EHB 351

ANALOG HABERLEŞME

(Arasınav 1 Çözümleri)

05.11.2015

36P (1) $x(t)=\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}=\sum_{n=-\infty}^{\infty}\frac{e^{j500n\pi t}}{n^2+1}$, $\omega_0=\frac{2\pi}{T}=2\pi f_0$

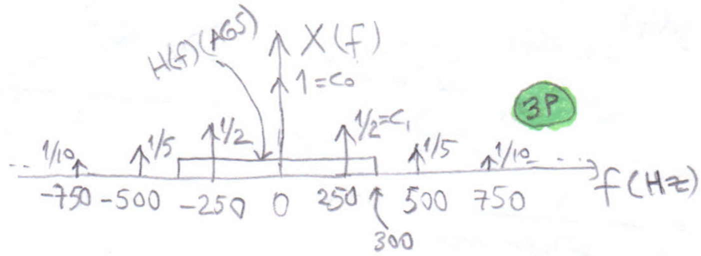
a) $c_n=\frac{1}{n^2+1}$, n tamsayı (4P)

$n\omega_0 t=500n\pi t \Rightarrow \frac{n2\cancel{\pi}}{T}\cancel{t}=500n\cancel{\pi}\cancel{t} \Rightarrow T=\frac{2}{500}=\frac{4}{1000}=4\times10^{-3}\,sn=4\,msn$ (2P)

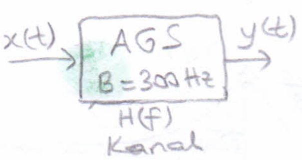
$\langle x(t)\rangle=\frac{1}{T}\int_T x(t)\,dt=c_n=\frac{1}{T}\int_T x(t)e^{-jn\omega_0 t}dt\Big|_{n=0}=c_0=\frac{1}{n^2+1}\Big|_{n=0}=1$ (2P)

b) $X(f)=\sum_n c_n\delta(f-nf_0)=\sum_n\frac{1}{n^2+1}\delta(f-nf_0)=\sum_n\frac{1}{n^2+1}\delta(f-250n)$ (4P)

$\frac{1}{T}=250\,Hz$

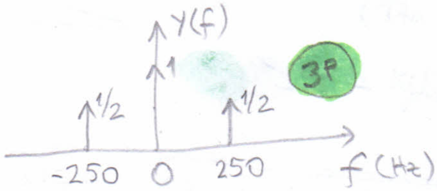


c)



$Y(f)=c_0\delta(f)+c_1\delta(f-f_0)+c_{-1}\delta(f+f_0)$ $\left(Y(f)=X(f)H(f)\right)$

$=\delta(f)+\frac{\delta(f-250)}{2}+\frac{\delta(f+250)}{2}$ (4P)



$y(t)=\mathcal{F}^{-1}\{Y(f)\}$

$=1+\cos 2\pi 250t$ (4P)

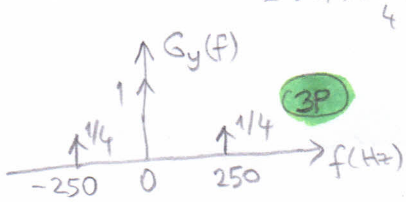
Kanalın transfer fonksiyonu $H(f)\neq Ke^{-j2\pi f t_0}$ $\forall f$ olduğu için bu sistem bozulmalıdır. (1P)

d)

$G_y(f)=G_x(f)\,|H(f)|^2$

$G_x(f)=\sum_{n=-\infty}^{\infty}|c_n|^2\delta(f-nf_0) \Rightarrow G_y(f)=\sum_n |y_n|^2\delta(f-nf_0)$, $y_n=\begin{cases}\frac{1}{n^2+1}, & n=-1,0,+1\\ 0, & \text{dışında}\end{cases}$

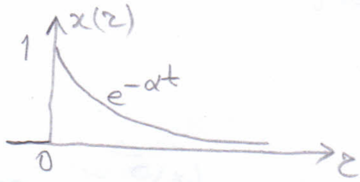
$=\delta(f)+\frac{1}{4}\delta(f-f_0)+\frac{1}{4}\delta(f+f_0)$ (4P)

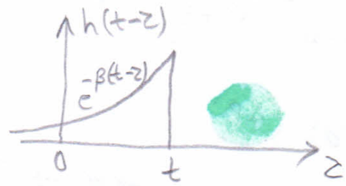


$P_y=\int_{-\infty}^{\infty}G_y(f)\,df=\frac{1}{4}+1+\frac{1}{4}=\frac{3}{2}$ W (2P)

34P (2) a)

$$x(t)=e^{-\alpha t}u(t) \longrightarrow \boxed{h(t)=e^{-\beta t}u(t)} \longrightarrow y(t)=x(t)*h(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau$$





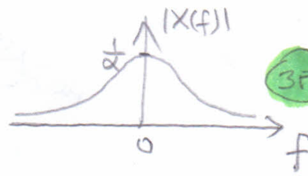
$t<0$ ise $y(t)=0$

$t\geq 0$ ise $y(t)=\int_0^t e^{-\alpha\tau}e^{-\beta(t-\tau)}d\tau = e^{-\beta t}\int_0^t e^{-(\alpha-\beta)\tau}d\tau = \begin{cases} te^{-\beta t}, & \alpha=\beta \\ \dfrac{e^{-\beta t}-e^{-\alpha t}}{\alpha-\beta}, & \alpha\neq\beta \end{cases}$ (10P)

b) $x(t)=e^{-\alpha t}u(t) \Rightarrow$

$X(f)=\int_{-\infty}^{\infty}x(t)e^{-j2\pi ft}dt=\int_0^{\infty}e^{-\alpha t}e^{-j2\pi ft}dt=\int_0^{\infty}e^{-(\alpha+j2\pi f)t}dt=\dfrac{1}{\alpha+j2\pi f}$ (5P)

$X(f)=|X(f)|e^{j\arg X(f)} \Rightarrow |X(f)|=\dfrac{1}{\sqrt{\alpha^2+(2\pi f)^2}}$

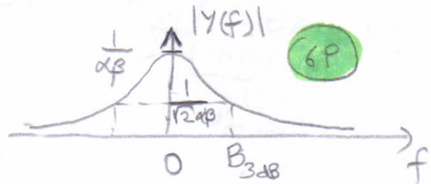


(3P)

c) $Y(f)=X(f)H(f)=\dfrac{1}{(\alpha+j2\pi f)(\beta+j2\pi f)}$

$H(f)=\dfrac{1}{\beta+j2\pi f}$

$|Y(f)|=\dfrac{1}{\sqrt{(\alpha^2+(2\pi f)^2)(\beta^2+(2\pi f)^2)}}$



(6P)

$y(t)$'nin mutlak band genişliği $B_{mutlak}=\infty$ (2P)

$|Y(f=B_{3dB})|=\dfrac{1}{\sqrt{(\alpha^2+(2\pi B_{3dB})^2)(\beta^2+(2\pi B_{3dB})^2)}}=\dfrac{1}{\alpha\beta\sqrt{2}} \Rightarrow B_{dB}=\dfrac{1}{2\pi}\sqrt{\dfrac{-(\alpha^2+\beta^2)+\sqrt{(\alpha^2+\beta^2)^2+4\alpha^2\beta^2}}{2}}$ (3P)

↑ Not: Pozitif olan alındı.

d) $\alpha=\beta \Rightarrow$

$y(t)=te^{-\alpha t} \Rightarrow Y(f)=\dfrac{1}{(\alpha+j2\pi f)^2}$ (5P)

30p (3) a)

$x(t) \longleftrightarrow X(f)$

$x(t)e^{j2\pi f_c t} \longleftrightarrow X(f-f_c)$ (Frekansta öteleme teoremi)

<u>İspat:</u>

$y(t) = x(t)e^{j2\pi f_c t}$

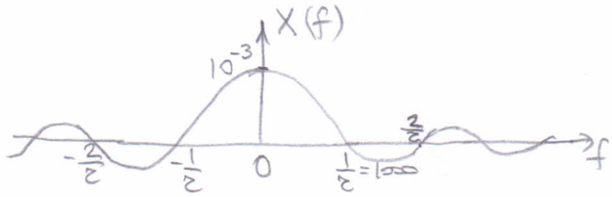
$$Y(f) = \int_{-\infty}^{\infty} y(t)e^{-j2\pi f t}\,dt = \int_{-\infty}^{\infty} x(t)e^{j2\pi f_c t} \cdot e^{-j2\pi f t}\,dt = \int_{-\infty}^{\infty} x(t)e^{-j2\pi (f-f_c)t}\,dt = X(f-f_c)$$

8p

b)

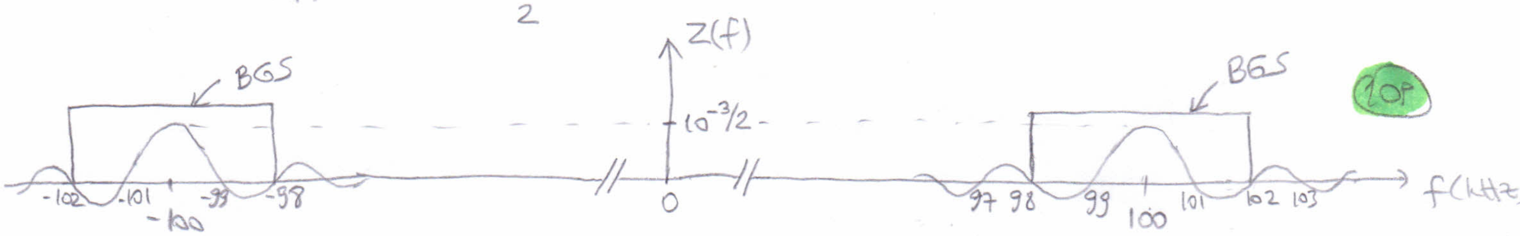
$x(t) = A\Pi(t/\tau) \Rightarrow X(f) = A\tau\,\mathrm{sinc}(f\tau) = 10^{-3}\,\mathrm{sinc}(10^{-3}f)$

$A=1,\ \tau = 10^{-3}$



$z(t) = x(t)\cos 2\pi 10^5 t$

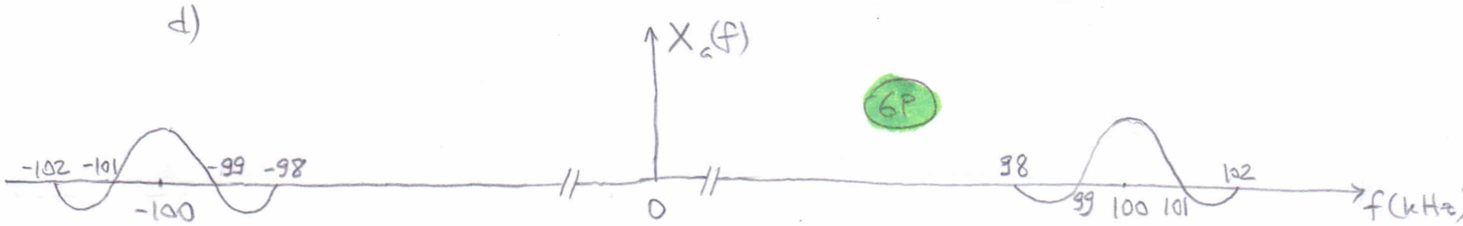
$$Z(f) = \frac{X(f-10^5) + X(f+10^5)}{2}$$



10p

c) BGS'in band genişliği $B = 4\,\text{kHz}$, merkez frekansı $f_c = 100$ kHz olmalı.
3p 3p

d)



6p