Theozette $\underline{VI}$

1)

a) $x = x(y, w(y,z))$

Bilde dx:

$$dx = \frac{\partial x}{\partial y} dy + \frac{\partial x}{\partial w} dw$$

$$= \frac{\partial x}{\partial y} dy + \frac{\partial x}{\partial w}\left(\frac{\partial w}{\partial y} dy + \frac{\partial w}{\partial z} dz\right)$$

Betrachte ~~dx~~ x an Stelle $dz = 0$

$$\Rightarrow dx = \left(\frac{\partial x}{\partial y}\right)_w dy + \left(\frac{\partial x}{\partial w}\right)_y \left(\frac{\partial w}{\partial y}\right)_z dy$$

$$\Leftrightarrow \left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial x}{\partial y}\right)_w + \left(\frac{\partial x}{\partial w}\right)_y \left(\frac{\partial w}{\partial y}\right)_z$$

b) für $x = x(w(y,z))$ gilt:

$$dx = \frac{\partial x}{\partial w}\left(\frac{\partial w}{\partial y} dy + \frac{\partial w}{\partial z} dz\right)$$

wieder ~~bei~~ $dz = 0$

$$\Rightarrow \left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial x}{\partial w}\right)_y \left(\frac{\partial w}{\partial y}\right)_z$$

c) für $\left(\frac{\partial x}{\partial z}\right)_y$

betrachte wieder dx bei $dy \equiv 0$

$$\Rightarrow dx = \left(\frac{\partial x}{\partial w}\right)_y \left(\frac{\partial w}{\partial z}\right)_y dz$$

$$\Leftrightarrow \left(\frac{\partial x}{\partial z}\right)_y = \left(\frac{\partial x}{\partial w}\right)_y \left(\frac{\partial w}{\partial z}\right)_y$$

d) $x = x(y,z)$

$$dx = \frac{\partial x}{\partial y}\,dy + \frac{\partial x}{\partial z}\,dz$$

Betrachte ~~x(y,z)~~ ~~bei~~ $dx \equiv 0$

$$\Rightarrow \quad 0 = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz$$

$$\Leftrightarrow \quad -\left(\frac{\partial x}{\partial y}\right)_z dy = \left(\frac{\partial x}{\partial z}\right)_y dz$$

$$\Leftrightarrow \quad -\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x = \left(\frac{\partial x}{\partial z}\right)_y$$

mit: $\left(\frac{\partial z}{\partial x}\right)_y = \frac{1}{\left(\frac{\partial x}{\partial z}\right)_y}$

$$\Rightarrow \quad \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

kein Widerspruch zu c), da ich hier $dx = 0$ betrachte und nicht $dy = 0$.

e) $\cancel{dE}$

$\cancel{dS = \left(\frac{\partial S}{\partial E}\right) dE + \left(\frac{\partial S}{\partial V}\right)}$

$$dE = \left(\frac{\partial E}{\partial S}\right)_V dS + \left(\frac{\partial E}{\partial V}\right)_S dV$$

$$dE = T\,dS - p\,dV \quad \Leftrightarrow$$

Unter der Annahme:

$dE = 0$

$\Longrightarrow \quad T\,dS = p\,dV$

bzw $\left(\frac{\partial E}{\partial S}\right)_V dS = p\,dV$

$\Longleftrightarrow \quad p = \left(\frac{\partial E}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_E$

$\Rightarrow$ Vermeintlicher Widerspruch, löst sich auf wenn klar ist:

$$\left(\frac{\partial E}{\partial S}\right)\left(\frac{\partial S}{\partial V}\right) = \left(\frac{\partial E}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_E$$

ii) a)

1) mit $dE = T\,dS - p\,dV$

Für C_V betrachte dE bei $dV = 0$
da Volumen konstand

$\Rightarrow \quad dE = T\,dS$

$\Rightarrow \quad \left(\frac{\partial E}{\partial S}\right)_V = T$

also

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial E}{\partial S}\right)_V \left(\frac{\partial S}{\partial T}\right)_V$$

$$= \left(\frac{\partial E}{\partial T}\right)_V$$

2) $E = E(S,V) = E(S(T,p), V(T,p))$

$$\Rightarrow \left(\frac{\partial E}{\partial T}\right)_p = \left(\frac{\partial E}{\partial S}\right)_V \left(\frac{\partial S}{\partial T}\right)_p + \left(\frac{\partial E}{\partial V}\right)_S \left(\frac{\partial V}{\partial T}\right)_p$$

umformen:

$$\underbrace{\left(\frac{\partial E}{\partial S}\right)_V}_{=T} \left(\frac{\partial S}{\partial T}\right)_p = \left(\frac{\partial E}{\partial T}\right)_p - \left(\frac{\partial E}{\partial V}\right)_S \left(\frac{\partial V}{\partial T}\right)_p$$

↙ mit Relation (1)

$$\Rightarrow T\left(\frac{\partial S}{\partial T}\right)_p = \left(\frac{\partial E}{\partial T}\right)_V + \left(\frac{\partial E}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p - \underbrace{\left(\frac{\partial E}{\partial V}\right)_S}_{=-p} \left(\frac{\partial V}{\partial T}\right)_p$$

$$\Rightarrow \underbrace{T\left(\frac{\partial S}{\partial T}\right)_p}_{=C_p} = \left(\frac{\partial E}{\partial T}\right)_V + \left(\left(\frac{\partial E}{\partial V}\right)_T + p\right)\left(\frac{\partial V}{\partial T}\right)_p$$

b)

1) $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

kalorische Zustandsgleichung:

$$\frac{E}{N} = \frac{3}{2}kT - \frac{aN}{V}$$

$$\Rightarrow \frac{3}{2}kT - \frac{E}{N} = \frac{aN}{V}$$

$$V = \frac{aN}{\left(\frac{3}{2}kT - \frac{E}{N}\right)}$$

$$\left(\frac{\partial V}{\partial T}\right)_p = -\frac{aN}{\left(\frac{3}{2}kT - \frac{E}{N}\right)^2} \cdot \frac{3}{2}k = \frac{-V}{\left(\frac{3}{2}kT - \frac{E}{N}\right)} \frac{3}{2}k$$

$$\Rightarrow \alpha = -\frac{3}{2}k \cdot \frac{1}{\left(\frac{3}{2}kT - \frac{E}{N}\right)}$$

2) $\beta = \frac{1}{p} \left(\frac{\partial p}{\partial T} \right)_V$

Thermische Zustandsgleichung

$$\left(p + \frac{a N^2}{V^2} \right) \left(\frac{V}{N} - V_0 \right) = kT$$

$$\Rightarrow p = \frac{kT}{\left(\frac{V}{N} - V_0 \right)} - \frac{a N^2}{V^2}$$

$$\left(\frac{\partial p}{\partial T} \right)_V = - \cancel{\frac{kT}{N\left(\frac{V}{N} - V_0\right)^2} + 2a\frac{N^2}{V^3}}$$

$$= \frac{k}{\left(\frac{V}{N} - V_0 \right)}$$

$$\beta = \frac{1}{p} \left(\frac{\partial p}{\partial T} \right)_V = \frac{k}{kT + \frac{N^2 V_0}{V^2} + \frac{Na}{V}}$$