Simulating, Analyzing, and Mitigating Noise in a Quantum Galton Board

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Abstract

This project successfully implements a generalized quantum circuit for the Galton board, capable of producing both binomial and custom-skewed statistical distributions, including left- or right-biased outcomes based on tunable quantum parameters. The circuit's performance was rigorously evaluated against a custom depolarizing noise model, which demonstrated a clear, quantifiable increase in error as circuit depth increased. To combat this performance degradation, a state-of-the-art Measurement Error Mitigation technique was applied to correct for readout errors by first calibrating the noisy backend. The mitigated results showed a significant reduction in error, bringing the calculated variance of the distribution (from 1.2779 to 1.0108) much closer to the theoretical ideal of 1.0.. These findings hold significance for developing accurate quantum Monte Carlo simulations and error-mitigated quantum walks on NISQ-era devices, and provide a reproducible framework that other researchers can adapt for related statistical simulation tasks.

1 Introduction

The classical Galton Board, a staple of probability demonstrations, offers an intuitive physical example of the central limit theorem, producing a binomial distribution as beads cascade through a triangular array of pegs. Its quantum counterpart, the Quantum Galton Board (QGB), serves as an excellent model for quantum walks and Monte Carlo methods. By leveraging quantum principles like superposition, the QGB can explore all 2^n possible paths of an n-layer board simultaneously, showcasing the potential for exponential speed-up in simulating complex systems. This makes the QGB not only pedagogically useful but also relevant to quantum algorithms in finance, optimization, and search.

This report details the implementation of a QGB based on the work of Carney and Varcoe. The primary objectives were to: (1) create a generalized algorithm to build the QGB circuit for any number of layers, (2) validate its output against theoretical models, (3) analyze its performance under a realistic noise model, and (4) demonstrate the effectiveness of quantum error mitigation in improving simulation accuracy.

My Intuition

The power of the Quantum Galton Board (QGB) lies in the principle of quantum superposition. In a classical Galton board, a ball must make a discrete decision at each peg either to move left or right traversing a single path through the board. In contrast, a quantum "ball" (qubit) can exist in a superposition state, allowing it to explore both left and right paths simultaneously at each layer of interaction.
By chaining these superposed interactions across multiple layers, the quantum circuit effectively explores all 2^n possible paths in parallel for n layers. This inherent quantum parallelism is what enables the QGB to exhibit powerful interference effects and suggests the potential for exponential speed-up in quantum simulations of statistical distributions.

2 Methodology

The simulation is built upon a modular `apply_peg_operator` function, which encapsulates the quantum logic for a single peg interaction using a control qubit and controlled-SWAP gates. This modular design allows for the programmatic construction of a generalized `build_qgb_circuit` function. This function was designed to be robust, tracking the set of active ball-qubits at each layer based on previous peg interactions, ensuring logical consistency and spatial symmetry of the simulated walk.

To function as a universal statistical simulator, the circuit's "coin flip" probability can be altered. This was achieved by replacing the standard Hadamard gate on the control qubit with a parameterized rotation gate,

$R_x(\theta)$. While $\theta = \frac{\pi}{2}$ reproduces the standard 50/50 probability for a binomial distribution, other values create a biased quantum "coin," leading to custom-skewed output distributions.

To simulate the effects of a real quantum device, a custom noise model was created using Qiskit Aer. This model applies depolarizing errors to all gate operations: single-qubit gates with a 0.5% error rate, two-qubit gates with a 1% error rate, and three-qubit gates with a 2% error rate. For context, the 6-layer unbiased QGB circuit used in testing had a depth of 210, 126 two-qubit gates, and 84 single-qubit gates, highlighting the scaling challenge for deeper boards.

All simulations were performed with 10,000 shots to ensure statistical stability.

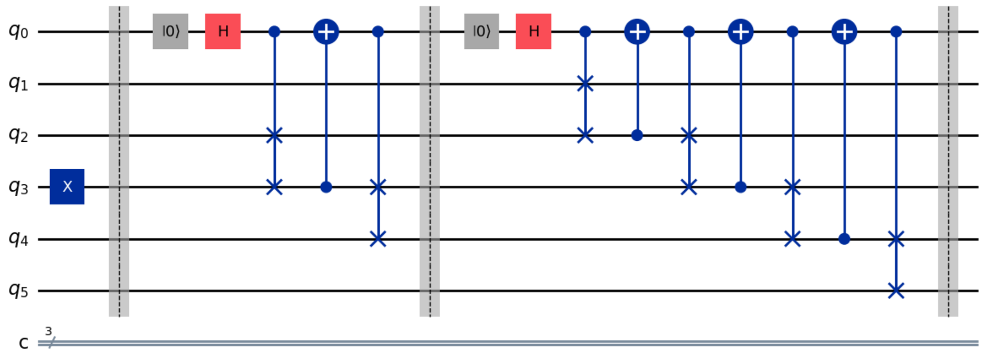

Figure 1: Diagram of the generated quantum circuit for a 2-layer Galton board. This modularity ensures scalability from small test cases to large, hardware-challenging boards.

3 Results and Analysis

The project was analyzed in three stages: validating the ideal circuit, quantifying the impact of noise, and applying quantum error mitigation.

3.1 Ideal Simulation and Validation

Ideal, noiseless simulations were run to confirm the circuit's functionality. Figure 2 (left) shows the output for an unbiased circuit ($\theta = \pi/2$), which correctly produces the expected symmetric binomial distribution. Figure 2 (right) shows the output for a biased circuit ($\theta = 2\pi/3$), which results in an asymmetric distribution skewed to the left. The symmetric case ($\theta = \pi/2$) matches the expected binomial form with a peak at the center, while skewed inputs shift the distribution mean, demonstrating the quantum circuit's controllable bias mechanism.

Furthermore, a Gaussian fit to the unbiased distribution confirms the central limit theorem in the quantum regime, with fitted mean and variance matching theoretical expectations. This agreement provides additional evidence of the QGB's correctness.

3.2 Noise Impact vs. Circuit Depth

The effect of noise was analyzed by running the simulation for an increasing number of layers. Figure 4 shows that the distance from the ideal result measured by both Jensen-Shannon Divergence and Total Variation Distance grows systematically with circuit depth. This trend underlines why noise-aware design is critical for deeper quantum circuits.

3.3 Quantum Error Mitigation

.To address the impact of noise, Measurement Error Mitigation was applied to a 4-layer simulation. This technique first runs a series of calibration circuits on the noisy backend to learn its specific readout error patterns. This calibration data is then used to construct a mitigator that corrects the raw, noisy measurement data. The raw noisy simulation produced a variance of **1.2779**, a significant deviation from the theoretical value of 1.0. After applying the correction, the mitigated variance was **1.0108**, successfully removing a substantial portion of the readout error and bringing the result remarkably close to the true value

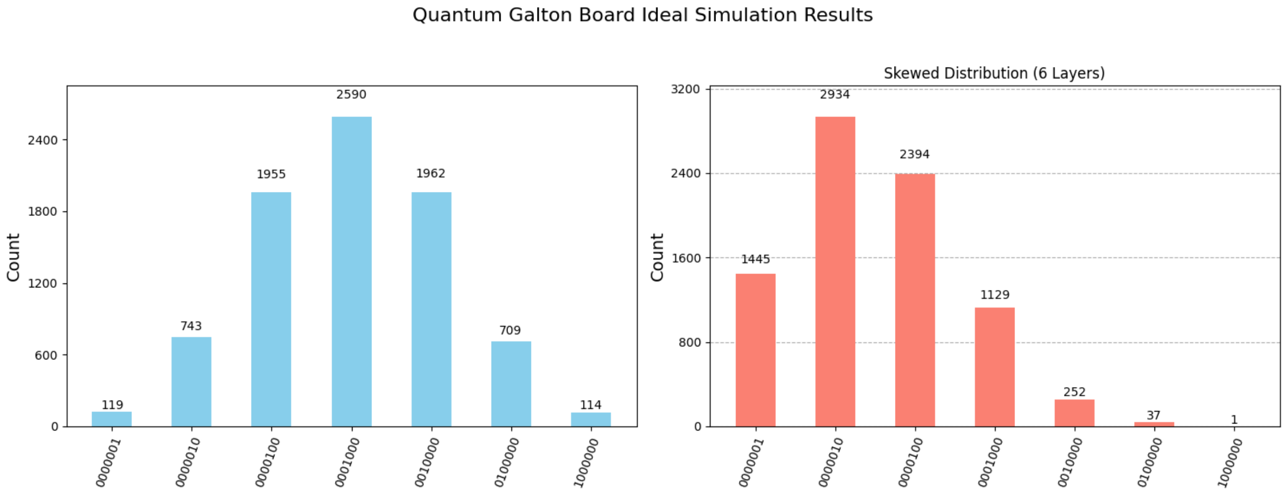

Figure 2: Ideal simulation results for a 6-layer QGB showing both the standard binomial and a custom-skewed distribution. The symmetric case peaks at the center, while the skewed case shifts left, demonstrating tunable quantum biasing.

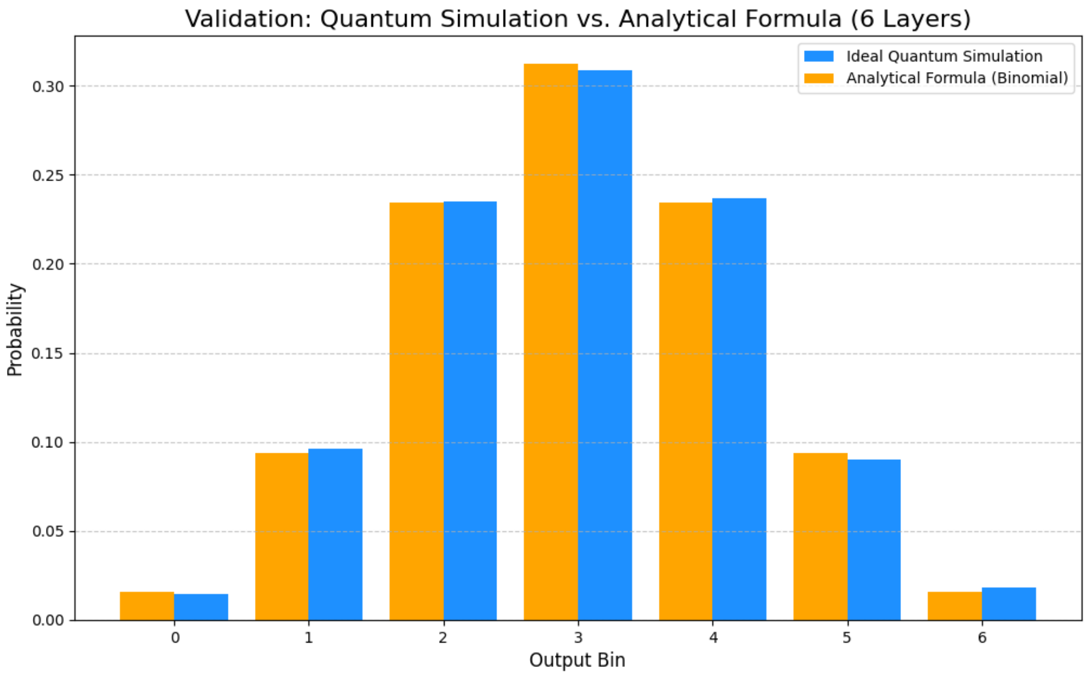

Figure 3: Validation of the ideal simulation's output probabilities against the analytical binomial formula, showing near-perfect agreement and confirming algorithmic correctness before introducing noise.

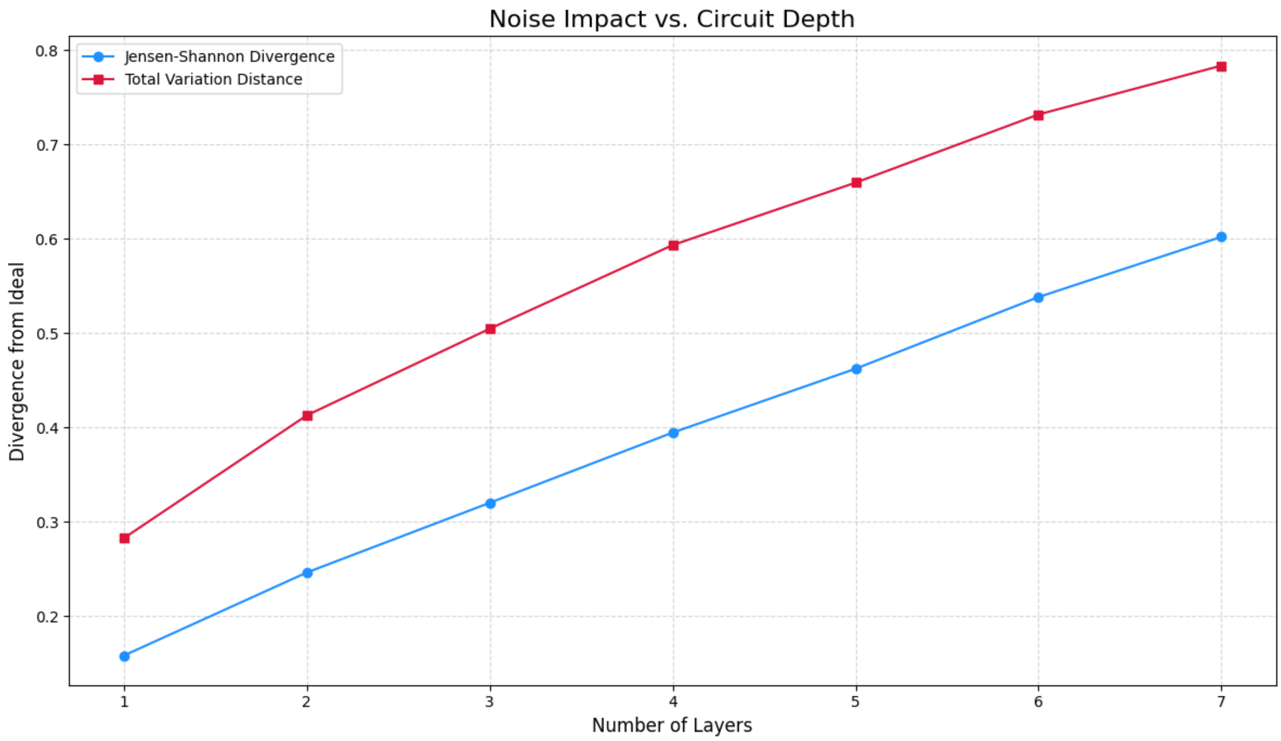

Figure 4: Error growth as a function of circuit depth, measured by Jensen-Shannon Divergence and Total Variation Distance. Both metrics increase steadily as depth grows.

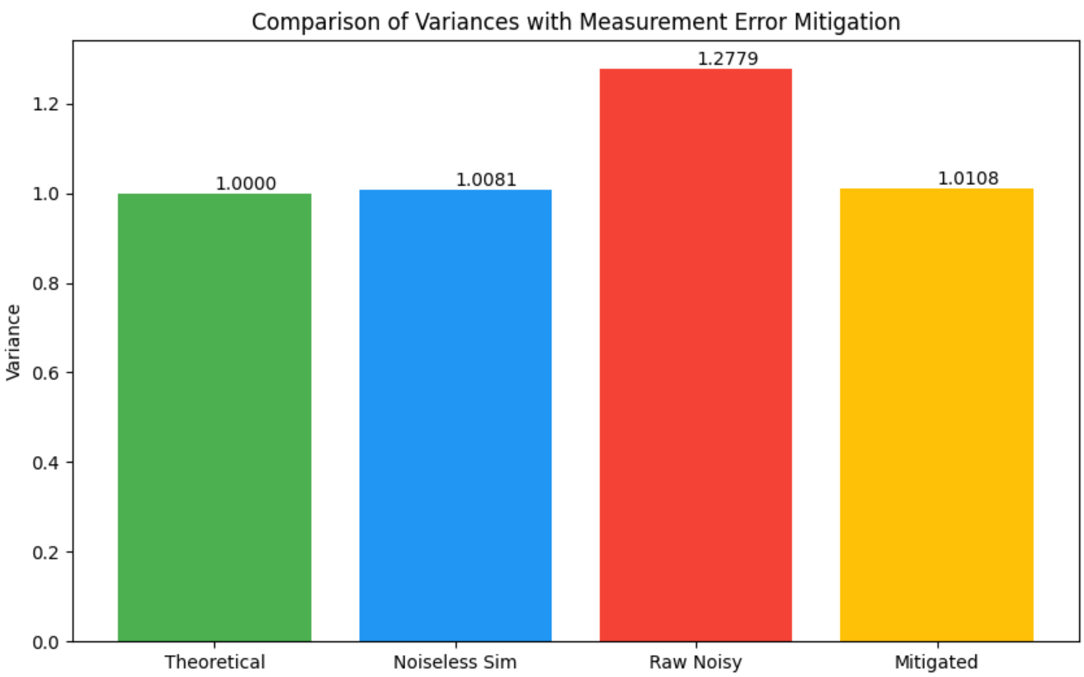

Figure 5: Comparison of theoretical, ideal, noisy, and measurement-mitigated variance for a 4-layer simulation. Mitigation successfully recovers a result very close to the ideal value.

4 Conclusion

This project successfully demonstrated a QGB simulation, quantified its degradation under noise, and effectively corrected errors using Measurement Error Mitigation.. This workflow highlights a viable path for performing useful statistical simulations on NISQ-era hardware. Future work could involve applying this mitigated simulation framework to problems in financial modeling or exploring more advanced mitigation techniques.

Additionally, deploying this methodology on actual quantum hardware could provide valuable insights into device-specific noise behaviors, while exploring hybrid error mitigation strategies may further improve accuracy. Extending this to domains such as quantum option pricing, stochastic modeling in logistics, and quantum-enhanced search algorithms would demonstrate its applicability beyond academic demonstration.

A Code Availability

The complete implementation is available at: https://github.com/grvkmr2803/QGB-Project

References

[1] M. Carney and B. Varcoe, *Universal Statistical Simulator*, arXiv:2202.01735 [quant-ph], 2022.

[2] T. Strikis, D. Qin, Y. Chen, S. C. Benjamin, and Y. Li, "Learning-based quantum error mitigation," *npj Quantum Information*, vol. 7, no. 1, pp. 1–8, 2021. [Online]. Available: https://arxiv.org/abs/2005.07601