LDA

$$\log P(y=k|X) = -\frac{1}{2}(X-\mu_k)^t \Sigma^{-1}(X-\mu_k) + \log P(y=k) + C_{ST}$$

CONSTANT TERMS

$$\mu_k = \frac{\sum_{i=0}^{n-1} x_i}{n}$$

$$\Sigma = \begin{bmatrix} S_{0,0} & S_{1,0} & \cdots & S_{c-1,0} \\ S_{0,1} & S_{1,1} & \cdots & S_{c-1,1} \\ \vdots & \vdots & \ddots & \vdots \\ S_{0,c-1} & S_{1,c-1} & \cdots & S_{c-1,c-1} \end{bmatrix}$$

$$S_{x,y} = \frac{1}{n-1}\sum_{i=0}^{n-1}(x_i - \bar{x}_i)(y_i - \bar{y}_i)$$

KRIGGING

$$\sum \lambda_i X_i = X$$

$$\sum \lambda_i = 1$$

$$\min_{\lambda_i} \sum \lambda_i^2 + q|\lambda_i|$$

DATABASE: $X^T = \begin{bmatrix} X_0 & X_1 & \cdots & X_{n-1} \end{bmatrix} = \begin{bmatrix} X_{0,0} & X_{1,0} & \cdots & X_{n-1,0} \\ X_{0,1} & X_{1,1} & \cdots & X_{n-1,1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{0,c-1} & X_{1,c-1} & \cdots & X_{n-1,c-1} \end{bmatrix}$

SAMPLE

NUMBER OF SAMPLES

NUMBER OF CHARACTERISTICS

DATA TO BE CLASSIFIED: $X_c = \begin{bmatrix} X_{c,0} \\ X_{c,1} \\ \vdots \\ X_{c,n-1} \end{bmatrix}$

LDA

$$\mu_k = \begin{bmatrix} \bar{X}_0 \\ \bar{X}_1 \\ \vdots \\ \bar{X}_{c-1} \end{bmatrix} = \begin{bmatrix} \frac{\sum_{i=0}^{n} X_{i,0}}{n} \\ \frac{\sum_{i=0}^{n} X_{i,1}}{n} \\ \vdots \\ \frac{\sum_{i=0}^{n} X_{i,c-1}}{n} \end{bmatrix} = X \underbrace{\begin{bmatrix} \frac{1}{n} \\ \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}}_{n-1}$$

$$S = \frac{1}{n-1} X X^T$$

$$\log P(y=k|x) = -\frac{1}{2}(X_c - \mu_k)^T S^{-1}(X_c - \mu_k) + \log P(y=k) + C_{ST}$$

$$y = k \Leftarrow \log P(y=k|x) > \log P(y=l|x)\ \forall l \Leftrightarrow \log P'(y=k|x) > \log P'(y=l|x)\ \forall l$$

LDA: SAME S FOR ALL CLASSES

$$\log P'(y=k|x) = -\frac{1}{2}(X_c - \mu_k)^T S^{-1}(X_c - \mu_k) + \log P(y=k)$$

BASE PROBABILITY OF THE CLASS

CAN BE REMOVED ON BALANCED SETTINGS

CAN BE ESTIMATED AS $\frac{n_{k}}{n}$

$$\log P'(y=k|x) = -\frac{1}{2}\left(X_c - X_k\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}\right)^T \left(\frac{1}{n-1} X X^T\right)^{-1}\left(X_c - X_k\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}\right) + \boxed{\log P(y=k)}$$

REMOVING CONSTANTS;

$\sum P' \neq 1$

SUBSET OF X INCLUDING ONLY THOSE FROM CLASS k

KRIGGING $\varphi = 0$

$$\min_{\lambda_i} \sum \lambda_i^2$$

$$\text{s.t.} \quad \sum \lambda_i X_i = X$$

$$\sum \lambda_i = 1$$

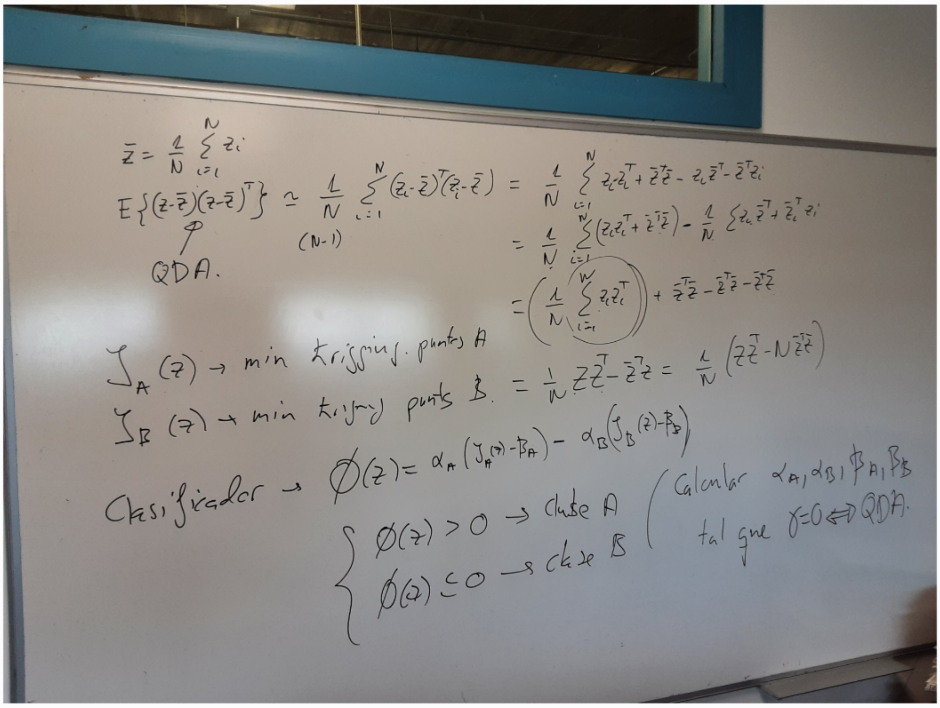
$$\lambda = \begin{bmatrix} \lambda_0 \\ \lambda_1 \\ \vdots \\ \lambda_{m-1} \end{bmatrix}$$

ONLY WORKS IF $\varphi = 0$

MOORE-PENROSE LEFT INVERSE

$\begin{bmatrix} X \\ 1 \end{bmatrix} \in \mathbb{R}$

$$\begin{bmatrix} X \\ 1 \end{bmatrix} \lambda = \begin{bmatrix} X_c \\ 1 \end{bmatrix} \rightarrow \lambda = \begin{bmatrix} X \\ 1 \end{bmatrix}^{\dagger} \begin{bmatrix} X_c \\ 1 \end{bmatrix} = \left(\begin{bmatrix} X \\ 1 \end{bmatrix}^T \begin{bmatrix} X \\ 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} X \\ 1 \end{bmatrix}^T \begin{bmatrix} X_c \\ 1 \end{bmatrix}$$



BASADO EN FUNCION

$$\delta_k = \sum \lambda_{k_i}^2 = \lambda_k^T \lambda_k = \left[\left(\begin{bmatrix} X_k \\ 1 \end{bmatrix}^T \begin{bmatrix} X_k \\ 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} X_k \\ 1 \end{bmatrix}^T \begin{bmatrix} X_c \\ 1 \end{bmatrix} \right]^T \left(\begin{bmatrix} X_k \\ 1 \end{bmatrix}^T \begin{bmatrix} X_k \\ 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} X_k \\ 1 \end{bmatrix}^T \begin{bmatrix} X_c \\ 1 \end{bmatrix}$$

$$\delta = \begin{bmatrix} X_c^T & 1 \end{bmatrix} \begin{bmatrix} X_k \\ 1 \end{bmatrix} \left(\begin{bmatrix} X_k^T & 1 \end{bmatrix} \begin{bmatrix} X_k \\ 1 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} X_k^T & 1 \end{bmatrix} \begin{bmatrix} X_k \\ 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} X_k^T & 1 \end{bmatrix} \begin{bmatrix} X_c \\ 1 \end{bmatrix}$$

$$\delta = \left(X_c^T X_k + \begin{bmatrix} 1 & \dots & 1 \end{bmatrix} \right) \left(X_k^T X_k + \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \right)^{-1} \left(X_k^T X_k + \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \right)^{-1} \left(X_k^T X_c + \begin{bmatrix} 1 & \dots & 1 \end{bmatrix} \right)$$

$$\delta = \left(X_c^T X_k + \begin{bmatrix} 1 & \dots & 1 \end{bmatrix} \right) \left(\left(X_k^T X_k + 1_{N \times N} \right) \left(X_k^T X_k + 1_{N \times N} \right) \right)^{-1} \left(X_k^T X_c + \begin{bmatrix} 1 & \dots & 1 \end{bmatrix} \right)$$

$$\delta = \left(X_c^T X_k + \begin{bmatrix} 1 & \dots & 1 \end{bmatrix} \right) \left(X_k^T X_k X_k^T X_k + X_k^T X_k 1_{N \times N} + 1_{N \times N} X_k^T X_k + 1_{N \times N} 1_{N \times N} \right)^{-1} \left(X_k^T X_c + \begin{bmatrix} 1 & \dots & 1 \end{bmatrix} \right)$$

$$\delta = \left(X_c^T X_k + \begin{bmatrix} 1 & \dots & 1 \end{bmatrix} \right) \left(X_k^T X_k X_k^T X_k + X_k^T X_k 1_{N \times N} + 1_{N \times N} X_k^T X_k + N_{N \times N} \right)^{-1} \left(X_k^T X_c + \begin{bmatrix} 1 & \dots & 1 \end{bmatrix} \right)$$

DATABASE: $X = \left[\underset{\text{SAMPLE}}{x_0}\ x_1 \ldots x_{\underset{\text{NUMBER OF SAMPLES}}{n-1}}\right] = \begin{bmatrix} x_{0,0} & x_{1,0} & \cdots & x_{n-1,0} \\ x_{0,1} & x_{1,1} & \cdots & x_{n-1,1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{0,\underset{\text{NUMBER OF CHARACTERISTICS}}{c-1}} & x_{1,c-1} & \cdots & x_{n-1,c-1} \end{bmatrix}$

DATA TO BE CLASSIFIED: $X_c = \begin{bmatrix} x_{c,0} \\ x_{c,1} \\ \vdots \\ x_{c,n-1} \end{bmatrix}$

QDA

CLASS K SCORE VALUE

$$\delta(K) = -\frac{1}{2}\log|S_K| - \frac{1}{2}(x_c-\mu_K)^T S_K^{-1}(x_c-\mu_K) + \log P(y=K) + \cancel{C_K}$$

CONSTANTS

HAS NO COMPARATIVE VALUE

X ROWS IN CLASS K

$$\mu_K = X_K \begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}$$

NUMBER OF SAMPLES IN CLASS K

$$S_K = \begin{bmatrix} VAR(X_{i,0}) & COV(X_{i,1},X_{i,0}) & \cdots & COV(X_{i,c-1},X_{i,0}) \\ COV(X_{i,0},X_{i,1}) & VAR(X_{i,1}) & \cdots & COV(X_{i,c-1},X_{i,1}) \\ \vdots & \vdots & \ddots & \vdots \\ COV(X_{i,0},X_{i,c-1}) & COV(X_{i,1},X_{i,c-1}) & \cdots & VAR(X_{i,c-1}) \end{bmatrix}$$

$$COV(A,B) = \frac{\sum_{i=0}^{n-1}(A_i-\mu_A)(B_i-\mu_B)}{n}$$

$$COV(A,B) = COV(B,A)$$

$$VAR(A) = COV(A,A)$$

$$S_K = \frac{1}{n}\left[\left(X_K - X_K\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}[1 \ldots 1]\right)\left(\underset{n,c}{X_K} - \underset{n,c}{X_K}\underset{1,n}{\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}}\underset{1,n}{[1 \ldots 1]}\right)^T\right]$$

$$\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}[1 \ldots 1] = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}\left[\frac{1}{n} \cdots \frac{1}{n}\right] = \left[\frac{1}{n}\right]_{n\times n}$$

$$S_K = \frac{1}{n}\left(X_K X_K^T - X_K\left[\frac{1}{n}\right]_{n\times n}X_K^T - \cancel{X_K\left[\frac{1}{n}\right]_{n\times n}}X_K^T + \cancel{X_K\left[\frac{1}{n}\right]_{n\times n}\left[\frac{1}{n}\right]_{n\times n}}X_K^T\right)$$

$$\left[\frac{1}{n}\right]_{n\times n}\left[\frac{1}{n}\right]_{n\times n} = \left[\frac{n}{n^2}\right]_{n\times n} = \left[\frac{1}{n}\right]_{n\times n}$$

$$\underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} n & n \\ n & n \end{bmatrix}}_{n\begin{bmatrix} A+D & A+D \\ C+D & C+D \end{bmatrix}}\begin{bmatrix} A & C \\ B & D \end{bmatrix} = n\begin{bmatrix} A^2+2AD+D^2 & AC+BC+AD+BD \\ AC+BC+AD+BD & C^2+2CD+D^2 \end{bmatrix}$$

$$S_K = \frac{1}{n}\left(X_K X_K^T - X_K\left[\frac{1}{n}\right]_{n\times n}X_K^T\right)$$

$$\delta(K) = -\frac{1}{2}\log|S_K| - \frac{1}{2}\left(x_c - X_K\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}\right)^T\frac{1}{n}\left(X_K X_K^T - X_K\left[\frac{1}{n}\right]_{n\times n}X_K^T\right)\left(x_c - X_K\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}\right) + \log P(y=K)$$

$$\delta(K) = -\frac{1}{2}\log|S_K| - \frac{1}{2n}\left(x_c^T - \left[\frac{1}{n} \cdots \frac{1}{n}\right]X_K^T\right)\left(X_K X_K^T - X_K\left[\frac{1}{n}\right]_{n\times n}X_K^T\right)\left(x_c - X_K\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}\right) + \log P(y=K)$$

$$\delta(K) = -\frac{1}{2}\log|S_K| - \frac{1}{2n}\left(x_c^T X_K X_K^T - x_c^T X_K\left[\frac{1}{n}\right]_{n\times n}X_K^T - \left[\frac{1}{n} \cdots \frac{1}{n}\right]X_K^T X_K X_K^T + \left[\frac{1}{n} \cdots \frac{1}{n}\right]X_K^T X_K\left[\frac{1}{n}\right]_{n\times n}X_K^T\right)\left(x_c - X_K\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}\right) + \log P(y=K)$$

$$\delta(K) = -\frac{1}{2}\log|S_K| - \frac{1}{2n}\Bigg(x_c^T X_K X_K^T x_c - x_c^T X_K\left[\frac{1}{n}\right]_{n\times n}X_K^T x_c - \left[\frac{1}{n} \cdots \frac{1}{n}\right]X_K^T X_K X_K^T x_c + \left[\frac{1}{n} \cdots \frac{1}{n}\right]X_K^T X_K\left[\frac{1}{n}\right]_{n\times n}X_K^T x_c -$$

$$x_c^T X_K X_K^T X_K\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix} + x_c^T X_K\left[\frac{1}{n}\right]_{n\times n}X_K^T X_K\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix} + \left[\frac{1}{n} \cdots \frac{1}{n}\right]X_K^T X_K X_K^T X_K\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix} - \left[\frac{1}{n} \cdots \frac{1}{n}\right]X_K^T X_K\left[\frac{1}{n}\right]_{n\times n}X_K^T X_K\begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}\Bigg) + \log P(y=K)$$

$$S(K) = -\frac{1}{2}\log|S_K| - \frac{1}{2n}\Big(X_c^T X_K X_K^T X_c - X_c^T X_K \left[\tfrac{1}{n}\right]_{n\times n} X_K^T X_c - \left[\tfrac{1}{n} \cdots \tfrac{1}{n}\right] X_K^T X_K X_K^T X_c + \left[\tfrac{1}{n} \cdots \tfrac{1}{n}\right] X_K^T X_K \left[\tfrac{1}{n}\right]_{n\times n} X_K^T X_c -$$

$$X_c^T X_K X_K^T X_K \begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix} + X_c^T X_K \left[\tfrac{1}{n}\right]_{n\times n} X_K^T X_K \begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix} + \left[\tfrac{1}{n} \cdots \tfrac{1}{n}\right] X_K^T X_K X_K^T X_K \begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix} - \left[\tfrac{1}{n} \cdots \tfrac{1}{n}\right] X_K^T X_K \left[\tfrac{1}{n}\right]_{n\times n} X_K^T X_K \begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix} \Big) + \log P(y=K)$$

1

$$\begin{bmatrix} x_{c_0} & \cdots & x_{c_{c-1}} \end{bmatrix} \begin{bmatrix} X_{K0,0} & \cdots & X_{Kn-1,0} \\ \vdots & \ddots & \vdots \\ X_{K0,c-1} & \cdots & X_{Kn-1,c-1} \end{bmatrix} \begin{bmatrix} X_{K0,0} & \cdots & X_{K0,c-1} \\ \vdots & \ddots & \vdots \\ X_{Kn-1,0} & \cdots & X_{Kn-1,c-1} \end{bmatrix} \begin{bmatrix} x_{c0} \\ \vdots \\ x_{c\,c-1} \end{bmatrix}$$

$$\begin{bmatrix} \sum_{i=0}^{c-1} x_{ci} X_{K0,i} & \cdots & \sum_{i=0}^{c-1} x_{ci} X_{Kn-1,i} \end{bmatrix}$$

$$\begin{bmatrix} \sum_{j=0}^{n-1} X_{Kj,0} \sum_{i=0}^{c-1} x_{ci} X_{Kj,i} & \cdots & \sum_{j=0}^{n-1} X_{Kj,c-1} \sum_{i=0}^{c-1} x_{ci} X_{Kj,i} \end{bmatrix}$$

$$\sum_{i=0}^{c-1} \sum_{j=0}^{c-1} \sum_{h=0}^{n-1} x_{c_j} X_{K_{h,j}} x_{ci} X_{K_{h,i}}$$

2

$$\begin{bmatrix} x_{c_0} & \cdots & x_{c_{c-1}} \end{bmatrix} \begin{bmatrix} X_{K0,0} & \cdots & X_{Kn-1,0} \\ \vdots & \ddots & \vdots \\ X_{K0,c-1} & \cdots & X_{Kn-1,c-1} \end{bmatrix} \begin{bmatrix} \frac{1}{n} & \cdots & \frac{1}{n} \\ \vdots & \ddots & \vdots \\ \frac{1}{n} & \cdots & \frac{1}{n} \end{bmatrix} \begin{bmatrix} X_{K0,0} & \cdots & X_{K0,c-1} \\ \vdots & \ddots & \vdots \\ X_{Kn-1,0} & \cdots & X_{Kn-1,c-1} \end{bmatrix} \begin{bmatrix} x_{c0} \\ \vdots \\ x_{c\,c-1} \end{bmatrix}$$

$$\begin{bmatrix} \sum_{i=0}^{c-1} x_{ci} X_{K0,i} & \cdots & \sum_{i=0}^{c-1} x_{ci} X_{Kn-1,i} \end{bmatrix}$$

$$\frac{1}{n}\begin{bmatrix} \sum_{j=0}^{n-1} \sum_{i=0}^{c-1} x_{ci} X_{Kj,i} & \cdots & \sum_{j=0}^{n-1} \sum_{i=0}^{c-1} x_{ci} X_{Kj,i} \end{bmatrix}$$

$$\frac{1}{n}\begin{bmatrix} \sum_{h=0}^{n-1} X_{Kh,0} \sum_{j=0}^{n-1} \sum_{i=0}^{c-1} x_{ci} X_{Kj,i} & \cdots & \sum_{h=0}^{n-1} X_{Kh,c-1} \sum_{j=0}^{n-1} \sum_{i=0}^{c-1} x_{ci} X_{Kj,i} \end{bmatrix}$$

$$\frac{1}{n} \sum_{l=0}^{n-1} \sum_{h=0}^{n-1} \sum_{j=0}^{c-1} \sum_{i=0}^{c-1} x_{c_j} X_{Kh,j} x_{ci} X_{Kl,i}$$

$$\delta_{QDA} = -\frac{1}{2}\log|S_k| - \frac{1}{2}\left(x_c - X_k\begin{bmatrix}\frac{1}{n}\\ \vdots \\ \frac{1}{n}\end{bmatrix}\right)^T \frac{1}{n}\left(X_k X_k^T - X_k\left[\frac{1}{n}\right]_{n\times n} X_k^T\right)\left(x_c - X_k\begin{bmatrix}\frac{1}{n}\\ \vdots \\ \frac{1}{n}\end{bmatrix}\right) + \log P(y=k)$$

$$\delta_k = \left(X_c^T X_k + [1 \dots 1]\right)\left(\left(X_k^T X_k + I_{N\times N}\right)\left(X_k^T X_k + I_{N\times N}\right)\right)^{-1}\left(X_k^T X_c + [1 \dots 1]\right)$$