6. Cascode Amplifiers and Cascode Current Mirrors

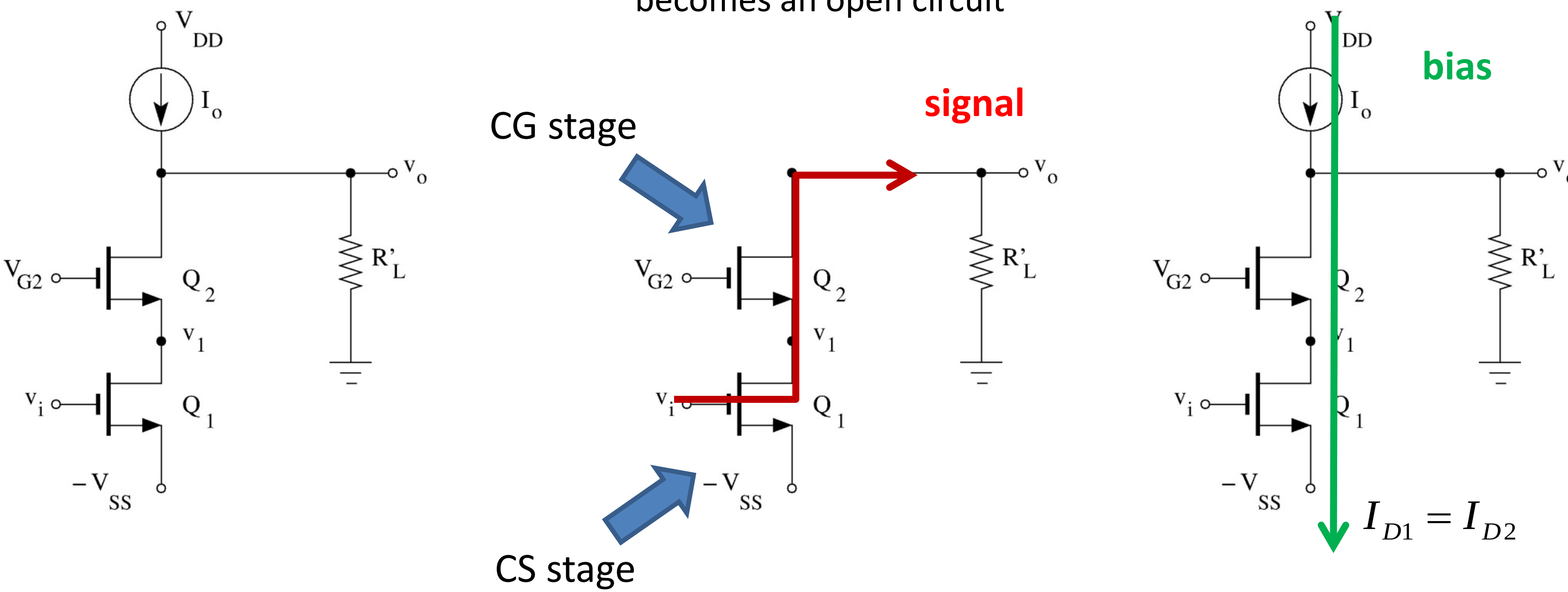
Sedra & Smith Sec. 7 (MOS portion)

(S&S 5th Ed: Sec. 6 MOS portion & ignore frequency response)

Cascode amplifier is a popular building block of ICs

Cascode Configuration

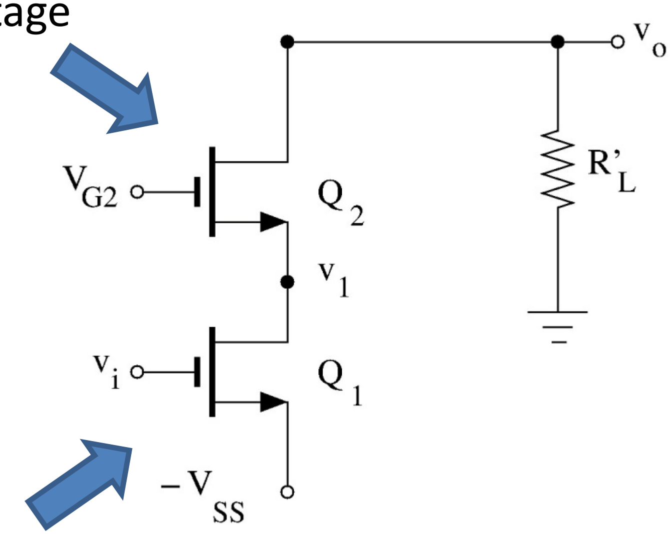
Signal circuit: Current source becomes an open circuit



Cascode amplifier is a two-stage, CS-CG configuration

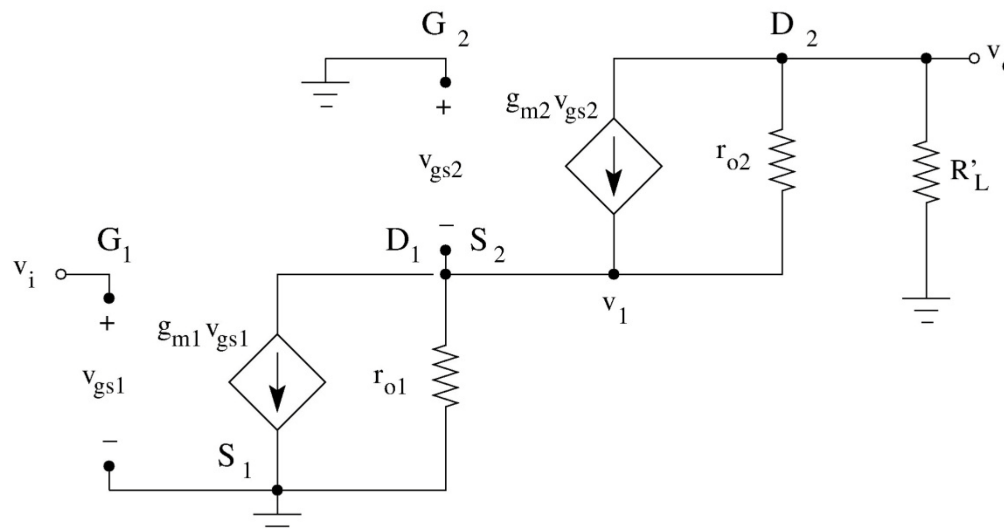
Small Signal Model of a Cascode Amplifier

CG stage



CS stage

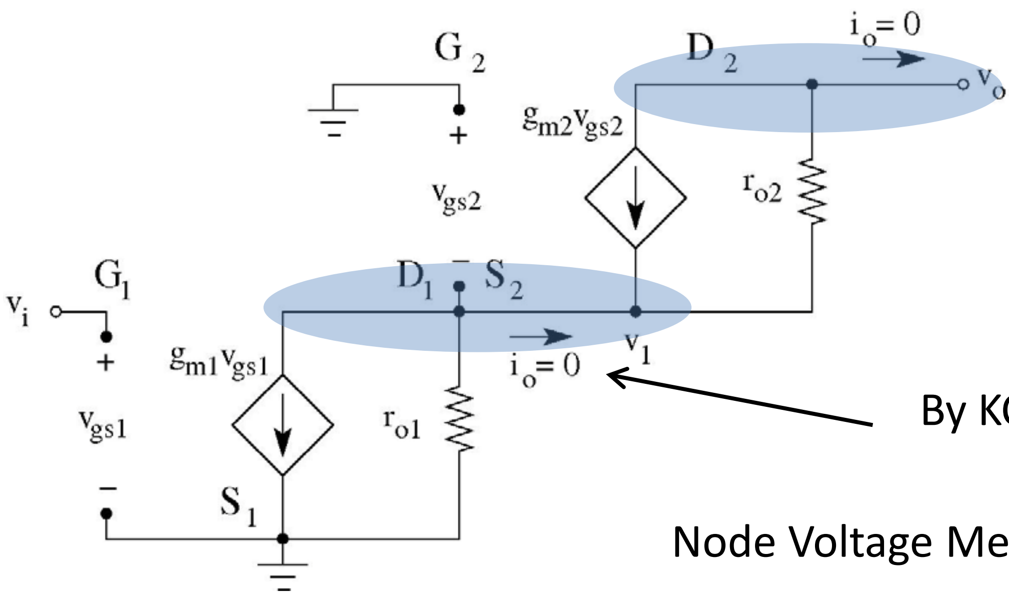
Small Signal Model



- Lengthy analysis to find A_v (and a complicated equation). Simpler to compute open-loop gain (A_{vo}) and R_o.
- Text book introduces G_m method to find A_{vo} (See S&S Sec. 1)
- Here will find A_{vo} directly from the small signal model.
- **However, the solution of and insight into Cascode amplifiers are best obtained using fundamental MOS configurations!**

Note that A_{vo} and R_o calculated here are meant to find A_v and guide the choice of the active load. A_{vo} and R_o should be re-calculated for a practical circuit (see slides 14 & 15)

Open-Loop gain of a Cascode amplifier (using small signal model)



Open Loop Gain
($R'_L \to \infty$, $i_o = 0$):

By KCL around Q2

Node Voltage Method:

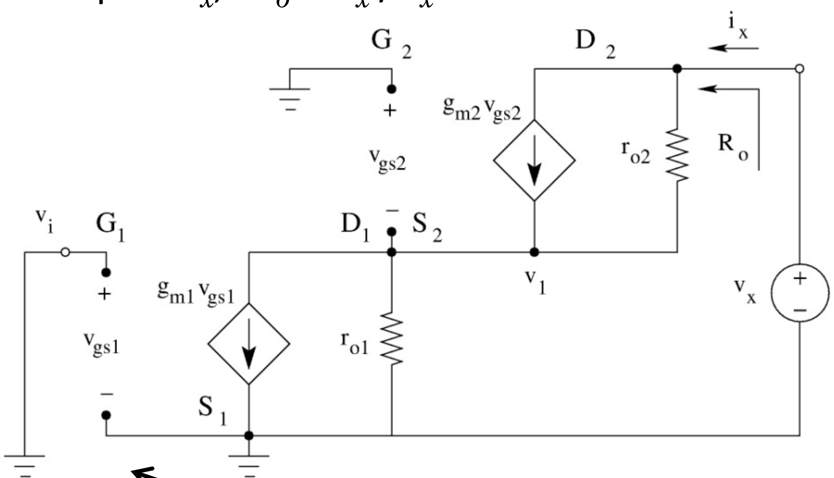
Node v_o: $\dfrac{v_o - v_1}{r_{o2}} - g_{m2}v_1 = 0 \quad \Rightarrow \quad v_o = (1 + g_{m2}r_{o2})\, v_1$

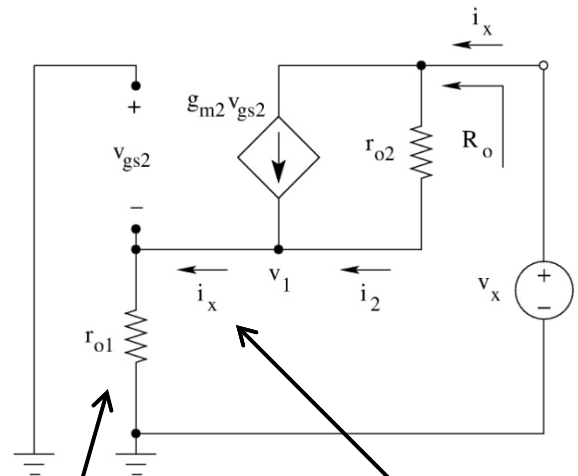
Node v_1: $\dfrac{v_1}{r_{o1}} + g_{m1}v_i + 0 = 0 \quad \Rightarrow \quad v_1 = -g_{m1}r_{o1}\, v_i$

$$\boxed{A_{vo} = \frac{v_o}{v_i} = -g_{m1}r_{o1} \times (1 + g_{m2}r_{o2}) \approx -g_{m1}r_{o1}g_{m2}r_{o2}}$$

Output Resistance of a Cascode amplifier (using small signal model)

Set $v_i = 0$, attach a voltage source v_x ,
compute i_x, $R_o = v_x / i_x$





$v_i = v_{gs1} = 0 \ \rightarrow g_{m1} \ v_{gs1}$ current source becomes open circuit

By KCL around Q2

$$\text{KVL}: \quad v_{gs2} = -i_x r_{o1}$$

$$\text{KCL}: \quad i_2 = i_x - g_{m2} v_{gs2} = i_x + i_x g_{m2} r_{o1} = i_x (1 + g_{m2} r_{o1})$$

$$\text{KVL}: \quad v_x = i_2 r_{o2} + i_x r_{o1} = i_x (1 + g_{m2} r_{o1}) r_{o2} + i_x r_{o1}$$

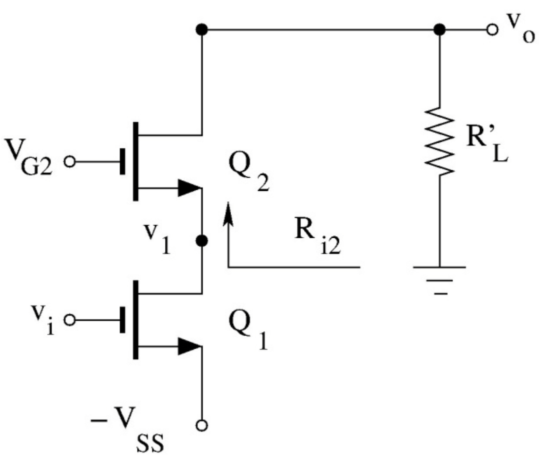
$$v_x = i_x [(1 + g_{m2} r_{o1}) r_{o2} + r_{o1}]$$

$$\boxed{R_o = \frac{v_x}{i_x} = r_{o1} + r_{o2} + g_{m2} \ r_{o1} \ r_{o2}}$$

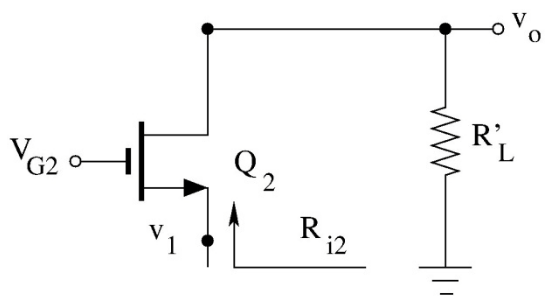
Note: $A_v = A_{vo} \times \dfrac{R_L' + R_o}{R_L'}$

Gain of a Cascode Amplifier (using MOS Fundamental Configurations)

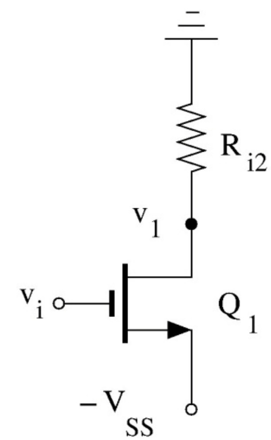
Cascode (signal circuit)



CG stage



CS stage



$$\boxed{A_{v2} = v_o / v_1 \approx g_{m2}(r_{o2} \,\|\, R'_L)}$$

$$\boxed{A_{v1} = v_1 / v_i = -g_{m1}(r_{o1} \,\|\, R_{i2})}$$

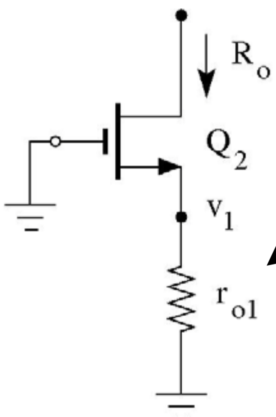
CG stages "reduces" the load seen by the CS stage by $g_{m2} r_{o2}$ →

$$\boxed{R_{L1} = R_{i2} = \frac{r_{o2} + R'_L}{1 + g_{m2} r_{o2}}}$$

$$\boxed{A_v = v_o / v_i = A_{v1} A_{v2} \;= -g_{m1} g_{m2} (r_{o1} \,\|\, R_{i2})\,(r_{o2} \,\|\, R'_L)}$$

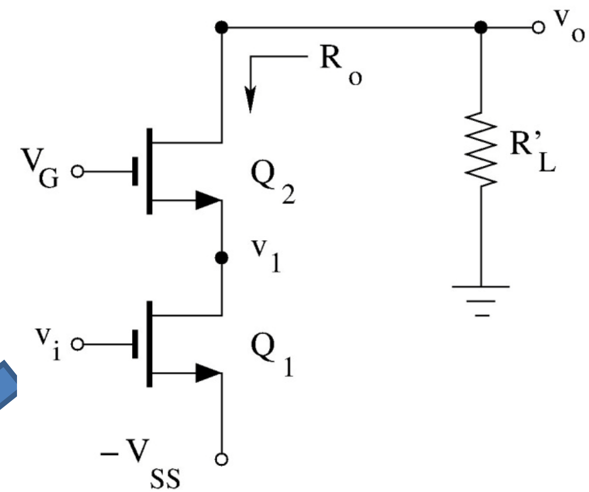
Note: Open Loop Gain: $(R'_L \to \infty)$ $\quad R_{L1} = R_{i2} = \dfrac{r_{o2} + R'_L}{1 + g_{m2} r_{o2}} = \infty \quad \to \quad A_{vo} = -g_{m1} r_{o1} g_{m2} r_{o2}$

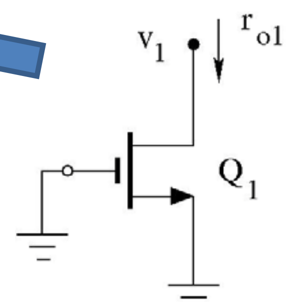
Output Resistance of a Cascode amplifier (from Elementary R forms)



$= r_o(1+g_m R)+R$

R





$$R_o = r_{o2}(1+g_{m2}r_{o1}) + r_{o1}$$

$$\boxed{R_o = r_{o1} + r_{o2} + g_{m2}\, r_{o1}\, r_{o2}}$$

$$R_o = r_{o2}(1+g_{m2}r_{o1}) + r_{o1}$$

$$R_o \approx g_{m2}r_{o2}r_{o1} + r_{o1} = r_{o1}(1+g_{m2}r_{o2})$$

$$\boxed{R_o \approx g_{m2}\, r_{o1}\, r_{o2}}$$

Cascode Amplifier needs a large load

$$\boxed{A_{v2} = g_{m2}(r_{o2} \parallel R'_L)} \quad \boxed{R_{L1} = R_{i2} = \frac{r_{o2} + R'_L}{1 + g_{m2} r_{o2}}} \quad \boxed{A_{v1} = -g_{m1}(r_{o1} \parallel R_{i2})} \quad \boxed{R_o \approx g_{m2}\, r_{o1}\, r_{o2}}$$

For simplicity assume $r_{o1} = r_{o2} = r_o$ and $g_{m1} = g_{m2} = g_m$

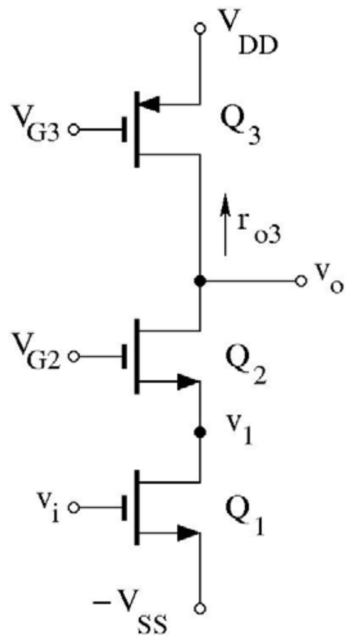
R'_L	A_{v2}(CG)	$R_{i2} = R_{L1}$	A_{v1} (CS)	$A_v = A_{v1} A_{v2}$	
∞	$g_m r_o$	∞	$-g_m r_o$	$-(g_m r_o)^2$	**Max. Gain**
$(g_m r_o)\, r_o = R_o$	$g_m r_o$	r_o	$-0.5 g_m r_o$	$-0.5 (g_m r_o)^2$	**Practical Gain**
r_o	$0.5\, g_m r_o$	$2/g_m$	-2	$-g_m r_o$	**Same gain as a single CS Amp.**

- For comparison, a two-stage CS-amplifier (CS-CS) has a gain of $0.5\,(g_m r_o)^2$ for $R'_L = r_o$ and a gain of $(g_m r_o)^2$ for $R'_L = g_m r_o^2$.
  - Cascode amplifier needs a large load ($R'_L = g_m r_o^2$).

Cascode amplifier needs a large load to get a high gain

Current Mirror

Cascode

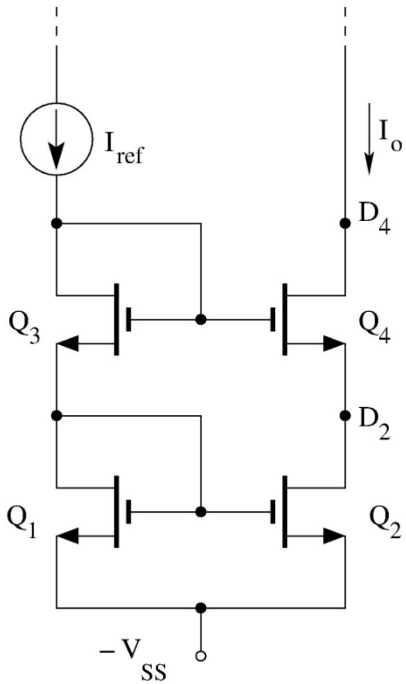


$R'_L = r_{o3}$

$A_v \approx -g_m r_o$

- Gain did not increase compared to a CS amplifier.
- This is still a useful circuit because of its high gain-bandwidth (we see this later).
- To get a high gain, $A_v = -0.5(g_m r_o)^2$, we need to increase the small-signal resistance of the current mirror to $\approx (g_m r_o)\ r_o$
  - Cascode current mirror

Cascode Current mirror



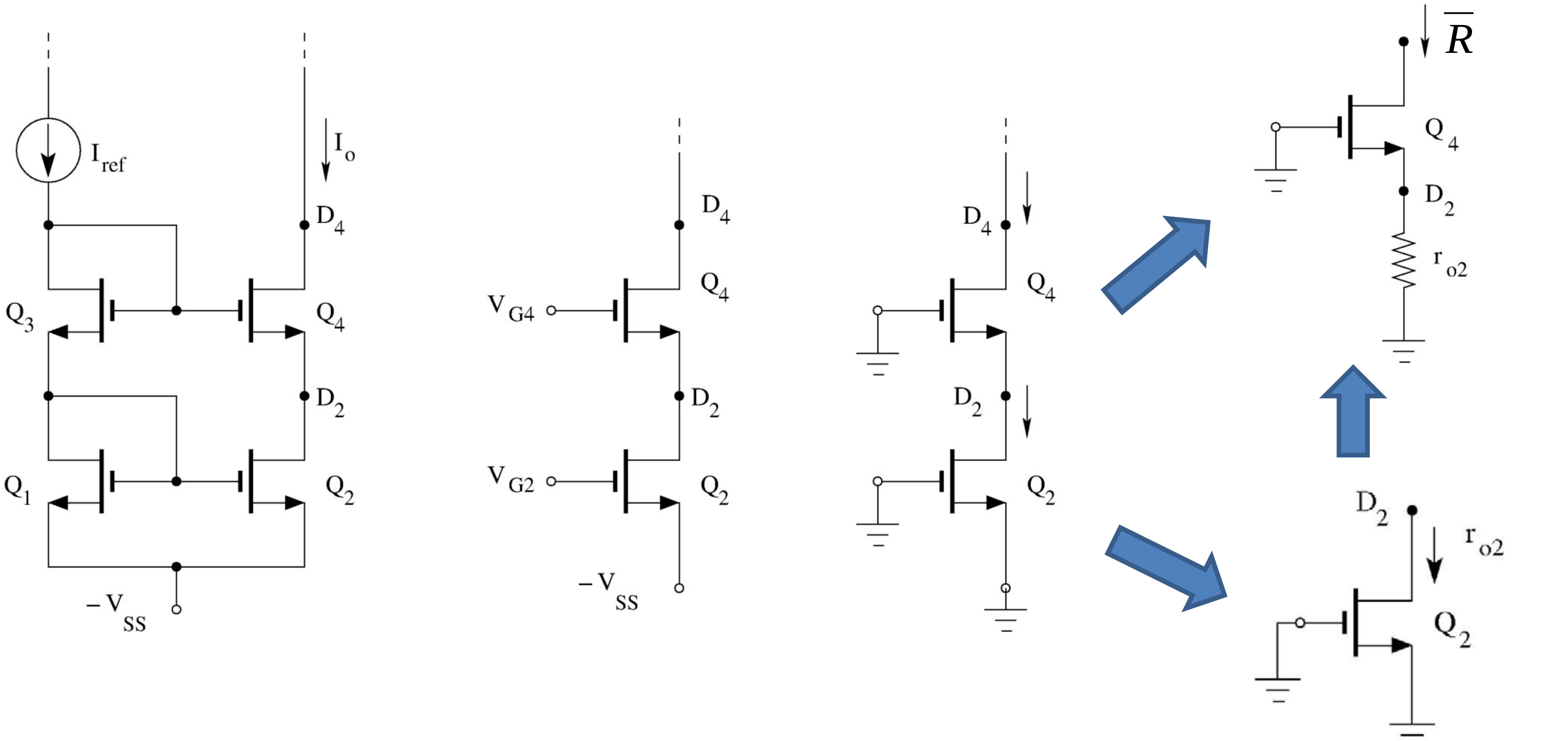
- Identical MOS: Same μC_{ox} and V_t , & $\dfrac{(W/L)_4}{(W/L)_3} = \dfrac{(W/L)_2}{(W/L)_1}$
  - o $v_{GS1} = v_{GS2}$ & $v_{GS3} = v_{GS4}$
- Usually: $(W/L)_1 = (W/L)_3$ and $(W/L)_2 = (W/L)_4$
  - o $v_{GS1} = v_{GS2} = v_{GS3} = v_{GS4} = v_{GS}$

- Q1 and Q3 are <u>always</u> in saturation
- <u>Q2 and Q4 both have to be in saturation for current mirror to work</u>
  - o $V_{DS2} > V_{GS} - V_t$
  - o $V_{DS4} > V_{GS} - V_t$
- Straight forward to show $\quad I_o = \dfrac{(W/L)_2}{(W/L)_1} I_{ref}$

Exercise: Show that a single current mirror (no cascoding) works only if $V_{D2} > V_{OV} - V_{SS}$ and a cascode current mirror requires $V_{D4} > 2V_{OV} - V_{SS}$

Small signal resistance of a cascode current mirror is quite large

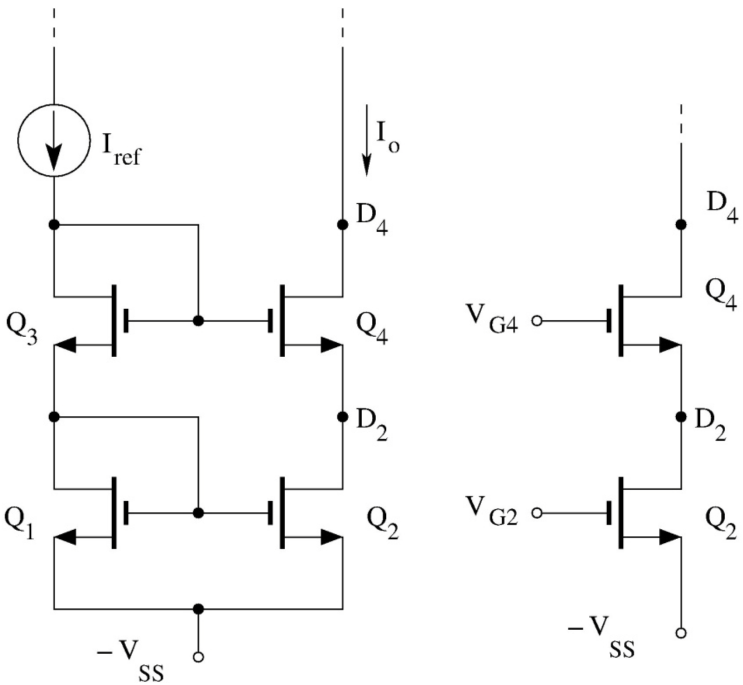
$$\overline{R} = r_{o4}(1+g_{m4}r_{o2}) + r_{o2}$$



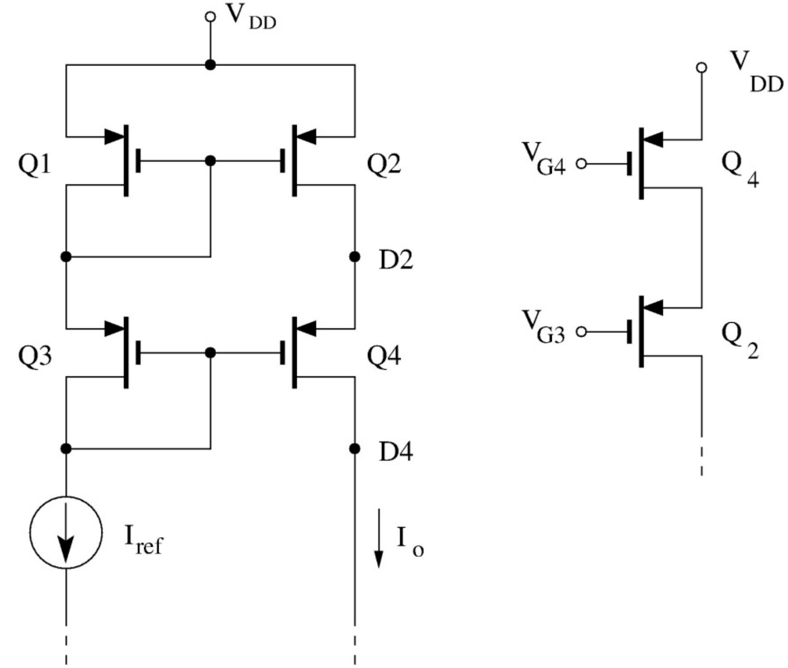
Transistor numbering is different in different circuits
Be careful in applying formulas!
It is best to use elementary R forms to find $\overline{R}$ instead of formula above.

PMOS cascode current mirror is similar to NMOS version

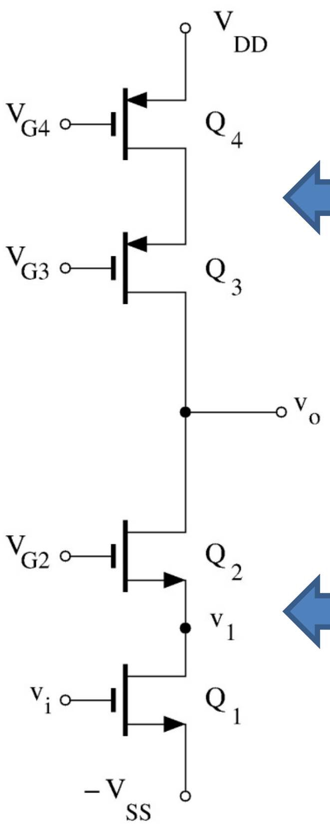
NMOS Cascode current mirror



PMOS Cascode current mirror

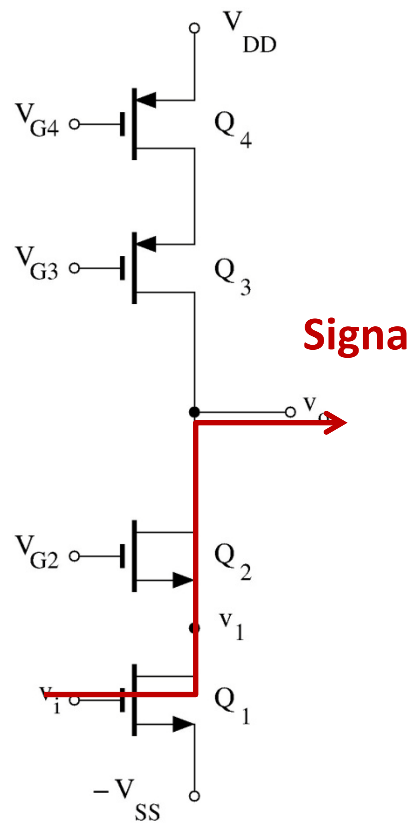


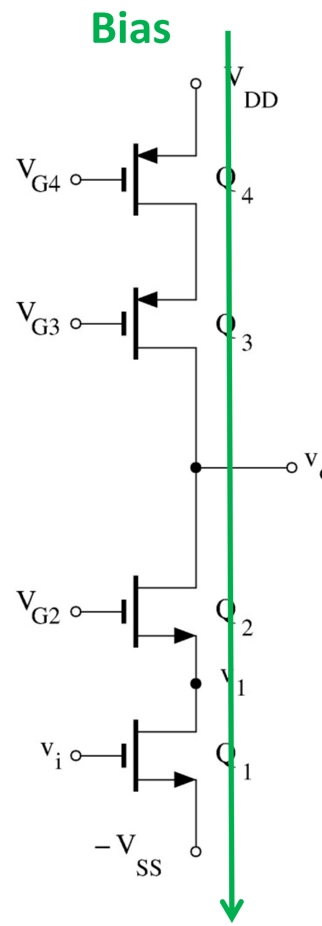
Cascode amplifier with a cascode current mirror/active load



Cascode current mirror

Cascode amplifier

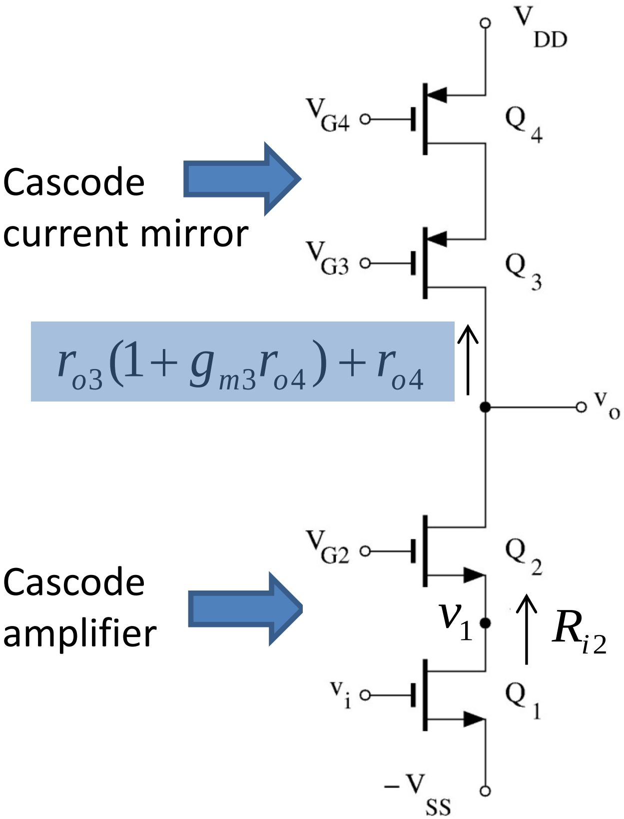




$$I_{D1} = I_{D2} = I_{D3} = I_{D4}$$

Exercise: Draw the circuit for a PMOS cascode with a cascode current mirror (cascode current mirror would be made of NMOS).

Gain of a Cascode amplifier with a cascode current mirror/active load



Q2 (CG Amp)

$$A_{v2}=v_o/v_1 \approx g_{m2}(r_{o2}\,\|\,R'_L)\approx g_{m2}r_{o2}$$

$$R_{L1}=R_{i2}=\frac{r_{o2}+r_{o3}(1+g_{m3}r_{o4})+r_{o4}}{1+g_{m2}r_{o2}}$$

$$\approx\frac{g_{m3}r_{o3}r_{o4}}{g_{m2}r_{o2}}$$

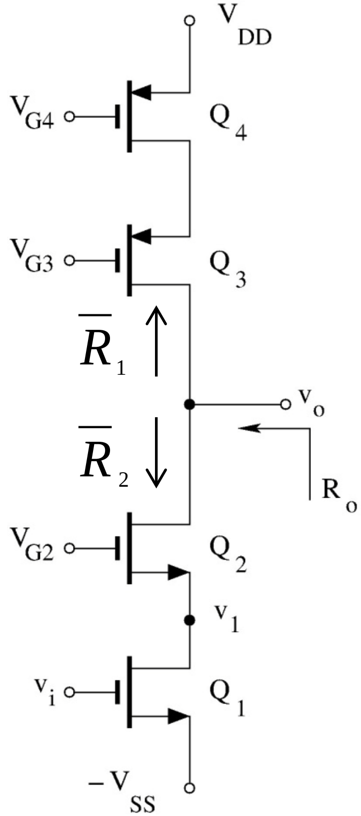
Q1 (CS Amp)

$$A_{v1}=v_1/v_i=-g_{m1}\left(r_{o1}\,\|\,\frac{g_{m3}r_{o3}r_{o4}}{g_{m2}r_{o2}}\right)$$

$$A_{vo}=A_{v1}A_{v2}\;=-\frac{g_{m1}g_{m2}g_{m3}r_{o1}r_{o2}r_{o3}r_{o4}}{g_{m2}r_{o1}r_{o2}+g_{m3}r_{o3}r_{o4}}$$

Value for the same g_m and r_o
$A_{v2}\approx g_m r_o$
$R_{L1}=R_{i2}\approx r_o$
$A_{v1}=-0.5g_m r_o$
$A_{vo}=-0.5\,(g_m r_o)^2$

Output Resistance of a Cascode amplifier with a cascode current mirror/active load



$$\overline{R}_1 = r_{o3}(1 + g_{m3} r_{o4}) + r_{o4}$$

$$\overline{R}_2 = r_{o2}(1 + g_{m2} r_{o1}) + r_{o1}$$

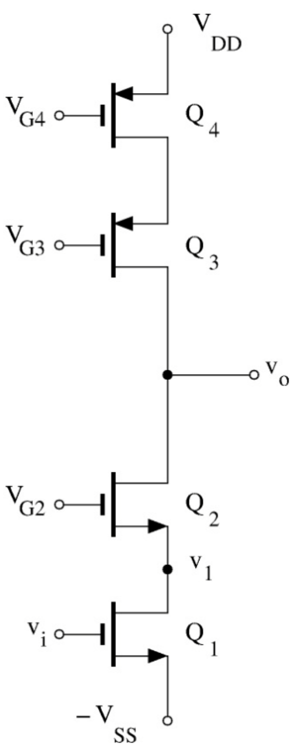
$$R_o = \overline{R}_1 \parallel \overline{R}_2$$

Value for the same g_m and r_o
$\overline{R}_1 \approx g_m r_o^2$
$\overline{R}_2 \approx g_m r_o^2$
$R_o = 0.5 g_m r_o^2$

Why Cascode Amplifiers are popular?

Cascode Amp.

$$A_{vo} = -0.5\,(g_m r_o)^2$$
$$R_o = 0.5 g_m r_o^2$$



Drawbacks:

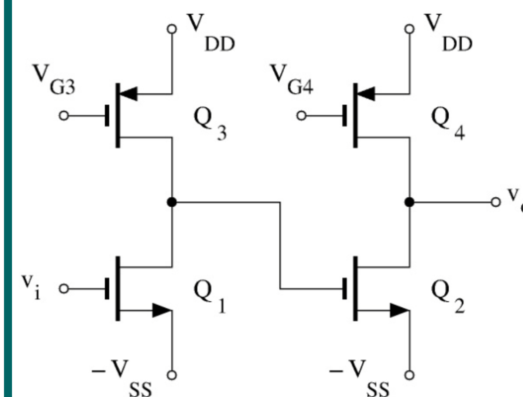
- While A_{vo} are similar, Cascode has a very R_o (MΩ level).
  - o should be followed with a CS or CD stage (infinite load for cascode)
  - o BJT cascodes are not useful.
- Low voltage headroom (V_{DD} across 4 MOS)
  - o Folded cascodes solve this.

Benefits:

- **Much better high-frequency response (high gain-bandwidth).***
- Simpler biasing.

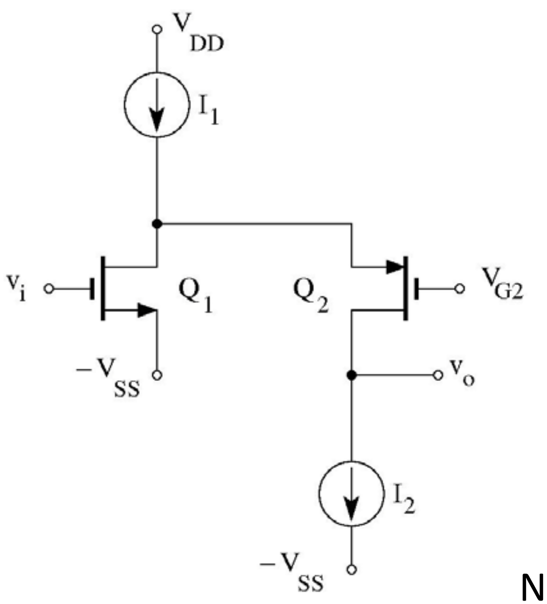
2-stage CS Amp.

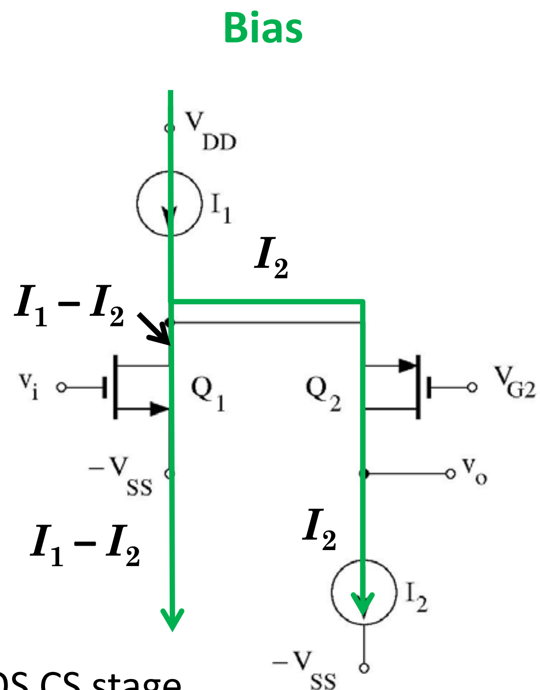
$$A_{vo} = +0.25(g_m r_o)^2$$
$$R_o = r_o$$



* We will see this later in our discussion of frequency response.

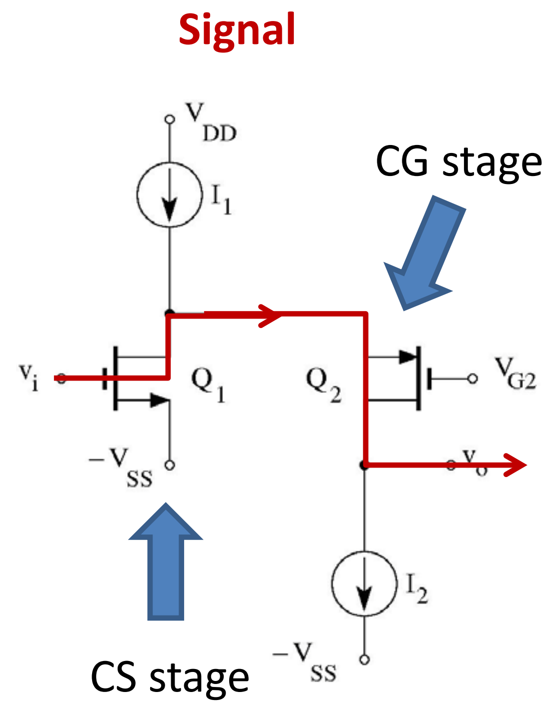
Folded Cascode increases voltage overhead*





NMOS CS stage
Biased with $I_1 - I_2$

PMOS CG stage
Biased with I_2



* Folded cascode only helps the voltage overhead issue for difference amplifiers (see S&S pages 999-1000)