Protokoll zum FP-Versuch

SQUID

Michael Fix Thomas Selmaier

March 30, 2009

Contents

1 Object of the Experiment

The aim of this experiment is to understand how a SQUID works. For this, we will

- optimize the signal to get the typical SQUID-Pattern.
- observe the magnetic field of a loop of a conductor with different currents.
- obeserve the magnetic field of various objects.

Further, we'll calculate the magnetic dipol moment of the loop, which we will compare with the theoretical value, and the magnetic dipol moment of the objects used in the experiment.

2 Theory

In general, a SQUID (**S**uperconducting **QU**antum **I**nterference **D**evice) consists of a superconducting ring, which has one or two weak links (gaps, filled with an insulating material, see figure 1). The advantage of a SQUID is, that it can measure very small magnetic fields. Lower bounds are only set by quantum effects.

2.1 Superconductors

All materials, who's electrical resistance vanishs below an characteristic temperature (the so called critical temperature T_c), are called superconductors In this state they are also perfect diamagnets ($\chi = -1$). This means that all magnetic field lines are pushed out of the material, inside the material there won't be any magnetic field left. This ist called the Meissner effect.
There are two types of superconductors: normal superconductors (like mercury) with a critical temperature of about $T_c = 4K$ and high temperature superconductors (some ceramics) with critical temperatures of $T_c = 80 \ldots 100K$. Above this critical temperature, these materials behave like normal resistences (following Ohm's law), below this temperature their behaviour is described by the BCS-theory.

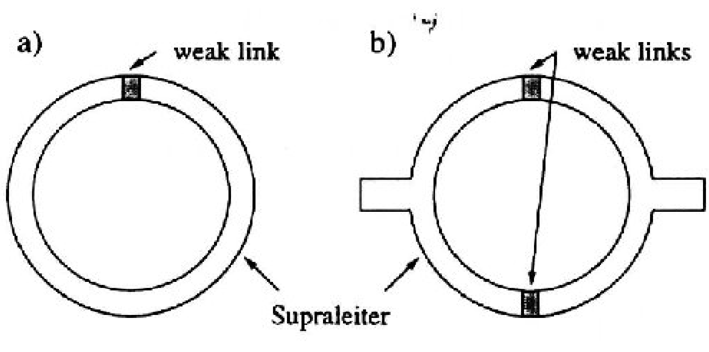

Figure 1: *General construction of a SQUID: a) RF-SQUID, b) DC-SQUID, From [1].*

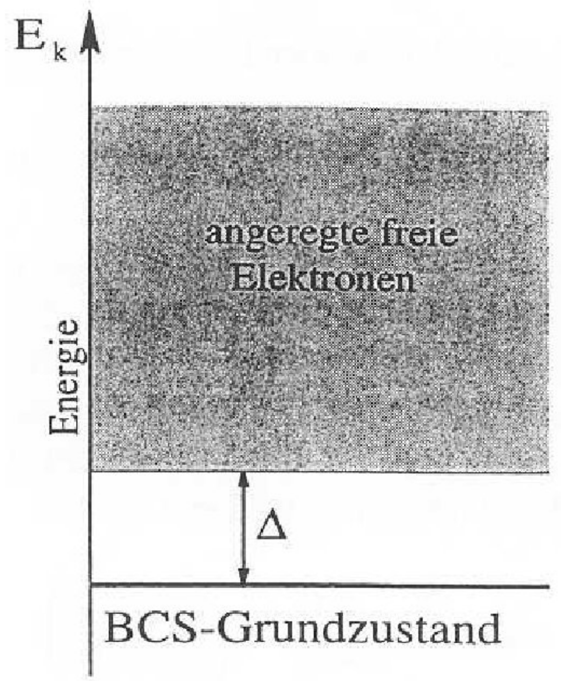

Figure 2: *The by Δ decreased BCS ground state; we need an energy of 2Δ to break up a Cooper pair, from [1].*

2.2 BCS-Theory and Cooper Pairs

The BCS-theory was advanced by Bardeen, Cooper and Schriefer in 1957 and explains the superconductivity by Cooper pairs. A Cooper pair is a pair of electrons of different spin, it develops in the following way: when an electron is moving through a crystal, it deforms the lattice by attracting the positiv nuclei with his negativ charge. Because of his velocity and the inertia of the nuclei the electron leavs a positiv trace, that is able to attract a further electron. So it is possible, that two electrons can form a pair over a distance of $100nm$, the repulsion because of the Coulomb force is small enough at this distance.
Because of it's contrary momentum and spin, the new particle has properties of bosons, what means that it is no more dominated by the Pauli principle, e.g. many of these particles can be in the same quantummecanical state. This leads to a decrease of the energy level of the hole system (compared with the former Fermi distribution). This new ground state, called *BCS ground state*, is Δ lower than the Fermi distribution (see figure 2) and stable against outer stimulation energies. Because of this there is no electrical resistence, because the energy, that is transfered to the Cooper pair by interaction with the lattice, is not big enough to get over the energy gap Δ and break up the Cooper pair. If we introduce a current, the Cooper pairs recive more energy, i.e. an addictional momentum. If the current is that strong that the Cooper pairs will have an energy greater than 2Δ after the interaction with the lattice, they will break up. So we have a

critical current that can destroy the superconductivity. Maxwell's equation

$$\nabla \times H = j \tag{1}$$

implies that there exists a critical magnetic field, too. But this equation is not only important in the case of a critical current/field, in general it conects the current in a loop with the magnetic field through this loop. In the next section we will se, that this is very important to understand the SQUID.

2.3 Flux Quantisation

If we introduce an permanent current in a superconductive loop (by cooling it down below his critical temperature within an external field and turning then the field off), for the current in the loop we have the quantisation condition of Bohr-Sommerfeld:

$$\oint pc_0 \cdot ds = nh$$

This means, that not every arbitrary current con flow in the loop, it can only take discreet values: multiples of Planck's constant h.
It's also possible, to describe this phenomenom with the wavefunction of an quantum particle: we describe the junction of all Cooper pairs by one wavefunktion as one particle. In the loop, there can't be waves with arbitrary wavelenth; to form a closed wave, a multiple of the wavelenght has to be the circumference of the loop. So there are only discreet values of the wavelenght and for this only discreet currents.
Because of equation (1) also the flux of the magnetic field through the loop can take discreet values: the flux is quantisized. Suposing, that this superconductive current only flows in a thin region of the surface of the superconductor, Maxwell's equation $B = \nabla \times A$ leads us to

$$\frac{nh}{q} = \oint A \cdot d\mathbf{s} = \iint B \cdot d\mathbf{F} = n\Phi_0, \tag{2}$$

where $q = 2e$ is the charge of the Cooper pair and Φ_0 ist the magnetic flux quantum: $\Phi_0 = 2,07 \cdot 10^{-15} Vs$.

2.4 Josephson Effect

One can observe the Josephson effect in a thin insulatin layer (namend Josephson contact or weak-link) between two superconductors. Normally, the thickness of this layer is smaller than $3nm$, what allows the Cooper pairs in the superconductor to tunnel through this weak-link. To talk in quantummecanical terms, the Cooper pairs have a finite prabability to tunnel through the potencial barrier, represented by the isolator (see figure 3). Now we have a tunnel current, that depends on the phase difference $\varphi_2 - \varphi_1$ of the wave funktion, that represents all the Cooper pairs:

$$I_{C_0} = I_{C_0}^{max} \sin(\varphi_2 - \varphi_1),$$

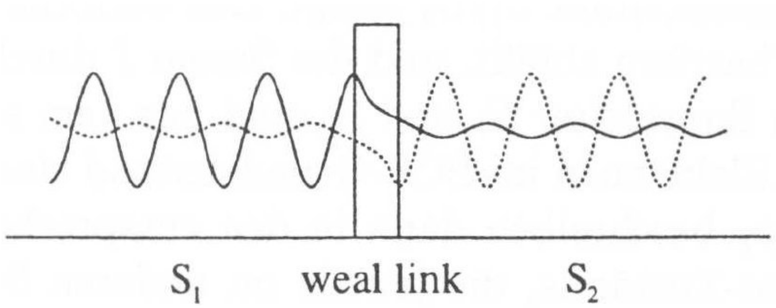

Figure 3: *Behavier of the many particles wave funktion in the weak link, from [1].*

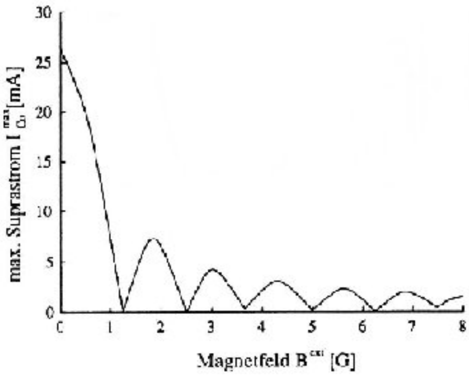

Figure 4: *Dependence of the maximum current* $I_{C_0}^{max}$ *of the exterior magnetic field* B^{ext}*, from [1].*

i.e. we have a DC, (without an applied voltage!), the so called Josephson-DC. The phase difference in the weak link is constant in time, if the maximum current is less than the critical current: $I_{C_0}^{max} < I_c$. If we exceed the critical current the phase difference is no more constant in time and we obtain an AC, the so called Josephson-AC. In this case one can measure a voltage between the ends of the weak-link and the superconductor changes into the normal conducting state.

The critical current depends on the magnetic flux through the loop. As described in 2.1, the flux can't get inside the loop, but it can pass through the weak-link and affect the phase difference in a characteristical way. Figure 4 shows this dependence.

2.5 The SQUID

The SQUID is a very sensitiv magnetometer. It's used to measure very small magnetic fields ($< 5 \cdot 10^{-15} T$). There are two posible realisations: the DC-SQUID and the RF-SQUID (see figure 1).

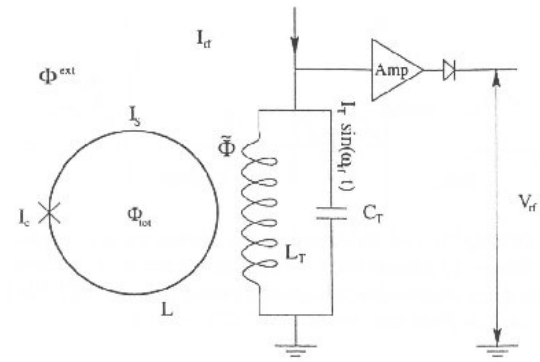

Figure 5: *Circuit for the use of the RF-SQUID, from [1].*

DC-SQUID The DC-SQUID has two weak-links and is directly connected with an electric circuit and converts the magnetic field we will measure in a voltage. In this experiment we will use a RF-SQUID, so we won't explain more about this type of SQUID.

RF-SQUID The RF-SQUID has only one weak-link. It isn't conected directly to the circuit but by induction to an high frequency oscillating circuit (see figure 5). In this high frequency oscillating circuit, we have an AC with a frequency of about $19MHz$, that produces an oszilating magnetic flux through the SQUID, that produces an AC in the loop of the SQUID. If this current is big enough to exceed the critical current I_c, the SQUID changes from the superconducting to the normal conducting state (see 2.4). Once the critical current ist exceeded, the SQUID extracts energy from the circuit by his resistance what leads to a decrease of the current and because of this to a decrease of the voltage. Now the SQUID is again in the superconducting state. The circuit will need some periods to reach his previous amplitude again. Then the current will exceed the critical current I_c again and the process will start again. Increasing the amplitude of excitacion I_{rf} of the oscillating circuit, at first the maximal voltage amplitude V_{rf} won't rise but later, when the current suffices to produce a flux quantum it will. So, if we make an V_{rf} - I_{rf} diagram we'll get a step function. If we apply an outer magnetic field, the critical current I_c will vary and the step funktion will move along the horizontal axis. So we can see, that the voltage V_{rf} depends periodicly on the magnetic flux Φ_{ext} and that's what we can measure (see figure 6).

2.6 The Lock-In Method

To improve the proportion signal - noise in this experiment we use the lock-in method. We feed the lock-in-amplifier with a reference signal of a defined frequency. The amplifier lets pass only those signals, which have the same frequency as the reference signal. For this, the flux through the SQUID will be modulated periodicly with a high frequency ($\sim 50kHz$) by a synchronal detector, what yields to a modulated voltage of the signal.

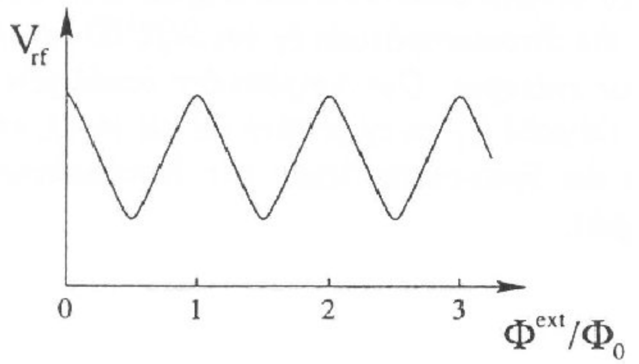

Figure 6: *Periodical dependency of V_{rf} and Φ_0, from [1].*

The synchronal detector multiplies the voltage of the signal with the reference signal and integrates the result for a selectable time. In this way, the noise is minimized, because many of it will be averaged out.

3 Experimental Set-Up

Firstly we had to fill the Dewar with the liquid nitrogen and then to putt the SQUID into the liquid. The cabinet has an opening to put different samples directly below the SQUID. Oposite to this opening a motor is located, at which is connected an holding device for the samples. The motor has different rotary speeds. The SQUID is connected with a control unit, which is connected with an oscilloscope and a computer (see figure 7). Via a special program, it's possible to vary the offset, the sensitivness and the integration time and to take pictures from the oszilloscope.

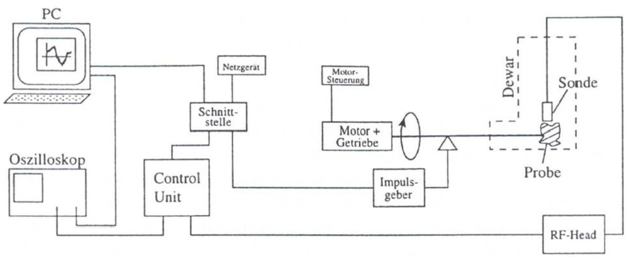

Figure 7: *The experimental set-up*

4 Experimental Procedure

Firstly we used the SQUID in the set-mode to search for the SQUID-pattern. We optimized the signal by changing the integration time an the FB-R. The best signal we got with integration time $2200pF$ and FB-R $10k$ (see figure 8). The caracteristical form ist good observable. In the points, where the triangular voltage changes the direction, a phase jump is clearly visible.
Now we changed into the measure-mode and measured the signal of the loop. We varied the current in the loop by changing the resistances R1 to R5. With R1 and R2 we got clearly signals, but from R3 up it wasn't possible to get somewhat of a sine-function so we didn't measure this resistances. Firstly we thought, that the batteries are low, but changing back to R1 and R2 gave us the same results as bevor. To have more data from the resistances, we measured the signal of the loop with some of the resistances: with R1 to R4, R2 to R4 and R2 to R5 and evaluated this measurements, too. By variing the integration time and the FB-R and the offset, we optimized the signal (see table 1 for details).
To measure the magnetic field of some objects we acted in the same way. The only difficulty was to fix the objects on the holding device, because there were no more screws to fix them. The settings are in table 1, too.

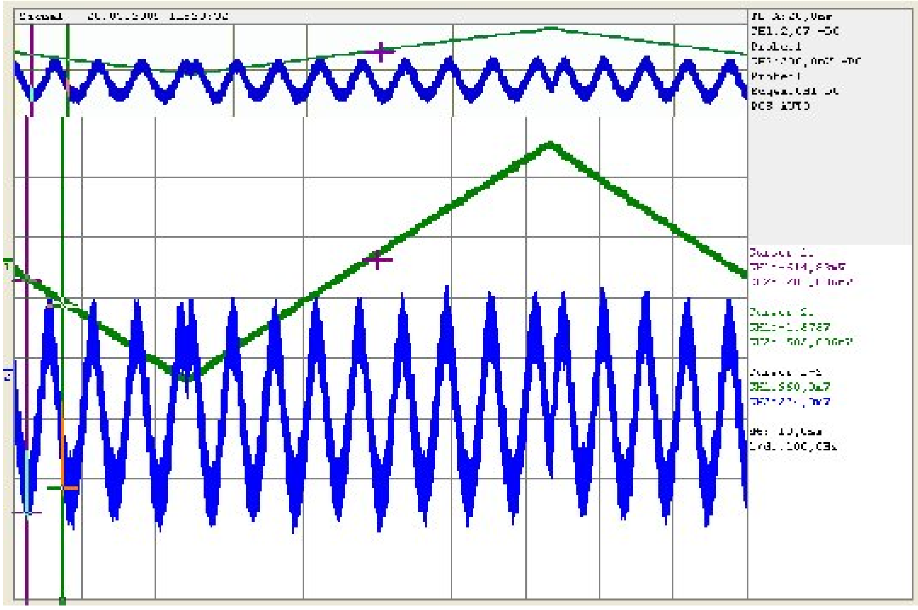

Figure 8: *The SQUID - pattern.*

Resistences / Object	Integration Time [F]	FB-R [kΩ]	Offset [mV]
R1	$2200 \cdot 10^{-12}$	10	2205
R2	$100 \cdot 10^{-9}$	100	2108
R1 - R4	$100 \cdot 10^{-9}$	100	2142
R2 - R4	$100 \cdot 10^{-9}$	100	2142
R2 - R5	$100 \cdot 10^{-9}$	100	2142
Magnet	$2200 \cdot 10^{-12}$	20	2204
Stick of Iron	$2200 \cdot 10^{-12}$	20	2113
Cylinder	$2200 \cdot 10^{-12}$	100	2108
Red Stone	$2200 \cdot 10^{-12}$	100	2090
White Stone	$2200 \cdot 10^{-12}$	100	2098
Piece of Gold	$2200 \cdot 10^{-12}$	100	2142
Piece of Iron	$2200 \cdot 10^{-12}$	100	2113

Table 1: *Settings, measuring the magnetic field of the loop and some objects*

5 Evaluation of the Measurements

Amplitude and Magnetic Field The variation of the voltage ΔU follows the following formula:

$$\Delta U = s_i \Delta\Phi,$$

where s_i is the factor of sensibility and $\Delta\Phi$ is the variation of the flux of the magnet field Φ_m, which is defined by

$$\Phi_m = \int \mathbf{B} \cdot d\mathbf{A}.$$

Because the magnetic field lines don't go parallel through the SQUID, we don't integrate over the real area of the SQUID but over a new area F_{eff}, that is penetrated by the same number of magnectic field lines as the real area but now these lines are parallel. In our case we have the new area $F_{eff} = 10^{-8} m^2$ (given in the instruction). With this new area we get the variation of the flux

$$\Delta\Phi = \Delta B_z F_{eff}$$

an with $\Delta B_z = 2B_2$ the variation of the voltage is

$$\Delta U = 2 s_i B_z F_{eff}. \tag{3}$$

5.1 Dipol Moment of the Loop

The dipol moment of a loop, that is flown by a current is defined by

$$\mathbf{p}_m = I \cdot \mathbf{A},$$

where I is the current and $\mathbf{A}$ the normal vector of the area of the loop. So, for the loop in the SQUID we have the theoretical value

$$p_{theo} = \pi r^2 I. \tag{4}$$

Disregarding the error of the battery current, the error of p_{theo} is

$$s_{p_{theo}} = \left| \frac{\partial p_{theo}}{\partial r} s_r \right| = 2\pi r I s_r$$

It's also possible to determine the dipol moment by measuring the magnetic field B_z on the dipol axis. We have:

$$B_z = \frac{\mu_0}{2\pi} \frac{p_m}{z^3} \quad \Longleftrightarrow \quad p_m = \frac{2\pi B_z z^3}{\mu_0} \tag{5}$$

The error is given by

$$s_{p_m} = \sqrt{\left(\frac{\partial p_m}{\partial B_z}\right)^2 s_{B_z}^2 + \left(\frac{\partial p_m}{\partial z}\right)^2 s_z^2} = p \sqrt{\left(\frac{s_{B_z}}{B_z}\right)^2 + \left(\frac{3 s_z}{z}\right)^2}$$

z was given in the instruction: $z = (2,70 \pm 0,25)cm$. For every fitted measurement, we calculated the dipol moment by formula 5 and the theoretical value by formula 4. In table 2 one can see the results. The theoretical results match good with the values, that were determined in the experiment.
To get these experimental data, we fitted the measured curves in Origin with the fit function

$$f(x) = y_0 + A\sin(\pi\frac{x - x_c}{\omega}) + mx,$$

where A is the amplitude, y_0 the offset and mx an extra linear term we added, because we were not sure, whether the batteries are able to provide a constant current while measuring is in process or not.

Resistences	$p_{theo}/10^{-7}Am^2$	$s_{p_{theo}}/10^{-7}Am^2$	$p_m/10^{-7}Am^2$	$s_{p_m}/10^{-7}Am^2$
R1	3,4347	0,7106	3,7882	1,0571
R2	0,6735	0,1393	0,6402	0,1780
R1-R4	4,5193	0,9350	4,1212	1,1448
R2-R4	1,0842	0,2243	1,0549	0,2931
R2-R5	1,1188	0,2315	1,0134	0,2816

Table 2: *Theoretical and experimental values for the dipol moment of the loop.*

5.2 Magnetic Field of Different Objects

For the later calculation of the magnetic fields of the different meassured objects, it's important to know, whether the holding device also has a magnetic field or not. As one can see in figure 9, there is nothing to observe so it wasn't necessary to subtract any underground. Where it was possible, we calculated the magnetic field and the dipol moment of an object by formulas (3) and (5). In section 7 are all grafical representations: the fits and also the vector diagrams of some of the objects.

The stick of iron The stick of iron showed a nice and clear magnetic field (see figures 17 and 18). We calculated the following values:

$$B_z = (1,268 \pm 0,003) \cdot 10^{-7}T$$
$$p_m = (1,2 \pm 0,3) \cdot 10^{-5}Am^2$$

The red stone The red stone showed a magnetic field, too (see figures 19 and 20):

$$B_z = (1,28 \pm 0,01) \cdot 10^{-9}T$$
$$p_m = (1,3 \pm 0,3) \cdot 10^{-7}Am^2$$

The vector diagramm doesn't show that clear structure as the other ones. We think this is a result of the position of the stone in the holding device, because he has no defined orientation (like the other objects).

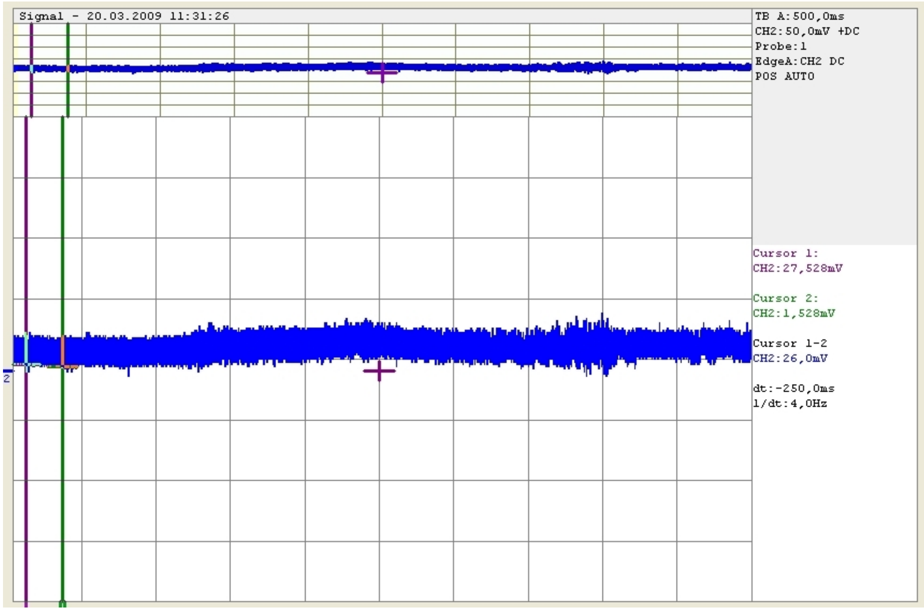

Figure 9: *Measurement of the underground (magnetic field of the holding device).*

The white stone The white stone showed no magnetic field. In figure 10 one can observe, that there is no periodicity in the signal, so we didn't calculate the magnetic field of this stone (as well it's not posible to fit here with a sine-function).

The Magnet The magnet showed a very nice and impressive dipol moment (see figures 21 and 22). We calculated the following values:

$$B_z = (2,084 \pm 0,004) \cdot 10^{-7} T$$
$$p_m = (2,1 \pm 0,5) \cdot 10^{-5} Am^2$$

The cylinder The cylinder was supposed to have no dipol moment but there was some periodicity in the measured signal (see figures 23 and 24), so we calculated the following values:

$$B_z = (3,71 \pm 0,08) \cdot 10^{-10} T$$
$$p_m = (3,7 \pm 1,0) \cdot 10^{-8} Am^2$$

We think that there was some (magnetic) dirt on this cylinder. This is a good example how the SQUID can be use to detect magnetic fields.

The pieces of gold and iron Like the white stone, the pieces of gold and iron didn't show any periodicity in their signals. The signals we got were like the signal of the white stone (see figure 10). For this, we neither calculated the magnetic fields and the dipol moments of this objects.

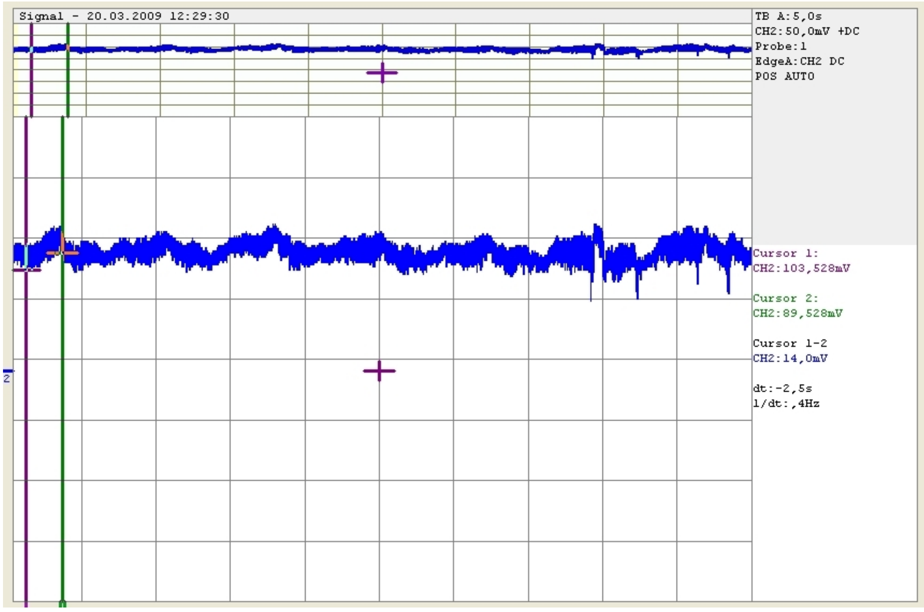

Figure 10: *Measurement of the white stone, there is no observable periodicity.*

6 Summary

In the first part of the experiment, we optimized the signal to get the SQUID-pattern. It showed the expected course (see figure 8).
In the second part of the experiment, we measured the magnetic field of a conduction loop (with different currents) and of some objects. If possible, we calculated the magnetic field an the magnetic dipol moment (see 5.1 and 5.2).

Literature:

[1] Bange, Volker: Einrichtung des Versuches "SQUID", Freiburg, Mai 2000

7 Fits and Dipol Moments

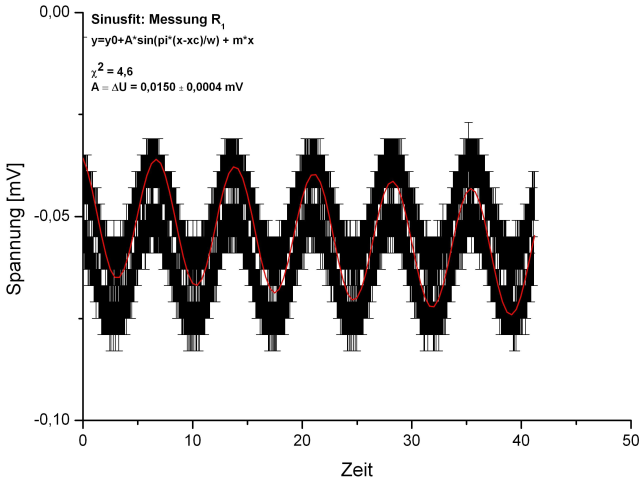

Figure 11: *Sinus fit of the measured curve with resistance R1.*

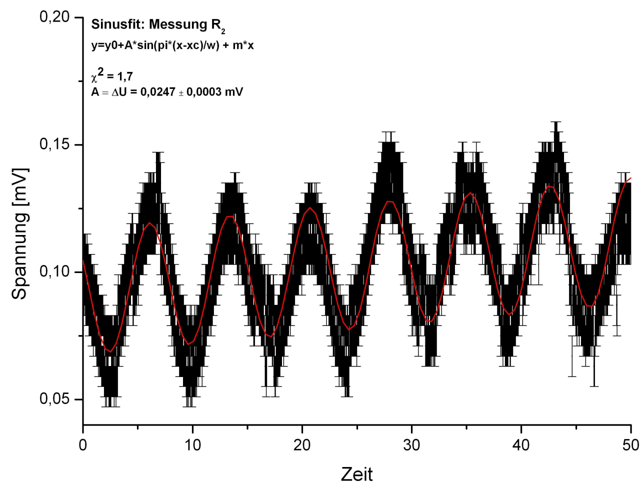

Figure 12: *Sinus fit of the measured curve with resistance R2.*

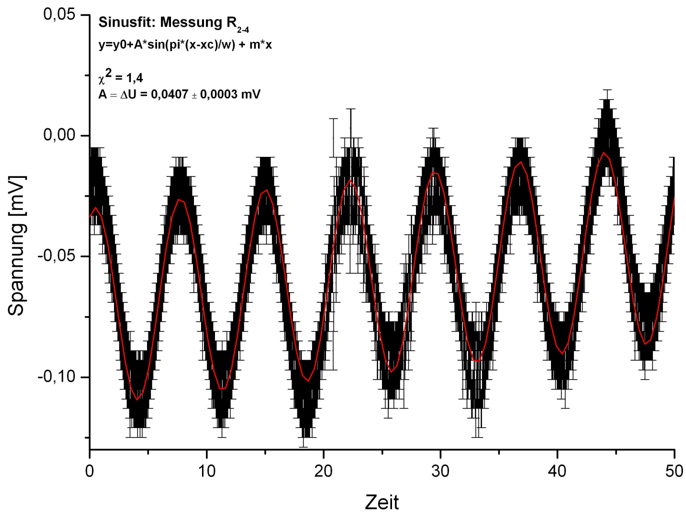

Figure 13: *Sinus fit of the measured curve with resistances R2 to R4.*

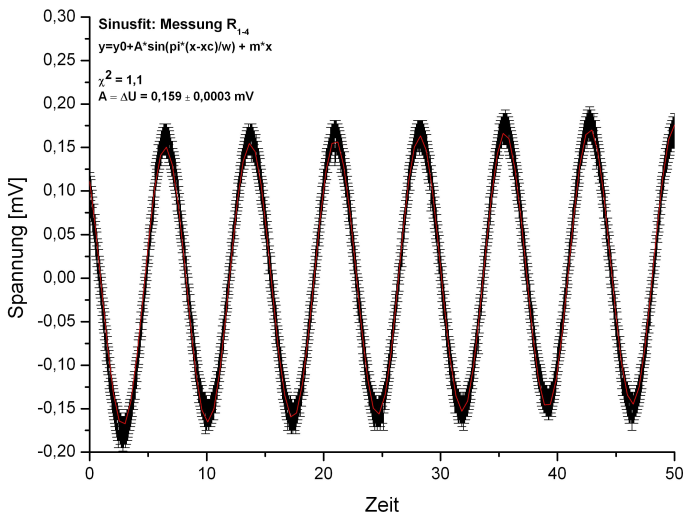

Figure 14: *Sinus fit of the measured curve with resistance R1 to R4.*

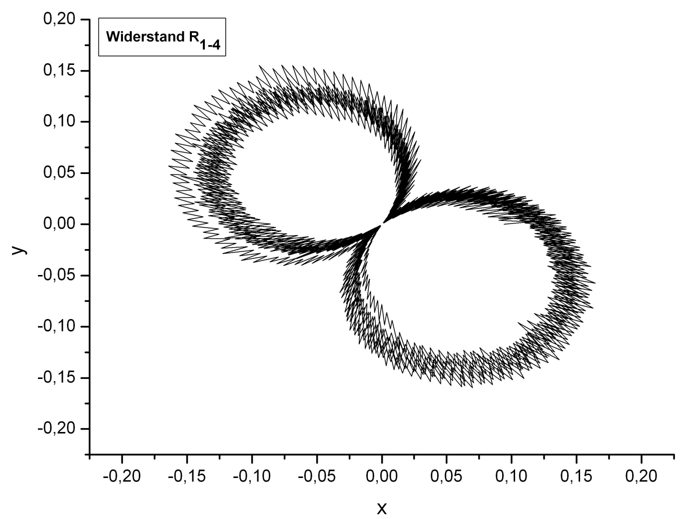

Figure 15: *Dipol moment of the loop with resistance R1 to R4.*

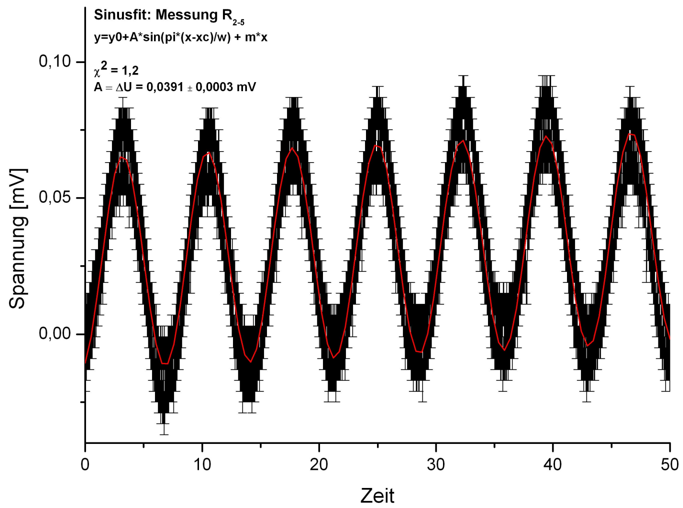

Figure 16: *Sinus fit of the measured curve with resistances R2 to R5.*

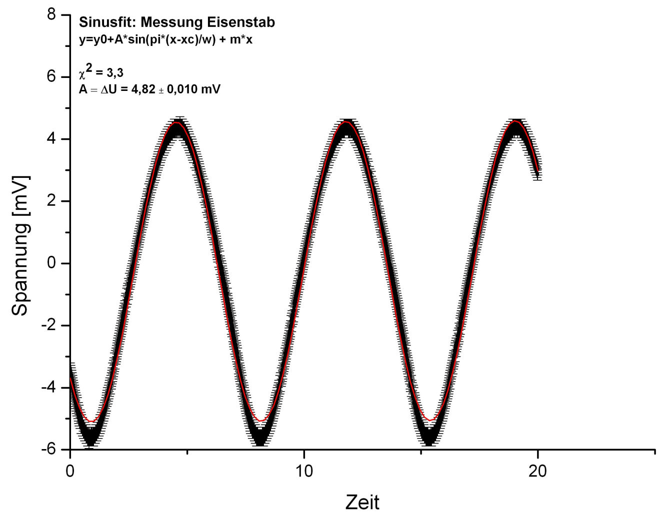

Figure 17: *Sinus fit of the measured curve of the piece of iron.*

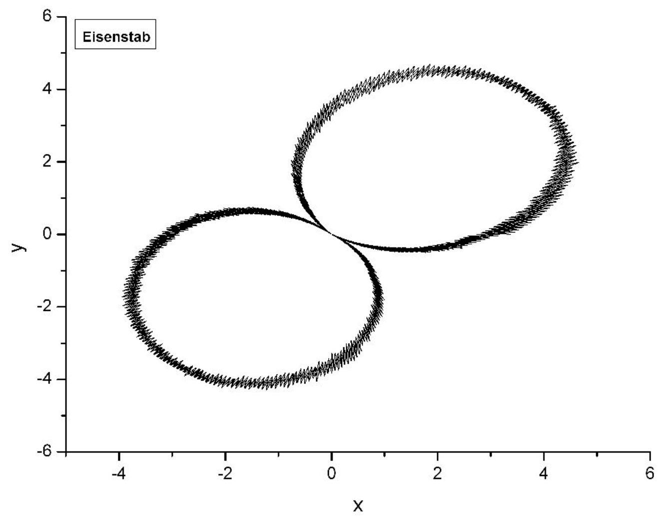

Figure 18: *Dipol moment of the piece of iron.*

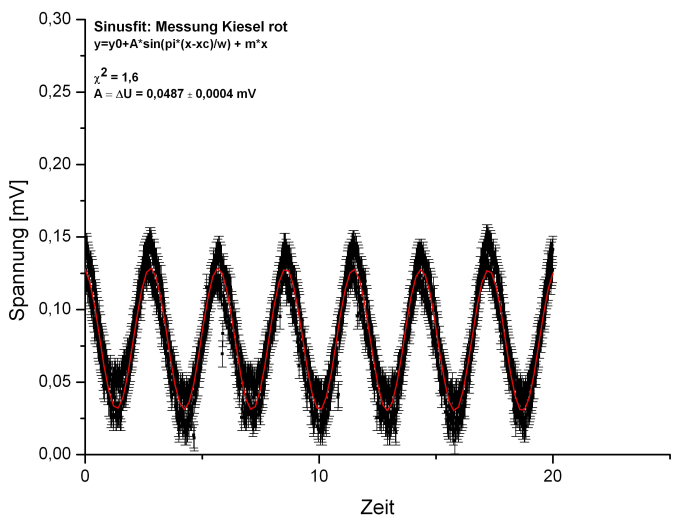

Figure 19: *Sinus fit of the measured curve of the red stone.*

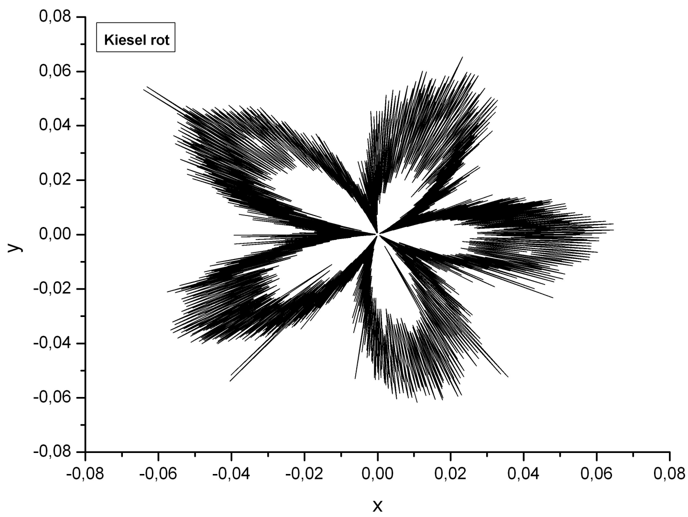

Figure 20: *Dipol moment of the red stone.*

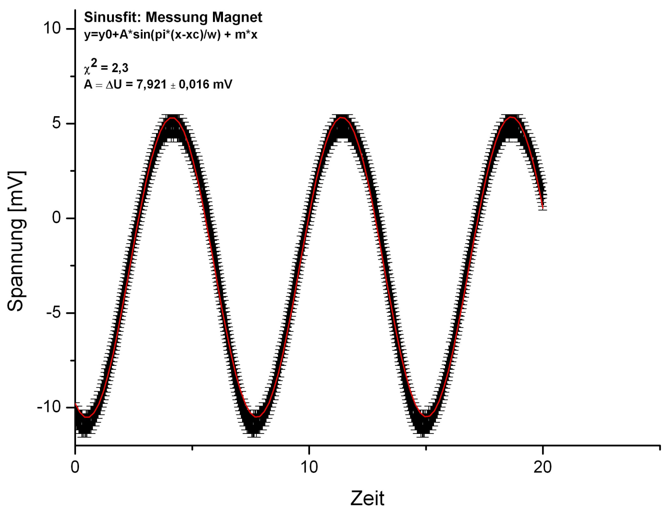

Figure 21: *Sinus fit of the measured curve of the magnet.*

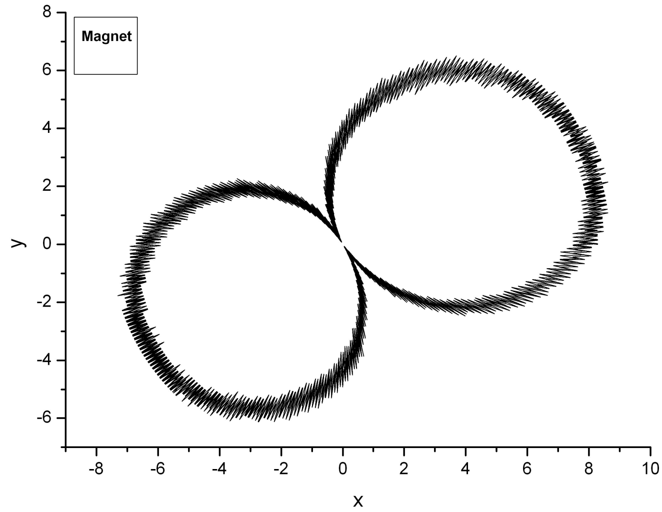

Figure 22: *Dipol moment of the magnet.*

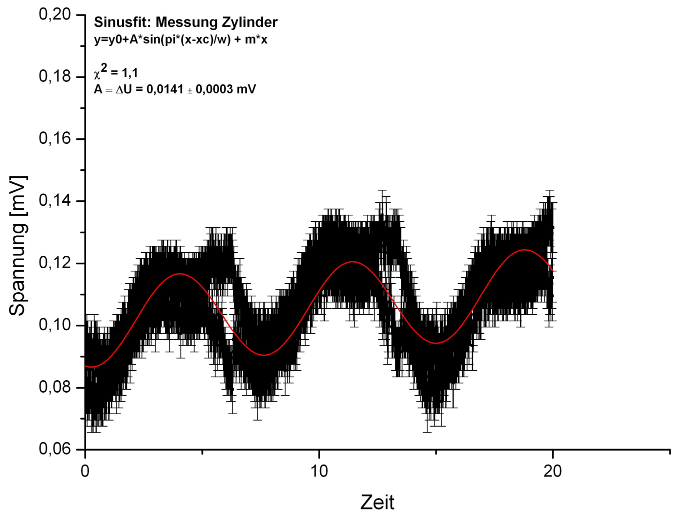

Figure 23: *Sinus fit of the measured curve of the cylinder.*

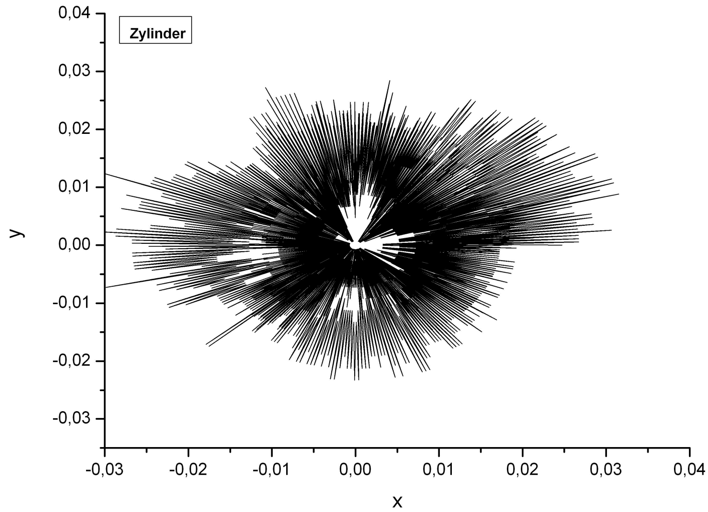

Figure 24: *Dipol moment of the cylinder.*