

# Optimality of Error Dynamics in Missile Guidance

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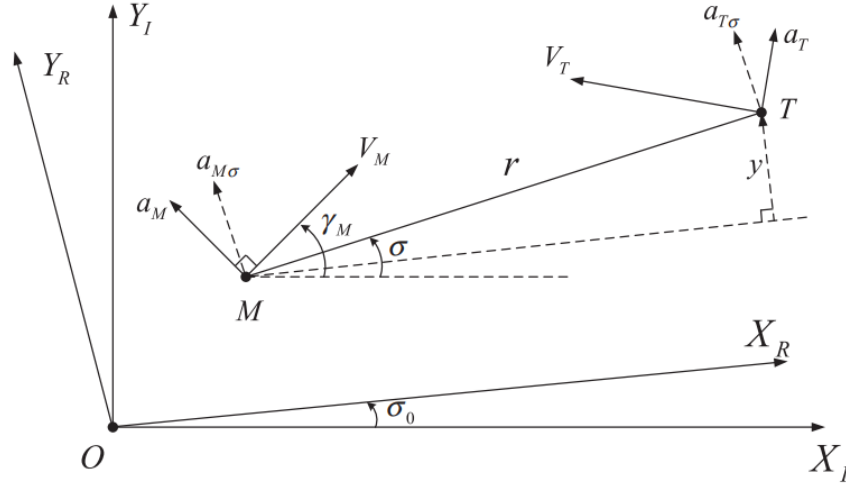
## I. Introduction

Missile guidance problem holds rich history with extensive research study. Development of missile guidance laws for successful interception of target is hold motivation due to its application in many fields: from heavy object deflection to protection against attacking vehicles/missiles. Traditionally proportional guidance navigation (PNG) law was used, where acceleration command was based on derivative of LOS angle, chosen so as to nullify it. Recent developed non-linear controllers use tracking error for developing guidance laws. The aim is to asymptotically converge error to zero in finite time. To regulate the tracking error to be zero, various control theories such as SMC, the Lyapunov function, and feedback linearization have been applied to system in previous works. In these approaches, the desired error dynamics is first selected, and then appropriate control input is calculated to ensure that the system equation shown in follows the selected error dynamics. All these method only focuses on asymptotic convergence. The attention on how fast the error converges, a major constraint of finite time missile guidance, seems to be missing. Finite time convergence is not guaranteed. Also, the optimality of the developed law with respect to meaningful cost function/performance index is not considered.

## II. System Model

Fig 1 below shows the system model for the problem. M and T denote the missile and target, respectively. The notation of  $(X_I, Y_I)$  represents the inertial frame. For the purpose of introducing the linearized kinematics, a new frame called the reference frame  $(X_R, Y_R)$  is also defined. This frame is rotated from the inertial frame by  $\sigma_0$ , which is the reference angle. The variables of  $\sigma$  and  $\gamma$  stand for the LOS angle and flight-path angle, respectively;  $r$  denotes the relative distance between

the target and the missile;  $y$  is the relative distance between the target and the missile perpendicular to the  $X_R$  direction; and  $a_M$  and  $a_T$  are the missile and target accelerations normal to the velocity vectors, respectively. The variables of  $a_{M\sigma}$  and  $a_{T\sigma}$  denote the missile and target accelerations normal to the LOS direction, respectively.



**Fig. 1 The homing engagement geometry and parameter definitions.**

### III. Goals and Improvement

This project investigate the optimal convergence of the tracking error and propose an optimal error dynamics that achieves this optimal pattern as well as guarantees the finite-time convergence for guidance law design. Improvements to be done:

- Reproduce the results given in paper: application to various missile guidance scenarios.
- Extend the given law to n-dimensional error dynamics with proof.
- Use the above extension to apply optimal error dynamics to spacecraft attitude control.

### References

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