Lesson 1
Multivariable Equations of Higher Degree and Absolute Values

Meet Teacher Dennis!

Courses taught at Think Academy:

From Pre-Algebra to Precalculus

AMC8, AMC10, and AIME coach

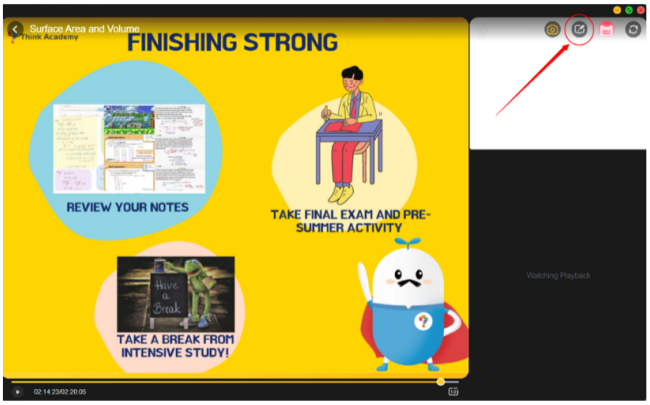
Fun Facts:

① Secret identity: coffee barista

② Have a husky

③ Visited 20 states

How to Reach Out to Your VIP Teacher During Class



Send a "feedback" to your VIP Teacher to initiate a chat

- The VIP Teacher may also initiate a chat with you, see if you have messages in the chatbox
- The VIP Teacher may initiate a temporary video chat with you when you need help

Meet Teacher Axell!

University of Texas at Austin

Major: Economics

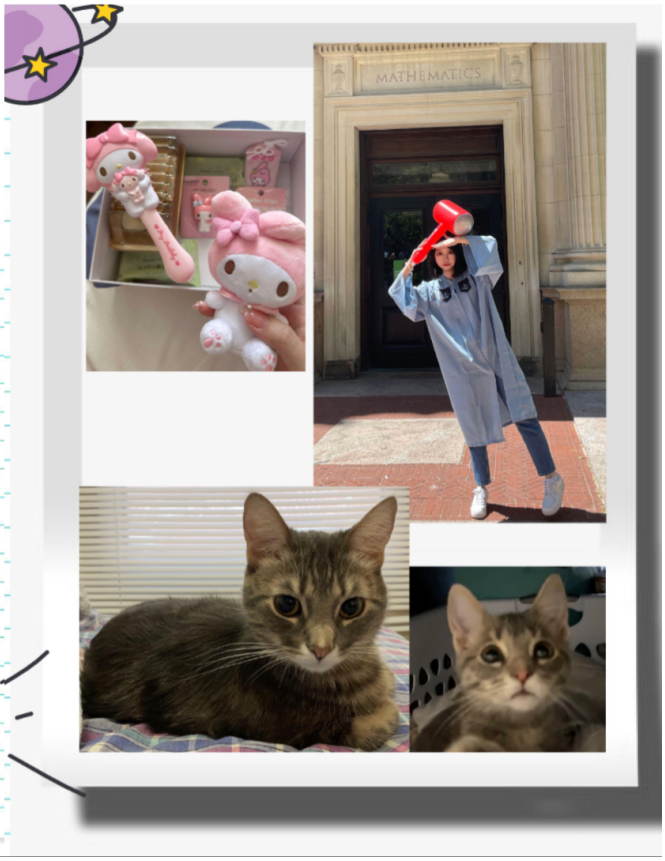
Fun Facts:

① Been to more than 10 countries

② K-pop dancer

Meet Teacher Elena!

- Applied Mathematics at Columbia University
- Teaching Assistants & Worked in Math Help Room
- Favorate Cartoon Character: "My Melody"
- A Cat Named "Tiger"
- Chocolate Ice Cream



Meet Teacher Lucy!

M.A. Learning Sciences and Educational Psychology

B.S. Education & Human Sciences

with a minor in Educational Psychology

Fun Facts:

① I have a little cat that is 0.5 years old now

Class Rules

1. Participate in this class and interact with me :)

2. Turn on your camera all the time!

3. Submit your homework on time

4. Review your mistakes often

Let's all qualify for AIME!

Concept 1

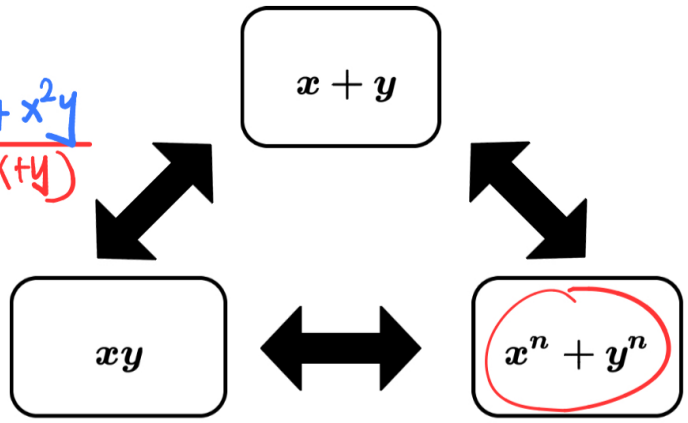
Substitution Method

Relationship between $x + y$ and xy

If you know any two of the following expressions, you can always calculate the other one!

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)(x^2+y^2) = x^3+y^3+\frac{xy^2+x^2y}{xy(x+y)}$

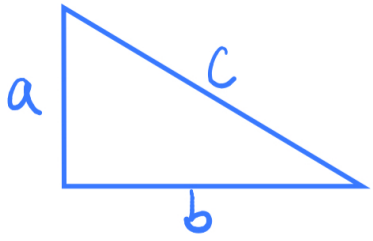


Also Found in Geometry!

$a+b = P_\triangle - c$

$ab = 2A_\triangle$

$a^2+b^2 = c^2$



MC(1)

Math Exploration 1.1

Two non-zero real numbers, a and b, satisfy $ab = a - b$. Which of the following is a possible value of $\frac{a}{b} + \frac{b}{a} - ab$?

A. -2 B. $\frac{-1}{2}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. 2

$\Rightarrow \frac{a^2+b^2}{ab} - ab$

$= \frac{a^2b^2+2ab}{ab} - ab$

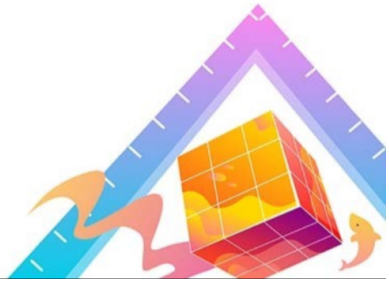
$= ab + 2 - ab$

$= 2$

$a^2b^2 = (a-b)^2$

$a^2b^2 = a^2 - 2ab + b^2$

$a^2 + b^2 = a^2b^2 + 2ab$



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Math Exploration 1.2

If $\frac{1}{a} - \frac{1}{b} = 4$, find the value of $\frac{a - 2ab - b}{2a + 7ab - 2b}$.

$\frac{b-a}{ab} = 4$

$b - a = 4ab$

$-(b-a) = -4ab$

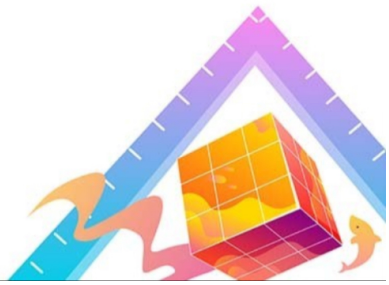
$a - b = -4ab$

$\frac{(a-b) - 2ab}{(2a-2b) + 7ab} = \frac{-4ab - 2ab}{-8ab + 7ab}$

$2(a-b)$

$= \frac{-6ab}{-ab}$

$= 6$



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Practice 1.1

$\frac{1\cdot 2b}{a\cdot 2b} + \frac{1\cdot a}{2b\cdot a}$

Given that $\frac{1}{a} + \frac{1}{2b} = 3$, find the value of $\frac{2a - 5ab + 4b}{4ab - 3a - 6b}$.

$\Rightarrow \frac{a+2b}{2ab} = 3$

$a + 2b = 6ab$

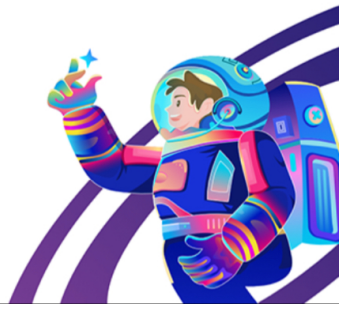
$\frac{2a+4b-5ab}{-3a-6b+4ab}$

$= \frac{2(a+2b)-5ab}{-3(a+2b)+4ab}$

$= \frac{2\cdot 6ab - 5ab}{-3\cdot 6ab + 4ab}$

$= \frac{7}{-14}$

$= -\frac{1}{2}$



MC(1)

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

Practice 1.2

Let a and b be relatively prime integers with $a > b > 0$ and $\frac{a^3 - b^3}{(a-b)^3} = \frac{73}{3}$.

What is $a - b$? (

$a^2+ab+b^2 = a^2-2ab+b^2+3ab = (a-b)^2+3ab$

2012 AMC10A Problem, Question#17)

A. 1 B. 2 C. 3 D. 4 E. 5

$\Rightarrow \frac{(a-b)(a^2+ab+b^2)}{(a-b)^3}$

$= \frac{a^2+ab+b^2-2ab+2ab}{(a-b)^2}$

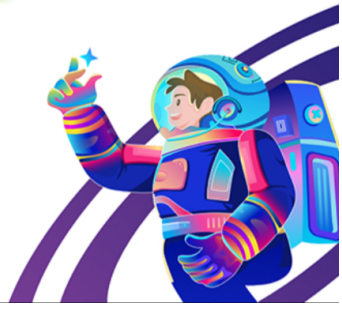
$= \frac{(a-b)^2+3ab}{(a-b)^2}$

$= 1 + \frac{3ab}{(a-b)^2}$

$\Rightarrow 1 + \frac{3ab}{(a-b)^2} = \frac{73}{3}$

$\frac{3ab}{(a-b)^2} = \frac{70}{3} = \frac{3\cdot 70}{9}$

$\begin{cases} ab = 70 \\ a-b = 3 \end{cases} \Rightarrow \begin{cases} a = 10 \\ b = 7 \end{cases}$



Concept 2

System of Multivariable Equations

MC(1)

Math Exploration 2.1

if $y+4=(x-2)^2$, $x+4=(y-2)^2$, and $x \neq y$, what is the value of x^2+y^2 ? (2015 AMC 10A Problem, Question#16)

A. **10** B. **15** C. **20** D. **25** E. **30**

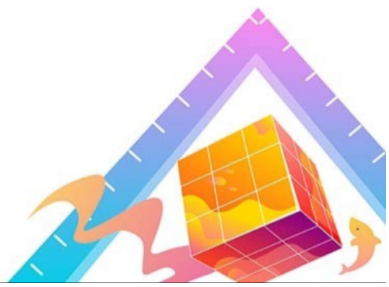
$$\begin{cases} y+4=x^2-4x+4 \\ x+4=y^2-4y+4 \end{cases} \Rightarrow \begin{cases} x^2=4x+y \\ y^2=4y+x \end{cases} \Rightarrow x^2+y^2=4x+y+4y+x = 5(x+y)$$

Subtract: $x^2-y^2=(4x+y)-(4y+x)$

$$(x-y)(x+y)=3x-3y=3(x-y)$$

$$x+y=3$$

Ans: $x^2+y^2=5(x+y)=5\cdot 3=15$



MC(1)

Practice 2.1

If a, b, and c are positive real numbers such that $a(b+c) = 152$, $b(c+a) = 162$, and $c(a+b) = 170$, then abc is _____ .

A. **672** B. **688** C. **704** D. **720** E. **750**

$\begin{cases} ab+ac=152 & ① \\ bc+ab=162 & ② \\ ac+bc=170 & ③ \end{cases}$

② − ①: $bc-ac=10$

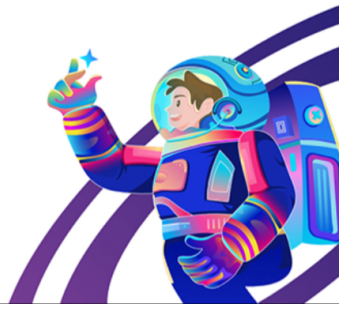
③: $bc+ac=170$

$\Rightarrow \begin{cases} ab=72 \\ ac=80 \\ bc=90 \end{cases}$

$\Rightarrow ab\cdot ac\cdot bc = 72\cdot 80\cdot 90$

$a^2b^2c^2 = (36\cdot 2)\cdot(8\cdot 10)\cdot(9\cdot 10)$

$abc = \sqrt{36\cdot 16\cdot 10^2\cdot 9} = 720$



MC(1)

Practice 2.2

There are **3** numbers A, B, and C, such that $1001C - 2002A = 4004$, and $1001B + 3003A = 5005$. What is the average of A, B, and C? ( ) $\frac{A+B+C}{3}$

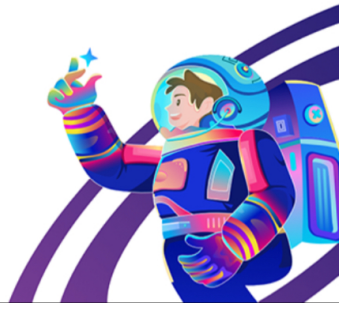
A. **1** B. **3** C. **6** D. **9** E. Not uniquely determined

$1001C - 2002A + 1001B + 3003A = 5005 + 4004$

$1001(A+B+C) = 9009$

$A+B+C = 9$

$\text{Average} = \frac{A+B+C}{3} = 3$



Concept 3

Discriminant of Quadratic Equations

Concept 3

A quadratic equation $ax^2 + bx + c = 0$ has 3 parameters

Methods to solve AMC10 problems about the parameters:

(1) **Factorization**

$ax^2 + bx + c = a(x - r_1)(x - r_2)$

Double root: $a(x-r)^2$

(2) **Discriminant**

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

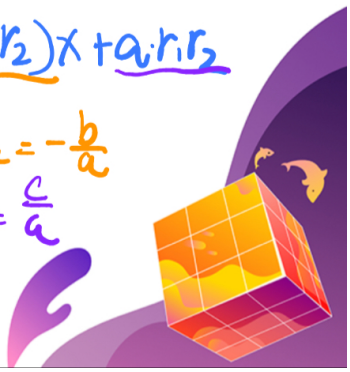
(3) **Vieta's Formula**

$$r_1 + r_2 = \frac{-b}{a}, r_1 r_2 = \frac{c}{a}$$

$ax^2+bx+c = ax^2 - a(r_1+r_2)x + a r_1 r_2$

$b = -a(r_1+r_2) \Rightarrow r_1+r_2 = -\frac{b}{a}$

$c = a r_1 \cdot r_2 \Rightarrow r_1 \cdot r_2 = \frac{c}{a}$



MC(1)

Math Exploration 3.1

The quadratic equation $x^2 + mx + n = 0$ has roots that are twice those of $x^2 + px + m = 0$, and none of m, n, and p is zero. What is the value of $\frac{n}{p}$?

A. 1 B. 2 C. 4 D. 8 E. 16

$\text{Sum}_1 = 2 \cdot \text{Sum}_2 \Rightarrow -m = 2(-p)$

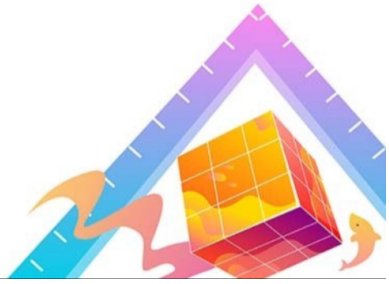
$m = 2p$

$\text{Product}_1 = 4\,\text{Product}_2 \Rightarrow n = 4m$

$\Rightarrow n = 4m = 4 \cdot 2p$

$\Rightarrow n = 8p$

$\frac{n}{p} = 8$



MC(1)

Math Exploration 3.2

How many ordered pairs of positive integers (b, c) exist where both $x^2 + bx + c = 0$ and $x^2 + cx + b = 0$ do not have distinct, real solutions? (2021 AMC Fall 10A, Question #20)

A. 4 B. 6 C. 8 D. 10 E. 12

$\begin{cases} b^2 - 4c \le 0 \\ c^2 - 4b \le 0 \end{cases} \Rightarrow \begin{cases} b^2 \le 4c \\ c^2 \le 4b \end{cases} \Rightarrow b^4 \le 16c^2 \Rightarrow c^2 \ge \frac{b^4}{16}$

$\Rightarrow \frac{b^4}{16} \le c^2 \le 4b$

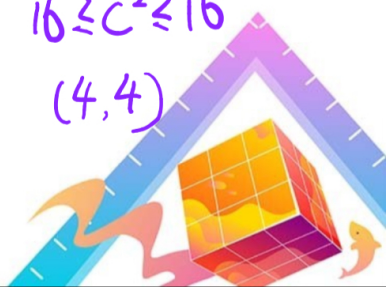
① $b = 1$: $\frac{1}{16} \le c^2 \le 4$ → $(1,1), (1,2)$

② $b = 2$: $1 \le c^2 \le 8$ → $(2,1), (2,2)$

③ $b = 3$: $\frac{81}{16} \le c^2 \le 12$ → $(3,3)$

④ $b = 4$: $16 \le c^2 \le 16$ → $(4,4)$

6 solutions



MC(1)

$b-a=c-b$

Practice 3.1

The real numbers c, b, a form an arithmetic sequence with $a \geqslant b \geqslant c \geqslant 0$.

The quadratic ax^2+bx+c has exactly one root. What is this root? (2013 AMC 10B Problem, Question #19)

A. $-7-4\sqrt{3}$ B. $-2-\sqrt{3}$ C. -1 D. $-2+\sqrt{3}$ E. $-7+4\sqrt{3}$

$ax^2+bx+c=a(x-r)^2$

$\frac{a}{a}\cdot x^2+\frac{b}{a}x+\frac{c}{a}=x^2-2rx+r^2$

Observe: $\frac{b}{a}-\frac{a}{a}=\frac{c}{a}-\frac{b}{a}$

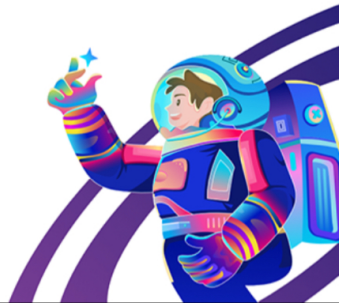
$\Rightarrow 1, -2r$ and r^2 are arithmetic sequence.

$\Rightarrow -2r-1=r^2-(-2r)$

$r^2+4r+1=0$

$r=\frac{-4\pm\sqrt{4^2-4}}{2}=-2\pm\sqrt{3}$

Check sign: $-2+\sqrt{3}$



Concept 4

The Geometric Meaning of Absolute Values

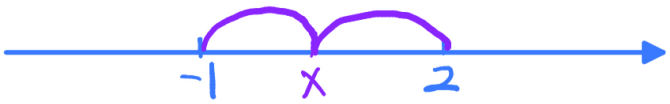
Concept 4

Geometric definition of $|a|$: the distance between point a and the origin on the number line. $|-2|$

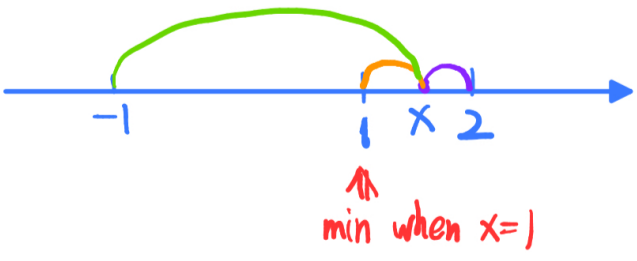
Geometric definition of $|a-b|$: the distance between points a and b on the number line. $|-3-(-1)|$

Find the minimum value

2 absolute values: $|x+1|+|x-2|$



3 absolute values: $|x+1|+|x-1|+|x-2|$



Find the minimum value

On a number line, there are n points representing the numbers $a_1, a_2, \ldots, a_n$, where

$a_1 \leqslant a_2 \leqslant a_3 \leqslant \cdots \leqslant a_n$

Let P be a point on the number line representing the number x.

Considering the algebraic expression:

$S = |x - a_1| + |x - a_2| + \cdots + |x - a_n|$

From the geometric meaning of absolute value, we can conclude:

① If n is odd, the minimum value of S is achieved when

$x = a_{\frac{n+1}{2}}$

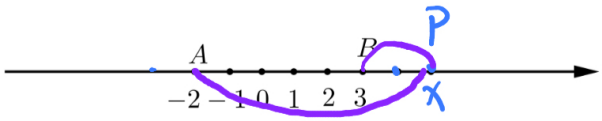
② If n is even, the minimum value of S is achieved when

$a_{\frac{n}{2}} \leqslant x \leqslant a_{\frac{n}{2}+1}$

Combination

Math Exploration 4.1

As an example, we know that $|5 - (-3)|$ defines the absolute value of $5 - (-3)$, which is also the distance between 5 and -3 on the number line. If the points A and B on the number line represent -2 and 3, respectively, and P is a moving point that represents the number x, answer the following questions:



(1) Use an expression including x to represent the distance between points P and B.

$|X-3|$

(2) Find all integers such that $|x+2| + |x-3| = 7$

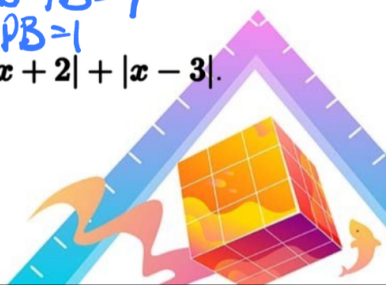
$PA + PB = 7$ ① If $PA > PB$, $PA = AB + PB$ $x = 4$ or -3

$AB + PB + PB = 7$

$PB = 1$

(3) Based on the previous discoveries, find the minimum value of $|x+2| + |x-3|$.

$\boxed{5}$



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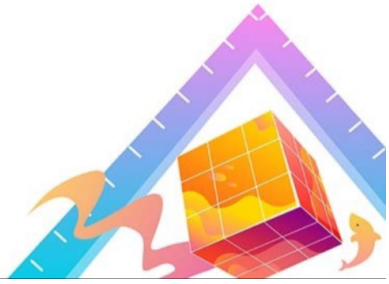
Math Exploration 4.2

Find the minimum value of $2|x-1|+|x-2|$.

$\Rightarrow |x-1|+|x-1|+|x-2|$

when $x=1$, $\min = 0+0+1$

$= 1$



MC(1)

Practice 4.2

Let a, b, c, and d be real numbers with $|a-b|=2$, $|b-c|=3$, and $|c-d|=4$. what is the sum of all possible values of $|a-d|$? (2009 AMC 10A Problem, Question#16)

A. **9** B. **12** C. **15** D. **18** E. **24**

$\begin{cases} a-b=\pm 2 \\ b-c=\pm 3 \\ c-d=\pm 4 \end{cases}$

$\Rightarrow a-d=\pm 2\pm 3\pm 4$

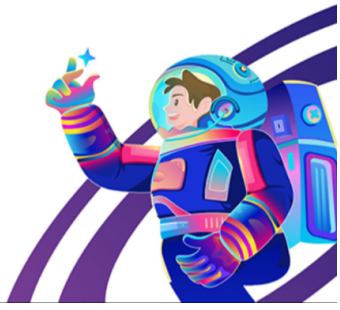
① $2+3+4=9$

② $-2+3+4=5$

③ $2-3+4=3$

④ $2+3-4=1$

Ans $=9+5+3+1=18$



PhotoPost

Math Exploration 4.3

when $x=\frac{1}{4}$, min $= \frac{3}{4}+\frac{2}{4}+\frac{1}{4}+\frac{1}{4}$
$= \frac{7}{4}$

Find the minimum value of

$|x-1|+|2x-1|+|3x-1|+|4x-1|+|5x-1|$ and the value of x on the case.

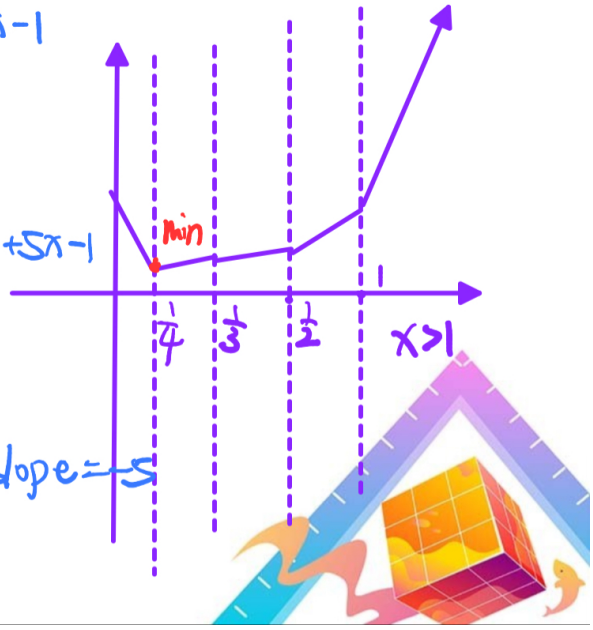
① $x>1$: $x-1+2x-1+3x-1+4x-1+5x-1$
$= 15x-5$
slope $= 15$

② $\frac{1}{2}<x\leq 1$: $-(x-1)+2x-1+3x-1+4x-1+5x-1$
$= 13x-3$
slope $= 13$

③ $\frac{1}{3}<x\leq\frac{1}{2}$: slope $= 9$

④ $\frac{1}{4}<x\leq\frac{1}{3}$: slope $= 3$

⑤ $\frac{1}{5}<x\leq\frac{1}{4}$, slope $= -5$



MC(1)

Practice 4.1

What is the minimum value of

$|x-1|+|2x-1|+|3x-1|+\cdots+|119x-1|$? ( )

A. **49** B. **50** C. **51** D. **52** E. **53**

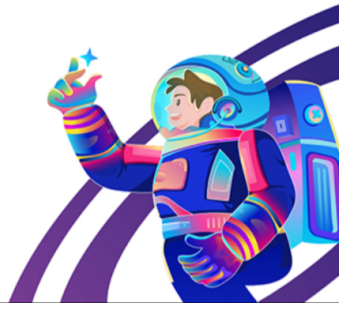
Find x to make slope become negative.

$\Rightarrow 1+2+3+\cdots+k \geq \frac{1}{2}(1+2+3+\cdots+119)$

$\frac{(k+1)k}{2} \geq \frac{1}{2}\cdot\frac{(1+119)\cdot 119}{2}$

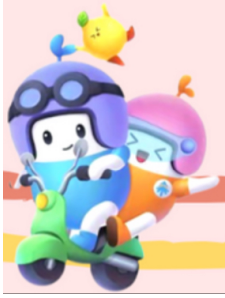
$k \geq 84$

$\Rightarrow x=\frac{1}{84}$: $\left|\frac{1}{84}-1\right|+\left|\frac{2}{84}-1\right|+\cdots+\left|\frac{119}{84}-1\right|$
$= \frac{83+82+\cdots+1+1+2+35}{84} = 49$



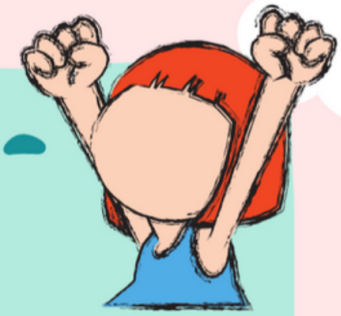
Welcome Parents & Students

2025 Summer Kick-off Parent Meeting



Today's Agenda

- Special Learning Theme for the term: Homework
- How to learn with us?



WHY HOMEWORK MATTER?

It shows how well student understood the lesson!

It builds great habits for the future!

It helps me help student!

What I will do to encourage my students to complete their homework is to.....

- Consistently recognize their efforts
- Remind them of the rewards and progress that come with every submission.

Loop for Building Strong HW Habits

📌 **1. Motivation Plan**

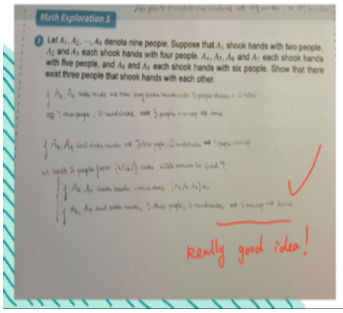
- Weekly coin rewards for submission
- Special recognition for the class with the highest completion rate

📣 **2. Public Recognition**

- Weekly Homework Leaderboard (Submission / Accuracy / On-time attendance)
- Showcase outstanding work in class

📝 **3. Personalized Comments (Parent app)**

- Every homework gets a meaningful comment
- Help students identify learning gaps and improve



🤝 **4. Long-Term Support**

- Teachers track growth through weekly submissions, stage tests and others
- 10 lessons = solid foundation for the entire school year

Every step is recognized and encouraged to motivate consistent progress.

1、Submitting homework: 200 coins

2、Submitting within one week earns an extra 100 coins

3、Showcase Last Week's Outstanding Homework to my students

4、Homework Submission Leaderboard : 100 coins

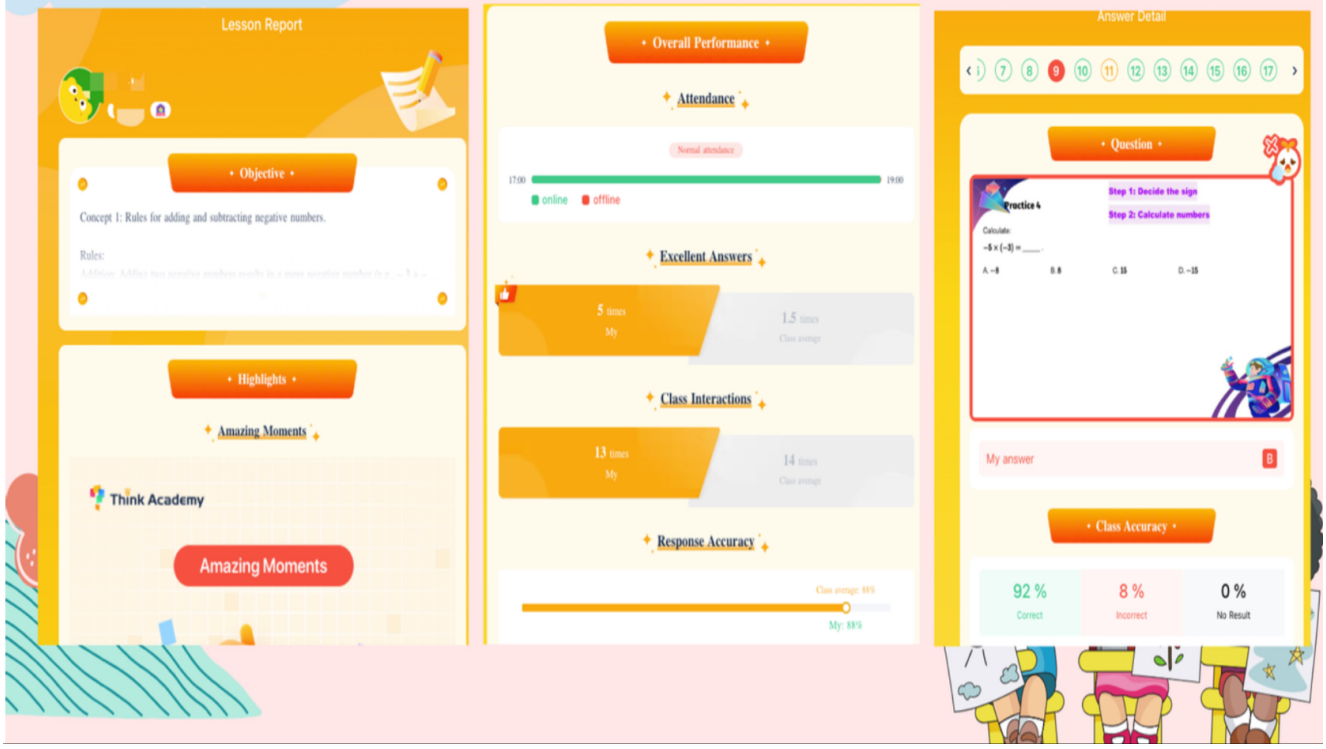
5、Perfect Homework Leaderboard : 100 coins

6、🏆 Highest Submission Class Reward within the grade will be announced the following class. Top performing class earns 500 coins the next following week.

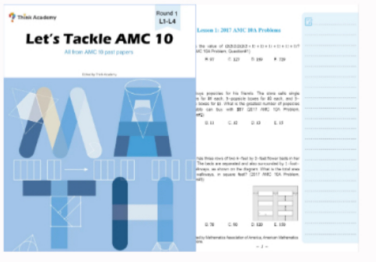
Strengthen a sense of class community and collective honor

Get Great In-Class Performance Report & HW feedback

You can receive it on the Parent App every week



Free AMC10 Workshop



- Each workshop is 2 hours, every two weeks
- Class contents:
  - Timed practice on Problem 1-12
  - Study important/challenging problems
  - Get used to the exam time management
  - Understand the efficient ways to solve problems
- Electronic handouts available in the app
- Summer Class Schedule:
  - 6/14-8/9 Sat 1-3PM PDT **(Added by Default)**
  - 6/19-8/14 Thu 5-7PM PDT **(You can transfer in APP)**

Office Hour Schedule

- Tuesday 6-7PM PDT
- Teacher will explain homework problems and answer questions.
- Join Office Hour via Student app
- **Playback available!**