2025 AMC10 必刷 20 题

前言： AMC10 是整个 AMC 系列考试中，考试题目难度和题目范围都最克制的一场。AMC10 相比于国际学科课程，知识在深度和广度上都有着较大的提升。可选择的题目和题目类型都相当的多。从传统数学竞赛的角度来看，一般会被分为代数，几何，数论和组合。我们在近几年考过的典型题目中，挑选了最有代表性的 20 个题，拆分到了四个模块，对于 2025 年需要准备 AMC10 的学生来说，这些近几年的典型题型是最合适的来准备 2025 年 AMC10 的材料。

板块一： Algebra

AMC10 里的代数内容涵盖了三角函数，复数，指数对数函数前的所有高中以前知识。涉及到重要的代数变化技巧，变形公式以及一系列高级数学问题所必备的代数基础，这样的基础不仅对于解决单独出现的代数类问题，典型的方程问题有帮助，还会悄悄出现在勾股定理，高阶数论问题中，是同学们必须掌握的基本内容。

题目 1：2023 AMC 12B Q14

Problem

For how many ordered pairs (a, b) of integers does the polynomial $x^3 + ax^2 + bx + 6$ have 3 distinct integer roots?

$\textbf{(A)}\ 5 \quad \textbf{(B)}\ 6 \quad \textbf{(C)}\ 8 \quad \textbf{(D)}\ 7 \quad \textbf{(E)}\ 4$

题目 2：2023 AMC12A Q12

Problem

What is the value of

$$2^3 - 1^3 + 4^3 - 3^3 + 6^3 - 5^3 + \cdots + 18^3 - 17^3?$$

$\textbf{(A)}\ 2023 \qquad \textbf{(B)}\ 2679 \qquad \textbf{(C)}\ 2941 \qquad \textbf{(D)}\ 3159 \qquad \textbf{(E)}\ 3235$

题目 3：2024 AMC12A Q9 / AMC10A Q15

Problem

Let M be the greatest integer such that both $M + 1213$ and $M + 3773$ are perfect squares. What is the units digit of M?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

题目 4：2023 AMC12B Q9/ AMC10B Q13

Problem

What is the area of the region in the coordinate plane defined by

$||x| - 1| + ||y| - 1| \leq 1?$

$\text{(A)}\ 2 \qquad \text{(B)}\ 8 \qquad \text{(C)}\ 4 \qquad \text{(D)}\ 15 \qquad \text{(E)}\ 12$

题目 5：2024 AMC12A Q10

Problem

Let α be the radian measure of the smallest angle in a $3-4-5$ right triangle. Let β be the radian measure of the smallest angle in a $7-24-25$ right triangle. In terms of α, what is β?

$\textbf{(A)}\ \frac{\alpha}{3} \qquad \textbf{(B)}\ \alpha-\frac{\pi}{8} \qquad \textbf{(C)}\ \frac{\pi}{2}-2\alpha \qquad \textbf{(D)}\ \frac{\alpha}{2} \qquad \textbf{(E)}\ \pi-4\alpha$

题目 6：2023 AMC10B Q22

Problem

How many distinct values of x satisfy $\lfloor x \rfloor^2 - 3x + 2 = 0$, where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x?

$\textbf{(A)}\ \text{an infinite number} \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 0$

题目 7：2021 AMC10B Q15

Problem

The real number x satisfies the equation $x + \frac{1}{x} = \sqrt{5}$. What is the value of $x^{11} - 7x^7 + x^3$?

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \sqrt{5}$

题目 8：2021 AMC 12A Q18/ AMC 10A Q18

Problem

Let f be a function defined on the set of positive rational numbers with the property that $f(a \cdot b) = f(a) + f(b)$ for all positive rational numbers a and b. Suppose that f also has the property that $f(p) = p$ for every prime number p. For which of the following numbers x is $f(x) < 0$?

$\textbf{(A)}\ \frac{17}{32} \qquad \textbf{(B)}\ \frac{11}{16} \qquad \textbf{(C)}\ \frac{7}{9} \qquad \textbf{(D)}\ \frac{7}{6} \qquad \textbf{(E)}\ \frac{25}{11}$

题目 9：2021 AMC12A Q16/AMC10A Q16

Problem

In the following list of numbers, the integer n appears n times in the list for $1 \leq n \leq 200$.

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots, 200, 200, \ldots, 200$$

What is the median of the numbers in this list?

$\textbf{(A)}\ 100.5 \qquad \textbf{(B)}\ 134 \qquad \textbf{(C)}\ 142 \qquad \textbf{(D)}\ 150.5 \qquad \textbf{(E)}\ 167$

板块二：Number Theory

数论是很完整的竞赛知识体系，且这些知识互相关联穿插。一方面，数论内容如果没有进行专门的，深入的学习，很多知识在标准的校内课程里学不到；另一方面，数论内容几乎不会单独考察某一方面，通常是综合进行数论的考察。所以，对于数论部分的处理应该是综合的。同学们需要注意的是，我们应当把数论的知识体系作为一个整体，理解它们之间的相互联系。

题目 10：2024 AMC12A Q12/ AMC10A Q19

Problem

The first three terms of a geometric sequence are the integers a, 720, and b, where $a < 720 < b$. What is the sum of the digits of the least possible value of b?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 21$

题目 11：2024 AMC12B Q14/AMC10B Q18

Problem

How many different remainders can result when the 100th power of an integer is divided by 125?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 125$

题目 12：2023 AMC12B Q15/AMC10B Q18

Problem

Suppose a, b, and c are positive integers such that

$$\frac{a}{14}+\frac{b}{15}=\frac{c}{210}.$$

Which of the following statements are necessarily true?

I. If $\gcd(a,14)=1$ or $\gcd(b,15)=1$ or both, then $\gcd(c,210)=1$.

II. If $\gcd(c,210)=1$, then $\gcd(a,14)=1$ or $\gcd(b,15)=1$ or both.

III. $\gcd(c,210)=1$ if and only if $\gcd(a,14)=\gcd(b,15)=1$.

$\textbf{(A)}$ I, II, and III $\quad$ $\textbf{(B)}$ I only $\quad$ $\textbf{(C)}$ I and II only $\quad$ $\textbf{(D)}$ III only $\quad$ $\textbf{(E)}$ II and III only

题目 13：2021 AMC10B Q13

Problem

Let n be a positive integer and d be a digit such that the value of the numeral $\underline{32d}$ in base n equals 263, and the value of the numeral $\underline{324}$ in base n equals the value of the numeral $\underline{11d1}$ in base six. What is $n+d$?

$\textbf{(A)}$ 10 $\quad$ $\textbf{(B)}$ 11 $\quad$ $\textbf{(C)}$ 13 $\quad$ $\textbf{(D)}$ 15 $\quad$ $\textbf{(E)}$ 16

题目 14：2021 AMC12B Q7/AMC10B Q12

Let $N=34\cdot 34\cdot 63\cdot 270$. What is the ratio of the sum of the odd divisors of N to the sum of the even divisors of N?

$\textbf{(A)}$ $1:16$ $\quad$ $\textbf{(B)}$ $1:15$ $\quad$ $\textbf{(C)}$ $1:14$ $\quad$ $\textbf{(D)}$ $1:8$ $\quad$ $\textbf{(E)}$ $1:3$

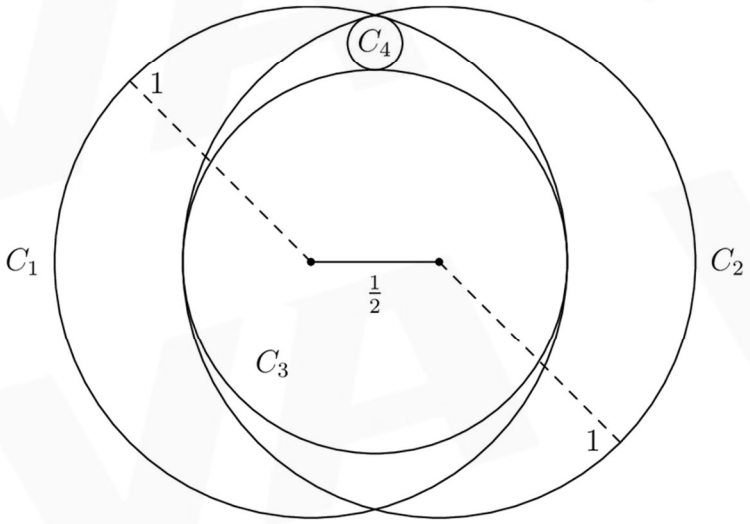
板块三：Geometry

AMC10 里的几何可以理解为是平面几何的过关考试。因为在这个年龄阶段之后，大部分的平面几何都会被转化为解析几何或和三角函数关联起来的解三角形等内容。所以 AMC10 的考察重点，还在几何本身，无论是三角形的勾股定理，相似，圆和不规则图形的面积计算，甚至解析几何中，都在频繁考察几何性质和对图形的理解。另外，AMC10 对于几何的考察也相对较广，还会涉及到立体几何等大家不太熟悉的内容。

题目 15：2023 AMC12A Q18/ AMC10A Q22

Problem

Circle C_1 and C_2 each have radius 1, and the distance between their centers is $\frac{1}{2}$. Circle C_3 is the largest circle internally tangent to both C_1 and C_2. Circle C_4 is internally tangent to both C_1 and C_2 and external tangent to C_3. What is the radius of C_4?

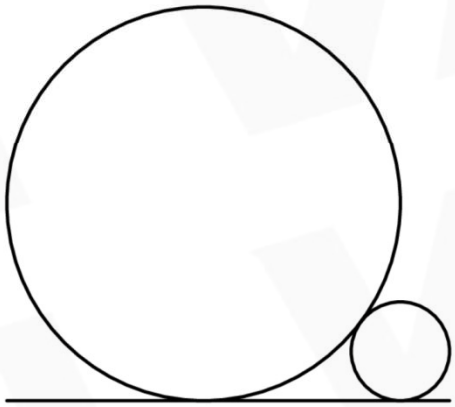


(A) $\frac{1}{14}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{3}{28}$ **(E)** $\frac{1}{9}$

题目 16：2024AMC10B Q21

Problem

Two straight pipes (circular cylinders), with radii 1 and $\frac{1}{4}$, lie parallel and in contact on a flat floor. The figure below shows a head-on view. What is the sum of the possible radii of a third parallel pipe lying on the same floor and in contact with both?



$\textbf{(A)}\ \frac{1}{9}$ $\quad$ $\textbf{(B)}\ 1$ $\quad$ $\textbf{(C)}\ \frac{10}{9}$ $\quad$ $\textbf{(D)}\ \frac{11}{9}$ $\quad$ $\textbf{(E)}\ \frac{19}{9}$

板块四：Probability

好消息：知识点比较少，

坏消息：题目不按常理出牌。

好消息：题不多，

坏消息：题有点难。

题目 17：2023 AMC12A Q7/ AMC10A Q9

Problem

A digital display shows the current date as an 8-digit integer consisting of a 4-digit year, followed by a 2-digit month, followed by a 2-digit date within the month. For example, Arbor Day this year is displayed as 20230428. For how many dates in 2023 will each digit appear an even number of times in the 8-digital display for that date?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

题目 18：2024 AMC12A Q16

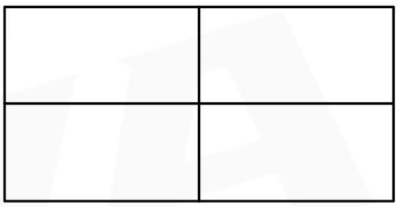
Problem

A set of 12 tokens — 3 red, 2 white, 1 blue, and 6 black — is to be distributed at random to 3 game players, 4 tokens per player. The probability that some player gets all the red tokens, another gets all the white tokens, and the remaining player gets the blue token can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

$\textbf{(A)}\ 387 \qquad \textbf{(B)}\ 388 \qquad \textbf{(C)}\ 389 \qquad \textbf{(D)}\ 390 \qquad \textbf{(E)}\ 391$

题目 19：2021 Fall AMC 10A Q18

A farmer's rectangular field is partitioned into 2 by 2 grid of 4 rectangular sections as shown in the figure. In each section the farmer will plant one crop: corn, wheat, soybeans, or potatoes. The farmer does not want to grow corn and wheat in any two sections that share a border, and the farmer does not want to grow soybeans and potatoes in any two sections that share a border. Given these restrictions, in how many ways can the farmer choose crops to plant in each of the four sections of the field?

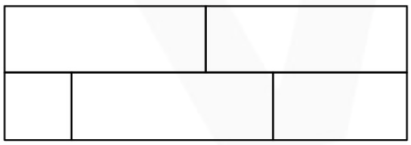


$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 64 \qquad \textbf{(C)}\ 84 \qquad \textbf{(D)}\ 90 \qquad \textbf{(E)}\ 144$

题目 20：2022 AMC12A Q7/AMC10A Q9

Problem

A rectangle is partitioned into 5 regions as shown. Each region is to be painted a solid color - red, orange, yellow, blue, or green - so that regions that touch are painted different colors, and colors can be used more than once. How many different colorings are possible?



$\textbf{(A)}\ 120 \qquad \textbf{(B)}\ 270 \qquad \textbf{(C)}\ 360 \qquad \textbf{(D)}\ 540 \qquad \textbf{(E)}\ 720$