第 2 次作业

危国锐　120034910021
（上海交通大学海洋学院，上海 200030）

摘　要：本文使用的程序和文档发布于 https://grwei.github.io/SJTU_2021-2022-2-MS8402/.

关键词：词 1，词 2

Homework 2

Guorui Wei　120034910021
(*School of Oceanography*, *Shanghai Jiao Tong University*, *Shanghai* 200030, *China*)

Abstract: The programs and documents used in this article are published at https://grwei.github.io/SJTU_2021-2022-2-MS8402/.

Keywords: keyword 1, keyword 2

目 录

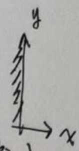
1 Question 1

Show that in the southern hemisphere, the Kelvin wave propagates with the boundary on the left.

1.1 Solution

MS8402   Homework 2   2022.3.15 (due date)

1. 解：控制方程 (Kelvin waves)



$$\begin{cases} x: & +fv - g\dfrac{\partial \eta}{\partial x} = 0, & (1) \quad (\text{地转平衡}) \\ y: & \dfrac{\partial v}{\partial t} = -g\dfrac{\partial \eta}{\partial y}, & (2) \quad (\text{PGF 驱动}) \\ C: & \dfrac{\partial \eta}{\partial t} + H\dfrac{\partial v}{\partial y} = 0. & (3) \end{cases}$$

$$\frac{\partial (2)}{\partial t} \xRightarrow{(3)} \frac{\partial^2 v}{\partial t^2} = gH\frac{\partial^2 v}{\partial y^2}. \quad (4)$$

d'Alembert
(通解：$v = F(y-\sqrt{gH}t)+G(x+\sqrt{gH}t)$)

设 $v = v^+(x)e^{i(ly-\omega t)} + v^-(x)e^{i(ly+\omega t)}$, $\Rightarrow \dfrac{\omega}{l} = \sqrt{gH}$. (5)

又设 $\eta = \eta^+(x)e^{i(ly-\omega t)} + \eta^-(x)e^{i(ly+\omega t)}$, $\dfrac{\omega}{l} = \sqrt{gH}$. (6)

$$\text{由}(2) \xRightarrow{(5)(6)} -i\omega v^+(x)e^{i(ly-\omega t)} + i\omega v^-(x)e^{i(ly+\omega t)}$$
$$= -gil\eta^+(x)e^{i(ly-\omega t)} - gil\eta^-(x)e^{i(ly+\omega t)}$$

$$\Rightarrow \eta^+(x) = \frac{\omega}{gl}v^+(x), \quad \eta^-(x) = -\frac{\omega}{gl}v^-(x). \quad (7)$$

将(5)(6)~~(7)~~代入(1) $\Rightarrow (fv^+ - g\dfrac{d\eta^+}{dx})e^{i(ly-\omega t)} + (fv^- - g\dfrac{d\eta^-}{dx})e^{i(ly+\omega t)} = 0$

$$\xRightarrow{(7)} \begin{cases} fv^+ = g\dfrac{\omega}{gl}\dfrac{dv^+}{dx}, \\ |v^+| < +\infty \text{ as } x \to +\infty, \end{cases} (8) \qquad \begin{cases} fv^- = -g\dfrac{\omega}{gl}\dfrac{dv^-}{dx}, \\ |v^-| < +\infty \text{ as } x \to -\infty. \end{cases} (9)$$

$$\Rightarrow v^+ = \begin{cases} c^+e^{(fl/\omega)x}, & f \leqslant 0, \\ 0, & f > 0, \end{cases} \qquad v^- = \begin{cases} c^-e^{-(fl/\omega)x}, & f \geqslant 0, \\ 0, & f < 0. \end{cases}$$

-1-

可见，在南半球（$f<0$），有 $\eta^-=V^-=0$, 从而无向负 y 方向的行波，只有向正 y 方向的行波，即边界在相速度方向的左侧.

2 Question 2

Using the principle of potential vorticity conservation and volume transport conservation, solve the "Analytical Problems" 7-8 (Page 213) in the book *Introduction to Geophysical Fluid Dynamics* by Cushman-Roison and Beckers (2011).

> In Utopia, a narrow 200-m-deep channel empties in a broad bay of varying bottom topography (Fig. 7.14). Trace the path to the sea and the velocity profile of the channel outflow. Take $f = 10^{-4}$ s^{-1}. Solve only for straight stretches of the flow and ignore corners. (Cushman-Roisin & Beckers, 2011, p. 232)

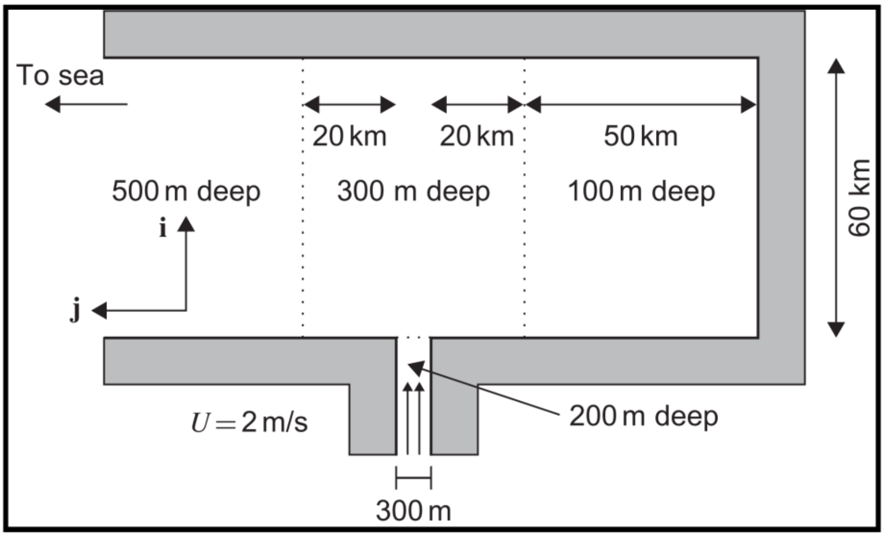


图 2.1 Geometry of the idealized bay and channel mentioned in Analytical Problem 7.8. (图片来自 Cushman-Roisin & Beckers, 2011, p. 233, Fig. 7.14)

2.1 Solution

将图 2.1 中的左、中、右三区分别记为 N（North）、M（Middle）、S（South）区. 若成立位涡守恒且 f 为常数，则有：

（1）若水流在 M 区向 $\mp\mathbf{j}$ 方向流动，则流速沿 $\pm\mathbf{i}$ 方向递减；

（2）若水流在 M 区向 $+\mathbf{i}$ 方向流动，则流速沿 $+\mathbf{j}$ 方向递减；

（3）若水流在 S 区向 $+\mathbf{i}$ 方向流动，则流速沿 $-\mathbf{j}$ 方向递减；

（4）若水流在 N 区向 $+\mathbf{j}$ 方向流动，则流速沿 $-\mathbf{i}$ 方向递减.

为进行定量计算，对以上四种情况，进一步假定：流速是线性递减的，例如：情况（3）的流速由 U_S 沿 $-\mathbf{j}$ 方向在流幅 L_S 内均匀递减至 0；情况（4）的流速由 V_N 沿 $-\mathbf{i}$ 方向在流幅 L_N 内均匀递减至 0.

由体积（质量）守恒

$$U_0L_0H_0 = V_\mathrm{M}L_\mathrm{M}H_\mathrm{M}/2 = U_\mathrm{S}L_\mathrm{S}H_\mathrm{S}/2 = V_\mathrm{N}L_\mathrm{N}H_\mathrm{N}/2 \tag{2.1}$$

和位涡守恒

$$\frac{f}{H_0} = \frac{f + V_\mathrm{M}/L_\mathrm{M}}{H_\mathrm{M}} = \frac{f - U_\mathrm{S}/L_\mathrm{S}}{H_\mathrm{S}} = \frac{f + V_\mathrm{N}/L_\mathrm{N}}{H_\mathrm{N}} \tag{2.2}$$

得

$$\begin{aligned}
V_\mathrm{M}^2 &= 2U_0L_0\left(1 - \frac{H_0}{H_\mathrm{M}}\right)f, \qquad & L_\mathrm{M}^2 &= \frac{2U_0L_0H_0^2}{H_\mathrm{M}(H_\mathrm{M} - H_0)f}, \\
U_\mathrm{S}^2 &= 2U_0L_0\left(\frac{H_0}{H_\mathrm{S}} - 1\right)f, \qquad & L_\mathrm{S}^2 &= \frac{2U_0L_0H_0^2}{H_\mathrm{S}(H_0 - H_\mathrm{S})f}, \\
V_\mathrm{N}^2 &= 2U_0L_0\left(1 - \frac{H_0}{H_\mathrm{N}}\right)f, \qquad & L_\mathrm{N}^2 &= \frac{2U_0L_0H_0^2}{H_\mathrm{N}(H_\mathrm{N} - H_0)f}.
\end{aligned}$$

代入数据，得

$$\begin{aligned}
V_\mathrm{M} &= 0.20\,\mathrm{m/s}, \qquad & L_\mathrm{M} &= 4000\ \mathrm{m}, \\
U_\mathrm{S} &= 0.35\,\mathrm{m/s}, \qquad & L_\mathrm{S} &= 6928\ \mathrm{m}, \\
V_\mathrm{N} &= 0.27\,\mathrm{m/s}, \qquad & L_\mathrm{N} &= 1789\ \mathrm{m}.
\end{aligned}$$

若规定流速要从近岸（或区域边界）到外海减弱，则一种可能的出海路径是（依次）：

1. 水流出 channel 后，以流幅 L_M 和最大流速 V_M 沿 M 区西边界向南流动；
2. 接近 M 区南边界时，折向东而不进入 S 区，以流幅 L_M 和最大流速 V_M 沿 M 区南边界向东流动；
3. 接近 M 区东边界时，折向北，以流幅 L_M 和最大流速 V_M 沿 M 区东边界向北流动；
4. 进入 N 区，以流幅 L_N 和最大流速 V_N 沿 N 区东边界向北流入海.

References

Cushman-Roisin, B., & Beckers, J.-M. (2011). Chapter 7 - Geostrophic Flows and Vorticity Dynamics. In B. Cushman-Roisin & J.-M. Beckers (Eds.), *International Geophysics* (Vol. 101, pp. 205-238). Academic Press. https://doi.org/https://doi.org/10.1016/B978-0-12-088759-0.00007-9