Optimal Error Dynamics for Flight Guidance

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AE691A Course Project
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This project is based on paper:

He, Shaoming and Lee, Chang-Hun. (2018). Optimality of Error Dynamics in Missile Guidance Problems. Journal of Guidance, Control, and Dynamics. 41. 1-10. 10.2514/1.G003343.

Motivation

General form of system dynamics:

$$\dot{\epsilon}(t) = g(t)u(t)$$

Error dynamics used by conventional controllers:

$$\dot{\epsilon}(t) + k\epsilon(t) = 0$$

- Ensures asymptotic convergence with exponential rate k

This error dynamics has two major drawbacks:

- finite-time convergence is not strictly guaranteed.
- only focuses on how to drive the tracking error to 0.
- Never considers how to achieve zero tracking error optimally with respect to a meaningful performance index.

Optimal Error Dynamics

The cost function $J = \frac{1}{2}\int_t^{t_f} R(\tau)u^2(\tau)d\tau$ is minimised if the error dynamics is chosen as:

$$\dot{\epsilon}(t) + \frac{\Gamma(t)}{t_{go}} = 0$$

where:

$$\Gamma(t) = \frac{t_{go}R^{-1}(\tau)g^2(\tau)}{\int_t^{t_f} R^{-1}(\tau)g^2(\tau)d\tau}$$

Extension to n-dimension

The cost function $J = \frac{1}{2}\int_{t}^{t_f} U^T(\tau)R(\tau)U(\tau)d\tau$ is minimised if **R is symmetric matrix** and the error dynamics is chosen as:

$$\dot{\epsilon}(t) + \frac{\Gamma(t)}{t_{go}} = 0$$

where:

$$\Gamma(t) = \frac{t_{go}G(\tau)R^{-1}(\tau)G^T(\tau)}{\int_t^{t_f} G(\tau)R^{-1}(\tau)G^T(\tau)d\tau}$$

Proof

Integrating the system dynamics equation:

$$-\epsilon(t) = \int_t^{t_f} G(\tau)U(\tau)d\tau$$
$$= \int_t^{t_f} G(\tau)e^{-\frac{1}{2}\log R(\tau)}e^{\frac{1}{2}\log R(\tau)}U(\tau)d\tau$$

where:

$$\log R = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{(R-I)^k}{k}$$

Applying Schwarz's inequality:

$$
\begin{aligned}
[-\epsilon(t)]^2 &\leq \int_t^{t_f} G(\tau)e^{-\frac{1}{2}\log R(\tau)}e^{-\frac{1}{2}(\log R(\tau))^T}G^T(\tau)d\tau \int_t^{t_f} U^T(\tau)e^{\frac{1}{2}(\log R(\tau))^T}e^{\frac{1}{2}\log R(\tau)}U(\tau)d\tau \\
&\leq \int_t^{t_f} G(\tau)e^{-\frac{1}{2}\log R(\tau)}e^{-\frac{1}{2}\log R^T(\tau)}G^T(\tau)d\tau \int_t^{t_f} U^T(\tau)e^{\frac{1}{2}\log R^T(\tau)}e^{\frac{1}{2}\log R(\tau)}U(\tau)d\tau
\end{aligned}
$$

Where we use property:

$$(e^A)^T = e^{A^T}$$

$$(\log(A))^T = \log(A^T)$$

Now using the Baker–Campbell–Hausdorff formula:

$$e^X e^Y = e^Z$$

$$Z = X + Y + \frac{1}{2}[X, Y] + \frac{1}{12}[X, [X, Y]] - \frac{1}{12}[Y, [X, Y]] + \ldots$$

The inequality reduces to:

$$[-\epsilon(t)]^2 \leq \int_t^{t_f} G(\tau)R^{-1}(\tau)G^T(\tau)d\tau \int_t^{t_f} U^T(\tau)R(\tau)U(\tau)d\tau$$

If R satisfies:

$$[R, R^T] = 0$$

$$\Rightarrow R = R^T$$

Application to Missile Guidance

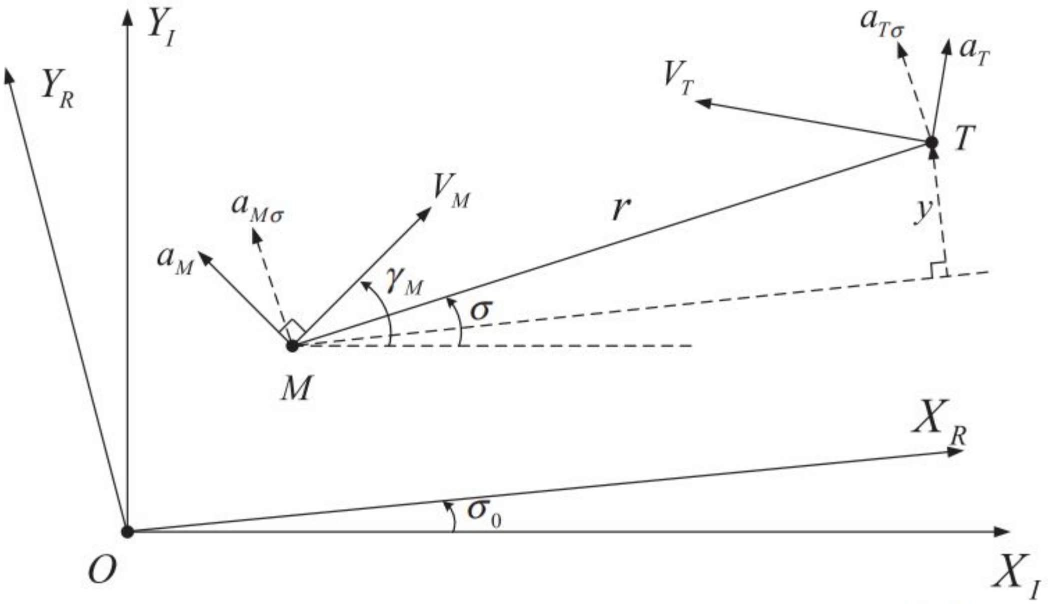


Fig. 1 The homing engagement geometry and parameter definitions.

Kinematics Equation

$$\dot{y} = v$$

$$\dot{v} = a_{T_\sigma} \cos\left(\sigma - \sigma_0\right) - a_{M_\sigma} \cos\left(\sigma - \sigma_0\right)$$

$$a_{M_\sigma} = a_M \cos\left(\gamma_M - \sigma\right)$$

$$\dot{\sigma} = \frac{y + v t_{go}}{V_c t_{go}^2}$$

Error dynamics

ZEM control:

$$a_{M_\sigma} = NV_c\dot{\sigma} + \frac{N}{2}a_{T_\sigma}$$

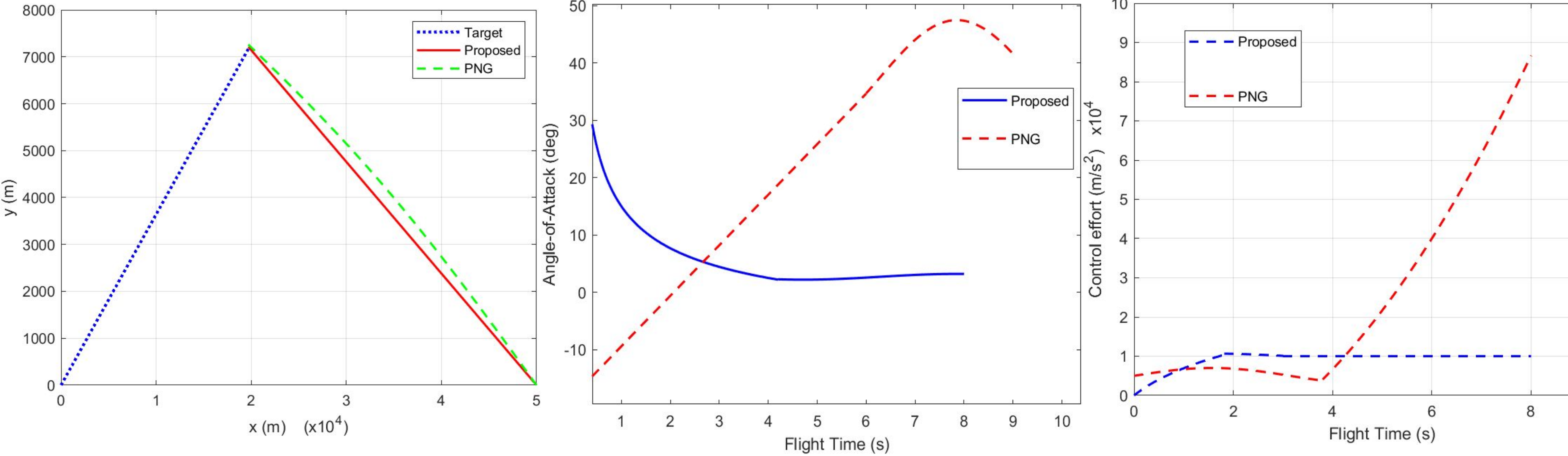
Guidance To Collision Control:

$$a_M \sin\alpha = \frac{KV_M(t)\epsilon}{t_{go}}$$

Numerical Simulation

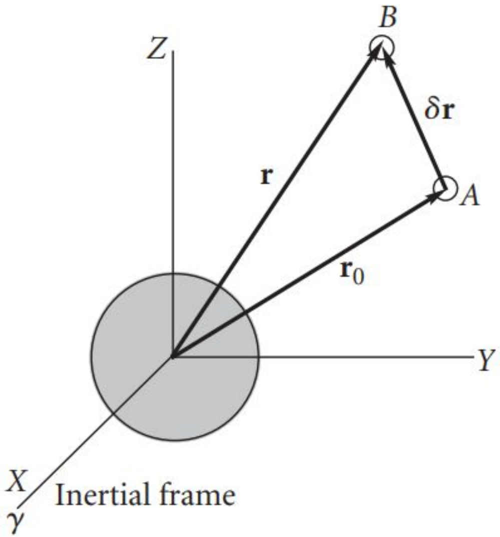
Table 1

Parameters	Values
Missile–target initial relative range r(0), km	5
Initial LOS angle $\sigma(0)$, *deg*	0
Missile initial velocity $V_M(0)$, m/s	2500
Missile initial flight-path angle $\gamma_M(0)$, deg	160
Target velocity V_T, m/s	3000
Target initial flight-path angle $\gamma_M(0)$, deg	20



Target
Proposed
PNG
y (m)
x (m) (x10^4)
Angle-of-Attack (deg)
Flight Time (s)
Proposed
PNG
Control effort (m/s^2) x10^4
Flight Time (s)
Proposed
PNG

Application to Space Docking



$$\delta\ddot{x} - 3n^2\delta x - 2n\delta\dot{y} = \frac{F_x}{m} = u_x$$

$$\delta\ddot{y} + 2n\delta\dot{x} = \frac{F_y}{m} = u_y$$

$$\delta\ddot{z} + n^2\delta z = \frac{F_z}{m} = u_z$$

$$\Rightarrow \dot{X} = AX + BU$$

$$\dot{\epsilon} = A\epsilon + BU$$

$$\dot{\epsilon} + \frac{kI_6}{t_{go}} = 0$$

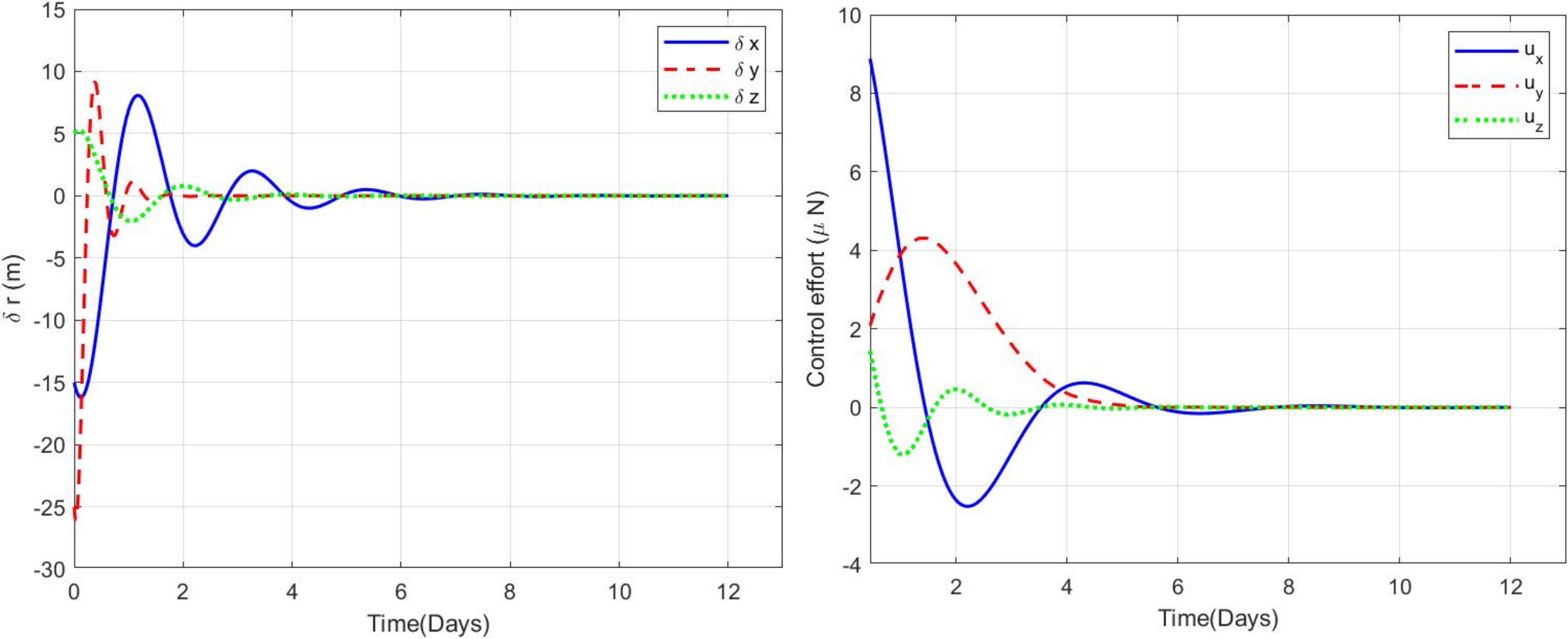
$$\Rightarrow BU = -\left(\frac{kI_6}{t_{go}} + A\right)\epsilon$$

$$J = \frac{1}{2}\int_t^{t_f} \frac{U^T(\tau)U(\tau)}{t_{go}^{k-1}} d\tau$$

Numerical Simulation

Table 2 Simulation Parameters

Parameter	Value	Unit
m_1	200	kg
m_2	200	kg
n	7.29×10^{-5}	rad/sec
$\delta x(0)$	-15	m
$\delta y(0)$	-25	m
$\delta z(0)$	5	m
$\dot{\delta} x(0)$	0.5	m/s
$\dot{\delta} y(0)$	0.5	m/s
$\dot{\delta} z(0)$	0	m/s
t_f	12	days


δ x
δ y
δ z
δ r (m)
Time(Days)
15
10
5
0
-5
-10
-15
-20
-25
-30
0
2
4
6
8
10
12
u_x
u_y
u_z
Control effort (μ N)
Time(Days)
10
8
6
4
2
0
-2
-4
2
4
6
8
10
12

Conclusion

- Finite time convergence is achieved
- Control effort required is optimal and less than PNG
- Can be applied to n-dimensional systems

Thank You