Let a firm have a standard production function of $y(k,l;z) = zk^{\alpha}l^{\beta}$. Capital is quasi-fixed and labor hired on the spot. For now firm productivity z is considered constant. Assume that the firm has a maximum capacity determined by installed capital. In particular, for every machine the firm owns it can usefully employ at most $\bar{c}$ workers, so that $l/k \leq \bar{c}$. Let the firm compete monopolistically such that the relationship between price and quantity produced is given by the standard demand function $q^{d}(b_i, p) = b_i P^{\sigma-1} Y / p^{\sigma}$ where P and Y are aggregate variables exogenous to the firm, and b_i is a random variable shifting demand in a proportional fashion. Inverting the production function yields labor input required for any level of production y with given z, k as $l(y;z,k) = [y/(zk^{\alpha})]^{1/\beta}$. Finally, due to the capacity constraint, maximum production happens when $l = \bar{c}k$ and amounts to $\bar{y}(z,k) \equiv zk^{\alpha+\beta}\bar{c}^{\beta}$.

Due to the capacity constraint, actual quantity produced is given by $q^{s}(b_i;z,k,p) = \min\left\{\bar{y}(z,k), q^{d}(b_i,p)\right\}$. The cutoff value $\bar{b}$ for which the constraint binds is hence given by$\bar{b}P^{\sigma-1}Y/p^{\sigma} = zk^{\alpha+\beta}\bar{c}^{\beta}$or

$$\bar{b} = \frac{zk^{\alpha+\beta}\bar{c}^{\beta}p^{\sigma}}{P^{\sigma-1}Y}.$$

Flow returns in any period are then given by

$$\begin{aligned}
E[ret] &= \int_0^{\bar{b}} \left[pq^{d}(b_i,p) - wl\left(q^{d}(b_i,p);z,k\right)\right] dF(b_i) + \left[1 - F\left(\bar{b}\right)\right]\left[p\bar{y}(z,k) - wl\left(\bar{y}(z,k);z,k\right)\right] \\
&= \int_0^{\bar{b}} \left[\frac{b_i P^{\sigma-1}Y}{p^{\sigma-1}} - w\left(\frac{b_i P^{\sigma-1}Y}{p^{\sigma}zk^{\alpha}}\right)^{1/\beta}\right] dF(b_i) + \left[1 - F\left(\bar{b}\right)\right]\left[pzk^{\alpha+\beta}\bar{c}^{\beta} - wk\bar{c}\right] \\
&= \int_0^{\frac{zk^{\alpha+\beta}\bar{c}^{\beta}p^{\sigma}}{P^{\sigma-1}Y}} \left[\frac{b_i P^{\sigma-1}Y}{p^{\sigma-1}} - w\left(\frac{b_i P^{\sigma-1}Y}{p^{\sigma}zk^{\alpha}}\right)^{1/\beta}\right] dF(b_i) + \left[1 - F\left(\frac{zk^{\alpha+\beta}\bar{c}^{\beta}p^{\sigma}}{P^{\sigma-1}Y}\right)\right]\left[pzk^{\alpha+\beta}\bar{c}^{\beta} - w\right.
\end{aligned}$$

Now let's look at a period in which the firm has to pick its capacity and price before realization of b_i. The firm's economic profit, assuming it can resell a portion $1-\delta$ of capital is given by

$$\begin{aligned}
\pi = \max_{k,p} & \left[(1-\delta) - R\right]k + \\
& \int_0^{\frac{zk^{\alpha+\beta}\bar{c}^{\beta}p^{\sigma}}{P^{\sigma-1}Y}} \frac{b_i P^{\sigma-1}Y}{p^{\sigma-1}} - w\left(\frac{b_i P^{\sigma-1}Y}{p^{\sigma}zk^{\alpha}}\right)^{1/\beta} dF(b_i) + \\
& \left[1 - F\left(\frac{zk^{\alpha+\beta}\bar{c}^{\beta}p^{\sigma}}{P^{\sigma-1}Y}\right)\right]\left[pzk^{\alpha+\beta}\bar{c}^{\beta} - wk\bar{c}\right].
\end{aligned}$$

This can be rewritten as

$$\pi = \max_{k,p}\left[(1-\delta) - R\right]k + \frac{P^{\sigma-1}Y}{p^{\sigma-1}}\int_0^{\bar{b}} b_i dF(b_i) - w\left(\frac{P^{\sigma-1}Y}{p^{\sigma}zk^{\alpha}}\right)^{\frac{1}{\beta}}\int_0^{\bar{b}} b_i^{\frac{1}{\beta}} dF(b_i) + \left[1 - F\left(\bar{b}\right)\right]\left[pzk^{\alpha+\beta}\bar{c}^{\beta} - wk\bar{c}\right.$$

and if b_i is lognormal then the partial expectations can be written as

$$\int_0^{\bar{b}} b_i dF(b_i) = \frac{e^{\mu+\sigma^2/2}}{2}\left[1 - \operatorname{erf}\left(\frac{\sigma^2 + \mu - \ln b_i}{\sqrt{2}\sigma}\right)\right]$$

where $\mu$and σ are mean and standard deviation of the associated normal distribution. Similarly,

$$\begin{aligned}\int_0^{\bar{b}} b_i^{\frac{1}{\beta}} dF(b_i) &= \int_0^{\bar{b}} c dF(c) \\ &= \frac{e^{\mu/\beta+\sigma^2/(2\beta^2)}}{2}\left[1-\operatorname{erf}\left(\frac{\sigma^2/\beta+\mu-\beta\ln\bar{b}}{\sqrt{2}\sigma}\right)\right]\end{aligned}$$

as follows by a change of variable with $c = b_i^{\frac{1}{\beta}}$ and noting that $\ln c \sim N\left(\mu/\beta, \sigma^2/\beta^2\right)$.

The first-order conditions are, for p,

$$\begin{aligned}&\int_0^{\frac{zk^{\alpha+\beta}\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y}} \frac{b_i P^{\sigma-1}Y}{p^\sigma} + \frac{w\sigma}{\beta}\left(\frac{b_i P^{\sigma-1}Y}{zk^\alpha}\right)^{\frac{1}{\beta}} \frac{1}{p^{\frac{\sigma}{\beta}-1}} dF(b_i) + \\ &+ \left[zk^{\alpha+\beta}\bar{c}^\beta p - wk\bar{c}\right]\sigma\frac{zk^{\alpha+\beta}\bar{c}^\beta p^{\sigma-1}}{P^{\sigma-1}Y} f\left(\frac{zk^{\alpha+\beta}\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y}\right) + \\ &+ \left[1-F\left(\frac{zk^{\alpha+\beta}\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y}\right)\right] zk^{\alpha+\beta}\bar{c}^\beta - \\ &- f\left(\frac{zk^{\alpha+\beta}\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y}\right)\sigma\frac{zk^{\alpha+\beta}\bar{c}^\beta p^{\sigma-1}}{P^{\sigma-1}Y}\left[pzk^{\alpha+\beta}\bar{c}^\beta - wk\bar{c}\right] = 0\end{aligned}$$

and, for k,

$$\begin{aligned}R-(1-\delta) = &\int_0^{\frac{zk^{\alpha+\beta}\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y}} \frac{w\alpha}{\beta}\left(\frac{b_i P^{\sigma-1}Y}{p^\sigma z}\right)^{1/\beta} \frac{1}{k^{1+\frac{\alpha}{\beta}}} dF(b_i) + \\ &+ \left[zk^{\alpha+\beta}\bar{c}^\beta p - wk\bar{c}\right](\alpha+\beta)\frac{zk^{\alpha+\beta-1}\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y} f\left(\frac{zk^{\alpha+\beta}\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y}\right) + \\ &+ \left[1-F\left(\frac{zk^{\alpha+\beta}\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y}\right)\right]\left[(\alpha+\beta)pzk^{\alpha+\beta-1}\bar{c}^\beta - w\bar{c}\right] - \\ &- f\left(\frac{zk^{\alpha+\beta}\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y}\right)(\alpha+\beta)\frac{zk^{\alpha+\beta-1}\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y}\left[pzk^{\alpha+\beta}\bar{c}^\beta - wk\bar{c}\right]\end{aligned}$$

or, in case of linear homogeneity $\alpha+\beta=1$,

$$\begin{aligned}&\int_0^{\frac{zk\bar{c}^\beta p^\sigma}{P^{\sigma-1}Y}} \frac{b_i P^{\sigma-1}Y}{p^\sigma} + \frac{w\sigma}{1-\alpha}\left(\frac{b_i P^{\sigma-1}Y}{zk^\alpha}\right)^{\frac{1}{1-\alpha}} \frac{1}{p^{\frac{\sigma}{1-\alpha}-1}} dF(b_i) + \\ &+ \left[zk\bar{c}^{1-\alpha}p - wk\bar{c}\right]\sigma\frac{zk\bar{c}^{1-\alpha}p^{\sigma-1}}{P^{\sigma-1}Y} f\left(\frac{zk\bar{c}^{1-\alpha}p^\sigma}{P^{\sigma-1}Y}\right) + \\ &+ \left[1-F\left(\frac{zk\bar{c}^{1-\alpha}p^\sigma}{P^{\sigma-1}Y}\right)\right] zk\bar{c}^{1-\alpha} - \\ &- f\left(\frac{zk\bar{c}^{1-\alpha}p^\sigma}{P^{\sigma-1}Y}\right)\sigma\frac{zk\bar{c}^{1-\alpha}p^{\sigma-1}}{P^{\sigma-1}Y}\left[pzk\bar{c}^{1-\alpha} - wk\bar{c}\right] = 0\end{aligned}$$

and

$$
\begin{aligned}
R-(1-\delta) = & \int_0^{\frac{zk\bar{c}^{1-\alpha}p^{\sigma}}{P^{\sigma-1}Y}} \frac{w\alpha}{1-\alpha}\left(\frac{b_i P^{\sigma-1}Y}{p^{\sigma}z}\right)^{1/(1-\alpha)} \frac{1}{k^{\frac{1}{1-\alpha}}} dF(b_i) + \\
& + \left[zk\bar{c}^{1-\alpha}p - wk\bar{c}\right] \frac{z\bar{c}^{1-\alpha}p^{\sigma}}{P^{\sigma-1}Y} f\left(\frac{zk\bar{c}^{1-\alpha}p^{\sigma}}{P^{\sigma-1}Y}\right) + \\
& + \left[1 - F\left(\frac{zk\bar{c}^{1-\alpha}p^{\sigma}}{P^{\sigma-1}Y}\right)\right]\left[pz\bar{c}^{1-\alpha} - w\bar{c}\right] - \\
& - f\left(\frac{zk\bar{c}^{1-\alpha}p^{\sigma}}{P^{\sigma-1}Y}\right) \frac{z\bar{c}^{1-\alpha}p^{\sigma}}{P^{\sigma-1}Y}\left[pzk\bar{c}^{1-\alpha} - wk\bar{c}\right]
\end{aligned}
$$

With the realization of the demand shock marginal cost of production are determined by wage and labor productivity. In particular, in order to produce one additional (marginal) unit of output, a firm needs to hire

$$
\begin{aligned}
\frac{\partial l(y;k,z)}{\partial y} &= \frac{1}{\beta} y^{\frac{1-\beta}{\beta}} (zk^{\alpha})^{-1/\beta} \\
&= \frac{1}{\beta}\left(\frac{y^{1-\beta}}{zk^{\alpha}}\right)^{1/\beta} \\
&= \frac{1}{\beta}\left(\frac{\left(zk^{\alpha}l^{\beta}\right)^{1-\beta}}{zk^{\alpha}}\right)^{1/\beta} \\
&= \frac{1}{\beta}\frac{l^{1-\beta}}{zk^{\alpha}}
\end{aligned}
$$

so that marginal cost are just $\mathrm{MC} = w\beta^{-1}z^{-1}k^{-\alpha}l^{1-\beta}$ or if $\beta = 1-\alpha$

$$
\begin{aligned}
\mathrm{MC} &= \frac{w}{1-\alpha}\frac{l^{\alpha}}{zk^{\alpha}} \\
&= \frac{1}{z}\frac{w}{1-\alpha}\left(\frac{l}{k}\right)^{\alpha}.
\end{aligned}
$$

Analogously, average cost per unit produced is given by $\left[(R-1+\delta)k + wl\right]/\left(zk^{\alpha}l^{\beta}\right)$ and in case that $\beta = 1-\alpha$

$$
\begin{aligned}
\mathrm{AC} &= \frac{[R-1+\delta]\,k}{zk^{\alpha}l^{1-\alpha}} + \frac{wl}{zk^{\alpha}l^{1-\alpha}} \\
&= \frac{1}{z}\left[(R-1+\delta)\left(\frac{l}{k}\right)^{\alpha-1} + w\left(\frac{l}{k}\right)^{\alpha}\right]
\end{aligned}
$$