u(z,t)}{\Delta z}$$

According to the Kirchhoff's voltage law

$$u(z,t) = i(z,t)\,R\Delta z + L\Delta z\,\frac{\partial i(z,t)}{\partial t} + \overbrace{u(z,t) + \frac{\partial u(z,t)}{\partial z}\Delta z}^{u(z+\Delta z,t)}$$

dividing the equation by Δz and omitting the argument (z,t), we obtain

$$-\frac{\partial u}{\partial z} = Ri + L\frac{\partial i}{\partial t}$$

Using Kirchhoff's current law at point B,

$$i(z,t) = \left[u(z,t) + \frac{\partial u(z,t)}{\partial z}\Delta z\right] G\Delta z + C\Delta z\,\frac{\partial}{\partial t}\left[u(z,t) + \frac{\partial u(z,t)}{\partial z}\Delta z\right] + \underbrace{i(z,t) + \frac{\partial i(z,t)}{\partial z}\Delta z}_{i(z+\Delta z,t)}$$

dividing this equation by Δz and letting $\Delta z = 0$, we have

$$-\frac{\partial i}{\partial z} = Gu + C\frac{\partial u}{\partial t}$$

Then by differentiating the first equation with respect to 'z' and the second equation with respect to 't' and combining the results, we obtain,

$$\frac{\partial^2 v}{\partial z^2} = RGv + (RC+LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2}$$ ← includes only voltage term

Also by differentiating the first eq. to 't' and the other eq. to 'z'

$$\frac{\partial^2 i}{\partial z^2} = RGi + (RC+LG)\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2}$$ ← includes only current term

These two equations are the general wave equations of the voltage and current on a uniform lossy line and they are known as "the telegrapher's equations"

For the lossless case ($R=G=0$) travelling wave equations reduce to:

$$\frac{\partial^2 v}{\partial z^2} = LC\frac{\partial^2 v}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 i}{\partial z^2} = LC\frac{\partial^2 i}{\partial t^2}$$

Transmission-Line Equation in the Frequency Domain

Remember that $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$ (Euler's formula) and $E\cos(\omega t) = Re[E e^{j\omega t}] = e$. It is a common practice to omit the symbol Re and simply to write $e = Ee^{j\omega t}$, then,

$$\frac{\partial e}{\partial t} = E j\omega e^{j\omega t} = j\omega e.$$

This means that for a function of $e^{j\omega t}$, $\partial/\partial t$ is equivalent to $j\omega$, and similarly for $e^{j\beta z}$, $\partial/\partial z$ is equivalent to $j\beta$. By substituting $j\omega$ for $\partial/\partial t$ in the first and second order diff. equations given above, and dividing each eq. by $e^{j\omega t}$ we obtain the equations in phasor form of the frequency domain:

$$\frac{dV}{dz} = -ZI, \quad \frac{dI}{dz} = -YV, \quad \frac{d^2V}{dz^2} = \gamma^2 V \text{ and } \frac{d^2I}{dz^2} = \gamma^2 I$$

in which the following substitutions were made:

$Z = R + j\omega L$, $Y = G + j\omega C$ and $\gamma = \sqrt{ZY} = \alpha + j\beta$

For a lossless line ($R = G = 0$), these equations reduce to,

$$\frac{dV}{dz} = -j\omega LI, \quad \frac{dI}{dz} = -j\omega CV, \quad \frac{d^2V}{dz^2} = -\omega^2 LCV \text{ and } \frac{d^2I}{dz^2} = -\omega^2 LCI$$