BIRLA INSTITUTE OF TECHNOLOGY, MESRA

OPTIMAL CONTROL OF INVERTED PENDULUM & ITS APPLICATION

Guided by: **Mrs. Shradha Kishore**

Team members: **Sompurna Modi** (BE/15225/16)
Gaurav Thakur (BE/15088/16)

INTRODUCTION

- A pendulum cart system is a non-linear and an unstable system, also known as Inverted Pendulum system which is designed with control methods and state observer to make the system stable. This standard design has a vast application in the field of Control Theory.
- All have the basic idea to sense tilt and drive the wheels in the required direction to keep the system erect i.e. wheels rotate in the forward direction when the cart tilts forward and in the backward direction when the cart tilts backwards. The tilting is due to the fact that the center of mass of the cart lies above the pivot point and hence the cart is initially unstable. Furthermore, external disturbances can also make the system lose balance. A linear mathematical method cannot balance the non linear system, hence, many different control theory methods are applied.
- To stabilize the proposed cart system, State-space Observer based Linear Quadratic Regulator optimal control has been implied.
- This presentation describes the dynamical model of the system, followed by pole-placement based state feedback controller and using controllability and observability of the system, an optimal control has been obtained to keep the system at upright position and stable.
- Moreover, both the alternating frameworks with respect to time; Continuous time and Discrete time control system has been attained.
- At the end, a two wheel self balancing Robot is designed as an application for the Inverted Pendulum Concept

SIMPLE PENDULUM

When a body with some mass is suspended from a light weighted string(negligible) that can oscillate when displaced from its rest position.

$T_P = 2\pi\sqrt{L/g}$

T_P is period, L is the pendulum length and g is acceleration due to gravity.

The angular velocity ω and the rate change of angular velocity dω/dt, is given by

$$\frac{d\theta}{dt} = \omega \qquad (1)$$

$$\frac{d\omega}{dt} = \frac{-MgL}{J\sin\theta} \qquad (2)$$

J is the moment of inertia, and g is the gravitational force.

At rest, $\omega = 0$ and $\frac{d\omega}{dt} = 0$

The equilibrium points are $\theta = 0^\circ$ and $\theta_1 = 0^\circ$ and $\theta_2 = 180^\circ$

θ_1 is stable (Not applicable for robots)

θ_2 is the target position for inverted pendulum systems. But at this point, the system is unstable and can move away from equilibrium position due to any external forces or disturbances.

Hence, actively balancing is the major requirement to keep the pendulum at equilibrium point.

This is why state feedback is used in the project so that it sends the error from the desired tilt and the designed controller is actively balances the system.

A pendulum that has its center of mass above its pivot point is known as Inverted Pendulum. It is unstable and fall over if not supported.

In this project, the stabilization problem around the upper position of the pendulum is fixed. A dynamical system has been created using MATLAB. Analyzing the system, poles are obtained and a state feedback controller is designed. With the help of MATLAB Simulink, state space and optimal controllers are designed for continuous time control system. Thereafter a State Observer is designed and closed loop of the system is investigated including an observer. At the end, a time discrete controller and an observer is designed for the Pendulum- cart system to stabilize and move horizontally as desired.

OBJECTIVES

- At first, the dynamical system of Inverted Pendulum Cart using MATLAB is created.
- Then, System Analysis controller is designed using MATLAB tools and its simulation.
- Design of a State space model with optimal control and its simulation is done.
- Then, a State Observer is designed and the closed loop is investigated using an Observer.
- Time discrete Controller and Observer is designed and simulated.
- System analysis is done using different parameters value to check their effect on the system behavior.
- At the end designing a prototype of Self balancing Robot.

DYNAMIC MODEL OF THE SYSTEM

In the figure, Pendulum is connected to a pair of only-rolling wheels with one translation degree of freedom where Mp is the mass of Pendulum, Mc is the mass of the cart and L is the length of the rod.

Model of the system has two coordinates:

a) q1 is the displacement of the cart.

b) q2 is the angle of pendulum

Using Lagrange-Formalism or Newton Euler for non-linear system is in the following form:

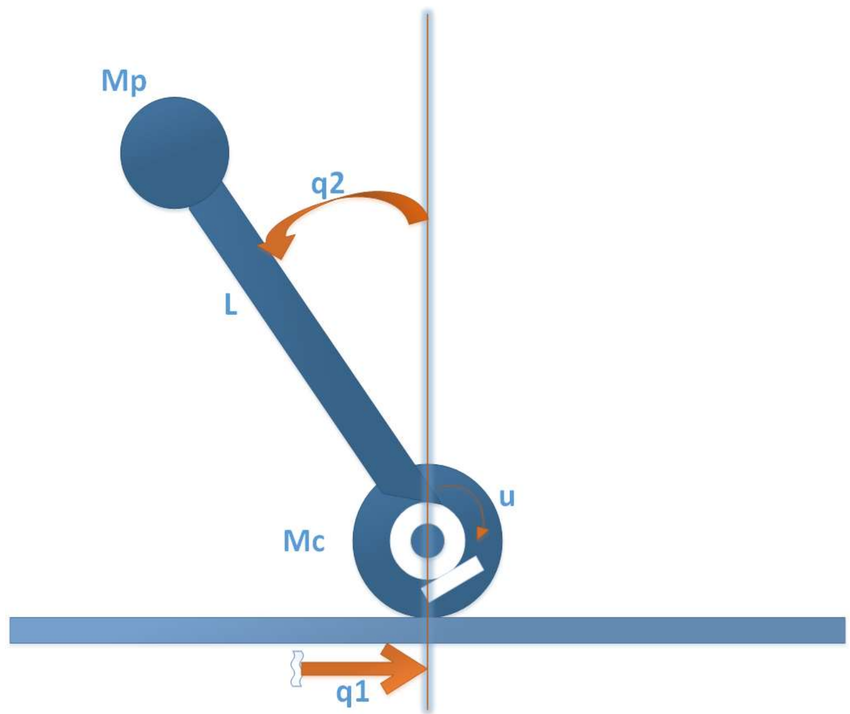
$$\mathrm{M(q)\ddot{q} + h\,(q, \dot{q}) = gc\,u} \qquad (3)$$

where, $M(q) = \begin{bmatrix} Me + Mp & -L\ Mp\ Cos\ q2 \\ -L\ Mp\ Cos\ q2 & L^2 Mp \end{bmatrix}$

$$h(q, \dot{q}) = \begin{bmatrix} L\ Mp\ q2\ Sin\ q2 \\ -L\ g\ Mp\ Sin\ q2 \end{bmatrix} + \begin{bmatrix} d1\ \dot{q1} \\ d2\ \dot{q2} \end{bmatrix}$$

$$g_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Pendulum Cart system



where d1 and d2 are damping factors. These represents friction in the cart displacement and joint displacement respectively. This defines the motion of the system.

For state space representation,

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \tag{4}$$

which are q1 and q2 respectively with their corresponding velocities.

$$\dot{x} = F(x) + G(x)u \tag{5}$$

$$\dot{x} = \begin{bmatrix} \dot{q1} \\ \dot{q2} \\ -M^{-1}(q)\, h(q, \dot{q}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M^{-1}(q)gc \end{bmatrix} u \tag{6}$$

Using the Taylor Method, we could linearize this non- linear equation around (q1)̇ =q2̇ =0 about the origin basically to get the linear differential equation or system equation or state-space equation for the system.

$$\dot{x} = Ax + Bu \tag{7}$$

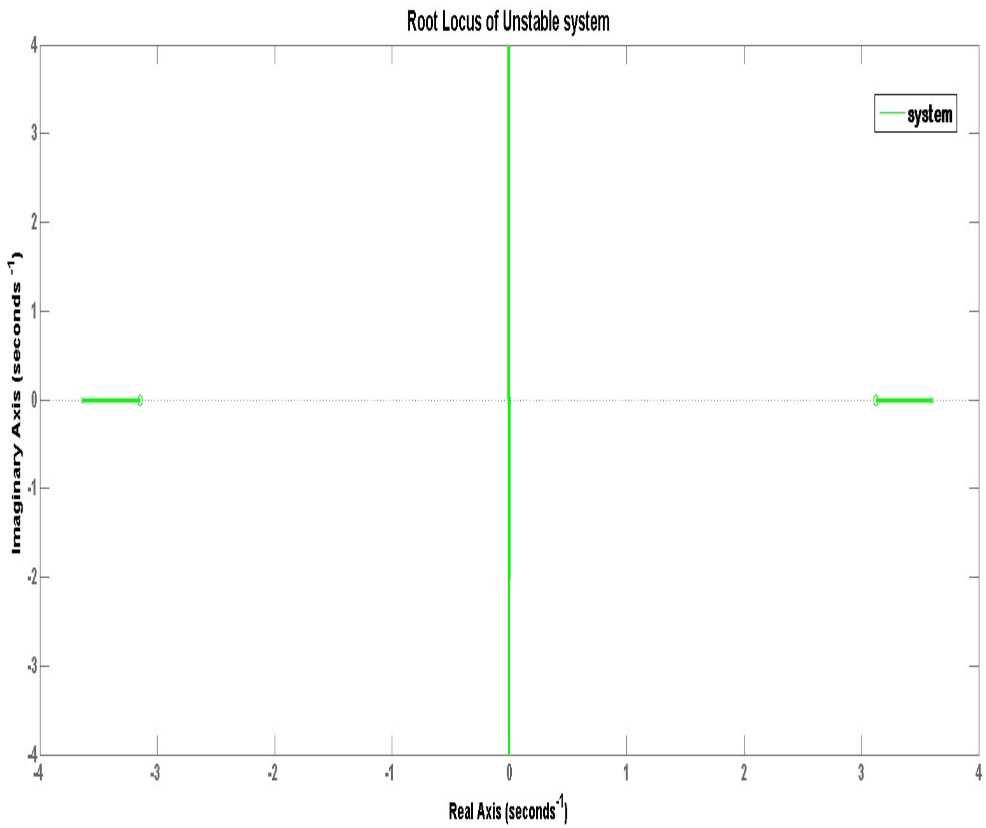
where,

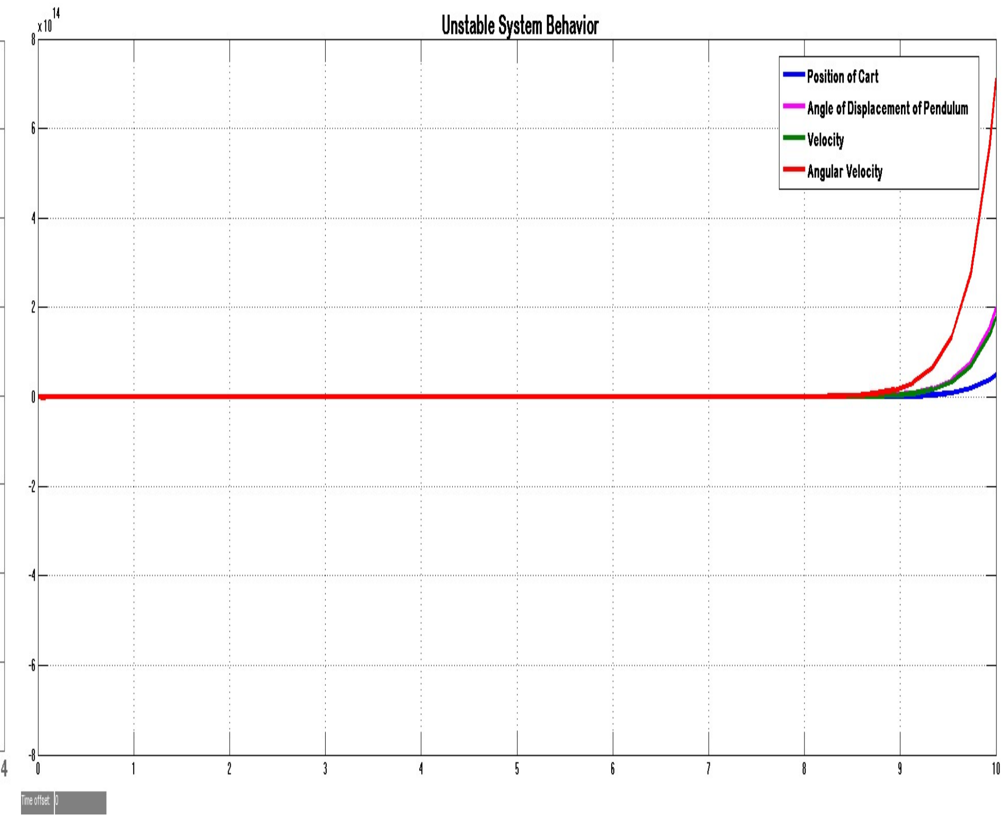
$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{g\,Mp}{Mc} & \frac{-d1}{Mc} & \frac{-d2}{L\,Mc} \\ 0 & \frac{g\,(Mc+Mp)}{L\,Mc} & \frac{-d1}{L\,Mc} & \frac{-d2\,(Mc+Mp)}{L^2 Mc\,Mp} \end{bmatrix};\ B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{Mc} \\ \frac{1}{L\,Mc} \end{bmatrix}$$

PARAMETERS OF THE CART SYSTEM

COMPONENT	ABBREVIATION	VALUE
Mass of Cart	M_C	1.5 kg
Mass of Pendulum	M_P	0.5 kg
Length of Pendulum from hinge point	L	1 m
Angle of displacement of Pendulum	q2	will be given
Displacement of the Cart	q1	will be given
Force applied to the Cart	u	will be given
Damping Factor during the cart displacement	d1	0.01
Damping Factor during the joint displacement	d2	0.01
Gravitational constant	g	9.82 m/s^2

ROOT LOCUS OF THE SYSTEM





The state space representation for the system is:

$$A = \begin{bmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0 & 3.2733 & -0.0067 & -0.0067 \\ 0 & 13.0933 & -0.0067 & -0.0267 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0.6667 \\ 0.6667 \end{bmatrix}$$

When obtaining the poles of the system, the result says that there is one pole at the right-hand side which means that the resistance is not stable.

Root Locus of the system is plotted. Poles of the system are always on the right-hand side, hence there is no peak control that can stabilize the system. There is no gain that can bring all of the poles on the left side of the system. Hence, peak control is not enough for the system.

At initial condition,

Position of the cart = 0; Angle of displacement of pendulum to the cart = 5°; Velocity = 0; Angular Velocity = 0;

x_0 = [0; 5*pi/180; 0; 0]

Since the system is not stable, in fig every state goes to infinity.

CONTROLABILITY AND OBSERVABILITY OF THE SYSTEM

The necessary and sufficient condition for a feedback-based controller is that it must be completely state controllable. For the proposed Linear invariant system, it is said to be controllable if the Rank of controllability matrix equals 4.

Taking C = [1;0;0;0] q1 as output

$$Sc = [B\ AB\ A^2B \dots\dots\dots..A^{n-1}B] \tag{8}$$

$$\mathrm{Sc} = \begin{bmatrix} 0 & 0.6667 & -0.0089 & 2.1824 \\ 0 & 0.6667 & -0.0222 & 8.7295 \\ 0.6667 & -0.0089 & 2.1824 & -0.1455 \\ 0.6667 & -0.0222 & 8.7295 & -0.5383 \end{bmatrix} \tag{9}$$

rank (Sc) = 4

The necessary and sufficient condition for an observer-based controller is that it must be observable. For the proposed Linear invariant system, it is said to be observable if the Rank of observability matrix equals to 4.

$$\mathrm{So} = [C^T\ A^TC^T \dots\dots\dots\dots (A^T)^{n-1}C^T] \tag{10}$$

$$So = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 3.2733 & -0.0067 & -0.0067 \\ 0 & -0.1091 & 0.0001 & 3.2736 \end{bmatrix} \tag{11}$$

rank (So) = 4

OPTIMAL CONTROL

This expresses to have an effective degrees of freedom that allows to choose poles that enhances the closed loop performance rather than just specifying the positions. This uses a performance index J. Small J implies to have a good performance. Performance is defined using Rise time, settling time, Overshoot, Oscillation and damping ratios, offset, peak values of signals (especially inputs)

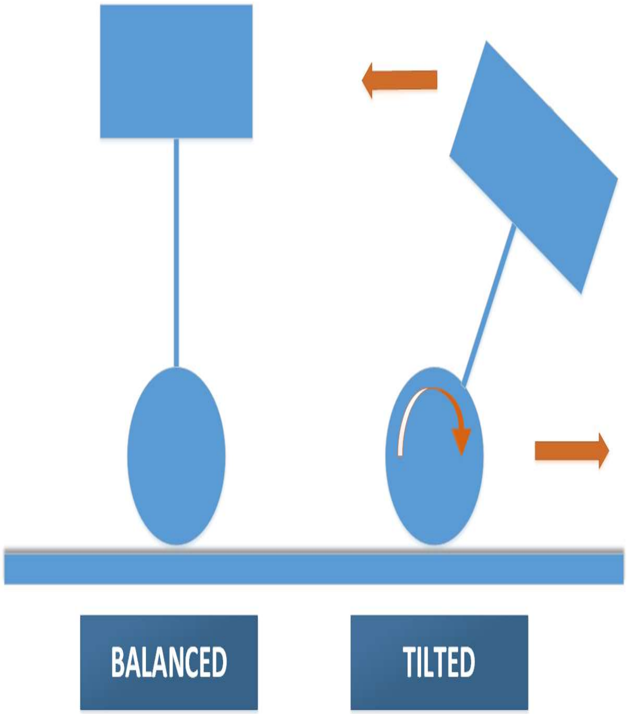
$$J = \int_{0}^{\infty} \left(x^T Q x + u^T R u\right) dt \tag{12}$$

where Q is defined as weighted squares of deviation of states from target and R is defined as weighted squares of Control activity.

Here, xTQx terms implicitly measures convergence rate (i.e rise time and settling time) and uTRu term penalizes aggressive use of the input. Due to use of squares, the positive and negative errors penalize equally and covers Overshoot, Oscillation and damping ratios. Due to use of Infinite horizons, implicitly it derives the asymptotic errors to zero. Peak values of the signal is also covered because of the use of squares. Because of squares, large values are penalized disproportionally as compared to smaller values. For discrete system, increase in R gives slower state behavior and poor controllability.

TWO-WHEEL SELF BALANCING ROBOT

Self-balancing robot is based on the principle of Inverted pendulum, which is a two wheel vehicle balances itself up in the vertical position with reference to the ground. It consist both hardware and software implementation. Mechanical model based on the state space design of the cart, pendulum system. According to the situation we have to control both angel of pendulum and position of cart. Mechanical design consist of two dc gear motor with encoder, one Arduino microcontroller, IMU (inertial mass unit) sensor and motor driver as a basic need. IMU sensor which consists of accelerometer and gyroscope gives the reference acceleration and angle with respect to ground (vertical), When encoder which is attached with the motor gives the speed of the motor. These parameters are taken as the system parameter and determine the external force needed to balance the robot up. It will be prevented from falling by giving acceleration to the wheels according to its inclination from the vertical. If the bot gets tilts by an angle, than in the frame of the wheels; the center of mass of the bot will experience a pseudo force which will apply a torque opposite to the direction of tilt. When the robot starts to fall in one direction, the wheels should move in the inclined direction with a speed proportional to angle and acceleration of falling to correct the inclination angle.

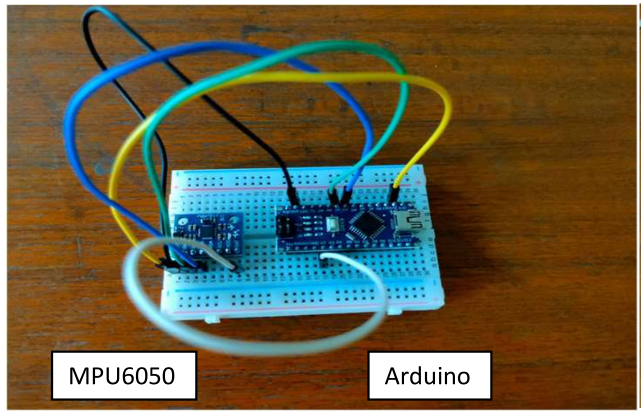


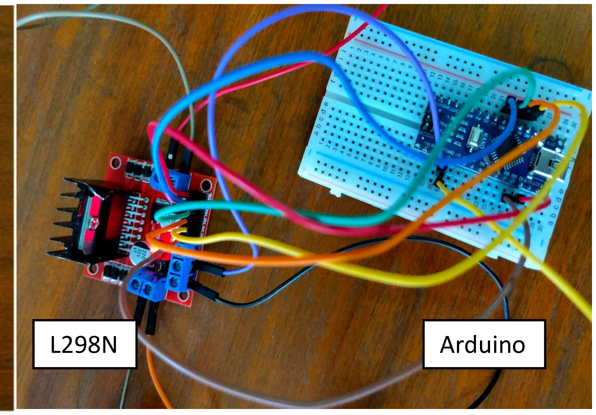
CONNECTIONS FOR THE MODEL

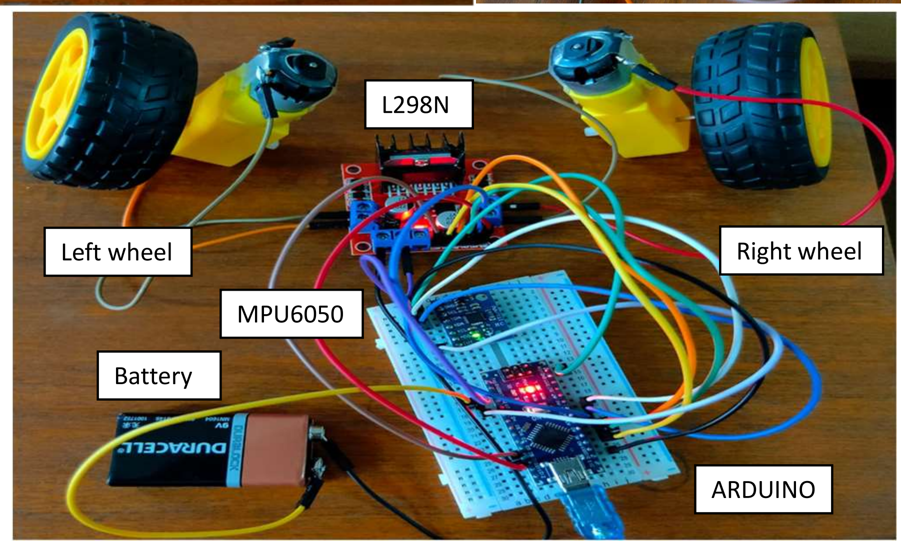
MPU6050	**Arduino**
V_{cc}	5V or 3V3
GND	GND
SCL	A5
SDA	A4
INT	D2

L298N	**Arduino**
GND	GND
ENA	D9
ENB	D10
INT1	A0
INT2	A1
INT3	A2
INT4	A3

L298N	**Battery**
+12V	+ve terminal
GND	-ve terminal



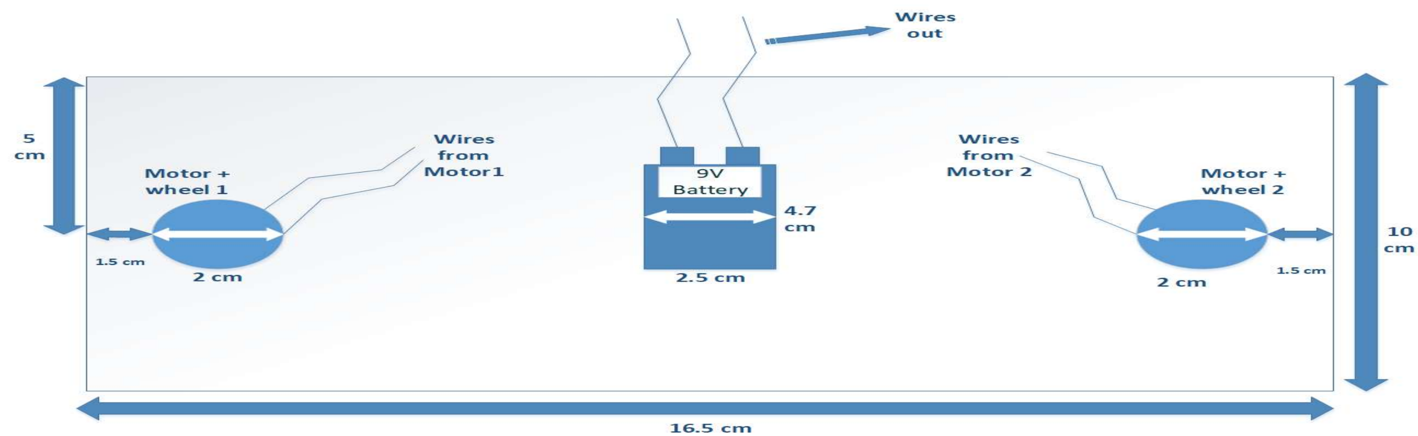




DESIGN OVERVIEW

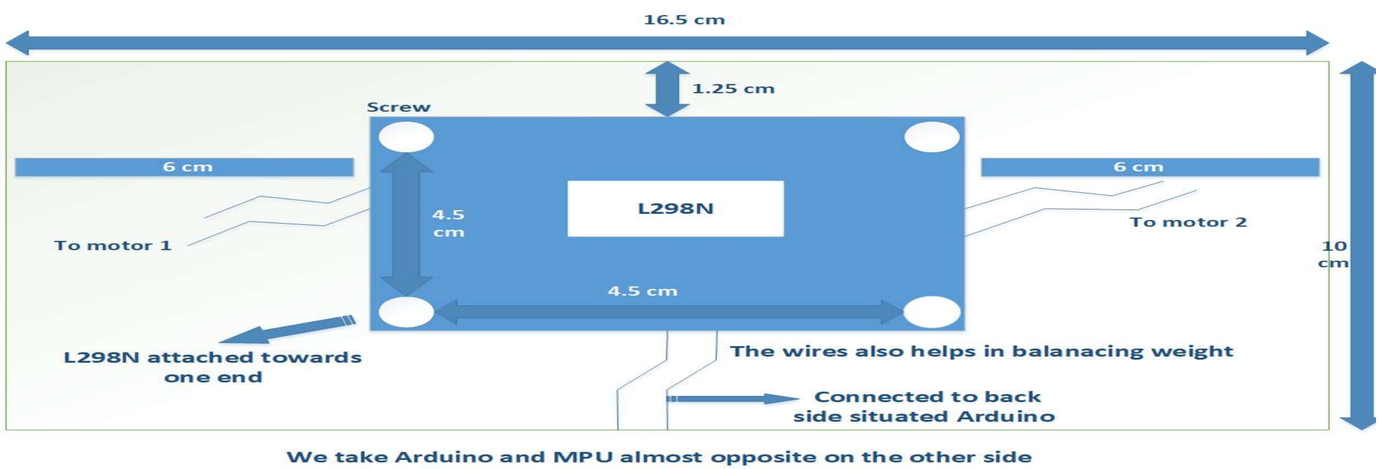
The self-balancing robot is essentially an inverted pendulum. It can be balanced better if the center of mass is higher relative to the wheel axles. A higher center of mass means a higher mass moment of inertia, which corresponds to lower angular acceleration (slower fall). This is why the different components are placed keeping in mind the balancing of the center of mass. The height of the robot is chosen depending on the number of components.

For our case we take three layers so as to place all the components.

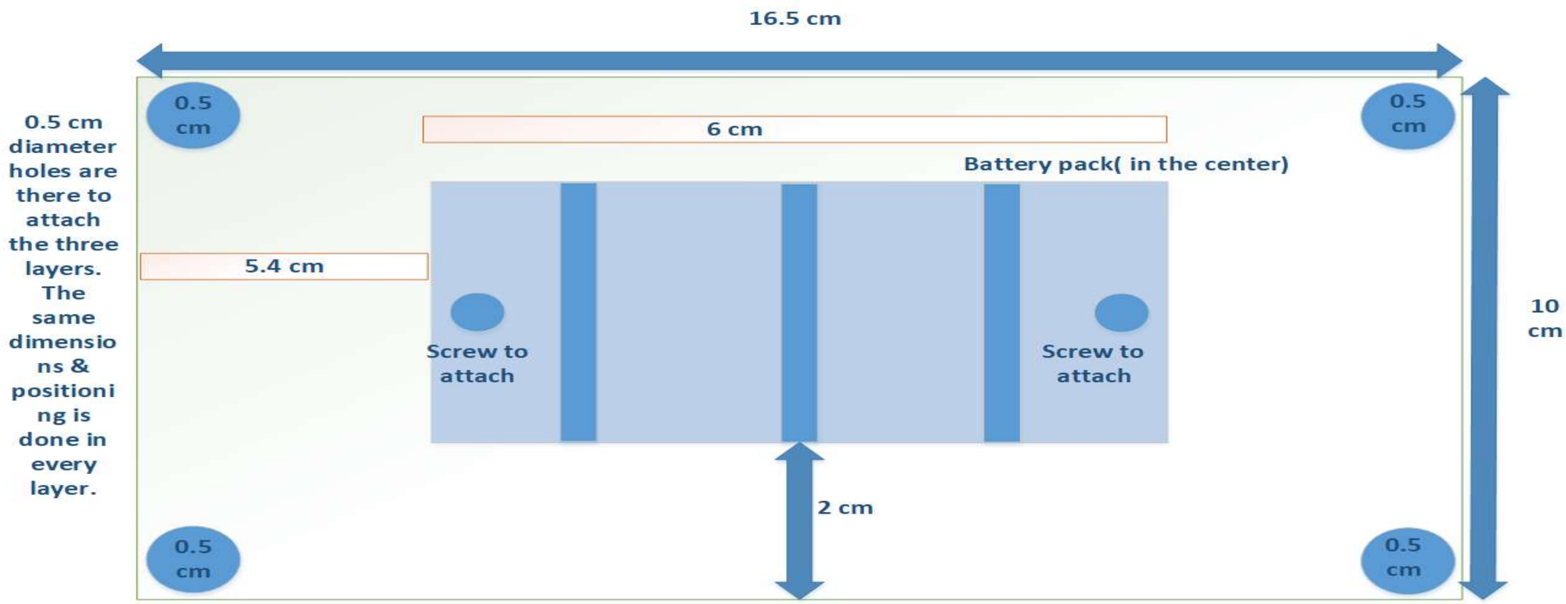


1st Layer

2nd Layer (Top view)

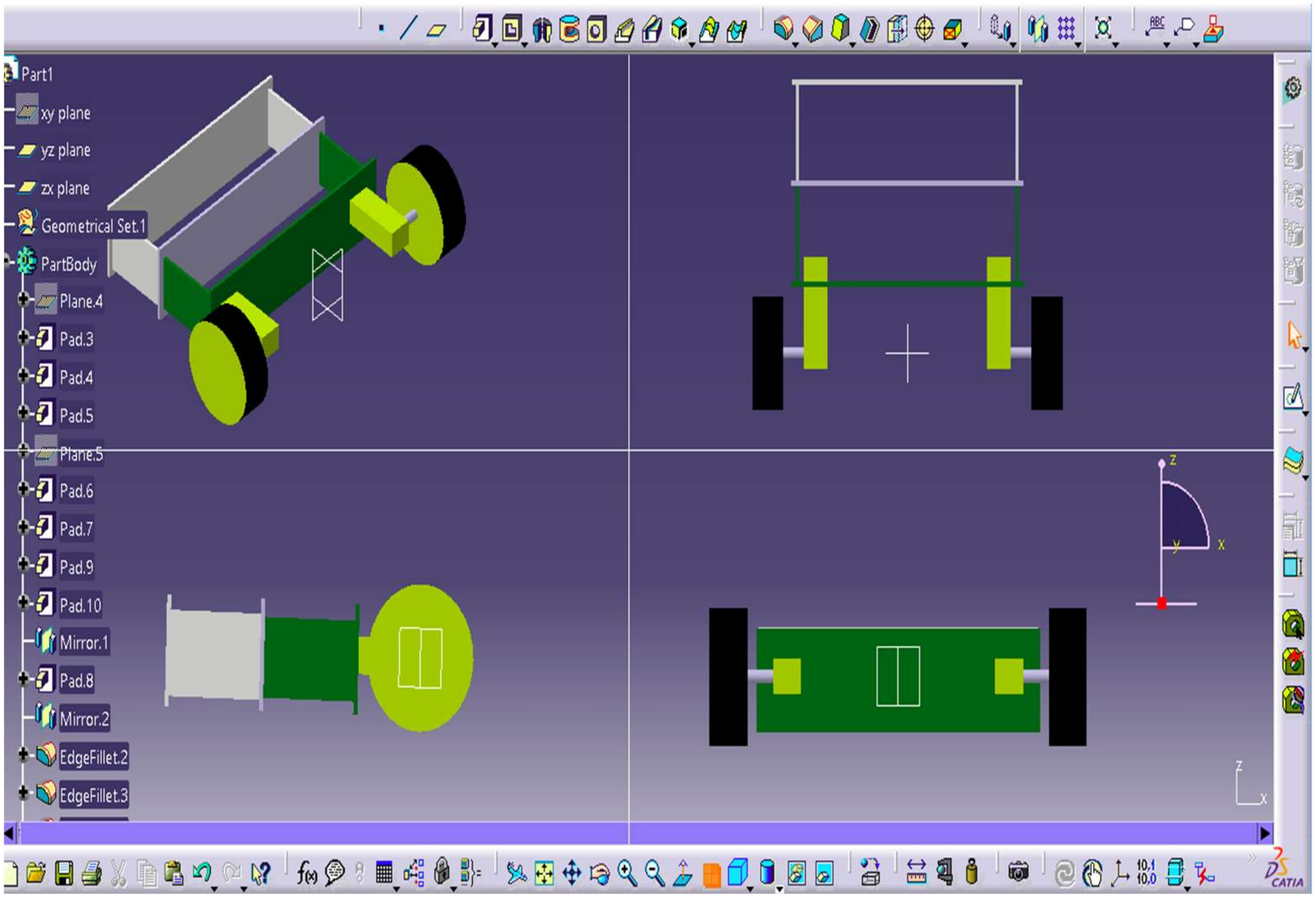


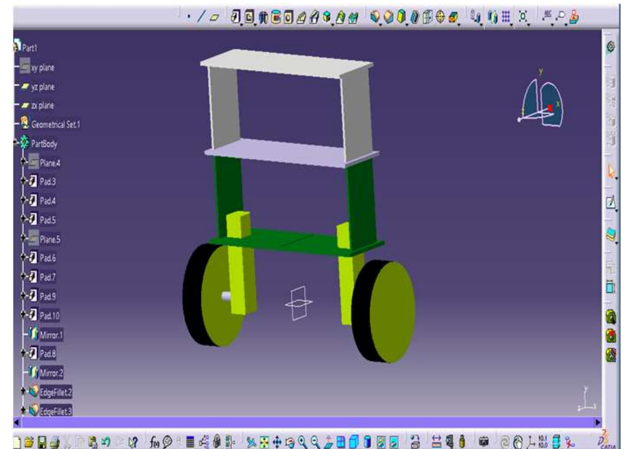
2nd Layer (Bottom view)

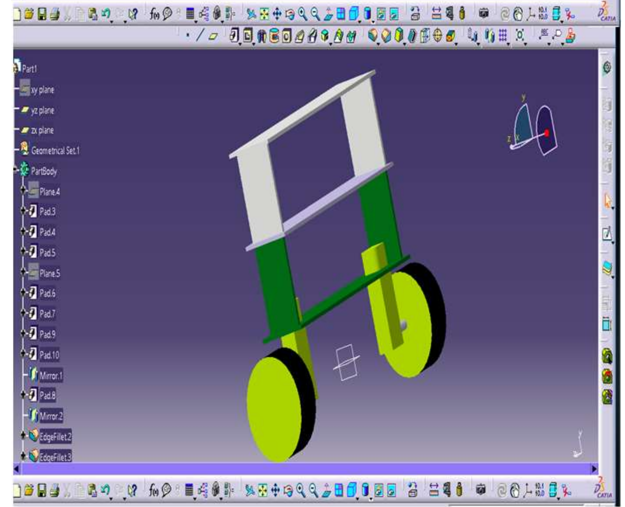

16.5 cm
0.5 cm
0.5 cm
6 cm
Battery pack(in the center)
0.5 cm diameter holes are there to attach the three layers. The same dimensions & positioning is done in every layer.
5.4 cm
Screw to attach
Screw to attach
10 cm
2 cm
0.5 cm
0.5 cm

3rd Layer

STRUCTURE DESIGN USING CAD

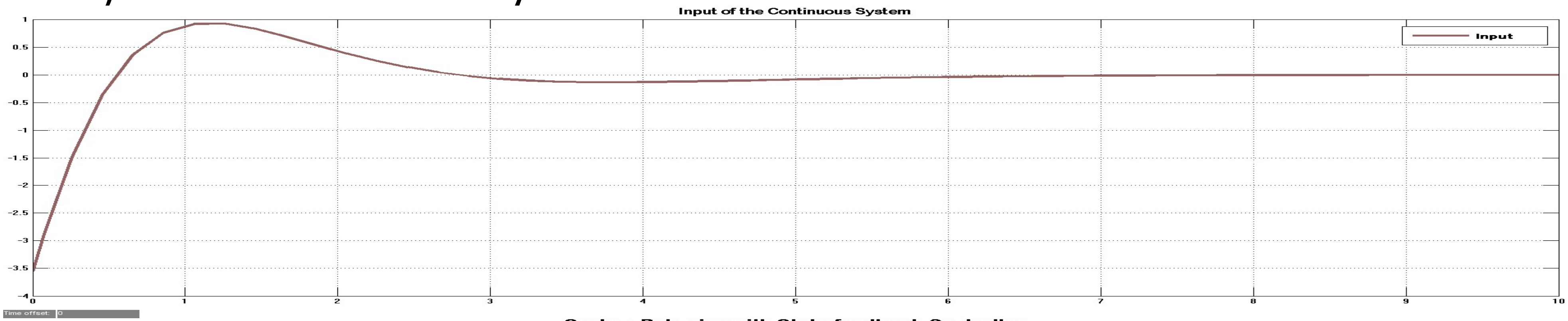


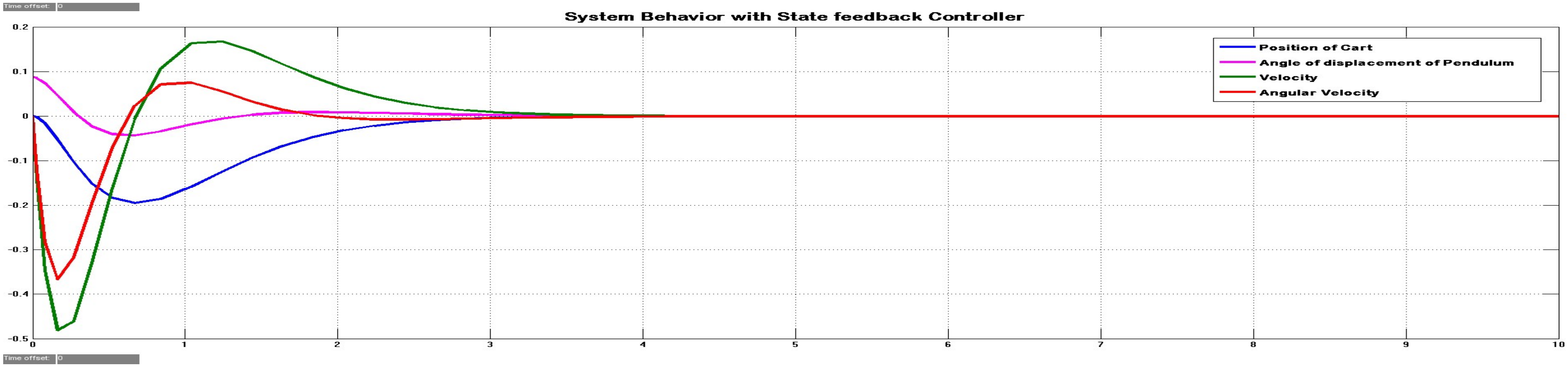




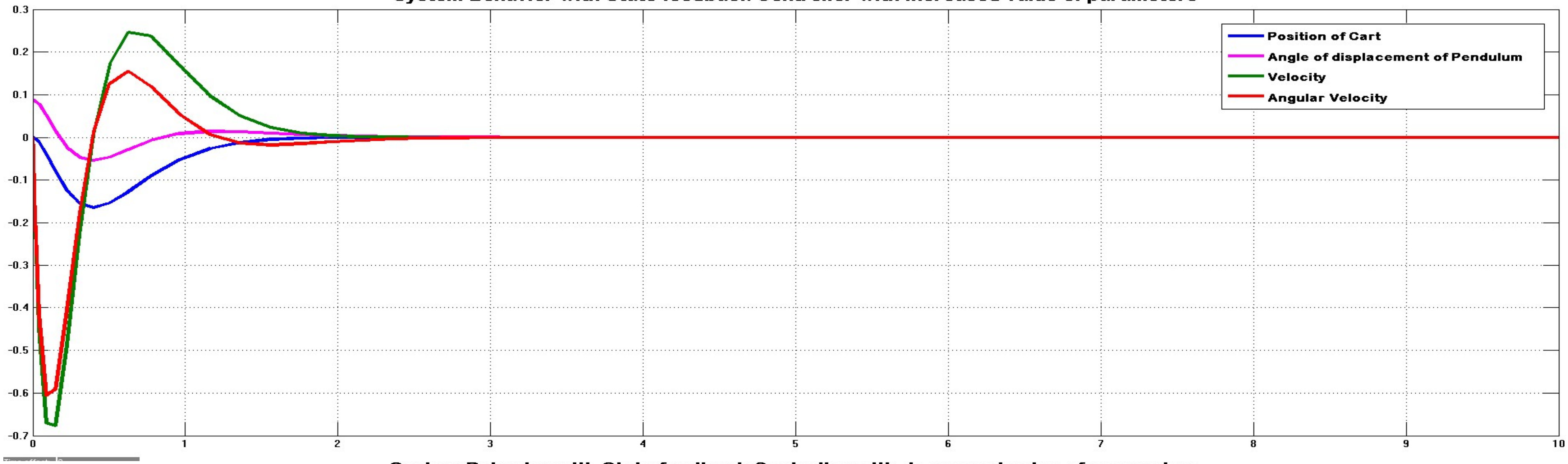
RESULTS AND GRAPHS

1) Continuous time system with State feedback Controller

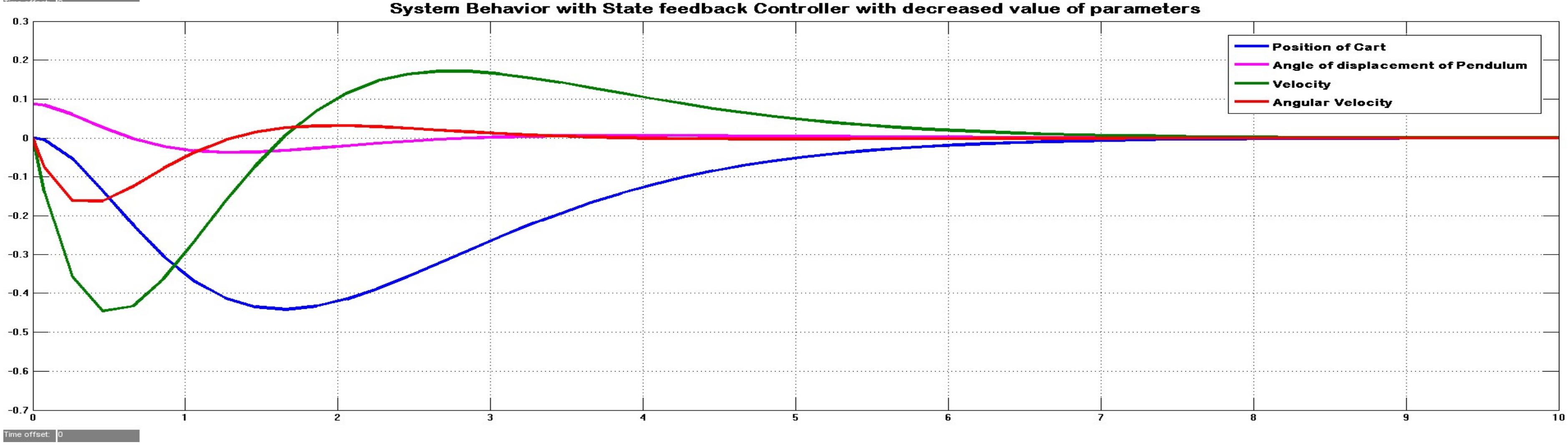




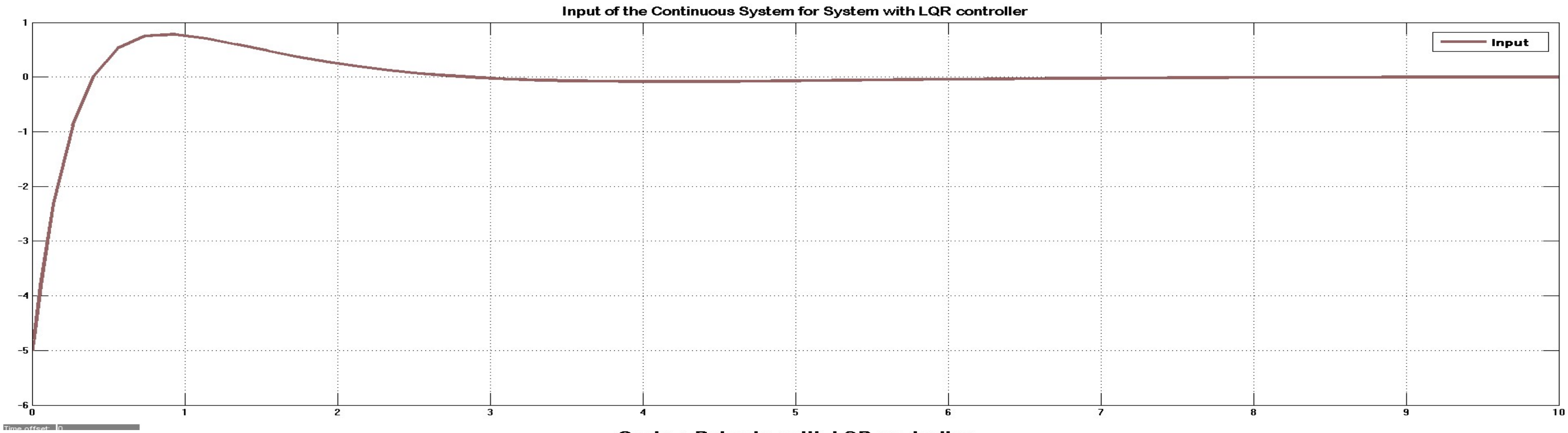
System Behavior with State feedback Controller with increased value of parameters

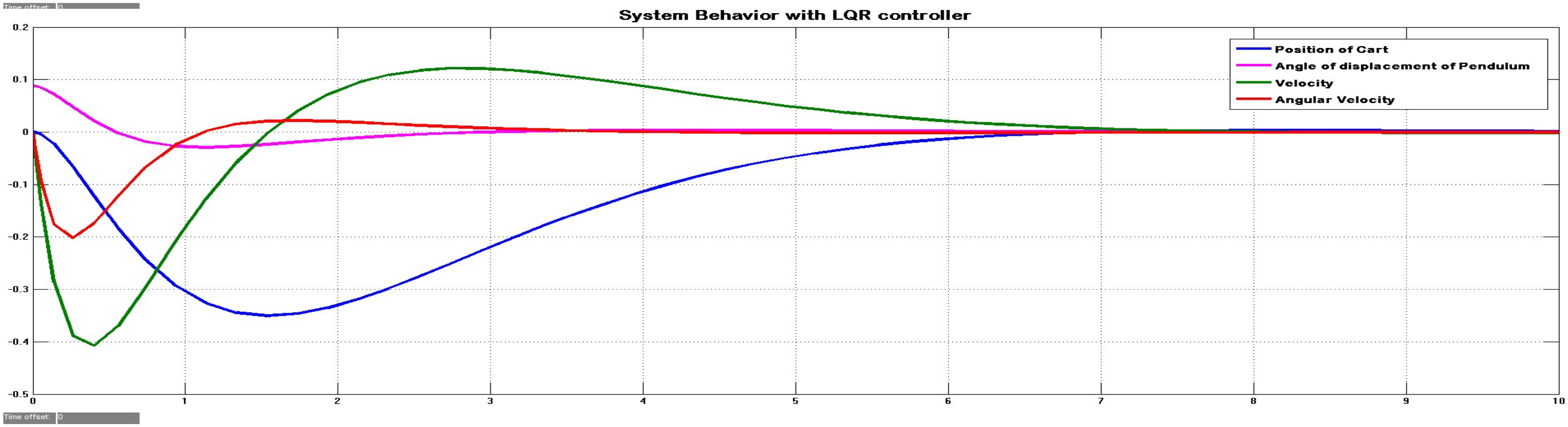


System Behavior with State feedback Controller with decreased value of parameters

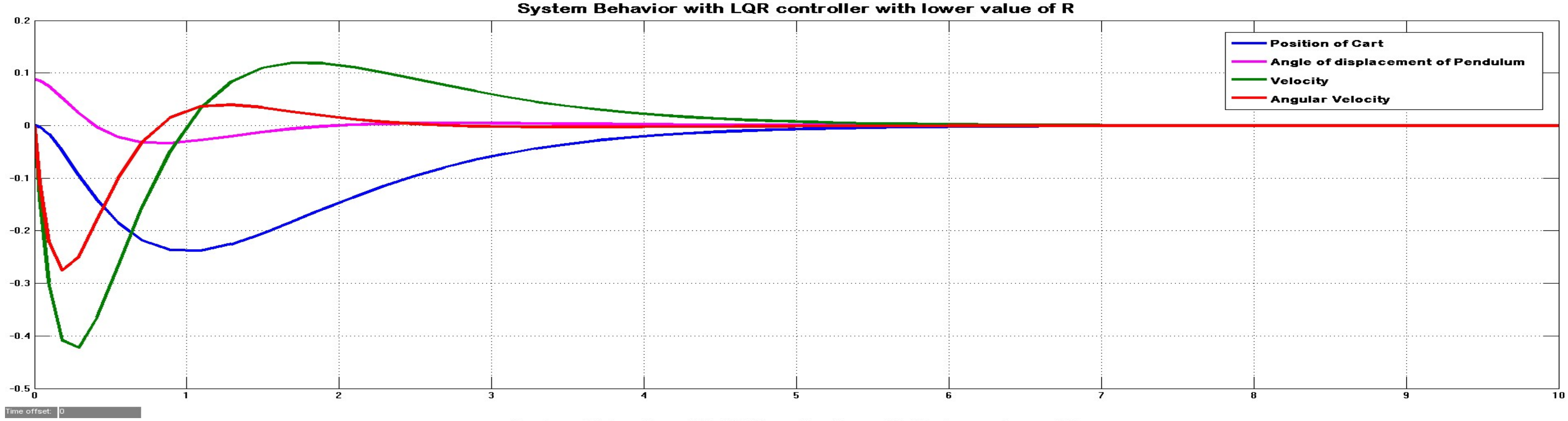


2) System behavior with LQR controller

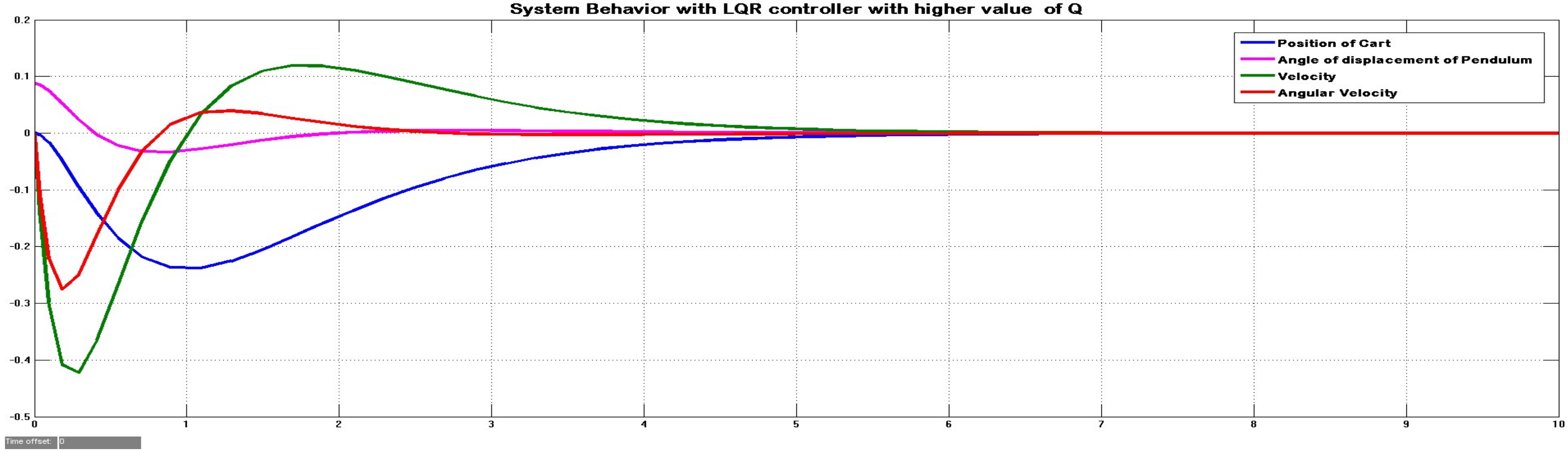




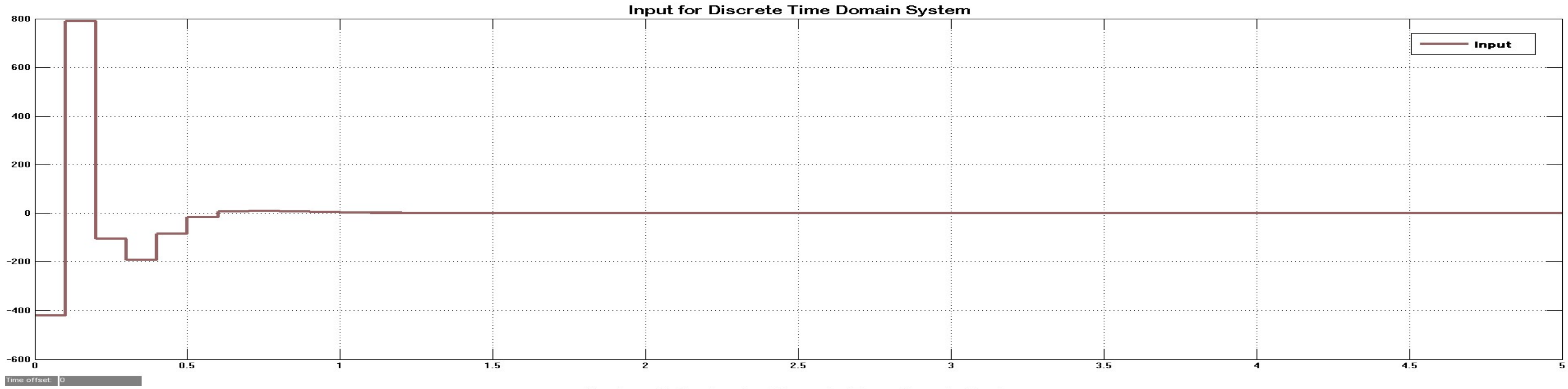
System Behavior with LQR controller with lower value of R

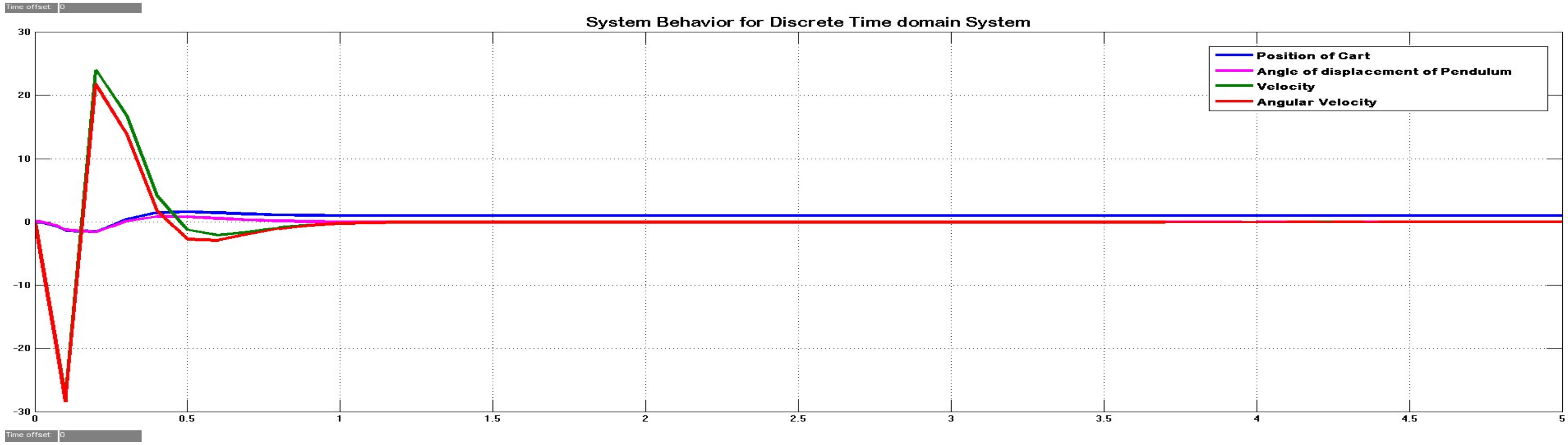


System Behavior with LQR controller with higher value of Q

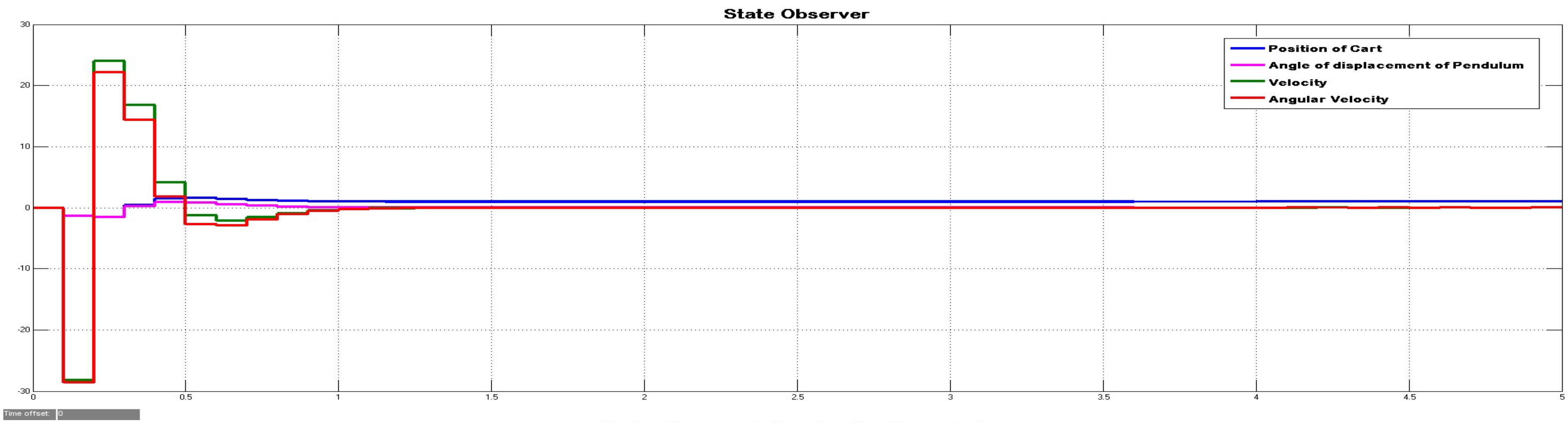


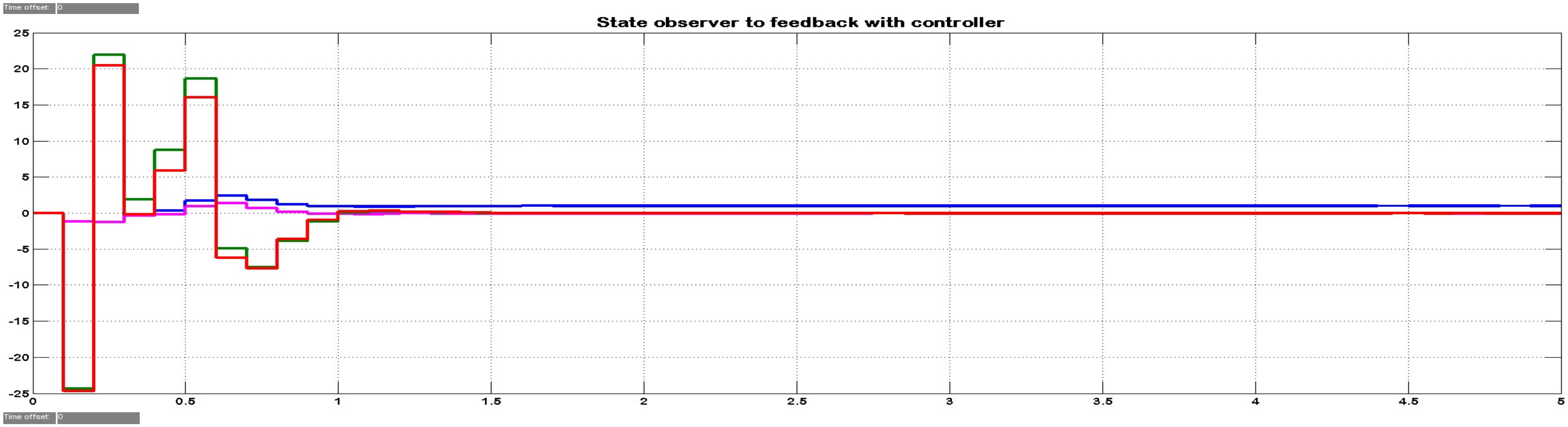
3) Discrete time State-space model

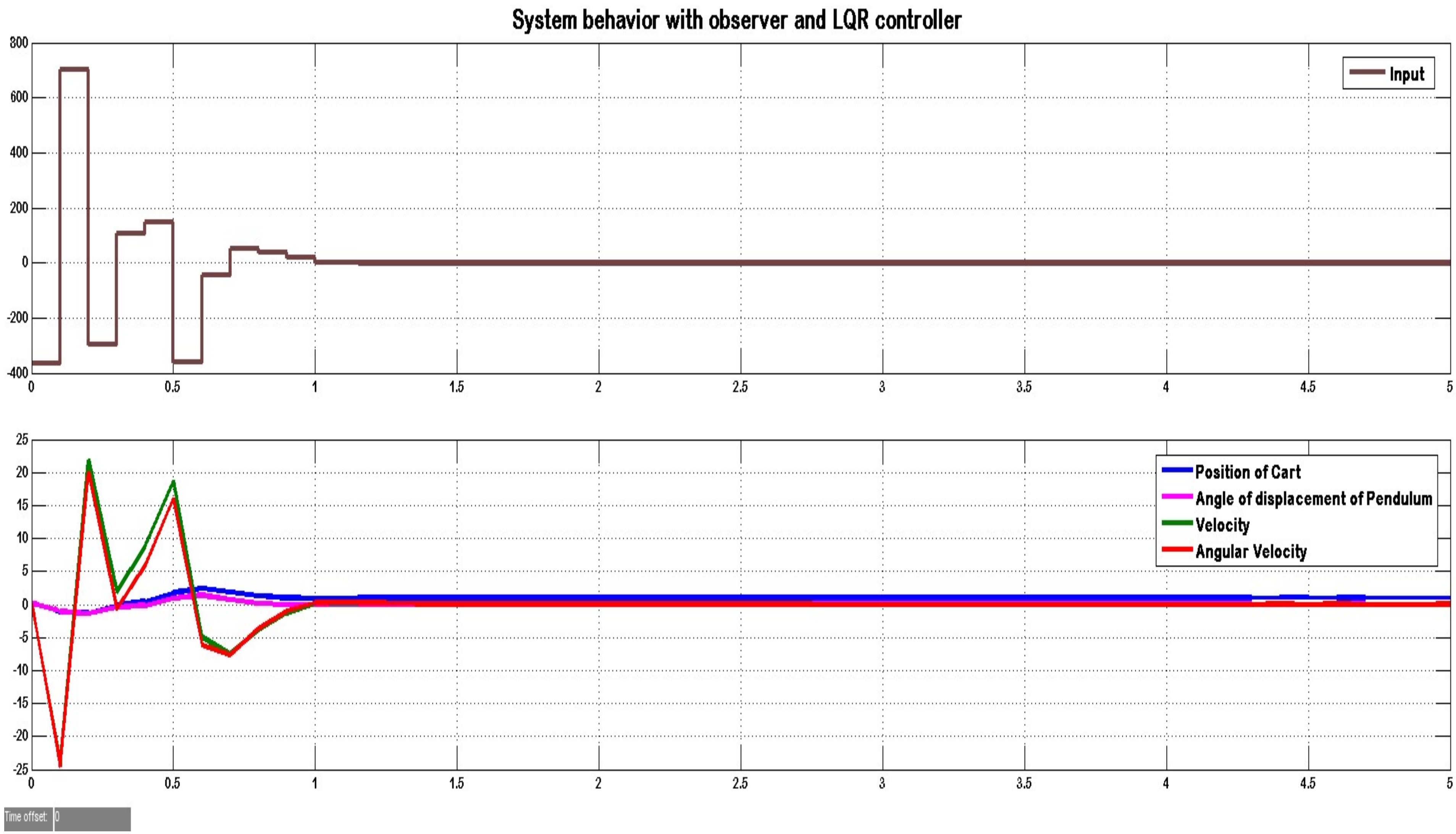




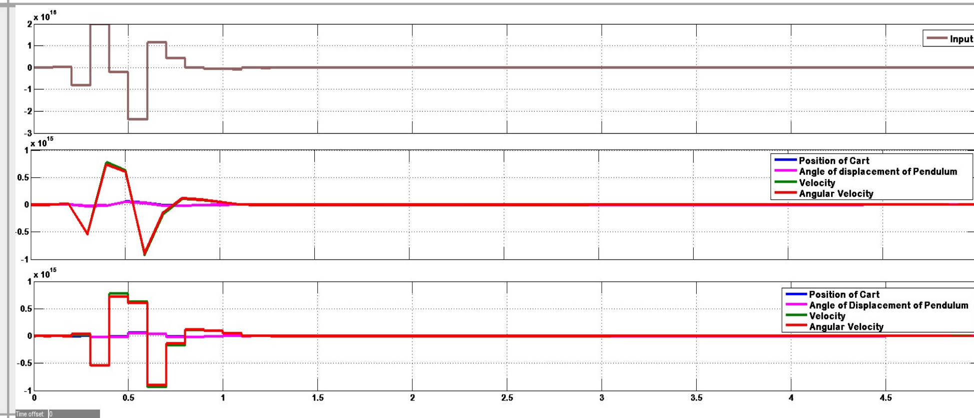
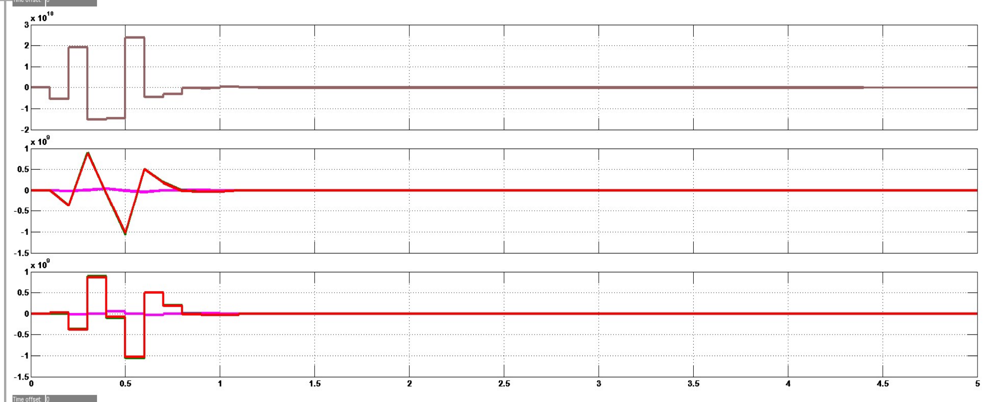
4) System Behavior with State Observer

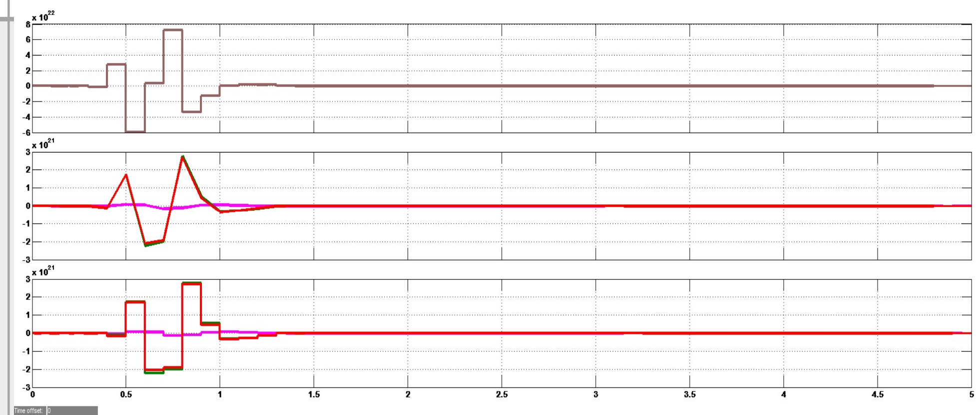
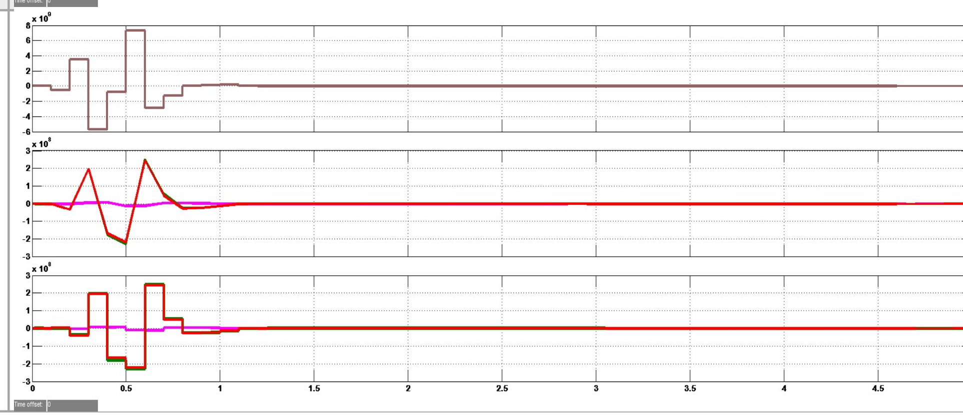





System behavior with observer and LQR controller
Input
Position of Cart
Angle of displacement of Pendulum
Velocity
Angular Velocity
Time offset: 0

5) System Analysis with respect to time

Sl. no	Position of Cart	Angle of displacement of Pendulum	Velocity	Angular Velocity	System behavior w.r.t time
1	0	5°	0	0	
2	0.2	15°	0.3	0.2	

Sl. no	Position of Cart	Angle of displacement of Pendulum	Velocity	Angular Velocity	System behavior w.r.t time
3	1	45°	0	0	
4	8	60°	4	6	

CONCLUSION

Inverted Pendulum is very difficult system to control due its intrinsic non linearity and instability. State feedback based control techniques namely pole placement and linear quadratic regulator is simulated with the help of MATLAB. The state feedback gain matrices are obtained and then closed loop responses for both cart position and angle of Inverted Pendulum are found to be satisfactory. Then the system states are estimated and observer based controller is designed. The observer based controller response is found to be same as state feedback closed loop response. Thus the system states are estimated successfully.

In this project, a dynamical model of the pendulum cart system is represented. The balancing and tracking control of the cart system is developed and analyzed. From graphs, it is analyzed that in a Controller based continuous time system, the graph converges fast and gets stable with increase in its parameters. With the use of LQR controller, when we go lower with R value, the input becomes less and behavior of the system becomes better. With increase in Q value, the performance of the system becomes better. For discrete state space model, ZOH (Zero order hold) is used for sampling the states of the system. A controller is efficient when it has access to all the states of the system. Therefore, an observer is constructed to get the output of the real system.

As a conclusion, the completion of the project managed to achieve all the objectives mentioned above slides.

LIMITATIONS AND FUTURE SCOPE

- Ackermann's approach for state feedback is not appropriate for large dimensional system.
- Many Robots have been developed with different methods of control to keep the Pendulum stable. Many controllers such as Proportional derivative controller, Proportional derivative controllers, fuzzy control are applied to make the system balance. There comes a drawback to these methods that all the states of the system is unable to get controlled. Hence , a state feedback controller is designed to overcome this drawback. An optimal control has been obtained to keep the system at upright position and stable.
- Therefore, the concept and method of stabilization of Inverted Pendulum could be implemented for a working hardware model.

ACKNOWLEDGMENT

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