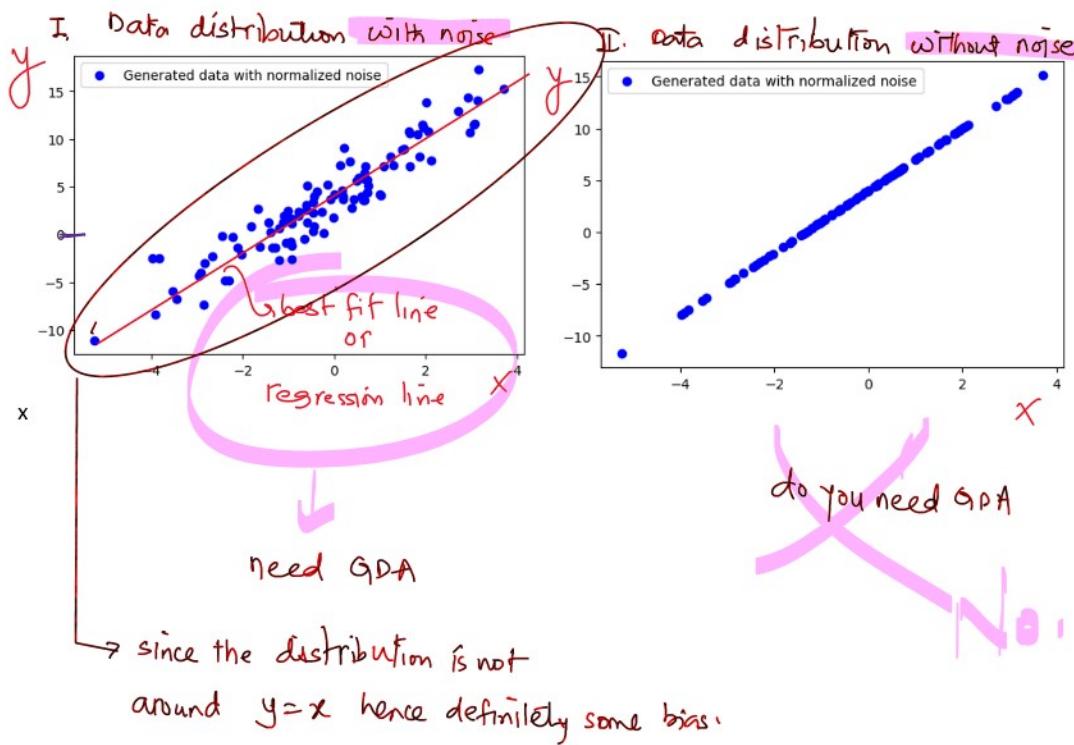
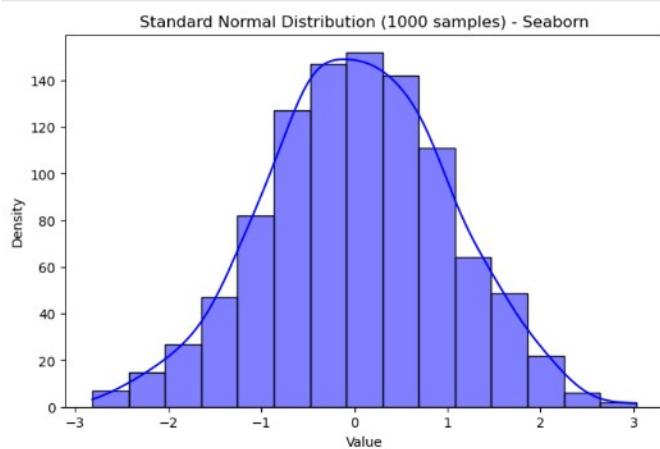
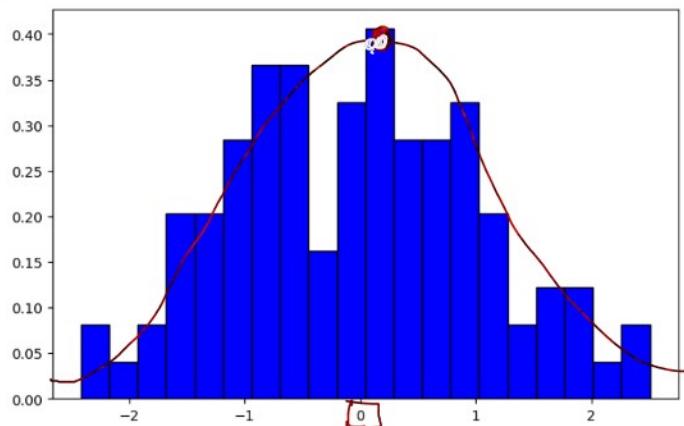


GDA Code Explanation

27 September 2025 11:55



number of data points to be generated

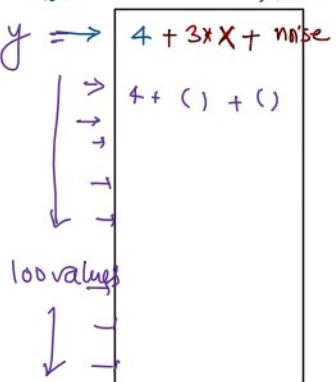
```

def generate_data(n_samples = 100, noise = 0.1, seed = 42):
    """
    Generate random samples (synthetic) linear data: y=4 + 3*x + noise
    """
    np.random.seed(seed) # to ensure the reproducibility of the same random data --> to fix the random numbers generated
    X = 2 * np.random.randn(n_samples, 1) # generates random numbers from 'standard normal distribution'
    y = 4 + 3*X + noise*np.random.randn(n_samples, 1) # Creates a random distribution around a Line having some random noise added to it as well
    return X, y

```

$0.1 \times (100 \text{ random})$

2x random std.
normal values
values coming
from std. normal distribution)



$$y = 4 + 3x$$

intercept coefficient/slope
(bias) (weight)

random values ← same →
 $X = 2 * np.random.randn(n_samples, 1)$ # generates random numbers from 'standard normal distribution'
 $y = 4 + 3*X + noise*np.random.randn(n_samples, 1)$ # Creates a random distribution around a Line having some random noise added to it as well

generate_data(n_samples = 10, noise = 0, seed = 42)

Random (same)

$X = \begin{bmatrix} 0.99342831 \\ -0.2765286 \\ 1.29537708 \\ 3.04605971 \\ -0.46830675 \\ -0.46827391 \\ 3.15842563 \\ 1.53486946 \\ -0.93894877 \\ 1.08512009 \end{bmatrix}$

$y = \begin{bmatrix} 6.98028492 \end{bmatrix}$

$x_1 = 0.9934$

$y_1 = 4 + 3 \times 0.9934 + 0$

$4 + 3 \times 0.99342831 = 6.98028493$

Mean Squared Error - Cost Function for regression problems

For m training examples:

$$J(w, b) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2$$

cost function

error $\overset{\textcircled{1}}{\rightarrow}$
 Predicted $\overset{\textcircled{2}}{\rightarrow}$ actual
 $(P-A)^2$

where:

- $y^{(i)}$ = actual output for sample i .
- $\hat{y}^{(i)}$ = predicted output.
- w, b = parameters (weights, bias).
- m = number of samples.
- $i \rightarrow i^{\text{th}}$ row/sample

Why square??

- to prevent positive and negative errors cancelling / nullifying each other
- squaring makes the error +ve
- penalizes large errors more strongly than small errors

Division by m gives the mean | avg. making it independent of dataset size
 ↓
 to get the avg. model error.

Factor $\frac{1}{2}$ is to simplify the derivative output (2 cancels when differentiating)

$$\begin{aligned} y &= xe^2 & y &= \frac{1}{2}x^2 \\ \frac{dy}{dx} &= 2x & \frac{dy}{dx} &= \frac{1}{2}(2x) = x. \end{aligned}$$

Task

Do the below derivations:

$$\frac{\partial J}{\partial w} = \frac{1}{m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)}) \cdot x^{(i)} \quad \checkmark$$

$$\frac{\partial J}{\partial b} = \frac{1}{m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)}) \quad \checkmark$$

Cost Function \rightarrow MSE

Cost Function \rightarrow MSE

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \rightarrow$$

\hat{y} (Predicted)

$$\hat{y} = \hat{\beta}_0 \cdot x^0 + \hat{\beta}_1 x$$

$$\hat{y} = [\hat{\beta}_0 \quad \hat{\beta}_1]_{1 \times 2} x \begin{bmatrix} 1 \\ x \end{bmatrix}_{2 \times 1}$$

$$\hat{y} = X \cdot \theta$$

$$J(w, b) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2$$

$$\left[\hat{\beta}_0 \quad \hat{\beta}_1 \quad \hat{\beta}_2 \dots \right] \rightarrow \theta \rightarrow \text{parameters}$$

```

: def compute_cost(x, y, theta):
    """
    Compute the mean squared error cost function
    """
    m = len(y) #no. of rows in the data
    return np.sum((x.dot(theta) - y)**2)/(2*m)

```

Linear Algebra Videos:

<https://www.khanacademy.org/math/linear-algebra>

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m \underbrace{(h_\theta(x^{(i)}) - y^{(i)})^2}_{P-A}$$

$$h_\theta(x) = \theta^T x = \hat{\beta}_0 + \hat{\beta}_1 x_1$$

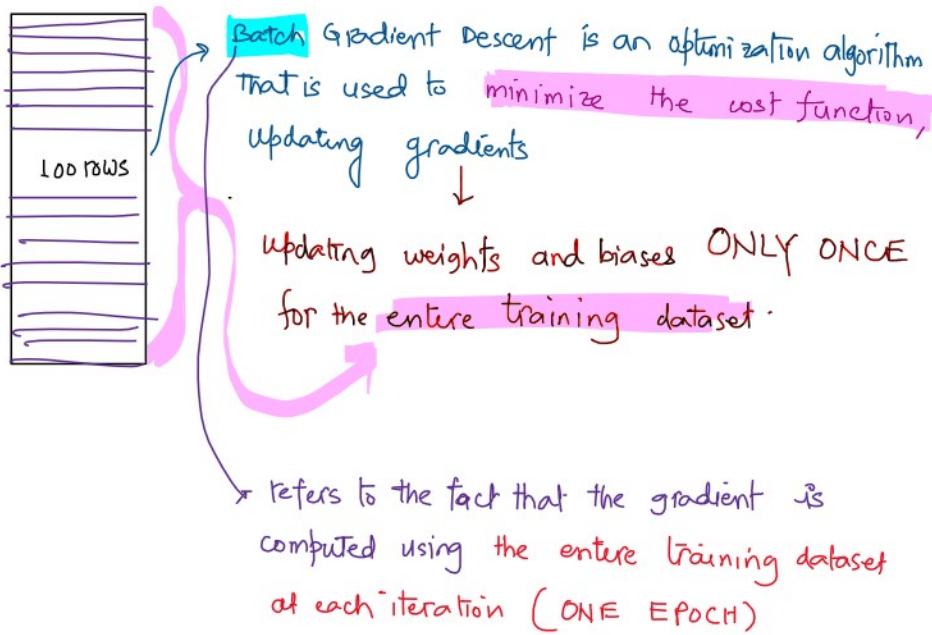
$$\begin{bmatrix} \hat{\beta}_0 & \hat{\beta}_1 \end{bmatrix}$$

$y = f(x)$

BATCH GRADIENT DESCENT ALGORITHM (BGD)

(Vanilla Gradient Descent)

↳ by default \rightarrow BGD



What is an Epoch?

An epoch → one complete pass through the entire training dataset by the model.

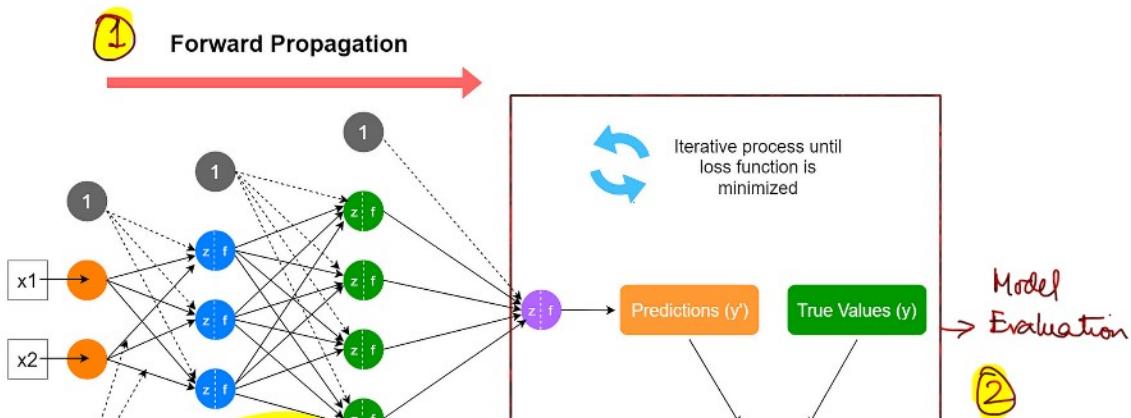
During one epoch, every training sample has been used once to update the model's parameters

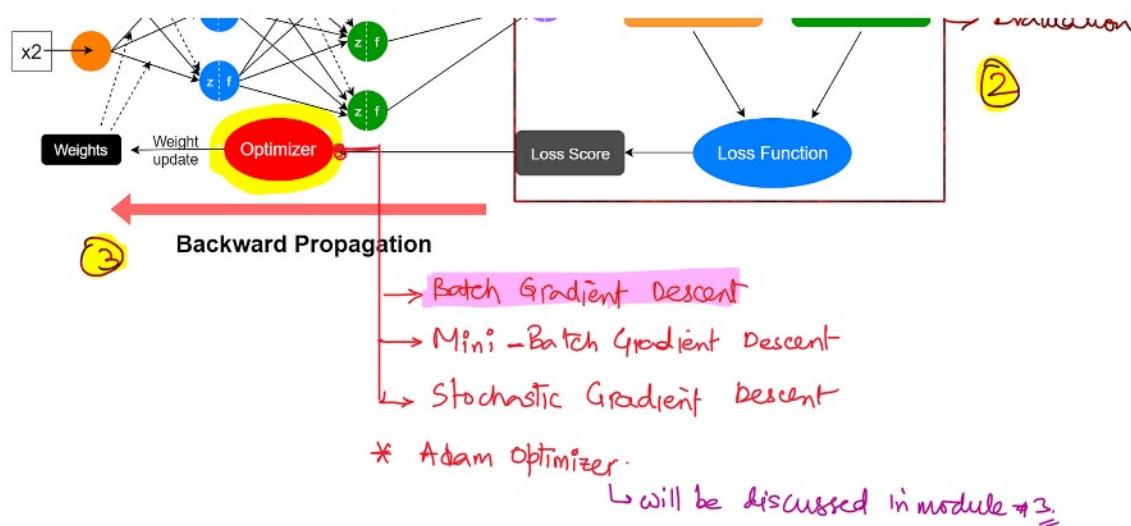
Weights biases

100 rows 1 epoch.

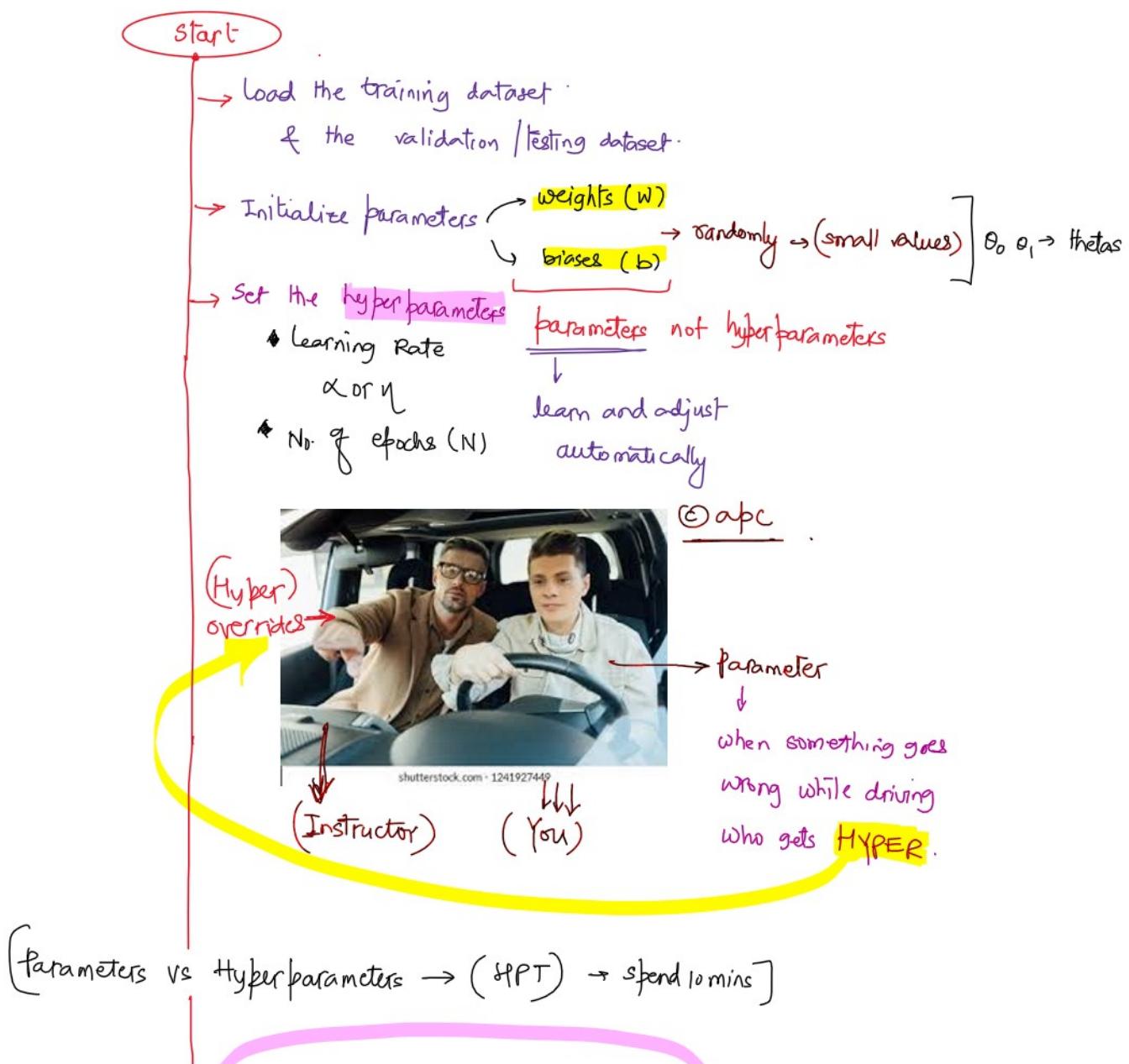
* *
pro-tip

Batch Gradient Descent: → uses all training samples / rows → the entire training dataset to compute the gradient of the **cost function** - (loss)





Flowchart : Batch Gradient Descent (from scratch)



For each epoch in range(N)

epochs

• Forward Pass

$$Y_{\text{pred}} = (W \cdot X + b)$$

weight matrix
bias

as many times
as many epochs.

• Compute Loss & model evaluation

$$L = \text{loss}(Y_{\text{pred}}, y)$$

Regression # MSE

Classification # cross-entropy

• Backward Pass

$$dW = \nabla_W L - \text{gradient of loss w.r.t. weight}$$

$$db = \nabla_b L - \text{gradient of loss w.r.t. bias}$$

• Update weights and biases

$$W_{\text{new}} = W_{\text{old}} - \frac{\alpha \times dW}{\text{Step size}}$$

$$b_{\text{new}} = b_{\text{old}} - \frac{\alpha \times db}{\text{Step size}}$$

• Plotting the loss function vs epoch

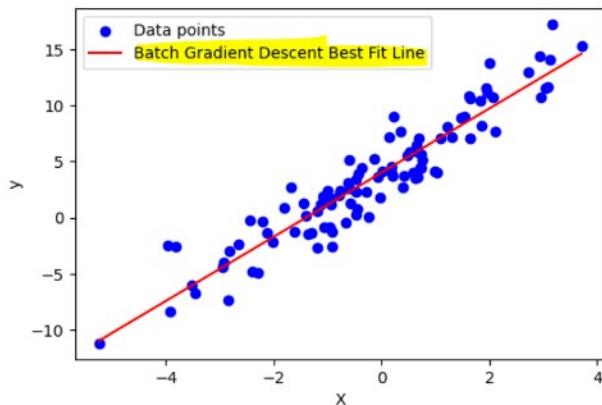
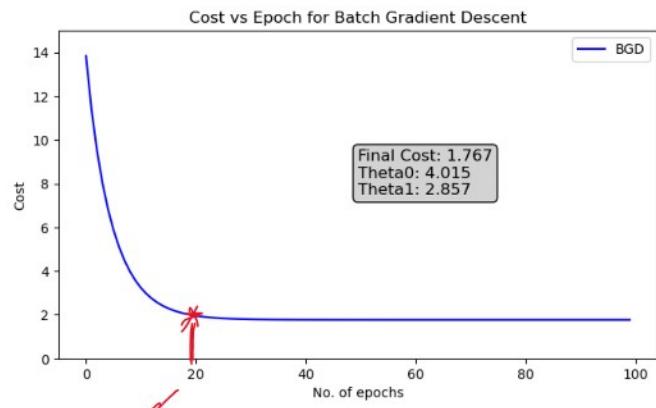
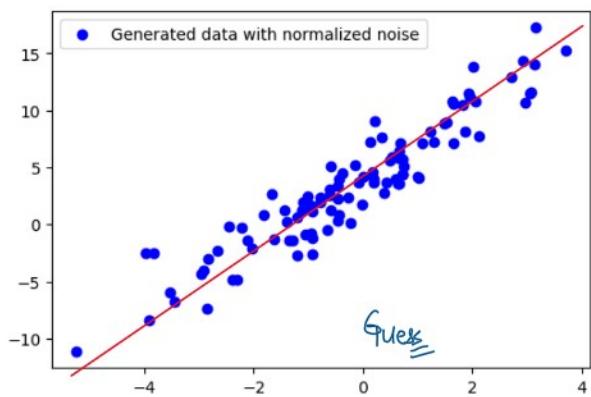
accuracy vs epoch

error vs epoch

Best fit line on the distribution

$$J(w, b) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2$$

```
: def compute_cost(X, y, theta):
    """
    Compute the mean squared error cost function
    """
    m = len(y) #no. of rows in the data
    return np.sum((X.dot(theta) - y)**2)/(2*m)
```



<https://github.com/scikit-learn/scikit-learn/tree/main/sklearn>

https://github.com/scikit-learn/scikit-learn/blob/main/sklearn/neural_network/_stochastic_optimizers.py

TASK: Create the documentation for the BGD concept