Predefined-Time Nonlinear Sliding Mode Attitude Tracking of Spacecraft

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***Abstract*—In this report, we consider the attitude tracking control problem for rigid spacecraft with bounded external disturbances. We implement and analyze a predefined-time predefined-bounded attitude tracking control scheme based on sliding mode control Two mathematical formulations are discussed which can be used to derive control law and their proof of stability is given. The attitude tracking errors are driven to a predefined-bounded region around the origin within a predefined time, which can be set as a tuning parameter during the controller design, independently of initial conditions. Finally, numerical simulations are carried out to evaluate the performance of both the control law and compare them.**

***Index Terms*—predefined stability, sliding mode control, attitude tracking, spacecraft**

I. Introduction

Attitude control of rigid spacecraft has been studied by many researchers in the literature, see, for example, the works in [1]–[3]. In particular, finite-time attitude control has attracted a significant amount of attention from researchers in virtue of its ability to provide better disturbance rejection property as well as higher precision attitude control performance. Advancements in mission complexity require more stringent constraints to be satisfied. This requires fast convergence characteristics that should be tunable according to mission requirements. Although the concept of fixed-time stability represents a significant advantage over the concept of finite-time stability because of the boundedness of the settling time, it cannot be guaranteed in general that the convergence time can be arbitrarily selected through the system tunable parameters. To overcome this mentioned drawback, it is necessary to consider another class of dynamical systems that exhibit the property of predefined-time stability. For these systems, an upper bound of the settling-time function can be arbitrarily chosen through an appropriate selection of the system parameters. Predefined time stability is a form of finite time stability which guarantees an upper bound on convergence time that can be predefined beforehand. This report uses Lyapunov like methods to design predefined time controllers using sliding mode control. Sliding mode control (SMC) is a non- linear control method that alters the dynamics of a nonlinear system by applying a discontinuous control signal (or more rigorously, a set-valued control signal) that forces the system to ”slide” along a cross-section of the system’s normal behavior. Two mathematical formulations of this control are discussed and compared. Simulation are carried out in MATLAB to demonstrate the results.

II. System Dynamics

We will use modified Rodriguez parameter (MRP) to model spacecraft dynamics.

$$\dot{q} = T(q)\omega = \frac{1}{2}\left(\frac{(1-q^Tq)}{2}I_3 + \tilde{q} + qq^T\right)\omega \tag{1}$$

$$J\dot{\omega} = -\tilde{\omega}J\omega + \tau + d \tag{2}$$

where where $\omega \in \mathbb{R}^3$ is the angular velocity of the spacecraft in a body-fixed frame, $\tilde{} \in \mathbb{R}^{3\times3}$ denoted skew-symmetric matrix associated with corresponding vector, $\tau \in \mathbb{R}^3$ is the control torque, $J \in \mathbb{R}^{3\times3}$ is the inertia matrix, $d \in \mathbb{R}^3$ is the bounded external disturbance, $q \in \mathbb{R}^3$ denotes the MRPs representing the spacecraft attitude with respect to an inertial frame. For reference, MRP are related to Euler angle representation as

$$q(t) = \eta(t)\tan\theta(t)) \tag{3}$$

where $\theta \in [0, 2\pi)$ with η and θ being the Euler eigenaxis and eigenangle, respectively.
Suppose the desired attitude and angular velocity are denoted by q_d and ω_d respectively. Then tracking error are calculated are:

$$q_e = q \otimes q_d = \frac{q_d(q^Tq-1) + q(1-q_d^Tq_d - 2\tilde{q_d}q)}{1+q_d^Tq_d+q^Tq+2q_d^Tq} \tag{4}$$

$$\omega_e = \omega - C(q_e)\omega_d = I_3 + \frac{8(\tilde{q_e})^2 - 4(1-q_e^Tq_e)\tilde{q_e}}{(1+q_e^Tq_e)^2} \tag{5}$$

Finally the error dynamics of a spacecraft (used for tracking) can be written as

$$\dot{q}_e = T(q_e)\omega_e \tag{6}$$

$$\begin{aligned} J\dot{\omega}_e &= -\tilde{\omega}J\omega - JC(q_e)\dot{\omega}_d + J\tilde{\omega}_eC(q_e)\omega_d + \tau + d \\ &= f(q_e, \omega, \omega_d, \dot{\omega}_d) + \tau + d \end{aligned} \tag{7}$$

We suppose the following assumptions holds:

Assumption 1. *The desired angular velocity ω_d and its derivative $\dot{\omega}_d$ are supposed to be uniformly bounded.*

Assumption 2. *There exists a positive constant d_M such that the disturbance d satisfies $||d||_\infty \leq d_M$, where $||\cdot||_\infty$ denotes the infinity norm of a vector.*

III. Predefined-time Sliding mode control

Consider a general system:

$$\dot{x} = f(x, t; \rho) \tag{8}$$

Definition 1 (Finite Time Stability)**.** The origin of system is finite time stable if it is globally asymptotic stable and for any arbitrary initial condition x_0, there exist a finite time $0 \leq \tau < \infty$ such that $x(t, x_0) = 0, \forall t \geq \tau$.

Definition 2 (Settling Time Set)**.** Settling time set for a system is defined as

$$T = \{T(x_0) : \inf\{\tau \geq 0 : x(t, x_0) = 0 \ , \forall t \geq \tau\} \ , \forall x_0 \in \mathbb{R}^n\} \tag{9}$$

Definition 3 (Fixed Time Stability)**.** The origin of system is fixed time stable if it is finite time stable and the settling time set is bounded: $\exists\ 0 \leq T_{\max} < \infty$ such that $T(x_0) \leq T_{\max}\ \forall x_0 \in \mathbb{R}^n$. Define the tightest bound on $T(x_0)$ as T_s, i.e, sup $T(x_0) = T_s$

Definition 4 (Predefined Time Stability)**.** The origin of system is predefined time stable if it is fixed time stable and the settling set bound $T_{\max}$ is tunable, i.e, it is a function of system parameter ρ.

A. Sliding mode controller

In control systems, sliding mode control (SMC) is a nonlinear control method that alters the dynamics of a nonlinear system by applying a discontinuous control signal (or more rigorously, a set-valued control signal) that forces the system to ”slide” along a cross-section of the system's normal behavior. The state-feedback control law is not a continuous function of time. Instead, it can switch from one continuous structure to another based on the current position in the state space. Hence, sliding mode control is a variable structure control method. The multiple control structures are designed so that trajectories always move toward an adjacent region with a different control structure, and so the ultimate trajectory will not exist entirely within one control structure. Instead, it will slide along the boundaries of the control structures. The motion of the system as it slides along these boundaries is called a sliding mode and the geometrical locus consisting of the boundaries is called the sliding (hyper)surface. In the context of modern control theory, any variable structure system, like a system under SMC, may be viewed as a special case of a hybrid dynamical system as the system both flows through a continuous state space but also moves through different discrete control modes.

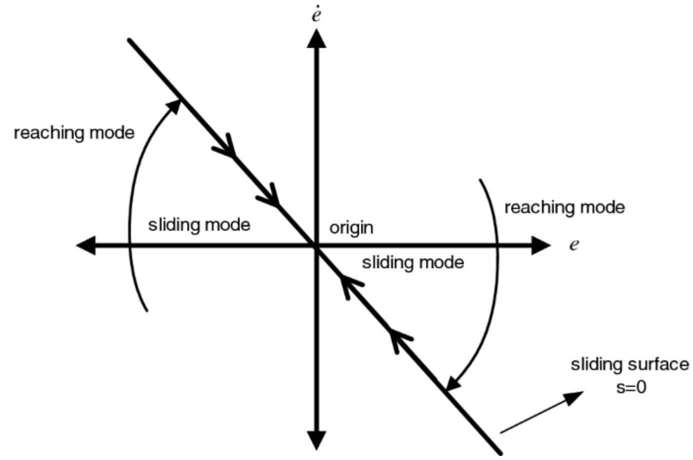

Fig. 1. Sliding mode control

IV. Control 1

Lemma 1. For the system defined by Eq. (8), if a radially unbounded Lyapunov function $V(x, t)$ exists such that

$$\dot{V}(x, t) \leq -\frac{\pi}{\alpha T_p}(V^{1-\frac{\alpha}{2}} + V^{1+\frac{\alpha}{2}}) \tag{10}$$

with $T_p > 0$ and $\alpha \in (0, 1)$ then the system is globally predefined time stable with predefined time T_p.

Proof. The inequality can be rearranged as

$$-\frac{\pi}{\alpha T_p}dt \geq \frac{dV}{(V^{1-\frac{\alpha}{2}} + V^{1+\frac{\alpha}{2}})} \tag{11}$$

Integrating by setting $V(x, t) = 0$ for $t = T(x_0)$ and $V(x, t) = V(x_0)$ for $t = t_0$,

$$-\frac{\pi}{\alpha T_p}T(x_0) \geq \int_{V(x_0)}^{0} \frac{dV}{(V^{1-\frac{\alpha}{2}} + V^{1+\frac{\alpha}{2}})} \tag{12}$$

$$\geq \frac{2}{\alpha}\arctan(V(x_0)^{\frac{\alpha}{2}}) \tag{13}$$

Hence,

$$T(x_0) \leq T_p\left[\frac{2}{\pi}\arctan(V(x_0)^{\frac{\alpha}{2}})\right] \tag{14}$$

Since $T(x_0)$ is upper bounded by T_p, the system is global predefined time stable with predefined time T_p. □

Define the following terms:

$$V_1 = \frac{q_e^T q_e}{2} \tag{15}$$

$$\Omega = \frac{2\pi}{\alpha(1 + q_e^T q_e)T_{p1}}(V_1^{-\frac{\alpha}{2}} + V_1^{\frac{\alpha}{2}})q_e \tag{16}$$

Next for sliding mode control, we define a sliding manifold as

$$s = \omega_e + \Omega \tag{17}$$

Theorem 1. *Consider the spacecraft system described by (3) and (4) and suppose that Assumptions 1 and 2 are satisfied. Then the control torque*

$$\tau = -J\dot{\Omega} - f - \frac{\pi}{2\alpha T_{p2}}(V_2^{-\frac{\alpha}{2}} + V_2^{\frac{\alpha}{2}})Js - \psi \tag{18}$$

where $V_2 = \frac{s^T J s}{2}$ and $\psi = ksgn(s)$ with k as a positive constant makes the sliding-mode manifold s converges to zero *within predefined time T_{p2} and the attitude tracking error q_e converges to zero within predefined time $T = T_{p1} + T_{p2}$, respectively.*

Proof. The dynamics of sliding manifold can be written as

$$J\dot{s} = J(\dot{\omega}_e + \Omega) \tag{19}$$

$$= J\dot{\Omega} + f + \tau + d \tag{20}$$

Applying the control law,

$$J\dot{s} = -\frac{\pi}{2\alpha T_{P2}}(V_2^{-\frac{\alpha}{2}} + V_2^{\frac{\alpha}{2}})Js - \psi + d \tag{21}$$

Now taking time derivative of V_2,

$$\dot{V}_2 = s^T J\dot{s} \tag{22}$$

$$= -\frac{\pi}{2\alpha T_{P2}}(V_2^{-\frac{\alpha}{2}} + V_2^{\frac{\alpha}{2}})s^T Js - s^T\psi + s^T d \tag{23}$$

$$= -\frac{\pi}{\alpha T_{P2}}(V_2^{1-\frac{\alpha}{2}} + V_2^{1+\frac{\alpha}{2}}) - s^T\psi + s^T d \tag{24}$$

$$\leq -\frac{\pi}{\alpha T_{P2}}(V_2^{1-\frac{\alpha}{2}} + V_2^{1+\frac{\alpha}{2}}) \tag{25}$$

Hence using Lemma 1, s will converge to zero within predefined time T_{p2}.
Now, taking time derivative of V_1,

$$\dot{V}_1 = q_e^T\dot{q}_e \tag{26}$$

$$= q_e^T T(q_e)\omega_e \tag{27}$$

$$= \frac{q_e^T}{2}\left(\frac{(1 - q^T q)}{2}I_3 + \tilde{q} + qq^T\right)\omega_e \tag{28}$$

$$= \frac{(1 - q^T q)}{4}q_e^T\omega_e + \frac{q_e^T q_e}{2}q_e^T\omega_e \tag{29}$$

$$= \frac{(1 + q^T q)}{4}q_e^T\omega_e \tag{30}$$

After $s = 0$ is reached,

$$\omega_e + \Omega = 0 \tag{31}$$

$$\omega_e = -\frac{2\pi}{\alpha(1 + q_e^T q_e)T_{p1}}(V_1^{-\frac{\alpha}{2}} + V_1^{\frac{\alpha}{2}})q_e \tag{32}$$

Hence $\dot{V}_1$ becomes,

$$\dot{V}_1 = -\frac{\pi}{2\alpha T_{p1}}(V_1^{-\frac{\alpha}{2}} + V_1^{\frac{\alpha}{2}})q_e^T q_e \tag{33}$$

$$= -\frac{\pi}{2\alpha T_{p1}}(V_1^{1-\frac{\alpha}{2}} + V_1^{1+\frac{\alpha}{2}}) \tag{34}$$

Thus again from Lemma 1, q_e will converge to zero within predefined time T_{p1}.
Hence, sliding-mode manifold s converges to zero within predefined time T_{p2} and the attitude tracking error q_e converges to zero within predefined time $T = T_{p1} + T_{p2}$, respectively. □

V. CONTROL 2

Lemma 2. Let $V : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ be a continuous, positive definite and radially unbounded function with $V(x) = 0$ iff $x = 0$. If there exists a $T_c \in \mathbb{R}_+$ such that along the system trajectories

$$\dot{V} = -\frac{1}{pT_c}e^{V^p}V^{1-p} \tag{35}$$

$\forall x \neq 0$ and $0 < p \leq 1$, then the system is predefined time stable with $T_c = T_{\max}$. For equality, $T_c = T_s$

Proof. Integrating Eq. (35), the solution is

$$V(t) = \left[\ln\left(\frac{1}{\frac{t-t_0}{T_c} + e^{-V(x_0)^p}}\right)\right]^{\frac{1}{p}} \tag{36}$$

Now for $V(t) = 0$, $\frac{t-t_0}{T_c} + e^{-V(x_0)^p} = 1$, hence the settling time function for the system satisfies,

$$T(x_0) = T_c[1 - e^{-V(x_0)^p}] \tag{37}$$

Since $0 < e^{-V(x_0)^p} \leq 1$, T_c is an upper bound for the settling-time function and hence system is predefined time stable with predefined time T_c. □

Theorem 2. *For any initial condition $x_0 \in \mathbb{R}^n$, the system*

$$\dot{x} = -\frac{1}{T_c}\Phi_{m,q}(x) \tag{38}$$

is global predefined time stable with $T_c = T_s$ where,

$$\Phi_{m,q}(x) = \frac{1}{mq}e^{||x||^{mq}}\frac{x}{||x||^{mq}} \tag{39}$$

with $m \geq 1$ and $0 < q \leq \frac{1}{m}$.

Proof. Take $V(x) = ||x||^m$ where $x \in \mathbb{R}^n$. Taking derivative along system trajectories,

$$\dot{V}(x) = m||x||^{m-2}x^T\dot{x} \tag{40}$$

$$= -\frac{1}{qT_c}e^{||x||^{mq}}\frac{x^T x}{||x||^{m(1-q)}} \tag{41}$$

$$= -\frac{1}{qT_c}e^{V^q}V^{1-q} \tag{42}$$

Hence using Lemma 3, we conclude that the system is global predefined time stable with settling time $T_s = T_c$. □

Remark 1. It should be noted that convergence characteristic of the system is affected by choice of the product mq. It is essential for a dynamic system to have a smooth convergence and produce a least steep initial response so that abrupt changes in dynamical properties can be avoided. [Refer] showed that for smooth convergence $mq < \frac{1}{2}$. Fig (1) shows the optimal product for a range of initial conditions.

Next consider a general system affine in control:

$$\dot{x} = f(x) + B(x)u + \delta(x, t) \tag{43}$$

Let the sliding manifold chosen for the system be $s(x, t)$. Then $s(x, t) = 0$ is the desired behavior of the system. Define

$$G(x, t) = \frac{\partial s}{\partial x} \tag{44}$$

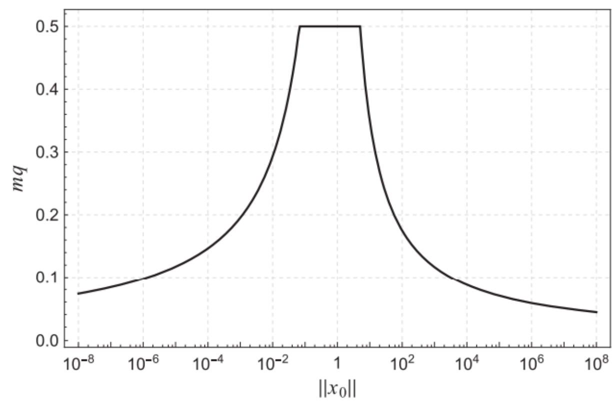

Fig. 2. Suggested value for mq as a function of the initial condition.

Defining $D(x,t) = G(x,t)B(x)$, then time derivative of $s(x,t)$ is

$$\dot{s} = \frac{\partial s}{\partial x}\dot{x} + \frac{\partial s}{\partial t} \tag{45}$$

$$= G(x,t)(f(x) + B(x)u + \delta(x,t)) + \frac{\partial s}{\partial t} \tag{46}$$

Lemma 3. Let a system be defined as

$$\dot{x} = u + \Delta(x,t) \tag{47}$$

such that non-vanishing bounded disturbance satisfies $||\Delta(x,t)|| \leq \gamma$ and $0 < \gamma < \infty$. Then selection of control input

$$u = -\gamma\frac{x}{||x||} - \frac{1}{T_c}\Phi_{m,q}(x) \tag{48}$$

leads to a closed loop robust globally predefined stable system with predefined time T_c.

Proof. Consider Lyapunov function $V(x) = ||x||^m$ where $x \in \mathbb{R}^n$. Taking derivative along system trajectories,

$$\dot{V}(x) = m||x||^{m-2}x^T\dot{x} \tag{49}$$

$$= m||x||^{m-2}x^T\left[\Delta(x,t) - \gamma\frac{x}{||x||} - \frac{1}{T_c}\Phi_{m,q}(x)\right] \tag{50}$$

$$= -\frac{1}{qT_c}e^{||x||^{mq}}||x||^{m(1-q)} + m||x||^{m-2}(x^T\Delta(x,t) - \gamma||x||) \tag{51}$$

Since $x^T\Delta(x,t) \leq ||x||||\Delta(x,t)|| \leq \gamma||x||$, so the term $m||x||^{m-2}(x^T\Delta(x,t) - \gamma||x||)$ is non-positive with

$$\textbf{sup}\ \ m||x||^{m-2}(x^T\Delta(x,t) - \gamma||x||) = 0 \tag{52}$$

Hence,

$$\textbf{sup}\ \ \dot{V} = -\frac{1}{qT_c}e^{||x||^{mq}}||x||^{m(1-q)} = -\frac{1}{qT_c}e^{V^q}V^{1-q} \tag{53}$$

From Lemma 3, the system is predefined time stable with predefined time T_c. $\square$

This Lemma can be used to find robust controller for sliding manifold convergence.

Theorem 3. *Let system be defined by Eq. (43) with the function $\delta(x,t)$ be considered as a matched and vanishing perturbation term. Hence, there exists a function $\hat{\delta}(x,t)$ such that $\delta(x,t) = B(x,t)\hat{\delta}(x,t)$ and $\|D(x)\hat{\delta}(x,t)\| \leq \gamma$, where $0 < \gamma < \infty$ is a known constant. Then, the control input*

$$u = -D^{-1}(x,t)\left[G(x,t)f(x) + \gamma\frac{s}{\|s\|} + \frac{1}{T_c}\Phi_{m,q}(s) + \frac{\partial s}{\partial t}\right] \tag{54}$$

induces a strong predefined-time SM in $s(x,t) = 0$ with T_c as the least upper bound for the settling time.

Proof. The dynamics of $s(x,t)$ is given by,

$$\dot{s} = G(x,t)f(x) + D(x)(u + \hat{\delta}(x,t)) + \frac{\partial s}{\partial t} \tag{55}$$

Substituting the value of u in above,

$$\dot{s} = -\gamma\frac{s}{\|s\|} - \frac{1}{T_c}\Phi_{m,q}(s) + D(x)\hat{\delta}(x,t) \tag{56}$$

Hence from Lemma 5, this control induces strong predefined-time SM in $s(x,t) = 0$ with T_c as the least upper bound for the settling time. $\square$

Next we use the above lemma to derive a control for spacecraft attitude control.

Theorem 4. *Consider the spacecraft dynamics given in Eq. (6-7). The control law :*

$$\tau = -D^{-1}(x,t)\left[G(x,t)f(x) + \gamma\frac{s}{\|s\|} + \frac{1}{T_{c1}}\Phi_{m,q}(s) + \frac{\partial s}{\partial t}\right] \tag{57}$$

where

$$s(q_e,t) = T(q_e)\omega_e + \frac{1}{T_{c2}}\Phi_{m,q}(q_e) \tag{58}$$

attitude tracking error q_e converges to zero within predefined time $T = T_{c1} + T_{c2}$, respectively.

Proof. Using Theorem 6, the control law will make the the sliding manifold s to converge to zero in predefined time T_{c1}. Now after reaching the manifold, the dynamics followed is

$$s = 0 \tag{59}$$

$$T(q_e)\omega_e + \frac{1}{T_{c2}}\Phi_{m,q}(q_e) = 0 \tag{60}$$

$$\dot{q}_e = -\frac{1}{T_{c2}}\Phi_{m,q}(q_e) \tag{61}$$

Now from Theorem 4, this converges in predefined time T_{c2}. Hence the attitude tracking error q_e converges to zero within predefined time $T = T_{c1} + T_{c2}$, respectively. $\square$

VI. Simulation Results

In this section, numerical simulations are examined to illustrate the performance of the designed attitude control law. The inertia matrix used is

$$J = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix} kg - m^2 \tag{62}$$

The reference attitude is $q_d = 0.2[\cos(0.1t), \sin(0.1t), \sqrt{3}]^T$. The external disturbance d is assumed to be $d = 10(\sin(t/2), \cos(t/3), \sin(2t/5))^T m - N - m$. The initial conditions are set as $q(0) = [1, -0.5, -1.5]^T$ and $\omega(0) = 0$ rad/s. The reference attitude, the external disturbance, and the controller parameters are considered the same in all simulations.

A. Control 1

The controller parameters are chosen as $\alpha = 0.3$, $T_{p1} = 200$, and $T_{p2} = 200$. Fig (3-5) represents the time variation of attitude error, angular velocity error and applied control torque respectively. We can observe that the attitude and angular velocity converges within predefined time $T_{p1} + T_{p2}$, thus verifying Theorem 2.

B. Control 2

The controller parameters are chosen as $\alpha = 0.3$, $T_{c1} = 200$, and $T_{c2} = 200$. Fig (6-8) represents the time variation of attitude error, angular velocity error and applied control torque respectively. We can observe that the attitude and angular velocity converges within predefined time $T_{c1} + T_{c2}$, thus verifying Theorem 7.

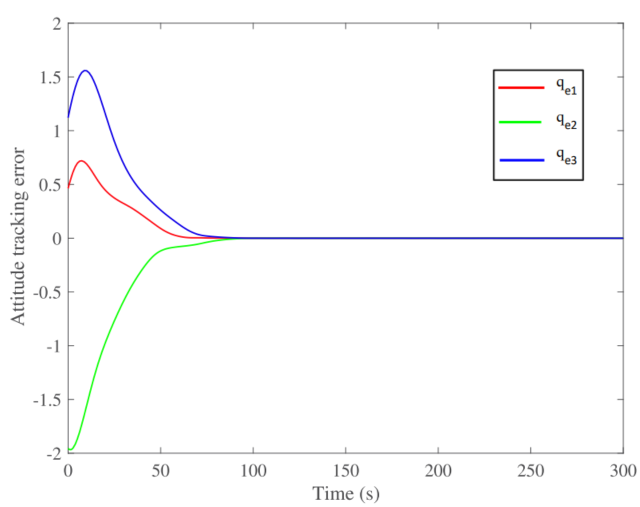

Fig. 3. Attitude tracking error for Control 1.

VII. Comparison of Controllers

Both controllers: Control 1 and Control 2 are predefined time sliding mode control but uses different mathematical formulations to handle control law design which affects the overall performance and efficiency of the closed loop system.

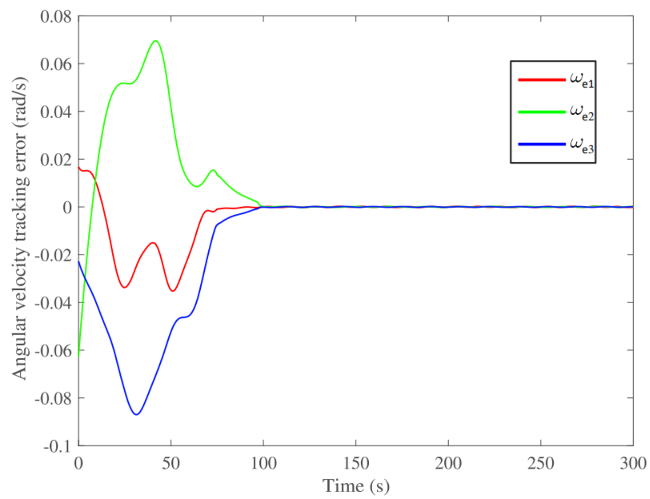

Fig. 4. Angular velocity tracking error for Control 1.

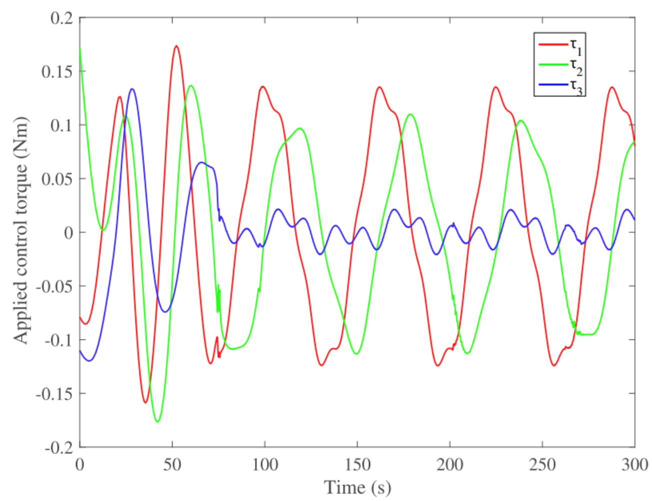

Fig. 5. Applied control torque for Control 1.

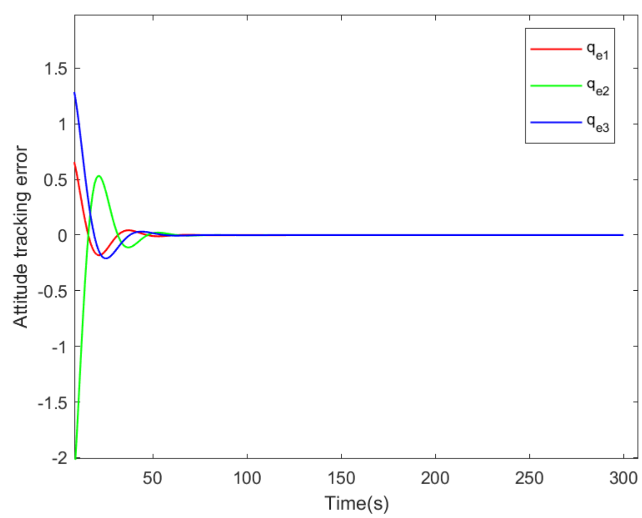

Fig. 6. Attitude tracking error for Control 2.

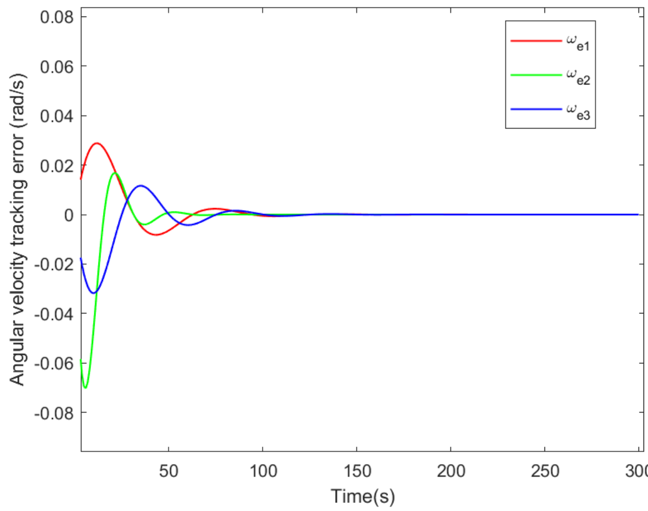

Fig. 7. Angular velocity tracking error for Control 2.

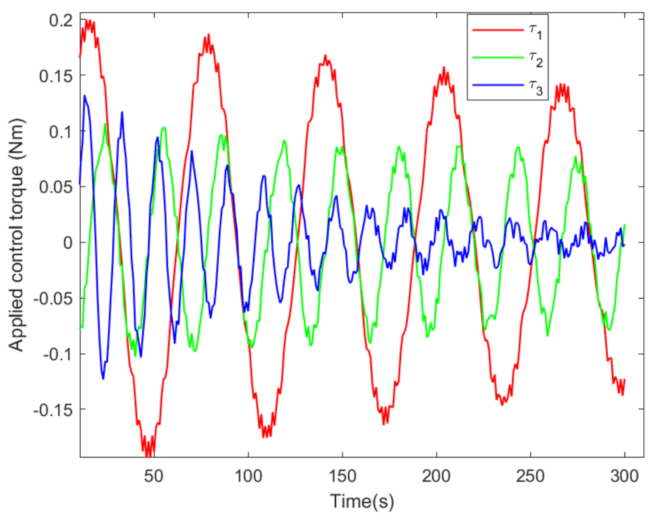

Fig. 8. Applied control torque for Control 2.

From Figs(3-8), we can observe that Control 2 performs better in terms of convergence of attitude and angular velocity error. In spite of both being predefined time stable with fixed predefined time, the Control 2 performs better in terms of convergence time. Also in terms of torque effort, Control 2 performs better as less fuel usage is there. This can be due to the fact that the value of mq greatly affects the performance of Control 2. By choosing the value dictated in Fig (2), performance can be increased to a great extent. But in Control 2, existence of inverse of $D(x,t)$ is essential for design, which is not true in all of the general system. Hence Control 2, although better can only be applied to restricted systems.

VIII. Conclusion

This paper presents attitude tracking of spacecraft described from MRP(modified Rodriguez parameter) using predefined time sliding mode control. This gurantees a predefined time convergence that can be tuned. Two control formulations are discussed and compared. Simulation are carried out in MATLAB to demonstrate the results.

References

[1] B.-S. Chen, C.-S. Wu, and Y.-W. Jan Adaptive fuzzy mixed H2/H attitude control of spacecraft IEEE Trans. Aerosp. Electron. Syst., vol. 36, no. 4, pp. 1343–1359, Oct. 2000.

[2] H. Wong, M. S. de Queiroz, and V. Kapila Adaptive tracking control using synthesized velocity from attitude measurements Automatica, vol. 37, no. 6, pp. 947–953, Jun. 2001.

[3] A. Tayebi Unit quaternion-based output feedback for the attitude tracking problem IEEE Trans. Autom. Control, vol. 53, no. 6, pp. 1516–1520, Jul. 2008.

[4] D. Ye, A. -M. Zou and Z. Sun, ”Predefined-Time Predefined-Bounded Attitude Tracking Control for Rigid Spacecraft,” in IEEE Transactions on Aerospace and Electronic Systems, vol. 58, no. 1, pp. 464-472, Feb. 2022, doi: 10.1109/TAES.2021.3103258.

[5] E. Jiménez-Rodríguez, A. J. Muñoz-Vázquez, J. D. Sánchez-Torres, M. Defoort and A. G. Loukianov, ”A Lyapunov-Like Characterization of Predefined-Time Stability,” in IEEE Transactions on Automatic Control, vol. 65, no. 11, pp. 4922-4927, Nov. 2020, doi: 10.1109/TAC.2020.2967555.

[6] Juan Diego Sánchez-Torres, David Gómez-Gutiérrez, Esteban López, Alexander G Loukianov, A class of predefined-time stable dynamical systems, IMA Journal of Mathematical Control and Information, Volume 35, Issue Supplement1, April 2018, Pages i1–i29