Lecture 16: Thermal Wind

Group: Luo Tong; Liu Yihang; Cheng Rui;
Li Hengyu; Wang Yan

上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

Derivation

Assumptions:

1. $f = c$

2. $-\frac{1}{\rho}\frac{\partial p}{\partial z} = g$

3. $\rho = \rho(x, y)$

Geostrophic flow:

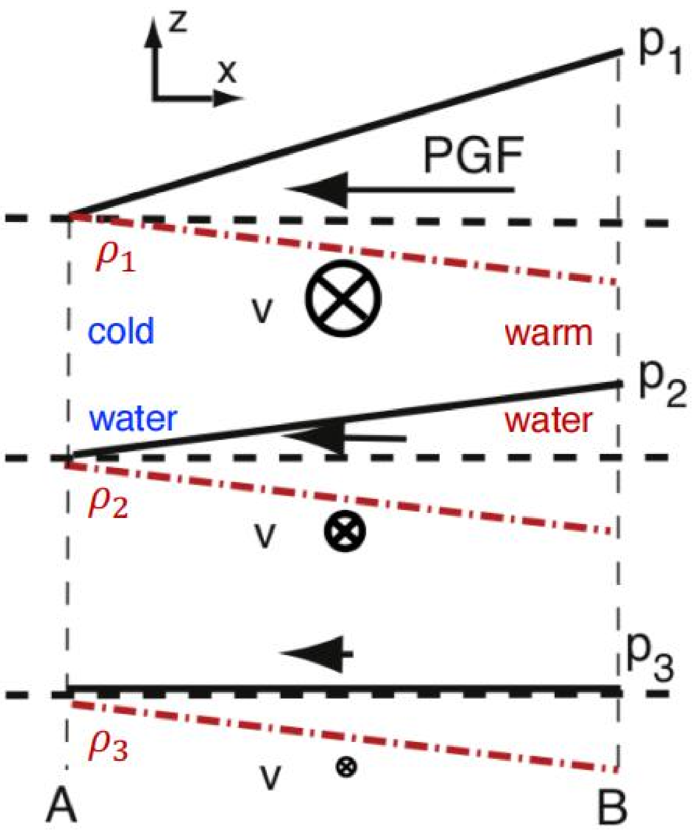
$$fv = \frac{1}{\rho}\frac{\partial p}{\partial x}$$

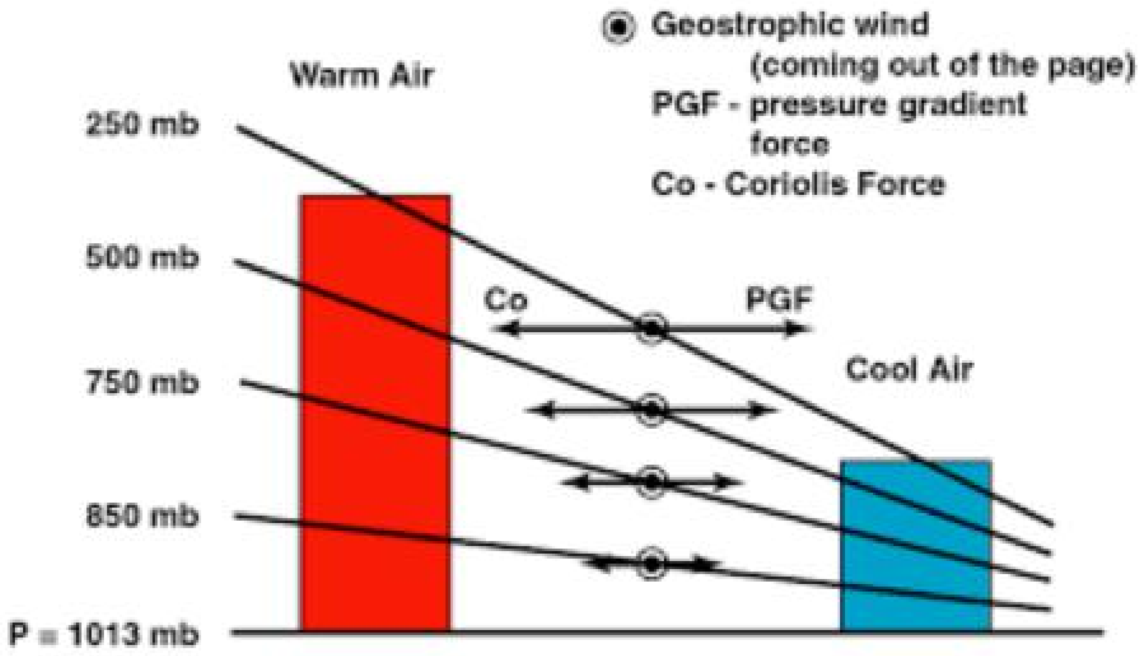
$$fu = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

Thermal wind balance:

$$f\frac{\partial v}{\partial z} = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial x} \qquad f\frac{\partial u}{\partial z} = \frac{g}{\rho_0}\frac{\partial \rho}{\partial y}$$

Explanations of Equations





An equatorial thermal wind equation

$$f_0 \sin\theta \frac{\partial \mathbf{v}_\perp(r,\theta,\phi)}{\partial r} = \frac{g}{T}\hat{\mathbf{r}} \times \nabla_\perp T\Big|_P,$$ TWE in the text book (written in terms of spherical coordinates)

↓

$$\frac{\nabla P}{P} = \frac{\nabla T}{T} + \frac{\nabla \rho}{\rho},$$ ideal gas equation

↓

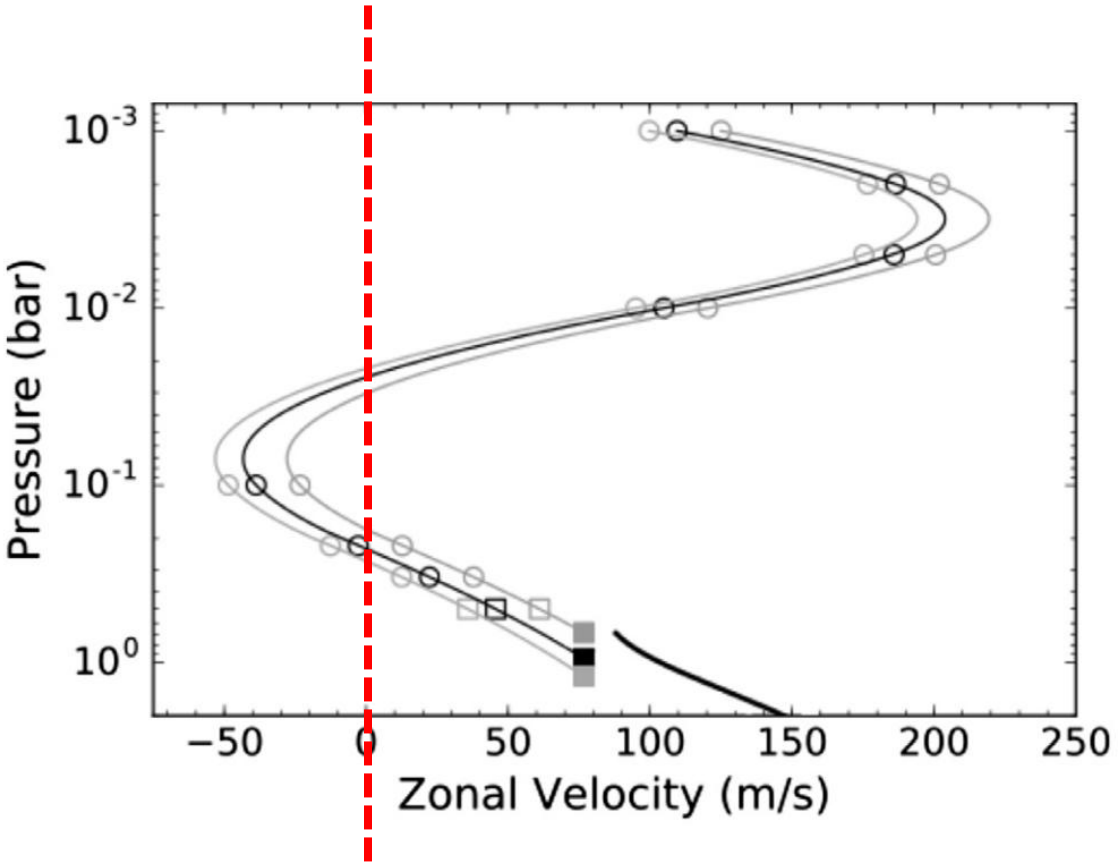
$$-\frac{g'}{r_0 T}\frac{\partial^2 T}{\partial \theta^2}\Big|_P = f_0 \frac{\partial v_\phi}{\partial r},$$ EQTWE

Values of some characteristic variables on four planets

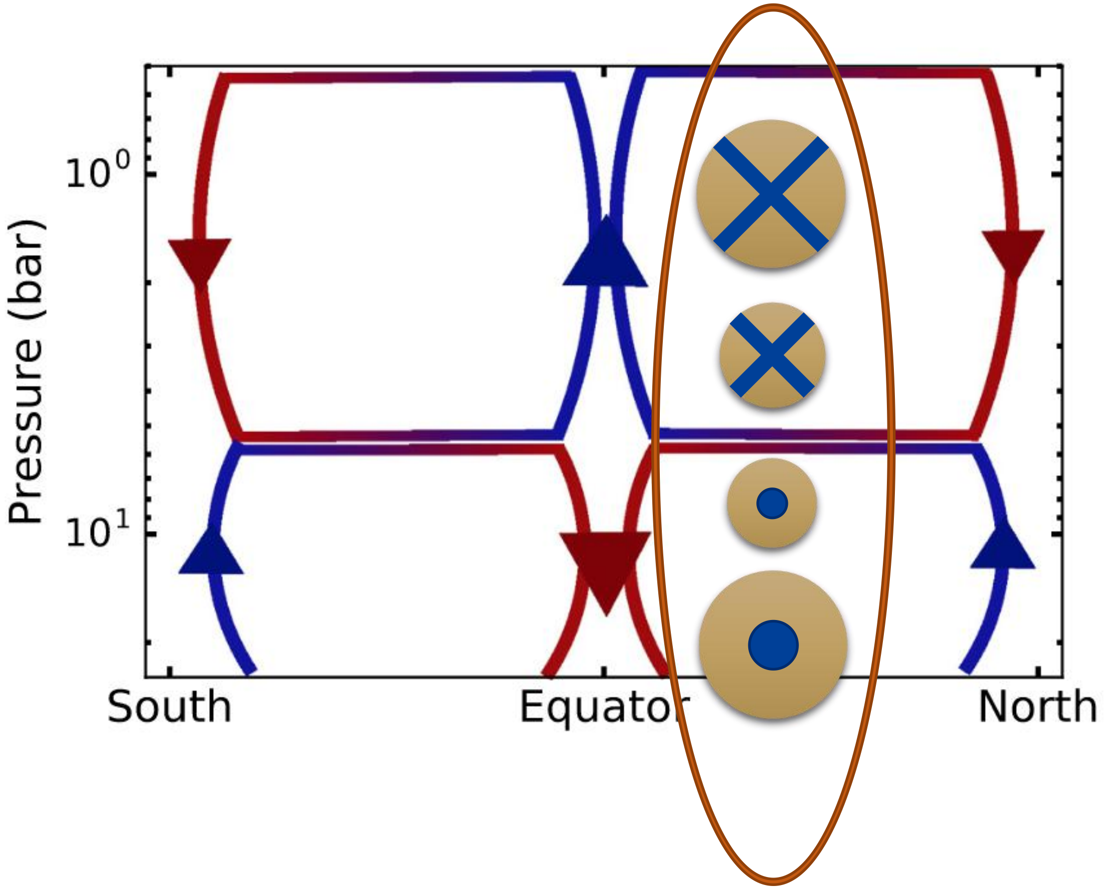
Parameter and description	Jupiter	Saturn	Uranus	Neptune
r_0 Equatorial Radius	7.0×10^7	5.8×10^7	2.5×10^7	2.5×10^7
g Gravitational Acceleration	25	10	9	11
Ω_0 Angular velocity	1.7×10^{-4}	1.6×10^{-4}	1.0×10^{-4}	1.1×10^{-4}
D Vertical Length Scale	2.7×10^4	6.0×10^4	2.8×10^4	2.0×10^4
L_ϕ Longitudinal Length Scale	10^8	10^8	10^8	10^8
L_θ Latitudinal Length Scale	10^7	10^7	10^7	10^7
V_r Characteristic Vertical Velocity	1–10	1–10	1–100	1–100
V_ϕ Characteristic Zonal Velocity	100	300	100	300
V_θ Characteristic Latitudinal Velocity	1–10	1–10	1–100	1–100
$\overline{Ro} \equiv V_\phi/(f_0 L_\theta)$	0.03	0.09	0.05	0.14
$\overline{Ro}\ (r_0/L_\theta)\ (V_\theta/V_\phi)^2$	0.002	0.0005	0.1	0.04
$\overline{Ro}\ (r_0/L_\phi)\ (V_\theta/V_\phi)$	0.002	0.002	0.01	0.01
$\overline{Ro}\ r_0/L_\phi$	0.02	0.05	0.01	0.04
$\overline{Ro}\ D/L_\theta$	8×10^{-5}	5×10^{-4}	1×10^{-4}	3×10^{-4}
$DV_\phi^2/(gL_\phi^2)$	1×10^{-9}	5×10^{-8}	3×10^{-9}	2×10^{-8}
$DV_\theta^2/(gL_\theta^2)$	1×10^{-9}	6×10^{-9}	3×10^{-7}	2×10^{-7}
$DV_\theta V_\phi/(gL_\phi L_\theta)$	1×10^{-9}	2×10^{-8}	3×10^{-8}	5×10^{-8}
Dr_0/L_θ^2	2×10^{-2}	3×10^{-2}	7×10^{-3}	5×10^{-3}
$\widetilde{Ro} \equiv V_\phi/(f_0 r_0)$	0.004	0.02	0.02	0.05

(Marcus P , Tollefson J , Depater I , et al. An Equatorial Thermal Wind Equation: Applications to Jupiter[C]// Aps Meeting. APS Meeting Abstracts, 2017.)

An equatorial thermal wind equation



Zonal velocity derived with the EQTWE at the equator



2-layer model

(Marcus P , Tollefson J , Depater I , et al. An Equatorial Thermal Wind Equation: Applications to Jupiter[C]// Aps Meeting. APS Meeting Abstracts, 2017.)

Jet stream in upper atmosphere

Geopotential height: $Z = \frac{\Psi}{g_0} = \frac{1}{g_0}\int_0^z g dz$

Hypsometric equation

$$Z_2 - Z_1 = \frac{R \cdot \overline{T_v}}{g_0} ln\left(\frac{p_1}{p_2}\right)$$

Mathematical expression of thermal wind balance

$$\vec{V_T} = \frac{1}{f}\vec{k} \times \nabla_p\left(\Psi_1 - \Psi_0\right) \quad (1)$$

R the specific gas constant for dry air

$\overline{T_v}$ the vertically-averaged temperature

u-component (geostrophic):

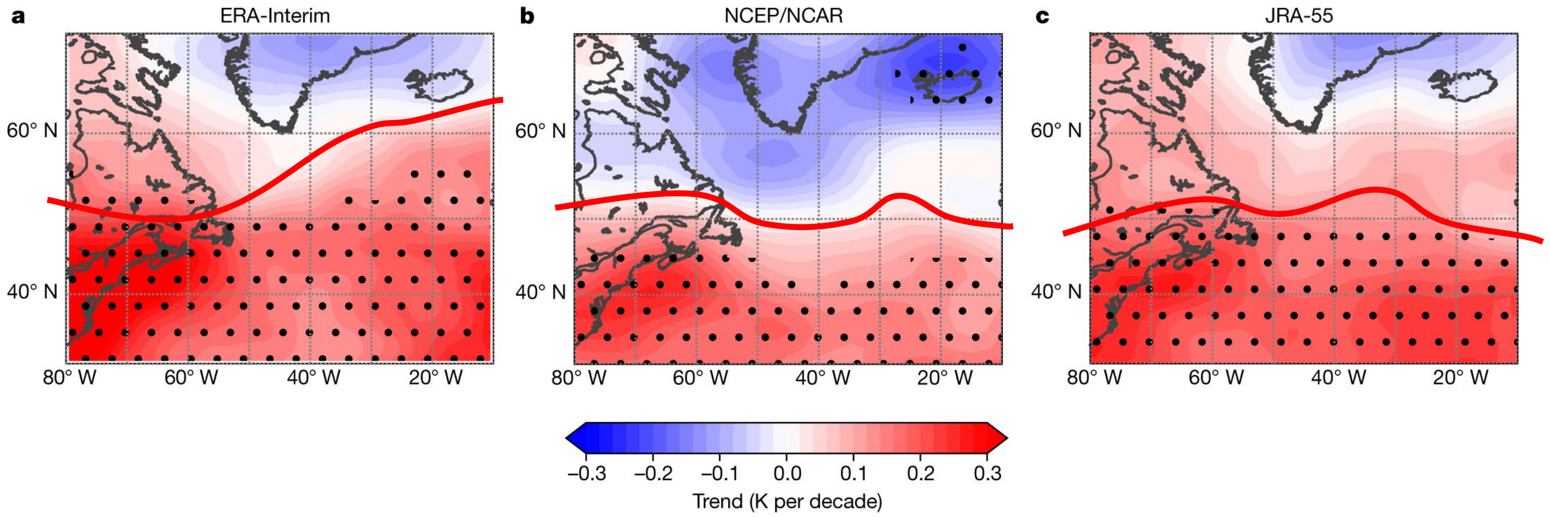
$$u_2 - u_1 = -\frac{g_0}{f}\frac{\partial\left(Z_2 - Z_1\right)}{\partial y} \quad (2)$$

The vertical shear in the zonal wind

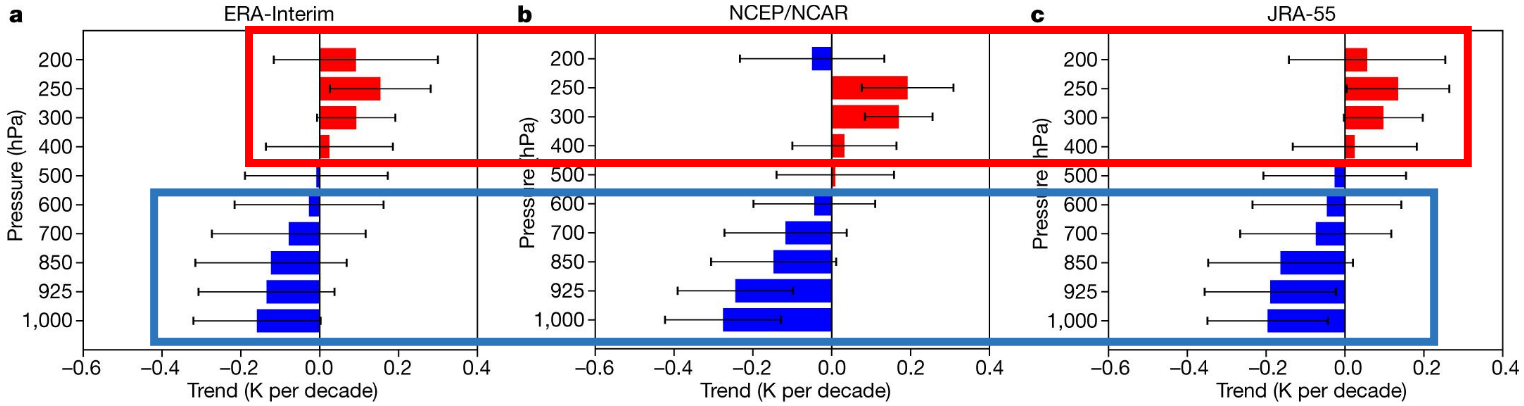
$$-\frac{\partial u}{\partial p} = -\frac{R}{fp}\frac{\partial T}{\partial y} \quad (3)$$

Temperature gradient generates westerly winds
(strengthen with height)

The impact from climate change



Annual-mean temperature trend over the period 1979-2017 (250 hPa)



(Simon H. Lee et al., Nature, 2019)

Vertical profiles of trends in the annual-mean north-south temperature difference

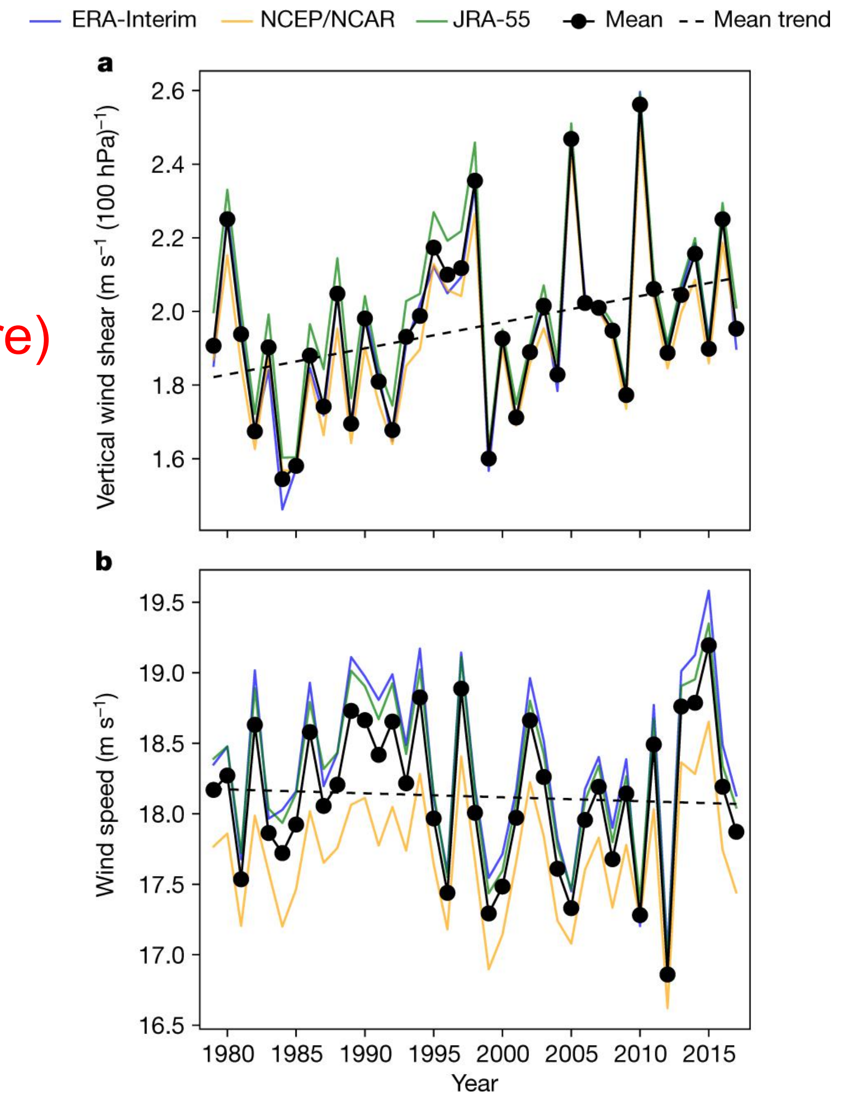
Increased wind shear

Conclusion of analysis on thermal wind balance:

- A statistically significant weakening of $\frac{\partial T}{\partial y}$ (lower atmosphere)
- A statistically significant strengthening of $\frac{\partial T}{\partial y}$ (upper atmosphere)
- No clear annual change in wind speed
- A 15 per cent increase of vertical shear (over 39-year period)

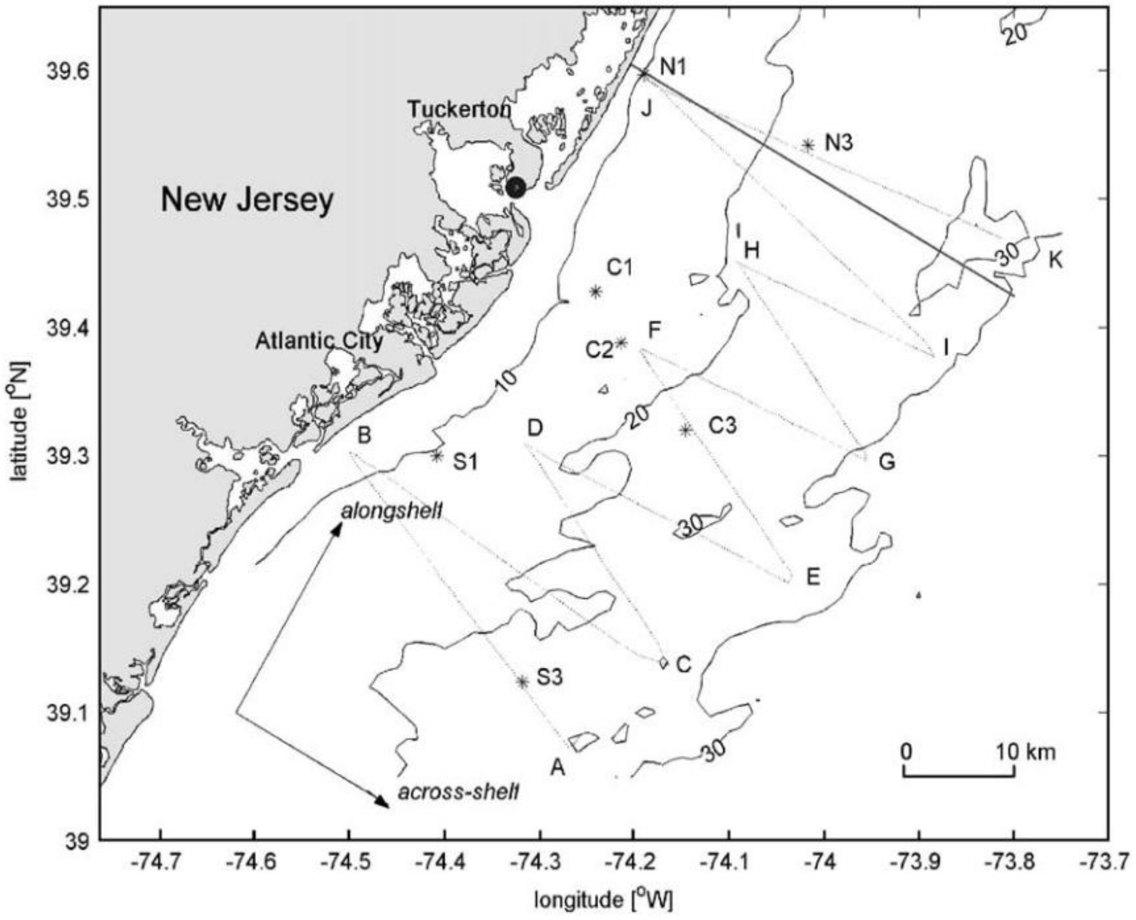
Potential effects:

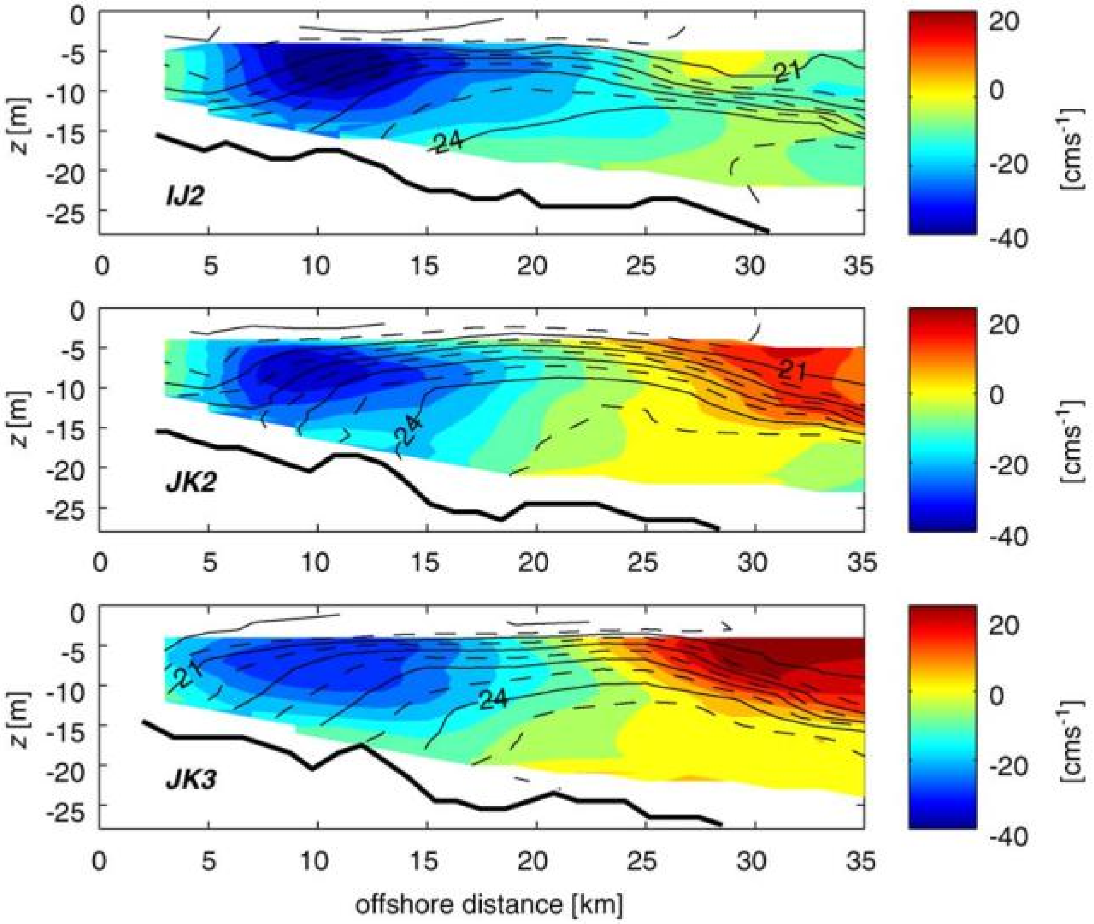
- Variability on the upper-level jet stream
- A more turbulent environment aloft



Time series of vertical shear (a) and zonal wind speed (b)

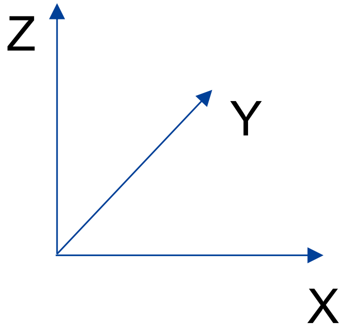
Example

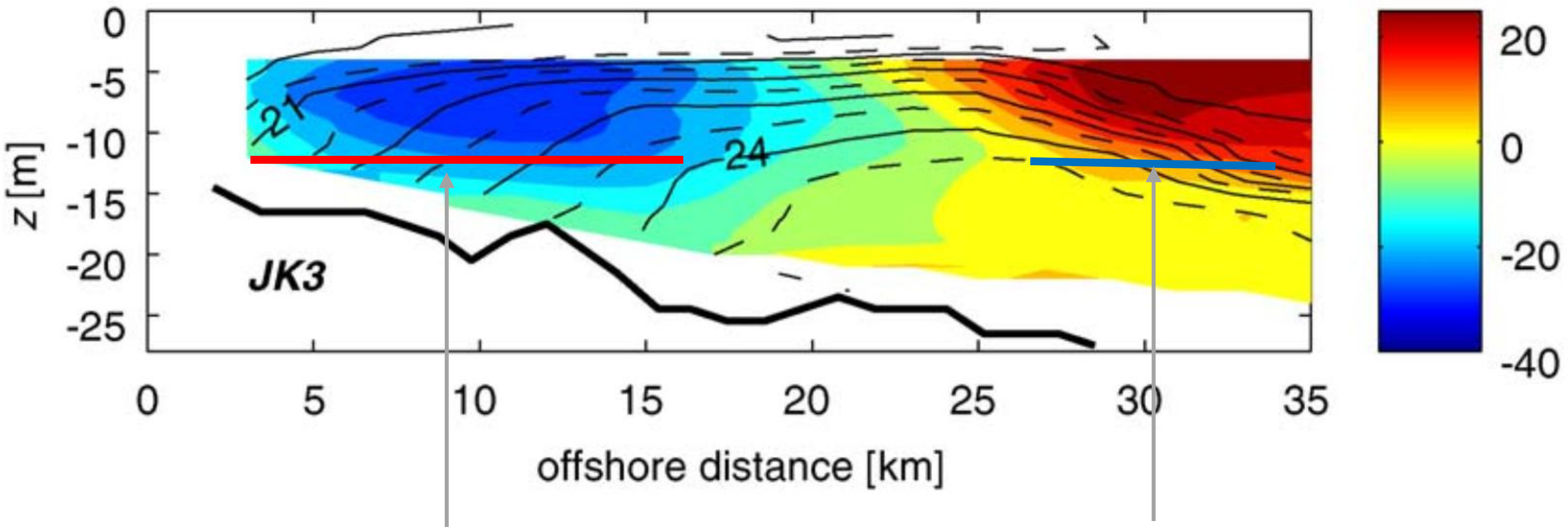




(Alexander E. Yankovsky, Continental Shelf Research,2006)

Example





$$f\frac{\partial v}{\partial z} = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial x}$$

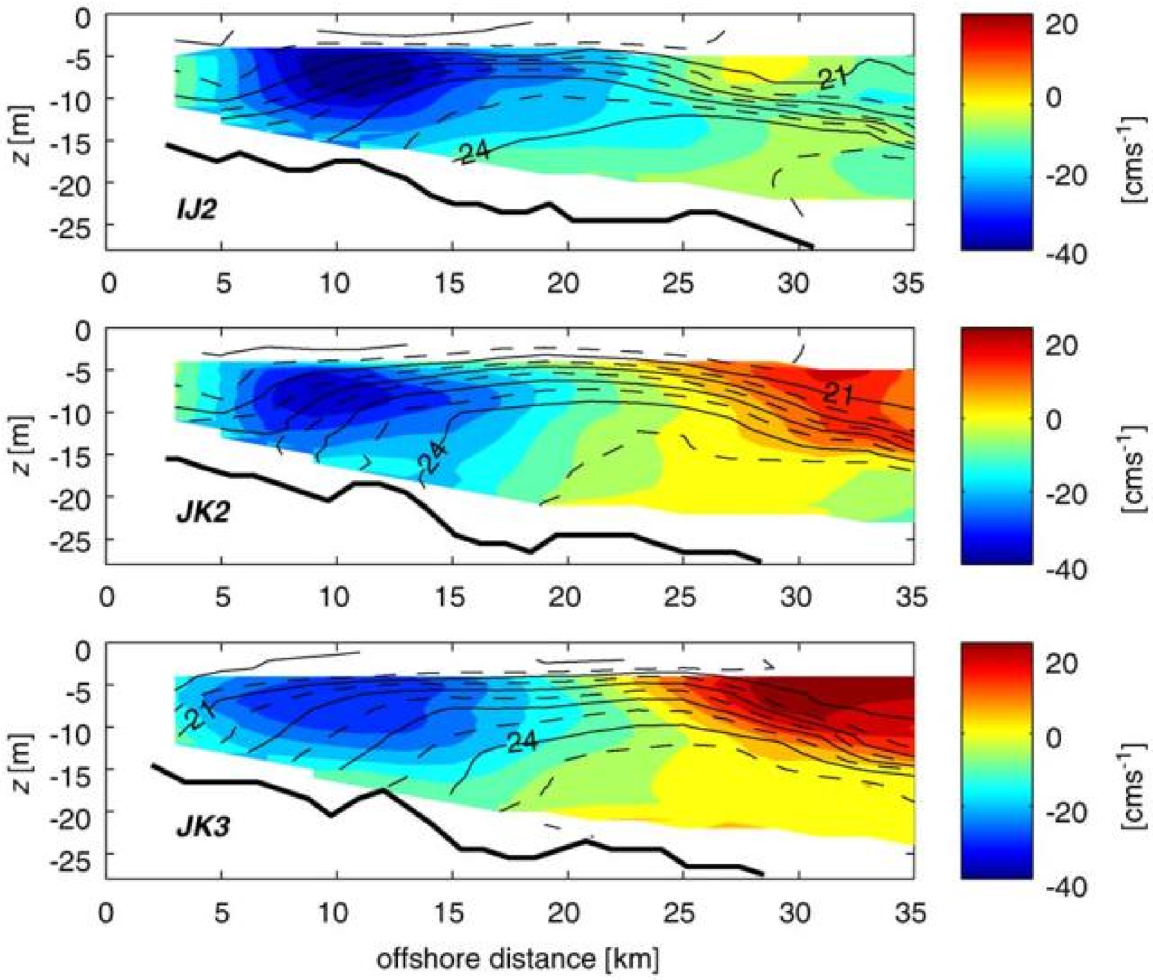
$$\frac{\partial \rho}{\partial x} > 0$$

$$\frac{\partial v}{\partial z} < 0$$

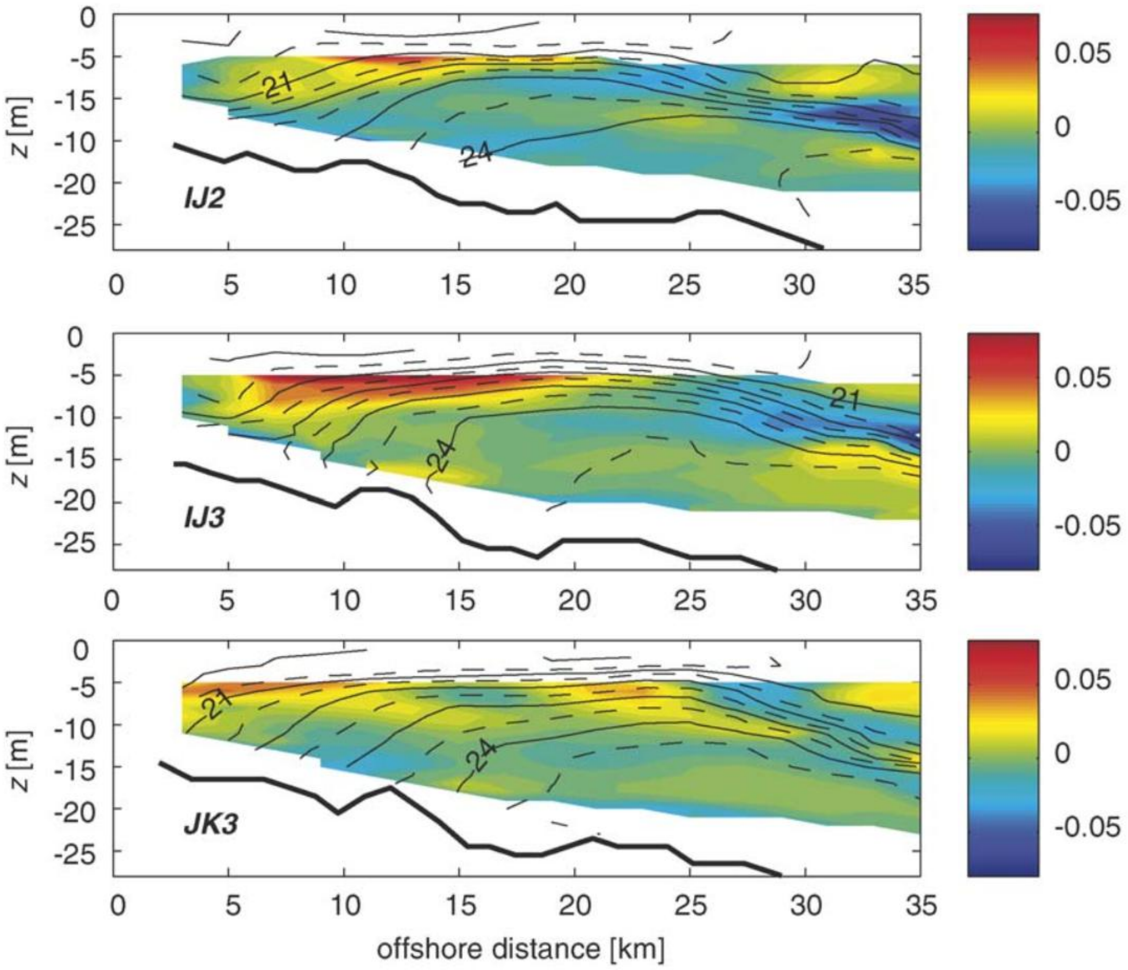
$$\frac{\partial \rho}{\partial x} < 0$$

$$\frac{\partial v}{\partial z} > 0$$

Example

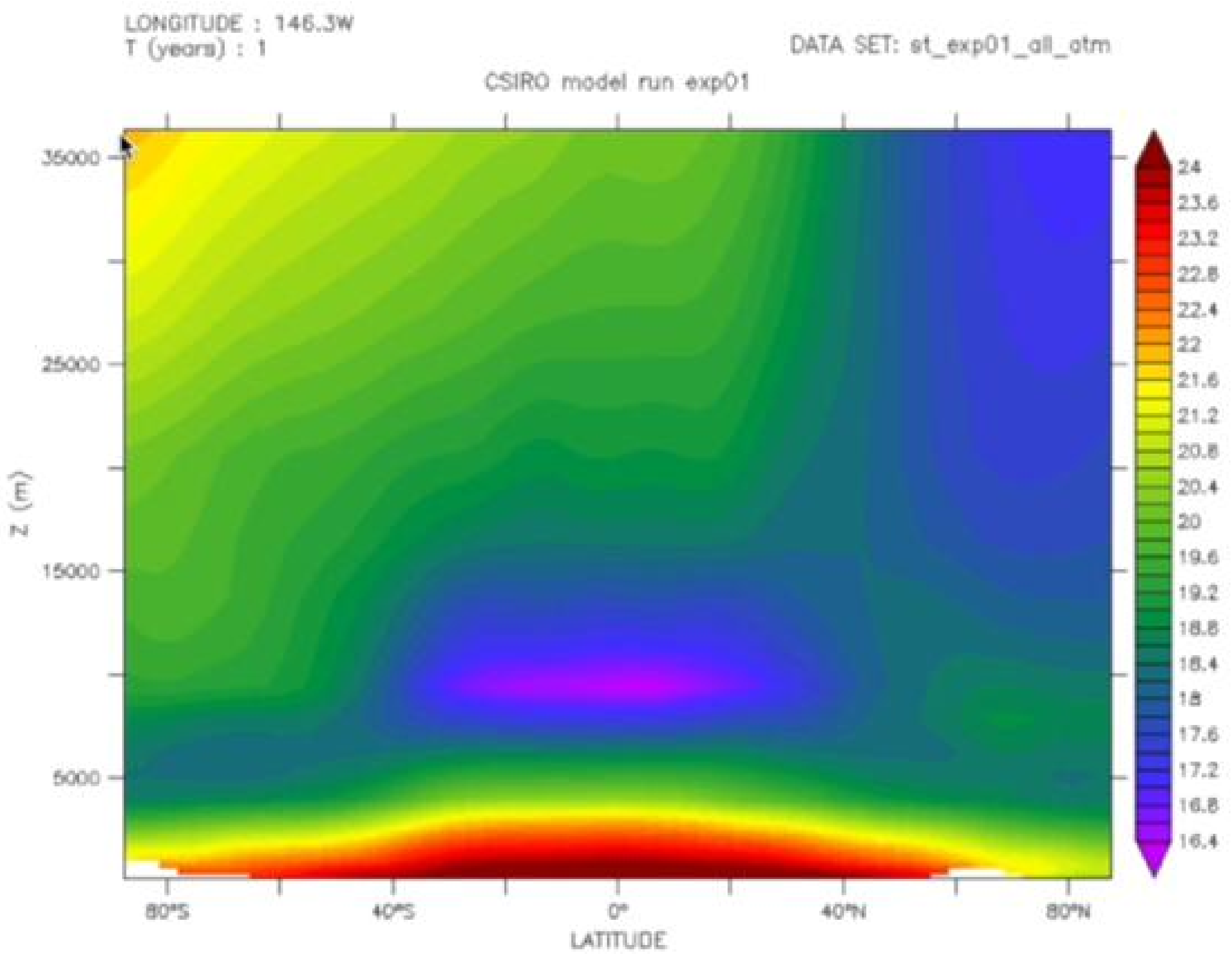


Observed vertical shear

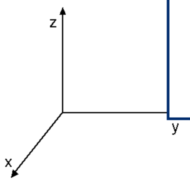


Observed vertical shear minus estimated TWE shear

Atmosphere meridional profile example



Zonally averaged air Temp (oC)



$$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial z} = -\rho g.$$

$$\frac{\partial u}{\partial z} = +\frac{g}{\rho_0 f}\frac{\partial \rho}{\partial y}.$$

$$\frac{\partial v}{\partial z} = -\frac{g}{\rho_0 f}\frac{\partial \rho}{\partial x}.$$

Video example