Linear Algebra

Sunday, 8 June 2025 10:01

Used for
↓
Machine Learning
&
Deep Learning

Scalars (0th order Tensor)

(34)

vectors (1st order Tensors)

[34, 21, 91, 87]

Matrix (2nd order Tensors)

Higher Order Tensors (3rd or higher order tensors)

Uses of Linear Algebra in Machine Learning →

① Generalize/represent the relationship b/w input & output cols in higher dimension data.

② Helps us in data representation numerically

- → Tabular
- → Images
- → Audio
- → Video
- → Text
- → Signal

⟶ Help us represent all these data numerically ⟶ Then these numerical data can be given to ML/DL.

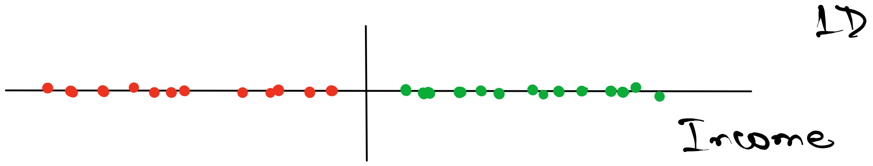
I/P O/P

	Income	Loan Approved?
0	—	No
1	—	Yes
2	—	Yes
3	—	No
4	—	Yes
5	—	No
6	—	No
7	—	No

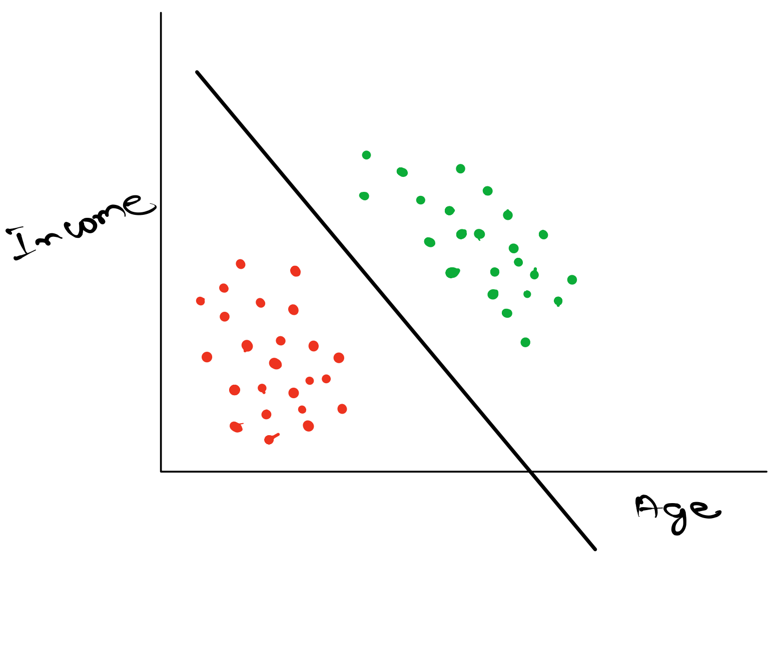
↓

ML

↓



	Age	Income	Loan Approved?
0	—	—	No
1	—	—	Yes
⋮			



Maximum number of cols we can represent as a graph visually is $\underline{3}$.

3 input cols
↓
3D

	Rooms	Floors	Age	Area	Price
0					
1					
2					
3					
4					
5					

↓
ML
↓

$$\text{Price} = \underline{0.98} \times \text{Rooms} + \underline{0.87} \times \text{Floors} + \underline{(-1.2)} \times \text{Age} + \underline{1.35} \times \text{Area} + \underbrace{\underline{1.8}}_{\text{Base Price}}$$

$$y = m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4 + c$$

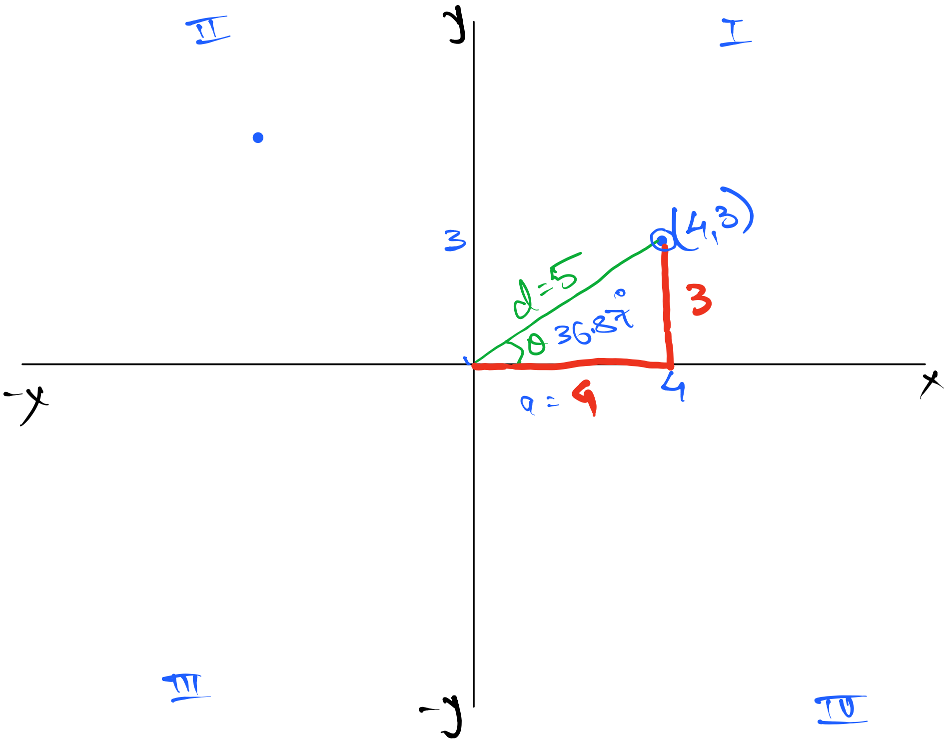
↓
Equation of a line

$y = mx + c$

Scalar → 22 47 99.51 12

vectors → [38, 47, 99.51, 72]

Vectors : Mathematical Approach



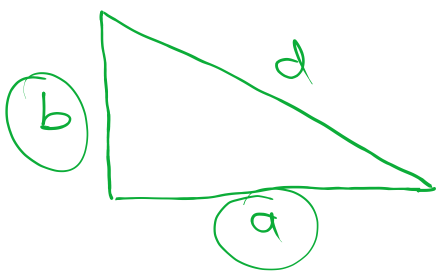
vector

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix} \begin{array}{l} \rightarrow x \\ \rightarrow y \end{array}$$

2D vector

vector has a magnitude and a direction

Pythagoras Theorem



$$d^2 = a^2 + b^2$$

$$d = \sqrt{a^2 + b^2}$$

$$d = \sqrt{4^2 + 3^2}$$

$$= 5$$

Direction of vector $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ is

theta

$$\tan\theta = \frac{b}{a}$$

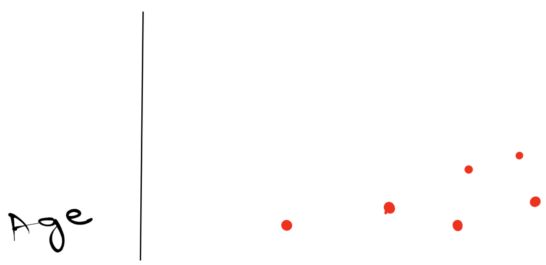
$$\tan\theta = \frac{3}{4}$$

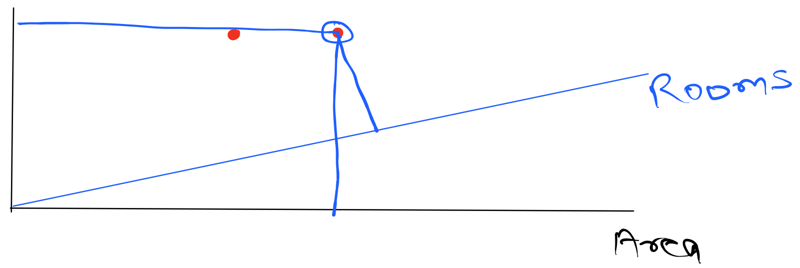
$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\theta = 36.87^\circ$$

Dimensions of a vector →

Area	Age
—	
—	
—	
—	





vector 1 → [1500, 2.8, 4, 2] ← 1 datapoint

vector 2 → [2200, 5, 6, 3]

How do we interpret a vector →

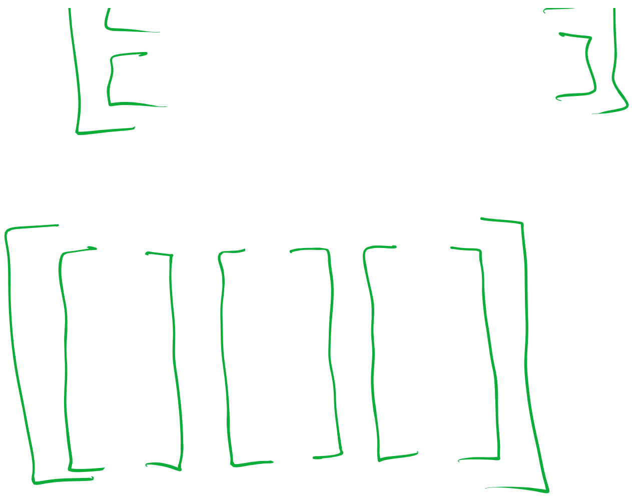
1. Row vectors
2. column vectors.

House Prices dataset

	Rooms	Age	Floors	Area	Price	
0	4	1.5	2	1500	$200k	→ V1
1	7	2	3	2200	$480k	→ V2
2						
3						
4						

vector 1 (House 1) = [4 1.5 2 1500]

vector 2 (H2) =



Matrices

Matrix : Collection of vectors

Operations on Matrices →

- → Addition
- → Subtraction
- → Multiplication
- → Division
- → Transpose
- → Determinant
- → Inverse
- → Rank of a Matrix

same shape

$$\overset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 7 & 6 \end{bmatrix}} + \overset{B}{\begin{bmatrix} 4 & 3 & 8 \\ 7 & 2 & 9 \end{bmatrix}}$$

$$\begin{bmatrix} 9 & 8 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 5 & 11 \end{bmatrix}$$

Multiplication of Matrices :

→ Matrix Multiplication.

→ Hadamard Multiplication.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 6 & 7 \\ 1 & 9 & 8 \end{bmatrix} * \begin{bmatrix} 9 & 7 & 1 \\ 2 & 8 & 5 \\ 4 & 3 & 0 \end{bmatrix}$$

Image

+

*

−

Filter

Transpose of a Matrix →

$$A = \begin{bmatrix} 23 & 31 & 47 \\ 92 & 39 & 11 \\ 44 & 63 & 58 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 23 & 92 & 44 \\ 31 & 39 & 63 \\ 47 & 11 & 58 \end{bmatrix}$$

Determinant of a Matrix →

$$A = \begin{array}{c} \begin{array}{ccc} (1,1) & (1,2) & (1,3) \end{array} \\ \begin{bmatrix} 2 & 2 & -3 \\ 1 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix} \end{array}$$

$\det(A)$ or $||A||$

$$\det(A) = 2(2\times2 - 1\times2) - 2(1\times2 - 1\times2) + (-3)(1\times1 - 1\times2)$$

$$= 7$$

Ex →

$$\text{Price} = \overset{w_1}{0.98} \times \text{Rooms} + \overset{w_2}{0.87} \times \text{Floors} + \overset{w_3}{(-1.2)} \times \text{Age} + \underset{w_4}{1.35} \times \text{Area}$$

$$\underline{\hat{w}} = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix}$$

$$\hat{w} = (X^T X)^{-1} X^T y$$

input cols

output

Rank of a matrix →

$$A = \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{ccc|c} C_1 & C_2 & C_3 & \\ 1 & 2 & 3 & R_1 \\ 2 & 1 & 4 & R_2 \\ 3 & 0 & 5 & R_3 \end{array}$$

We can apply some elementary operation on the rows and cols to reduce it

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & -6 & -4 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{bmatrix} \leftarrow \text{Duplicate / redundant}$$

Rank of Matrix : 2

$10 \longrightarrow 6$

$$\begin{bmatrix} 2 & 3 & 0 \\ 4 & -2 & 0 \\ 6 & -7 & 0 \end{bmatrix}$$

~~Area~~ Length width

$A = L \times W$

Feature Selection