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PROJECT REPORT

Optimal Control for High Precision Rendezvous and Docking of Coulomb Satellites

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Contents

I. Acknowledgement

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II. Abstract

As the complexity of space missions are progressing, the need for efficient backup system for space architecture like servicing, refuelling and their optimal maintenance increases to extend the lifetime and reduce cost. Therefore developing a robust and efficient docking system: joining one spacecraft or space station module to another is necessary.

Here, in this project we develop dynamics and control for on-track docking for Coulomb actuated satellites. First we derive the system dynamics by considering the chaser and the target as a rigid system perturbed from the equilibrium axis which can be quantified by 3-2-1 Euler angle sequence. The rotations of chaser is also taken into account with respect to the target as docking requires very precise attitude configurations. Then charge control is transformed into voltage control, which is more practical in physical systems using capacitance matrix of the system. This gives us the system dynamics governing the length and the attitude configurations in the form of coupled differential equations.

Next optimal control technique, Linear Quadratic Tracking(LQT) is applied for control of the system state based on the dynamics of the derived system. Reference trajectories are derived from HCW equations for the system to track. The system will then be simulated on MATLAB to confirm the efficiency of the derived control on various mission scenarios for stability and robustness analysis of the system.

III. Introduction

Using inter-vehicle electrostatic Coulomb forces for satellite formation flying is a relatively new and emerging concept. Pioneering work in developing this Coulomb formation flying concept is presented in References [7,8,17]. Coulomb formation flying works on the principle that by controlling the charge of the spacecraft the inter-craft Coulomb forces can be changed, which in turn can be used to control the relative motion of the spacecraft. With high Isp fuel efficiencies [7,8] ranging between 108 1013 seconds and low Watt-level power requirements, this method of propulsion is considered to be virtually propellant less. The other advantage of this method over conventional thrusters includes clean propulsion without thruster plume contamination issues with neighbouring satellites. However, the Coulomb propulsion method also has certain inherent limitations. The Coulomb forces are formation internal forces that can not be used to reorient the satellite formation as such. Secondly, The Coulomb electrostatic force magnitude is inversely proportional to the square of the separation distance, resulting in the increase of the nonlinear coupling of spacecraft equations of motion. Additionally, the Coulomb force effectiveness is diminished in a space plasma environment due to the presence of charged plasma particles. The electric field strength drops off exponentially with increasing separation distance. The severity of this drop is characterized using the Debye length[13,4]. For low earth orbits (LEO), the Debye length is of the order of millimeters to centimeters, making the Coulomb formation flying concept impractical at these low orbit altitudes.[15] At high to geostationary orbit (GEO) altitudes the plasma environment is hotter and less dense. As a result the Debye length is much larger and varies between 100-1000 meters depending on the solar activity cycles. Further, the electrostatic charging data of the SCATHA spacecraft[10] confirms that spacecraft can charge at least to kilovolt levels in GEO environments, and that the spacecraft charge can be actively controlled through charge emission devices. Thus, Coulomb formation flying concept appears to be feasible at GEO. The currently flying CLUSTER spacecraft also use active charge control.[18] However, the charge emission is applied to zero out the spacecraft potential and not to control relative motion. References [11] and [12] introduce the concept of a Coulomb tether. Here a conventional mechanical tether cable connecting two crafts is replaced by an electrostatic force which acts as a virtual tether. Conventional tethers are limited to tensile forces whereas Coulomb tethers allow both tensile and compressive forces. However, while traditional spacecraft tether missions consider very large separation distances of multiple kilometers, the Coulomb tether concept is only viable for separation distances up to about 100 meters because of the electrical field strength drop off. Reference [11] studies the stabilization of the simple nadir-aligned static 2-craft Coulomb tether structure. Compared to the previous works on static Coulomb structures,[8,1,2,17] Reference [11] is the first study to introduce a charge feedback law to stabilize a charged spacecraft cluster to a specific shape and orientation. Coulomb forces are inter-spacecraft forces and cannot control the inertial angular momentum of the formation. Hence, stability characteristics of orbital rigid body motion under a differential gravity field are applied to a Coulomb tethered two-spacecraft system to develop an active charge feedback control. With this control the spacecraft separation distance is maintained at a fixed value, while the coupled formation gravity gradient torque is exploited to stabilize the tether attitude about the orbit radial direction. Further, Reference [12] investigates the reconfiguring of a nadir-aligned 2-craft Coulomb tether formation by forcing the craft to move apart or come closer using the Coulomb force and again using the gravity gradient to stabilize the formation orientation relative to the orbit radial direction. Gravity gradient rigid satellites or conventional tethers have only bounded stability along the orbit radial direction.[16] Similarly, mechanical tether deployment studies in References [19] and [9] develop length rate laws that guarantee only bounded stability for attitudes. In comparison, the feedback control laws for the Coulomb tether regulation problem in Reference [11] and reconfiguration problem in Reference [12] guarantee asymptotic stability for separation distance and in-plane angle. This asymptotic stability is achieved by exploiting the charged relative motion of the spacecraft and varying the separation distance (virtual tether length). Similar to the study of rigid axially symmetric body under the influence of the gravity gradient torque, we know that there are two other relative equilibriums of the charged two-craft problem other than the orbit radial or nadir direction. These equilibriums are along the orbit normal and the along-track direction[2] shown in Figure 1. In particular, zero tension is required between the two craft aligned with the along-track direction to maintain the static unperturbed formation. On the other hand, repulsive forces are required to maintaining the cluster along the orbit normal direction. It is worth noting that both zero tension and compression cases considered are not possible with conventional cable tethers. This paper studies the stability of a two craft formation about along-track and orbit-normal relative equilibrium configurations. A feedback control law is introduced to asymptotically stabilize both the shape and orientation of this cluster. While the charged two-craft formation aligned along the orbit radial direction could stabilize the cluster using only Coulomb forces, this study investigates a hybrid feedback control strategy where both conventional thrusters and Coulomb forces are used. The References [6] and [14] have introduced a similar hybrid actuation system using conventional propulsion and Coulomb actuation, for navigating satellites in a cluster and for the self assembly of large structures in a formation. The control laws developed in these papers effectively use Coulomb forces for maneuvers that require internal forces. This results in significant reduction in fuel consumption. The goal of the present study is to use the thrusters as little as possible and make the Coulomb forces provide the bulk of the actuation requirement. However, to employ small-force thrusters like ion-engines in close proximity to other spacecraft, great care must be taken that the thruster exhaust plume does not impinge on the neighboring craft. These plumes can be very caustic and cause damage to on-board sensors. The control strategy must be designed such that the thruster is never directed at the 2nd craft. The formation is studied at GEO where the Debye lengths are large enough to consider Coulomb spacecraft missions. Reference [15] establishes that the differential solar drag is the largest disturbance acting on a Coulomb formation at GEO. Therefore, the effects of differential solar drag on the formation and the ability of the controller to withstand this disturbance are also studied.

IV. System Dynamics

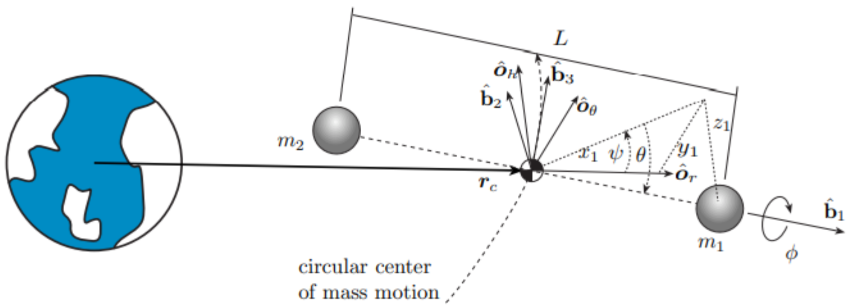

Fig. 1 System Configuration

The relative trajectory equation of the relative vector from the target satellite is developed in this section.

Let

$$[x\ y\ z]^T = \text{Relative position in radial,tangential and normal direction}$$

$$[\dot{x}\ \dot{y}\ \dot{z}]^T = \text{Relative velocity in radial,tangential and normal direction}$$

$$\theta = \text{True anamoly motion}$$

The non-linear equation of the chaser satellite with respect of target can be written as:

$$\ddot{x} + 2\dot{\theta}\dot{y} + \ddot{\theta}y - \dot{\theta}^2 x + \frac{\mu(R_t + x)}{[(R_t + x)^2 + y^2 + z^2]^{\frac{3}{2}}} = a_x \tag{1}$$

$$\ddot{y} - 2\dot{\theta}\dot{x} - \ddot{\theta}x - \dot{\theta}^2 y + \frac{\mu y}{[(R_t + x)^2 + y^2 + z^2]^{\frac{3}{2}}} = a_y \tag{2}$$

$$\ddot{z} + \frac{\mu z}{[(R_t + x)^2 + y^2 + z^2]^{\frac{3}{2}}} = a_z \tag{3}$$

The x and y equation of motion is coupled and z motion is uncoupled. For Kinematic state of the system X can be represented as:

$$X = [x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}]^T$$

And the system satisfies the following control law:

$$\dot{X} = f(X, U) \text{ where } U = [a_x\ a_y\ a_z]^T \tag{4}$$

V. Relative Attitude Dynamics

In this section, the relative attitude dynamical equations are developed step by step.
First let us see the dynamics of the target satellite:

$$I_t\dot{\omega}_t + \omega_t \times (I_t\omega_t) = 0 \quad \text{(Euler Equation)} \tag{5}$$

Now for attitude description of the the target satellite we use quaternion, which will satisfy the differential equation:

$$\dot{q}_t = \frac{1}{2}\Omega(\omega_t)q_t \tag{6}$$

Now for the dynamics of Chaser satellite: The quaternion differential equation for the satellite is:

$$\dot{q}_c = \frac{1}{2}\Omega(\omega_c)q_c \tag{7}$$

The system dynamic equation for the Chaser satellite is:

$$I_c\dot{\omega}_c + \omega_c \times (I_c\omega_c) = T_c + T_d \quad \text{(Euler Equation)} \tag{8}$$

where, T_c and T_d is control and disturbance torque respectively.
Now from equation (8), the differential equation of ω_c can be written as:

$$\dot{\omega}_c = I_c^{-1}(T_c + T_d - \omega_c \times (I_c\omega_c)) \tag{9}$$

Now the relative kinematics and dynamic equation can be developed for controlling the dynamics of the docking process with respect to the target satellite:
Relative Kinematics:

$$q = q_c \otimes q_t \tag{10}$$

Now, q will satisfy the differential equation:

$$\dot{q} = \frac{1}{2}\Omega(\omega_e)q \tag{11}$$

where ω_e will be

$$\omega_e^I = \omega_c^I - A_t^I \omega_t^I \tag{12}$$

where

$$A_t^c = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \tag{13}$$

Now,for $\omega_e = [\omega_1\, \omega_2\, \omega_3]^T$,

$$\Omega(\omega_e) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \tag{14}$$

So finally, the differential equation for relative attitude dynamics comes out to be:

$$\dot{\omega}_e = I_c^{-1}(T_c + T_d - (\omega_e + A_t^c\omega_t) \times I_c(\omega_e + A_t^c\omega_t) - A_t^c\omega_t - A_t^c\omega_t \times \omega_e) \tag{15}$$

VI. Optimal Control Design

RHMPC deals with the optimization problem at each point with the goal of updating the new control input vector by considering the process variable and inequality constraint.

The RHMPC is implemented in the following steps:

A. Step 1: Prediction of Step

The control problem at hand is:

$$\dot{X} = AX + BU \tag{16}$$

Now in discrete from,

$$X(k+1) = A_d X(k) + B_d U(k)$$

where,

$$A_d = e^{AT} \; and \; B_d = \int_0^T e^{A(T-\tau)} B d\tau$$

$\therefore$ for a general discrete time j

$$X(k+j) = A_d^j X(k) + \sum_{i=0}^{j-1} A_d^{j-i-1} B_d U(k+i)$$

So finally the state space system becomes;

$$X_s(k+1) = A_s X(k) + B_s U_s(k) \tag{17}$$

where,

$$A_s = \begin{bmatrix} A_d \\ A_d^2 \\ \vdots \\ A_d^{N_p} \end{bmatrix} \; B_s = \begin{bmatrix} B_d & 0 & \dots & 0 \\ A_d B_d & B_d & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_d^{N_p-1} B_d & A_d^{N_p-2} B_d & \dots & B_d \end{bmatrix} \; X_s = \begin{bmatrix} x(k+1) \\ x(k+2) \\ \vdots \\ x(k+N_p) \end{bmatrix} \; U_s = \begin{bmatrix} u(k+1) \\ u(k+2) \\ \vdots \\ u(k+N_p) \end{bmatrix}$$

B. Step 2: Objective Function

Now we develop the cost Function or the objective function of the problem.

$$J(k) = \sum_{i=1}^{N_p} [\hat{x}^T(k+i|k) Q(k+i) \hat{x}(k+1|k)] + \sum_{i=1}^{N_p} [u^T(k+i-1) R u(k+i-1)]$$

$$= ((A_s X_s(k) + B_s U_s(k)^T Q_s (A_s X_s(k) + B_s U_s(k)) + U_s^T(k) R_s U_s(k)$$

$\therefore$ Simplified J is:

$$J(k) = U_s^T(k) R_{ss} U_s(k) + 2X_s^T(k) M_{ss} U_s(k) + X_s^T Q_{ss} X_s(k) \tag{18}$$

where,

$$Q_{ss} = A_s^T Q_s A_s \qquad R_{ss} = B_s^T Q_s B_s + R_s \qquad M_{ss} = A_s^T Q_s B_s$$

C. Step 3a: Unconstrained Optimization Solution

For optimal solution we minimise J(k);

$$U(k) = \min J(k)$$

Now to minimise J(k),

$$\nabla_{U_s} J = 0$$

or,

$$2R_{ss} U_s + 2M_{ss}^T X_s = 0$$

$$\therefore U_s^*(k) = -R_{ss} M_{ss}^T X_s(k) \tag{19}$$

D. Step 3b: Constrained Optimization

For constrained optimization , the problem is:

$$\min_{U_s} \frac{1}{2} U_s^T R_{ss} U_s + H^T U_s$$

Subjected to

$$V_{ineq} U_s = W_{ineq} \text{ and } V_{eq} U_s = W_{eq}$$

where,

$$H = X_s^T(k) M_{ss}$$

This problem can be solved by Quadratic Program with the following constraints:

$$A_{ineq}B_sU_s < b_c - A_cA_sX_s : \text{ Inequality or path constraint on state}$$

$$A_{eq}B_sU_s = -A_{eq}A_sX_0 : \text{ Equality or terminal constraint on state}$$

VII. Simulation

The System is simulated for position and velocity error and are plotted for Obstacle avoidance and docking mission. The initial and desired coordinates are mentioned in fig 2.

States	Initial Conditions	Desired Conditions
Position (m)	(50,-200,0)	(0 0 0)
Velocity(m/s)	(0,0,0)	(0, 0, 0)

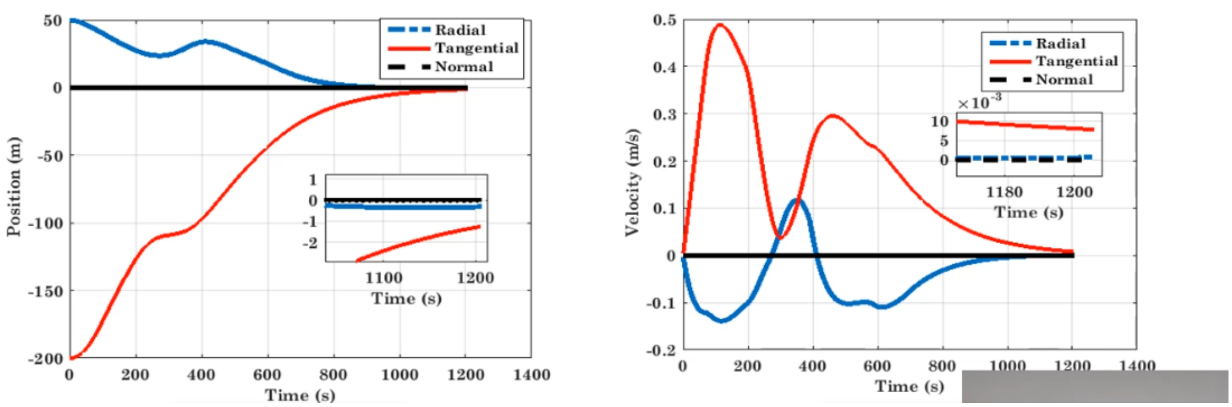

Fig. 2 Simulation Result: Position and Velocity

The 3-D view of the simulation parameters showing Obstacles and The required Chaser Thrust is shown below:

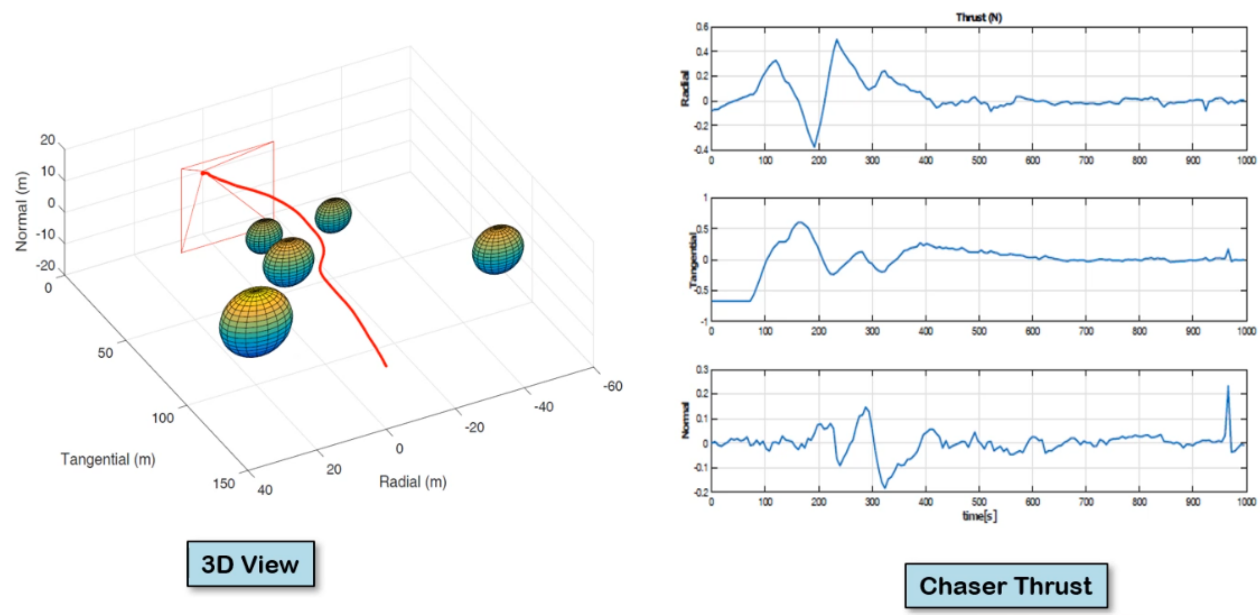

Fig. 3 Simulation Result: 3-D view and Chaser Thrust

A. Forward Kinematics Using Dual Quaternion

For a general link system

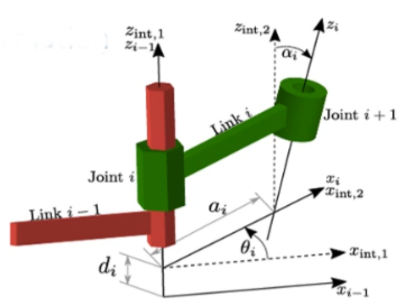

Fig. 4 DH Transformation

For describing the motion of robotic arm, we use DH transformation. From the figure, the transformation from link i-1 to link i is:

$$^{i-1}T_i = T_Z(d_i)R_Z(\theta_i)T_X(a_i)R_X(\alpha_i)$$

$$\therefore \ {}^{i}T_{i-1} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So for a 7 link system:

$$[{}^{0}A_7] = [{}^{0}A_1][{}^{1}A_2][{}^{2}A_3][{}^{3}A_4][{}^{4}A_5][{}^{5}A_6][{}^{6}A_7]$$

For computational efficiency we use Dual quaternion as matrix multiplication and inversion is computational heavy. **Dual Quaternion** is defined as dual number with quaternion component(i.e rotation and translation) as single parameter.

Composite Dual parameter is written as:

$$q_{(i/i-1)} = (q_{(int,2/int,1)} + \epsilon r^{int,1}_{int,1/i-1} q_{(int,2/int,1)})(q_{(i/int,2)} + \epsilon r^{int,2}_{int,2/int,1} q_{(i/int,2)})$$

Where,

$$q_{(i/int,2)} = (\cos\theta_i/2, [0, 0, \sin\theta_i/2]^T)$$

$$r^{int,1}_{int,1/i-1} = (0, [0, 0, d_i]^T)$$

$$q_{(i/int,2)} = (\cos(\alpha_i/2), [\sin(\alpha_i/2), 0, 0]^T$$

$$r^{int,2}_{int,2/int,1} = (0, [a_i, 0, 0]^T)$$

So for a seven link system:

$$[q^7_0] = [q^1_0][q^2_1][q^3_2][q^4_3][q^5_4][q^6_5][q^7_6]$$

This will give the position and orientation of the spacecraft required for docking.

VIII. Results

In this section Position Guidance using MPC and Attitude control using PID is carried out on the Robotic Arm Docking System and results are plotted.

A. Docking Motion from 15m to 3m

This is the closing phase for the docking:

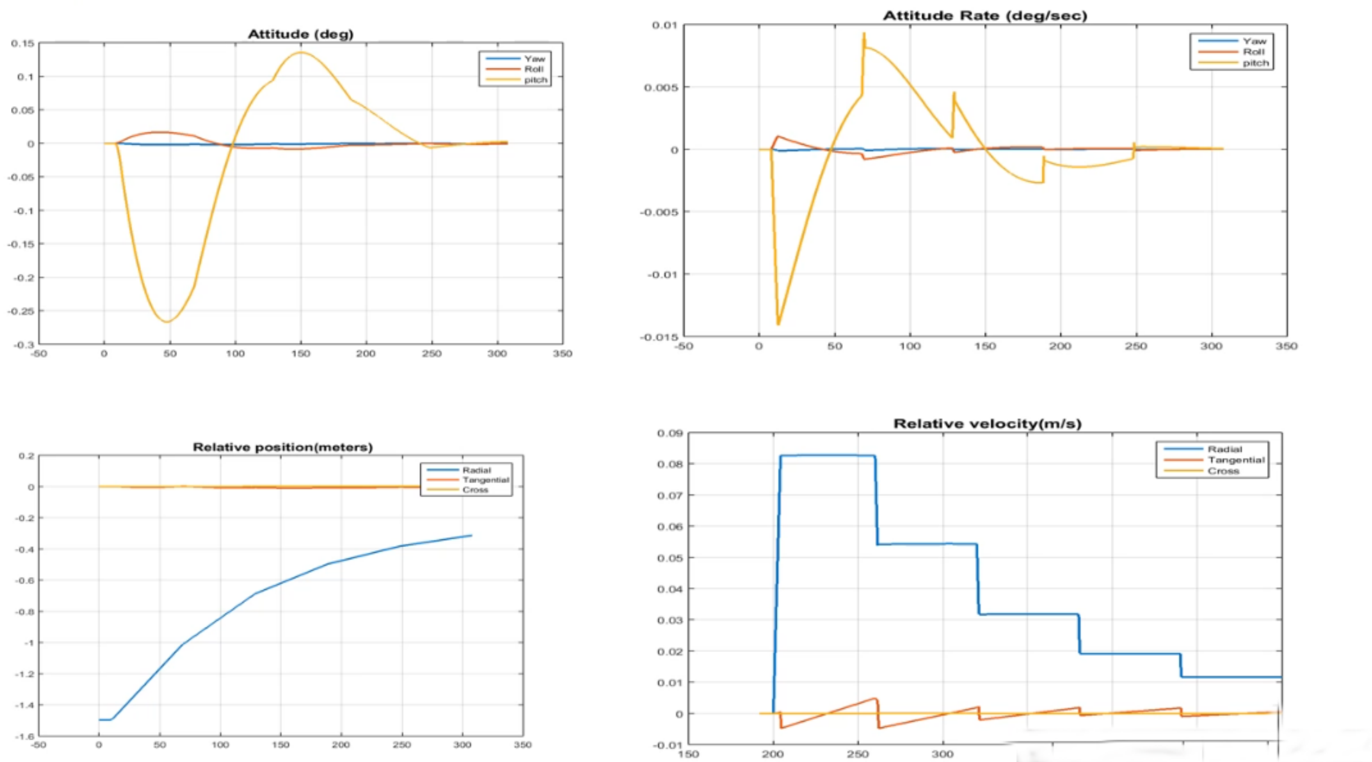

Fig. 5 Attitude simulation

B. Docking motion from 3m to 80mm

This is the terminal phase for docking.

IX. Conclusion

From the Simulation results it is seen that use of Dual quaternion reduces computational complexity by a huge amount to meet cycle delay. The terminal accuracy is maintained from 15m distance in ground environment. Cross axis relative attitude rates of 0.001 deg/s are observed during motion.

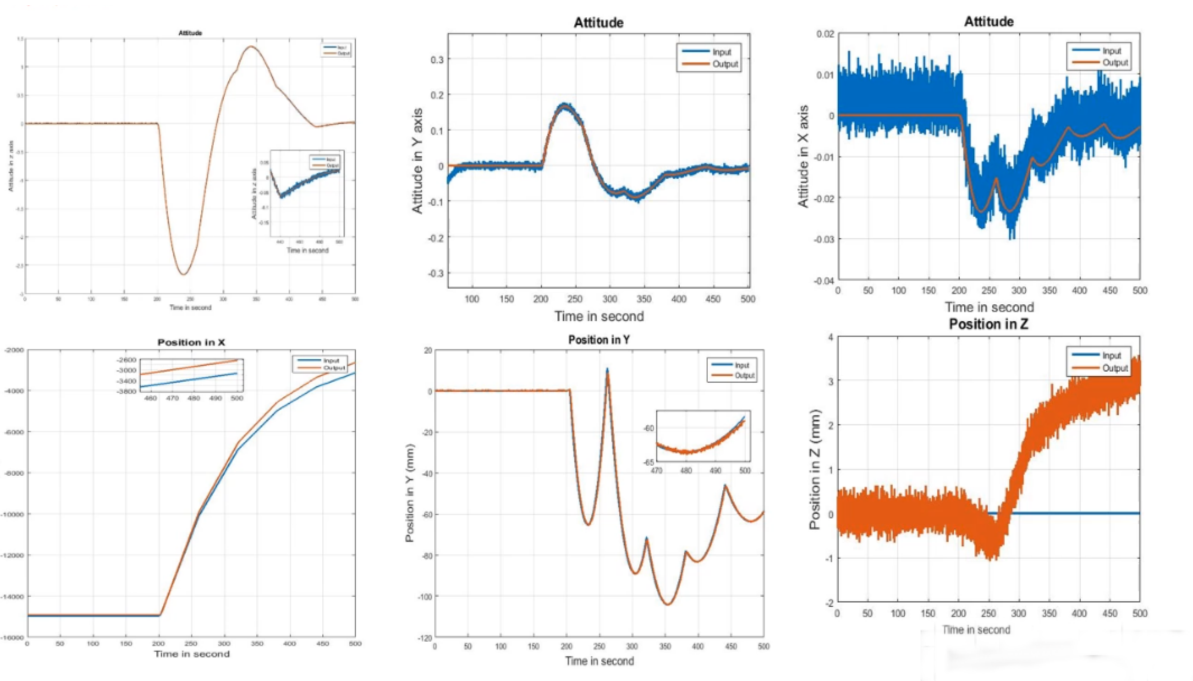

Fig. 6 Overall motion simulation

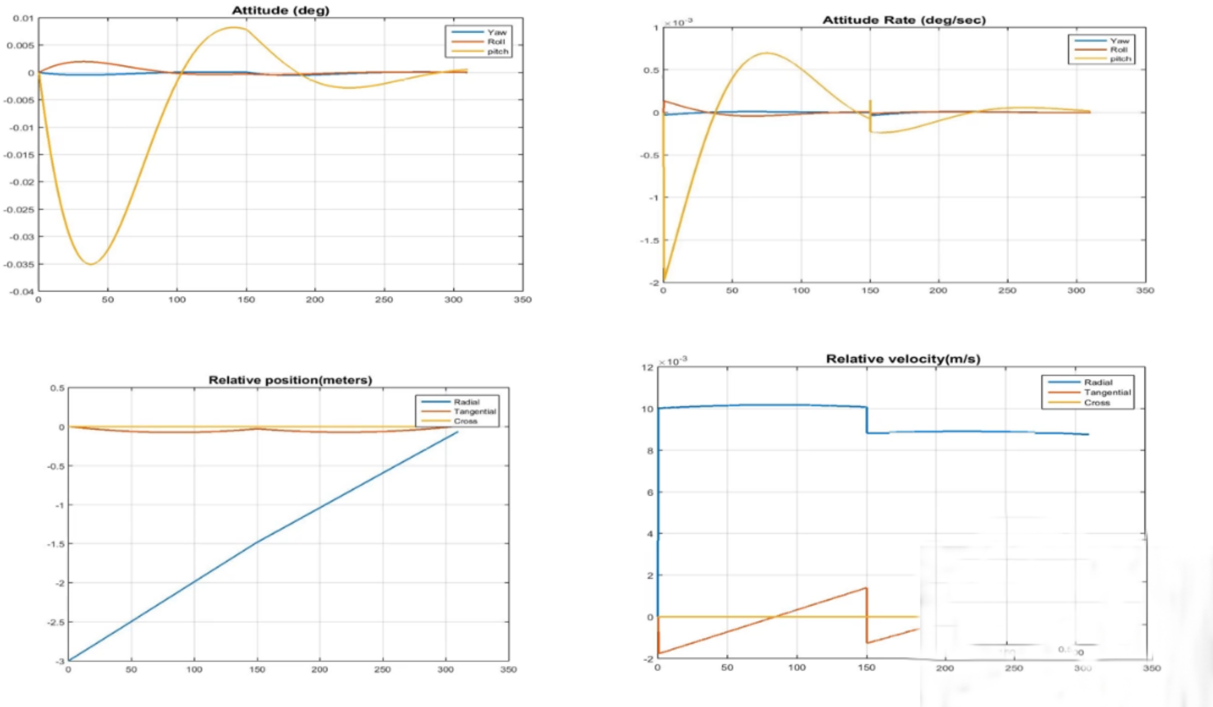

Fig. 7 Attitude Simulation

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