Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.

a) $x[n] = \sin\left(\frac{6\pi}{7}n + 1\right)$ b) $x[n] = \cos\left(\frac{n}{8} - \pi\right)$

Solution

a) If the signal is periodic, then

$$x[n] = x[n + mN]$$

where N is period and m, N are integer.

$$x[n+mN] = \sin\left(\frac{6\pi}{7}(n+mN)+1\right) = \sin\left(\frac{6\pi}{7}n + \frac{6\pi mN}{7} + 1\right)$$

The signal is periodic if $\frac{6\pi mN}{7} = 2\pi k$

When $m=1$ and $k=3$ (makes N smallest integer) $N=7$.

→ The signal is periodic with fundamental period $N=7$

b) $x[n] = x[n+mN]$

$$x[n+mN] = \cos\left(\frac{n+mN}{8} - \pi\right) = \cos\left(\frac{n}{8} + \frac{mN}{8} - \pi\right)$$

The signal is periodic if $\frac{mN}{8} = 2\pi k$

There is no integer m and k values that can make N an integer.

→ The signal is NOT periodic

Determine whether each of the following system is

i) linear ii) time-invariant iii) causal

iv) memoryless v) bounded-input bounded-output stable

a) $y[n] = x[-n]$

b) $y[n] = e^{x[n]}$

Solution

a) i)

$$x_1[n] \longrightarrow \boxed{\text{System}} \longrightarrow y_1[n] = x_1[-n]$$

$$x_2[n] \longrightarrow \boxed{\text{System}} \longrightarrow y_2[n] = x_2[-n]$$

$$x_3[n] = ax_1[n] + bx_2[n] \longrightarrow \boxed{\text{System}} \longrightarrow y_3[n] \stackrel{?}{=} ay_1[n] + by_2[n]$$

$$y_3[n] = x_3[-n] = a\underbrace{x_1[-n]}_{y_1[n]} + b\underbrace{x_2[-n]}_{y_2[n]}$$

$$y_3[n] = ay_1[n] + by_2[n]$$

This system is linear.

ii)

$$x_1[n] \longrightarrow \boxed{\text{System}} \longrightarrow y_1[n] = x_1[-n]$$

$$x_2[n] = x_1[n-n_0] \longrightarrow \boxed{\text{System}} \longrightarrow y_2[n] \stackrel{?}{=} y_1[n-n_0]$$

$$\left.\begin{array}{l} y_2[n] = x_2[-n] = x_1[-n-n_0] \\ y_1[n-n_0] = x_1[-(n-n_0)] = x_1[-n+n_0] \end{array}\right\} y_2[n] \neq y_1[n-n_0]$$

This system is time-variant.

iii) The value of $y[n]$ at $n=-1$ depends on $x[n]$ at $n=1$ ($y[-1] = x[1]$). $y[n]$ depends on future values of $x[n]$, so it is noncausal.

iv) It is not memoryless because $y[n]$ depends on a value of $x[\cdot]$ other than nth value.

v) $|x[n]| \le B < \infty \qquad |y[n]| = |x[-n]| \le B < \infty$

It is BIBO stable.

$$x_1[n] \rightarrow \boxed{\text{System}} \rightarrow y_1[n] = e^{x_1[n]}$$
$$x_2[n] \rightarrow \boxed{\text{System}} \rightarrow y_2[n] = e^{x_2[n]}$$

$$x_3[n] = a x_1[n] + b x_2[n] \rightarrow \boxed{\text{System}} \rightarrow y_3[n] = e^{x_3[n]} \stackrel{?}{=} a y_1[n] + b y_2[n]$$

$$y_3[n] = e^{(a x_1[n] + b x_2[n])} = e^{a x_1[n]} e^{b x_2[n]}$$
$$\neq a e^{x_1[n]} + b e^{x_2[n]}$$

The system is not linear.

$$x_1[n] \rightarrow \boxed{\text{System}} \rightarrow y_1[n] = e^{x_1[n]}$$

$$x_2[n] = x_1[n-n_0] \rightarrow \boxed{\text{System}} \rightarrow y_2[n] \stackrel{?}{=} y_1[n-n_0]$$

$$\left.\begin{aligned} y_2[n] &= e^{x_2[n]} = e^{x_1[n-n_0]} \\ y_1[n-n_0] &= e^{x_1[n-n_0]} \end{aligned}\right\} y_2[n] = y_1[n-n_0]$$

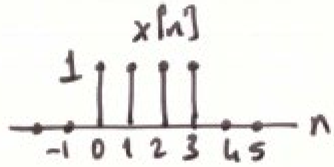
The system is time-invariant.

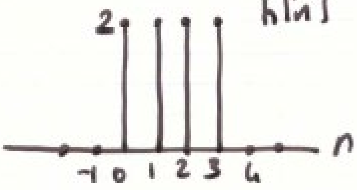
$y[n]$ depends on the n^{th} value of x only, so it is memoryless. Because of the system is memoryless, it is also causal.

$|x[n]| < B < \infty \qquad |y[n]| = |e^{x[n]}| = e^{|x[n]|} \leq e^{B} < \infty$

The system is BIBO stable.

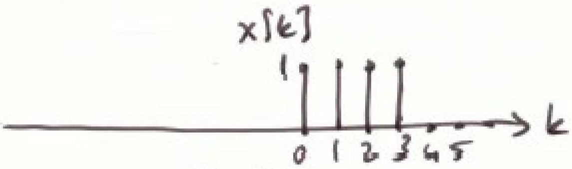
Determine the discrete-time convolution of $x[n]$ and $h[n]$ for following case.

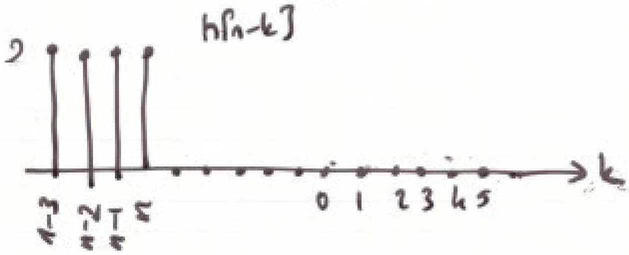




Solution

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$





$y[-1]=0 \quad y[0]=2 \quad y[1]=2+2=4 \quad y[2]=2+2+2=6$
$y[3]=2+2+2+2=8 \quad y[4]=2+2+2=6 \quad y[5]=2+2=4$
$y[6]=2 \quad y[7]=0$