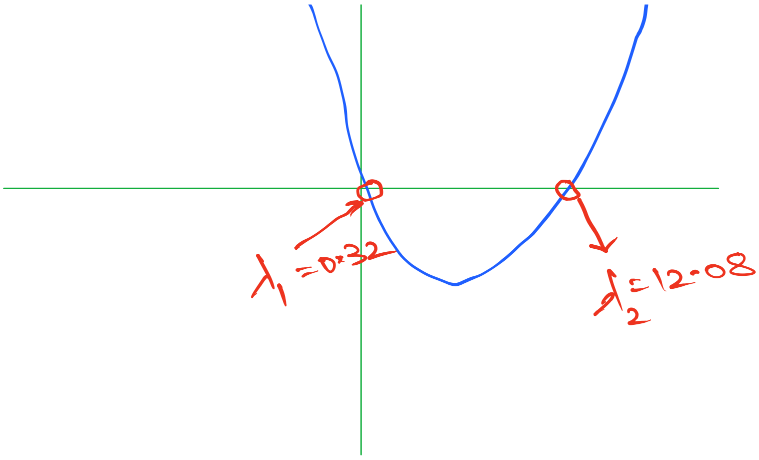


$\left.\begin{array}{l}\lambda_1 = 0.32 \\ \lambda_2 = 12.08\end{array}\right]$ Eigenvalues

We will now calculate their eigenvectors.

Starting with $\lambda_2 = 12.08$,

This equation is true for an eigenvector

$$Av = \lambda v$$

$$\begin{bmatrix} 4.4 & 5.6 \\ 5.6 & 8.0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 12.08 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\left[\begin{array}{l} 4.4x + 5.6y = 12.08x \\ 5.6x + 8.0y = 12.08y \end{array}\right.$$

$$5.6y = 7.68x$$

$$5.6x = 4.08y$$

$y = 1.37x$

$1.37x = y$

so the eigenvector here is

$$V_2 = \begin{bmatrix} 1 \\ 1.37 \end{bmatrix} \text{ or } \begin{bmatrix} 0.59 \\ 0.81 \end{bmatrix}$$ after normalizing

Now finding the eigenvector for $\lambda_1 = 0.32$

$$\begin{bmatrix} 4.4 & 5.6 \\ 5.6 & 8.0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0.32 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$V_1 = \begin{bmatrix} -0.81 \\ 0.59 \end{bmatrix}$$ after normalizing

(4) Arranging the eigenvectors in order, we get

$$\lambda_1 = 12.08 \qquad V_1 = \begin{bmatrix} 0.59 \\ 0.81 \end{bmatrix}$$

$$\lambda_2 = 0.32 \qquad V_2 = \begin{bmatrix} -0.81 \\ 0.59 \end{bmatrix}$$

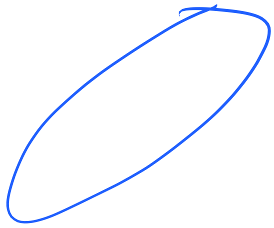
Putting both vectors in a matrix, we get

$$V = \begin{array}{cc} v_1 & v_2 \\ \end{array}$$
$$V = \begin{bmatrix} 0.59 & -0.81 \\ 0.81 & 0.59 \end{bmatrix}$$

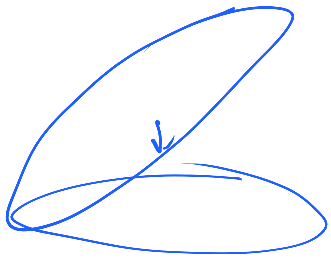
(5) Calculate the principal components →

We will multiply our data matrix (X) with matrix (V), that we got above

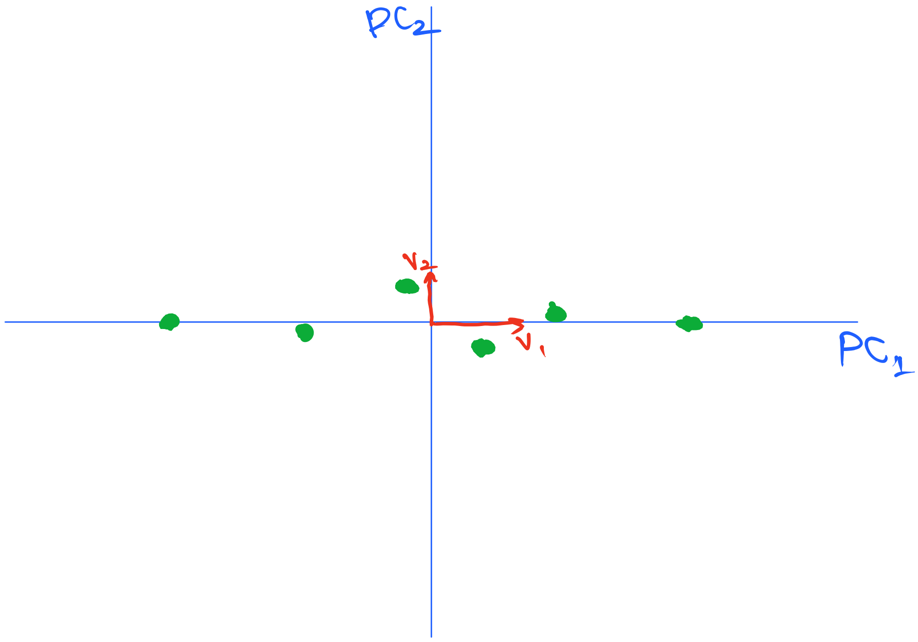
$$XV = \begin{bmatrix} -3 & -4 \\ -1 & -2 \\ -1 & 0 \\ 1 & 0 \\ 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0.59 & -0.81 \\ 0.81 & 0.59 \end{bmatrix}$$



$$= \begin{bmatrix} -5.0 & 0.1 \\ -2.2 & -0.4 \\ -0.6 & 0.8 \\ 0.6 & -0.8 \\ 2.2 & 0.4 \\ 5.0 & -0.1 \end{bmatrix}$$



This matrix represents our transformed data.
It can be represented as:



It is like we rotated the data until the eigenvectors point in the same direction as x & y axis.

⑥ Interpret the principal component

PC_1	PC_2
-5	0.1
-2.2	-0.4
-0.6	0.8
0.6	-0.8
2.2	0.4
5.0	-0.1

$$CM = \begin{bmatrix} \underline{12.08} & 0 \\ 0 & \underline{0.32} \end{bmatrix}$$

12.08	0.32

variance of PC_1 = eigenvalue 1 (λ_1)

$= 12.08$

variance of PC_2 = eigenvalue 2 (λ_2)

$= 0.32$

$$\%\,\text{Var}(PC_1) = \frac{12.08}{12.08 + 0.32}$$

$$= \underline{97.4\%}$$

$$\%\,\text{Var}(PC_2) = \frac{0.32}{12.08 + 0.32}$$

$$= 2.6\%$$

~~C1 C2~~ → PC1 97.4% ~~PC2 2.6%~~

10 Cols → 10 PCs

6 PCs

$\downarrow$

$>95\%$

~~4 PCs~~

~~$<5\%$~~

1. M Centered
2. CM, EVec, EValue
   2 2
3. Arranged EVectors

$V_1 \to 12.08$

$V_2 \to 0.32$

4.

50k cols X

$\downarrow$

PCA

$\downarrow$

1000 cols ✓

PCA

Resume at 11:48