User Guide for SuiteSparse:GraphBLAS

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Abstract

SuiteSparse:GraphBLAS is a full implementation of the Graph-BLAS standard, which defines a set of sparse matrix operations on an extended algebra of semirings using an almost unlimited variety of operators and types. When applied to sparse adjacency matrices, these algebraic operations are equivalent to computations on graphs. GraphBLAS provides a powerful and expressive framework for creating high-performance graph algorithms based on the elegant mathematics of sparse matrix operations on a semiring.

When compared with MATLAB R2021a, some methods in Graph-BLAS are up to a million times faster than MATLAB, even when using the same syntax. Typical speedups are in the range 2x to 30x. The statement C(M)=A when using MATLAB sparse matrices takes $O(e^2)$ time where e is the number of entries in C. GraphBLAS can perform the same computation with the exact same syntax, but in $O(e\log e)$ time (or O(e) in some cases), and in practice that means GraphBLAS can compute C(M)=A for a large problem in under a second, while MATLAB takes about 4 to 5 days.

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1 Introduction

The GraphBLAS standard defines sparse matrix and vector operations on an extended algebra of semirings. The operations are useful for creating a wide range of graph algorithms.

For example, consider the matrix-matrix multiplication, $\mathbf{C} = \mathbf{AB}$. Suppose \mathbf{A} and \mathbf{B} are sparse n-by-n Boolean adjacency matrices of two undirected graphs. If the matrix multiplication is redefined to use logical AND instead of scalar multiply, and if it uses the logical OR instead of add, then the matrix \mathbf{C} is the sparse Boolean adjacency matrix of a graph that has an edge (i,j) if node i in \mathbf{A} and node j in \mathbf{B} share any neighbor in common. The OR-AND pair forms an algebraic semiring, and many graph operations like this one can be succinctly represented by matrix operations with different semirings and different numerical types. GraphBLAS provides a wide range of built-in types and operators, and allows the user application to create new types and operators without needing to recompile the GraphBLAS library.

For more details on SuiteSparse:GraphBLAS, and its use in LAGraph, see [Dav19, Dav23, Dav18, DAK19, ACD+20, MDK+19].

A full and precise definition of the GraphBLAS specification is provided in *The GraphBLAS C API Specification* by Aydın Buluç, Timothy Mattson, Scott McMillan, José Moreira, Carl Yang, and Benjamin Brock [BMM+17a, BMM+17b, BBM+21], based on *GraphBLAS Mathematics* by Jeremy Kepner [Kep17]. The GraphBLAS C API Specification is available at http://graphblas.org. This version of SuiteSparse:GraphBLAS conforms to Version 2.1.0 (Dec 22, 2023) of *The GraphBLAS C API specification*.

In this User Guide, aspects of the GraphBLAS specification that would be true for any GraphBLAS implementation are simply called "GraphBLAS." Details unique to this particular implementation are referred to as Suite-Sparse:GraphBLAS.

All functions, objects, and macros with a name of the form GxB_* are SuiteSparse-specific extensions to the specification.

SPEC: Non-obvious deviations or additions to the GraphBLAS C API Specification are highlighted in a box like this one, except for GxB* methods. They are not highlighted since their name makes it clear that they are extensions to the GraphBLAS C API.

2 Basic Concepts

Since the *GraphBLAS C API Specification* provides a precise definition of GraphBLAS, not every detail of every function is provided here. For example, some error codes returned by GraphBLAS are self-explanatory, but since a specification must precisely define all possible error codes a function can return, these are listed in detail in the *GraphBLAS C API Specification*. However, including them here is not essential and the additional information on the page might detract from a clearer view of the essential features of the GraphBLAS functions.

This User Guide also assumes the reader is familiar with MATLAB/Octave. MATLAB supports only the conventional plus-times semiring on sparse double and complex matrices, but a MATLAB-like notation easily extends to the arbitrary semirings used in GraphBLAS. The matrix multiplication in the example in the Introduction can be written in MATLAB notation as C=A*B, if the Boolean OR-AND semiring is understood. Relying on a MATLAB-like notation allows the description in this User Guide to be expressive, easy to understand, and terse at the same time. The GraphBLAS C API Specification also makes use of some MATLAB-like language, such as the colon notation.

MATLAB notation will always appear here in fixed-width font, such as C=A*B(:,j). In standard mathematical notation it would be written as the matrix-vector multiplication $C=Ab_j$ where b_j is the jth column of the matrix B. The GraphBLAS standard is a C API and SuiteSparse:GraphBLAS is written in C, and so a great deal of C syntax appears here as well, also in fixed-width font. This User Guide alternates between all three styles as needed.

2.1 Graphs and sparse matrices

Graphs can be huge, with many nodes and edges. A dense adjacency matrix \mathbf{A} for a graph of n nodes takes $O(n^2)$ memory, which is impossible if n is, say, a million. Let $|\mathbf{A}|$ denote the number of entries in a matrix. Most graphs arising in practice are sparse, however, with only $|\mathbf{A}| = O(n)$ edges, where $|\mathbf{A}|$ denotes the number of edges in the graph, or the number of explicit entries present in the data structure for the matrix \mathbf{A} . Sparse graphs with millions of nodes and edges can easily be created by representing them as sparse matrices, where only explicit values need to be stored. Some graphs

are hypersparse, with $|\mathbf{A}| << n$. SuiteSparse:GraphBLAS supports three kinds of sparse matrix formats: a regular sparse format, taking $O(n + |\mathbf{A}|)$ space, a hypersparse format taking only $O(|\mathbf{A}|)$ space, and a bitmap form, taking $O(n^2)$ space. Full matrices are also represented in $O(n^2)$ space. Using its hypersparse format, creating a sparse matrix of size n-by-n where $n = 2^{60}$ (about 10^{18}) can be done on quite easily on a commodity laptop, limited only by $|\mathbf{A}|$. To the GraphBLAS user application, all matrices look alike, since these formats are opaque, and SuiteSparse:GraphBLAS switches between them at will.

A sparse matrix data structure only stores a subset of the possible n^2 entries, and it assumes the values of entries not stored have some implicit value. In conventional linear algebra, this implicit value is zero, but it differs with different semirings. Explicit values are called *entries* and they appear in the data structure. The *pattern* (also called the *structure*) of a matrix defines where its explicit entries appear. It will be referenced in one of two equivalent ways. It can be viewed as a set of indices (i, j), where (i, j) is in the pattern of a matrix **A** if $\mathbf{A}(i, j)$ is an explicit value. It can also be viewed as a Boolean matrix **S** where $\mathbf{S}(i, j)$ is true if (i, j) is an explicit entry and false otherwise. In MATLAB notation, $\mathbf{S=spones}(\mathbf{A})$ or $\mathbf{S=(A^{\sim}=0)}$, if the implicit value is zero. The (\mathbf{i},\mathbf{j}) pairs, and their values, can also be extracted from the matrix via the MATLAB expression $[\mathbf{I},\mathbf{J},\mathbf{X}]=\mathbf{find}(\mathbf{A})$, where the kth tuple $(\mathbf{I}(\mathbf{k}),\mathbf{J}(\mathbf{k}),\mathbf{X}(\mathbf{k}))$ represents the explicit entry $\mathbf{A}(\mathbf{I}(\mathbf{k}),\mathbf{J}(\mathbf{k}))$, with numerical value $\mathbf{X}(\mathbf{k})$ equal to a_{ij} , with row index $i=\mathbf{I}(\mathbf{k})$ and column index $j=\mathbf{J}(\mathbf{k})$.

The entries in the pattern of **A** can take on any value, including the implicit value, whatever it happens to be. This differs slightly from MATLAB, which always drops all explicit zeros from its sparse matrices. This is a minor difference but GraphBLAS cannot drop explicit zeros. For example, in the max-plus tropical algebra, the implicit value is negative infinity, and zero has a different meaning. Here, the MATLAB notation used will assume that no explicit entries are ever dropped because their explicit value happens to match the implicit value.

Graph Algorithms in the Language on Linear Algebra, Kepner and Gilbert, eds., provides a framework for understanding how graph algorithms can be expressed as matrix computations [KG11]. For additional background on sparse matrix algorithms, see also [Dav06] and [DRSL16].

2.2 Overview of GraphBLAS methods and operations

GraphBLAS provides a collection of *methods* to create, query, and free its of objects: sparse matrices, sparse vectors, scalars, types, operators, monoids, semirings, and a descriptor object used for parameter settings. Details are given in Section 6. Once these objects are created they can be used in mathematical *operations* (not to be confused with the how the term *operator* is used in GraphBLAS). A short summary of these operations and their nearest MATLAB/Octave analog is given in the table below.

| operation | approximate MATLAB/Octave analog |
|---------------------------------|----------------------------------|
| matrix multiplication | C=A*B |
| element-wise operations | C=A+B and C=A.*B |
| reduction to a vector or scalar | s=sum(A) |
| apply unary operator | C=-A |
| transpose | C=A' |
| submatrix extraction | C=A(I,J) |
| submatrix assignment | C(I,J)=A |
| select | C=tril(A) |

GraphBLAS can do far more than what MATLAB/Octave can do in these rough analogs, but the list provides a first step in describing what GraphBLAS can do. Details of each GraphBLAS operation are given in Section 12. With this brief overview, the full scope of GraphBLAS extensions of these operations can now be described.

SuiteSparse:GraphBLAS has 13 built-in scalar types: Boolean, single and double precision floating-point (real and complex), and 8, 16, 32, and 64-bit signed and unsigned integers. In addition, user-defined scalar types can be created from nearly any C typedef, as long as the entire type fits in a fixed-size contiguous block of memory (of arbitrary size). All of these types can be used to create GraphBLAS sparse matrices, vectors, or scalars.

The scalar addition of conventional matrix multiplication is replaced with a monoid. A monoid is an associative and commutative binary operator z=f(x,y) where all three domains are the same (the types of x, y, and z), and where the operator has an identity value id such that f(x,id)=f(id,x)=x. Performing matrix multiplication with a semiring uses a monoid in place of the "add" operator, scalar addition being just one of many possible monoids. The identity value of addition is zero, since x + 0 = 0 + x = x. Graph-BLAS includes many built-in operators suitable for use as a monoid: min

(with an identity value of positive infinity), max (whose identity is negative infinity), add (identity is zero), multiply (with an identity of one), four logical operators: AND, OR, exclusive-OR, and Boolean equality (XNOR), four bitwise operators (AND, OR, XOR, and XNOR), and the ANY operator See Section 6.3.6 for more details on the unusual ANY operator. User-created monoids can be defined with any associative and commutative operator that has an identity value.

Finally, a semiring can use any built-in or user-defined binary operator z=f(x,y) as its "multiply" operator, as long as the type of its output, z matches the type of the semiring's monoid. The user application can create any semiring based on any types, monoids, and multiply operators, as long these few rules are followed.

Just considering built-in types and operators, GraphBLAS can perform C=A*B in thousands of unique semirings. With typecasting, any of these semirings can be applied to matrices C, A, and B of 13 predefined types, in any combination. This results in millions of possible kinds of sparse matrix multiplication supported by GraphBLAS, and this is counting just built-in types and operators. By contrast, MATLAB provides just two semirings for its sparse matrix multiplication C=A*B: plus-times-double and plus-times-complex, not counting the typecasting that MATLAB does when multiplying a real matrix times a complex matrix.

A monoid can also be used in a reduction operation, like s=sum(A) in MATLAB. MATLAB provides the plus, times, min, and max reductions of a real or complex sparse matrix as s=sum(A), s=prod(A), s=min(A), and s=max(A), respectively. In GraphBLAS, any monoid can be used (min, max, plus, times, AND, OR, exclusive-OR, equality, bitwise operators, or any user-defined monoid on any user-defined type).

Element-wise operations are also expanded from what can be done in MATLAB. Consider matrix addition, C=A+B in MATLAB. The pattern of the result is the set union of the pattern of A and B. In GraphBLAS, any binary operator can be used in this set-union "addition." The operator is applied to entries in the intersection. Entries in A but not B, or visa-versa, are copied directly into C, without any application of the binary operator. The accumulator operation for $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$ described in Section 2.3 is one example of this set-union application of an arbitrary binary operator.

Consider element-wise multiplication, C=A.*B in MATLAB. The operator (multiply in this case) is applied to entries in the set intersection, and the pattern of C just this set intersection. Entries in A but not B, or visa-versa,

do not appear in C. In GraphBLAS, any binary operator can be used in this manner, not just scalar multiplication. The difference between element-wise "add" and "multiply" is not the operators, but whether or not the pattern of the result is the set union or the set intersection. In both cases, the operator is only applied to the set intersection.

Finally, GraphBLAS includes a *non-blocking* mode where operations can be left pending, and saved for later. This is very useful for submatrix assignment (C(i,j)=A where I and J are integer vectors), or scalar assignment (C(i,j)=x where i and j are scalar integers). Because of how MATLAB stores its matrices, adding and deleting individual entries is very costly. For example, this is very slow in MATLAB, taking $O(nz^2)$ time:

```
A = sparse (m,n);  % an empty sparse matrix
for k = 1:nz
    compute a value x, row index i, and column index j
    A (i,j) = x;
end
```

The above code is very easy read and simple to write, but exceedingly slow. In MATLAB, the method below is preferred and is far faster, taking at most $O(|\mathbf{A}| \log |\mathbf{A}| + n)$ time. It can easily be a million times faster than the method above. Unfortunately the second method below is a little harder to read and a little less natural to write:

GraphBLAS can do both methods. SuiteSparse:GraphBLAS stores its matrices in a format that allows for pending computations, which are done later in bulk, and as a result it can do both methods above equally as fast as the MATLAB sparse function, allowing the user to write simpler code.

2.3 The accumulator and the mask

Most GraphBLAS operations can be modified via transposing input matrices, using an accumulator operator, applying a mask or its complement, and by clearing all entries the matrix C after using it in the accumulator operator but before the final results are written back into it. All of these steps are optional, and are controlled by a descriptor object that holds parameter settings (see Section 6.16) that control the following options:

- the input matrices A and/or B can be transposed first.
- an accumulator operator can be used, like the plus in the statement C=C+A*B. The accumulator operator can be any binary operator, and an element-wise "add" (set union) is performed using the operator.
- an optional mask can be used to selectively write the results to the output. The mask is a sparse Boolean matrix Mask whose size is the same size as the result. If Mask(i,j) is true, then the corresponding entry in the output can be modified by the computation. If Mask(i,j) is false, then the corresponding in the output is protected and cannot be modified by the computation. The Mask matrix acts exactly like logical matrix indexing in MATLAB, with one minor difference: in GraphBLAS notation, the mask operation is $C\langle M \rangle = Z$, where the mask M appears only on the left-hand side. In MATLAB, it would appear on both sides as C(Mask) = Z(Mask). If no mask is provided, the Mask matrix is implicitly all true. This is indicated by passing the value GrB_NULL in place of the Mask argument in GraphBLAS operations.

This process can be described in mathematical notation as:

```
\mathbf{A} = \mathbf{A}^\mathsf{T}, if requested via descriptor (first input option)
```

 $\mathbf{B} = \mathbf{B}^\mathsf{T},$ if requested via descriptor (second input option)

 ${f T}$ is computed according to the specific operation

 $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot\mathbf{T}$, accumulating and writing the results back via the mask

The application of the mask and the accumulator operator is written as $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot\mathbf{T}$ where $\mathbf{Z} = \mathbf{C}\odot\mathbf{T}$ denotes the application of the accumulator operator, and $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$ denotes the mask operator via the Boolean matrix \mathbf{M} . The Accumulator Phase, $\mathbf{Z} = \mathbf{C}\odot\mathbf{T}$, is performed as follows:

```
 \begin{aligned} \textbf{Accumulator Phase} \colon & \text{compute } \mathbf{Z} = \mathbf{C} \odot \mathbf{T} ; \\ & \text{if accum is NULL} \\ & \mathbf{Z} = \mathbf{T} \\ & \text{else} \\ & \mathbf{Z} = \mathbf{C} \odot \mathbf{T} \end{aligned}
```

The accumulator operator is \odot in GraphBLAS notation, or accum in the code. The pattern of $\mathbf{C} \odot \mathbf{T}$ is the set union of the patterns of \mathbf{C} and \mathbf{T} , and the operator is applied only on the set intersection of \mathbf{C} and \mathbf{T} . Entries in neither the pattern of \mathbf{C} nor \mathbf{T} do not appear in the pattern of \mathbf{Z} . That is:

```
for all entries (i, j) in \mathbf{C} \cap \mathbf{T} (that is, entries in both \mathbf{C} and \mathbf{T})
z_{ij} = c_{ij} \odot t_{ij}
for all entries (i, j) in \mathbf{C} \setminus \mathbf{T} (that is, entries in \mathbf{C} but not \mathbf{T})
z_{ij} = c_{ij}
for all entries (i, j) in \mathbf{T} \setminus \mathbf{C} (that is, entries in \mathbf{T} but not \mathbf{C})
z_{ij} = t_{ij}
```

The Accumulator Phase is followed by the Mask/Replace Phase, $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{Z}$ as controlled by the $\mathtt{GrB_REPLACE}$ and $\mathtt{GrB_COMP}$ descriptor options:

```
\begin{split} \mathbf{Mask/Replace\ Phase:\ compute\ C\langle M\rangle = Z:} \\ & \text{if\ } (\mathtt{GrB\_REPLACE}) \ delete\ all\ entries\ in\ C} \\ & \text{if\ } (\mathtt{Mask\ is\ NULL} \\ & \text{if\ } (\mathtt{GrB\_COMP}) \\ & \mathbf{C} \ \text{is\ not\ modified} \\ & \text{else} \\ & \mathbf{C} = \mathbf{Z} \\ & \text{else} \\ & \text{if\ } (\mathtt{GrB\_COMP}) \\ & \mathbf{C\langle \neg M\rangle = Z} \\ & \text{else} \\ & \mathbf{C\langle M\rangle = Z} \end{split}
```

Both phases of the accum/mask process are illustrated in MATLAB notation in Figure 1.

A GraphBLAS operation starts with its primary computation, producing a result T; for matrix multiply, T=A*B, or if A is transposed first, T=A'*B, for example. Applying the accumulator, mask (or its complement) to obtain the final result matrix C can be expressed in the MATLAB accum_mask function

```
function C = accum_mask (C, Mask, accum, T, C_replace, Mask_complement)
[m n] = size (C.matrix) ;
Z.matrix = zeros (m, n) ;
Z.pattern = false (m, n) ;
if (isempty (accum))
   Z = T;
               % no accum operator
  % Z = accum (C,T), like Z=C+T but with an binary operator, accum
  p = C.pattern & T.pattern; Z.matrix (p) = accum (C.matrix (p), T.matrix (p));
  p = C.pattern & ~T.pattern; Z.matrix (p) = C.matrix (p);
  p = ~C.pattern & T.pattern ; Z.matrix (p) = T.matrix (p) ;
  Z.pattern = C.pattern | T.pattern ;
end
% = 1000 apply the mask to the values and pattern
C.matrix = mask (C.matrix, Mask, Z.matrix, C_replace, Mask_complement);
C.pattern = mask (C.pattern, Mask, Z.pattern, C_replace, Mask_complement) ;
end
function C = mask (C, Mask, Z, C_replace, Mask_complement)
% replace C if requested
if (C_replace)
   C(:,:) = 0;
if (isempty (Mask))
                                % if empty, Mask is implicit ones(m,n)
  % implicitly, Mask = ones (size (C))
   if (~Mask_complement)
     C = Z;
                                % this is the default
   else
                                % Z need never have been computed
     C = C;
   end
else
   % apply the mask
   if (~Mask_complement)
     C (Mask) = Z (Mask);
   else
     C (^{\sim}Mask) = Z (^{\sim}Mask);
   end
end
end
```

Figure 1: Applying the mask and accumulator, $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot \mathbf{T}$

shown in the figure. This function is an exact, fully functional, and nearly-complete description of the GraphBLAS accumulator/mask operation. The only aspects it does not consider are typecasting (see Section 2.4), and the value of the implicit identity (for those, see another version in the Test folder).

One aspect of GraphBLAS cannot be as easily expressed in a MATLAB sparse matrix: namely, what is the implicit value of entries not in the pattern? To accommodate this difference in the accum_mask MATLAB function, each sparse matrix A is represented with its values A.matrix and its pattern, A.pattern. The latter could be expressed as the sparse matrix A.pattern=spones(A) or A.pattern=(A~=0) in MATLAB, if the implicit value is zero. With different semirings, entries not in the pattern can be 1, +Inf, -Inf, or whatever is the identity value of the monoid. As a result, Figure 1 performs its computations on two MATLAB matrices: the values in A.matrix and the pattern in the logical matrix A.pattern. Implicit values are untouched.

The final computation in Figure 1 with a complemented Mask is easily expressed in MATLAB as C(~Mask)=Z(~Mask) but this is costly if Mask is very sparse (the typical case). It can be computed much faster in MATLAB without complementing the sparse Mask via:

```
R = Z; R (Mask) = C (Mask); C = R;
```

A set of MATLAB functions that precisely compute the $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}$ operation according to the full GraphBLAS specification is provided in Suite-Sparse:GraphBLAS as $\mathtt{GB_spec_accum.m}$, which computes $\mathbf{Z}=\mathbf{C}\odot\mathbf{T}$, and $\mathtt{GB_spec_mask.m}$, which computes $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{Z}$. SuiteSparse:GraphBLAS includes a complete list of $\mathtt{GB_spec_*}$ functions that illustrate every GraphBLAS operation.

The methods in Figure 1 rely heavily on MATLAB's logical matrix indexing. For those unfamiliar with logical indexing in MATLAB, here is short summary. Logical matrix indexing in MATLAB is written as A(Mask) where A is any matrix and Mask is a logical matrix the same size as A. The expression x=A(Mask) produces a column vector x consisting of the entries of A where Mask is true. On the left-hand side, logical submatrix assignment A(Mask)=x does the opposite, copying the components of the vector x into the places in A where Mask is true. For example, to negate all values greater than 10 using logical indexing in MATLAB:

```
>> A = magic (4)
                    3
     16
             2
                           13
            11
                   10
                           8
             7
      9
                    6
                           12
            14
                   15
                            1
>> A (A>10) =
                   A (A>10)
             2
                    3
                         -13
    -16
      5
           -11
                   10
                           8
      9
             7
                    6
                         -12
           -14
                  -15
                            1
```

In MATLAB, logical indexing with a sparse matrix A and sparse logical matrix Mask is a built-in method. The Mask operator in GraphBLAS works identically as sparse logical indexing in MATLAB, but is typically far faster in SuiteSparse:GraphBLAS than the same operation using MATLAB sparse matrices.

2.4 Typecasting

If an operator z=f(x) or z=f(x,y) is used with inputs that do not match its inputs x or y, or if its result z does not match the type of the matrix it is being stored into, then the values are typecasted. Typecasting in Graph-BLAS extends beyond just operators. Almost all GraphBLAS methods and operations are able to typecast their results, as needed.

If one type can be typecasted into the other, they are said to be *compatible*. All built-in types are compatible with each other. GraphBLAS cannot typecast user-defined types thus any user-defined type is only compatible with itself. When GraphBLAS requires inputs of a specific type, or when one type cannot be typecast to another, the GraphBLAS function returns an error code, Grb_DOMAIN_MISMATCH (refer to Section 5.6 for a complete list of error codes). Typecasting can only be done between built-in types, and it follows the rules of the ANSI C language (not MATLAB) wherever the rules of ANSI C are well-defined.

However, unlike MATLAB, the C11 language specification states that the results of typecasting a float or double to an integer type is not always defined. In SuiteSparse:GraphBLAS, whenever C leaves the result undefined the rules used in MATLAB are followed. In particular +Inf converts to the largest integer value, -Inf converts to the smallest (zero for unsigned in-

tegers), and NaN converts to zero. Positive values outside the range of the integer are converted to the largest positive integer, and negative values less than the most negative integer are converted to that most negative integer. Other than these special cases, SuiteSparse:GraphBLAS trusts the C compiler for the rest of its typecasting.

Typecasting to bool is fully defined in the C language specification, even for NaN. The result is false if the value compares equal to zero, and true otherwise. Thus NaN converts to true. This is unlike MATLAB, which does not allow a typecast of a NaN to the MATLAB logical type.

SPEC: the GraphBLAS API C Specification states that typecasting follows the rules of ANSI C. Yet C leaves some typecasting undefined. All typecasting between built-in types in SuiteSparse:GraphBLAS is precisely defined, as an extension to the specification.

SPEC: Some functions do not make use of all of their inputs; in particular the binary operators FIRST, SECOND, and ONEB, and many of the index unary operators. The Specification requires that the inputs to these operators must be compatible with (that is, can be typecasted to) the inputs to the operators, even if those inputs are not used and no typecasting would ever occur. As an extension to the specification, SuiteSparse:GraphBLAS does not perform this error check on unused inputs of built-in operators. For example, the GrB_FIRST_INT64 operator can be used in GrB_eWiseMult(C,..,A,B,...) on a matrix B of any type, including user-defined types. For this case, the matrix A must be compatible with GrB_INT64.

2.5 Notation and list of GraphBLAS operations

As a summary of what GraphBLAS can do, the following table lists all Graph-BLAS operations. Upper case letters denote a matrix, lower case letters are vectors, and **AB** denote the multiplication of two matrices over a semiring.

Each operation takes an optional $GrB_Descriptor$ argument that modifies the operation. The input matrices A and B can be optionally transposed, the mask M can be complemented, and C can be cleared of its entries after it is used in $Z = C \odot T$ but before the $C\langle M \rangle = Z$ assignment. Vectors are never transposed via the descriptor.

Let $\mathbf{A} \oplus \mathbf{B}$ denote the element-wise operator that produces a set union pattern (like A+B in MATLAB). Any binary operator can be used this way in GraphBLAS, not just plus. Let $\mathbf{A} \otimes \mathbf{B}$ denote the element-wise operator that produces a set intersection pattern (like A.*B in MATLAB); any binary operator can be used this way, not just times.

Reduction of a matrix **A** to a vector reduces the *i*th row of **A** to a scalar w_i . This is like w=sum(A') since by default, MATLAB reduces down the columns, not across the rows.

| GrB_mxm | matrix-matrix multiply | $\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot\mathbf{AB}$ | |
|------------------------|--|---|--|
| <pre>GrB_vxm</pre> | vector-matrix multiply | $\mathbf{w}^{T} \langle \mathbf{m}^{T} \rangle = \mathbf{w}^{T} \odot \mathbf{u}^{T} \mathbf{A}$ | |
| <pre>GrB_mxv</pre> | matrix-vector multiply | $\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot\mathbf{A}\mathbf{u}$ | |
| GrB_eWiseMult | element-wise, | $\mathbf{C}\langle \mathbf{M} angle = \mathbf{C} \odot (\mathbf{A} \otimes \mathbf{B})$ | |
| | set intersection | $\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot(\mathbf{u}\otimes\mathbf{v})$ | |
| GrB_eWiseAdd | element-wise, $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot(\mathbf{A}\oplus\mathbf{B})$ | | |
| | set union | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot(\mathbf{u}\oplus\mathbf{v})$ | |
| GxB_eWiseUnion | element-wise, | $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot (\mathbf{A} \oplus \mathbf{B})$ | |
| | set union | $\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot(\mathbf{u}\oplus\mathbf{v})$ | |
| GrB_extract | extract submatrix | $\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot\mathbf{A}(\mathbf{I},\mathbf{J})$ | |
| | | $\mathbf{w}\langle\mathbf{m} angle=\mathbf{w}\odot\mathbf{u}(\mathbf{i})$ | |
| ${\tt GxB_subassign}$ | assign submatrix | $\mathbf{C}(\mathbf{I},\mathbf{J})\langle\mathbf{M} angle = \mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$ | |
| | (with submask for $\mathbf{C}(\mathbf{I}, \mathbf{J})$) | $\mathbf{w}(\mathbf{i})\langle\mathbf{m}\rangle = \mathbf{w}(\mathbf{i})\odot\mathbf{u}$ | |
| GrB_assign | assign submatrix | $\mathbf{C}\langle\mathbf{M} angle(\mathbf{I},\mathbf{J})=\mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$ | |
| | (with mask for \mathbf{C}) | $\mathbf{w}\langle\mathbf{m} angle(\mathbf{i})=\mathbf{w}(\mathbf{i})\odot\mathbf{u}$ | |
| GrB_apply | apply unary operator | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A})$ | |
| | | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u})$ | |
| | apply binary operator | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A}, y)$ | |
| | | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(x,\mathbf{A})$ | |
| | | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u},y)$ | |
| | | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(x, \mathbf{u})$ | |
| | apply index-unary op | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A},i,j,k)$ | |
| | | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u}, i, 0, k)$ | |
| <pre>GrB_select</pre> | select entries | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot\operatorname{select}(\mathbf{A}, i, j, k)$ | |
| | | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \operatorname{select}(\mathbf{u}, i, 0, k)$ | |
| <pre>GrB_reduce</pre> | reduce to vector | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot[\oplus_{j}\mathbf{A}(:,j)]$ | |
| - | reduce to scalar | $s = s \odot [\oplus_{ij} \mathbf{A}(i,j)]$ | |
| GrB_transpose | transpose | $\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot\mathbf{A}^T$ | |
| GrB_kronecker | Kronecker product | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot \mathrm{kron}(\mathbf{A},\mathbf{B})$ | |

3 Changes in GraphBLAS v10: 32/64 bit integers

GraphBLAS v10 adds a new feature that improves performance and reduces memory requirements for GraphBLAS matrices and vectors: the use of any mix of 32-bit and 64-bit integers in the internal data structures for the GrB_Matrix, GrB_Vector, and GrB_Scalar (the latter is nominally revised, but only because GraphBLAS stores its GrB_Scalar as a 1-by-1 matrix).

All prior methods work without any modification to the user application, so v10 is upward-compatible with prior versions of GraphBLAS.

3.1 Controlling the sizes of integers

Different integers are used for different parts of the matrix/vector data structure. The decision as to which integers to use is determined by the dimensions and number of entries in the matrix. The decisions can also be modified by GrB_set and queried by GrB_get . A matrix can have up to three different kinds of integers. If a matrix is m-by-n with e entries, with default settings:

- if $m > 2^{31}$: 64-bit integers must be used for the row indices of a matrix; otherwise, 32-bit integers may be used.
- if $n > 2^{31}$: 64-bit integers must be used for the column indices of a matrix; otherwise, 32-bit integers may be used.
- if $e > 2^{32}$: 64-bit integers must be used for the row/column offsets of a matrix; otherwise 32-bit integers may be used.

See Section 10.13 for details.

3.2 Passing arrays to/from GraphBLAS

Several of the methods in the GraphBLAS v2.1 C API use plain C arrays of type uint64_t to pass lists of integers to GraphBLAS. These are extended to allow any integers arrays to be passed, by adding new methods where all plain C arrays (including pointers to numerical values for build and extractTuples) are replaced with GrB_Vectors. The new methods are given a name that matches the method they revised, with a _Vector appended on

the end. All of the new methods are accessible via the polymorphic interface, using the existing polymorphic name. In the methods below, all I, J, and X parameters become GrB_Vector objects.

- GrB_Vector_build: build a vector from (I,X) or (I,scalar) tuples.
 - GxB_Vector_build_Vector (w, I, X, dup, desc) (see Section 6.9.8).
 - GxB_Vector_build_Scalar_Vector (w, I, scalar, desc) (see Section 6.9.10).
- GrB_Matrix_build: build a matrix from (I,J,X) or (I,J,scalar) tuples.
 - GxB_Matrix_build_Vector (C, I, J, X, dup, desc) (see Section 6.10.9).
 - GxB_Matrix_build_Scalar_Vector (C, I, J, scalar, desc) (see Section 6.10.11).
- ullet GrB_Vector_extractTuples: extract (I,X) tuples from a vector.
 - GxB_Vector_extractTuples_Vector (I, X, v, desc) (see Section 6.9.16).
- GrB_Matrix_extractTuples: extract (I,J,X) tuples from a matrix.
 - GxB_Matrix_extractTuples_Vector (I, J, X, A, desc) (see Section 6.10.17).
- ullet GrB_assign: $\mathbf{C}\langle\mathbf{M}
 angle(\mathbf{I},\mathbf{J})=\mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$
 - GxB_Vector_assign_Vector (w, mask, accum, u, I, desc) (see Section 12.8.2).
 - GxB_Matrix_assign_Vector (C, Mask, accum, A, I, J, desc) (see Section 12.8.4).
 - GxB_Col_assign_Vector (C, mask, accum, u, I, j, desc) (see Section 12.8.6).
 - GxB_Row_assign_Vector (C, mask, accum, u, i, J, desc) (see Section 12.8.8).

- GxB_Vector_assign_Scalar_Vector (w, mask, accum, scalar, I, desc) (see Section 12.8.10).
- GxB_Matrix_assign_Scalar_Vector (C, Mask, accum, scalar, I, J, desc) (see Section 12.8.12).
- ullet GrB_subassign: $\mathbf{C}(\mathbf{I},\mathbf{J})\langle\mathbf{M}
 angle = \mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$
 - GxB_Vector_subassign_Vector (w, mask, accum, u, I, desc) (see Section 12.9.2).
 - GxB_Matrix_subassign_Vector (C, Mask, accum, A, I, J, desc) (see Section 12.9.4).
 - GxB_Col_subassign_Vector (C, mask, accum, u, I, j, desc) (see Section 12.9.6).
 - GxB_Row_subassign_Vector (C, mask, accum, u, i, J, desc) (see Section 12.9.8).
 - GxB_Vector_subassign_Scalar_Vector (w, mask, accum, scalar, I, desc) (see Section 12.9.10).
 - GxB_Matrix_subassign_Scalar_Vector (C, Mask, accum, scalar, I, J, desc) (see Section 12.9.12).
- GrB_extract: $C\langle M \rangle = C \odot A(I,J)$
 - GxB_Vector_extract_Vector (w, mask, accum, u, I, desc) (see Section 12.7.2).
 - GxB_Matrix_extract_Vector (C, Mask, accum, A, I, J, desc) (see Section 12.7.4).
 - GxB_Col_extract_Vector (w, mask, accum, A, I, j, desc) (see Section 12.7.6).

In each of the above methods where I, J, and X are GrB_Vector inputs to the method (all but extractTuples), the vectors can be interpretted in up to 3 different ways. For the first two ways, suppose extractTuples is used to extract two lists from a vector I, J, or X: values and indices. Then either of those two lists can then be used by the method. The default is to use the values, but the indices can be selected by changing the following descriptor fields:

- GxB_ROWINDEX_LIST: how the GrB_Vector I is intrepretted.
- GxB_COLINDEX_LIST: how the GrB_Vector J is intrepretted.
- GxB_VALUE_LIST: how GrB_Vector X is intrepretted (for GrB_build only).

These can be set to one of the following values:

- Grb_Default or Gxb_use_values: use the values of the vector (default).
- GxB_USE_INDICES: use the indices of the vector.
- GxB_IS_STRIDE: use the values, of size 3, for a strided range, or lo:inc:hi in MATLAB notation. This usage is limited to the I and J vectors (except this option may not be used for GrB_build). The vector must have exactly three entries, lo, hi, and inc, in that order.

3.3 Container methods: loading/unloading data to/from a matrix or vector

The new methods described in the previous Section 3.2 provide GrB_Vector inputs and outputs. This section gives an overview of how data is moved between these opaque objects and user-visible C arrays, using the new load/unload methods.

See Section 6.12 for all of the details of the new load/unload methods, using a new *Container* object.

Methods in the v2.1 GraphBLAS C Specification can be used, but these methods require a copy (GrB_*_build, GrB_*_extractTuples, GrB_*_import and GrB_*_export). The following two methods accomplish this task when the vectors are dense, with all possible entries present.

• GxB_Vector_load: this method moves data in O(1) time from a user-visible C array into a GrB_Vector. The vector length and type are revised to match the new data from the C array. Ownership is normally transferred to the GrB_Vector, but this can be revised with a handling parameter. The C array is passed in as a void * pointer, and its type is indicated by a GrB_Type parameter. See Section 6.12.1 for details.

• GxB_Vector_unload: this method moves data in O(1) time from a GrB_Vector into a user-visible C array. The length of the GrB_Vector is reduced to zero, to denote that it no longer holds any content. The vector must be dense; it must have the same number of entries as its size (that is GrB_Vector_nvals and GrB_Vector_size must return the same value). The C array is returned as a void * pointer, and its type is indicated by a GrB_Type parameter. See Section 6.12.2 for details.

To move data between a GrB_Matrix or GrB_Vector in all other cases, a new GxB_Container object is introduced. This object is non-opaque but contains opaque objects. Its primary components are five dense GrB_Vectors that hold the contents of the matrix/vector. The data in these dense vectors can then be loaded/unloaded via GxB_Vector_load and GxB_Vector_unload.

- GxB_Container_new: creates a container. (see Section 6.12.3).
- GxB_Container_free: frees a container. (see Section 6.12.4).
- GxB_load_Matrix_from_Container: moves all of the data from a GxB_Container into a GrB_Matrix in O(1) time. (see Section 6.12.5).
- GxB_load_Vector_from_Container: moves all of the data from a GxB_Container into a GrB_Vector in O(1) time. (see Section 6.12.6).
- GxB_unload_Matrix_into_Container: moves all of the data from a GrB_Matrix into a GxB_Container in O(1) time. (see Section 6.12.7).
- GxB_unload_Vector_into_Container: moves all of the data from a GrB_Vector into a GxB_Container in O(1) time. (see Section 6.12.8).

3.4 Historical methods: pack/unpack

GraphBLAS v5.1.0 (released in June 2021) added a suite of $\texttt{GxB_pack}$ and $\texttt{GxB_unpack}$ methods to move data between opaque objects ($\texttt{GrB_Matrix}$ and $\texttt{GrB_Vector}$) and user-visible C arrays, in O(1) time unless the data format needed to be revised.

These methods are now declared *historical*, which means they will be kept in working order but will no longer be documented.

In GraphBLAS v10, these methods still pack/unpack their contents into uint64_t * user arrays. If the matrix has 32-bit integers, this requires a

copy and typecast. Thus, performance will be degraded for existing user codes that expect O(1) time to pack/unpack their matrices/vectors.

Rather than extend pack/unpack to handle a wide range of integer types, new methods using the $GxB_Container$ are introduced instead (see Section 3.3), to rapidly move data into/out of a GrB_Matrix or GrB_Vector in O(1) time and space.

4 Interfaces to MATLAB, Octave, Python, Julia, Go, Java, ...

The MATLAB/Octave interface to SuiteSparse:GraphBLAS is included with this distribution, described in Section 4.1. Python, Julia, Go, and Java interfaces are available. These are not part of the SuiteSparse:GraphBLAS distribution. See the links below.

4.1 MATLAB/Octave Interface

An easy-to-use MATLAB/Octave interface for SuiteSparse:GraphBLAS is available; see the documentation in the GraphBLAS/GraphBLAS folder for details. Start with the README.md file in that directory. An easy-to-read output of the MATLAB demos can be found in GraphBLAS/GraphBLAS/demo/html.

The MATLAB/Octave interface adds the @GrB class, which is an opaque MATLAB/Octave object that contains a GraphBLAS matrix, either double or single precision (real or complex), boolean, or any of the built-in integer types. MATLAB/Octave sparse and full matrices can be arbitrarily mixed with GraphBLAS matrices. The following overloaded operators and methods all work as you would expect for any matrix. The matrix multiplication A*B uses the conventional PLUS_TIMES semiring.

A+B A-B A*B A.*B A./B A.\B A.\b A.\b C=A(I,J)
-A +A
$$^{^{\prime}}$$
A A' A.' A&B A|B b\A C(I,J)=A
A^{^{\prime}}B A>B A=B A<=B A>=B A

For a list of overloaded operations and static methods, type methods GrB in MATLAB/Octave, or help GrB for more details.

Limitations: Some features for MATLAB/Octave sparse matrices are not yet available for GraphBLAS matrices. Some of these may be added in future releases.

- GrB matrices with dimension larger than 2⁵³ do not display properly in the whos command. The size is displayed correctly with disp or display.
- Non-blocking mode is not exploited.
- Linear indexing: A(:) for a 2D matrix, and I=find(A).
- Singleton expansion.

- Dynamically growing arrays, where C(i)=x can increase the size of C.
- Saturating element-wise binary and unary operators for integers. For C=A+B with MATLAB uint8 matrices, results saturate if they exceed 255. This is not compatible with a monoid for C=A*B, and thus MATLAB does not support matrix-matrix multiplication with uint8 matrices. In GraphBLAS, uint8 addition acts in a modulo fashion.
- Solvers, so that $x=A\b$ could return a GF(2) solution, for example.
- Sparse matrices with dimension higher than 2.

4.2 Performance of MATLAB versus GraphBLAS

MATLAB R2021a SuiteSparse:GraphBLAS as a built-in library, but uses it only for C=A*B when both A and B are sparse. In prior versions of MATLAB, C=A*B relied on the SFMULT and SSMULT packages in SuiteSparse, which are single-threaded (also written by this author). The GraphBLAS GrB_mxm is up to 30x faster on a 20-core Intel Xeon, compared with C=A*B in MATLAB R2020b and earlier. With MATLAB R2021a and later, the performance of C=A*B when using MATLAB sparse matrices is identical to the performance for GraphBLAS matrices, since the same code is being used by both (GrB_mxm).

Other methods in GraphBLAS are also faster, some *extremely* so, but are not yet exploited as built-in operations MATLAB. In particular, the statement C(M)=A (where M is a logical matrix) takes under a second for a large sparse problem when using GraphBLAS via its @GrB interface. By stark contrast, MATLAB would take about 4 or 5 days, a speedup of about 500,000x. For a smaller problem, GraphBLAS takes 0.4 seconds while MATLAB takes 28 hours (a speedup of about 250,000x). Both cases use the same statement with the same syntax (C(M)=A) and compute exactly the same result. Below are the results for n-by-n matrices in GraphBLAS v5.0.6 and MATLAB R2020a, on a Dell XPS13 laptop (16GB RAM, Intel(R) Core(TM) i7-8565U CPU @ 1.80GHz with 4 hardware cores). GraphBLAS is using 4 threads.

| n | nnz(C) | nnz(M) | GraphBLAS (sec) | MATLAB (sec) | speedup |
|------------|-------------|-----------|-----------------|--------------|----------|
| 2,048 | 20,432 | 2,048 | 0.005 | 0.024 | 4.7 |
| 4,096 | 40,908 | 4,096 | 0.003 | 0.115 | 39 |
| 8,192 | 81,876 | 8,191 | 0.009 | 0.594 | 68 |
| 16,384 | 163,789 | 16,384 | 0.009 | 2.53 | 273 |
| 32,768 | 327,633 | 32,767 | 0.014 | 12.4 | 864 |
| $65,\!536$ | $655,\!309$ | 65,536 | 0.025 | 65.9 | 2,617 |
| 131,072 | 1,310,677 | 131,070 | 0.055 | 276.2 | 4,986 |
| 262,144 | 2,621,396 | 262,142 | 0.071 | 1,077 | 15,172 |
| 524,288 | 5,242,830 | 524,288 | 0.114 | 5,855 | 51,274 |
| 1,048,576 | 10,485,713 | 1,048,576 | 0.197 | 27,196 | 137,776 |
| 2,097,152 | 20,971,475 | 2,097,152 | 0.406 | 100,799 | 248,200 |
| 4,194,304 | 41,942,995 | 4,194,304 | 0.855 | 4 to 5 days? | 500,000? |

The assignment C(I, J)=A in MATLAB, when using @GrB objects, is up to 1000x faster than the same statement with the same syntax, when using MATLAB sparse matrices instead. Matrix concatenation C = [A B] is about 17 times faster in GraphBLAS, on a 20-core Intel Xeon. For more details, see the GraphBLAS/GraphBLAS/demo folder and its contents.

Below is a comparison of other methods in SuiteSparse:GraphBLAS, compared with MATLAB 2021a. SuiteSparse:GraphBLAS: v6.1.4 (Jan 12, 2022), was used, compiled with gcc 11.2.0. The system is an Intel(R) Xeon(R) CPU E5-2698 v4 @ 2.20GHz (20 hardware cores, 40 threads), Ubuntu 20.04, 256GB RAM. Full details appear in the GraphBLAS/GraphBLAS/demo/benchmark folder. For this matrix, SuiteSparse:GraphBLAS is anywhere from 3x to 17x faster than the built-in methods in MATLAB. This matrix is not special, but is typical of the relative performance of many large matrices. Note that two of these (C=L*S and C=S*R) rely on an older version of SuiteSparse:GraphBLAS (v3.3.3) built into MATLAB R2021a.

Legend:

```
S: large input sparse matrix (n-by-n), the GAP-twitter matrix
```

GAP/GAP-twitter: n: 61.5784 million nnz: 1468.36 million (run time in seconds):

```
9.49
y=S*x:
         MATLAB: 22.8012 GrB:
                                 2.4018 speedup:
y=x*S:
         MATLAB: 16.1618 GrB:
                                 1.1610 speedup:
                                                     13.92
C=S*F:
         MATLAB: 30.6121 GrB:
                                 9.7052 speedup:
                                                      3.15
C=F*S:
         MATLAB: 26.4044 GrB:
                                 1.5245 speedup:
                                                     17.32
C=L*S:
         MATLAB: 19.1228 GrB:
                                 2.4301 speedup:
                                                     7.87
```

x: dense vector (1-by-n or n-by-1)

F: dense matrix (4-by-n or n-by-4)

L: 8-by-n sparse matrix, about 1000 entries

R: n-by-8 sparse matrix, about 1000 entries

B: n-by-n sparse matrix, about nnz(S)/10 entries

p,q: random permutation vectors

```
C=S*R:
         MATLAB:
                   0.0087 GrB:
                                 0.0020 speedup:
                                                      4.40
C=S'
         MATLAB: 224.7268 GrB:
                                22.6855 speedup:
                                                      9.91
C=S+S:
                                  1.5539 speedup:
                                                      9.23
         MATLAB:
                  14.3368 GrB:
C=S+B:
         MATLAB: 15.5600 GrB:
                                  1.5098 speedup:
                                                      10.31
C=S(p,q) MATLAB: 95.6219 GrB: 15.9468 speedup:
                                                      6.00
```

4.3 Python Interface

See Michel Pelletier's Python interface at https://github.com/michelp/pygraphblas; it also appears at https://anaconda.org/conda-forge/pygraphblas.

See Jim Kitchen and Erik Welch's (both from Anaconda, Inc.) Python interface at https://github.com/python-graphblas/python-graphblas (formerly known as grblas). See also https://anaconda.org/conda-forge/graphblas.

4.4 Julia Interface

The Julia interface is at https://github.com/JuliaSparse/SuiteSparseGraphBLAS. jl, developed by Will Kimmerer, Abhinav Mehndiratta, Miha Zgubic, and Viral Shah. Unlike the MATLAB/Octave interface (and like the Python interfaces) the Julia interface can keep pending work (zombies, pending tuples, jumbled state) in a GraphBLAS. This makes Python and Julia the best highlevel interfaces for SuiteSparse:GraphBLAS. MATLAB is not as well suited, since it does not allow inputs to a function or mexFunction to be modified, so any pending work must be finished before a matrix can be used as input.

4.5 Go Interface

Pascal Costanza (Intel) has a Go interface to GraphBLAS and LAGraph:

- forGraphBLASGo: https://github.com/intel/forGraphBLASGo, which is almost a complete wrapper for SuiteSparse:GraphBLAS. Documentation is at https://pkg.go.dev/github.com/intel/forGraphBLASGo.
- forLAGraphGo: https://github.com/intel/forLAGraphGo, which is in progress. Documentation is at https://pkg.go.dev/github.com/intel/forLAGraphGo.

4.6 Java Interface

Fabian Murariu is working on a Java interface. See https://github.com/fabianmurariu/graphblas-java-native.

5 GraphBLAS Initialization/Finalization

A user application that directly relies on GraphBLAS must include the GraphBLAS.h header file:

```
#include "GraphBLAS.h"
```

The GraphBLAS.h file defines functions, types, and macros prefixed with GrB_ and GxB_ that may be used in user applications. The prefix GrB_ denotes items that appear in the official *GraphBLAS C API Specification*. The prefix GxB_ refers to SuiteSparse-specific extensions to the GraphBLAS API.

The GraphBLAS.h file includes all the definitions required to use Graph-BLAS, including the following macros that can assist a user application in compiling and using GraphBLAS.

There are two version numbers associated with SuiteSparse:GraphBLAS: the version of the *GraphBLAS C API Specification* it conforms to, and the version of the implementation itself. These can be used in the following manner in a user application:

```
#if GxB_SPEC_VERSION >= GxB_VERSION (2,0,3)
... use features in GraphBLAS specification 2.0.3 ...
#else
... only use features in early specifications
#endif

#if GxB_IMPLEMENTATION >= GxB_VERSION (5,2,0)
... use features from version 5.2.0 (or later)
of a specific GraphBLAS implementation
#endif
```

SuiteSparse:GraphBLAS also defines the following strings with #define. Refer to the GraphBLAS.h file for details.

| Macro | purpose |
|----------------------------|---|
| GxB_IMPLEMENTATION_ABOUT | this particular implementation, copyright, and URL |
| GxB_IMPLEMENTATION_DATE | the date of this implementation |
| GxB_SPEC_ABOUT | the GraphBLAS specification for this implementation |
| GxB_SPEC_DATE | the date of the GraphBLAS specification |
| GxB_IMPLEMENTATION_LICENSE | the license for this particular implementation |

Finally, SuiteSparse:GraphBLAS gives itself a unique name of the form GxB_SUITESPARSE_GRAPHBLAS that the user application can use in #ifdef tests. This is helpful in case a particular implementation provides non-standard features that extend the GraphBLAS specification, such as additional predefined built-in operators, or if a GraphBLAS implementation does not yet fully implement all of the GraphBLAS specification.

For example, SuiteSparse:GraphBLAS predefines additional built-in operators not in the specification. If the user application wishes to use these in any GraphBLAS implementation, an #ifdef can control when they are used. Refer to the examples in the GraphBLAS/Demo folder.

As another example, the GraphBLAS API states that an implementation need not define the order in which <code>GrB_Matrix_build</code> assembles duplicate tuples in its <code>[I,J,X]</code> input arrays. As a result, no particular ordering should be relied upon in general. However, SuiteSparse:GraphBLAS does guarantee an ordering, and this guarantee will be kept in future versions of SuiteSparse:GraphBLAS as well. Since not all implementations will ensure a particular ordering, the following can be used to exploit the ordering returned by SuiteSparse:GraphBLAS.

```
#ifdef GxB_SUITESPARSE_GRAPHBLAS
// duplicates in I, J, X assembled in a specific order;
// results are well-defined even if op is not associative.
GrB_Matrix_build (C, I, J, X, nvals, op);
#else
// duplicates in I, J, X assembled in no particular order;
// results are undefined if op is not associative.
GrB_Matrix_build (C, I, J, X, nvals, op);
#endif
```

The remainder of this section describes GraphBLAS functions that start or finalize GraphBLAS, error handling, and the GraphBLAS integer.

| GraphBLAS function/type | purpose | Section |
|---------------------------|---|---------|
| GrB_Index | the GraphBLAS integer | 5.1 |
| <pre>GrB_init</pre> | start up GraphBLAS | 5.2 |
| <pre>GrB_getVersion</pre> | C API supported by the library | 5.3 |
| <pre>GxB_init</pre> | start up GraphBLAS with different malloc | 5.4 |
| <pre>GrB_Info</pre> | status code returned by GraphBLAS functions | 5.5 |
| GrB_error | get more details on the last error | 5.6 |
| GrB_finalize | finish GraphBLAS | 5.7 |

5.1 GrB_Index: the GraphBLAS integer

Matrix and vector dimensions and indexing rely on a specific integer, GrB_Index, which is defined in GraphBLAS.h as

```
typedef uint64_t GrB_Index ;
```

Row and column indices of an nrows-by-ncols matrix range from zero to the nrows-1 for the rows, and zero to ncols-1 for the columns. Indices are zero-based, like C, and not one-based, like MATLAB/Octave. In Suite-Sparse:GraphBLAS, the largest permitted index value is GrB_INDEX_MAX , defined as $2^{60} - 1$. The largest permitted matrix or vector dimension is 2^{60} (that is, GrB_INDEX_MAX+1). The largest GrB_Matrix that SuiteSparse: GraphBLAS can construct is thus 2^{60} -by- 2^{60} . An n-by-n matrix \mathbf{A} that size can easily be constructed in practice with $O(|\mathbf{A}|)$ memory requirements, where $|\mathbf{A}|$ denotes the number of entries that explicitly appear in the pattern of \mathbf{A} . The time and memory required to construct a matrix that large does not depend on n, since SuiteSparse:GraphBLAS can represent \mathbf{A} in hypersparse form (see Section 10.10.2). The largest GrB_Vector that can be constructed is 2^{60} -by-1.

Internally, GraphBLAS may store its integer indices using 32-bit integers (as of GraphBLAS v10.0.0). See Section 10.13 for details.

5.2 GrB_init: initialize GraphBLAS

GrB_init must be called before any other GraphBLAS operation. It defines the mode that GraphBLAS will use: blocking or non-blocking. With

blocking mode, all operations finish before returning to the user application. With non-blocking mode, operations can be left pending, and are computed only when needed. Non-blocking mode can be much faster than blocking mode, by many orders of magnitude in extreme cases. Blocking mode should be used only when debugging a user application. The mode cannot be changed once it is set by GrB_init.

GraphBLAS objects are opaque. This allows GraphBLAS to postpone operations and then do them later in a more efficient manner by rearranging them and grouping them together. In non-blocking mode, the computations required to construct an opaque GraphBLAS object might not be finished when the GraphBLAS method or operation returns to the user. However, user-provided arrays are not opaque, and GraphBLAS methods and operations that read them (such as GrB_Matrix_build) or write to them (such as GrB_Matrix_extractTuples) always finish reading them, or creating them, when the method or operation returns to the user application.

All methods and operations that extract values from a GraphBLAS object and return them into non-opaque user arrays always ensure that the user-visible arrays are fully populated when they return: GrB_*_reduce (to scalar), GrB_*_nvals, GrB_*_extractElement, and GrB_*_extractTuples. These functions do *not* guarantee that the opaque objects they depend on are finalized. To do that, use GrB_wait instead.

SuiteSparse:GraphBLAS is multithreaded internally, via OpenMP, and it is also safe to use in a multithreaded user application. See Section 18 for details. User threads must not operate on the same matrices at the same time, with one exception. Multiple user threads can use the same matrices or vectors as inputs to GraphBLAS operations or methods, but only if they have no pending operations (use GrB_wait first). User threads cannot simultaneously modify a matrix or vector via any GraphBLAS operation or method.

It is safe to use the internal parallelism in SuiteSparse:GraphBLAS on matrices, vectors, and scalars that are not yet completed. The library handles this on its own. The <code>GrB_wait</code> function is only needed when a user application makes multiple calls to GraphBLAS in parallel, from multiple user threads.

With multiple user threads, exactly one user thread must call <code>GrB_init</code> before any user thread may call any <code>GrB_*</code> or <code>GxB_*</code> function. When the user application is finished, exactly one user thread must call <code>GrB_finalize</code>, after which no user thread may call any <code>GrB_*</code> or <code>GxB_*</code> function. The mode

of a GraphBLAS session can be queried with GrB_get; see Section 10 for details.

5.3 GrB_getVersion: determine the C API Version

```
GrB_Info GrB_getVersion // run-time access to C API version number

(
    unsigned int *version, // returns GRB_VERSION
    unsigned int *subversion // returns GRB_SUBVERSION
);
```

GraphBLAS defines two compile-time constants that define the version of the C API Specification that is implemented by the library: GRB_VERSION and GRB_SUBVERSION. If the user program was compiled with one version of the library but linked with a different one later on, the compile-time version check with GRB_VERSION would be stale. GrB_getVersion thus provides a run-time access of the version of the C API Specification supported by the library.

5.4 GxB_init: initialize with alternate malloc

GxB_init is identical to GrB_init, except that it also redefines the memory management functions that SuiteSparse:GraphBLAS will use. Giving the user application control over this is particularly important when using the GxB_*serialize and GxB_Container methods described in Section 6.11 and 6.12, since they require the user application and GraphBLAS to use the same memory manager. user_calloc_func and user_realloc_func are optional, and may be NULL. If NULL, then the user_malloc_func is relied on instead, for all memory allocations. These functions can only be set once, when GraphBLAS starts. They can be queried using GrB_get (see

Section 10.2). Either GrB_init or GxB_init must be called before any other GraphBLAS operation, but not both. The functions passed to GxB_init must be thread-safe. The following usage is identical to GrB_init(mode):

GxB_init (mode, malloc, calloc, realloc, free);

5.5 GrB_Info: status code returned by GraphBLAS

Each GraphBLAS method and operation returns its status to the caller as its return value, an enumerated type (an enum) called GrB_Info. The first two values in the following table denote a successful status, the rest are error codes.

Not all GraphBLAS methods or operations can return all status codes. In the discussions of each method and operation in this User Guide, most of the obvious error code returns are not discussed. For example, if a required input is a NULL pointer, then GrB_NULL_POINTER is returned. Only error codes specific to the method or that require elaboration are discussed here. For a full list of the status codes that each GraphBLAS function can return, refer to *The GraphBLAS C API Specification* [BMM+17b, BBM+21].

| Error | value | description |
|------------------------------|-------|--|
| GrB_SUCCESS | 0 | the method or operation was successful |
| GrB_NO_VALUE | 1 | the method was successful, but the entry |
| | | does not appear in the matrix or vector. |
| GxB_EXHAUSTED | 2 | the iterator is exhausted |
| GrB_UNINITIALIZED_OBJECT | -1 | object has not been initialized |
| GrB_NULL_POINTER | -2 | input pointer is NULL |
| GrB_INVALID_VALUE | -3 | generic error code; some value is bad |
| <pre>GrB_INVALID_INDEX</pre> | -4 | a row or column index is out of bounds |
| GrB_DOMAIN_MISMATCH | -5 | object domains are not compatible |
| GrB_DIMENSION_MISMATCH | -6 | matrix dimensions do not match |
| GrB_OUTPUT_NOT_EMPTY | -7 | output matrix already has values in it |
| GrB_NOT_IMPLEMENTED | -8 | not implemented in SS:GrB |
| GrB_ALREADY_SET | -9 | field already written to |
| GrB_PANIC | -101 | unrecoverable error |
| GrB_OUT_OF_MEMORY | -102 | out of memory |
| GrB_INSUFFICIENT_SPACE | -103 | output array not large enough |
| GrB_INVALID_OBJECT | -104 | object is corrupted |
| GrB_INDEX_OUT_OF_BOUNDS | -105 | a row or column index is out of bounds |
| GrB_EMPTY_OBJECT | -106 | a input scalar has no entry |
| GxB_JIT_ERROR | -7001 | JIT compiler error |
| GxB_OUTPUT_IS_READONLY | -7003 | output has read-only components |

5.6 GrB_error: get more details on the last error

Each GraphBLAS method and operation returns a GrB_Info error code. The GrB_error function returns additional information on the error for a particular object in a null-terminated string. The string returned by GrB_error is never a NULL string, but it may have length zero (with the first entry being the '\0' string-termination value). The string must not be freed or modified.

```
info = GrB_some_method_here (C, ...) ;
if (! (info == GrB_SUCCESS || info == GrB_NO_VALUE))
```

```
{
    char *err ;
    GrB_error (&err, C) ;
    printf ("info: %d error: %s\n", info, err) ;
}
```

If C has no error status, or if the error is not recorded in the string, an empty non-null string is returned. In particular, out-of-memory conditions result in an empty string from GrB_error.

SuiteSparse:GraphBLAS reports many helpful details via GrB_error. For example, if a row or column index is out of bounds, the report will state what those bounds are. If a matrix dimension is incorrect, the mismatching dimensions will be provided. Refer to the output of the example programs in the Demo and Test folder, which intentionally generate errors to illustrate the use of GrB_error.

The only functions in GraphBLAS that return an error string are functions that have a single input/output argument C, as a GrB_Matrix, GrB_Vector, GrB_Scalar, or GrB_Descriptor. Methods that create these objects (such as GrB_Matrix_new) return a NULL object on failure, so these methods cannot also return an error string in C.

Any subsequent GraphBLAS method that modifies the object C clears the error string.

Note that GrB_NO_VALUE is an not error, but an informational status. GrB_*_extractElment(&x,A,i,j), which does x=A(i,j), returns this value to indicate that A(i,j) is not present in the matrix. That method does not have an input/output object so it cannot return an error string.

5.7 GrB_finalize: finish GraphBLAS

```
GrB_Info GrB_finalize ( ) ; // finish GraphBLAS
```

GrB_finalize must be called as the last GraphBLAS operation, even after all calls to GrB_free. All GraphBLAS objects created by the user application should be freed first, before calling GrB_finalize since GrB_finalize will not free those objects. In non-blocking mode, GraphBLAS may leave some computations as pending. These computations can be safely abandoned if the user application frees all GraphBLAS objects it has created and then calls GrB_finalize. When the user application is finished, exactly one user thread must call GrB_finalize.

6 GraphBLAS Objects and their Methods

GraphBLAS defines twelve different objects to represent matrices, vectors, scalars, data types, operators (binary, unary, and index-unary), monoids, semirings, a *descriptor* object used to specify optional parameters that modify the behavior of a GraphBLAS operation, and a *context* object for controlling computational resources.

The GraphBLAS API makes a distinction between *methods* and *operations*. A method is a function that works on a GraphBLAS object, creating it, destroying it, or querying its contents. An operation (not to be confused with an operator) acts on matrices and/or vectors in a semiring.

| GrB_Type | a scalar data type |
|----------------------------|---|
| <pre>GrB_UnaryOp</pre> | a unary operator $z = f(x)$, where z and x are scalars |
| <pre>GrB_BinaryOp</pre> | a binary operator $z = f(x, y)$, where z, x, and y are scalars |
| ${\tt GrB_IndexUnaryOp}$ | an index-unary operator |
| ${\tt GxB_IndexBinaryOp}$ | an index-binary operator |
| <pre>GrB_Monoid</pre> | an associative and commutative binary operator |
| | and its identity value |
| GrB_Semiring | a monoid that defines the "plus" and a binary operator |
| | that defines the "multiply" for an algebraic semiring |
| <pre>GrB_Matrix</pre> | a 2D sparse matrix of any type |
| <pre>GrB_Vector</pre> | a 1D sparse column vector of any type |
| <pre>GrB_Scalar</pre> | a scalar of any type |
| <pre>GrB_Descriptor</pre> | a collection of parameters that modify an operation |
| GxB_Context | allocating computational resources |
| <pre>GxB_Container</pre> | for moving data between GraphBLAS and a user application |

Most of these objects are implemented in C as an opaque handle, which is a pointer to a data structure held by GraphBLAS. User applications may not examine the content of the object directly; instead, they can pass the handle back to GraphBLAS which will do the work. Assigning one handle to another is valid but it does not make a copy of the underlying object.

The GxB_Container object is non-opaque but contains opaque objects as its contents. All other objects are entirely opaque.

6.1 The GraphBLAS type: GrB_Type

A GraphBLAS GrB_Type defines the type of scalar values that a matrix or vector contains, and the type of scalar operands for a unary or binary operator. There are 13 built-in types, and a user application can define any types of its own as well. The built-in types correspond to built-in types in C (in the #include files stdbool.h, stdint.h, and complex.h) as listed in the following table.

| GraphBLAS | C type | description | range |
|------------|----------------|-------------------------|---------------------------|
| type | | | |
| GrB_BOOL | bool | Boolean | true (1) , false (0) |
| GrB_INT8 | int8_t | 8-bit signed integer | -128 to 127 |
| GrB_INT16 | int16_t | 16-bit integer | -2^{15} to $2^{15}-1$ |
| GrB_INT32 | int32_t | 32-bit integer | -2^{31} to $2^{31}-1$ |
| GrB_INT64 | int64_t | 64-bit integer | -2^{63} to $2^{63} - 1$ |
| GrB_UINT8 | uint8_t | 8-bit unsigned integer | 0 to 255 |
| GrB_UINT16 | uint16_t | 16-bit unsigned integer | 0 to $2^{16} - 1$ |
| GrB_UINT32 | uint32_t | 32-bit unsigned integer | 0 to $2^{32} - 1$ |
| GrB_UINT64 | uint64_t | 64-bit unsigned integer | 0 to $2^{64} - 1$ |
| GrB_FP32 | float | 32-bit IEEE 754 | -Inf to +Inf |
| GrB_FP64 | double | 64-bit IEEE 754 | -Inf to +Inf |
| GxB_FC32 | float complex | 32-bit complex | -Inf to +Inf |
| GxB_FC64 | double complex | 64-bit complex | -Inf to +Inf |

The C11 definitions of float complex and double complex are not always available. The GraphBLAS.h header defines them as GxB_FC32_t and GxB_FC64_t, respectively.

The user application can also define new types based on any typedef in the C language whose values are held in a contiguous region of memory of fixed size. For example, a user-defined GrB_Type could be created to hold any C struct whose content is self-contained. A C struct containing pointers might be problematic because GraphBLAS would not know to dereference the pointers to traverse the entire "scalar" entry, but this can be done if the objects referenced by these pointers are not moved. A user-defined complex type with real and imaginary types can be defined, or even a "scalar" type containing a fixed-sized dense matrix (see Section 6.1.1). The possibilities are endless. GraphBLAS can create and operate on sparse matrices and vectors in any of these types, including any user-defined ones. For user-defined types, GraphBLAS simply moves the data around itself (via memcpy), and then

passes the values back to user-defined functions when it needs to do any computations on the type. The next sections describe the methods for the GrB_Type object:

| GraphBLAS function | purpose | Section |
|-------------------------------|--|---------|
| GrB_Type_new | create a user-defined type | 6.1.1 |
| GxB_Type_new | create a user-defined type, with name and definition | 6.1.2 |
| <pre>GrB_Type_wait</pre> | wait for a user-defined type | 6.1.3 |
| GrB_get | get properties of a type | 10.3 |
| GrB_set | set the type name/definitition | 10.3 |
| <pre>GxB_Type_from_name</pre> | return the type from its name | 6.1.4 |
| <pre>GrB_Type_free</pre> | free a user-defined type | 6.1.5 |

6.1.1 GrB_Type_new: create a user-defined type

GrB_Type_new creates a new user-defined type. The type is a handle, or a pointer to an opaque object. The handle itself must not be NULL on input, but the content of the handle can be undefined. On output, the handle contains a pointer to a newly created type. The ctype is the type in C that will be used to construct the new GraphBLAS type. It can be either a built-in C type, or defined by a typedef. The second parameter should be passed as sizeof(ctype). The only requirement on the C type is that sizeof(ctype) is valid in C, and that the type reside in a contiguous block of memory so that it can be moved with memcpy. For example, to create a user-defined type called Complex for double-precision complex values using the C11 double complex type, the following can be used. A complete example can be found in the usercomplex.c and usercomplex.h files in the Demo folder.

```
#include <math.h>
#include <complex.h>
GrB_Type Complex ;
GrB_Type_new (&Complex, sizeof (double complex)) ;
```

To demonstrate the flexibility of the GrB_Type, consider a "scalar" consisting of 4-by-4 floating-point matrix and a string. This type might be useful

for the 4-by-4 translation/rotation/scaling matrices that arise in computer graphics, along with a string containing a description or even a regular expression that can be parsed and executed in a user-defined operator. All that is required is a fixed-size type, where sizeof(ctype) is a constant.

```
typedef struct
{
    float stuff [4][4];
    char whatstuff [64];
}
wildtype;
GrB_Type WildType;
GrB_Type_new (&WildType, sizeof (wildtype));
```

With this type a sparse matrix can be created in which each entry consists of a 4-by-4 dense matrix stuff and a 64-character string whatstuff. GraphBLAS treats this 4-by-4 as a "scalar." Any GraphBLAS method or operation that simply moves data can be used with this type without any further information from the user application. For example, entries of this type can be assigned to and extracted from a matrix or vector, and matrices containing this type can be transposed. A working example (wildtype.c in the Demo folder) creates matrices and multiplies them with a user-defined semiring with this type.

Performing arithmetic on matrices and vectors with user-defined types requires operators to be defined. Refer to Section 17.5 for more details on these example user-defined types.

User defined types created by GrB_Type_new will not work with the JIT; use GxB_Type_new instead.

6.1.2 GxB_Type_new: create a user-defined type (with name and definition)

GxB_Type_new creates a type with a name and definition that are known to GraphBLAS, as strings. The type_name is any valid string (max length of

128 characters, including the required null-terminating character) that may appear as the name of a C type created by a C typedef statement. It must not contain any white-space characters. For example, to create a type of size 16*4+1=65 bytes, with a 4-by-4 dense float array and a 32-bit integer:

The type_name and type_defn are both null-terminated strings. The two strings are optional, but are required to enable the JIT compilation of kernels that use this type. At most GxB_MAX_NAME_LEN characters are accessed in type_name; characters beyond that limit are silently ignored.

If the sizeof_ctype is zero, and the strings are valid, a JIT kernel is compiled just to determine the size of the type. This is feature useful for interfaces in languages other than C, which could create valid strings for C types but would not have a reliable way to determine the size of the type.

The above example is identical to the following usage, except that GrB_Type_new requires sizeof_ctype to be nonzero, and equal to the size of the C type.

6.1.3 GrB_Type_wait: wait for a type

After creating a user-defined type, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. Currently, SuiteSparse:GraphBLAS currently does nothing except to ensure that type is valid.

6.1.4 GxB_Type_from_name: return the type from its name

Returns the built-in type from the corresponding name of the type. The following examples both return type as GrB_BOOL.

```
GxB_Type_from_name (&type, "bool") ;
GxB_Type_from_name (&type, "GrB_BOOL") ;
```

If the name is from a user-defined type, the type is returned as NULL. This is not an error condition. The user application must itself do this translation since GraphBLAS does not keep a registry of all user-defined types.

With this function, a user application can manage the translation for both built-in types and its own user-defined types, as in the following example.

```
typedef struct { double x ; char stuff [16] ; } myfirsttype ;
typedef struct { float z [4][4] ; int color ; } myquaternion ;
GrB_Type MyType1, MyQType ;
GxB_Type_new (&MyType1, sizeof (myfirsttype), "myfirsttype",
    "typedef struct { double x ; char stuff [16] ; } myfirsttype ;") ;
GxB_Type_new (&MyQType, sizeof (myquaternion), "myquaternion",
    "typedef struct { float z [4][4] ; int color ; } myquaternion ;") ;
GrB_Matrix A ;
// ... create a matrix A of some built-in or user-defined type
// later on, to query the type of A:
size_t typesize ;
GrB_Scalar_new (s, GrB_UINT64) ;
GrB_get (type, s, GrB_SIZE) ;
GrB_Scalar_extractElement (&typesize, GrB_UINT64) ;
GrB_Type atype ;
char atype_name [GxB_MAX_NAME_LEN] ;
GrB_get (A, atype_name, GrB_EL_TYPE_STRING) ;
GxB_Type_from_name (&atype, atype_name) ;
if (atype == NULL)
    // This is not yet an error. It means that A has a user-defined type.
    if ((strcmp (atype_name, "myfirsttype")) == 0) atype = MyType1;
```

```
else if ((strcmp (atype_name, "myquaternion")) == 0) atype = MyQType;
else { ... this is now an error ... the type of A is unknown. }
}
```

6.1.5 GrB_Type_free: free a user-defined type

GrB_Type_free frees a user-defined type. Either usage:

```
GrB_Type_free (&type) ;
GrB_free (&type) ;
```

frees the user-defined type and sets type to NULL. It safely does nothing if passed a NULL handle, or if type == NULL on input.

It is safe to attempt to free a built-in type. SuiteSparse:GraphBLAS silently ignores the request and returns GrB_SUCCESS. A user-defined type should not be freed until all operations using the type are completed. Suite-Sparse:GraphBLAS attempts to detect this condition but it must query a freed object in its attempt. This is hazardous and not recommended. Operations on such objects whose type has been freed leads to undefined behavior.

It is safe to first free a type, and then a matrix of that type, but after the type is freed the matrix can no longer be used. The only safe thing that can be done with such a matrix is to free it.

The function signature of GrB_Type_free uses the generic name GrB_free, which can free any GraphBLAS object. See Section 6.17 details. GraphBLAS includes many such generic functions. When describing a specific variation, a function is described with its specific name in this User Guide (such as GrB_Type_free). When discussing features applicable to all specific forms, the generic name is used instead (such as GrB_free).

6.2 GraphBLAS unary operators: GrB_UnaryOp, z = f(x)

A unary operator is a scalar function of the form z = f(x). The domain (type) of z and x need not be the same.

In the notation in the tables below, T is any of the 13 built-in types and is a place-holder for BOOL, INT8, UINT8, ... FP32, FP64, FC32, or FC64. For example, GrB_AINV_INT32 is a unary operator that computes z=-x for two values x and z of type GrB_INT32 .

The notation R refers to any real type (all but FC32 and FC64), I refers to any integer type (INT* and UINT*), F refers to any real or complex floating point type (FP32, FP64, FC32, or FC64), Z refers to any complex floating point type (FC32 or FC64), and N refers to INT32 or INT64.

The logical negation operator $\mathtt{GrB_LNOT}$ only works on Boolean types. The $\mathtt{GxB_LNOT}_R$ functions operate on inputs of type R, implicitly typecasting their input to Boolean and returning result of type R, with a value 1 for true and 0 for false. The operators $\mathtt{GxB_LNOT_BOOL}$ and $\mathtt{GrB_LNOT}$ are identical.

| Unary operators for all types | | | | |
|-------------------------------|-----------------|----------|------------------------|--|
| GraphBLAS name | types (domains) | z = f(x) | description | |
| $\mathtt{GxB_ONE_}T$ | $T \to T$ | z = 1 | one | |
| ${\tt GrB_IDENTITY_}T$ | $T \to T$ | z = x | identity | |
| ${\tt GrB_AINV_}T$ | $T \to T$ | z = -x | additive inverse | |
| ${\tt GrB_MINV_}T$ | $T \to T$ | z = 1/x | multiplicative inverse | |

| Unary operators for real and integer types | | | | |
|---|-----------|--------|------------------|--|
| GraphBLAS name types (domains) $z = f(x)$ description | | | | |
| ${\tt GrB_ABS_}T$ | $R \to R$ | z = x | absolute value | |
| GrB_LNOT bool $z=\neg x$ logical negation | | | | |
| GxB_LNOT_ R $R 	o R$ $z = \neg(x \neq 0)$ | | | logical negation | |
| $\texttt{GrB_BNOT_}I \hspace{1cm} I \rightarrow I \hspace{1cm} z = \neg x \hspace{1cm} \text{bitwise negation}$ | | | | |

| Index-based unary operators for any type (including user-defined) | | | |
|---|-----------------|-----------------|------------------------|
| GraphBLAS name | types (domains) | $z = f(a_{ij})$ | description |
| $\texttt{GxB_POSITIONI}_N$ | $\rightarrow N$ | z = i | row index (0-based) |
| ${\tt GxB_POSITIONI1_}N$ | $\rightarrow N$ | z = i + 1 | row index (1-based) |
| ${\tt GxB_POSITIONJ_}N$ | $\rightarrow N$ | z = j | column index (0-based) |
| ${\tt GxB_POSITIONJ1_}N$ | $\rightarrow N$ | z = j + 1 | column index (1-based) |

| Unary operators for floating-point types (real and complex) | | | | |
|---|---------------------|-------------------------------|----------------------------|--|
| GraphBLAS name | types (domains) | z = f(x) | description | |
| $\texttt{GxB_SQRT_}F$ | $F \to F$ | $z = \sqrt{(x)}$ | square root | |
| $\mathtt{GxB_LOG_}F$ | $F \to F$ | $z = \log_e(x)$ | natural logarithm | |
| $\mathtt{GxB}_\mathtt{EXP}_F$ | $F \to F$ | $z = e^x$ | natural exponent | |
| $\texttt{GxB_LOG10}_F$ | $F \to F$ | $z = \log_{10}(x)$ | base-10 logarithm | |
| $\mathtt{GxB_LOG2_}F$ | $F \to F$ | $z = \log_2(x)$ | base-2 logarithm | |
| $\mathtt{GxB}_\mathtt{EXP2}_F$ | $F \to F$ | $z = 2^x$ | base-2 exponent | |
| $\texttt{GxB_EXPM1_}F$ | $F \to F$ | $z = e^x - 1$ | natural exponent - 1 | |
| ${\tt GxB_LOG1P_}F$ | $F \to F$ | $z = \log(x+1)$ | natural log of $x+1$ | |
| $\mathtt{GxB_SIN_}F$ | $F \to F$ | $z = \sin(x)$ | sine | |
| $\mathtt{GxB_COS_}F$ | $F \to F$ | $z = \cos(x)$ | cosine | |
| $\mathtt{GxB_TAN_}F$ | $F \to F$ | $z = \tan(x)$ | tangent | |
| ${\tt GxB_ASIN_}F$ | $F \to F$ | $z = \sin^{-1}(x)$ | inverse sine | |
| $\mathtt{GxB_ACOS_}F$ | $F \to F$ | $z = \cos^{-1}(x)$ | inverse cosine | |
| $\mathtt{GxB_ATAN_}F$ | $F \to F$ | $z = \tan^{-1}(x)$ | inverse tangent | |
| $\mathtt{GxB_SINH_}F$ | $F \to F$ | $z = \sinh(x)$ | hyperbolic sine | |
| $\mathtt{GxB_COSH_}F$ | $F \to F$ | $z = \cosh(x)$ | hyperbolic cosine | |
| $\mathtt{GxB_TANH_}F$ | $F \to F$ | $z = \tanh(x)$ | hyperbolic tangent | |
| $\mathtt{GxB_ASINH_}F$ | $F \to F$ | $z = \sinh^{-1}(x)$ | inverse hyperbolic sine | |
| $\mathtt{GxB_ACOSH_}F$ | $F \to F$ | $z = \cosh^{-1}(x)$ | inverse hyperbolic cosine | |
| $\mathtt{GxB_ATANH_}F$ | $F \to F$ | $z = \tanh^{-1}(x)$ | inverse hyperbolic tangent | |
| $\texttt{GxB_SIGNUM_}F$ | $F \to F$ | $z = \operatorname{sgn}(x)$ | sign, or signum function | |
| ${\tt GxB_CEIL_}F$ | $F \to F$ | $z = \lceil x \rceil$ | ceiling function | |
| ${\tt GxB_FLOOR_}F$ | $F \to F$ | $z = \lfloor x \rfloor$ | floor function | |
| ${\tt GxB_ROUND_}F$ | $F \to F$ | z = round(x) | round to nearest | |
| $\mathtt{GxB_TRUNC}_F$ | $F \to F$ | $z = \operatorname{trunc}(x)$ | round towards zero | |
| $\texttt{GxB_ISINF}_F$ | $F 	o 	exttt{bool}$ | z = isinf(x) | true if $\pm \infty$ | |
| ${\tt GxB_ISNAN_}F$ | $F 	o 	exttt{bool}$ | $z = i\operatorname{snan}(x)$ | true if NaN | |
| $\texttt{GxB_ISFINITE_}F$ | $F 	o 	exttt{bool}$ | z = isfinite(x) | true if finite | |

| Unary operators for floating-point types (real only) | | | |
|--|-----------------|------------------------------|------------------------------|
| GraphBLAS name | types (domains) | z = f(x) | description |
| ${\tt GxB_LGAMMA_}R$ | $R \to R$ | $z = \log(\Gamma(x))$ | log of gamma function |
| ${\tt GxB_TGAMMA_}R$ | $R \to R$ | $z = \Gamma(x)$ | gamma function |
| ${\tt GxB_ERF_}R$ | $R \to R$ | $z = \operatorname{erf}(x)$ | error function |
| $\mathtt{GxB_ERFC_}R$ | $R \to R$ | $z = \operatorname{erfc}(x)$ | complimentary error function |
| ${\tt GxB_CBRT_}R$ | $R \to R$ | $z = x^{1/3}$ | cube root |
| $\texttt{GxB_FREXPX_}R$ | $R \to R$ | z = frexpx(x) | normalized fraction |
| ${\tt GxB_FREXPE_}R$ | $R \to R$ | z = frexpe(x) | normalized exponent |

| Unary operators for complex types | | | | |
|-----------------------------------|-----------------|------------------------------|-------------------|--|
| GraphBLAS name | types (domains) | z = f(x) | description | |
| $\mathtt{GxB_CONJ_}Z$ | Z 	o Z | $z = \overline{x}$ | complex conjugate | |
| $\mathtt{GxB_ABS_}Z$ | $Z \to F$ | z = x | absolute value | |
| $\mathtt{GxB_CREAL_}Z$ | $Z \to F$ | z = real(x) | real part | |
| ${\tt GxB_CIMAG_}Z$ | $Z \to F$ | z = imag(x) | imaginary part | |
| ${\tt GxB_CARG_}Z$ | $Z \to F$ | $z = \operatorname{carg}(x)$ | angle | |

Built-in index-based unary operators return the row or column index of an entry. For a matrix $z = f(a_{ij})$ returns z = i or z = j, or +1 for 1-based indices. The latter is useful in the MATLAB/Octave interface, where row and column indices are 1-based. When applied to a vector, j is always zero, and i is the index in the vector. These built-in unary operators come in two types: INT32 and INT64, which is the type of the output, z. The functions are agnostic to the type of their inputs; they only depend on the position of the entries, not their values. User-defined index-based operators cannot be defined by $grb_uraryOp_new$; use $grb_uraryOp_new$ instead; see Section 6.4.

GxB_FREXPX and GxB_FREXPE return the mantissa and exponent, respectively, from the C11 frexp function. The exponent is returned as a floating-point value, not an integer.

The operators GxB_EXPM1_FC* and GxB_LOG1P_FC* for complex types are currently not accurate. They will be revised in a future version.

The functions casin, casinf, casinh, and casinhf provided by Microsoft Visual Studio for computing $\sin^{-1}(x)$ and $\sinh^{-1}(x)$ when x is complex do not compute the correct result. Thus, the unary operators GxB_ASIN_FC32 , GxB_ASIN_FC64 GxB_ASINH_FC32, and GxB_ASINH_FC64 do not work properly if the MS Visual Studio compiler is used. These functions work properly if the gcc, icc, or clang compilers are used on Linux or MacOS.

Integer division by zero normally terminates an application, but this is avoided in SuiteSparse:GraphBLAS. For details, see the binary $\mathtt{GrB_DIV_}T$ operators.

SPEC: The definition of integer division by zero is an extension to the specification.

The next sections define the following methods for the GrB_UnaryOp object:

| GraphBLAS function | purpose | Section |
|-----------------------------|--|---------|
| GrB_UnaryOp_new | create a user-defined unary operator | 6.2.1 |
| <pre>GxB_UnaryOp_new</pre> | create a named user-defined unary operator | 6.2.2 |
| <pre>GrB_UnaryOp_wait</pre> | wait for a user-defined unary operator | 6.2.3 |
| <pre>GrB_get</pre> | get properties of an operator | 10.4 |
| GrB_set | set the operator name/definition | 10.4 |
| <pre>GrB_UnaryOp_free</pre> | free a user-defined unary operator | 6.2.4 |

6.2.1 GrB_UnaryOp_new: create a user-defined unary operator

```
GrB_Info GrB_UnaryOp_new  // create a new user-defined unary operator (

GrB_UnaryOp *unaryop,  // handle for the new unary operator  // pointer to the unary function  // type of output z  // type of input x );
```

GrB_UnaryOp_new creates a new unary operator. The new operator is returned in the unaryop handle, which must not be NULL on input. On output, its contents contains a pointer to the new unary operator.

The two types xtype and ztype are the GraphBLAS types of the input x and output z of the user-defined function z = f(x). These types may be built-in types or user-defined types, in any combination. The two types need not be the same, but they must be previously defined before passing them to $GrB_UnaryOp_new$.

The function argument to GrB_UnaryOp_new is a pointer to a user-defined function with the following signature:

```
void (*f) (void *z, const void *x);
```

When the function f is called, the arguments z and x are passed as (void *) pointers, but they will be pointers to values of the correct type, defined by ztype and xtype, respectively, when the operator was created.

NOTE: The pointers passed to a user-defined operator may not be unique. That is, the user function may be called with multiple pointers that point to the same space, such as when z=f(z,y) is to be computed by a binary operator, or z=f(z) for a unary operator. Any parameters passed to the user-callable function may be aliased to each other.

6.2.2 GxB_UnaryOp_new: create a named user-defined unary operator

Creates a named GrB_UnaryOp. Only the first 127 characters of unop_name are used. The unop_defn is a string containing the entire function itself. For example:

The two strings unop_name and unop_defn are optional, but are required to enable the JIT compilation of kernels that use this operator.

If JIT compilation is enabled, or if the corresponding JIT kernel has been copied into the PreJIT folder, the function may be NULL. In this case, a JIT kernel is compiled that contains just the user-defined function. If the JIT is disabled and the function is NULL, this method returns GrB_NULL_POINTER.

The above example is identical to the following usage, except that GrB_UnaryOp_new requires a non-NULL function pointer.

6.2.3 GrB_UnaryOp_wait: wait for a unary operator

```
GrB_Info GrB_wait  // wait for a user-defined unary operator (

GrB_UnaryOp unaryop,  // unary operator to wait for int mode  // GrB_COMPLETE or GrB_MATERIALIZE

);
```

After creating a user-defined unary operator, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. Currently, Suite-Sparse:GraphBLAS currently does nothing except to ensure that the unaryop is valid.

6.2.4 GrB_UnaryOp_free: free a user-defined unary operator

GrB_UnaryOp_free frees a user-defined unary operator. Either usage:

```
GrB_UnaryOp_free (&unaryop) ;
GrB_free (&unaryop) ;
```

frees the unaryop and sets unaryop to NULL. It safely does nothing if passed a NULL handle, or if unaryop == NULL on input. It does nothing at all if passed a built-in unary operator.

6.3 GraphBLAS binary operators: GrB_BinaryOp, z = f(x,y)

A binary operator is a scalar function of the form z=f(x,y). The types of z, x, and y need not be the same. The built-in binary operators are listed in the tables below. The notation T refers to any of the 13 built-in types, but two of those types are SuiteSparse extensions (GxB_FC32 and GxB_FC64). For those types, the operator name always starts with GxB, not GrB). The notation R refers to any real type (all but FC32 and FC64).

The six GxB_IS* comparators and the GxB_* logical operators all return a result one for true and zero for false, in the same domain T or R as their inputs. These six comparators are useful as "multiply" operators for creating semirings with non-Boolean monoids.

| Binary operators for all 13 types | | | | |
|-----------------------------------|--------------------|------------------|-----------------------------|--|
| GraphBLAS name | types (domains) | z = f(x, y) | description | |
| ${\tt GrB_FIRST_}T$ | $T \times T \to T$ | z = x | first argument | |
| ${\tt GrB_SECOND_}T$ | $T \times T \to T$ | z = y | second argument | |
| $\mathtt{GxB_ANY_}T$ | $T\times T\to T$ | z = x or y | pick x or y arbitrarily | |
| ${\tt GrB_ONEB_}T$ | $T\times T\to T$ | z = 1 | one | |
| ${\tt GxB_PAIR_}T$ | $T\times T\to T$ | z = 1 | one (historical) | |
| ${\tt GrB_PLUS_}T$ | $T\times T\to T$ | z = x + y | addition | |
| ${\tt GrB_MINUS_}T$ | $T\times T\to T$ | z = x - y | subtraction | |
| ${\tt GxB_RMINUS_}T$ | $T\times T\to T$ | z = y - x | reverse subtraction | |
| ${\tt GrB_TIMES_}T$ | $T\times T\to T$ | z = xy | multiplication | |
| ${\tt GrB_DIV_}T$ | $T\times T\to T$ | z = x/y | division | |
| ${\tt GxB_RDIV_}T$ | $T\times T\to T$ | z = y/x | reverse division | |
| ${\tt GxB_POW_}T$ | $T\times T\to T$ | $z = x^y$ | power | |
| $\mathtt{GxB_ISEQ_}T$ | $T \times T \to T$ | z = (x == y) | equal | |
| $\mathtt{GxB_ISNE_}T$ | $T\times T\to T$ | $z = (x \neq y)$ | not equal | |

The GxB_POW_* operators for real types do not return a complex result, and thus $z = f(x, y) = x^y$ is undefined if x is negative and y is not an integer. To compute a complex result, use GxB_POW_FC32 or GxB_POW_FC64 .

Operators that require the domain to be ordered (MIN, MAX, less-than, greater-than, and so on) are not defined for complex types. These are listed in the following table:

| Binary operators for all non-complex types | | | |
|--|--------------------|-------------------------------------|-----------------------|
| GraphBLAS name | types (domains) | z = f(x, y) | description |
| ${\tt GrB_MIN_}R$ | $R \times R \to R$ | $z = \min(x, y)$ | minimum |
| ${\tt GrB_MAX_}R$ | $R \times R \to R$ | $z = \max(x, y)$ | maximum |
| $\texttt{GxB_ISGT_}R$ | $R \times R \to R$ | z = (x > y) | greater than |
| ${\tt GxB_ISLT_}R$ | $R \times R \to R$ | z = (x < y) | less than |
| ${\tt GxB_ISGE_}R$ | $R \times R \to R$ | $z = (x \ge y)$ | greater than or equal |
| ${\tt GxB_ISLE_}R$ | $R \times R \to R$ | $z = (x \le y)$ | less than or equal |
| ${\tt GxB_LOR_}R$ | $R \times R \to R$ | $z = (x \neq 0) \lor (y \neq 0)$ | logical OR |
| ${\tt GxB_LAND_}R$ | $R \times R \to R$ | $z = (x \neq 0) \land (y \neq 0)$ | logical AND |
| ${\tt GxB_LXOR_}R$ | $R\times R\to R$ | $z = (x \neq 0) \veebar (y \neq 0)$ | logical XOR |

Another set of six kinds of built-in comparators have the form $T \times T \to bool$. Note that when T is bool, the six operators give the same results as the six GxB_IS*_B00L operators in the table above. These six comparators are useful as "multiply" operators for creating semirings with Boolean monoids.

| Binary comparators for all 13 types | | | |
|-------------------------------------|-------------------------------|------------------|-------------|
| GraphBLAS name | types (domains) | z = f(x, y) | description |
| ${\tt GrB_EQ_}T$ | $T \times T 	o \mathtt{bool}$ | z = (x == y) | equal |
| ${\tt GrB_NE_}T$ | $T\times T\to \texttt{bool}$ | $z = (x \neq y)$ | not equal |

| Binary comparators for non-complex types | | | |
|--|-------------------------------|-----------------|-----------------------|
| GraphBLAS name | types (domains) | z = f(x, y) | description |
| ${\tt GrB_GT_}R$ | $R \times R 	o \mathtt{bool}$ | z = (x > y) | greater than |
| ${\tt GrB_LT_}R$ | $R\times R\to \texttt{bool}$ | z = (x < y) | less than |
| ${\tt GrB_GE_}R$ | $R\times R\to \texttt{bool}$ | $z = (x \ge y)$ | greater than or equal |
| $\mathtt{GrB_LE}_R$ | $R\times R\to \texttt{bool}$ | $z = (x \le y)$ | less than or equal |

GraphBLAS has four built-in binary operators that operate purely in the Boolean domain. The first three are identical to the <code>GxB_L*_BOOL</code> operators described above, just with a shorter name. The <code>GrB_LXNOR</code> operator is the same as <code>GrB_EQ_BOOL</code>.

| Binary operators for the boolean type only | | | |
|--|--|----------------------|--------------|
| | types (domains) | V - V | description |
| GrB_LOR | $\mathtt{bool} \times \mathtt{bool} \to \mathtt{bool}$ | $z = x \vee y$ | logical OR |
| GrB_LAND | $\texttt{bool} \times \texttt{bool} \to \texttt{bool}$ | $z = x \wedge y$ | logical AND |
| GrB_LXOR | $\texttt{bool} \times \texttt{bool} \to \texttt{bool}$ | $z = x \veebar y$ | logical XOR |
| GrB_LXNOR | $\texttt{bool} \times \texttt{bool} \to \texttt{bool}$ | $z = \neg(x \vee y)$ | logical XNOR |

The following operators are defined for real floating-point types only (GrB_FP32 and GrB_FP64). They are identical to the C11 functions of the same name. The last one in the table constructs the corresponding complex type.

| Binary operators for the real floating-point types only | | | |
|---|--------------------|------------------------|-------------------------------|
| GraphBLAS name | types (domains) | z = f(x, y) | description |
| $\mathtt{GxB_ATAN2_}F$ | $F \times F \to F$ | $z = \tan^{-1}(y/x)$ | 4-quadrant arc tangent |
| $\mathtt{GxB_HYPOT_}F$ | $F\times F\to F$ | $z = \sqrt{x^2 + y^2}$ | hypotenuse |
| ${\tt GxB_FMOD_}F$ | $F\times F\to F$ | · | $\mathrm{C}11\ \mathtt{fmod}$ |
| ${\tt GxB_REMAINDER_}F$ | $F\times F\to F$ | | $\mathrm{C}11$ remainder |
| $\mathtt{GxB_LDEXP}_F$ | $F\times F\to F$ | | C11 ldexp |
| ${\tt GxB_COPYSIGN_}F$ | $F\times F\to F$ | | $\mathrm{C}11$ copysign |
| $\mathtt{GxB_CMPLX_}F$ | $F \times F \to Z$ | $z = x + y \times i$ | complex from real & imag |

Eight bitwise operators are predefined for signed and unsigned integers.

| Binary operators for signed and unsigned integers | | | |
|---|-------------------------------|-------------|----------------------|
| GraphBLAS name | types (domains) | z = f(x, y) | description |
| ${\tt GrB_BOR_\it I}$ | $I \times I \to I$ | z=x y | bitwise logical OR |
| ${\tt GrB_BAND_}I$ | $I \times I \to I$ | z=x&y | bitwise logical AND |
| ${\tt GrB_BXOR_}I$ | $I \times I \to I$ | z=x^y | bitwise logical XOR |
| ${\tt GrB_BXNOR_}I$ | $I \times I \to I$ | z=~(x^y) | bitwise logical XNOR |
| GxB_BGET_I | $I \times I \to I$ | | get bit y of x |
| $\mathtt{GxB_BSET_}I$ | $I \times I \to I$ | | set bit y of x |
| $\mathtt{GxB_BCLR}_I$ | $I \times I \to I$ | | clear bit y of x |
| ${\tt GxB_BSHIFT_}I$ | $I{	imes}{	ext{int8}}{	o}\;I$ | | bit shift |

There are two sets of built-in comparators in SuiteSparse:GraphBLAS, but they are not redundant. They are identical except for the type (domain) of their output, z. The GrB_EQ_T and related operators compare their inputs of type T and produce a Boolean result of true or false. The GxB_ISEQ_T and related operators compute the same thing and produce a result with same type T as their input operands, returning one for true or zero for false. The IS* comparators are useful when combining comparators with other non-Boolean operators. For example, a PLUS-ISEQ semiring counts how many terms are true. With this semiring, matrix multiplication C = AB for two weighted undirected graphs A and B computes c_{ij} as the number of edges node i and j have in common that have identical edge weights. Since the output type of the "multiplier" operator in a semiring must match the type

of its monoid, the Boolean EQ cannot be combined with a non-Boolean PLUS monoid to perform this operation.

Likewise, SuiteSparse:GraphBLAS has two sets of logical OR, AND, and XOR operators. Without the $_T$ suffix, the three operators $\tt GrB_LOR$, $\tt GrB_LAND$, and $\tt GrB_LXOR$ operate purely in the Boolean domain, where all input and output types are $\tt GrB_BOOL$. The second set $\tt (GxB_LOR_T GxB_LAND_T$ and $\tt GxB_LXOR_T$) provides Boolean operators to all 11 real domains, implicitly typecasting their inputs from type T to Boolean and returning a value of type T that is 1 for true or zero for false. The set of $\tt GxB_L*_T$ operators are useful since they can be combined with non-Boolean monoids in a semiring.

Floating-point operations follow the IEEE 754 standard. Thus, computing x/0 for a floating-point x results in +Inf if x is positive, -Inf if x is negative, and NaN if x is zero. The application is not terminated. However, integer division by zero normally terminates an application. Suite-Sparse:GraphBLAS avoids this by adopting the same rules as MATLAB, which are analogous to how the IEEE standard handles floating-point division by zero. For integers, when x is positive, x/0 is the largest positive integer, for negative x it is the minimum integer, and 0/0 results in zero. For example, for an integer x of type GrB_INT32 , 1/0 is $2^{31} - 1$ and (-1)/0 is -2^{31} . Refer to Section 6.1 for a list of integer ranges.

6.3.1 GraphBLAS binary operators based on index binary operators

Eight binary operators based on underlying index binary operators are predefined. They differ when used in a semiring and when used in GrB_eWise* and GrB_apply. These index-based binary operators cannot be used in GrB_build, nor can they be used as the accum operator for any operation.

The index-based binary operators do not depend on the type or numerical value of their inputs, just their position in a matrix or vector. For a vector, j is always 0, and i is the index into the vector. There are two types N available: INT32 and INT64, which is the type of the output z. User-defined index-based operators are not defined by $GrB_BinaryOp_new$, but by $GxB_BinaryOp_new_IndexOp$ instead. See Section 6.5 for details.

| Index-based binary operators for any type (including user-defined) | | | |
|--|---------------------|-------------------------|------------------------------------|
| wh | nen used as a multi | plicative operator | r in a semiring |
| GraphBLAS name | types (domains) | $z = f(a_{ik}, b_{kj})$ | description |
| $\texttt{GxB_FIRSTI}_N$ | $\rightarrow N$ | z = i | row index of a_{ik} (0-based) |
| ${\tt GxB_FIRSTI1_}N$ | $\rightarrow N$ | z = i + 1 | row index of a_{ik} (1-based) |
| ${\tt GxB_FIRSTJ_}N$ | $\rightarrow N$ | z = k | column index of a_{ik} (0-based) |
| ${\tt GxB_FIRSTJ1_}N$ | $\rightarrow N$ | z = k + 1 | column index of a_{ik} (1-based) |
| ${\tt GxB_SECONDI_}N$ | $\rightarrow N$ | z = k | row index of b_{kj} (0-based) |
| ${\tt GxB_SECONDI1_}N$ | $\rightarrow N$ | z = k + 1 | row index of b_{kj} (1-based) |
| ${\tt GxB_SECONDJ_}N$ | $\rightarrow N$ | z = j | column index of b_{kj} (0-based) |
| ${\tt GxB_SECONDJ1_}N$ | $\rightarrow N$ | z = j + 1 | column index of b_{kj} (1-based) |

| Index-based binary operators for any type (including user-defined) | | | | |
|--|--------------------------------|-------------------------|------------------------------------|--|
| | when used in all other methods | | | |
| GraphBLAS name | types (domains) | $z = f(a_{ij}, b_{ij})$ | description | |
| ${\tt GxB_FIRSTI_}N$ | $\rightarrow N$ | z = i | row index of a_{ij} (0-based) | |
| ${\tt GxB_FIRSTI1_}N$ | $\rightarrow N$ | z = i + 1 | row index of a_{ij} (1-based) | |
| ${\tt GxB_FIRSTJ_}N$ | $\rightarrow N$ | z = j | column index of a_{ij} (0-based) | |
| ${\tt GxB_FIRSTJ1_}N$ | $\rightarrow N$ | z = j + 1 | column index of a_{ij} (1-based) | |
| ${\tt GxB_SECONDI_}N$ | $\rightarrow N$ | z = i | row index of b_{ij} (0-based) | |
| ${\tt GxB_SECONDI1_}N$ | $\rightarrow N$ | z = i + 1 | row index of b_{ij} (1-based) | |
| ${\tt GxB_SECONDJ_}N$ | $\rightarrow N$ | z = j | column index of b_{ij} (0-based) | |
| ${\tt GxB_SECONDJ1_}N$ | $\rightarrow N$ | z = j + 1 | column index of b_{ij} (1-based) | |

Finally, one special binary operator can only be used as input to <code>GrB_Matrix_build</code> or <code>GrB_Vector_build</code>: the <code>GxB_IGNORE_DUP</code> operator. If <code>dup</code> is <code>NULL</code>, any duplicates in the <code>GrB*build</code> methods result in an error. If <code>dup</code> is the special binary operator <code>GxB_IGNORE_DUP</code>, then any duplicates are ignored. If duplicates appear, the last one in the list of tuples is taken and the prior ones ignored. This is not an error.

The next sections define the following methods for the GrB_BinaryOp object:

| GraphBLAS function | purpose | Section |
|------------------------------|---|---------|
| GrB_BinaryOp_new | create a user-defined binary operator | 6.3.2 |
| <pre>GxB_BinaryOp_new</pre> | create a named user-defined binary operator | 6.3.3 |
| <pre>GrB_BinaryOp_wait</pre> | wait for a user-defined binary operator | 6.3.4 |
| GrB_get | get properties of an operator | 10.6 |
| GrB_set | set the operator name/definition | 10.6 |
| <pre>GrB_BinaryOp_free</pre> | free a user-defined binary operator | 6.3.5 |

6.3.2 GrB_BinaryOp_new: create a user-defined binary operator

GrB_BinaryOp_new creates a new binary operator. The new operator is returned in the binaryop handle, which must not be NULL on input. On output, its contents contains a pointer to the new binary operator.

The three types xtype, ytype, and ztype are the GraphBLAS types of the inputs x and y, and output z of the user-defined function z = f(x, y). These types may be built-in types or user-defined types, in any combination. The three types need not be the same, but they must be previously defined before passing them to $GrB_BinaryOp_new$.

The final argument to GrB_BinaryOp_new is a pointer to a user-defined function with the following signature:

```
void (*f) (void *z, const void *x, const void *y) ;
```

When the function f is called, the arguments z, x, and y are passed as (void *) pointers, but they will be pointers to values of the correct type, defined by ztype, xtype, and ytype, respectively, when the operator was created.

NOTE: SuiteSparse:GraphBLAS may call the function with the pointers z and x equal to one another, in which case z=f(z,y) should be computed. Future versions may use additional pointer aliasing.

6.3.3 GxB_BinaryOp_new: create a named user-defined binary operator

```
GrB_Info GxB_BinaryOp_new
    GrB_BinaryOp *op,
                                    // handle for the new binary operator
    GxB_binary_function function,
                                    // pointer to the binary function
    GrB_Type ztype,
                                    // type of output z
    GrB_Type xtype,
                                    // type of input x
   GrB_Type ytype,
                                    // type of input y
                                    // name of the user function
    const char *binop_name,
    const char *binop_defn
                                    // definition of the user function
);
```

Creates a named GrB_BinaryOp. Only the first 127 characters of binop_name are used. The binop_defn is a string containing the entire function itself. For example:

```
void absdiff (double *z, double *x, double *y) { (*z) = fabs ((*x) - (*y)) ; } ;
...
GrB_Type AbsDiff;
GxB_BinaryOp_new (&AbsDiff, absdiff, GrB_FP64, GrB_FP64, GrB_FP64, "absdiff",
   "void absdiff (double *z, double *x, double *y) { (*z) = fabs ((*x) - (*y)) ; }");
```

The two strings binop_name and binop_defn are optional, but are required to enable the JIT compilation of kernels that use this operator.

If the JIT is enabled, or if the corresponding JIT kernel has been copied into the PreJIT folder, the function may be NULL. In this case, a JIT kernel is compiled that contains just the user-defined function. If the JIT is disabled and the function is NULL, this method returns GrB_NULL_POINTER.

The above example is identical to the following usage, except that GrB_BinaryOp_new requires a non-NULL function pointer.

```
void absdiff (double *z, double *x, double *y) { (*z) = fabs ((*x) - (*y)) ; } ;
...
GrB_Type AbsDiff;
GrB_BinaryOp_new (&AbsDiff, absdiff, GrB_FP64, GrB_FP64, GrB_FP64) ;
GrB_set (AbsDiff, "absdiff", GxB_JIT_C_NAME) ;
GrB_set (AbsDiff,
   "void absdiff (double *z, double *x, double *y) { (*z) = fabs ((*x) - (*y)) ; }",
   GxB_JIT_C_DEFINITION) ;
```

6.3.4 GrB_BinaryOp_wait: wait for a binary operator

After creating a user-defined binary operator, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. Currently, Suite-Sparse:GraphBLAS currently does nothing for except to ensure that the binaryop is valid.

6.3.5 GrB_BinaryOp_free: free a user-defined binary operator

GrB_BinaryOp_free frees a user-defined binary operator. Either usage:

```
GrB_BinaryOp_free (&op) ;
GrB_free (&op) ;
```

frees the op and sets op to NULL. It safely does nothing if passed a NULL handle, or if op == NULL on input. It does nothing at all if passed a built-in binary operator.

6.3.6 ANY and PAIR (ONEB) operators

The GxB_PAIR operator (also called GrB_ONEB) is simple to describe: just f(x,y)=1. It is called the PAIR operator since it returns 1 in a semiring when a pair of entries a_{ik} and b_{kj} is found in the matrix multiply. This operator is simple yet very useful. It allows purely structural computations to be performed on matrices of any type, without having to typecast them to Boolean with all values being true. Typecasting need not be performed on the inputs to the PAIR operator, and the PAIR operator does not need to access the values of the matrix. This cuts memory accesses, so it is a very fast operator to use.

The GxB_PAIR_T operator is a SuiteSparse:GraphBLAS extension. It has since been added to the v2.0 C API Specification as GrB_ONEB_T. They are identical, but the latter name should be used for compatibility with other GraphBLAS libraries.

The ANY operator is very unusual, but very powerful. It is the function $f_{\text{any}}(x,y) = x$, or y, where GraphBLAS has to freedom to select either x, or y, at its own discretion. Do not confuse the ANY operator with the any function in MATLAB/Octave, which computes a reduction using the logical OR operator.

The ANY function is associative and commutative, and can thus serve as an operator for a monoid. The selection of x are y is not randomized. Instead, SuiteSparse:GraphBLAS uses this freedom to compute as fast a result as possible. When used as the monoid in a dot product,

$$c_{ij} = \sum_{k} a_{ik} b_{kj}$$

for example, the computation can terminate as soon as any matching pair of entries is found. When used in a parallel saxpy-style computation, the ANY operator allows for a relaxed form of synchronization to be used, resulting in a fast benign race condition.

Because of this benign race condition, the result of the ANY monoid can be non-deterministic, unless it is coupled with the PAIR multiplicative operator. In this case, the ANY_PAIR semiring will return a deterministic result, since $f_{\text{any}}(1,1)$ is always 1.

When paired with a different operator, the results are non-deterministic. This gives a powerful method when computing results for which any value selected by the ANY operator is valid. One such example is the breadth-first-search tree. Suppose node j is at level v, and there are multiple nodes i at level v-1 for which the edge (i,j) exists in the graph. Any of these nodes i can serve as a valid parent in the BFS tree. Using the ANY operator, GraphBLAS can quickly compute a valid BFS tree; if it used again on the same inputs, it might return a different, yet still valid, BFS tree, due to the non-deterministic nature of intra-thread synchronization.

6.4 GraphBLAS IndexUnaryOp operators: GrB_IndexUnaryOp

An index-unary operator is a scalar function of the form $z = f(a_{ij}, i, j, y)$ that is applied to the entries a_{ij} of an m-by-n matrix. It can be used in $\mathtt{GrB_apply}$ (Section 12.12) or in $\mathtt{GrB_select}$ (Section 12.13) to select entries from a matrix or vector.

The signature of the index-unary function f is as follows:

The following built-in operators are available. Operators that do not depend on the value of A(i,j) can be used on any matrix or vector, including those of user-defined type. In the table, y is a scalar whose type matches the suffix of the operator. The VALUEEQ and VALUENE operators are defined for any built-in type. The other VALUE operators are defined only for real (not complex) built-in types. Any index computations are done in int64_t arithmetic; the result is typecasted to int32_t for the *INDEX_INT32 operators.

| GraphBLAS name | MATLAB/Octave | description |
|--------------------------------|--|---|
| | analog | |
| GrB_ROWINDEX_INT32 | z=i+y | row index of A(i,j), as int32 |
| GrB_ROWINDEX_INT64 | z=i+y | row index of A(i,j), as int64 |
| GrB_COLINDEX_INT32 | z=j+y | column index of A(i,j), as int32 |
| GrB_COLINDEX_INT64 | z=j+y | column index of A(i,j), as int64 |
| <pre>GrB_DIAGINDEX_INT32</pre> | z=j-(i+y) | column diagonal index of A(i,j), as int32 |
| GrB_DIAGINDEX_INT64 | z=j-(i+y) | column diagonal index of A(i,j), as int64 |
| GrB_TRIL | z=(j<=(i+y)) | true for entries on or below the yth diagonal |
| GrB_TRIU | z=(j>=(i+y)) | true for entries on or above the yth diagonal |
| GrB_DIAG | z=(j==(i+y)) | true for entries on the yth diagonal |
| GrB_OFFDIAG | z=(j!=(i+y)) | true for entries not on the yth diagonal |
| GrB_COLLE | z=(j<=y) | true for entries in columns 0 to y |
| GrB_COLGT | z=(j>y) | true for entries in columns $y+1$ and above |
| GrB_ROWLE | $z=(i \le y)$ | true for entries in rows 0 to y |
| GrB_ROWGT | z=(i>y) | true for entries in rows $y+1$ and above |
| GrB_VALUENE_T | z=(aij!=y) | true if A(i,j) is not equal to y |
| GrB_VALUEEQ_T | z=(aij==y) | true if A(i,j) is equal to y |
| GrB_VALUEGT_T | z=(aij>y) | true if A(i,j) is greater than y |
| GrB_VALUEGE_T | z=(aij>=y) | true if A(i,j) is greater than or equal to y |
| GrB_VALUELT_T | z=(aij <y)< td=""><td>true if A(i,j) is less than y</td></y)<> | true if A(i,j) is less than y |
| GrB_VALUELE_T | z=(aij<=y) | true if A(i,j) is less than or equal to y |

The following methods operate on the ${\tt GrB_IndexUnaryOp}$ object:

| GraphBLAS function | purpose | Section |
|----------------------------------|--|---------|
| GrB_IndexUnaryOp_new | create a user-defined index-unary operator | 6.4.1 |
| <pre>GxB_IndexUnaryOp_new</pre> | create a named user-defined index-unary operator | 6.4.2 |
| <pre>GrB_IndexUnaryOp_wait</pre> | wait for a user-defined index-unary operator | 6.4.3 |
| GrB_get | get properties of an operator | 10.5 |
| GrB_set | set the operator name/definition | 10.5 |
| ${\tt GrB_IndexUnaryOp_free}$ | free a user-defined index-unary operator | 6.4.4 |

6.4.1 GrB_IndexUnaryOp_new: create a user-defined index-unary operator

GrB_IndexUnaryOp_new creates a new index-unary operator. The new operator is returned in the op handle, which must not be NULL on input. On output, its contents contains a pointer to the new index-unary operator.

The function argument to $GrB_IndexUnaryOp_new$ is a pointer to a user-defined function whose signature is given at the beginning of Section 6.4. Given the properties of an entry a_{ij} in a matrix, the function should return z as true if the entry should be kept in the output of GrB_select , or false if it should not appear in the output. If the return value is not GrB_BOOL , it is typecasted to GrB_BOOL by GrB_select .

The type **xtype** is the GraphBLAS type of the input x of the user-defined function z = f(x, i, j, y), which is used for the entry A(i, j) of a matrix or v(i) of a vector. The type may be built-in or user-defined.

The type ytype is the GraphBLAS type of the scalar input y of the user-defined function z = f(x, i, j, y). The type may be built-in or user-defined.

6.4.2 GxB_IndexUnaryOp_new: create a named user-defined index-unary operator

```
GrB_Info GxB_IndexUnaryOp_new // create a named user-created IndexUnaryOp
    GrB_IndexUnaryOp *op,
                                    // handle for the new IndexUnary operator
   GxB_index_unary_function function,
                                          // pointer to index_unary function
   GrB_Type ztype,
                                   // type of output z
    GrB_Type xtype,
                                   // type of input x
                                   // type of scalar input y
    GrB_Type ytype,
    const char *idxop_name,
                                   // name of the user function
                                   // definition of the user function
    const char *idxop_defn
);
```

Creates a named GrB_IndexUnaryOp. Only the first 127 characters of idxop_name are used. The ixdop_defn is a string containing the entire function itself.

The two strings <code>idxop_name</code> and <code>idxop_defn</code> are optional, but are required to enable the JIT compilation of kernels that use this operator. The strings can also be set the <code>GrB_set</code> after the operator is created with <code>GrB_IndexUnaryOp_new</code>. For example:

```
void banded_idx
(
    bool *z,
    const int64_t *x, // unused
    int64_t i,
    int64_t j,
    const int64_t *thunk
)
{
    // d = abs (j-i)
    int64_t d = j-i;
    if (d < 0) d = -d;
    (*z) = (d \le *thunk) ;
}
#define BANDED_IDX_DEFN
                                         \n" \
"void banded_idx
"(
                                         \n" \
                                         \n" \
     bool *z,
     const int64_t *x,
                         // unused
                                         \n" \
     int64_t i,
                                         \n" \
                                         \n" \
     int64_t j,
```

```
11
     const int64_t *thunk
                                          \n" \
")
                                          \n" \
"{
                                          \n" \
                                          \n" \
     int64_t d = j-i;
11
                                          \n" \
     if (d < 0) d = -d;
                                          \n" \
     (*z) = (d \le *thunk);
11711
GxB_IndexUnaryOp_new (&Banded,
    (GxB_index_unary_function) banded_idx,
    GrB_BOOL, GrB_INT64, GrB_INT64,
    "banded_idx", BANDED_IDX_DEFN));
```

If JIT compilation is enabled, or if the corresponding JIT kernel has been copied into the PreJIT folder, the function may be NULL. In this case, a JIT kernel is compiled that contains just the user-defined function. If the JIT is disabled and the function is NULL, this method returns GrB_NULL_POINTER.

The above example is identical to the following usage except that GrB_IndexUnaryOp_new requires a non-NULL function pointer. The banded_idx function is defined the same as above.

6.4.3 GrB_IndexUnaryOp_wait: wait for an index-unary operator

After creating a user-defined index-unary operator, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. Currently, SuiteSparse:GraphBLAS currently does nothing except to ensure that the op is valid.

$\textbf{6.4.4} \quad \mathsf{GrB_IndexUnaryOp_free: free \ a \ user-defined \ index-unary \ operator}$

 ${\tt GrB_IndexUnaryOp_free}$ frees a user-defined index-unary operator. Either usage:

```
GrB_IndexUnaryOp_free (&op) ;
GrB_free (&op) ;
```

frees the op and sets op to NULL. It safely does nothing if passed a NULL handle, or if op == NULL on input. It does nothing at all if passed a built-in index-unary operator.

6.5 GraphBLAS index-binary operators: GxB_IndexBinaryOp

An index-binary operator is a scalar function of the following form:

$$z = f(x, i_x, j_x, y, i_y, j_y, \Theta),$$

where the value x appears at row i_x and column j_x in its matrix, and the value y appears at row i_y and column j_y in its matrix. The value Θ is a scalar that is the same for all uses of the operator. See our IEEE HPEC'24 paper for more details ([MBM⁺24]).

When used in an element-wise method for $C = A \oplus B$ and related methods (GrB_eWiseAdd, GxB_eWiseUnion, GrB_eWiseMult, or GrB_Kronecker), the operator is used for a pair of entries a_{ij} and b_{ij} , as

$$z = f(a_{ij}, i, j, b_{ij}, i, j, \Theta).$$

When used as the multiplicative operator in a semiring, to compute $C = A \oplus . \otimes B$, the operator is used as

$$z = f(a_{ik}, i, k, b_{kj}, k, j, \Theta)$$

to compute an entry to be summed by the monoid of the semiring.

No GraphBLAS operations directly use the GxB_IndexBinaryOp. Instead, the operator is coupled with a scalar Theta value to create a new index-based binary operator, which is simply a special case of a GrB_BinaryOp. The resulting GrB_BinaryOp can then be passed to element-wise methods and as the multiplicative operator of a new semiring.

The signature of the index-binary function f is as follows:

```
void f
(
                        // output value z, of type ztype
   void *z,
    const void *x,
                        // input value x of type xtype; value of v(ix) or A(ix,jx)
   GrB_Index ix,
                        // row index of v(ix) or A(ix,jx)
   GrB_Index jx,
                        // column index of A(ix,jx), or zero for v(ix)
    const void *y,
                        // input value y of type ytype; value of w(iy) or B(iy,jy)
    GrB_Index iy,
                        // row index of w(iy) or B(iy,jy)
   GrB_Index jy,
                        // column index of B(iy,jy), or zero for w(iy)
    const void *theta
                        // input scalar theta of type theta_type
);
```

The following binary operators ($\texttt{GrB_Binary0p}$ objects) are pre-defined, where N can be INT32 or INT64. These operators do not use theta. Instead, the offset of 1 in $\texttt{GxB_FIRSTI1}$ is fixed into the operator itself.

| Built-in index-based binary operators for any type | | | | |
|--|-----------------|---------------|-------------------------------|--|
| GraphBLAS name | types (domains) | z = f(x, y) | description | |
| $\texttt{GxB_FIRSTI_}N$ | $\rightarrow N$ | $z = i_x$ | row index of x (0-based) | |
| ${\tt GxB_FIRSTI1_}N$ | $\rightarrow N$ | $z = i_x + 1$ | row index of x (1-based) | |
| ${\tt GxB_FIRSTJ_}N$ | $\rightarrow N$ | $z = j_x$ | column index of x (0-based) | |
| ${\tt GxB_FIRSTJ1_}N$ | $\rightarrow N$ | $z = j_x + 1$ | column index of x (1-based) | |
| ${\tt GxB_SECONDI_}N$ | $\rightarrow N$ | $z = i_y$ | row index of y (0-based) | |
| ${\tt GxB_SECONDI1_}N$ | $\rightarrow N$ | $z = i_y + 1$ | row index of y (1-based) | |
| $\mathtt{GxB_SECONDJ_}N$ | $\rightarrow N$ | $z = j_y$ | column index of y (0-based) | |
| ${\tt GxB_SECONDJ1_}N$ | $\rightarrow N$ | $z = j_y + 1$ | column index of y (1-based) | |

The following methods operate on the ${\tt GxB_IndexBinaryOp}$ object:

| GraphBLAS function | purpose | Section |
|-------------------------------------|---|---------|
| GxB_IndexBinaryOp_new | create a named user-defined index-binary operator | 6.5.1 |
| ${\tt GxB_IndexBinary0p_wait}$ | wait for a user-defined index-binary operator | 6.5.2 |
| GrB_get | get properties of an operator | 10.7 |
| GrB_set | set the operator name/definition | 10.7 |
| ${\tt GxB_IndexBinary0p_free}$ | free a user-defined index-binary operator | 6.5.3 |
| <pre>GxB_BinaryOp_new_IndexOp</pre> | create a new index-based GrB_BinaryOp | 6.5.4 |

6.5.1 GxB_IndexBinaryOp_new: create a user-defined index-binary operator

```
GrB_Info GxB_IndexBinaryOp_new
                                    // handle for the new index binary operator
    GxB_IndexBinaryOp *op,
    GxB_index_binary_function function, // pointer to the index binary function
    GrB_Type ztype,
                                    // type of output z
    GrB_Type xtype,
                                    // type of input x
    GrB_Type ytype,
                                    // type of input y
                                    // type of input theta
    GrB_Type theta_type,
    const char *idxbinop_name,
                                    // name of the user function
    const char *idxbinop_defn
                                    // definition of the user function
);
```

Creates a named GxB_IndexBinaryOp. Only the first 127 characters of idxbinop_name are used. The ixdbinop_defn is a string containing the entire function itself.

The two strings $idxbinop_name$ and $idxbinop_defn$ are optional, but are required to enable the JIT compilation of kernels that use this operator. For example, the following operator can be used to compute the argmax of a matrix with a single call to GrB_mxv . It returns a vector c where c(i) = (k,v), where the largest value in the ith row of A has value v and appears in column k. If multiple values in the ith row have the same largest value, the one with the smallest column index is returned.

```
z->k = y->k;
       z->v = y->v;
    }
}
#define MAX_FP64 (a string containing the max_fp64 function above)
// create the types and operators:
                                    // unused, but cannot be NULL
GrB_Scalar Theta ;
GrB_Scalar_new (&Theta, GrB_BOOL) ;
GrB_Scalar_setElement_BOOL (Theta, 0) ;
GxB_IndexBinaryOp Iop ;
GrB_BinaryOp Bop, MonOp ;
GrB_Type Tuple ;
GxB_Type_new (&Tuple, sizeof (tuple_fp64), "tuple_fp64", FP64_K) ;
GxB_IndexBinaryOp_new (&Iop, make_fp64, Tuple, GrB_FP64, GrB_B00L, GrB_B00L,
    "make_fp64", MAKE_FP64));
GxB_BinaryOp_new_IndexOp (&Bop, Iop, Theta) ;
tuple_fp64 id;
memset (&id, 0, sizeof (tuple_fp64));
id.k = INT64_MAX ;
id.v = (double) (-INFINITY) ;
GxB_BinaryOp_new (&MonOp, max_fp64, Tuple, Tuple, Tuple, "max_fp64", MAX_FP64);
GrB_Monoid MonOp ;
GrB_Semiring Semiring ;
GrB_Monoid_new_UDT (&Monoid, MonOp, &id) ;
GrB_Semiring_new (&Semiring, Monoid, Bop) ;
// compute the argmax of each row of a GrB_FP64 matrix A:
// y = zeros (ncols,1);
GrB_Vector y ;
GrB_Matrix_new (&y, GrB_BOOL, ncols, 1));
GrB_Matrix_assign_BOOL (y, NULL, NULL, 0, GrB_ALL, ncols, GrB_ALL, 1, NULL));
// c = A*y using the argmax semiring
GrB_Vector_new (&c, Tuple, nrows, 1));
GrB_mxv (c, NULL, NULL, Semiring, A, y, NULL) ;
```

6.5.2 GxB_IndexBinaryOp_wait: wait for an index-binary operator

After creating a user-defined index-binary operator, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. Currently, SuiteSparse:GraphBLAS currently does nothing except to ensure that the op is valid.

6.5.3 GxB_IndexBinaryOp_free: free a user-defined index-binary operator

GxB_IndexBinaryOp_free frees a user-defined index-binary operator. Either usage:

```
GxB_IndexBinaryOp_free (&op) ;
GrB_free (&op) ;
```

frees the op and sets op to NULL. It safely does nothing if passed a NULL handle, or if op == NULL on input. No built-in index-binary operators exist, but if they did, the method does nothing at all if passed a built-in index-binary operator.

6.5.4 GxB_BinaryOp_new_IndexOp: create a index-based binary operator

The GxB_IndexBinaryOp cannot be directly used in any GraphBLAS operation such as GrB_mxm. Instead, it must be used to create a new index-based GrB_BinaryOp. The resulting binary operator can then be used to as the multiplicative operator in a new user-defined semiring, or as the primary binary operator of the element-wise operations (eWiseAdd, eWiseUnion, eWiseMult, or kronecker).

The resulting binary operator cannot be used as the accum operator in any GraphBLAS operation. It also cannot be used in other places where a binary operator appears, including GrB_*_build, GrB_apply, GrB_reduce and GrB_*_sort.

The GxB_BinaryOp_new_IndexOp method creates this index-based binary operator. It takes two input parameters: an index-binary operator, and a scalar Theta. The value of Theta is copied into this new binary operator, and the value cannot be changed. To change Theta, the binary operator must be freed, and any semiring that would like to use the new value of Theta must also be recreated.

An example of its use is given in Section 6.5.1.

6.6 GraphBLAS monoids: GrB_Monoid

A monoid is defined on a single domain (that is, a single type), T. It consists of an associative binary operator z = f(x, y) whose three operands x, y, and z are all in this same domain T (that is $T \times T \to T$). The operator must also have an identity element, or "zero" in this domain, such that f(x,0) = f(0,x) = x. Recall that an associative operator f(x,y) is one for which the condition f(a,f(b,c)) = f(f(a,b),c) always holds. That is, operator can be applied in any order and the results remain the same. If used in a semiring, the operator must also be commutative.

The 77 predefined monoids are listed in the table below, which includes nearly all monoids that can be constructed from built-in binary operators. A few additional monoids can be defined with $\mathtt{GrB_Monoid_new}$ using built-in operators, such as bitwise monoids for signed integers. Recall that T denotes any built-in type (including boolean, integer, floating point real, and complex), R denotes any non-complex type (including bool), I denotes any integer type, and I denotes any complex type. Let I denote the 10 non-boolean real types. Let I denote all unsigned integer types.

The table lists the GraphBLAS monoid, its type, expression, identity value, and terminal value (if any). For these built-in monoids, the terminal values are the annihilators of the function, which is the value z so that z = f(z, y) regardless of the value of y. For example $\min(-\infty, y) = -\infty$ for any y. For integer domains, $+\infty$ and $-\infty$ are the largest and smallest integer in their range. With unsigned integers, the smallest value is zero, and thus Grb_MIN_MONOID_UINT8 has an identity of 255 and a terminal value of 0.

When computing with a monoid, the computation can terminate early if the terminal value arises. No further work is needed since the result will not change. This value is called the terminal value instead of the annihilator, since a user-defined operator can be created with a terminal value that is not an annihilator. See Section 6.6.3 for an example.

The GxB_ANY_* monoid can terminate as soon as it finds any value at all.

| GraphBLAS | types (domains) | expression | identity | terminal |
|--------------------------------------|--|-------------------|---------------|----------------------|
| operator | | z = f(x, y) | | |
| ${	t Grb_PLUS_MONOID_S}$ | $S \times S \to S$ | z = x + y | 0 | none |
| ${\tt GrB_TIMES_MONOID_}S$ | $S \times S \to S$ | z = xy | 1 | 0 or none (see note) |
| ${	t Grb_MIN_MONOID_}S$ | $S \times S \to S$ | $z = \min(x, y)$ | $+\infty$ | $-\infty$ |
| ${\tt GrB_MAX_MONOID_}S$ | $S \times S \to S$ | $z = \max(x, y)$ | $-\infty$ | $+\infty$ |
| ${\tt GxB_PLUS_}Z_{\tt MONOID}$ | $Z \times Z \to Z$ | z = x + y | 0 | none |
| ${\tt GxB_TIMES_} Z_{\tt MONOID}$ | $Z \times Z \to Z$ | z = xy | 1 | none |
| ${\tt GxB_ANY_}T{\tt _MONOID}$ | $T \times T \to T$ | z = x or y | any | any |
| GrB_LOR_MONOID | $\texttt{bool} \times \texttt{bool} \to \texttt{bool}$ | $z = x \vee y$ | false | true |
| <pre>GrB_LAND_MONOID</pre> | $\mathtt{bool} \times \mathtt{bool} \to \mathtt{bool}$ | $z = x \wedge y$ | true | false |
| GrB_LXOR_MONOID | $\mathtt{bool} \times \mathtt{bool} \to \mathtt{bool}$ | $z = x \veebar y$ | false | none |
| GrB_LXNOR_MONOID | $\texttt{bool} \times \texttt{bool} \to \texttt{bool}$ | z = (x == y) | true | none |
| ${\tt GxB_BOR_}U{\tt _MONOID}$ | $U \times U \to U$ | z=x y | all bits zero | all bits one |
| ${\tt GxB_BAND_}U_{\tt MONOID}$ | $U \times U \to U$ | z=x&y | all bits one | all bits zero |
| ${\tt GxB_BXOR_}U_{\tt MONOID}$ | $U \times U \to U$ | z=x^y | all bits zero | none |
| ${\tt GxB_BXNOR_}U{\tt _MONOID}$ | $U \times U \to U$ | z=~(x^y) | all bits one | none |

The C API Specification includes 44 predefined monoids, with the naming convention <code>GrB_op_MONOID_type</code>. Forty monoids are available for the four operators MIN, MAX, PLUS, and TIMES, each with the 10 non-boolean real types. Four boolean monoids are predefined: <code>GrB_LOR_MONOID_BOOL</code>, <code>GrB_LAND_MONOID_BOOL</code>, <code>GrB_LXOR_MONOID_BOOL</code>, and <code>GrB_LXNOR_MONOID_BOOL</code>.

These all appear in SuiteSparse:GraphBLAS, which adds 33 additional predefined GxB* monoids, with the naming convention GxB_op_type_MONOID. The ANY operator can be used for all 13 types (including complex). The PLUS and TIMES operators are provided for both complex types, for 4 additional complex monoids. Sixteen monoids are predefined for four bitwise operators (BOR, BAND, BXOR, and BNXOR), each with four unsigned integer types (UINT8, UINT16, UINT32, and UINT64).

NOTE: The GrB_TIMES_FP* operators do not have a terminal value of zero, since they comply with the IEEE 754 standard, and O*NaN is not zero, but NaN. Technically, their terminal value is NaN, but this value is rare in practice and thus the terminal condition is not worth checking.

The next sections define the following methods for the GrB_Monoid object:

| GraphBLAS function | purpose | Section |
|------------------------------------|---|---------|
| GrB_Monoid_new | create a user-defined monoid | 6.6.1 |
| <pre>GrB_Monoid_wait</pre> | wait for a user-defined monoid | 6.6.2 |
| <pre>GxB_Monoid_terminal_new</pre> | create a monoid that has a terminal value | 6.6.3 |
| GrB_get | get properties of a monoid | 10.8 |
| GrB_set | set the monoid name | 10.8 |
| <pre>GrB_Monoid_free</pre> | free a monoid | 6.6.4 |

6.6.1 GrB_Monoid_new: create a monoid

```
GrB_Info GrB_Monoid_new  // create a monoid

(

GrB_Monoid *monoid,  // handle of monoid to create

GrB_BinaryOp op,  // binary operator of the monoid

<type> identity  // identity value of the monoid

);
```

GrB_Monoid_new creates a monoid. The operator, op, must be an associative binary operator, either built-in or user-defined.

In the definition above, <type> is a place-holder for the specific type of the monoid. For built-in types, it is the C type corresponding to the built-in type (see Section 6.1), such as bool, int32_t, float, or double. In this case, identity is a scalar value of the particular type, not a pointer. For user-defined types, <type> is void *, and thus identity is a not a scalar itself but a void * pointer to a memory location containing the identity value of the user-defined operator, op.

If op is a built-in operator with a known identity value, then the identity parameter is ignored, and its known identity value is used instead. The op cannot be a binary operator created by GxB_BinaryOp_new_IndexOp.

6.6.2 GrB_Monoid_wait: wait for a monoid

After creating a user-defined monoid, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. Currently, SuiteSparse:GraphBLAS currently does nothing except to ensure that the monoid is valid.

6.6.3 GxB_Monoid_terminal_new: create a monoid with terminal

GxB_Monoid_terminal_new is identical to GrB_Monoid_new, except that it allows for the specification of a *terminal value*. The <type> of the terminal value is the same as the identity parameter; see Section 6.6.1 for details.

The terminal value of a monoid is the value z for which z = f(z, y) for any y, where z = f(x, y) is the binary operator of the monoid. This is also called the *annihilator*, but the term *terminal value* is used here. This is because all annihilators are terminal values, but a terminal value need not be an annihilator, as described in the MIN example below.

If the terminal value is encountered during computation, the rest of the computations can be skipped. This can greatly improve the performance of GrB_reduce , and matrix multiply in specific cases (when a dot product method is used). For example, using GrB_reduce to compute the sum of all entries in a GrB_FP32 matrix with e entries takes O(e) time, since a monoid based on GrB_PLUS_FP32 has no terminal value. By contrast, a reduction using GrB_LOR on a GrB_BOOL matrix can take as little as O(1) time, if a true value is found in the matrix very early.

Monoids based on the built-in GrB_MIN_* and GrB_MAX_* operators (for any type), the boolean GrB_LOR, and the boolean GrB_LAND operators all have terminal values. For example, the identity value of GrB_LOR is false, and its terminal value is true. When computing a reduction of a set of boolean values to a single value, once a true is seen, the computation can exit early since the result is now known.

If op is a built-in operator with known identity and terminal values, then the identity and terminal parameters are ignored, and its known identity and terminal values are used instead.

There may be cases in which the user application needs to use a non-standard terminal value for a built-in operator. For example, suppose the matrix has type GrB_FP32, but all values in the matrix are known to be non-negative. The annihilator value of MIN is -INFINITY, but this will never be seen. However, the computation could terminate when finding the value

zero. This is an example of using a terminal value that is not actually an annihilator, but it functions like one since the monoid will operate strictly on non-negative values.

In this case, a monoid created with <code>GrB_MIN_FP32</code> will not terminate early, because the identity and terminal inputs are ignored when using <code>GrB_Monoid_new</code> with a built-in operator as its input. To create a monoid that can terminate early, create a user-defined operator that computes the same thing as <code>GrB_MIN_FP32</code>, and then create a monoid based on this user-defined operator with a terminal value of zero and an identity of <code>+INFINITY</code>. The <code>op</code> cannot be a binary operator created by <code>GxB_BinaryOp_new_IndexOp</code>.

6.6.4 GrB_Monoid_free: free a monoid

```
GrB_Info GrB_free // free a user-created monoid (
GrB_Monoid *monoid // handle of monoid to free
);
```

GrB_Monoid_frees frees a monoid. Either usage:

```
GrB_Monoid_free (&monoid) ;
GrB_free (&monoid) ;
```

frees the monoid and sets monoid to NULL. It safely does nothing if passed a NULL handle, or if monoid == NULL on input. It does nothing at all if passed a built-in monoid.

6.7 GraphBLAS semirings: GrB_Semiring

A semiring defines all the operators required to define the multiplication of two sparse matrices in GraphBLAS, $\mathbf{C} = \mathbf{AB}$. The "add" operator is a commutative and associative monoid, and the binary "multiply" operator defines a function z = fmult(x,y) where the type of z matches the exactly with the monoid type. SuiteSparse:GraphBLAS includes 1,473 predefined built-in semirings. The next sections define the following methods for the GrB_Semiring object:

| GraphBLAS function | purpose | Section |
|------------------------------|----------------------------------|---------|
| GrB_Semiring_new | create a user-defined semiring | 6.7.1 |
| <pre>GrB_Semiring_wait</pre> | wait for a user-defined semiring | 6.7.2 |
| GrB_get | get properties of a semiring | 10.9 |
| GrB_set | set the semiring name | 10.9 |
| <pre>GrB_Semiring_free</pre> | free a semiring | 6.7.3 |

6.7.1 GrB_Semiring_new: create a semiring

GrB_Semiring_new creates a new semiring, with add being the additive monoid and multiply being the binary "multiply" operator. In addition to the standard error cases, the function returns GrB_DOMAIN_MISMATCH if the output (ztype) domain of multiply does not match the domain of the add monoid.

The v2.0 C API Specification for GraphBLAS includes 124 predefined semirings, with names of the form $GrB_add_mult_SEMIRING_type$, where add is the operator of the additive monoid, mult is the multiply operator, and type is the type of the input x to the multiply operator, f(x,y). The name of the domain for the additive monoid does not appear in the name, since it always matches the type of the output of the mult operator. Twelve kinds of GrB* semirings are available for all 10 real, non-boolean types: $PLUS_TIMES$, $PLUS_MIN$, MIN_PLUS , MIN_TIMES , MIN_FIRST , MIN_SECOND , MIN_MAX , MAX_PLUS , MAX_TIMES , MAX_FIRST , MAX_SECOND , and

MAX_MIN. Four semirings are for boolean types only: LOR_LAND, LAND_LOR, LXOR_LAND, and LXNOR_LOR.

SuiteSparse:GraphBLAS pre-defines 1,553 semirings from built-in types and operators, listed below. The naming convention is <code>GxB_add_mult_type</code>. The 124 <code>GrB*</code> semirings are a subset of the list below, included with two names: <code>GrB*</code> and <code>GxB*</code>. If the <code>GrB*</code> name is provided, its use is preferred, for portability to other <code>GraphBLAS</code> implementations.

- 1000 semirings with a multiplier $T \times T \to T$ where T is any of the 10 non-Boolean, real types, from the complete cross product of:
 - 5 monoids (MIN, MAX, PLUS, TIMES, ANY)
 - 20 multiply operators (FIRST, SECOND, PAIR (same as ONEB), MIN, MAX, PLUS, MINUS, RMINUS, TIMES, DIV, RDIV, ISEQ, ISNE, ISGT, ISLT, ISGE, ISLE, LOR, LAND, LXOR).
 - 10 non-Boolean types, T
- 300 semirings with a comparator $T \times T \to \mathsf{bool}$, where T is non-Boolean and real, from the complete cross product of:
 - 5 Boolean monoids (LAND, LOR, LXOR, EQ, ANY)
 - 6 multiply operators (EQ, NE, GT, LT, GE, LE)
 - 10 non-Boolean types, T
- 55 semirings with purely Boolean types, bool × bool → bool, from the complete cross product of:
 - 5 Boolean monoids (LAND, LOR, LXOR, EQ, ANY)
 - 11 multiply operators (FIRST, SECOND, PAIR (same as ONEB), LOR, LAND, LXOR, EQ, GT, LT, GE, LE)
- 54 complex semirings, $Z \times Z \to Z$ where Z is GxB_FC32 (single precision complex) or GxB_FC64 (double precision complex):
 - 3 complex monoids (PLUS, TIMES, ANY)
 - 9 complex multiply operators (FIRST, SECOND, PAIR (same as ONEB), PLUS, MINUS, TIMES, DIV, RDIV, RMINUS)
 - -2 complex types, Z

- 64 bitwise semirings, $U \times U \to U$ where U is an unsigned integer.
 - 4 bitwise monoids (BOR, BAND, BXOR, BXNOR)
 - 4 bitwise multiply operators (the same list)
 - 4 unsigned integer types
- 80 index-based semirings, $X \times X \to N$ where N is INT32 or INT64:
 - 5 monoids (MIN, MAX, PLUS, TIMES, ANY)
 - 8 index-based operators (FIRSTI, FIRSTI1, FIRSTJ, FIRSTJ1, SECONDI, SECONDI1, SECONDJ, SECONDJ1)
 - 2 integer types (INT32, INT64)

The multiply operator can be any a binary operator, including one created by GxB_BinaryOp_new_IndexOp.

6.7.2 GrB_Semiring_wait: wait for a semiring

```
GrB_Info GrB_wait  // wait for a user-defined semiring

(
GrB_Semiring semiring,  // semiring to wait for  
int mode  // GrB_COMPLETE or GrB_MATERIALIZE
);
```

After creating a user-defined semiring, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. Currently, SuiteSparse:GraphBLAS currently does nothing except to ensure that the semiring is valid.

6.7.3 GrB_Semiring_free: free a semiring

GrB_Semiring_free frees a semiring. Either usage:

```
GrB_Semiring_free (&semiring) ;
GrB_free (&semiring) ;
```

frees the semiring and sets semiring to NULL. It safely does nothing if passed a NULL handle, or if semiring == NULL on input. It does nothing at all if passed a built-in semiring.

6.8 GraphBLAS scalars: GrB_Scalar

This section describes a set of methods that create, modify, query, and destroy a GraphBLAS scalar, GrB_Scalar:

| GraphBLAS function | purpose | Section |
|--------------------------------------|--------------------------------------|---------|
| GrB_Scalar_new | create a scalar | 6.8.1 |
| <pre>GrB_Scalar_wait</pre> | wait for a scalar | 6.8.2 |
| <pre>GrB_Scalar_dup</pre> | copy a scalar | 6.8.3 |
| <pre>GrB_Scalar_clear</pre> | clear a scalar of its entry | 6.8.4 |
| <pre>GrB_Scalar_nvals</pre> | return number of entries in a scalar | 6.8.5 |
| GrB_get | get properties of a scalar | 10.12 |
| GrB_set | set properties of a scalar | 10.12 |
| <pre>GrB_Scalar_setElement</pre> | set the single entry of a scalar | 6.8.6 |
| <pre>GrB_Scalar_extractElement</pre> | get the single entry from a scalar | 6.8.7 |
| <pre>GxB_Scalar_memoryUsage</pre> | memory used by a scalar | 6.8.8 |
| <pre>GxB_Scalar_type</pre> | type of a scalar | 6.8.9 |
| <pre>GrB_Scalar_free</pre> | free a scalar | 6.8.10 |

6.8.1 GrB_Scalar_new: create a scalar

```
GrB_Info GrB_Scalar_new  // create a new GrB_Scalar with no entry
(
    GrB_Scalar *s,  // handle of GrB_Scalar to create
    GrB_Type type  // type of GrB_Scalar to create
);
```

GrB_Scalar_new creates a new scalar with no entry in it, of the given type. This is analogous to MATLAB/Octave statement s = sparse(0), except that GraphBLAS can create scalars any type. The pattern of the new scalar is empty.

6.8.2 GrB_Scalar_wait: wait for a scalar

```
GrB_Info GrB_wait  // wait for a scalar (

GrB_Scalar s,  // scalar to wait for int mode  // GrB_COMPLETE or GrB_MATERIALIZE );
```

In non-blocking mode, the computations for a GrB_Scalar may be delayed. In this case, the scalar is not yet safe to use by multiple independent user threads. A user application may force completion of a scalar s via GrB_Scalar_wait(&s) (in v5.2.0), or GrB_Scalar_wait(s,mode) (in v6.0.0). With a mode of GrB_MATERIALIZE, all pending computations are finished, and different user threads may simultaneously call GraphBLAS operations that use the scalar s as an input parameter. See Section 10.2.2 if GraphBLAS is compiled without OpenMP.

6.8.3 GrB_Scalar_dup: copy a scalar

GrB_Scalar_dup makes a deep copy of a scalar. In GraphBLAS, it is possible, and valid, to write the following:

Then **s** and **t** can be used interchangeably. However, only a pointer reference is made, and modifying one of them modifies both, and freeing one of them leaves the other as a dangling handle that should not be used. If two different scalars are needed, then this should be used instead:

Then **s** and **t** are two different scalars that currently have the same value, but they do not depend on each other. Modifying one has no effect on the other. The Grb_NAME is copied into the new scalar.

6.8.4 GrB_Scalar_clear: clear a scalar of its entry

GrB_Scalar_clear clears the entry from a scalar. The pattern of s is empty, just as if it were created fresh with GrB_Scalar_new. Analogous with s = sparse (0) in MATLAB/Octave. The type of s does not change. Any pending updates to the scalar are discarded.

6.8.5 GrB_Scalar_nvals: return the number of entries in a scalar

GrB_Scalar_nvals returns the number of entries in a scalar, which is either 0 or 1. Roughly analogous to nvals = nnz(s) in MATLAB/Octave, except that the implicit value in GraphBLAS need not be zero and nnz (short for "number of nonzeros") in MATLAB is better described as "number of entries" in GraphBLAS.

6.8.6 GrB_Scalar_setElement: set the single entry of a scalar

GrB_Scalar_setElement sets the single entry in a scalar, like s = sparse(x) in MATLAB notation. For further details of this function, see GrB_Matrix_setElement in Section 6.10.12. If an error occurs, GrB_error(&err,s) returns details about the error. The scalar x can be any non-opaque C scalar corresponding to a built-in type, or void * for a user-defined type. It cannot be a GrB_Scalar.

6.8.7 GrB_Scalar_extractElement: get the single entry from a scalar

GrB_Scalar_extractElement extracts the single entry from a sparse scalar, like x = full(s) in MATLAB. Further details of this method are discussed in Section 6.10.13, which discusses GrB_Matrix_extractElement. NOTE: if no entry is present in the scalar s, then x is not modified, and the return value of GrB_Scalar_extractElement is GrB_NO_VALUE.

6.8.8 GxB_Scalar_memoryUsage: memory used by a scalar

```
GrB_Info GxB_Scalar_memoryUsage // return # of bytes used for a scalar
(
    size_t *size, // # of bytes used by the scalar s
    const GrB_Scalar s // GrB_Scalar to query
);
```

Returns the memory space required for a scalar, in bytes. By default, any read-only components are not included in the total memory. This can be changed with via GrB_get; see Section 10.2.

6.8.9 GxB_Scalar_type: type of a scalar

Returns the type of a scalar. See GxB_Matrix_type for details (Section 6.10.26).

6.8.10 GrB_Scalar_free: free a scalar

```
GrB_Info GrB_free // free a GrB_Scalar (
GrB_Scalar *s // handle of GrB_Scalar to free
```

```
) ;
```

```
GrB_Scalar_free frees a scalar. Either usage:
   GrB_Scalar_free (&s) ;
   GrB_free (&s) ;
```

frees the scalar s and sets s to NULL. It safely does nothing if passed a NULL handle, or if s == NULL on input. Any pending updates to the scalar are abandoned.

6.9 GraphBLAS vectors: GrB_Vector

This section describes a set of methods that create, modify, query, and destroy a GraphBLAS sparse vector, $\mathtt{GrB_Vector}$:

| GraphBLAS function | purpose | Section |
|--|----------------------------------|---------|
| GrB_Vector_new | create a vector | 6.9.1 |
| <pre>GrB_Vector_wait</pre> | wait for a vector | 6.9.2 |
| <pre>GrB_Vector_dup</pre> | copy a vector | 6.9.3 |
| <pre>GrB_Vector_clear</pre> | clear a vector of all entries | 6.9.4 |
| <pre>GrB_Vector_size</pre> | size of a vector | 6.9.5 |
| <pre>GrB_Vector_nvals</pre> | number of entries in a vector | 6.9.6 |
| GrB_get | get properties of a vector | 10.11 |
| GrB_set | set properties of a vector | 10.11 |
| <pre>GrB_Vector_build</pre> | build a vector from tuples | 6.9.7 |
| <pre>GxB_Vector_build_Scalar</pre> | build a vector from tuples | 6.9.9 |
| <pre>GxB_Vector_build_Vector</pre> | build a vector from tuples | 6.9.8 |
| <pre>GxB_Vector_build_Scalar_Vector</pre> | build a vector from tuples | 6.9.10 |
| <pre>GrB_Vector_setElement</pre> | add an entry to a vector | 6.9.11 |
| <pre>GrB_Vector_extractElement</pre> | get an entry from a vector | 6.9.12 |
| <pre>GxB_Vector_isStoredElement</pre> | check if entry present in vector | 6.9.13 |
| <pre>GrB_Vector_removeElement</pre> | remove an entry from a vector | 6.9.14 |
| <pre>GrB_Vector_extractTuples</pre> | get all entries from a vector | 6.9.15 |
| <pre>GxB_Vector_extractTuples_Vector</pre> | get all entries from a vector | 6.9.16 |
| <pre>GrB_Vector_resize</pre> | resize a vector | 6.9.17 |
| <pre>GxB_Vector_diag</pre> | extract a diagonal from a matrix | 6.9.18 |
| <pre>GxB_Vector_memoryUsage</pre> | memory used by a vector | 6.9.19 |
| GxB_Vector_type | type of the matrix | 6.9.20 |
| GrB_Vector_free | free a vector | 6.9.21 |
| GxB_Vector_serialize | serialize a vector | 6.11.1 |
| GxB_Vector_deserialize | deserialize a vector | 6.11.2 |
| GxB_Vector_sort | sort a vector | 6.15.1 |

Refer to Section 6.11 for serialization/deserialization methods and to Section 6.15 for sorting methods.

6.9.1 GrB_Vector_new: create a vector

GrB_Vector_new creates a new n-by-1 sparse vector with no entries in it, of the given type. This is analogous to MATLAB/Octave statement v = sparse (n,1), except that GraphBLAS can create sparse vectors any type. The pattern of the new vector is empty.

SPEC: n may be zero, as an extension to the specification.

6.9.2 GrB_Vector_wait: wait for a vector

```
GrB_Info GrB_wait  // wait for a vector

(
GrB_Vector w,  // vector to wait for
int mode  // GrB_COMPLETE or GrB_MATERIALIZE
);
```

In non-blocking mode, the computations for a GrB_Vector may be delayed. In this case, the vector is not yet safe to use by multiple independent user threads. A user application may force completion of a vector w via GrB_Vector_wait(&w) (in v5.2.0), or GrB_Vector_wait(w,mode) (in v6.0.0). With a mode of GrB_MATERIALIZE, all pending computations are finished, and different user threads may simultaneously call GraphBLAS operations that use the vector w as an input parameter. See Section 10.2.2 if GraphBLAS is compiled without OpenMP.

6.9.3 GrB_Vector_dup: copy a vector

GrB_Vector_dup makes a deep copy of a sparse vector. In GraphBLAS, it is possible, and valid, to write the following:

Then w and u can be used interchangeably. However, only a pointer reference is made, and modifying one of them modifies both, and freeing one of them leaves the other as a dangling handle that should not be used. If two different vectors are needed, then this should be used instead:

```
GrB_Vector u, w ;
GrB_Vector_new (&u, GrB_FP64, n) ;
GrB_Vector_dup (&w, u) ;  // like w = u, but making a deep copy
```

Then w and u are two different vectors that currently have the same set of values, but they do not depend on each other. Modifying one has no effect on the other. The Grb_NAME is copied into the new vector.

6.9.4 GrB_Vector_clear: clear a vector of all entries

GrB_Vector_clear clears all entries from a vector. All values v(i) are now equal to the implicit value, depending on what semiring ring is used to perform computations on the vector. The pattern of v is empty, just as if it were created fresh with GrB_Vector_new. Analogous with v (:) = sparse(0) in MATLAB. The type and dimension of v do not change. Any pending updates to the vector are discarded.

6.9.5 GrB_Vector_size: return the size of a vector

 GrB_Vector_size returns the size of a vector (the number of rows). Analogous to n = length(v) or n = size(v, 1) in MATLAB.

6.9.6 GrB_Vector_nvals: return the number of entries in a vector

GrB_Vector_nvals returns the number of entries in a vector. Roughly analogous to nvals = nnz(v) in MATLAB, except that the implicit value in GraphBLAS need not be zero and nnz (short for "number of nonzeros") in MATLAB is better described as "number of entries" in GraphBLAS.

6.9.7 GrB_Vector_build: build a vector from a set of tuples

GrB_Vector_build constructs a sparse vector w from a set of tuples, I and X, each of length nvals. The vector w must have already been initialized with GrB_Vector_new, and it must have no entries in it before calling GrB_Vector_build. This function is just like GrB_Matrix_build (see Section 6.10.8), except that it builds a sparse vector instead of a sparse matrix. For a description of what GrB_Vector_build does, refer to GrB_Matrix_build.

For a vector, the list of column indices J in GrB_Matrix_build is implicitly a vector of length nvals all equal to zero. Otherwise the methods are identical.

If dup is NULL, any duplicates result in an error. If dup is the special binary operator GxB_IGNORE_DUP, then any duplicates are ignored. If duplicates appear, the last one in the list of tuples is taken and the prior ones ignored. This is not an error. The dup operator cannot be a binary operator created by GxB_BinaryOp_new_IndexOp.

SPEC: Results are defined even if dup is non-associative and/or non-commutative.

6.9.8 GrB_Vector_build_Vector: build a vector from a set of tuples

GxB_Vector_build_Vector is identical to GrB_Vector_build, except that the inputs I and X are GrB_Vector objects, each with nvals entries. The interpretation of I_vector and X_vector are controlled by descriptor settings GxB_ROWINDEX_LIST and GxB_VALUE_LIST, respectively. The method can use either the indices or values of each of the input vectors; the default is to use the values. See Section 6.16.4 for details.

6.9.9 GxB_Vector_build_Scalar: build a vector from a set of tuples

GxB_Vector_build_Scalar constructs a sparse vector w from a set of tuples defined by the index array I of length nvals, and a scalar. The

scalar is the value of all of the tuples. Unlike GrB_Vector_build, there is no dup operator to handle duplicate entries. Instead, any duplicates are silently ignored (if the number of duplicates is desired, simply compare the input nvals with the value returned by GrB_Vector_nvals after the vector is constructed). All entries in the sparsity pattern of w are identical, and equal to the input scalar value.

6.9.10 GxB_Vector_build_Scalar_Vector: build a vector from a set of tuples

GxB_Vector_build_Scalar_Vector is identical to GxB_Vector_build_Scalar, except that the inputs I and X are GrB_Vector objects, each with nvals entries. The interpretation of I_vector is controlled by the descriptor setting GxB_ROWINDEX_LIST. The method can use either the indices or values of the I_input vector; the default is to use the values. See Section 6.16.4 for details.

6.9.11 GrB_Vector_setElement: add an entry to a vector

GrB_Vector_setElement sets a single entry in a vector, w(i) = x. The operation is exactly like setting a single entry in an n-by-1 matrix, A(i,0) = x, where the column index for a vector is implicitly j=0. For further details of this function, see GrB_Matrix_setElement in Section 6.10.12. If an error occurs, GrB_error(&err,w) returns details about the error.

6.9.12 GrB_Vector_extractElement: get an entry from a vector

```
GrB_Info GrB_Vector_extractElement // x = v(i)
                              // scalar extracted (non-opaque, C scalar)
    <type> *x,
    <type> *x,
const GrB_Vector v,
                           // vector to extract an entry from
    GrB_Index i
                               // index
) ;
GrB_Info GrB_Vector_extractElement // x = v(i)
    GrB_Scalar x,
                              // GrB_Scalar extracted
    const GrB_Vector v,
                              // vector to extract an entry from
    GrB_Index i
                               // index
);
```

GrB_Vector_extractElement extracts a single entry from a vector, x = v(i). The method is identical to extracting a single entry x = A(i,0) from an n-by-1 matrix; see Section 6.10.13.

6.9.13 GxB_Vector_isStoredElement: check if entry present in vector

GxB_Vector_isStoredElement checks if a single entry v(i) is present, returning GrB_SUCCESS if the entry is present or GrB_NO_VALUE otherwise. The value of v(i) is not returned. See also Section 6.10.14.

6.9.14 GrB_Vector_removeElement: remove an entry from a vector

GrB_Vector_removeElement removes a single entry w(i) from a vector. If no entry is present at w(i), then the vector is not modified. If an error occurs, GrB_error(&err,w) returns details about the error.

6.9.15 GrB_Vector_extractTuples: get all entries from a vector

GrB_Vector_extractTuples extracts all tuples from a sparse vector, analogous to [I,~,X] = find(v) in MATLAB/Octave. This function is identical to its GrB_Matrix_extractTuples counterpart, except that the array of column indices J does not appear in this function. Refer to Section 6.10.16 where further details of this function are described.

6.9.16 GxB_Vector_extractTuples_Vector: get all entries from a vector

GxB_Vector_extractTuples_Vector is identical to GrB_Vector_extractTuples except that its two outputs are GrB_Vector objects. The vectors I_vector and X_vector objects must exist on input. On output, any prior content is erased and their type, dimensions, and values are revised to contain dense vectors of length nvals.

6.9.17 GrB_Vector_resize: resize a vector

```
GrB_Info GrB_Vector_resize // change the size of a vector

(
GrB_Vector u, // vector to modify
GrB_Index nrows_new // new number of rows in vector
);
```

GrB_Vector_resize changes the size of a vector. If the dimension decreases, entries that fall outside the resized vector are deleted.

6.9.18 GxB_Vector_diag: extract a diagonal from a matrix

GxB_Vector_diag extracts a vector v from an input matrix A, which may be rectangular. If k = 0, the main diagonal of A is extracted; k > 0 denotes diagonals above the main diagonal of A, and k < 0 denotes diagonals below the main diagonal of A. Let A have dimension m-by-n. If k is in the range 0 to n - 1, then v has length $\min(m, n - k)$. If k is negative and in the range -1 to -m + 1, then v has length $\min(m + k, n)$. If k is outside these ranges, v has length 0 (this is not an error). This function computes the same thing as the MATLAB/Octave statement v=diag(A,k) when A is a matrix, except that GxB_Vector_diag can also do typecasting.

The vector \mathbf{v} must already exist on input, and $\mathtt{GrB_Vector_size}$ (&len,v) must return $\mathtt{len} = 0$ if $\mathtt{k} \geq n$ or $\mathtt{k} \leq -m$, $\mathtt{len} = \min(m, n-k)$ if \mathtt{k} is in the range 0 to n-1, and $\mathtt{len} = \min(m+k,n)$ if \mathtt{k} is in the range -1 to -m+1. Any existing entries in \mathbf{v} are discarded. The type of \mathbf{v} is preserved, so that if the type of \mathtt{A} and \mathtt{v} differ, the entries are typecasted into the type of \mathtt{v} . Any settings made to \mathtt{v} by $\mathtt{GrB_set}$ (bitmap switch and sparsity control) are unchanged.

6.9.19 GxB_Vector_memoryUsage: memory used by a vector

```
GrB_Info GxB_Vector_memoryUsage // return # of bytes used for a vector
(
    size_t *size, // # of bytes used by the vector v
    const GrB_Vector v // vector to query
);
```

Returns the memory space required for a vector, in bytes. By default, any read-only components are not included in the total memory. This can be changed with via GrB_get; see Section 10.2.

6.9.20 GxB_Vector_type: type of a vector

Returns the type of a vector. See GxB_Matrix_type for details (Section 6.10.26).

6.9.21 GrB_Vector_free: free a vector

GrB_Vector_free frees a vector. Either usage:

```
GrB_Vector_free (&v) ;
GrB_free (&v) ;
```

frees the vector v and sets v to NULL. It safely does nothing if passed a NULL handle, or if v == NULL on input. Any pending updates to the vector are abandoned.

$\mathbf{6.10} \quad \mathbf{GraphBLAS} \ \mathbf{matrices:} \ \mathsf{GrB}_\mathsf{Matrix}$

This section describes a set of methods that create, modify, query, and destroy a GraphBLAS sparse matrix, <code>GrB_Matrix</code>:

| GraphBLAS function | purpose | Section |
|--|----------------------------------|---------|
| GrB_Matrix_new | create a matrix | 6.10.1 |
| <pre>GrB_Matrix_wait</pre> | wait for a matrix | 6.10.2 |
| <pre>GrB_Matrix_dup</pre> | copy a matrix | 6.10.3 |
| <pre>GrB_Matrix_clear</pre> | clear a matrix of all entries | 6.10.4 |
| <pre>GrB_Matrix_nrows</pre> | number of rows of a matrix | 6.10.5 |
| <pre>GrB_Matrix_ncols</pre> | number of columns of a matrix | 6.10.6 |
| <pre>GrB_Matrix_nvals</pre> | number of entries in a matrix | 6.10.7 |
| <pre>GrB_Matrix_build</pre> | build a matrix from tuples | 6.10.8 |
| <pre>GxB_Matrix_build_Vector</pre> | build a matrix from tuples | 6.10.9 |
| <pre>GxB_Matrix_build_Scalar</pre> | build a matrix from tuples | 6.10.10 |
| <pre>GxB_Matrix_build_Scalar_Vector</pre> | build a matrix from tuples | 6.10.11 |
| <pre>GrB_Matrix_setElement</pre> | add an entry to a matrix | 6.10.12 |
| <pre>GrB_Matrix_extractElement</pre> | get an entry from a matrix | 6.10.13 |
| <pre>GxB_Matrix_isStoredElement</pre> | check if entry present in matrix | 6.10.14 |
| <pre>GrB_Matrix_removeElement</pre> | remove an entry from a matrix | 6.10.15 |
| <pre>GrB_Matrix_extractTuples</pre> | get all entries from a matrix | 6.10.16 |
| <pre>GxB_Matrix_extractTuples_Vector</pre> | get all entries from a matrix | 6.10.17 |
| <pre>GrB_Matrix_resize</pre> | resize a matrix | 6.10.18 |
| <pre>GxB_Matrix_reshape</pre> | reshape a matrix | 6.10.19 |
| <pre>GxB_Matrix_reshapeDup</pre> | reshape a matrix | 6.10.20 |
| <pre>GxB_Matrix_concat</pre> | concatenate matrices | 6.10.21 |
| <pre>GxB_Matrix_split</pre> | split a matrix into matrices | 6.10.22 |
| <pre>GrB_Matrix_diag</pre> | diagonal matrix from vector | 6.10.23 |
| <pre>GxB_Matrix_diag</pre> | diagonal matrix from vector | 6.10.24 |
| <pre>GxB_Matrix_memoryUsage</pre> | memory used by a matrix | 6.10.25 |
| <pre>GxB_Matrix_type</pre> | type of the matrix | 6.10.26 |
| <pre>GrB_Matrix_free</pre> | free a matrix | 6.10.27 |
| GrB_Matrix_serializeSize | return size of serialized matrix | 6.11.3 |
| GrB_Matrix_serialize | serialize a matrix | 6.11.4 |
| GxB_Matrix_serialize | serialize a matrix | 6.11.5 |
| GrB_Matrix_deserialize | deserialize a matrix | 6.11.6 |
| GxB_Matrix_deserialize | deserialize a matrix | 6.11.7 |
| GrB_get | get properties of a matrix | 10.10 |
| GrB_set | set properties of a matrix | 10.10 |
| | | |

| GraphBLAS function | purpose | Section |
|----------------------------------|---------------------------|---------|
| GrB_Matrix_import | import in various formats | 6.14.1 |
| <pre>GrB_Matrix_export</pre> | export in various formats | 6.14.2 |
| <pre>GrB_Matrix_exportSize</pre> | array sizes for export | 6.14.3 |
| <pre>GrB_Matrix_exportHint</pre> | hint best export format | 6.14.4 |
| GxB_Matrix_sort | sort a matrix | 6.15.2 |

Refer to Section 6.11 for serialization/deserialization methods, Section 6.14 for GrB import/export methods, Section 6.15 for sorting methods, and Section 10.10 for get/set methods.

6.10.1 GrB_Matrix_new: create a matrix

```
GrB_Info GrB_Matrix_new // create a new matrix with no entries

(
GrB_Matrix *A, // handle of matrix to create
GrB_Type type, // type of matrix to create
GrB_Index nrows, // matrix dimension is nrows-by-ncols
GrB_Index ncols
);
```

GrB_Matrix_new creates a new nrows-by-ncols sparse matrix with no entries in it, of the given type. This is analogous to the MATLAB statement A = sparse (nrows, ncols), except that GraphBLAS can create sparse matrices of any type.

By default, matrices of size nrows-by-1 are held by column, regardless of the global setting controlled by GrB_set (GrB_GLOBAL, ..., GrB_STORAGE_ORIENTATION_HINT), for any value of nrows. Matrices of size 1-by-ncols with ncols not equal to 1 are held by row, regardless of this global setting. The global setting only affects matrices with both m > 1 and n > 1. Empty matrices (0-by-0) are also controlled by the global setting.

Once a matrix is created, its format (by-row or by-column) can be arbitrarily changed with GrB_set (A, fmt, GrB_STORAGE_ORIENTATION_HINT) with fmt equal to GrB_COLMAJOR or GrB_ROWMAJOR.

SPEC: nrows and/or ncols may be zero. as an extension to the specification.

6.10.2 GrB Matrix wait: wait for a matrix

```
GrB_Info GrB_wait  // wait for a matrix
(
GrB_Matrix C,  // matrix to wait for int mode  // GrB_COMPLETE or GrB_MATERIALIZE
);
```

In non-blocking mode, the computations for a GrB_Matrix may be delayed. In this case, the matrix is not yet safe to use by multiple independent user threads. A user application may force completion of a matrix C via GrB_Matrix_wait(&C) (in v5.2.0), or GrB_Matrix_wait(C,mode) (in v6.0.0). With a mode of GrB_MATERIALIZE, all pending computations are finished, and different user threads may simultaneously call GraphBLAS operations that use the matrix C as an input parameter. See Section 10.2.2 if GraphBLAS is compiled without OpenMP.

6.10.3 GrB_Matrix_dup: copy a matrix

GrB_Matrix_dup makes a deep copy of a sparse matrix. In GraphBLAS, it is possible, and valid, to write the following:

Then C and A can be used interchangeably. However, only a pointer reference is made, and modifying one of them modifies both, and freeing one of them leaves the other as a dangling handle that should not be used. If two different matrices are needed, then this should be used instead:

```
GrB_Matrix A, C ;
GrB_Matrix_new (&A, GrB_FP64, n) ;
GrB_Matrix_dup (&C, A) ;  // like C = A, but making a deep copy
```

Then C and A are two different matrices that currently have the same set of values, but they do not depend on each other. Modifying one has no effect on the other. The Grb_NAME is copied into the new matrix.

6.10.4 GrB_Matrix_clear: clear a matrix of all entries

```
GrB_Info GrB_Matrix_clear // clear a matrix of all entries;
( // type and dimensions remain unchanged
GrB_Matrix A // matrix to clear
);
```

GrB_Matrix_clear clears all entries from a matrix. All values A(i,j) are now equal to the implicit value, depending on what semiring ring is used to perform computations on the matrix. The pattern of A is empty, just as if it were created fresh with GrB_Matrix_new. Analogous with A (:,:) = 0 in MATLAB. The type and dimensions of A do not change. Any pending updates to the matrix are discarded.

6.10.5 GrB_Matrix_nrows: return the number of rows of a matrix

```
GrB_Info GrB_Matrix_nrows // get the number of rows of a matrix

(
GrB_Index *nrows, // matrix has nrows rows
const GrB_Matrix A // matrix to query
);
```

GrB_Matrix_nrows returns the number of rows of a matrix (nrows=size(A,1) in MATLAB).

6.10.6 GrB_Matrix_ncols: return the number of columns of a matrix

GrB_Matrix_ncols returns the number of columns of a matrix (ncols=size(A,2) in MATLAB).

6.10.7 GrB_Matrix_nvals: return the number of entries in a matrix

GrB_Matrix_nvals returns the number of entries in a matrix. Roughly analogous to nvals = nnz(A) in MATLAB, except that the implicit value in GraphBLAS need not be zero and nnz (short for "number of nonzeros") in MATLAB is better described as "number of entries" in GraphBLAS.

6.10.8 GrB_Matrix_build: build a matrix from a set of tuples

GrB_Matrix_build constructs a sparse matrix C from a set of tuples, I, J, and X, each of length nvals. The matrix C must have already been initialized with GrB_Matrix_new, and it must have no entries in it before calling GrB_Matrix_build. Thus the dimensions and type of C are not changed by this function, but are inherited from the prior call to GrB_Matrix_new or GrB_matrix_dup.

An error is returned (GrB_INDEX_OUT_OF_BOUNDS) if any row index in I is greater than or equal to the number of rows of C, or if any column index in J is greater than or equal to the number of columns of C

Any duplicate entries with identical indices are assembled using the binary dup operator provided on input. All three types (x, y, z for z=dup(x,y)) must be identical. The types of dup, C and X must all be compatible. See Section 2.4 regarding typecasting and compatibility. The values in X are typecasted, if needed, into the type of dup. Duplicates are then assembled into a matrix T of the same type as dup, using T(i,j) = dup (T(i,j), X(k)).

After T is constructed, it is typecasted into the result C. That is, typecasting does not occur at the same time as the assembly of duplicates.

If dup is NULL, any duplicates result in an error. If dup is the special binary operator <code>GxB_IGNORE_DUP</code>, then any duplicates are ignored. If duplicates appear, the last one in the list of tuples is taken and the prior ones ignored. This is not an error.

SPEC: As an extension to the specification, results are defined even if dup is non-associative and/or non-commutative.

The GraphBLAS API requires dup to be associative so that entries can be assembled in any order, and states that the result is undefined if dup is not associative. However, SuiteSparse:GraphBLAS guarantees a well-defined order of assembly. Entries in the tuples $[\mathtt{I},\mathtt{J},\mathtt{X}]$ are first sorted in increasing order of row and column index, with ties broken by the position of the tuple in the $[\mathtt{I},\mathtt{J},\mathtt{X}]$ list. If duplicates appear, they are assembled in the order they appear in the $[\mathtt{I},\mathtt{J},\mathtt{X}]$ input. That is, if the same indices \mathtt{i} and \mathtt{j} appear in positions $\mathtt{k1}$, $\mathtt{k2}$, $\mathtt{k3}$, and $\mathtt{k4}$ in $[\mathtt{I},\mathtt{J},\mathtt{X}]$, where $\mathtt{k1} < \mathtt{k2} < \mathtt{k3} < \mathtt{k4}$, then the following operations will occur in order:

```
T (i,j) = X (k1);
T (i,j) = dup (T (i,j), X (k2));
T (i,j) = dup (T (i,j), X (k3));
T (i,j) = dup (T (i,j), X (k4));
```

This is a well-defined order but the user should not depend upon it when using other GraphBLAS implementations since the GraphBLAS API does not require this ordering.

However, SuiteSparse:GraphBLAS guarantees this ordering, even when it compute the result in parallel. With this well-defined order, several operators become very useful. In particular, the SECOND operator results in the last tuple overwriting the earlier ones. The FIRST operator means the value of the first tuple is used and the others are discarded.

The acronym dup is used here for the name of binary function used for assembling duplicates, but this should not be confused with the _dup suffix in the name of the function GrB_Matrix_dup. The latter function does not apply any operator at all, nor any typecasting, but simply makes a pure deep copy of a matrix.

The parameter X is a pointer to any C equivalent built-in type, or a void * pointer. The GrB_Matrix_build function uses the _Generic feature

of C11 to detect the type of pointer passed as the parameter X. If X is a pointer to a built-in type, then the function can do the right typecasting. If X is a void * pointer, then it can only assume X to be a pointer to a user-defined type that is the same user-defined type of C and dup. This function has no way of checking this condition that the void * X pointer points to an array of the correct user-defined type, so behavior is undefined if the user breaks this condition.

The GrB_Matrix_build method is analogous to C = sparse (I,J,X) in MATLAB, with several important extensions that go beyond that which MATLAB can do. In particular, the MATLAB sparse function only provides one option for assembling duplicates (summation), and it can only build double, double complex, and logical sparse matrices. The dup operator cannot be a binary operator created by GxB_BinaryOp_new_IndexOp.

6.10.9 GxB_Matrix_build_Vector: build a matrix from a set of tuples

GxB_Matrix_build_Vector is identical to GrB_Matrix_build, except that the inputs I, J, and X are GrB_Vector objects, each with nvals entries. The interpretation of I_vector, J_vector, and X_vector are controlled by descriptor settings GxB_ROWINDEX_LIST, GxB_COLINDEX_LIST, and GxB_VALUE_LIST, respectively. The method can use either the indices or values of each of the input vectors; the default is to use the values. See Section 6.16.4 for details.

6.10.10 GxB_Matrix_build_Scalar: build a matrix from a set of tuples

GxB_Matrix_build_Scalar constructs a sparse matrix C from a set of tuples defined the index arrays I and J of length nvals, and a scalar. The scalar is the value of all of the tuples. Unlike GrB_Matrix_build, there is no dup operator to handle duplicate entries. Instead, any duplicates are silently ignored (if the number of duplicates is desired, simply compare the input nvals with the value returned by GrB_Vector_nvals after the matrix is constructed). All entries in the sparsity pattern of C are identical, and equal to the input scalar value.

6.10.11 GxB_Matrix_build_Scalar_Vector: build a matrix from a set of tuples

GxB_Matrix_build_Scalar_Vector is identical to GxB_Matrix_build_Scalar, except that the inputs I, J, and X are GrB_Vector objects, each with nvals entries. The interpretation of I_vector and J_vector are controlled by descriptor settings GxB_ROWINDEX_LIST and GxB_VALUE_LIST, respectively. The method can use either the indices or values of the I_input and J_vector vectors; the default is to use the values. See Section 6.16.4 for details.

6.10.12 GrB_Matrix_setElement: add an entry to a matrix

GrB_Matrix_setElement sets a single entry in a matrix, C(i,j)=x. If the entry is already present in the pattern of C, it is overwritten with the new value. If the entry is not present, it is added to C. In either case, no entry is ever deleted by this function. Passing in a value of x=0 simply creates an explicit entry at position (i,j) whose value is zero, even if the implicit value is assumed to be zero.

An error is returned (Grb_INVALID_INDEX) if the row index i is greater than or equal to the number of rows of C, or if the column index j is greater than or equal to the number of columns of C. Note that this error code differs from the same kind of condition in Grb_Matrix_build, which returns Grb_INDEX_OUT_OF_BOUNDS. This is because Grb_INVALID_INDEX is an API error, and is caught immediately even in non-blocking mode, whereas Grb_INDEX_OUT_OF_BOUNDS is an execution error whose detection may wait until the computation completes sometime later.

The scalar x is typecasted into the type of C. Any value can be passed to this function and its type will be detected, via the _Generic feature of C11. For a user-defined type, x is a void * pointer that points to a memory space holding a single entry of this user-defined type. This user-defined type must exactly match the user-defined type of C since no typecasting is done between user-defined types. If x is a GrB_Scalar and contains no entry, then the entry C(i,j) is removed (if it exists). The action taken is identical to GrB_Matrix_removeElement(C,i,j) in this case.

Performance considerations: SuiteSparse:GraphBLAS exploits the non-blocking mode to greatly improve the performance of this method. Refer to the example shown in Section 2.2. If the entry exists in the pattern already, it is updated right away and the work is not left pending. Otherwise, it is placed in a list of pending updates, and the later on the updates are done all at once, using the same algorithm used for GrB_Matrix_build. In other words, setElement in SuiteSparse:GraphBLAS builds its own internal list of

tuples [I,J,X], and then calls GrB_Matrix_build whenever the matrix is needed in another computation, or whenever GrB_Matrix_wait is called.

As a result, if calls to setElement are mixed with calls to most other methods and operations (even extractElement) then the pending updates are assembled right away, which will be slow. Performance will be good if many setElement updates are left pending, and performance will be poor if the updates are assembled frequently.

A few methods and operations can be intermixed with setElement, in particular, some forms of the GrB_assign and GxB_subassign operations are compatible with the pending updates from setElement. Section 12.11 gives more details on which GxB_subassign and GrB_assign operations can be interleaved with calls to setElement without forcing updates to be assembled. Other methods that do not access the existing entries may also be done without forcing the updates to be assembled, namely GrB_Matrix_clear (which erases all pending updates), GrB_Matrix_free, GrB_Matrix_ncols, GrB_Matrix_nrows, GrB_get, and of course GrB_Matrix_setElement itself. All other methods and operations cause the updates to be assembled. Future versions of SuiteSparse:GraphBLAS may extend this list.

See Section 17.2 for an example of how to use GrB_Matrix_setElement. If an error occurs, GrB_error(&err,C) returns details about the error.

6.10.13 GrB_Matrix_extractElement: get an entry from a matrix

```
GrB_Info GrB_Matrix_extractElement
                                          // x = A(i,j)
    \langle type \rangle *x,
                                 // extracted scalar (non-opaque C scalar)
    const GrB_Matrix A,
                                 // matrix to extract a scalar from
    GrB_Index i,
                                 // row index
    GrB_Index j
                                 // column index
);
GrB_Info GrB_Matrix_extractElement
                                          // x = A(i,j)
                                 // extracted GrB_Scalar
    GrB_Scalar x,
    const GrB_Matrix A,
                                 // matrix to extract a scalar from
    GrB_Index i,
                                 // row index
    GrB_Index j
                                 // column index
);
```

GrB_Matrix_extractElement extracts a single entry from a matrix x=A(i,j). An error is returned (GrB_INVALID_INDEX) if the row index i is greater than

or equal to the number of rows of C, or if column index j is greater than or equal to the number of columns of C. If the entry is present, x=A(i,j) is performed and the scalar x is returned with this value. The method returns GrB_SUCCESS. If no entry is present at A(i,j), and x is a non-opaque C scalar, then x is not modified, and the return value of GrB_Matrix_extractElement is GrB_NO_VALUE. If x is a GrB_Scalar, then x is returned as an empty scalar with no entry, and GrB_SUCCESS is returned.

The function knows the type of the pointer x, so it can do typecasting as needed, from the type of A into the type of x. User-defined types cannot be typecasted, so if A has a user-defined type then x must be a void * pointer that points to a memory space the same size as a single scalar of the type of A.

Currently, this method causes all pending updates from GrB_setElement, GrB_assign, or GxB_subassign to be assembled, so its use can have performance implications. Calls to this function should not be arbitrarily intermixed with calls to these other two functions. Everything will work correctly and results will be predictable, it will just be slow.

6.10.14 GxB_Matrix_isStoredElement: check if entry present in matrix

GxB_Matrix_isStoredElement check if the single entry A(i,j) is present in the matrix A. It returns GrB_SUCCESS if the entry is present, or GrB_NO_VALUE otherwise. The value of A(i,j) is not returned. It is otherwise identical to GrB_Matrix_extractElement.

6.10.15 GrB_Matrix_removeElement: remove an entry from a matrix

GrB_Matrix_removeElement removes a single entry A(i,j) from a matrix. If no entry is present at A(i,j), then the matrix is not modified. If an error occurs, GrB_error(&err,A) returns details about the error.

6.10.16 GrB_Matrix_extractTuples: get all entries from a matrix

GrB_Matrix_extractTuples extracts all the entries from the matrix A, returning them as a list of tuples, analogous to [I,J,X]=find(A) in MAT-LAB. Entries in the tuples [I,J,X] are unique. No pair of row and column indices (i,j) appears more than once.

The GraphBLAS API states the tuples can be returned in any order. If GrB_wait is called first, then SuiteSparse:GraphBLAS chooses to always return them in sorted order, depending on whether the matrix is stored by row or by column. Otherwise, the indices can be returned in any order.

The number of tuples in the matrix A is given by GrB_Matrix_nvals(&anvals,A). If anvals is larger than the size of the arrays (nvals in the parameter list), an error GrB_INSUFFICIENT_SIZE is returned, and no tuples are extracted. If nvals is larger than anvals, then only the first anvals entries in the arrays I J, and X are modified, containing all the tuples of A, and the rest of I J, and X are left unchanged. On output, nvals contains the number of tuples extracted.

SPEC: As an extension to the specification, the arrays I, J, and/or X may be passed in as NULL pointers. GrB_Matrix_extractTuples does not return a component specified as NULL. This is not an error condition.

6.10.17 GxB_Matrix_extractTuples_Vector: get all entries from a matrix

```
GrB_Info GrB_Matrix_extractTuples  // [I,J,X] = find (A)
(
    GrB_Vector I_vector,  // row indices
    GrB_Vector J_vector,  // col indices
    GrB_Vector X_vector,  // values
    const GrB_Matrix A,  // matrix to extract tuples from
    const GrB_Descriptor desc  // currently unused; for future expansion
);
```

GxB_Matrix_extractTuples_Vector is identical to GrB_Matrix_extractTuples except that its three outputs are GrB_Vector objects. The vectors I_vector, J_vector, and X_vector objects must exist on input. On output, any prior content is erased and their type, dimensions, and values are revised to contain dense vectors of length nvals.

6.10.18 GrB_Matrix_resize: resize a matrix

GrB_Matrix_resize changes the size of a matrix. If the dimensions decrease, entries that fall outside the resized matrix are deleted. Unlike GxB_Matrix_reshape* (see Sections 6.10.19 and 6.10.20), entries remain in their same position after resizing the matrix.

6.10.19 GxB_Matrix_reshape: reshape a matrix

GxB_Matrix_reshape changes the size of a matrix C, taking entries from the input matrix either column-wise or row-wise. If matrix C on input is nrows-by-ncols, and the requested dimensions of C on output are nrows_new-by-nrows_cols, then the condition nrows*ncols == nrows_new*nrows_cols must hold. The matrix C is modified in-place, as both an input and output for this method. To create a new matrix, use GxB_Matrix_reshapeDup instead (Section 6.10.20).

For example, if C is 3-by-4 on input, and is reshaped column-wise to have dimensions 2-by-6:

```
C on input C on output (by_col true) 00 01 02 03 00 20 11 02 22 13 10 11 12 13 10 01 21 12 03 23 20 21 22 23
```

If the same C on input is reshaped row-wise to dimensions 2-by-6:

NOTE: because an intermediate linear index must be computed for each entry, GxB_Matrix_reshape cannot be used on matrices for which nrows*ncols exceeds 2⁶⁰.

6.10.20 GxB_Matrix_reshapeDup: reshape a matrix

GxB_Matrix_reshapeDup is identical to GxB_Matrix_reshape (see Section 6.10.19), except that creates a new output matrix C that is reshaped from the input matrix A.

6.10.21 GxB_Matrix_concat: concatenate matrices

GxB_Matrix_concat concatenates an array of matrices (Tiles) into a single GrB_Matrix C.

Tiles is an m-by-n dense array of matrices held in row-major format, where Tiles [i*n+j] is the (i,j)th tile, and where m > 0 and n > 0 must hold. Let $A_{i,j}$ denote the (i,j)th tile. The matrix C is constructed by concatenating these tiles together, as:

$$C = \begin{bmatrix} A_{0,0} & A_{0,1} & A_{0,2} & \cdots & A_{0,n-1} \\ A_{1,0} & A_{1,1} & A_{1,2} & \cdots & A_{1,n-1} \\ \vdots & \vdots & & & & \\ A_{m-1,0} & A_{m-1,1} & A_{m-1,2} & \cdots & A_{m-1,n-1} \end{bmatrix}$$

On input, the matrix C must already exist. Any existing entries in C are discarded. C must have dimensions nrows by ncols where nrows is the sum

of the number of rows in the matrices $A_{i,0}$ for all i, and **ncols** is the sum of the number of columns in the matrices $A_{0,j}$ for all j. All matrices in any given tile row i must have the same number of rows (that is, and all matrices in any given tile column j must have the same number of columns).

The type of C is unchanged, and all matrices $A_{i,j}$ are typecasted into the type of C. Any settings made to C by GrB_set (format by row or by column, bitmap switch, hyper switch, and sparsity control) are unchanged.

6.10.22 GxB_Matrix_split: split a matrix

GxB_Matrix_split does the opposite of GxB_Matrix_concat. It splits a single input matrix A into a 2D array of tiles. On input, the Tiles array must be a non-NULL pointer to a previously allocated array of size at least m*n where both m and n must be greater than zero. The Tiles_nrows array has size m, and Tiles_ncols has size n. The (i, j)th tile has dimension Tiles_nrows[i]-by-Tiles_ncols[j]. The sum of Tiles_nrows [0:m-1] must equal the number of rows of A, and the sum of Tiles_ncols [0:n-1] must equal the number of columns of A. The type of each tile is the same as the type of A; no typecasting is done.

6.10.23 GrB_Matrix_diag: construct a diagonal matrix

GrB_Matrix_diag constructs a matrix from a vector. Let n be the length of the v vector, from GrB_Vector_size (&n, v). If k = 0, then C is an

n-by-n diagonal matrix with the entries from v along the main diagonal of C, with C(i,i)=v(i). If k is nonzero, C is square with dimension n+|k|. If k is positive, it denotes diagonals above the main diagonal, with C(i,i+k)=v(i). If k is negative, it denotes diagonals below the main diagonal of C, with C(i-k,i)=v(i). This behavior is identical to the MATLAB statement C=diag(v,k), where v is a vector.

The output matrix C is a newly-constructed square matrix with the same type as the input vector v. No typecasting is performed.

6.10.24 GxB_Matrix_diag: build a diagonal matrix

Identical to GrB_Matrix_diag, except for the extra parameter (a descriptor to provide control over the number of threads used), and this method is not a constructor.

The matrix C must already exist on input, of the correct size. It must be square of dimension n + |k| where the vector \mathbf{v} has length n. Any existing entries in C are discarded. The type of C is preserved, so that if the type of C and \mathbf{v} differ, the entries are typecasted into the type of C. Any settings made to C by GrB_set (format by row or by column, bitmap switch, hyper switch, and sparsity control) are unchanged.

6.10.25 GxB_Matrix_memoryUsage: memory used by a matrix

```
GrB_Info GxB_Matrix_memoryUsage // return # of bytes used for a matrix
(
    size_t *size, // # of bytes used by the matrix A
    const GrB_Matrix A // matrix to query
);
```

Returns the memory space required for a matrix, in bytes. By default, any read-only components are not included in the total memory. This can be changed with via GrB_get; see Section 10.2.

6.10.26 GxB_Matrix_type: type of a matrix

Returns the type of a matrix. The type parameter is not allocated. Calling GxB_Matrix_type is identical to making a shallow pointer copy of the type used to create a matrix. In particular, suppose a matrix is created, and a copy of its type is saved at the same time:

```
GrB_Matrix_new (&A, atype, m, n) ;
GrB_Type save_type = atype ;
```

Sometime later, while the matrix A and its type atype have not been freed, the following two code fragments are identical:

```
// using GxB_Matrix_type:
GrB_Type atype2;
GxB_Matrix_type (&atype2, A);
assert (atype2 == save_type);
// without GxB_Matrix_type:
GrB_Type atype2 = save_type;
```

As a result, freeing atype2 would be the same as freeing the original atype.

6.10.27 GrB_Matrix_free: free a matrix

GrB_Matrix_free frees a matrix. Either usage:

```
GrB_Matrix_free (&A) ;
GrB_free (&A) ;
```

frees the matrix A and sets A to NULL. It safely does nothing if passed a NULL handle, or if A == NULL on input. Any pending updates to the matrix are abandoned.

6.11 Serialize/deserialize methods

Serialization takes an opaque GraphBLAS object (a vector or matrix) and encodes it in a single non-opaque array of bytes, the blob. The blob can only be deserialized by the same library that created it (SuiteSparse:GraphBLAS in this case). The array of bytes can be written to a file, sent to another process over an MPI channel, or operated on in any other way that moves the bytes around. The contents of the array cannot be interpreted except by deserialization back into a vector or matrix, by the same library (and sometimes the same version) that created the blob.

All versions of SuiteSparse:GraphBLAS that implement serialization/deserialization use essentially the same format for the blob, so the library versions are compatible with each other. Version v9.0.0 adds the GrB_NAME and GrB_EL_TYPE_STRING to the blob in an upward compatible manner, so that older versions of SS:GraphBLAS can read the blobs created by v9.0.0; they simply ignore those components. SS:GraphBLAS v10 adds 32/64-bit integers, and can read the blobs created by any prior version of GraphBLAS (they be deserialized with all 64-bit integers however). If an older version of SS:GraphBLAS (v9 or earlier) attempts to deserialize a blob containing a matrix with 32-bit integers, it will safely report that the blob is invalid and refuse to deserialize it. If SS:GraphBLAS v10 creates a serialized blob with all-64-bit integers, then it can be read correctly by SS:GraphBLAS v9, and likely also be earlier versions.

There are two forms of serialization: GrB*serialize and GxB*serialize. For the GrB form, the blob must first be allocated by the user application, and it must be large enough to hold the matrix or vector.

By default, ZSTD (level 1) compression is used for serialization, but other options can be selected via the descriptor: GrB_set (desc, method, GxB_COMPRESSION), where method is an integer selected from the following options:

| method | description |
|-------------------------|-----------------------------|
| GxB_COMPRESSION_NONE | no compression |
| GxB_COMPRESSION_DEFAULT | ZSTD, with default level 1 |
| GxB_COMPRESSION_LZ4 | LZ4 |
| GxB_COMPRESSION_LZ4HC | LZ4HC, with default level 9 |
| GxB_COMPRESSION_ZSTD | ZSTD, with default level 1 |

The LZ4HC method can be modified by adding a level of zero to 9, with 9 being the default. Higher levels lead to a more compact blob, at the cost

of extra computational time. This level is simply added to the method, so to compress a vector with LZ4HC with level 6, use:

```
GrB_set (desc, GxB_COMPRESSION_LZ4HC + 6, GxB_COMPRESSION) ;
```

The ZSTD method can be specified as level 1 to 19, with 1 being the default. To compress with ZSTD at level 6, use:

```
GrB_set (desc, GxB_COMPRESSION_ZSTD + 6, GxB_COMPRESSION) ;
```

Descrialization of untrusted data is a common security problem; see https://cwe.mitre.org/data/definitions/502.html. The descrialization methods do a few basic checks so that no out-of-bounds access occurs during descrialization, but the output matrix or vector itself may still be corrupted. If the data is untrusted, use <code>GxB_*_fprint</code> to check the matrix or vector after descrializing it:

```
info = GxB_Vector_fprint (w, "w deserialized", GrB_SILENT, NULL);
if (info != GrB_SUCCESS) GrB_free (&w);
info = GxB_Matrix_fprint (A, "A deserialized", GrB_SILENT, NULL);
if (info != GrB_SUCCESS) GrB_free (&A);
```

The following methods are described in this Section:

| GraphBLAS function | purpose | Section |
|-----------------------------------|----------------------------------|---------|
| GxB_Vector_serialize | serialize a vector | 6.11.1 |
| <pre>GxB_Vector_deserialize</pre> | deserialize a vector | 6.11.2 |
| GrB_Matrix_serializeSize | return size of serialized matrix | 6.11.3 |
| <pre>GrB_Matrix_serialize</pre> | serialize a matrix | 6.11.4 |
| <pre>GxB_Matrix_serialize</pre> | serialize a matrix | 6.11.5 |
| <pre>GrB_Matrix_deserialize</pre> | deserialize a matrix | 6.11.6 |
| <pre>GxB_Matrix_deserialize</pre> | deserialize a matrix | 6.11.7 |
| GrB_get | get blob properties | 10.16 |

6.11.1 GxB_Vector_serialize: serialize a vector

GxB_Vector_serialize serializes a vector into a single array of bytes (the blob), which is malloc'ed and filled with the serialized vector. By default, ZSTD (level 1) compression is used, but other options can be selected via the descriptor. Serializing a vector is identical to serializing a matrix; see Section 6.11.5 for more information.

6.11.2 GxB_Vector_deservative: deservative a vector

This method creates a vector w by descrializing the contents of the blob, constructed by GxB_Vector_serialize. Descrializing a vector is identical to descrializing a matrix; see Section 6.11.7 for more information.

The blob is allocated with the malloc function passed to GxB_init, or the C11 malloc if GrB_init was used to initialize GraphBLAS. The blob must be freed by the matching free method, either the free function passed to GxB_init or the C11 free if GrB_init was used.

6.11.3 GrB_Matrix_serializeSize: return size of serialized matrix

```
GrB_Info GrB_Matrix_serializeSize  // estimate the size of a blob

(
    // output:
    GrB_Index *blob_size_handle,  // upper bound on the required size of the  // blob on output.

    // input:
    GrB_Matrix A  // matrix to serialize

);
```

GrB_Matrix_serializeSize returns an upper bound on the size of the blob needed to serialize a GrB_Matrix with GrB_Matrix_serialize. After the matrix is serialized, the actual size used is returned, and the blob may be realloc'd to that size if desired. This method is not required for GxB_Matrix_serialize.

6.11.4 GrB_Matrix_serialize: serialize a matrix

GrB_Matrix_serialize serializes a matrix into a single array of bytes (the blob), which must be already allocated by the user application. On input, &blob_size is the size of the allocated blob in bytes. On output, it is reduced to the numbed of bytes actually used to serialize the matrix. After calling GrB_Matrix_serialize, the blob may be realloc'd to this revised size if desired (this is optional). ZSTD (level 1) compression is used to construct a compact blob.

6.11.5 GxB Matrix serialize: serialize a matrix

GxB_Matrix_serialize is identical to GrB_Matrix_serialize, except that it does not require a pre-allocated blob. Instead, it allocates the blob internally, and fills it with the serialized matrix. By default, ZSTD (level 1) compression is used, but other options can be selected via the descriptor.

The blob is allocated with the malloc function passed to GxB_init, or the C11 malloc if GrB_init was used to initialize GraphBLAS. The blob must be freed by the matching free method, either the free function passed to GxB_init or the C11 free if GrB_init was used.

6.11.6 GrB Matrix deservative: deservative a matrix

```
GrB_Info GrB_Matrix_deserialize
                                    // deserialize blob into a GrB_Matrix
    // output:
    GrB_Matrix *C,
                        // output matrix created from the blob
    // input:
    GrB_Type type,
                        // type of the matrix C. Required if the blob holds a
                        // matrix of user-defined type. May be NULL if blob
                        // holds a built-in type; otherwise must match the
                        // type of C.
                            // the blob
    const void *blob,
    GrB_Index blob_size
                            // size of the blob
);
```

This method creates a matrix A by descrializing the contents of the blob, constructed by either GrB_Matrix_serialize or GxB_Matrix_serialize.

The type may be NULL if the blob holds a serialized matrix with a built-in type. In this case, the type is determined automatically. For user-defined types, the type must match the type of the matrix in the blob. The GrB_get method can be used to query the blob for the name of this type.

6.11.7 GxB_Matrix_deserialize: deserialize a matrix

```
GrB_Info GxB_Matrix_deserialize
                                    // deserialize blob into a GrB_Matrix
    // output:
   GrB_Matrix *C,
                       // output matrix created from the blob
    // input:
   GrB_Type type,
                       // type of the matrix C. Required if the blob holds a
                        // matrix of user-defined type. May be NULL if blob
                        // holds a built-in type; otherwise must match the
                       // type of C.
                            // the blob
    const void *blob,
                           // size of the blob
   GrB_Index blob_size,
   const GrB_Descriptor desc
);
```

Identical to GrB_Matrix_deserialize.

6.12 The GxB_Container object and its methods

A new set of load/unload methods are introduced in GraphBLAS v10 to move data between a GrB_Matrix or GrB_Vector and a a new GxB_Container object. This object is non-opaque but contains opaque objects. Its primary components are five dense GrB_Vectors that hold the contents of the matrix/vector. The data in these dense vectors can then be loaded/unloaded via GxB_Vector_load and GxB_Vector_unload.

Moving data from a GrB_Matrix into user-visible C arrays is a two-step process. The data is first moved into a GxB_Container using GxB_unload_Matrix_into_Container, and then from the Container into C arrays with GxB_Vector_unload. Moving data in the opposite direction is also a two-step process: first load the C array into a GrB_Vector component of a GxB_Container with GxB_Vector_load, and then from the GxB_Container into a GrB_Matrix using the GxB_load_Matrix_from_Container method.

The following methods are available. The first two do not use the Container object, but instead move data to/from a dense GrB_Vector:

- GxB_Vector_load: this method moves data in O(1) time from a user-visible C array into a GrB_Vector. The vector length and type are revised to match the new data from the C array. Ownership is normally transferred to the GrB_Vector, but this can be revised with a handling parameter. The C array is passed in as a void * pointer, and its type is indicated by a GrB_Type parameter. See Section 6.12.1 for details.
- GxB_Vector_unload: this method moves data in O(1) time from a GrB_Vector into a user-visible C array. The length of the GrB_Vector is reduced to zero, to denote that it no longer holds any content. The vector must be dense; it must have the same number of entries as its size (that is GrB_Vector_nvals and GrB_Vector_size must return the same value). The C array is returned as a void * pointer, and its type is indicated by a GrB_Type parameter. See Section 6.12.2 for details.

The next six methods rely on the GxB_Container object:

- GxB_Container_new: creates a container. (see Section 6.12.3).
- GxB_Container_free: frees a container. (see Section 6.12.4).

- GxB_load_Matrix_from_Container: moves all of the data from a GxB_Container into a GrB_Matrix in O(1) time. (see Section 6.12.5).
- GxB_load_Vector_from_Container: moves all of the data from a GxB_Container into a GrB_Vector in O(1) time. (see Section 6.12.6).
- GxB_unload_Matrix_into_Container: moves all of the data from a GrB_Matrix into a GxB_Container in O(1) time. (see Section 6.12.7).
- GxB_unload_Vector_into_Container: moves all of the data from a GrB_Vector into a GxB_Container in O(1) time. (see Section 6.12.8).

6.12.1 GxB_Vector_load: load data into a vector

```
GrB_Info GxB_Vector_load
    // input/output:
    GrB_Vector V,
                           // vector to load from the C array X
                           // numerical array to load into V
    void **X,
    // input:
                           // type of X
    GrB_Type type,
    uint64_t n,
                            // # of entries in X
    uint64_t X_size,
                            // size of X in bytes (at least n*(sizeof the type))
    int handling,
                            // GrB_DEFAULT (0): transfer ownership to GraphBLAS
                            // GxB_IS_READONLY: X treated as read-only;
                            //
                                    ownership kept by the user application
    const GrB_Descriptor desc // currently unused; for future expansion
);
```

 ${\tt GxB_Vector_load}$ loads data from a C array into a ${\tt GrB_Vector}$ in O(1) time.

On input, the GrB_Vector V must already exist, but its content (type, size, and entries) are ignored. On output, any prior content of V is freed, and its data is replaced with the C array X of length n entries, whose type is given by the type parameter. The size of V becomes n, and its type is changed to match the type parameter.

After this method returns, GrB_Vector_size and GrB_Vector_nvals will both return n. That is, the vector V is a dense vector. It is held in the GxB_FULL data format, in GrB_COLMAJOR orientation.

The size in bytes of X is also provided on input as X_size; this must be at least n times the size of the given type, but it can be larger. This size

is maintained and returned to the user application by GxB_Vector_unload. The error code GrB_INVALID_VALUE is returned if X_size is too small.

The handling parameter provides control over the ownership of the array X. By default, ownership of X is handed over to the GrB_Vector V. In this case, the vector V acts as a normal GraphBLAS vector. It can be modified or freed as usual. Freeing V with GrB_Vector_free will also free X. The array X is returned as NULL to denote this change of ownership.

If the handling parameter is GxB_IS_READONLY, then X is still owned by the user application. It remains non-NULL when this method returns. The resulting vector V can be used as an input to any GraphBLAS method, but it cannot be modified (except that it can be freed). If a call is made that attempts to modify a matrix with read-only components, an error is returned (GxB_OUTPUT_IS_READONLY). Freeing the vector V does not modify X, however. It simply frees the rest of the object V.

Note that this method does not rely on the GxB_Container object. Instead, it loads a C array X directly into a dense GrB_Vector V.

6.12.2 GxB_Vector_unload: unload data from a vector

```
GrB_Info GxB_Vector_unload
    // input/output:
    GrB_Vector V,
                            // vector to unload
    void **X,
                            // numerical array to unload from V
    // output:
                            // type of X
    GrB_Type *type,
                            // # of entries in X
    uint64_t *n,
                            // size of X in bytes (at least n*(sizeof the type))
    uint64_t *X_size,
                            // see GxB_Vector_load
    int *handling,
    const GrB_Descriptor desc // currently unused; for future expansion
);
```

GxB_Vector_unload unloads data from GrB_Vector into a C array in O(1) time (unless the vector has pending work that must be finished first).

On input, the vector V must have all possible entries present (that is, GrB_Vector_nvals and GrB_Vector_size must return the same value). The vector can be in any internal data format; it does not have to be in the GxB_FULL format on input, but it must be in GrB_COLMAJOR orientation. If any entries are missing, the unload is not performed and GrB_INVALID_OBJECT is returned.

On output, the size of V is reduced to zero, and it holds no entries but its type is unchanged. The array X is returned to the called with the entries from the vector. The type of X is given by the type parameter. The number of entries in V is returned as n. The size of X in bytes is returned as X_size.

The handling parameter informs the user application of the ownership of the array X. If it was created by GraphBLAS, or passed into GraphBLAS via GxB_Vector_load with handling set to GrB_DEFAULT, then this is returned to the user as handling set to GrB_DEFAULT. This denotes that ownership of the array X has been transferred from GraphBLAS The user application is now responsible for freeing the array X.

If the handling parameter is returned as GxB_IS_READONLY, then this array X was originally passed to GraphBLAS via GxB_Vector_load with a handling parameter of GxB_IS_READONLY. The ownership of the array X is not changed; it remains owned by the user application. The user application is still responsible for freeing the array X, but caution must be observed so that it is not freed twice.

Note that this method does not rely on the GxB_Container object. Instead, it unloads a dense GrB_Vector V directly into a C array X.

6.12.3 GxB_Container_new: create a container

```
GrB_Info GxB_Container_new (GxB_Container *Container) ;
```

GxB_Container_new creates a new container. It has the following definition (extra space for future expansion excluded for clarity):

The Container holds all of the data from a single GraphBLAS matrix or vector, with any data type and any data format. It contains extra space for

future data formats (not shown above). Each scalar component is described below:

- nrows: the number of rows of a matrix, or the size of a vector.
- ncols: the number of columns of a matrix, or 1 for a vector.
- nrows_nonempty: if known, this value must give the exact number of non-empty rows of a matrix or vector. If not known, this value must be set to -1.
- ncols_nonempty: if known, this value must give the exact number of non-empty columns of a matrix or vector. If not known, this value must be set to -1.
- nvals: the number of entries in the matrix or vector.
- format: this defines the data format of a matrix or vector. Currently, GraphBLAS supports four formats, described in Section 6.13, each with two orientations.
- orientation: whether the matrix is held by row or by column. This is always GrB_COLMAJOR for a container holding data for a GrB_Vector.
- iso: if true, all of the entries in the matrix have the same value, and only a single value is stored in the x component of the Container.
- jumbled: if true, the indices in any given row (if in row-major orientation) or column (if column-major) may appear out of order. Otherwise, they must appear in ascending order. A jumbled matrix or vector must not have any read-only components.

The Container holds five dense GrB_Vector objects that hold the primary data for the matrix or vector, and a single GrB_Matrix Y that holds an optional optimization structure called the hyper-hash. These components are fully described in Section 6.13.

6.12.4 GxB Container free: free a container

```
GrB_Info GrB_free (GxB_Container *Container) ;
```

GxB_Container_free frees a container. It also frees all its opaque components. Any read-only array inside these opaque objects is not freed.

6.12.5 GxB_load_Matrix_from_Container: load a matrix from a container

 $\texttt{GxB_load_Matrix_from_Container}$ moves all of the data from a $\texttt{GxB_Container}$ into a $\texttt{GrB_Matrix}$ in O(1) time.

6.12.6 GxB_load_Vector_from_Container: load a vector from a container

 $\texttt{GxB_load_Vector_from_Container}$ moves all of the data from a $\texttt{GxB_Container}$ into a $\texttt{GrB_Vector}$ in O(1) time.

6.12.7 GxB_unload_Matrix_into_Container: unload a matrix into a container

 $GxB_unload_Matrix_into_Container$: moves all of the data from a GrB_Matrix into a $GxB_Container$ in O(1) time.

6.12.8 GxB_unload_Vector_into_Container: unload a vector into a container

```
GrB_Info GxB_unload_Vector_into_Container // GrB_Vector -> GxB_Container
(
    GrB_Vector V, // vector to unload into the Container
    GxB_Container Container, // Container to hold the contents of V
    const GrB_Descriptor desc // currently unused
);
```

 $GxB_unload_Vector_into_Container$: moves all of the data from a GrB_Vector into a $GxB_Container$ in O(1) time.

6.12.9 Container example: unloading/loading an entire matrix into C arrays

The following example unloads a GrB_Matrix A of size nrows-by-ncols, with nvals entries, of type xtype. The example takes will take O(1) time, and the only mallocs are in GxB_Container_new (which can be reused for an arbitrary number of load/unload cycles), and the only frees are in GxB_Container_free.

Note that getting C arrays from a GrB_Matrix is a 2-step process: First unload the matrix A into a Container, giving GrB_Vector objects Container->p, Container->i, Container->x, etc, and then unload those dense vectors into C arrays. This may seem tedious but it allows everything to be done in O(1) time and space (often no new malloc'd space), and it allows support for arbitrary integers for the p, h, and i components of a matrix. It also makes for a simple API overall.

```
GxB_Container_new (&Container) ; // requires several O(1)-sized mallocs
// no malloc/free will occur below, until GxB_Container_free.
for (as many times as you like)
   GxB_unload_Matrix_into_Container (A, Container, desc);
   // A is now 0-by-0 with nvals(A)=0. Its type is unchanged.
   // All of the following is optional; if any item in the Container is not
   // needed by the user, it can be left as-is, and then it will be put
    // back into A at the end. (This is done for the Container->Y).
   // to extract numerical values from the Container:
    void *x = NULL;
   uint64_t nvals = 0, nheld = 0;
   GrB_Type xtype = NULL ;
    int x_handling, p_handling, h_handling, i_handling, b_handling;
   uint64_t x_size, p_size, h_size, i_size, b_size ;
    GxB_Vector_unload (Container->x, &x, &xtype, &nheld, &x_size, &x_handling,
        desc);
   // The C array {\bf x} now has size nheld and contains the values of the original
    // GrB_Matrix A, with type xtype being the original type of the matrix A.
    // The Container->x GrB_Vector still exists but it now has length 0.
   // If the matrix A was iso-valued, nheld == 1.
   // to extract the sparsity pattern from the Container:
   GrB_Type ptype = NULL, htype = NULL, itype = NULL, btype = NULL;
   void *p = NULL, *h = NULL, *i = NULL, *b = NULL;
   uint64_t plen = 0, plen1 = 0, nheld = 0;
   switch (Container->format)
       case GxB_HYPERSPARSE :
           // The Container->Y matrix can be unloaded here as well,
           // if desired. Its use is optional.
            GxB_Vector_unload (Container->h, &h, &htype, &plen, &h_size,
                &h_handling, desc);
        case GxB_SPARSE :
           GxB_Vector_unload (Container->p, &p, &ptype, &plen1, &p_size,
                &p_handling, desc);
            GxB_Vector_unload (Container->i, &i, &itype, &nvals, &i_size,
                &i_handling, desc);
```

```
case GxB_BITMAP :
            GxB_Vector_unload (Container->b, &b, &btype, &nheld, &b_size,
                &b_handling, desc);
           break ;
   }
   // Now the C arrays (p, h, i, b, and x) are all populated and owned by the
   // user application. They can be modified here, if desired. Their C type
   // is (void *), and their actual types correspond to ptype, htype, itype,
   // btype, and xtype).
   // to load them back into A, first load them into the Container->[phbix]
    // vectors:
    switch (Container->format)
       case GxB_HYPERSPARSE :
           // The Container->Y matrix can be loaded here as well,
           // if desired. Its use is optional.
           GxB_Vector_load (Container->h, &h, htype, plen, h_size,
                h_handling, desc);
        case GxB_SPARSE :
           GxB_Vector_load (Container->p, &p, ptype, plen1, p_size,
                p_handling, desc);
            GxB_Vector_load (Container->i, &i, itype, nvals, i_size,
                i_handling, desc);
           break;
       case GxB_BITMAP :
            GxB_Vector_load (Container->b, &b, btype, nheld, b_size,
                b_handling, desc);
           break ;
   GxB_Vector_load (Container->x, &x, xtype, nheld, x_size,
       x_handling, desc) ;
   // Now the C arrays p, h, i, b, and x are all NULL. They are in the
    // Container->p,h,b,i,x GrB_Vectors. Load A from the non-opaque Container:
   GxB_load_Matrix_from_Container (A, Container, desc) ;
   // A is now back to its original state. The Container and its p,h,b,i,x
    // GrB_Vectors exist but its vectors all have length 0.
}
GxB_Container_free (&Container) ;
                                  // does several O(1)-sized free's
```

break ;

6.12.10 Container example: unloading/loading, but not using C arrays

Using the container is very simple if the resulting Container GrB_Vector components are used directly by GraphBLAS, with no need for C arrays. For example, in a push/relabel maxflow algorithm, there is a need to extract the tuples from a GrB_Vector Delta, followed by a call to GrB_Matrix_build to create a matrix from that data. In GraphBLAS v9 and earlier, extracting the tuples requires a copy. In v10, it can be done using the container, without requiring a copy of the contents of Delta.

The contents of the Delta vector can be used unloaded in to the container for use by GxB_Matrix_build_Vector, in O(1) time, and then loaded back afterwards, also in O(1) time. The construction of the DeltaMatrix takes the same time as GrB_Matrix_build, but the extra copy that would be required for GrB_Vector_extractTuples is entirely avoided.

6.13 SuiteSparse:GraphBLAS data formats

SuiteSparse:GraphBLAS uses four distinct data formats: sparse, hypersparse, bitmap, and full, each in row-major or column-major orientations, for eight different variants (each of which are listed below). Each of these eight total variants can be iso-valued, where if Container->iso is true the numerical values are all the same, and Container->x holds a single entry with this value. Each of the sparse and hypersparse formats can appear in *jumbled* form, where the indices within any given row (if the orientation is row-major) or column may be out of order. If Container->jumbled is false, then the indices appear in ascending order.

6.13.1 Sparse, held by row

A sparse matrix in CSR format, held by row, has a Container->format value of GxB_SPARSE and a Container->orientation of GrB_ROWMAJOR. It requires three arrays:

- integer Ap [nrows+1]; The Ap array is the row "pointer" array. It does not actual contain pointers, but integer offsets. More precisely, it is an integer array that defines where the column indices and values appear in Aj and Ax, for each row. The number of entries in row i is given by the expression Ap [i+1] Ap [i].
- integer Aj [nvals]; The Aj array defines the column indices of entries in each row.
- type Ax [nvals] ; The Ax array defines the values of entries in each row

The content of the three arrays Ap Aj, and Ax is very specific. This content is not checked, since this function takes only O(1) time. Results are undefined if the following specification is not followed exactly.

The column indices of entries in the ith row of the matrix are held in Aj [Ap [i] ... Ap[i+1]], and the corresponding values are held in the same positions in Ax. Column indices must be in the range 0 to ncols-1. If jumbled is false, column indices must appear in ascending order within each row. If jumbled is true, column indices may appear in any order within each row. No duplicate column indices may appear in any row. Ap [0] must

equal zero, and Ap [nrows] must equal nvals. The Ap array must be of size nrows+1 (or larger), and the Aj and Ax arrays must have size at least nvals.

An example of the CSR format is shown below. Consider the following matrix with 10 nonzero entries, and suppose the zeros are not stored.

$$A = \begin{bmatrix} 4.5 & 0 & 3.2 & 0 \\ 3.1 & 2.9 & 0 & 0.9 \\ 0 & 1.7 & 3.0 & 0 \\ 3.5 & 0.4 & 0 & 1.0 \end{bmatrix}$$
 (1)

The Ap array has length 5, since the matrix is 4-by-4. The first entry must always zero, and Ap [5] = 10 is the number of entries. The content of the arrays is shown below:

```
int64_t Ap [] = { 0, 2, 5, 7, 10 };
int64_t Aj [] = { 0, 2, 0, 1, 3, 1, 2, 0, 1, 3 };
double Ax [] = { 4.5, 3.2, 3.1, 2.9, 0.9, 1.7, 3.0, 3.5, 0.4, 1.0 };
```

Spaces have been added to the Ap array, just for illustration. Row zero is in Aj [0..1] (column indices) and Ax [0..1] (values), starting at Ap [0] = 0 and ending at Ap [0+1]-1 = 1. The list of column indices of row one is at Aj [2..4] and row two is in Aj [5..6]. The last row (three) appears Aj [7..9], because Ap [3] = 7 and Ap [4]-1 = 10-1 = 9. The corresponding numerical values appear in the same positions in Ax.

To iterate over the rows and entries of this matrix, the following code can be used (assuming it has type GrB_FP64):

In the container, the three arrays Ap, Aj and Ax are held in three GrB_Vector objects: Container->p, Container->i, and Container->x, respectively.

6.13.2 Sparse, held by column

This format is the transpose of sparse-by-row. A sparse matrix in CSC format, held by column, has a Container->format value of GxB_SPARSE and a Container->orientation of GrB_COLMAJOR. It requires three arrays: Ap, Ai, and Ax.

The column "pointer" array Ap has size ncols+1. The row indices of the columns are in Ai, and if jumbled is false, they must appear in ascending order in each column. The corresponding numerical values are held in Ax. The row indices of column j are held in Ai [Ap [j]...Ap [j+1]-1], and the corresponding numerical values are in the same locations in Ax.

The same matrix from Equation 1 in the last section (repeated here):

$$A = \begin{bmatrix} 4.5 & 0 & 3.2 & 0 \\ 3.1 & 2.9 & 0 & 0.9 \\ 0 & 1.7 & 3.0 & 0 \\ 3.5 & 0.4 & 0 & 1.0 \end{bmatrix}$$
 (2)

is held in CSC form as follows:

That is, the row indices of column 1 (the second column) are in Ai [3..5], and the values in the same place in Ax, since Ap [1] = 3 and Ap [2]-1 = 5.

To iterate over the columns and entries of this matrix, the following code can be used (assuming it has type GrB_FP64):

In the container, the three arrays Ap, Ai and Ax are held in three GrB_Vector objects: Container->p, Container->i, and Container->x, respectively.

6.13.3 Hypersparse, held by row

The hypersparse HyperCSR format is identical to the CSR format, except that the Ap array itself becomes sparse, if the matrix has rows that are completely empty. An array Ah of size nvec provides a list of rows that appear in the data structure. For example, consider Equation 3, which is a sparser version of the matrix in Equation 1. Row 2 and column 1 of this matrix are all empty.

$$A = \begin{bmatrix} 4.5 & 0 & 3.2 & 0 \\ 3.1 & 0 & 0 & 0.9 \\ 0 & 0 & 0 & 0 \\ 3.5 & 0 & 0 & 1.0 \end{bmatrix}$$
 (3)

The conventional CSR format would appear as follows. Since the third row (row 2) is all zero, accessing Ai [Ap [2] ... Ap [3]-1] gives an empty set ([2..1]), and the number of entries in this row is Ap [i+1] - Ap [i] = Ap [3] - Ap [2] = 0.

```
int64_t Ap [] = { 0, 2, 2, 4, 5 }
int64_t Aj [] = { 0, 2, 0, 3, 0 3 }
double Ax [] = { 4.5, 3.2, 3.1, 0.9, 3.5, 1.0 };
```

A hypersparse CSR format for this same matrix would discard these duplicate integers in Ap. Doing so requires another array, Ah, that keeps track of the rows that appear in the data structure.

Note that the Aj and Ax arrays are the same in the CSR and HyperCSR formats. If jumbled is false, the row indices in Ah must appear in ascending order, and no duplicates can appear. To iterate over this data structure (assuming it has type GrB_FP64):

This is more complex than the CSR format, but it requires at most O(e) space, where A is m-by-n with $e = \mathtt{nvals}$ entries. The CSR format requires O(m+e) space. If e << m, then the size m+1 of Ap can dominate the memory required. In the hypersparse form, Ap takes on size $\mathtt{nvec+1}$, and Ah has size \mathtt{nvec} , where \mathtt{nvec} is the number of rows that appear in the data structure. The CSR format can be viewed as a dense array (of size \mathtt{nrows}) of sparse row vectors. By contrast, the hypersparse CSR format is a sparse array (of size \mathtt{nvec}) of sparse row vectors.

In the container, the four arrays Ap, Ah, Aj and Ax are held in four GrB_Vector objects: Container->p, Container->h, Container->i, and Container->x, respectively.

In addition, the container may hold an optional optimization structure, Container->Y, called the hyper-hash. This is a GrB_Matrix that holds the inverse of the Container->h array (called Y because it looks like an upside-down h). If a matrix is being loaded from raw data, the hyper-hash is not yet constructed, so the Container->Y matrix should be set to NULL. GraphBLAS will compute it when needed.

When a matrix is unload into a container, GraphBLAS will place the hyper-hash matrix there if it has been computed. If the matrix is subsequently loaded from the container, and Container->h is unchanged, then leaving the hyper-hash unmodified will preserve this optional optimization data structure. If instead Container->h is revised, the hyper-hash in Container->Y should be freed.

A GrB_Vector is never held in hypersparse format.

6.13.4 Hypersparse, held by column

The hypersparse-by-column format is the transpose of the hypersparse-by-row format. The Container->format is GxB_HYPERSPARSE and the Container->orientation is GrB_COLMAJOR. In the container, the four arrays Ap, Ah, Ai and Ax are held in four GrB_Vector objects: Container->p,

Container->h, Container->i, and Container->x, respectively. A GrB_Vector is never held in hypersparse format.

6.13.5 Bitmap, held by row

The Container->format is GxB_BITMAP and the Container->orientation is GrB_ROWMAJOR. This format requires two arrays, Ab and Ax, each of which are size nrows*ncols. They correspond to Container->b and Container->x in the GxB_Container object. These arrays define the pattern and values of the matrix A:

- int8_t Ab [nrows*ncols]; The Ab array defines which entries of A are present. If Ab[i*ncols+j]=1, then the entry A(i,j) is present, with value Ax[i*ncols+j]. If Ab[i*ncols+j]=0, then the entry A(i,j) is not present. The Ab array must contain only 0s and 1s. The nvals input must exactly match the number of 1s in the Ab array.
- type Ax [nrows*ncols]; The Ax array defines the values of entries in the matrix. If Ab[p] is zero, the value of Ax[p] is ignored.

6.13.6 Bitmap, held by column

This is the transpose of the bitmap-by-row format. The Container->format is GxB_BITMAP and the Container->orientation is $GrB_COLMAJOR$. The value of the entry A(i,j) is held in Ax [i+j*nrows]. It is present if Ab [i+j*nrows] is 1, and not present if zero.

6.13.7 Full, held by row

The Container->format is GxB_FULL and the Container->orientation is $GrB_ROWMAJOR$. This format is held in a single GrB_Vector , Container->x. The A(i,j) entry is in position i*ncols+j in this array. All entries are present.

6.13.8 Full, held by column

This is the transpose of the full-by-row format. The Container->format is GxB_FULL and the Container->orientation is $GrB_COLMAJOR$. This format is held in a single GrB_Vector , Container->x. The A(i,j) entry is in position i+j*nrows in this array. All entries are present.

6.14 GraphBLAS import/export: using copy semantics

The v2.0 C API includes import/export methods for matrices (not vectors) using a different strategy as compared to the $GxB_Container$ methods. The $GxB_Container$ methods are based on move semantics, in which ownership of arrays is passed between SuiteSparse:GraphBLAS and the user application. This allows the $GxB_Container$ methods to work in O(1) time, and require no additional memory, but it requires that GraphBLAS and the user application agree on which memory manager to use. This is done via GxB_init . This allows GraphBLAS to malloc an array that can be later freed by the user application, and visa versa.

The GrB import/export methods take a different approach. The data is always copied in and out between the opaque GraphBLAS matrix and the user arrays. This takes $\Omega(e)$ time, if the matrix has e entries, and requires more memory. It has the advantage that it does not require GraphBLAS and the user application to agree on what memory manager to use, since no ownership of allocated arrays is changed.

The format for GrB_Matrix_import and GrB_Matrix_export is controlled by the following enum:

6.14.1 GrB_Matrix_import: import a matrix

```
GrB_Info GrB_Matrix_import // import a matrix
    GrB_Matrix *A,
                            // handle of matrix to create
    GrB_Type type,
                            // type of matrix to create
    GrB_Index nrows,
                            // number of rows of the matrix
                            // number of columns of the matrix
   GrB_Index ncols,
    const GrB_Index *Ap,
                            // pointers for CSR, CSC, column indices for COO
    const GrB_Index *Ai,
                            // row indices for CSR, CSC
    const <type> *Ax,
                            // values
    GrB_Index Ap_len,
                            // number of entries in Ap (not # of bytes)
    GrB_Index Ai_len,
                            // number of entries in Ai (not # of bytes)
    GrB_Index Ax_len,
                            // number of entries in Ax (not # of bytes)
    int format
                            // import format (GrB_Format)
);
```

The GrB_Matrix_import method copies from user-provided arrays into an opaque GrB_Matrix and GrB_Matrix_export copies data out, from an opaque GrB_Matrix into user-provided arrays.

The suffix TYPE in the prototype above is one of BOOL, INT8, INT16, etc, for built-n types, or UDT for user-defined types. The type of the Ax array must match this type. No typecasting is performed.

Unlike the GxB_Container methods, memory is not handed off between the user application and GraphBLAS. The three arrays Ap, Ai. and Ax are not modified, and are still owned by the user application when the method finishes.

The matrix can be imported in one of three different formats:

- GrB_CSR_FORMAT: Compressed-row format. Ap is an array of size nrows+1. The arrays Ai and Ax are of size nvals = Ap [nrows], and Ap[0] must be zero. The column indices of entries in the ith row appear in Ai[Ap[i]...Ap[i+1]-1], and the values of those entries appear in the same locations in Ax. The column indices need not be in any particular order. See Section 6.13.1 for details of the sparse-by-row (CSR) format.
- GrB_CSC_FORMAT: Compressed-column format. Ap is an array of size ncols+1. The arrays Ai and Ax are of size nvals = Ap [ncols], and Ap[0] must be zero. The row indices of entries in the jth column appear in Ai[Ap[j]...Ap[j+1]-1], and the values of those entries appear in the same locations in Ax. The row indices need not be in any

- particular order. See Section 6.13.2 for details of the sparse-by-column (CSC) format.
- GrB_COO_FORMAT: Coordinate format. This is the same format as GrB_Matrix_build. The three arrays Ap, Ai, and Ax have the same size. The kth tuple has row index Ai[k], column index Ap[k], and value Ax[k]. The tuples can appear any order, but no duplicates are permitted.

6.14.2 GrB_Matrix_export: export a matrix

```
GrB_Info GrB_Matrix_export // export a matrix
    GrB_Index *Ap,
                            // pointers for CSR, CSC, column indices for COO
   GrB_Index *Ai,
                            // col indices for CSR/COO, row indices for CSC
    <type> *Ax,
                            // values (must match the type of A_input)
    GrB_Index *Ap_len,
                            // number of entries in Ap (not # of bytes)
    GrB_Index *Ai_len,
                            // number of entries in Ai (not # of bytes)
    GrB_Index *Ax_len,
                            // number of entries in Ax (not # of bytes)
    int format,
                            // export format (GrB_Format)
    GrB_Matrix A
                            // matrix to export
);
```

GrB_Matrix_export copies the contents of a matrix into three user-provided arrays, using any one of the three different formats described in Section 6.14.1. The size of the arrays must be at least as large as the lengths returned by GrB_Matrix_exportSize. The matrix A is not modified.

On input, the size of the three arrays Ap, Ai, and Ax is given by Ap_len, Ai_len, and Ax_len, respectively. These values are in terms of the number of entries in these arrays, not the number of bytes. On output, these three value are adjusted to report the number of entries written to the three arrays.

The suffix TYPE in the prototype above is one of BOOL, INT8, INT16, etc, for built-n types, or UDT for user-defined types. The type of the Ax array must match this type. No typecasting is performed.

6.14.3 GrB_Matrix_exportSize: determine size of export

Returns the required sizes of the arrays Ap, Ai, and Ax for exporting a matrix using GrB_Matrix_export, using the same format.

6.14.4 GrB_Matrix_exportHint: determine best export format

This method suggests the most efficient format for the export of a given matrix. For SuiteSparse:GraphBLAS, the hint depends on the current format of the GrB_Matrix:

- GxB_SPARSE, GrB_ROWMAJOR: export as GrB_CSR_FORMAT
- GxB_SPARSE, GrB_COLMAJOR: export as GrB_CSC_FORMAT
- GxB_HYPERSPARSE: export as GrB_COO_FORMAT
- GxB_BITMAP, GrB_ROWMAJOR: export as GrB_CSR_FORMAT
- GxB_BITMAP, GrB_COLMAJOR: export as GrB_CSC_FORMAT
- GxB_FULL, GrB_ROWMAJOR: export as GrB_CSR_FORMAT
- GxB_FULL, GrB_COLMAJOR: export as GrB_CSC_FORMAT

6.15 Sorting methods

GxB_Matrix_sort provides a mechanism to sort all the rows or all the columns of a matrix, and GxB_Vector_sort sorts all the entries in a vector.

6.15.1 GxB_Vector_sort: sort a vector

GxB_Vector_sort is identical to sorting the single column of an n-by-1 matrix. Refer to Section 6.15.2 for details. The op cannot be a binary operator created by GxB_BinaryOp_new_IndexOp.

6.15.2 GxB_Matrix_sort: sort the rows/columns of a matrix

GxB_Matrix_sort sorts all the rows or all the columns of a matrix. Each row (or column) is sorted separately. The rows are sorted by default. To sort the columns, use GrB_DESC_TO. A comparator operator is provided to define the sorting order (ascending or descending). For example, to sort a GrB_FP64 matrix in ascending order, use GrB_LT_FP64 as the op, and to sort in descending order, use GrB_GT_FP64.

The op must have a return value of GrB_BOOL, and the types of its two inputs must be the same. The entries in A are typecasted to the inputs of

the op, if necessary. Matrices with user-defined types can be sorted with a user-defined comparator operator, whose two input types must match the type of A, and whose output is GrB_BOOL.

The two matrix outputs are C and P. Any entries present on input in C or P are discarded on output. The type of C must match the type of A exactly. The dimensions of C, P, and A must also match exactly (even with the GrB_DESC_TO descriptor).

With the default sort (by row), suppose A(i,:) contains k entries. In this case, C(i,0:k-1) contains the values of those entries in sorted order, and P(i,0:k-1) contains their corresponding column indices in the matrix A. If two values are the same, ties are broken according column index.

If the matrix is sorted by column, and A(:,j) contains k entries, then C(0:k-1,j) contains the values of those entries in sorted order, and P(0:k-1,j) contains their corresponding row indices in the matrix A. If two values are the same, ties are broken according row index.

The outputs C and P are both optional; either one (but not both) may be NULL, in which case that particular output matrix is not computed. The op cannot be a binary operator created by GxB_BinaryOp_new_IndexOp.

6.16 GraphBLAS descriptors: GrB_Descriptor

A GraphBLAS descriptor modifies the behavior of a GraphBLAS operation. If the descriptor is GrB_NULL, defaults are used.

The access to these parameters and their values is governed by two enum types, GrB_Desc_Field and GrB_Desc_Value:

```
typedef enum
    GrB_OUTP = 0,
                   // descriptor for output of a method
    GrB_MASK = 1, // descriptor for the mask input of a method
    GrB_INPO = 2,
                   // descriptor for the first input of a method
    GrB_INP1 = 3,
                   // descriptor for the second input of a method
    GxB_AxB_METHOD = 1000, // descriptor for selecting C=A*B algorithm
    GxB\_SORT = 35
                  // control sort in GrB_mxm
    GxB\_COMPRESSION = 36,
                           // select compression for serialize
    GxB_ROWINDEX_LIST = 7062,
                                   // how GrB_Vector I is intrepretted
    GxB\_COLINDEX\_LIST = 7063,
                                   // how GrB_Vector J is intrepretted
    GxB_VALUE_LIST = 7064,
                                   // how GrB_Vector X is intrepretted
}
GrB_Desc_Field ;
typedef enum
    // for all GrB_Descriptor fields:
    GrB_DEFAULT = 0,
                        // default behavior of the method
    // for GrB_OUTP only:
    GrB_REPLACE = 1,
                        // clear the output before assigning new values to it
    // for GrB_MASK only:
    GrB\_COMP = 2,
                       // use the complement of the mask
    GrB_STRUCTURE = 4, // use the structure of the mask
    // for GrB_INPO and GrB_INP1 only:
    GrB_TRAN = 3,
                        // use the transpose of the input
    // for GxB_AxB_METHOD only:
    GxB_AxB_GUSTAVSON = 1001,
                               // gather-scatter saxpy method
    GxB_AxB_DOT
                      = 1003,
                               // dot product
    GxB_AxB_HASH
                      = 1004,
                               // hash-based saxpy method
                      = 1005
                               // saxpy method (any kind)
    GxB_AxB_SAXPY
    // for GxB_ROWINDEX_LIST, GxB_COLINDEX_LIST, and GxB_VALUE_LIST:
    // GxB\_USE\_VALUES = ((int) GrB\_DEFAULT) // use the values of the vector
    GxB_USE_INDICES = 7060, // use the indices of the vector
    GxB_IS_STRIDE = 7061,
                            // use the values, of size 3, for lo:hi:inc
}
GrB_Desc_Value ;
```

• GrB_OUTP is a parameter that modifies the output of a GraphBLAS operation. In the default case, the output is not cleared, and $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$ then $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{Z}$ are computed as-is, where \mathbf{T} is the results of the particular GraphBLAS operation.

In the non-default case, $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$ is first computed, using the results of \mathbf{T} and the accumulator \odot . After this is done, if the $\mathtt{GrB_OUTP}$ descriptor field is set to $\mathtt{GrB_REPLACE}$, then the output is cleared of its entries. Next, the assignment $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{Z}$ is performed.

• GrB_MASK is a parameter that modifies the Mask, even if the mask is not present.

If this parameter is set to its default value, and if the mask is not present (Mask==NULL) then implicitly Mask(i,j)=1 for all i and j. If the mask is present then Mask(i,j)=1 means that C(i,j) is to be modified by the $C\langle M \rangle = Z$ update. Otherwise, if Mask(i,j)=0, then C(i,j) is not modified, even if Z(i,j) is an entry with a different value; that value is simply discarded.

If the GrB_MASK parameter is set to GrB_COMP, then the use of the mask is complemented. In this case, if the mask is not present (Mask==NULL) then implicitly Mask(i,j)=0 for all i and j. This means that none of C is modified and the entire computation of Z might as well have been skipped. That is, a complemented empty mask means no modifications are made to the output object at all, except perhaps to clear it in accordance with the GrB_OUTP descriptor. With a complemented mask, if the mask is present then Mask(i,j)=0 means that C(i,j) is to be modified by the $C\langle M \rangle = Z$ update. Otherwise, if Mask(i,j)=1, then C(i,j) is not modified, even if Z(i,j) is an entry with a different value; that value is simply discarded.

If the GrB_MASK parameter is set to GrB_STRUCTURE, then the values of the mask are ignored, and just the pattern of the entries is used. Any entry M(i,j) in the pattern is treated as if it were true.

The GrB_COMP and GrB_STRUCTURE settings can be combined, either by setting the mask option twice (once with each value), or by setting the mask option to GrB_COMP+GrB_STRUCTURE (the latter is an extension to the specification).

Using a parameter to complement the Mask is very useful because constructing the actual complement of a very sparse mask is impossible since it has too many entries. If the number of places in C that should be modified is very small, then use a sparse mask without complementing it. If the number of places in C that should be protected from modification is very small, then use a sparse mask to indicate those places, and use a descriptor GrB_MASK that complements the use of the mask.

• GrB_INPO and GrB_INP1 modify the use of the first and second input matrices A and B of the GraphBLAS operation.

If the GrB_INPO is set to GrB_TRAN, then A is transposed before using it in the operation. Likewise, if GrB_INP1 is set to GrB_TRAN, then the second input, typically called B, is transposed.

Vectors and scalars are never transposed via the descriptor. If a method's first parameter is a matrix and the second a vector or scalar, then GrB_INPO modifies the matrix parameter and GrB_INP1 is ignored. If a method's first parameter is a vector or scalar and the second a matrix, then GrB_INP1 modifies the matrix parameter and GrB_INPO is ignored.

To clarify this in each function, the inputs are labeled as first input: and second input: in the function signatures.

- GxB_AxB_METHOD suggests the method that should be used to compute C=A*B. All the methods compute the same result, except they may have different floating-point roundoff errors. This descriptor should be considered as a hint; SuiteSparse:GraphBLAS is free to ignore it.
 - Grb_DEFAULT means that a method is selected automatically.
 - GxB_AxB_SAXPY: select any saxpy-based method: GxB_AxB_GUSTAVSON, and/or GxB_AxB_HASH, or any mix of the two, in contrast to the dot-product method.
 - GxB_AxB_GUSTAVSON: an extended version of Gustavson's method [Gus78], which is a very good general-purpose method, but sometimes the workspace can be too large. Assuming all matrices are stored by column, it computes C(:,j)=A*B(:,j) with a sequence of saxpy operations (C(:,j)+=A(:,k)*B(k:,j) for each nonzero

- B(k,j)). In the *coarse Gustavson* method, each internal thread requires workspace of size m, to the number of rows of C, which is not suitable if the matrices are extremely sparse or if there are many threads. For the *fine Gustavson* method, threads can share workspace and update it via atomic operations. If all matrices are stored by row, then it computes C(i,:)=A(i,:)*B in a sequence of sparse saxpy operations, and using workspace of size n per thread, or group of threads, corresponding to the number of columns of C.
- GxB_AxB_HASH: a hash-based method, based on [NMAB18]. It is very efficient for hypersparse matrices, matrix-vector-multiply, and when |B| is small. SuiteSparse:GraphBLAS includes a coarse hash method, in which each thread has its own hash workspace, and a fine hash method, in which groups of threads share a single hash workspace, as concurrent data structure, using atomics.
- GxB_AxB_DOT: computes C(i,j)=A(i,:)*B(j,:)', for each entry C(i,j). If the mask is present and not complemented, only entries for which M(i,j)=1 are computed. This is a very specialized method that works well only if the mask is present, very sparse, and not complemented, when C is small, or when C is bitmap or full. For example, it works very well when A and B are tall and thin, and C<M>=A*B' or C=A*B' are computed. These expressions assume all matrices are in CSR format. If in CSC format, then the dot-product method used for A'*B. The method is impossibly slow if C is large and the mask is not present, since it takes Ω(mn) time if C is m-by-n in that case. It does not use any workspace at all. Since it uses no workspace, it can work very well for extremely sparse or hypersparse matrices, when the mask is present and not complemented.
- GxB_SORT provides a hint to GrB_mxm, GrB_mxv, GrB_vxm, and GrB_reduce (to vector). These methods can leave the output matrix or vector in a jumbled state, where the final sort is left as pending work. This is typically fastest, since some algorithms can tolerate jumbled matrices on input, and sometimes the sort can be skipped entirely. However, if the matrix or vector will be immediately exported in unjumbled form, or provided as input to a method that requires it to not be jumbled, then

sorting it during the matrix multiplication is faster. By default, these methods leave the result in jumbled form (a *lazy sort*), if GxB_SORT is set to zero (GrB_DEFAULT). A nonzero value will inform the matrix multiplication to sort its result, instead.

• GxB_COMPRESSION selects the compression method for serialization. The default is ZSTD (level 1). See Section 6.11 for other options.

The next sections describe the methods for a GrB_Descriptor:

| GraphBLAS function | purpose | Section |
|--------------------------------|-----------------------------------|---------|
| GrB_Descriptor_new | create a descriptor | 6.16.1 |
| <pre>GrB_Descriptor_wait</pre> | wait for a descriptor | 6.16.2 |
| GrB_get | get a parameter from a descriptor | 10.14 |
| GrB_set | set a parameter in a descriptor | 10.14 |
| <pre>GrB_Descriptor_free</pre> | free a descriptor | 6.16.3 |

6.16.1 GrB_Descriptor_new: create a new descriptor

```
GrB_Info GrB_Descriptor_new  // create a new descriptor
(
    GrB_Descriptor *descriptor // handle of descriptor to create
);
```

GrB_Descriptor_new creates a new descriptor, with all fields set to their defaults (output is not replaced, the mask is not complemented, the mask is valued not structural, neither input matrix is transposed, the method used in C=A*B is selected automatically, and GrB_mxm leaves the final sort as pending work).

6.16.2 GrB_Descriptor_wait: wait for a descriptor

After creating a user-defined descriptor, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. Currently, SuiteSparse:GraphBLAS does nothing except to ensure that d is valid.

6.16.3 GrB_Descriptor_free: free a descriptor

GrB_Descriptor_free frees a descriptor. Either usage:

```
GrB_Descriptor_free (&descriptor) ;
GrB_free (&descriptor) ;
```

frees the descriptor and sets descriptor to NULL. It safely does nothing if passed a NULL handle, or if descriptor == NULL on input.

6.16.4 Descriptor settings for GrB_Vector parameters

Several methods GraphBLAS v10 accept GrB_Vector parameters for their index lists I and J, which appear only as uint64_t * C arrays in the v2.1 C Specification. Likewise, several methods accept a GrB_Vector parameter X, where the related method in the Specification accepts only a raw C array of a given type.

By default, GrB_Vector inputs I, J, and X are interpretted as if their values are first extracted with GrB_Vector_extractTuples, where the values are extracted in order (with ascending indices), and their values are then passed to the method. The actual method is much faster; GraphBLAS uses the values directly.

This behavior can be revised via the descriptor for the method. Three settings are available:

- GxB_ROWINDEX_LIST: how the GrB_Vector I is intrepretted.
- GxB_COLINDEX_LIST: how the GrB_Vector J is intrepretted.
- GxB_VALUE_LIST: how GrB_Vector X is intrepretted (for GrB_build only).

These can be set to one of the following values:

• Grb_Default or Gxb_use_values: use the values of the vector (default).

- GxB_USE_INDICES: use the indices of the vector. This acts as if the indices are first extracted into a C array with GrB_Vector_extractTuples, where the indices are extracted in ascending order, and then this C array is then passed to the method. The actual method is much faster; GraphBLAS uses the indices directly.
- GxB_IS_STRIDE: use the values, of size 3, for a strided range, or lo:inc:hi in MATLAB notation. This usage is limited to the I and J vectors (except this option may not be used for GrB_build). The vector must have exactly three entries, lo, hi, and inc, in that order.

The GxB_IS_STRIDE option is fully described in Section 11. In that section, there are many options available. Here, the GrB_Vector I or J must have length exactly three. The first entry present is the start of the sequence (10), the second entry is the end of the sequence (hi) and the third entry is the stride (inc). This corresponds to the GxB_STRIDE option when passing a uint64_t * array. To use a stride of one, simply set the third entry to 1; this corresponds to the GxB_RANGE option when passing a uint64_t * array. To use a negative stride, simply pass in the vector with a signed data type (GrB_INT32 or GrB_INT64 as appropriate; this corresponds to the GxB_BACKWARDS option desribed in Section 11). These three values appear in this order to be consistent GxB_BEGIN (0), GxB_END (1), GxB_INC (2).

When using the _Vector methods, the GrB_Vector objects I, J, and X may be sparse. If the vectors are sparse, GrB_Vector_extractTuples returns a dense list of indices or values, and this is how the I,J,X vectors may be used in the new methods in GraphBLAS v10 with the _Vector suffix at then end of their name.

To use the GrB_ALL option, specifying all the rows or columns of a matrix or all indices of a vector, pass in the corresponding GrB_Vector I or J as a NULL pointer.

6.16.5 GrB_DESC_*: built-in descriptors

Built-in descriptors are listed in the table below. A dash in the table indicates the default. These descriptors may not be modified or freed. Attempts to modify them result in an error (Grb_INVALID_VALUE); attempts to free them are silently ignored.

| Descriptor | OUTP | MASK | MASK | INPO | INP1 |
|------------------|-------------|--------------------------|------------|----------|----------|
| | | structural | complement | | |
| GrB_NULL | - | = | - | - | - |
| GrB_DESC_T1 | - | - | - | - | GrB_TRAN |
| GrB_DESC_TO | _ | - | - | GrB_TRAN | _ |
| GrB_DESC_TOT1 | _ | - | - | GrB_TRAN | GrB_TRAN |
| GrB_DESC_C | - | - | GrB_COMP | - | - |
| GrB_DESC_CT1 | - | - | GrB_COMP | - | GrB_TRAN |
| GrB_DESC_CTO | _ | - | GrB_COMP | GrB_TRAN | - |
| GrB_DESC_CTOT1 | _ | - | GrB_COMP | GrB_TRAN | GrB_TRAN |
| GrB_DESC_S | - | GrB_STRUCTURE | - | - | - |
| GrB_DESC_ST1 | _ | <pre>Grb_STRUCTURE</pre> | - | - | GrB_TRAN |
| GrB_DESC_STO | - | <pre>Grb_STRUCTURE</pre> | - | GrB_TRAN | - |
| GrB_DESC_STOT1 | - | <pre>GrB_STRUCTURE</pre> | - | GrB_TRAN | GrB_TRAN |
| GrB_DESC_SC | - | GrB_STRUCTURE | GrB_COMP | - | - |
| GrB_DESC_SCT1 | _ | <pre>GrB_STRUCTURE</pre> | GrB_COMP | - | GrB_TRAN |
| GrB_DESC_SCTO | - | <pre>Grb_STRUCTURE</pre> | GrB_COMP | GrB_TRAN | - |
| GrB_DESC_SCTOT1 | _ | <pre>Grb_STRUCTURE</pre> | GrB_COMP | GrB_TRAN | GrB_TRAN |
| GrB_DESC_R | GrB_REPLACE | - | - | - | - |
| GrB_DESC_RT1 | GrB_REPLACE | - | - | - | GrB_TRAN |
| GrB_DESC_RTO | GrB_REPLACE | - | - | GrB_TRAN | - |
| GrB_DESC_RTOT1 | GrB_REPLACE | - | - | GrB_TRAN | GrB_TRAN |
| GrB_DESC_RC | GrB_REPLACE | = | GrB_COMP | - | - |
| GrB_DESC_RCT1 | GrB_REPLACE | - | GrB_COMP | - | GrB_TRAN |
| GrB_DESC_RCTO | GrB_REPLACE | - | GrB_COMP | GrB_TRAN | - |
| GrB_DESC_RCTOT1 | GrB_REPLACE | - | GrB_COMP | GrB_TRAN | GrB_TRAN |
| GrB_DESC_RS | GrB_REPLACE | GrB_STRUCTURE | - | - | - |
| GrB_DESC_RST1 | GrB_REPLACE | <pre>Grb_STRUCTURE</pre> | - | - | GrB_TRAN |
| GrB_DESC_RSTO | GrB_REPLACE | <pre>Grb_STRUCTURE</pre> | - | GrB_TRAN | - |
| GrB_DESC_RSTOT1 | GrB_REPLACE | GrB_STRUCTURE | - | GrB_TRAN | GrB_TRAN |
| GrB_DESC_RSC | GrB_REPLACE | GrB_STRUCTURE | GrB_COMP | - | - |
| GrB_DESC_RSCT1 | GrB_REPLACE | GrB_STRUCTURE | GrB_COMP | - | GrB_TRAN |
| GrB_DESC_RSCTO | GrB_REPLACE | GrB_STRUCTURE | GrB_COMP | GrB_TRAN | - |
| GrB_DESC_RSCTOT1 | GrB_REPLACE | GrB_STRUCTURE | GrB_COMP | GrB_TRAN | GrB_TRAN |

6.17 GrB_free: free any GraphBLAS object

Each of the ten objects has GrB_*_new and GrB_*_free methods that are specific to each object. They can also be accessed by a generic function, GrB_free, that works for all ten objects. If G is any of the ten objects, the statement

```
GrB_free (&G) ;
```

frees the object and sets the variable G to NULL. It is safe to pass in a NULL handle, or to free an object twice:

```
GrB_free (NULL); // SuiteSparse:GraphBLAS safely does nothing GrB_free (&G); // the object G is freed and G set to NULL GrB_free (&G); // SuiteSparse:GraphBLAS safely does nothing
```

However, the following sequence of operations is not safe. The first two are valid but the last statement will lead to undefined behavior.

```
H = G;  // valid; creates a 2nd handle of the same object GrB\_free (&G);  // valid; G is freed and set to NULL; H now undefined GrB\_some\_method (H);  // not valid; H is undefined
```

Some objects are predefined, such as the built-in types. If a user application attempts to free a built-in object, SuiteSparse:GraphBLAS will safely do nothing. The GrB_free function in SuiteSparse:GraphBLAS always returns GrB_SUCCESS.

7 The mask, accumulator, and replace option

After a GraphBLAS operation computes a result \mathbf{T} , (for example, $\mathbf{T} = \mathbf{AB}$ for $\mathtt{GrB_mxm}$), the results are assigned to an output matrix \mathbf{C} via the mask/accumulator phase, written as $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}$. This phase is affected by the $\mathtt{GrB_REPLACE}$ option in the descriptor, the presence of an optional binary accumulator operator (\odot) , the presence of the optional mask matrix \mathbf{M} , and the status of the mask descriptor. The interplay of these options is summarized in Table 1.

The mask \mathbf{M} may be present, or not. It may be structural or valued, and it may be complemented, or not. These options may be combined, for a total of 8 cases, although the structural/valued option as no effect if \mathbf{M} is not present. If \mathbf{M} is not present and not complemented, then m_{ij} is implicitly true. If not present yet complemented, then all m_{ij} entries are implicitly zero; in this case, \mathbf{T} need not be computed at all. Either \mathbf{C} is not modified, or all its entries are cleared if the replace option is enabled. If \mathbf{M} is present, and the structural option is used, then m_{ij} is treated as true if it is an entry in the matrix (its value is ignored). Otherwise, the value of m_{ij} is used. In both cases, entries not present are implicitly zero. These values are negated if the mask is complemented. All of these various cases are combined to give a single effective value of the mask at position ij.

The combination of all these options are presented in the Table 1. The first column is the $GrB_REPLACE$ option. The second column lists whether or not the accumulator operator is present. The third column lists whether or not c_{ij} exists on input to the mask/accumulator phase (a dash means that it does not exist). The fourth column lists whether or not the entry t_{ij} is present in the result matrix \mathbf{T} . The mask column is the final effective value of m_{ij} , after accounting for the presence of \mathbf{M} and the mask options. Finally, the last column states the result of the mask/accum step; if no action is listed in this column, then c_{ij} is not modified.

Several important observations can be made from this table. First, if no mask is present (and the mask-complement descriptor option is not used), then only the first half of the table is used. In this case, the <code>Grb_REPLACE</code> option has no effect. The entire matrix ${\bf C}$ is modified.

Consider the cases when c_{ij} is present but t_{ij} is not, and there is no mask or the effective value of the mask is true for this ij position. With no accumulator operator, c_{ij} is deleted. If the accumulator operator is present and the replace option is not used, c_{ij} remains unchanged.

| repl | accum | \mathbf{C} | \mathbf{T} | mask | action taken by $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot \mathbf{T}$ |
|------|-------|--------------|--------------|------|--|
| - | - | c_{ij} | t_{ij} | 1 | $c_{ij} = t_{ij}$, update |
| - | - | - | t_{ij} | 1 | $c_{ij} = t_{ij}$, insert |
| - | - | c_{ij} | - | 1 | delete c_{ij} because t_{ij} not present |
| - | - | - | - | 1 | |
| - | - | c_{ij} | t_{ij} | 0 | |
| - | - | - | t_{ij} | 0 | |
| - | - | c_{ij} | - | 0 | |
| - | - | - | - | 0 | |
| yes | - | c_{ij} | t_{ij} | 1 | $c_{ij} = t_{ij}$, update |
| yes | - | - | t_{ij} | 1 | $c_{ij} = t_{ij}$, insert |
| yes | - | c_{ij} | - | 1 | delete c_{ij} because t_{ij} not present |
| yes | - | - | - | 1 | |
| yes | - | c_{ij} | t_{ij} | 0 | delete c_{ij} (because of $GrB_REPLACE$) |
| yes | - | - | t_{ij} | 0 | |
| yes | - | c_{ij} | - | 0 | delete c_{ij} (because of GrB_REPLACE) |
| yes | - | - | - | 0 | |
| - | yes | c_{ij} | t_{ij} | 1 | $c_{ij} = c_{ij} \odot t_{ij}$, apply accumulator |
| - | yes | - | t_{ij} | 1 | $c_{ij} = t_{ij}$, insert |
| - | yes | c_{ij} | - | 1 | |
| - | yes | - | - | 1 | |
| - | yes | c_{ij} | t_{ij} | 0 | |
| - | yes | - | t_{ij} | 0 | |
| - | yes | c_{ij} | - | 0 | |
| | yes | - | - | 0 | |
| yes | yes | c_{ij} | t_{ij} | 1 | $c_{ij} = c_{ij} \odot t_{ij}$, apply accumulator |
| yes | yes | - | t_{ij} | 1 | $c_{ij} = t_{ij}$, insert |
| yes | yes | c_{ij} | - | 1 | |
| yes | yes | - | - | 1 | |
| yes | yes | c_{ij} | t_{ij} | 0 | delete c_{ij} (because of $GrB_REPLACE$) |
| yes | yes | - | t_{ij} | 0 | |
| yes | yes | c_{ij} | - | 0 | delete c_{ij} (because of $GrB_REPLACE$) |
| yes | yes | - | _ | 0 | |

Table 1: Results of the mask/accumulator phase.

When there is no mask and the mask GrB_COMP option is not selected, the table simplifies (Table 2). The $GrB_REPLACE$ option no longer has any effect. The GrB_SECOND_T binary operator when used as the accumulator unifies the first cases, shown in Table 3. The only difference now is the behavior when c_{ij} is present but t_{ij} is not. Finally, the effect of GrB_FIRST_T as the accumulator is shown in Table 4.

| accum | \mathbf{C} | \mathbf{T} | action taken by $\mathbf{C} = \mathbf{C} \odot \mathbf{T}$ |
|-------|--------------|--------------|--|
| - | c_{ij} | t_{ij} | $c_{ij} = t_{ij}$, update |
| - | - | t_{ij} | $c_{ij} = t_{ij}$, insert |
| - | c_{ij} | - | delete c_{ij} because t_{ij} not present |
| - | - | - | |
| yes | c_{ij} | t_{ij} | $c_{ij} = c_{ij} \odot t_{ij}$, apply accumulator |
| yes | - | t_{ij} | $c_{ij} = t_{ij}$, insert |
| yes | c_{ij} | - | |
| yes | - | - | |

Table 2: When no mask is present (and not complemented).

| accum | \mathbf{C} | \mathbf{T} | action taken by $\mathbf{C} = \mathbf{C} \odot \mathbf{T}$ |
|-------|--------------|--------------|--|
| yes | c_{ij} | t_{ij} | $c_{ij} = t_{ij}$, apply GrB_SECOND accumulator |
| yes | - | t_{ij} | $c_{ij} = t_{ij}$, insert |
| yes | c_{ij} | - | |
| yes | - | - | |

Table 3: No mask, with the SECOND operator as the accumulator.

| accum | \mathbf{C} | \mathbf{T} | action taken by $\mathbf{C} = \mathbf{C} \odot \mathbf{T}$ |
|-------|--------------|--------------|--|
| yes | c_{ij} | t_{ij} | |
| yes | - | t_{ij} | $c_{ij} = t_{ij}$, insert |
| yes | c_{ij} | - | |
| yes | - | - | |

Table 4: No Mask, with the FIRST operator as the accumulator.

8 GxB_Context: controlling computational resources

SuiteSparse:GraphBLAS v8.0.0 adds a new object, the GxB_Context, which controls the number of threads used by OpenMP. In the future, this same object will control the number of GPUs used.

The GxB_Context object is not needed if the user application is itself single threaded, with all parallelism is inside GraphBLAS itself. The object is also not needed if the user application is multi-threaded, but all user threads create the same number of threads inside GraphBLAS (say each using a single thread). In that case, GrB_set(GrB_GLOBAL,1,GxB_NTHREADS) can be used (for example).

However, suppose the user application creates 5 threads of its own, on a machine with 16 cores, and each thread wants to use a different number of threads inside GraphBLAS (one user thread uses 8 OpenMP threads and the other four use 2 each, for example). This is where the GxB_Context object becomes essential.

The default context is GxB_CONTEXT_WORLD, which is not created by the user application but it can be modified. If a user thread does not create its own context, then its computational resources are determine by this GxB_CONTEXT_WORLD object. The following GrB_set/get methods access this global object without naming it directly (where chunk is a GrB_Scalar of type GrB_FP64 or GrB_FP32):

- GrB_set (GrB_GLOBAL, nthreads, GxB_NTHREADS)
- GrB_get (GrB_GLOBAL, &nthreads, GxB_NTHREADS)
- GrB_set (GrB_GLOBAL, chunk, GxB_CHUNK)
- GrB_get (GrB_GLOBAL, chunk, GxB_CHUNK)

The above methods control the OpenMP threads used by all user threads in the user application. To allow each user thread to control its own OpenMP threading, each user thread needs to create its own Context object via GxB_Context_new. Next, the user thread must engage this context via GxB_Context_engage; all subsequent calls to GraphBLAS from this particular user thread will then use the number of OpenMP threads dictated by this particular context.

Engaging a GxB_Context object assigns to a threadprivate space accessible only by this particular user thread, so that any calls to GraphBLAS can access the settings in this object.

The opposite operation is to *disengage* a context. This removes a particular object from the **threadprivate** space of the user thread that is disengaging its context.

After a context object is created, the user thread that owns it can modify its settings in this object. An example appears in the GraphBLAS/Demo folder, part of which is listed below.

```
#pragma omp parallel for num_threads (nouter) schedule (dynamic, 1)
for (int k = 0; k < nmat; k++)
    // each user thread constructs its own context
    GxB_Context Context = NULL ;
    GxB_Context_new (&Context) ;
    GrB_set (Context, ninner, GxB_NTHREADS) ;
    GxB_Context_engage (Context) ;
    // kth user thread builds kth matrix with ninner threads
    GrB_Matrix A = NULL ;
    GrB_Matrix_new (&A, GrB_FP64, n, n);
    GrB_Matrix_build (A, I, J, X, nvals, GrB_PLUS_FP64) ;
    // free the matrix just built
    GrB_Matrix_free (&A) ;
    // each user thread frees its own context
    GxB_Context_disengage (Context) ;
    GxB_Context_free (&Context) ;
}
```

In this example, nouter user threads are created. Inside the parallel loop, each user thread creates and engages its own context object. In this simple example, each user thread then uses ninner threads to do some work, although in principle each user thread to request a different number of threads for each of its calls to GraphBLAS. This leads to nested parallelism, so to use this context object effectively, the nested parallelism feature of OpenMP must be enabled.

The next sections describe the methods for a GxB_Context:

| GraphBLAS function | purpose | Section |
|----------------------------------|----------------------------|---------|
| GxB_Context_new | create a context | 8.1 |
| <pre>GxB_Context_engage</pre> | engage a context | 8.2 |
| <pre>GxB_Context_disengage</pre> | disengage a context | 8.3 |
| <pre>GxB_Context_free</pre> | free a context | 8.4 |
| GxB_Context_wait | wait for a context | 8.5 |
| GrB_get | get a value from a context | 10.15 |
| GrB_set | set a value in a context | 10.15 |
| GxB_Context_fprint | check/print a context | 13.11 |

8.1 GxB_Context_new: create a new context

A new context is created and initialized with the current global settings for GxB_NTHREADS and GxB_CHUNK. See GrB_get. The context object will not have an effect on any calls to GraphBLAS until it is *engaged* by a user thread.

8.2 GxB_Context_engage: engaging context

GxB_Context_engage sets the provided Context object as the Context for this user thread. Multiple user threads can share a single Context. Any prior Context for this user thread is superseded by the new Context (the prior one is not freed). GrB_SUCCESS is returned, and future calls to GraphBLAS by this user thread will use the provided Context.

If the Context on input is the <code>GxB_CONTEXT_WORLD</code> object, then the current Context is disengaged. That is, the following calls have the same effect, setting the Context of this user thread to <code>GxB_CONTEXT_WORLD</code>:

```
GxB_Context_engage (GxB_CONTEXT_WORLD) ;
GxB_Context_disengage (NULL) ;
```

The result for both cases above is Grb_SUCCESS.

Error cases: If Context is NULL on input, GrB_NULL_POINTER is returned. If a non-NULL Context is provided but it is faulty in some way, then an error code is returned (GrB_INVALID_OBJECT or GrB_UNINITIALIZED_OBJECT). If an error code is returned, the current Context for this user thread is unmodified.

8.3 GxB_Context_disengage: disengaging context

```
GrB_Info GxB_Context_disengage // disengage a Context
(
GxB_Context Context // Context to disengage
);
```

If a NULL Context is provided or if the Context input parameter is GxB_CONTEXT_WORLD, then any current Context for this user thread is disengaged. If a valid non-NULL Context is provided and it matches the current Context for this user thread, it is disengaged. In all of these cases, GrB_SUCCESS is returned. The user thread has no Context object and any subsequent calls to GraphBLAS functions will use the world Context, GxB_CONTEXT_WORLD.

Error cases: If a non-NULL Context is provided but it is faulty in some way, then an error code is returned (Grb_INVALID_OBJECT or Grb_UNINITIALIZED_OBJECT). If a non-NULL Context is provided on input that doesn't match the current Context for this thread, then Grb_INVALID_VALUE is returned. If an error code is returned, the current Context for this user thread is unmodified.

8.4 GxB_Context_free: free a context

GxB_Context_free frees a descriptor. Either usage:

```
GxB_Context_free (&Context) ;
GrB_free (&Context) ;
```

frees the Context and sets Context to NULL. It safely does nothing if passed a NULL handle, or if Context == NULL on input.

8.5 GxB_Context_wait: wait for a context

```
GrB_Info GrB_wait  // wait for a context

(
GxB_Context Context,  // context to wait for  int mode  // GrB_COMPLETE or GrB_MATERIALIZE
);
```

After creating or modifying a context, a GraphBLAS library may choose to exploit non-blocking mode to delay its creation. Currently, SuiteSparse:GraphBLAS currently does nothing except to ensure that Context is valid.

9 The SuiteSparse:GraphBLAS JIT

SuiteSparse:GraphBLAS v8.0 adds a new JIT feature that greatly improves performance of user-defined types and operators, and improves the performance of built-in operators as well. The JIT can compile kernels that are specific to the matrix type and the operators that work on it. In version v7.4.4 and prior versions, user-defined types and operators were handled by generic kernels that used function pointers for each operator and for any type-casting required. Even built-in types and operators were sometimes handled by the generic kernels, if any typecasting was done, or if the specific operator, monoid, or semiring was disabled when GraphBLAS was compiled.

9.1 Using the JIT

Using the JIT in a user application is simple: by default, there is nothing to do. LAGraph v1.0.1 can use the JIT (and PreJIT) kernels without changing a single line of code.¹

Currently, the JIT compiles kernels for the CPU only, but a CUDA JIT is in progress to exploit NVIDIA GPUs, in collaboration with Joe Eaton and Corey Nolet, with NVIDIA.

When GraphBLAS is compiled, the cmake build system creates a cache folder where it will keep any kernels created and compiled by the JIT (both source code and compiled libraries for each kernel). The default folder is ~/.SuiteSparse/Grbs.o.o for SuiteSparse:GraphBLAS version v8.o.o, where the tilde refers to the user's home directory. The version numbers in the folder name are set automatically, so that a new version will ignore kernels compiled by an older version of GraphBLAS. If the GRAPHBLAS_CACHE_PATH environment variable is set when GraphBLAS is compiled, that variable defines the folder. If the user's home directory cannot be determined and the GRAPHBLAS_CACHE_PATH environment variable is not set, then JIT compilation is disabled and only PreJIT kernels can be used. The optional environment variable, GRAPHBLAS_CACHE_PATH, is also read by Grb_init when the user application runs. See Section 9.1.11 for a description of the valid characters that can appear in the cache path.

¹LAGraph v1.0.1 does not work with GraphBLAS v8.0.0 but not because of the JIT. LAGraph must be updated because it uses two deprecated GxB_SelectOp operators; these are replaced with GrB_IndexUnaryOp operators in the dev branch of LAGraph, to appear in a future release of LAGraph.

When the user application starts, it can modify the location of the cache folder after calling <code>GrB_init</code>. It can also modify the C compiler and its flags, and can control when and how the JIT is used. These changes are made via <code>GrB_set</code>, and can be queried via <code>GrB_get</code>; refer to Section 10 for details, and the <code>GxB_JIT_*</code> settings:

| field | value | description |
|--------------------------|-----------|--|
| GxB_JIT_C_COMPILER_NAME | char * | C compiler for JIT kernels |
| GxB_JIT_C_COMPILER_FLAGS | char * | flags for the C compiler |
| GxB_JIT_C_LINKER_FLAGS | char * | link flags for the C compiler |
| GxB_JIT_C_LIBRARIES | char * | libraries to link against (no cmake) |
| GxB_JIT_C_CMAKE_LIBS | char * | libraries to link against (with cmake) |
| GxB_JIT_C_PREFACE | char * | C code as preface to JIT kernels |
| GxB_JIT_C_CONTROL | see below | CPU JIT control |
| GxB_JIT_USE_CMAKE | see below | CPU JIT control |
| GxB_JIT_ERROR_LOG | char * | error log file |
| GxB_JIT_CACHE_PATH | char * | folder with compiled kernels |

To control the JIT in the MATLAB @GrB interface, use the GrB.jit method. Refer to help GrB.jit for details.

Kernels compiled during one run of a user application are kept in the cache folder, so that when the user application runs again, the kernels do not have to be compiled. If the kernel relies on user-defined types and/or operators, a check is made the first time the compiled kernel is loaded. If the current definition of the user-defined type or operator does not exactly match the definition when the kernel was compiled, then the compiled kernel is discarded and recompiled. The stale kernel is overwritten with the new one, so there is no need to for the user to take any action to delete the stale kernel from the cache path. If the cache path is changed via GrB_set, compiled kernels in the old cache folder are not copied over. New ones are compiled instead.

9.1.1 GxB_JIT_C_CONTROL

The usage of the CPU JIT can be controlled via GrB_get/set using the GxB_JIT_C_CONTROL setting. If the JIT is enabled at compile time, the initial setting is GxB_JIT_ON. If the JIT is disabled at compile time (by setting the cmake variable GRAPHBLAS_USE_JIT to OFF), the initial setting is GxB_JIT_RUN, so that any PreJIT kernels can be run. This setting can be modified; for example to disable the JIT and clear all loaded JIT kernels

from memory, use:

```
GrB_set (GrB_GLOBAL, GxB_JIT_OFF, GxB_JIT_C_CONTROL) ;
```

The above call to <code>GrB_set</code> does not clear any PreJIT kernels, however, since those are integral components of the single compiled GraphBLAS library and cannot be cleared (see Section 9.3). It also does not clear any compiled user functions, created by the JIT for <code>GxB_*Op_new</code> when the input function pointer is <code>NULL</code>.

The following settings are available for GxB_JIT_C_CONTROL. For examples on how to use it, see GraphBLAS/Demo/Program/gauss_demo.c.

If the JIT is disabled at compile time via setting the GRAPHBLAS_USE_JIT option OFF, PreJIT kernels are still available, and can be controlled via the GxB_JIT_OFF, GxB_JIT_PAUSE, or GxB_JIT_RUN settings listed above. If the application tries to set the control to GxB_JIT_LOAD or GxB_JIT_ON, the setting is changed to GxB_JIT_RUN instead. This is not an error condition. The resulting setting can be queried via GrB_get, if desired.

If your copy of GraphBLAS has many PreJIT kernels compiled into it, or uses many run-time JIT kernels, turning of the JIT with GxB_JIT_OFF can be costly. This setting clears the entire JIT hash table. Renabling the JIT and using it will require the JIT table to be repopulated, including a check of each PreJIT kernel the first time they are used. If you wish to temporarily disable the JIT, consider switching the JIT control to GxB_JIT_PAUSE and then back to GxB_JIT_RUN to reenable the JIT.

9.1.2 JIT error handling

The JIT control setting can be changed by GraphBLAS itself, based on following error conditions. These changes affect all kernels, not just the kernel

causing the error. If any of these cases occur, the call to GraphBLAS returns <code>GxB_JIT_ERROR</code>, unless <code>GraphBLAS</code> runs out of memory, in which case it returns <code>GrB_OUT_OF_MEMORY</code> instead. If the <code>JIT</code> is disabled through any of these errors, it can be detected by <code>GrB_get</code> to read the <code>GxB_JIT_C_CONTROL</code> state.

- When a kernel is loaded that relies on user-defined types and/or operators, the definitions in the previously compiled kernel are checked against the current definitions. If they do not match, the old one is discarded, and a new kernel will be compiled. However, if the control is set to GxB_JIT_LOAD, no new kernels may be compiled. To avoid a continual reloading and checking of stale kernels, the control is changed from GxB_JIT_LOAD to GxB_JIT_RUN. To solve this problem, delete the compiled kernel with the stale definition, or enable the full JIT by setting the control to GxB_JIT_ON so that the kernel can recompiled with the current definitions.
- If a new kernel is to be compiled with the control set to GxB_JIT_ON but the source file cannot be created in the cache folder, or a compiler error occurs, further compilation is disabled. The control is changed from GxB_JIT_ON to GxB_JIT_LOAD. To solve this problem, make sure your application has write permission to the cache path and that any user-defined types and operators are defined properly so that no syntax error is detected by the compiler.
- If a kernel is loaded but the lookup of the kernel function itself in the compiled library fails, the control is changed to GxB_JIT_RUN to prevent this error from occuring again. To solve this problem, delete the corrupted compiled kernel from the cache folder. This case is unlikely to occur since no user action can trigger it. It could indicate a system problem with loading the kernel, or some kind of compiler error that allows the kernel to be compiled but not loaded.
- If an out-of-memory condition occurs in the JIT, the JIT control is set to GxB_JIT_PAUSE. To solve this, try freeing up memory, use a larger system, or solve smaller problems.

As a result of this automatic change in the JIT control setting, after the first JIT error is returned, subsequent calls to GraphBLAS will likely succeed. GraphBLAS will use a generic kernel instead. To re-enable the JIT for subsequent calls to GraphBLAS, the user application must reset the GxB_JIT_C_CONTROL back to GxB_JIT_ON.

In many use cases of GraphBLAS (such as LAGraph), a function will create a type or operator, use it, and then free it just before returning. It would be far to costly to clear the loaded kernel and reload it each time the LAGraph function is called, so any kernels that use this type or operator are kept loaded when the type or operator is freed. The typical case is that when the LAGraph function is called again, it will recreate the type or operator with the identical name and definition. The kernels that use these types or operators will still be loaded and can thus be used with no overhead.

However, if a user-defined type or operator is freed and then redefined with the same name but a different definition, any loaded kernels should be freed. This case is not detected by GraphBLAS since it would be far too costly to check each time a previously loaded kernel is called. As a result, this condition is only checked when the kernel is first loaded. To avoid this issue, if the user application frees a user-defined type or operator and creates a new one with a different definition but with the same name, clear all prior kernels by setting the control to GxB_JIT_OFF. Then turn the JIT back on with GxB_JIT_ON. This clears all run-time JIT kernels so that they will be checked when reloaded, and recompiled if their definitions changed. All PreJIT kernels are flagged as unchecked, just as they were flagged by GrB_init, so that they will be checked the next time they run.

9.1.3 GxB_JIT_C_COMPILER_NAME

The GxB_JIT_C_COMPILER_NAME string is the name of the C compiler to use, or its full path. By default it is set to the C compiler used to compile GraphBLAS itself.

9.1.4 GxB_JIT_C_COMPILER_FLAGS

The GxB_JIT_C_COMPILER_FLAGS string is the C compiler flags. By default it is set to the C compiler flags used to compile GraphBLAS itself.

9.1.5 GxB_JIT_C_LINKER_FLAGS

The GxB_JIT_C_LINKER_FLAGS string only affects the kernel compilation when cmake is not used to compile the kernels (see Section 9.1.9). By default

it is set to the C link flags used to compile GraphBLAS itself. If cmake is used to compile the kernels, then it determines the linker flags itself, and this cannot be modified.

9.1.6 GxB_JIT_C_LIBRARIES

The GxB_JIT_C_LIBRARIES string is used to set the libraries to link against when cmake is not being used to compile the kernels (see Section 9.1.9). For example, on Linux it is set by default to the -lm, -ld, and OpenMP libraries used to link GraphBLAS itself. Any standalone library name is prepended with -l. If cmake is used to compile the kernels, this string is ignored.

9.1.7 GxB_JIT_C_CMAKE_LIBS

The GxB_JIT_C_LIBRARIES string is used to set the libraries to link against when cmake is being used to compile the kernels (see Section 9.1.9). For example, on Linux it is set by default to the m, dl, and OpenMP libraries used to link GraphBLAS itself. Libraries in the string should normally be separated by semicolons. If cmake is not used to compile the kernels, this string is ignored.

9.1.8 GxB_JIT_C_PREFACE

The GxB_JIT_C_PREFACE string is added at the top of each JIT kernel. It is useful for providing additional #include files that GraphBLAS does not provide. It can also be useful for diagnostics and for configuring the PreJIT. For example, suppose you wish to tag specific kernels as having been constructed for particular parts of an application. The application can modify this string to some unique comment, and then run some benchmarks that call GraphBLAS. Any JIT kernels created will be tagged with this unique comment, which may be helpful to select specific kernels to copy into the PreJIT folder.

9.1.9 GxB_JIT_USE_CMAKE

Two methods are provided for compiling the JIT kernels: cmake, and a direct compiler/link command. On Windows, only cmake may be used, and this setting is ignored (it is always true). On Linux or Mac, the default is false since a direct compile/link is faster. However, it is possible that some

compilers are not handled properly with this method, so cmake can also be used on those platforms by setting the value of GxB_JIT_USE_CMAKE to true.

Normally the same version of cmake should be used to compile both GraphBLAS and the JIT kernels. However, compiling GraphBLAS itself requires cmake v3.16 or later (v3.19 for some options), while compiling the JIT kernels only requires cmake v3.13 or later.

9.1.10 GxB_JIT_ERROR_LOG

The GxB_JIT_ERROR_LOG string is the filename of the optional error log file. By default, this string is empty, which means that any compiler errors are routed to the stderr output of the user process. If set to a non-empty string, any compiler errors are appended to this file. The string may be NULL, which means the same as an empty string.

9.1.11 GxB_JIT_CACHE_PATH

The GxB_JIT_CACHE_PATH string is the full path to the user's cache folder (described above). The default on Linux/Mac is ~/.SuiteSparse/GrB8.0.0 for GraphBLAS version 8.0.0. On Windows, the cache folder is created inside the user's LOCALAPPDATA folder, called SuiteSparse/GrB8.0.0. When GraphBLAS starts, GrB_init checks if the GRAPHBLAS_CACHE_PATH environment variable exists, and initializes the cache path with that value instead of using the default.

Restrictions: the cache path is sanitized for security reasons. No spaces are permitted. Backslashes are converted into forward slashes. It can contain only characters in the following list:

abcdefghijklmnopqrstuvwxyz ABCDEFGHIJKLMNOPQRSTUVWXYZ 0123456789.-_/

In addition, the second character in the string is allowed to be the colon character (:) to allow for the use of Windows drive letters. Any character outside of these rules is converted into an underscore (_).

9.2 Compilation options: GRAPHBLAS_USE_JIT and GRAPH-BLAS_COMPACT

The CPU JIT can be disabled at compile time by setting the GRAPHBLAS_USE_JIT option OFF in the cmake build options. Good performance will be obtained only by using the FactoryKernels or the PreJIT kernels that are compiled into GraphBLAS when it is first compiled with cmake. By default, GRAPHBLAS_USE_JIT is ON, to enable the CPU JIT.

With the introduction of the JIT kernels, it is now possible to obtain good performance in GraphBLAS without compiling the many factory kernels that appear in the GraphBLAS/Source/FactoryKernels directory. If the JIT is enabled, GraphBLAS will still be fast, once the JIT kernels are compiled, or by using any PreJIT kernels. To compile GraphBLAS without its FactoryKernels, enable the COMPACT option in the cmake build options. By default, COMPACT is off, to enable the FactoryKernels.

When GraphBLAS is compiled with GRAPHBLAS_USE_JIT set to OFF, the GxB_JIT_C_CONTROL may be set to GxB_JIT_OFF, GxB_JIT_PAUSE, or GxB_JIT_RUN. No kernels will be loaded at run-time (the GxB_JIT_LOAD setting is disabled and treated as GxB_JIT_RUN), and no new kernels will be compiled at run-time (the GxB_JIT_ON is disabled and treated as GxB_JIT_RUN). Only pre-existing PreJIT kernels can be run, described in Section 9.3.

If both GRAPHBLAS_USE_JIT is set OFF and GRAPHBLAS_COMPACT is set ON, all features of GraphBLAS will be functional. The only fast kernels available will be the PreJIT kernels (if any). Otherwise, generic kernels will be used, in which every single operator is implemented with a function pointer, and every scalar assignment requires a memcpy. Generic kernels are slow, so using this combination of options is not recommended when preparing GraphBLAS for production use, benchmarking, or for a Linux distro or other widely-used distribution, unless you are able to run your application in advance and create all the JIT kernels you need, and then copy them into GraphBLAS/PreJIT. This would be impossible to do for a general-purpose case such as a Linux distro, but feasible for a more targetted application such as FalkorDB.

9.3 Adding PreJIT kernels to GraphBLAS

When GraphBLAS runs, it constructs JIT kernels in the user's cache folder, which by default is ~/.SuiteSparse/GrB8.0.0 for v8.0.0. The kernels placed in a subfolder (c) and inside that folder they are further subdivided arbitrar-

ily into subfolders (via an arbitary hash). The files are split into subfolders because a single folder may grow too large for efficient access. Once Graph-BLAS has generated some kernels, some or all of them kernels can then incorporated into the compiled GraphBLAS library by copying them into the GraphBLAS/PreJIT folder. Be sure to move any *.c files into the single GraphBLAS/PreJIT folder; do not keep the subfolder structure.

If GraphBLAS is then recompiled via cmake, the build system will detect these kernels, compile them, and make them available as pre-compiled JIT kernels. The kernels are no longer "Just-In-Time" kernels since they are not compiled at run-time. They are referred to as PreJIT kernels since they were at one time created at run-time by the GraphBLAS JIT, but are now compiled into GraphBLAS before it runs.

It's that simple. Just copy the source files for any kernels you want from your cache folder (typically ~/.SuiteSparse/GrB8.0.0/c) into GraphBLAS/PreJIT, and recompile GraphBLAS. There's no need to change any other cmake setting, and no need to do anything different in any applications that use GraphBLAS. Do not copy the compiled libraries; they are not needed and will be ignored. Just copy the *.c files.

If the resulting GraphBLAS library is installed for system-wide usage (say in a Linux distro, Python, RedisGraph, etc), the GraphBLAS/PreJIT kernels will be available to all users of that library. They are not disabled by the GRAPHBLAS_USE_JIT option.

Once these kernels are moved to GraphBLAS/PreJIT and GraphBLAS is recompiled, they can be deleted from the cache folder. However, even if they are left there, they will not be used since GraphBLAS will find these kernels as PreJIT kernels inside the compiled library itself (libgraphblas.so on Linux, libgraphblas.dylib on the Mac). GraphBLAS will not be any slower if these kernels are left in the cache folder, and the compiled library size will not be affected.

If the GraphBLAS version is changed at all (even in the last digit), all GB_jit_*.c files in the GraphBLAS/PreJIT folder should be deleted. The version mismatch will be detected during the call to GrB_init, and any stale kernels will be safely ignored. Likewise, if a user-defined type or operator is changed, the relevant kernels should also be deleted from GraphBLAS/PreJIT. For example, the GraphBLAS/Demo/Program/gauss_demo.c program creates a user-defined gauss type, and two operators, addgauss and multgauss. It then intentionally changes one of the operators just to test this feature. If the type and/or operators are changed, then the *gauss*.c files in the

GraphBLAS/PreJIT folder should be deleted.

GraphBLAS will safely detect any stale PreJIT kernels by checking them the first time they are run after calling GrB_init and will not use them if they are found to be stale. If the JIT control is set to GxB_JIT_OFF all PreJIT kernels are flagged as unchecked. If the JIT is then renabled by setting the control to GxB_JIT_RUN or GxB_JIT_ON, all PreJIT kernels will be checked again and any stale kernels will be detected.

If a stale PreJIT kernel is found, GraphBLAS will use its run-time JIT to compile new ones with the current definitions, or it will punt to a generic kernel if JIT compilation is disabled. GraphBLAS will be functional, and fast if it can rely on a JIT kernel, but the unusable stale PreJIT kernels take up space inside the compiled GraphBLAS library. The best practice is to delete any stale kernels from the GraphBLAS/PreJIT folder, or replace them with newly compiled JIT kernels from the cache folder, and recompile GraphBLAS.

It is safe to copy only a subset of the JIT kernels from the cache folder into GraphBLAS/PreJIT. You may also delete any files in GraphBLAS/PreJIT and recompile GraphBLAS without those kernels. If GraphBLAS encounters a need for a particular kernel that has been removed from GraphBLAS/PreJIT, it will create it at run-time via the JIT, if permitted. If not permitted, by either compiling GraphBLAS with the GRAPHBLAS_USE_JIT option set ot OFF, or by using GxB_JIT_C_CONTROL at run-time, the factory kernel or generic kernel will be used instead. The generic kernel will be slower than the PreJIT or JIT kernel, but GraphBLAS will still be functional.

In addition to a single README.txt file, the GraphBLAS/PreJIT folder includes a .gitignore file that prevents any files in the folder from being synced via git. If you wish to add your PreJIT kernels to a fork of Graph-BLAS, you will need to revise this .gitignore file.

9.4 JIT and PreJIT performance considerations

To create a good set of PreJIT kernels for a particular user application, it is necessary to run the application with many different kinds of workloads. Each JIT or PreJIT kernel is specialized to the particular matrix format, data type, operators, and descriptors of its inputs. GraphBLAS can change a matrix format (from sparse to hypersparse, for example), at its discretion, thus triggering the use of a different kernel. Some GraphBLAS methods use heuristics to select between different methods based upon the sparsity

structure or estimates of the kind or amount of work required. In these cases, entirely different kernels will be compiled. As a result, it's very difficult to predict which kernels GraphBLAS will find the need to compile, and thus a wide set of test cases should be used in an application to allow GraphBLAS to generate as many kernels as could be expected to appear in production use.

GraphBLAS can encounter very small matrices, and it will often select its bitmap format to store them. This change of format will trigger a different kernel than the sparse or hypersparse cases. There are many other cases like that where specific kernels are only needed for small problems. In this case, compiling an entirely new kernel is costly, since using a compiled kernel will be no faster than the generic kernel. When benchmarking an application to allow GraphBLAS to compile its JIT kernels, it may be useful to pause the JIT via GxB_JIT_PAUSE, GxB_JIT_RUN, or GxB_JIT_LOAD, when the application knows it is calling GraphBLAS for tiny problems. These three settings keep any loaded JIT kernels in memory, but pauses the compilation of any new JIT kernels. Then the control can be reset to GxB_JIT_ON once the application finishes with its tiny problems and moves to larger ones where the JIT will improve performance. A future version of GraphBLAS may allow this heuristic to be implemented inside GraphBLAS itself, but for now, the JIT does not second guess the user application; if it wants a new kernel, the JIT will compile it if the control is set to GxB_JIT_ON.

9.5 Mixing JIT kernels: MATLAB and Apple Silicon

In general, the JIT kernels compiled by the C interface and the kernels compiled while using GraphBLAS in MATLAB are interchangable, and the same cache folder can be used for both. This is the default.

However, when using the <code>@GrB</code> MATLAB interface to GraphBLAS on Apple Silicon, using an older version of MATLAB, the MATLAB JIT kernels are compiled as x86 binaries and executed inside MATLAB via Rosetta. The pure C installation may compile native Arm64 binaries for its JIT kernels. Do not mix the two. In this case, set another cache path for MATLAB using <code>GrB.jit</code> in MATLAB, or using <code>GrB_set</code> in the C interface for your native Arm64 binaries.

This issue does not apply to for recent MATLAB versions on the Mac, which are native.

9.6 Updating the JIT when GraphBLAS source code changes

If you edit the GraphBLAS source code itself or add any files to GraphBLAS/PreJIT, read the instructions in GraphBLAS/JITpackage/README.txt for details on how to update the JIT source code.

If your cache folder (~/.SuiteSparse/GrBx.y.z) changes in any way except via GraphBLAS itself, simply delete your cache folder. GraphBLAS will then reconstruct the kernels there as needed.

9.7 Future plans for the JIT and PreJIT

9.7.1 Kernel fusion

The introduction of the JIT and its related PreJIT kernels allow for the future exploitation of kernel fusion via an aggressive exploitation of the GraphBLAS non-blocking mode. In that mode, multiple calls to GraphBLAS can be fused into a single kernel. There are far to many possible variants to allow a fused kernel to appear in the GraphBLAS/Source/FactoryKernels folder, but specific fused kernels could be created by the JIT.

9.7.2 Heuristics for controlling the JIT

As mentioned in Section 9.4, GraphBLAS may compile JIT kernels that are used for only tiny problems where the compile time of a single kernel will dominate any performance gains from using the compiled kernel. A heuristic could be introduced so that it compiles them only for larger problems. The possible downside of this approach is that the same JIT kernels might be needed later for larger problems.

9.7.3 CUDA / SYCL / OpenCL kernels

The CUDA JIT will enable NVIDIA GPUs to be exploited. There are simply too many kernels to create at compile time as the "factory kernels." This CUDA JIT is in progress. A related JIT for SYCL / OpenCL kernels is under consideration.

9.7.4 Better performance for multithreaded user programs:

This version is thread-safe when used in a multithread user application, but a better JIT critical section (many readers, one writer) might be needed. The current critical section may be sufficiently fast since the typical case of work done inside the critical section is a single hash table lookup. However, the performance issues related to this have not been tested. This has no effect if all parallelism is exploited only within GraphBLAS. It only affects the case when multiple user threads each call GraphBLAS in parallel (using the GxB_Context; see Section 8).

10 GraphBLAS Options (GrB_get and GrB_set)

GraphBLAS includes two methods, GrB_get and GrB_set that allow the user to query GraphBLAS objects and change their state. These two methods are polymorphic wrappers for a suite of methods for each object.

The general polymorphic signatures of these methods are given below:

```
GrB_Info GrB_get (object, value, int field) ;
GrB_Info GrB_set (object, value, int field) ;
GrB_Info GrB_set (object, void *value, int field, size_t s) ;
```

where object can be any GraphBLAS object. The value can be a GrB_Scalar, an int32_t (or a pointer to int32_t for GrB_get), a string (char *), or a void * pointer. In the latter case, GrB_set requires an additional parameter (size_t s) that specifies the size of the object pointed to by void *value.

The non-polymorphic names have the following format, where [OBJ] is any GraphBLAS object.

```
GrB_Info GrB_[OBJ]_get_[KIND] (object, value, int field) ;
GrB_Info GrB_[OBJ]_set_[KIND] (object, value, int field) ;
GrB_Info GrB_[OBJ]_set_VOID (object, void *value, int field, size_t s) ;
```

The [KIND] suffix defines the type of second parameter, and can be: Scalar (for GrB_Scalar), String (for char *), INT32 (for int32_t), and SIZE (for size_t * for GrB_get only), and VOID (for void *).

The tables below list all the valid fields that can be used for each object. Each table contains four columns: (1) the int, (2) a column labelled R/W, (3) a column defining the C type, and a description. For the R/W column:

- If the R/W column of a table is R, then the value can be read by GrB_get but not written by GrB_set.
- R/W means that both GrB_get and GrB_set can be used any number of times, with different values.
- R/W1 means that GrB_get can be done multiple times, but GrB_set can be used only once. Subsequent calls to GrB_set return the error code GrB_ALREADY_SET.
- W means that only GrB_set can be used (any number of times), but GrB_get cannot be done.

The second parameter (value) of GrB_get and GrB_set can take on several different C types, and it can also be a GrB_Scalar that holds a value with the given C type (or that can be typecasted to the given C type):

• int32_t: For GrB_set the value parameter is int32_t. For GrB_get the value parameter is int32_t *. The following example sets the global number of threads, and then retrieves that value into nthreads.

```
GrB_set (GrB_GLOBAL, 8, GxB_NTHREADS) ;
int32_t nthreads ;
GrB_get (GrB_GLOBAL, &nthreads, GxB_NTHREADS) ;
printf ("nthreads: %d\n", nthreads) ;
```

A GrB_Scalar can also be used for an int32_t value. For GrB_set, the scalar must not be empty. Here is the same example but using a GrB_Scalar instead:

```
GrB_Scalar s ;
GrB_Scalar_new (s, GrB_INT32) ;
GrB_Scalar_setElement (s, 8) ;
GrB_set (GrB_GLOBAL, s, GxB_NTHREADS) ;
GrB_get (GrB_GLOBAL, s, GxB_NTHREADS) ;
int32_t nthreads ;
GrB_Scalar_extractElement (&nthreads, s) ;
printf ("nthreads: %d\n", nthreads) ;
```

• char *: The value parameter is char * for both GrB_get and GrB_set.

A GrB_Scalar cannot be used. The size of the string required for GrB_get is given by using a size_t * parameter with the same field. For example:

```
size_t len ;
GrB_get (GrB_GLOBAL, &len, GrB_NAME) ;
char *name = malloc (len) ;
GrB_get (GrB_GLOBAL, name, GrB_NAME) ;
printf ("The library is: %s\n", name) ;
free (name) ;
```

To get the current JIT C compiler and then set it to something else:

```
size_t len ;
GrB_get (GrB_GLOBAL, &len, GxB_JIT_C_COMPILER_NAME) ;
char *compiler = malloc (len) ;
GrB_get (GrB_GLOBAL, compiler, GxB_JIT_C_COMPILER_NAME) ;
printf ("The current JIT compiler is: %s\n", compiler) ;
GrB_set (GrB_GLOBAL, "gcc", GxB_JIT_C_COMPILER_NAME) ;
GrB_get (GrB_GLOBAL, &len, GxB_JIT_C_COMPILER_NAME) ;
char *new_compiler = malloc (len) ;
GrB_get (GrB_GLOBAL, new_compiler, GxB_JIT_C_COMPILER_NAME) ;
printf ("The new JIT compiler is: %s\n", new_compiler) ;
free (compiler) ;
```

• Other scalar data types (typically double): Only a GrB_Scalar can be used. When using GrB_set with a GrB_Scalar, the scalar cannot be empty. For example, to get then set the global GxB_HYPER_SWITCH parameter to 0.3:

```
GrB_Scalar s ;
GrB_Scalar_new (s, GrB_FP64) ;
GrB_get (GrB_GLOBAL, s, GxB_HYPER_SWITCH) ;
double hs ;
GrB_Scalar_extractElement (&hs, s) ;
printf ("current hyper_switch: %g\n", hs) ;
GrB_Scalar_setElement (s, 0.3) ;
GrB_set (GrB_GLOBAL, s, GxB_HYPER_SWITCH) ;
```

• void *: This type is used for all other cases. For GrB_get, the array must have the right size, just like a char * string. Use the same field first, but with size_t *value as the second parameter to obtain the size of the void * array, then use GrB_get with a void * array of the right size. In some cases, the size is always the same. For example, to query the operator of a monoid:

```
GrB_BinaryOp op ;
GrB_get (GrB_PLUS_MONOID_FP64, (void *) &op, GxB_MONOID_OPERATOR) ;
assert (op == GrB_PLUS_FP64) ;
```

For GrB_set, a fourth parameter is required to tell GraphBLAS the size of the input array.

10.1 Enum types for get/set

The get/set methods share a **int** (enum) type to specify which component of the object is to be set or retrieved.

```
typedef enum
   // GrB_Descriptor only:
                       // descriptor for output of a method
   GrB_OUTP_FIELD = 0,
   GrB_INPO_FIELD = 2, // descriptor for the first input of a method
   GrB_INP1_FIELD = 3,
                        // descriptor for the second input of a method
   // all objects, including GrB_GLOBAL:
   GrB_NAME = 10,
                         // name of the object, as a string
   // GrB_GLOBAL only:
   GrB_LIBRARY_VER_MAJOR = 11,
                                 // SuiteSparse:GraphBLAS version
   GrB_LIBRARY_VER_MINOR = 12,
   GrB_LIBRARY_VER_PATCH = 13,
   GrB_API_VER_MAJOR = 14,
                                 // C API version
   GrB_API_VER_MINOR = 15,
   GrB\_API\_VER\_PATCH = 16,
                                 // GrB_Mode
   GrB_BLOCKING_MODE = 17,
   // GrB_GLOBAL, GrB_Matrix, GrB_Vector, GrB_Scalar (and void * serialize?):
   GrB_STORAGE_ORIENTATION_HINT = 100, // GrB_Orientation
   // GrB_Matrix, GrB_Vector, GrB_Scalar (and void * serialize):
   GrB_EL_TYPE_CODE = 102,
                             // a GrB_Type_code (see below)
   GrB_EL_TYPE_STRING = 106,
                              // name of the type
   // GrB_*Op, GrB_Monoid, and GrB_Semiring:
   GrB_INP1_TYPE_CODE = 103,
                                // GrB_Type_code
   GrB_INP2_TYPE_CODE = 104,
   GrB_OUTP_TYPE_CODE = 105,
   GrB_INP1_TYPE_STRING = 107,
                                 // name of the type, as a string
   GrB_INP2_TYPE_STRING = 108,
   GrB_OUTP_TYPE_STRING = 109,
   // GrB_Type (readable only):
   GrB\_SIZE = 110,
                                  // size of the type
   // GrB_Type, GrB_UnaryOp, GrB_BinaryOp, GrB_IndexUnaryOp,
   // and GxB_IndexBinaryOp
   GxB_JIT_C_NAME = 7041,
                                 // C type or function name
```

```
GxB_JIT_C_DEFINITION = 7042,
                             // C typedef or function definition
// GrB_Monoid and GrB_Semiring:
GxB_MONOID_IDENTITY = 7043,
                            // monoid identity value
                            // monoid terminal value
GxB_MONOID_TERMINAL = 7044,
GxB_MONOID_OPERATOR = 7045,  // monoid binary operator
// GrB_Semiring only:
GxB\_SEMIRING\_MONOID = 7046,
                             // semiring monoid
GxB\_SEMIRING\_MULTIPLY = 7047,
                             // semiring multiplicative op
// GrB_BinaryOp and GxB_IndexBinaryOp:
GxB\_THETA\_TYPE\_CODE = 7050,
                            // for binary and index binary ops
GxB\_THETA\_TYPE\_STRING = 7051,
// GrB_BinaryOp or GrB_Semiring:
GxB_THETA = 7052,
                             // to get the value of theta
// GrB_get/GrB_set for GrB_Matrix, GrB_Vector, GrB_Scalr and GrB_GLOBAL:
GxB_ROWINDEX_INTEGER_HINT = 7053,  // hint for row indices
                                 // hint for column indices
GxB_COLINDEX_INTEGER_HINT = 7054,
GxB_OFFSET_INTEGER_HINT = 7056,
                                 // hint for offsets
// hyper_hash control (global int64 value,
GxB_HYPER_HASH = 7048,
                         // or bool per matrix)
GxB_BITMAP_SWITCH = 7001, // switch to bitmap (double value)
// GrB_get/GrB_set for GrB_Matrix, GrB_Vector, GrB_Scalar:
GxB ISO = 7079.
                  // get: returns the current iso status
                  // set true: make the matrix iso-valued, if possible.
                  // set false: make the matrix non-iso-valued.
GxB\_SPARSITY\_CONTROL = 7036,
                           // sparsity control: 0 to 15; see below
// GrB_get for GrB_Matrix, GrB_Vector, GrB_Scalar:
GxB_ROWINDEX_INTEGER_BITS = 7057,  // # bits for row indices
GxB_COLINDEX_INTEGER_BITS = 7058,  // # bits for column indices
GxB_SPARSITY_STATUS = 7034,  // hyper, sparse, bitmap or full (1,2,4,8)
GxB_IS_READONLY = 7078,
                            // true if it has any read-only components
GxB_WILL_WAIT = 7076,
                             // true if GrB_wait(A) will do anything
// GrB_get for GrB_GLOBAL:
GxB_LIBRARY_DATE = 7006, // date of the library (char *)
GxB_LIBRARY_ABOUT = 7007,
                            // about the library (char *)
                             // URL for the library (char *)
GxB_LIBRARY_URL = 7008,
```

```
GxB_LIBRARY_LICENSE = 7009,
                                     // license of the library (char *)
    GxB_LIBRARY_COMPILE_DATE = 7010, // date library was compiled (char *)
    GxB_LIBRARY_COMPILE_TIME = 7011, // time library was compiled (char *)
                                    // date of the API (char *)
    GxB\_API\_DATE = 7013,
                                    // about the API (char *)
    GxB\_API\_ABOUT = 7014,
                                    // URL for the API (char *)
    GxB\_API\_URL = 7015,
    GxB\_COMPILER\_VERSION = 7016,
                                    // compiler version (3 int's)
    GxB\_COMPILER\_NAME = 7017,
                                    // compiler name (char *)
    GxB_LIBRARY_OPENMP = 7018,
                                    // library compiled with OpenMP
    GxB_MALLOC_FUNCTION = 7037,
                                    // malloc function pointer
    GxB_CALLOC_FUNCTION = 7038,
                                    // calloc function pointer
    GxB_REALLOC_FUNCTION = 7039,
                                    // realloc function pointer
   GxB_FREE_FUNCTION = 7040,
                                     // free function pointer
    // GrB_get/GrB_set for GrB_GLOBAL:
    #define GxB_NTHREADS 7086
    #define GxB_CHUNK 7087
    GxB_GLOBAL_NTHREADS = GxB_NTHREADS, // max number of threads to use
    GxB_GLOBAL_CHUNK = GxB_CHUNK,
                                         // chunk size for small problems.
                                     // diagnostic output (bool *)
    GxB_BURBLE = 7019,
    GxB_PRINTF = 7020,
                                     // printf function diagnostic output
    GxB_FLUSH = 7021,
                                     // flush function diagnostic output
    GxB_PRINT_1BASED = 7023,
                                     // print matrices as 0-based or 1-based
    GxB_INCLUDE_READONLY_STATISTICS = 7077, // include read-only memory in
                                     // memory usage statistics
   GxB_JIT_C_COMPILER_NAME = 7024, // CPU JIT C compiler name
    GxB_JIT_C_COMPILER_FLAGS = 7025, // CPU JIT C compiler flags
   GxB_JIT_C_LINKER_FLAGS = 7026, // CPU JIT C linker flags
    GxB_JIT_C_LIBRARIES = 7027,
                                    // CPU JIT C libraries
    GxB_JIT_C_PREFACE = 7028,
                                    // CPU JIT C preface
    GxB_JIT_C_CONTROL = 7029,
                                    // CPU JIT C control
    GxB_JIT_CACHE_PATH = 7030,
                                    // CPU/CUDA JIT path for compiled kernels
    GxB_JIT_C_CMAKE_LIBS = 7031,
                                    // CPU JIT C libraries when using cmake
    GxB_JIT_USE_CMAKE = 7032,
                                    // CPU JIT: use cmake or direct compile
    GxB_JIT_ERROR_LOG = 7033,
                                    // CPU JIT: error log file
} GxB_Option_Field ;
typedef enum
   GrB_ROWMAJOR = 0,
   GrB\_COLMAJOR = 1,
   GrB_BOTH
                 = 2,
   GrB_UNKNOWN = 3,
```

}

```
GrB_Orientation ;
typedef enum
{
    GrB_UDT_CODE
                    = 0,
                                // user-defined type
                                // GraphBLAS: GrB_BOOL
                                                             C: bool
   GrB_BOOL_CODE
                    = 1,
   GrB_INT8_CODE
                    = 2,
                                // GraphBLAS: GrB_INT8
                                                             C: int8_t
   GrB_UINT8_CODE = 3,
                                // GraphBLAS: GrB_UINT8
                                                             C: uint8_t
   GrB_INT16_CODE
                   = 4,
                                // GraphBLAS: GrB_INT16
                                                             C: int16_t
   GrB_UINT16_CODE = 5,
                                // GraphBLAS: GrB_UINT16
                                                             C: uint16_t
   GrB_INT32_CODE = 6,
                                // GraphBLAS: GrB_INT32
                                                             C: int32_t
    GrB_UINT32_CODE = 7,
                                // GraphBLAS: GrB_UINT32
                                                             C: uint32_t
   GrB_INT64_CODE = 8,
                                // GraphBLAS: GrB_INT64
                                                             C: int64_t
   GrB_UINT64_CODE = 9,
                                // GraphBLAS: GrB_UINT64
                                                             C: uint64_t
   GrB_FP32_CODE
                   = 10,
                                // GraphBLAS: GrB_FP32
                                                             C: float
   GrB_FP64_CODE
                    = 11,
                                // GraphBLAS: GrB_FP64
                                                             C: double
   GxB_FC32_CODE
                    = 7070,
                                // GraphBLAS: GxB_FC32
                                                             C: float complex
    GxB_FC64_CODE
                    = 7071,
                                // GraphBLAS: GxB_FC64
                                                             C: double complex
GrB_Type_Code ;
```

10.2 Global Options (GrB_Global)

A single object GrB_GLOBAL whose type is GrB_Global is used to denote global settings to read or modify. To use it with GrB_get and GrB_set, pass in GrB_GLOBAL as the first parameter.

```
GrB_Info GrB_get (GrB_Global g, GrB_Scalar value, int f);
GrB_Info GrB_get (GrB_Global g, char * value, int f);
GrB_Info GrB_get (GrB_Global g, int32_t * value, int f);
GrB_Info GrB_get (GrB_Global g, size_t * value, int f);
GrB_Info GrB_get (GrB_Global g, void * value, int f);
GrB_Info GrB_set (GrB_Global g, GrB_Scalar value, int f);
GrB_Info GrB_set (GrB_Global g, char * value, int f);
GrB_Info GrB_set (GrB_Global g, int32_t value, int f);
GrB_Info GrB_set (GrB_Global g, void * value, int f);
```

| int field | R/W | C type | description |
|---------------------------------|-----|---------|--|
| GrB_LIBRARY_VER_MAJOR | R | int32_t | major version of the library |
| GrB_LIBRARY_VER_MINOR | R | int32_t | minor version of the library |
| GrB_LIBRARY_VER_PATCH | R | int32_t | patch version of the library |
| GrB_API_VER_MAJOR | R | int32_t | major version of the API |
| GrB_API_VER_MINOR | R | int32_t | major version of the API |
| GrB_API_VER_PATCH | R | int32_t | major version of the API |
| GrB_BLOCKING_MODE | R | int32_t | blocking mode (GrB_BLOCKING |
| | | | or GrB_NONBLOCKING) |
| GxB_LIBRARY_OPENMP | R | int32_t | if OpenMP is in use (true/false) |
| int field | R/W | C type | description |
| GrB_STORAGE_ORIENTATION_HINT | R/W | int32_t | see GrB_Orientation: default format |
| | | | for matrices. |
| GxB_NTHREADS | R/W | int32_t | number of OpenMP threads used. |
| | | | See Section 10.2.2. |
| GxB_BURBLE | R/W | int32_t | diagnostic output (true/false). |
| | | | See Section 10.2.1. |
| GxB_PRINT_1BASED | R/W | int32_t | matrices printed as 1-based or 0-based |
| GxB_INCLUDE_READONLY_STATISTICS | R/W | int32_t | include read-only memory in statistics |
| GxB_JIT_C_CONTROL | R/W | int32_t | see Section 9 |
| GxB_JIT_USE_CMAKE | R/W | int32_t | " |
| GxB_HYPER_SWITCH | R/W | double | global hypersparsity control. |
| | | | See Section 10.10.2. |
| GxB_HYPER_HASH | R/W | int64_t | global hypersparsity (hyper-hash) |
| | | | control |
| GxB_CHUNK | R/W | double | global chunk size for parallel task cre- |
| | | | ation. See Section 10.2.2. |
| GrB_NAME | R | char * | name of the library |
| | | | ("SuiteSparse:GraphBLAS") |
| GxB_LIBRARY_DATE | R | char * | library release date |
| GxB_LIBRARY_ABOUT | R | char * | details about the library |
| GxB_LIBRARY_LICENSE | R | char * | license of the library |
| GxB_LIBRARY_COMPILE_DATE | R | char * | date the library was compiled |
| GxB_LIBRARY_COMPILE_TIME | R | char * | time the library was compiled |
| GxB_LIBRARY_URL | R | char * | URL for the library |
| GxB_API_DATE | R | char * | C API release date |
| GxB_API_ABOUT | R | char * | about the C API |
| GxB_API_URL | R | char * | URL for the C API |
| GxB_COMPILER_NAME | R | char * | name of the compiler used to compile |
| | | | the library |

| int field | R/W | C type | description |
|--------------------------|-----|--------|---|
| GxB_JIT_C_COMPILER_NAME | R/W | char * | See Section 9 |
| GxB_JIT_C_COMPILER_FLAGS | R/W | char * | " |
| GxB_JIT_C_LINKER_FLAGS | R/W | char * | " |
| GxB_JIT_C_LIBRARIES | R/W | char * | " |
| GxB_JIT_C_CMAKE_LIBS | R/W | char * | " |
| GxB_JIT_C_PREFACE | R/W | char * | " |
| GxB_JIT_ERROR_LOG | R/W | char * | " |
| GxB_JIT_CACHE_PATH | R/W | char * | " |
| GxB_BITMAP_SWITCH | R/W | void * | double array of size |
| | | | GxB_NBITMAP_SWITCH. |
| | | | See Section 10.10.3. |
| GxB_COMPILER_VERSION | R | void * | int32_t array of size 3. The version |
| | | | of the compiler used to compile the li- |
| | | | brary. |
| GxB_PRINTF | W | void * | pointer to printf function for diag- |
| | | | nostic output. See Section 10.2.1. |
| GxB_FLUSH | W | void * | pointer to flush function for diagnos- |
| | | | tic output. See Section 10.2.1. |
| GxB_MALLOC_FUNCTION | R | void * | malloc function |
| GxB_CALLOC_FUNCTION | R | void * | calloc function |
| GxB_REALLOC_FUNCTION | R | void * | realloc function |
| GxB_FREE_FUNCTION | R | void * | free function |

10.2.1 Global diagnostic settings

GrB_set (GrB_GLOBAL, ..., GxB_BURBLE) controls the burble setting. It can also be controlled via GrB.burble(b) in the MATLAB/Octave interface.

```
GrB_set (GrB_GLOBAL, true, GxB_BURBLE) ; // enable burble
GrB_set (GrB_GLOBAL, false, GxB_BURBLE) ; // disable burble
```

If enabled, SuiteSparse:GraphBLAS reports which internal kernels it uses, and how much time is spent. If you see the word <code>generic</code>, it means that SuiteSparse:GraphBLAS was unable to use its JIT kernels, or its faster kernels in <code>Source/FactoryKernels</code>, but used a generic kernel that relies on function pointers. This is done for user-defined types and operators when they cannot be used in the JIT, and when typecasting is performed. Generic kernels are typically slower than the JIT kernels or kernels in <code>Source/FactoryKernels</code>.

If you see a lot of wait statements, it may mean that a lot of time is spent finishing a matrix or vector. This may be the result of an inefficient use of the setElement and assign methods. If this occurs you might try changing the sparsity format of a vector or matrix to GxB_BITMAP, assuming there's enough space for it.

The following setting allows the user application to change the function used to print diagnostic output:

```
GrB_set (GrB_GLOBAL, (void *) printf, GxB_PRINTF, sizeof (void *));
```

This also controls the output of the GxB_*print functions. By default this parameter is NULL, in which case the C11 printf function is used. The parameter is a function pointer with the same signature as the C11 printf function. The MATLAB/Octave interface to GraphBLAS sets it to mexPrintf so that GraphBLAS can print to the MATLAB/Octave Command Window.

After each call to the printf function, an optional flush function is called, which is NULL by default. If NULL, the function is not used. This can be changed with:

```
GrB_set (GrB_GLOBAL, (void *) flush, GxB_FLUSH, sizeof (void *));
```

The flush function takes no arguments, and returns an int which is 0 if successful, or any nonzero value on failure (the same output as the C11 fflush function, except that flush has no inputs).

10.2.2 OpenMP parallelism

SuiteSparse:GraphBLAS is a parallel library, based on OpenMP. By default, all GraphBLAS operations will use up to the maximum number of threads specified by the omp_get_max_threads OpenMP function. For small problems, GraphBLAS may choose to use fewer threads, using two parameters: the maximum number of threads to use (which may differ from the omp_get_max_threads value), and a parameter called the chunk. Suppose work is a measure of the work an operation needs to perform (say the number of entries in the two input matrices for GrB_eWiseAdd). No more than floor(work/chunk) threads will be used (or one thread if the ratio is less than 1).

GxB_NTHREADS controls how many threads a method uses. By default (if set to zero, or GrB_DEFAULT), all available threads are used. The maximum

available threads is controlled by the global setting, which is omp_get_max_threads () by default. If set to some positive integer nthreads less than this maximum, at most nthreads threads will be used.

GxB_CHUNK is a double value that controls how many threads a method uses for small problems. The default chunk value is 65,536, but this may change in future versions, or it may be modified when GraphBLAS is installed on a particular machine.

Both parameters can be set in two ways:

• Globally: If the following methods are used, then all subsequent Graph-BLAS operations will use these settings. Note the typecast, (double) chunk. This is necessary if a literal constant such as 20000 is passed as this argument. The type of the constant must be double.

```
int32_t nthreads_max = 40 ;
GrB_set (GrB_GLOBAL, nthreads_max, GxB_NTHREADS) ;
GrB_Scalar_new (&s, GrB_FP64) ;
GrB_Scalar_setElement (s, (double) 20000) ;
GrB_set (GrB_GLOBAL, s, GxB_CHUNK) ;
```

• Context: this object can be used to choose a different number of threads used in calls to GraphBLAS from different user threads, exploiting nested parallelism. Refer to Section 8. If a thread has engaged a context object, it ignores the global settings for GxB_NTHREADS and GxB_CHUNK, and uses the settings in its own context instead.

The smaller of nthreads_max and floor(work/chunk) is used for any given GraphBLAS operation, except that a single thread is used if this value is zero or less.

If either parameter is set to GrB_DEFAULT, then default values are used. The default for nthreads_max is the return value from omp_get_max_threads, and the default chunk size is currently 65,536.

If a descriptor value for either parameter is left at its default, or set to GrB_DEFAULT, then the global setting is used. This global setting may have been modified from its default, and this modified value will be used.

For example, suppose omp_get_max_threads reports 8 threads. If GrB_set (GrB_GLOBAL, 4, GxB_NTHREADS) is used, then the global setting is four threads, not eight.

GraphBLAS may be compiled without OpenMP, by setting <code>-DNOPENMP=1</code>. The library will be thread-safe, with one exception. <code>GrB_wait</code> is intended to provide thread-safety by flushing the cache of one user thread so the object can be safely read by another thread. This is accomplished with <code>pragma omp flush</code>, but if OpenMP is not available, this does nothing. If OpenMP is not available or <code>-DNOPEMP=1</code> is used, then user applications need to ensure their own thread safety when one user thread computes a result that is then read by another thread.

You can query GraphBLAS at run-time to ask if it was compiled with OpenMP:

```
bool have_openmp ;
GrB_get (GrB_GLOBAL, &have_openmp, GxB_LIBRARY_OPENMP) ;
if (!have_openmp) printf ("GraphBLAS not compiled with OpenMP\n") :
```

Compiling GraphBLAS without OpenMP is not recommended for installation in a package manager (Linux, conda-forge, spack, brew, vcpkg, etc).

10.2.3 Other global options

GrB_BLOCKING_MODE can only be queried by GrB_get; it cannot be modified by GrB_set. The mode is the value passed to GrB_init (blocking or non-blocking).

All threads in the same user application share the same global options, including hypersparsity, bitmap options, and CSR/CSC format determined by GrB_set, and the blocking mode determined by GrB_init. Specific format and hypersparsity parameters of each matrix are specific to that matrix and can be independently changed.

The Grb_Library_* and Gxb_Library_* options can be used to query the current implementation of SuiteSparse:GraphBLAS. The Grb_API_* and Gxb_API_* options can be used to query the current GraphBLAS C API Specification.

10.3 GrB_Type Options

```
GrB_Info GrB_get (GrB_Type t, GrB_Scalar value, int f) ;
GrB_Info GrB_get (GrB_Type t, char * value, int f) ;
GrB_Info GrB_get (GrB_Type t, int32_t * value, int f) ;
GrB_Info GrB_get (GrB_Type t, size_t * value, int f) ;
GrB_Info GrB_set (GrB_Type t, char * value, int f) ;
```

| int field | R/W | C type | description |
|----------------------|------|---------|--|
| GrB_EL_TYPE_CODE | R | int32_t | type code (see GrB_Type_Code) |
| GrB_SIZE | R | size_t | sizeof the type |
| GrB_NAME | R/W1 | char * | name of the type. For built-in types, this |
| | | | returns the GraphBLAS name ("GrB_FP32" |
| | | | for GrB_FP32, for example). For user- |
| | | | defined types, the name can be any string |
| | | | of any length. It is not used by the JIT. It |
| | | | can be set at most once. |
| GxB_JIT_C_NAME | R/W1 | char * | This must be a valid name of a C type |
| | | | to enable its use in the JIT. For built- |
| | | | in types, this returns the C name of the |
| | | | type ("float" for GrB_FP32, for example). |
| | | | The length of the name can be at most |
| | | | GxB_MAX_NAME_LEN, including the nul ter- |
| | | | minating byte. It can be set at most once. |
| GxB_JIT_C_DEFINITION | R/W1 | char * | type definition, as a C typedef; built-in |
| | | | types return an empty string. It can be set |
| | | | at most once. |

Built-in types cannot be modified by GrB_set. User-defined types can be used without setting their name or definition, but they can be used in JIT kernels only when both the JIT C name and the definition are set.

To use the JIT, all operators, monoids, and semirings that access this type must be defined after the user-defined type has been given both a name and a definition. GraphBLAS can use an operator that uses a type without a name, but it cannot use the JIT, even if the type is given a name later on after the operator is created.

The size of the type can be returned as a size_t C scalar, or as a GrB_Scalar, normally of type GrB_UINT64, with the examples below.

```
size_t size ;
```

```
GrB_get (GrB_FP32, &size, GrB_SIZE) ;
assert (size == sizeof (float)) ;

GrB_Scalar s ;
GrB_Scalar_new (&s, GrB_UINT64) ;
GrB_get (GrB_FP32, s, GrB_SIZE) ;
GrB_Scalar_extractElement (&size, s) ;
assert (size == sizeof (float)) ;
```

10.4 GrB_UnaryOp Options

```
GrB_Info GrB_get (GrB_UnaryOp op, GrB_Scalar value, int f);
GrB_Info GrB_get (GrB_UnaryOp op, char * value, int f);
GrB_Info GrB_get (GrB_UnaryOp op, int32_t * value, int f);
GrB_Info GrB_get (GrB_UnaryOp op, size_t * value, int f);
GrB_Info GrB_set (GrB_UnaryOp op, char * value, int f);
```

| int field | R/W | C type | description |
|----------------------|------|---------|---|
| GrB_INPO_TYPE_CODE | R | int32_t | input type code (see GrB_Type_code) |
| GrB_OUTP_TYPE_CODE | R | int32_t | output type code |
| GrB_INPO_TYPE_STRING | R | char * | name of the input type |
| GrB_OUTP_TYPE_STRING | R | char * | name of the output type |
| GrB_NAME | R/W1 | char * | name of the operator. For built-in oper- |
| | | | ators, this returns the GraphBLAS name |
| | | | ("GrB_LNOT" for GrB_LNOT, for example). |
| | | | For user-defined operators, the name can |
| | | | be any string of any length. It is not used |
| | | | by the JIT. It can be set at most once. |
| GxB_JIT_C_NAME | R/W1 | char * | This must be a valid name of a C |
| | | | function to enable its use in the JIT. |
| | | | The length of the name can be at most |
| | | | GxB_MAX_NAME_LEN, including the nul ter- |
| | | | minating byte. It can be set at most once. |
| GxB_JIT_C_DEFINITION | R/W1 | char * | definition for a user-defined operator, as |
| | | | a C function; built-in operators return an |
| | | | empty string. It can be set at most once. |

Built-in operators cannot be modified by GrB_set. User-defined operators can be used without setting their name or definition, but they can be used in JIT kernels only when both the JIT C name and the definition are set.

10.5 GrB_IndexUnaryOp Options

```
GrB_Info GrB_get (GrB_IndexUnaryOp op, GrB_Scalar value, int f);
GrB_Info GrB_get (GrB_IndexUnaryOp op, char * value, int f);
GrB_Info GrB_get (GrB_IndexUnaryOp op, int32_t * value, int f);
GrB_Info GrB_get (GrB_IndexUnaryOp op, size_t * value, int f);
GrB_Info GrB_set (GrB_IndexUnaryOp op, char * value, int f);
```

| int field | R/W | C type | description |
|----------------------|------|---------|---|
| GrB_INPO_TYPE_CODE | R | int32_t | 1st input type code (see GrB_Type_code) |
| GrB_INP1_TYPE_CODE | R | int32_t | 2nd input type code |
| GrB_OUTP_TYPE_CODE | R | int32_t | output type code |
| GrB_INPO_TYPE_STRING | R | char * | name of the 1st input type |
| GrB_INP1_TYPE_STRING | R | char * | name of the 2nd input type |
| GrB_OUTP_TYPE_STRING | R | char * | name of the output type |
| GrB_NAME | R/W1 | char * | name of the operator. For built-in oper- |
| | | | ators, this returns the GraphBLAS name |
| | | | ("GrB_TRIL" for GrB_TRIL, for example). |
| | | | For user-defined operators, the name can |
| | | | be any string of any length. It is not used |
| | | | by the JIT. It can be set at most once. |
| GxB_JIT_C_NAME | R/W1 | char * | This must be a valid name of a C |
| | | | function to enable its use in the JIT. |
| | | | The length of the name can be at most |
| | | | GxB_MAX_NAME_LEN, including the nul ter- |
| | | | minating byte. It can be set at most once. |
| GxB_JIT_C_DEFINITION | R/W1 | char * | definition for a user-defined operator, as |
| | | | a C function; built-in operators return an |
| | | | empty string. It can be set at most once. |

Built-in operators cannot be modified by GrB_set. User-defined operators can be used without setting their name or definition, but they can be used in JIT kernels only when both the JIT C name and the definition are set.

10.6 GrB_BinaryOp Options

```
GrB_Info GrB_get (GrB_BinaryOp op, GrB_Scalar value, int f);
GrB_Info GrB_get (GrB_BinaryOp op, char * value, int f);
GrB_Info GrB_get (GrB_BinaryOp op, int32_t * value, int f);
GrB_Info GrB_get (GrB_BinaryOp op, size_t * value, int f);
GrB_Info GrB_set (GrB_BinaryOp op, char * value, int f);
```

| int field | R/W | C type | description |
|-----------------------|------|------------|--|
| GrB_INPO_TYPE_CODE | R | int32_t | 1st input type code (see GrB_Type_code) |
| GrB_INP1_TYPE_CODE | R | int32_t | 2nd input type code |
| GrB_OUTP_TYPE_CODE | R | int32_t | output type code |
| GxB_THETA_TYPE_CODE | R | int32_t | Θ type code, if any |
| GrB_INPO_TYPE_STRING | R | char * | name of the 1st input type |
| GrB_INP1_TYPE_STRING | R | char * | name of the 2nd input type |
| GrB_OUTP_TYPE_STRING | R | char * | name of the output type |
| GxB_THETA_TYPE_STRING | R | char * | name of the Θ type, if any |
| GrB_NAME | R/W1 | char * | name of the operator. For built-in oper- |
| | | | ators, this returns the GraphBLAS name |
| | | | ("GrB_LOR" for GrB_LOR, for example). For |
| | | | user-defined operators, the name can be |
| | | | any string of any length. It is not used |
| | | | by the JIT. It can be set at most once. |
| GxB_JIT_C_NAME | R/W1 | char * | This must be a valid name of a C |
| | | | function to enable its use in the JIT. |
| | | | The length of the name can be at most |
| | | | GxB_MAX_NAME_LEN, including the nul ter- |
| | | | minating byte. It can be set at most once. |
| GxB_JIT_C_DEFINITION | R/W1 | char * | definition for a user-defined operator, as |
| | | | a C function; built-in operators return an |
| | | | empty string. It can be set at most once. |
| GxB_THETA | R | GrB_Scalar | value of Theta, if any. The type of the |
| | | | GrB_Scalar must match the Theta type of |
| | | | the underlying index-binary operator ex- |
| | | | actly. |

Built-in operators cannot be modified by GrB_set. User-defined operators can be used without setting their name or definition, but they can be used in JIT kernels only when both the JIT C name and the definition are set.

To use the JIT, all monoids and semirings that access this binary operator

must be defined after the user-defined operator has been given both a name and a definition. GraphBLAS can use a monoid or semiring that uses a binary operator without a name, but it cannot use the JIT, even if the operator is given a name later on after the operator is created.

The *THETA* options can only be used in the binary operator was created by GxB_BinaryOp_new_IndexOp.

10.7 GxB_IndexBinaryOp Options

```
GrB_Info GrB_get (GxB_IndexBinaryOp op, GrB_Scalar value, int f);
GrB_Info GrB_get (GxB_IndexBinaryOp op, char * value, int f);
GrB_Info GrB_get (GxB_IndexBinaryOp op, int32_t * value, int f);
GrB_Info GrB_get (GxB_IndexBinaryOp op, size_t * value, int f);
GrB_Info GrB_set (GxB_IndexBinaryOp op, char * value, int f);
```

| int field | R/W | C type | description |
|-----------------------|------|---------|--|
| GrB_INPO_TYPE_CODE | R | int32_t | 1st input type code (see GrB_Type_code) |
| GrB_INP1_TYPE_CODE | R | int32_t | 2nd input type code |
| GrB_OUTP_TYPE_CODE | R | int32_t | output type code |
| GxB_THETA_TYPE_CODE | R | int32_t | Θ type code |
| GrB_INPO_TYPE_STRING | R | char * | name of the 1st input type |
| GrB_INP1_TYPE_STRING | R | char * | name of the 2nd input type |
| GrB_OUTP_TYPE_STRING | R | char * | name of the output type |
| GxB_THETA_TYPE_STRING | R | char * | name of the Θ type |
| GrB_NAME | R/W1 | char * | name of the operator. For user-defined op- |
| | | | erators, the name can be any string of any |
| | | | length. It is not used by the JIT. It can be |
| | | | set at most once. |
| GxB_JIT_C_NAME | R/W1 | char * | This must be a valid name of a C |
| | | | function to enable its use in the JIT. |
| | | | The length of the name can be at most |
| | | | GxB_MAX_NAME_LEN, including the nul ter- |
| | | | minating byte. It can be set at most once. |
| GxB_JIT_C_DEFINITION | R/W1 | char * | definition for a user-defined operator, as |
| | | | a C function; built-in operators return an |
| | | | empty string. It can be set at most once. |

There are no built-in index-binary operators, but if there are in the future, they will not be modified by GrB_set. User-defined operators can be used without setting their name or definition, but they can be used in JIT kernels only when both the JIT C name and the definition are set.

To use the JIT, all semirings that access this index-binary operator must be defined after the user-defined operator has been given both a name and a definition. GraphBLAS can use a semiring that uses a binary operator without a name, but it cannot use the JIT, even if the operator is given a name later on after the operator is created.

10.8 GrB_Monoid Options

```
GrB_Info GrB_get (GrB_Monoid monoid, GrB_Scalar value, int f);
GrB_Info GrB_get (GrB_Monoid monoid, char * value, int f);
GrB_Info GrB_get (GrB_Monoid monoid, int32_t * value, int f);
GrB_Info GrB_get (GrB_Monoid monoid, size_t * value, int f);
GrB_Info GrB_get (GrB_Monoid monoid, void * value, int f);
GrB_Info GrB_set (GrB_Monoid monoid, char * value, int f);
```

| int field | R/W | C type | description |
|----------------------|------|------------|--|
| GrB_INPO_TYPE_CODE | R | int32_t | 1st input type code |
| | | | (see GrB_Type_code) |
| GrB_INP1_TYPE_CODE | R | int32_t | 2nd input type code |
| GrB_OUTP_TYPE_CODE | R | int32_t | output type code |
| GrB_INPO_TYPE_STRING | R | char * | name of the 1st input type |
| GrB_INP1_TYPE_STRING | R | char * | name of the 2nd input type |
| GrB_OUTP_TYPE_STRING | R | char * | name of the output type |
| GrB_NAME | R/W1 | char * | name of the monoid. For built-in monoids, this returns the GraphBLAS name ("GrB_LOR_MONOID_BOOL" for GrB_LOR_MONOID_BOOL, for example). For user-defined monoids, the name can be any string of any length. It is not used by the JIT. It can be set at most once. |
| GxB_MONOID_IDENTITY | R | GrB_Scalar | identity value of the monoid. The type of the GrB_Scalar must match the monoid type exactly. |
| GxB_MONOID_TERMINAL | R | GrB_Scalar | terminal value of a terminal monoid. The type of the GrB_Scalar must match the monoid type exactly. If the monoid is not terminal, the GrB_Scalar is returned with no entry. |
| GxB_MONOID_OPERATOR | R | void * | binary operator of the monoid, as a GrB_BinaryOp |

Built-in monoids cannot be modified by GrB_set.

For GxB_MONOID_OPERATOR, the op is returned as an alias, not as a new object. For example, if a monoid is created with a user-defined binary operator, the following usage returns a shallow copy of the operator:

```
GrB_BinaryOp binop ;
GrB_BinaryOp_new (&binop, func, GrB_BOOL, GrB_BOOL, GrB_BOOL) ;
GrB_Monoid monoid ;
GrB_Monoid_new (&monoid, binop, (bool) false) ;
```

With the above objects defined, the following two code snippets do the same thing:

```
// getting an alias to the binary operator directly:
GrB_BinaryOp op ;
op = binop ;

// getting an alias to the binary operator using GrB_get:
GrB_BinaryOp op ;
GrB_get (monoid, (void *) &op, GxB_MONOID_OPERATOR) ;
assert (op == binop) ;
```

As a result, it is not valid to free both the op and the binop, since they are the same object. This usage returns the built-in GrB_LOR operator of the corresponding built-in monoid:

```
GrB_BinaryOp op ;
GrB_get (GrB_LOR_MONOID, (void *) &op, GxB_MONOID_OPERATOR) ;
assert (op == GrB_LOR) ;
```

10.9 GrB_Semiring Options

```
GrB_Info GrB_get (GrB_Semiring semiring, GrB_Scalar value, int f);
GrB_Info GrB_get (GrB_Semiring semiring, char * value, int f);
GrB_Info GrB_get (GrB_Semiring semiring, int32_t * value, int f);
GrB_Info GrB_get (GrB_Semiring semiring, size_t * value, int f);
GrB_Info GrB_get (GrB_Semiring semiring, void * value, int f);
GrB_Info GrB_set (GrB_Semiring semiring, GrB_Scalar value, int f);
```

| int field | R/W | C type | description |
|-----------------------|------|------------|--|
| GrB_INPO_TYPE_CODE | R | int32_t | 1st input type code (see GrB_Type_code) |
| | | | of the multiplicative operator |
| GrB_INP1_TYPE_CODE | R | int32_t | 2nd input type code of the multiplicative |
| | | | operator |
| GrB_OUTP_TYPE_CODE | R | int32_t | output type code of the multiplicative op- |
| | | | erator, and the monoid type. |
| GxB_THETA_TYPE_CODE | R | int32_t | Θ type code, if any |
| GrB_INPO_TYPE_STRING | R | char * | name of the 1st input type of the multi- |
| | | | plicative operator |
| GrB_INP1_TYPE_STRING | R | char * | name of the 2nd input type of the multi- |
| | | | plicative operator |
| GrB_OUTP_TYPE_STRING | R | char * | name of the output type of the multiplica- |
| | | | tive operator, and the monoid type. |
| GxB_THETA_TYPE_STRING | R | char * | name of the Θ type, if any |
| GrB_NAME | R/W1 | char * | name of the semiring. For built-in |
| | | | semirings, this returns the GraphBLAS |
| | | | name ("GrB_LOR_LAND_SEMIRING_BOOL" |
| | | | for GrB_LOR_LAND_SEMIRING_BOOL, for |
| | | | example). For user-defined semirings, the |
| | | | name can be any string of any length. It |
| | | | is not used by the JIT. It can be set at |
| | | | most once. |
| GxB_THETA | R | GrB_Scalar | value of Theta, if any. The type of the |
| | | | GrB_Scalar must match the Theta type of |
| | | | the underlying index-binary operator ex- |
| | | | actly. |

| int field | R/W | C type | description |
|-----------------------|-----|------------|--|
| GxB_MONOID_IDENTITY | R | GrB_Scalar | identity value of the monoid. The type |
| | | | of the GrB_Scalar must match the |
| | | | monoid type exactly. |
| GxB_MONOID_TERMINAL | R | GrB_Scalar | terminal value of a terminal monoid. |
| | | | The type of the GrB_Scalar must |
| | | | match the monoid type exactly. If |
| | | | the monoid is not terminal, the |
| | | | GrB_Scalar is returned with no entry. |
| GxB_MONOID_OPERATOR | R | void * | binary operator of the monoid, as a |
| | | | GrB_BinaryOp; See Section 10.8 |
| GxB_SEMIRING_MONOID | R | void * | monoid of the semiring, as a |
| | | | GrB_Monoid |
| GxB_SEMIRING_MULTIPLY | R | void * | multiplicative operator of the semir- |
| | | | ing, as a GrB_BinaryOp |

Built-in semirings cannot be modified by GrB_set.

The GxB_SEMIRING_MONOID option returns the GrB_Monoid of the semiring. The GxB_SEMIRING_MULTIPLY option returns the GrB_BinaryOp for the multiplicative operator of the semiring. For example:

```
// getting an alias to the monoid and multiply operator using GrB_get:
GrB_BinaryOp op ;
GrB_Monoid mon ;
GrB_Semiring semiring = GrB_PLUS_TIMES_FP32 ;
GrB_get (semiring, (void *) &mon, GxB_SEMIRING_MONOID) ;
GrB_get (semiring, (void *) &op, GxB_SEMIRING_MULTIPLY) ;
assert (op == GrB_TIMES_FP32) ;
assert (mon == GrB_PLUS_MONOID_FP32) ;
```

The binary op and monoid returned are aliases, not new objects.

The *THETA* options can only be used in the multiplicative binary operator of the semiring was created by GxB_BinaryOp_new_IndexOp.

10.10 GrB_Matrix Options

```
GrB_Info GrB_get (GrB_Matrix A, GrB_Scalar value, int f);
GrB_Info GrB_get (GrB_Matrix A, char * value, int f);
GrB_Info GrB_get (GrB_Matrix A, int32_t * value, int f);
GrB_Info GrB_get (GrB_Matrix A, size_t * value, int f);

GrB_Info GrB_set (GrB_Matrix A, GrB_Scalar value, int f);
GrB_Info GrB_set (GrB_Matrix A, char * value, int f);
GrB_Info GrB_set (GrB_Matrix A, int32_t value, int f);
```

| int field | R/W | C type | description |
|------------------------------|-----|---------|---------------------------------------|
| GrB_STORAGE_ORIENTATION_HINT | R/W | int32_t | See GrB_Orientation, |
| | | | and Section 10.10.1. |
| GrB_EL_TYPE_CODE | R | int32_t | matrix type |
| GxB_SPARSITY_CONTROL | R/W | int32_t | See Section 10.10.4 |
| GxB_SPARSITY_STATUS | R | int32_t | See Section 10.10.4 |
| GxB_IS_READONLY | R | int32_t | true if it has any read-only com- |
| | | | ponents |
| GxB_WILL_WAIT | R | int32_t | will GrB_wait do anything (Sec- |
| | | | tion 10.10.6) |
| GxB_ISO | R/W | int32_t | iso status (Section 10.10.5) |
| GxB_ROWINDEX_INTEGER_BITS | R | int32_t | number of bits for row indices (32 |
| | | | or 64) |
| GxB_COLINDEX_INTEGER_BITS | R | int32_t | number of bits for column indices |
| | | | (32 or 64) |
| GxB_OFFSET_INTEGER_BITS | R | int32_t | number of bits for offsets (32 or |
| | | | 64) |
| GxB_ROWINDEX_INTEGER_HINT | R/W | int32_t | hint for row indices (0, 32, 64) |
| GxB_COLINDEX_INTEGER_HINT | R/W | int32_t | hint for column indices $(0, 32, 64)$ |
| GxB_OFFSET_INTEGER_HINT | R/W | int32_t | hint for offsets (0, 32, 64) |
| GrB_NAME | R/W | char * | name of the matrix. This can be |
| | | | set any number of times. |
| GrB_EL_TYPE_STRING | R | char * | name of the type of the matrix. |
| GxB_JIT_C_NAME | R | char * | JIT C name of the type of the ma- |
| | | | trix. |
| GxB_HYPER_SWITCH | R/W | double | See Section 10.10.2 |
| GxB_BITMAP_SWITCH | R/W | double | See Section 10.10.3 |

10.10.1 Storing a matrix by row or by column

The GraphBLAS GrB_Matrix is entirely opaque to the user application, and the GraphBLAS API does not specify how the matrix should be stored. However, choices made in how the matrix is represented in a particular implementation, such as SuiteSparse:GraphBLAS, can have a large impact on performance.

Many graph algorithms are just as fast in any format, but some algorithms are much faster in one format or the other. For example, suppose the user application stores a directed graph as a matrix A, with the edge (i, j) represented as the value A(i,j), and the application makes many accesses to the *i*th row of the matrix, with $GrB_Col_extract(w, ..., A, GrB_ALL, ..., i, desc)$ with the transposed descriptor (GrB_INPO set to GrB_TRAN). If the matrix is stored by column this can be extremely slow, just like the expression w=A(i,:) in MATLAB, where i is a scalar. Since this is a typical usecase in graph algorithms, the default format in SuiteSparse:GraphBLAS is to store its matrices by row, in Compressed Sparse Row format (CSR).

MATLAB stores its sparse matrices by column, in "non-hypersparse" format, in what is called the Compressed Sparse Column format, or CSC for short. An m-by-n matrix in MATLAB is represented as a set of n column vectors, each with a sorted list of row indices and values of the nonzero entries in that column. As a result, w=A(:,j) is very fast in MATLAB, since the result is already held in the data structure a single list, the jth column vector. However, w=A(i,:) is very slow in MATLAB, since every column in the matrix has to be searched to see if it contains row i. In MATLAB, if many such accesses are made, it is much better to transpose the matrix (say AT=A') and then use w=AT(:,i) instead. This can have a dramatic impact on the performance of MATLAB.

Likewise, if u is a very sparse column vector and A is stored by column, then $w=u^*A$ (via GrB_vxm) is slower than w=A*u (via GrB_mxv). The opposite is true if the matrix is stored by row.

SuiteSparse:GraphBLAS stores its matrices by row, by default (with one exception described below). However, it can also be instructed to store any selected matrices, or all matrices, by column instead (just like MATLAB), so that w=A(:,j) (via GrB_Col_extract) is very fast. The change in data format has no effect on the result, just the time and memory usage. To use a column-oriented format by default, the following can be done in a user application that tends to access its matrices by column.

```
GrB_init (...);
// just after GrB_init: do the following:
GrB_set (GrB_GLOBAL, GrB_COLMAJOR, GrB_STORAGE_ORIENTATION_HINT);
```

If this is done, and no other <code>GrB_set</code> calls are made with <code>GrB_STORAGE_ORIENATION_HINT</code>, all matrices will be stored by column. The default format is <code>GrB_ROWMAJOR</code>.

All vectors (GrB_Vector) are held by column, and this cannot be changed. By default, matrices of size m-by-1 are held by column, regardless of the global setting described above. Matrices of size 1-by-n with n not equal to 1 are held by row, regardless of the global setting. The global setting only affects matrices with both m > 1 and n > 1. Empty matrices (0-by-0) are also controlled by the global setting.

After creating a matrix with GrB_Matrix_new (&A, ...), its format can be changed arbitrarily with:

```
GrB_set (A, GrB_COLMAJOR, GrB_STORAGE_ORIENTATION_HINT) ;
GrB_set (A, GrB_ROWMAJOR, GrB_STORAGE_ORIENTATION_HINT) ;
```

If set to other values (GrB_BOTH or GrB_UNKNOWN), the format is changed to GrB_ROWMAJOR.

With this setting, even an m-by-1 matrix can then be changed to be held by row, for example. Likewise, once a 1-by-n matrix is created, it can be converted to column-oriented format.

10.10.2 Hypersparse matrices

MATLAB can store an m-by-n matrix with a very large value of m, since a CSC data structure takes $O(n+|\mathbf{A}|)$ memory, independent of m, where $|\mathbf{A}|$ is the number of nonzeros in the matrix. It cannot store a matrix with a huge n, and this structure is also inefficient when $|\mathbf{A}|$ is much smaller than n. In contrast, SuiteSparse:GraphBLAS can store its matrices in *hypersparse* format, taking only $O(|\mathbf{A}|)$ memory, independent of how it is stored (by row or by column) and independent of both m and n [BG08, BG12].

In both the CSR and CSC formats, the matrix is held as a set of sparse vectors. In non-hypersparse format, the set of sparse vectors is itself dense; all vectors are present, even if they are empty. For example, an m-by-n matrix in non-hypersparse CSC format contains n sparse vectors. Each column vector takes at least one integer to represent, even for a column with no entries. This

allows for quick lookup for a particular vector, but the memory required is $O(n+|\mathbf{A}|)$. With a hypersparse CSC format, the set of vectors itself is sparse, and columns with no entries take no memory at all. The drawback of the hypersparse format is that finding an arbitrary column vector \mathbf{j} , such as for the computation $\mathbf{C}=\mathbf{A}(:,\mathbf{j})$, takes $O(\log k)$ time if there $k\leq n$ vectors in the data structure. One advantage of the hypersparse structure is the memory required for an m-by-n hypersparse CSC matrix is only $O(|\mathbf{A}|)$, independent of m and n. Algorithms that must visit all non-empty columns of a matrix are much faster when working with hypersparse matrices, since empty columns can be skipped.

The hyper_switch parameter controls the hypersparsity of the internal data structure for a matrix. The parameter is typically in the range 0 to 1. The default is hyper_switch = GxB_HYPER_DEFAULT, which is an extern const double value, currently set to 0.0625, or 1/16. This default ratio may change in the future.

The hyper_switch determines how the matrix is converted between the hypersparse and non-hypersparse formats. Let n be the number of columns of a CSC matrix, or the number of rows of a CSR matrix. The matrix can have at most n non-empty vectors.

Let k be the actual number of non-empty vectors. That is, for the CSC format, $k \leq n$ is the number of columns that have at least one entry. Let h be the value of hyper_switch.

If a matrix is currently hypersparse, it can be converted to non-hypersparse if the either condition $n \leq 1$ or k > 2nh holds, or both. Otherwise, it stays hypersparse. Note that if $n \leq 1$ the matrix is always stored as non-hypersparse.

If currently non-hypersparse, it can be converted to hypersparse if both conditions n>1 and $k\leq nh$ hold. Otherwise, it stays non-hypersparse. Note that if $n\leq 1$ the matrix always remains non-hypersparse.

The default value of hyper_switch is assigned at startup by GrB_init, and can then be modified globally with GrB_set. All new matrices are created with the same hyper_switch, determined by the global value. Once a particular matrix A has been constructed, its hypersparsity ratio can be modified from the default with:

```
double hyper_switch = 0.2 ;
GrB_set (A, hyper_switch, GxB_HYPER_SWITCH) ;
```

To force a matrix to always be non-hypersparse, use hyper_switch equal

to GxB_NEVER_HYPER. To force a matrix to always stay hypersparse, set hyper_switch to GxB_ALWAYS_HYPER.

A GrB_Matrix can thus be held in one of four formats: any combination of hyper/non-hyper and CSR/CSC. All GrB_Vector objects are always stored in non-hypersparse CSC format.

A new matrix created via $\mathtt{GrB_Matrix_new}$ starts with k=0 and is created in hypersparse form by default unless $n\leq 1$ or if h<0, where h is the global hyper_switch value. The matrix is created in either $\mathtt{GrB_ROWMAJOR}$ or $\mathtt{GrB_COLMAJOR}$ format, as determined by the last call to $\mathtt{GrB_set}(\mathtt{GrB_GLOBAL},\ldots,\mathtt{GrB_STORAGE_ORIENTATION_HINT,\ldots})$ or $\mathtt{GrB_init}$.

A new matrix C created via GrB_dup (&C,A) inherits the CSR/CSC format, hypersparsity format, and hyper_switch from A.

10.10.3 Bitmap matrices

By default, SuiteSparse:GraphBLAS switches between all four formats (hypersparse, sparse, bitmap, and full) automatically. Let $d = |\mathbf{A}|/mn$ for an m-by-n matrix \mathbf{A} with $|\mathbf{A}|$ entries. If the matrix is currently in sparse or hypersparse format, and is modified so that d exceeds a given threshold, it is converted into bitmap format. The default threshold is controlled by the GxB_BITMAP_SWITCH setting, which can be set globally, or for a particular matrix or vector.

The default value of the switch to bitmap format depends on $\min(m, n)$, for a matrix of size m-by-n. For the global setting, the bitmap switch is a double array of size $\texttt{GxB_NBITMAP_SWITCH}$. The defaults are given below:

| parameter | default | matrix sizes |
|-------------------|---------|------------------------------------|
| bitmap_switch [0] | 0.04 | $\min(m, n) = 1$ (and all vectors) |
| bitmap_switch [1] | 0.05 | $\min(m,n)=2$ |
| bitmap_switch [2] | 0.06 | $\min(m,n) = 3 \text{ to } 4$ |
| bitmap_switch [3] | 0.08 | $\min(m,n) = 5 \text{ to } 8$ |
| bitmap_switch [4] | 0.10 | $\min(m,n) = 9 \text{ to } 16$ |
| bitmap_switch [5] | 0.20 | $\min(m,n) = 17 \text{ to } 32$ |
| bitmap_switch [6] | 0.30 | $\min(m,n) = 33 \text{ to } 64$ |
| bitmap_switch [7] | 0.40 | $\min(m, n) > 64$ |

That is, by default a GrB_Vector is held in bitmap format if its density exceeds 4%. To change the global settings, do the following:

If the matrix is currently in bitmap format, it is converted to full if all entries are present, or to sparse/hypersparse if d drops below b/2, if its bitmap switch is b. A matrix or vector with d between b/2 and b remains in its current format.

10.10.4 Sparsity status

The sparsity status of a matrix can be queried with the following, which returns a value of GxB_HYPERSPARSE (1), GxB_SPARSE (2), GxB_BITMAP (4), or GxB_FULL (8).

```
int32_t sparsity ;
GrB_get (A, &sparsity, GxB_SPARSITY_STATUS) ;
```

The sparsity format of a matrix can be controlled with the field set to GxB_SPARSITY_CONTROL, for which the value can be any mix (a sum or bitwise or) of GxB_HYPERSPARSE, GxB_SPARSE, GxB_BITMAP, and GxB_FULL. By default, a matrix or vector can be held in any format, with the default setting GxB_AUTO_SPARSITY, which is equal to GxB_HYPERSPARSE + GxB_SPARSE + GxB_BITMAP + GxB_FULL (15). To enable a matrix to take on just GxB_SPARSE or GxB_FULL formats, but not GxB_HYPERSPARSE or GxB_BITMAP, for example, use the following:

```
GrB_set (A, GxB_SPARSE + GxB_FULL, GxB_SPARSITY_CONTROL) ;
```

In this case, SuiteSparse:GraphBLAS will hold the matrix in sparse format (CSR or CSC, depending on its Grb_STORAGE_ORIENTATION_HINT), unless all entries are present, in which case it will be converted to full format.

Only the least significant 4 bits of the sparsity control are considered, so the formats can be bitwise negated. For example, to allow for any format except full:

```
GrB_set (A, ~GxB_FULL, GxB_SPARSITY_CONTROL) ;
```

10.10.5 iso status

The GxB_ISO option allows the iso status of a matrix or vector to be queried. The option can be set, as well. If set true, this asks GraphBLAS to attempt to revise the storage of the matrix to make it iso-valued. GraphBLAS will check all the values, and if they are all the same, the matrix is converted to an iso-valued format. If set false, the matrix is revised so that it is held in a non-iso format, if it is stored in iso-valued form.

10.10.6 wait status

The GxB_WILL_WAIT option can be queried with GrB_get to determine if a call to GrB_wait on the matrix, vector, or scalar will do any work.

10.11 GrB_Vector Options

```
GrB_Info GrB_get (GrB_Vector v, GrB_Scalar value, int f);
GrB_Info GrB_get (GrB_Vector v, char * value, int f);
GrB_Info GrB_get (GrB_Vector v, int32_t * value, int f);
GrB_Info GrB_get (GrB_Vector v, size_t * value, int f);

GrB_Info GrB_set (GrB_Vector v, GrB_Scalar value, int f);
GrB_Info GrB_set (GrB_Vector v, char * value, int f);
GrB_Info GrB_set (GrB_Vector v, int32_t value, int f);
```

| int field | R/W | C type | description |
|---------------------------|-----|---------|--|
| GrB_EL_TYPE_CODE | R | int32_t | vector type |
| GxB_SPARSITY_CONTROL | R/W | int32_t | See Section 10.10.4 |
| GxB_SPARSITY_STATUS | R | int32_t | See Section 10.10.4 |
| GxB_IS_READONLY | R | int32_t | true if it has any read-only components |
| GxB_WILL_WAIT | R | int32_t | will GrB_wait do anything (Section 10.10.6) |
| GxB_ISO | R/W | int32_t | iso status (Section 10.10.5) |
| GxB_ROWINDEX_INTEGER_BITS | R | int32_t | number of bits for row indices (32 or 64) |
| GxB_COLINDEX_INTEGER_BITS | R | int32_t | number of bits for column indices (32 or 64) |
| GxB_OFFSET_INTEGER_BITS | R | int32_t | number of bits for offsets (32 or 64) |
| GxB_ROWINDEX_INTEGER_HINT | R/W | int32_t | hint for row indices (0, 32, 64) |
| GxB_COLINDEX_INTEGER_HINT | R/W | int32_t | hint for column indices (0, 32, 64) |
| GxB_OFFSET_INTEGER_HINT | R/W | int32_t | hint for offsets (0, 32, 64) |
| GrB_NAME | R/W | char * | name of the vector. |
| GrB_EL_TYPE_STRING | R | char * | name of the type of the vector. |
| GxB_JIT_C_NAME | R | char * | JIT C name of the type of the vector. |
| GxB_BITMAP_SWITCH | R/W | double | See Section 10.10.3 |

See Section 10.10; a GrB_Vector is treated as if it were an n-by-1 matrix, and is always in column major form. It is never hypersparse.

10.12 GrB_Scalar Options

```
GrB_Info GrB_get (GrB_Scalar s, GrB_Scalar value, int f);
GrB_Info GrB_get (GrB_Scalar s, char * value, int f);
GrB_Info GrB_get (GrB_Scalar s, int32_t * value, int f);
GrB_Info GrB_get (GrB_Scalar s, size_t * value, int f);
GrB_Info GrB_set (GrB_Scalar s, GrB_Scalar value, int f);
GrB_Info GrB_set (GrB_Scalar s, char * value, int f);
GrB_Info GrB_set (GrB_Scalar s, int32_t value, int f);
```

| int field | R/W | C type | description |
|---------------------|-----|---------|---|
| GrB_EL_TYPE_CODE | R | int32_t | scalar type |
| GxB_SPARSITY_STATUS | R | int32_t | See Section 10.10.4 |
| GxB_IS_READONLY | R | int32_t | true if it has any read-only components |
| GxB_WILL_WAIT | R | int32_t | will GrB_wait do anything (Section 10.10.6) |
| GxB_ISO | R/W | int32_t | iso status (Section 10.10.5) |
| GrB_NAME | R/W | char * | name of the scalar. |
| GrB_EL_TYPE_STRING | R | char * | name of the type of the scalar. |
| GxB_JIT_C_NAME | R | char * | JIT C name of the type of the scalar. |

See Section 10.10; a GrB_Scalar is treated as if it were a 1-by-1 matrix, and is always in column major form. It is never hypersparse.

10.13 Controlling the sizes of integers

Different integers are used for different parts of the matrix/vector/scalar data structure. The decision as to which integers to use is determined by the dimensions and number of entries in the matrix. The decisions can also be modified by GrB_set and queried by GrB_get. A matrix can have up to three different kinds of integers. If a matrix is m-by-n with e entries, with default settings:

- if $m > 2^{31}$: 64-bit integers must be used for the row indices of a matrix; otherwise, 32-bit integers may be used.
- if $n > 2^{31}$: 64-bit integers must be used for the column indices of a matrix; otherwise, 32-bit integers may be used.
- if $e > 2^{32}$: 64-bit integers must be used for the row/column offsets of a matrix; otherwise 32-bit integers may be used.

This gives up to 8 different matrix types when the matrix is hypersparse. Sparse matrices use just two of these integer types (a sparse CSR matrix, held by row, does not store any row indices). example, if a matrix held in sparse CSR format (GxB_SPARSE), of size m-by-n with e entries, then the value of m does not affect the integers used to store the matrix. Bitmap and full matrices use no arrays of integers at all, and are not affected by these settings. For

These decisions can be revised on a global basis and a per matrix/vector basis. Three fields can be used for GrB_get and GrB_get, for the GrB_GLOBAL, GrB_Matrix, GrB_Vector, and GrB_Scalar objects:

- GxB_ROWINDEX_INTEGER_HINT: for the global setting, this can be set to 32 (the default) or 64. The default (32) means that 32-bit integers are used if the number of rows of a matrix or length of a vector is not too large (> 2³¹), and 64-bit integers are used otherwise. On a per matrix/vector basis, setting this hint changes the integers used for row indices in the vector or matrix. The setting of zero is the default for individual matrices/vectors, which means the global setting is used.
- GxB_COLINDEX_INTEGER_HINT: the same as the row index hint above, except for column indices. This setting has little effect on a GrB_Vector, since it is held internally by GraphBLAS as an *m*-by-1 matrix.
- GxB_OFFSET_INTEGER_HINT: Internally, a sparse or hypersparse matrix or vector holds an offset array with the cumulative sum of the number of entries in each row (if held by row) or column (if held by column). A global setting of 32 is the default, so that 32-bit integers are used if the matrix has fewer than 2³² entries. If this setting is changed to 64, then 64-bit integers are always used.

Once an object is created, the sizes of its three integers can be queried by GrB_get with the following fields. Each query returns the result of 32 or 64, as an integer, to denote if 32-bit or 64-bit integers are used for that component of the matrix/vector. Bitmap and full matrices always return 64.

- GxB_ROWINDEX_INTEGER_BITS: query the number of bits in the row index integer
- GxB_COLINDEX_INTEGER_BITS: query the number of bits in the column index integer
- GxB_OFFSET_INTEGER_BITS: query the number of bits in the offset integer

10.14 GrB_Descriptor Options

```
GrB_Info GrB_get (GrB_Descriptor desc, GrB_Scalar value, int f);
GrB_Info GrB_get (GrB_Descriptor desc, char * value, int f);
GrB_Info GrB_get (GrB_Descriptor desc, int32_t * value, int f);
GrB_Info GrB_get (GrB_Descriptor desc, size_t * value, int f);
GrB_Info GrB_set (GrB_Descriptor desc, GrB_Scalar value, int f);
GrB_Info GrB_set (GrB_Descriptor desc, char * value, int f);
GrB_Info GrB_set (GrB_Descriptor desc, int32_t value, int f);
```

| int field | R/W | C type | description | |
|-------------------|-----|---------|---|--|
| GrB_OUTP | R/W | int32_t | GrB_DEFAULT or GrB_REPLACE | |
| GrB_MASK | R/W | int32_t | GrB_DEFAULT, GrB_COMP, GrB_STRUCTURE, or | |
| | | | GrB_COMP_STRUCTURE | |
| GrB_INPO | R/W | int32_t | GrB_DEFAULT or GrB_TRAN | |
| GrB_INP1 | R/W | int32_t | GrB_DEFAULT or GrB_TRAN | |
| GxB_AxB_METHOD | R/W | int32_t | Method used by GrB_mxm (GrB_DEFAULT, | |
| | | | GxB_AxB_GUSTAVSON. GxB_AxB_HASH, | |
| | | | GxB_AxB_SAXPY, or GxB_AxB_DOT). | |
| GxB_SORT | R/W | int32_t | if true, GrB_mxm returns its output in sorted form. | |
| GxB_COMPRESSION | R/W | int32_t | compression method for serialize methods. | |
| GxB_ROWINDEX_LIST | R/W | int32_t | how the GrB_Vector I is intrepretted. | |
| GxB_COLINDEX_LIST | R/W | int32_t | how the GrB_Vector J is intrepretted. | |
| GxB_VALUE_LIST | R/W | int32_t | how GrB_Vector X is intrepretted (for GrB_build | |
| | | | only). | |
| GrB_NAME | R/W | char * | name of the descriptor. This can be set any num- | |
| | | | ber of times for user-defined descriptors. Built-in | |
| | | | descriptors have the same name as the variable | |
| | | | name ("GrB_DESC_T1" for the GrB_DESC_T1 de- | |
| | | | scriptor, for example) | |

The following table describes each option. See Section 6.16 for more details.

| Descriptor | Default | Non-default |
|-------------------|--|--|
| field | | |
| GrB_OUTP | GrB_DEFAULT: The output matrix is not cleared. The operation computes $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}.$ | GrB_REPLACE: After computing $\mathbf{Z} = \mathbf{C} \odot \mathbf{T}$, the output \mathbf{C} is cleared of all entries. Then $\mathbf{C} \langle \mathbf{M} \rangle = \mathbf{Z}$ is performed. |
| GrB_MASK | GrB_DEFAULT: The Mask is not complemented. Mask(i,j)=1 means the value C_{ij} can be modified by the operation, while Mask(i,j)=0 means the value C_{ij} shall not be modified by the operation. | GrB_COMP: The Mask is complemented. Mask(i,j)=0 means the value C_{ij} can be modified by the operation, while Mask(i,j)=1 means the value C_{ij} shall not be modified by the operation. GrB_STRUCTURE: The values of the Mask are ignored. If Mask(i,j) is an entry in the Mask matrix, it is treated as if Mask(i,j)=1. The two options GrB_COMP and GrB_STRUCTURE can be combined, with two subsequent calls, or with a single call with the setting GrB_COMP+GrB_STRUCTURE. |
| GrB_INPO | GrB_DEFAULT: The first input is not transposed prior to using it in the operation. | GrB_TRAN: The first input is transposed prior to using it in the operation. Only matrices are transposed, never vectors. |
| GrB_INP1 | GrB_DEFAULT: The second input is not transposed prior to using it in the operation. | GrB_TRAN: The second input is transposed prior to using it in the operation. Only matrices are transposed, never vectors. |
| GxB_AxB_METHOD | GrB_DEFAULT: The method for C=A*B is selected automatically. | GxB_AxB_method: The selected method is used to compute C=A*B. |
| GxB_SORT | GrB_DEFAULT: The computation of C=A*B may leave C in a jumbled state; GrB_wait will finalize the matrix. | any nonzero value: C=A*B always returns C in final, sorted form. |
| GxB_COMPRESSION | GrB_DEFAULT: Serialize methods will use the default method, ZSTD (level 1) | See Section 6.11 |
| GxB_ROWINDEX_LIST | GrB_DEFAULT: use the values of I | GxB_USE_INDICES: use the indices of I; GxB_IS_STRIDE: I is a strided range (lo:inc:hi) |
| GxB_COLINDEX_LIST | GrB_DEFAULT: use the values of J | GxB_USE_INDICES: use the indices of J; GxB_IS_STRIDE: J is a strided range (lo:inc:hi) |
| GxB_VALUE_LIST | GrB_DEFAULT: use the values of X | GxB_USE_INDICES: use the indices |

10.15 GxB_Context Options

```
GrB_Info GrB_get (GxB_Context Context, GrB_Scalar value, int f);
GrB_Info GrB_get (GxB_Context Context, char * value, int f);
GrB_Info GrB_get (GxB_Context Context, int32_t * value, int f);
GrB_Info GrB_get (GxB_Context Context, size_t * value, int f);
GrB_Info GrB_set (GxB_Context Context, GrB_Scalar value, int f);
GrB_Info GrB_set (GxB_Context Context, char * value, int f);
GrB_Info GrB_set (GxB_Context Context, int32_t value, int f);
```

| int field | R/W | C type | description | |
|--------------|-----|---------|---|--|
| GxB_NTHREADS | R/W | int32_t | number of OpenMP threads to use; See Section 10.2.2 | |
| GxB_CHUNK | R/W | double | chunk factor for task creation; See Section 10.2.2 | |
| GrB_NAME | R/W | char * | name of the context. This can be set any num- | |
| | | | ber of times for user-defined contexts. Built-in con- | |
| | | | texts have the same name as the variable name | |
| | | | ("GxB_CONTEXT_WORLD" for the GxB_CONTEXT_WORLD | |
| | | | context, for example) | |

NOTE: the non-polymorphic names for this method are GxB_Context_get_[KIND] and GxB_Context_set_[KIND], where KIND can be: Scalar (for GrB_Scalar), String (for char *), INT (for int32_t), and SIZE (for size_t * for GrB_get only), and VOID (for void *). The non-polymorphic suffix of INT is used here instead of INT32 because GxB_Context_*_INT32 appear as historical methods in version v8.0 and earlier, which are now deprecated.

For the int32_t type, the use of the polymorphic GrB_set and GrB_get accesses the correct version of this method. When using non-polymorphic methods, the use of GxB_Context_get_INT and GxB_Context_set_INT is recommended.

10.16 Options for inspecting a serialized blob

```
GrB_Info GrB_get (const void *blob, GrB_Scalar value, int f, size_t blobsize);
GrB_Info GrB_get (const void *blob, char * value, int f, size_t blobsize);
GrB_Info GrB_get (const void *blob, int32_t * value, int f, size_t blobsize);
GrB_Info GrB_get (const void *blob, size_t * value, int f, size_t blobsize);
```

| int field | R/W | C type | description |
|------------------------------|-----|---------|-----------------------------------|
| GrB_STORAGE_ORIENTATION_HINT | R | int32_t | See GrB_Orientation, |
| | | | and Section 10.10.1. |
| GrB_EL_TYPE_CODE | R | int32_t | type of matrix in the blob |
| GxB_SPARSITY_CONTROL | R | int32_t | See Section 10.10.4 |
| GxB_SPARSITY_STATUS | R | int32_t | See Section 10.10.4 |
| GrB_NAME | R | char * | name of the matrix in the blob. |
| GrB_EL_TYPE_STRING | R | char * | name of the type of the matrix in |
| | | | the blob. |
| GxB_JIT_C_NAME | R | char * | JIT C name of the type of the ma- |
| | | | trix in the blob. |
| GxB_HYPER_SWITCH | R | double | See Section 10.10.2 |
| GxB_BITMAP_SWITCH | R | double | See Section 10.10.3 |

The GrB_Matrix_serialize and GxB_Matrix_serialize methods create a *blob* as a single array of bytes that contains all content of a GrB_Matrix. These GrB_get methods can query a blob for the same values that can be queried for a GrB_Matrix. The blob cannot be modified by GrB_set.

Note that these <code>GrB_get</code> methods add a fourth parameter, the size of the blob. All other <code>GrB_get</code> methods have just three parameters: the object, the value, and the field.

11 SuiteSparse:GraphBLAS Colon and Index Notation

MATLAB/Octave uses a colon notation to index into matrices, such as C=A(2:4,3:8), which extracts C as 3-by-6 submatrix from A, from rows 2 through 4 and columns 3 to 8 of the matrix A. A single colon is used to denote all rows, C=A(:,9), or all columns, C=A(12,:), which refers to the 9th column and 12th row of A, respectively. An arbitrary integer list can be given as well, such as the MATLAB/Octave statements:

```
I = [2 1 4];

J = [3 5];

C = A (I,J);
```

which creates the 3-by-2 matrix C as follows:

$$C = \left[\begin{array}{cc} a_{2,3} & a_{2,5} \\ a_{1,3} & a_{1,5} \\ a_{4,3} & a_{4,5} \end{array} \right]$$

The GraphBLAS API can do the equivalent of C=A(I,J), C=A(:,J), C=A(I,:), and C=A(:,:), by passing a parameter const GrB_Index *I as either an array of size ni, or as the special value GrB_ALL, which corresponds to the stand-alone colon C=A(:,J), and the same can be done for J.. To compute C=A(2:4,3:8) in GraphBLAS requires the user application to create two explicit integer arrays I and J of size 3 and 5, respectively, and then fill them with the explicit values [2,3,4] and [3,4,5,6,7,8]. This works well if the lists are small, or if the matrix has more entries than rows or columns.

However, particularly with hypersparse matrices, the size of the explicit arrays I and J can vastly exceed the number of entries in the matrix. When using its hypersparse format, SuiteSparse:GraphBLAS allows the user application to create a GrB_Matrix with dimensions up to 2^{60} , with no memory constraints. The only constraint on memory usage in a hypersparse matrix is the number of entries in the matrix.

For example, creating a n-by-n matrix A of type GrB_FP64 with $n = 2^{60}$ and one million entries is trivial to do in Version 2.1 (and later) of Suite-Sparse:GraphBLAS, taking at most 24MB of space. SuiteSparse:GraphBLAS

Version 2.1 (or later) could do this on an old smartphone. However, using just the pure GraphBLAS API, constructing C=A(0:(n/2),0:(n/2)) in SuiteSparse Version 2.0 would require the creation of an integer array I of size 2⁵⁹, containing the sequence 0, 1, 2, 3,, requiring about 4 ExaBytes of memory (4 million terabytes). This is roughly 1000 times larger than the memory size of the world's largest computer in 2018.

SuiteSparse:GraphBLAS Version 2.1 and later extends the GraphBLAS API with a full implementation of the MATLAB colon notation for integers, I=begin:inc:end. This extension allows the construction of the matrix C=A(0:(n/2),0:(n/2)) in this example, with dimension 2⁵⁹, probably taking just milliseconds on an old smartphone.

The GrB_extract, GrB_assign, and GxB_subassign operations (described in the Section 12) each have parameters that define a list of integer indices, using two parameters:

```
const GrB\_Index *I ; // an array, or a special value GrB\_ALL GrB\_Index ni ; // the size of I, or a special value
```

These two parameters define five kinds of index lists, which can be used to specify either an explicit or implicit list of row indices and/or column indices. The length of the list of indices is denoted |I|. This discussion applies equally to the row indices I and the column indices J. The five kinds are listed below.

- 1. An explicit list of indices, such as I = [2 1 4 7 2] in MATLAB notation, is handled by passing in I as a pointer to an array of size 5, and passing ni=5 as the size of the list. The length of the explicit list is ni=|I|. Duplicates may appear, except that for some uses of GrB_assign and GxB_subassign, duplicates lead to undefined behavior according to the GraphBLAS C API Specification. Suite-Sparse:GraphBLAS specifies how duplicates are handled in all cases, as an addition to the specification. See Section 12.10 for details.
- 2. To specify all rows of a matrix, use I = GrB_ALL. The parameter ni is ignored. This is equivalent to C=A(:,J) in MATLAB. In GraphBLAS, this is the sequence 0:(m-1) if A has m rows, with length |I|=m. If J is used the columns of an m-by-n matrix, then J=GrB_ALL refers to all columns, and is the sequence 0:(n-1), of length |J|=n.

SPEC: If I or J are GrB_ALL, the specification requires that ni be passed in as m (the number of rows) and nj be passed in as n. Any other value is an error. SuiteSparse:GraphBLAS ignores these scalar inputs and treats them as if they are equal to their only possible correct value.

3. To specify a contiguous range of indices, such as I=10:20 in MATLAB, the array I has size 2, and ni is passed to SuiteSparse:GraphBLAS as the special value ni = GxB_RANGE. The beginning index is I[GxB_BEGIN] and the ending index is I[GxB_END]. Both values must be non-negative since GrB_Index is an unsigned integer (uint64_t). The value of I[GxB_INC] is ignored.

Let $b = I[GxB_BEGIN]$, let $e = I[GxB_END]$, The sequence has length zero if b > e; otherwise the length is |I| = (e - b) + 1.

4. To specify a strided range of indices with a non-negative stride, such as I=3:2:10, the array I has size 3, and ni has the special value GxB_STRIDE. This is the sequence 3, 5, 7, 9, of length 4. Note that 10 does not appear in the list. The end point need not appear if the increment goes past it.

The GxB_STRIDE sequence is the same as the List generated by the following for loop:

```
int64_t k = 0 ;
GrB_Index *List = (a pointer to an array of large enough size)
for (int64_t i = I [GxB_BEGIN] ; i <= I [GxB_END] ; i += I [GxB_INC])
{
    // i is the kth entry in the sequence
    List [k++] = i ;
}</pre>
```

Then passing the explicit array List and its length ni=k has the same effect as passing in the array I of size 3, with ni=GxB_STRIDE. The latter is simply much faster to produce, and much more efficient for SuiteSparse:GraphBLAS to process.

Let $b = I[GxB_BEGIN]$, let $e = I[GxB_END]$, and let $\Delta = I[GxB_INC]$. The sequence has length zero if b > e or $\Delta = 0$. Otherwise, the length of the sequence is

$$|I| = \left\lfloor \frac{e - b}{\Delta} \right\rfloor + 1$$

5. In MATLAB notation, if the stride is negative, the sequence is decreasing. For example, 10:-2:1 is the sequence 10, 8, 6, 4, 2, in that order. In SuiteSparse:GraphBLAS, use ni = GxB_BACKWARDS, with an array I of size 3. The following example specifies defines the equivalent of the MATLAB expression 10:-2:1 in SuiteSparse:GraphBLAS:

The value -2 cannot be assigned to the GrB_Index array I, since that is an unsigned type. The signed increment is represented instead with the special value ni = GxB_BACKWARDS. The GxB_BACKWARDS sequence is the same as generated by the following for loop:

```
int64_t k = 0 ;
GrB_Index *List = (a pointer to an array of large enough size)
for (int64_t i = I [GxB_BEGIN] ; i >= I [GxB_END] ; i -= I [GxB_INC])
{
    // i is the kth entry in the sequence
    List [k++] = i ;
}
```

Let $b = I[GxB_BEGIN]$, let $e = I[GxB_END]$, and let $\Delta = I[GxB_INC]$ (note that Δ is not negative). The sequence has length zero if b < e or $\Delta = 0$. Otherwise, the length of the sequence is

$$|I| = \left\lfloor \frac{b-e}{\Lambda} \right\rfloor + 1$$

Since GrB_Index is an unsigned integer, all three values I[GxB_BEGIN], I[GxB_INC], and I[GxB_END] must be non-negative.

Just as in MATLAB, it is valid to specify an empty sequence of length zero. For example, I = 5:3 has length zero in MATLAB and the same is true for a GxB_RANGE sequence in SuiteSparse:GraphBLAS, with $I[GxB_BEGIN]=5$ and $I[GxB_END]=3$. This has the same effect as array I with ni=0.

12 GraphBLAS Operations

The next sections define each of the GraphBLAS operations, also listed in the table below.

| GrB_mxm | matrix-matrix multiply | $\mathbf{C}\langle\mathbf{M} angle=\mathbf{C}\odot\mathbf{AB}$ |
|-----------------------|---|---|
| <pre>GrB_vxm</pre> | vector-matrix multiply | $\mathbf{w}^{T} \langle \mathbf{m}^{T} \rangle = \mathbf{w}^{T} \odot \mathbf{u}^{T} \mathbf{A}$ |
| <pre>GrB_mxv</pre> | matrix-vector multiply | $\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot\mathbf{A}\mathbf{u}$ |
| GrB_eWiseMult | element-wise, | $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot (\mathbf{A} \otimes \mathbf{B})$ |
| | set intersection | $\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot(\mathbf{u}\otimes\mathbf{v})$ |
| GrB_eWiseAdd | element-wise, | $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot (\mathbf{A} \oplus \mathbf{B})$ |
| | set union | $\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot(\mathbf{u}\oplus\mathbf{v})$ |
| GxB_eWiseUnion | element-wise, | $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot (\mathbf{A} \oplus \mathbf{B})$ |
| | set union | $\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot(\mathbf{u}\oplus\mathbf{v})$ |
| GrB_extract | extract submatrix | $\mathbf{C}\langle\mathbf{M} angle = \mathbf{C}\odot\mathbf{A}(\mathbf{I},\mathbf{J})$ |
| | | $\mathbf{w}\langle\mathbf{m}\rangle=\mathbf{w}\odot\mathbf{u}(\mathbf{i})$ |
| GrB_assign | assign submatrix | $\mathbf{C}\langle\mathbf{M} angle(\mathbf{I},\mathbf{J})=\mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$ |
| | with submask for ${\bf C}$ | $\mathbf{w} \langle \mathbf{m} \rangle (\mathbf{i}) = \mathbf{w}(\mathbf{i}) \odot \mathbf{u}$ |
| GxB_subassign | assign submatrix, | $\mathbf{C}(\mathbf{I},\mathbf{J})\langle\mathbf{M} angle = \mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$ |
| | with submask for $\mathbf{C}(\mathbf{I}, \mathbf{J})$ | $\mathbf{w}(\mathbf{i})\langle\mathbf{m} angle=\mathbf{w}(\mathbf{i})\odot\mathbf{u}$ |
| <pre>GrB_apply</pre> | apply unary operator | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A})$ |
| | | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u})$ |
| | apply binary operator | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(x,\mathbf{A})$ |
| | | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A}, y)$ |
| | | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(x, \mathbf{x})$ |
| | | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u}, y)$ |
| | apply index-unary op | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot f(\mathbf{A},i,j,k)$ |
| | | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot f(\mathbf{u}, i, 0, k)$ |
| <pre>GrB_select</pre> | select entries | $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C} \odot \operatorname{select}(\mathbf{A}, i, j, k)$ |
| | | $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w} \odot \operatorname{select}(\mathbf{u}, i, 0, k)$ |
| <pre>GrB_reduce</pre> | reduce to vector | $\mathbf{w}\langle\mathbf{m} angle=\mathbf{w}\odot[\oplus_{j}\mathbf{A}(:,j)]$ |
| | reduce to scalar | $s = s \odot [\oplus_{ij} \mathbf{A}(I,J)]$ |
| GrB_transpose | transpose | $\mathbf{C}\langle\mathbf{M} angle = \mathbf{C}\odot\mathbf{A}^T$ |
| GrB_kronecker | Kronecker product | $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathrm{kron}(\mathbf{A},\mathbf{B})$ |
| | | |

If an error occurs, GrB_error(&err,C) or GrB_error(&err,w) returns details about the error, for operations that return a modified matrix C or vector w. The only operation that cannot return an error string is reduction to a C scalar with GrB_reduce.

12.1 GrB_mxm: matrix-matrix multiply

```
GrB_Info GrB_mxm
                                    // C<Mask> = accum (C, A*B)
                                    // input/output matrix for results
    GrB_Matrix C,
   const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
    const GrB_Semiring semiring,
                                    // defines '+' and '*' for A*B
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Matrix B,
                                    // second input: matrix B
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, A, and B
);
```

GrB_mxm multiplies two sparse matrices A and B using the semiring. The input matrices A and B may be transposed according to the descriptor, desc (which may be NULL) and then typecasted to match the multiply operator of the semiring. Next, T=A*B is computed on the semiring, precisely defined in the GB_spec_mxm.m script in GraphBLAS/Test. The actual algorithm exploits sparsity and does not take $O(n^3)$ time, but it computes the following:

```
[m s] = size (A.matrix) ;
[s n] = size (B.matrix) ;
T.matrix = zeros (m, n, multiply.ztype) ;
T.pattern = zeros (m, n, 'logical') ;
T.matrix (:,:) = identity ;
                                        % the identity of the semiring's monoid
T.class = multiply.ztype ;
                                        % the ztype of the semiring's multiply op
A = cast (A.matrix, multiply.xtype);
                                       % the xtype of the semiring's multiply op
                                        % the ytype of the semiring's multiply op
B = cast (B.matrix, multiply.ytype);
for j = 1:n
   for i = 1:m
       for k = 1:s
            % T (i,j) += A (i,k) * B (k,j), using the semiring
            if (A.pattern (i,k) && B.pattern (k,j))
                z = multiply (A (i,k), B (k,j));
                T.matrix (i,j) = add (T.matrix (i,j), z);
                T.pattern(i,j) = true;
            end
        end
    end
end
```

Finally, T is typecasted into the type of C, and the results are written back into C via the accum and Mask, $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}$. The latter step is reflected in the MATLAB function $\mathtt{GB_spec_accum_mask.m}$, discussed in Section 2.3.

Performance considerations: Suppose all matrices are in GrB_COLMAJOR format, and B is extremely sparse but A is not as sparse. Then computing C=A*B is very fast, and much faster than when A is extremely sparse. For example, if A is square and B is a column vector that is all nonzero except for one entry B(j,0)=1, then C=A*B is the same as extracting column A(:,j). This is very fast if A is stored by column but slow if A is stored by row. If A is a sparse row with a single entry A(0,i)=1, then C=A*B is the same as extracting row B(i,:). This is fast if B is stored by row but slow if B is stored by column.

If the user application needs to repeatedly extract rows and columns from a matrix, whether by matrix multiplication or by <code>GrB_extract</code>, then keep two copies: one stored by row, and other by column, and use the copy that results in the fastest computation.

By default, GrB_mxm, GrB_mxv, GrB_vxm, and GrB_reduce (to vector) can return their result in a jumbled state, with the sort left pending. It can sometimes be faster for these methods to do the sort as they compute their result. Use the GxB_SORT descriptor setting to select this option. Refer to Section 6.16 for details.

12.2 GrB_vxm: vector-matrix multiply

```
GrB_Info GrB_vxm
                                    // w'<mask> = accum (w, u'*A)
   GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w.t)
                                    // defines '+' and '*' for u'*A
    const GrB_Semiring semiring,
    const GrB_Vector u,
                                    // first input: vector u
    const GrB_Matrix A,
                                    // second input: matrix A
    const GrB_Descriptor desc
                                    // descriptor for w, mask, and A
);
```

GrB_vxm multiplies a row vector u' times a matrix A. The matrix A may be first transposed according to desc (as the second input, GrB_INP1); the column vector u is never transposed via the descriptor. The inputs u and A are typecasted to match the xtype and ytype inputs, respectively, of the multiply operator of the semiring. Next, an intermediate column vector t=A'*u is computed on the semiring using the same method as GrB_mxm. Finally, the column vector t is typecasted from the ztype of the multiply operator of the semiring into the type of w, and the results are written back into w using the optional accumulator accum and mask.

The last step is $\mathbf{w}\langle \mathbf{m} \rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

Performance considerations: If the Grb_Storage_Orientation_Hint of A is Grb_Rowmajor, and the default descriptor is used (A is not transposed), then Grb_vxm is faster than than Grb_mxv with its default descriptor, when the vector u is very sparse. However, if the Grb_Storage_Orientation_Hint of A is Grb_Colmajor, then Grb_mxv with its default descriptor is faster than Grb_vxm with its default descriptor, when the vector u is very sparse. Using the non-default Grb_Tran descriptor for A makes the Grb_vxm operation equivalent to Grb_mxv with its default descriptor (with the operands reversed in the multiplier, as well). The reverse is true as well; Grb_mxv with Grb_Tran is the same as Grb_vxm with a default descriptor.

12.3 GrB_mxv: matrix-vector multiply

```
GrB_Info GrB_mxv
                                    // w<mask> = accum (w, A*u)
   GrB_Vector w,
                                    // input/output vector for results
   const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
                                    // defines '+' and '*' for A*B
    const GrB_Semiring semiring,
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Vector u,
                                    // second input: vector u
    const GrB_Descriptor desc
                                    // descriptor for w, mask, and A
);
```

GrB_mxv multiplies a matrix A times a column vector u. The matrix A may be first transposed according to desc (as the first input); the column vector u is never transposed via the descriptor. The inputs A and u are typecasted to match the xtype and ytype inputs, respectively, of the multiply operator of the semiring. Next, an intermediate column vector t=A*u is computed on the semiring using the same method as GrB_mxm. Finally, the column vector t is typecasted from the ztype of the multiply operator of the semiring into the type of w, and the results are written back into w using the optional accumulator accum and mask.

The last step is $\mathbf{w}\langle \mathbf{m} \rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

Performance considerations: Refer to the discussion of GrB_vxm. In SuiteSparse:GraphBLAS, GrB_mxv is very efficient when u is sparse or dense, when the default descriptor is used, and when the matrix is GrB_COLMAJOR. When u is very sparse and GrB_INPO is set to its non-default GrB_TRAN, then this method is not efficient if the matrix is in GrB_COLMAJOR format. If an application needs to perform A'*u repeatedly where u is very sparse, then use the GrB_ROWMAJOR format for A instead.

12.4 GrB_eWiseMult: element-wise operations, set intersection

Element-wise "multiplication" is shorthand for applying a binary operator element-wise on two matrices or vectors A and B, for all entries that appear in the set intersection of the patterns of A and B. This is like A.*B for two sparse matrices in MATLAB, except that in GraphBLAS any binary operator can be used, not just multiplication.

The pattern of the result of the element-wise "multiplication" is exactly this set intersection. Entries in A but not B, or visa versa, do not appear in the result.

Let \otimes denote the binary operator to be used. The computation $\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$ is given below. Entries not in the intersection of \mathbf{A} and \mathbf{B} do not appear in the pattern of \mathbf{T} . That is:

for all entries
$$(i, j)$$
 in $\mathbf{A} \cap \mathbf{B}$
 $t_{ij} = a_{ij} \otimes b_{ij}$

Depending on what kind of operator is used and what the implicit value is assumed to be, this can give the Hadamard product. This is the case for A.*B in MATLAB since the implicit value is zero. However, computing a Hadamard product is not necessarily the goal of the eWiseMult operation. It simply applies any binary operator, built-in or user-defined, to the set intersection of A and B, and discards any entry outside this intersection. Its usefulness in a user's application does not depend upon it computing a Hadamard product in all cases. The operator need not be associative, commutative, nor have any particular property except for type compatibility with A and B, and the output matrix C.

The generic name for this operation is **GrB_eWiseMult**, which can be used for both matrices and vectors.

12.4.1 GrB_Vector_eWiseMult: element-wise vector multiply

```
GrB_Info GrB_eWiseMult
                                    // w<mask> = accum (w, u.*v)
    GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const <operator> multiply,
                                    // defines '.*' for t=u.*v
    const GrB_Vector u,
                                    // first input: vector u
    const GrB_Vector v,
                                    // second input: vector v
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB_Vector_eWiseMult computes the element-wise "multiplication" of two vectors \mathbf{u} and \mathbf{v} , element-wise using any binary operator (not just times). The vectors are not transposed via the descriptor. The vectors \mathbf{u} and \mathbf{v} are first typecasted into the first and second inputs of the multiply operator. Next, a column vector \mathbf{t} is computed, denoted $\mathbf{t} = \mathbf{u} \otimes \mathbf{v}$. The pattern of \mathbf{t} is the set intersection of \mathbf{u} and \mathbf{v} . The result \mathbf{t} has the type of the output ztype of the multiply operator.

The operator is typically a GrB_BinaryOp, but the method is type-generic for this parameter. If given a monoid (GrB_Monoid), the additive operator of the monoid is used as the multiply binary operator. If given a semiring (GrB_Semiring), the multiply operator of the semiring is used as the multiply binary operator. The multiply operator may be a binary operator created by GxB_BinaryOp_new_IndexOp.

The next and final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot\mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices. Note for all GraphBLAS operations, including this one, the accumulator $\mathbf{w}\odot\mathbf{t}$ is always applied in a set union manner, even though $\mathbf{t} = \mathbf{u}\otimes\mathbf{v}$ for this operation is applied in a set intersection manner.

12.4.2 GrB_Matrix_eWiseMult: element-wise matrix multiply

```
GrB_Info GrB_eWiseMult
                                    // C<Mask> = accum (C, A.*B)
    GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C, unused if NULL
    const GrB_Matrix Mask,
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
    const <operator> multiply,
                                    // defines '.*' for T=A.*B
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Matrix B,
                                    // second input: matrix B
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, A, and B
);
```

GrB_Matrix_eWiseMult computes the element-wise "multiplication" of two matrices A and B, element-wise using any binary operator (not just times). The input matrices may be transposed first, according to the descriptor desc. They are then typecasted into the first and second inputs of the multiply operator. Next, a matrix T is computed, denoted $T = A \otimes B$. The pattern of T is the set intersection of A and B. The result T has the type of the output ztype of the multiply operator.

The multiply operator is typically a GrB_BinaryOp, but the method is type-generic for this parameter. If given a monoid (GrB_Monoid), the additive operator of the monoid is used as the multiply binary operator. If given a semiring (GrB_Semiring), the multiply operator of the semiring is used as the multiply binary operator. The multiply operator may be a binary operator created by GxB_BinaryOp_new_IndexOp.

The operation can be expressed in MATLAB notation as:

```
[nrows, ncols] = size (A.matrix);
T.matrix = zeros (nrows, ncols, multiply.ztype);
T.class = multiply.ztype;
p = A.pattern & B.pattern;
A = cast (A.matrix (p), multiply.xtype);
B = cast (B.matrix (p), multiply.ytype);
T.matrix (p) = multiply (A, B);
T.pattern = p;
```

The final step is $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot \mathbf{T}$, as described in Section 2.3. Note for all GraphBLAS operations, including this one, the accumulator $\mathbf{C} \odot \mathbf{T}$ is always applied in a set union manner, even though $\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$ for this operation is applied in a set intersection manner.

12.5 GrB_eWiseAdd: element-wise operations, set union

Element-wise "addition" is shorthand for applying a binary operator element-wise on two matrices or vectors A and B, for all entries that appear in the set intersection of the patterns of A and B. This is like A+B for two sparse matrices in MATLAB, except that in GraphBLAS any binary operator can be used, not just addition. The pattern of the result of the element-wise "addition" is the set union of the pattern of A and B. Entries in neither in A nor in B do not appear in the result.

Let \oplus denote the binary operator to be used. The computation $\mathbf{T} = \mathbf{A} \oplus \mathbf{B}$ is exactly the same as the computation with accumulator operator as described in Section 2.3. It acts like a sparse matrix addition, except that any operator can be used. The pattern of $\mathbf{A} \oplus \mathbf{B}$ is the set union of the patterns of \mathbf{A} and \mathbf{B} , and the operator is applied only on the set intersection of \mathbf{A} and \mathbf{B} . Entries not in either the pattern of \mathbf{A} or \mathbf{B} do not appear in the pattern of \mathbf{T} . That is:

```
for all entries (i, j) in \mathbf{A} \cap \mathbf{B}

t_{ij} = a_{ij} \oplus b_{ij}

for all entries (i, j) in \mathbf{A} \setminus \mathbf{B}

t_{ij} = a_{ij}

for all entries (i, j) in \mathbf{B} \setminus \mathbf{A}

t_{ij} = b_{ij}
```

The only difference between element-wise "multiplication" ($\mathbf{T} = \mathbf{A} \otimes \mathbf{B}$) and "addition" ($\mathbf{T} = \mathbf{A} \oplus \mathbf{B}$) is the pattern of the result, and what happens to entries outside the intersection. With \otimes the pattern of \mathbf{T} is the intersection; with \oplus it is the set union. Entries outside the set intersection are dropped for \otimes , and kept for \oplus ; in both cases the operator is only applied to those (and only those) entries in the intersection. Any binary operator can be used interchangeably for either operation.

Element-wise operations do not operate on the implicit values, even implicitly, since the operations make no assumption about the semiring. As a result, the results can be different from MATLAB, which can always assume the implicit value is zero. For example, C=A-B is the conventional matrix subtraction in MATLAB. Computing A-B in GraphBLAS with eWiseAdd will apply the MINUS operator to the intersection, entries in A but not B will be unchanged and appear in C, and entries in neither A nor B do not appear in C. For these cases, the results matches the MATLAB C=A-B. Entries in B but not A do appear in C but they are not negated; they cannot be subtracted

from an implicit value in A. This is by design. If conventional matrix subtraction of two sparse matrices is required, and the implicit value is known to be zero, use GrB_apply to negate the values in B, and then use eWiseAdd with the PLUS operator, to compute A+(-B).

The generic name for this operation is GrB_eWiseAdd, which can be used for both matrices and vectors.

There is another minor difference in two variants of the element-wise functions. If given a semiring, the eWiseAdd functions use the binary operator of the semiring's monoid, while the eWiseMult functions use the multiplicative operator of the semiring.

12.5.1 GrB_Vector_eWiseAdd: element-wise vector addition

```
// w<mask> = accum (w, u+v)
GrB_Info GrB_eWiseAdd
    GrB_Vector w,
                                    // input/output vector for results
   const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
                                    // optional accum for z=accum(w.t)
    const GrB_BinaryOp accum,
    const <operator> add,
                                    // defines '+' for t=u+v
    const GrB_Vector u,
                                    // first input: vector u
    const GrB_Vector v,
                                    // second input: vector v
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB_Vector_eWiseAdd computes the element-wise "addition" of two vectors u and v, element-wise using any binary operator (not just plus). The vectors are not transposed via the descriptor. Entries in the intersection of u and v are first typecasted into the first and second inputs of the add operator. Next, a column vector t is computed, denoted $t = u \oplus v$. The pattern of t is the set union of u and v. The result t has the type of the output ztype of the add operator.

The add operator is typically a GrB_BinaryOp, but the method is type-generic for this parameter. If given a monoid (GrB_Monoid), the additive operator of the monoid is used as the add binary operator. If given a semiring (GrB_Semiring), the additive operator of the monoid of the semiring is used as the add binary operator. The add operator may be a binary operator created by GxB_BinaryOp_new_IndexOp.

The final step is $\mathbf{w}\langle \mathbf{m} \rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

12.5.2 GrB_Matrix_eWiseAdd: element-wise matrix addition

```
GrB_Info GrB_eWiseAdd
                                    // C<Mask> = accum (C, A+B)
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
                                    // defines '+' for T=A+B
    const <operator> add,
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Matrix B,
                                    // second input: matrix B
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, A, and B
);
```

GrB_Matrix_eWiseAdd computes the element-wise "addition" of two matrices A and B, element-wise using any binary operator (not just plus). The input matrices may be transposed first, according to the descriptor desc. Entries in the intersection then typecasted into the first and second inputs of the add operator. Next, a matrix T is computed, denoted $\mathbf{T} = \mathbf{A} \oplus \mathbf{B}$. The pattern of T is the set union of A and B. The result T has the type of the output ztype of the add operator.

The add operator is typically a GrB_BinaryOp, but the method is type-generic for this parameter. If given a monoid (GrB_Monoid), the additive operator of the monoid is used as the add binary operator. If given a semiring (GrB_Semiring), the additive operator of the monoid of the semiring is used as the add binary operator. The add operator may be a binary operator created by GxB_BinaryOp_new_IndexOp.

The operation can be expressed in MATLAB notation as:

```
[nrows, ncols] = size (A.matrix) ;
T.matrix = zeros (nrows, ncols, add.ztype) ;
p = A.pattern & B.pattern ;
A = GB_mex_cast (A.matrix (p), add.xtype) ;
B = GB_mex_cast (B.matrix (p), add.ytype) ;
T.matrix (p) = add (A, B) ;
p = A.pattern & ~B.pattern ; T.matrix (p) = cast (A.matrix (p), add.ztype) ;
p = ~A.pattern & B.pattern ; T.matrix (p) = cast (B.matrix (p), add.ztype) ;
T.pattern = A.pattern | B.pattern ;
T.class = add.ztype ;
```

Except for when typecasting is performed, this is identical to how the accum operator is applied in Figure 1.

The final step is $C(M) = C \odot T$, as described in Section 2.3.

12.6 GxB_eWiseUnion: element-wise operations, set union

GxB_eWiseUnion computes a result with the same pattern GrB_eWiseAdd, namely, a set union of its two inputs. It differs in how the binary operator is applied.

Let \oplus denote the binary operator to be used. The operator is applied to every entry in **A** and **B**. A pair of scalars, α and β (alpha and beta in the API, respectively) define the inputs to the operator when entries are present in one matrix but not the other.

```
for all entries (i, j) in \mathbf{A} \cap \mathbf{B}

t_{ij} = a_{ij} \oplus b_{ij}

for all entries (i, j) in \mathbf{A} \setminus \mathbf{B}

t_{ij} = a_{ij} \oplus \beta

for all entries (i, j) in \mathbf{B} \setminus \mathbf{A}

t_{ij} = \alpha \oplus b_{ij}
```

 $GxB_eWiseUnion$ is useful in contexts where $GrB_eWiseAdd$ cannot be used because of the typecasting rules of GraphBLAS. In particular, suppose A and B are matrices with a user-defined type, and suppose < is a user-defined operator that compares two entries of this type and returns a Boolean value. Then C=A<B can be computed with $GxB_eWiseUnion$ but not with $GrB_eWiseAdd$. In the latter, if A(i,j) is present but B(i,j) is not, then A(i,j) must typecasted to the type of $C(GrB_eBOOL)$ in this case, and the assignment C(i,j) = (bool) A(i,j) would be performed. This is not possible because user-defined types cannot be typecasted to any other type.

Another advantage of GxB_eWiseUnion is its performance. For example, the MATLAB/Octave expression C=A-B computes C(i,j)=-B(i,j) when A(i,j) is not present. This cannot be done with a single call GrB_eWiseAdd, but it can be done with a single call to GxB_eWiseUnion, with the GrB_MINUS_FP64 operator, and with both alpha and beta scalars equal to zero. It is possible to compute this result with a temporary matrix, E=-B, computed with GrB_apply and GrB_AINV_FP64, followed by a call to GrB_eWiseAdd to compute C=A+E, but this is slower than a single call to GxB_eWiseUnion, and uses more memory.

12.6.1 GxB_Vector_eWiseUnion: element-wise vector addition

```
// w<mask> = accum (w, u+v)
GrB_Info GxB_eWiseUnion
    GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
   const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
                                    // defines '+' for t=u+v
   const GrB_BinaryOp add,
   const GrB_Vector u,
                                    // first input: vector u
    const GrB_Scalar alpha,
    const GrB_Vector v,
                                    // second input: vector v
    const GrB_Scalar beta,
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

Identical to GrB_Vector_eWiseAdd except that two scalars are used to define how to compute the result when entries are present in one of the two input vectors (u and v), but not the other. Each of the two input scalars, alpha and beta must contain an entry. When computing the result t=u+v, if u(i) is present but v(i) is not, then t(i)=u(i)+beta. Likewise, if v(i) is present but u(i) is not, then t(i)=alpha+v(i), where + denotes the binary operator, add. The add operator may be a binary operator created by GxB_BinaryOp_new_IndexOp.

12.6.2 GxB_Matrix_eWiseUnion: element-wise matrix addition

```
// C<M> = accum (C, A+B)
GrB_Info GxB_eWiseUnion
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
                                    // defines '+' for T=A+B
   const GrB_BinaryOp add,
    const GrB_Matrix A,
                                    // first input: matrix A
   const GrB_Scalar alpha,
    const GrB_Matrix B,
                                    // second input: matrix B
    const GrB_Scalar beta,
    const GrB_Descriptor desc
                                    // descriptor for C, M, A, and B \,
);
```

Identical to GrB_Matrix_eWiseAdd except that two scalars are used to define how to compute the result when entries are present in one of the two input matrices (A and B), but not the other. Each of the two input scalars, alpha and beta must contain an entry. When computing the result T=A+B, if A(i,j) is present but B(i,j) is not, then T(i,j)=A(i,j)+beta. Likewise, if B(i,j) is present but A(i,j) is not, then T(i,j)=alpha+B(i,j), where + denotes the binary operator, add. The add operator may be a binary operator created by GxB_BinaryOp_new_IndexOp.

12.7 GrB_extract: submatrix extraction

The GrB_extract function is a generic name for three specific functions: GrB_Vector_extract, GrB_Col_extract, and GrB_Matrix_extract. The generic name appears in the function signature, but the specific function name is used when describing what each variation does.

12.7.1 GrB_Vector_extract: extract subvector from vector

```
// w<mask> = accum (w, u(I))
GrB_Info GrB_extract
   GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const GrB_Vector u,
                                    // first input: vector u
                                    // row indices
    const GrB_Index *I,
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB_Vector_extract extracts a subvector from another vector, identical to $\mathbf{t} = \mathbf{u}$ (I) in MATLAB where I is an integer vector of row indices. Refer to GrB_Matrix_extract for further details; vector extraction is the same as matrix extraction with n-by-1 matrices. See Section 11 for a description of I and ni. The final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot\mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

12.7.2 GxB_Vector_extract_Vector: extract subvector from vector

GxB_Vector_extract_Vector is identical to GrB_Vector_extract, except that the row indices are given by the GrB_Vector I with ni entries. The interpretation of I_vector is controlled by descriptor setting GxB_ROWINDEX_LIST.

The method can use either the indices or values of the input vector, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.7.3 GrB_Matrix_extract: extract submatrix from matrix

```
GrB_Info GrB_extract
                                    // C<Mask> = accum (C, A(I,J))
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
                                    // column indices
    const GrB_Index *J,
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, and A
);
```

<code>GrB_Matrix_extract</code> extracts a submatrix from another matrix, identical to T = A(I,J) in MATLAB where I and J are integer vectors of row and column indices, respectively, except that indices are zero-based in Graph-BLAS and one-based in MATLAB. The input matrix A may be transposed first, via the descriptor. The type of T and A are the same. The size of C is |I|-by-|J|. Entries outside A(I,J) are not accessed and do not take part in the computation. More precisely, assuming the matrix A is not transposed, the matrix T is defined as follows:

If duplicate indices are present in I or J, the above method defines the result in T. Duplicates result in the same values of A being copied into different places in T. See Section 11 for a description of the row indices I and ni, and the column indices J and nj. The final step is $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}$, as described in Section 2.3.

Performance considerations: If A is not transposed via input descriptor: if |I| is small, then it is fastest if A is GrB_ROWMAJOR; if |J| is small, then it is fastest if A is GrB_COLMAJOR. The opposite is true if A is transposed.

12.7.4 GxB_Matrix_extract_Vector: extract submatrix from matrix

```
// C<Mask> = accum (C, A(I,J))
GrB_Info GrB_extract
                                    // input/output matrix for results
    GrB_Matrix C,
                                    // optional mask for C, unused if NULL
    const GrB_Matrix Mask,
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Vector I_vector,
                                    // row indices
    const GrB_Vector J_vector,
                                    // column indices
                                    // descriptor for C, M, and A
    const GrB_Descriptor desc
);
```

GxB_Matrix_extract_Vector is identical to GrB_Matrix_extract, except that the row indices are given by the GrB_Vector I with ni entries, and the column indices are given by the GrB_Vector J with nj entries. The interpretation of I_vector and J_vector are controlled by descriptor setting GxB_ROWINDEX_LIST and GxB_COLINDEX_LIST, respectively. The method can use either the indices or values of each of the input vectors, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.7.5 GrB_Col_extract: extract column vector from matrix

```
// w<mask> = accum (w, A(I,j))
GrB_Info GrB_extract
                                    // input/output matrix for results
    GrB_Vector w,
                                    // optional mask for w, unused if NULL
    const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Index j,
                                    // column index
    const GrB_Descriptor desc
                                    // descriptor for w, mask, and A
);
```

GrB_Col_extract extracts a subvector from a matrix, identical to t = A (I,j)
in MATLAB where I is an integer vector of row indices and where j is a single

column index. The input matrix A may be transposed first, via the descriptor, which results in the extraction of a single row j from the matrix A, the result of which is a column vector w. The type of t and A are the same. The size of w is |I|-bv-1.

See Section 11 for a description of the row indices I and ni. The final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot\mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

Performance considerations: If A is not transposed: it is fastest if the format of A is Grb_COLMAJOR. The opposite is true if A is transposed.

12.7.6 GxB_Col_extract_Vector: extract column vector from matrix

```
// w<mask> = accum (w, A(I,j))
GrB_Info GrB_extract
    GrB_Vector w,
                                    // input/output matrix for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Vector I_vector,
                                    // row indices
    const GrB_Index j,
                                    // column index
    const GrB_Descriptor desc
                                    // descriptor for w, mask, and A
);
```

GxB_Col_extract_Vector is identical to GrB_Col_extract, except that the row indices are given by the GrB_Vector I with ni entries. The interpretation of I_vector is controlled by descriptor setting GxB_ROWINDEX_LIST. The method can use either the indices or values of the input vector, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.8 GrB_assign: submatrix assignment

The methods described in this section are all variations of the form C(I,J)=A, which modifies a submatrix of the matrix C. All methods can be used in their generic form with the single name GrB_assign. These methods are very similar to their GxB_subassign counterparts in Section 12.9. They differ primarily in the size of the Mask, and how the GrB_REPLACE option works. Section 12.11 compares GxB_subassign and GrB_assign.

See Section 11 for a description of I, ni, J, and nj.

12.8.1 GrB_Vector_assign: assign to a subvector

```
// w<mask>(I) = accum (w(I),u)
GrB_Info GrB_assign
    GrB_Vector w,
                                    // input/output vector for results
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w(I),t)
    const GrB_Vector u,
                                    // first input: vector u
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB_Vector_assign operates on a subvector w(I) of w, modifying it with the vector u. The mask vector has the same size as w. The method is identical to GrB_Matrix_assign described in Section 12.8.3, where all matrices have a single column each. The only other difference is that the input u in this method is not transposed via the GrB_INPO descriptor.

12.8.2 GxB_Vector_assign_Vector: assign to a subvector

GxB_Vector_assign_Vector is identical to GrB_Vector_assign, except that the row indices are given by the GrB_Vector I with ni entries. The interpretation of I_vector is controlled by descriptor setting GxB_ROWINDEX_LIST. The method can use either the indices or values of the input vector, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.8.3 GrB_Matrix_assign: assign to a submatrix

```
GrB_Info GrB_assign
                                    // C<Mask>(I,J) = accum (C(I,J),A)
                                    // input/output matrix for results
    GrB_Matrix C,
                                    // optional mask for C, unused if NULL
   const GrB_Matrix Mask,
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C(I,J),T)
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Index *J,
                                    // column indices
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, and A
);
```

GrB_Matrix_assign operates on a submatrix S of C, modifying it with the matrix A. It may also modify all of C, depending on the input descriptor desc and the Mask.

| Step | GraphBLAS | description |
|------|---|---|
| | notation | |
| 1 | $\mathbf{S} = \mathbf{C}(\mathbf{I}, \mathbf{J})$ | extract $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix |
| 2 | $\mathbf{S} = \mathbf{S} \odot \mathbf{A}$ | apply the accumulator (but not the mask) to ${\bf S}$ |
| 3 | $\mathbf{Z} = \mathbf{C}$ | make a copy of C |
| 4 | $\mathbf{Z}(\mathbf{I},\mathbf{J}) = \mathbf{S}$ | put the submatrix into $\mathbf{Z}(\mathbf{I}, \mathbf{J})$ |
| 5 | $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{Z}$ | apply the mask/replace phase to all of ${\bf C}$ |

In contrast to GxB_subassign, the Mask has the same as C.

Step 1 extracts the submatrix and then Step 2 applies the accumulator (or S = A if accum is NULL). The Mask is not yet applied.

Step 3 makes a copy of the C matrix, and then Step 4 writes the submatrix S into Z. This is the same as Step 3 of GxB_subassign, except that it operates on a temporary matrix Z.

Finally, Step 5 writes Z back into C via the Mask, using the Mask/Replace

Phase described in Section 2.3. If $GrB_REPLACE$ is enabled, then all of C is cleared prior to writing Z via the mask. As a result, the $GrB_REPLACE$ option can delete entries outside the C(I, J) submatrix.

Performance considerations: If A is not transposed: if |I| is small, then it is fastest if the format of C is GrB_ROWMAJOR; if |J| is small, then it is fastest if the format of C is GrB_COLMAJOR. The opposite is true if A is transposed.

12.8.4 GxB_Matrix_assign_Vector: assign to a submatrix

```
GrB_Info GrB_assign
                                    // C<Mask>(I,J) = accum (C(I,J),A)
                                    // input/output matrix for results
    GrB_Matrix C,
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C(I,J),T)
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Vector I_vector,
                                    // row indices
    const GrB_Vector J_vector,
                                    // column indices
    const GrB_Descriptor desc
);
```

GxB_Matrix_assign_Vector is identical to GrB_Matrix_assign, except that the row indices are given by the GrB_Vector I with ni entries, and the column indices are given by the GrB_Vector J with nj entries. The interpretation of I_vector and J_vector are controlled by descriptor setting GxB_ROWINDEX_LIST and GxB_COLINDEX_LIST, respectively. The method can use either the indices or values of each of the input vectors, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.8.5 GrB_Col_assign: assign to a sub-column of a matrix

```
GrB_Info GrB_assign
                                    // C<mask>(I,j) = accum (C(I,j),u)
    GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C(:,j), unused if NULL
    const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(C(I,j),t)
    const GrB_Vector u,
                                    // input vector
    const GrB_Index *I,
                                    // row indices
                                    // number of row indices
    const GrB_Index ni,
    const GrB_Index j,
                                    // column index
    const GrB_Descriptor desc
                                    // descriptor for C(:,j) and mask
);
```

GrB_Col_assign modifies a single sub-column of a matrix C. It is the same as GrB_Matrix_assign where the index vector J[0]=j is a single column index, and where all matrices in GrB_Matrix_assign (except C) consist of a single column.

Unlike GrB_Matrix_assign, the mask is a vector with the same size as a single column of C.

The input descriptor GrB_INPO is ignored; the input vector **u** is not transposed. Refer to GrB_Matrix_assign for further details.

Performance considerations: GrB_Col_assign is much faster than GrB_Row_assign if the format of C is GrB_COLMAJOR. GrB_Row_assign is much faster than GrB_Col_assign if the format of C is GrB_ROWMAJOR.

12.8.6 GxB_Col_assign_Vector: assign to a sub-column of a matrix

```
// C < mask > (I,j) = accum (C(I,j),u)
GrB_Info GrB_assign
                                     // input/output matrix for results
    GrB_Matrix C,
    const GrB_Vector mask,
                                     // optional mask for C(:,j), unused if NULL
    const GrB_BinaryOp accum,
                                     // optional accum for z=accum(C(I,j),t)
    const GrB_Vector u,
                                     // input vector
    const GrB_Vector I_vector,
                                    // row indices
                                     // column index
    GrB_Index j,
    const GrB_Descriptor desc
);
```

GxB_Col_assign_Vector is identical to GrB_Col_assign, except that the row indices are given by the GrB_Vector I with ni entries. The interpretation of I_vector is controlled by descriptor setting GxB_ROWINDEX_LIST.

The method can use either the indices or values of the input vector, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.8.7 GrB_Row_assign: assign to a sub-row of a matrix

```
// C < mask' > (i,J) = accum (C(i,J),u')
GrB_Info GrB_assign
                                    // input/output matrix for results
    GrB_Matrix C,
                                    // optional mask for C(i,:), unused if NULL
    const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(C(i,J),t)
    const GrB_Vector u,
                                    // input vector
    const GrB_Index i,
                                    // row index
    const GrB_Index *J,
                                    // column indices
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C(i,:) and mask
);
```

GrB_Row_assign modifies a single sub-row of a matrix C. It is the same as GrB_Matrix_assign where the index vector I[0]=i is a single row index, and where all matrices in GrB_Matrix_assign (except C) consist of a single row.

Unlike GrB_Matrix_assign, the mask is a vector with the same size as a single row of C.

The input descriptor GrB_INPO is ignored; the input vector **u** is not transposed. Refer to GrB_Matrix_assign for further details.

Performance considerations: GrB_Col_assign is much faster than GrB_Row_assign if the format of C is GrB_COLMAJOR. GrB_Row_assign is much faster than GrB_Col_assign if the format of C is GrB_ROWMAJOR.

12.8.8 GxB_Row_assign_Vector: assign to a sub-row of a matrix

```
GrB_Info GrB_assign
                                     // C < mask' > (i,J) = accum (C(i,J),u')
    GrB_Matrix C,
                                     // input/output matrix for results
                                     // mask for C(i,:), unused if NULL
    const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                     // optional accum for z=accum(C(i,J),t)
    const GrB_Vector u,
                                     // input vector
    GrB_Index i,
                                     // row index
                                     // column indices
    const GrB_Vector J_vector,
    const GrB_Descriptor desc
);
```

GxB_Row_assign_Vector is identical to GrB_Row_assign, except that the column indices are given by the GrB_Vector J with nj entries. The interpretation of J_vector is controlled by descriptor setting GxB_COLINDEX_LIST. The method can use either the indices or values of the input vector, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.8.9 GrB_Vector_assign_<type>: assign a scalar to a subvector

```
// w<mask>(I) = accum (w(I),x)
GrB_Info GrB_assign
                                    // input/output vector for results
    GrB_Vector w,
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w(I),x)
    const <type> x,
                                    // scalar to assign to w(I)
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB_Vector_assign_<type> assigns a single scalar to an entire subvector of the vector w. The operation is exactly like setting a single entry in an n-by-1 matrix, A(I,0) = x, where the column index for a vector is implicitly j=0. The mask vector has the same size as w. If x is a GrB_Scalar, the nonpolymorphic name of the method is GrB_Vector_assign_Scalar. For further details of this function, see GrB_Matrix_assign_<type> in the next section (12.8.11). Following the C API Specification, results are well-defined if I contains duplicate indices. Duplicate indices are simply ignored. See Section 12.10 for more details.

12.8.10 GxB_Vector_assign_Scalar_Vector: assign a scalar to a subvector

GxB_Vector_assign_Scalar_Vector is identical to GrB_Vector_assign_Scalar, except that the row indices are given by the GrB_Vector I with ni entries. The interpretation of I_vector is controlled by descriptor setting GxB_ROWINDEX_LIST. The method can use either the indices or values of the input vector, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.8.11 GrB_Matrix_assign_<type>: assign a scalar to a submatrix

```
GrB_Info GrB_assign
                                    // C<Mask>(I,J) = accum (C(I,J),x)
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C(I,J),x)
                                    // scalar to assign to C(I,J)
    const <type> x,
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Index *J,
                                    // column indices
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C and Mask
);
```

GrB_Matrix_assign_<type> assigns a single scalar to an entire submatrix of C, like the scalar expansion C(I,J)=x in MATLAB. The scalar x is implicitly expanded into a matrix A of size ni by nj, and then the matrix A is assigned to C(I,J) using the same method as in GrB_Matrix_assign. Refer to that function in Section 12.8.3 for further details.

The Mask has the same size as C.

For the accumulation step, the scalar x is typecasted directly into the type of C when the accum operator is not applied to it, or into the ytype

of the accum operator, if accum is not NULL, for entries that are already present in C.

The <type> x notation is otherwise the same as GrB_Matrix_setElement (see Section 6.10.12). Any value can be passed to this function and its type will be detected, via the _Generic feature of C11. For a user-defined type, x is a void * pointer that points to a memory space holding a single entry of a scalar that has exactly the same user-defined type as the matrix C. This user-defined type must exactly match the user-defined type of C since no typecasting is done between user-defined types.

If a void * pointer is passed in and the type of the underlying scalar does not exactly match the user-defined type of C, then results are undefined. No error status will be returned since GraphBLAS has no way of catching this error.

If x is a GrB_Scalar, the nonpolymorphic name of the method is GrB_Matrix_assign_Scalar. In this case, if x has no entry, then it is implicitly expanded into a matrix A of size ni by nj, with no entries present.

Following the C API Specification, results are well-defined if I or J contain duplicate indices. Duplicate indices are simply ignored. See Section 12.10 for more details.

Performance considerations: If A is not transposed: if |I| is small, then it is fastest if the format of C is GrB_ROWMAJOR; if |J| is small, then it is fastest if the format of C is GrB_COLMAJOR. The opposite is true if A is transposed.

12.8.12 GxB_Matrix_assign_Scalar_Vector: assign a scalar to a submatrix

```
GrB_Info GrB_assign
                                    // C<Mask>(I,J) = accum (C(I,J),x)
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C(I,J),x)
    const GrB_Scalar x,
                                    // scalar to assign to C(I,J)
    const GrB_Vector I_vector,
                                    // row indices
    const GrB_Vector J_vector,
                                    // column indices
    const GrB_Descriptor desc
);
```

GxB_Matrix_assign_Scalar_Vector is identical to GrB_Matrix_assign_Scalar, except that the row indices are given by the GrB_Vector I with ni entries,

and the column indices are given by the GrB_Vector J with nj entries. The interpretation of I_vector and J_vector are controlled by descriptor setting GxB_ROWINDEX_LIST and GxB_COLINDEX_LIST, respectively. The method can use either the indices or values of each of the input vectors, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.9 GxB_subassign: submatrix assignment

The methods described in this section are all variations of the form C(I,J)=A, which modifies a submatrix of the matrix C. All methods can be used in their generic form with the single name GxB_subassign. This is reflected in the prototypes. However, to avoid confusion between the different kinds of assignment, the name of the specific function is used when describing each variation. If the discussion applies to all variations, the simple name GxB_subassign is used.

See Section 11 for a description of the row indices I and ni, and the column indices J and nj.

GxB_subassign is very similar to GrB_assign, described in Section 12.8. The two operations are compared and contrasted in Section 12.11. For a discussion of how duplicate indices are handled in I and J, see Section 12.10.

12.9.1 GxB_Vector_subassign: assign to a subvector

```
// w(I) < mask > = accum (w(I),u)
GrB_Info GxB_subassign
    GrB_Vector w,
                                     // input/output vector for results
    const GrB_Vector mask,
                                     // optional mask for w(I), unused if NULL
    const GrB_BinaryOp accum,
                                     // optional accum for z=accum(w(I),t)
    const GrB_Vector u,
                                     // first input: vector u
    const GrB_Index *I,
                                     // row indices
    const GrB_Index ni,
                                     // number of row indices
    const GrB_Descriptor desc
                                     // descriptor for w(I) and mask
);
```

GxB_Vector_subassign operates on a subvector w(I) of w, modifying it with the vector u. The method is identical to GxB_Matrix_subassign described in Section 12.9.3, where all matrices have a single column each. The mask has the same size as w(I) and u. The only other difference is that the input u in this method is not transposed via the GrB_INPO descriptor.

12.9.2 GxB_Vector_subassign_Vector: assign to a subvector

GxB_Vector_subassign_Vector is identical to GxB_Vector_subassign, except that the row indices are given by the GrB_Vector I with ni entries. The interpretation of I_vector is controlled by descriptor setting GxB_ROWINDEX_LIST. The method can use either the indices or values of the input vector, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.9.3 GxB_Matrix_subassign: assign to a submatrix

```
GrB_Info GxB_subassign
                                    // C(I,J) < Mask > = accum (C(I,J),A)
    GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C(I,J), unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C(I,J),T)
                                    // first input: matrix A
    const GrB_Matrix A,
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Index *J,
                                    // column indices
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C(I,J), Mask, and A
);
```

GxB_Matrix_subassign operates only on a submatrix S of C, modifying it with the matrix A. For this operation, the result is not the entire matrix C, but a submatrix S=C(I,J) of C. The steps taken are as follows, except that A may be optionally transposed via the GrB_INPO descriptor option.

| Step | GraphBLAS | description |
|------|---|---|
| | notation | |
| 1 | S = C(I, J) | extract the $C(I, J)$ submatrix |
| 2 | $\mathbf{S}\langle\mathbf{M} angle=\mathbf{S}\odot\mathbf{A}$ | apply the accumulator/mask to the submatrix ${f S}$ |
| 3 | $\mathbf{C}(\mathbf{I},\mathbf{J}) = \mathbf{S}$ | put the submatrix S back into $C(I, J)$ |

The accumulator/mask step in Step 2 is the same as for all other Graph-BLAS operations, described in Section 2.3, except that for $GxB_subassign$, it is applied to just the submatrix S = C(I, J), and thus the Mask has the same size as A, S, and C(I, J).

The GxB_subassign operation is the reverse of matrix extraction:

- For submatrix extraction, GrB_Matrix_extract, the submatrix A(I,J) appears on the right-hand side of the assignment, C=A(I,J), and entries outside of the submatrix are not accessed and do not take part in the computation.
- For submatrix assignment, GxB_Matrix_subassign, the submatrix C(I, J) appears on the left-hand-side of the assignment, C(I, J)=A, and entries outside of the submatrix are not accessed and do not take part in the computation.

In both methods, the accumulator and mask modify the submatrix of the assignment; they simply differ on which side of the assignment the submatrix resides on. In both cases, if the Mask matrix is present it is the same size as the submatrix:

- For submatrix extraction, $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{A}(\mathbf{I},\mathbf{J})$ is computed, where the submatrix is on the right. The mask \mathbf{M} has the same size as the submatrix $\mathbf{A}(\mathbf{I},\mathbf{J})$.
- For submatrix assignment, $\mathbf{C}(\mathbf{I}, \mathbf{J})\langle \mathbf{M} \rangle = \mathbf{C}(\mathbf{I}, \mathbf{J}) \odot \mathbf{A}$ is computed, where the submatrix is on the left. The mask \mathbf{M} has the same size as the submatrix $\mathbf{C}(\mathbf{I}, \mathbf{J})$.

In Step 1, the submatrix S is first computed by the GrB_Matrix_extract operation, S=C(I,J).

Step 2 accumulates the results $S(M) = S \odot T$, exactly as described in Section 2.3, but operating on the submatrix S, not C, using the optional

Mask and accum operator. The matrix T is simply T = A, or $T = A^T$ if A is transposed via the desc descriptor, GrB_INPO. The GrB_REPLACE option in the descriptor clears S after computing Z = T or $Z = C \odot T$, not all of C since this operation can only modify the specified submatrix of C.

Finally, Step 3 writes the result (which is the modified submatrix S and not all of C) back into the C matrix that contains it, via the assignment C(I,J)=S, using the reverse operation from the method described for matrix extraction:

Performance considerations: If A is not transposed: if |I| is small, then it is fastest if the format of C is GrB_ROWMAJOR; if |J| is small, then it is fastest if the format of C is GrB_COLMAJOR. The opposite is true if A is transposed.

12.9.4 GxB_Matrix_subassign_Vector: assign to a submatrix

```
GrB_Info GxB_subassign
                                     // C(I,J) < Mask > = accum (C(I,J),A)
    GrB_Matrix C,
                                     // input/output matrix for results
    const GrB_Matrix Mask,
                                     // mask for C(I,J), unused if NULL
    const GrB_BinaryOp accum,
                                     // accum for Z=accum(C(I,J),T)
    const GrB_Matrix A,
                                     // first input: matrix A
    const GrB_Vector I_vector,
                                     // row indices
    const GrB_Vector J_vector,
                                     // column indices
    const GrB_Descriptor desc
) ;
```

GxB_Matrix_subassign_Vector is identical to GxB_Matrix_subassign, except that the row indices are given by the GrB_Vector I with ni entries, and the column indices are given by the GrB_Vector J with nj entries. The interpretation of I_vector and J_vector are controlled by descriptor setting GxB_ROWINDEX_LIST and GxB_COLINDEX_LIST, respectively. The method can use either the indices or values of each of the input vectors, or it can use

the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.9.5 GxB_Col_subassign: assign to a sub-column of a matrix

```
GrB_Info GxB_subassign
                                    // C(I,j) < mask > = accum (C(I,j),u)
   GrB_Matrix C.
                                    // input/output matrix for results
    const GrB_Vector mask,
                                    // optional mask for C(I,j), unused if NULL
   const GrB_BinaryOp accum,
                                    // optional accum for z=accum(C(I,j),t)
    const GrB_Vector u,
                                    // input vector
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Index j,
                                    // column index
    const GrB_Descriptor desc
                                    // descriptor for C(I,j) and mask
);
```

GxB_Col_subassign modifies a single sub-column of a matrix C. It is the same as GxB_Matrix_subassign where the index vector J[0]=j is a single column index (and thus nj=1), and where all matrices in GxB_Matrix_subassign (except C) consist of a single column. The mask vector has the same size as u and the sub-column C(I,j). The input descriptor GrB_INPO is ignored; the input vector u is not transposed. Refer to GxB_Matrix_subassign for further details.

Performance considerations: GxB_Col_subassign is much faster than GxB_Row_subassign if the format of C is GrB_COLMAJOR. GxB_Row_subassign is much faster than GxB_Col_subassign if the format of C is GrB_ROWMAJOR.

12.9.6 GxB_Col_subassign_Vector: assign to a sub-column of a matrix

```
GrB_Info GxB_subassign
                                    // C(I,j) < mask > = accum (C(I,j),u)
                                    // input/output matrix for results
    GrB_Matrix C,
    const GrB_Vector mask,
                                    // optional mask for C(I,j), unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(C(I,j),t)
    const GrB_Vector u,
                                    // input vector
                                    // row indices
    const GrB_Vector I_vector,
    const GrB_Index j,
                                    // column index
    const GrB_Descriptor desc
);
```

GxB_Col_subassign_Vector is identical to GxB_Col_subassign, except that the row indices are given by the GrB_Vector I with ni entries. The interpretation of I_vector is controlled by descriptor setting GxB_ROWINDEX_LIST. The method can use either the indices or values of the input vector, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.9.7 GxB_Row_subassign: assign to a sub-row of a matrix

```
GrB_Info GxB_subassign
                                    // C(i,J) < mask' > = accum (C(i,J),u')
                                    // input/output matrix for results
    GrB_Matrix C,
                                    // optional mask for C(i,J), unused if NULL
   const GrB_Vector mask,
   const GrB_BinaryOp accum,
                                    // optional accum for z=accum(C(i,J),t)
    const GrB_Vector u,
                                    // input vector
    const GrB_Index i,
                                    // row index
    const GrB_Index *J,
                                    // column indices
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C(i,J) and mask
);
```

GxB_Row_subassign modifies a single sub-row of a matrix C. It is the same as GxB_Matrix_subassign where the index vector I[0]=i is a single row index (and thus ni=1), and where all matrices in GxB_Matrix_subassign (except C) consist of a single row. The mask vector has the same size as u and the sub-column C(I,j). The input descriptor GrB_INPO is ignored; the input vector u is not transposed. Refer to GxB_Matrix_subassign for further details.

Performance considerations: GxB_Col_subassign is much faster than GxB_Row_subassign if the format of C is GrB_COLMAJOR. GxB_Row_subassign is much faster than GxB_Col_subassign if the format of C is GrB_ROWMAJOR.

12.9.8 GxB_Row_subassign_Vector: assign to a sub-row of a matrix

```
GrB_Info GxB_Row_subassign_Vector
                                    // C(i,J) < mask' > = accum (C(i,J),u')
    GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C(i,J), unused if NULL
    const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(C(i,J),t)
    const GrB_Vector u,
                                    // input vector
    const GrB_Index i,
                                    // row index
                                    // column indices
    const GrB_Vector J_vector,
    const GrB_Descriptor desc
);
```

GxB_Row_subassign_Vector is identical to GrB_Row_subassign, except that the column indices are given by the GrB_Vector J with nj entries. The interpretation of J_vector is controlled by descriptor setting GxB_COLINDEX_LIST. The method can use either the indices or values of the input vector, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.9.9 GxB_Vector_subassign_<type>: assign a scalar to a subvector

```
// w(I) < mask > = accum (w(I),x)
GrB_Info GxB_subassign
                                    // input/output vector for results
    GrB_Vector w,
    const GrB_Vector mask,
                                    // optional mask for w(I), unused if NULL
                                    // optional accum for z=accum(w(I),x)
    const GrB_BinaryOp accum,
    const <type> x,
                                    // scalar to assign to w(I)
    const GrB_Index *I,
                                    // row indices
    const GrB_Index ni,
                                    // number of row indices
    const GrB_Descriptor desc
                                    // descriptor for w(I) and mask
);
```

GxB_Vector_subassign_<type> assigns a single scalar to an entire subvector of the vector w. The operation is exactly like setting a single entry in an n-by-1 matrix, A(I,0) = x, where the column index for a vector is implicitly j=0. If x is a GrB_Scalar, the nonpolymorphic name of the method is GxB_Vector_subassign_Scalar. For further details of this function, see GxB_Matrix_subassign_<type> in Section 12.9.11.

12.9.10 GxB_Vector_subassign_Scalar_Vector: assign a scalar to a subvector

GxB_Vector_subassign_Scalar_Vector is identical to GxB_Vector_subassign_Scalar, except that the row indices are given by the GrB_Vector I with ni entries. The interpretation of I_vector is controlled by descriptor setting GxB_ROWINDEX_LIST. The method can use either the indices or values of the input vector, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.9.11 GxB_Matrix_subassign_<type>: assign a scalar to a submatrix

```
// C(I,J) < Mask > = accum (C(I,J),x)
GrB_Info GxB_subassign
                                    // input/output matrix for results
    GrB_Matrix C,
    const GrB_Matrix Mask,
                                    // optional mask for C(I,J), unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C(I,J),x)
                                    // scalar to assign to C(I,J)
    const <type> x,
    const GrB_Index *I,
                                    // row indices
                                    // number of row indices
    const GrB_Index ni,
    const GrB_Index *J,
                                    // column indices
    const GrB_Index nj,
                                    // number of column indices
    const GrB_Descriptor desc
                                    // descriptor for C(I,J) and Mask
);
```

GxB_Matrix_subassign_<type> assigns a single scalar to an entire submatrix of C, like the scalar expansion C(I,J)=x in MATLAB. The scalar x is implicitly expanded into a matrix A of size ni by nj, with all entries present and equal to x, and then the matrix A is assigned to C(I,J) using the same method as in GxB_Matrix_subassign. Refer to that function in Section 12.9.3 for further details. For the accumulation step, the scalar x is typecasted directly into the type of C when the accum operator is not applied

to it, or into the ytype of the accum operator, if accum is not NULL, for entries that are already present in C.

The <type> x notation is otherwise the same as GrB_Matrix_setElement (see Section 6.10.12). Any value can be passed to this function and its type will be detected, via the _Generic feature of C11. For a user-defined type, x is a void * pointer that points to a memory space holding a single entry of a scalar that has exactly the same user-defined type as the matrix C. This user-defined type must exactly match the user-defined type of C since no typecasting is done between user-defined types.

If a void * pointer is passed in and the type of the underlying scalar does not exactly match the user-defined type of C, then results are undefined. No error status will be returned since GraphBLAS has no way of catching this error.

If x is a GrB_Scalar, the nonpolymorphic name of the method is GxB_Matrix_subassign_Scalar. In this case, if x has no entry, then it is implicitly expanded into a matrix A of size ni by nj, with no entries present.

Performance considerations: If A is not transposed: if |I| is small, then it is fastest if the format of C is GrB_ROWMAJOR; if |J| is small, then it is fastest if the format of C is GrB_COLMAJOR. The opposite is true if A is transposed.

12.9.12 GxB_Matrix_subassign_Scalar_Vector: assign a scalar to a submatrix

```
GrB_Info GxB_subassign
                                     // C(I,J) < Mask > = accum (C(I,J),x)
    GrB_Matrix C,
                                     // input/output matrix for results
                                     // optional mask for C(I,J), unused if NULL
    const GrB_Matrix Mask,
    const GrB_BinaryOp accum,
                                     // optional accum for Z=accum(C(I,J),x)
    const GrB_Scalar scalar,
                                     // scalar to assign to C(I,J)
    const GrB_Vector I_vector,
                                     // row indices
    const GrB_Vector J_vector,
                                     // column indices
    const GrB_Descriptor desc
) ;
```

GxB_Matrix_subassign_Scalar_Vector is identical to GxB_Matrix_subassign_Scalar, except that the row indices are given by the GrB_Vector I with ni entries, and the column indices are given by the GrB_Vector J with nj entries. The interpretation of I_vector and J_vector are controlled by descriptor setting GxB_ROWINDEX_LIST and GxB_COLINDEX_LIST, respectively. The method can

use either the indices or values of each of the input vectors, or it can use the values as a stride (lo:inc:hi); the default is to use the values. See Section 6.16.4 for details.

12.10 Duplicate indices in GrB_assign and GxB_subassign

According to the GraphBLAS C API Specification if the index vectors I or J contain duplicate indices, the results are undefined for GrB_Matrix_assign, GrB_Matrix_assign, GrB_Col_assign, and GrB_Row_assign. Only the scalar assignment operations (GrB_Matrix_assign_TYPE and GrB_Matrix_assign_TYPE) are well-defined when duplicates appear in I and J. In those two functions, duplicate indices are ignored.

As an extension to the specification, SuiteSparse:GraphBLAS provides a definition of how duplicate indices are handled in all cases. If I has duplicate indices, they are ignored and the last unique entry in the list is used. When no mask and no accumulator is present, the results are identical to how MAT-LAB handles duplicate indices in the built-in expression C(I,J)=A. Details of how this is done is shown below.

```
function C = subassign (C, I, J, A)
% submatrix assignment with pre-sort of I and J; and remove duplicates
% delete duplicates from I, keeping the last one seen
[I2 I2k] = sort(I);
Idupl = [(I2 (1:end-1) == I2 (2:end)), false];
I2 = I2 ( (Idupl) ;
I2k = I2k (~Idupl);
assert (isequal (I2, unique (I)))
% delete duplicates from J, keeping the last one seen
[J2 \ J2k] = sort(J);
Jdupl = [(J2 (1:end-1) == J2 (2:end)), false];
J2 = J2 (~Jdupl);
J2k = J2k (~Jdupl);
assert (isequal (J2, unique (J)))
% do the submatrix assignment, with no duplicates in I2 or J2
C (I2,J2) = A (I2k,J2k);
```

If a mask is present, then it is replaced with M = M (I2k, J2k) for $GxB_subassign$, or with M = M (I2, J2) for GrB_assign . If an accumulator operator is present, it is applied after the duplicates are removed, as (for example):

```
C(I2,J2) = C(I2,J2) + A(I2k,J2k);
```

These definitions allow the MATLAB/Octave interface to GraphBLAS to return the same results for C(I,J)=A for a GrB object as they do for built-in MATLAB/Octave matrices. They also allow the assignment to be done in parallel.

Results are always well-defined in SuiteSparse:GraphBLAS, but they might not be what you expect. For example, suppose the MIN operator is being used the following assignment to the vector \mathbf{x} , and suppose I contains the entries $[0\ 0]$. Suppose \mathbf{x} is initially empty, of length 1, and suppose \mathbf{y} is a vector of length 2 with the values $[5\ 7]$.

```
#include "GraphBLAS.h"
#undef I
            /* complex.h #define's I, but I is used an array below */
#include <stdio.h>
int main (void)
{
    GrB_init (GrB_NONBLOCKING) ;
    GrB_Vector x, y ;
    GrB_Vector_new (&x, GrB_INT32, 1);
    GrB_Vector_new (&y, GrB_INT32, 2);
    GrB_Index I [2] = \{0, 0\};
    GrB_Vector_setElement (y, 5, 0);
    GrB_Vector_setElement (y, 7, 1);
    GrB_Vector_wait (&y) ;
    GxB_print (x, 3);
    GxB_print (y, 3);
    GrB_assign (x, NULL, GrB_MIN_INT32, y, I, 2, NULL) ;
    GrB_Vector_wait (&y) ;
    GxB_print (x, 3);
    GrB_finalize ( ) ;
}
```

You might (wrongly) expect the result to be the vector $\mathbf{x}(0)=5$, since two entries seem to be assigned, and the min operator might be expected to take the minimum of the two. This is not how SuiteSparse:GraphBLAS handles duplicates.

Instead, the first duplicate index of I is discarded (I [0] = 0, and y(0)=5). and only the second entry is used (I [1] = 0, and y(1)=7). The output of the above program is:

```
1x1 GraphBLAS int32_t vector, sparse by col:
x, no entries

2x1 GraphBLAS int32_t vector, sparse by col:
y, 2 entries

(0,0) 5
(1,0) 7

1x1 GraphBLAS int32_t vector, sparse by col:
x, 1 entry

(0,0) 7
```

You see that the result is x(0)=7, since the y(0)=5 entry has been ignored because of the duplicate indices in I.

SPEC: Providing a well-defined behavior for duplicate indices with matrix and vector assignment is an extension to the specification. The specification only defines the behavior when assigning a scalar into a matrix or vector, and states that duplicate indices otherwise lead to undefined results.

12.11 Comparing GrB_assign and GxB_subassign

The GxB_subassign and GrB_assign operations are very similar, but they differ in two ways:

- 1. The Mask has a different size: The mask in GxB_subassign has the same dimensions as w(I) for vectors and C(I,J) for matrices. In GrB_assign, the mask is the same size as w or C, respectively (except for the row/col variants). The two masks are related. If M is the mask for GrB_assign, then M(I,J) is the mask for GxB_subassign. If there is no mask, or if I and J are both GrB_ALL, the two masks are the same. For GrB_Row_assign and GrB_Col_assign, the mask vector is the same size as a row or column of C, respectively. For the corresponding GxB_Row_subassign and GxB_Col_subassign operations, the mask is the same size as the sub-row C(i,J) or subcolumn C(I,j), respectively.
- 2. GrB_REPLACE is different: They differ in how C is affected in areas outside the C(I,J) submatrix. In GxB_subassign, the C(I,J) submatrix is the only part of C that can be modified, and no part of C outside the submatrix is ever modified. In GrB_assign, it is possible to delete entries in C outside the submatrix, but only in one specific manner. Suppose the mask M is present (or, suppose it is not present but GrB_COMP is true). After (optionally) complementing the mask, the value of M(i,j) can be 0 for some entry outside the C(I,J) submatrix. If the GrB_REPLACE descriptor is true, GrB_assign deletes this entry.

GxB_subassign and GrB_assign are identical if GrB_REPLACE is set to its default value of false, and if the masks happen to be the same. The two masks can be the same in two cases: either the Mask input is NULL (and it is not complemented via GrB_COMP), or I and J are both GrB_ALL. If all these conditions hold, the two algorithms are identical and have the same performance. Otherwise, GxB_subassign is much faster than GrB_assign when the latter must examine the entire matrix C to delete entries (when GrB_REPLACE is true), and if it must deal with a much larger Mask matrix. However, both methods have specific uses.

Consider using C(I,J)+=F for many submatrices F (for example, when assembling a finite-element matrix). If the Mask is meant as a specification for which entries of C should appear in the final result, then use GrB_assign.

If instead the Mask is meant to control which entries of the submatrix C(I,J) are modified by the finite-element F, then use GxB_subassign. This is particularly useful is the Mask is a template that follows along with the finite-element F, independent of where it is applied to C. Using GrB_assign would be very difficult in this case since a new Mask, the same size as C, would need to be constructed for each finite-element F.

In GraphBLAS notation, the two methods can be described as follows:

| matrix and vector subassign | $\mathbf{C}(\mathbf{I},\mathbf{J})\langle\mathbf{M} angle = \mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$ |
|-----------------------------|---|
| matrix and vector assign | $\mathbf{C}\langle\mathbf{M} angle(\mathbf{I},\mathbf{J})=\mathbf{C}(\mathbf{I},\mathbf{J})\odot\mathbf{A}$ |

This notation does not include the details of the GrB_COMP and GrB_REPLACE descriptors, but it does illustrate the difference in the Mask. In the subassign, Mask is the same size as C(I,J) and A. If I[0]=i and J[0]=j, Then Mask(0,0) controls how C(i,j) is modified by the subassign, from the value A(0,0). In the assign, Mask is the same size as C, and Mask(i,j) controls how C(i,j) is modified.

The GxB_subassign and GrB_assign functions have the same signatures; they differ only in how they consider the Mask and the GrB_REPLACE descriptor

Details of each step of the two operations are listed below:

| Step | <pre>GrB_Matrix_assign</pre> | GxB_Matrix_subassign |
|------|--|--|
| 1 | $\mathbf{S} = \mathbf{C}(\mathbf{I}, \mathbf{J})$ | $\mathbf{S} = \mathbf{C}(\mathbf{I}, \mathbf{J})$ |
| 2 | $\mathbf{S} = \mathbf{S} \odot \mathbf{A}$ | $\mathbf{S}\langle \mathbf{M} \rangle = \mathbf{S} \odot \mathbf{A}$ |
| 3 | $\mathbf{Z} = \mathbf{C}$ | $\mathbf{C}(\mathbf{I},\mathbf{J})=\mathbf{S}$ |
| 4 | $\mathbf{Z}(\mathbf{I},\mathbf{J}) = \mathbf{S}$ | |
| 5 | $\mathbf{C}\langle \mathbf{M} angle = \mathbf{Z}$ | |

Step 1 is the same. In the Accumulator Phase (Step 2), the expression $\mathbf{S} \odot \mathbf{A}$, described in Section 2.3, is the same in both operations. The result is simply \mathbf{A} if accum is NULL. It only applies to the submatrix \mathbf{S} , not the whole matrix. The result $\mathbf{S} \odot \mathbf{A}$ is used differently in the Mask/Replace phase.

The Mask/Replace Phase, described in Section 2.3 is different:

• For Grb_assign (Step 5), the mask is applied to all of C. The mask has the same size as C. Just prior to making the assignment via the mask, the Grb_REPLACE option can be used to clear all of C first. This is the only way in which entries in C that are outside the C(I, J) submatrix can be modified by this operation.

• For GxB_subassign (Step 2b), the mask is applied to just S. The mask has the same size as C(I, J), S, and A. Just prior to making the assignment via the mask, the GrB_REPLACE option can be used to clear S first. No entries in C that are outside the C(I, J) can be modified by this operation. Thus, GrB_REPLACE has no effect on entries in C outside the C(I, J) submatrix.

The differences between GrB_assign and $GxB_subassign$ can be seen in Tables 5 and 6. The first table considers the case when the entry c_{ij} is in the C(I, J) submatrix, and it describes what is computed for both GrB_assign and $GxB_subassign$. They perform the exact same computation; the only difference is how the value of the mask is specified. Compare Table 5 with Table 1 in Section 7.

The first column of Table 5 is yes if $GrB_REPLACE$ is enabled, and a dash otherwise. The second column is yes if an accumulator operator is given, and a dash otherwise. The third column is c_{ij} if the entry is present in \mathbf{C} , and a dash otherwise. The fourth column is $a_{i'j'}$ if the corresponding entry is present in \mathbf{A} , where $i = \mathbf{I}(i')$ and $j = \mathbf{J}(i')$.

The mask column is 1 if the effective value of the mask mask allows C to be modified, and 0 otherwise. This is m_{ij} for $\mathtt{GrB_assign}$, and $m_{i'j'}$ for $\mathtt{GrB_subassign}$, to reflect the difference in the mask, but this difference is not reflected in the table. The value 1 or 0 is the value of the entry in the mask after it is optionally complemented via the $\mathtt{GrB_COMP}$ option.

Finally, the last column is the action taken in this case. It is left blank if no action is taken, in which case c_{ij} is not modified if present, or not inserted into \mathbf{C} if not present.

| repl | accum | \mathbf{C} | \mathbf{A} | $\max k$ | action taken by GrB_assign and GxB_subassign |
|------|-------|--------------|--------------|----------|--|
| - | - | c_{ij} | $a_{i'j'}$ | 1 | $c_{ij} = a_{i'j'}$, update |
| - | - | - | $a_{i'j'}$ | 1 | $c_{ij} = a_{i'j'}$, insert |
| - | - | c_{ij} | - | 1 | delete c_{ij} because $a_{i'j'}$ not present |
| - | - | - | - | 1 | |
| - | - | c_{ij} | $a_{i'j'}$ | 0 | |
| - | - | - | $a_{i'j'}$ | 0 | |
| - | - | c_{ij} | - | 0 | |
| _ | - | - | - | 0 | |
| yes | - | c_{ij} | $a_{i'j'}$ | 1 | $c_{ij} = a_{i'j'}$, update |
| yes | - | - | $a_{i'j'}$ | 1 | $c_{ij} = a_{i'j'}$, insert |
| yes | - | c_{ij} | - | 1 | delete c_{ij} because $a_{i'j'}$ not present |
| yes | - | - | - | 1 | |
| yes | - | c_{ij} | $a_{i'j'}$ | 0 | delete c_{ij} (because of GrB_REPLACE) |
| yes | - | - | $a_{i'j'}$ | 0 | |
| yes | - | c_{ij} | - | 0 | delete c_{ij} (because of $GrB_REPLACE$) |
| yes | - | - | - | 0 | |
| - | yes | c_{ij} | $a_{i'j'}$ | 1 | $c_{ij} = c_{ij} \odot a_{i'j'}$, apply accumulator |
| - | yes | - | $a_{i'j'}$ | 1 | $c_{ij} = a_{i'j'}$, insert |
| - | yes | c_{ij} | - | 1 | |
| - | yes | - | - | 1 | |
| - | yes | c_{ij} | $a_{i'j'}$ | 0 | |
| - | yes | - | $a_{i'j'}$ | 0 | |
| - | yes | c_{ij} | - | 0 | |
| _ | yes | - | - | 0 | |
| yes | yes | c_{ij} | $a_{i'j'}$ | 1 | $c_{ij} = c_{ij} \odot a_{i'j'}$, apply accumulator |
| yes | yes | - | $a_{i'j'}$ | 1 | $c_{ij} = a_{i'j'}$, insert |
| yes | yes | c_{ij} | - | 1 | |
| yes | yes | - | - | 1 | |
| yes | yes | c_{ij} | $a_{i'j'}$ | 0 | delete c_{ij} (because of $GrB_REPLACE$) |
| yes | yes | - | $a_{i'j'}$ | 0 | |
| yes | yes | c_{ij} | - | 0 | delete c_{ij} (because of GrB_REPLACE) |
| yes | yes | - | - | 0 | |

Table 5: Results of assign and subassign for entries in the $\mathbf{C}(\mathbf{I},\mathbf{J})$ submatrix

| repl | accum | \mathbf{C} | $\mathbf{C} = \mathbf{Z}$ | mask | action taken by GrB_assign |
|------|-------|--------------|---------------------------|------|---|
| - | - | c_{ij} | c_{ij} | 1 | |
| - | - | - | - | 1 | |
| - | - | c_{ij} | c_{ij} | 0 | |
| - | - | - | - | 0 | |
| yes | - | c_{ij} | c_{ij} | 1 | |
| yes | - | - | - | 1 | |
| yes | - | c_{ij} | c_{ij} | 0 | delete c_{ij} (because of $GrB_REPLACE$) |
| yes | - | - | - | 0 | |
| - | yes | c_{ij} | c_{ij} | 1 | |
| - | yes | - | - | 1 | |
| - | yes | c_{ij} | c_{ij} | 0 | |
| - | yes | - | - | 0 | |
| yes | yes | c_{ij} | c_{ij} | 1 | |
| yes | yes | - | - | 1 | |
| yes | yes | c_{ij} | c_{ij} | 0 | delete c_{ij} (because of $GrB_REPLACE$) |
| yes | yes | - | - | 0 | |

Table 6: Results of assign for entries outside the C(I, J) submatrix. Subassign has no effect on these entries.

Table 6 illustrates how GrB_assign and $GxB_subassign$ differ for entries outside the submatrix. $GxB_subassign$ never modifies any entry outside the C(I, J) submatrix, but GrB_assign can modify them in two cases listed in Table 6. When the $GrB_REPLACE$ option is selected, and when the Mask(i,j) for an entry c_{ij} is false (or if the Mask(i,j) is true and GrB_COMP is enabled via the descriptor), then the entry is deleted by GrB_assign .

The fourth column of Table 6 differs from Table 5, since entries in **A** never affect these entries. Instead, for all index pairs outside the $I \times J$ submatrix, **C** and **Z** are identical (see Step 3 above). As a result, each section of the table includes just two cases: either c_{ij} is present, or not. This in contrast to Table 5, where each section must consider four different cases.

The GrB_Row_assign and GrB_Col_assign operations are slightly different. They only affect a single row or column of C. For GrB_Row_assign, Table 6 only applies to entries in the single row C(i, J) that are outside the list of indices, J. For GrB_Col_assign, Table 6 only applies to entries in the single column C(I,j) that are outside the list of indices, I.

12.11.1 Example

The difference between $GxB_subassign$ and GrB_assign is illustrated in the following example. Consider the 2-by-2 matrix C where all entries are present.

$$\mathbf{C} = \left[\begin{array}{cc} 11 & 12 \\ 21 & 22 \end{array} \right]$$

Suppose GrB_REPLACE is true, and GrB_COMP is false. Let the Mask be:

$$\mathbf{M} = \left[\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right].$$

Let $\mathbf{A} = 100$, and let the index sets be $\mathbf{I} = 0$ and $\mathbf{J} = 1$. Consider the computation $\mathbf{C}\langle\mathbf{M}\rangle(0,1) = \mathbf{C}(0,1) + \mathbf{A}$, using the GrB_assign operation. The result is:

$$\mathbf{C} = \left[\begin{array}{cc} 11 & 112 \\ - & 22 \end{array} \right].$$

The (0,1) entry is updated and the (1,0) entry is deleted because its Mask is zero. The other two entries are not modified since $\mathbf{Z} = \mathbf{C}$ outside the submatrix, and those two values are written back into \mathbf{C} because their Mask values are 1. The (1,0) entry is deleted because the entry $\mathbf{Z}(1,0) = 21$ is prevented from being written back into \mathbf{C} since Mask(1,0)=0.

Now consider the analogous GxB_subassign operation. The Mask has the same size as A, namely:

$$\mathbf{M} = [1].$$

After computing $\mathbf{C}(0,1)\langle \mathbf{M} \rangle = \mathbf{C}(0,1) + \mathbf{A}$, the result is

$$\mathbf{C} = \left[\begin{array}{cc} 11 & 112 \\ 21 & 22 \end{array} \right].$$

Only the $\mathbf{C}(\mathbf{I}, \mathbf{J})$ submatrix, the single entry $\mathbf{C}(0, 1)$, is modified by $\mathtt{GxB_subassign}$. The entry $\mathbf{C}(1, 0) = 21$ is unaffected by $\mathtt{GxB_subassign}$, but it is deleted by $\mathtt{GrB_assign}$.

12.11.2 Performance of GxB_subassign, GrB_assign and GrB_*_setElement

When SuiteSparse:GraphBLAS uses non-blocking mode, the modifications to a matrix by GxB_subassign, GrB_assign, and GrB_*_setElement can postponed, and computed all at once later on. This has a huge impact on performance.

A sequence of assignments is fast if their completion can be postponed for as long as possible, or if they do not modify the pattern at all. Modifying the pattern can be costly, but it is fast if non-blocking mode can be fully exploited.

Consider a sequence of t submatrix assignments C(I,J)=C(I,J)+A to an n-by-n matrix C where each submatrix A has size a-by-a with s entries, and where C starts with s entries. Assume the matrices are all stored in non-hypersparse form, by row ($GrB_ROWMAJOR$).

If blocking mode is enabled, or if the sequence requires the matrix to be completed after each assignment, each of the t assignments takes $O(a + s \log n)$ time to process the A matrix and then $O(n + c + s \log s)$ time to complete C. The latter step uses GrB_*build to build an update matrix and then merge it with C. This step does not occur if the sequence of assignments does not add new entries to the pattern of C, however. Assuming in the worst case that the pattern does change, the total time is $O(t [a + s \log n + n + c + s \log s])$.

If the sequence can be computed with all updates postponed until the end of the sequence, then the total time is no worse than $O(a+s\log n)$ to process each A matrix, for t assignments, and then a single build at the end, taking $O(n+c+st\log st)$ time. The total time is $O(t\left[a+s\log n\right]+(n+c+st\log st))$. If no new entries appear in C the time drops to $O(t\left[a+s\log n\right])$, and in this case, the time for both methods is the same; both are equally efficient.

A few simplifying assumptions are useful to compare these times. Consider a graph of n nodes with O(n) edges, and with a constant bound on the degree of each node. The asymptotic bounds assume a worst-case scenario where C has a least some dense rows (thus the $\log n$ terms). If these are not present, if both t and c are O(n), and if a and s are constants, then the total time with blocking mode becomes $O(n^2)$, assuming the pattern of C changes at each assignment. This very high for a sparse graph problem. In contrast, the non-blocking time becomes $O(n \log n)$ under these same assumptions, which is asymptotically much faster.

The difference in practice can be very dramatic, since n can be many millions for sparse graphs with n nodes and O(n), which can be handled on a commodity laptop.

The following guidelines should be considered when using GxB_subassign, GrB_assign and GrB_*_setElement.

- 1. A sequence of assignments that does not modify the pattern at all is fast, taking as little as $\Omega(1)$ time per entry modified. The worst case time complexity is $O(\log n)$ per entry, assuming they all modify a dense row of C with n entries, which can occur in practice. It is more common, however, that most rows of C have a constant number of entries, independent of n. No work is ever left pending when the pattern of C does not change.
- 2. A sequence of assignments that modifies the entries that already exist in the pattern of a matrix, or adds new entries to the pattern (using the same accum operator), but does not delete any entries, is fast. The matrix is not completed until the end of the sequence.
- 3. Similarly, a sequence that modifies existing entries, or deletes them, but does not add new ones, is also fast. This sequence can also repeatedly delete pre-existing entries and then reinstate them and still be fast. The matrix is not completed until the end of the sequence.
- 4. A sequence that mixes assignments of types (2) and (3) above can be costly, since the matrix may need to be completed after each assignment. The time complexity can become quadratic in the worst case.
- 5. However, any single assignment takes no more than $O(a+s\log n+n+c+s\log s)$ time, even including the time for a matrix completion, where C is n-by-n with c entries and A is a-by-a with s entries. This time is essentially linear in the size of the matrix C, if A is relatively small and sparse compared with C. In this case, n+c are the two dominant terms.
- 6. In general, GxB_subassign is faster than GrB_assign. If GrB_REPLACE is used with GrB_assign, the entire matrix C must be traversed. This is much slower than GxB_subassign, which only needs to examine the C(I,J) submatrix. Furthermore, GrB_assign must deal with a much larger Mask matrix, whereas GxB_subassign has a smaller mask. Since

its mask is smaller, GxB_subassign takes less time than GrB_assign to access the mask.

Submatrix assignment in SuiteSparse:GraphBLAS is extremely efficient, even without considering the advantages of non-blocking mode discussed in Section 12.11. It can be up to 1000x faster than MATLAB R2019b, or even higher depending on the kind of matrix assignment. MATLAB logical indexing (the mask of GraphBLAS) is extremely faster with GraphBLAS as compared in MATLAB R2019b; differences of up to 250,000x have been observed (0.4 seconds in GraphBLAS versus 28 hours in MATLAB).

All of the algorithmic variants of assign/subassign in SuiteSparse:GraphBLAS are either asymptotically optimal, or to within a log factor of being asymptotically optimal. The methods are also fully parallel. For hypersparse matrices, the term n in the expressions in the above discussion is dropped, and is replaced with $h \log h$, at the worst case, where h << n is the number of non-empty columns of a hypersparse matrix stored by column, or the number of non-empty rows of a hypersparse matrix stored by row. In many methods, n is replaced with h, not $h \log h$.

12.12 GrB_apply: apply a unary, binary, or index-unary operator

GrB_apply is the generic name for 92 specific functions:

- GrB_Vector_apply and GrB_Matrix_apply apply a unary operator to the entries of a matrix (two variants).
- GrB_*_apply_BinaryOp1st_* applies a binary operator where a single scalar is provided as the x input to the binary operator. There are 30 variants, depending on the type of the scalar: (matrix or vector) x (13 built-in types, one for user-defined types, and a version for GrB_Scalar).
- GrB_*_apply_BinaryOp2nd_* applies a binary operator where a single scalar is provided as the *y* input to the binary operator. There are 30 variants, depending on the type of the scalar: (matrix or vector) x (13 built-in types, one for user-defined types, and a version for GrB_Scalar).
- GrB_*_apply_IndexOp_* applies a GrB_IndexUnaryOp, single scalar is provided as the scalar y input to the index-unary operator. There are 30 variants, depending on the type of the scalar: (matrix or vector) x (13 built-in types, one for user-defined types, and a version for GrB_Scalar).

The generic name appears in the function prototypes, but the specific function name is used when describing each variation. When discussing features that apply to all versions, the simple name GrB_apply is used.

12.12.1 GrB_Vector_apply: apply a unary operator to a vector

GrB_Vector_apply applies a unary operator to the entries of a vector, analogous to t = op(u) in MATLAB except the operator op is only applied

to entries in the pattern of \mathbf{u} . Implicit values outside the pattern of \mathbf{u} are not affected. The entries in \mathbf{u} are typecasted into the **xtype** of the unary operator. The vector \mathbf{t} has the same type as the **ztype** of the unary operator. The final step is $\mathbf{w}\langle\mathbf{m}\rangle = \mathbf{w}\odot\mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

12.12.2 GrB_Matrix_apply: apply a unary operator to a matrix

GrB_Matrix_apply applies a unary operator to the entries of a matrix, analogous to T = op(A) in MATLAB except the operator op is only applied to entries in the pattern of A. Implicit values outside the pattern of A are not affected. The input matrix A may be transposed first. The entries in A are typecasted into the xtype of the unary operator. The matrix T has the same type as the ztype of the unary operator. The final step is $C\langle M \rangle = C \odot T$, as described in Section 2.3.

The built-in $GrB_IDENTITY_T$ operators (one for each built-in type T) are very useful when combined with this function, enabling it to compute $C\langle M \rangle = C \odot A$. This makes GrB_apply a direct interface to the accumulator/mask function for both matrices and vectors. The $GrB_IDENTITY_T$ operators also provide the fastest stand-alone typecasting methods in Suite-Sparse:GraphBLAS, with all $13 \times 13 = 169$ methods appearing as individual functions, to typecast between any of the 13 built-in types.

To compute $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{A}$ or $\mathbf{C}\langle \mathbf{M} \rangle = \mathbf{C} \odot \mathbf{A}$ for user-defined types, the user application would need to define an identity operator for the type. Since GraphBLAS cannot detect that it is an identity operator, it must call the operator to make the full copy T=A and apply the operator to each entry of the matrix or vector.

The other GraphBLAS operation that provides a direct interface to the accumulator/mask function is GrB_transpose, which does not require an operator to perform this task. As a result, GrB_transpose can be used as

an efficient and direct interface to the accumulator/mask function for both built-in and user-defined types. However, it is only available for matrices, not vectors.

12.12.3 GrB_Vector_apply_BinaryOp1st: apply a binary operator to a vector; 1st scalar binding

```
GrB_Info GrB_apply
                                    // w<mask> = accum (w, op(x,u))
                                    // input/output vector for results
   GrB_Vector w,
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const GrB_BinaryOp op,
                                    // operator to apply to the entries
    <type> x,
                                    // first input: scalar x
    const GrB_Vector u,
                                    // second input: vector u
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

 ${\tt GrB_Vector_apply_Binary0p1st_<type>}$ applies a binary operator z=f(x,y) to a vector, where a scalar x is bound to the first input of the operator. The scalar x can be a non-opaque C scalar corresponding to a built-in type, a void * for user-defined types, or a ${\tt GrB_Scalar}$. It is otherwise identical to ${\tt GrB_Vector_apply}$.

The op can be any binary operator except that it cannot be a user-defined <code>GrB_BinaryOp</code> created by <code>GxB_BinaryOp_new_IndexOp</code>. For backward compatibility with prior versions of <code>SuiteSparse:GraphBLAS</code>, built-in index-based binary operators such as <code>GxB_FIRSTI_INT32</code> may be used, however. The equivalent index-unary operators are used in their place.

12.12.4 GrB_Vector_apply_BinaryOp2nd: apply a binary operator to a vector; 2nd scalar binding

```
GrB_Info GrB_apply
                                    // w<mask> = accum (w, op(u,y))
    GrB_Vector w,
                                    // input/output vector for results
                                    // optional mask for w, unused if NULL
    const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const GrB_BinaryOp op,
                                    // operator to apply to the entries
    const GrB_Vector u,
                                    // first input: vector u
                                    // second input: scalar y
    <type> y,
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB_Vector_apply_BinaryOp2nd_<type> applies a binary operator z = f(x,y) to a vector, where a scalar y is bound to the second input of the operator. The scalar x can be a non-opaque C scalar corresponding to a built-in type, a void * for user-defined types, or a GrB_Scalar. It is otherwise identical to GrB_Vector_apply.

The op can be any binary operator except that it cannot be a user-defined GrB_BinaryOp created by GxB_BinaryOp_new_IndexOp. For backward compatibility with prior versions of SuiteSparse:GraphBLAS, built-in index-based binary operators such as GxB_FIRSTI_INT32 may be used, however. The equivalent index-unary operators are used in their place.

12.12.5 GrB_Vector_apply_IndexOp: apply an index-unary operator to a vector

```
GrB_Info GrB_apply
                                    // w<mask> = accum (w, op(u,y))
    GrB_Vector w,
                                    // input/output vector for results
                                    // optional mask for w, unused if NULL
    const GrB_Vector mask,
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
                                    // operator to apply to the entries
    const GrB_IndexUnaryOp op,
    const GrB_Vector u,
                                    // first input: vector u
    const <type> y,
                                    // second input: scalar y
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB_Vector_apply_IndexOp_<type> applies an index-unary operator z = f(x, i, 0, y) to a vector. The scalar y can be a non-opaque C scalar corresponding to a built-in type, a void * for user-defined types, or a GrB_Scalar. It is otherwise identical to GrB_Vector_apply.

12.12.6 GrB_Matrix_apply_BinaryOp1st: apply a binary operator to a matrix; 1st scalar binding

```
GrB_Info GrB_apply
                                     // C < M > = accum(C, op(x, A))
    GrB_Matrix C,
                                     // input/output matrix for results
                                     // optional mask for C, unused if NULL
    const GrB_Matrix Mask,
    const GrB_BinaryOp accum,
                                     // optional accum for Z=accum(C,T)
    const GrB_BinaryOp op,
                                     // operator to apply to the entries
    <type> x,
                                     // first input: scalar x
    const GrB_Matrix A,
                                     // second input: matrix A
    const GrB_Descriptor desc
                                     // descriptor for C, mask, and A
);
```

 ${\tt GrB_Matrix_apply_Binary0p1st_<type>}$ applies a binary operator z=f(x,y) to a matrix, where a scalar x is bound to the first input of the operator. The scalar x can be a non-opaque C scalar corresponding to a built-in type, a void * for user-defined types, or a ${\tt GrB_Scalar}$. It is otherwise identical to ${\tt GrB_Matrix_apply}$.

The op can be any binary operator except that it cannot be a user-defined GrB_BinaryOp created by GxB_BinaryOp_new_IndexOp. For backward compatibility with prior versions of SuiteSparse:GraphBLAS, built-in index-based binary operators such as GxB_FIRSTI_INT32 may be used, however. The equivalent index-unary operators are used in their place.

12.12.7 GrB_Matrix_apply_BinaryOp2nd: apply a binary operator to a matrix; 2nd scalar binding

```
GrB_Info GrB_apply
                                    // C<M>=accum(C,op(A,y))
   GrB_Matrix C,
                                    // input/output matrix for results
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
    const GrB_BinaryOp op,
                                    // operator to apply to the entries
    const GrB_Matrix A,
                                    // first input: matrix A
    <type> y,
                                    // second input: scalar y
    const GrB_Descriptor desc
                                    // descriptor for C, mask, and A
);
```

GrB_Matrix_apply_BinaryOp2nd_<type> applies a binary operator z = f(x, y) to a matrix, where a scalar x is bound to the second input of the

operator. The scalar y can be a non-opaque C scalar corresponding to a builtin type, a void * for user-defined types, or a GrB_Scalar. It is otherwise identical to GrB_Matrix_apply.

The op can be any binary operator except that it cannot be a user-defined <code>GrB_BinaryOp</code> created by <code>GxB_BinaryOp_new_IndexOp</code>. For backward compatibility with prior versions of <code>SuiteSparse:GraphBLAS</code>, built-in index-based binary operators such as <code>GxB_FIRSTI_INT32</code> may be used, however. The equivalent index-unary operators are used in their place.

12.12.8 GrB_Matrix_apply_IndexOp: apply an index-unary operator to a matrix

```
// C<M>=accum(C,op(A,y))
GrB_Info GrB_apply
                                    // input/output matrix for results
    GrB_Matrix C,
    const GrB_Matrix Mask,
                                    // optional mask for C, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
                                    // operator to apply to the entries
    const GrB_IndexUnaryOp op,
    const GrB_Matrix A,
                                    // first input: matrix A
    const <type> y,
                                    // second input: scalar y
    const GrB_Descriptor desc
                                    // descriptor for C, mask, and A
);
```

 ${\tt GrB_Matrix_apply_IndexOp_<type>}$ applies an index-unary operator z=f(x,i,j,y) to a matrix. The scalar y can be a non-opaque C scalar corresponding to a built-in type, a void * for user-defined types, or a ${\tt GrB_Scalar}$. It is otherwise identical to ${\tt GrB_Matrix_apply}$.

12.13 GrB_select: select entries based on an index-unary operator

The GrB_select function is the generic name for 30 specific functions, depending on whether it operates on a matrix or vector, and depending on the type of the scalar y: (matrix or vector) x (13 built-in types, void * for user-defined types, and a GrB_Scalar). The generic name appears in the function prototypes, but the specific function name is used when describing each variation. When discussing features that apply to both versions, the simple name GrB_select is used.

12.13.1 GrB_Vector_select: select entries from a vector

```
// w<mask> = accum (w, op(u))
GrB_Info GrB_select
                                    // input/output vector for results
    GrB_Vector w,
    const GrB_Vector mask,
                                    // optional mask for w, unused if NULL
    const GrB_BinaryOp accum,
                                    // optional accum for z=accum(w,t)
    const GrB_IndexUnaryOp op,
                                    // operator to apply to the entries
    const GrB_Vector u,
                                    // first input: vector u
    const <type> y,
                                    // second input: scalar y
    const GrB_Descriptor desc
                                    // descriptor for w and mask
);
```

GrB_Vector_select_* applies a GrB_IndexUnaryOp operator to the entries of a vector. If the operator evaluates as true for the entry u(i), it is copied to the vector t, or not copied if the operator evaluates to false. The vector t is then written to the result w via the mask/accumulator step. This operation operates on vectors just as if they were m-by-1 matrices, except that GraphBLAS never transposes a vector via the descriptor. Refer to the next section (12.13.2) on GrB_Matrix_select for more details.

12.13.2 GrB_Matrix_select: apply a select operator to a matrix

```
GrB_Info GrB_select
                                    // C<M>=accum(C,op(A))
    GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C, unused if NULL
    const GrB_Matrix Mask,
    const GrB_BinaryOp accum,
                                    // optional accum for Z=accum(C,T)
    const GrB_IndexUnaryOp op,
                                    // operator to apply to the entries
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Scalar y,
                                    // second input: scalar y
    const GrB_Descriptor desc
                                    // descriptor for C, mask, and A
);
```

GrB_Matrix_select_* applies a GrB_IndexUnaryOp operator to the entries of a matrix. If the operator evaluates as true for the entry A(i,j), it is copied to the matrix T, or not copied if the operator evaluates to false. The input matrix A may be transposed first. The entries in A are typecasted into the xtype of the select operator. The final step is $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathbf{T}$, as described in Section 2.3.

The matrix T has the same size and type as A (or the transpose of A if the input is transposed via the descriptor). The entries of T are a subset of those of A. Each entry A(i,j) of A is passed to the op, as $z=f(a_{ij},i,j,y)$. If A is transposed first then the operator is applied to entries in the transposed matrix, A'. If z is returned as true, then the entry is copied into T, unchanged. If it returns false, the entry does not appear in T.

The action of GrB_select with the built-in index-unary operators is described in the table below. The MATLAB analogs are precise for tril and triu, but shorthand for the other operations. The MATLAB diag function returns a column with the diagonal, if A is a matrix, whereas the matrix T in GrB_select always has the same size as A (or its transpose if the GrB_INPO is set to GrB_TRAN). In the MATLAB analog column, diag is as if it operates like GrB_select, where T is a matrix.

The following operators may be used on matrices with a user-defined type: GrB_ROWINDEX_*, GrB_COLINDEX_*, GrB_DIAGINDEX_*, GrB_TRIL, GrB_TRIU, GrB_DIAG, GrB_OFFIAG, GrB_COLLE, GrB_COLGT, GrB_ROWLE, and GrB_ROWGT.

For floating-point values, comparisons with NaN always return false. The GrB_VALUE* operators should not be used with a scalar y that is equal to NaN. For this case, create a user-defined index-unary operator that performs the test with the ANSI C isnan function instead.

| G I DI LO | 3.5.4.EET. 4.D. / O | 1 |
|----------------------------|--|--|
| GraphBLAS name | MATLAB/Octave | description |
| | analog | |
| <pre>GrB_ROWINDEX_*</pre> | z=i+y | select A(i,j) if i != -y |
| <pre>GrB_COLINDEX_*</pre> | z=j+y | select A(i,j) if j != -y |
| <pre>GrB_DIAGINDEX_*</pre> | z=j-(i+y) | select A(i,j) if j != i+y |
| GrB_TRIL | z=(j<=(i+y)) | select entries on or below the yth diagonal |
| GrB_TRIU | z=(j>=(i+y)) | select entries on or above the yth diagonal |
| GrB_DIAG | z=(j==(i+y)) | select entries on the yth diagonal |
| <pre>GrB_OFFDIAG</pre> | z=(j!=(i+y)) | select entries not on the yth diagonal |
| GrB_COLLE | z=(j<=y) | select entries in columns 0 to y |
| GrB_COLGT | z=(j>y) | select entries in columns y+1 and above |
| GrB_ROWLE | $z=(i \le y)$ | select entries in rows 0 to y |
| <pre>GrB_ROWGT</pre> | z=(i>y) | select entries in rows $y+1$ and above |
| GrB_VALUENE_T | z=(aij!=y) | select A(i,j) if it is not equal to y |
| GrB_VALUEEQ_T | z=(aij==y) | select A(i,j) is it equal to y |
| <pre>GrB_VALUEGT_T</pre> | z=(aij>y) | select A(i,j) is it greater than y |
| <pre>GrB_VALUEGE_T</pre> | z=(aij>=y) | select A(i,j) is it greater than or equal to y |
| GrB_VALUELT_T | z=(aij <y)< td=""><td>select A(i,j) is it less than y</td></y)<> | select A(i,j) is it less than y |
| <pre>Grb_VALUELE_T</pre> | $z=(aij \le y)$ | select A(i,j) is it less than or equal to y |

12.14 GrB reduce: reduce to a vector or scalar

The generic function name <code>GrB_reduce</code> may be used for all specific functions discussed in this section. When the details of a specific function are discussed, the specific name is used for clarity.

12.14.1 GrB_Matrix_reduce_Monoid reduce a matrix to a vector

GrB_Matrix_reduce_Monoid reduces a matrix to a column vector using a monoid, roughly analogous to t = sum (A') in MATLAB, in the default case, where t is a column vector. By default, the method reduces across the rows to obtain a column vector; use GrB_TRAN to reduce down the columns.

The input matrix A may be transposed first. Its entries are then typecast into the type of the reduce operator or monoid. The reduction is applied to all entries in A (i,:) to produce the scalar t (i). This is done without the use of the identity value of the monoid. If the ith row A (i,:) has no entries, then (i) is not an entry in t and its value is implicit. If A (i,:) has a single entry, then that is the result t (i) and reduce is not applied at all for the ith row. Otherwise, multiple entries in row A (i,:) are reduced via the reduce operator or monoid to obtain a single scalar, the result t (i).

The final step is $\mathbf{w}\langle \mathbf{m} \rangle = \mathbf{w} \odot \mathbf{t}$, as described in Section 2.3, except that all the terms are column vectors instead of matrices.

12.14.2 GrB_Vector_reduce_<type>: reduce a vector to a scalar

```
// c = accum (c, reduce_to_scalar (u))
GrB_Info GrB_reduce
    <type> *c,
                                    // result scalar
                                    // optional accum for c=accum(c,t)
    const GrB_BinaryOp accum,
    const GrB_Monoid monoid,
                                    // monoid to do the reduction
    const GrB_Vector u,
                                    // vector to reduce
    const GrB_Descriptor desc
                                    // descriptor (currently unused)
);
GrB_Info GrB_reduce
                                    // c = accum (c, reduce_to_scalar (u))
   GrB_Scalar c,
                                    // result scalar
    const GrB_BinaryOp accum,
                                    // optional accum for c=accum(c,t)
    const GrB_Monoid monoid,
                                    // monoid to do the reduction
    const GrB_Vector u,
                                    // vector to reduce
    const GrB_Descriptor desc
                                    // descriptor (currently unused)
);
```

GrB_Vector_reduce_<type> reduces a vector to a scalar, analogous to
t = sum (u) in MATLAB, except that in GraphBLAS any commutative and
associative monoid can be used in the reduction.

The scalar c can be a pointer C type: bool, int8_t, ... float, double, or void * for a user-defined type, or a GrB_Scalar. If c is a void * pointer to a user-defined type, the type must be identical to the type of the vector u. This cannot be checked by GraphBLAS and thus results are undefined if the types are not the same.

If the vector u has no entries, that identity value of the monoid is copied into the scalar t (unless c is a GrB_Scalar, in which case t is an empty GrB_Scalar, with no entry). Otherwise, all of the entries in the vector are reduced to a single scalar using the monoid.

The descriptor is unused, but it appears in case it is needed in future versions of the GraphBLAS API. This function has no mask so its accumulator/mask step differs from the other GraphBLAS operations. It does not use the methods described in Section 2.3, but uses the following method instead.

If accum is NULL, then the scalar t is typecast into the type of c, and c = t is the final result. Otherwise, the scalar t is typecast into the ytype of the accum operator, and the value of c (on input) is typecast into the xtype of the accum operator. Next, the scalar z = accum (c,t) is computed, of the ztype of the accum operator. Finally, z is typecast into the final result, c.

If c is a non-opaque scalar, no error message can be returned by GrB_error. If c is a GrB_Scalar, then GrB_error(&err,c) can be used to return an error string, if an error occurs.

12.14.3 GrB_Matrix_reduce_<type>: reduce a matrix to a scalar

```
// c = accum (c, reduce_to_scalar (A))
GrB_Info GrB_reduce
                                    // result scalar
    <type> *c,
    const GrB_BinaryOp accum,
                                    // optional accum for c=accum(c,t)
    const GrB_Monoid monoid,
                                    // monoid to do the reduction
                                    // matrix to reduce
    const GrB_Matrix A,
                                    // descriptor (currently unused)
    const GrB_Descriptor desc
);
GrB_Info GrB_reduce
                                    // c = accum (c, reduce_to_scalar (A))
                                    // result scalar
    GrB_Scalar c,
                                    // optional accum for c=accum(c,t)
   const GrB_BinaryOp accum,
    const GrB_Monoid monoid,
                                    // monoid to do the reduction
    const GrB_Matrix A,
                                    // matrix to reduce
    const GrB_Descriptor desc
                                    // descriptor (currently unused)
);
```

GrB_Matrix_reduce_<type> reduces a matrix A to a scalar, roughly analogous to t = sum (A (:)) in MATLAB. This function is identical to reducing a vector to a scalar, since the positions of the entries in a matrix or vector have no effect on the result. Refer to the reduction to scalar described in the previous Section 12.14.2.

12.15 GrB_transpose: transpose a matrix

GrB_transpose transposes a matrix A, just like the array transpose T = A.' in MATLAB. The internal result matrix T = A' (or merely T = A if A is transposed via the descriptor) has the same type as A. The final step is $\mathbf{C}\langle\mathbf{M}\rangle = \mathbf{C}\odot\mathbf{T}$, as described in Section 2.3, which typecasts T as needed and applies the mask and accumulator.

To be consistent with the rest of the GraphBLAS API regarding the descriptor, the input matrix A may be transposed first by setting the GrB_INPO setting to GrB_TRAN. This results in a double transpose, and thus A is not transposed is computed.

12.16 GrB_kronecker: Kronecker product

```
GrB_Info GrB_kronecker
                                    // C<Mask> = accum (C, kron(A,B))
    GrB_Matrix C,
                                    // input/output matrix for results
                                    // optional mask for C, unused if NULL
   const GrB_Matrix Mask,
                                    // optional accum for Z=accum(C,T)
    const GrB_BinaryOp accum,
    const <operator> op,
                                    // defines '*' for T=kron(A,B)
    const GrB_Matrix A,
                                    // first input: matrix A
    const GrB_Matrix B,
                                    // second input: matrix B
    const GrB_Descriptor desc
                                    // descriptor for C, Mask, A, and B
);
```

GrB_kronecker computes the Kronecker product, $\mathbf{C}\langle\mathbf{M}\rangle=\mathbf{C}\odot\mathrm{kron}(\mathbf{A},\mathbf{B})$ where

$$\operatorname{kron}(\mathbf{A}, \mathbf{B}) = \begin{bmatrix} a_{00} \otimes \mathbf{B} & \dots & a_{0,n-1} \otimes \mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{m-1,0} \otimes \mathbf{B} & \dots & a_{m-1,n-1} \otimes \mathbf{B} \end{bmatrix}$$

The \otimes operator is defined by the op parameter. It is applied in an element-wise fashion (like $GrB_eWiseMult$), where the pattern of the submatrix $a_{ij}\otimes B$ is the same as the pattern of B if a_{ij} is an entry in the matrix A, or empty otherwise. The input matrices A and B can be of any dimension, and both matrices may be transposed first via the descriptor, desc. Entries in A and B are typecast into the input types of the op. The matrix T=kron(A,B) has the same type as the ztype of the binary operator, op. The final step is $C\langle M \rangle = C \odot T$, as described in Section 2.3.

The operator op may be a GrB_BinaryOp, a GrB_Monoid, or a GrB_Semiring. In the latter case, the multiplicative operator of the semiring is used. The op may be a binary operator created by GxB_BinaryOp_new_IndexOp.

13 Printing GraphBLAS objects

The ten different objects handled by SuiteSparse:GraphBLAS are all opaque, although nearly all of their contents can be extracted via methods such as GrB_Matrix_extractTuples, GrB_Matrix_extractElement, GrB_get, and so on. The GraphBLAS C API has no mechanism for printing all the contents of GraphBLAS objects, but this is helpful for debugging. Ten type-specific methods and two type-generic methods are provided:

| GxB_Type_fprint | print and check a GrB_Type |
|------------------------------------|-------------------------------------|
| <pre>GxB_UnaryOp_fprint</pre> | print and check a GrB_UnaryOp |
| <pre>GxB_BinaryOp_fprint</pre> | print and check a GrB_BinaryOp |
| <pre>GxB_IndexUnaryOp_fprint</pre> | print and check a GrB_IndexUnaryOp |
| ${\tt GxB_IndexBinary0p_fprint}$ | print and check a GxB_IndexBinaryOp |
| <pre>GxB_Monoid_fprint</pre> | print and check a GrB_Monoid |
| GxB_Semiring_fprint | print and check a GrB_Semiring |
| <pre>GxB_Descriptor_fprint</pre> | print and check a GrB_Descriptor |
| <pre>GxB_Context_fprint</pre> | print and check a GxB_Context |
| <pre>GxB_Matrix_fprint</pre> | print and check a GrB_Matrix |
| <pre>GxB_Vector_fprint</pre> | print and check a GrB_Vector |
| <pre>GxB_Scalar_fprint</pre> | print and check a GrB_Scalar |
| GxB_fprint | print/check any object to a file |
| GxB_print | print/check any object to stdout |
| | |

These methods do not modify the status of any object, and thus they cannot return an error string for use by GrB_error.

If a matrix or vector has not been completed, the pending computations are guaranteed to *not* be performed. The reason is simple. It is possible for a bug in the user application (such as accessing memory outside the bounds of an array) to mangle the internal content of a GraphBLAS object, and the <code>GxB_*print</code> methods can be helpful tools to track down this bug. If <code>GxB_*print</code> attempted to complete any computations prior to printing or checking the contents of the matrix or vector, then further errors could occur, including a segfault.

By contrast, GraphBLAS methods and operations that return values into user-provided arrays or variables might finish pending operations before the return these values, and this would change their state. Since they do not change the state of any object, the <code>GxB_*print</code> methods provide a useful method for debugging, and for a quick understanding of what GraphBLAS is computing while developing a user application.

Each of the methods has a parameter of type GxB_Print_Level that specifies the amount to print:

The ten type-specific functions include an additional argument, the name string. The name is printed at the beginning of the display (assuming the print level is not GxB_SILENT) so that the object can be more easily identified in the output. For the type-generic methods GxB_fprint and GxB_print, the name string is the variable name of the object itself.

If the file f is NULL, stdout is used. If name is NULL, it is treated as the empty string. These are not error conditions.

The methods check their input objects carefully and extensively, even when pr is equal to GxB_SILENT. The following error codes can be returned:

- GrB_SUCCESS: object is valid
- Grb_UNINITIALIZED_OBJECT: object is not initialized
- Grb_INVALID_OBJECT: object is not valid
- Grb_NULL_POINTER: object is a NULL pointer
- GrB_INVALID_VALUE: fprintf returned an I/O error.

The content of any GraphBLAS object is opaque, and subject to change. As a result, the exact content and format of what is printed is implementation-dependent, and will change from version to version of SuiteSparse:GraphBLAS. Do not attempt to rely on the exact content or format by trying to parse the resulting output via another program. The intent of these functions is to produce a report of an object for visual inspection. If the user application needs to extract content from a GraphBLAS matrix or vector, use GrB_*cextractTuples or the import/export methods instead.

GraphBLAS matrices and vectors are zero-based, where indices of an n-by-n matrix are in the range 0 to n-1. However, MATLAB, Octave,

and Julia prefer to print their matrices and vectors as one-based. To enable 1-based printing, use GrB_set (GrB_GLOBAL, true, GxB_PRINT_1BASED). Printing is done as zero-based by default.

13.1 GxB_fprint: Print a GraphBLAS object to a file

The GxB_fprint function prints the contents of any of the ten Graph-BLAS objects to the file f. If f is NULL, the results are printed to stdout. For example, to print the entire contents of a matrix A to the file f, use GxB_fprint (A, GxB_COMPLETE, f).

13.2 GxB_print: Print a GraphBLAS object to stdout

GxB_print is the same as GxB_fprint, except that it prints the contents of the object to stdout instead of a file f. For example, to print the entire contents of a matrix A, use GxB_print (A, GxB_COMPLETE).

13.3 GxB_Type_fprint: Print a GrB_Type

For example, GxB_Type_fprint (GrB_BOOL, "boolean type", GxB_COMPLETE, f) prints the contents of the GrB_BOOL object to the file f.

13.4 GxB_UnaryOp_fprint: Print a GrB_UnaryOp

For example, GxB_UnaryOp_fprint (GrB_LNOT, "not", GxB_COMPLETE, f) prints the GrB_LNOT unary operator to the file f.

13.5 GxB_BinaryOp_fprint: Print a GrB_BinaryOp

For example, GxB_BinaryOp_fprint (GrB_PLUS_FP64, "plus", GxB_COMPLETE, f) prints the GrB_PLUS_FP64 binary operator to the file f.

13.6 GxB_IndexUnaryOp_fprint: Print a GrB_IndexUnaryOp

```
GrB_Info GxB_IndexUnaryOp_fprint  // print and check a GrB_IndexUnaryOp
(
    GrB_IndexUnaryOp op,  // object to print and check
    const char *name,  // name of the object
    int pr,  // print level (GxB_Print_Level)
    FILE *f  // file for output
);
```

For example, GrB_IndexUnaryOp_fprint (GrB_TRIL, "tril", GxB_COMPLETE, f) prints the GrB_TRIL index-unary operator to the file f.

13.7 GxB_IndexBinaryOp_fprint: Print a GxB_IndexBinaryOp

```
GrB_Info GxB_IndexBinaryOp_fprint  // print and check a GxB_IndexBinaryOp
(
    GxB_IndexBinaryOp op,  // object to print and check
    const char *name,  // name of the object
    int pr,  // print level (GxB_Print_Level)
    FILE *f  // file for output
);
```

13.8 GxB_Monoid_fprint: Print a GrB_Monoid

For example, GxB_Monoid_fprint (GxB_PLUS_FP64_MONOID, "plus monoid", GxB_COMPLETE, f) prints the predefined GxB_PLUS_FP64_MONOID (based on the binary operator GrB_PLUS_FP64) to the file f.

13.9 GxB_Semiring_fprint: Print a GrB_Semiring

For example, GxB_Semiring_fprint (GxB_PLUS_TIMES_FP64, "standard", GxB_COMPLETE, f) prints the predefined GxB_PLUS_TIMES_FP64 semiring to the file f.

13.10 GxB_Descriptor_fprint: Print a GrB_Descriptor

For example, GxB_Descriptor_fprint (d, "descriptor", GxB_COMPLETE, f) prints the descriptor d to the file f.

13.11 GxB_Context_fprint: Print a GxB_Context

This method can be used to print the context created for a user thread, or the contents of the GxB_CONTEXT_WORLD object.

13.12 GxB_Matrix_fprint: Print a GrB_Matrix

For example, GxB_Matrix_fprint (A, "my matrix", GxB_SHORT, f) prints about 30 entries from the matrix A to the file f.

13.13 GxB_Vector_fprint: Print a GrB_Vector

For example, GxB_Vector_fprint (v, "my vector", GxB_SHORT, f) prints about 30 entries from the vector v to the file f.

13.14 GxB_Scalar_fprint: Print a GrB_Scalar

For example, GxB_Scalar_fprint (s, "my scalar", GxB_SHORT, f) prints a short description of the scalar s to the file f.

13.15 Performance and portability considerations

Even when the print level is GxB_SILENT, these methods extensively check the contents of the objects passed to them, which can take some time. They should be considered debugging tools only, not for final use in production.

The return value of the GxB_*print methods can be relied upon, but the output to the file (or stdout) can change from version to version. If these methods are eventually added to the GraphBLAS C API Specification, a conforming implementation might never print anything at all, regardless of the pr value. This may be essential if the GraphBLAS library is installed in a dedicated device, with no file output, for example.

Some implementations may wish to print nothing at all if the matrix is not yet completed, or just an indication that the matrix has pending operations and cannot be printed, when non-blocking mode is employed. In this case, use GrB_Matrix_wait, GrB_Vector_wait, or GxB_Scalar_wait to finish all

pending computations first. If a matrix or vector has pending operations, SuiteSparse:GraphBLAS prints a list of the *pending tuples*, which are the entries not yet inserted into the primary data structure. It can also print out entries that remain in the data structure but are awaiting deletion; these are called *zombies* in the output report.

Most of the rest of the report is self-explanatory.

14 Matrix and Vector iterators

The GxB_Iterator is an object that allows user applications to iterate over the entries of a matrix or vector, one entry at a time. Iteration can be done in a linear manner (analogous to reading a file one entry at a time, from start to finish), or in a random-access pattern (analogous to the fseek method for repositioning the access to file to a different position).

Multiple iterators can be used on a single matrix or vector, even in parallel by multiple user threads. While a matrix or vector is being used with an iterator, the matrix or vector must not be modified. Doing so will lead to undefined results.

Since accessing a matrix or vector via an iterator requires many calls to the iterator methods, they must be very fast. Error checking is skipped, except for the methods that create, attach, or free an iterator. Methods that advance an iterator or that access values or indices from a matrix or vector do not return error conditions. Instead, they have well-defined preconditions that must be met (and which should be checked by the user application). If those preconditions are not met, results are undefined.

The iterator methods are implemented in SuiteSparse:GraphBLAS as both macros (via #define) and as functions of the same name that appear in the compiled libgraphblas.so library. This requires that the opaque contents of the iterator object be defined in GraphBLAS.h itself. The user application must not access these contents directly, but can only do so safely via the iterator methods provided by SuiteSparse:GraphBLAS.

The iterator object can be used in one of four sets of methods, for four different access patterns:

- 1. row iterator: iterates across the rows of a matrix, and then within each row to access the entries in a given row. Accessing all the entries of a matrix using a row iterator requires an outer loop (for the rows) and an inner loop (for the entries in each row). A matrix can be accessed via a row iterator only if its format (determined by GrB_get (A, &fmt, GrB_STORAGE_ORIENTATION_HINT)) is by-row (that is, GrB_ROWMAJOR). See Section 10.
- 2. column iterator: iterates across the columns of a matrix, and then within each column to access the entries in a given column. Accessing all the entries of a matrix using a column iterator requires an outer loop

(for the columns) and an inner loop (for the entries in each column). A matrix can be accessed via a column iterator only if its format (determined by GrB_get (A, &fmt, GrB_STORAGE_ORIENTATION_HINT)) is by-column (that is, GrB_COLMAJOR). See Section 10.

- 3. *entry iterator*: iterates across the entries of a matrix. Accessing all the entries of a matrix using an entry iterator requires just a single loop. Any matrix can be accessed with an entry iterator.
- 4. vector iterator: iterates across the entries of a vector. Accessing all the entries of a vector using a vector iterator requires just a single loop. Any vector can be accessed with a vector iterator.

14.1 Creating and destroying an iterator

The process for using an iterator starts with the creation of an iterator, with GxB_Iterator_new. This method creates an iterator object but does not attach it to any specific matrix or vector:

```
GxB_Iterator iterator ;
GxB_Iterator_new (&iterator) ;
When finished, the iterator is freed with either of these methods:
GrB_free (&iterator) ;
GxB_Iterator_free (&iterator) ;
```

14.2 Attaching an iterator to a matrix or vector

This new iterator object can be *attached* to any matrix or vector, and used as a row, column, or entry iterator for any matrix, or as an iterator for any vector. The iterator can be used in any of these methods before it is freed, but with just one access method at a time.

Once it is created, the iterator must be attached to a matrix or vector. This process also selects the method by which the iterator will be used for a matrix. Each of the four GxB_*Iterator_attach methods returns a GrB_Info result.

1. row iterator:

```
GrB_Info info = GxB_rowIterator_attach (iterator, A, desc) ;
```

2. column iterator:

```
GrB_Info info = GxB_colIterator_attach (iterator, A, desc) ;
```

3. entry iterator:

```
GrB_Info info = GxB_Matrix_Iterator_attach (iterator, A, desc) ;
```

4. vector iterator:

```
GrB_Info info = GxB_Vector_Iterator_attach (iterator, v, desc) ;
```

On input to GxB_*Iterator_attach, the iterator must already exist, having been created by GxB_Iterator_new. If the iterator is already attached to a matrix or vector, it is detached and then attached to the given matrix A or vector v.

The return values for row/column methods are:

- GrB_SUCCESS: if the iterator is successfully attached to the matrix A.
- GrB_NULL_POINTER: if the iterator or A are NULL.
- Grb_INVALID_OBJECT: if the matrix A is invalid.
- GrB_NOT_IMPLEMENTED: if the matrix A cannot be iterated in the requested access method (row iterators require the matrix to be held by-row, and column iterators require the matrix to be held by-column).
- Grb_Out_of_memory: if the method runs out of memory.

The other two methods (entry iterator for matrices, or the vector iterator) return the same error codes, except that they do not return GrB_NOT_IMPLEMENTED.

14.3 Seeking to an arbitrary position

Attaching the iterator to a matrix or vector does not define a specific position for the iterator. To use the iterator, a single call to the corresponding *seek* method is required. These GxB*_Iterator_*seek* methods may also be used later on to change the position of the iterator arbitrarily.

1. row iterator:

```
GrB_Info info = GxB_rowIterator_seekRow (iterator, row) ;
GrB_Index kount = GxB_rowIterator_kount (iterator) ;
GrB_Info info = GxB_rowIterator_kseek (iterator, k) ;
```

These methods move a row iterator to a specific row, defined in one of two ways: (1) the row index itself (in range 0 to nrows-1), or (2) by specifying k, which moves the iterator to the kth explicit row (in the range 0 to kount-1). For sparse, bitmap, or full matrices, these two methods are identical. For hypersparse matrices, not all rows are present in the data structure; these implicit rows are skipped and not included in the kount. Implicit rows contain no entries. The GxB_rowIterator_kount method returns the kount of the matrix, where kount is equal to nrows for sparse, bitmap, and matrices, and kount \leq nrows for hypersparse matrices. All three methods listed above can be used for any row iterator.

The GxB_rowIterator_*seek* methods return GrB_SUCCESS if the iterator has been moved to a row that contains at least one entry, GrB_NO_VALUE if the row has no entries, or GxB_EXHAUSTED if the row is out of bounds (row \geq nrows or if k \geq kount). None of these return conditions are errors; they are all informational.

For sparse, bitmap, and full matrices, GxB_rowIterator_seekRow always moves to the given row. For hypersparse matrices, if the requested row is implicit, the iterator is moved to the first explicit row following it. If no such row exists, the iterator is exhausted and GxB_EXHAUSTED is returned. The GxB_rowIterator_kseek method always moves to the kth explicit row, for any matrix. Use GxB_rowIterator_getRowIndex, described below, to determine the row index of the current position.

Precondition: on input, the iterator must have been successfully attached to a matrix via a prior call to GxB_rowIterator_attach. Results are undefined if this precondition is not met.

2. column iterator:

```
GrB_Info info = GxB_colIterator_seekCol (iterator, col) ;
GrB_Index kount = GxB_colIterator_kount (iterator) ;
GrB_Info info = GxB_colIterator_kseek (iterator, k) ;
```

These methods move a column iterator to a specific column, defined in one of two ways: (1) the column index itself (in range 0 to ncols-1), or

(2) by specifying k, which moves the iterator to the kth explicit column (in the range 0 to kount-1). For sparse, bitmap, or full matrices, these two methods are identical. For hypersparse matrices, not all columns are present in the data structure; these implicit columns are skipped and not included in the kount. Implicit columns contain no entries. The $GxB_collerator_kount$ method returns the kount of the matrix, where kount is equal to ncols for sparse, bitmap, and matrices, and kount \leq ncols for hypersparse matrices. All three methods listed above can be used for any column iterator.

The $GxB_collterator_*seek*$ methods return $GrB_SUCCESS$ if the iterator has been moved to a column that contains at least one entry, GrB_NO_VALUE if the column has no entries, or $GxB_EXHAUSTED$ if the column is out of bounds ($col \ge ncols$ or $k \ge kount$). None of these return conditions are errors; they are all informational.

For sparse, bitmap, and full matrices, GxB_colIterator_seekCol always moves to the given column. For hypersparse matrices, if the requested column is implicit, the iterator is moved to the first explicit column following it. If no such column exists, the iterator is exhausted and GxB_EXHAUSTED is returned. The GxB_colIterator_kseek method always moves to the kth explicit column, for any matrix. Use GxB_colIterator_getColIndex, described below, to determine the column index of the current position.

Precondition: on input, the iterator must have been successfully attached to a matrix via a prior call to GxB_collterator_attach. Results are undefined if this precondition is not met.

3. entry iterator:

```
GrB_Info info = GxB_Matrix_Iterator_seek (iterator, p) ;
GrB_Index pmax = GxB_Matrix_Iterator_getpmax (iterator) ;
GrB_Index p = GxB_Matrix_Iterator_getp (iterator);
```

The GxB_Matrix_Iterator_seek method moves the iterator to the given position p, which is in the range 0 to pmax-1, where the value of pmax is obtained from GxB_Matrix_Iterator_getpmax. For sparse, hypersparse, and full matrices, pmax is the same as nvals returned by GrB_Matrix_nvals. For bitmap matrices, pmax is equal to nrows*ncols.

If $p \ge pmax$, the iterator is exhausted and $GxB_EXHAUSTED$ is returned. Otherwise, $GrB_SUCCESS$ is returned.

All entries in the matrix are given an ordinal position, p. Seeking to position p will either move the iterator to that particular position, or to the next higher position containing an entry if there is entry at position p. The latter case only occurs for bitmap matrices. Use GxB_Matrix_Iterator_getp to determine the current position of the iterator.

Precondition: on input, the iterator must have been successfully attached to a matrix via a prior call to GxB_Matrix_Iterator_attach. Results are undefined if this precondition is not met.

4. vector iterator:

```
GrB_Info info = GxB_Vector_Iterator_seek (iterator, p) ;
GrB_Index pmax = GxB_Vector_Iterator_getpmax (iterator) ;
GrB_Index p = GxB_Vector_Iterator_getp (iterator);
```

The GxB_Vector_Iterator_seek method is identical to the entry iterator of a matrix, but applied to a GrB_Vector instead.

Precondition: on input, the iterator must have been successfully attached to a vector via a prior call to <code>GxB_Vector_Iterator_attach</code>. Results are undefined if this precondition is not met.

14.4 Advancing to the next position

For best performance, the *seek* methods described above should be used with care, since some of them require $O(\log n)$ time. The fastest method for changing the position of the iterator is the corresponding *next* method, described below for each iterator:

1. row iterator: To move to the next row.

```
GrB_Info info = GxB_rowIterator_nextRow (iterator) ;
```

The row iterator is a 2-dimensional iterator, requiring an outer loop and an inner loop. The outer loop iterates over the rows of the matrix, using GxB_rowIterator_nextRow to move to the next row. If the matrix

is hypersparse, the next row is always an explicit row; implicit rows are skipped. The return conditions are identical to GxB_rowIterator_seekRow.

Preconditions: on input, the row iterator must already be attached to a matrix via a prior call to GxB_rowIterator_attach, and the iterator must be at a specific row, via a prior call to GxB_rowIterator_*seek* or GxB_rowIterator_nextRow. Results are undefined if these conditions are not met.

2. row iterator. To move to the next entry within a row.

```
GrB_Info info = GxB_rowIterator_nextCol (iterator) ;
```

The row iterator is moved to the next entry in the current row. The method returns <code>GrB_NO_VALUE</code> if the end of the row is reached. The iterator does not move to the next row in this case. The method returns <code>GrB_SUCCESS</code> if the iterator has been moved to a specific entry in the current row.

Preconditions: the same as GxB_rowIterator_nextRow.

3. column iterator: To move to the next column

```
GrB_Info info = GxB_colIterator_nextCol (iterator) ;
```

The column iterator is a 2-dimensional iterator, requiring an outer loop and an inner loop. The outer loop iterates over the columns of the matrix, using GxB_collterator_nextCol to move to the next column. If the matrix is hypersparse, the next column is always an explicit column; implicit columns are skipped. The return conditions are identical to GxB_collterator_seekCol.

Preconditions: on input, the column iterator must already be attached to a matrix via a prior call to GxB_collterator_attach, and the iterator must be at a specific column, via a prior call to GxB_collterator_*seek* or GxB_collterator_nextCol. Results are undefined if these conditions are not met.

4. column iterator: To move to the next entry within a column.

```
GrB_Info info = GxB_colIterator_nextRow (iterator) ;
```

The column iterator is moved to the next entry in the current column. The method returns <code>GrB_NO_VALUE</code> if the end of the column is reached. The iterator does not move to the next column in this case. The method returns <code>GrB_SUCCESS</code> if the iterator has been moved to a specific entry in the current column.

Preconditions: the same as GxB_collterator_nextCol.

5. entry iterator: To move to the next entry.

```
GrB_Info info = GxB_Matrix_Iterator_next (iterator) ;
```

This method moves an iterator to the next entry of a matrix. It returns GrB_SUCCESS if the iterator is at an entry that exists in the matrix, or GrB_EXHAUSTED otherwise.

Preconditions: on input, the entry iterator must be already attached to a matrix via GxB_Matrix_Iterator_attach, and the position of the iterator must also have been defined by a prior call to GxB_Matrix_Iterator_seek or GxB_Matrix_Iterator_next. Results are undefined if these conditions are not met.

6. vector iterator: To move to the next entry.

```
GrB_Info info = GxB_Vector_Iterator_next (iterator) ;
```

This method moves an iterator to the next entry of a vector. It returns Grb_Success if the iterator is at an entry that exists in the vector, or Grb_EXHAUSTED otherwise.

Preconditions: on input, the iterator must be already attached to a vector via GxB_Vector_Iterator_attach, and the position of the iterator must also have been defined by a prior call to GxB_Vector_Iterator_seek or GxB_Vector_Iterator_next. Results are undefined if these conditions are not met.

14.5 Accessing the indices of the current entry

Once the iterator is attached to a matrix or vector, and is placed in position at an entry in the matrix or vector, the indices and value of this entry can be obtained. The methods for accessing the value of the entry are described in Section 14.6. Accessing the indices is performed with four different sets of methods, depending on which access pattern is in use, described below:

1. row iterator: To get the current row index.

```
GrB_Index i = GxB_rowIterator_getRowIndex (iterator) ;
```

The method returns nrows(A) if the iterator is exhausted, or the current row index i otherwise. There need not be any entry in the current row. Zero is returned if the iterator is attached to the matrix but GxB_rowIterator_*seek* has not been called, but this does not mean the iterator is positioned at row zero.

Preconditions: on input, the iterator must be already successfully attached to matrix as a row iterator via GxB_rowIterator_attach. Results are undefined if this condition is not met.

2. row iterator: To get the current column index.

```
GrB_Index j = GxB_rowIterator_getColIndex (iterator) ;
```

Preconditions: on input, the iterator must be already successfully attached to matrix as a row iterator via GxB_rowIterator_attach, and in addition, the row iterator must be positioned at a valid entry present in the matrix. That is, the last call to GxB_rowIterator_*seek* or GxB_rowIterator_*next*, must have returned GrB_SUCCESS. Results are undefined if these conditions are not met.

3. column iterator: To get the current column index.

```
GrB_Index j = GxB_colIterator_getColIndex (iterator) ;
```

The method returns ncols(A) if the iterator is exhausted, or the current column index j otherwise. There need not be any entry in the current column. Zero is returned if the iterator is attached to the matrix but GxB_collterator_*seek* has not been called, but this does not mean the iterator is positioned at column zero.

Precondition: on input, the iterator must be already successfully attached to matrix as a column iterator via GxB_colIterator_attach. Results are undefined if this condition is not met.

4. column iterator: To get the current row index.

```
GrB_Index i = GxB_colIterator_getRowIndex (iterator) ;
```

Preconditions: on input, the iterator must be already successfully attached to matrix as a column iterator via GxB_collterator_attach, and in addition, the column iterator must be positioned at a valid entry present in the matrix. That is, the last call to GxB_collterator_*seek* or GxB_collterator_*next*, must have returned GrB_SUCCESS. Results are undefined if these conditions are not met.

5. entry iterator: To get the current row and column index.

```
GrB_Index i, j ;
GxB_Matrix_Iterator_getIndex (iterator, &i, &j) ;
```

Returns the row and column index of the current entry.

Preconditions: on input, the entry iterator must be already attached to a matrix via GxB_Matrix_Iterator_attach, and the position of the iterator must also have been defined by a prior call to GxB_Matrix_Iterator_seek or GxB_Matrix_Iterator_next, with a return value of GrB_SUCCESS. Results are undefined if these conditions are not met.

6. vector iterator: To get the current index.

```
GrB_Index i = GxB_Vector_Iterator_getIndex (iterator) ;
```

Returns the index of the current entry.

Preconditions: on input, the entry iterator must be already attached to a matrix via GxB_Vector_Iterator_attach, and the position of the iterator must also have been defined by a prior call to GxB_Vector_Iterator_seek or GxB_Vector_Iterator_next, with a return value of GrB_SUCCESS. Results are undefined if these conditions are not met.

14.6 Accessing the value of the current entry

So far, all methods that create or use an iterator have been split into four sets of methods, for the row, column, or entry iterators attached to a matrix, or for a vector iterator. Accessing the value is different. All four iterators use the same set of methods to access the value of their current entry. These methods return the value of the current entry at the position determined by the iterator. The return value can of course be typecasted using standard C syntax once the value is returned to the caller.

Preconditions: on input, the prior call to <code>GxB_*Iterator_*seek*</code>, or <code>GxB_*Iterator_*next*</code> must have returned <code>GrB_SUCCESS</code>, indicating that the iterator is at a valid current entry for either a matrix or vector. No typecasting is permitted, in the sense that the method name must match the type of the matrix or vector. Results are undefined if these conditions are not met.

```
// for built-in types:
          value = GxB_Iterator_get_BOOL (iterator) ;
int8_t
          value = GxB_Iterator_get_INT8 (iterator) ;
int16_t value = GxB_Iterator_get_INT16 (iterator) ;
int32_t
          value = GxB_Iterator_get_INT32 (iterator) ;
int64_t value = GxB_Iterator_get_INT64 (iterator);
uint8_t
          value = GxB_Iterator_get_UINT8 (iterator) ;
uint16_t value = GxB_Iterator_get_UINT16 (iterator) ;
uint32_t value = GxB_Iterator_get_UINT32 (iterator) ;
uint64_t value = GxB_Iterator_get_UINT64 (iterator) ;
          value = GxB_Iterator_get_FP32 (iterator) ;
float
double
          value = GxB_Iterator_get_FP64 (iterator) ;
GxB_FC32_t value = GxB_Iterator_get_FC32 (iterator) ;
GxB_FC64_t value = GxB_Iterator_get_FC64 (iterator) ;
// for user-defined types:
<type> value ;
GxB_Iterator_get_UDT (iterator, (void *) &value) ;
```

14.7 Example: row iterator for a matrix

The following example uses a row iterator to access all of the entries in a matrix A of type GrB_FP64. Note the inner and outer loops. The outer loop iterates over all rows of the matrix. The inner loop iterates over all entries in the row i. This access pattern requires the matrix to be held by-row, but otherwise it works for any matrix. If the matrix is held by-column, then use the column iterator methods instead.

```
// create an iterator
GxB_Iterator iterator ;
GxB_Iterator_new (&iterator) ;
// attach it to the matrix A, known to be type GrB_FP64
GrB_Info info = GxB_rowIterator_attach (iterator, A, NULL) ;
if (info < 0) { handle the failure ... }</pre>
// seek to A(0,:)
info = GxB_rowIterator_seekRow (iterator, 0);
while (info != GxB_EXHAUSTED)
    // iterate over entries in A(i,:)
    GrB_Index i = GxB_rowIterator_getRowIndex (iterator) ;
    while (info == GrB_SUCCESS)
        // get the entry A(i,j)
        GrB_Index j = GxB_rowIterator_getColIndex (iterator) ;
        double aij = GxB_Iterator_get_FP64 (iterator) ;
        // move to the next entry in A(i,:)
        info = GxB_rowIterator_nextCol (iterator) ;
    // move to the next row, A(i+1,:), or a subsequent one if i+1 is implicit
    info = GxB_rowIterator_nextRow (iterator) ;
GrB_free (&iterator) ;
```

14.8 Example: column iterator for a matrix

The column iterator is analgous to the row iterator.

The following example uses a column iterator to access all of the entries in a matrix A of type GrB_FP64. The outer loop iterates over all columns of the matrix. The inner loop iterates over all entries in the column j. This access pattern requires the matrix to be held by-column, but otherwise it works for any matrix. If the matrix is held by-row, then use the row iterator methods instead.

```
// create an iterator
GxB_Iterator iterator ;
GxB_Iterator_new (&iterator) ;
// attach it to the matrix A, known to be type GrB_FP64
GrB_Info info = GxB_colIterator_attach (iterator, A, NULL) ;
// seek to A(:,0)
info = GxB_colIterator_seekCol (iterator, 0);
while (info != GxB_EXHAUSTED)
    // iterate over entries in A(:,j)
    GrB_Index j = GxB_colIterator_getColIndex (iterator) ;
    while (info == GrB_SUCCESS)
        // get the entry A(i,j)
        GrB_Index i = GxB_colIterator_getRowIndex (iterator) ;
        double aij = GxB_Iterator_get_FP64 (iterator) ;
        // move to the next entry in A(:,j)
        info = GxB_colIterator_nextRow (iterator) ;
    // move to the next column, A(:,j+1), or a subsequent one if j+1 is implicit
    info = GxB_colIterator_nextCol (iterator) ;
GrB_free (&iterator) ;
```

14.9 Example: entry iterator for a matrix

The entry iterator allows for a simpler access pattern, with a single loop, but using a row or column iterator is faster. The method works for any matrix.

```
// create an iterator
GxB_Iterator iterator ;
GxB_Iterator_new (&iterator) ;
// attach it to the matrix A, known to be type GrB_FP64
GrB_Info info = GxB_Matrix_Iterator_attach (iterator, A, NULL) ;
if (info < 0) { handle the failure ... }</pre>
// seek to the first entry
info = GxB_Matrix_Iterator_seek (iterator, 0);
while (info != GxB_EXHAUSTED)
    // get the entry A(i,j)
    GrB_Index i, j ;
    GxB_Matrix_Iterator_getIndex (iterator, &i, &j) ;
    double aij = GxB_Iterator_get_FP64 (iterator) ;
    // move to the next entry in A
    info = GxB_Matrix_Iterator_next (iterator) ;
GrB_free (&iterator) ;
```

14.10 Example: vector iterator

A vector iterator is used much like an entry iterator for a matrix.

```
// create an iterator
GxB_Iterator iterator;
GxB_Iterator_new (&iterator);
// attach it to the vector v, known to be type GrB_FP64
GrB_Info info = GxB_Vector_Iterator_attach (iterator, v, NULL);
if (info < 0) { handle the failure ... }
// seek to the first entry
info = GxB_Vector_Iterator_seek (iterator, 0);
while (info != GxB_EXHAUSTED)
{
    // get the entry v(i)
    GrB_Index i = GxB_Vector_Iterator_getIndex (iterator);
    double vi = GxB_Iterator_get_FP64 (iterator);
    // move to the next entry in v
    info = GxB_Vector_Iterator_next (iterator);
}
GrB_free (&iterator);</pre>
```

14.11 Performance

I have benchmarked the performance of the row and column iterators to compute y=0 and then y+=A*x where y is a dense vector and A is a sparse matrix, using a single thread. The row and column iterators are very fast, sometimes only 1% slower than calling GrB_mxv to compute the same thing (also assuming a single thread), for large problems. For sparse matrices that average just 1 or 2 entries per row, the row iterator can be about 30% slower than GrB_mxv, likely because of the slightly higher complexity of moving from one row to the next using these methods.

It is possible to split up the problem for multiple user threads, each with its own iterator. Given the low overhead of the row and column iterator for a single thread, this should be very fast. Care must be taken to ensure a good load balance. Simply spliting up the rows of a matrix and giving the same number of rows to each user thread can result in imbalanced work. This is handled internally in <code>GrB_*</code> methods, but enabling parallelism when using iterators is the responsibility of the user application.

The entry iterators are easier to use but harder to implement. The methods must internally fuse both inner and outer loops so that the user application can use a single loop. As a result, the computation y+=A*x can be up to 4x slower (about 2x typical) than when using GrB_mxv with a single thread.

To obtain the best performace possible, many of the iterator methods are implemented as macros in GraphBLAS.h. Using macros is the default, giving typical C and C++ applications access to the fastest methods possible.

To ensure access to these methods when not using the macros, these methods are also defined as regular functions that appear in the compiled libgraphblas.so library with the same name as the macros. Applications that cannot use the macro versions can #undef the macros after the #include <GraphBLAS.h> statement, and then they would access the regular compiled functions in libgraphblas.so. This non-macro approach is not the default, and the iterator methods may be slightly slower.

15 Iso-Valued Matrices and Vectors

The GraphBLAS C API states that the entries in all GrB_Matrix and GrB_Vector objects have a numerical value, with either a built-in or user-defined type. Representing an unweighted graph requires a value to be placed on each edge, typically $a_{ij} = 1$. Adding a structure-only data type would not mix well with the rest of GraphBLAS, where all operators, monoids, and semirings need to operate on a value, of some data type. And yet unweighted graphs are very important in graph algorithms.

The solution is simple, and exploiting it in SuiteSparse:GraphBLAS requires nearly no extensions to the GraphBLAS C API. SuiteSparse:GraphBLAS can often detect when the user application is creating a matrix or vector where all entries in the sparsity pattern take on the same numerical value.

For example, $\mathbf{C}\langle\mathbf{C}\rangle=1$, when the mask is structural, sets all entries in \mathbf{C} to the value 1. SuiteSparse:GraphBLAS detects this, and performs this assignment in O(1) time. It stores a single copy of this "iso-value" and sets an internal flag in the opaque data structure for \mathbf{C} , which states that all entries in the pattern of \mathbf{C} are equal to 1. This saves both time and memory and allows for the efficient representation of sparse adjacency matrices of unweighted graphs, yet does not change the \mathbf{C} API. To the user application, it still appears that \mathbf{C} has $\mathtt{nvals}(\mathbf{C})$ entries, all equal to 1.

Creating and operating on iso-valued matrices (or just iso matrices for short) is significantly faster than creating matrices with different data values. A matrix that is iso requires only O(1) space for its numerical values. The sparse and hypersparse formats require an additional O(n+e) or O(e) integer space to hold the pattern of an n-by-n matrix C, respectively, and a matrix C in bitmap format requires $O(n^2)$ space for the bitmap. A full matrix requires no integer storage, so a matrix that is both iso and full requires only O(1) space, regardless of its dimension.

The sections below a describe the methods that can be used to create iso matrices and vectors. Let a, b, and c denote the iso values of A, B, and C, respectively.

15.1 Using iso matrices and vectors in a graph algorithm

There are two primary useful ways to use iso-valued matrices and vectors: (1) as iso sparse/hypersparse adjacency matrices for unweighted graphs, and

(2) as iso full matrices or vectors used with operations that do not need to access all of the content of the iso full matrix or vector.

In the first use case, simply create a GrB_Matrix with values that are all the same (those in the sparsity pattern). The $GxB_Matrix_build_Scalar$ method can be used for this, since it guarantees that the time and work spent on the numerical part of the array is only O(1). The method still must spend O(e) or $O(e \log e)$ time on the integer arrays that represent the sparsity pattern, but the reduction in time and work on the numerical part of the matrix will improve performance.

The use of GxB_Matrix_build_Scalar is optional. Matrices can also be constructed with GrB* methods. In particular, GrB_Matrix_build_* can be used. It first builds a non-iso matrix and then checks if all of the values are the same, after assembling any duplicate entries. This does not save time or memory for the construction of the matrix itself, but it will lead to savings in time and memory later on, when the matrix is used.

To ensure a matrix C is iso-valued, simply use GrB_assign to compute C<C,struct>=1, or assign whatever value of scalar you wish. It is essential to use a structural mask. Otherwise, it is not clear that all entries in C will be assigned the same value. The following code takes O(1) time, and it resets the size of the numerical part of the C matrix to be O(1) in size:

The MATLAB/Octave analog of the code above is C=spones(C).

The second case for where iso matrices and vectors are useful is to use them with operations that do not necessarily access all of their content. Suppose you have a matrix A of arbitrarily large dimension (say n-by-n where n=2^60, of type GrB_FP64). A matrix this large can be represented by Suite-Sparse:GraphBLAS, but only in a hypersparse form.

Now, suppose you wish to compute the maximum value in each row, reducing the matrix to a vector. This can be done with GrB_reduce:

```
GrB_Vector_new (&v, GrB_FP64, n) ;
GrB_reduce (v, NULL, GrB_MAX_MONOID_FP64, A, NULL) ;
```

It can also be done with $\mathtt{GrB_mxv}$, by creating an iso full vector \mathbf{x} . The creation of \mathbf{x} takes O(1) time and memory, and the $\mathtt{GrB_mxv}$ computation takes O(e) time (with modest assumptions; if \mathbf{A} needs to be transposed the time would be $O(e \log e)$).

```
GrB_Vector_new (&v, GrB_FP64, n);
GrB_Vector_new (&x, GrB_FP64, n);
GrB_assign (x, NULL, NULL, 1, GrB_ALL, n, NULL);
GrB_mxv (v, NULL, NULL, GrB_MAX_FIRST_SEMIRING_FP64, A, x, NULL);
```

The above computations are identical in SuiteSparse:GraphBLAS. Internally, GrB_reduce creates x and calls GrB_mxv. Using GrB_mxm directly gives the user application additional flexibility in creating new computations that exploit the multiplicative operator in the semiring. GrB_reduce always uses the FIRST operator in its semiring, but any other binary operator can be used instead when using GrB_mxv.

15.2 Iso matrices from matrix multiplication

Consider GrB_mxm , GrB_mxv , and GrB_vxm , and let C=A*B, where no mask is present, or C<M>=A*B where C is initially empty. If C is not initially empty, then these rules apply to a temporary matrix T<M>=A*B, which is initially empty and is then assigned to C via C<M>=T.

The iso property of C is determined with the following rules, where the first rule that fits defines the property and value of C.

- If the semiring includes a index-based multiplicative operator (GxB_FIRSTI, GrB_SECONDI, and related operators), then C is never iso.
- Define an iso-monoid as a built-in monoid with the property that reducing a set of n > 1 identical values x returns the same value x. These are the MIN MAX LOR LAND BOR BAND and ANY monoids. All other monoids are not iso monoids: PLUS, TIMES, LXNOR, EQ, BXOR, BXNOR, and all user-defined monoids. Currently, there is no mechanism for telling SuiteSparse:GraphBLAS that a user-defined monoid is an iso-monoid.
- If the multiplicative op is PAIR (same as ONEB), and the monoid is an iso-monoid, or the EQ or TIMES monoids, then C is iso with a value of 1.
- If both B and the monoid are iso, and the multiplicative op is SECOND or ANY, then C is iso with a value of b.
- If both A and the monoid are iso, and the multiplicative op is FIRST or ANY, then C is iso with a value of a.

- If A, B, and the monoid are all iso, then C is iso, with a value c = f(a, b), where f is any multiplicative op (including user-defined, which assumes that a user-defined f has no side effects).
- If A and B are both iso and full (all entries present, regardless of the format of the matrices), then C is iso and full. Its iso value is computed in $O(\log(n))$ time, via a reduction of n copies of the value t = f(a, b) to a scalar. The storage required to represent C is just O(1), regardless of its dimension. Technically, the PLUS monoid could be computed as c = nt in O(1) time, but the log-time reduction works for any monoid, including user-defined ones.
- Otherwise, C is not iso.

15.3 Iso matrices from eWiseMult and kronecker

Consider $GrB_eWiseMult$. Let C=A.*B, or C<M>=A.*B with any mask and where C is initially empty, where .* denotes a binary operator f(x,y) applied with eWiseMult. These rules also apply to $GrB_kronecker$.

- If the operator is index-based (GxB_FIRSTI and related) then C is not iso.
- If the op is PAIR (same as ONEB), then C is iso with c=1.
- If B is iso and the op is SECOND or ANY, then C is iso with c = b.
- If A is iso and the op is FIRST or ANY, then C is iso with c=a.
- If both A and B are iso, then C is iso with c = f(a, b).
- Otherwise, C is not iso.

15.4 Iso matrices from eWiseAdd

Consider $GrB_eWiseAdd$, and also the accumulator phase of C<M>+=T when an accumulator operator is present. Let C=A+B, or C<M>=A+B with any mask and where C is initially empty.

• If both A and B are full (all entries present), then the rules for eWiseMult in Section 15.3 are used instead.

- If the operator is index-based (GxB_FIRSTI and related) then C is not iso.
- If a and b differ (when typecasted to the type of C), then C is not iso.
- If c = f(a, b) = a = b holds, then C is iso, where f(a, b) is the operator.
- Otherwise, C is not iso.

15.5 Iso matrices from eWiseUnion

GxB_eWiseUnion is very similar to GrB_eWiseAdd, but the rules for when the result is iso-valued are very different.

- If both A and B are full (all entries present), then the rules for eWiseMult in Section 15.3 are used instead.
- If the operator is index-based (GxB_FIRSTI and related) then C is not iso.
- If the op is PAIR (same as ONEB), then C is iso with c = 1.
- If B is iso and the op is SECOND or ANY, and the input scalar beta matches b (the iso-value of B), then C is iso with c = b.
- If A is iso and the op is FIRST or ANY, and the input scalar alpha matches a (the iso-value of A), then C is iso with c = a.
- If both A and B are iso, and $f(a,b) = f(\alpha,b) = f(a,\beta)$, then C is iso with c = f(a,b).
- Otherwise, C is not iso.

15.6 Reducing iso matrices to a scalar or vector

If A is iso with e entries, reducing it to a scalar takes $O(\log(e))$ time, regardless of the monoid used to reduce the matrix to a scalar. Reducing A to a vector c is the same as the matrix-vector multiply c=A*x or c=A*x, depending on the descriptor, where x is an iso full vector (refer to Section 15.2).

15.7 Iso matrices from apply

Let C=f(A) denote the application of a unary operator f, and let C=f(A,s) and C=f(s,A) denote the application of a binary operator with s a scalar.

- If the operator is index-based (GxB_POSITION*, GxB_FIRSTI, and related) then C is not iso.
- If the operator is ONE or PAIR (same as ONEB), then C iso with c=1.
- If the operator is FIRST or ANY with C=f(s,A), then C iso with c=s.
- If the operator is SECOND or ANY with C=f(A,s), then C iso with c=s.
- If A is iso then C is iso, with the following value of c:
 - If the op is IDENTITY, then c = a.
 - If the op is unary with C=f(A), then c=f(a).
 - If the op is binary with C=f(s,A), then c=f(s,a).
 - If the op is binary with C=f(A,s), then c = f(a,s).
- Otherwise, C is not iso.

15.8 Iso matrices from select

Let C=select(A) denote the application of a GrB_IndexUnaryOp operator in GrB_select.

- If A is iso, then C is iso with c = a.
- If the operator is any GrB_VALUE*_BOOL operator, with no typecasting, and the test is true only for a single boolean value, then C is iso.
- If the operator is $GrB_VALUEEQ_*$, with no typecasting, then C is iso, with c=t where t is the value of the scalar y.
- If the operator is $GrB_VALUELE_UINT*$, with no typecasting, and the scalar y is zero, then C is iso with c=0.
- Otherwise, C is not iso.

15.9 Iso matrices from assign and subassign

These rules are somewhat complex. Consider the assignment C<M>(I,J)=... with GrB_assign . Internally, this assignment is converted into C(I,J)<M(I,J)>=... and then $GxB_subassign$ is used. Thus, all of the rules below assume the form C(I,J)<M>=... where M has the same size as the submatrix C(I,J).

15.9.1 Assignment with no accumulator operator

If no accumulator operator is present, the following rules are used.

- For matrix assignment, A must be iso. For scalar assignment, the single scalar is implicitly expanded into an iso matrix A of the right size. If these rules do not hold, C is not iso.
- If A is not iso, or if C is not iso on input, then C is not iso on output.
- If C is iso or empty on input, and A is iso (or scalar assignment is begin performed) and the iso values c and a (or the scalar s) match, then the following forms of assignment result in an iso matrix C on output:

```
- C(I,J) = scalar
- C(I,J) < M > = scalar
- C(I,J) < M > = scalar
- C(I,J) < M, replace > = scalar
- C(I,J) < M, replace > = scalar
- C(I,J) = A
- C(I,J) < M > = A
- C(I,J) < M > = A
- C(I,J) < M, replace > = A
- C(I,J) < M, replace > = A
- C(I,J) < M, replace > = A
```

• For these forms of assignment, C is always iso on output, regardless of its iso property on input:

```
- C = scalar
```

- C<M, struct>=scalar; C empty on input.
- C<C,struct>=scalar
- For these forms of assignment, C is always iso on output if A is iso:
 - C = A
 - C<M,str> = A; C empty on input.

15.9.2 Assignment with an accumulator operator

If an accumulator operator is present, the following rules are used. Index-based operators (GxB_FIRSTI and related) cannot be used as accumulator operators, so these rules do not consider that case.

- For matrix assignment, A must be iso. For scalar assignment, the single scalar is implicitly expanded into an iso matrix A of the right size. If these rules do not hold, C is not iso.
- For these forms of assignment ${\tt C}$ is iso if ${\tt C}$ is empty on input, or if c=c+a for the where a is the iso value of ${\tt A}$ or the value of the scalar for scalar assignment.
 - C(I,J) += scalar
 - C(I,J) < M > += scalar
 - C(I,J) < !M> += scalar
 - C(I,J) < M, replace > += scalar
 - C(I,J)<!M,replace> += scalar
 - C(I,J) < M, replace > += A
 - C(I,J) < !M,replace > += A
 - C(I,J) += A
 - C(I,J) < M > += A
 - C(I,J) < !M> += A
 - C += A

15.10 Iso matrices from build methods

GxB_Matrix_build_Scalar and GxB_Vector_build_Scalar always construct an iso matrix/vector.

GrB_Matrix_build and GrB_Vector_build can also construct iso matrices and vectors. A non-iso matrix/vector is constructed first, and then the entries are checked to see if they are all equal. The resulting iso-valued matrix/vector will be efficient to use and will use less memory than a non-iso matrix/vector. However, constructing an iso matrix/vector with GrB_Matrix_build and GrB_Vector_build will take more time and memory than constructing the matrix/vector with GxB_Matrix_build_Scalar or GxB_Vector_build_Scalar.

15.11 Iso matrices from other methods

- For GrB_Matrix_dup and GrB_Vector_dup, the output matrix/vector has the same iso property as the input matrix/vector.
- GrB_*_setElement_* preserves the iso property of the matrix/vector it modifies, if the input scalar is equal to the iso value of the matrix/vector. If the matrix or vector has no entries, the first call to setElement makes it iso. This allows a sequence of setElement calls with the same scalar value to create an entire iso matrix or vector, if starting from an empty matrix or vector.
- GxB_Matrix_concat constructs an iso matrix as its result if all input tiles are either empty or iso.
- GxB_Matrix_split constructs its output tiles as iso if its input matrix is iso.
- GxB_Matrix_diag and GrB_Matrix_diag construct an iso matrix if its input vector is iso.
- GxB_Vector_diag constructs an iso vector if its input matrix is iso.
- GrB_*extract constructs an iso matrix/vector if its input matrix/vector is iso.
- GrB_transpose constructs an iso matrix if its input is iso.
- The GxB_Container methods preserve the iso property of their matrices/vectors.

15.12 Iso matrices not exploited

There are many cases where an matrix may have the iso property but it is not detected by SuiteSparse:GraphBLAS. For example, if A is non-iso, C=A(I,J) from GrB_extract may be iso, if all entries in the extracted submatrix have the same value. Future versions of SuiteSparse:GraphBLAS may extend the rules described in this section to detect these cases.

16 Performance

Getting the best performance out of an algorithm that relies on GraphBLAS can depend on many factors. This section describes some of the possible performance pitfalls you can hit when using SuiteSparse:GraphBLAS, and how to avoid them (or at least know when you've encountered them).

16.1 The burble is your friend

Turn on the burble with GrB_set (GrB_GLOBAL, true, GxB_BURBLE). You will get a single line of output from each (significant) call to GraphBLAS. The burble output can help you detect when you are likely using sub-optimal methods, as described in the next sections. When the JIT is in use the burble reports when a JIT kernel is run (which is quick), loaded for the first time (which takes a small amount of time), and when a JIT kernels is compiled (which can take a few tenths of a second or more). The compiler command is printed in full. If you encounter a compiler error, you can cut-and-paste the compiler command while outside of your application to help track down the compiler error.

16.2 Data types and typecasting: use the JIT

If the JIT is disabled, avoid mixing data types and relying on typecasting as much as possible. SuiteSparse:GraphBLAS has a set of highly-tuned kernels for each data type, and many operators and semirings, but there are too many combinations to generate ahead of time. If typecasting is required, or if SuiteSparse:GraphBLAS does not have a kernel for the specific operator or semiring, the word generic will appear in the burble. The generic methods rely on function pointers for each operation on every scalar, so they are slow. Enabling the JIT avoids this problem, since GraphBLAS can then compile kernel specific to the types used.

Without the JIT, the only time that typecasting is fast is when computing C=A via GrB_assign or GrB_apply , where the data types of C and A can differ. In this case, one of $13^2=169$ kernels are called, each of which performs the specific typecasting requested, without relying on function pointers.

16.3 Matrix data structures: sparse, hypersparse, bitmap, or full

SuiteSparse:GraphBLAS tries to automatically determine the best data structure for your matrices and vectors, selecting between sparse, hypersparse, bitmap, and full formats. By default, all 4 formats can be used. A matrix typically starts out hypersparse when it is created by <code>GrB_Matrix_new</code>, and then changes during its lifetime, possibly taking on all four different formats at different times. This can be modified via <code>GrB_set</code>. For example, this line of code:

```
GrB_set (A, GxB_SPARSE + GxB_BITMAP, GxB_SPARSITY_CONTROL) ;
```

tells SuiteSparse that the matrix A can be held in either sparse or bitmap format (at its discretion), but not hypersparse or full. The bitmap format will be used if the matrix has enough entries, or sparse otherwise. Sometimes this selection is best controlled by the user algorithm, so a single format can be requested:

```
GrB_set (A, GxB_SPARSE, GxB_SPARSITY_CONTROL) ;
```

This ensures that SuiteSparse will primarily use the sparse format. This is still just a hint, however. The data structure is opaque and SuiteSparse is free to choose otherwise. In particular, if you insist on using only the $\texttt{GxB_FULL}$ format, then that format is used when all entries are present. However, if the matrix is not actually full with all entries present, then the bitmap format is used instead. The full format does not preserve the sparsity structure in this case. Any GraphBLAS library must preserve the proper structure, per the C Specification. This is critical in a graph algorithm, since an edge (i,j) of weight zero, say, is not the same as no edge (i,j) at all.

16.4 Matrix formats: by row or by column, or using the transpose of a matrix

By default, SuiteSparse uses a simple rule: all matrices are held by row, unless the consist of a single column, in which case they are held by column. All vectors are treated as if they are *n*-by-1 matrices with a single column. Changing formats from row-oriented to column-oriented can have

significant performance implications, so SuiteSparse never tries to outguess the application. It just uses this simple rule.

However, there are cases where changing the format can greatly improve performance. There are two ways to handle this, which in the end are equivalent in the SuiteSparse internals. You can change the format (row to column oriented, or visa versa), or work with the explicit transpose of a matrix in the same storage orientation.

There are cases where SuiteSparse must explicitly transpose an input matrix, or the output matrix, in order to perform a computation. For example, if all matrices are held in row-oriented fashion, SuiteSparse does not have a method for computing C=A'*B, where A is transposed. Thus, SuiteSparse either computes a temporary transpose of its input matrix AT=A and then C=AT*B, or it swaps the computations, performing C=(B'*A)', which requires an explicit transpose of BT=B, and a transpose of the final result to obtain C.

These temporary transposes are costly to compute, taking time and memory. They are not kept, but are discarded when the method returns to the user application. If you see the term transpose in the burble output, and if you need to perform this computation many times, try constructing your own explicit transpose, say AT=A', via GrB_transpose, or create a copy of A but held in another orientation via GrB_set. For example, assuming the default matrix format is by-row, and that A is m-by-n of type GrB_FP32:

```
// method 1: AT = A'
GrB_Matrix_new (AT, GrB_FP32, n, m);
GrB_transpose (AT, NULL, NULL, A, NULL);

// method 2: A2 = A but held by column instead of by row
// note: doing the set before the assign is faster than the reverse
GrB_Matrix_new (A2, GrB_FP32, m, n);
GrB_set (A2, GrB_COLMAJOR, GrB_STORAGE_ORIENTATION_HINT);
GrB_assign (A2, NULL, NULL, A, GrB_ALL, m, GrB_ALL, n, NULL);
```

Internally, the data structure for AT and A2 are nearly identical (that is, the tranpose of A held in row format is the same as A held in column format). Using either of them in subsequent calls to GraphBLAS will allow SuiteSparse to avoid computing an explicit transpose. The two matrices AT and A2 do differ in one very significant way: their dimensions are different, and they behave differement mathematically. Computing C=A'*B using these matrices would differ:

```
// method 1: C=A'*B using AT
GrB_mxm (C, NULL, NULL, semiring, AT, B, NULL);
// method 2: C=A'*B using A2
GrB_mxm (C, NULL, NULL, semiring, A2, B, GrB_DESC_TO);
```

The first method computes C=AT*B. The second method computes C=A2'*B, but the result of both computations is the same, and internally the same kernels will be used.

16.5 Push/pull optimization

Closely related to the discussion above on when to use a matrix or its transpose is the exploitation of "push/pull" direction optimization. In linear algebraic terms, this is simply deciding whether to multiply by the matrix or its transpose. Examples can be see in the BFS and Betweeness-Centrality methods of LAGraph. Here is the BFS kernel:

```
int sparsity = do_push ? GxB_SPARSE : GxB_BITMAP ;
GrB_set (q, sparsity, GxB_SPARSITY_CONTROL) ;
if (do_push)
{
    // q'{!pi} = q'*A
    GrB_vxm (q, pi, NULL, semiring, q, A, GrB_DESC_RSC) ;
}
else
{
    // q{!pi} = AT*q
    GrB_mxv (q, pi, NULL, semiring, AT, q, GrB_DESC_RSC) ;
}
```

The call to GrB_set is optional, since SuiteSparse will likely already determine that a bitmap format will work best when the frontier q has many entries, which is also when the pull step is fastest. The push step relies on a sparse vector times sparse matrix method originally due to Gustavson. The output is computed as a set union of all rows A(i,:) where q(i) is present on input. This set union is very fast when q is very sparse. The pull step relies on a sequence of dot product computations, one per possible entry in the output q, and it uses the matrix AT which is a row-oriented copy of the explicit transpose of the adjacency matrix A.

Mathematically, the results of the two methods are identical, but internally, the data format of the input matrices is very different (using A held

by row, or AT held by row which is the same as a copy of A that is held by column), and the algorithms used are very different.

16.6 Computing with full matrices and vectors

Sometimes the best approach to getting the highest performance is to use dense vectors, and occassionaly dense matrices are tall-and-thin or short-and-fat. Packages such as Julia, Octave, or MATLAB, when dealing with the conventional plus-times semirings, assume that multiplying a sparse matrix A times a dense vector \mathbf{x} , $\mathbf{y}=\mathbf{A}*\mathbf{x}$, will result in a dense vector \mathbf{y} . This is not always the case, however. GraphBLAS must always return a result that respects the sparsity structure of the output matrix or vector. If the ith row of A has no entries then $\mathbf{y}(\mathbf{i})$ must not appear as an entry in the vector \mathbf{y} , so it cannot be held as a full vector. As a result, the following computation can be slower than it could be:

```
GrB_mxv (y, NULL, NULL, semiring, A, x, NULL) ;
```

SuiteSparse must do extra work to compute the sparsity of this vector y, but if this is not needed, and y can be padded with zeros (or the identity value of the monoid, to be precise), a faster method can be used, by relying on the accumulator. Instead of computing y=A*x, set all entries of y to zero first, and then compute y+=A*x where the accumulator operator and type matches the monoid of the semiring. SuiteSparse has special kernels for this case; you can see them in the burble as F+=S*F for example.

```
// y = 0  
   GrB_assign (y, NULL, NULL, 0, GrB_ALL, n, NULL) ;  
   // y += A*x  
   GrB_mxv (y, NULL, GrB_PLUS_FP32, GrB_PLUS_TIMES_SEMIRING_FP32, A, x, NULL) ;
```

You can see this computation in the LAGraph PageRank method, where all entries of r are set to the teleport scalar first.

```
for (iters = 0 ; iters < itermax && rdiff > tol ; iters++)
{
    // swap t and r ; now t is the old score
    GrB_Vector temp = t ; t = r ; r = temp ;
    // w = t ./ d
    GrB_eWiseMult (w, NULL, NULL, GrB_DIV_FP32, t, d, NULL) ;
    // r = teleport
```

```
GrB_assign (r, NULL, NULL, teleport, GrB_ALL, n, NULL);
// r += A'*w
GrB_mxv (r, NULL, GrB_PLUS_FP32, LAGraph_plus_second_fp32, AT, w, NULL);
// t -= r
GrB_assign (t, NULL, GrB_MINUS_FP32, r, GrB_ALL, n, NULL);
// t = abs (t)
GrB_apply (t, NULL, NULL, GrB_ABS_FP32, t, NULL);
// rdiff = sum (t)
GrB_reduce (&rdiff, NULL, GrB_PLUS_MONOID_FP32, t, NULL);
}
```

SuiteSparse exploits the iso-valued property of the scalar-to-vector assignment of y=0, or r=teleport, and performs these assignments in O(1) time and space. Because the r vector start out as full on input to GrB_mxv, and because there is an accumulatr with no mask, no entries in the input/output vector r will be deleted, even if A has empty rows. The call to GrB_mxv exploits this, and is able to use a fast kernel for this computation. SuiteSparse does not need to compute the sparsity pattern of the vector r.

16.7 Iso-valued matrices and vectors

Using iso-valued matrices and vectors is always faster than using matrices and vectors whose entries can have different values. Iso-valued matrices are very important in graph algorithms. For example, an unweighted graph is best represented as an iso-valued sparse matrix, and unweighted graphs are very common. The burble output, <code>GxB_print</code>, or <code>GrB_get</code> can all be used to report whether or not your matrix or vector is iso-valued.

Sometimes a matrix or vector may have values that are all the same, but SuiteSparse hasn't detected this. If this occurs, you can force a matrix or vector to be iso-valued by assigning a single scalar to all its entries.

```
// C < s(C) > = 3.14159

GrB_assign(C, C, NULL, 3.14159, GrB_ALL, m, GrB_ALL, n, GrB_DESC_S);
```

The matrix C is used as its own mask. The descriptor is essential here, telling the mask to be used in a structural sense, without regard to the values of the entries in the mask. This assignment sets all entries that already exist in C to be equal to a single value, 3.14159. The sparsity structure of C does not change. Of course, any scalar can be used; the value 1 is common for unweighted graphs. SuiteSparse:GraphBLAS performs the above assignment

in O(1) time and space, independent of the dimension of C or the number of entries in contains.

16.8 User-defined types and operators: use the JIT

If the JIT is disabled, these will be slow. With the JIT enabled, data types and operators are just as fast as built-in types and operators. A CUDA JIT for the GPU is in progress, collaboration with Joe Eaton and Corey Nolet. A SYCL/OpenCL JIT is under consideration, but work has not yet been started.

16.9 About NUMA systems

I have tested this package extensively on multicore single-socket systems, but have not yet optimized it for multi-socket systems with a NUMA architecture. That will be done in a future release. If you publish benchmarks with this package, please state the SuiteSparse:GraphBLAS version, and a caveat if appropriate. If you see significant performance issues when going from a single-socket to multi-socket system, I would like to hear from you so I can look into it.

17 Examples

Several examples of how to use GraphBLAS are listed below. They all appear in the Demo folder of SuiteSparse:GraphBLAS. Programs in the Demo folder are meant as simple examples; for the fastest methods, see LAgraph (Section 17.1).

- 1. creating a random matrix
- 2. creating a finite-element matrix
- 3. reading a matrix from a file
- 4. complex numbers as a user-defined type
- 5. matrix import/export

Additional examples appear in the newly created LAGraph project, currently in progress.

17.1 LAGraph

The LAGraph project is a community-wide effort to create graph algorithms based on GraphBLAS (any implementation of the API, not just SuiteSparse: GraphBLAS). Some of the algorithms and utilities in LAGraph are listed in the table below. Many additional algorithms are planned. Refer to https://github.com/GraphBLAS/LAGraph for a current list of algorithms. All functions in the Demo/ folder in SuiteSparse:GraphBLAS will eventually be translated into algorithms or utilities for LAGraph, and then removed from GraphBLAS/Demo.

To use LAGraph with SuiteSparse:GraphBLAS, place the two folders LAGraph and GraphBLAS in the same parent directory. This allows the cmake script in LAGraph to find the copy of GraphBLAS. Alternatively, the GraphBLAS source could be placed anywhere, as long as sudo make install is performed.

17.2 Creating a random matrix

The random_matrix function in the Demo folder generates a random matrix with a specified dimension and number of entries, either symmetric or unsymmetric, and with or without self-edges (diagonal entries in the matrix).

It relies on simple_rand* functions in the Demo folder to provide a portable random number generator that creates the same sequence on any computer and operating system.

random_matrix can use one of two methods: GrB_Matrix_setElement and GrB_Matrix_build. The former method is very simple to use:

```
GrB_Matrix_new (&A, GrB_FP64, nrows, ncols) ;
for (int64_t k = 0 ; k < ntuples ; k++)
{
    GrB_Index i = simple_rand_i () % nrows ;
    GrB_Index j = simple_rand_i () % ncols ;
    if (no_self_edges && (i == j)) continue ;
    double x = simple_rand_x () ;
    // A (i,j) = x
    GrB_Matrix_setElement (A, x, i, j) ;
    if (make_symmetric)
    {
        // A (j,i) = x
        GrB_Matrix_setElement (A, x, j, i) ;
    }
}</pre>
```

The above code can generate a million-by-million sparse double matrix with 200 million entries in 66 seconds (6 seconds of which is the time to generate the random i, j, and x), including the time to finish all pending computations. The user application does not need to create a list of all the tuples, nor does it need to know how many entries will appear in the matrix. It just starts from an empty matrix and adds them one at a time in arbitrary order. GraphBLAS handles the rest. This method is not feasible in MATLAB.

The next method uses GrB_Matrix_build. It is more complex to use than setElement since it requires the user application to allocate and fill the tuple lists, and it requires knowledge of how many entries will appear in the matrix, or at least a good upper bound, before the matrix is constructed. It is slightly faster, creating the same matrix in 60 seconds, 51 seconds of which is spent in GrB_Matrix_build.

```
GrB_Index *I, *J;
double *X;
int64_t s = ((make_symmetric) ? 2 : 1) * nedges + 1;
I = malloc (s * sizeof (GrB_Index));
J = malloc (s * sizeof (GrB_Index));
```

```
X = malloc (s * sizeof (double
if (I == NULL || J == NULL || X == NULL)
    // out of memory
    if (I != NULL) free (I);
    if (J != NULL) free (J) ;
    if (X != NULL) free (X);
    return (GrB_OUT_OF_MEMORY) ;
int64_t ntuples = 0 ;
for (int64_t k = 0 ; k < nedges ; k++)
    GrB_Index i = simple_rand_i ( ) % nrows ;
    GrB_Index j = simple_rand_i ( ) % ncols ;
    if (no_self_edges && (i == j)) continue ;
    double x = simple_rand_x ( ) ;
    // A (i,j) = x
    I [ntuples] = i ;
    J [ntuples] = j ;
    X [ntuples] = x ;
    ntuples++;
    if (make_symmetric)
        // A (j,i) = x
        I [ntuples] = j ;
        J [ntuples] = i ;
        X [ntuples] = x ;
        ntuples++;
    }
GrB_Matrix_build (A, I, J, X, ntuples, GrB_SECOND_FP64) ;
```

The equivalent sprandsym function in MATLAB takes 150 seconds, but sprandsym uses a much higher-quality random number generator to create the tuples [I,J,X]. Considering just the time for sparse(I,J,X,n,n) in sprandsym (equivalent to GrB_Matrix_build), the time is 70 seconds. That is, each of these three methods, setElement and build in Suite-Sparse:GraphBLAS, and sparse in MATLAB, are equally fast.

17.3 Creating a finite-element matrix

Suppose a finite-element matrix is being constructed, with k=40,000 finite-element matrices, each of size 8-by-8. The following operations (in pseudo-

MATLAB notation) are very efficient in SuiteSparse:GraphBLAS.

```
A = sparse (m,n) ; % create an empty n-by-n sparse GraphBLAS matrix
for i = 1:k
    construct a 8-by-8 sparse or dense finite-element F
    I and J define where the matrix F is to be added:
    I = a list of 8 row indices
    J = a list of 8 column indices
    % using GrB_assign, with the 'plus' accum operator:
    A (I,J) = A (I,J) + F
end
```

If this were done in MATLAB or in GraphBLAS with blocking mode enabled, the computations would be extremely slow. A far better approach is to construct a list of tuples [I,J,X] and to use sparse(I,J,X,n,n). This is identical to creating the same list of tuples in GraphBLAS and using the GrB_Matrix_build, which is equally fast.

In SuiteSparse:GraphBLAS, the performance of both methods is essentially identical, and roughly as fast as sparse in MATLAB. Inside SuiteSparse:GraphBLAS, GrB_assign is doing the same thing. When performing A(I,J)=A(I,J)+F, if it finds that it cannot quickly insert an update into the A matrix, it creates a list of pending tuples to be assembled later on. When the matrix is ready for use in a subsequent GraphBLAS operation (one that normally cannot use a matrix with pending computations), the tuples are assembled all at once via GrB_Matrix_build.

GraphBLAS operations on other matrices have no effect on when the pending updates of a matrix are completed. Thus, any GraphBLAS method or operation can be used to construct the F matrix in the example above, without affecting when the pending updates to A are completed.

The MATLAB wathen.m script is part of Higham's gallery of matrices [Hig02]. It creates a finite-element matrix with random coefficients for a 2D mesh of size nx-by-ny, a matrix formulation by Wathen [Wat87]. The pattern of the matrix is fixed; just the values are randomized. The GraphBLAS equivalent can use either GrB_Matrix_build, or GrB_assign. Both methods have good performance. The GrB_Matrix_build version below is about 15% to 20% faster than the MATLAB wathen.m function, regardless of the problem size. It uses the identical algorithm as wathen.m.

```
int64_t ntriplets = nx*ny*64 ;
I = malloc (ntriplets * sizeof (int64_t)) ;
```

```
J = malloc (ntriplets * sizeof (int64_t));
X = malloc (ntriplets * sizeof (double )) ;
if (I == NULL || J == NULL || X == NULL)
    FREE_ALL ;
    return (GrB_OUT_OF_MEMORY) ;
ntriplets = 0 ;
for (int j = 1; j \le ny; j++)
    for (int i = 1 ; i <= nx ; i++)
       nn [0] = 3*j*nx + 2*i + 2*j + 1;
        nn [1] = nn [0] - 1;
        nn [2] = nn [1] - 1;
       nn [3] = (3*j-1)*nx + 2*j + i - 1;
        nn [4] = 3*(j-1)*nx + 2*i + 2*j - 3;
        nn [5] = nn [4] + 1;
        nn [6] = nn [5] + 1;
        nn [7] = nn [3] + 1;
        for (int krow = 0 ; krow < 8 ; krow++) nn [krow]-- ;</pre>
        for (int krow = 0; krow < 8; krow++)
        {
            for (int kcol = 0; kcol < 8; kcol++)
                I [ntriplets] = nn [krow] ;
                J [ntriplets] = nn [kcol] ;
                X [ntriplets] = em (krow,kcol) ;
                ntriplets++ ;
            }
        }
   }
}
// A = sparse (I,J,X,n,n);
GrB_Matrix_build (A, I, J, X, ntriplets, GrB_PLUS_FP64) ;
```

The GrB_assign version has the advantage of not requiring the user application to construct the tuple list, and is almost as fast as using GrB_Matrix_build. The code is more elegant than either the MATLAB wathen.m function or its GraphBLAS equivalent above. Its performance is comparable with the other two methods, but slightly slower, being about 5% slower than the MATLAB wathen, and 20% slower than the GraphBLAS method above.

```
GrB_Matrix_new (&F, GrB_FP64, 8, 8);
```

```
for (int j = 1; j \le ny; j++)
   for (int i = 1; i \le nx; i++)
       nn [0] = 3*j*nx + 2*i + 2*j + 1;
       nn [1] = nn [0] - 1;
       nn [2] = nn [1] - 1;
       nn [3] = (3*j-1)*nx + 2*j + i - 1;
       nn [4] = 3*(j-1)*nx + 2*i + 2*j - 3;
       nn [5] = nn [4] + 1;
       nn [6] = nn [5] + 1;
       nn [7] = nn [3] + 1;
       for (int krow = 0; krow < 8; krow++) nn [krow]--;
       for (int krow = 0; krow < 8; krow++)
       {
           for (int kcol = 0; kcol < 8; kcol++)
               // F (krow,kcol) = em (krow, kcol)
               GrB_Matrix_setElement (F, em (krow,kcol), krow, kcol) ;
        // A (nn,nn) += F
       GrB_assign (A, NULL, GrB_PLUS_FP64, F, nn, 8, nn, 8, NULL) ;
   }
}
```

Since there is no Mask, and since GrB_REPLACE is not used, the call to GrB_assign in the example above is identical to GxB_subassign. Either one can be used, and their performance would be identical.

Refer to the wathen.c function in the Demo folder, which uses GraphBLAS to implement the two methods above, and two additional ones.

17.4 Reading a matrix from a file

See also LAGraph_mmread and LAGraph_mmwrite, which can read and write any matrix in Matrix Market format, and LAGraph_binread and LAGraph_binwrite, which read/write a matrix from a binary file. The binary file I/O functions are much faster than the read_matrix function described here, and also much faster than LAGraph_mmread and LAGraph_mmwrite.

The read_matrix function in the Demo reads in a triplet matrix from a file, one line per entry, and then uses GrB_Matrix_build to create the matrix. It creates a second copy with GrB_Matrix_setElement, just to test that method and compare the run times. Section 17.2 has already compared build versus setElement.

The function can return the matrix as-is, which may be rectangular or unsymmetric. If an input parameter is set to make the matrix symmetric, read_matrix computes $A=(A+A^{\prime})/2$ if A is square (turning all directed edges into undirected ones). If A is rectangular, it creates a bipartite graph, which is the same as the augmented matrix, $A=[0\ A\ ;\ A^{\prime}\ 0]$. If C is an n-by-n matrix, then $C=(C+C^{\prime})/2$ can be computed as follows in GraphBLAS, (the scale2 function divides an entry by 2):

```
GrB_Descriptor_new (&dt2) ;
GrB_set (dt2, GrB_TRAN, GrB_INP1) ;
GrB_Matrix_new (&A, GrB_FP64, n, n) ;
GrB_eWiseAdd (A, NULL, NULL, GrB_PLUS_FP64, C, C, dt2) ;  // A=C+C'
GrB_free (&C) ;
GrB_Matrix_new (&C, GrB_FP64, n, n) ;
GrB_UnaryOp_new (&scale2_op, scale2, GrB_FP64, GrB_FP64) ;
GrB_apply (C, NULL, NULL, scale2_op, A, NULL) ;  // C=A/2
GrB_free (&A) ;
GrB_free (&scale2_op) ;
```

This is of course not nearly as elegant as A=(A+A')/2 in MATLAB, but with minor changes it can work on any type and use any built-in operators instead of PLUS, or it can use any user-defined operators and types. The above code in SuiteSparse:GraphBLAS takes 0.60 seconds for the Freescale2 matrix, slightly slower than MATLAB (0.55 seconds).

Constructing the augmented system is more complicated using the Graph-BLAS C API Specification since it does not yet have a simple way of specifying a range of row and column indices, as in A(10:20,30:50) in MATLAB (GxB_RANGE is a SuiteSparse:GraphBLAS extension that is not in the Specification). Using the C API in the Specification, the application must instead build a list of indices first, I=[10, 11...20].

Thus, to compute the MATLAB equivalent of $A = [0 \ A \ ; \ A' \ 0]$, index lists I and J must first be constructed:

```
int64_t n = nrows + ncols ;
I = malloc (nrows * sizeof (int64_t)) ;
J = malloc (ncols * sizeof (int64_t)) ;
// I = 0:nrows-1
// J = nrows:n-1
if (I == NULL || J == NULL)
{
```

```
if (I != NULL) free (I);
if (J != NULL) free (J);
return (GrB_OUT_OF_MEMORY);
}
for (int64_t k = 0; k < nrows; k++) I [k] = k;
for (int64_t k = 0; k < ncols; k++) J [k] = k + nrows;</pre>
```

Once the index lists are generated, however, the resulting GraphBLAS operations are fairly straightforward, computing A=[0 C; C'0].

```
GrB_Descriptor_new (&dt1) ;
GrB_set (dt1, GrB_TRAN, GrB_INPO) ;
GrB_Matrix_new (&A, GrB_FP64, n, n) ;
// A (nrows:n-1, 0:nrows-1) = C'
GrB_assign (A, NULL, NULL, C, J, ncols, I, nrows, dt1) ;
// A (0:nrows-1, nrows:n-1) = C
GrB_assign (A, NULL, NULL, C, I, nrows, J, ncols, NULL) ;
```

This takes 1.38 seconds for the Freescale2 matrix, almost as fast as A=[sparse(m,m) C; C' sparse(n,n)] in MATLAB (1.25 seconds). The GxB_Matrix_concat function would be faster still (this example was written prior to GxB_Matrix_concat was added to SuiteSparse:GraphBLAS).

Both calls to GrB_assign use no accumulator, so the second one causes the partial matrix A=[0 0; C'0] to be built first, followed by the final build of A=[0 C; C'0]. A better method, but not an obvious one, is to use the GrB_FIRST_FP64 accumulator for both assignments. An accumulator enables SuiteSparse:GraphBLAS to determine that that entries created by the first assignment cannot be deleted by the second, and thus it need not force completion of the pending updates prior to the second assignment.

SuiteSparse:GraphBLAS also adds a GxB_RANGE mechanism that mimics the MATLAB colon notation. This speeds up the method and simplifies the code the user needs to write to compute $A=[0\ C\ ;\ C'\ 0]$:

Any operator will suffice because it is not actually applied. An operator is only applied to the set intersection, and the two assignments do not overlap. If an accum operator is used, only the final matrix is built, and the time in GraphBLAS drops slightly to 1.25 seconds. This is a very small improvement because in this particular case, SuiteSparse:GraphBLAS is able to detect that no sorting is required for the first build, and the second one is a simple concatenation. In general, however, allowing GraphBLAS to postpone pending updates can lead to significant reductions in run time.

17.5 User-defined types and operators

The Demo folder contains two working examples of user-defined types, first discussed in Section 6.1.1: double complex, and a user-defined typedef called wildtype with a struct containing a string and a 4-by-4 float matrix.

Double Complex: Prior to v3.3, GraphBLAS did not have a native complex type. It now appears as the GxB_FC64 predefined type, but a complex type can also easily added as a user-defined type. The Complex_init function in the usercomplex.c file in the Demo folder creates the Complex type based on the C11 double complex type. It creates a full suite of operators that correspond to every built-in GraphBLAS operator, both binary and unary. In addition, it creates the operators listed in the following table, where D is double and C is Complex.

| name | types | MATLAB/Octave equivalent | description |
|----------------------|--------------------|--------------------------|-----------------------------|
| Complex_complex | $D \times D \to C$ | z=complex(x,y) | complex from real and imag. |
| Complex_conj | $C \to C$ | z=conj(x) | complex conjugate |
| Complex_real | $C \to D$ | z=real(x) | real part |
| Complex_imag | $C \to D$ | z=imag(x) | imaginary part |
| Complex_angle | $C \to D$ | z=angle(x) | phase angle |
| Complex_complex_real | $D \to C$ | z=complex(x,0) | real to complex real |
| Complex_complex_imag | $D \to C$ | z=complex(0,x) | real to complex imag. |

The Complex_init function creates two monoids (Complex_add_monoid and Complex_times_monoid) and a semiring Complex_plus_times that corresponds to the conventional linear algebra for complex matrices. The include file usercomplex.h in the Demo folder is available so that this user-

defined Complex type can easily be imported into any other user application. When the user application is done, the Complex_finalize function frees the Complex type and its operators, monoids, and semiring. NOTE: the Complex type is not supported in this Demo in Microsoft Visual Studio.

Struct-based: In addition, the wildtype.c program creates a user-defined typedef of a struct containing a dense 4-by-4 float matrix, and a 64-character string. It constructs an additive monoid that adds two 4-by-4 dense matrices, and a multiplier operator that multiplies two 4-by-4 matrices. Each of these 4-by-4 matrices is treated by GraphBLAS as a "scalar" value, and they can be manipulated in the same way any other GraphBLAS type can be manipulated. The purpose of this type is illustrate the endless possibilities of user-defined types and their use in GraphBLAS.

17.6 User applications using OpenMP or other threading models

An example demo program (context_demo) is included that illustrates how a multi-threaded user application can use GraphBLAS, where each user thread calls GraphBLAS simultaneously, with nested parallelism.

GraphBLAS can also be combined with user applications that rely on MPI, the Intel TBB threading library, POSIX pthreads, Microsoft Windows threads, or any other threading library. If GraphBLAS itself is compiled with OpenMP, it will be thread safe when combined with other libraries. See Section 10.2.2 for thread-safety issues that can occur if GraphBLAS is compiled without OpenMP.

18 Compiling and Installing SuiteSparse:GraphBLAS

18.1 Quick Start

GraphBLAS requires **cmake** version 3.20 or later. It optionally can use OpenMP for best performance. For OpenMP on the Mac, see Section 18.3.6. Without OpenMP, GraphBLAS will be significantly slower since it is a highly parallel package.

A cmake build system is available for Linux, Mac, and Windows. For Linux or Mac, a simple Makefile wrapper is available that accesses this cmake build system. Simply do:

```
make
sudo make install
```

For Windows, open CMake and use the provided CMakeLists.txt file to build GraphBLAS in the GraphBLAS/build folder.

Next, try the demos with make demos. The output of the demos will be compared with expected output files in Demo/Output.²

18.2 Quick Start for MATLAB/Octave

As of GraphBLAS 9.2.0, a new and simpler method for compiling Graph-BLAS and its MATLAB interface has been added. In the MATLAB/Octave Command Window, simply type:

```
cd GraphBLAS/GraphBLAS
graphblas_install
```

This will use cmake to compile the GraphBLAS library. You can skip the details in the remainder of this section. Next, add your GraphBLAS/GraphBLAS folder to your path, by editting your startup.m script (usually in your Documents/MATLAB folder). Add this line:

²NOTE: DO NOT publish benchmarks of these demos, and do not link against the demo library in any user application. These codes are sometimes slow, and are meant as simple illustrations only, not for performance. The fastest methods are in LAGraph, not in GraphBLAS/Demo. Benchmark LAGraph instead. Eventually, all GraphBLAS/Demos methods will be removed, and LAGraph will serve all uses: for illustration, benchmarking, and production uses.

```
addpath ('/home/me/GraphBLAS/GraphBLAS');
```

where /home/me/GraphBLAS is the top-level folder containing your copy of GraphBLAS.

The graphblas_install MATLAB script may fail to run cmake. If it does, it will print the following workaround, where the commands it tells you to use will differ depending on the platform:

Building GraphBLAS with cmake failed. Try this outside of MATLAB:

```
cd /home/me/GraphBLAS/GraphBLAS/build
cmake ..
cmake --build . --config Release -j40
```

Then do this inside MATLAB:

cd /home/me/GraphBLAS/GraphBLAS/@GrB/private
gbmake

where /home/me/GraphBLAS is your copy of GraphBLAS.

18.3 More details

18.3.1 On Linux and Mac

GraphBLAS makes extensive use of features in the C11 standard, and thus a C compiler supporting this version of the C standard is required to use all features of GraphBLAS.

Any version of the Intel icx compiler is highly recommended. In most cases, the Intel icx and the Intel OpenMP library (libiomp) result in the best performance. The gcc and the GNU OpenMP library (libgomp) generally gives good performance: typically on par with icx but in a few special cases significantly slower. The Intel icc compiler is not recommended; it results in poor performance for #pragma omp atomic.

If you are using a C compiler that does not support the C11 standard, such as c1 in Microsoft Visual Studio, then the _Generic keyword is not available. SuiteSparse:GraphBLAS will still compile, but you will not have access to polymorphic functions such as GrB_assign. You will need to use the non-polymorphic functions instead.

To compile SuiteSparse:GraphBLAS, simply type make in the main Graph-BLAS folder, which compiles the library with your default system compiler. This compile GraphBLAS using 8 threads, which will take a long time. To compile with more threads (40, for this example), use:

```
make JOBS=40
```

To use a non-default compiler with 4 threads:

```
make CC=icx CXX=icpx JOBS=4
```

GraphBLAS v6.1.3 and later use the cpu_features package by Google to determine if the target architecture supports AVX2 and/or AVX512F (on Intel x86_64 architectures only). In case you have build issues with this package, you can compile without it (and then AVX2 and AVX512F acceleration will not be used):

```
make CMAKE_OPTIONS='-DGBNCPUFEAT=1'
```

Without cpu_features, it is still possible to enable AVX2 and AVX512F. Rather than relying on run-time tests, you can use these flags to enable both AVX2 and AVX512F, without relying on cpu_features:

```
make CMAKE_OPTIONS='-DGBNCPUFEAT=1 -DGBAVX2=1 -DGBAVX512F=1'
```

To use multiple options, separate them by a space. For example, to build just the library but not cpu_features, and to enable AVX2 but not AVX512F, and use 40 threads to compile:

```
make CMAKE_OPTIONS='-DGBNCPUFEAT=1 -DGBAVX2=1' JOBS=40
```

After compiling the library, you can compile the demos with make all and then make demos while in the top-level GraphBLAS folder.

If cmake or make fail, it might be that your default compiler does not support C11. Try another compiler. For example, try one of these options. Go into the build directory and type one of these:

```
CC=gcc cmake ..
CC=gcc-11 cmake ..
CC=xlc cmake ..
CC=icx cmake ..
```

You can also do the following in the top-level GraphBLAS folder instead:

```
CC=gcc make
CC=gcc-11 make
CC=xlc make
CC=icx make
```

For faster compilation, you can specify a parallel make. For example, to use 32 parallel jobs and the gcc compiler, do the following:

```
JOBS=32 CC=gcc make
```

18.3.2 On the Mac (Intel or ARM)

GraphBLAS requires cmake v3.20 or later, and it optionally uses make to simplify the use of the cmake build system. It also needs other Apple Command Line Tools from Xcode. First install Xcode (see https://developer.apple.com/xcode), and then install the command line tools for Xcode:

```
xcode-select --install
```

Next, install brew, at https://brew.sh, or spack. Use brew or spack to install cmake version 3.20 or later.

18.3.3 On the Intel-based Mac

If you have the Intel compiler and its OpenMP library, then use the following in the top-level GraphBLAS folder. The Intel OpenMP library will be found automatically:

```
make CC=icc CXX=icpc
```

18.3.4 MATLAB on the Mac (Apple Silicon based)

MATLAB on the Apple-Silicon-based Mac is now a native ARM64 application (as of R2023b). GraphBLAS is not supported for earlier versions of MATLAB on Apple Silicon.

Note that GraphBLAS must use the same OpenMP library as MATLAB. This is handled by the graphblas_install.m script.

18.3.5 On Microsoft Windows

SuiteSparse:GraphBLAS can be compiled by the Microsoft C compiler (c1) using Microsoft Visual Studio. However, that compiler is not C11 compliant. As a result, GraphBLAS on Windows will have a few minor limitations.

- The MS Visual Studio compiler does not support the _Generic keyword, required for the polymorphic GraphBLAS functions. So for example, you will need to use GrB_Matrix_free instead of just GrB_free.
- Variable-length arrays are not supported, so user-defined types are limited to 1024 bytes in size. This can be changed by editing GB_VLA_MAXSIZE in Source/GB_compiler.h, and recompiling SuiteSparse:GraphBLAS.
- AVX acceleration is not enabled.
- You must compile with 64-bit computing enabled (x64). Otherwise, a compiler error will occur (InterlockedExchange64, InterlockdOr64 and other methods will not be found).

If you use a recent gcc or icx compiler on Windows other than the Microsoft Compiler (cl), these limitations can be avoided.

The following instructions apply to Windows 10, CMake 3.16, and Visual Studio 2019, but may work for earlier versions.

- 1. Install CMake 3.16 or later, if not already installed. See https://cmake.org/ for details.
- 2. Install Microsoft Visual Studio, if not already installed. See https://visualstudio.microsoft.com/ for details. Version 2019 is preferred, but earlier versions may also work.
- 3. Open a terminal window and type this in the GraphBLAS/build folder:

cmake ..

Alternatively, use the cmake gui program to configure the cmake build system for GraphBLAS.

4. The cmake command generates many files in GraphBLAS/build, and the file graphblas.sln in particular. Open the generated graphblas.sln file in Visual Studio.

- 5. Optionally: right-click graphblas in the left panel (Solution Explorer) and select properties; then navigate to Configuration Properties, C/C++, General and change the parameter Multiprocessor Compilation to Yes (/MP). Click OK. This will significantly speed up the compilation of GraphBLAS.
- 6. Select the Build menu item at the top of the window and select Build Solution. This should create a folder called Release and place the compiled graphblas.dll, graphblas.lib, and graphblas.exp files there. Please be patient; some files may take a while to compile and sometimes may appear to be stalled. Just wait.
- 7. Alternatively, instead of opening Visual Studio, type this command in the terminal window while in the build folder:

```
cmake --build . --config Release
```

- 8. Add the GraphBLAS/build/Release folder to the Windows System path:
 - Open the Start Menu and type Control Panel.
 - Select the Control Panel app.
 - When the app opens, select System and Security.
 - Under System and Security, select System.
 - From the top left side of the System window, select Advanced System Settings. You may have to authenticate at this step. If you cannot authenticate, try setting the User Environment Variables instead.
 - The Systems Properties window should appear with the Advanced tab selected; select Environment Variables.
 - The Environment Variables window displays 2 sections, one for User variables and the other for System variables. Under the Systems variable section, scroll to and select Path, then select Edit. A editor window appears allowing to add, modify, delete or re-order the parts of the Path.

- Add the full path of the GraphBLAS\build\Release folder (typically starting with C:\Users\you\..., where you is your Windows username) to the Path. To use the MATLAB interface, add the full path of the GraphBLAS\GraphBLAS\build\Release folder as well.
- If the above steps do not work, you can instead copy the graphblas.* files from GraphBLAS\build\Release into any existing folder listed in your Path.
- 9. The GraphBLAS/Include/GraphBLAS.h file must be included in user applications via #include "GraphBLAS.h". This is already done for you in the MATLAB/Octave interface discussed in the next section.

18.3.6 Mac using clang

To use OpenMP with clang on the Mac, try installing it from the https://mac.r-project.org/openmp/ project. Be sure to check that page for the OpenMP version that matches your version of Apple Xcode. For example, if using Xcode 13.3 to 13.4.1, use:

```
curl -0 https://mac.r-project.org/openmp/openmp-13.0.0-darwin21-Release.tar.gz
sudo tar fvxz openmp-13.0.0-darwin21-Release.tar.gz -C /
```

These commands will install universal binaries (ARM and x86) for libomp.dylib, and the following files:

```
/usr/local/lib/libomp.dylib
/usr/local/include/ompt.h
/usr/local/include/omp.h
/usr/local/include/omp-tools.h
```

Once you do this, the GraphBLAS cmake build system should find this copy of OpenMP for clang.

18.3.7 Linking issues after installation

My Linux distro (Ubuntu 18.04) includes a copy of libgraphblas.so.1, which is SuiteSparse:GraphBLAS v1.1.2. After installing SuiteSparse:GraphBLAS

in /usr/local/lib (with sudo make install), compiling a simple standalone program links against libgraphblas.so.1 instead of the latest version, while at the same time accessing the latest version of the include file as /usr/local/include/GraphBLAS.h. This command fails:

```
gcc prog.c -lgraphblas
```

Revising my LD_LIBRARY_PATH to put /usr/local/lib first in the library directory order didn't help. If you encounter this problem, try one of the following options (all four work for me, and link against the proper version, /usr/local/lib/libgraphblas.so.9.2.0 for example):

```
gcc prog.c -1:libgraphblas.so.9
gcc prog.c -1:libgraphblas.so.9.2.0
gcc prog.c /usr/local/lib/libgraphblas.so
gcc prog.c -Wl,-v -L/usr/local/lib -lgraphblas
This prog.c test program is a trivial one, which works in v1.0 and later:
#include <GraphBLAS.h>
int main (void)
{
    GrB_init (GrB_NONBLOCKING) ;
    GrB_finalize ( ) ;
}
```

Compile the program above, then use this command to ensure libgraphblas.so.9 appears:

ldd a.out

18.3.8 Running the tests

To run a short test, type make demo at the top-level GraphBLAS folder. This will run all the demos in GraphBLAS/Demos. MATLAB is not required.

To perform the extensive tests in the Test folder, and the statement coverage tests in Tcov, MATLAB R2018a or later is required. See the README.txt files in those two folders for instructions on how to run the tests. The tests in the Test folder have been ported to MATLAB on Linux, MacOS, and Windows. The Tcov tests do not work on Windows. The MATLAB interface test (gbtest) works on all platforms; see the GraphBLAS/GraphBLAS folder for more details.

18.3.9 Cleaning up

To remove all compiled files, type make distclean in the top-level Graph-BLAS folder.

19 Release Notes

- Mar 1, 2025: version 10.0.0
 - 32/64 bit matrices and vectors: the GrB_Matrix and GrB_Vector now exploit 32-bit integers when possible. New methods added to pass 32-bit integer arrays to/from GrB_build, extract, assign, subassign, and extractTuples. New object, the GxB_Container added for fast import/export of matrices/vectors with arbitrary integer content.
 - GrB_Field: this enum is strongly deprecated, and replaced with typedef int GrB_Field. This is an upward-compatible change to the API, and will allow the creation of a future mathematical field object in GraphBLAS. This type should not be used; use an int instead. It will be replaced in a future version of GraphBLAS.
 - enum parameters: replaced all enum parameters with int, to simplify future updates to enum parameters, including the GrB_Field.
 - GxB_JIT_ERROR: added in 9.4.x, changed value to avoid conflict with LAGraph error codes.
 - workaround for an AppleClang compiler bug: On the Mac, the Source/mask/GB_masker.c file triggers a bug in AppleClang 16.0.0 with -O3 in (MacOS 14.6.1 (23G93), Xcode 16.2, Apple clang version 16.0.0, clang-1600.0.26.6). It also fails in MacOS 15.2 (Target: arm64-apple-darwin23.6.0). The bug is triggered by these tests in LAGraph (v1.2 branch, unreleased, Jan 4, 2025):
 - 39 LAGraphX_BF (SIGTRAP)
 - 40 LAGraphX_Coarsen_Matching (Failed)
 - 41 LAGraphX_FastGraphletTransform (SIGTRAP)
 - 49 LAGraphX_PageRankGX (SIGTRAP)
 - 54 LAGraphX_SquareClustering (SIGTRAP)
 - 61 LAGraphX_msf (Failed)

When using clang, optimization is turned off for this file. This has no impact on performance since GB_masker.c is very simple, consisting of a single sequence of calls to other methods.

- pack/unpack: these are declared historical; they still work but use 64-bit integers only. Use the new GxB_Container methods instead.
- GxB_Matrix_iso and GxB_Vector_iso: declared historical; use
 GrB_get with the new GxB_ISO enum.
- GxB_Matrix_type, GxB_Vector_type, GxB_Scalar_type: no longer historical; added back to the user guide.
- Summary: the API is upward-compatible with 9.4.x, but only after the user application is recompiled with GraphBLAS v10.0.0. As a result, the SO version must increase from 9 to 10.
- Feb 18, 2025: version 9.4.5
 - (59) bug fix: the GxB_NO_INT32 and GxB_NOT_INT64 flags in GB_control.h did not completely remove some of the INT32 and INT64 factory kernels. Caught by Erik Welch, NVIDIA.
- Feb 17, 2024: version 9.4.4
 - (58) bug fix: semirings with user-defined monoids and the GrB_ONEB or GxB_PAIR operators caused a JIT compiler error. Caught by Gabe Gomez.
 - (57) bug fix: GraphBLAS.h header: remove duplicate definitions of GxB_MAX_FIRST_* semirings (incompletely moved to 'historical' section in 9.4.2).
- Nov 20, 2024: version 9.4.2
 - clarified User Guide: regarding when the hyper-hash is built
 - JIT: reduced JIT kernel encodings
 - (also includes the updates from 9.4.0.beta and 9.4.1.beta listed below).

- Nov 15, 2024: version 9.4.1 (only released as BETA)
 - More JIT kernels: all JIT kernels for GrB_assign , $GxB_subassign$, $GrB_extract$, $GxB_sort\ GrB_kronecker$, the stand-alone mask phase (an internal method that computes $C\langle M\rangle = Z$), and utilities have been created. All kernels formerly tagged in the code as JIT: needed are now finished.
 - removed Factory kernels for: types int8 and uint8, and semirings:
 max_min, max_plus, max_times, min_max, min_times, plus_min,
 plus_max, non-Boolean land/lor/lxor/lxnor, and integer times_first/second,
 to reduce size of compiled library. JIT kernels will be used instead
 for these types and semirings.
 - GxB_IndexBinaryOp: finalized and named as GxB_*.
- Oct 15, 2024: version 9.4.0 (only released as BETA)
 - new operator and associated methods: added the draft G*B_IndexBinaryOp.
 - JIT error-handling behavior changed: if a compiler error occurs in the JIT, GxB_JIT_ERROR is now returned. Previously, GraphBLAS would fall back to a generic method if such an error occurred.
- Aug 12, 2024: version 9.3.1
 - (56) bug fix: wrong type for fgetc return value in JITpackage; leads to infinite loop on some systems when building GraphBLAS.
- Aug 2, 2024: version 9.3.0
 - code restructuring: Source folder split into many subfolders, and some files and internal functions renamed. No visible external change.
 - (55) bug fix: **GrB_apply** with user-defined index-unary op and generic kernel.
 - (54) bug fix: reducing a huge iso full matrix to a scalar resulted in integer overflow if nrows*ncols was larger than about 2⁶⁰.
 - reduced size of compiled library: int16 and uint16 types and operators for FactoryKernels are disabled in GB_control.h. The JIT will always be used instead.

- May 22, 2024: version 9.2.0
 - Added graphblas_install.m for a simpler method of compiling the MATLAB/Octave interface for GraphBLAS.
 - JIT: sanitizing the JIT cache path, better burble for compiler errors.
 - GrB_get/GrB_set: better handling of concurrent get/set between different user threads.
- Mar 22, 2024: version 9.1.0
 - minor updates to build system
 - C11 complex type detection: this is now detected and configured by cmake, instead of using an #if ... in the GraphBLAS.h header. This change was required to port GraphBLAS to the clang-cl compiler on Windows when it simulates the MSVC compiler. Also added a new feature (thus the minor version update to 9.1.0): GxB_HAVE_COMPLEX* to GraphBLAS.h to indicate which kind of complex data types are available in C11 or MSVC. Contributed by Markus Mützel.
 - (53) bug fix: eWiseAdd C<M>=A+B when M, A, and B are all hypersparse; access to M was incorrect (also affects C<M>+=T for any operation, if M and T are both hypersparse).
- Mar 1, 2024: version 9.0.3
 - (52) performance bug fix: JIT kernels since v8.3.1 were not compiled with OpenMP.
- Feb 26, 2024: version 9.0.2
 - GraphBLAS/Makefile make static was incorrect.
- Jan 20, 2024: version 9.0.1
 - minor updates to build system
- Version 9.0.0, Jan 10, 2024

- GrB_get/GrB_set: new functions from the v2.1 C API.
- GrB_Type_new, GrB_UnaryOp_new, GrB_IndexUnaryOp_new: no longer macros, since GrB_set can be used to set the names of the operators. These methods no longer extract the name, so the default name is now the empty string. This is because GrB_get/set can only set these names once. This is a non-compatible change of behavior for these 3 methods, so SuiteSparse:GraphBLAS must become v9.0.0.
- historical methods: many methods are replaced by GrB_get and GrB_set. They remain in SuiteSparse:GraphBLAS but have been declared historical. Terse prototypes exist in GraphBLAS.h, and any discussion is removed from the User Guide: GxB_get, GxB_set, and the methods they call, and many more. Use GrB_get/set in place those methods, and for: GxB_*type_name, GxB_*type, GxB_Monoid_operator, GxB_Monoid_identity, GxB_Monoid_terminal, GxB_Semiring_add, GxB_Semiring_multiply. Use GrB_STORAGE_ORIENTATION_HINT in place of GxB_FORMAT.
- hyper_hash: constructed only if the number of non-empty vectors in a hypersparse matrix is large (> 1024, by default).
- minor updates to build system: *.pc files for pkgconfig
- Dec 30, 2023: version 8.3.1
 - remove #undef I from GraphBLAS.h, so as not to conflict with the definition of I from complex.h.
 - major change to build system: by Markus Mützel
- Oct 7, 2023: version 8.2.1
 - (49) bug fix: GrB_mxm saxpy4 and saxpy5 had incorrectly handling of typecasting in v8.0.0 to v8.2.0 (caught by Erik Welch)
 - cross-compiler support: replace check_c_source_runs with _compiles for GraphBLAS and SuiteSparse_config, and remove check for getenv("HOME").
 - cmake update: add "None" build type, from Antonio Rojas, for Arch Linux

- Version 8.2.0, Sept 8, 2023
 - cmake updates: SuiteSparse:: namespace by Markus Mützel.
- Version 8.0.2, June 16, 2023
 - added -DJITINIT=option: use -DJITINIT to set the initial state of the GxB_JIT_C_CONTROL (4:on, 3:load, 2:run, 1:pause, 0:off).
 The default is 4 (on) if the JIT is enabled, or 2 (run) if -DNJIT=1 is set.
 - xxHash: upgraded to latest version as of June 16, 2023
- Version 8.0.1, May 27, 2023
 - (48) bug fix: GrB_*_nvals returned UINT64_MAX ('infinity') for a GrB_Vector of size n-by-2⁶⁰; it should return 2⁶⁰. Caught by Erik Welch, NVIDIA.
 - added GxB_Context_error and GxB_Context_wait
 - C++: changed complex typedefs for C++ that include GraphBLAS.h.
 Update from Markus Mützel.
- Version 8.0.0 (May 18, 2023)
 - version 8: This version is a major SO version increase, since it removes a few minor user-visible features from SuiteSparse:GraphBLAS: the GrB_Descriptor no longer supports threading control, and some features of the GxB_SelectOp are removed (see below). Enum values have been changed for compatibility with the upcoming GrB_set/get features in the V2.1 C API.
 - The JIT: GraphBLAS v8.0.0 includes a JIT for the CPU kernels, which can compile kernels at run-time. Added GxB_set/get options and environment variables to control the JIT. The GxB_*Op_new methods can accept NULL function pointers, if the strings are provided, valid, and the JIT is enabled.
 - GxB_Type_new: the size of the type can be given as zero, in which case the size is determined via a JIT kernel.

- GxB_UnaryOp_new, GxB_BinaryOp_new, and GxB_IndexUnaryOp_new: the function pointer can be given as NULL, in which case the function is created by the JIT.
- math kernels: revised for CUDA JIT. More accurate complex floating-point for Mac OS on Apple Silicon.
- Demo/wildtype_demo: change to double so that CPU and GPU versions compute the same result.
- GxB_get: can return malloc/calloc/realloc/free
- GxB_Context: an object for controlling computational resources: number of OpenMP threads, the chunk factor, and (draft) GPU id.
- GrB_Descriptor: removed ability to control the number of OpenMP threads from the descriptor (a rarely used feature). Replaced with the GxB_Context object.
- GxB_SelectOp: GraphBLAS no longer supports user-defined GxB_SelectOps. Use a GrB_IndexUnaryOp instead. The GxB_SelectOp_new and GxB_SelectOp_free functions are removed entirely. The built-in GxB_SelectOps, GxB_Matrix_select, GxB_Vector_select, and GxB_select still work. However, the GxB_EQ_THUNK, GxB_EQ_ZERO, GxB_NE_THUNK, and GxB_NE_ZERO operators no longer work on user-defined types, as they did in v7.4.4 and earlier. Create a user-defined GrB_IndexUnaryOp to compute these operations instead, and use GrB_select.
- alternative/Makefile: removed; not compatible with the JIT.
- zstd: upgraded to v1.5.5 (Apr 4, 2023)
- Version 7.4.4 (Mar 25, 2023)
 - (47) bug fix: OpenMP atomics require seq_cst on the ARM. Revised GB_atomics.h accordingly, and added them for all architectures (caught by Gábor Szárnyas).
- Version 7.4.3 (Jan 20, 2023)
 - debug: turned on in GrB_Matrix_removeElement by mistake.

- Version 7.4.2 (Jan 17, 2023)
 - minor change to build system: for SuiteSparse v7.0.0.
 - deprecation notice: in GraphBLAS v8.0.0, the ability to set the number of threads, and chunk size, in the descriptor will be removed. It still appears in v7.x, but will be replaced by a Context object in v8.0.0.
- Version 7.4.1 (Dec 29, 2022)
 - global free pool: disabled. Benefit for single-thread user applications was modest, and it causes too much contention in a critical section when the user application is multi-threaded.
 - GrB_mxm: revised task creation heuristics for sparse-times-sparse for better performance.
- Version 7.4.0 (Dec 23, 2022)
 - add non-va_arg methods: va_arg-based GxB_get/set methods are C11 but cause issues for cffi in Python. As a temporary workaround, new methods have been added that do not use va_arg. The existing GxB_get/set methods are not changed. The new methods are not in the user guide, since all of the GxB_get/set methods will be superceded with GrB_get/set in the v2.1 C API. At that point, all GxB_get/set methods will become historical (kept, not deprecated, but removed from the user guide).
- Version 7.3.3 (Dec 9, 2022)
 - stdatomic.h: using #include <stdatomic.h> and atomic_compare_exchange_weak instead of GCC/clang/icx __atomic_* variants. Added -latomic if required.
 - chunk factor for C=A*B (saxpy3 method): revised for non-builtinsemirings
- Version 7.3.2 (Nov 12, 2022)
 - cmake_modules: minor revision to build system, to sync with SuiteSparse v6.0.0

- Added option -DNOPENMP=1 to disable OpenMP parallelism.
- Version 7.3.1 (Oct 21, 2022)
 - workaround for a bug in the Microsoft Visual Studio Compiler, MSC 19.2x (in vs2019).
- Version 7.3.0 (Oct 14, 2022)
 - GrB_Matrix: changes to the internal data structure
 - minor internal changes: A->nvals for sparse/hypersparse
 - more significant changes: added hyper-hash for hypersparse case, speeds up many operations on hypersparse matrices. Based on [Gre21].
 - GxB_unpack_HyperHash and GxB_pack_HyperHash: to pack/unpack the hyper-hash
 - @GrB MATLAB/Octave interface: changed license to Apache-2.0.
 - MATLAB library: renamed to libgraphblas_matlab.so
 - performance: faster C=A*B when using a single thread and B is a sparse vector with many entries.
- Version 7.2.0 (Aug 8, 2022)
 - added ZSTD as a compression option for serialize/deserialize: Version 1.5.3 by Yann Collet, https://github.com/facebook/zstd.git.
 Copyright (c) 2016-present, Facebook, Inc. All rights reserved. Included in SuiteSparse:GraphBLAS via its BSD-3-clause license. The default method is now ZSTD, level 1.
 - GxB_Matrix_reshape* added.
 - MATLAB interface: reshape, C(:)=A, C=A(:) are faster. Better error messages.
- Version 7.1.2 (July 8, 2022)
 - MATLAB interface: linear indexing added for C(:)=A, C=A(:), and single-output I=find(C). Faster bandwidth, istriu, istril, isbanded, isdiag. C(I,J)=A can now grow the size of A.
- Version 7.1.1 (June 3, 2022)

- minor updates to documentation and error messages
- MATLAB interface: minor revision of GrB.deserialize
- Version 7.1.0 (May 20, 2022)
 - added cube root: GxB_CBRT_FP32 and GxB_CBRT_FP64 unary operators
 - added GxB_Matrix_isStoredElement and GxB_Vector_isStoredElement
- Version 7.0.4 (Apr 25, 2022)
 - (46) bug fix: user-defined type size was incorrectly limited to 128 bytes. Caught by Erik Welch.
- Version 7.0.3 (Apr 8, 2022)
 - faster transpose when using 2 threads
- Version 7.0.2 (Apr 5, 2022)
 - (45) bug fix: vector iterator was broken for iterating across a vector in bitmap format. Caught by Erik Welch.
- Version 7.0.1 (Apr 3, 2022)
 - added revised ACM TOMS submission to the Doc folder
- Version 7.0.0 (Apr 2, 2022)
 - (44) spec bug: GrB_Matrix_diag was implemented in v5.2.x and v6.x with the wrong signature. This fix requires the major release to change, from v6.x to v7.x.
 - (43) performance bug fix for GrB_mxm: auto selection for saxpy method (Hash vs Gustavson) revised.
 - GrB_assign: better performance for C(i,j)=scalar and C(i,j)+=scalar when i and j have length 1 (scalar assignment with no scalar expansion).
- Version 6.2.5 (Mar 14, 2022)

- For SuiteSparse v5.11.0.
- Version 6.2.4 (Mar 8, 2022)
 - (42) bug fix: GrB_mxm with 0-by-0 iso full matrices. Caught by Henry Amuasi in the Python grblas interface, then triaged and isolated by Erik Welch.
- Version 6.2.3 (Mar 5, 2022)
 - minor update to documentation in GrB.build: no change to any code
- Version 6.2.2 (Feb 28, 2022)
 - revised output of GxB_*_sort to return newly created matrices
 C and P as full or bitmap matrices, as appropriate, instead of sparse/hypersparse, following their sparsity control settings.
- Version 6.2.1 (Feb 14, 2022)
 - (41) bug fix: GxB_Iterator_get used (void *) + size arithmetic
- Version 6.2.0 (Feb 14, 2022)
 - added the GxB_Iterator object and its methods. See Section 14.
 - @GrB interface: revised sparse-times-full rule for the conventional semiring (the syntax C=A*B), so that sparse-times-full results in C as full, but hypersparse-times-sparse is not full (typically sparse or hypersparse).
- Version 6.1.4 (Jan 12, 2022)
 - added Section 16 to User Guide: how to get the best performance out of algorithms based on GraphBLAS.
 - cpu_features: no longer built as a separate library, but built directly into libgraphblas.so and libgraphblas.a. Added compiletime flags to optionally disable the use of cpu_features completely.

- Octave 7: port to Apple Silicon (thanks to Gábor Szárnyas).
- min/max monoids: real case (FP32 and FP64) no longer terminal
- @GrB interface: overloaded C=A*B syntax where one matrix is full always results in a full matrix C.
- Faster C=A*B for sparse-times-full and full-times-sparse for @GrB interface.
- Version 6.1.3 (Jan 1, 2022)
 - performance: task creation for GrB_mxm had a minor flaw (results were correct but parallelism suffered). Performance improvement of up to 10x when nnz(A);;nnz(B).
- Version 6.1.2 (Dec 31, 2021)
 - performance: revised swap_rule in GrB_mxm, which decides whether to compute C=A*B or C=(B'*A')', and variants, resulting in up to 3x performance gain over v6.1.1 for GrB_mxm (observed; could be higher in other cases).
- Version 6.1.1 (Dec 28, 2021)
 - minor revision to AVX2 and AVX512f selection
 - cpu_features/Makefile: remove test of list_cpu_features so that the package can be built when cross-compiling
- Versions 6.1.0 (Dec 26, 2021)
 - added GxB_get options: compiler name and version.
 - added package: https://github.com/google/cpu_features, Nov 30, 2021 version.
 - performance: faster C+=A*B when C is full, A is bitmap/full, and
 B is sparse/hyper. Faster C+=A*B when A is sparse/hyper, and B is bitmap/full.
 - (40) bug fix: deserialization of iso and empty matrices/vectors was incorrect
- Versions 6.0.2 and 5.2.2 (Nov 30, 2021)

- (39) bug fix: GrB_Matrix_export: numerical values not properly exported
- Versions 6.0.1 and 5.2.1 (Nov 27, 2021)
 - v6.0.x and v5.2.x (for the same x): differ only in GrB_wait, GrB_Info, GrB_SCMP, and GxB_init.
 - (38) bug fix: C+=A'*B when the accum operator is the same as the monoid and C is iso-full, and A or B are hypersparse. (dot4 method).
 - performance: GrB_select with user-defined GrB_IndexUnaryOp about 2x faster.
 - performance: faster (MIN, MAX)_(FIRSTJ, SECONDI) semirings
- Version 6.0.0 (Nov 15, 2021)
 - this release contains only a few changes that cause a break with backward compatibility. It is otherwise identical to v5.2.0.
 - v6.0.0 is fully compliant with the v2.0 C API Specification. Three changes from the v2.0 C API Spec are not backward compatible (GrB_*wait, GrB_Info, GrB_SCMP). GxB_init has also changed.
 - * GrB_wait (object, mode): was GrB_wait (&object).
 - * GrB_Info: changed enum values
 - * GrB_SCMP: removed
 - * GxB_init (mode, malloc, calloc, realloc, free, is_thread_safe): the last parameter, is_thread_safe, is deleted. The malloc, calloc, realloc, and free functions must be thread-safe.
- Version 5.2.0 (Nov 15, 2021)
 - Added for the v2.0 C API Specification: only features that are backward compatible with SuiteSparse:GraphBLAS v5.x have been added to v5.2.0:
 - * GrB_Scalar: replaces GxB_Scalar, GxB_Scalar_* functions renamed GrB
 - * GrB_IndexUnaryOp: new, free, fprint, wait
 - * GrB_select: selection via GrB_IndexUnaryOp
 - * GrB_apply: with GrB_IndexUnaryOp

- * GrB_reduce: reduce matrix or vector to GrB_Scalar
- * GrB_assign, GrB_subassion: with GrB_Scalar input
- * GrB_*_extractElement_Scalar: get GrB_Scalar from a matrix or vector
- * GrB*build: when dup is NULL, duplicates result in an error.
- * GrB import/export: import/export from/to user-provided arrays
- * Grb_EMPTY_OBJECT, Grb_NOT_IMPLEMENTED: error codes added
- * GrB_*_setElement_Scalar: set an entry in a matrix or vector, from a GrB_Scalar
- * GrB_Matrix_diag: same as GxB_Matrix_diag (C,v,k,NULL)
- * GrB_*_serialize/deserialize: with compression
- * GrB_ONEB_T: binary operator, f(x,y) = 1, the same as GxB_PAIR_T.
- GxB*import* and GxB*export*: now historical.
- GxB_select: is now historical; use GrB_select instead.
- GxB_IGNORE_DUP: special operator for build methods only; if dup is this operator, then duplicates are ignored (not an error)
- GxB_IndexUnaryOp_new: create a named index-unary operator
- GxB_BinaryOp_new: create a named binary operator
- GxB_UnaryOp_new: create a named unary operator
- GxB_Type_new: to create a named type
- GxB_Type_name: to query the name of a type
- added GxB_*type_name methods to query the name of a type as a string.
- GxB_Matrix_serialize/deserialize: with compression; optional descriptor.
- GxB_Matrix_sort, GxB_Vector_sort: sort a matrix or vector
- GxB_eWiseUnion: like GrB_eWiseAdd except for how entries in $\mathbf{A} \setminus \mathbf{B}$ and $\mathbf{B} \setminus \mathbf{A}$ are computed.
- added LZ4/LZ4HC: compression library, http://www.lz4.org (BSD 2), v1.9.3, Copyright (c) 2011-2016, Yann Collet.
- MIS and pagerank demos: removed; MIS added to LAGraph/experimental
- disabled free memory pool if OpenMP not available
- (37) bug fix: ewise C=A+B when all matrices are full, GBCOMPACT not used, but GB_control.h disabled the operator or type. Caught by Roi Lipman, Redis.

- (36) bug fix: C<M>=Z not returning C as iso if Z iso and C initially empty. Caught by Erik Welch, Anaconda.
- performance improvements: C=A*B: sparse/hyper times bitmap/full,
 and visa versa, including C += A*B when C is full.
- Version 5.1.10 (Oct 27, 2021)
 - (35) bug fix: GB_selector; A->plen and C->plen not updated correctly. Caught by Jeffry Lovitz, Redis.
- Version 5.1.9 (Oct 26, 2021)
 - (34) bug fix: in-place test incorrect for C+=A'*B using dot4
 - (33) bug fix: disable free pool if OpenMP not available
- Version 5.1.8 (Oct 5, 2021)
 - (32) bug fix: C=A*B when A is sparse and B is iso and bitmap. Caught by Mark Blanco, CMU.
- Version 5.1.7 (Aug 23, 2021)
 - (31) bug fix: GrB_apply, when done in-place and matrix starts non-iso and becomes iso, gave the wrong iso result. Caught by Fabian Murariu.
- Version 5.1.6 (Aug 16, 2021)
 - one-line change to C=A*B: faster symbolic analysis when a vector
 C(:,j) is dense (for CSC) or C(i,:) for CSR.
- Version 5.1.5 (July 15, 2021)
 - submission to ACM Transactions on Mathematical Software as a Collected Algorithm of the ACM.
- Version 5.1.4 (July 6, 2021)
 - faster Octave interface. Octave v7 or later is required.

- (30) bug fix: 1-based printing not enabled for pending tuples. Caught by Will Kimmerer, while working on the Julia interface.

• Version 5.1.3 (July 3, 2021)

- added GxB_Matrix_iso and GxB_Vector_iso: to query if a matrix or vector is held as iso-valued
- (29) bug fix: Matrix_pack_*R into a matrix previously held by column, or Matrix_pack*C into a matrix by row, would flip the dimensions. Caught by Erik Welch, Anaconda.
- (28) bug fix: kron(A,B) with iso input matrices A and B fixed.
 Caught by Michel Pelletier, Graphegon.
- (27) bug fix: v5.1.0 had a wrong version of a file; posted by mistake. Caught by Michel Pelletier, Graphegon.

• Version 5.1.2 (June 30, 2021)

- iso matrices added: these are matrices and vectors whose values in the sparsity pattern are all the same. This is an internal change to the opaque data structures of the GrB_Matrix and GrB_Vector with very little change to the API.
- added GxB_Matrix_build_Scalar and GxB_Vector_build_Scalar, which always build iso matrices and vectors.
- import/export methods can now import/export iso matrices and vectors.
- added GrB.argmin/argmax to MATLAB/Octave interface
- added GxB_*_pack/unpack methods in place of import/export.
- added GxB_PRINT_1BASED to the global settings.
- added GxB_*_memoryUsage
- port to Octave: gbmake and gbtest work in Octave 7 to build and test the @GrB interface to GraphBLAS. Octave 7.0.0 is required.

• Version 5.0.6 (May 24, 2021)

 BFS and triangle counting demos removed from GraphBLAS/Demo: see LAGraph for these algorithms. Eventually, all of Graph-BLAS/Demo will be deleted, once LAGraph includes all the methods included there.

- Version 5.0.5 (May 17, 2021)
 - (26) performance bug fix: reduce-to-vector where A is hypersparse CSR with a transposed descriptor (or CSC with no transpose), and some cases for GrB_mxm/mxv/vxm when computing C=A*B with A hypersparse CSC and B bitmap/full (or A bitmap/full and B hypersparse CSR), the wrong internal method was being selected via the auto-selection strategy, resulting in a significant slowdown in some cases.
- Version 5.0.4 (May 13, 2021)
 - @GrB MATLAB/Octave interface: changed license to GNU General Public License v3.0 or later. It was licensed under Apache-2.0 in Version 5.0.3 and earlier. Changed back to Apache-2.0 for Version 7.3.0; see above.
- Version 5.0.3 (May 12, 2021)
 - (25) bug fix: disabling ANY_PAIR semirings by editing Source/GB_control.h would cause a segfault if those disabled semirings were used.
 - demos are no longer built by default
 - (24) bug fix: new functions in v5.0.2 not declared as extern in GraphBLAS.h.
 - GrB_Matrix_reduce_BinaryOp reinstated from v4.0.3; same limit on built-in ops that correspond to known monoids.
- Version 5.0.2 (May 5, 2021)
 - (23) bug fix: GrB_Matrix_apply_BinaryOp1st and 2nd were using the wrong descriptors for GrB_INPO and GrB_INP1. Caught by Erik Welch, Anaconda.
 - memory pool added for faster allocation/free of small blocks
 - QGrB interface ported to MATLAB R2021a.
 - GxB_PRINTF and GxB_FLUSH global options added.
 - GxB_Matrix_diag: construct a diagonal matrix from a vector
 - GxB_Vector_diag: extract a diagonal from a matrix
 - concat/split: methods to concatenate and split matrices.

- import/export: size of arrays now in bytes, not entries. This change is required for better internal memory management, and it is not backward compatible with the GxB*import/export functions in v4.0. A new parameter, is_uniform, has been added to all import/export methods, which indicates that the matrix values are all the same.
- (22) bug fix: SIMD vectorization was missing reduction(+,task_cnvals) in GB_dense_subassign_06d_template.c. Caught by Jeff Huang, Texas A&M, with his software package for race-condition detection.
- GrB_Matrix_reduce_BinaryOp: removed. Use a monoid instead,
 with GrB_reduce or GrB_Matrix_reduce_Monoid.
- Version 4.0.3 (Jan 19, 2021)
 - faster min/max monoids
 - G=GrB(G) converts G from v3 object to v4
- Version 4.0.2 (Jan 13, 2021)
 - ability to load *.mat files saved with the v3 GrB
- Version 4.0.1 (Jan 4, 2021)
 - significant performance improvements: compared with v3.3.3, up to 5x faster in breadth-first-search (using LAGraph_bfs_parent2), and 2x faster in Betweenness-Centrality (using LAGraph_bc_batch5).
 - GrB_wait(void), with no inputs: removed
 - GrB_wait(&object): polymorphic function added
 - GrB_*_nvals: no longer guarantees completion; use GrB_wait(&object)
 or non-polymorphic GrB_*_wait (&object) instead
 - GrB_error: now has two parameters: a string (char **) and an object.
 - GrB_Matrix_reduce_BinaryOp limited to built-in operators that correspond to known monoids.
 - GrB_*_extractTuples: may return indices out of order
 - removed internal features: GBI iterator, slice and hyperslice matrices

- bitmap/full matrices and vectors added
- index-based operators and semirings: GxB_FIRSTI_INT32 and related ops
- jumbled matrices: sort left pending, like zombies and pending tuples
- GxB_get/set: added GxB_SPARSITY_* (hyper, sparse, bitmap, or full) and GxB_BITMAP_SWITCH.
- GxB_HYPER: enum renamed to GxB_HYPER_SWITCH
- GxB*import/export: API modified
- GxB_SelectOp: nrows and ncols removed from function signature.
- OpenMP tasking removed from mergesort and replaced with parallel for loops. Just as fast on Linux/Mac; now the performance ports to Windows.
- GxB_BURBLE added as a supported feature. This was an undocumented feature of prior versions.
- bug fix: A({lo,hi})=scalar A(lo:hi)=scalar was OK
- Version 3.3.3 (July 14, 2020). Bug fix: w<m>=A*u with mask non-empty and u empty.
- Version 3.3.2 (July 3, 2020). Minor changes to build system.
- Version 3.3.1 (June 30, 2020). Bug fix to GrB_assign and GxB_subassign when the assignment is simple (C=A) but with typecasting.
- Version 3.3.0 (June 26, 2020). Compliant with V1.3 of the C API (except that the polymorphic GrB_wait(&object) doesn't appear yet; it will appear in V4.0).

Added complex types (GxB_FC32 and GxB_FC64), many unary operators, binary operators, monoids, and semirings. Added bitwise operators, and their monoids and semirings. Added the predefined monoids and semirings from the v1.3 specification. @GrB interface: added complex matrices and operators, and changed behavior of integer operations to more closely match the behavior on built-in integer matrices. The rules for typecasting large floating point values to integers has changed. The specific object-based GrB_Matrix_wait, GrB_Vector_wait, etc,

functions have been added. The no-argument GrB_wait() is deprecated. Added GrB_getVersion, GrB_Matrix_resize, GrB_Vector_resize, GrB_kronecker, GrB_*_wait, scalar binding with binary operators for GrB_apply,

GrB_Matrix_removeElement, and GrB_Vector_removeElement.

- Version 3.2.0 (Feb 20, 2020). Faster GrB_mxm, GrB_mxv, and GrB_vxm, and faster operations on dense matrices/vectors. Removed compiletime user objects (GxB_*_define), since these were not compatible with the faster matrix operations. Added the ANY and PAIR operators. Added the predefined descriptors, GrB_DESC_*. Added the structural mask option. Changed default chunk size to 65,536. @GrB interface modified: GrB.init is now optional.
- Version 3.1.2 (Dec, 2019). Changes to allow SuiteSparse:GraphBLAS to be compiled with the Microsoft Visual Studio compiler. This compiler does not support the _Generic keyword, so the polymorphic functions are not available. Use the equivalent non-polymorphic functions instead, when compiling GraphBLAS with MS Visual Studio. In addition, variable-length arrays are not supported, so user-defined types are limited to 128 bytes in size. These changes have no effect if you have an C11 compliant compiler.

QGrB interface modified: **GrB**.init is now required.

- Version 3.1.0 (Oct 1, 2019). @GrB interface added. See the GraphBLAS/GraphBLAS folder for details and documentation, and Section 4.1.
- Version 3.0 (July 26, 2019), with OpenMP parallelism.

The version number is increased to 3.0, since this version is not backward compatible with V2.x. The GxB_select operation changes; the Thunk parameter was formerly a const void * pointer, and is now a GxB_Scalar. A new parameter is added to GxB_SelectOp_new, to define the expected type of Thunk. A new parameter is added to GxB_init, to specify whether or not the user-provided memory management functions are thread safe.

The remaining changes add new features, and are upward compatible with V2.x. The major change is the addition of OpenMP paral-

lelism. This addition has no effect on the API, except that round-off errors can differ with the number of threads used, for floating-point types. GxB_set can optionally define the number of threads to use (the default is $omp_get_max_threads$). The number of threads can also defined globally, and/or in the $GrB_Descriptor$. The RDIV and RMINUS operators are added, which are defined as f(x,y) = y/x and f(x,y) = y-x, respectively. Additional options are added to GxB_get .

- Version 2.3.3 (May 2019): Collected Algorithm of the ACM. No changes from V2.3.2 other than the documentation.
- Version 2.3 (Feb 2019) improves the performance of many Graph-BLAS operations, including an early-exit for monoids. These changes have a significant impact on breadth-first-search (a performance bug was also fixed in the two BFS Demo codes). The matrix and vector import/export functions were added, in support of the new LA-Graph project (https://github.com/GraphBLAS/LAGraph, see also Section 17.1). LAGraph includes a push-pull BFS in GraphBLAS that is faster than two versions in the Demo folder. GxB_init was added to allow the memory manager functions (malloc, etc) to be specified.
- Version 2.2 (Nov 2018) adds user-defined objects at compile-time, via user *.m4 files placed in GraphBLAS/User, which use the GxB_*_define macros (NOTE: feature removed in v3.2). The default matrix format is now GxB_BY_ROW. Also added are the GxB_*print methods for printing the contents of each GraphBLAS object (Section 13). PageRank demos have been added to the Demos folder.
- Version 2.1 (Oct 2018) was a major update with support for new matrix formats (by row or column, and hypersparse matrices), and colon notation (I=begin:end or I=begin:inc:end). Some graph algorithms are more naturally expressed with matrices stored by row, and this version includes the new GxB_BY_ROW format. The default format in Version 2.1 and prior versions is by column. New extensions to Graph-BLAS in this version include GxB_get, GxB_set, and GxB_AxB_METHOD, GxB_RANGE, GxB_STRIDE, and GxB_BACKWARDS, and their related definitions, described in Sections 6.16, 10, and 11.
- Version 2.0 (March 2018) addressed changes in the GraphBLAS C API Specification and added GxB_kron and GxB_resize.

- Version 1.1 (Dec 2017) primarily improved the performance.
- Version 1.0 was released on Nov 25, 2017.

19.1 Regarding historical and deprecated functions and symbols

When a GxB* function or symbol is added to the C API Specification with a GrB* name, the new GrB* name should be used instead, if possible. However, the old GxB* name will be kept as long as possible for historical reasons. Historical functions and symbols will not always be documented here in the SuiteSparse:GraphBLAS User Guide, but they will be kept in GraphbBLAS.h and kept in good working order in the library. Historical functions and symbols would only be removed in the very unlikely case that they cause a serious conflict with future methods.

The following methods have been fully deprecated and removed. The older versions of GrB_wait and GrB_error methods have been removed since they are incompatible with the latest versions, per the C API Specification. The GxB_SelectOp_new and GxB_SelectOp_free methods have been removed, and some of the built-in operators have been revised (specifically, the GxB_EQ_THUNK, GxB_EQ_ZERO, GxB_NE_THUNK, and GxB_NE_ZERO operators no longer work on user-defined types).

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21 Additional Resources

See http://graphblas.org for the GraphBLAS community page. See https://github.com/GraphBLAS/GraphBLAS-Pointers for an up-to-date list of additional resources on GraphBLAS, maintained by Gábor Szárnyas.

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From the preface: Graphs are among the most important abstract data types in computer science, and the algorithms that operate on them are critical to modern life. Graphs have been shown to be powerful tools for modeling complex problems because of their simplicity and generality. Graph algorithms are one of the pillars of mathematics, informing research in such diverse areas as combinatorial optimization, complexity theory, and topology. Algorithms on graphs are applied in many ways in today's world – from Web rankings to metabolic networks, from finite element meshes to semantic graphs. The current exponential growth in graph data has forced a shift to parallel computing for executing graph algorithms. Implementing parallel graph algorithms and achieving good parallel performance have proven difficult. This book addresses these challenges by exploiting the well-known duality between a canonical representation of graphs as abstract collections of vertices and edges and a sparse adjacency matrix representation. This linear algebraic approach is widely accessible to scientists and engineers who may not be formally trained in computer science. The authors show how to leverage existing parallel matrix computation techniques and the large amount of software infrastructure that exists for these computations to implement efficient and scalable parallel graph algorithms. The benefits of this approach are reduced algorithmic complexity, ease of implementation, and improved performance. DOI: https://dx.doi.org/10.1137/1.9780898719918

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