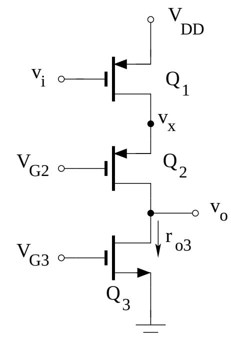
# ECE102 (Winter 2011), Solution of the Final

**Problem 1.** Find the mid-frequency gain of the circuit below with identical transistors with $V_{OV} = 0.5$ V, $I_D = 50\ \mu$A, and $V_A = 10$ V.

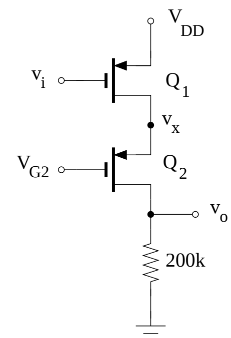



This is a PMOS Cascode amplifier (Q1 and Q2) with an active load (Q3).

$$g_{m1} = g_{m2} = g_{m3} = g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 50 \times 10^{-6}}{0.5} = 2 \times 10^{-4}\ \text{A/V}$$

$$r_{o1} = r_{o2} = r_{o3} = r_o = \frac{V_A}{I_D} = \frac{10}{50 \times 10^{-6}} = 200\ \text{k}$$

Replacing Q3 (active load) with its value: $r_{o3} = 20$ k, we get the circuit to the right above. For a Cacode amplifier:



$$A_{vo} \approx -g_{m1} g_{m2} r_{o1} r_{o2} = -1,600$$

$$R_o = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2} = 8,400\ \text{k}$$

$$A_v = A_{v0}\ \frac{R_L}{R_o + R_L} = -1,600 \times \frac{200}{200 + 8,400} = -37.2$$

Alternatively, Q2 is a CG amplifier and CS is a CS amplifier:

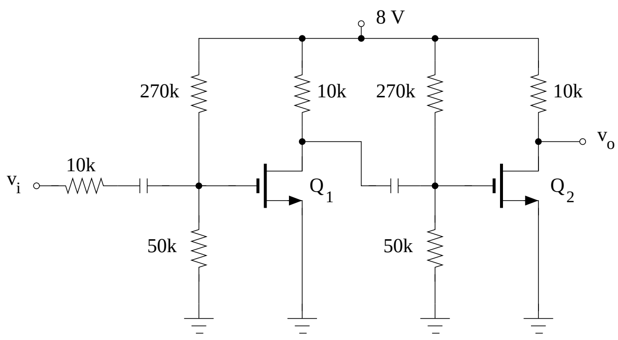
$$\frac{v_o}{v_x} = A_{vo} = g_{m2}(r_{o2} \parallel R_D) = 2 \times 10^{-4}(200\ \text{k} \parallel 200\ \text{k}) = -20$$

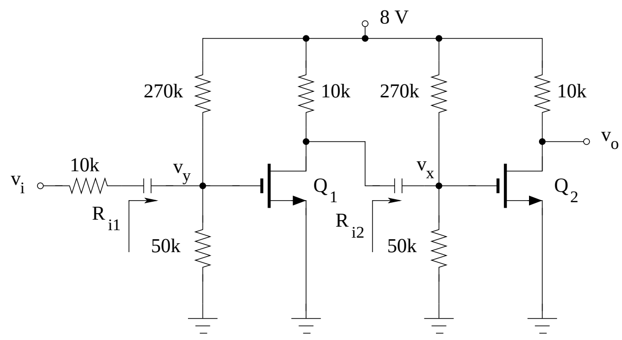
$$R_{L1} = R_{i2} = \frac{r_{o2} + r_{o3}}{1 + g_{m2} r_{o2}} = 9.76\ \text{k}$$

$$\frac{v_x}{v_i} = A_{v1} = -g_{m1}(r_{o1} \parallel R_{L1}) = -18.6$$

$$\frac{v_o}{v_i} = \frac{v_o}{v_x} \times \frac{v_x}{v_i} = -37.2$$

**Problem 2.** Find $v_o/v_i$ in the circuit below with large capacitors and identical transistors with $\mu_n C_{ox}(W/L) = 2.4\ \text{mA/V}^2$, $V_t = 0.75$ V and $\lambda = 0$.




**Bias:** We need to calaculte $I_D$ and $V_{OV}$ frist from bias values. Note that the bias circuits for both transsistors are the same:

$$V_{G1} = V_{G2} = \frac{50\ \text{k}}{270\ \text{k} + 50\ \text{k}} \times 8 = 1.25\ \text{V}$$

$$V_{GS1} = V_{GS2} = V_G - 0 = 1.25\ \text{V}$$

$$V_{OV1} = V_{OV2} = V_{GS1} - V_t = 0.5\ \text{V}$$

$$I_{D1} = I_{D2} = 0.5\mu_n C_{ox}(W/L)V_{ov}^2 = 0.5 \times 2.4 \times 10^{-3} \times (0.5)^2 = 0.3\ \text{mA}$$

$$g_{m1} = g_{m2} = \frac{2I_{D1}}{V_{OV1}} = 1.2\ \text{mA/V} \qquad \text{and} \qquad r_o \to \infty$$

Since $V_{OV} > 0$ and $V_{DS} = 8 - I_D R_D = 5 > V_{OV}$, both transistors are in saturation.

To find the gain, we start with Q2 (load side). Q2 is a CS amplifier with:

$$\frac{v_o}{v_x} = A_{v2} = -g_m(r_o \parallel R_D \parallel R_L) = -g_m R_D = -1.2 \times 10^{-3} \times 10 \times 10^3 = -12$$

$$R_{i2} = 50\ \text{k} \parallel 270\ \text{k} = 42.2\ \text{k}$$

$R_{i2}$ is the load for Q1. Q1 is also a CS amplifier:

$$\frac{v_x}{v_y} = A_{v1} = -g_m(r_o \parallel R_D \parallel R_L) = -g_m(R_D \parallel R_{i2})$$

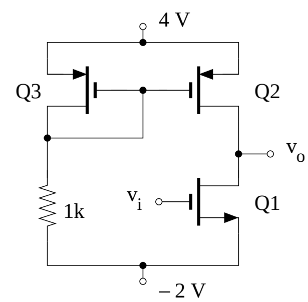
$$\frac{v_x}{v_y} = -1.2 \times 10^{-3} \times (10 \times 10^3 \parallel 42.2 \times 10^3) = -9.7$$

$$R_{i1} = 50\ \text{k} \parallel 270\ \text{k} = 42.2\ \text{k}$$

Finally, $v_y/v_i$ can be found from the voltage divider formula:

$$\frac{v_y}{v_i} = \frac{R_{i1}}{R_{i1} + 10\text{ k}} = 0.81$$
$$\frac{v_o}{v_i} = \frac{v_o}{v_x} \times \frac{v_x}{v_y} \times \frac{v_y}{v_i} = (-12)(-9.7)(0.81) = 94.3$$

**Problem 3.** Find $v_o/v_i$ in the circuit below with $\mu_n C_{ox}(W/L) = \mu_p C_{ox}(W/L) = 8\text{ mA/V}^2$ and $|V_t| = 1$ V, and $\lambda = 0.1$ 1/V. Ignore channel-width modulation in biasing calculations.




Q1 is a CS amplifier biased by a current mirror (Q2 and Q3). Q2 is also the active load for Q1. Bias values are needed to compute $g_m$ and $r_o$ of the transistors.

$$I_{D3} = 0.5\mu_p C_{ox}(W/L)V_{OV3}^2 = 4 \times 10^{-3} V_{OV3}^2$$

GS3-KVL $\quad 4 = V_{SG3} + 10^3 I_{D3} - 2 = V_{OV3} + |V_t| + 10^3 I_{D3} - 2$

$$5 = V_{OV3} + 10^3 I_{D3}$$
$$5 = V_{OV3} + 10^3 \times 4 \times 10^{-3} V_{OV3}^2$$
$$4V_{OV3}^2 + V_{OV3} - 5 = 0 \qquad \rightarrow \qquad V_{OV3} = 1\text{ V}$$
$$I_{D3} = 4 \times 10^{-3} V_{OV3}^2 = 4\text{ mA}$$

Since Q2 and Q3 form a current mirror and $(W/L)_2 = (W/L)_3$, $I_{D2} = I_{D3} = 4$ mA and $V_{OV3} = V_{OV2} = 1$ V.

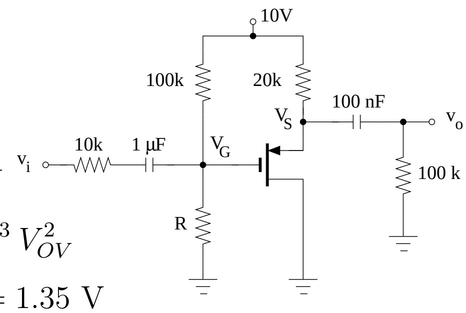
Furthermore, $I_{D1} = I_{D2} = 4$ mA and since $\mu_n C_{ox}(W/L) = \mu_p C_{ox}(W/L)$, $V_{OV1} = V_{OV2} = 1$ V. Therefore,

$$g_{m1} = g_{m2} = \frac{2I_{D1}}{V_{OV}} = \frac{2 \times 4 \times 10^{-3}}{1} = 8 \times 10^{-3}\text{A/V}$$
$$r_{o1} = r_{o2} = \frac{V_A}{I_{D1}} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 4 \times 10^{-3}} = 2.5\text{ k}$$

Q1 is a CS amplifier with Q2 as its active load ($R_D = r_{o2} = 2.5$ k). Thus,

$$\frac{v_o}{v_i} = A_v = -g_m(r_{o1} \parallel R_D) = 8 \times 10^{-3} \times (2.5\text{ k} \parallel 2.5\text{ k}) = -10$$

**Problem 4.** Consider the circuit below $\mu_p C_{ox}(W/L) = 3.2$ mA/V$^2$, $V_t = -1$ V and $\lambda = 0$. A) Find $R$ such that $I_D = 200$ $\mu$A. B) Find the mid-frequency gain. C) Find $f_L$.




Starting with the Ohm's Law across the 20k resistor:

$$10 - V_S = 20 \times 10^3 I_D$$

$$V_S = 10 - 20 \times 10^3 \times 200 \times 10^{-6} = 10 - 4 = 6 \text{ V}$$

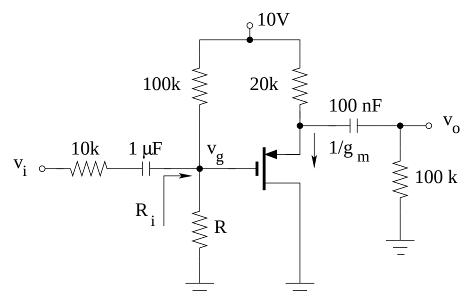
$$200 \times 10^{-6} = I_D = 0.5\mu_p C_{ox}(W/L)V_{OV}^2 = 1.6 \times^{-3} V_{OV}^2$$

$$V_{OV} = 0.354 \text{ V} \qquad \rightarrow \qquad V_{SG} = V_{OV} + |V_t| = 1.35 \text{ V}$$

$$V_{SG} = V_S - V_G \qquad \rightarrow \qquad V_G = 6 - 1.35 = 4.65 \text{ V}$$

$$V_G = 4.65 = \frac{R}{R + 100 \text{ k}} \times 10 \qquad \rightarrow \qquad R = 87 \text{ k}$$

This is a common drain amplifier (source follower):

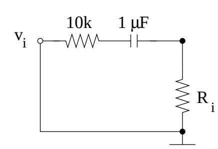



$$g_m = \frac{2I_D}{V_{OV}} = 1.13 \text{ mA/V} \qquad r_o \rightarrow \infty$$

$$\frac{v_o}{v_g} = A_v = \frac{g_m(R_S \parallel R_L)}{1 + g_m(R_S \parallel R_L)}$$

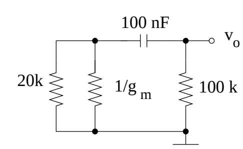
$$R_S \parallel R_L = 20 \text{ k} \parallel 100 \text{ k} = 16.7 \text{ k}$$

$$\frac{v_o}{v_g} = \frac{1.13 \times 16.7}{1 + 1.13 \times 16.7} = 0.95$$




$$R_i = R \parallel 100 \text{ k} = 87 \text{ k} \parallel 100 \text{ k} = 46.5 \text{ k}$$

$$\frac{v_g}{v_i} = \frac{R_i}{R_i + 10 \text{ k}} = 0.82$$




$$\frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = (0.95)(0.82) = 0.78$$

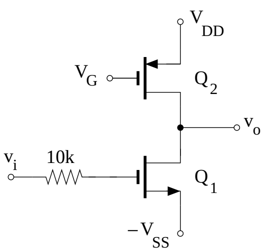
To find $f_L$, we compute resistances seen across the terminals of the two capacitors (see circuits with $1/g_m \parallel 20$ k $= 885 \parallel 20$ k $= 850$ $\Omega$)

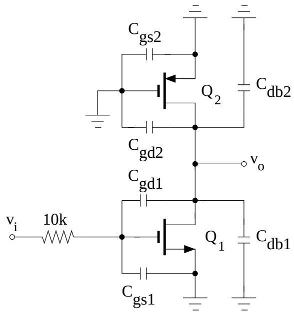
$$f_{p1} = \frac{1}{2\pi(10 \text{ k} + R_i) \times 10^{-6}} = 2.82 \text{ Hz}$$

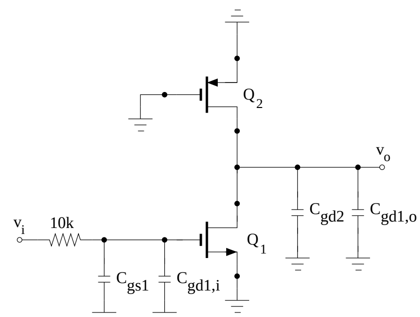
$$f_{p2} = \frac{1}{2\pi(100 \text{ k} + 850) \times 100 \times 10^{-9}} = 15.9 \text{ Hz}$$

$$f_L = f_{p1} + f_{p2} = 18.7 \text{ Hz}$$

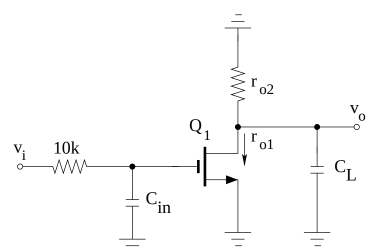
**Problem 5.** Find $f_H$ in the circuit below with identical transistors. Use $V_{OV} = 0.5$ V, $I_D = 50\ \mu$A, $V_A = 10$ V, $C_{gs} = 250$ fF, $C_{gd} = 80$ fF, and ignore $C_{db}$. (10 pts).




Including internal capacitances of transistors (middle circuit above with $C_{db}$ also shown), we see that both $C_{gs2}$ terminals are connected to the ground, $C_{gd2}$ is between output and the ground, $C_{gs1}$ is between input and the ground and $C_{gd1}$ is between input and output. Using Miller's Theorem to replace $C_{gd1}$ we arrive at the circuit above, right. Comibing capacitors, we get:

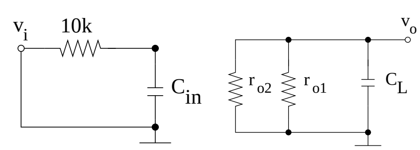



$$g_{m1} = g_{m2} = \frac{2I_D}{V_{OV}} = 0.2 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I_D} = 200 \text{ k}$$

$$A = \frac{v_{d1}}{v_{g1}} = -g_{m1}(r_{o1} \parallel r_{o2}) = -g_{m1}(200 \text{ k} \parallel 200 \text{ k})$$

$$A = -0.2 \times 10^{-3} \times 100 \times 10^3 = -20$$




$$C_{gd1,i} = C_{gd1}(1 - A) = 80(1 + 20) = 1,680 \text{ fF}$$

$$C_{gd1,o} = C_{gd1}(1 - 1/A) = 80(1 + 1/20) = 84 \text{ fF}$$

$$C_{in} = C_{gd1,i} + C_{gs1} = 1,680 + 250 = 1,930 \text{ fF}$$

$$C_L = C_{gd1,o} + C_{gd2} = 84 + 80 = 164 \text{ fF}$$

Computing the time constants assocaited with each capacitor (see circuits above):

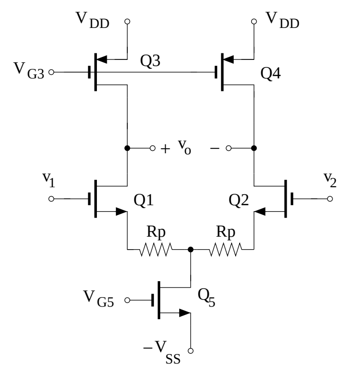
$$\tau_{in} = 10^4 C_{in} = 10^4 \times 1,930^{-15} = 1.93 \times 10^{-8}$$

$$\tau_L = (r_{o1} \parallel r_{o2})C_L = 100 \times 10^3 \times 164^{-15} = 1.64 \times 10^{-8}$$

$$\tau = \tau_{in} + \tau_L = 3.57 \times 10^{-8}$$

$$f_H = 1/(2\pi\tau) = 4.4 \text{ MHz}$$

**Problem 6.** Consider the circuti below with identical transistors having $V_{OV} = 0.5$ V and $V_A = 10$ V. Drin current of Q5 is 200 $\mu$A. Find the differential gain for A) $R_p = 0$, and B) $R_p = 1$ k.



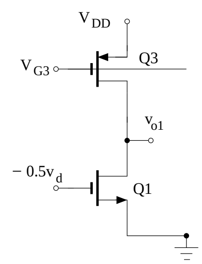

Because of symmetry, the drain current of Q5 is equally divided between the two half-circuits:

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = 100\ \mu\text{A}$$

$$g_{m1} = g_{m2} = g_{m3} = g_{m4} = \frac{2I_D}{V_{OV}} = 0.4\ \text{mA/V}$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{V_A}{I_D} = 100\ \text{k}$$

*Part A, $R_p = 0$:* Using half circuit concept, we arrrive at a CS amplifier (Q1) with Q3 acting as its active load:

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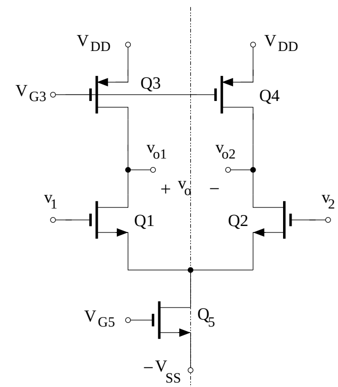



$$\frac{v_{o1}}{-0.5v_d} = -g_{m1}(r_{o1} \parallel r_{o3})$$

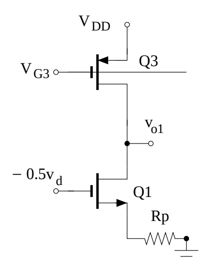
$$\frac{v_{o1}}{-0.5v_d} = -g_{m1}(100\ \text{k} \parallel 100\ \text{k})$$

$$\frac{v_{o1}}{-0.5v_d} = -0.4 \times 10^{-3} \times 50 \times 10^3 = -20$$

$$v_{o1} = -10(-0.5v_d) = 10v_d$$

$$v_{o2} = -v_{o1} = -10v_d \qquad \longrightarrow v_o = v_{o1} - v_{o2} = -20v_d \qquad \longrightarrow A_d = v_o/v_d = -20$$

*Part B, $R_p = 1$ k:* Using half circuit concept, we arrrive at a CS amplifier with source resistance (Q1) with Q3 acting as its active load:

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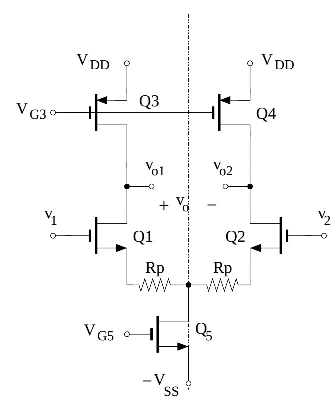


$$\frac{v_{o1}}{-0.5v_d} = -\frac{g_{m1}r_{o3}}{1 + g_{m1}R_p + r_{o3}/r_{o1}}$$

$$\frac{v_{o1}}{-0.5v_d} = -\frac{0.4 \times 10^{-3} \times 10^5}{2 + 0.4 \times 10^{-3} \times 1 \times 10^3} = -16.7$$

$$v_{o1} = -10(-0.5v_d) = 8.3v_d$$

$$v_{o2} = -v_{o1} = -8.3v_d \qquad \longrightarrow v_o = v_{o1} - v_{o2} = -16.7v_d \qquad \longrightarrow A_d = v_o/v_d = -16.7$$