Formula Sheet

You may detach this page, but it must be turned in with your exam.

Error in Numerical Approximations

Suppose you are approximating the integral $\int_a^b f(x)\mathrm{d}x$.

- Assume that $|f''(x)| \leq M$ for all $a \leq x \leq b$. Then the total error introduced by the trapezoidal rule is bounded by $\frac{M}{12}\frac{(b-a)^3}{n^2}$.
- Assume that $|f^{(4)}(x)| \leq L$ for all $a \leq x \leq b$. Then the total error introduced by Simpson's rule is bounded by $\frac{L}{180}\frac{(b-a)^5}{n^4}$.

Common Taylor Series

$$
\begin{aligned}
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} &&= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots && \text{for all } -\infty < x < \infty \\
\sin(x) &= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!}x^{2n+1} &&= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots && \text{for all } -\infty < x < \infty \\
\cos(x) &= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!}x^{2n} &&= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots && \text{for all } -\infty < x < \infty \\
\frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n &&= 1 + x + x^2 + x^3 + \cdots && \text{for all } -1 < x < 1 \\
\log(1+x) &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} &&= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots && \text{for all } -1 < x \leq 1 \\
\arctan x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} &&= x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots && \text{for all } -1 \leq x \leq 1
\end{aligned}
$$

Taylor Remainder

The Lagrange Remainder Formula guarantees that for any evaluation point x, if M_{n+1} is a constant such that $|f^{(n+1)}(c)| \leq M_{n+1}$ for all c between x and a, then

$$|E_n| = |f(x) - T_n(x)| \leq \frac{M_{n+1}}{(n+1)!}|x-a|^{n+1}.$$