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The "Salt-Fountain" and Thermohaline Convection

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Abstract

A "gravitationally stable" stratification of salinity and temperature, such as is observed in the oceans, is actually unstable due to the fact that the *molecular* diffusivity of heat is much greater than the diffusivity of salt. We discuss this stability characteristic and the form of the convective motion in the laminar regime. Future studies of this model relative to the amplitude of the motion and the subsequent transition to turbulence should lead to the formulation of critical observational questions, which will determine whether the proposed mechanism is significant in the vertical mixing of the sea.

STOMMEL, ARONS and BLANCHARD (1956) have described an "oceanographical curiosity" by noting that if a long vertical tube was lowered into the ocean, in such a manner that its bottom was exposed to cold fresh water and its top to warm saline water, a continuous motion could be maintained therein after priming the fountain. Their explanation is that the ascending (or descending) water in the tube would exchange heat but not salinity with the ambient ocean and would be accelerated due to its deficit in salt and density relative to fluid at the same level outside the tube. The purpose of this note, stemming from conversations with Henry Stommel, is to point out that in view of the great difference between the molecular diffusivity of salt ($K_S = 1.3 \times 10^{-5}$ cm^2 sec^{-1} for salinity of 35 ‰ at 20° C) and temperature ($K_T = 1.5 \times \times 10^{-3}$ cm^2 sec^{-1}) nature provides her own convective fountains. If a parcel of small radius is given a small vertical displacement it will lose its temperature excess much more rapidly than its salinity excess and the resulting buoyant force may be sufficient to maintain a free convection, despite the fact that the mean density field increases in the direction of gravity. The decrease of mean salinity in this direction provides the energy source for the convective motions, and if this salt gradient is maintained by climatological factors then it furnishes a mechanism, whose relative importance must be determined, for the vertical transport of salt and heat in the ocean.

In order to forestall premature discussion of eddy excange coefficients let us first consider an isolated system consisting of a horizontal convection chamber whose top surface is maintained at a higher temperature and salinity than the bottom surface. An equilibrium state consisting of linear variations of temperature and salinity may be produced, with the denser liquid below, and we shall inquire into the stability of this arrangement. This system is subjected to small perturbations and it will be shown that there is a direct, but superficial, analogy with the classical Rayleigh-Benard convection where the denser liquid is above. In either problem elimination of the horizontal velocity perturbations and the pressure from the four dynamical equations gives the following relation between the vertical velocity (w) and the buoyancy force ($g\varrho/\varrho_0$):

$$(\lambda - \nu\nabla^2)\nabla^2 w = -\nabla_2^2 g\varrho/\varrho_0 \qquad (1)$$

where ν is the kinematic viscosity, ∇_2^2 is the horizontal laplacian, and λ the complex growth rate of the eigen-modes replaces the local time derivative. It will be assumed that all molecular parameters are constant, in particular the coefficients of temperature and salinity expansion, so that the equation of state is linear in the temperature (T) and salinity (S in gms/c.c.) perturbations.[1] Let $\frac{\partial \overline{T}}{\partial z} = -\beta_T$, $\frac{\partial \overline{S}}{\partial z} = \beta_S$ denote the mean temperature and salinity gradients in a direction (z) opposite to gravity. Local changes in temperature and salinity are brought about by advection of these mean fields and by diffusion, these relations being expressed by (2) and (3) below:

$$(\lambda - K_T\nabla^2)T = \beta_T w \qquad (2)$$

$$(\lambda - K_S\nabla^2)S = -\beta_S w \qquad (3)$$

$$-g\varrho/\varrho_0 = g(\alpha T - \sigma S) \qquad (4)$$

Equation (4) is the equation of state where $\sigma(\approx 1$ c g.s.) is a function of the volumetric expansion of water due to a dissolved mass of salt. We first investigate the condition of marginal stability and later consider internal waves. Thus, when $\lambda = 0$ eqs. (1)—(4) combine to give:

$$\nu\nabla^6 w = g\nabla_2^2\left[\frac{\alpha\beta_T}{K_T} + \frac{\sigma\beta_S}{K_S}\right]w \qquad (5)$$

In the case $\beta_S = 0$, $\beta_T > 0$ (5) applies to thermal convection. For the thermohaline convection problem we shall explicitly state that $\alpha\beta_T$ is negative and greater in magnitude than $\sigma\beta_S$ so that the mean density field decreases with z. However if $\sigma\beta_S/|\alpha\beta_T| > \frac{K_S}{K_T} \sim 1/100$ then the bracketed term in (5) is again positive and when the bounding surfaces are perfect conductors of heat and salinity there is a direct correspondence with Rayleigh's problem. The analogy is made by identifying the field $\left[\frac{\sigma\beta_S}{K_S} + \frac{\alpha\beta_T}{K_T}\right]$ with a new thermal profile $\frac{\alpha\beta'_T}{K_T}$, and we therefore conclude that the condition for marginal stability depends on the numerical value of the following thermohaline Rayleigh number:

$$R_S + R_T = \frac{gH^4}{\nu}\left[\frac{\sigma\beta_S}{K_S} + \frac{\alpha\beta_T}{K_T}\right] \qquad (6)$$

where H is the vertical height of the cell or the chamber. The minimum value of this quantity depends on the kinematical boundary conditions and in general is of order 10^3. If this number is exceeded then the diffusive equilibrium state is unstable. It will be noted in the formal argument that we need not assume β_S and β_T to be independent of z and therefore the marginal stability analogy ought to hold when these basic fields are non-linear, representing quasi-steady diffusive states. An example of this occurs when warm saline water is placed above cold, fresh and dense water. Henry Stommel and Alan Faller have performed such an experiment and have kindly made available the notes of the results.

Approximately 250 c. c. of warm water (38° C) were added to a graduate above 250 c.c. of water at (18° C), care being taken to avoid excessive mixing at the interface. The salt gradient was then produced by the addition of 2.5 c. c. of sea water to the upper layer, with a small quantity of menthylene blue dye solution added as a tracer. A fairly even distribution of the additive was produced by the mixing in the upper layer. It was immediately noticed that small elements of the dyed saline water began to extend downward into the denser region, taking the form of thin vertical and continuous filaments that eventually reached the bottom. There were four cells per linear inch while the speed of descent of the columns was about 1×10^{-2} cm sec^{-1}. These regular structures were completely disrupted when a thermometer was inserted but re-established themselves within a few minutes. The experiment was repeated in a small aquarium (10″ × 30″ × 11″ deep). Cold water (15° C) filled the bottom half and warm water (49° C) the top. A mixture of salt water and dye was introduced to give about the same density contrast as in the previous case. The qualitative results were the same as above.

The following points may be noted from these observations. The stabilizing thermal contrast was about twenty times greater than the destabilizing salt field, measured in terms of density alone. The mixed layer separating the

[1] N. P. Fofnoff in an unpublished paper entitled "Energy Transformations and Stability in the Ocean" discusses the effect of the non-linearity in the equation of state on the mixing of water masses of different temperature and salinity.

two fluids was of the order of three to six centimeters and therefore the thermohaline Rayleigh number was highly supercritical. The horizontal size and shape of the convective pattern are different from what one would expect by analogy with the thermal problem. A qualitative explanation of this latter point will be sought by comparing growth rates of disturbances of differing sizes at supercritical values of $R_S + R_T$. It will also appear that internal gravity waves are damped so that the criterion for marginal stability gives the minimum critical value for self-excited (infinitesimal) disturbances. In this calculation β_S and β_T are constant and "free-free" boundaries are assumed so that the vertical velocity, temperature and salinity perturbations are of the form $\sin(m\pi z)\exp \pi i(kx + ly)$, where m is an integral multiple of the reciprocal height of the chamber. Substituting these in (1)—(4) gives the following cubic equation for λ:

$$\left(\frac{\lambda}{K_S a^2} + 1\right)\left(\frac{\lambda}{K_S a^2} + r\right)\left(\frac{\lambda}{K_S a^2 r N_p} + 1\right) = \\ = f\left[(r - D) - \frac{\lambda}{K_S a^2}(D - 1)\right] \tag{7}$$

where

$$f = \frac{g\sigma\beta_S a_2^2}{\nu K_S a^4 a^2} \qquad D = -\frac{\alpha\beta_T}{\sigma\beta_S} \qquad N_P = \frac{\nu}{K_T}$$

$$r = \frac{K_T}{K_S} \qquad a_2^2 = \pi^2(k^2 + l^2) = a^2 - \pi^2 m^2$$

To determine whether neutral gravity waves can exist substitute $\lambda = i\lambda_i K_s a^2$ into (7). Equating real and imaginary parts to zero gives

$$\lambda_i^2\left(1 + \frac{1 + r}{rN_P}\right) = r - f(r - D) \tag{8}$$

$$\lambda_i^2 = N_P[f(D - 1) + 1 + r] + r \tag{9}$$

By eliminating the frequency (λ_i) it may readily be shown that these equations are incompatible if $D > 1$ and $f > 0$, from which it is concluded that internal waves are damped[1] and there can only be one root whose real part is positive and this root must be a real number.

The approximate value of this root is now computed under the conditions: $r \gg D \gg 1$, $N_P > 1$. In the experiment mentioned above $N_P = 7$, $r = 120$, $D \sim 20$. First note that the left hand side of (7) is positive and therefore the right side implies the upper bound $\frac{\lambda}{K_S a^2} < \frac{r - D}{D - 1} \sim \frac{r}{D}$. Let $O(1/D)$ denote a term which is small compared to unity by order $1/D$. Then we approximate the cubical expression as follows:

$$\left(\frac{\lambda}{K_S a^2} + 1\right)\left(\frac{\lambda}{K_S a^2} + r\right)\left(\frac{\lambda}{K_S a^2 r N_P} + 1\right) =$$

$$= \left[r + \frac{\lambda}{K_S a^2}(1 + r)\left(1 + \frac{\lambda/K_S a^2}{1 + r}\right)\right][1 + O(1/D)]$$

$$= \left[r + \frac{\lambda}{K_S a^2}(1 + r)[1 + O(1/D)]\right][1 + O(1/D)]$$

$$\sim r + \frac{\lambda}{K_S a^2}(1 + r)$$

Equating this to the right side of (7) gives

$$\frac{\lambda}{K_S a^2} \sim \frac{f(r - D) - r}{r + 1 + f(D - 1)} \sim \frac{f(1 - D/r) - 1}{1 + fD/r}$$

and when the definitions of D, f, r are substituted above one obtains

$$\frac{\lambda}{K_S} \sim \frac{(a_2^2 + m^2\pi^2)\,(g\sigma\beta_S/\nu K_S)(1 - D/r)}{a_2^{-2}(a_2^2 + m^2\pi^2)^3 + |g\alpha\beta_T/\nu K_T|} - \\ - \frac{a_2^{-2}(a_2^2 + m^2\pi^2)^3}{a_2^{-2}(a_2^2 + m^2\pi^2)^3 + |g\alpha\beta_T/\nu K_T|} \tag{10}$$

We already know that at marginal stability the size of the cell is $m = \frac{1}{H}$, $a_2 = O(m)$. At supercritical conditions it follows from (10) that disturbances of this size grow at the rate $\lambda H^2/K_S \sim R_S|R_T|^{-1} = rD^{-1}$. Compare this to disturbances which have the much larger horizontal wave number: $a_2^2 = c|g\alpha\beta_T/\nu K_T|^{1/3}$ where c is an arbitrary number of order unity. Since $D \gg 1$ and $R_T \gg 10^3$ eq. (10) simplifies to $\frac{\lambda H^2}{K_S} \sim \frac{c}{1 + c^2} R_S|R_T|^{-\frac{1}{3}}$, which has a maxi-

[1] Compare the interesting case of a "solute" which is distributed so as to stabilize a super-adiabatic lapse rate of temperature. For exemple, if $f < 0$, $r > D > 1$ (i.e. $\beta_S < 0$, $\beta_T > 0$) the salt can stabilize the stationary mode and destabilize the oscillatory mode, thereby releasing the potential energy in the thermal stratification.

mum at $c = 1$. Amongst the class of disturbances whose horizontal plan-form consists of close-packed squares, the one which grows the fastest has a side of length

$$L \sim \pi \left| \frac{g \alpha \beta_T}{4 \nu K_T} \right|^{-\frac{1}{4}} \tag{11}$$

In the first experiment which was cited the temperature difference between top and bottom was 20° C and the transition region between the two fluids was 3—6 cm. Substituting the mean gradient into (11) gives a value of L between 0.2 and 0.3 cm, as compared with an observed value of about 0.3 cm.

It appears then that although cells having a horizontal dimension comparable with H are preferred just above the critical point, their large size requires that the vertical velocity be kept small enough so that the temperature excess may be diffused. As the Rayleigh numbers are increased, with the salt-temperature ratio held constant, the thin columns can reach greater velocities and more effectively release the potential energy in the salt stratification. This explanation was suggested by Willem Malkus and I should like to acknowledge the fruitful discussions with him and with Bert Bolin.

REFERENCE

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