Ibrahim Yıldırım

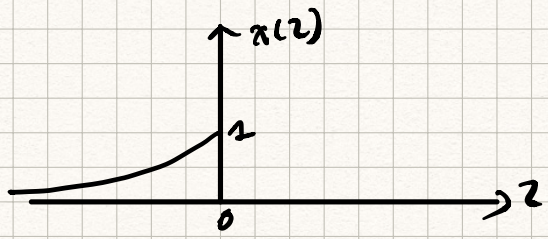
EHB 206 - Uygulama 2

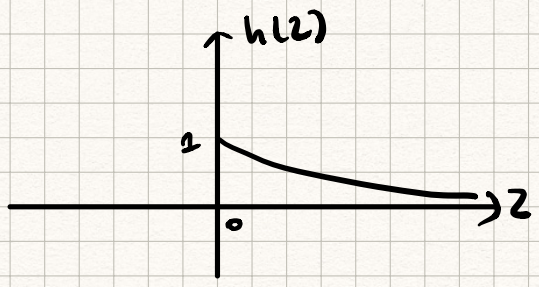
1) $h(t)$ dürtü yanıtına sahip DZD bir sistemin girişine $x(t)$ uygulanmaktadır. Bu girişe karşı düşen $y(t)$ çıkışını bulunuz.

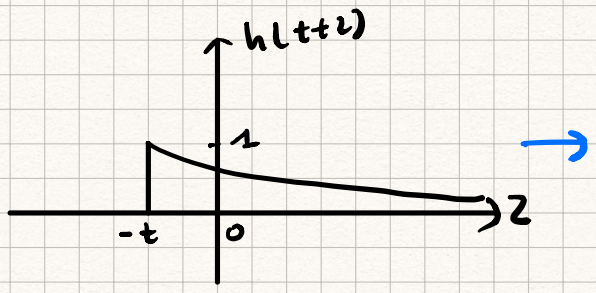
$$h(t) = e^{at} \cdot u(t),$$

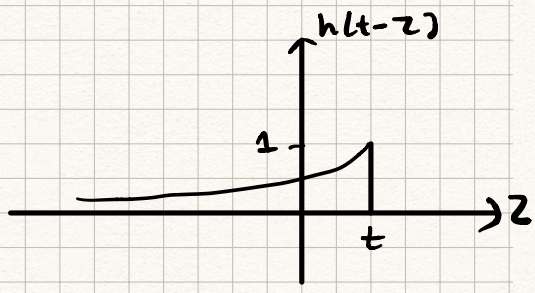
$$x(t) = e^{at} \cdot u(-t), \quad a > 0.$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(z)\, h(t-z)\, dz$$

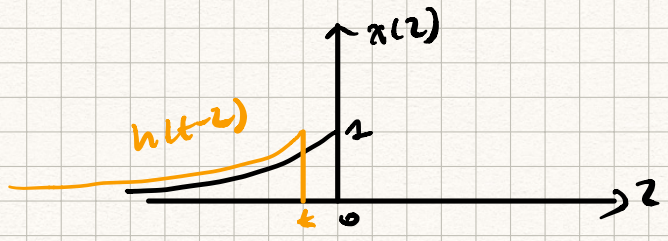








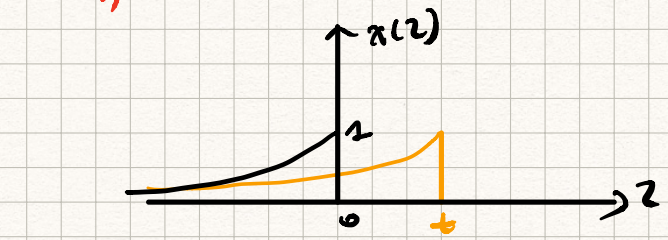
(*) $t<0$



$$y(t)=\int_{-\infty}^{t} e^{az}e^{-a(t-z)}\,dz = e^{-at}\int_{-\infty}^{t} e^{2az}\,dz = \frac{e^{-at}}{2a}\left(e^{2az}\Big|_{-\infty}^{t}\right)$$

$$= \frac{e^{-at}}{2a}\left(e^{2at}-0\right) = \frac{e^{at}}{2a}$$

(*) $t\geq 0$

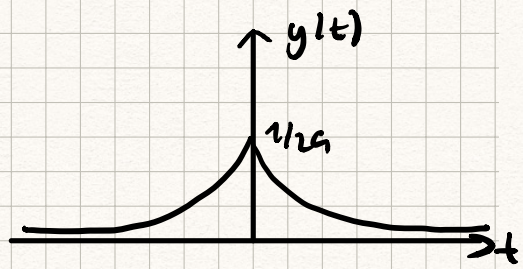


$$y(t) = \int_{-\infty}^{0} e^{az}e^{-a(t-z)}\,dz = e^{-at}\int_{-\infty}^{0} e^{2az}\cdot dz$$

$$= \frac{e^{-at}}{2a}\left(1-0\right) = \frac{e^{-at}}{2a}$$

(*)

$$y(t) = \begin{cases} e^{at}/2a\,, & t<0 \\ e^{-at}/2a\,, & t\geq 0 \end{cases}$$



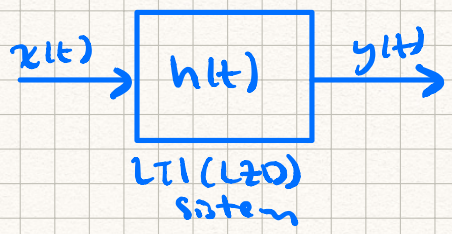
(2)

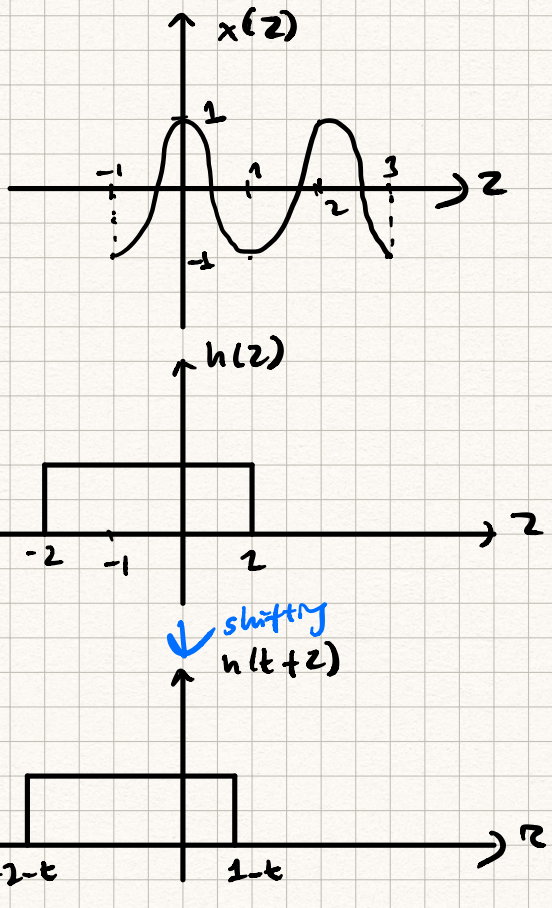
$x(t) = \cos(\pi t)[u(t+1) - u(t-3)]$ ve

$h(t) = u(t+2) - u(t-1)$ olmak üzere,

$y(t) = x(t) * h(t)$'yi bulunuz.

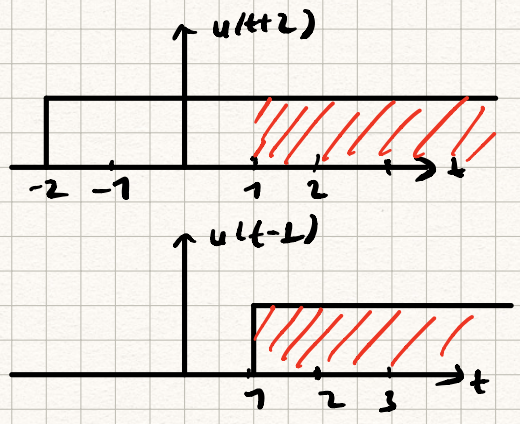
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$



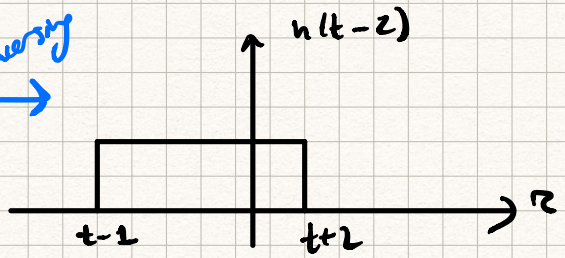


$\cos(\pi t) = \cos(2\pi f_0 t)$

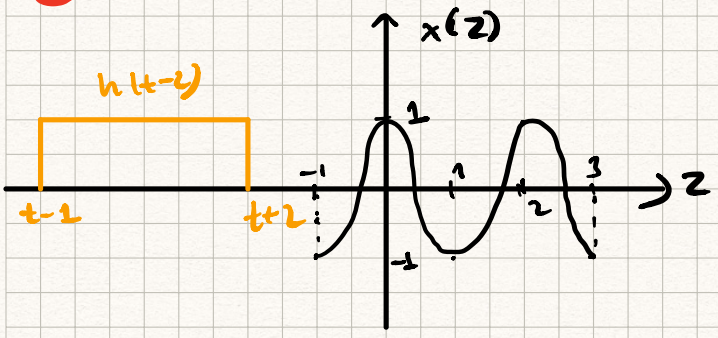
$f_0 = 1/2$, $T_0 = \frac{1}{f_0} = 2$



reversing

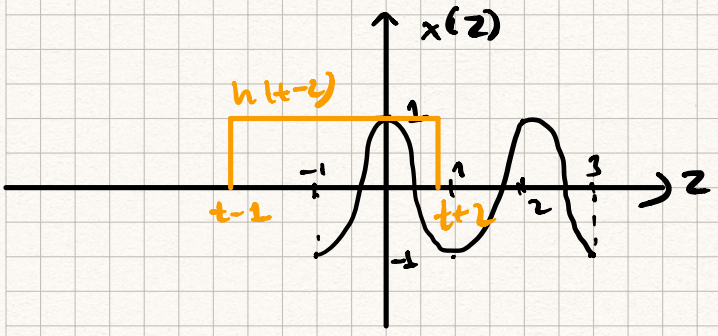


① $t+2 < -1 \Rightarrow t < -3$



$$y(t) = 0$$

② $t+2 \geq -1$, $t-1 < -1 \Rightarrow -3 \leq t < 0$



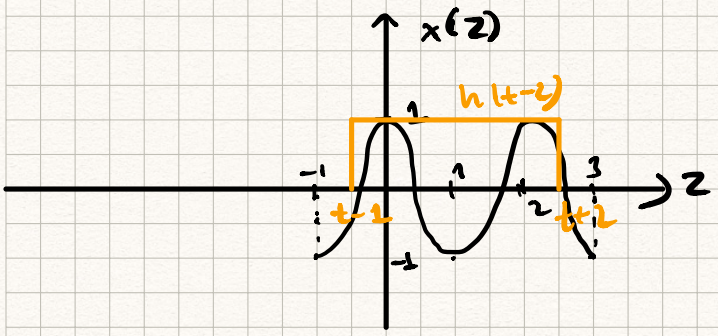
$$y(t) = \int_{-1}^{t+2} \cos(\pi z) \cdot 1 \, dz$$

$$= \frac{1}{\pi} \sin \pi z \Big|_{-1}^{t+2}$$

$$= \frac{1}{\pi} \sin \pi (t+2) - \frac{1}{\pi} \sin(-\pi)$$

$$= \frac{1}{\pi} \sin(\pi t), \quad -3 \leq t < 0$$

③ $t-1 \geq -1$, $t+2 < 3 \Rightarrow 0 \leq t < 1$

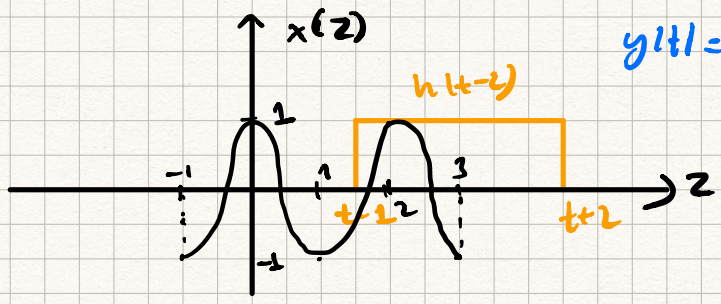


$$y(t) = \int_{t-1}^{t+2} \cos(\pi z) \, dz$$

$$= \frac{1}{\pi} \sin(\pi t + 2\pi) - \frac{1}{\pi} \sin(\pi t - \pi)$$

$$= \frac{1}{\pi} \sin(\pi t) + \frac{1}{\pi} \sin(\pi t) = \frac{2}{\pi} \sin \pi t$$

④ $t+2 \geq 3$, $t-1 < 3 \Rightarrow 1 \leq t < 4$



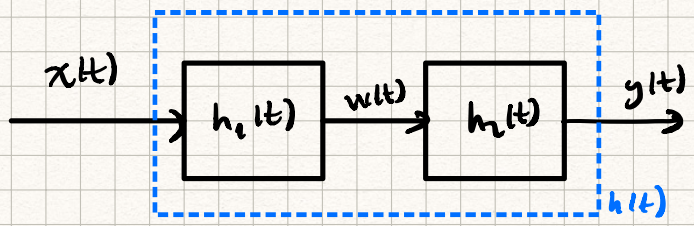
$$y(t) = \int_{t-1}^{3} \cos(\pi z) \, dz$$

$$= \frac{1}{\pi} \sin(3\pi) - \frac{1}{\pi} \sin(\pi t - \pi)$$

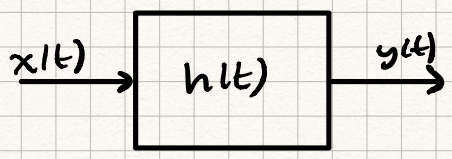
$$= \frac{1}{\pi} \sin(\pi t)$$

$$y(t) = \begin{cases} 0 & , \ t<-3 \text{ ve } t\geq 4 \\ \frac{1}{\pi}\sin(\pi t) & , \ -3\leq t<0 \text{ ve } 1\leq t<4 \\ \frac{2}{\pi}\sin(\pi t) & , \ 0\leq t<1 \end{cases}$$

(3)



Yukarıdaki kaskat DZD sistem için $h_1(t) = e^{-2t}u(t)$ ve $h_2(t) = 2e^{-t}u(t)$ olacak şekilde verilmektedir. Tüm sistemin dürtü yanıtı $h(t)$'yi bulunuz.



$$\left.\begin{aligned} w(t) &= x(t) * h_1(t) \\ y(t) &= w(t) * h_2(t) \end{aligned}\right\} \quad y(t) = [x(t) * h_1(t)] * h_2(t)$$

$$y(t) = x(t) * \underbrace{[h_1(t) * h_2(t)]}_{h(t)}$$

Tüm sistemin dürtü yanıtı:

$$h(t) = h_1(t) * h_2(t)$$

$$h(t) = \int_{-\infty}^{\infty} h_1(\tau)\, h_2(t-\tau)\, d\tau = \int_{-\infty}^{\infty} e^{-2\tau}u(\tau)\, 2e^{-(t-\tau)}u(t-\tau)\, d\tau$$

$$= 2\left(e^{-t} - e^{2t}\right)u(t)$$