OPTIMAL CONTROL OF INVERTED PENDULUM AND ITS APPLICATION

A Project Report
Submitted in partial fulfillment of the requirements
for the award of the Degree of

BACHELOR OF ENGINEERING

IN

ELECTRICAL & ELECTRONICS ENGINEERING

BY:

SOMPURNA MODI (BE/15225/16)

GAURAV THAKUR (BE/15088/16)

Under the Guidance of: Mrs. SHRADHA KISHORE

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

BIRLA INSTITUTE OF TECHNOLOGY, MESRA

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DECLERATION CERTIFICATE

This is to certify that the work presented in the project report entitled "**OPTIMAL CONTROL OF INVERTED PENDULUM AND ITS APPLICATION**" in partial fulfillment of the requirement for the award of Degree of **Bachelor of Engineering in Electrical and Electronics Engineering** of Birla Institute of Technology Mesra, Ranchi is an authentic work carried out under my supervision and guidance.

To the best of my knowledge, the content of this project does not form a basis for the award of any previous Degree to anyone else.

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CERTIFICATE OF APPROVAL

The following project report entitled "**OPTIMAL CONTROL OF INVERTED PENDULUM AND ITS APPLICATION**", is hereby approved as a creditable study of research topic and has been presented in satisfactory manner to warrant its acceptance as prerequisite to the degree for which it has been submitted.

It is understood that by this approval, the undersigned do not necessarily endorse any conclusion drawn or opinion expressed therein, but approve the thesis for the purpose for which it is submitted.

(Internal Examiner) (External Examiner)

(Chairman)

Head of the Department

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ABSTRACT

The primary incentive of the project is to develop general understanding of control theory. For the last few decades, "the Inverted Pendulum" has been the most popular topic, among others, for teaching and research in control theory and robotics. The material and methods learnt have a wide array of applications; for example, inverted pendulums have been used to model human locomotion, which then was used to develop bipedal robots. A pendulum cart system is a non-linear and an unstable system, also known as Inverted Pendulum system which is designed with control methods and state observer to make the system stable. This standard design has a vast application in the field of Control Theory. To stabilize the proposed cart system, State-space Observer based Linear Quadratic Regulator optimal control has been implied. This report describes the dynamical model of the system, followed by pole-placement based state feedback controller and using controllability and observability of the system, optimal control has been obtained. Moreover, both the alternating frameworks with respect to time; Continuous time and Discrete time control system has been attained. There are many applications of Inverted Pendulum Cart system. One of them is Two-wheel Self balancing Robot. In recent years, the flexible robot has become one of the famous and most interesting part in the control field for researchers as well as engineers. Many works are done on stabilizing the robot since the classic problem is the difficulty in its control and stabilization. Many different optimal controls have been researched and practiced for developing a stable two wheeled robot. Here, some work has been done upon this interesting field. The report describes the mechanical overview of the Two-wheel Self Balancing Robot. This overview is based on the balancing properties for the robot by selecting appropriate sensors. We have here designed a small prototype of Segway so we selected sensors that are effective as well as compatible to the weight decided for the robot. The layers have been designed and the structure using CAD. The measurements have been done considering the Center of Mass and the placement of the Components have also been described. The connections have been shown among the sensors, wheels and the battery for the setup and working of the robot. Moreover, the usability of such prototype has also been explained. The overview of the design of the prototype of Segway aims to carry a weight maximum of 25kg upon itself.

1. Introduction

The chapter aims to provide an introductory information about the project. At first, the requirement of the project is described, followed by the evolution of the concept of Inverted Pendulum and afterwards, the objectives of the project are defined.

1.1. Introduction

Pendulum-cart system proves to have a vast concept where researchers [1] could find different controlling strategies which has been used in many applications such as Two wheels self-balancing Robot [2], Segway [3], Hoverboard, etc. This concept is used to design a self-balancing robot and was also recognized as a human transporter. A feedback controller used prototype HTV was also developed which can move both on ground or on slope surfaces [4]. All have the basic idea to sense tilt and drive the wheels in the required direction to keep the system erect i.e. wheels rotate in the forward direction when the cart tilts forward and in the backward direction when the cart tilts backwards. The tilting is due to the fact that the center of mass of the cart lies above the pivot point and hence the cart is initially unstable. Furthermore, external disturbances can also make the system lose balance. A linear mathematical method cannot balance the non-linear system, hence, many different control theory methods are applied. Many controllers such as Proportional derivative controller [5], Proportional derivative controllers, [6]fuzzy control [7] are applied to make the system balance. There comes a drawback to these methods that all the states of the system is unable to get controlled. Hence , a state feedback controller is designed to overcome this drawback. An optimal control has been obtained to keep the system at upright position and stable.

Moreover, one of the Application based on the concept of Inverted Pendulum i.e Two-wheel Self balancing Robot is also represented with its mechanical overview. The connections between the sensors for the Robot has also been shown in this thesis. The Robot will be prevented from falling by giving acceleration to the wheels according to its inclination from the vertical. If the bot gets tilts by an angle, than in the frame of the wheels; the center of mass of the bot will experience a pseudo force which will apply a torque opposite to the direction of tilt.

1.2. History of Evolution

An inverted pendulum is the base of all self -balancing mechanisms. It requires assistance to stay perpendicular to the base attached to it. An simple pendulum is stable because it hangs downwards but an inverted one, being forced against gravity, is quite naturally unstable and needs to be kept upright by some mechanism. The mass, it is attached to, when moves forward or backward horizontally, the pendulum fails to stay upright in the same perpendicular position to the mass. To keep the pendulum in that state we need to apply a certain amount of torque at the center of it and make sure it stays upright by moving the whole thing forward or backward directions. This is the principle used for a self-balancing mechanism. In this section, the evolvement of the concept of Inverted pendulum is described. The fundamentals of inverted pendulum systems are described such as determination of the equilibrium point and what makes the system interesting to control engineers.

1.2.1. Inverted Pendulum System

A pendulum that has its center of mass above its pivot point known as Inverted Pendulum. It is unstable and will fall over if not supported. It can be suspended stably in an inverted position by using a control system to monitor the angle of the pole and move the pivot point horizontally back under the center of mass when it starts to fall over, to keep it in an upright position and balanced. The inverted pendulum is an interesting topic in dynamics and control theory and is used as a benchmark for testing control strategies. It is often implemented with the pivot point mounted on a cart that can move horizontally under control of an electronic servo system. Most applications limit the pendulum to 1 degree of freedom by affixing the pole to an axis of rotation, whereas a normal pendulum is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain at upright position ; this can be done either by applying a torque at the pivot point, by moving the pivot point horizontally as part of a feedback system, changing the rate of rotation of a mass mounted on the pendulum on an axis parallel to the pivot axis and thereby generating a net torque on the pendulum, or by oscillating the pivot point vertically. A simple demonstration can be observed of moving the pivot point in a feedback system by balancing an upturned broomstick on the end of one's finger.

Tiltmeter is a second type of inverted pendulum for tall structures, which consists of a wire anchored to the bottom of the foundation and attached to a float in a pool of oil at the top of the structure that has devices for measuring movement of the neutral position of the float away from its original position.

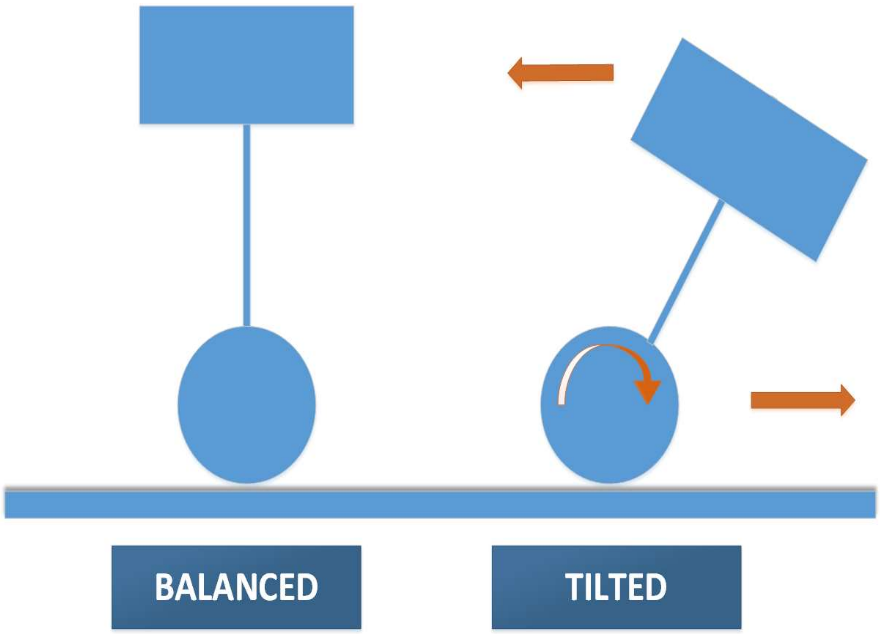


fig1: Inverted Pendulum

1.2.2. Simple Pendulum

When a body with some mass is suspended from a light weighted string(negligible) that can oscillate when displaced from its rest position.

$T_P = 2\pi\sqrt{\frac{L}{g}}$

T_P is period, L is the pendulum length and g is acceleration due to gravity.

The angular velocity ω and the rate change of angular velocity $\frac{d\omega}{dt}$, is given by

$$\frac{d\theta}{dt} = \omega \tag{1}$$

$$\frac{d\omega}{dt} = \frac{-MgL}{J}\sin\theta \tag{2}$$

J is the moment of inertia, and g is the gravitational force.

At rest, $\omega = 0$ and $\frac{d\omega}{dt} = 0$

The equilibrium points are $\theta = 0°$ and $\theta_1 = 0°$ and $\theta_2 = 180°$

θ_1 is stable (Not applicable for robots)

θ_2 is the target position for inverted pendulum systems. But at this point, the system is unstable and can move away from equilibrium position due to any external forces or disturbances.

Hence, actively balancing is the major requirement to keep the pendulum at equilibrium point.

1.2.3. Non minimum Phase zeros

Perhaps the most fascinating aspect of inverted pendulum systems is the notion of non-minimum phase zeros. Through [8], a summary of the consequences of the non-minimum phase zeros is given, When the system is casual and stable but the inverses are unstable and casual, these are called non minimum phase systems. This is opposite of linear time invariant system.

- The zeros of the discrete time system are outside the unit circle.
- The zeros of the continuous time system are in right hand side of the complex plane.

Given the transfer function G(s),

$$G(s) = N(s)\ D(s) \tag{3}$$

A zero is the root in the numerator, N(s), of G(s). A non-minimum phase zero is a positive zero that is in the right half of the pole-zero plot. To exhibit the behavior, in which direction reversals occurs, has to be an odd number of zeros in the transfer function.

This gives a much larger contribution to phase shift though the magnitude response which is equivalent to a minimum phase system. From the transfer function, further conclusions can be made that if there are positive poles, the plant is unstable and based on parity interlacing principle. If the plant has odd number of positive real poles to the right of the non-minimum poles the plant cannot be controlled by a stable controller. This is actually present in a linearized transfer function of an inverted pendulum on a cart. Further discussion in context of the its importance is given in chapter 4.

1.3. Objectives

The objectives of this thesis are as follows:

- At first, the dynamical system of Inverted Pendulum Cart using MATLAB is created.
- Then, System Analysis controller is designed using MATLAB tools and its simulation.
- Design of a State space model with optimal control and its simulation is done.
- Then, a State Observer is designed and the closed loop is investigated using an Observer.
- Time discrete Controller and Observer is designed and simulated.
- At the end, the mechanical Overview of one of the Applications of Pendulum Cart System is developed.

2. Literature Review

In this chapter, the literature review of the work done on this field till date is discussed. Moreover, the controlling technique used in this project has also been described, followed by the properties of the sensor used for building up the Robot.

2.1. Scope of Work

A pendulum with its bob hanging directly below the support pivot is at a stable equilibrium point; there is no torque on the pendulum so it will remain stable and will not move, and if disturbed from this position will experience a restoring torque which returns it toward the equilibrium position. A pendulum with its bob in an inverted position, supported on a rigid rod directly above the pivot, 180° from its stable equilibrium position, it is at an unstable equilibrium point. At this point there is no torque on the pendulum, but the slightest displacement away from this position will cause a gravitation torque on the pendulum which will accelerate it away from equilibrium, and it will fall over and get unstable.

In order to stabilize a pendulum at this inverted position, a feedback control system is used, which monitors the angle of displacement of pendulum and moves the position of the pivot point sideways when the pendulum starts to fall over, to keep it balanced. The inverted pendulum has a vast part in the dynamics and control theory and is widely used for testing control algorithms (PID controllers, state space representation, neural networks, fuzzy control, genetic algorithms, etc.). Variations on this concept include multiple links, allowing the motion of the cart to be commanded while maintaining the pendulum, and like balancing the cart-pendulum system on a see-saw. The inverted pendulum is related to rocket, where the center of gravity is located behind the center of drag causing aerodynamic instability. The understanding of a similar topic can be shown by simple robotics in the form of a balancing cart. Some researches done on this concept is implied in many applications such as self-balancing personal transporters such as the Segway PT, the self-balancing hoverboard and the self-balancing unicycle, etc

Another way that an inverted pendulum may be stabilized, without any feedback or control mechanism, is by oscillating the pivot rapidly in up and down positions. This is called Kapitza's pendulum. If the oscillation is sufficiently strong (in terms of its acceleration and amplitude) then the inverted pendulum can recover from perturbations in a strikingly counterintuitive manner and can get stable. If the driving point moves in simple harmonic motion, the pendulum's motion is described by the Mathieu equation.

Moreover, in these recent years, the self-balancing robot has been a very fascinating topic for researchers. This topic holds a vast concept of Inverted pendulum in it, which makes the work interesting for researchers as well as engineers. JOE, a mobile, inverted pendulum is introduced [9], which has two wheels coupled with DC motors and a control system made up of two decoupled state space controllers. More work is done in this field where the two wheeled robot [10] is stabilized using the Linear Quadratic Regulator and Pole Placement Controller and a comparison is made to determine which of the two is better. Using DC motors, Arduino, Single-axis Gyroscope, 2-axis Accelerometer and a complimentary filter; PID and LQR based PI-PD control methods are performed and comparisons are made [11]

A Segway was introduced [12] a two wheeled self-balancing transport mode for Humans. It makes use of MPU6050, 6-axis Gyroscope and Accelerometer, Arduino Microcontroller, a motor driver and DC Motors. Here, a rider commands the Segway to move backward or forward, by tilting his/her body along with the handlebar in that direction. A Two-wheeled self-balancing vehicle commercially known as "Segway" and also the Segway can never stay upright. Besides the development of Segway, studies of two wheeled self-balancing robots have been widely reported.

Arduino is an open prototyping platform based up on AT mega processors .And It will be a fast becoming popular platform for both education [13] and product development, with applications ranging from robotics [14], [15]to process control [16], [17]and networked control [18]. More work has been done to make the robot remote sensing [19]

Work has been done on the stability and evaluates the performance of a PID controller in the self-balancing robot [20]. A PID controller is incorporated in the hardware section of the robot to improve and ensure a good performance for the system [21]

Segway is readily available in market since 2011 and is also termed as a "Human Transporter" [22]. Some has discussed the studies between different control systems that could be used.

2.2. Methodology

When a body with some mass is suspended from a light weighted string (massless), that mass oscillates when displaced from its rest position. Hence, the equilibrium points are $\theta 1 = 0°$ or $\theta_2 = 180°$ where θ is angle of the suspended mass.

Here, θ_2 is the target position for inverted pendulum systems. But at this point, the system is unstable and can move away from equilibrium position due to any external forces or disturbances. Hence, actively balancing is the key requirement to keep the pendulum at equilibrium point. This is why state feedback is used so that it sends the error from the desired tilt and the designed controller is actively balances the system.

In this report, the stabilization problem around the upper position of the pendulum is fixed. A dynamical system has been created using MATLAB. Analyzing the system, poles are obtained and a state feedback controller is designed. With the help of MATLAB Simulink, state space and optimal controllers are designed for continuous time control system. Thereafter a State Observer is designed and closed loop of the system is investigated including an observer. At the end, a time discrete controller and an observer is designed for the Pendulum- cart system to stabilize and move horizontally as desired.

For the application, an overview of Two-wheel Self balancing Robot has been represented with limited resources possible. The measurements for the structure set up has been done considering the Center of Gravity. The structure of the robot is explained in different layers with the at most information required.

2.3. Frameworks with respect to time

In this section, both the time domain systems are described which is used in further system stabilization with both types of frameworks as system input. The further description regarding the context is discussed in Chapter 8.

2.3.1. Continuous time domain System

The concept of system, basic in engineering analysis and synthesis, is introduced in this chapter. Considering the properties of linearity, time-invariance, causality and stability allows us to obtain useful models for actual devices. Each of these properties are defined and their application in the analysis of continuous-time systems illustrated. Linearity between the system's input and output, as well as the constancy of the parameters (time-invariance) simplify the mathematical models. Causality provides the cause-and-effect relationship between the input and the output of the system, while stability considers a desirable behavior of systems. Using the generic characterization of a signal in terms of impulse functions, linearity and time-invariance and the concept of the impulse response of the system, permits us to obtain the response of a linear time-invariant (LTI) system as a convolution integral. This convolution integral, although difficult to compute, has significant theoretical value. It allows us not only to determine the response of LTI systems, but also to characterize causal and stable systems.

A continuous-time system is a system in which the signals at input and output are continuous-time signals. The report considers signals with systems, especially the study of linear time-invariant dynamic systems. Simple examples of systems, ranging from the vocal system to simple circuits, illustrate the use of the linear time-invariant (LTI) model and point to its practical application. At the same time, modulators also show that more complicated systems need to be explored to be able to communicate wirelessly. Although a system is viewed as a mathematical transformation of an input signal (or signals) into an output signal (or signals), it is important to understand that such transformation results from an idealized model of the physical device. A system's approach to the theory of differential equations and some features of transforms are also considered. The general characteristics attributed to systems are classified such as static or dynamic systems, lumped- or distributed-parameter systems, and passive or active systems are also considered. The Laplace transform allows transient as well as steady-state analysis and converts the solution of differential equations into an algebraic problem and is very significant in the area of classic control. The report also provides the concept of transfer function that connects with the impulse response and the convolution integral. The analysis of systems with continuous-time signals by means of transforms is presented. When developing a mathematical model for a continuous-time system it is important to contrast the accuracy of the model with its simplicity and practicality.

2.3.2. Discrete time domain System

Digital control systems are time-discrete systems. The fundamental difference between continuous and time-discrete systems comes from the need to convert analog signals into digital numbers, and from the time a computer system needs to compute the corrective action and apply it to the output.

The theory of discrete-time signals and systems, whose basic theory is very much like that for continuous-time signals and systems. Characteristics such as energy, power, and symmetry of continuous-time signals are conceptually the same for discrete-time signals. Integrals are replaced by sums, derivatives by finite differences, and differential equations by difference equations. The discrete approximation of derivatives and integrals provides an approximation of differential equations, representing dynamic continuous-time systems by difference equations. A computationally significant difference with continuous-time systems is that the solution of difference equations can be recursively obtained and that the convolution sum provides a class of systems that do not have a counterpart in the analog domain. The relation that exists between the Z-transform and the Fourier representations of discrete-time signals and systems, not only with each other but with the Laplace and Fourier transforms could also be seen in the report. There is a great deal of connection among all of these transforms and a clear understanding of this would help with the analysis and synthesis of discrete-time signals and systems. In this report, the discrete-time systems such as linearity time invariance, stability, and causality have also been considered.

2.4. Controller

To maintain the robot upright, the most commonly used controllers are Proportional Integral-Derivative (PID) and the Linear Quadratic Regulator. Other have also explored the use of Linear-Gaussian Control (LQG), Fuzzy Logic and Pole Placement; In these where the robot displacement is also controlled, either LQG is used or combination of controllers. For example; LQR to maintain the robot upright and PID for controlling displacement, or a cascaded PID controller [23]. LQR is the explored controller in this thesis, thus further details will only be provided for it.

LQR is a form of optimal control that aims to minimize the performance index whist taking into account the control effort, as often, higher input effort would imply higher energy consumption [24]. LQR control requires derivation of the state-space model of the system, thus it is more challenging to implement. A great advantage of LQR is that unlike PID it can be applied to Multiple Input, Multiple Output (MIMO) systems.

To get rid of some problems that are faced by conventional PID controller, the another control strategy is used, which is Linear-Quadratic Regulator (LQR) optimal control, which we have described in this thesis. LQR is a control strategy which operates the system with minimum cost when the system dynamics is expressed by differential equations which are linear in nature. The performance measurement is expressed with a cost function which is quadratic in nature and made of state vector and control input control defines the pole location which are optimal in location depends on two cost function of the system. For obtaining the gains, which are optimal, the optimal performance index must be expressed first and then solve the State Dependent Algebraic Riccati Equation (ARE) for the system. The LQR design method consists of obtaining a state feedback controller K such a way that the quadratic performance index J is minimized. In the control strategy a feedback gain matrix is obtained which aids to minimize the quadratic performance index and makes the system stable.

Some requirement for the LQR controller are:

- Q must be positive definite or positive semi-definite symmetric matrix. R must be positive definite symmetric matrix.
- Weighting matrices should be diagonal matrix.
- The value of the elements of the weighting matrices are linked to the impact on the performance index J.

LQR is an automatic method to find out an appropriate state. Different preferable methods such as full state feedback controller can also be used to find a controller over the LQR controller. This gives a much clearer link between adjusted parameters and the resulting changes in the behavior of the controller. Difficulties are faced in finding out the right weighting matrices, which limits the application of the LQR Controller design for the system [25].

2.5. Gyroscope Fundamentals

Gyroscope devices are primary units for navigation and control systems in aviation, space, ships, and other industries. The main property of the gyroscope device is maintaining the axis of a spinning rotor for which mathematical models have been formulated on the law of kinetic energy conservation and the changes in the angular momentum. However, known mathematical models for the gyroscope effects do not match actual forces and motions underway. The gyroscope measures angular velocity, in radians per second or degrees per second. Intuitively, by integrating the angular velocity the tilt angle can be calculated. Since the gyroscope readings are taken at discrete time intervals, numerical integration is performed using the Euler method.

The nature of the gyroscope properties is more complex than is represented by contemporary theories. Recent investigations have demonstrated that gyroscopes have four inertial forces interdependently and simultaneously acting on them. These forces are internal kinetic energies generated by the mass-elements and center-mass of the spinning rotor and represented by centrifugal, coriolis, and common inertial forces as well as changes in angular momentum. The applied torque generates internal resistance torques that are based on action of centrifugal and coriolis forces; and the precession torques generated by common inertial forces and by the change in the angular momentum. Apart these, the friction forces acting on the gyroscope supports play considerable role in decreasing the internal kinetic energy of the spinning rotor. The new mathematical models for gyroscope effects describe clearly and exactly all known and new gyroscope properties. Mathematical models for the most unsolvable motions of the gyroscope with one side support are validated by practical tests. Formulated models for motions of the gyroscope represent fundamental principles of gyroscope theory based on the actions of internal centrifugal, coriolis and inertial forces and the change in angular momentum, and external applied and friction forces. This new theoretical approach for the gyroscope problems represents new challenge in engineering science.

2.6. Accelerometer Basic Principles

The accelerometer measures the acceleration relative to free fall. The acceleration is often measured in gs, which is based on earth's gravitational pull (9.81m/s). To determine the orientation of the accelerometer, it is assumed that the only force acting on the object is earth's gravitation pull. Gravity always acts 'down', thus when the object is tilted, the force is divided into components in the x, y, and z directions of the object. Since the axes are orthogonal to one another, Pythagoras theorem can be used to show the relationship between the forces.

The basic underlying working principle of an accelerometer is such as a dumped mass on a spring. When acceleration is experienced by this device, the mass gets displaced till the spring can easily move the mass, with the same rate equal to the acceleration it sensed. Then this displacement value is used to measure the give the acceleration.

Accelerometers are available as digital devices and analog devices. Accelerometers are designed using different methods. Piezoelectric, piezoresistive and capacitive components are generally used to convert the mechanical motion caused in accelerometer into an electrical signal.

Piezoelectric accelerometers are made up of single crystals. These use the piezoelectric effect to measure the acceleration. When applied to stress, these crystals generate a voltage which is interpreted to determine the velocity and orientation.

Capacitive accelerometers use a silicon micro-machined element. Here capacitance is generated when acceleration is sensed and this capacitance is translated into a voltage to measure the velocity values.

Modern accelerometers are the smallest MEMS, consisting of a cantilever beam with proof mass. Accelerometers are available as two-dimensional and three-dimensional forms to measure velocity along with orientation. When the upper-frequency range, high-temperature range, and low packaged weight are required, piezoelectric accelerometers are the best choice.

3. Mathematical Background

3.1. Dynamic Equations

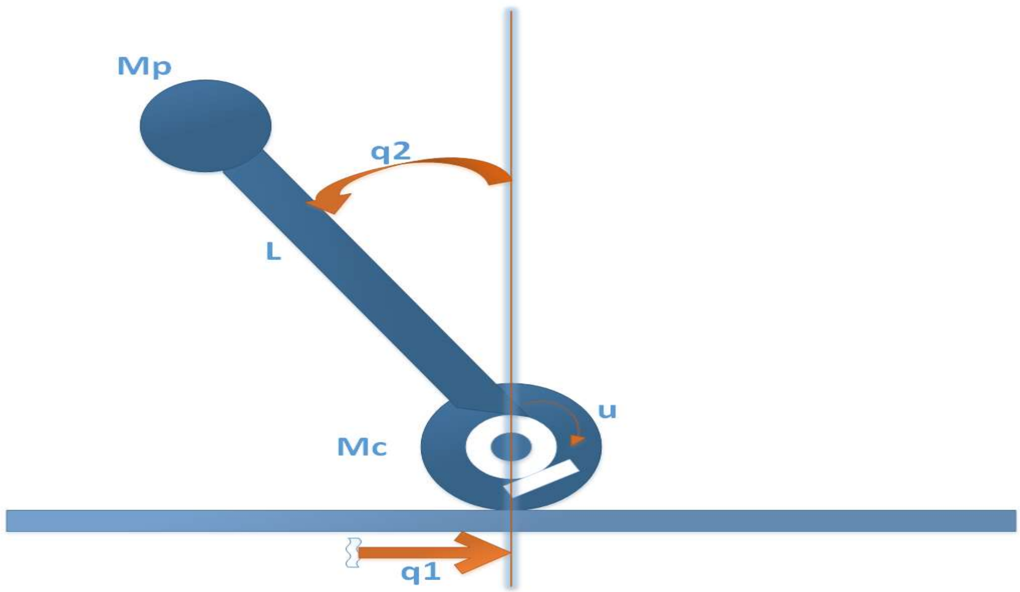


fig2: pendulum cart system

In fig2, Pendulum is connected to a pair of only-rolling wheels with one translation degree of freedom

Mp is the mass of the pendulum, Mc is the mass of the cart and L is the length of the massless rod.

Model of the system has two coordinates:

a) q_1 is the displacement of the cart.

b) q_2 is the angle of pendulum

Using Lagrange-Formalism [6] or Newton Euler for non-linear system is in the following form:

$$M(q)\ddot{q} + h\,(q, \dot{q}) = gc\,u \qquad (4)$$

where

$$M(q) = \begin{bmatrix} Me + Mp & -L\,Mp\,Cos\,q2 \\ -L\,Mp\,Cos\,q2 & L^2 Mp \end{bmatrix}$$

$$h(q,\dot{q}) = \begin{bmatrix} L\ Mp\ q2\ Sin\ q2 \\ -L\ g\ Mp\ Sin\ q2 \end{bmatrix} + \begin{bmatrix} d1\ \dot{q1} \\ d2\ \dot{q2} \end{bmatrix}$$

$$g_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where d1 and d2 are damping factors.
These represents friction in the cart displacement and joint displacement respectively. This defines the motion of the system.
For state space representation,

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \tag{5}$$

which are q1 and q2 respectively with their corresponding velocities.

$$\dot{x} = F(x) + G(x)u \tag{6}$$

$$\dot{x} = \begin{bmatrix} \dot{q1} \\ \dot{q2} \\ -M^{-1}(q)\ h(q,\dot{q}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M^{-1}(q)gc \end{bmatrix} u \tag{7}$$

Using the Taylor Method, we could linearize this non- linear equation (3) around $\dot{q1} = \dot{q2} = 0$ about the origin basically to get the linear differential equation or system equation or state-space equation for the system.

$$\dot{x} = Ax + Bu \tag{8}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{g\ Mp}{Mc} & \frac{-d1}{Mc} & \frac{-d2}{L\ Mc} \\ 0 & \frac{g\ (Mc+Mp)}{L\ Mc} & \frac{-d1}{L\ Mc} & \frac{-d2\ (Mc+Mp)}{L^2 Mc\ Mp} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{Mc} \\ \frac{1}{L\ Mc} \end{bmatrix}$$

3.2. Force Analysis

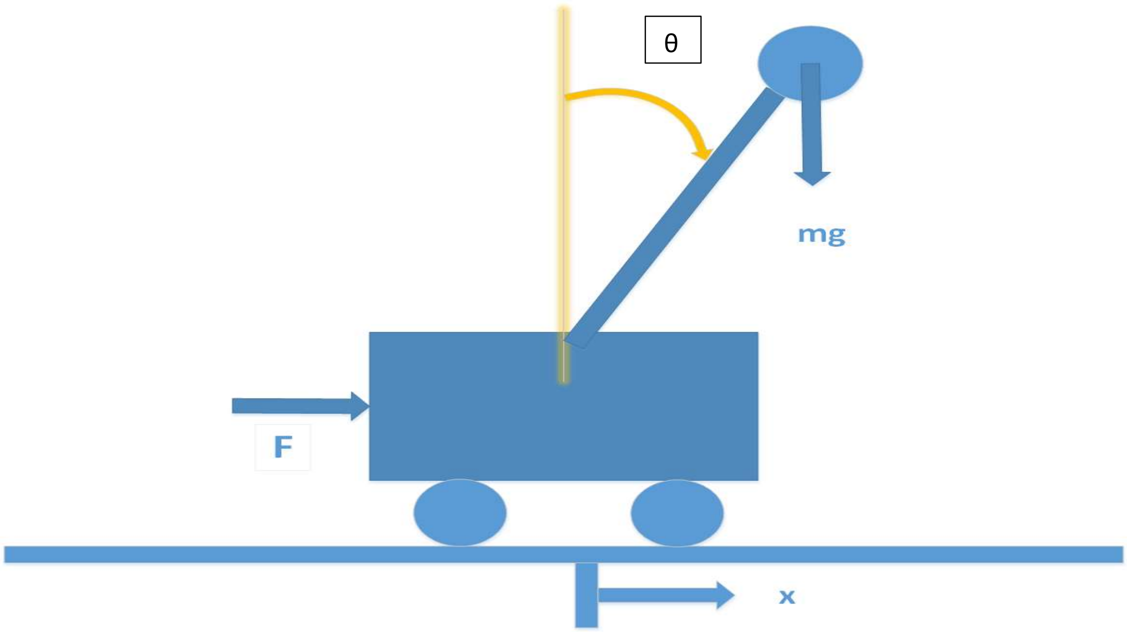


Fig3: free body diagram

Here, F is the force applied to the cart externally, x is the displacement of the cart, mg is the force at downward direction and θ is the angle of displacement of the pendulum.

Initial condition of the system, θ=0, x=0

Using Newtonian dynamics method,

Total force along the horizontal axis of the cart,

$$\mathrm{M}\dot{x} + N = F \tag{9}$$

$$\mathrm{N} = \mathrm{m}\ddot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta \tag{10}$$

$$\mathrm{P} - \mathrm{mg} = -\mathrm{ml}\ddot{\theta}sin\theta - ml\dot{\theta}^2 - ml\dot{\theta}^2 cos\theta \tag{11}$$

$$\mathrm{Plsin\theta} - \mathrm{Nl\ cos\theta} = \frac{1}{3}\, ml^2\ddot{\theta} \tag{12}$$

$\ddot{x}, \ddot{\theta}$ is the cart's acceleration and the acceleration of the angle of pendulum rod respectively.

$\dot{x}, \dot{\theta}$ is the speed of the cart and angular velocity of the swinging pendulum respectively.

And g is the acceleration of gravity.

Equation of Motion:

$$(M+m)\ddot{x} + ml\,\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta = F \tag{13}$$

$$\frac{4}{3}\mathrm{ml}^2\theta + \mathrm{ml}\ddot{x}cos\theta - mglsin\theta = 0 \tag{14}$$

Considering swinging rob to give small vibration near the set point θ =0, to localize the type linearization which means cosθ ≈1, sinθ≈θ

When θ= 0,$\dot{\theta}^2$as higher order infinitesimal to make appropriate treatment available. We get:

$$(\mathrm{M}+\mathrm{m})\ddot{x} + ml\ddot{\theta} = F \tag{15}$$

$$\frac{4}{3}\mathrm{ml}^2\ddot{\theta} + \mathrm{ml}\ddot{x} - mgl\theta = 0 \tag{16}$$

Equation of state of Inverted Pendulum Cart System :

$$\ddot{\theta} = \frac{3\mathrm{g}(\mathrm{M}+\mathrm{m})}{\mathrm{l}(4\mathrm{M}+\mathrm{m})}\theta + \frac{-3}{l(4M+m)}F \tag{17}$$

$$\ddot{x} = \frac{3\mathrm{mg}}{4\mathrm{M}+\mathrm{m}}\theta + \frac{4}{4M+m}F \tag{18}$$

4. Architecture of the Inverted Pendulum Cart System

In this chapter, the base of the simulation modeling has been described. The controller for attaining the optimal control, the parameters for the cart system and the state space model for the system are shown.

4.1. Inverted Pendulum Cart system model

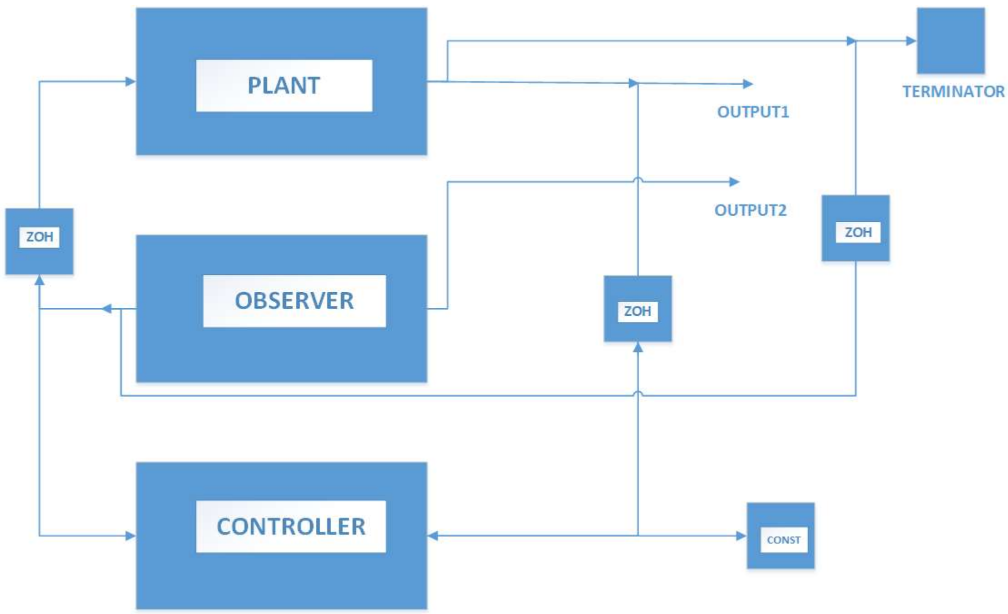


Fig4 : Model considering States of the System

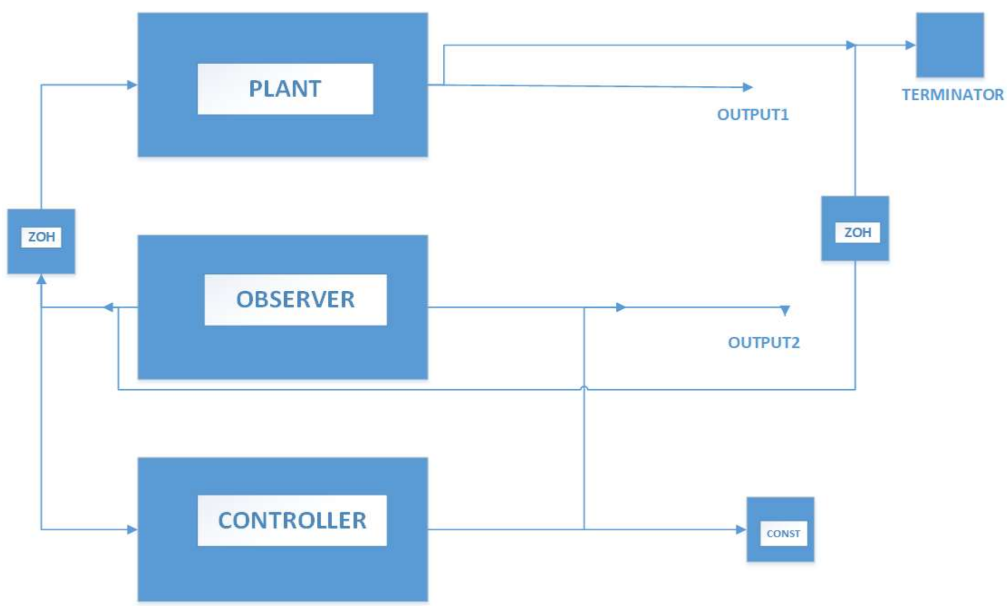


Fig5: Model of Inverted Pendulum Cart System

4.2. Parameters of the system

COMPONENT	ABBREVIATION	VALUE
Mass of Cart	M_C	1.5 kg
Mass of Pendulum	M_P	0.5 kg
Length of Pendulum from hinge point	L	1 m
Angle of displacement of Pendulum	q2	will be given
Displacement of the Cart	q1	will be given
Force applied to the Cart	u	will be given
Damping Factor during the cart displacement	d1	0.01
Damping Factor during the joint displacement	d2	0.01
Gravitational constant	g	9.82 m/s^2

Table1: Parameters of the Pendulum cart system

The state space representation for the system is:

$$A = \begin{bmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0 & 3.2733 & -0.0067 & -0.0067 \\ 0 & 13.0933 & -0.0067 & -0.0267 \end{bmatrix} \tag{19}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0.6667 \\ 0.6667 \end{bmatrix} \tag{20}$$

When obtaining the poles of the system, the result says that there is one pole at the right-hand side which means that the resistance is not stable.

System eigen values were computed to be (21)

$$= \begin{bmatrix} -3.6327 \\ 3.6043 \\ -0.0050 \\ 0 \end{bmatrix}$$

4.3. Root Locus of the system

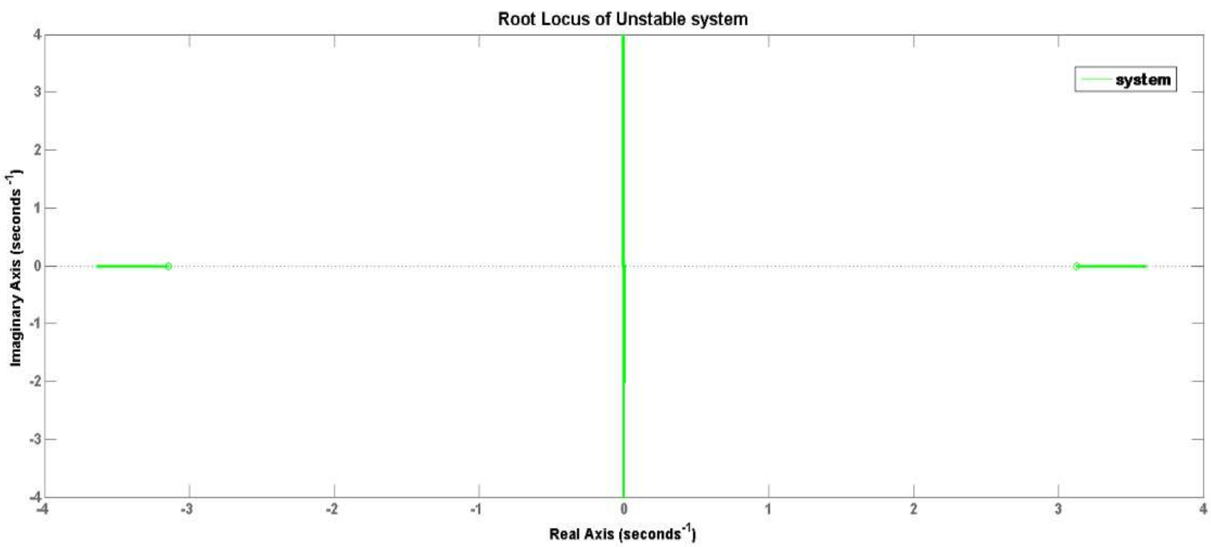


Fig6: Root locus of the Unstable Pendulum System

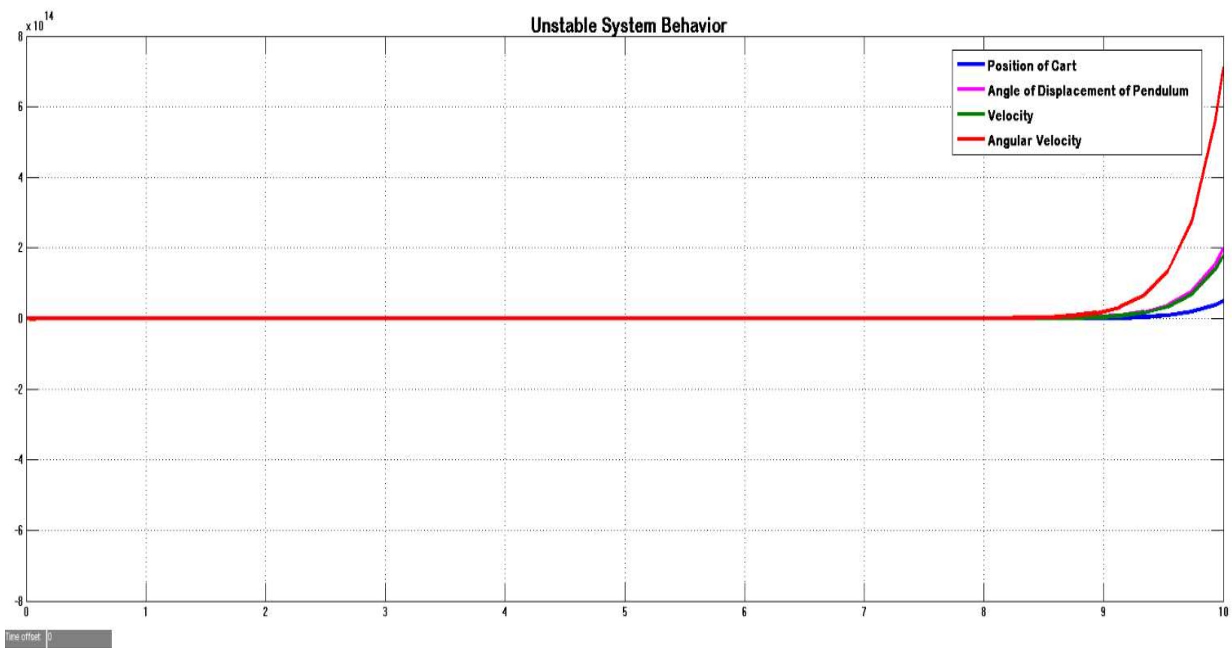


Fig7: Unstable System Behavior

In fig6, Root Locus of the system is plotted. Poles of the system are always on the right-hand side, hence there is no peak control that can stabilize the system. There is no gain that can bring all of the poles on the left side of the system. Hence, peak control is not enough for the system.

At initial condition,

Position of the cart = 0; Angle of displacement of pendulum to the cart = 5°; Velocity = 0; Angular Velocity = 0;

x_0 = [0; 5*pi/180; 0; 0]

Since the system is not stable, in fig every state goes to infinity.

4.4. Controllability and Observability

The necessary and sufficient condition for a feedback-based controller is that it must be completely state controllable [26]. For the proposed Linear invariant system, it is said to be controllable if the Rank of controllability matrix equals 4.

Taking C = [1;0;0;0] q1 as output

$$Sc = [B\ AB\ A^2B \ldots\ldots\ldots A^{n-1}B] \tag{22}$$

$$\mathrm{Sc} = \begin{bmatrix} 0 & 0.6667 & -0.0089 & 2.1824 \\ 0 & 0.6667 & -0.0222 & 8.7295 \\ 0.6667 & -0.0089 & 2.1824 & -0.1455 \\ 0.6667 & -0.0222 & 8.7295 & -0.5383 \end{bmatrix} \tag{23}$$

rank (Sc) = 4

The necessary and sufficient condition for an observer-based controller is that it must be observable. For the proposed Linear invariant system, it is said to be observable if the Rank of observability matrix equals to 4.

$$\mathrm{So} = [C^T\ A^TC^T \ldots\ldots\ldots\ (A^T)^{n-1}C^T] \tag{24}$$

$$\mathrm{So} = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 3.2733 & -0.0067 & -0.0067 \\ 0 & -0.1091 & 0.0001 & 3.2736 \end{bmatrix} \tag{25}$$

rank (So) = 4

4.5. State feedback

State feedback is to determine the control actions taking state measurements in consideration. Here, the aim is to give the input by any linear combination of the states.

$$u = -Kx \tag{26}$$

where K is the state feedback matrix

State space model expression is $\dot{x} = Ax + Bu$ (27)

Combining both; $\dot{x} = (A - BK)x$ (28)

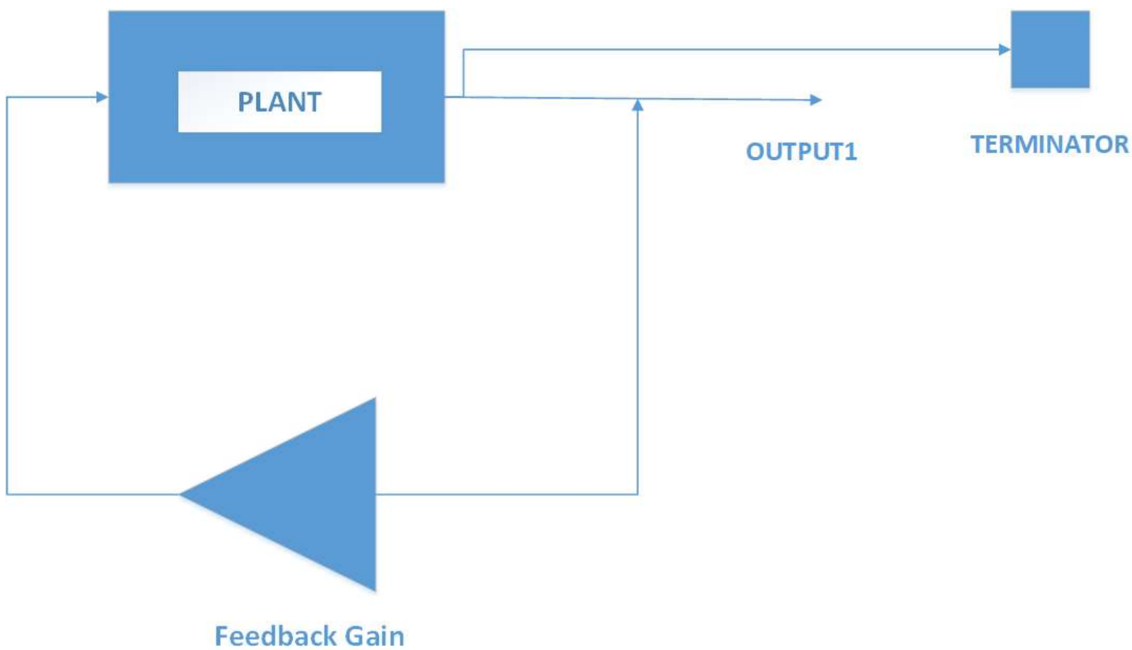


Fig8 : Model of State Feedback Controller System

Closed loop behavior is realized by the eigen values of equation (14). It allows the poles of the eigen values to move.

For continuous time Controller,
In open loop, equation (13) poles are realized by the eig(A)
In closed loop, equation (14) poles are realized by eig(A-BK)

For discrete time Controller,

In open loop, Poles are derived by eig(A) $x_{k+1} = Ax_k + Bu_k$ (29)

In closed loop, Poles are derived by eig(A-BK) $x_{k+1} = (A-BK)x_k$ (30)

Here, Ackermann's approach is used to find the required state feedback.

Desired pole for the system is [-3; -3; -3; -3]

Hence the K attained using MATLAB is

$$K = -12.3727 \quad 113.3432 \quad -16.5321 \quad 34.4821 \tag{31}$$

4.6. Optimal control

This expresses to have an effective degrees of freedom that allows to choose poles that enhances the closed loop performance rather than just specifying the positions. This uses a performance index J. Small J implies to have a good performance. Performance is defined using Rise time, settling time, Overshoot, Oscillation and damping ratios, offset, peak values of signals (especially inputs) [27].

$$J = \int_0^{\infty} (x^T Q x + u^T R u)dt \tag{32}$$

where Q is defined as weighted squares of deviation of states from target and R is defined as weighted squares of Control activity.
Here, x^TQx terms implicitly measures convergence rate (i.e rise time and settling time) and u^TRu term penalizes aggressive use of the input. Due to use of squares, the positive and negative errors penalize equally and covers Overshoot, Oscillation and damping ratios. Due to use of Infinite horizons, implicitly it derives the asymptotic errors to zero. Peak values of the signal is also covered

because of the use of squares. Because of squares, large values are penalized disproportionally as compared to smaller values. For discrete system, increase in R gives slower state behavior and poor controllability.

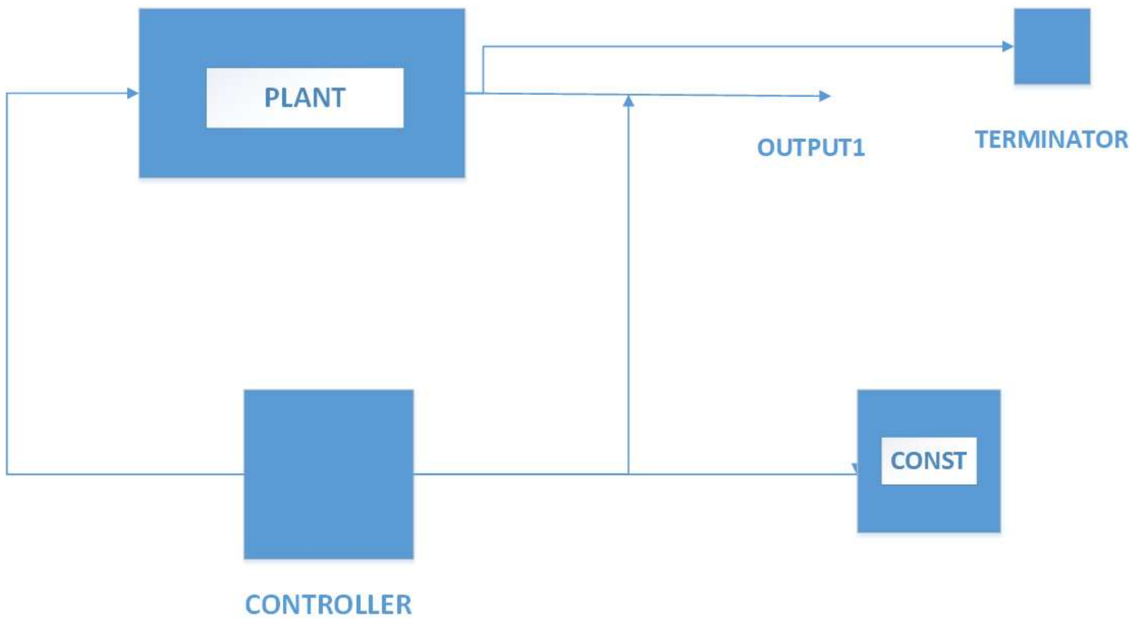


Fig9 : Model of the Continuous time domain system

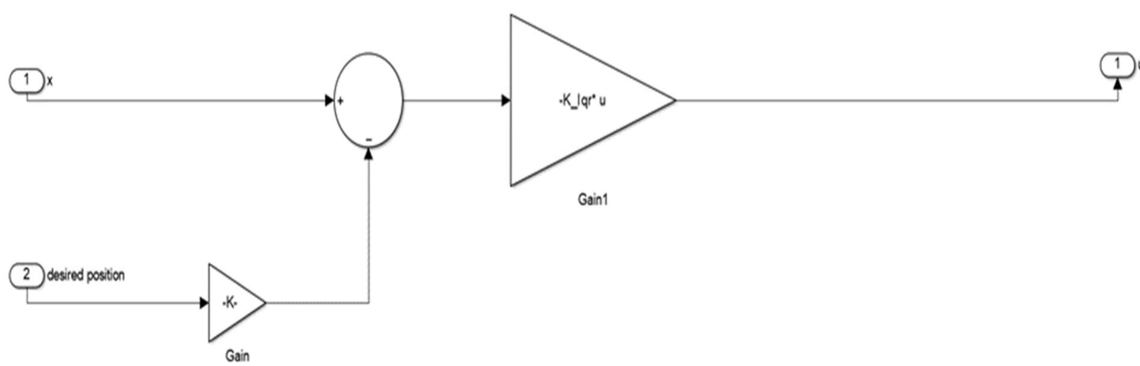


Fig10: Simulink model of LQR controller

4.7. State Observer

While designing pole placement based state feedback controller, we have assumed the availability of all the state variables for feedback but, in practice, this is not the case (a few seems to be unavailable). We need to estimate these unavailable state variables. The state observer estimates the state variables based on the measurement of outputs and control variables of the system.

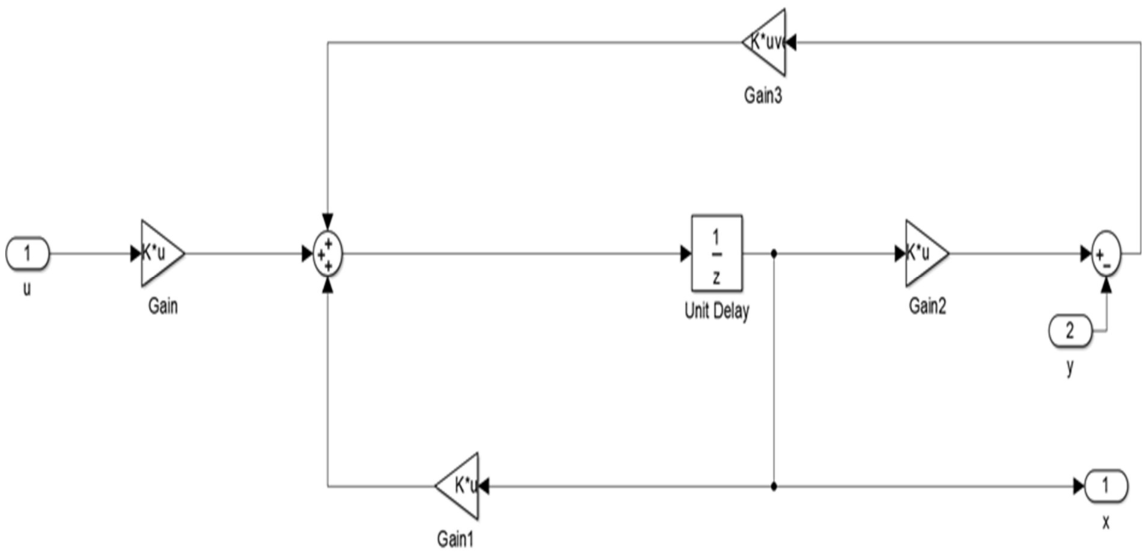


Fig11: Simulink model of State Observer

5. Application of Inverted Pendulum System

Traditional robots consisted of four wheels, were easily stabilized and comparatively bigger in size. A traditional robot uses four wheels and four motors for movement, while a self-balancing robot uses only two wheels and motors for movement. A very famous application of the Inverted Pendulum is a self balancing robot which is a prototype of Segway. As the robot tilts forward or backward, the new orientation is sensed by the MPU and the gyroscope and accelerometer data are combined within it using the Digital Motion Processing (DMP) and then this data is sent to the Arduino. The Arduino processes the data and if it feels that the robot is losing balance, then it gives an instruction to the L298N motor driver module to drive the motor as per the requirement. The wheels are in turn driven by the motor and then the required correction is provided to the robot and it regains its balance.

The wheels turn forward if the robot is falling forward and backward if the robot is falling backward.

5.1. Two-wheel Self balancing Robot

The field of robotics is the playground of creative minds of modern age. Dreams turned into reality with the development in this field. Two wheel self-balancing robot is an example of advance development in the field of robotics. This robot is based on the concept of inverted pendulum. Here, a control system is used to stabilize an unstable system using appropriate and feasible microcontrollers and sensors. The system architecture comprises a pair of a dc motor and an Arduino microcontroller board; a single- axis gyroscope and a 2- axis accelerometer. As the robot tilts forward or backward, the new orientation is sensed by the MPU and the gyroscope and accelerometer data are combined within it using the Digital Motion Processing (DMP) and then this data is sent to the Arduino. The Arduino processes the data and if it feels that the robot is losing balance, then it gives an instruction to the L298N motor driver module to drive the motor as per the requirement. The wheels are in turn driven by the motor and then the required correction is provided to the robot and it regains its balance.

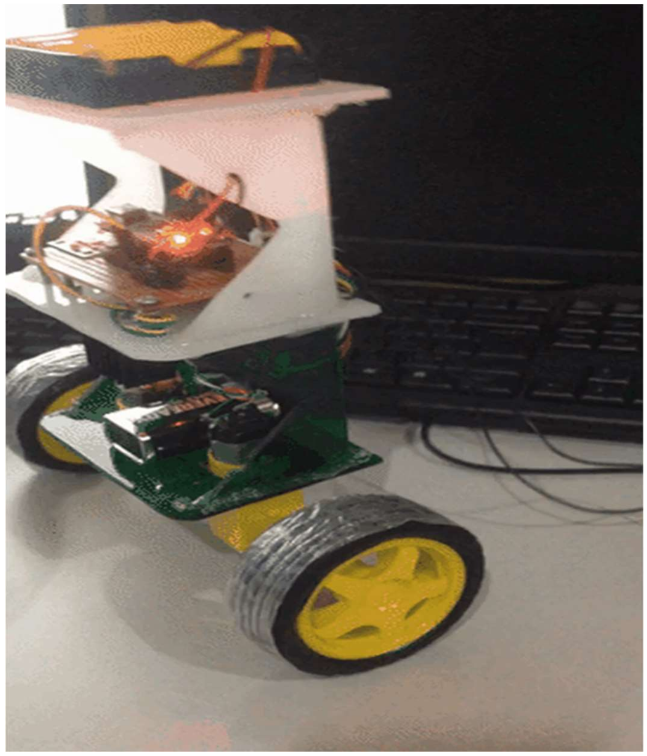


Fig12: Two wheel self-balancing robot

5.2. Working of the Robot

Self-balancing robot is based on the principle of Inverted pendulum, which is a two wheel vehicle balances itself up in the vertical position with reference to the ground. It consist both hardware and software implementation. Mechanical model based on the state space design of the cart, pendulum system. According to the situation we have to control both angel of pendulum and position of cart. Mechanical design consist of two dc gear motor with encoder, one Arduino microcontroller, IMU (inertial mass unit) sensor and motor driver as a basic need. IMU sensor which consists of accelerometer and gyroscope gives the reference acceleration and angle with respect to ground (vertical), When encoder which is attached with the motor gives the speed of the motor. These parameters are taken as the system parameter and determine the external force needed to balance the robot up. It will be prevented from falling by giving acceleration to the wheels according to its inclination from the vertical. If the bot gets tilts by an angle, than in the frame of the wheels; the center of mass of the bot will experience a pseudo force which will apply a torque opposite to the direction of tilt. When the robot starts to fall in one direction, the wheels should move in the inclined direction with a speed proportional to angle and acceleration of falling to correct the inclination angle.

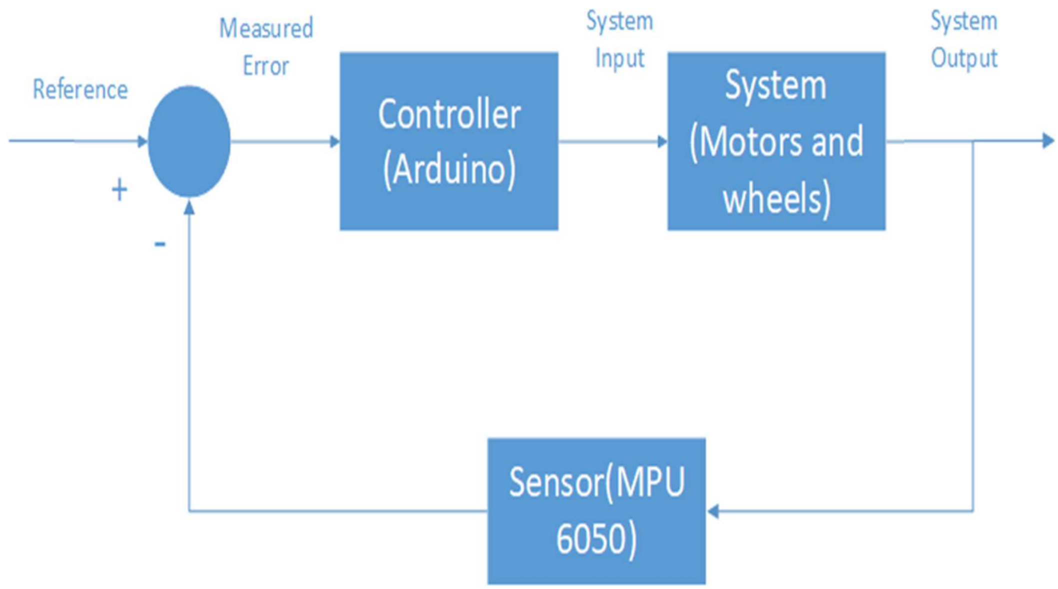


Fig13: Block diagram of robot

So the idea is that when the deviation from equilibrium is small, we should move "gently" and when the deviation is large we should move more quickly.

5.3. Selection and application of Sensors

In order to maintain the robot upright, knowing the tilt angle is imperative. There are a wide array of sensors that can be used, such as inclinometers, light sensors, accelerometer or gyroscopes. However, each these sensors have their shortcomings, the inclinometer takes a long time to converge to the angle it is currently at, light sensors are highly susceptible to background noise (ambient light and the reflective index of the surface it is operating in), gyroscopes have a bias and accelerometers are relatively noisy.

Most often, a combination of a gyroscope and an accelerometer are used.

- <u>MPU6050 Sensor</u>

It has Micro-Electro-Mechanical Systems (MEMS), 3-axis accelerometer and 3-axis gyroscope, the data of which are combined by using the Digital Motion

Processing (DMP), which is present within the MPU. The MPU is responsible for sensing the current orientation and acceleration of robot and then informing the Arduino about it.

It requires a power supply of 3V-5V. It has a built in 16-bit AD Converter and gives 16-bit data output.

- Gyroscope: Gyroscope measures the angular rate around an axes. Tilt angle can be obtained by integrating angular rate over sampled time. An estimate of angular displacement is obtained by integrating velocity signal over time.
- Accelerometer: This meter is used to measures the total external acceleration of the balancing robot, and also which includes the gravitational and motion accelerations. Accelerometer can measure the force of gravity and with that information, the angle of robot can be obtained.
- Encoders: In generally the encoders return the rotation angles of individual motor shafts as digital signals, which are sent to the processor. Next, after conversion based on gear ratio and wheel radius, the distance traveled can be calculated. The encoder chosen for this project is the 64 cycle-per revolution, which provides a resolution of 64 counts per revolution of the motor shaft.

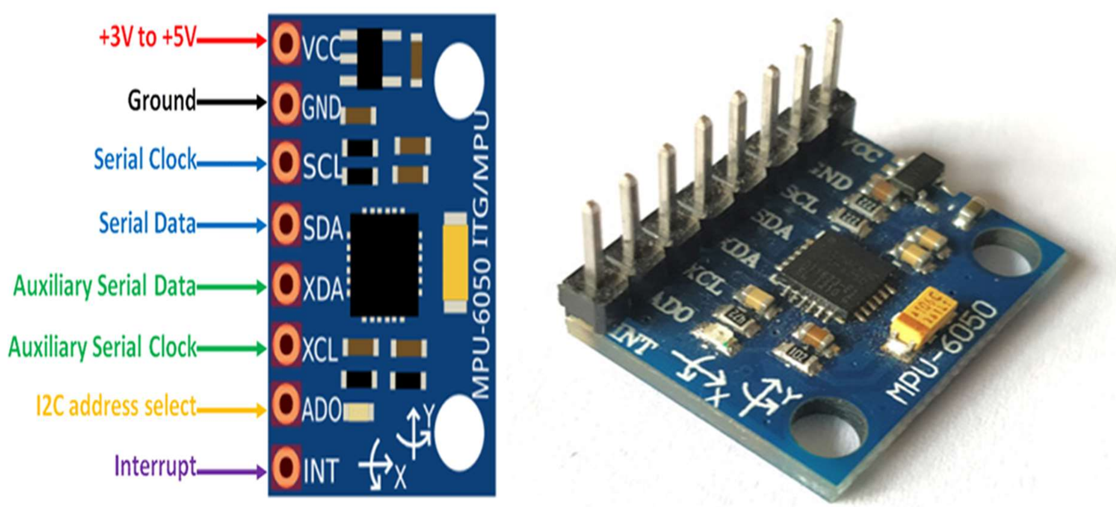


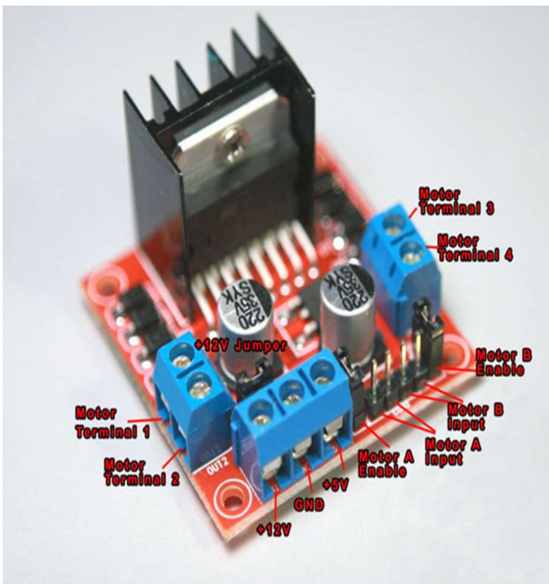
Fig14: MPU6050 sensor

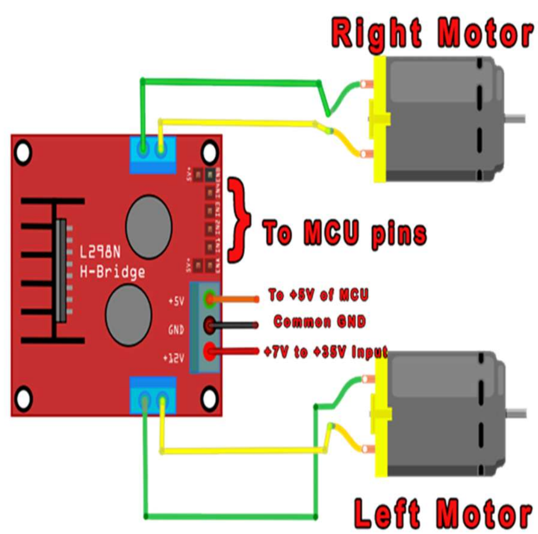
5.4. Motor and Motor Control Board

- <u>L298N Motor Control Board</u>

The L298N is a dual H-bridge motor driver which allows speed and direction control of two DC motors at the same time. The module can drive motors that have voltages between 5V and 35V, with a peak current up to 2A. It also has an in built 5V voltage regulator.

In the robot the L298N drives the motor when instructed by the Arduino and as per the instructions (i.e. depending upon whether the robot is falling in the forward or backward direction).





(a) (b)

Fig15: (a)L298N dual H-bridge (b) connection with DC motors

- <u>DC Motors</u>

Here, we use two geared DC motors which implies that when driven by the motor driver, it can turn both clockwise and anticlockwise and in turn rotates the wheels. So, depending on the movement of the robot, the L298N drives the motor, which in turn rotates the wheel.

They have an operating voltage 12V each and an rpm of 100. They can even work for a voltage anywhere between 4V to 12V. The DC motors along with the wheels attached are shown in fig.

(a) (b)

Fig16:(a) DC motor (b) wheels

5.5. Arduino Nano Development Board

It is a small and breadboard friendly microcontroller board. It is based on ATmega328p. The operating voltage of the Arduino nano is 5V and the input voltage is between 7-12V. It has 14 digital I/O pins out of which 6 provide PWM output and it has 8 analog input pins. There are 2 reset pins and 6 power pins. The Arduino has a clock speed of 16MHz. It has a flash memory of 32KB. There are 3 ways to power the Arduino: One is through the mini USB, second is through the 5V pin which can take a regulated supply of 5V and the third one is through the Vin pin which can take a supply of 7-12V.

According to Performance: Normally, the self-balancing robot needs almost real-time response to estimate and correct its tilt angle. Hence, the development of board must provide a processing speed that is sufficiently fast to perform the processing tasks, including data acquisition, control computation and signal output, within the sampling time. The Arduino Mega development board is equipped with the ATmega328 processor, which features a maximum clock rate of 16 MHz.

On the robot, sensors are deployed to obtain measurements of its motion: a gyroscope and an accelerometer are used to estimate the tilt angle,encoders are used to obtained the measurements of the robot.In our model, we will be using the Arduino to interface the MPU6050 and the L298N motor driver. The Arduino will also act as a decider on whether the robot is losing balance or not and a command is to be given for the motor and wheels to turn or not respectively.

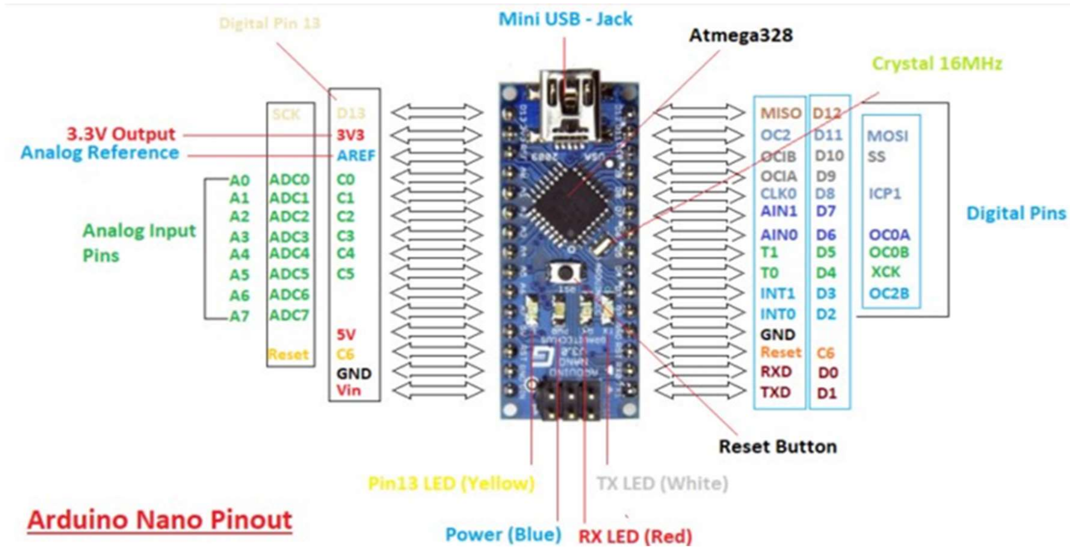


Fig17: Arduino nano

5.6. Price and Expansion

As we all know Arduino boards are low-cost and expandable, where optional peripherals called shields can be purchased as and when needed.

5.7. Power Supply

There are two sources of power. One is a 9V battery which powers the L298N motor driver and the other which powers the Arduino and consists of 4 AA batteries of 1.5V each.

6. Mechanical Design of the Robot

In this chapter, the model of the structure of a self-balancing robot is expressed. The robot is able to afford a maximum 25kg load. If a big load is added to the robot, it will have a large inertial. The layers of the bot is described with measurements in metric unit.

6.1. Design

The self-balancing robot is essentially an inverted pendulum. It can be balanced better if the center of mass is higher relative to the wheel axles. A higher center of mass means a higher mass moment of inertia, which corresponds to lower angular acceleration (slower fall). This is why the different components are placed keeping in mind the balancing of the center of mass. The height of the robot is chosen depending on the number of components.

For our case we take three layers so as to place all the components.

6.2. Layers of the Structure

Here, there is layer by layer discussion of the robot structure.

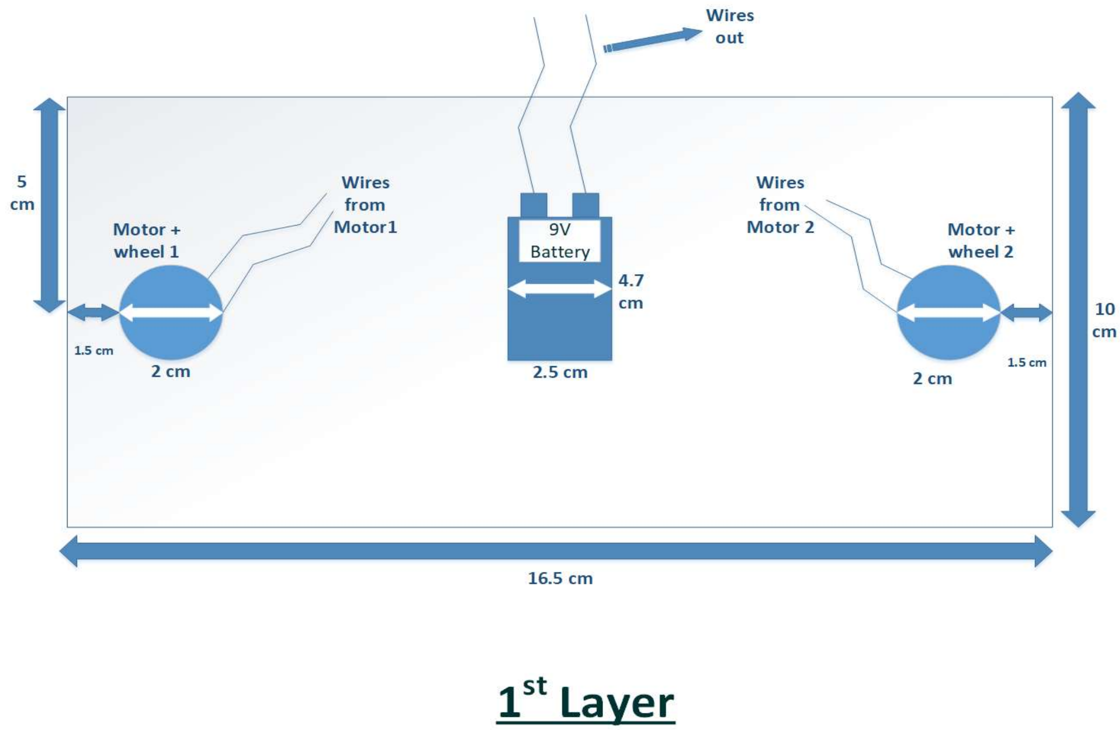


Fig18:1st layer of bot

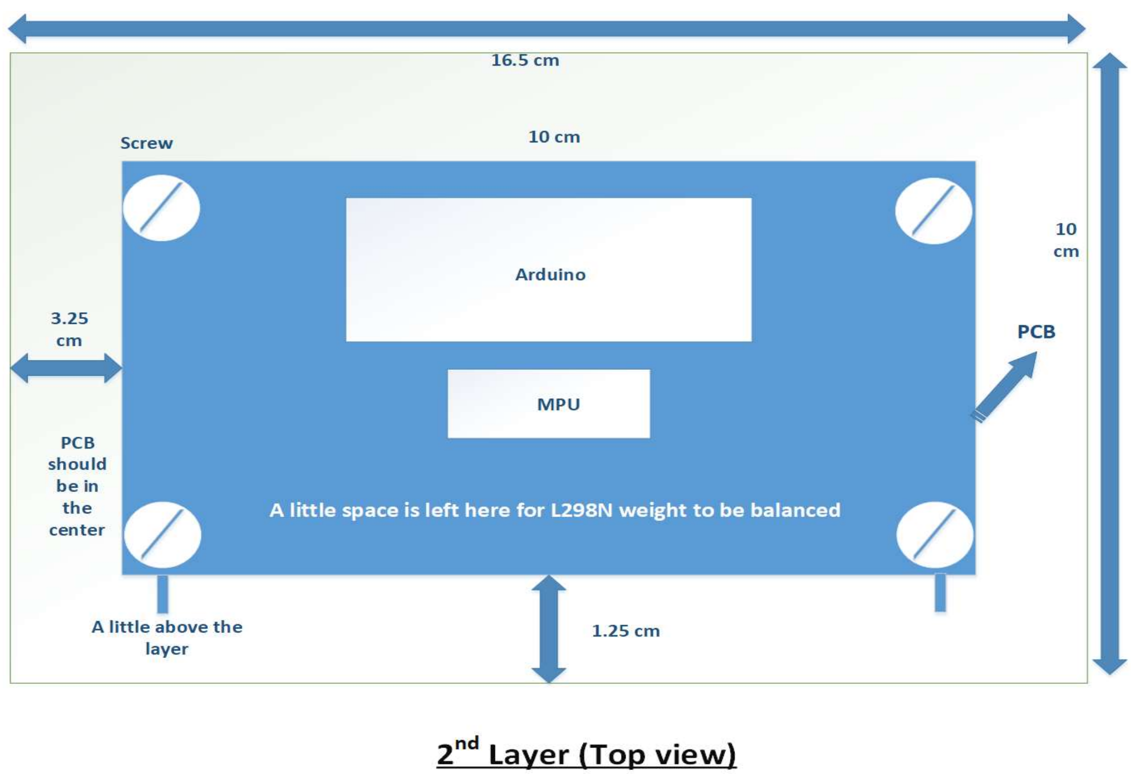


Fig19: 2nd top layer of bot

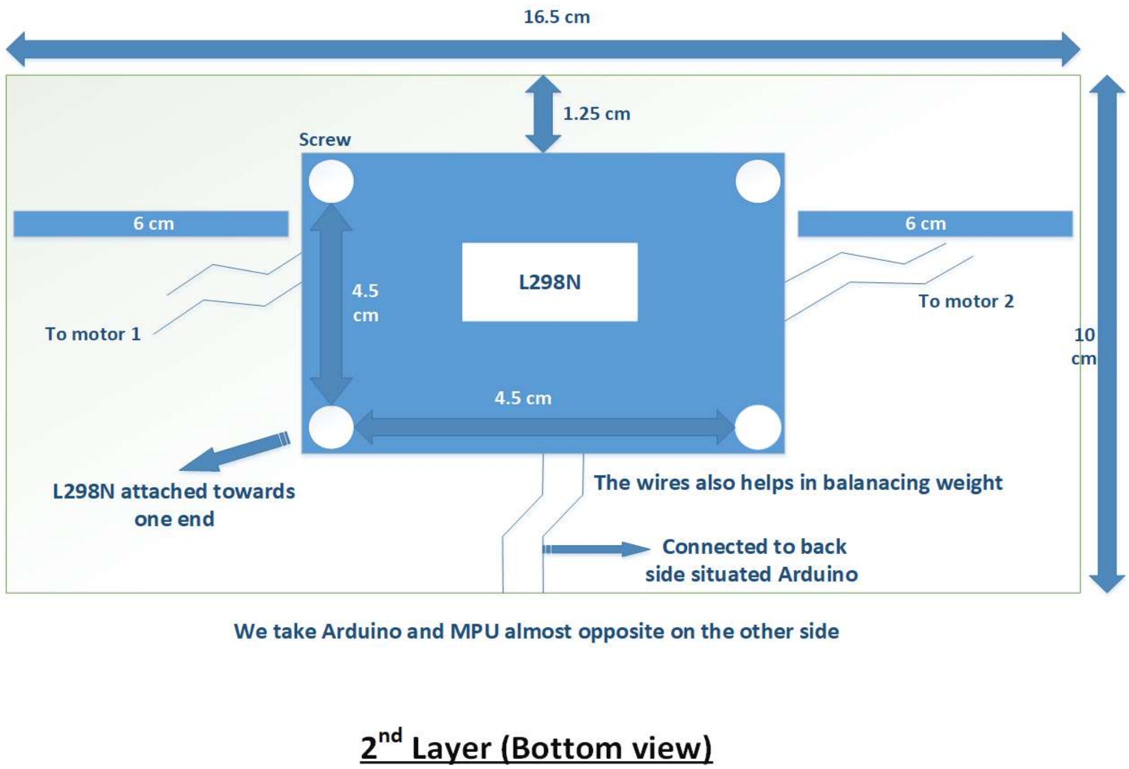


Fig20: 2^{nd} bottom layer of bot

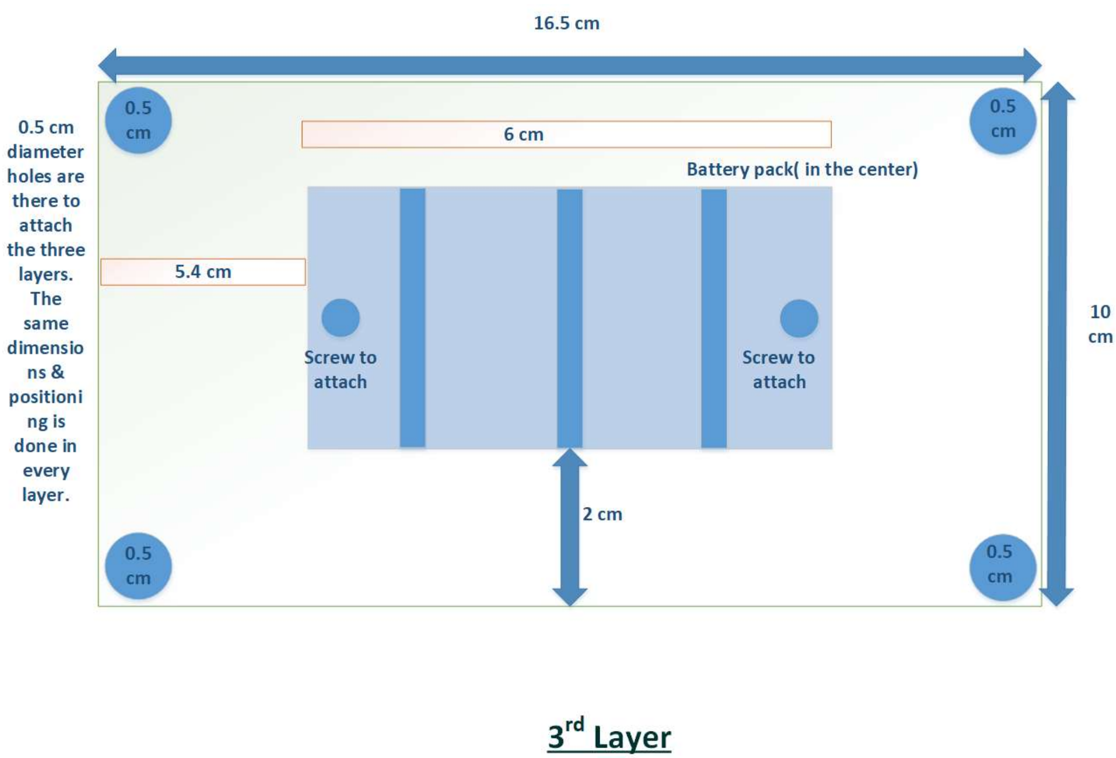


Fig20: 3^{rd} layer of bot

STEPS FOR CONSTRUCTION:

- We will take three rectangular acrylic or wooden sheet of size 16.5cm×10cm.
- Then we will mount DC motor at the bottom sheet of the bot and place a 9V battery in the center of the sheet. This is layer 1 and this is shown in Fig 18.
- The middle layer has the Arduino, the MPU6050 in the upper side of sheet mounted on a PCB board (Fig 19) and L298N motor driver module in the bottom side of sheet (Fig 20) (the PCB board is placed along the center of the sheet). This is layer 2 and is shown in Fig19 and Fig20.
- Four 1.5V(AA) batteries are placed on the topmost layer along the centre of the sheet. This is layer 3 and is shown in Fig21.
- The three layers are connected to each other by using wooden links in the diameter of 0.5cm and the length of 8cm.
- The distance between each of the layers is 7.5 cm.
- We will use some nuts and bolts to assemble the robot hardware like the ones shown in Fig

7. Design Overview of the Robot

In this chapter, the connections of the robot have been described.

7.1. Architecture of the Robot

There are many types of two wheel balancing robot being introduced in the market, modeling are mostly various depend on personal criteria on personal transporter. Modeling mostly refers to Segway liked personal transport that was the first and based design for first prototype of two wheel balancing vehicle. Typical two wheel balancing vehicle is made up of base with two parallel positioned wheels on both left and right side while a steer functioned rod is positioned at forward front part of the base, enabling directional controller for end user to determine rotation angle.

7.2. Connections of the Robot

The basic connections of the system are discussed below with pictures.

1. <u>Arduino and MPU6050</u>

Table2: Arduino to MPU6050 connection

MPU6050	**Arduino**
V_{cc}	5V or 3V3
GND	GND
SCL	A5
SDA	A4
INT	D2

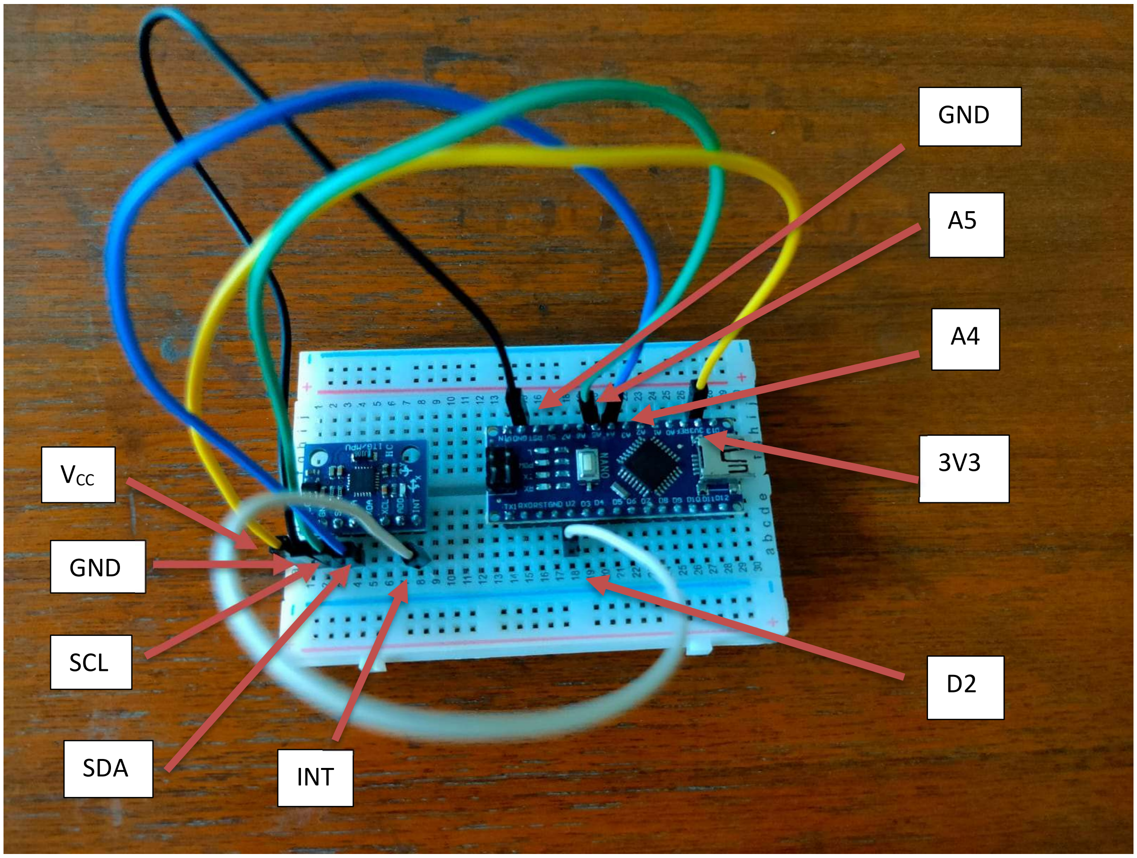


Fig21: Arduino to MPU6050 connection

2. Arduino and L298N

Table3: L298N to Arduino connection

L298N	Arduino
GND	GND
ENA	D9
ENB	D10
INT1	A0
INT2	A1
INT3	A2
INT4	A3

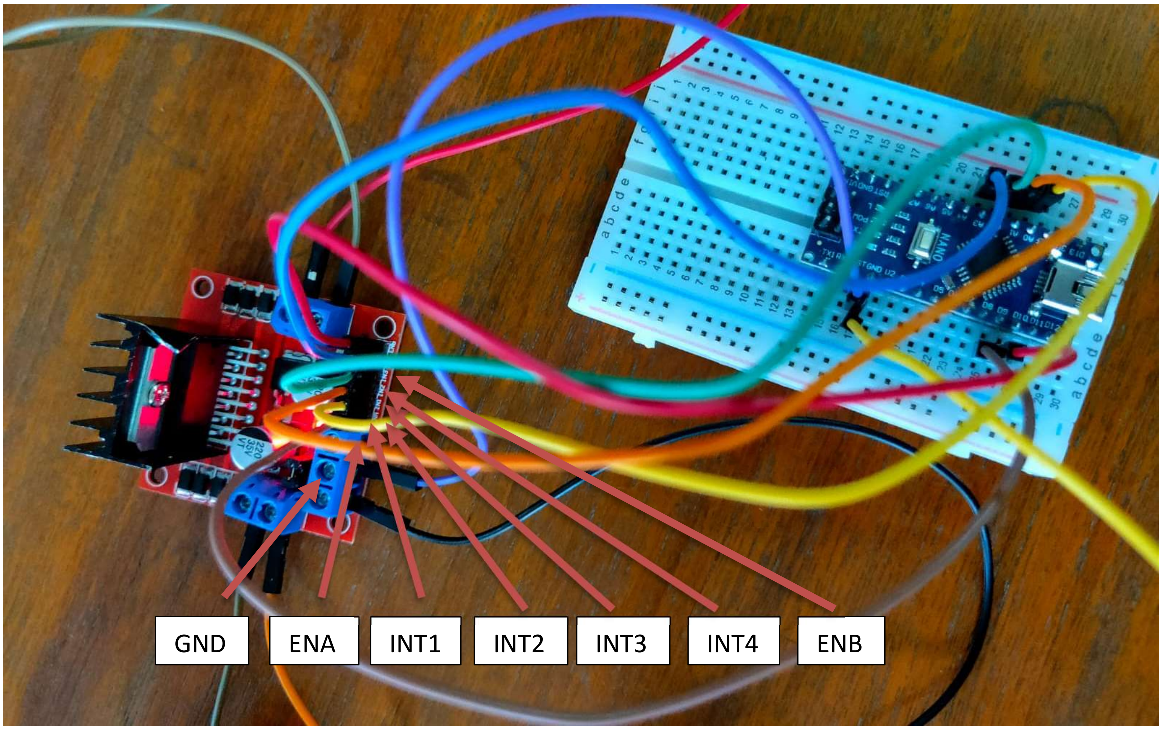


Fig22: L298N to MPU6050 connection

3. <u>L298N and Battery</u>

Table4: L298N to battery connection

L298N	**Battery**
+12V	+ve terminal
GND	-ve terminal

The MPU6050, the Arduino, the motors and the battery are connected together to check the connections and the complete connection is shown in Fig 23.

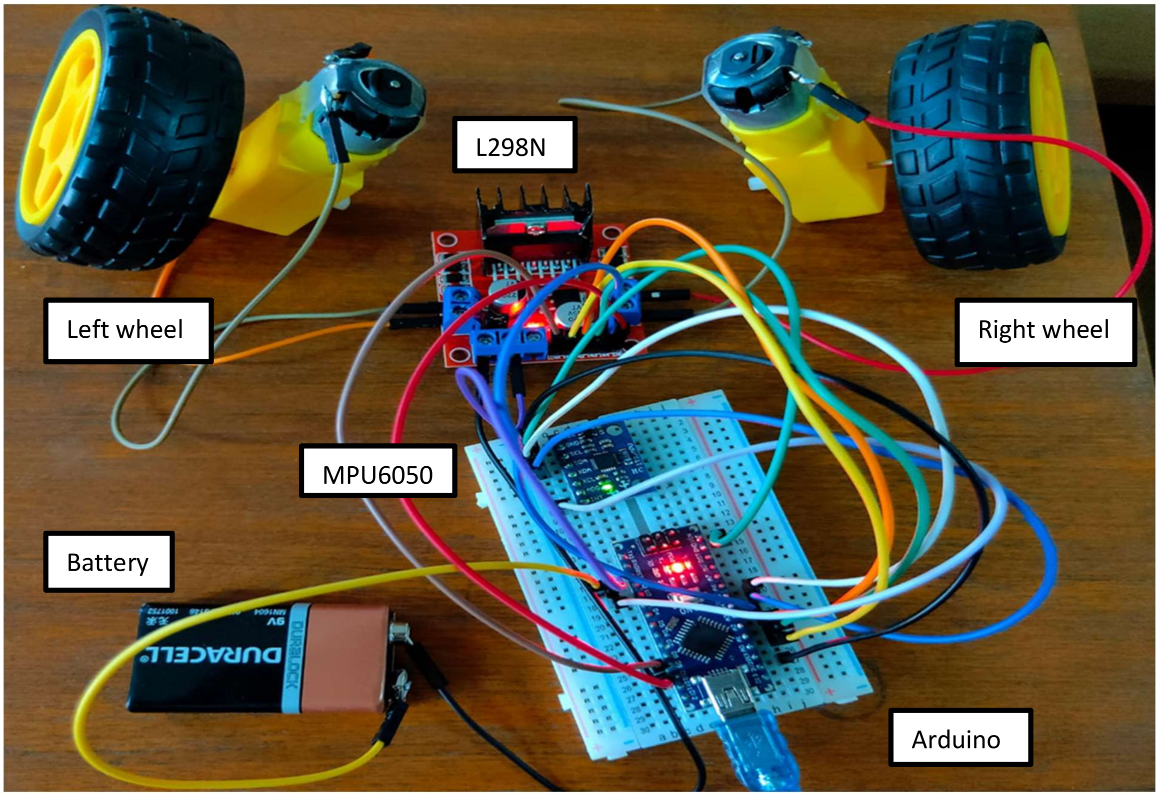


Fig23: total connection

7.3. Final Assembly

The structure of the robot is designed using CAD

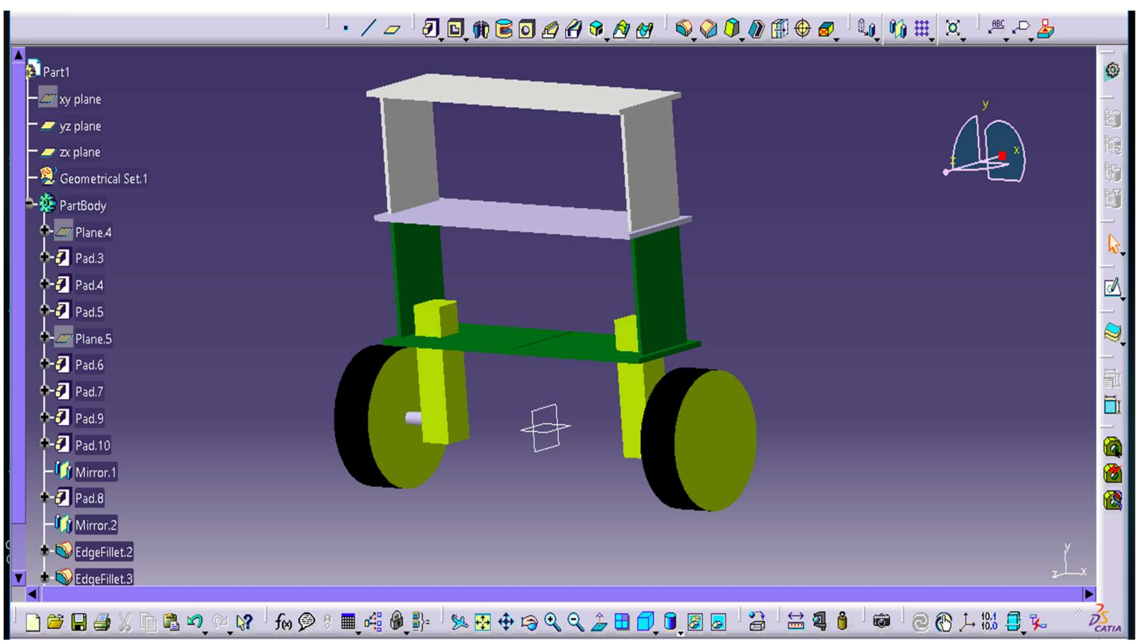
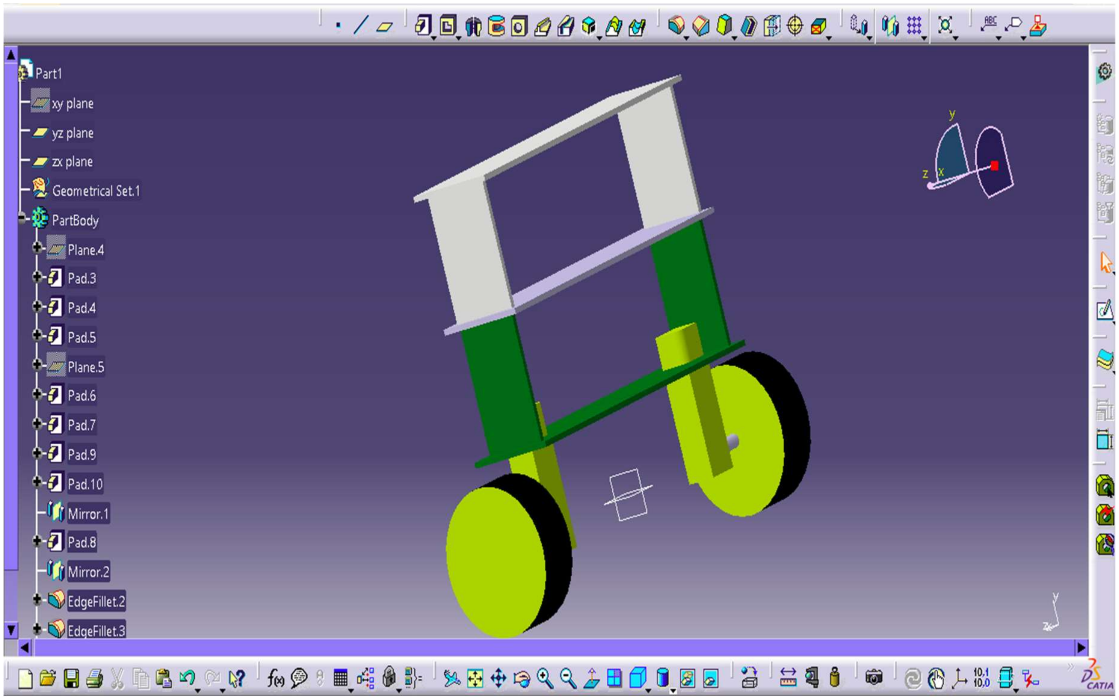
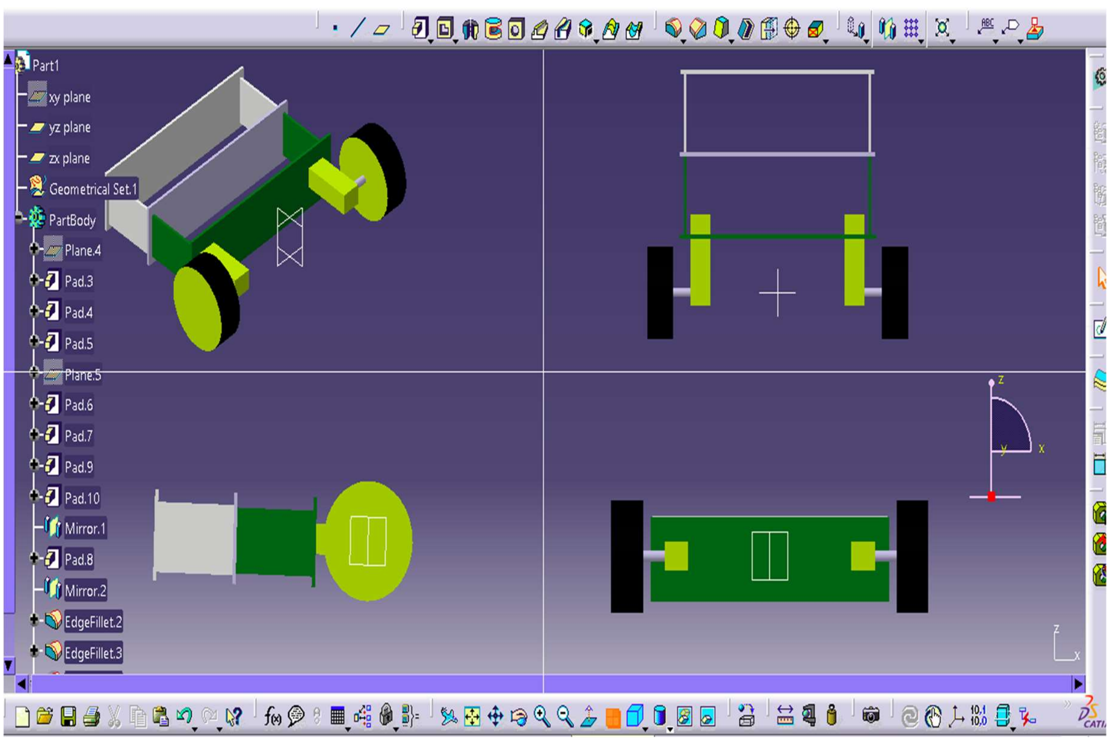


Fig24(a)



(b)



(c)

Fig24: CAD view of the structure of robot

8. Results and graphs

In this chapter, all the results and the output graphs have been plotted through MATLAB.

8.1. Continuous time system with State feedback Controller

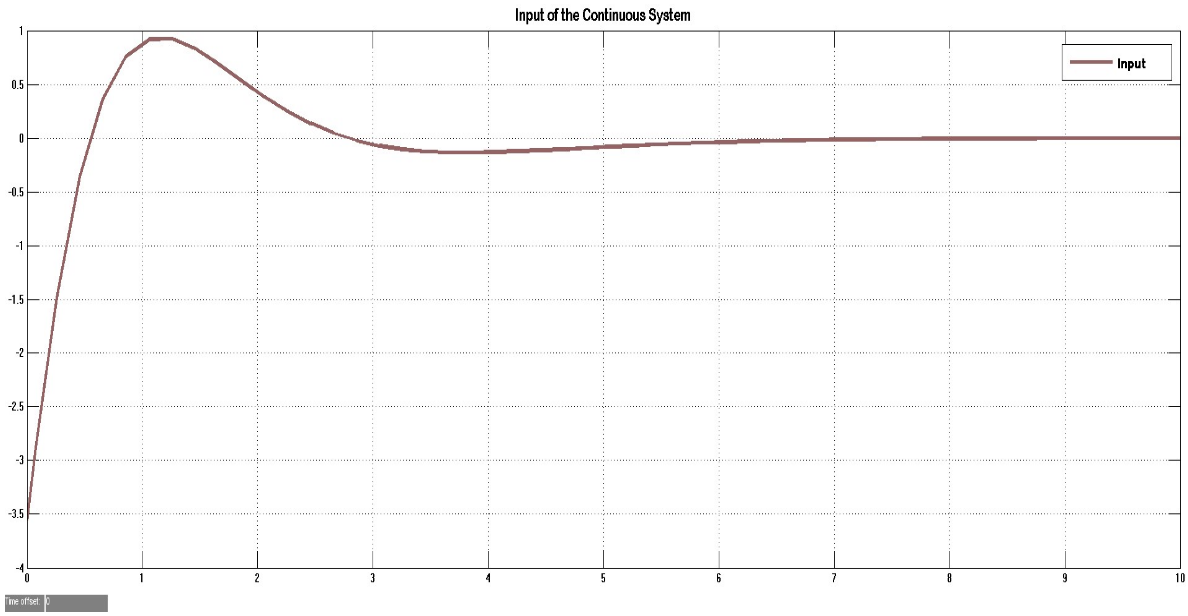
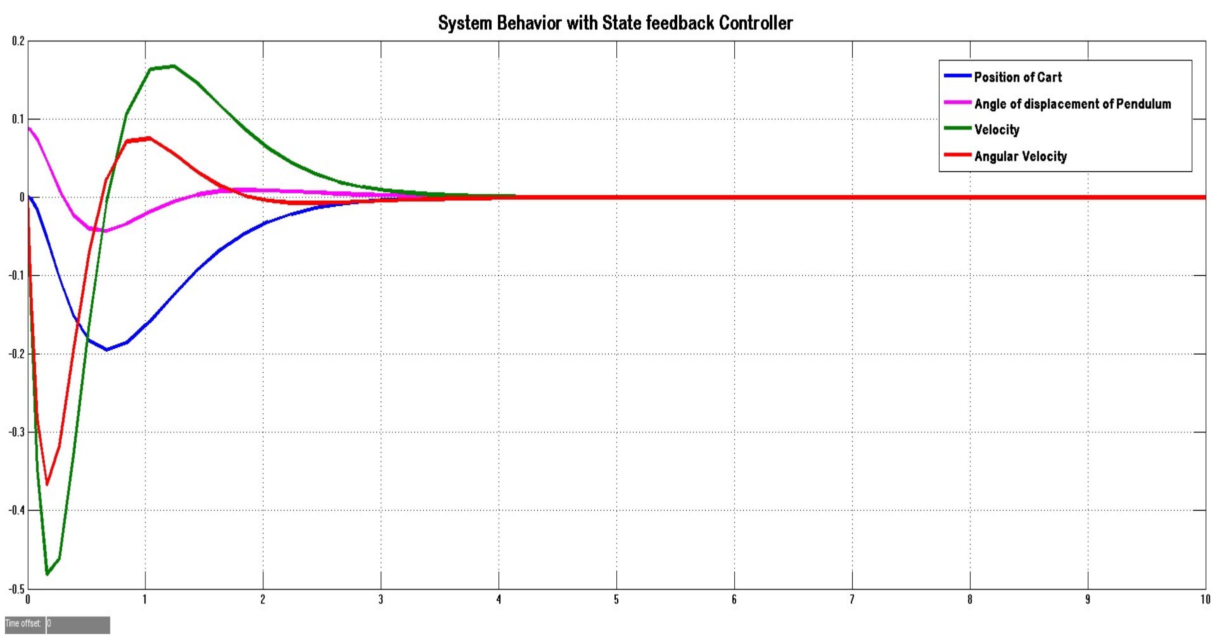
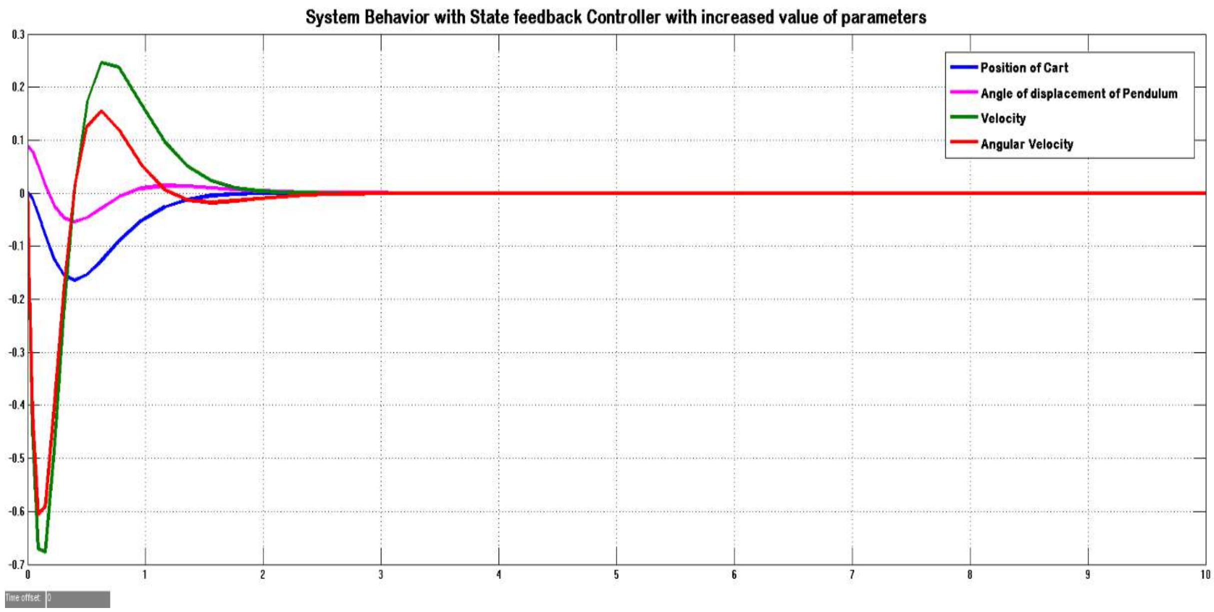


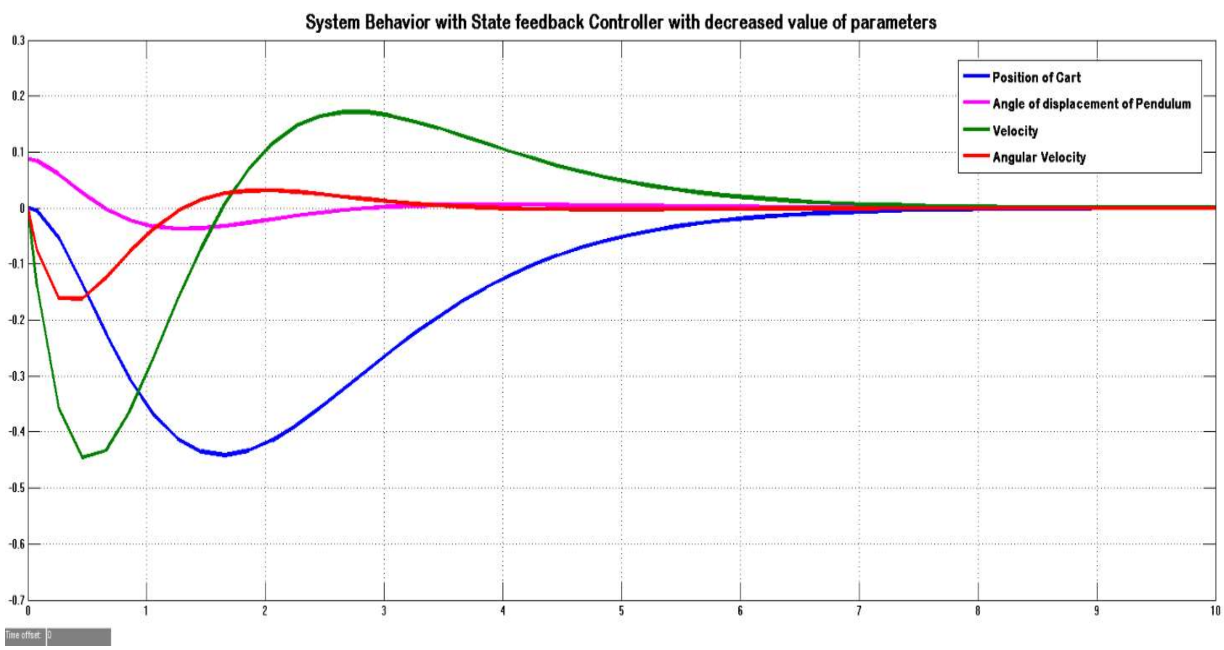
Fig25(a)



(b)



(c)



(d)

Fig25: System behavior (a) input (b) at ideal case (c) with increased value of parameters (d) with decreased value of parameters

8.2. System behavior with LQR Controller

For the initial condition,

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}; R = 0.5; \tag{33}$$

Using MATLAB,

K_lqr = -4.4721   84.2561   -8.8900   (34)
25.5379

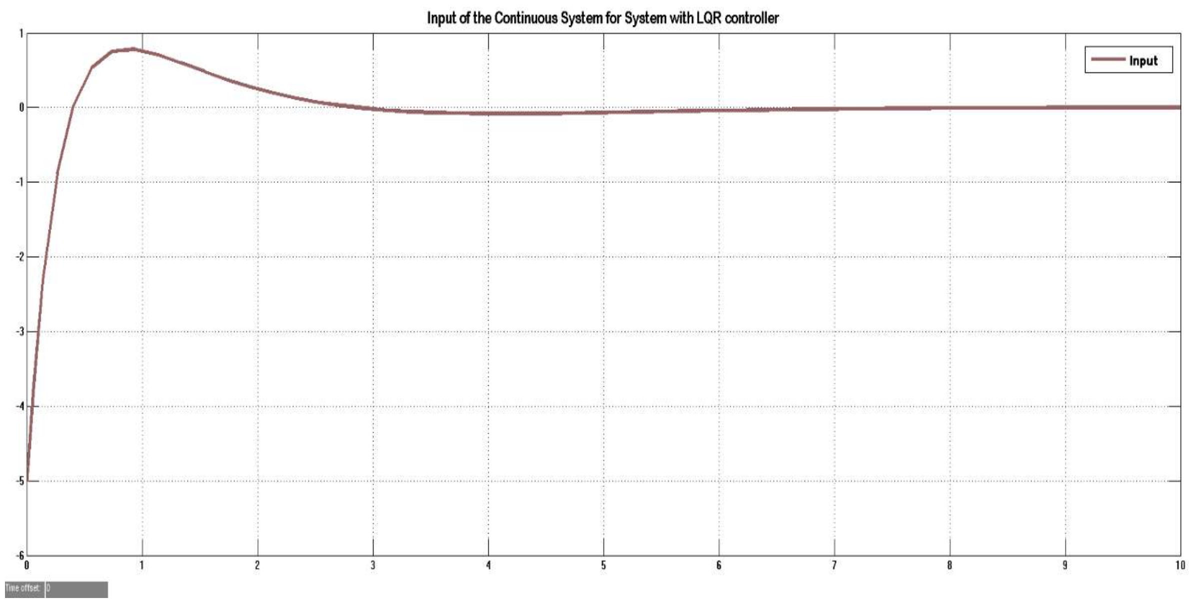
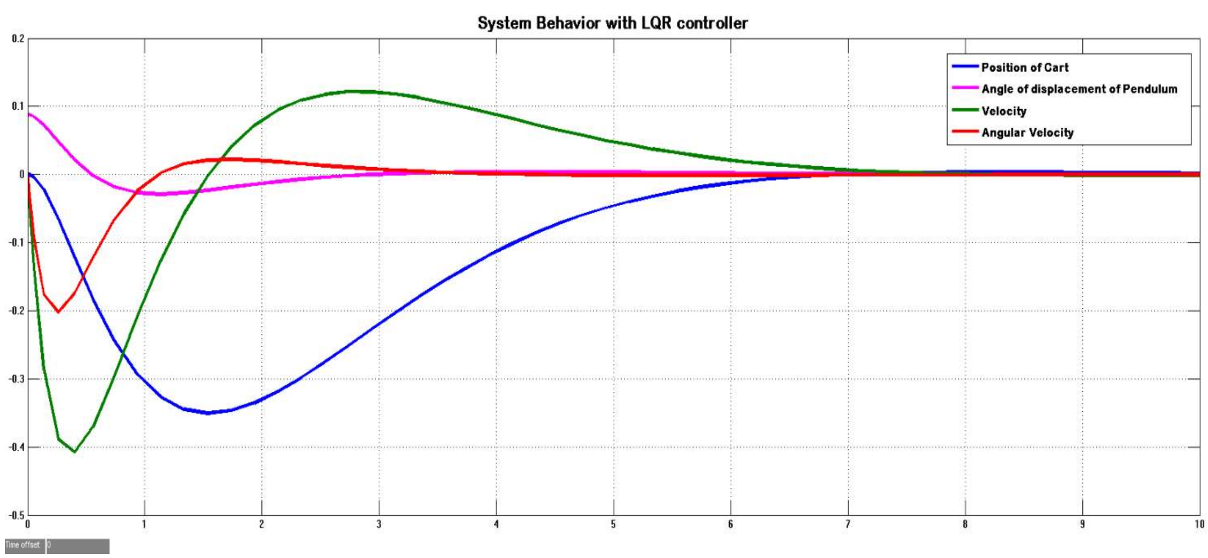
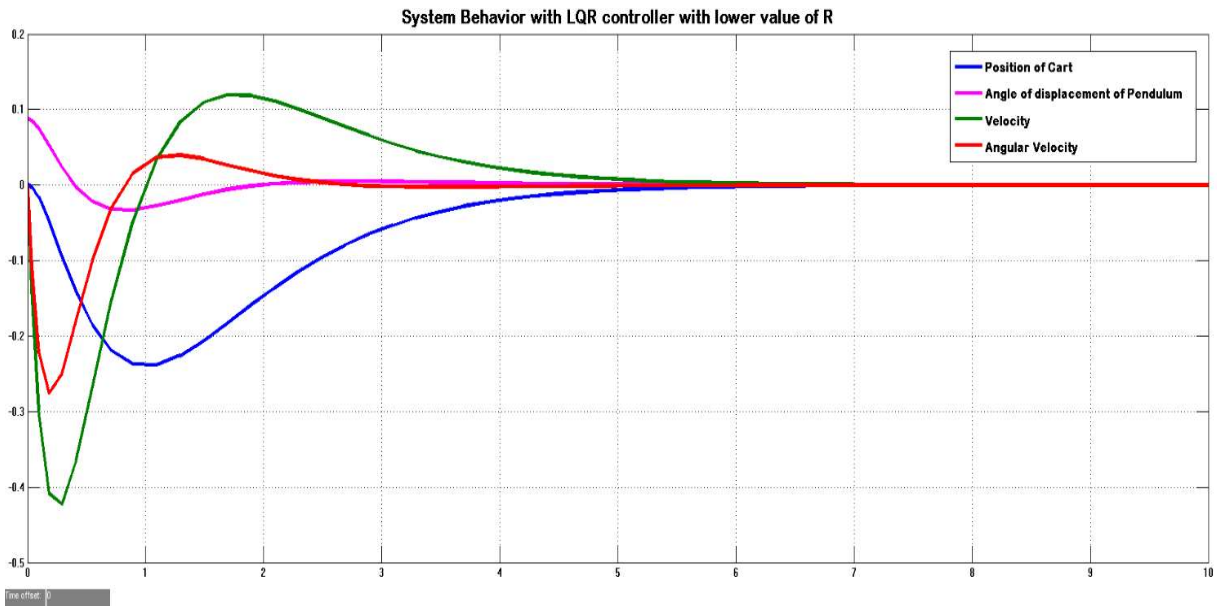


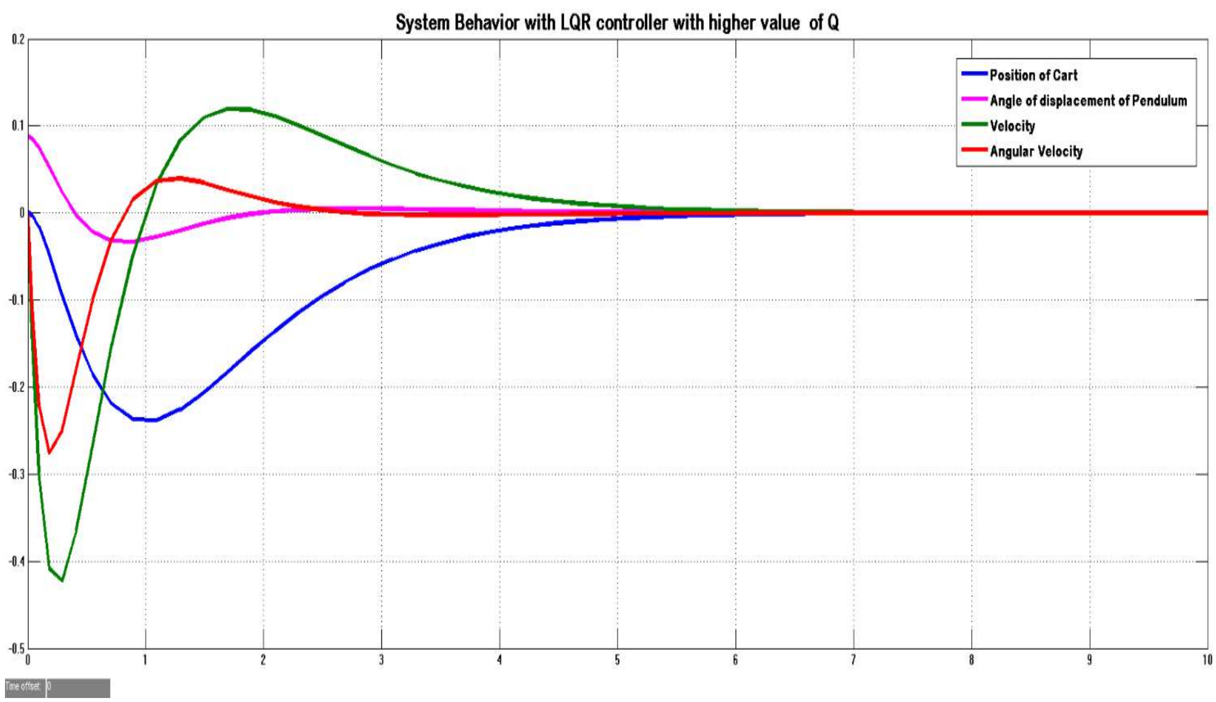
Fig26: (a)



(b)



(c)



(d)

Fig26: System behavior with LQR controller (a)input (b)at ideal case (c)low R value (d)high Q vlaue

8.3. Discrete time state space model

For sampling time = 0.1 second,
The element matrices are:

a =

	x1	x2	x3	x4
x1	1	0.01653	0.09997	0.000515
x2	0	1.066	-3.366e-05	0.1021
x3	0	0.334	0.9993	0.01585
x4	0	1.336	-0.0006802	1.063

b =

	u1
x1	0.003341
x2	0.003366
x3	0.06699
x4	0.06802

c =

	x1	x2	x3	x4
y1	1	0	0	0

d =

	u1
y1	0

The obtained K for the system is

$$K_{\text{discrete}} = -363.3809 \quad 651.8284 \quad -153.8791 \quad 180.1997 \tag{35}$$

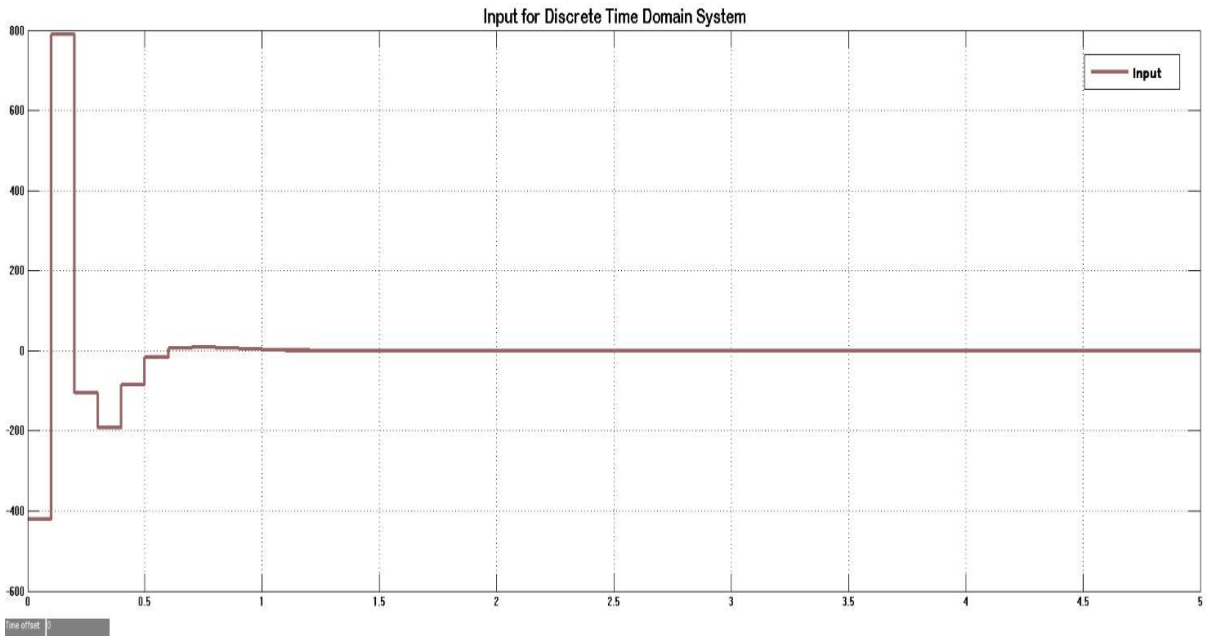
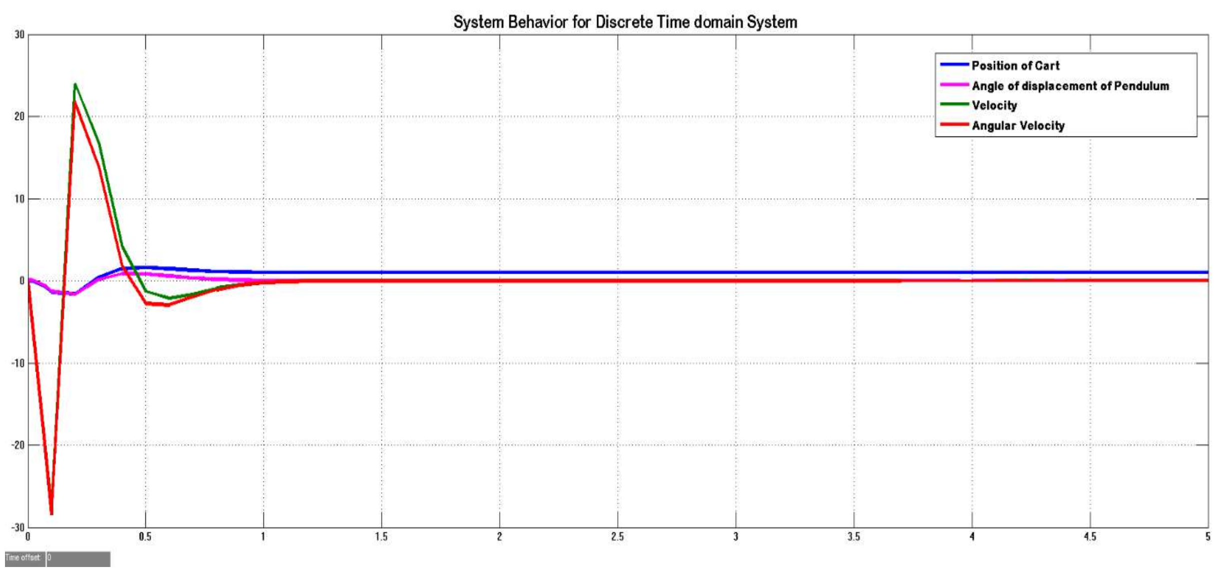


Fig27: (a)



(b)

Fig27: System behavior for discrete time domain (a)input (b) output

8.4. System behavior with State Observer

Using MATLAB, the observer matrix for the initial condition was derived according to the desired poles for the system

Ob = 4.0089 100.2853 44.0930 444.4227 (36)

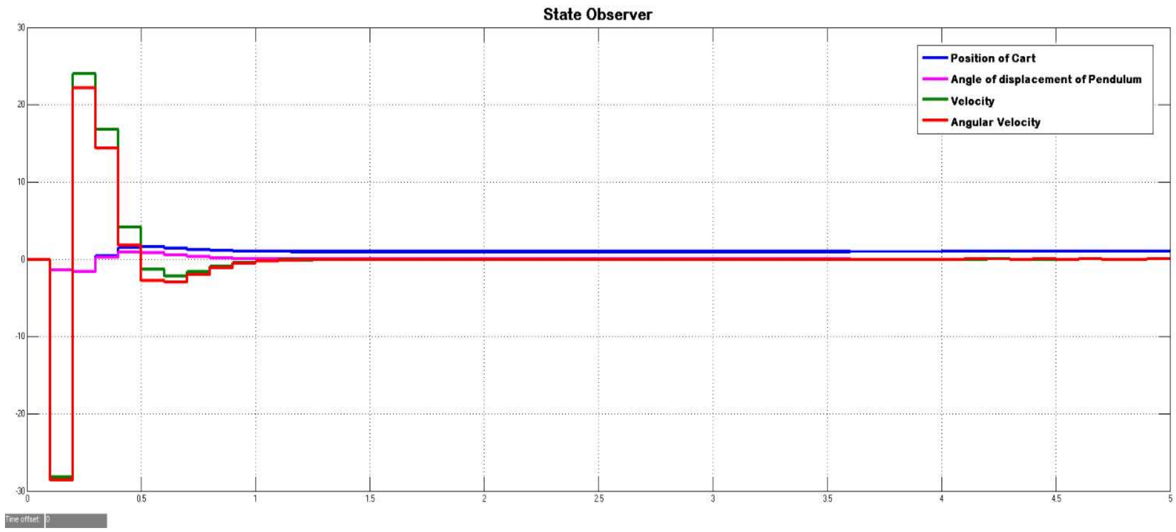
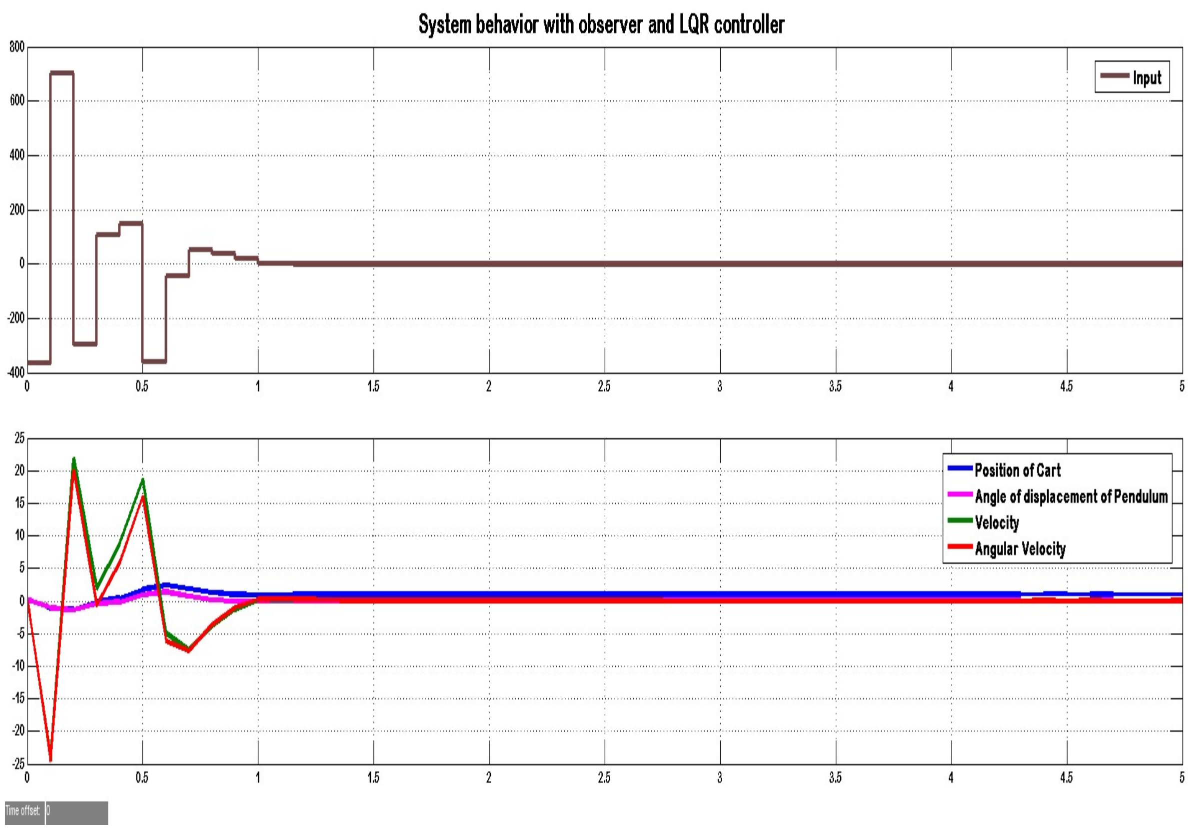
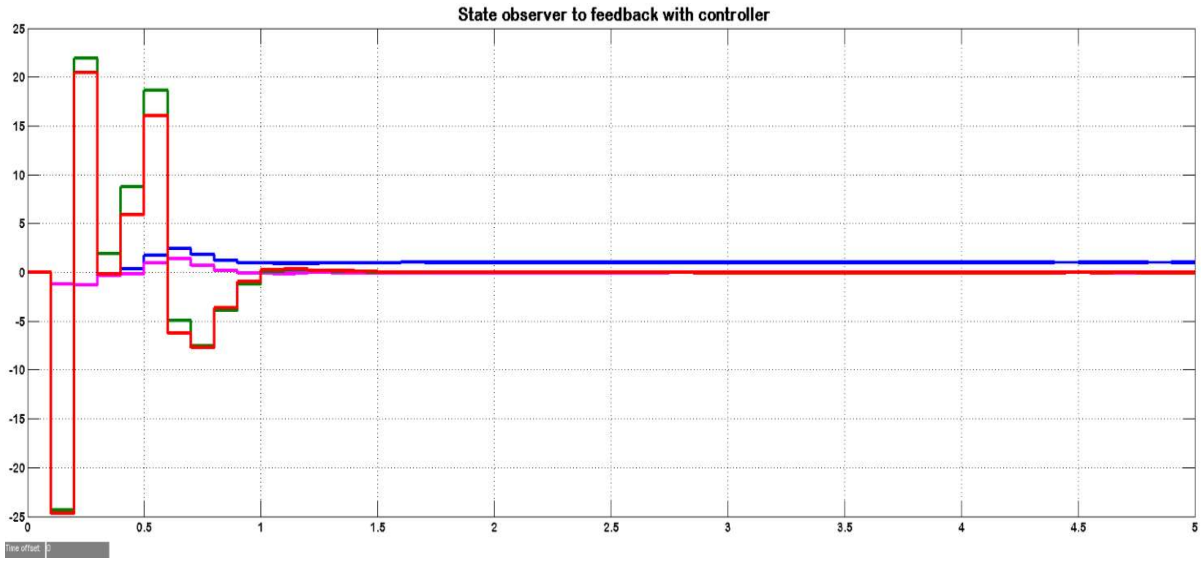


Fig28 (a)



(b)



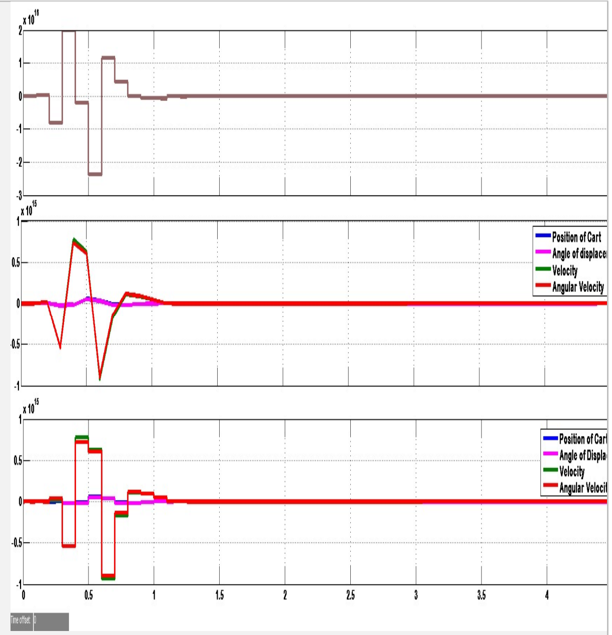
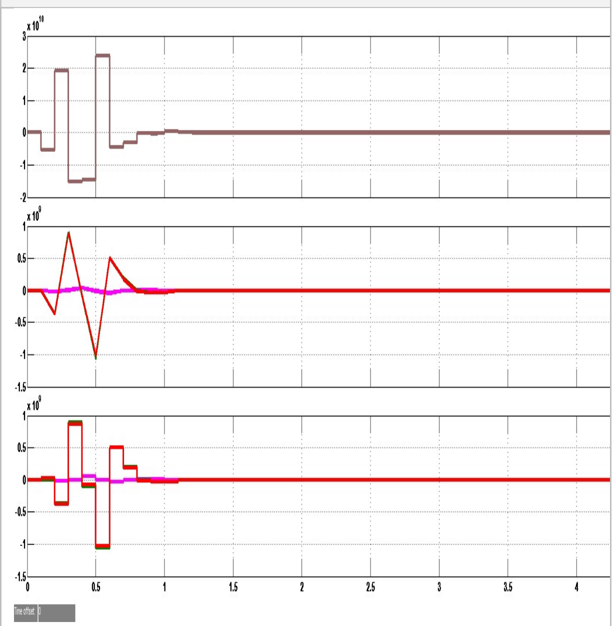
(c)

Fig28: (a) State observer (b)System behavior with observer and LQR (c)System behavior of State observer to feedback with controller

8.5. System Analysis with respect to time

Here, different parameters are taken to check the effect of their change in the system behavior.

Sl. no	Position of the cart (m)	Angle of displacement of the cart (degree)	Velocity (m/s)	Angular velocity (rad/sec)	Graphs with respect to time

1	0	5°	0	0	
2	0.2	15°	0.3	0.2	

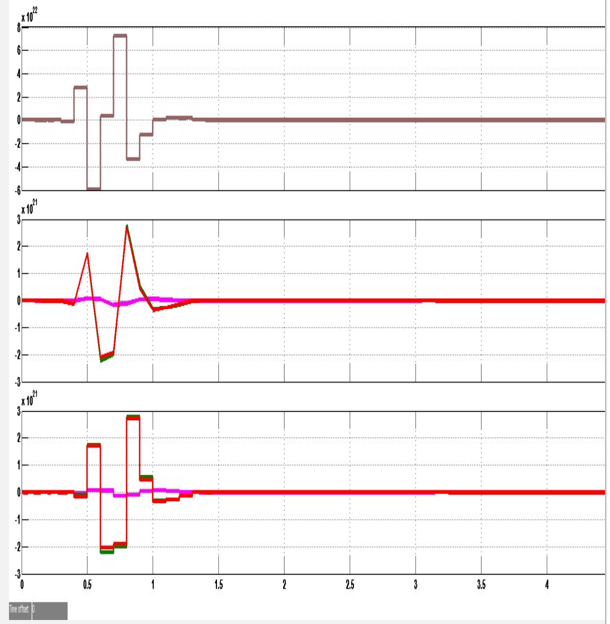
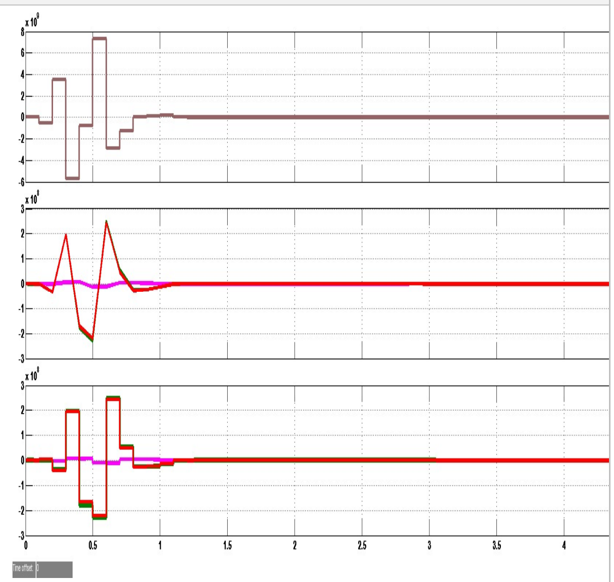
3	1	45°	0	0	
4	8	60°	4	6	

Table5: System analysis with different parameters

8.6. Usability of the Robot

Two-wheeled mobile robots are able to achieve better mobility and rotation in small spaces and to move faster than legged robots such as humanoid type robots. For this reason, the two-wheeled mobile robot is generally used as a mobile robot platform. However, to maintain its balance, the two-wheeled robot needs to use movements of its two wheels. When an unexpected disturbance affects the robot, the robot maintains its balance with movements of the wheels and tilting of the body. If the disturbance exceeds the response capability of the robot, the robot will lose its stability. At the same time, the safety of the robot may be put at risk by movements to maintain balance. To address these issues, a robot was designed with a control moment gyroscope module to improve balance while minimizing movement. When a disturbance is applied to the robot, the disturbance is estimated by a disturbance observer and the control moment gyroscope controller compensates the disturbance. Using the control moment gyroscope module, the robot can maintain balance with just small movements of its wheels.

The robot will only be run and tested indoors on flat surfaces. The pendulum is assumed to have one degree of freedom. It will therefore only be controlled in one direction, regarding both the angle of tilt and the position of the robot.

8.7. Safety of the Robot

Because of the way the chassis is designed, if the robot falls over, the wheels are raised off the ground, and the robot becomes immobile until it is again placed upright. As an additional precaution, we could have also had an automatic motor shutoff past a certain tilt angle, but we never took the time to implement it. But regardless, this device is not a toy and has powerful motors and batteries. It should be used by kids only under adult supervision. Always use in a dry environment, and never expose the robot to water or moisture. Always remove the batteries before storing for extended periods of time.

9. Conclusion

9.1. Evaluation

Inverted Pendulum is very difficult system to control due its intrinsic non linearity and instability. State feedback based control techniques namely pole placement and linear quadratic regulator is simulated with the help of MATLAB. The state feedback gain matrices are obtained and then closed loop responses for both cart position and angle of Inverted Pendulum are found to be satisfactory. Then the system states are estimated and observer based controller is designed. The observer based controller response is found to be same as state feedback closed loop response. Thus the system states are estimated successfully.

In this thesis, a dynamical model of the pendulum cart system is represented. The balancing and tracking control of the cart system is developed and analyzed. In figure25, it is analyzed that in a Controller based continuous time system, the graph converges fast and gets stable with increase in its parameters. With the use of LQR controller in figure 26, when we go lower with R value, the input becomes less and behavior of the system becomes better. With increase in Q value, the performance of the system becomes better. In figure27, ZOH (Zero order hold) is used for sampling the states of the system. A controller is efficient when it has access to all the states of the system. Therefore, an observer is constructed to get the output of the real system. Ackermann's approach for state feedback is not appropriate for large dimensional system.

Moreover, a protype of Segway which is based on the concept of Inverted Pendulum is designed. The measurements are done considering Center of Mass as priority. The structure of the bot has been designed using CAD. The connections between the components have also been specified. Development of Self Balancing Two Wheel self balancing robot using Arduino is a new approach for the personal transporter [28]. Besides from transport human as loads, the vehicle also designed with compact and lighter weight that consumes less space and energy to carry. In terms of short moving distances, having a two wheel self-balancing vehicle does supports travel within short distances as it does not consume energy and eco-friendly. Meanwhile, the cost of production is considered low compared to real actual product. The implementation of this low cost self-balancing bot is encouraged where it leads to eco-friendly by the battery used without harming environment and able to own by everyone with a smaller compact sized vehicle.

As a conclusion, the completion of this thesis managed to achieve all the objectives mentioned above.

9.2. Limitations

Ackermann’s approach for state feedback is not appropriate for large dimensional system.

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Appendices

Appendix-1

Input code

```
%%
clc
clear
 close all

%% Define the parameter
Mc = 1.5; % Mass of the cart
Mp = 0.5; % Mass of the pendulum
g = 9.82; % Earth gravity
 L = 1; % Length of the Pendulum
d1 = 1e-2;% Damping of the cart displacement
d2 = 1e-2;% Damping in the joint

%% Define the matrices
%First we define the matrices to use their elements easily
A = [0, 0, 1, 0; 0, 0, 0, 1; 0,(g*Mp)/Mc, -d1/Mc, -d2/(L*Mc); 0,(g*(Mc +
Mp))/(L*Mc), -d1/(L*Mc), -(d2*Mc + d2*Mp)/(L^2*Mc*Mp)];
 B = [0 ; 0 ; 1/Mc ; 1/(L*Mc)];

%% Output
%C = [0;1;0;0];
```

```
% q_2 as output C = [1;0;0;0];
% q_1 as output D = 0;

%% Build system sys = ss(A,B,C',D);
x0 = [0; 5*pi/180; 0;0];

%% Controller
des_pole = [-3;-3;-3;-3];
K = acker(A,B,des_pole
) Q = 10*eye(4);
R = 0.5; K_lqr = lqr(A,B,Q,R);

%% discrete time Ts = 0.1;
sys_d = c2d(sys,Ts)
Ad = sys_d.a;
Bd = sys_d.b;
Cd = sys_d.c;
Dd = sys_d.d;
des_pole_d = [0.3;0.3;0.3;0.3]*1;
K_d = acker(Ad,Bd,des_pole_d)

%% des_pole_d = [0.3;0.3;0.3;0.3]*0.1;
Ob = acker(Ad',Cd',des_pole_d)
```