The Journal of the Astronautical Sciences, Vol. 54, Nos. 3 & 4, July–December 2006, pp. 525–541

Necessary Conditions for Circularly-Restricted Static Coulomb Formations[1]

Hanspeter Schaub,[2] Christopher D. Hall,[3] and John Berryman[4]

Abstract

A noncanonical Hamiltonian formulation of the Coulomb formation dynamics is used to develop necessary conditions for static Coulomb formations with constant charges. With a static or frozen formation the satellites perform non-Keplerian orbits and maintain constant relative position vectors. As seen by an observer following the center of mass motion, the spacecraft formation would appear to behave equivalently to a rigid body in orbit. Previous research has demonstrated the existence of such static Coulomb formations analytically by employing symmetry simplifying assumptions with linearized relative motion dynamics, or by using numerical genetic search algorithms. These static solutions are used as reference geometries and charges for feedback law developments. This paper presents nonlinear static formation conditions for the circularly restricted problem. Hamiltonian formulations have been used to study equilibrium conditions of rigid bodies in orbit. Analogous techniques are employed to study necessary conditions to achieve a static Coulomb formation. Analytical results using the full and truncated formation gravity potential function are presented. Numerical results illustrate convergence performance improvements of an evolutionary search algorithm where the presented necessary conditions are enforced.

Introduction

A great variety of spacecraft formation flying missions are being considered to distribute sensors over a large area and control their relative positions. In these missions a set of spacecraft fly in formation with separation distances ranging from hundreds of meters to multiple kilometers. For example, the Techsat 21 program [1] considered satellites flying in Low Earth Orbits (LEO) to use radar interferometry to scan the Earth's surface. The spacecraft separation distances were about 1 km with the satellites performing coordinated in-plane and out-of-plane relative maneuvers. Other spacecraft formations include dynamically simpler along-track formations [2].

[1]Presented as paper AAS 05-472 at the AAS Malcolm D. Shuster Astronautics Symposium, State University of New York, Buffalo, New York, June 12–15, 2005.
[2]Assistant Professor, Aerospace and Ocean Engineering Department, Virginia Tech, Blacksburg, VA 24061-0203.
[3]Professor, Aerospace and Ocean Engineering Department, Virginia Tech, Blacksburg, VA 24061-0203.
[4]Graduate Research Assistant, Aerospace and Ocean Engineering Department, Virginia Tech, Blacksburg, VA 24061-0203.

Such formations are used to perform stereo Earth imaging or gravity field measurements [3]. In an along-track formation two satellites are essentially in the same orbit, but one satellite lags behind the second satellite. If the satellites are not of equal type and build, then they will experience different amounts of atmospheric drag. The formation control strategy must compensate for the different orbit perturbation forces. An example of a very large-scale spacecraft formation is the LISA mission [4], which has satellites flying millions of kilometers apart in heliocentric orbits. The scientific purpose of the LISA mission is to measure gravitational waves.

In all these spacecraft formation flying missions it is vital to develop fuel efficient control strategies. The craft are intended to orbit each other for several years. Small modeling errors in the relative motion dynamics will cause unnecessary fuel usage. Due to the long mission durations, this unwarranted fuel usage will quickly accumulate to significant amounts. Thus, when developing such traditional spacecraft formation missions, it is crucial that the reference formation geometries satisfy the naturally occurring orbital motion as much as possible.

In 2002 King and Parker proposed a novel method of spacecraft propulsion in their NIAC study [5]. Studying the mission data of the SCATHA spacecraft [6], they found that the naturally occurring electrostatic potentials of this High Earth Orbit (HEO) spacecraft can grow large enough to produce milli-Newton level forces onto a craft within a few dozen meters distance. King and Parker suggested exploiting this electrostatic (Coulomb) force and using it to control the relative motion of closely flying spacecraft. By emitting either positive ions or negative electrons, the spacecraft charge can be controlled. The SCATHA mission itself demonstrated this technology. Currently, the CLUSTERS mission [7] is using this charge emission technique to maintain a zero spacecraft charge relative to the local space plasma environment.

Referred to as Coulomb thrusting, this mode of propulsion has been shown to be essentially propellantless with I_{sp} values approaching 10^{10}–10^{13} seconds for relative motion control [5, 8, 9]. Furthermore, the associated electrical power requirements are typically less than a Watt, depending on the spacecraft separation distances and charge levels. However, the relative motion dynamics of charged vehicles is much more complex than the dynamics of traditional formations. The motion of any one craft is directly coupled though Coulomb force field interactions to the motion of any other neighboring spacecraft.

Due to the extreme fuel efficiency and low power consumption of Coulomb thrusting, unconventional formation control strategies can be considered. King and Parker discovered in reference [5] that it is possible to use constant Coulomb forces to cancel all relative motion of the charged spacecraft. As a result, these static Coulomb formations appear frozen to an external observer and move as if they were a single, continuous rigid body. Here the formation center of mass (chief location) is assumed to have a circular orbital motion, and the linearized relative motion is described relative to the formation center of mass LVLH coordinate frame. The associated non-Keplerian orbits of the spacecraft require continuous control efforts to remain in this static configuration. With conventional thrusting techniques, flying such static formations would quickly deplete the fuel supply. However, with the Coulomb thrusting concept, extended missions are possible where a small, continuous thrust is applied. Note that this discussion on static Coulomb formations pertains to using open-loop spacecraft charges. None of the static Coulomb formations considered so far have been found to be stable without any feedback. The charge required to achieve the static formation is useful in control law development. For the two-craft Coulomb tether formation considered in reference [10] it is used as a feed-forward control component along with a feedback control law.

The charged spacecraft equations of motion are highly nonlinear and coupled. Finding such static formation geometries, as well as the associated static charges, is a nontrivial matter. References [5], [8], and [9] show static analytical solutions of the Clohessy-Wiltshire equations for three-seven craft using symmetry assumptions. Here one satellite is always located in the formation center. In reference [11] another three-craft equilateral triangular formation is presented, which does not contain a craft in the formation center. Berryman and Schaub presented a genetic search algorithm in reference [12] to search for static Coulomb formations. This search yielded new two-craft formations used to perform Coulomb tether formations [10], as well as more general three-craft solutions and several larger formation solutions containing up to nine spacecraft.

This paper investigates necessary conditions to achieve such static Coulomb formations with constant charges, and presents the nonlinear static formation conditions for the circularly restricted problem. The results of the genetic algorithm search in reference [12] indicated some general patterns where the resulting formation symmetry axes tended to align with the chief coordinate frame axes. Using a noncanonical Hamiltonian formulation of the relative motion, equilibrium conditions for a static Coulomb formation are developed. In reference [13] the Coulomb formation dynamics is discussed by considering the formation to be a single continuous body, where the shape is determined through the individual spacecraft locations. The rotating chief LVLH coordinate frame origin is defined to be the Cartesian center of mass of the formation. Writing the relative position vector component in the chief LVLH frame, it is possible to formulate a formation inertia matrix. For a Coulomb formation to be static with respect to this chief LVLH frame, the inertia matrix must be a constant matrix. In essence, the frozen formation becomes equivalent to a rigid body, where the body internal forces which maintain a given shape are replaced with the electrostatic Coulomb forces. Achieving a static Coulomb formation is related to finding equilibrium conditions of a rigid body in a circular orbit. As such, techniques similar to finding zero-gravity gradient torques for satellites can be employed for the frozen Coulomb formation search. A common technique is to obtain a first order truncation of the gravity gradient torque acting on a spacecraft and determine linear equilibrium conditions [14]. Here the satellite motion is assumed to be decoupled from the attitude. A more rigorous approach is presented by Wang, Maddocks, and Krishnaprasad in reference [15] using a noncanonical Hamiltonian formulation. Here the equilibrium attitudes are determined numerically from the complete nonlinear formulation. Also, this approach allows the center of mass motion and attitude coupling to be retained. More recently Beck and Hall used this noncanonical Hamiltonian formulation in reference [16] to determine analytical first order conditions for general and axisymmetric rigid bodies in a circular orbit. This noncanonical Hamiltonian formulation is applied to the Coulomb spacecraft dynamics problem to determine necessary conditions for static formations with constant charges. The necessary conditions are developed with the full formation gravitational potential function. Further results are obtained after truncating this potential function to second order. Finally, the developed necessary conditions are incorporated into an evolutionary search algorithm. Numerical studies are performed to evaluate the convergence improvements.

Nonlinear Static Solution Conditions

Consider a formation of N spacecraft flying in close proximity to each other with typical separation distances ranging from 10–100 meters. As illustrated in Fig. 1, each craft is assumed to have an electrostatic (Coulomb) charge. The Coulomb forces cause a complex dynamic interaction between all charged spacecraft in the formation. Let $\mathbf{r}_i$ be the inertial position vector of a single craft of mass m_i. The vector $\mathbf{r}_c$ is the center of mass position vector of this formation defined as

$$\mathbf{r}_c = \frac{1}{M} \sum_{i=1}^{N} m_i \mathbf{r}_i \tag{1}$$

with $M = \Sigma_{i=1}^{N} m_i$ being the total formation mass. The relative position vector of the ith satellite with respect to this center of mass is

$$\boldsymbol{\rho}_i = \mathbf{r}_1 - \mathbf{r}_c \tag{2}$$

To describe the relative motion, a rotating orbit frame $\mathcal{O}:\{\hat{\mathbf{o}}_r, \hat{\mathbf{o}}_\theta, \hat{\mathbf{o}}_h\}$ is introduced where

$$\hat{\mathbf{o}}_r = \frac{\mathbf{r}_c}{r_c} \tag{3a}$$

$$\hat{\mathbf{o}}_\theta = \hat{\mathbf{o}}_h \times \hat{\mathbf{o}}_r \tag{3b}$$

$$\hat{\mathbf{o}}_h = \frac{\mathbf{r}_c \times \dot{\mathbf{r}}_c}{|\mathbf{r}_c \times \dot{\mathbf{r}}_c|} \tag{3c}$$

with $r_c = |\mathbf{r}_c|$ and $\dot{\mathbf{r}}_c$ being the inertial derivative of $\mathbf{r}_c$.

Each craft is assumed to have an electrostatic charge q_i relative to the ambient space plasma environment. The Coulomb force experienced by craft i due to the electrostatic field interaction with the jth craft is

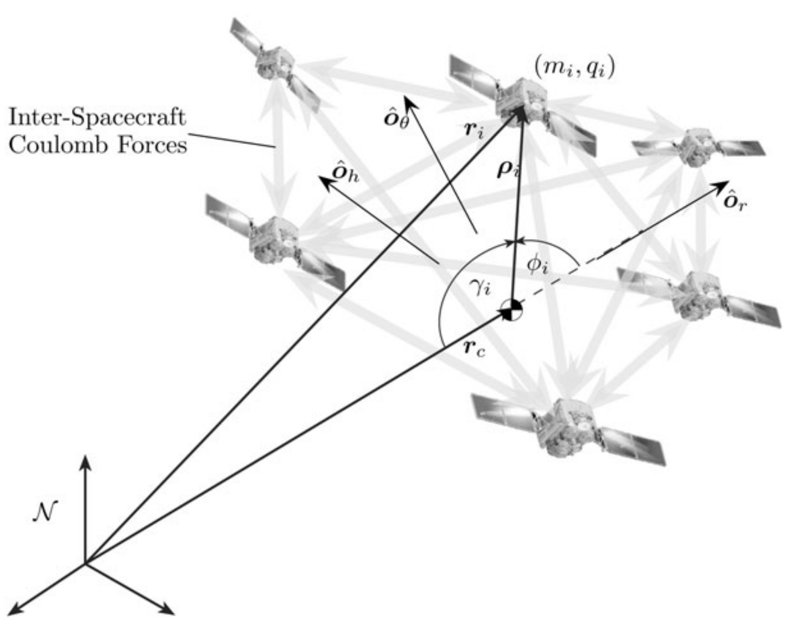

FIG. 1. Illustration of a Coulomb Spacecraft Formation.

$$\mathbf{F}_{ij} = k_c q_i q_j \frac{\boldsymbol{\rho}_{ji}}{\rho_{ji}^3} e^{-\frac{\rho_{ji}}{\lambda_d}} \quad \text{with } i \neq j \tag{4}$$

where $\boldsymbol{\rho}_{ji} = \boldsymbol{\rho}_i - \boldsymbol{\rho}_j$ and $k_c = 8.99 \times 10^9 \mathrm{Nm}^2/\mathrm{C}^2$ is the Coulomb's constant. Note that the typical Coulomb force magnitude is augmented with an additional exponential term depending on the Debye length λ_d [17, 18]. The electro-static field strength of a spacecraft flying through a space becomes absorbed by the plasma environment of free-flying electrons and ions. In low Earth orbits, this Debye length is on the order of centimeters, making the Coulomb thrusting concept difficult to implement. In geostationary orbits (GEO) the Debye length can vary between 140–1400 meters, depending on solar activity levels. The Coulomb spacecraft motions studied in the following sections are assumed to be flying on high Earth orbits where the Debye length does not degrade the Coulomb field too much.

The gravitational potential of a point mass in orbit about a planet with mass m is given by [14]

$$V_i(\mathbf{r}_i) = -\frac{Gmm_i}{r_i} = -\frac{\mu m_i}{r_i} \tag{5}$$

where G is the universal gravitational constant and $\mu = Gm$ is the gravitational constant of a particular planet. The equation of motion of the ith charged satellite in orbit about a planet is then given by

$$m_i \ddot{\mathbf{r}}_i = -\nabla_{\mathbf{r}_i} V_i + \sum_{j=1}^{N} \mathbf{F}_{ij} \quad \text{with } i \neq j \tag{6}$$

Earlier treatments on studying static Coulomb formations in References [5], [8], and [11] utilized the formation Hill coordinate frame. Note that no linearization is performed in equation (6). The full nonlinear orbital mechanics of a point mass are retained.

A static Coulomb formation consists of a set of N spacecraft with constant charges that do not vary their *relative positions* [5, 8, 11]. To an observer traveling with the orbit frame $\mathcal{O}$, the formation would appear to be rigid or frozen and not vary its shape. Several such solutions have been found using the linearized Clohessy-Wiltshire-Hill equations [19, 20]. Let us define the orbit frame derivative through

$$\frac{{}^{\mathcal{O}}\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}t} = \boldsymbol{\rho}' \tag{7}$$

For a static Coulomb formation $\boldsymbol{\rho}_i' = 0$ and the inertial derivative of $\boldsymbol{\rho}$ is given by

$$\dot{\boldsymbol{\rho}}_i = \frac{{}^{\mathcal{N}}\mathrm{d}\boldsymbol{\rho}_i}{\mathrm{d}t} = \boldsymbol{\rho}_i' + \boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} \times \boldsymbol{\rho}_i = \boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} \times \boldsymbol{\rho}_i \tag{8}$$

The inertial acceleration of the relative position vector must satisfy

$$\ddot{\boldsymbol{\rho}}_i = \boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} \times \boldsymbol{\rho}_i) + \dot{\boldsymbol{\omega}}_{\mathcal{O}/\mathcal{N}} \times \boldsymbol{\rho}_i \tag{9}$$

Using equation (2) the static formation condition is written as

$$m_i(\ddot{\mathbf{r}}_c + \boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} \times \boldsymbol{\rho}_i) + \dot{\boldsymbol{\omega}}_{\mathcal{O}/\mathcal{N}} \times \boldsymbol{\rho}_i) = -\nabla_{\mathbf{r}_i} V_i + \sum_{j=1}^{N} F_{ij} \qquad \text{with } i \neq j \tag{10}$$

To find a static Coulomb formation, spacecraft locations $\boldsymbol{\rho}_i = x_i\hat{\mathbf{o}}_r + y_i\hat{\mathbf{o}}_\theta + z_i\hat{\mathbf{o}}_h$ and charges q_i must be found such that equation (10) is satisfied for each spacecraft. If successful, then the electrostatic forces will perfectly cancel the relative orbital accelerations and freeze the formation as seen by the rotating orbit frame $\mathcal{O}$. Equation (10) is written for general orbit types. Restricting our search to cases where the formation center of mass motion is circular or nearly circular, the center of mass and associated orbit frame motion simplify to:

$$\boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} = \sqrt{\frac{\mu}{r_c^3}}\,\hat{\mathbf{o}}_h = n\hat{\mathbf{o}}_h \tag{11}$$

$$\ddot{\mathbf{r}}_c = -r_c n^2 \hat{\mathbf{o}}_r \tag{12}$$

$$\dot{\boldsymbol{\omega}}_{\mathcal{O}/\mathcal{N}} = 0 \tag{13}$$

Note that by making this simplification we are assuming that the formation center of mass motion is decoupled from the satellite relative motion [13]. This decoupling is justifiable as long as the spacecraft relative position vectors $\boldsymbol{\rho}_i$ remain small compared to the inertial center of mass position vector $\mathbf{r}_c$. The static formation condition in Eq. (10) simplifies for the circularly restricted case to

$$m_i(-r_c n^2\hat{\mathbf{o}}_r + n^2\hat{\mathbf{o}}_h \times (\hat{\mathbf{o}}_h \times \boldsymbol{\rho}_i)) = -\nabla_{\mathbf{r}_i} V_i + \sum_{j=1}^{N} F_{ij} \qquad \text{with } i \neq j \tag{14}$$

Expressing all vectors in orbit frame $\mathcal{O}$ components through $\boldsymbol{\rho} = (x, y, z)$, $\hat{\mathbf{o}}_r = (1, 0, 0)$, and $\hat{\mathbf{o}}_h = (0, 0, 1)$, the static formation condition is also expressed as

$$-m_i \,{}^{\mathcal{O}}\!\begin{pmatrix} n^2(x + r_c) \\ n^2 y \\ 0 \end{pmatrix} = -\nabla_{\mathbf{r}_i} V_i + \sum_{j=1}^{N} \mathbf{F}_{ij} \qquad \text{with } i \neq j \tag{15}$$

Because the formation is maintaining a fixed shape, it can be compared to a rigid body consisting of a discretely distributed set of masses. Studying attitude equilibrium solutions of orbiting satellites, the attitude-to-orbit coupling is typically neglected [14, 16]. However, this decoupling is an approximation. For satellites with very unevenly distributed mass components the center of mass motion coupling may need to be included. The resulting equilibrium attitudes can vary substantially from the decoupled center of mass motion case [15]. This paper focuses on formation cases where the mass distribution is reasonably even and the center of motion decoupling is justified.

Formation Forces and Torques

The force acting on a single satellite is defined as

$$\mathbf{F}_i = -\nabla_{\mathbf{r}_i} V_i + \sum_{j=1}^{N} \mathbf{F}_{ij} \tag{16}$$

Using $\mathbf{r}_i = \mathbf{r}_c + \boldsymbol{\rho}_i$, a change of coordinates can be introduced through

$$\nabla_{\mathbf{r}_i} V_i = \left[\frac{\partial V_i}{\partial \mathbf{r}_i}\right]^{\mathrm{T}} = \left[\frac{\partial V_i}{\partial \mathbf{r}_c}\frac{\partial \mathbf{r}_c}{\partial \mathbf{r}_i}\right]^{\mathrm{T}} = \left[\frac{\partial V_i}{\partial \mathbf{r}_c}\right]^{\mathrm{T}} = \nabla_{\mathbf{r}_c} V_i \tag{17}$$

Let the formation gravitational potential function $V(\mathbf{r}_c)$ be written as

$$V(\mathbf{r}_c) = \sum_{i=1}^{N} V_i = -\sum_{i=1}^{N} \frac{\mu m_i}{|\mathbf{r}_c + \boldsymbol{\rho}_i|} \tag{18}$$

Using Fig. 1, we can express

$$|\mathbf{r}_c + \boldsymbol{\rho}_i| = \sqrt{r_c^2 + 2\mathbf{r}_c \cdot \boldsymbol{\rho}_i + \rho_i^2} = r_c \sqrt{1 - 2\left(\frac{\rho_i}{r_c}\right) \cos \gamma_i + \left(\frac{\rho_i}{r_c}\right)^2} \tag{19}$$

Using the Legendre polynomial definition [14, 21] the $1/r_i$ term is written as a polynomial expansion as

$$\frac{1}{r_i} = \frac{1}{|\mathbf{r}_c + \rho_i|} = \frac{1}{r_c} \sum_{k=0}^{\infty} P_k(\cos \gamma_i) \left(\frac{\rho_i}{r_c}\right)^k \tag{20}$$

Thus, the total formation gravitational potential is given by

$$V(\mathbf{r}_c) = -\frac{\mu}{r_c} \sum_{i=1}^{N} \sum_{k=0}^{\infty} m_i P_k(\cos \gamma_i) \left(\frac{\rho_i}{r_c}\right)^k \tag{21}$$

This potential function can be conveniently truncated at desired orders k. To study the rigid body orbital attitude problem, the equivalent potential function is typically truncated at second order. The total force acting on the formation is then given by

$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i = -\nabla_{\mathbf{r}_c} V(\mathbf{r}_c) \tag{22}$$

The Coulomb forces mutually cancel each other here due to being formation internal forces. Because $\nabla_{\mathbf{r}_c} V_i$ is aligned with $\mathbf{r}_i$, the external torque being applied to the ith spacecraft about the formation center of mass can be written as [16]

$$\boldsymbol{\tau}_i = \boldsymbol{\rho}_i \times (-\nabla_{\mathbf{r}_c} V_i) = -(-\mathbf{r}_i + \mathbf{r}_c) \times \nabla_{\mathbf{r}_c} V_i = \mathbf{r}_c \times \nabla_{\mathbf{r}_c} V_i \tag{23}$$

The total torque applied to the formation about the center of mass is then

$$\boldsymbol{\tau}_c = \sum_{i=1}^{N} \boldsymbol{\tau}_i = \mathbf{r}_c \times \nabla_{\mathbf{r}_c} V \tag{24}$$

Using the orbit frame unit direction vectors, this torque can also be expressed as

$$\boldsymbol{\tau}_c = (r_c \hat{\mathbf{o}}_r) \times \left[\frac{\partial V}{\partial r_c}\right]^{\mathrm{T}} = r_c \hat{\mathbf{o}}_r \times \left[\frac{\partial V}{\partial \hat{\mathbf{o}}_r} \underbrace{\frac{\partial \hat{\mathbf{o}}_r}{\partial \mathbf{r}_c}}_{\frac{1}{r_c}[I_{3\times 3}]}\right]^{\mathrm{T}} = \hat{\mathbf{o}}_r \times \nabla_{\hat{\mathbf{o}}_r} V \tag{25}$$

Note that if the gradient of the formation potential function is aligned with the orbit radial direction $\hat{\mathbf{o}}_r$, then the gravity gradient torque will be zero.

Noncanonical Formulation

In this section a noncanonical formulation of a static Coulomb formation is developed. This development assumes that electrostatic charges can be produced to achieve a static formation, and develops necessary equilibrium conditions on where the satellite masses must be placed with the rotating orbit frame $\mathcal{O}$.

Because we are considering static Coulomb formations, the spacecraft formation can be represented as a discretely distributed rigid body. The formation inertia matrix about the center of mass is expressed as [13]

$$[I] = -\sum_{i=1}^{N} m_i[\tilde{\rho}_i][\tilde{\rho}_i] = \sum_{i=1}^{N} m_i \,{}^{\mathcal{O}}\!\begin{bmatrix} y_i^2 + z_i^2 & -x_i y_i & -x_i z_i \\ -x_i y_i & x_i^2 + z_i^2 & -y_i z_i \\ -x_i z_i & -y_i z_i & z_i^2 + y_i^2 \end{bmatrix} \tag{26}$$

Given that static formation shapes are considered, a body-fixed coordinate frame $\mathcal{B}$ is introduced. The orientation of this formation-fixed frame $\mathcal{B}$ relative to the center of mass orbit frame $\mathcal{O}$ is given through the rotation matrix

$$[BO] = [OB]^{\mathrm{T}} = [{}^{\mathcal{B}}\hat{\mathbf{o}}_r \quad {}^{\mathcal{B}}\hat{\mathbf{o}}_\theta \quad {}^{\mathcal{B}}\hat{\mathbf{o}}_h] \tag{27}$$

where the unit direction vectors of the $\mathcal{O}$ frame are expressed in $\mathcal{B}$ frame components. Using rigid body notations and conventions, the inertial angular momentum of the formation about its center of mass is written as

$$\mathbf{H}_c = [I]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \tag{28}$$

Given the inertial momentum vector, the formation body frame angular velocity vector can be found using

$$\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} = [I]^{-1}\mathbf{H}_c \tag{29}$$

For the following development in this section, let us use the shorthand notation

$$\frac{{}^{\mathcal{B}}\mathrm{d}\mathbf{H}_c}{\mathrm{d}t} = \mathbf{H}_c' \tag{30}$$

Taking the inertial derivative of equation (28) we find

$$\dot{\mathbf{H}}_c = \mathbf{H}_c' + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \mathbf{H}_c = \boldsymbol{\tau}_c \tag{31}$$

Solving for $\mathbf{H}_C'$, we find

$$\mathbf{H}_C' = \mathbf{H}_C \times [I]^{-1}\mathbf{H}_C + \hat{\mathbf{o}} \times \nabla_{\hat{\mathbf{o}}_r} V \tag{32}$$

The formation attitude motion relative to the orbit frame is of interest. Assuming a circular center of mass orbital motion, the relative angular velocity vector is expressed as

$$\boldsymbol{\omega}_{\mathcal{B}/\mathcal{O}} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} = \boldsymbol{\omega} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - n\hat{\mathbf{o}}_h \tag{33}$$

The formation angular momentum relative to the orbit frame is

$$\mathbf{H} = \mathbf{H}_c - [I]\boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} = \mathbf{H}_c - n[I]\hat{\mathbf{o}}_h \tag{34}$$

Also, note that

$$\mathbf{H} = [I]\boldsymbol{\omega} \quad \text{or} \quad \boldsymbol{\omega} = [I]^{-1}\mathbf{H} \tag{35}$$

Using equations (32) and (35), taking the formation body frame derivative of $\mathbf{H}$ yields

$$\begin{aligned} \mathbf{H}' = {} & \mathbf{H} \times [I]^{-1}\mathbf{H} + n\mathbf{H} \times \hat{\mathbf{o}}_h + n([I]\hat{\mathbf{o}}_h) \times ([I]^{-1}\mathbf{H}) + n^2([I]\hat{\mathbf{o}}_h) \times \hat{\mathbf{o}}_h \\ & - n[I]\hat{\mathbf{o}}_h' + \hat{\mathbf{o}}_r \nabla_{\hat{\mathbf{o}}_r} V \end{aligned} \tag{36}$$

The orbit normal vector derivative is written as

$$\hat{\mathbf{o}}_h' = -\boldsymbol{\omega} \times \hat{\mathbf{o}}_h = \hat{\mathbf{o}}_h \times \boldsymbol{\omega} = [\tilde{o}_h][I]^{-1}\mathbf{H} \tag{37}$$

$$\hat{\mathbf{o}}_h' = \hat{\mathbf{o}}_r \times \boldsymbol{\omega} = [\tilde{o}_r][I]^{-1}\mathbf{H} \tag{38}$$

where the tilde matrix notation $[\tilde{a}]\mathbf{b}$ being equivalent to $\mathbf{a} \times \mathbf{b}$ is used [14]. Let $\mathbf{A}$ be defined as

$$\mathbf{A} = (\mathrm{tr}([I])[I_{3\times 3}] - [I])\hat{\mathbf{o}}_h \tag{39}$$

Reference [22] proves the following useful identity

$$[\tilde{A}] = [\tilde{o}_h][I] + [I][\tilde{o}_h] \tag{40}$$

Substituting equations (37) and (40) into equation (36) yields

$$\mathbf{H}' = [\tilde{A}][I]^{-1}\mathbf{H} - n^2\hat{\mathbf{o}}_h \times ([I]\hat{\mathbf{o}}_h) + \hat{\mathbf{o}}_r \times \nabla_{\hat{\mathbf{o}}_r} V \tag{41}$$

To define the angular rate and orientation of the static Coulomb formation relative to the orbit frame, the state vector $\mathbf{x}$ is introduced as

$$\mathbf{x} = \begin{pmatrix} \mathbf{H} \\ \hat{\mathbf{o}}_h \\ \hat{\mathbf{o}}_r \end{pmatrix} \tag{42}$$

The vector $\mathbf{H} = [I]\boldsymbol{\omega}$ is a measure of the relative rotation rate, while the two unit direction vectors $\hat{\mathbf{o}}_h$ and $\hat{\mathbf{o}}_r$ uniquely determine the orientation of the formation fixed frame $\mathcal{B}$. The equations of motion of a static Coulomb formation are then written in the noncanonical form as

$$\mathbf{x}' = \begin{pmatrix} \mathbf{H}' \\ \hat{\mathbf{o}}_h' \\ \hat{\mathbf{o}}_r' \end{pmatrix} = \begin{bmatrix} \tilde{A} & \tilde{o}_h & \tilde{o}_r \\ \tilde{o}_h & 0_{3\times 3} & 0_{3\times 3} \\ \tilde{o}_r & 0_{3\times 3} & 0_{3\times 3} \end{bmatrix} \begin{pmatrix} [I]^{-1}\mathbf{H} \\ -n^2[I]\hat{\mathbf{o}}_h \\ \nabla_{\hat{\mathbf{o}}_r} V \end{pmatrix} \tag{43}$$

By defining the scalar Hamilton function $\mathcal{H}$ through [16]

$$\mathcal{H}(\mathbf{x}) = \frac{1}{2} H[I]^{-1}H - \frac{n^2}{2}\hat{\mathbf{o}}_h[I]\hat{\mathbf{o}}_n + V(\hat{\mathbf{o}}_r) \tag{44}$$

the equations of motion can be written as

$$\mathbf{x}' = \begin{bmatrix} \tilde{A} & \tilde{o}_h & \tilde{o}_r \\ \tilde{o}_h & 0_{3\times 3} & 0_{3\times 3} \\ \tilde{o}_r & 0_{3\times 3} & 0_{3\times 3} \end{bmatrix} \nabla_{\mathbf{x}} \mathcal{H} \tag{45}$$

Note that these equations of motion assume that electrostatic charges exist which will cause the formation to maintain a constant shape. This assumption is not true for all possible formation shapes. The noncanonical formulation in equation (45) allows for the formation to rotate or remain fixed relative to the orbit frame $\mathcal{O}$. Also, note that the formation gravity potential function V has not been truncated in this expression.

Necessary Equilibrium Conditions

This paper explores necessary conditions for fixed static Coulomb formations with constant charges. For the formation spacecraft to remain frozen relative to the orbit frame, it is necessary that $\mathbf{x}' = \mathbf{0}$. Studying the null space of the skew-symmetric 9×9 matrix in equation (45), three independent Casimir functions are found for this system as

$$C_1(\mathbf{x}) = \frac{1}{2}\hat{\mathbf{o}}_h \cdot \hat{\mathbf{o}}_h \quad C_2(\mathbf{x}) = \frac{1}{2}\hat{\mathbf{o}}_r \cdot \hat{\mathbf{o}}_r \quad C_3(\mathbf{x}) = \frac{1}{2}\hat{\mathbf{o}}_h \cdot \hat{\mathbf{o}}_r \tag{46}$$

Let us define the scalar function $\mathcal{F}$ through

$$\mathcal{F}(\mathbf{x}) = \mathcal{H}(\mathbf{x}) - \lambda_1 C_1(\mathbf{x}) - \lambda_2 C_2(\mathbf{x}) - \lambda_3 C_3(\mathbf{x}) \tag{47}$$

where λ_i are scalar unknown coefficients. Because the skew-symmetric matrix in equation (45) is not full rank, simply setting $\nabla_{\mathbf{x}}\mathcal{H} = \mathbf{0}$ is not sufficient. Instead, equilibrium conditions are determined by investigating

$$\nabla_{\mathbf{x}}\mathcal{F} = \mathbf{0} \tag{48}$$

The resulting three orbit frame equilibrium conditions of the static Coulomb formation are

$$[I]^{-1}\mathbf{H}^* = \mathbf{0} \tag{49}$$

$$-n^2[I]\hat{\mathbf{o}}_h^* - \lambda_1\hat{\mathbf{o}}_h^* - \lambda_3\hat{\mathbf{o}}_r^* = \mathbf{0} \tag{50}$$

$$\nabla_{\hat{\mathbf{o}}_r}V - \lambda_2\hat{\mathbf{o}}_2^* - \lambda_3\hat{\mathbf{o}}_h^* = \mathbf{0} \tag{51}$$

The condition in equation (49) is rather trivial and requires that $\boldsymbol{\omega}^* = [I]^{-1}\mathbf{H}^* = \mathbf{0}$. For an actual rigid body, the inertia matrix $[I]$ is always full rank and this condition would imply $\mathbf{H}^* = \mathbf{0}$ at equilibrium. However, in this study $[I]$ is the formation inertia matrix which can be rank-deficient if all spacecraft are in a colinear formation. Recall that the spacecraft are treated as point masses. Without loss of generality, let us assume that all craft are aligned along the first formation body axis $\hat{\mathbf{b}}_1$. In this case the equilibrium relative momentum $\mathbf{H}^*$ is expressed in $\mathcal{B}$ frame components as

$$\mathbf{H}^* = {}^{\mathcal{B}}\!\begin{pmatrix} H_1^* \\ H_2^* \\ H_3^* \end{pmatrix} = {}^{\mathcal{B}}\!\begin{bmatrix} 0 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_2 \end{bmatrix} {}^{\mathcal{B}}\!\begin{pmatrix} \omega_1^* \\ \omega_2^* \\ \omega_3^* \end{pmatrix} \tag{52}$$

Equation (49) requires that $\boldsymbol{\omega}^* = \mathbf{0}$, even though the inverse of $[I]$ does not exist in this case, the equilibrium $\mathbf{H}^*$ must still be $\mathbf{0}$. Substituting equations (25) and (49) into equation (41), the external formation torque at equilibrium must be

$$\boldsymbol{\tau}_c^* = n^2[\tilde{o}_h^*][I]\hat{\mathbf{o}}_h^* \tag{53}$$

This formation torque equilibrium condition is true for the full, untruncated formation gravitational potential function.

To determine the coefficients λ_i, we first take the vector dot product of equation (50) with $\hat{\mathbf{o}}_r^*$ and the vector dot product of equation (51) with $\hat{\mathbf{o}}_h^*$. Both steps yield an expression for λ_3 as

$$\hat{\mathbf{o}}_h^* \cdot \nabla_{\hat{\mathbf{o}}_r} V = \lambda_3 = -n^2 \hat{\mathbf{o}}_h^* \cdot [I] \hat{\mathbf{o}}_r^* \tag{54}$$

Taking the vector cross product between $\hat{\mathbf{o}}_r^*$ and equation (51), while using the torque definition in equation (25) yields

$$\boldsymbol{\tau}_c^* = -\lambda_3 \hat{\mathbf{o}}_\theta^* \tag{55}$$

The equilibrium torque condition expressions in equations (53) and (55) must both be true, namely

$$\boldsymbol{\tau}_c^* = n^2 [\tilde{o}_h^*][I] \hat{\mathbf{o}}_h^* = -\lambda_3 \hat{\mathbf{o}}_\theta^* \tag{56}$$

This equilibrium torque condition is only possible if (Case 1) $\lambda_3 \geq 0$ and $n^2 [I] \hat{\mathbf{o}}_h^* = -\lambda_3 \hat{\mathbf{o}}_r^*$, or (Case 2) $\lambda_3 = 0$ and $[\tilde{o}_h^*][I] \hat{\mathbf{o}}_h^* = \mathbf{0}$.

Let us first investigate Case 1. Here it is necessary that

$$-\lambda_3 \hat{\mathbf{o}}_r^* = n^2 [I] \hat{\mathbf{o}}_h^* \tag{57}$$

Expressing all vectors in the orbit frame $\mathcal{O}$ components, equation (57) is expressed as

$$-\lambda_3 \, {}^{\mathcal{O}}\!\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = n^2 \, {}^{\mathcal{O}}\!\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix} {}^{\mathcal{O}}\!\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{58}$$

Let $\boldsymbol{\rho}_i = x_i \hat{\mathbf{o}}_r + y_i \hat{\mathbf{o}}_\theta + z_i \hat{\mathbf{o}}_h$. The formation inertia matrix can be expressed in $\mathcal{O}$ frame components as

$${}^{\mathcal{O}}\!\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix} = \sum_{i=1}^{N} \begin{bmatrix} y_i^2 + z_i^2 & -x_i y_i & -x_i z_i \\ -x_i y_i & x_i^2 + z_i^2 & -y_i z_i \\ -x_i z_i & -y_i z_i & x_i^2 + y_i^2 \end{bmatrix} \tag{59}$$

Studying the third line in equation (58), we find that $I_{33} = 0$ must be true for this case. This I_{33} condition is only possible if $x_i = y_i = 0$ for all spacecraft. Thus, this case is only feasible if all satellites are aligned with the orbit normal vector. Thus, all off-diagonal inertia matrix elements will be zero here and $I_{11} = I_{22} = \Sigma_{i=1}^{N} z_i^2$. Studying the first line of equation (58) we then determine that

$$\lambda_3 = 0 \tag{60}$$

must be true as well. Using equation (57), $\lambda_3 = 0$ implies that $[I]\hat{\mathbf{o}}_h^* = \mathbf{0}$ for Case 1.

For Case 2 we determined that $\lambda_3 = 0$ and

$$[I] \hat{\mathbf{o}}_h^* \infty \, \hat{\mathbf{o}}_h^* \tag{61}$$

such that $[\tilde{o}_h^*][I] \hat{\mathbf{o}}_h^* = \mathbf{0}$. Note that the results of Case 1 actually satisfy these conditions as well. Thus there is no need to further distinguish between both cases. Equation (61) indicates that at equilibrium the vector $\hat{\mathbf{o}}_h$ must be an eigenvector of the formation inertia matrix $[I]$. Thus, $\hat{\mathbf{o}}_h^*$ must be aligned with a principal axis of $[I]$. Further, substituting $\lambda_3 = 0$ in the equilibrium condition in equation (51) yields

$$\nabla_{\hat{\mathbf{o}}_r} V = \lambda_2 \hat{\mathbf{o}}_r^* \tag{62}$$

Thus, the gradient of the formation gravitational potential function V must be exactly in the $\hat{\mathbf{o}}_r$ direction at equilibrium. The coefficient λ_2 can be determined through

$$\lambda_2 = \nabla_{\hat{\mathbf{o}}_r} V \cdot \hat{\mathbf{o}}_r^* \tag{63}$$

Finally, the λ_1 coefficient is determined by substituting $\lambda_3 = 0$ into equation (50) and solving for

$$\lambda_1 = -n^2 \hat{\mathbf{o}}_h^* \cdot [I] \hat{\mathbf{o}}_h^* = -n^2 I_{33} \tag{64}$$

where I_{33} is defined in equation (59). Also, using $\lambda_3 = 0$, the equilibrium formation torque is $\boldsymbol{\tau}_c^* = \mathbf{0}$. For the static Coulomb formation to remain fixed relative to the orbit frame $\mathcal{O}$, the external gravitational gradient torque acting on the formation must be zero. This result parallels equivalent results of rigid body attitude studies of circularly restricted orbital motion [15, 16].

Note that none of the above formation equilibrium results for λ_i or $\boldsymbol{\tau}_c$ required truncating the formation gravitational potential function V in equation (21). For this general case the formation attitude has to satisfy $\hat{\mathbf{o}}_h^*$ being aligned with a principal axis of the formation inertia matrix. However, the above conditions did not require that $\hat{\mathbf{o}}_r^*$ and $\hat{\mathbf{o}}_\theta^*$ be eigenvectors as well. A common second order approximation of the gravitational potential function of a body is [14]

$$V(\hat{\mathbf{o}}_r) \approx \frac{3}{2} n^2 \hat{\mathbf{o}}_r^{\mathrm{T}} [I] \hat{\mathbf{o}}_r \tag{65}$$

with the gradient being

$$\nabla_{\hat{\mathbf{o}}_r} V = 3n^2 [I] \hat{\mathbf{o}}_r^* \tag{66}$$

Substituting equation (66) into equation (62), we find that

$$3n^2 [I] \hat{\mathbf{o}}_r^* = \lambda_2 \hat{\mathbf{o}}_r^* \tag{67}$$

This can only be satisfied if $\hat{\mathbf{o}}_r^*$ is also an eigenvector, and thus a principal axis, of the formation inertia matrix. By approximating the gravitational potential function to second order, the necessary conditions for a static Coulomb formation require that the principal axes of the formation inertia matrix be aligned with the orbit frame $\mathcal{O}$ unit direction vectors $\hat{\mathbf{o}}_r$, $\hat{\mathbf{o}}_\theta$, and $\hat{\mathbf{o}}_h$.

Constrained Genetic Search Results

Parker and King searched for static Coulomb solutions in references [5] and [8] using analytical methods with simplifying symmetry assumptions. In reference [12] an evolutionary search method is outlined which will numerically search for static Coulomb formations with circular formation center of mass motions. Here a population of candidate solutions of spacecraft position and charge states is generated. The fitness J of each population member is evaluated by computing the weighted sum of all spacecraft accelerations within the rotating chief Hill frame. If these accelerations are zero, then the fitness J will be zero and a static Coulomb formation has been found. The Nth satellite position is not determined randomly in reference [12], but it is determined through the formation center of mass condition

$$\sum_{i=1}^{N} m_i \boldsymbol{\rho}_i = 0 \tag{68}$$

Given the $N - 1$ spacecraft position vectors $\boldsymbol{\rho}_i$, the Nth position is evaluated using

$$\boldsymbol{\rho}_N = -\frac{1}{m_N}\sum_{i=1}^{N-1} m_i\boldsymbol{\rho}_i \tag{69}$$

This constrained evolutionary search algorithm yielded candidate solutions which were all centered about the Hill frame origin. Also, applying this algorithm helped avoid duplicated formation solutions displaced in the along-track direction.

If the potential gravity field is approximated to second order in equation (65), then for the circularly restricted chief orbit problem any static Coulomb formation must have the principal axis of the formation inertia matrix in equation (59) be aligned with the orbit frame $\mathcal{O}$. Applying these constraints to the evolutionary search algorithm should improve the static formation search performance. Let $\boldsymbol{\rho}_i$ be the N spacecraft relative position vectors which are determined after the evolutionary matic algorithm. These N vectors will not satisfy either the center of mass or the principal axis conditions. To satisfy these constraints, appropriate position corrections $\delta\boldsymbol{\rho}_i$ must be determined. The center of mass condition requires that

$$\sum_{i=1}^{N} m_i(\boldsymbol{\rho}_i + \delta\boldsymbol{\rho}_i) = \mathbf{0} \tag{70}$$

Note that compared to the center of mass enforcement method in equation (69) where only the Nth spacecraft position is adjusted, this new method will cause *all* spacecraft positions to be adjusted to comply with the center of mass condition. To satisfy the formation principle axes condition, it is necessary that all off-diagonal formation inertia matrix components in equation (59) are zero, namely

$$\sum_{i=1}^{N} m_i(x_i + \delta x_i)(y_i + \delta y_i) = 0 \tag{71a}$$

$$\sum_{i=1}^{N} m_i(x_i + \delta x_i)(z_i + \delta z_i) = 0 \tag{71b}$$

$$\sum_{i=1}^{N} m_i(z_i + \delta z_i)(y_i + \delta y_i) = 0 \tag{71c}$$

The center of mass constraint condition in equation (70) has the position corrections $\delta\boldsymbol{\rho}_i$ appearing linearly. However, the formation principal axes conditions in equation (71) depend quadratically on the corrections $\delta\boldsymbol{\rho}_i$. A nonlinear least-squares optimization method could be applied to solve this system of $6N$ equations for the required $\delta\boldsymbol{\rho}_i$ corrections. With the evolutionary algorithm, computational efficiency for computing each new population generation is vitally important. Thus, an approximate method is investigated to satisfy the constraints in equations (70) and (71).

Assume that the evolutionary algorithm has begun to converge to a proper static Coulomb formation solution. After mating two parents to generate new children, the required spacecraft position corrections $\delta\boldsymbol{\rho}_i$ to satisfy the two constraints should be small. As the numerical search converges to a static Coulomb formation solution, the corrections will go to zero. Linearizing the formation principal axis constraint in equation (71), the $6N$ constraint equations can be written in a linear form and the desired corrections $\delta\boldsymbol{\rho}_i$ can be determined through a standard least-squares algorithm given by

$$-\sum_{i=1}^{N} m_i x_i = \sum_{i=1}^{N} m_i \delta x_i \qquad -\sum_{i=1}^{N} m_i x_i y_i \approx \sum_{i=1}^{N} m_i (y_i \delta x_i + x_i \delta y_i) \tag{72a}$$

$$-\sum_{i=1}^{N} m_i y_i = \sum_{i=1}^{N} m_i \delta y_i \qquad -\sum_{i=1}^{N} m_i x_i z_i \approx \sum_{i=1}^{N} m_i (z_i \delta x_i + x_i \delta z_i) \tag{72b}$$

$$-\sum_{i=1}^{N} m_i z_i = \sum_{i=1}^{N} m_i \delta z_i \qquad -\sum_{i=1}^{N} m_i z_i y_i \approx \sum_{i=1}^{N} m_i (z_i \delta y_i + y_i \delta z_i) \tag{72c}$$

This spacecraft position correction algorithm which enforces both the formation center of mass and principal axis constraint has been incorporated into the evolutionary algorithm described in reference [12]. The static Coulomb formation search performance is evaluated by running the algorithm several times and tracking the formation cost function J values at each evolutionary generation iteration step.

Method 1 is the unmodified algorithm outlined in reference [12]. Note that this version of the evolutionary algorithm is designed to run on a single processor only. Distributed versions of this strategy are under development.

Method 2 replaces the center of mass constraint application with a new method where equation (70) is solved using a fast least-squares algorithm. On average this change is expected to improve the convergence rate. By only correcting the last satellite position $\boldsymbol{\rho}_N$, its position can vary drastically. The associated charge (computed through the parent mating process) will then not be appropriate to cause zero acceleration on the neighboring satellites. By moving all the satellites by a common small amount $\delta\boldsymbol{\rho}_i$ (i.e. the difference between the Hill frame origin and the initial formation center of mass), the final convergence rate of the evolutionary algorithm is expected to improve.

Method 3 enforces both the formation center of mass and linearized principal axis constraint condition in equation (72). Note that it is possible to iterate locally on the principal axis constraint to satisfy the quadratic version exactly. However, the numerical test runs show that the linearized version is sufficient to provide noticeable convergence improvements. Iterating to satisfy the quadratic form could provide slight improvements to the convergence rate, but would also increase the computational time to evaluate a generation.

Figure 2 illustrates the evolutionary algorithm convergence performance with Methods 1–3. Test cases with three–nine spacecraft are evaluated. For each case with a particular number of craft, several runs were performed. The displayed performance is an average of all these runs. Typically Method 1 performs the poorest, followed by Method 2, while Method 3 is always the best performing method. For smaller number of spacecraft, the convergence improvements are about 200–300%. As the number of satellites increase, the overall convergence rates of all methods are reduced.

Several interesting behaviors are apparent in these results. Note that for the three and six craft case that Method 1 performs better than Method 2. The reason for this is still unclear, but it appears to be related to what formations are possible with certain number of satellites.

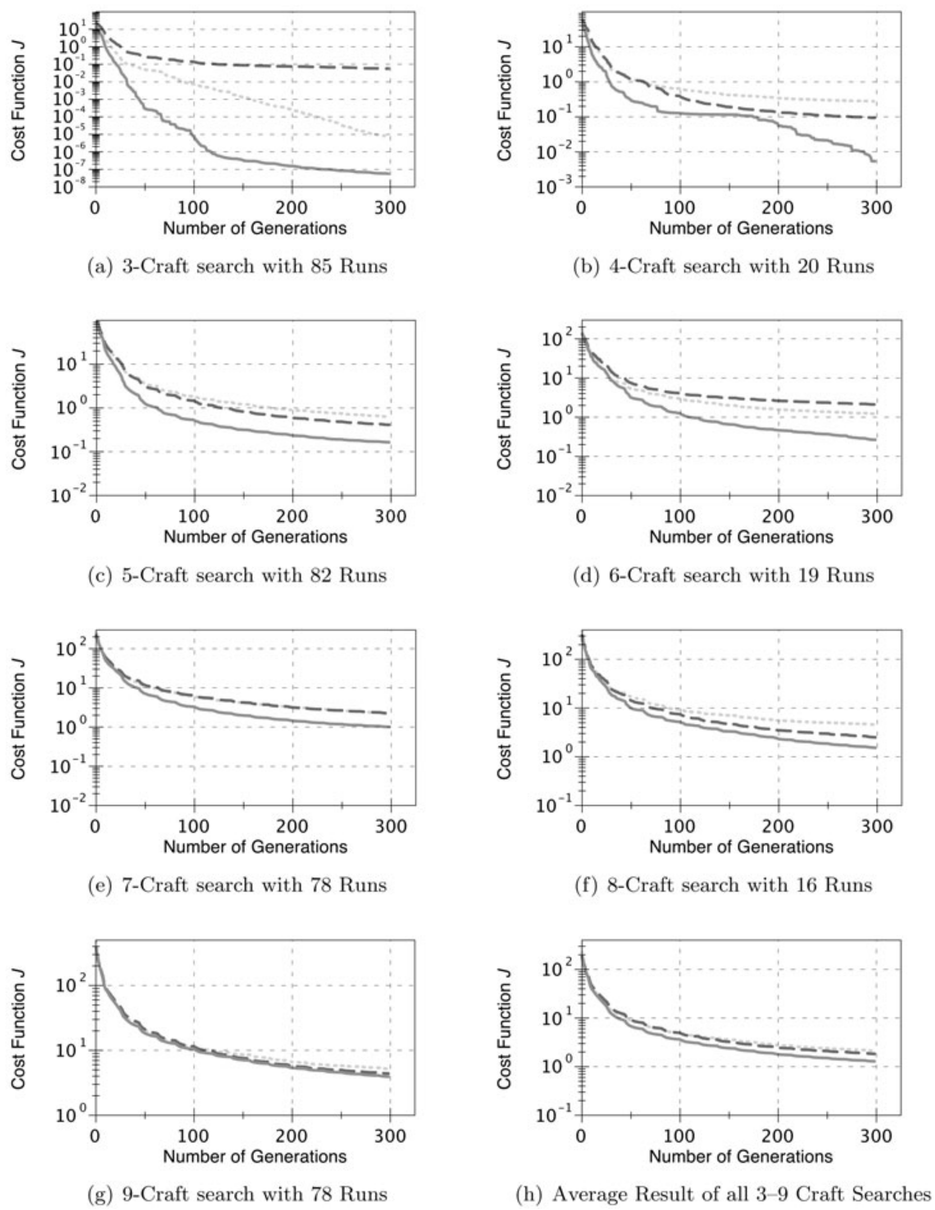

FIG. 2. Genetic Search Convergence Comparisons for Method 1 (Dotted Line • • •), Method 2 (Dashed Line (- - -), and Method 3 (Solid Line ——) for Static Coulomb Formations Sizes Ranging from Three-Nine Craft.

Conclusion

Necessary conditions to achieve static Coulomb formations with constant charges are formulated. By describing the formation as a continuous body where the Coulomb forces are internal forces maintaining a specific shape, a method is employed which is analogous to those used for finding rigid body equilibrium conditions. Specifically, a noncanonical Hamiltonian formulation is developed for the Coulomb formation. This description allows for various levels of gravitational potential function simplifications to be performed. Considering the full nonlinear gravitational potential function of the spacecraft formation, the orbit out-of-plane unit direction vector must be a principal vector of the formation inertia matrix. If the formation gravitational potential function is approximated to the common second-order form, then all three formation principal axes must be aligned with the rotating chief frame axes. Note that this is only a necessary, and not a sufficient condition for a static Coulomb formation with constant charges to exist. A numerical study illustrates how applying this constraint can improve the convergence rate of the evolutionary search algorithm by up to 200–300%.

Acknowledgment

The second author acknowledges the support of the Air Force Office of Scientific Research.

References

[1] BURNS, R., MCLAUGHLIN, C. A., LEITNER, J., et al. "TechSat 21: Formation Design, Control, and Simulation," *IEEE Aerospace Conference Proceedings,* Big Sky, Montana, Vol. 7, March 18–25, 2000, pp. 19–25.

[2] MIDDOUR, J. W. "Along Track Formationkeeping for Satellites With Low Eccentricity," *The Journal of the Astronautical Sciences,* Vol. 41, No. 1, Jan.-March 1993, pp. 19–33.

[3] TAPLEY, B.D., BETTADPUR, S., RIES, J. C., et al. "GRACE Measurements of Mass Variability in the Earth System," *Science,* Vol. 305, No. 5683, 2004, pp. 503–505.

[4] SWEETSER, T. H. "An End-to-End Trajectory Description of the LISA Mission," *Classical and Quantum Gravity,* Vol. 22, No. 10, 2005, pp. 429–435.

[5] KING, L. B., PARKER, G. G., DESHMUKH, S., et al. "Spacecraft Formation-Flying using Inter-Vehicle Coulomb Forces," technical report, NASA/NIAC, January 2002, http://www.niac.usra.edu.

[6] MULLEN, E. G., GUSSENHOVEN, M. S., and HARDY, D. A. "SCATHA Survey of High-Voltage Spacecraft Charging in Sunlight," *Journal of the Geophysical Sciences,* Vol. 91, 1986, pp. 1074–1090.

[7] TORKAR, K., et. al. "Active Spacecraft Potential Control for Cluster—Implementation and First Results," *Annales Geophysicae,* Vol. 19, 2001, pp. 1289–1302.

[8] KING, L.B., PARKER, G.G., DESHMUKH, S., and CHONG, J. "Study of Interspacecraft Coulomb Forces and Implications for Formation Flying," *AIAA Journal of Propulsion and Power,* Vol. 19, No. 3, May–June 2003, pp. 497–505.

[9] CHONG, J.-H., *Dynamic Behavior of Spacecraft Formation Flying using Coulomb Forces,* Masters thesis, Michigan Technological University, May 2002.

[10] NATARAJAN, A. and SCHAUB, H. "Linear Dynamics and Stability Analysis of a Coulomb Tether Formation," *Journal of Guidance, Control, and Dynamics,* Vol. 29, No. 4, July–Aug. 2006, pp. 831–839.

[11] SCHAUB, H., PARKER, G. G., and KING, L. B. "Challenges and Prospect of Coulomb Formations," *The Journal of the Astronautical Sciences,* Vol. 52, No. 1-2, Jan.-June 2004, pp. 169–193.

[12] BERRYMAN, J. and SCHAUB, H. "Static Equilibrium Configurations in GEO Coulomb Spacecraft Formations," presented as paper AAS 05-104 at the AAS Space Flight Mechanics Meeting, Copper Mountain, CO, Jan. 23–27, 2005.

[13] JOE, H., SCHAUB, H., and PARKER, G. G. "Formation Dynamics of Coulomb Satellites," presented at the 6th International Conference on Dynamics and Control of Systems and Structures in Space, Riomaggiore, Cinque Terre, Italy, July 2004.

[14] SCHAUB, H. and JUNKINS, J. L. *Analytical Mechanics of Space Systems,* AIAA Education Series, Reston, VA, October 2003.

[15] WANG, L.-S., MADDOCKS, J. H., and KRISHNAPRASAD, P. S. "Steady Rigid-Body Motions in a Central Gravitational Field," *The Journal of the Astronautical Sciences,* Vol. 40, No. 4, 1992, pp. 449–478.

[16] BECK, J. A. and HALL, C. D. "Relative Equilibria of a Rigid Satellite in a Circular Keplerian Orbit," *The Journal of the Astronautical Sciences,* Vol. 46, No. 3, 1998, pp. 215–247.

[17] NICHOLSON, D. R. *Introduction to Plasma Theory,* Krieger, 1992.

[18] GOMBOSI, T. I. *Physics of the Space Environment,* Cambridge University Press, 1998.

[19] CLOHESSY, W. H. and WILTSHIRE, R. S. "Terminal Guidance System for Satellite Rendezvous," *Journal of the Aerospace Sciences,* Vol. 27, No. 9, Sept. 1960, pp. 653–658.

[20] HILL, G. W. "Researches in the Lunar Theory," *American Journal of Mathematics,* Vol. 1, No. 1, 1878, pp. 5–26.

[21] BATTIN, R. H. *An Introduction to the Mathematics and Methods of Astrodynamics,* AIAA Education Series, New York, 1987.

[22] BECK, J. A. *Relative Equilibria of a Rigid Satellite in a Central Gravitational Field* , Ph.D. dissertation, Air Force Institute of Technology, Wright-Patterson AFB, OH, Sept. 1997, AFIT/DS/ENY/97-6.