Midterm answers

Geophysical Fluid Dynamics I, Spring 2022

Problem 1

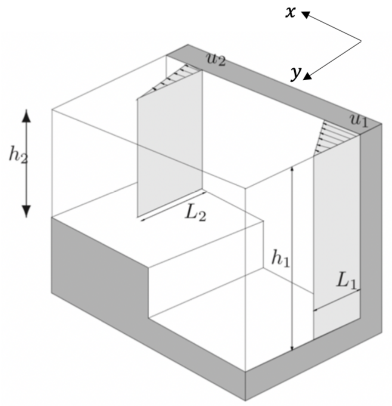

Figure 1. A sheared coastal jet negotiating a bottom escarpment.

1) The x and y axes are defined in Figure 1. According to the conservation of potential vorticity,

$$\frac{f+\zeta_1}{H_1}=\frac{f+\zeta_2}{H_2}, \tag{1}$$

i.e.

$$\frac{f+\frac{U_1}{L_1}}{H_1}=\frac{f+\frac{U_2}{L_2}}{H_2}. \tag{2}$$

$$\begin{aligned}\frac{U_2}{L_2}&=(f+\frac{U_1}{L_1})H_2/H_1-f\\&=(10^{-4}s^{-1}+\frac{0.5m/s}{10^4 m})\times 160m/200m-10^{-4}s^{-1}\\&=2\times 10^{-5}s^{-1}\end{aligned} \tag{3}$$

From the conservation of volume,

$$U_1L_1H_1=U_2L_2H_2 \tag{4}$$

$$U_2L_2=\frac{U_1L_1H_1}{H_2}=\frac{0.5m/s\times 10^4 m\times 200m}{160m}=6.25\times 10^3 m^2/s \tag{5}$$

Combine (3) and (5):

$$U_2=0.34m/s,\qquad L_2=17.68km \tag{6}$$

2) For $H_2=100m$, based on the equations above,

$$\frac{U_2}{L_2} = -2.5 \times 10^{-5} s^{-1} \tag{7}$$

The negative $\frac{U}{L}$ means that the direction of the velocity gradient reverses, i.e. the magnitude of the velocity will increase offshore to infinity, which is impossible. So the flow cannot climb on the escarpment. Then what can happen to the flow is that it turns offshore in the region of 200m, in which case it can generate a negative ζ.

Problem 2

The expression of the Raileigh's criterion is:

$$R(y) = \beta_0 - \frac{d^2\bar{u}}{dy^2} \quad (1)$$

Based on the flow profile, we can obtain:

$$R(y) = \beta_0 - \left(\frac{Uy^2}{L^4} - \frac{U}{L^2}\right) Exp\left(-\frac{y^2}{2L^2}\right) \quad (2)$$

Using the parameters provided in the problem, the pattern for $R(y)$ is shown in the left panel of Figure 2. You can see that in the jet domain $y \in [-L, L]$, $R(y)$ is always positive, which does not satisfy the Raileigh's criterion.

The expression of the FjØtoft's criterion is:

$$F(y) = (\bar{u} - \bar{u}_0)\left(\beta_0 - \frac{d^2\bar{u}}{dy^2}\right) \quad (3)$$

Take $\bar{u}_0 = \bar{u}(y = 0)$, and we have:

$$F(y) = \left[40 \cdot Exp\left(-\frac{y^2}{2 \cdot (5.7 \times 10^5)^2}\right) - 40\right] R(y) \quad (4)$$

The profile for $F(y)$ is shown in the right panel of Figure 2. In the jet domain $y \in [-L, L]$, $F(y)$ is always negative, which does not satisfy the FjØtoft's criterion. As neither the Raileigh's criterion nor the FjØtoft's criterion is satisfied, the jet is stable.

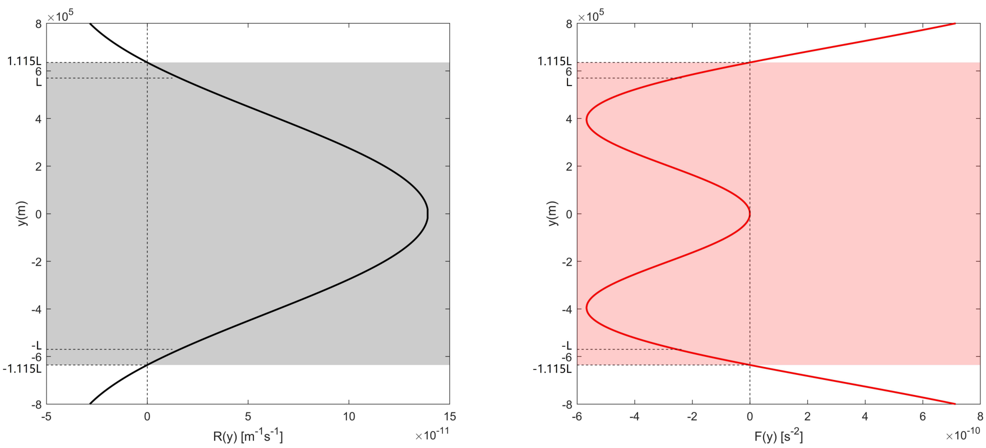

Figure 2. The profiles of $R(y)$ (left) and $F(y)$ (right).

Problem 3

1) Summarize the wave properties.

The waves exist at 10–20°N and are radiated from the instability of equatorial current systems;

the waves propagate southwestward;

the waves have a period range of 26–38 days and zonal wavelength of 9–13°;

the waves normally become discernable in August and the signals remain for about 6 months in a year;

the phase of the waves changes abruptly at 10°N.

2) Explain why these waves can be considered as Rossby waves.

This study uses the dispersion relation of barotropic Rossby waves to predict the meridional wave number, the orientation of the wave crests (or phase propagation) and the orientation of energy propagation, based on the observed zonal wavenumber and frequency. The predicted values are in good agreement with observations.

3) Discuss the similarities and differences between these waves and the Rossby waves mentioned in our class.

Your answers may not be limited to the following points.

Similarities:

They are both barotropic Rossby waves generated by the Beta effect following the potential vorticity conservation. Therefore, in the zonal direction they both propagate westward.

Differences:

In this study, the Rossby waves are short-wavelength waves, so the dispersion relation can be reduced to Equation (3) from the original dispersion relation introduced in our class.

In class, when plotting the dispersion relation diagram and analyze the energy propagation direction, we only considered the zonal direction (x-axis is the zonal wavenumber), so the energy propagates only in the zonal direction. In this study both zonal and meridional energy propagations are considered.