INPUT SIGNALS

```
In [0]:
```

```
import numpy as np
import scipy.signal as ss
import matplotlib.pyplot as plt
from matplotlib.ticker import AutoMinorLocator
from decimal import Decimal
import pdb
A = 1
```

```
In [0]:
```

```
def evaluate_periodic_sine_and_a_half(time_array : list, v_max):
    V_MAX = v_max
    res = []
    for t in time_array:
        t_in_oritginal_period = float(Decimal(str(t)) % Decimal(str(3*np.pi)))

    if t_in_oritginal_period > 0:
        y = V_MAX * np.sin(t_in_oritginal_period)
    else:
        y = V_MAX * np.sin(t_in_oritginal_period + 3*np.pi)
        res.append(y)
    return np.asarray(res)
```

```
In [0]: ▶
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```
def evaluate_periodic_exp(time_array : list, v_max):
    V_MAX = v_max
    res = []
    for t in time_array:
        t_in_oritginal_period = float(Decimal(str(t)) % Decimal('10'))

    if t_in_oritginal_period < 0:
        t_in_oritginal_period = 10 - t_in_oritginal_period

    if t_in_oritginal_period < 5:
        y = V_MAX * np.e**(-np.abs(t_in_oritginal_period))
    else:
        y = V_MAX * np.e**(-np.abs(t_in_oritginal_period - 10))

    res.append(y)
    return np.asarray(res)</pre>
```

```
def compute_fft(time_interval, signal, period, n_periods=1, window='boxcar'):
  ''' window can be:
      'boxcar' (rectangle),
      'barthann' (Bartlett-Hann),
      'bartlett',
      'hanning',
      'hamming',
      'tukey',
      'flattop',
      'hann',
      'nuttall',
      'parzen',
      'cosine',
      'blackman',
      'bohman',
      'blackmanharris' '''
  import numpy as np
 import scipy.signal as ss
 t_step = (max(time_interval) - min(time_interval)) / len(time_interval)
 points_in_period = int(np.rint(period / t_step))
 try:
   window = getattr(ss, window)((points_in_period * n_periods))
   amount_of_zeros_to_pad = len(signal) - len(window)
   window = np.append(window, [0] * amount_of_zeros_to_pad)
   signal_for_fft = np.multiply(signal, window)[:(points_in_period * n_periods)]
 except AttributeError:
     return
 fft = np.fft.fft(signal_for_fft)
 N = signal_for_fft.size
 f = np.fft.fftfreq(N, d=t_step)
 return (f, fft, N)
```

```
def fft(time_interval, signal, period, mode='fast'):
    ''' time_interval: array containing time values for the signal meant to be transformed.
        signal: array containing signal meant to be transformed.
        period: period of the signal to be transformed.
        If mode='fast', only one period will be used from the input signal.
        If mode='best', the max amount of periods will be used.
        'fast' is default.'''
    if mode == 'best':
        n_p = int(np.rint((max(time_interval) - min(time_interval)) / period))
    elif mode == 'fast':
        n_p = 1
    else:
        return
    window_types = ['boxcar', 'barthann', 'bartlett', 'hanning', 'hamming', 'tukey', 'hann'
    fft = []
    merits = []
    for w in window_types:
      f, X, N = compute_fft(time_interval, signal, period, n_periods=n_p, window=w)
      new_{fft} = [f, X, N, w]
      fft.append(new_fft)
      merit = 0
      max_bin = max(np.abs(X))
      for X_bin in X:
        if np.abs(X_bin) < (0.2*max_bin):</pre>
          merit = merit + np.abs(X_bin)
      merits.append(merit)
    return fft[merits.index(min(merits))]
```

```
In [0]:
```

```
periods_to_use = 5
period_A = 0.002
period_B1 = 3*np.pi
period_B2 = 10
period_C = 0.01

t_A = np.linspace(-0.002, 0.008, periods_to_use * 100)
t_B1 = np.linspace(-3*np.pi, 12*np.pi, periods_to_use * 1000)
t_B2 = np.linspace(-5, 45, periods_to_use * 1000)
t_C = np.linspace(-0.01, 0.04, periods_to_use * 1000)

x_A = A * np.cos(2 * np.pi * 0.5e3 * t_A)
x_B1 = evaluate_periodic_sine_and_a_half(t_B1, A)
x_B2 = evaluate_periodic_exp(t_B2, A)
x_C = A * (0.5 * np.cos(2*np.pi * 0.9e3 * t_C) + np.cos(2*np.pi * 1e3 * t_C) + 0.5 * np.cos
```

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Input signals in time domain.')
ax1.plot(t_A, x_A)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax1.set_ylabel('$ x_A $ (V)')
ax2.plot(t_B1, x_B1)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax2.set_ylabel('$ x_{B1} $ (V)')
ax3.plot(t_B2, x_B2)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax3.set_ylabel('$ x_{B2} $ (V)')
ax4.plot(t_C, x_C)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
ax4.set_ylabel('$ x_C $ (V)')
fig.show()
```

10 0.5 0.5 0.5 0.5 0.5 0.5 0.00 0.00

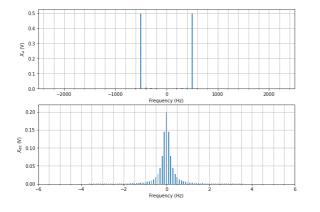
Input signals in time domain

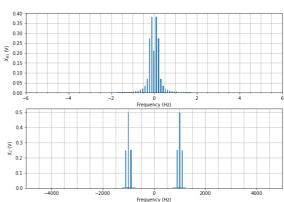
```
In [0]: ▶
```

```
f_A, X_A, N_A = compute_fft(t_A, x_A, period_A, 5)
f_B1, X_B1, N_B1 = compute_fft(t_B1, x_B1, period_B1, 5)
f_B2, X_B2, N_B2 = compute_fft(t_B2, x_B2, period_B2, 5)
f_C, X_C, N_C = compute_fft(t_C, x_C, period_C, 5)
```

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Input signals in frequency domain.')
ax1.bar(f_A, np.abs(X_A) * 1/N_A, width=20)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Frequency (Hz)')
ax1.set_ylabel('$ X_A $ (V)')
ax1.set_xlim(left=-2500, right=2500)
ax2.bar(f_B1, np.abs(X_B1) * 1/N_B1, width=0.05)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Frequency (Hz)')
ax2.set_ylabel('$ X_{B1} $ (V)')
ax2.set_xlim(left=-6, right=6)
ax2.set_ylim(top=0.4)
ax3.bar(f_B2, np.abs(X_B2) * 1/N_B2, width=0.04)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Frequency (Hz)')
ax3.set_ylabel('$ X_{B2} $ (V)')
ax3.set_xlim(left=-6, right=6)
ax3.set_ylim(top=0.22)
ax4.bar(f_C, np.abs(X_C) * 1/N_C, width=40)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Frequency (Hz)')
ax4.set_ylabel('$ X_C $ (V)')
ax4.set_xlim(left=-5000, right=5000)
fig.show()
```

Input signals in frequency domain





ANTI-ALIASING FILTER

In [0]:

```
In [0]:

def apply_aaf(input_time, input_signal_in_time, input_freq, input_signal_in_frequency, f_aa tau = 2 * f_aaf

aaf_pulse = [0] * int(len(input_signal_in_frequency))
for i in range(len(input_freq)):
    if np.abs(input_freq[i]) < (tau / 2):
        aaf_pulse[i] = 1

res_freq = np.multiply(input_signal_in_frequency, aaf_pulse)
res_time = np.fft.ifft(res_freq)

return (res_time, res_freq)</pre>
```

```
f_aaf_low = 2.5
f_aaf_high = 1.8e3

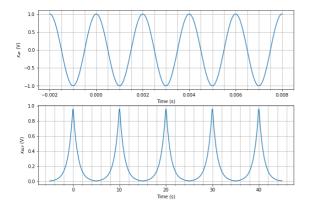
x_A_filtered, X_A_filtered = apply_aaf(t_A, x_A, f_A, X_A, f_aaf_high)
x_B1_filtered, X_B1_filtered = apply_aaf(t_B1, x_B1, f_B1, X_B1, f_aaf_low)
x_B2_filtered, X_B2_filtered = apply_aaf(t_B2, x_B2, f_B2, X_B2, f_aaf_low)
```

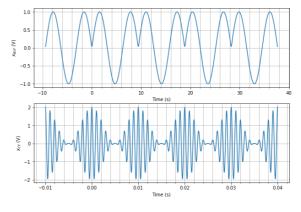
x_C_filtered, X_C_filtered = apply_aaf(t_C, x_C, f_C, X_C, f_aaf_high)

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after AAF in time domain.')
ax1.plot(t_A, x_A_filtered)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax1.set_ylabel('$x_{AF}$(V)')
ax2.plot(t_B1, x_B1_filtered)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax2.set_ylabel('$ x_{B1F} $ (V)')
ax3.plot(t_B2, x_B2_filtered)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax3.set_ylabel('$x_{B2F} $ (V)')
ax4.plot(t_C, x_C_filtered)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
ax4.set_ylabel('$x_{CF}$(V)')
fig.show()
```

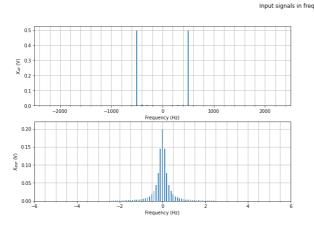
```
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
```

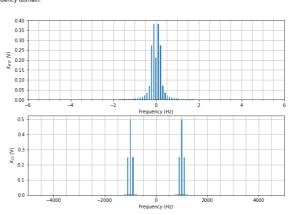






```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Input signals in frequency domain.')
ax1.bar(f_A, np.abs(X_A_filtered) * 1/N_A, width=20)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Frequency (Hz)')
ax1.set_ylabel('$ X_{AF} $ (V)')
ax1.set_xlim(left=-2500, right=2500)
ax2.bar(f_B1, np.abs(X_B1_filtered) * 1/N_B1, width=0.05)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Frequency (Hz)')
ax2.set_ylabel('$ X_{B1F} $ (V)')
ax2.set_xlim(left=-6, right=6)
ax2.set_ylim(top=0.4)
ax3.bar(f_B2, np.abs(X_B2_filtered) * 1/N_B2, width=0.04)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Frequency (Hz)')
ax3.set_ylabel('$ X_{B2F} $ (V)')
ax3.set_xlim(left=-6, right=6)
ax3.set_ylim(top=0.22)
ax4.bar(f_C, np.abs(X_C_filtered) * 1/N_C, width=40)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Frequency (Hz)')
ax4.set_ylabel('$ X_{CF} $ (V)')
ax4.set_xlim(left=-5000, right=5000)
fig.show()
```





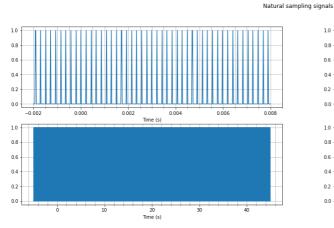
NATURAL SAMPLING

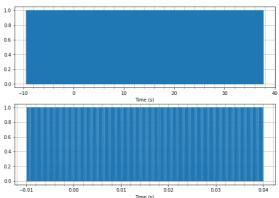
```
f_s_low = 600e3 / (2**16)
f_s_high = 600e3 / (2**7)
duty = 0.1

natural_sampling_signal_A = (ss.square(2 * np.pi * f_s_high * t_A, duty=duty) + 1) /2
natural_sampling_signal_B1 = (ss.square(2 * np.pi * f_s_low * t_B1, duty=duty) + 1) / 2
natural_sampling_signal_B2 = (ss.square(2 * np.pi * f_s_low * t_B2, duty=duty) + 1) / 2
natural_sampling_signal_C = (ss.square(2 * np.pi * f_s_high * t_C, duty=duty) + 1) / 2
```

In [0]: ▶

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Natural sampling signals.')
ax1.plot(t_A, natural_sampling_signal_A)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax2.plot(t_B1, natural_sampling_signal_B1)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax3.plot(t_B2, natural_sampling_signal_B2)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax4.plot(t_C, natural_sampling_signal_C)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
fig.show()
```





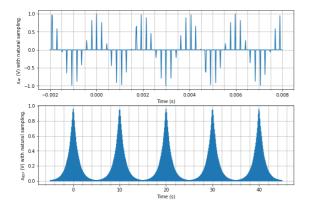
In [0]: ▶

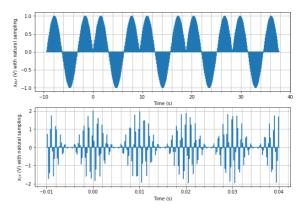
```
x_A_filtered_ns = np.multiply(x_A_filtered, natural_sampling_signal_A)
x_B1_filtered_ns = np.multiply(x_B1_filtered, natural_sampling_signal_B1)
x_B2_filtered_ns = np.multiply(x_B2_filtered, natural_sampling_signal_B2)
x_C_filtered_ns = np.multiply(x_C_filtered, natural_sampling_signal_C)
```

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after AAF and natural sampling in time domain.')
ax1.plot(t_A, x_A_filtered_ns)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax1.set_ylabel('$ x_{AF} $ (V) with natural sampling.')
ax2.plot(t_B1, x_B1_filtered ns)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax2.set_ylabel('$ x_{B1F} $ (V) with natural sampling.')
ax3.plot(t_B2, x_B2_filtered_ns)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax3.set_ylabel('$ x_{B2F} $ (V) with natural sampling.')
ax4.plot(t_C, x_C_filtered_ns)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
ax4.set_ylabel('$ x_{CF} $ (V) with natural sampling.')
fig.show()
```

/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)



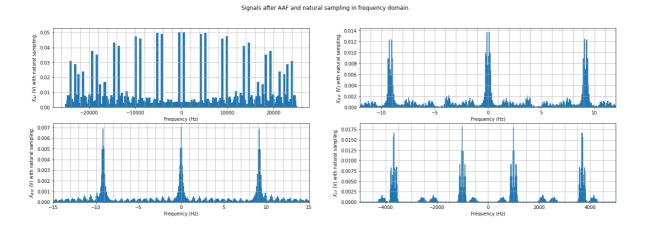




```
f_A_filtered_ns, X_A_filtered_ns, N_A_filtered_ns = compute_fft(t_A, x_A_filtered_ns, perio
# f_B1_filtered_ns, X_B1_filtered_ns, N_B1_filtered_ns = compute_fft(t_B1, x_B1_filtered_ns
# f_B2_filtered_ns, X_B2_filtered_ns, N_B2_filtered_ns = compute_fft(t_B2, x_B2_filtered_ns
# f_C_filtered_ns, X_C_filtered_ns, N_C_filtered_ns = compute_fft(t_C, x_C_filtered_ns, 0.0
# f_A_filtered_ns, X_A_filtered_ns, N_A_filtered_ns, window_used_A = fft(t_A, x_A_filtered_ns)
# f_B1_filtered_ns, X_B1_filtered_ns, N_B1_filtered_ns, window_used_B1 = fft(t_B1, x_B1_filtered_ns)
# f_B2_filtered_ns, X_B2_filtered_ns, N_B2_filtered_ns, window_used_B2 = fft(t_B2, x_B2_filtered_ns)
# f_C_filtered_ns, X_C_filtered_ns, N_C_filtered_ns, window_used_C = fft(t_C, x_C_filtered_ns)
```

/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:24: Deprecation Warning: `hanning` is deprecated, use `scipy.signal.windows.hann` instead!

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after AAF and natural sampling in frequency domain.')
ax1.bar(f_A_filtered_ns, np.abs(X_A_filtered_ns) * 1/N_A_filtered_ns, width=500)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Frequency (Hz)')
ax1.set_ylabel('$ X_{AF} $ (V) with natural sampling.')
ax2.bar(f_B1_filtered_ns, np.abs(X_B1_filtered_ns) * 1/N_B1_filtered_ns, width=0.08)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Frequency (Hz)')
ax2.set_ylabel('$ X_{B1F} $ (V) with natural sampling.')
ax2.set_xlim(left=-12, right=12)
ax3.bar(f B2_filtered_ns, np.abs(X_B2_filtered_ns) * 1/N_B2_filtered_ns, width=0.1)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Frequency (Hz)')
ax3.set_ylabel('$ X_{B2F} $ (V) with natural sampling.')
ax3.set_xlim(left=-15, right=15)
ax4.bar(f_C_filtered_ns, np.abs(X_C_filtered_ns) * 1/N_C_filtered_ns, width=40)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Frequency (Hz)')
ax4.set ylabel('$ X {CF} $ (V) with natural sampling.')
ax4.set_xlim(left=-5000, right=5000)
fig.show()
```



Using ideal frequency (multiple of the signal's frequency)

```
duty = 0.1

natural_sampling_signal_A = (ss.square(2 * np.pi * 5e3 * t_A, duty=duty) + 1) /2

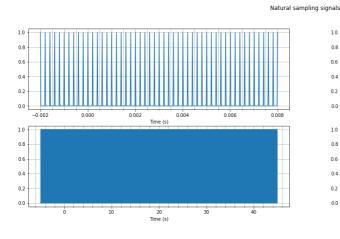
natural_sampling_signal_B1 = (ss.square(2 * np.pi * (40 * 1/(period_B1)) * t_B1, duty=duty)

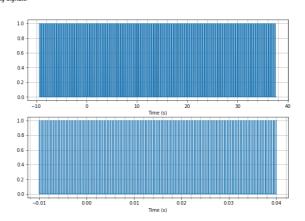
natural_sampling_signal_B2 = (ss.square(2 * np.pi * 10 * t_B2, duty=duty) + 1) / 2

natural_sampling_signal_C = (ss.square(2 * np.pi * 3e3 * t_C, duty=duty) + 1) / 2
```

```
In [0]: ▶
```

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Natural sampling signals.')
ax1.plot(t_A, natural_sampling_signal_A)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax2.plot(t_B1, natural_sampling_signal_B1)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax3.plot(t_B2, natural_sampling_signal_B2)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax4.plot(t_C, natural_sampling_signal_C)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
fig.show()
```





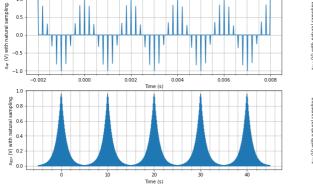
In [0]: ▶

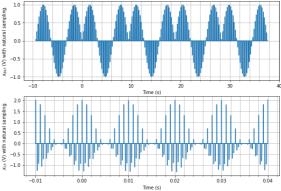
```
x_A_filtered_ns = np.multiply(x_A_filtered, natural_sampling_signal_A)
x_B1_filtered_ns = np.multiply(x_B1_filtered, natural_sampling_signal_B1)
x_B2_filtered_ns = np.multiply(x_B2_filtered, natural_sampling_signal_B2)
x_C_filtered_ns = np.multiply(x_C_filtered, natural_sampling_signal_C)
```

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after AAF and natural sampling in time domain.')
ax1.plot(t_A, x_A_filtered_ns)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax1.set_ylabel('$ x_{AF} $ (V) with natural sampling.')
ax2.plot(t_B1, x_B1_filtered ns)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax2.set_ylabel('$ x_{B1F} $ (V) with natural sampling.')
ax3.plot(t_B2, x_B2_filtered_ns)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax3.set_ylabel('$ x_{B2F} $ (V) with natural sampling.')
ax4.plot(t_C, x_C_filtered_ns)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
ax4.set_ylabel('$ x_{CF} $ (V) with natural sampling.')
fig.show()
```

/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)







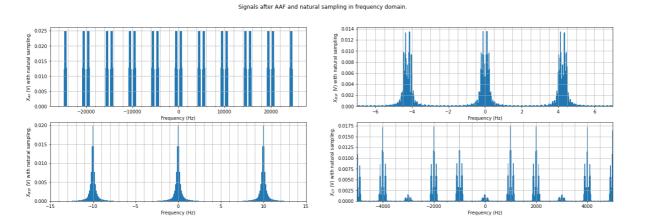
```
# f_A_filtered_ns, X_A_filtered_ns, N_A_filtered_ns = compute_fft(t_A, x_A_filtered_ns, per
# f_B1_filtered_ns, X_B1_filtered_ns, N_B1_filtered_ns = compute_fft(t_B1, x_B1_filtered_ns
# f_B2_filtered_ns, X_B2_filtered_ns, N_B2_filtered_ns = compute_fft(t_B2, x_B2_filtered_ns
# f_C_filtered_ns, X_C_filtered_ns, N_C_filtered_ns = compute_fft(t_C, x_C_filtered_ns, 0.0

f_A_filtered_ns, X_A_filtered_ns, N_A_filtered_ns, window_used_A = fft(t_A, x_A_filtered_ns
f_B1_filtered_ns, X_B1_filtered_ns, N_B1_filtered_ns, window_used_B1 = fft(t_B1, x_B1_filtered_ns)
```

f_B2_filtered_ns, X_B2_filtered_ns, N_B2_filtered_ns, window_used_B2 = fft(t_B2, x_B2_filte
f_C_filtered_ns, X_C_filtered_ns, N_C_filtered_ns, window_used_C = fft(t_C, x_C_filtered_ns)

/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:24: Deprecation Warning: `hanning` is deprecated, use `scipy.signal.windows.hann` instead!

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after AAF and natural sampling in frequency domain.')
ax1.bar(f_A_filtered_ns, np.abs(X_A_filtered_ns) * 1/N_A_filtered_ns, width=500)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Frequency (Hz)')
ax1.set_ylabel('$ X_{AF} $ (V) with natural sampling.')
ax2.bar(f_B1_filtered_ns, np.abs(X_B1_filtered_ns) * 1/N_B1_filtered_ns, width=0.06)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Frequency (Hz)')
ax2.set_ylabel('$ X_{B1F} $ (V) with natural sampling.')
ax2.set_xlim(left=-7, right=7)
ax3.bar(f_B2_filtered_ns, np.abs(X_B2_filtered_ns) * 1/N_B2_filtered_ns, width=0.1)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Frequency (Hz)')
ax3.set_ylabel('$ X_{B2F} $ (V) with natural sampling.')
ax3.set_xlim(left=-15, right=15)
ax4.bar(f_C_filtered_ns, np.abs(X_C_filtered_ns) * 1/N_C_filtered_ns, width=30)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Frequency (Hz)')
ax4.set ylabel('$ X {CF} $ (V) with natural sampling.')
ax4.set_xlim(left=-5000, right=5000)
fig.show()
```



SAMPLE & HOLD

Ideal sampling

```
def delta_train(f, time_array):
    res = np.zeros(int(np.rint(len(time_array))))

t_step = np.abs(time_array[0] - time_array[1])
    period = 1/f
    points_per_period = np.rint(period / t_step)
    res = [1 if (i%points_per_period == 0) else 0 for i in range(len(res))]

    return res
```

```
In [0]: ▶
```

```
ideal_sampling_signal_A = delta_train(f_s_high, t_A)
ideal_sampling_signal_B1 = delta_train(f_s_low, t_B1)
ideal_sampling_signal_B2 = delta_train(f_s_low, t_B2)
ideal_sampling_signal_C = delta_train(f_s_high, t_C)
```

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Ideal sampling signals.')
ax1.plot(t_A, ideal_sampling_signal_A)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax2.plot(t_B1, ideal_sampling_signal_B1)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax3.plot(t_B2, ideal_sampling_signal_B2)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax4.plot(t_C, ideal_sampling_signal_C)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
fig.show()
```

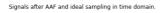

In [0]:

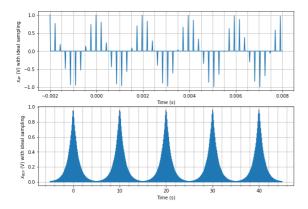
```
x_A_filtered_sample = np.multiply(x_A_filtered, ideal_sampling_signal_A)
x_B1_filtered_sample = np.multiply(x_B1_filtered, ideal_sampling_signal_B1)
x_B2_filtered_sample = np.multiply(x_B2_filtered, ideal_sampling_signal_B2)
x_C_filtered_sample = np.multiply(x_C_filtered, ideal_sampling_signal_C)
```

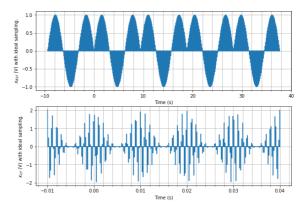
H

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after AAF and ideal sampling in time domain.')
ax1.plot(t_A, x_A_filtered_sample)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax1.set_ylabel('$ x_{AF} $ (V) with ideal sampling.')
ax2.plot(t_B1, x_B1_filtered_sample)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax2.set_ylabel('$ x_{B1F} $ (V) with ideal sampling.')
ax3.plot(t_B2, x_B2_filtered_sample)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax3.set_ylabel('$ x_{B2F} $ (V) with ideal sampling.')
ax4.plot(t_C, x_C_filtered_sample)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
ax4.set_ylabel('$ x_{CF} $ (V) with ideal sampling.')
fig.show()
```

```
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
```







Hold

In [0]:

 $tau_high = 0.3 * 1/f_s_high$

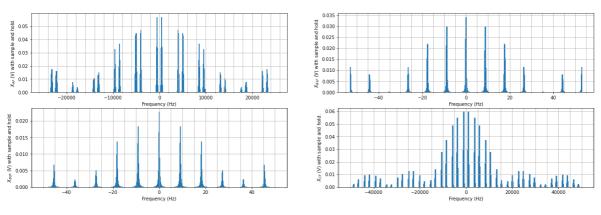
```
tau_low = 0.3 * 1/f_s_low
In [0]:
                                                                                           H
def apply_hold(time_array, signal, signal_period, tau,):
 t_step = (max(time_array) - min(time_array)) / len(time_array)
 points_in_tau = int(np.rint(tau / t_step))
 signal_hold = np.array(signal)
 last_non_zero_value = 0
 last_non_zero_index = 0
 for i in range(len(signal)):
   if signal[i] != 0:
     last_non_zero_value = signal[i]
      last_non_zero_index = i
      signal_hold[i] = last_non_zero_value
   else:
      if (i - last_non_zero_index) < points_in_tau:</pre>
        signal_hold[i] = last_non_zero_value
 f_filtered_hold, X_filtered_hold, N_filtered_hold, window_used_signal = fft(time_array, s
 return (f_filtered_hold, X_filtered_hold, N_filtered_hold, window_used_signal, signal_hol
In [0]:
                                                                                           H
f_A_filtered_hold, X_A_filtered_hold, N_A_filtered_hold, window_used_A, x_A_filtered_hold =
f B1 filtered hold, X B1 filtered hold, N B1 filtered hold, window used B1, x B1 filtered h
f_B2_filtered_hold, X_B2_filtered_hold, N_B2_filtered_hold, window_used_B2, x_B2_filtered_h
f_C_filtered_hold, X_C_filtered_hold, N_C_filtered_hold, window_used_C, x_C_filtered_hold =
```

/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:24: Deprecation Warning: `hanning` is deprecated, use `scipy.signal.windows.hann` instead!

H

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after AAF and sample and hold in frequency domain.')
ax1.bar(f_A_filtered_hold, np.abs(X_A_filtered_hold) * 1/N_A_filtered_hold, width=220)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Frequency (Hz)')
ax1.set_ylabel('$ X_{AF} $ (V) with sample and hold.')
ax2.bar(f_B1_filtered_hold, np.abs(X_B1_filtered_hold) * 1/N_B1_filtered_hold, width=0.4)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Frequency (Hz)')
ax2.set_ylabel('$ X_{B1F}) $ (V) with sample and hold.')
ax3.bar(f_B2_filtered_hold, np.abs(X_B2_filtered_hold) * 1/N_B2_filtered_hold, width=0.4)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Frequency (Hz)')
ax3.set_ylabel('$ X_{B2F} $ (V) with sample and hold.')
ax4.bar(f_C_filtered_hold, np.abs(X_C_filtered_hold) * 1/N_C_filtered_hold, width=600)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Frequency (Hz)')
ax4.set_ylabel('$ X_{CF} $ (V) with sample and hold.')
fig.show()
```

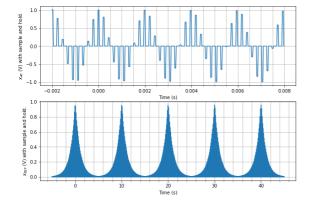


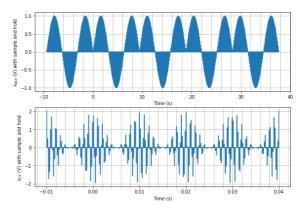


```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after AAF and sample and hold in time domain.')
ax1.plot(t_A, x_A_filtered_hold)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax1.set_ylabel('$ x_{AF} $ (V) with sample and hold.')
ax2.plot(t_B1, x_B1_filtered_hold)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax2.set_ylabel('$ x_{B1F} $ (V) with sample and hold.')
ax3.plot(t_B2, x_B2_filtered_hold)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax3.set_ylabel('$ x_{B2F} $ (V) with sample and hold.')
ax4.plot(t_C, x_C_filtered_hold)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
ax4.set_ylabel('$ x_{CF} $ (V) with sample and hold.')
fig.show()
```

/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)







SUBNYQUIST SAMPLING

Both types of sampling will be performed but now using subnyquist frequencies, and this will only be used for the x_C signal, since it's the only one shaped like band pass.

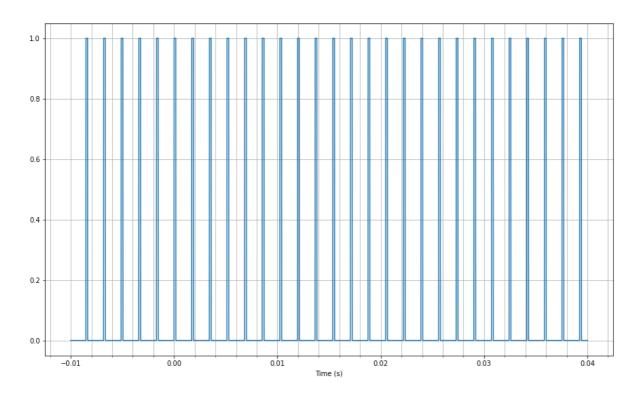
Natural sampling

```
In [0]:

f_sny = 600e3 / 2**10
duty = 0.1
natural_sampling_sny_signal_C = (ss.square(2 * np.pi * f_sny * t_C, duty=duty) + 1) / 2
```

```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Natural sampling signals for subnyquist.')
ax.plot(t_C, natural_sampling_sny_signal_C)
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Time (s)')
fig.show()
```

Natural sampling signals for subnyquist.



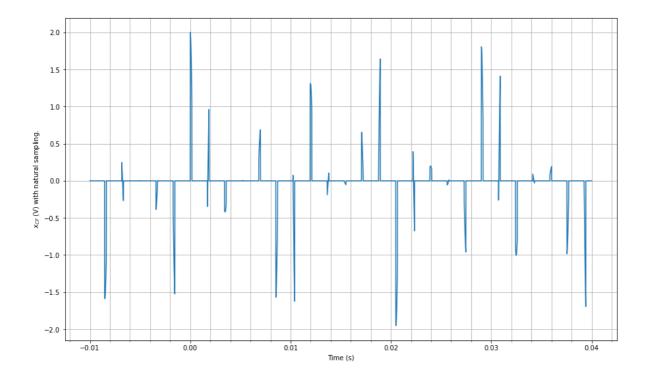
In [0]: ▶

x_C_filtered_sny_ns = np.multiply(x_C_filtered, natural_sampling_sny_signal_C)

```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Signals after AAF and natural sampling in time domain, with subnyquist freque
ax.plot(t_C, x_C_filtered_sny_ns)
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Time (s)')
ax.set_ylabel('$ x_{CF} $ (V) with natural sampling.')
fig.show()
```

/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)

Signals after AAF and natural sampling in time domain, with subnyquist frequency.



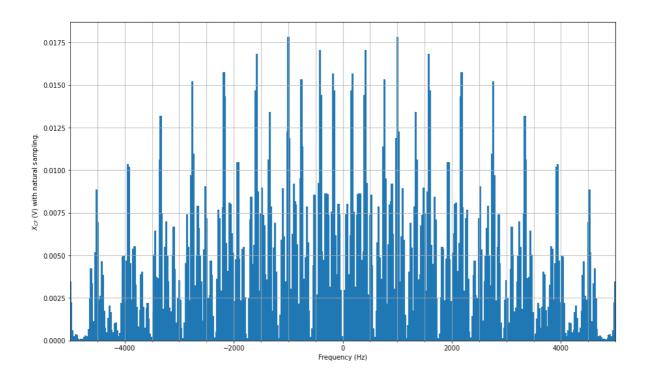
```
f_C_{filtered_sny_ns}, X_C_{filtered_sny_ns}, N_C_{filtered_sny_ns}, window_used_C = fft(t_C, x_C)
```

/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:24: Deprecation Warning: `hanning` is deprecated, use `scipy.signal.windows.hann` instead!

```
In [0]: ▶
```

```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Signals after AAF and natural sampling in frequency domain, with subnyquist f
ax.bar(f_C_filtered_sny_ns, np.abs(X_C_filtered_sny_ns) * 1/N_C_filtered_sny_ns, width=40)
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Frequency (Hz)')
ax.set_ylabel('$ X_{CF} $ (V) with natural sampling.')
ax.set_xlim(left=-5000, right=5000)
fig.show()
```

Signals after AAF and natural sampling in frequency domain, with subnyquist frequency.



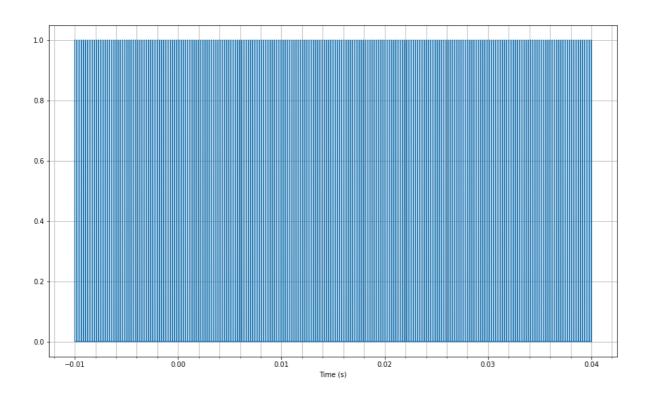
Sample & Hold

```
In [0]:

ideal_sampling_sny_signal_C = delta_train(f_sny, t_C)
```

```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Ideal sampling signals for subnyquist.')
ax.plot(t_C, ideal_sampling_signal_C)
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Time (s)')
fig.show()
```

Ideal sampling signals for subnyquist.



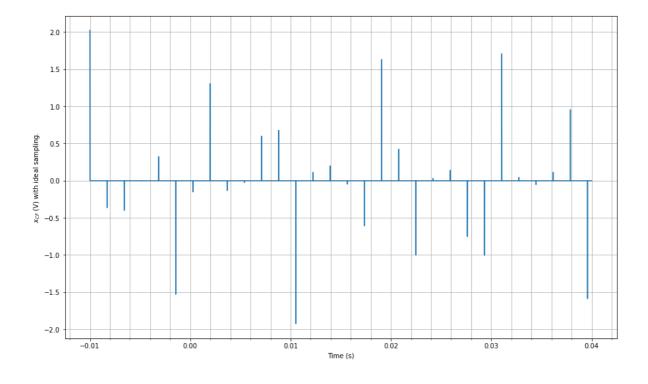
In [0]:

x_C_filtered_sample_sny = np.multiply(x_C_filtered, ideal_sampling_sny_signal_C)

```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Signals after AAF and ideal sampling subnyquist in time domain.')
ax.plot(t_C, x_C_filtered_sample_sny)
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Time (s)')
ax.set_ylabel('$ x_{CF} $ (V) with ideal sampling.')
fig.show()
```

/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)

Signals after AAF and ideal sampling subnyquist in time domain.



In [0]:

tau_sny = 0.3 * 1/f_sny

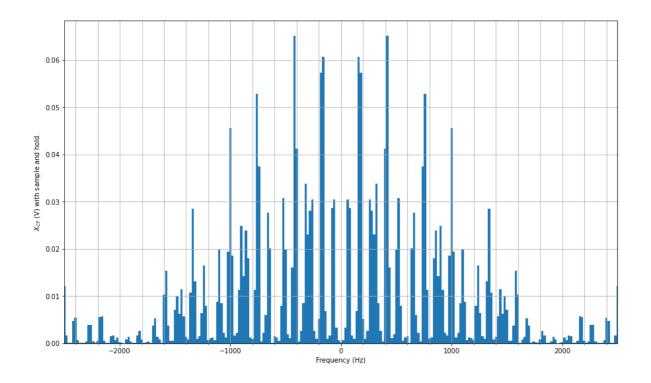
```
f_C_filtered_hold_sny, X_C_filtered_hold_sny, N_C_filtered_hold_sny, window_used_C, x_C_fil
```

/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:24: Deprecation Warning: `hanning` is deprecated, use `scipy.signal.windows.hann` instead!

```
In [0]: ▶
```

```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Signals after AAF and sample and hold subnyquist in frequency domain.')
ax.bar(f_C_filtered_hold_sny, np.abs(X_C_filtered_hold_sny) * 1/N_C_filtered_hold_sny, widt
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Frequency (Hz)')
ax.set_ylabel('$ X_{CF} $ (V) with sample and hold.')
ax.set_xlim(left=-2500, right=2500)
fig.show()
```

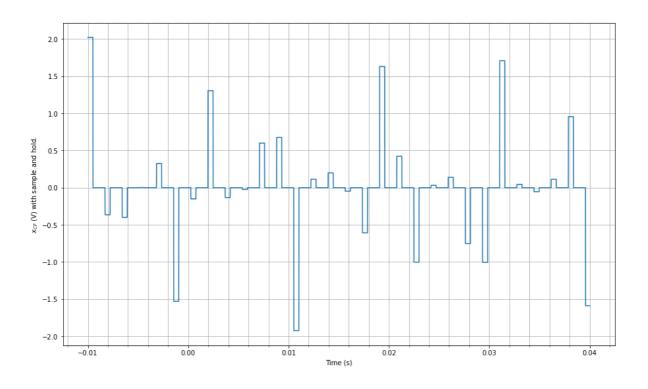
Signals after AAF and sample and hold subnyquist in frequency domain.



```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Signals after AAF and sample and hold subnyquist in time domain.')
ax.plot(t_C, x_C_filtered_hold_sny)
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Time (s)')
ax.set_ylabel('$ x_{CF} $ (V) with sample and hold.')
fig.show()
```

/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)

Signals after AAF and sample and hold subnyquist in time domain.



RECOVERY FILTER

```
In [0]: ▶
```

```
f_rec_low = 2.5
f_rec_high = 1.8e3
```

Natural sampling

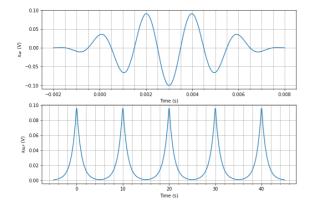
```
In [0]: ▶
```

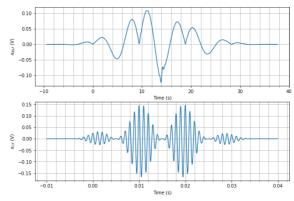
```
x_A_recovered_ns, X_A_recovered_ns = apply_aaf(t_A, x_A_filtered_ns, f_A_filtered_ns, X_A_f
x_B1_recovered_ns, X_B1_recovered_ns = apply_aaf(t_B1, x_B1_filtered_ns, f_B1_filtered_ns,
x_B2_recovered_ns, X_B2_recovered_ns = apply_aaf(t_B2, x_B2_filtered_ns, f_B2_filtered_ns,
x_C_recovered_ns, X_C_recovered_ns = apply_aaf(t_C, x_C_filtered_ns, f_C_filtered_ns, X_C_f
```

```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after RF and natural sampling in time domain.')
ax1.plot(t_A, x_A_recovered_ns)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax1.set_ylabel('$x_{AF}$(V)')
ax2.plot(t_B1, x_B1_recovered_ns)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax2.set_ylabel('$x_{B1F}$$(V)')
ax3.plot(t_B2, x_B2_recovered_ns)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax3.set_ylabel('$x_{B2F} $ (V)')
ax4.plot(t_C, x_C_recovered_ns)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
ax4.set_ylabel('$x_{CF} $(V)')
fig.show()
```

```
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
```

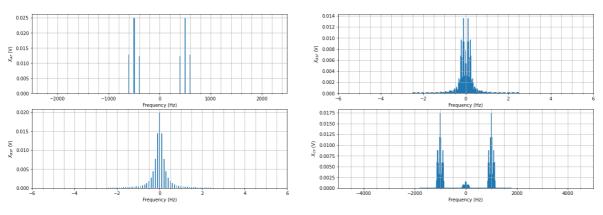






```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after RF and natural sampling in frequency domain.')
ax1.bar(f_A, np.abs(X_A_recovered_ns) * 1/N_A, width=20)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Frequency (Hz)')
ax1.set_ylabel('$ X_{AF} $ (V)')
ax1.set_xlim(left=-2500, right=2500)
ax2.bar(f_B1, np.abs(X_B1_recovered_ns) * 1/N_B1, width=0.05)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Frequency (Hz)')
ax2.set_ylabel('$ X_{B1F} $ (V)')
ax2.set_xlim(left=-6, right=6)
ax3.bar(f_B2, np.abs(X_B2_recovered_ns) * 1/N_B2, width=0.04)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Frequency (Hz)')
ax3.set_ylabel('$ X_{B2F} $ (V)')
ax3.set_xlim(left=-6, right=6)
ax4.bar(f_C, np.abs(X_C_recovered_ns) * 1/N_C, width=40)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Frequency (Hz)')
ax4.set ylabel('$ X {CF} $ (V)')
ax4.set_xlim(left=-5000, right=5000)
fig.show()
```





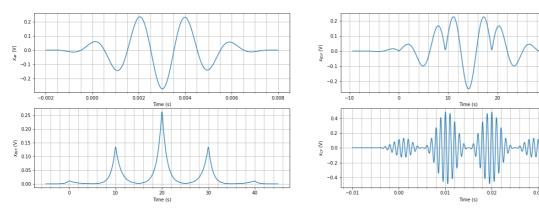
Sample & Hold

```
x_A_recovered_sh, X_A_recovered_sh = apply_aaf(t_A, x_A_filtered_hold, f_A_filtered_hold, X
x_B1_recovered_sh, X_B1_recovered_sh = apply_aaf(t_B1, x_B1_filtered_hold, f_B1_filtered_ho
x_B2_recovered_sh, X_B2_recovered_sh = apply_aaf(t_B2, x_B2_filtered_hold, f_B2_filtered_ho
x_C_recovered_sh, X_C_recovered_sh = apply_aaf(t_C, x_C_filtered_hold, f_C_filtered_hold, X
```

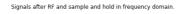
```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after RF and sample and hold in time domain.')
ax1.plot(t_A, x_A_recovered_sh)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Time (s)')
ax1.set_ylabel('$x_{AF}$(V)')
ax2.plot(t_B1, x_B1_recovered_sh)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Time (s)')
ax2.set_ylabel('$x_{B1F}$$(V)')
ax3.plot(t_B2, x_B2_recovered_sh)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Time (s)')
ax3.set_ylabel('$x_{B2F} $ (V)')
ax4.plot(t_C, x_C_recovered_sh)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Time (s)')
ax4.set_ylabel('$x_{CF} $(V)')
fig.show()
```

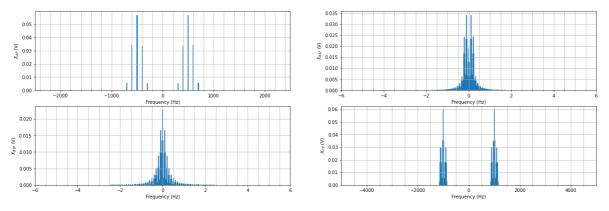
```
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)
```





```
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2, 2, figsize=(22, 7))
fig.suptitle('Signals after RF and sample and hold in frequency domain.')
ax1.bar(f_A, np.abs(X_A_recovered_sh) * 1/N_A, width=20)
ax1.xaxis.set_minor_locator(AutoMinorLocator())
ax1.grid(True, which='both')
ax1.set_xlabel('Frequency (Hz)')
ax1.set_ylabel('$ X_{AF} $ (V)')
ax1.set_xlim(left=-2500, right=2500)
ax2.bar(f_B1, np.abs(X_B1_recovered_sh) * 1/N_B1, width=0.05)
ax2.xaxis.set_minor_locator(AutoMinorLocator())
ax2.grid(True, which='both')
ax2.set_xlabel('Frequency (Hz)')
ax2.set_ylabel('$ X_{B1F} $ (V)')
ax2.set_xlim(left=-6, right=6)
ax3.bar(f_B2, np.abs(X_B2_recovered_sh) * 1/N_B2, width=0.04)
ax3.xaxis.set_minor_locator(AutoMinorLocator())
ax3.grid(True, which='both')
ax3.set_xlabel('Frequency (Hz)')
ax3.set_ylabel('$ X_{B2F} $ (V)')
ax3.set_xlim(left=-6, right=6)
ax4.bar(f_C, np.abs(X_C_recovered_sh) * 1/N_C, width=40)
ax4.xaxis.set_minor_locator(AutoMinorLocator())
ax4.grid(True, which='both')
ax4.set_xlabel('Frequency (Hz)')
ax4.set_ylabel('$ X_{CF} $ (V)')
ax4.set_xlim(left=-5000, right=5000)
fig.show()
```





Subnyquist

Natural sampling

```
In [0]:
```

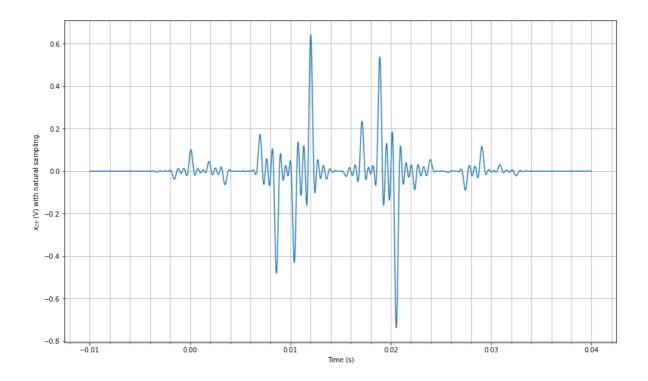
```
x_C_recovered_sny_ns, X_C_recovered_sny_ns = apply_aaf(t_C, x_C_filtered_sny_ns, f_C_filter
```

```
In [0]: ▶
```

```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Signals after RF and natural sampling in time domain, with subnyquist frequen
ax.plot(t_C, x_C_recovered_sny_ns)
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Time (s)')
ax.set_ylabel('$ x_{CF} $ (V) with natural sampling.')
fig.show()
```

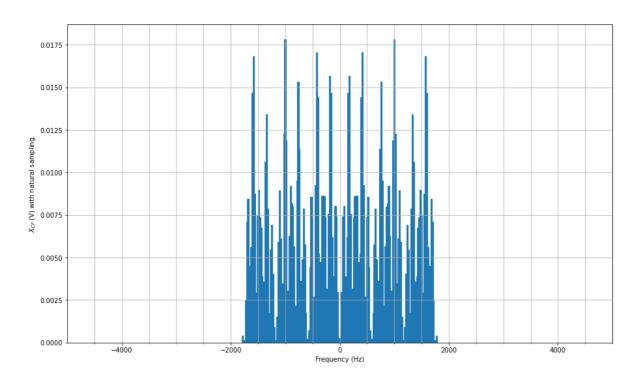
/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)





```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Signals after RF and natural sampling in frequency domain, with subnyquist fr
ax.bar(f_C_filtered_sny_ns, np.abs(X_C_recovered_sny_ns) * 1/N_C_filtered_sny_ns, width=40)
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Frequency (Hz)')
ax.set_ylabel('$ X_{CF} $ (V) with natural sampling.')
ax.set_xlim(left=-5000, right=5000)
fig.show()
```

Signals after RF and natural sampling in frequency domain, with subnyquist frequency.



Sample & Hold

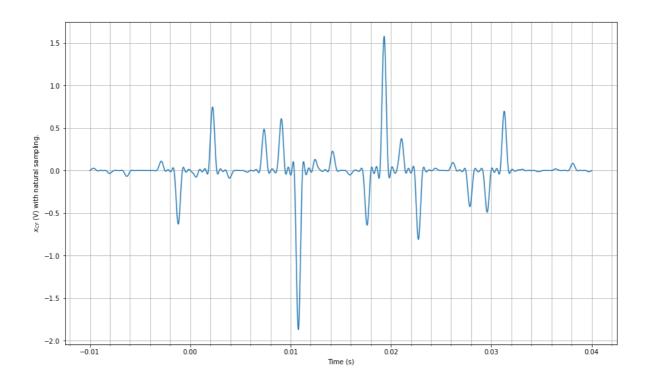
In [0]:

x_C_recovered_hold_sny, X_C_recovered_hold_sny = apply_aaf(t_C, x_C_filtered_hold_sny, f_C_

```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Signals after RF and natural sampling in time domain, with subnyquist frequen
ax.plot(t_C, x_C_recovered_hold_sny)
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Time (s)')
ax.set_ylabel('$ x_{CF} $ (V) with natural sampling.')
fig.show()
```

/usr/local/lib/python3.6/dist-packages/numpy/core/_asarray.py:85: ComplexWar ning: Casting complex values to real discards the imaginary part return array(a, dtype, copy=False, order=order)

Signals after RF and natural sampling in time domain, with subnyquist frequency.



```
fig, ax = plt.subplots(figsize=(15, 9))
fig.suptitle('Signals after RF and natural sampling in frequency domain, with subnyquist fr

ax.bar(f_C_filtered_hold_sny, np.abs(X_C_recovered_hold_sny) * 1/N_C_filtered_hold_sny, wid
ax.xaxis.set_minor_locator(AutoMinorLocator())
ax.grid(True, which='both')
ax.set_xlabel('Frequency (Hz)')
ax.set_ylabel('$ X_{CF} $ (V) with natural sampling.')
ax.set_xlim(left=-5000, right=5000)
fig.show()
```

Signals after RF and natural sampling in frequency domain, with subnyquist frequency.

