CSE4312F12 Project Solution ROI

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Note

- A customer elicitation session was held during class on Tuesday November 6, 2012. If you were not there sure to catch up with a fellow student who was there.
- This template is handed out *caveat emptor*. There may be errors and wrong information. It is ultimately your responsibility to elicit the correct requirements from the customer and to ensure that you satisfy the customer goals and specify correct output from the input.
- Your are required to correct any errors or ambiguities in this template and use this template to produce your final requirements document.

Revisions

Date	Revision	Description
10 October 2012	1.0	Initial customer elicitation
15 November 2012	2.0	Initial Student solution
1 December 2012	3.0	Final Student solution

Contents

1	Con	text Diagram	3
2	Dic	tionary	4
3	E/F 3.1 3.2	R-descriptions E-descriptions	
4	Mat 4.1	Function tables	18 19 20 21
\mathbf{A}	RE	GEXP	23
Li	st of	Figures	
	1 2 3 4	Context diagram for the ROI system	11 12
Li	st of	Tables	
	11 15 16 17 18	Mathematical model for the ROI system Function table for ROI system (calculation of the TWRs) Function table for ROI system (calculation of the ROIs) Function table for ROI system (calculation of the benchmarks) Function table for ROI system (name of the portfolio history)	19

1 Context Diagram

The following diagram is the context diagram for the ROI system.

The monitored variables (which are the content of the CSV file, provided by the user), are :

- an header, which is composed of a required name, an optional description of the file and optional information about the customer (account number, email, address and phone number)
- the evaluation dates (start and end)
- the tuple data (date, market value, cash flow, agent fees and benchmark).

The format of the output is the following (whole input = everything between the earliest date and the latest date in the sequence of tuple data):

Name: ??

Whole input: yyyy-mm-dd to yyyy-mm-dd

TWR: ?? % ROI: ?? %

Benchmark: ?? %

Evaluation Period: yyyy-mm-dd to yyyy-mm-dd

TWR: ?? % ROI: ?? %

Benchmark: ?? %

The controlled variables are also a warning (if a calculation is not possible, or if there is no name) and an error (if the CSV file is not valid).

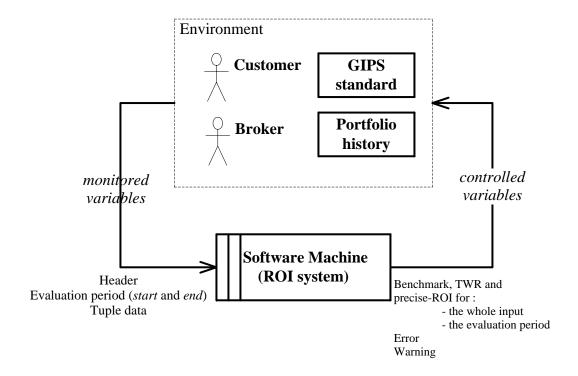


Figure 1: Context diagram for the ROI system

2 Dictionary

Agent fees: Money that the customer pays to the investment advisor to run the account.

Benchmark: Standard used as a point of reference for evaluating performance.

Cash Flow: Revenue or expense stream that changes a cash account over a given period.

CSV: Comma Separated Value file format used to store tabular data in which numbers and text are stored in plain-text form that can be easily written and read in a text editor.

Customer: The user of the software system.

Evaluation Period: a start and end date (provided by the user) for the portfolio history over which the return on investment is calculated.

GIPS: Global Investment Performance Standards

Investment broker: Runs the portfolio on behalf of the customer and supplies portfolio accounts.

Portfolio statement: List of all investments and current value.

Portfolio History: the historical data of investment performance over time that the customer stores about their investments as gleaned from their monthly or yearly investment accounts. Usually stored by customers in a CSV file (see Figure 1).

ROI: Return On Investment: Performance measure used to evaluate the efficiency of an investment.

TWR: Time Weighted Return: Measure of the compound rate of growth in a portfolio.

Tuple data: date, market value, cash flow, agent fees and benchmark.

3 E/R-descriptions

3.1 E-descriptions

ID	Description	Comment
E1	Customers create and store a portfolio history, i.e.	
	the historical data of their investment performance	
	as determined from portfolio statements.	
E2	Customers store their portfolio history as a CSV	
	text file. CSV files may be prepared on editors	
	of any operating system and encoded as ANSI or	
	UTF-8.	

	Header of the CSV file		
E3.1	Every portfolio history has a name.		
E3.2	Optionally, every portfolio history has a		
	description, account number, email, address,		
	and phone number fields.		

	Evaluation period in the CSV file				
E4.1	Optionally, every portfolio has an evaluation	See Invariant 1 of			
	period that is between the start and end date of	TWR_ROI_CALCULATION			
	the historical performance data.	(Fig. 11)			

E4.2	The start date and the end date must be in ISO				
	format (yyyy-mm-dd).				
E4.3	The evaluation period is in range.	See	Invariant	1	of
		TWR	_ROI_CALC	CULAT	TON
		(Fig.	11)		

	Data in the CSV file	
E5.1	A portfolio history records investment	See tr of
	performance in a non-empty sequence of tuple	TWR_ROI_CALCULATION
	data, each tuple having the fields: date (required),	(Fig. 11)
	market value (required), cash flow (optional),	
	agent fees (optional) and benchmark (optional).	
E5.2	For each tuple, the dates must be in ISO format	
	(yyyy-mm-dd).	
E5.3	When there is a customer contribution, the cash	
	flow is a positive number. For a withdrawal, the	
	number is negative.	
E5.4	Agent fees can be internal (deducted from within	
	the portfolio) or external (additional amounts paid	
	by the customer to the investment broker). The	
	portfolio history reflects only external agent fees,	
	always reported as a non-negative amount.	
E5.5	Every data tuple (row in the CSV file) has a date	See Invariant 2 of
	and a non-negative market value.	TWR_ROI_CALCULATION
		(Fig. 11)
E5.6	Dates in the tuples are unique and ordered.	See Invariant 3 of
		TWR_ROI_CALCULATION
		(Fig. 11)
E5.7	No withdrawal in the tuple data can be greater	See Invariant 4 of
	than the market value.	TWR_ROI_CALCULATION
		(Fig. 11)
E5.8	An account cannot grow from zero market value	See Invariant 5 of
	and cash flow.	TWR_ROI_CALCULATION
		(Fig. 11)

E5.9	For each tuple, the market value plus cash-flow	See	precon	dition	3
	plus agent-fees must be non-zero.	of	feature	twr	of
		TWI	R_ROI_CAI	CULA	ΓΙΟΝ
		(Fig.	11)		

3.2 R-descriptions

ID	Description	Comment
R1	All return on investment calculations shall follow	See twr, roi, benchmark
	the GIPS standard.	(Fig. 11)

	Evaluation period	
R2.1	If no evaluation period is provided, then the start	See Start_Valid and
	date is the earliest date and the end date the latest	End_Valid in Function
	date in the sequence of tuple data.	Table
R2.2	Warning message: If the evaluation dates are not	See Function Table
	valid, then the following error message shall be	
	displayed to the user: "Invalid evaluation period"	

	CSV file	
R3.1	Error message: If the CSV file is not valid (i.e.	See Function table
	if any of the conditions mentioned above do not	
	hold), then the following error message shall be	
	displayed to the user: "Invalid file".	
R3.2	Warning message: If the CSV file does not contain	See Function table
	a name, then the following error message shall be	
	displayed to the user: "Incomplete file: absence of	
	name".	

	Calculation of the TWR			
R4.1	The system shall provide two TWRs (if each one	See Function Table		
	is calculable): one for the evaluation period, and			
	one for the whole input.			
R4.2	The TWRs shall be rounded to two decimal places.			

R4.3	If the evaluation period is less than a year, then	See postcondition of
	the TWR shall be reported in absolute terms as	$annual_compounded_TWR$
	a percentage return (i.e. it is not annualized). If	of
	the evaluation period is a year or more, then the	TWR_ROI_CALCULATION
	TWR is annualized to a percentage per year.	(Fig. 11)
R4.4	The annualized TWR shall be reported as a	See
	percentage.	$annual_compounded_TWR$
		of
		TWR_ROI_CALCULATION
		(Fig. 11)
R4.5	Agent fees are treated like a deposit (the agent fees	See twr of
	are <u>added</u> to the market value and the cash flow).	TWR_ROI_CALCULATION
		(Fig. 11)
R4.6	Warning message: If the TWR is not calculable,	See Function Table
	Training message. If the 1 111 is not calculate,	Sec I director I diste
	then a warning message shall be displayed to the	See I discussi I diste

	Calculation of the ROI	
R5.1	The system shall provide two ROIs : one for the	See Function Table
	evaluation period, and one for the whole input.	
R5.2	The ROIs shall be rounded to two decimal places.	
R5.3	The ROI shall be reported as a percentage.	See roi of
		TWR_ROI_CALCULATION
		(Fig. 11)
R5.4	Agent fees are treated like a deposit (the agent fees	See roi of
	are <u>added</u> to the cash flow).	TWR_ROI_CALCULATION
		(Fig. 11)

	Calculation of the Benchma	rk
R6.1	The system shall provide two benchmarks (if each	See Function Table
	one is calculable): one for the evaluation period,	
	and one for the whole input.	
R6.2	The benchmarks shall be rounded to two decimal	
	places.	

R6.3	The benchmark shall be reported as a compounded	See benchmark of
	ROI, if the benchmark figures are available for the	TWR_ROI_CALCULATION
	evaluation period.	(Fig. 11)
R6.4	Warning message: If the benchmark is not	See Function Table
	calculable, then a warning message shall be	
	displayed to the user.	

4 Mathematical model

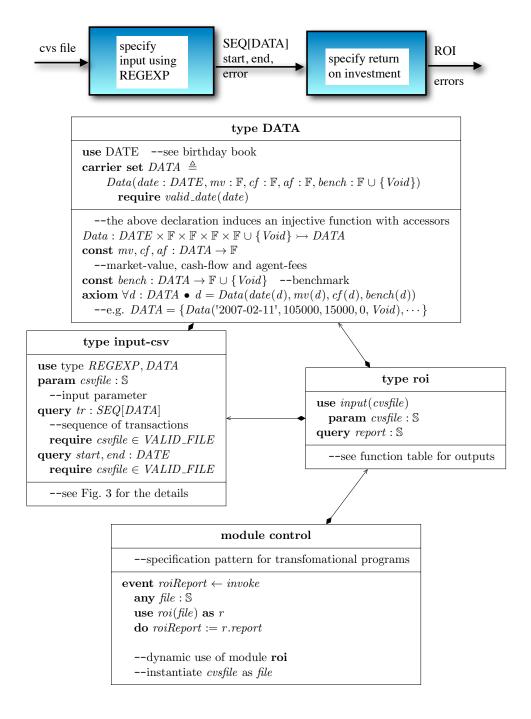


Figure 2: Module specification of return on investment

```
type input-csv
use type REGEXP, DATA, DATE --we let \epsilon = \{\text{""}\}, eol = \{\text{"\n"}\} etc.
carrier set DATA \triangleq Data(date : DATE, mv : \mathbb{F}, cf : \mathbb{F}, af : \mathbb{F}, bench : \mathbb{F} \cup \{Void\})
param csvfile : \mathbb{S} --input parameter
query tr: SEQ[DATA] --sequence of transactions defined by axiom below
  require csvfile \in VALID\_FILE
query start, end : DATE
  require csvfile \in VALID\_FILE
{f const}\ VALID\_FILE: REGEXP
 \triangleq HEADER \cdot PARAMETERS \cdot eol \cdot ROW \cdot *(eol \cdot ROW) \cdot *(","|eol)
const HEADER: REGEXP
  \triangleq *(HLINE · eol)
{f const} HLINE: REGEXP
  \triangleq *(\Sigma \setminus eol) \setminus (EV \_PER \cdot *\Sigma)
const PARAMETERS : REGEXP
  \triangleq EV\_PER \cdot DATE\_STR \cdot "\_to\_" \cdot DATE\_STR \cdot *", " \cdot eol \cdot COL\_HEAD
\mathbf{const}\ \mathit{COL\_HEAD} : \mathit{REGEXP}
 \triangleq + "," · eol·
      "Transaction_Date,Market_Value,Cash_Flow,Agent_Fees,Benchmark" · * ","
\mathbf{const}\ EV\_PER : REGEXP \triangleq \text{"Evaluation\_Period:\_"}
const ROW : REGEXP
  \triangleq (DATE\_STR \cdot "," \cdot FLOAT \cdot "," \cdot (FLOAT|\epsilon) \cdot "," \cdot (FLOAT|\epsilon)
      "," \cdot (FLOAT \cdot "\%" | \epsilon) \cdot *",")
const s2d: DATE\_STR \rightarrow DATE --see birthday book for DATE
const s2f: FLOAT \to \mathbb{F} --deferred, FLOAT is the string version of \mathbb{F}
const f2s: \mathbb{F} \to FLOAT --deferred, see your favourite programming language
\mathbf{const}\ d2s: DATE \to DATE\_STR \quad \text{--deferred}
const s2optf[G]: (FLOAT|\epsilon) \times G \to \mathbb{F} \cup G --string-to-optional float
   where \forall G \bullet s2optf \in (FLOAT|\epsilon) \times G \rightarrow \mathbb{F} \cup G
   --parameter G is a set such as \{Void\} or a default value such as \{0\}
const f: ROW \rightarrow DATA
dummy w : ROW and s_0, s_1, s_2, s_3 : \mathbb{S}
axiom 1: --definition of function f that maps a row string to data
            w \in (d2s(d) \cdot ", " \cdot s_0 \cdot ", " \cdot s_1 \cdot ", " \cdot s_2 \cdot ", " \cdot s_3 \cdot * ", ")
        \land (s_4 \cdot "\%" = s_3 \lor s_4 = s_3 = \epsilon)
      \Rightarrow f(w) = Data(d, s2f(s_0), s2optf(s_1, 0), s2optf(s_2, 0), s2optf(s_4, Void))
query error : \mathbb{B} \triangleq textfile \notin VALID\_FILE --definition of tr, start, end
axiom 2: --definition of tr, start, end
            csvfile \in VALID\_FILE \Rightarrow
            (\exists h, foot, s, e : \mathbb{S}; data : SEQ[ROW]
             | h \in HEADER \cdot EV\_PER \cdot s \cdot \text{``\_to\_''} \cdot e \cdot \text{*``,''} \cdot eol \cdot COL\_HEAD
               \wedge data \in SEQ[ROW]
               \land end \in *(', '|eol)
               \land textfile \in h \cdot (\cdot i | 0 \le i < \#data \bullet eol \cdot data(i)) \cdot foot
                 tr = (\cdot i | 0 \le i < \# data \bullet < f(data(i)) >
                 \land (start = s2d(s)) \land (end = s2d(e))
```

Figure 3: Type input-csv

TWR_ROI_CALCULATION

```
--input (input.csv)
tr: SEQ[DATA]
  --sequence of data [date, market_value, cash_flow, agent_fees, benchmark]
  --tr.domain = \{1,2,...,tr.count\}
count: INTEGER \triangleq tr.count
dates: SET[DATE] \triangleq \{t \in tr \bullet t.date\}
start: DATE \triangleq tr[1] --first date of the file
end : DATE \triangleq tr[count] --last date of the file
duration: VALUE \triangleq days(end - start) \div (365.2422)
  --years between start and end calculated by days
  --days(x) similar to Excel
  --output calculation (input.out.csv)
di (d:DATE): INTEGER
  --index into sequence of transaction for date d
  require d \in dates
  ensure Result \in tr.domain \land tr/Result/.date=d
TODO: DANS REQUIRE I VARIE DANS L'INTERVALLE D'INDICE DEFINIT PAR S ET E
IL FAUT VERIFIER QUE CE SOIT BIEN CA DANS LES E/R DESCRIPTIONS
  --TWR for the period s \dots e
twr (s, e: DATE): VALUE
  require
    s, e \in dates
    \forall i \in di(s)+1..di(e) \bullet tr[i-1].mv + tr[i-1].cf + tr[i-1].af \neq 0
  ensure
     Result \triangleq (\Pi i: INTEGER \mid di(s) < i \leq di(e) \bullet wealth(i)) - 1
```

```
where wealth(i) \triangleq tr[i].mv \div (tr[i-1].mv + tr[i-1].cf + tr[i-1].af)
annual_compounded_TWR (s, e: DATE): VALUE
  ensure
    (duration \ge 1) \Rightarrow Result = ((1 + twr(s, e))^{1 \div duration} - 1) * 100
    (duration < 1) \Rightarrow Result = twr(s, e) * 100
TODO: IL FAUT RETIRER DES E-R DESCRIPTION QUE
AF EST UN DEPOSIT AILLEUR QUE POUR LE TWR
TODO: VERIFIER QUE LE + POUR LES AGENT FEES
CI DESSOUS EST CORRECT
roi (s, e: DATE): VALUE
  require
    s, e \in dates
    e > s
  ensure
    (tr[m].mv + tr[m].cf) * (1 + Result \div 100)^{days(e-s) \div 365.2422}
         + (\Sigma i : \mathbb{N} | m < i < n \bullet (tr[i].cf + tr[i].af) *
         (1 + Result \div 100)^{days(e-tr[i].date) \div 365.2422}) - tr[n].mv = 0
  where
    m = di(s)
    n = di(e)
year(d : DATE): INTEGER
  require
    d = "yyyy - mm - dd"
  ensure
    Result = yyyy
mon(d : DATE): INTEGER
  require
    d = "yyyy - mm - dd"
  ensure
    Result = mm
day(d : DATE): INTEGER
```

```
require
    d = "yyyy - mm - dd"
  ensure
    Result = dd
TODO FAIRE LES FONCTION TABLE POUR MIN ET MAX
TODO VOIR POUR LE REQUIRE DE MIN ET MAX
\uparrow (f, s: DATE): DATE
  require
  ensure
    << see table...>>
\downarrow (f, s: DATE): DATE
  require
  ensure
    << see table...>>
TODO FAIRE ET NUMEROTER LA FUNCTION TABLE
DE LA FCT SUIVANTE
at (d: DATE): DATA
  ensure
    << see table... >>
bm_calculable (s, e: DATE): BOOL
  require
  ensure
    Result = at[end \downarrow a].bench \neq void
         \land (\forall d \in DATE | mon(d) = day(d) = 1 \land year(s) \leq year(d) \leq year(e)
             • at[d].bench \neq void
         \land ((C \land at[s].bench \neq void) \lor (\neg C \land at[end \downarrow b].bench \neq void))
  where
    a = "year(s) + 1 - 01 - 01"
    b = "year(e) + 1 - 01 - 01"
    C = mon(e) = 1 \land day(e) = 1
TODO RAJOUTER LA FUNCTION TABLE
  --the function below return a set of DATA with a benchmark \neq void
bm\_seq (s, e: DATE): SEQ[DATA]
```

```
require
     s, e \in dates
     bm_{-}calculable(s, e)
  ensure
     Result = (\bigoplus i | di(s) < i < di(e) \land tr[i].bench \neq void \bullet < tr[i] >) \oplus < t >
  where
     t = \ll \text{see table } \dots \gg
TODO DEFINIR LA FONCTION PO()
bm\_final\_value(s, e : DATE): VALUE
  require
     s, e \in dates
     bm_{-}calculable(s, e)
  ensure
     Result \triangleq
     tr[m].mv*(\Pi i, seq, t|seq = bm\_seq(s, e) \land a \oplus seq[0] \land i \ge 1
           \land t = seq[i] \bullet t.bench^{po(s,e,seq[i-1].date,t.date)})
     +(\Sigma k|m \le k \le n \bullet (tr[k].cf - tr[k].af)*
           (\Pi i, seq, t | seq = bm\_seq(tr[k].date, e) \land b(k) \oplus seq \land i \ge 1
           \wedge t = seq[i] \bullet t.bench^{po(s,e,seq[i-1].date,t.date)})
  where
     m = di(s)
     n = di(e)
      -a defines seq[0] in the first product
      -b(k) defines seq[0] in the second product
     a = < ("year(s) - 01 - 01", 0, 0, 0, void) >
     b(k) = \langle ("year(tr[k].date) - 01 - 01", 0, 0, 0, void) \rangle
benchmark(s, e: DATE): VALUE
  require
     s, e \in dates
     bm\_calculable(s, e)
  ensure
     tr[m].mv*(Result+1)^{days(e-s)\div 365.2422})
     +(\Sigma k|m \le k \le n \bullet tr[k].cf*
           (Result + 1)^{days(e-tr[k].date) \div 365.2422}) - FV = 0
  where
```

```
m = di(s)

n = di(e)

FV \triangleq bm\_final\_value(s, e)
```

Invariants

- (1) (start<end)∧(start,end∈dates)
 --metadata evaluation period is in range and valid
- (2) $\forall t \in tr \bullet t. date \neq Void \land t. mv \geq 0$ —every row has a date and a non-negative market value
- (3) $\forall i \in 2... count \bullet tr[i]. date > tr[i-1]. date$ --date are unique and ordered
- (4) $\forall t \in tr \bullet t.mv + t.cf \geq 0$ --Cannot withdraw more than the market value
- (5) $\forall i \in 2...count \mid tr[i-1].mv = 0 \land tr[i-1].cf = 0 \bullet tr[i].mv = 0$ --account coannot grow from zero market value and cash flow

Table 11: Mathematical model for the ROI system

4.1 Function tables

4.1.1 Abbreviations, conditions and messages

Abbreviation	Description
first	tr[1].date
last	tr[tr.count].date
a_c_TWR	annual_compounded_TWR
	(see TWR_ROI_CALCULATION (Fig. 11))
b	benchmark (see TWR_ROI_CALCULATION (Fig. 11))

Condition	Description
C1	$\forall i \in 2count \bullet tr[i-1].mv + tr[i-1].cf + tr[i-1].af \neq 0$
C2	$\forall i \in di(start) + 1di(end) \bullet tr[i-1].mv + tr[i-1].cf + tr[i-1].af \neq 0$
C3	benchmark_calculable(first,last)
	(see TWR_ROI_CALCULATION (Fig. 11))
C4	benchmark_calculable(start,end)
	(see TWR_ROI_CALCULATION (Fig. 11))
Start_Valid	$(start \in dates \cup \{null\}) \land (start in ISO format)$
	(if Start_Valid \land start=null, then start=tr[1].date)
End_Valid	$(end \in dates \cup \{null\}) \land (end in ISO format)$
	(if $End_Valid \land end=null$, then $end=tr[tr.count].date$)
Name	(csvfile.name = null)

Message	Description
Е	"Invalid file"
W	"Invalid evaluation period"
W1	"The TWR for the whole input is not calculable"
W2	"The TWR's are not calculable"
W3	W + W1
W4	"Benchmark for the whole input is not calculable"
W5	"The benchmarks are not calculable"
W6	W + W4
W7	"Incomplete file: absence of name"

4.1.2 Calculation of the TWRs

			Error	Error Warning TWR	TWR	TWR
					(whole input)	(evaluation period)
Valid_CSV	$\neg(Start_Valid) \lor $	C1		W	a_c_TWR(first,last)	
	$\neg (\text{End_Valid}) \lor $					
	$end \leq start$					
		¬C1		W3		
		C1			a_c_TWR(first,last)	$a_{-c_TWR(first, last)} \mid a_{-c_TWR(start, end)} \mid$
	End_Valid ∧					
	end>start					
		¬C1 C2		W1		a_c_TWR(start,end)
		$\neg C2$		W2		
Invalid_CSV	1		E			

Table 15: Function table for ROI system (calculation of the TWRs)

4.1.3 Calculation of the ROIs

		Error	Warning	ROI (whole input)	Error Warning ROI (whole input) ROI (evaluation period)
Valid_CSV	$Valid_CSV \mid \neg(Start_Valid) \lor \mid$		W		
	¬(End_Valid) ∨				
	$end \leq start$				
	Start_Valid \			roi(first, last)	roi(start,end)
	End_Valid ∧				
	end>start				
Invalid_CSV	7	H			

Table 16: Function table for ROI system (calculation of the ROIs)

4.1.4 Calculation of the benchmarks

				Error	Warning	Error Warning Benchmark Benchmark	Benchmark
						(whole input)	$(whole\ input)\ \ (evaluation\ period)\ $
Valid_CSV	$ \neg(Start_Valid) \lor C3$	C3			W	b(first, last)	
	$\neg(\operatorname{End}_{\operatorname{-}}\operatorname{Valid}) \vee$						
	$end \leq start$						
		¬C3			9M		
	Start_Valid \	C3				b(first,last)	b(start,end)
	End_Valid ∧						
	end>start						
		¬C3 C4	C4		W4		b(start,end)
			¬C4		W5		
Invalid_CSV	1			田			

Table 17: Function table for ROI system (calculation of the benchmarks)

4.1.5 Name of the portfolio history

Name	Warning —
¬Name	W7

Table 18: Function table for ROI system (name of the portfolio history)

A REGEXP

A set of strings is used as the model for regular expressions. We use prefix operators for the Kleene closure (e.g. *x where x is a regular expression such as $\{'hello'\}$) and iteration at least one or more (e.g. +x) rather than suffix operators. Note that where there is no confusion we use 'hello' instead of $\{'hello'\}$ where the set is a singleton.

We may use type REGEXP to specify a FLOAT_STRING as follows.

$$FLOAT_STRING = '+'Inf$$
 (1)

$$|'-'Inf|$$
 (2)

$$|NaN|$$
 (3)

$$|('-'|'+'|\epsilon) \cdot (*d \cdot '.'|\epsilon) \cdot *d \cdot (('e' \cdot ('-'|\epsilon) \cdot +d) |\epsilon)$$

$$(4)$$

$$d = 0'|1'| \cdots |9'$$
 (5)

In the above we use the convention that 'e', for example, really stands for the single set {'e'}.

```
type REGEXP
carrier set REGEXP --set of all regular string expressions
axiom REGEXP \subseteq \mathbb{P}(\mathbb{S})
carrier set \Sigma \triangleq \{\text{"0", "1", "2", \cdots, "a", "b", etc., all printing characters}\}
dummy x, y, z : REGEXP
dummy s, t, u : \mathbb{S}
axiom \forall s \in \Sigma \bullet \{s\} \in REGEXP
const 0: REGEXP \triangleq \{\} --zero is the unit element of alternation
const 1: REGEXP \triangleq {""} --1 is the unit element of concatenation
  --we also use \epsilon instead of 1
const infix "|" : REGEXP \times REGEXP \rightarrow REGEXP
  --alternation
const infix "\cdot": REGEXP \times REGEXP \rightarrow REGEXP
  --concatenation
const prefix "*": REGEXP \times REGEXP \rightarrow REGEXP
  --iteration zero or more times
const prefix "+" : REGEXP \times REGEXP \rightarrow REGEXP
  --iteration one or more times
axiom s \in x | y \equiv s \in x \lor s \in y
theorem x|0=0|x=x
axiom s \in x \cdot y \equiv (\exists t, u | s = t \cdot u \bullet t \in x \land u \in y)
  --note that t \cdot u is concatenation over SEQ[\mathbb{S}]
theorem 1 \cdot x = x \cdot 1 = 1 --1 is the identity of concatenation
const infix "^": REGEXP \times \mathbb{N} \to REGEXP
  --use this operator by raising the second argument like an exponent
axiom x^n = (i \mid 0 \le i \le n \bullet x) --concatenation quantifier
  --e.g. x^3 = x \cdot x \cdot x
theorem x^0 = 1
axiom s \in *x \equiv (\exists n : \mathbb{N} \bullet s \in x^n)
axiom s \in +x \equiv (\exists n : \mathbb{N}_1 \bullet s \in x^n)
```

Figure 4: Type REGEXP for regular expressions over printing characters