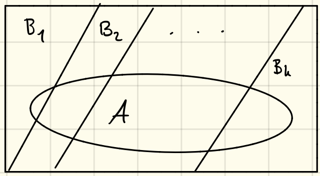
Forelesning 5 (22.01)

Bayes' regel:

$$P(B_j \mid A) = \frac{P(A \mid B_j)\,P(B_j)}{P(A)}$$



$B_1, \dots, B_k$ oppdeling av utfallsrommet

Total sannsynlighet: $P(A) = \sum_{i=1}^{n} \underbrace{P(A \mid B_i) \cdot P(B_i)}_{P(A \cap B_i)}$

- Stokastisk variabel:

3.2 Diskrete sannsynlighetsfordelinger

La x være en diskret stokastisk variabel. Funksjonen f definert ved at:

$f(x) = P(X = x)$ for alle mulige verdier x av X, kalles sannsynlighetsmassefunksjonen (pmf) (punkt sannsynligheten, sannsynlighetsfordelingen) for X. Den har egenskapene $f(x) \geq 0$ for alle x,

$$\sum_x f(x) = 1, \quad P(X \in A) = \sum_{x \in A} f(x).$$

Eks: Kast av to terninger. La X = antall seksere. Utfallsrom: $S = \{11, 12, \dots, 66\}$ (36 like sannsynlige utfall). La f være pmf for X.

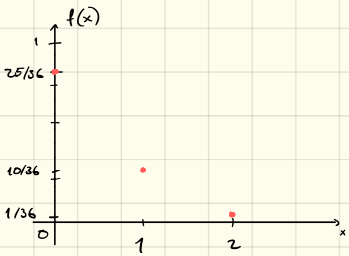
$$f(0) = P(X=0) = P(\{11, 12, \dots 15, 21, 22, \dots, 25, 31, 32, \dots, 35, 41, 42, \dots, 45, 51, 52, \dots 55\}) = \frac{25}{36}$$

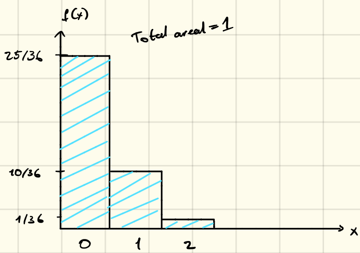
$$f(1) = P(X=1) = P(\{16, 26, \dots 56, 61, 62, \dots, 65\}) = \frac{10}{36}$$

$$f(2) = P(X=2) = P(\{66\}) = \frac{1}{36}$$

$$f(X) = 0 \text{ for } x \notin \{0, 1, 2\}.$$

Grafisk framstilling:



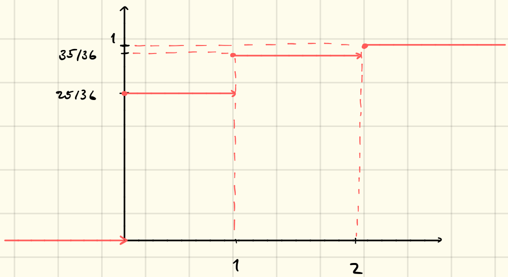


Kumulativ fordelingsfunksjon (cdf) F for en stokastisk variabel X er definert ved at $F(x) = P(X \leq x)$ for alle $x \in \mathbb{R}$

Eks forts.: $F(0) = P(X \leq 0) = P(X=0) = f(0) = \frac{25}{36}$

$F(1) = P(X \leq 1) = P(X=0) + P(X=1) = f(0) + f(1) = \frac{25}{36} + \frac{10}{36} = \frac{35}{36}$

$F(2) = P(X \leq 2) = f(0) + f(1) + f(2) = \frac{25}{36} + \frac{10}{36} + \frac{1}{36} = 1$



$F(-0{,}2) = P(X \leq -0{,}2)$

$F(1{,}9) = P(X \leq 1{,}9) = \frac{35}{36}$

$F(13) = P(X \leq 13) = \underline{1}$

$P(X > x) = 1 - P(X \leq x) = 1 - F(x)$ La $a < b$: $P(a < X < b) = 1 - P(X \leq a) - P(X \leq b) = F(b) - F(a)$

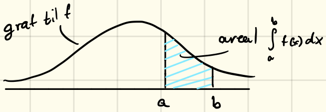
$F(x) = P(X \leq x) = \sum_{i \leq x} P(X = i) = \sum_{i \leq x} f(i)$

$P(a < X \leq b) = \sum_{a < x \leq b} f(x)$

3.3 Kontinuerlig sannsynlighetsfordeling

For en kontinuerlig variabel X: $P(X = x) = 0$ for hver x.

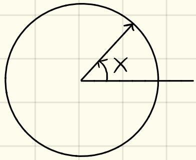
$f: \mathbb{R} \to \mathbb{R}$ er **sannsynlighetstettheten** (pdf) til X hvis $P(a < X < b) = \int_a^b f(x)\,dx$ for alle a, b der $a \leq b$



Har egenskapen $f(x) \geq 0$ for alle x, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Merk: For kontinuerlig variabel X: $P(a < X \leq b) = P(a < X \leq b) = P(a \leq X < b) = P(a \leq X \leq b)$

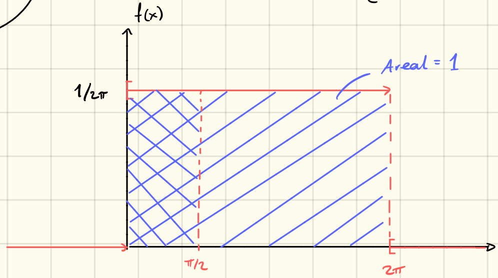
Eks: Lykkehjul



La X = vinkel på pil,

$X \in [0, 2\pi)$. Skal finne pdf f. Ønsker at alle intervall av samme lengde skal ha samme sannsynlighet, og at $\int_0^{2\pi} f(x)\,dx = 1$

Er tilfredstilt når $f(x) = \begin{cases} 1/2\pi, & 0 \le x \le 2\pi \\ 0, & \text{ellers} \end{cases}$



$$P\left(0 < X < \tfrac{\pi}{2}\right) = \int_0^{\pi/2} f(x)\,dx = \int_0^{\pi/2} \frac{1}{2\pi}\,dx = \left[\frac{x}{2\pi}\right]_0^{\pi/2} = \frac{1}{4}$$

Kummulativ fordelingsfunksjon

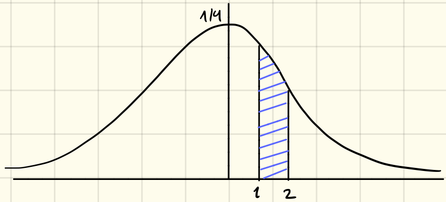
(cdf) F er definert ved: $F(x) = P(X \le x)$ (som diskret). Dvs: $F(x) = \int_{-\infty}^{x} f(t)\,dt$ for alle x $(f = \text{pdf})$

Merk: $P(a < X \le b) = \int_a^b f(x)\,dx = F(b) - F(a)$ (som diskret) og $F' = f$

Eks: Anta at X har tetthet (pdf) $f(x) = \dfrac{e^{-x}}{(1+e^{-x})^2}$

Kumulativ fordelingsfunksjon:

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{-\infty}^{x} \frac{e^{-t}}{(1+e^{-t})^2}\,dt = \dots = \frac{1}{1+e^{-x}}$$



$$P(X < 0) = F(0) = \frac{1}{1+e^{-0}} = \frac{1}{2}$$

$$P(1 < X < 2) = F(2) - F(1) = \frac{1}{1+e^{-2}} - \frac{1}{1+e^{-1}} = 0{,}5$$