Part II. Motions under rotation

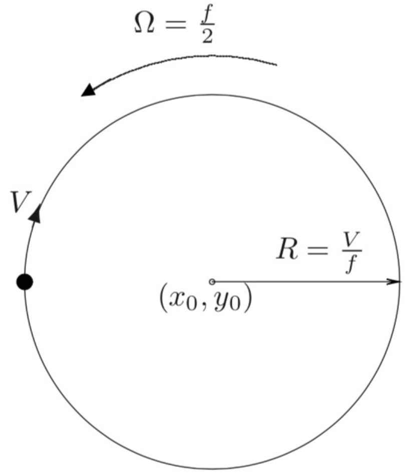
- homogeneous fluids, no stratification

Free motions – Inertial Oscillations

Not subject to real force:

$$\frac{du}{dt} - fv = 0$$

$$\frac{dv}{dt} + fu = 0$$



$$\underset{\frac{dx}{dt}}{u} = V \sin(ft + \phi), \quad \underset{\frac{dy}{dt}}{v} = V \cos(ft + \phi) \qquad V = \sqrt{u^2 + v^2}$$

Table 9.1 Inertial Oscillations

Latitude (φ)	T_i (hr) for V = 20 cm/s	D (km)
90°	11.97	2.7
35°	20.87	4.8
10°	68.93	15.8

$$x = x_0 - \frac{V}{f}\cos(ft + \phi)$$

$$y = y_0 + \frac{V}{f}\sin(ft + \phi),$$

$$\boxed{(x - x_0)^2 + (y - y_0)^2 = \left(\frac{V}{f}\right)^2}$$

f : inertial frequency

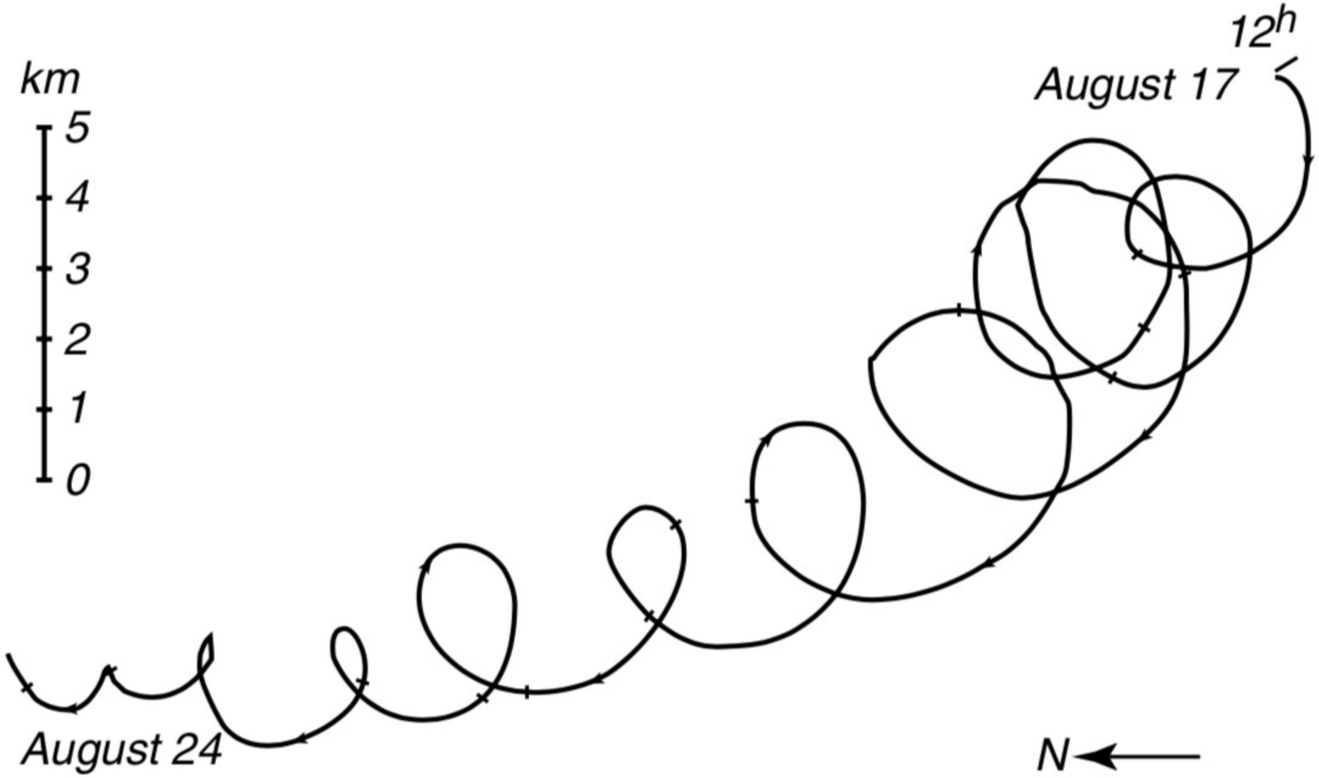
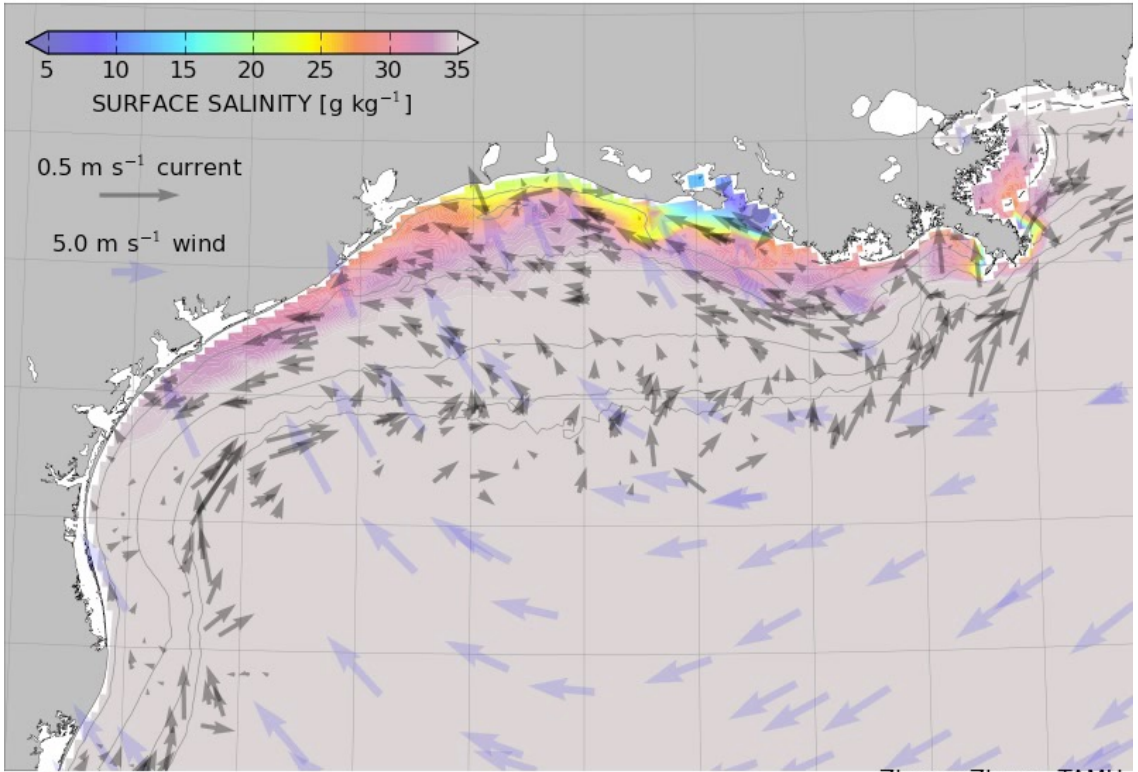


Figure 9.1 Trajectory of a water parcel calculated from current measured from August 17 to August 24, 1933 at 57°49'N and 17°49'E west of Gotland (From Sverdrup, Johnson, and Fleming, 1942).



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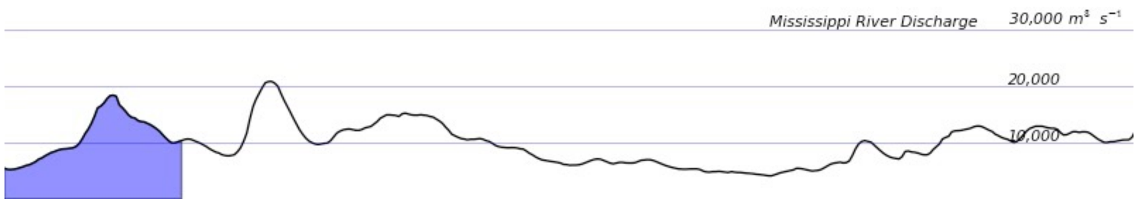


Geostrophic flow

Assumptions:

$R_o << 1$ and $E_k << 1$

Inertial acceleration, nonlinear advection and viscosity terms are neglected

Governing equations

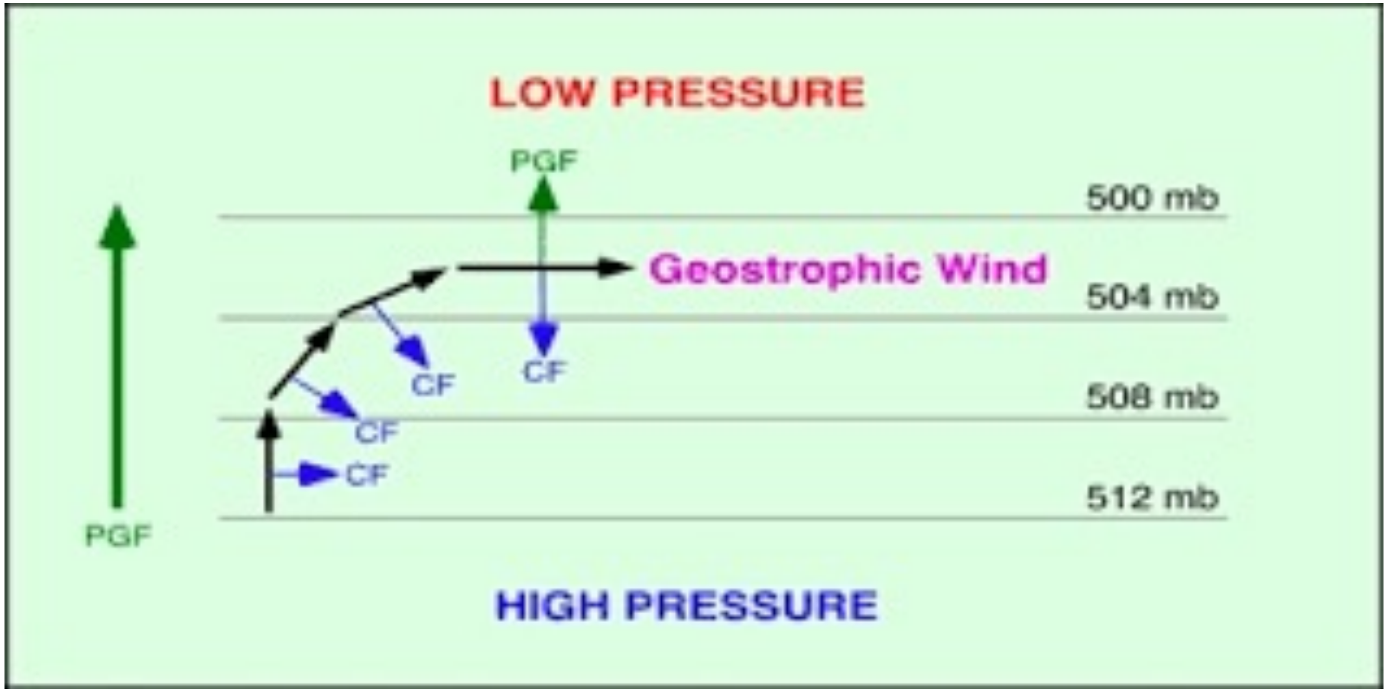
$$\boxed{\begin{aligned} -fv &= -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \\ +fu &= -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \end{aligned}}$$

Geostrophic balance

balance between the pressure gradient term and the Coriolis term

$$\boldsymbol{u} \cdot \nabla p = 0$$

Geostrophic flow is perpendicular to the pressure gradient (force).



High pressure is to the right (left) of the geostrophic flow in the northern (southern) hemisphere

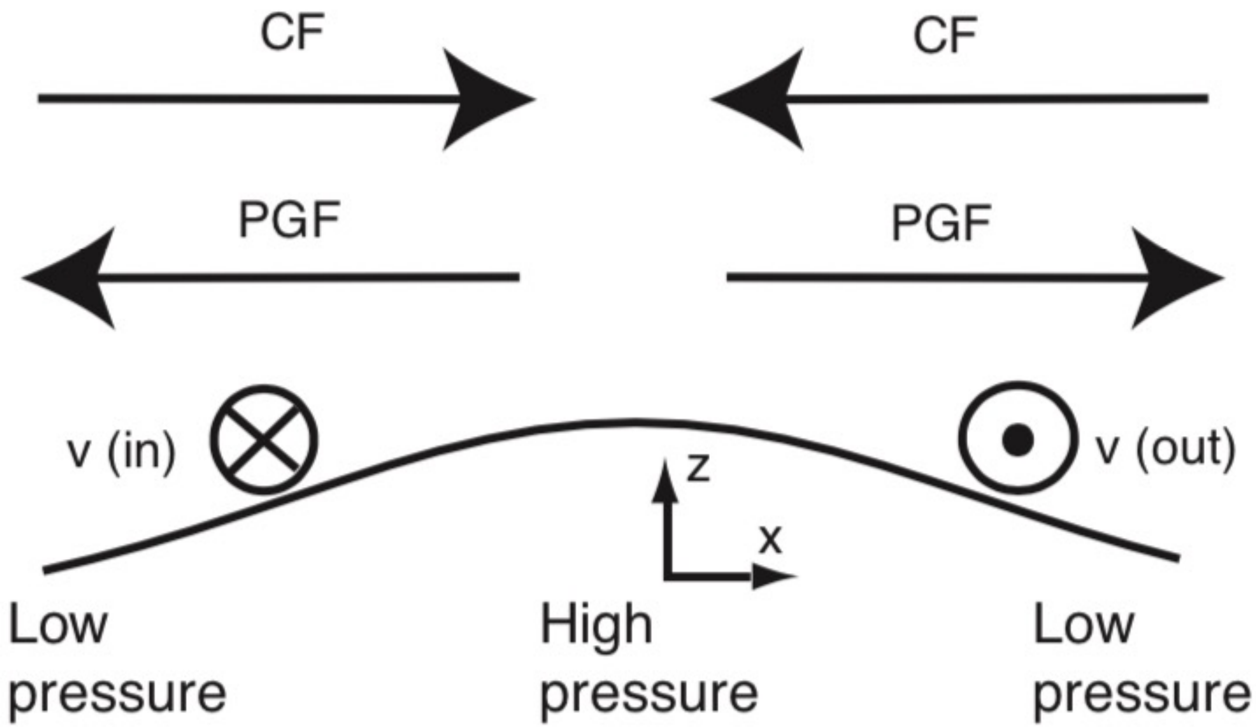


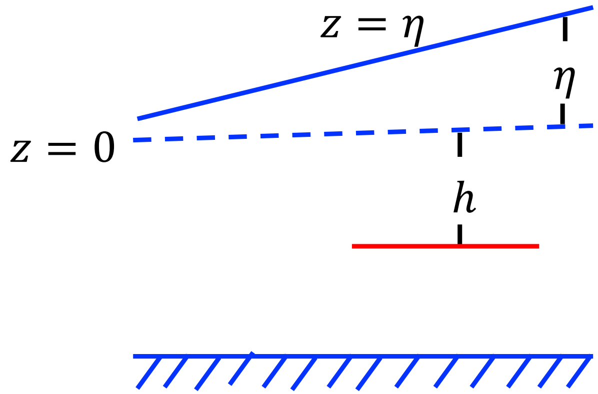
FIGURE 7.9 Geostrophic balance: horizontal forces and velocity. PGF = pressure gradient force. CF = Coriolis force. v = velocity (into and out of page). See also Figure S7.17.

Hydrostatic balance $\dfrac{\partial p}{\partial z} + \rho g = 0$

If $\rho=\rho_0$ everywhere, vertically integrating the equation from z=–h to the surface z= η :

$$\int_{z=-h}^{z=\eta} \frac{\partial p}{\partial z}\, dz + \rho_0 g(h+\eta) = 0$$

$$p|_{z=-h} = \overbrace{p|_{z=\eta}}^{P_o} + \rho_0 g(h+\eta)$$



The the horizontal momentum equations become:

$$-fv = -\frac{1}{\rho_0}\frac{\partial}{\partial x}(P_o + \rho_0 g h + \rho_0 g \eta) = -\mathrm{g}\frac{\partial \eta}{\partial x}$$

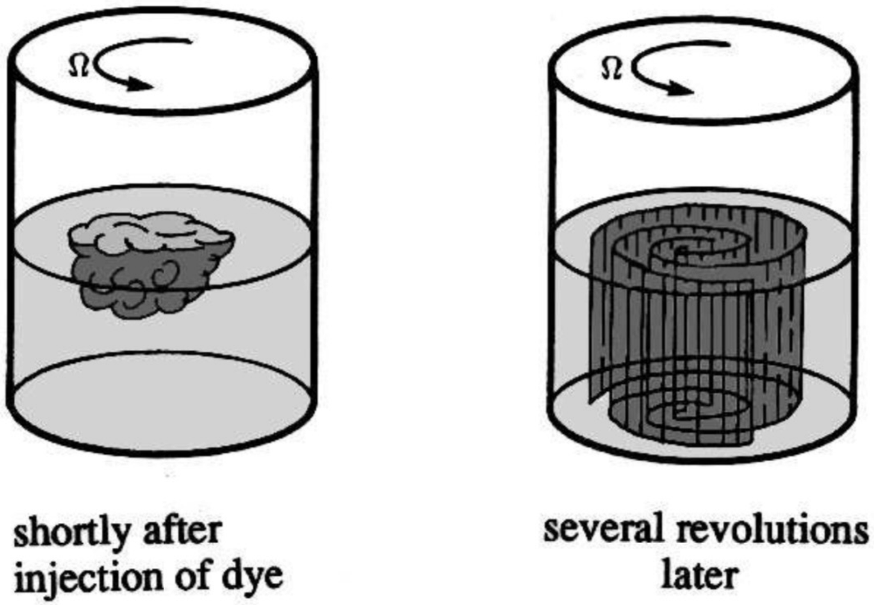
$$\frac{\partial h}{\partial x} = 0$$

$$\mathrm{f}u = -\frac{1}{\rho_0}\frac{\partial}{\partial y}(P_o + \rho_0 g h + \rho_0 g \eta) = -\mathrm{g}\frac{\partial \eta}{\partial y}$$

If for everywhere $\rho = \rho_0$

$$\frac{\partial u}{\partial z} = 0$$

$$\frac{\partial v}{\partial z} = 0$$



The flow has no vertical shear and the fluid moves like a slab –

Taylor column

$$\boxed{\begin{aligned} -fv &= -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \\ +fu &= -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \end{aligned}}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial}{\partial x}\left(\frac{1}{\rho_0 f}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho_0 f}\frac{\partial p}{\partial x}\right) = 0.$$

Geostrophic flows are horizontally non-divergent for a f-plane

From the continuity equation: $\frac{\partial w}{\partial z} = 0$

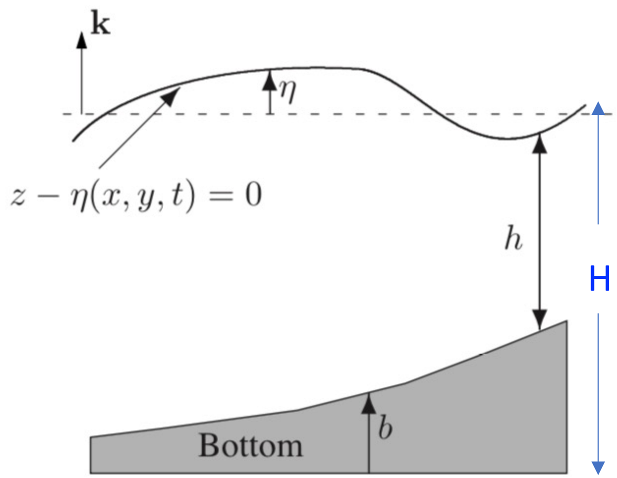
For flat surface or bottom, $w = 0$ through the water column

Streamfunction ψ

$$u = -\frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \psi}{\partial x}$$

Geostrophic flow over irregular bottom

Boundary conditions at the surface and bottom:



$$\boxed{\frac{\partial w}{\partial z} = 0}$$

$$w|_{z=\eta} = \frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y}$$

$$w|_{z=b} = u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y}$$

$$\frac{\partial \eta}{\partial t} + u\frac{\overset{h-H}{\partial(\eta - b)}}{\partial x} + v\frac{\overset{h-H}{\partial(\eta - b)}}{\partial y} = 0$$

$$\boxed{\eta = h + b - H}$$

For steady motion

$$\boldsymbol{u} \cdot \boldsymbol{\nabla} h = \boldsymbol{0}$$

Geostrophic flows must follow contant h