

$$\rho_0 \left\{ \frac{\partial(\bar{u} + u')}{\partial t} + \frac{\partial(\bar{u} + u')^2}{\partial x} + \frac{\partial(\bar{u} + u')(\bar{v} + v')}{\partial y} + \frac{\partial(\bar{u} + u')(\bar{w} + w')}{\partial z} \right\} + \rho_0(-f\bar{v} + f_*\bar{u})$$

$$= -\frac{\partial(\bar{p} + p')}{\partial x} + \mu \left( \frac{\partial^2(\bar{u} + u')}{\partial x^2} + \frac{\partial^2(\bar{u} + u')}{\partial y^2} + \frac{\partial^2(\bar{u} + u')}{\partial z^2} \right)$$

$$F^x = \frac{\partial \tau^{xx}}{\partial x} + \frac{\partial \tau^{yx}}{\partial y} + \frac{\partial \tau^{zx}}{\partial z}$$

$$= \frac{1}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) + \frac{1}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \frac{1}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right)$$

$$= \mu \nabla^2 u$$

$$\Delta = \nabla^2$$

$$\rho_0 \left( \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} \right) + \rho_0(-f\bar{v} + f_*\bar{u}) = -\frac{\partial \bar{p}}{\partial x} + \mu \Delta \bar{u} - \rho_0 \left( \frac{\partial \overline{u'u'}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z} \right)$$

$$= \rho_0 \frac{(v + v_E) \Delta \bar{u}}{A} \quad \text{Eddy viscosity}$$

$$-\overline{u'u'} = v_E \frac{\partial \bar{u}}{\partial x} \quad -\overline{u'v'} = v_E \frac{\partial \bar{u}}{\partial y} \quad -\overline{u'w'} = v_E \frac{\partial \bar{u}}{\partial z}$$

**Reynolds stress:** stress induced by turbulence acting on mean flow

## Reynolds-averaged equations

$$x: \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f v + f_* w = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} (A_H \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y} (A_H \frac{\partial u}{\partial y}) + \frac{\partial}{\partial z} (A_V \frac{\partial u}{\partial z})$$

$$y: \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + f u = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} (A_H \frac{\partial v}{\partial x}) + \frac{\partial}{\partial y} (A_H \frac{\partial v}{\partial y}) + \frac{\partial}{\partial z} (A_V \frac{\partial v}{\partial z})$$

$$z: \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + f_* u = -\frac{1}{\rho_0} \frac{\partial p}{\partial z} + \frac{\partial}{\partial x} (A_H \frac{\partial w}{\partial x}) + \frac{\partial}{\partial y} (A_H \frac{\partial w}{\partial y}) + \frac{\partial}{\partial z} (A_V \frac{\partial w}{\partial z}) - \frac{\rho}{\rho_0} g$$

# Scale analysis

**Characteristic scale:** a typical scale for a variable

**Table 4.1** TYPICAL SCALES OF ATMOSPHERIC AND OCEANIC FLOWS

Variable	Scale	Unit	Atmospheric value	Oceanic value
$x, y$	$L$	m	100 km = $10^5$ m	10 km = $10^4$ m <span style="color: red;"><math>10^4 \sim 10^6</math> km</span>
$z$	$H$	m	1 km = $10^3$ m	100 m = $10^2$ m <span style="color: red;"><math>10^1 \sim 10^3</math> km</span>
$t$	$T$	s	$\geq \frac{1}{2}$ day $\simeq 4 \times 10^4$ s	$\geq 1$ day $\simeq 9 \times 10^4$ s
$u, v$	$U$	m/s	10 m/s	0.1 m/s
$w$	$W$	m/s		
$p$	$P$	$\text{kg m}^{-1} \text{s}^{-2}$		variable
$\rho$	$\Delta\rho$	$\text{kg/m}^3$		

## The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial x} = \frac{\Delta u}{\Delta x} \sim \frac{U}{L}$$

scale

$$\frac{U}{L} \quad \frac{U}{L} \quad \frac{W}{H}$$

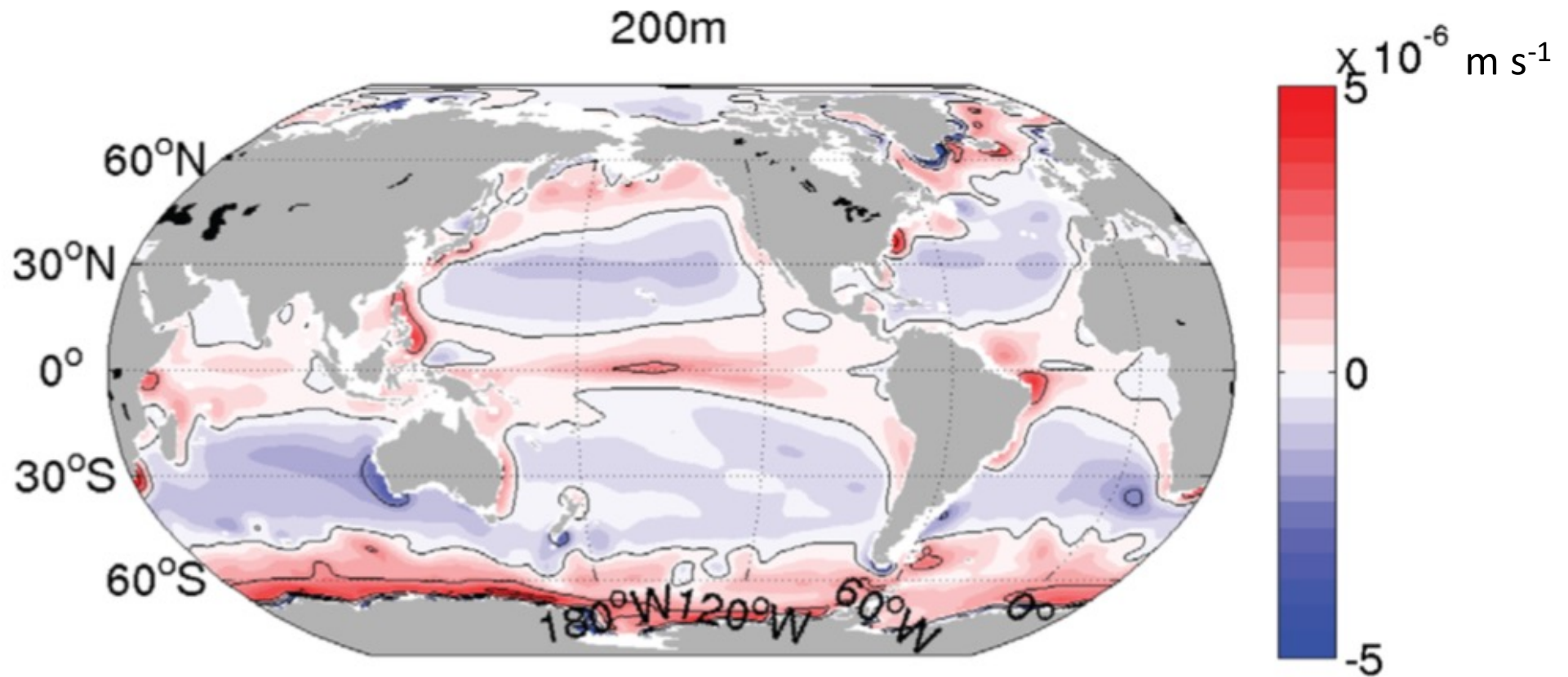
$$\frac{U}{L} \sim \frac{W}{H}$$

$$W \sim U \frac{H}{L} \sim U \delta$$

$$\delta = \frac{H}{L}: \text{ aspect ratio } (10^{-4} \sim 10^{-2})$$

$$W \ll U$$

# Vertical velocity in the global ocean



Liang et al. (2017, JGR)

## The momentum equation

x direction:  $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f v + \cancel{f_* w} =$

$$\frac{U}{T} \quad \frac{U^2}{L} \quad \frac{U^2}{L} \quad w \frac{U}{H} \quad fU \quad \cancel{fW}$$

$$W \ll U$$

$$-\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( A_H \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_H \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_V \frac{\partial u}{\partial z} \right)$$

$$\frac{P}{\rho_0 L} \quad A_H \frac{U}{L^2} \quad A_H \frac{U}{L^2} \quad A_V \frac{U}{H^2}$$

## Non-dimensional numbers

**Rossby number ( $R_o$ )** = scale of nonlinear advection term / scale of Coriolis term

$$= \frac{U^2}{L} / fU = U / fL \quad \text{measure of the role of Earth's rotation in motions}$$

$R_o \ll 1$ : rotation is important for the motion

**Ekman number ( $E_k$ )** = scale of vertical viscosity term / scale of Coriolis term

$$= A_V \frac{U}{H^2} / fU = \frac{A_V}{fH^2} \quad \text{measure of the importance of frictional force}$$

$E_k \ll 1$ : friction can be neglected

**Reynolds number ( $Re$ )** = scale of the nonlinear term / scale of viscous term

$$= \frac{U^2}{L} / \nu \frac{U}{L^2} = \frac{UL}{\nu}$$

small  $Re$ : viscous flow

large  $Re$ : inviscous, turbulent flow

# The momentum equation From the x-direction equation:

z direction:

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + f_* u =$$

$\frac{P}{\rho_0 L} \sim fU \sim \frac{U^2}{L} \sim A_H \frac{U}{L^2} \sim A_V \frac{U}{H^2}$

As  $H \ll L$ ,  $W \ll U$

$$-\frac{1}{\rho_0} \frac{\partial p}{\partial z} + \frac{\partial}{\partial x} \left( A_H \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_H \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_V \frac{\partial w}{\partial z} \right) - \frac{\rho}{\rho_0} g$$

$\frac{P}{\rho_0 H} \gg fU$   
 $\frac{P}{\rho_0 H} \gg \frac{UW}{L}$   
 $\frac{P}{\rho_0 H} \gg A_H \frac{W}{L^2}$   
 $\frac{P}{\rho_0 H} \gg A_V \frac{W}{H^2}$

$$\frac{\partial p}{\partial z} + \rho g = 0 \quad \text{Hydrostatic equation (balance)}$$



Hydrostatic balance:  $\frac{\partial p}{\partial z} + \rho g = 0$

$$p = \tilde{p}(z) + p'(x, y, z, t) \quad \tilde{p}(z) = P_0 - \rho_0 g z$$

$$\frac{\partial \tilde{p}}{\partial z} + \rho_0 g = 0$$

$$\frac{\partial p'}{\partial z} + \Delta \rho g = 0$$

$$\frac{P}{H} \quad g \Delta \rho$$

$$\frac{P}{\rho_0 L} \sim fU$$

$$\frac{gH\Delta\rho}{P} = \frac{gH\Delta\rho}{\rho_0 f L U} = \frac{U}{fL} \cdot \frac{gH\Delta\rho}{\rho_0 U^2}$$

$$= R_o \frac{N^2}{(U/H)^2} \quad \text{Richardson number}$$

## The momentum equations

$$\text{x direction: } \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f v = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( A_H \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_H \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_V \frac{\partial u}{\partial z} \right)$$

$$\text{y direction: } \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + f u = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( A_H \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_H \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_V \frac{\partial v}{\partial z} \right)$$

$$\text{z direction: } \frac{\partial p}{\partial z} + \rho g = 0 \quad \text{Hydrostatic equation}$$

$$\frac{\partial p'}{\partial z} + \Delta \rho g = 0$$

# The thermodynamic equation

1<sup>st</sup> Law of Thermodynamics (energy conservation, and for per unit mass):

$$\frac{dI}{dt} = Q - W$$

$I$ : internal energy

$Q$ : rate of heat gain

$W$ : rate of work done by pressure force  
onto surrounding fluids

$C_v$ : specific heat capacity       $I = C_v T$

Fourier's Law of heat conduction:

$$Q = \frac{k_T}{\rho} \nabla^2 T$$

Rate of work done by pressure force:      specific volume

$$W = p \frac{d\alpha}{dt} \quad \boxed{\alpha = 1/\rho}$$

$$C_v \frac{dT}{dt} = \frac{k_T}{\rho} \nabla^2 T - p \frac{d\alpha}{dt}$$

$$\alpha = 1/\rho$$

$$= \frac{k_T}{\rho} \nabla^2 T + \frac{p}{\rho^2} \frac{d\rho}{dt}$$

From the continuity equation:

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{v}$$

$$C_v \frac{dT}{dt} = \frac{k_T}{\rho} \nabla^2 T - \frac{p}{\rho} \nabla \cdot \mathbf{v}$$

For incompressible fluids:

$K_T$ : thermal diffusivity coefficient

$$\frac{dT}{dt} = \frac{k_T}{\rho C_v} \nabla^2 T$$

$$\frac{dS}{dt} = K_S \nabla^2 S$$

# Equation of State

Specific volume:

$$\alpha = 1/\rho$$

$$\alpha = \alpha(T, S, P)$$

For small variations of  $\alpha$  around a reference value:

$$d\alpha = \left(\frac{\partial \alpha}{\partial T}\right)_{S,p} dT + \left(\frac{\partial \alpha}{\partial S}\right)_{T,p} dS + \left(\frac{\partial \alpha}{\partial p}\right)_{T,S} dp = \alpha(\beta_T dT - \beta_S dS - \beta_p dp)$$

$\beta_T$ : thermal expansion coefficient

$$\alpha = \alpha_0 \left[ 1 + \beta_T(T - T_0) - \beta_S(S - S_0) - \beta_p(p - p_0) \right]$$

$\beta_S$ : salt contraction coefficient

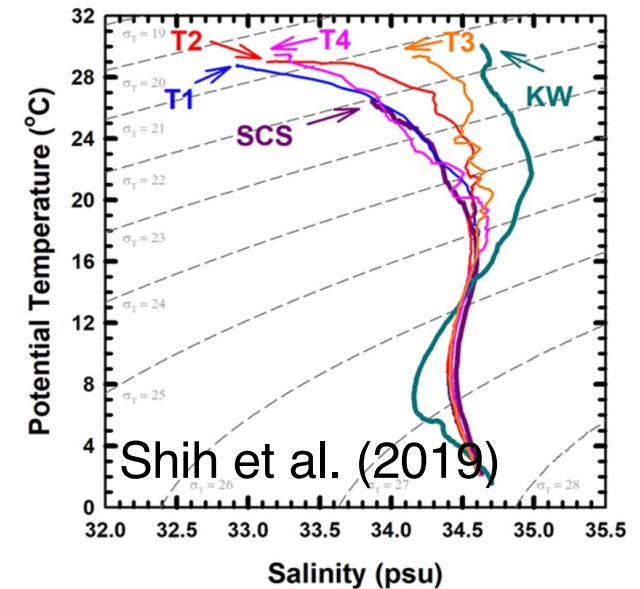
$$\rho = \rho_0 \left[ 1 - \beta_T(T - T_0) + \beta_S(S - S_0) + \beta_p(p - p_0) \right]$$

$\beta_p$ : compressibility coefficient

nonlinear

nonlinear

$$\alpha = \alpha_0 \left[ 1 + \beta_T(1 + \gamma^* p)(T - T_0) + \frac{\beta_T^*}{2}(T - T_0)^2 - \beta_S(S - S_0) - \beta_p(p - p_0) \right]$$



# TEOS-10

## Thermodynamic Equation Of Seawater - 2010

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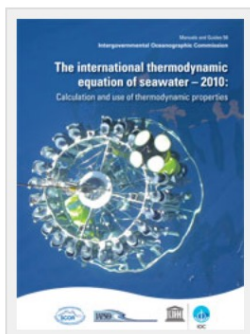
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### HOME

This site is the official source of information about the Thermodynamic Equation Of Seawater - 2010 (TEOS-10), and the way in which it should be used.

TEOS-10 is based on a Gibbs function formulation from which all thermodynamic properties of seawater (density, enthalpy, entropy sound speed, etc.) can be derived in a thermodynamically consistent manner. [TEOS-10 was adopted by the Intergovernmental Oceanographic Commission at its 25th Assembly in June 2009](#) to replace EOS-80 as the official description of seawater and ice properties in marine science.

A significant change compared with past practice is that TEOS-10 uses Absolute Salinity  $S_A$  (mass fraction of salt in seawater) as opposed to Practical Salinity  $S_p$  (which is essentially a measure of the conductivity of seawater) to describe the salt content of seawater. Ocean salinities now have units of g/kg.

Absolute Salinity (g/kg) is an SI unit of concentration. The thermodynamic properties of seawater, such as density and enthalpy, are now correctly expressed as functions of Absolute Salinity rather than being functions of the conductivity of seawater. Spatial variations of the composition of seawater mean that Absolute Salinity is not simply proportional to Practical Salinity; TEOS-10 contains procedures to correct for these effects.

## Summary of the governing equations

Momentum equation: 
$$\frac{d\mathbf{v}}{dt} + f\mathbf{k} \times \mathbf{v} = -\frac{1}{\rho_0} \nabla p + \frac{\rho}{\rho_0} \mathbf{g} + \nu_E \nabla^2 \mathbf{v}$$

Continuity equation: 
$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0$$

Tracer equations: 
$$\frac{dT}{dt} = K \nabla^2 T$$

$$\frac{dS}{dt} = K \nabla^2 S$$

Equation of state:

$$\rho = \rho(T, S, P)$$