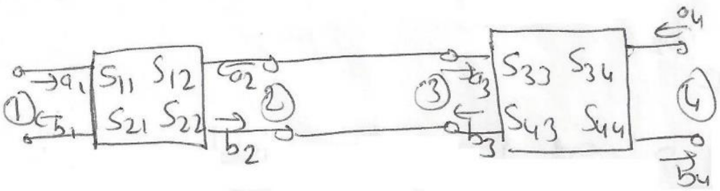
Cascaded Networks.

A microwave system does not consist of one two-port network. More generally one two-port will be followed by another two port connected in cascade, usually with a length of waveguide between them. The overall parameters of the system will then be required. The scattering coefficients of the two networks are known. The overall scattering coefficients, S_{11}', $S_{12}' = S_{21}'$, S_{22}' are to be found.



The formal equations for each 2-port are,

$b_1 = S_{11} a_1 + S_{12} a_2$ $\qquad$ $b_3 = S_{33} a_3 + S_{34} a_4$

$b_2 = S_{21} a_1 + S_{22} a_2$ $\qquad$ $b_4 = S_{43} a_3 + S_{44} a_4$

and for the overall system

$$b_1 = S_{11}' a_1 + S_{12}' a_4$$
$$b_4 = S_{21}' a_1 + S_{22}' a_4$$

Since ports 2 and 3 are connected together we have,

$$a_3 = b_2 e^{-j\beta l}$$
and $$a_2 = b_3 e^{-j\beta l}$$

β is the phase change coefficient of the connecting transmission line.

S_{11}' and S_{21}' are found by terminating port 4 by its characteristic impedance, so that $a_4 = 0$: then $S_{11}' = b_1/a_1$ and $S_{21}' = b_4/a_1$.

for S_{21}',

$$b_4 = S_{43} a_3 = S_{43} b_2 e^{-j\beta l} = S_{43}[S_{21} a_1 + S_{22} a_2] e^{-j\beta l}$$

$$= S_{43} S_{21} a_1 e^{-j\beta l} + S_{43} S_{22} a_2 e^{-j\beta l}$$

$$= S_{43} S_{21} a_1 e^{-j\beta l} + S_{43} S_{22} e^{-j2\beta l} b_3$$

$$= \quad '' \quad + \underline{S_{43}} S_{22} S_{33} e^{-j2\beta l} a_3 \qquad (\underset{a_4=0}{S_{43} a_3 = b_4},)$$

$$= \quad '' \quad + b_4 S_{22} S_{33} e^{-j2\beta l}$$

Then, $$S_{21}' = S_{12}' = \frac{S_{21} S_{43} e^{-j\beta l}}{1 - S_{22} S_{33} e^{-j2\beta l}}$$ and $$S_{11}' = S_{11} + \frac{S_{21} S_{12} S_{33} e^{-j2\beta l}}{1 - S_{22} S_{33} e^{-j2\beta l}}$$

Similarly S_{22}' is found by terminating port 1 by its characteristic impedance, so that $a_1 = 0$, when,

$$S_{22}' = \frac{b_4}{a_4} = S_{44} + \frac{S_{43} S_{34} S_{22} e^{-j2\beta l}}{1 - S_{22} S_{33} e^{-j2\beta l}}$$

By repeated application of this method the overall scattering coefficients for any number of cascaded two-port networks are obtained.

Scattering Transfer Parameters

In dealing with circuits in cascode, the scattering formalism in not the best description of the network. To overcome this difficulty, scattering transfer parameters are defined. This new matrix is obtained by rearranging the scattering relations so that the input waves a_1 and b_1 are the dependent variables and the output waves a_2 and b_2 are the independent ones. In the original S matrix, the backward waves b_1 and b_2 are

dependent variables and a_1 and a_2 are the independent variables. This new matrix is known as the T matrix. The standard S matrix relates a_1, a_2 to b_1, b_2 by

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Rearranging this matrix so that a_1 and b_1 are the dependent variables gives,

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

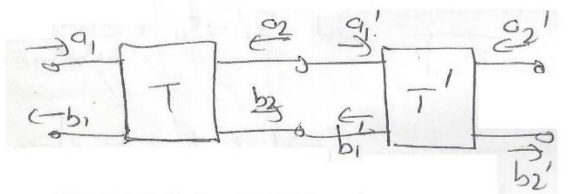
where

$$T_{11} = S_{12} - \frac{S_{11} S_{22}}{S_{21}}$$

$$T_{12} = S_{11} / S_{21}$$

$$T_{21} = - S_{22} / S_{21}$$

$$T_{22} = 1 / S_{21}$$



For this figure, the transfer matrices for the two network are

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} b_1' \\ a_1' \end{bmatrix} = \begin{bmatrix} T_{11}' & T_{12}' \\ T_{21}' & T_{22} \end{bmatrix} \begin{bmatrix} a_2' \\ b_2' \end{bmatrix}$$

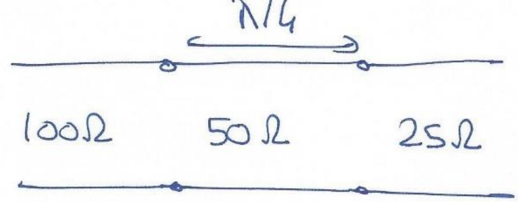
Using $\begin{bmatrix} b_1' \\ a_1' \end{bmatrix} = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$

gives

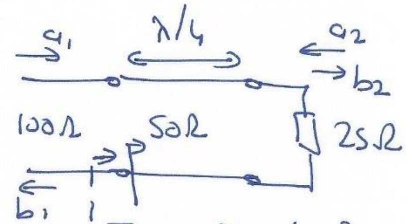
$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} T_{11}' & T_{12}' \\ T_{21}' & T_{22} \end{bmatrix} \begin{bmatrix} a_2' \\ b_2' \end{bmatrix}$$

Taking the ratio of b_1 / a_1 gives S_{11} for the overall network. Since matrix multiplication is not commutative, these T matrices must be multiplied in the proper order.

Ex.



$$S_{11} = \frac{b_1}{a_1}\bigg|_{a_2=0} \quad - - \rightarrow$$



$Z_1 = 100\Omega \Leftarrow (50^2 = Z_1 \cdot 25)$

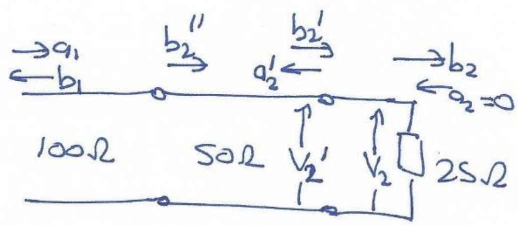
$\Gamma_1 = \frac{100-100}{100+100} = 0$

$$b_1 = \Gamma_1 a_1$$

$$\underline{\underline{S_{11}}} = \Gamma_1 = \underline{\underline{0}}$$

similarly $\underline{\underline{S_{22} = 0}}$

$$S_{21} = \frac{b_2}{a_1}\bigg|_{a_2=0} \quad - - \rightarrow$$



$$V_2' = V_2 \Rightarrow \overset{0\,(a_2=0)}{V_{a_2}} + V_{b_2} = V_{a_2'} + V_{b_2'}$$

$$b_2\sqrt{25} = (b_2' + a_2')\sqrt{50}$$

$$= b_2'\,(1+\Gamma_2')\sqrt{50}$$

$$= b_2'\,\frac{2}{3}\sqrt{50}$$

$a_2' = \Gamma_2' b_2'$

$\Gamma_2' = \frac{25-50}{25+50} = -1/3$

$$\Rightarrow \underline{b_2' = b_2 \frac{1}{\sqrt{2}} \frac{3}{2}}$$

$\alpha = 0 \quad \gamma = j\beta$

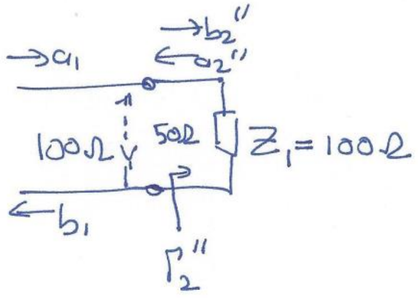
$\beta = \frac{2\pi}{\lambda}, \quad l = \frac{\lambda}{4}$

$\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$

$e^{j\beta l} = j$

$$b_2'' = b_2' e^{j\gamma l} = \frac{1}{\sqrt{2}}\frac{3}{2} e^{j\gamma l}\, b_2$$

$$\underline{b_2'' = j\frac{3}{2\sqrt{2}}\, b_2}$$



$$\Gamma_2'' = \frac{100-50}{100+50} = \frac{1}{3}$$

$$a_2'' = \Gamma_2'' b_2'' = \frac{1}{3} b_2''$$

$$V_1 = V_2'' \Rightarrow \sqrt{100}\,(a_1 + b_1) = \sqrt{50}\,(a_2'' + b_2'') = \frac{4}{3} b_2''$$

$$\Rightarrow \frac{4}{3} b_2'' \sqrt{\frac{1}{2}} = a_1 \qquad (b_1 = \Gamma_1 a_1,\ \Gamma_1 = 0 \Rightarrow b_1 = 0)$$

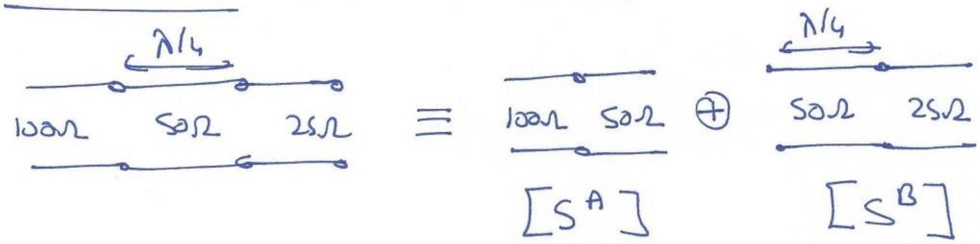
$$\Rightarrow a_1 = \frac{4}{3} \frac{1}{\sqrt{2}}\, j \frac{3}{2\sqrt{2}}\, b_2$$

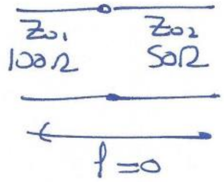
$$= j\,\frac{3}{4}\,\frac{4}{3}\, b_2 \Longrightarrow \underline{\underline{S_{21} = \frac{b_2}{a_1} = -j}} = S_{12}$$

(from symmetry prop.)

$$[S] = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix}$$

Solution-2 (Coscaded structure)





$$S_{11} = \Gamma_1 = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = \frac{50 - 100}{50 + 100} = -1/3$$

$$S_{22} = \Gamma_2 = \frac{Z_{01} - Z_{02}}{Z_{01} + Z_{02}} = 1/3$$

$$V_1 = V_2 \Rightarrow \sqrt{Z_{01}}\,(a_1 + b_1) = \sqrt{Z_{02}}\,(b_2 + 0) \qquad \left(S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0}\right)$$

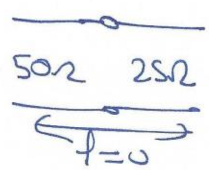
$$\sqrt{Z_{01}}\; a_1\,(1 + S_{11}) = \sqrt{Z_{02}}\; b_2$$

$$\Rightarrow S_{21} = \frac{b_2}{a_1} = \sqrt{\frac{Z_{01}}{Z_{02}}}\left(1 + \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}\right) = \sqrt{\frac{Z_{01}}{Z_{02}}}\left(\frac{2Z_{02}}{Z_{02} + Z_{01}}\right)$$

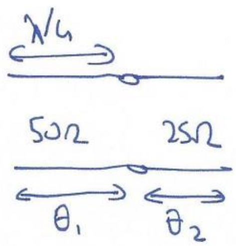
$$= 2\,\frac{\sqrt{Z_{01}Z_{02}}}{Z_{01} + Z_{02}} = S_{12} = \underline{\underline{\frac{2\sqrt{2}}{3}}}$$

$$\left(S_{11}^2 + S_{21}^2 = 1 \Rightarrow S_{21} = \sqrt{1 - S_{11}^2} = \sqrt{1 - \Gamma_1^2} = \sqrt{1 - (\tfrac{1}{3})^2} = \frac{2\sqrt{2}}{3} \right)$$

$$[S^A] = \begin{bmatrix} -\frac{1}{3} & \frac{2\sqrt{2}}{3} \\ \frac{2\sqrt{2}}{3} & +1/3 \end{bmatrix}$$



: $[S^{B'}]$ similar to above solution $[S^{B'}] = [S^A]$



using phase shifting property: $S_{ij}' = S_{ij}\, e^{-j(\theta_i + \theta_j)}$

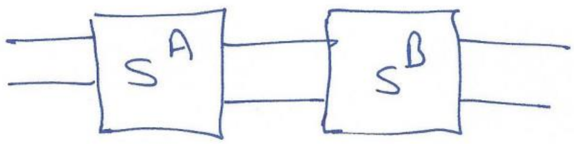
$$\theta_2 = 0\,, \quad \theta_1 = \beta \ell_1 = \frac{2\pi}{\lambda}\,\frac{\lambda}{4} = \underline{\underline{\frac{\pi}{2}}}$$

$$S_{11}' = S_{11}\, e^{-j(\theta_1 + \theta_1)} = S_{11}\, e^{-j2\theta_1} = \left(-\tfrac{1}{3}\right) e^{-j\frac{\pi}{2}\cdot 2} = +\frac{1}{3}$$

$$S_{22}' = S_{22}\, e^{-j(\theta_2 + \theta_2)} = S_{22} = -\frac{1}{3}$$

$$S_{21}' = S_{12}' = S_{21}\, e^{-j(\theta_2 + \theta_1)} = S_{21}\, e^{-j\frac{\pi}{2}} = -jS_{21} = -j\,\frac{2\sqrt{2}}{3}$$

$$\Rightarrow [S^B] = \begin{bmatrix} 1/3 & -j\frac{2\sqrt{2}}{3} \\ -j\frac{2\sqrt{2}}{3} & 1/3 \end{bmatrix}$$



$$S^A = \begin{bmatrix} -1/3 & \frac{2\sqrt{2}}{3} \\ \frac{2\sqrt{2}}{3} & 1/3 \end{bmatrix} \qquad S^B = \begin{bmatrix} 1/3 & -j\frac{2\sqrt{2}}{3} \\ -j\frac{2\sqrt{2}}{3} & 1/3 \end{bmatrix}$$

$$S_{11} = S_{11}^A + \frac{S_{11}^B S_{12}^A S_{21}^A}{1 - S_{11}^B S_{22}^A} = -\frac{1}{3} + \frac{\frac{1}{3}\frac{2\sqrt{2}}{3}\frac{2\sqrt{2}}{3}}{1 - \frac{1}{3}\frac{1}{3}} = -\frac{1}{3} + \frac{1}{3} = 0$$

$$S_{22} = S_{22}^B + \frac{S_{12}^B S_{21}^B S_{22}^A}{1 - S_{11}^B S_{22}^A} = \frac{1}{3} + \frac{-\frac{1}{3}\cdot\frac{2\sqrt{2}}{3}\frac{2\sqrt{2}}{3}}{8/9} = 0$$

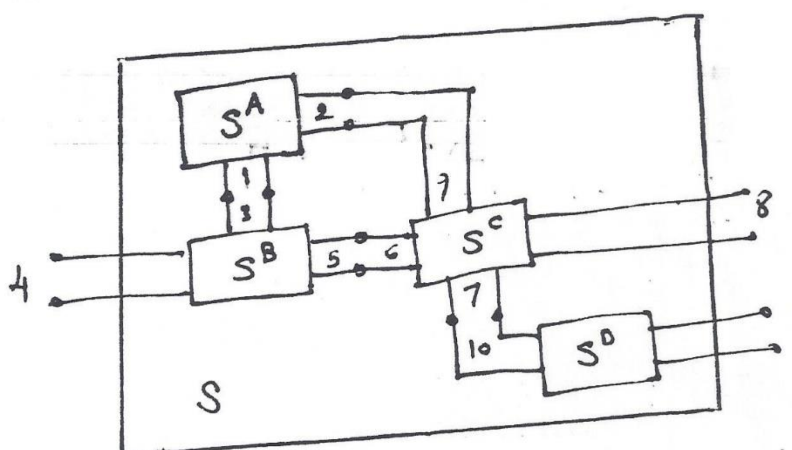
$$S_{12} = \frac{S_{12}^A S_{12}^B}{1 - S_{11}^B S_{22}^A} = \frac{-j\frac{2\sqrt{2}}{3}\frac{2\sqrt{2}}{3}}{8/9} = -j$$

$$S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{11}^B S_{22}^A} = \frac{-j\frac{2\sqrt{2}}{3}\frac{2\sqrt{2}}{3}}{8/9} = -j$$

$$[S] = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix}$$

Herhangi Bir Bicimde Bağlanmış N-Kapılılar

Kaskat bağlı 2-kapılılara ayrıştırılamayan mikrodalga devrelerinin analizinde d-tane dış kapısı olan bir devrenin i/2 jonksiyonla, her iç kapı yalnızca başka bir tek iç kapıya bağlanacak şekilde alt devrelere ayrıldığı varsayılmaktadır. Bu durumda sonuç devrenin d-dış kapısı ve i/2 jonksiyonda birleştirilmiş i-tane iç kapısı olacaktır. Şekildeki üç kapılı devrede



4 jonk.la birbirine bağlı 8 iç kapı vardır. Amaç alt devrelerin bilinen par.i yardımıyla 3-kapılının S-par.ini belirlemektir.

Giriş – çıkış büyüklükleri dış kapılarda $[a_d]$, $[b_d]$, iç kapılarda $[a_i]$, $[b_i]$ vektörleri ile gösterildiğinde

$$\begin{bmatrix} b_d \\ \cdots \\ b_i \end{bmatrix} = \begin{bmatrix} S_{dd} & \vdots & S_{di} \\ \cdots & & \cdots \\ S_{id} & \vdots & S_{ii} \end{bmatrix} \begin{bmatrix} a_d \\ \cdots \\ a_i \end{bmatrix}.$$

S_{dd}, $d \times d$; S_{di}, $d \times i$'lik matrislerdir. S matrisi

$$[b_d] = [S]\,[a_d]$$

aranan d-kapılıya ilişkin matristir. Devrenin alt devrelere bölünmesi sırasında ortaya çıkan iç kapılara ilişkin giriş çıkış büyüklükleri arasındaki geçiş koşulları Γ bağlantı matrisi yordamıyla

$$[b_i] = [\Gamma]\,[a_i]$$

şeklinde ifade edilebilir. $i \times i$'lik bir kare matris olan Γ'nın elemanları Γ_{kl}, k-kapısı l-kapısına bağlı ise 1, bağlı değilse 0 olacaktır.

$$[b_i] = [S_{id}]\,[a_d] + [S_{ii}]\,[a_i]$$

olduğundan

$$[a_i] = ([\Gamma] - [S_{ii}])^{-1}\,[S_{id}]\,[a_d]$$

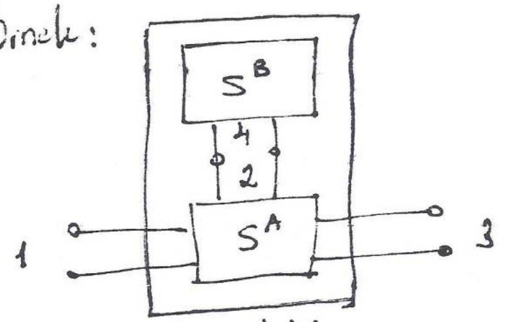
$$[b_d] = [S_{dd}]\,[a_d] + [S_{di}]\,[a_i]$$

ifadesinde $[a_i]$ yerine konursa,

$$[S] = [S_{dd}] + [S_{di}]\,([\Gamma] - [S_{ii}])^{-1}\,[S_{id}]$$

bulunur.

Örnek:



S matrisi

2 - dış kapı
1 - jonksiyon
2 - iç kapı
Toplam: 4 kapı

$$\begin{bmatrix} b_d \\ \hline b_i \end{bmatrix} = \begin{bmatrix} b_1 \\ b_3 \\ b_2 \\ b_4 \end{bmatrix} = \left[\begin{array}{cc|cc} S_{11}^A & S_{13}^A & S_{12}^A & 0 \\ S_{31}^A & S_{33}^A & S_{32}^A & 0 \\ \hline S_{21}^A & S_{23}^A & S_{22}^A & 0 \\ 0 & 0 & 0 & S_{44}^B \end{array}\right] \begin{bmatrix} a_1 \\ a_3 \\ \hline a_2 \\ a_4 \end{bmatrix}$$

$$\begin{bmatrix} b_2 \\ b_4 \end{bmatrix} = \begin{bmatrix} \Gamma \end{bmatrix} \begin{bmatrix} a_2 \\ a_4 \end{bmatrix} , \quad \Gamma = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$[S] = \begin{bmatrix} S_{11}^A & S_{13}^A \\ S_{31}^A & S_{33}^A \end{bmatrix} + \begin{bmatrix} S_{12}^A & 0 \\ S_{32}^A & 0 \end{bmatrix} \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} S_{22}^A & 0 \\ 0 & S^B \end{bmatrix} \right)^{-1} \begin{bmatrix} S_{21}^A & S_{23}^A \\ 0 & 0 \end{bmatrix}$$

$$S_{11} = S_{11}^A + \frac{S_{12}^A S_{21}^A S^B}{1 - S_{22}^A S^B}$$

$$S_{12} = S_{13}^A + \frac{S_{12}^A S_{23}^A S^B}{1 - S_{22}^A S^B}$$

$$S_{21} = S_{31}^A + \frac{S_{21}^A S_{32}^A S^B}{1 - S_{22}^A S^B}$$

$$S_{22} = S_{33}^A + \frac{S_{32}^A S_{23}^A S^B}{1 - S_{22}^A S^B}$$