## Logistic Regression

1

- The statisticians approached the problem - "how can we use linear regression to solve this?"
  - We could consider the following encoding
    - Dependent variable coded as 0 (Not Spam) or 1 (Spam)

1

## Logistic Regression

2

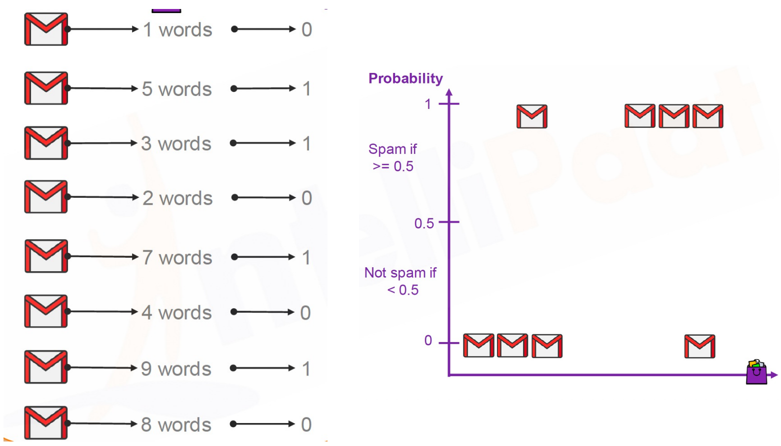
- We can fit a linear regression to this binary response
  - and classify as Spam if $\hat{y}$ > 0.5 and Not Spam otherwise, interpreting $\hat{y}$ as a probability that Email is Spam

2

# Logistic Regression

3

- Spam email classifier




3

# Logistic Regression

4

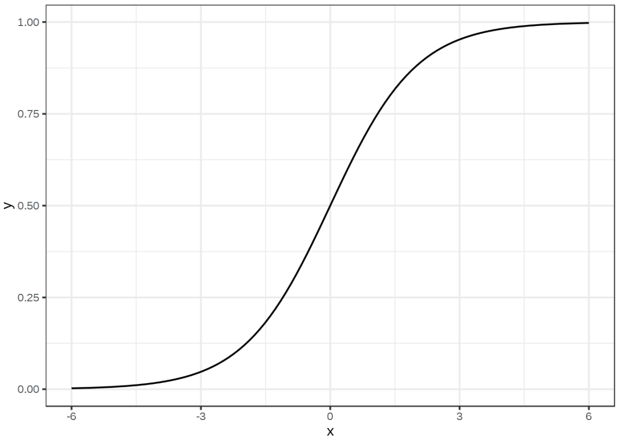
- A major problem with such an approach
- Linear regression models produce values in (-∞, +∞), which does not make sense as a probability
  - Employ a function that constrains the values between 0 and 1

4

# Logistic Regression

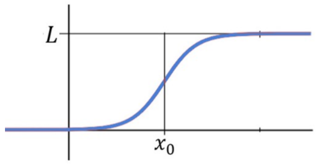
5

- Logistic function (Sigmoid)




$$f(x) = \frac{L}{1 + e^{-k(x-x_0)}}$$

$x_0$ = x value of midpoint
$L$ = maximum value
$k$ = growth rate




5

# Logistic Regression

6

- Logistic regression model
  - Let $Y$ be a binary outcome and $X$ a predictor

Sigmoid input

Linear regression output

$$\log\left(\frac{p_X}{1-p_X}\right) = \beta_0 + \beta_1 X$$

$$p_X = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}} \quad 0 \leq p_x \leq 1$$

6

# Logistic Regression

7

- Terminologv
  - The logit $\log\left(\frac{p_X}{1-p_X}\right)$ Log odds
  - The odds of an event is defined as

$$\text{odds}(Y=1) = \frac{P(Y=1)}{1-P(Y=1)} = \frac{p}{1-p}$$

    - Chance can be expressed either as a probability or as odds

7

# Logistic Regression

8

- Ranges

| Measure | Min | Max | Name |
|---|---|---|---|
| $P(Y=1)$ | 0 | 1 | "probability" |
| $\frac{P(Y=1)}{1-P(Y=1)}$ | 0 | $\infty$ | "odds" |
| $\log\left[\frac{P(Y=1)}{1-P(Y=1)}\right]$ | $-\infty$ | $\infty$ | "log-odds" or "logit" |

8

# Logistic Regression

9

- The cost/loss function
  - The likelihood function

$$\prod_{i=1}^{N} p_{x_i}^{y_i}(1 - p_{x_i})^{1-y_i}$$

  - The likelihood is a function of model parameters, and we can estimate them by maximizing the likelihood
    - Maximum likelihood estimates (MLE)

9

# Logistic Regression

10

- The cost/loss function
  - No closed form solution for MLE
  - We rely on numerical approximation to find the MLE
    - Most software uses the Newton Raphson algorithm or gradient descent algorithm

10

## Logistic Regression

11

- In practice, it is more convenient to maximize the log of the likelihood function
  - product of a large number of small probabilities can easily lead to underflow in computing machines

11

# Thank You!

12