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DOUBLE DIFFUSION IN OCEANOGRAPHY

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INTRODUCTION

The modern study of double-diffusive convection began with Melvin Stern's article on "The Salt Fountain and Thermohaline Convection" in 1960. In that paper, he showed how opposing stratifications of two component species could drive convection if their diffusivities differed. Stommel ct al (1956) had earlier noted that there was significant potential energy available in the decrease of salinity with depth found in much of the tropical and subtropical ocean. While they suggested that a flow (the salt fountain) would be driven in a thermally-conducting pipe, it was Stern who realized that the two orders of magnitude difference in heat and salt diffusivities allowed the ocean to form its own pipes. These later came to be known as "salt fingers." Stern also identified the potential for the oscillatory instability when cold, fresh water overlies warm, salty water in the 1960 paper, though only in a footnote. Turner & Stommel (1964) demonstrated the "diffusive-convection" process a few years later.

From these beginnings in oceanography over three decades ago, double diffusion has come to be recognized as an important convection process in a wide variety of fluid media, including magmas, metals, and stellar interiors (Schmitt 1983, Turner 1985). However, it is interesting to note that about one hundred years before Stern's paper, W. S. Jevons (1857) reported on the observation of long, narrow convection cells formed when warm, salty water was introduced over cold, fresh water. He correctly attributed the phenomenon to a difference in the diffusivities for heat and

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salt in a qualitative way, though his attempt to apply the results to cloud convection was misguided. It is a curious twist of history that Jevons' work motivated Rayleigh (1883) to first derive the expression for the frequency of internal waves in a stratified fluid. However, he ignored the effects of diffusion and thus missed the opportunity to initiate the theoretical study of double-diffusive phenomena. Thus, we could have had over a century of progress to report on, rather than only three decades!

Nonetheless, it is fitting that the ocean should be the impetus for the discovery of this fundamental convection process. As will be discussed below, the ocean is strongly unstable to double-diffusive processes and seems to be profoundly affected by their presence. Only recent advances in ocean instrumentation have enabled us to begin to understand the complicated nature of double-diffusive mixing in the large, under-explored fluid which covers most of the Earth. Given the vital role of mixing in control of ocean heat storage, the thermohaline circulation, climate, carbon dioxide absorption, and pollutant dispersal, it is increasingly important that we achieve a more complete understanding of oceanic double diffusion.

In this review, I attempt to summarize our current knowledge of oceanographic double diffusion for the salt finger, diffusive convection, and intrusive instabilities. Due to my own interests, there is a more complete discussion of salt fingers. In addition, I note some of the potential implications for large-scale modeling of the ocean thermohaline circulation.

SALT FINGERS

Early efforts to understand double-diffusive convection drew on extensive experience with classic Rayleigh-Bénard convection, in which a fluid confined between two boundaries is heated from below. However, at least for the salt finger case, boundaries are largely irrelevant, as the instability defines its own internal length scale. Down-flowing warm, salty water loses heat but not salt to surrounding cold, fresh water, thereby becoming denser and accelerating its flow. Cold, fresh plumes gain thermal buoyancy and are accelerated upward. The scale of the fingers is set by a balance between thermal diffusion (which releases energy in the salt stratification and is greatest at small scales) and viscous drag (which limits the smallest scales of motion). In the situation where opposing linear profiles (constant gradients) of temperature (or fast diffusing substance) and salinity (slow diffusing substance) are found, the simplest and most useful salt finger model is the unbounded, depth-independent similarity solution. Discussed for the asymptotic case of large Prandtl number and diffusivity ratio by Stern (1975), it has proven very valuable in understanding the length scale of ocean and laboratory salt fingers and the ratio of heat to salt transport. Here we examine the solutions given by Schmitt (1979a, 1983) which are valid for all Prandtl numbers and diffusivity ratios. Because of the purely vertical flow, the similarity solutions are valid at finite amplitude (it is a linear system, but not linearized). The evolution of the temperature (T), salinity (S), and vertical velocity (w) fields in salt fingers is described by the equations:

$$\frac{\partial \alpha T'}{\partial t}+w'\alpha\bar{T}_z = \kappa_T\nabla_2^2\alpha T'$$

$$\frac{\partial \beta S'}{\partial t}+w'\beta\bar{S}_z = \kappa_S\nabla_2^2\beta S'$$

$$\frac{\partial w'}{\partial t}+g(\beta S'-\alpha T') = \nu\nabla_2^2 w' \tag{1}$$

where the thermal expansion and haline contraction coefficients $\alpha[= -(1/\rho)(\partial\rho/\partial T)]$ and $\beta[= (1/\rho)(\partial\rho/\partial S)]$ are assumed constant, κ_T and κ_S are the molecular heat and salt diffusivities, ν is the kinematic viscosity, ($'$) represents the finger perturbation away from the horizontal average ($\bar{\ }$), and ∇_2^2 is the horizontal Laplacian. The requirement for instability is $\alpha\bar{T}_z/\beta\bar{S}_z \equiv R_\rho < \kappa_T/\kappa_S \simeq 100$ in the ocean. This requirement for the density ratio (R_ρ) to be less than the diffusivity ratio ($\kappa_T/\kappa_S = \tau$) means that the vertical salt gradient necessary for fingers is only 1/100 that of the temperature, when both are compared in density units. This criterion is satisfied over vast regions of the ocean. In order to maintain the larger scale static stability of the water column, we also require $R_\rho > 1$.

In an unbounded fluid, these equations have depth-independent solutions of the form:

$$(w', T', S') = (\hat{w}, \hat{T}, \hat{S})\exp(\lambda t)\phi(x, y), \tag{2}$$

where λ is the growth rate and ϕ is a horizontal planform function. Schmitt (1979a, 1983) provides a comprehensive look at the parametric dependence of these solutions.

In order to yield a separable problem, the function ϕ must satisfy the Helmholtz equation, $\nabla_2^2\phi+m^2\phi = 0$ (Proctor & Holyer 1986). A solution for rectangular fingers is $\phi = \sin(m_1x)\sin(m_2y)$; a solution for sheets is $\phi = \sin(my)$. Equal amounts of horizontal diffusion (and the same growth rates) are realized in squares ($m_1 = m_2$), rectangles ($m_1 \neq m_2$), and sheets, provided $m_1^2+m_2^2 = m^2$. Laboratory observations have confirmed the existence of the rectangular (Shirtcliffe & Turner 1970) and sheet (Linden 1974a) modes. Linden showed that parallel sheets are preferred when a vertical shear damps the downstream modes, leaving the cross-stream modes unaffected. The fact that horizontal diffusion in one dimension can compensate for the lack of it in the other dimension allows us to construct a variety of interesting planforms. The total horizontal wavenumber (m) is given by

$$m = \left(\frac{g\alpha \bar{T}_z}{\nu\kappa_{\mathrm{T}}}\right)^{1/4} M \tag{3}$$

where M is a nondimensional function of R_ρ, the heat/salt buoyancy flux ratio (γ), Prandtl number ($\nu/\kappa_{\mathrm{T}} = \sigma$), and diffusivity ratio ($\kappa_{\mathrm{T}}/\kappa_{\mathrm{S}} = \tau$).

The growth rate of the fingers is scaled with the local vertical temperature gradient,

$$\lambda = (g\alpha \bar{T}_z)^{1/2} G, \tag{4}$$

where G is the nondimensional growth rate. The dependence of G and M on σ, τ, γ, and R_ρ is given in Schmitt (1979a, 1983). The nondimensional growth rate can be expressed as:

$$G = \frac{M^2}{\sigma^{1/2}} \frac{(\gamma - R_\rho/\tau)}{(R_\rho - \gamma)}. \tag{5}$$

For heat–salt fingers ($\sigma = 10$, $\tau = 100$), an accurate expression for $\gamma(M)$ for $R_\rho > 1.15$ is: $\gamma = \frac{1}{2}[b - (b^2 - 4R_\rho)^{1/2}]$, where $b = R_\rho(1 + M^4) + 1$. This relation for $G(M)$ can be used to construct an evolving spectrum of salt fingers, given an initial "seed" spectrum, as in Schmitt (1979a) (Figure 1). Gargett & Schmitt (1982) compared such spectra with towed microstructure data from the Pacific. Certain portions of the data with narrow bandwidth, limited amplitude structure (distinct from the wide band, spiky nature of turbulence patches), displayed a remarkably good fit to the theoretical spectrum with 2–4 buoyancy periods of growth. Later observational work by Lueck (1987), Marmorino (1987), and Fleury & Lueck (1992) has also found a spectral peak in horizontal temperature gradient records at the predicted wavenumber of the fastest growing finger. Laboratory experiments by Taylor (1991) in which salt fingers grow after disruption by grid turbulence similarly reveal that the fastest growing finger dominates the structure. The fingers reform rapidly after the passage of the grid, within a few e-folding periods. He notes that exponential growth may be limited to a few e-folding times, before reaching a self limiting amplitude (which is expected to be a function of available salt contrast). In the ocean, where the time between turbulence events is long compared to the finger e-folding time (about one buoyancy period for $R_\rho = 2$), fingers should have sufficient time to establish modest fluxes. Shen

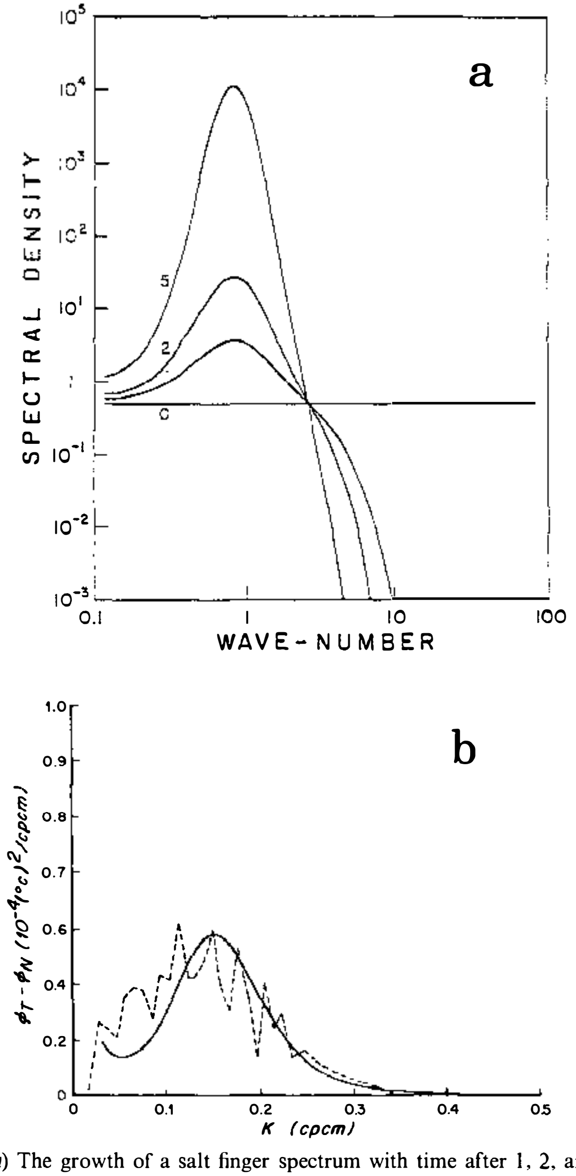

Figure 1 (*a*) The growth of a salt finger spectrum with time after 1, 2, and 5 buoyancy periods at $R_\rho = 2$. The "seed" spectrum is white. Both axes are nondimensional; when scaled to typical ocean conditions the spectral peak is usually at a wavelength of 5–7 cm. (*b*) Comparison of observed and theoretical salt finger spectra from Gargett & Schmitt (1982). Here a red seed spectrum ($-5/3$ slope) has been allowed to grow for 3.5 buoyancy periods at $R_\rho = 2.4$. The temperature microstructure data come from a towed body operating at a depth of 290 m in the North Pacific.

(1989) has performed numerical experiments on two-dimensional salt fingers which also show the dominance of the fastest growing finger.

The relationship between wavenumber and flux ratio in salt fingers is particularly strong. Wide fingers (low m) will lose less of their temperature anomaly than thin fingers, and thus advect more heat. The close dependence of flux ratio on wavenumber helps provide additional evidence that the fastest growing finger dominates. That is, the different flux ratios observed in laboratory experiments in both the heat/salt ($\gamma = 0.5$–0.7) and salt/sugar ($\gamma = 0.9$) systems agree well with those of the fastest growing finger (Schmitt 1979a). Similarly, ocean observations of the horizontal variations in layer properties within a thermohaline staircase require a substantial contribution from the fastest growing finger (Schmitt et al 1987).

In theory, any function satisfying the Helmholtz equation is a possible salt finger planform. Most work has focused on the square or sheet solutions. While squares appear to be the preferred mode in long-term laboratory experiments (Williams 1975), in the presence of vertical shear, sheets are observed (Linden 1974a). According to the theory of Proctor & Holyer (1986), sheets should be preferred over squares at finite amplitude, even in the absence of mean shear, contrary to the laboratory results. Schmitt (1993) has pointed out that a rich variety of finger planforms is possible when sheet and rectangular modes are combined. By choosing one component of the rectangular mode to be a subharmonic of the sheet wavenumber, interesting tesselations can be generated, including asymmetric $[\phi = 0.5\cos(my)+\sin(\sqrt{3}/2\,mx)\sin(\tfrac{1}{2}my)]$ and triangular $[\phi = 0.5\sin(my)+\sin(\sqrt{3}/2\,mx)\sin(\tfrac{1}{2}my)]$ modes. The asymmetric mode, in which a narrow jet of up- or down-going fluid is surrounded by a broader, weaker counterflow, appears to have been observed by Osborn (1991) in down-going, warm (and presumably salty) plumes observed from a submarine near the surface under calm conditions. Scales of the phenomena he discovered seem to be about right for the fastest growing asymmetric mode, though the density ratio was 10–20, much higher than typical salt finger observations. Perhaps exceptionally calm conditions permitted growth in this case. However, though the observations fit a possible mode of the system, the cause of the asymmetry remains obscure. It could result from vertical variations in the background gradients, asymmetries in the perturbations seeding the fingers or in the nonlinearity of the equation of state for seawater. No laboratory work has yet been done on this topic, though it seems ripe for exploration.

Another very puzzling aspect of salt finger planforms stems from microstructure observations during the Caribbean-Sheets and Layers Transects (C-SALT) experiment in 1985. Very strong temperature and salinity steps were observed in the main thermocline east of Barbados over an area of 1 million square kilometers (Schmitt et al 1987). Well mixed layers 5–30 m thick were separated by high gradient interfaces with temperature contrasts up to 1°C (Figure 2). Such "thermohaline staircases" are the expected finescale signature of active double diffusion, as demonstrated in the laboratory by Stern & Turner (1969). In a staircase, fingers within the high gradient interfaces provide an unstable buoyancy flux which drives large-scale overturning in the adjacent mixed layers. Marmorino et al (1987) documented both finger-scale microstructure within the interfaces and convective plumes in the mixed layers for the C-SALT staircase.

However, shadowgraph images from C-SALT (Kunze et al 1987) revealed small-scale laminae with a nearly horizontal orientation in the finger-favorable interfaces (Figure 3). This was surprising, as earlier shadowgraphs (Williams 1975, 1981; Schmitt & Georgi 1982) revealed more vertically oriented structure similar to that seen in the laboratory. A shadowgraph is an indicator of the Laplacian of the index of refraction. The images are sensitive to the focal length of the system; the C-SALT systcm was designcd to best resolve relativcly short focal-length structure. The earlier systems of Williams had focal lengths 2 to 10 times greater. It seems likely that the images represent fingers tilted by shear, or sheets initially aligned with the shear but tilted by the rotating shear vector (Kunze 1990). As discussed by Kunze, the centimeter scale laminae must represent primarily salt structure. The model he developed indicates that the temperature structure should have a more vertical orientation. It would also have a larger scale and thus a longer focal length. This perhaps explains the more vertical structure seen in previous shadowgraph imagery. Shear tilted fingers or sheets might also explain the low vertical coherence found in data from closely spaced temperature probes on the towed microstructure instrument described by Lueck (1987) and Fluery & Lueck (1992) and the low dissipations reported by Gregg & Sanford (1987). If most of the small-scale shear is due to inertial waves (as is usually found in the ocean), alignment of sheets with the shear should not be expected unless the time scale for finger growth is sufficiently short compared to the inertial period. This can only be achieved if the interfacial gradients are sharp. Most of the C-SALT interfaces were much thicker (2–5 m) than predicted from laboratory experiments (20–50 cm), so could not support very rapid finger growth. Marmorino (1989) and Fluery & Lueck (1991) find that the flux may be independent of the interface thickness, contrary to theory. However, the Kunze (1987, 1990) models do appear to explain most other features of the C-SALT microstructure: the low dissipation rate, the horizontal laminae, and the low vertical coherence of towed microstructure. In the Kunze (1990) model, even the slow turning of the inertial shear at this latitude is sufficient to limit sheet growth, so that fluxes are surprisingly sensitive to the Coriolis frequency. This predicted effect may help to rationalize some of the observed properties of ocean thermohaline staircases, discussed below.

Persistent staircases have been well documented in the Tyrrhenian Sea within the Mediterranean, below the warm, salty Mediterranean outflow water in the eastern North Atlantic, and in the western tropical North

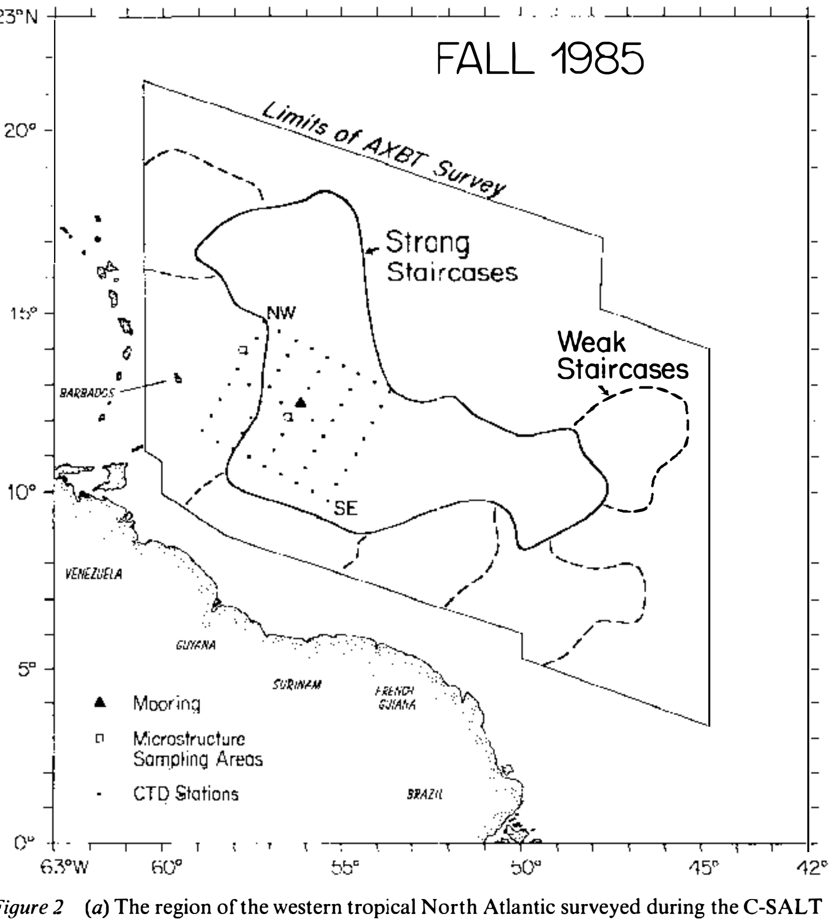

Figure 2 (*a*) The region of the western tropical North Atlantic surveyed during the C-SALT (Caribbean-Sheets and Layers Transects) field program. The AXBT is an Air deployable eXpendible BathyThermograph, a device used to obtain ocean temperature profiles from an airplane. An area of over 10^6 km^2 (roughly equivalent to the combined areas of Texas and California) was found to have significant thermohaline layering in the main thermocline.

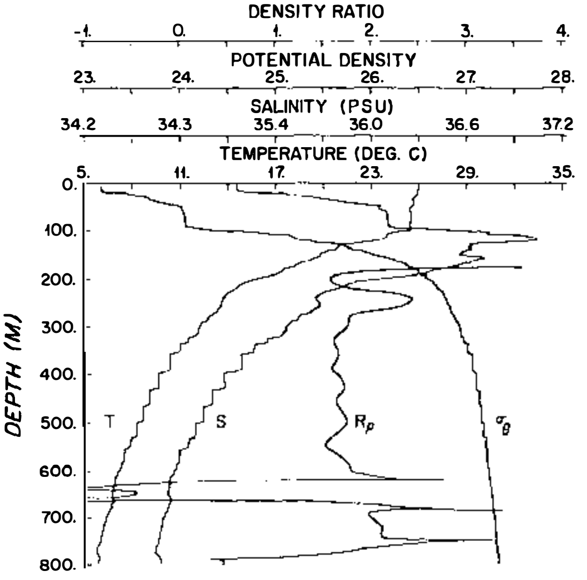

Figure 2 (*b*) The vertical profiles of temperature, salinity, potential density anomaly (σ_θ, kg/m^3), and R_ρ from a station in the C-SALT area, from the surface to 800 m depth. Temperature contrasts across the steps are typically 0.5–1.0°C. Mixed layers are 5–30 m thick. The layered structure in the 300–600 m depth range has $1.5 < R_\rho < 1.8$.

Atlantic. In the Tyrrhenian layers, the temperature and salinity of the layers are observed to remain constant for many years (Johannessen & Lee 1974, Molcard & Williams 1975, Molcard & Tait 1977). The Mediterranean Outflow steps appear to have a lateral coherence of 50–100 km, and have been identified several times (Tait & Howe 1968, Elliot et al 1974). In the western tropical North Atlantic, the layers appear to be a permanent feature of the thermocline, with observations available over the past 3 decades (Mazeika 1974, Lambert & Sturges 1977, Boyd & Perkins 1987, Boyd 1989, Schmitt et al 1987). Lambert & Sturges (1977) suggested that the vertical flux convergence due to salt fingers was sufficient to balance the lateral advection in a thermohaline staircase in a Caribbean passage. However, when Lee & Veronis (1991) assume an advective-diffusive balance for the C-SALT layers in an inverse model, they find tracer velocities that differ from geostrophic, and mixing rates well above those derived from the microstructure measurements. Schmitt et al (1987) found that the layer temperature and salinity properties changed horizontally, with a remarkably constant lateral density ratio of $\alpha T_x/\beta S_x = 0.85(\pm 0.03)$

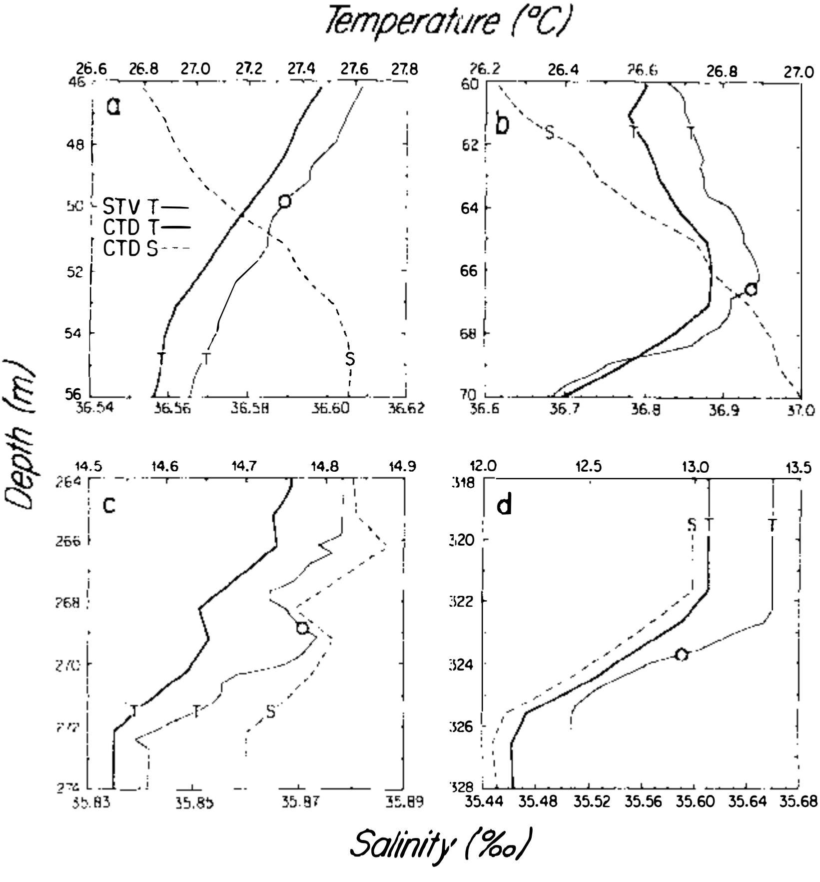

Figure 3 (*A*) Vertical thermohaline structure in the vicinity of shadowgraph images (*open circles*) shown in Figure 3*B*. Plotted are the shadowgraph temperature (offset) and temperature and salinity profiles from a simultaneous Conductivity-Temperature-Depth cast. Note that temperature and salinity scalings differ in the four panels. Both temperature and salinity profiles are gravitationally stable for the images in (*a*) and (*b*). In (*c*) the unstable temperature gradient is compensated by salinity, conditions which favor the "diffusive" form of double diffusion. In (*d*) an unstable salinity gradient across a 2-m thick interface between two homogeneous layers is compensated by temperature, conditions which are favorable to salt fingers.

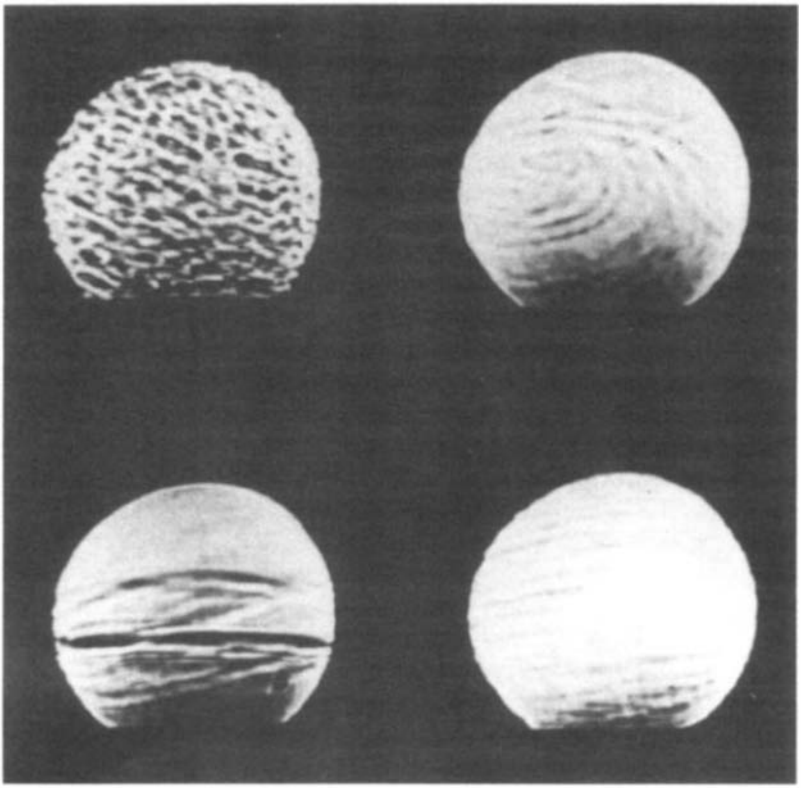

Figure 3 (*B*) Four images of optical microstructure from a free-fall shadowgraph profiler deployed in the C-SALT regions east of Barbados. Each image is 10 cm in diameter and represents a view through 60 cm of water. The four examples are thought to correspond to stratified turbulence (uppcr left, panel (*a*) of Figurc 3*A*), a shear billow (uppcr right, (*b*) of Figure 3*A*), a diffusive interface (lower left, (*c*) of Figure 3*A*), and shear tilted salt fingers (lower right, (*d*) of Figure 3*A*).

(Figure 4). Equally remarkable, historical data from the region show layer T/S properties falling along the same lines. The C-SALT layers seem immune to disruption by internal waves and the strong eddy field of the region, persisting for decades like geologic strata over about one million square kilometers. That such extensive features could be maintained by small-scale salt fingers is testament to the large amount of energy available in the unstable salt gradient; indeed, it is comparable to the available potential energy of the general circulation (Schmitt & Evans 1978).

The value of the lateral density ratio of 0.85 is worth some discussion. It represents a clear signature of salt fingers, as data on the time evolution

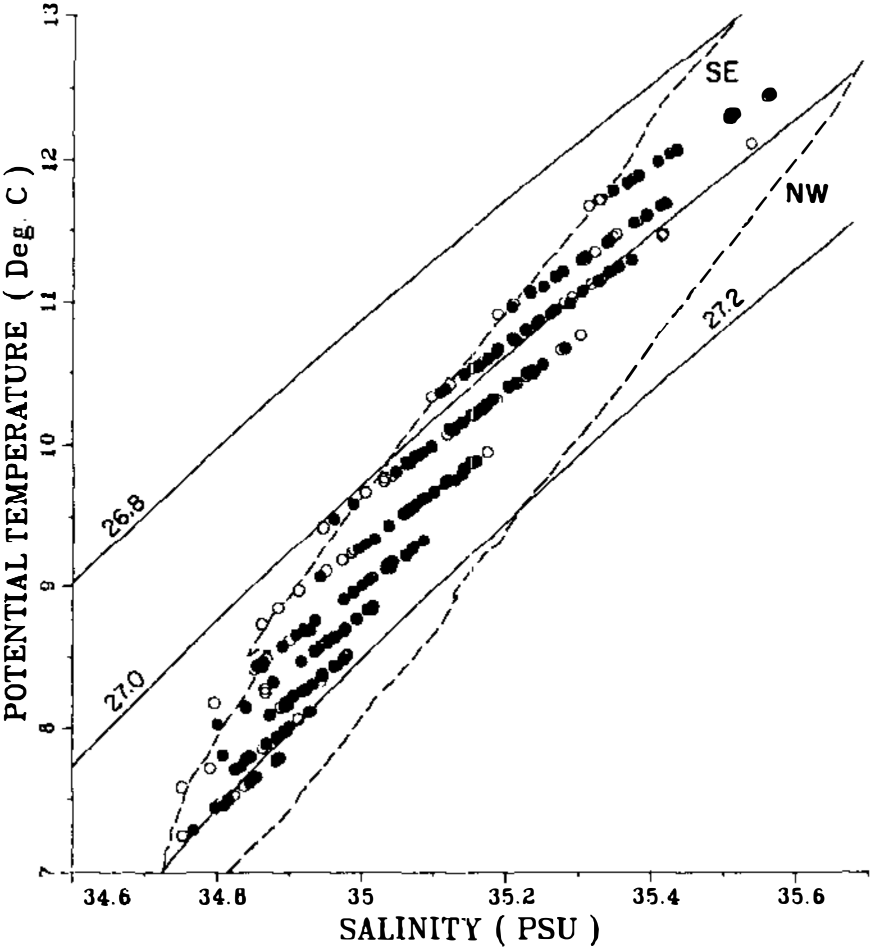

Figure 4 Potential temperature-salinity values of the mixed layers observed in the CTD stations occupied during the spring of 1985 for C-SALT. The solid circles are from mixed layers more than 10 m thick; the open circles are from layers 5–10 m thick. Also shown are the potential temperature-salinity relationships from stations at the northwest and southeast corners of the survey area (*dashed curves*). The evolution of properties across the region is such that individual layers become warmer, saltier, and denser from southeast to northwest as would be expected from the vertical convergence of salt finger fluxes. The layer properties cross isopycnals (the 26.8, 27.0, and 27.2 potential density surfaces are shown) with an apparent heat/salt density flux convergence ratio of 0.85 ($\pm$0.03).

of layer properties in laboratory experiments show variations strikingly similar to those of Figure 4 (Lambert & Demenkow 1972). The simplest interpretation is that the layer temperature and salinity values are given by a balance between along-layer advection and the divergence of the vertical heat and salt fluxes. The fact that the horizontal density ratio is 0.85 rules out a dominance of vertical turbulence, which would require heat and salt to change in the same ratio as the vertical density ratio, $R_\rho = 1.6$. It also rules out a purely isopycnal process, which would yield a horizontal density ratio of 1.0. It is consistent with a dominance of salt fingers, though slightly higher than might be expected from the salt finger flux ratio in theory (Schmitt 1979a) and experiments (Turner 1965, Schmitt 1979b). The small discrepancy may be due to the occasional patches of turbulence noted by Marmorino (1990) and Fleury & Lueck (1991). More likely though, the elevation above that expected for simple salt finger flux convergence is due to the upward migration of interfaces caused by the nonlinear equation of state (McDougall 1991). The "densification on mixing" causes asymmetric entrainment of the bottom of fingering interfaces, leading to a weak but significant contribution to the flux divergence ratio that is proportional to $R_\rho = 1.6$. The interface migration effect has been observed in the laboratory by Schmitt (1979b) and McDougall (1981).

One interesting aspect of the lateral variation in layer properties is that the changes are sometimes concentrated at internal fronts. These fronts take the form of tilted temperature inversions within the mixed layers. As documented by Marmorino (1991), the temperature inversions will support diffusive convection, with a vertical density ratio of 0.85. These intrusive structures may be an intrinsic part of the staircase system—perhaps a self-propagating mechanism for lateral exchange. The flux convergence between the finger and diffusive interfaces should set up lateral pressure gradients capable of driving the intrusions. Their origin and propagation mechanism remain obscure though, and the fronts are more simply explained by eddy stirring of the mean horizontal gradients in the region.

A full understanding of the detailed structure of thermohaline staircases must await further observational and theoretical developments. However, the long-term persistence of ocean staircases, and the relatively modest mixing rates observed within the C-SALT layers, lead me to propose the following rationalization of salt fingering staircases.

1. Persistent staircases must be maintained against the smoothing effects of shear-induced mixing by an "up-gradient" density flux due to double diffusion. Since turbulence due to breaking internal waves in the thermocline is generally weak (Gregg 1989) (and even weaker in the C-SALT staircase), the double-diffusive flux needed may be small, corresponding to a salt vertical eddy diffusivity of only 0.1 cm^2/s or so.

2. Once a staircase is formed, it is a simple matter for it to greatly increase its salt flux in response to increased advection of salt (Schmitt 1990), by thinning the interfaces and increasing the temperature and salinity contrasts. Such effects are suggested by some of the C-SALT steps (Schmitt 1988), where thin interfaces were found in association with lower R_ρ (though without accompanying microstructure measurements). The enhanced transfer of salt over heat would tend to increase R_ρ (Schmitt 1981), driving the staircase back to a state where the double-diffusive fluxes are just sufficient to maintain the layers. The actual flux in a staircase may be a stronger function of the salt forcing caused by differential advection than the instantaneous density ratio.
3. If we accept the Kunze (1990) model of salt sheet limitation by inertial waves, which seems to explain many of the C-SALT observations, then we might expect that the critical value of the density ratio necessary for maintenance of a staircase is a function of inertial-wave shear and latitude. That is, the Kunze model depends on a balance between sheet growth and disruption by inertial shear. If the inertial period decreases, then fingers or sheets have less growth time before being limited by the turning shear vector. Since the growth rate is a function of R_ρ, this suggests that a lower R_ρ would be required at higher latitudes, to maintain sufficient separation between the finger growth time and the local inertial period. Interestingly, density ratios in known staircases do seem to vary in this manner. The Tyrhennian Sea steps at 40°N have a density ratio of 1.2; the Mediterranean Outflow steps at 35°N have a density ratio of 1.3; and the C-SALT steps at 12°N have a density ratio of 1.6. The ratio of finger growth rate to inertial frequency is found to be roughly constant. While the data are sketchy at best, the correspondence with what could be expected from the Kunze model is certainly encouraging. Staircase observations from other latitudes, with careful shear and turbulence measurements, would be valuable for extending the comparison.

Two further issues concern the interpretation of ocean microstructure measurements. The first approach involves the analysis of towed temperature microstructure data; the second utilizes coincident measurements of the dissipations of both thermal and velocity variance.

The detection of salt fingers in data from a towed instrument was first achieved by Magnell (1976) in the Mediterranean Outflow steps at the time of Williams' optical detection of salt fingers. As mentioned previously, Gargett & Schmitt (1982) found good agreement between towed temperature spectra and theory. Further progress was reported by Mack (1985), who found that some thermal microstructure was well correlated with values of R_ρ near one. Holloway & Gargett (1987) noted that the narrow band, limited amplitude character of salt fingers compared to the broad band, high amplitude nature of turbulence leads to very different kurtosis signatures in towed temperature or conductivity gradient data. The kurtosis (4th moment) of the signal can be computed efficiently and thus is a useful discriminator between salt fingers (with low kurtosis) and turbulence (with high kurtosis). Marmorino (1987) noted the similarity of towed gradient spectra with the Schmitt (1979a) spectral model and suggested that a slope of $+2$ described the conductivity gradient spectra at wavenumbers below that of the fastest growing. Mack (1989) reported further evidence for an association of microstructure with R_ρ near 1 and also found gradient spectral slopes of $+2$, as contrasted with a slope of $+1$ in the turbulent patches. Marmorino & Greenewalt (1988) used both kurtosis and spectral slope to discriminate salt fingers from turbulence. They found it was easy to discriminate the two processes in data from the seasonal thermocline, but curiously, a less clear picture emerges from the C-SALT data. There the interfaces seem to contain both salt fingering and turbulent signatures, perhaps consistent with the elevation of the horizontal density ratio above the simple flux ratio for salt fingers. This turbulence could arise from convection in the layers, interface erosion due to the nonlinear equation of state (McDougall 1991), or shear instability across the interfaces (Marmorino 1990). Mack & Schoeberlein (1993) develop a discrimination technique, the log-likelihood ratio, based on the two-dimensional probability distributions of the slope and kurtosis. It appears that high slope, low kurtosis salt finger regions can be efficiently distinguished from low slope, high kurtosis turbulent patches with this approach. It would be useful to see such techniques applied to increasingly common towed CTD (Conductivity-Temperature-Depth) vehicles, if they can be fitted with fast response microstructure probes.

The second major approach to salt finger identification has been applied to data from vertically profiling vehicles. From these, measurements of the dissipation rates of turbulent kinetic energy (ε) and thermal variance (χ) can be made. Such measurements can be combined with information on background gradients to provide estimates of the vertical eddy diffusivity. In the case of ordinary turbulence, such as that caused by breaking internal waves, the effective vertical diffusivity is given by the relations

$$K_{\mathrm{T}} = \frac{\chi}{2(\bar{T}_z)^2}, \tag{6}$$

due to Osborn & Cox (1972) and

$$K_\rho = \left[\frac{R_{\mathrm{f}}}{(1-R_{\mathrm{f}})}\right]\frac{\varepsilon}{N^2} = \Gamma^{\mathrm{t}}\frac{\varepsilon}{N^2}, \tag{7}$$

due to Osborn (1980), in which R_f is the efficiency of conversion of kinetic to potential energy, and N is the local buoyancy frequency, $N = \sqrt{-g(1/\rho)(\partial\rho/\partial z)}$. Laboratory data suggest that R_f is about 0.15–0.2, giving an expected Γ^t of about 0.18–0.25.

However, for salt fingers, the relation between diffusivity and ε/N^2 is much different. McDougall (1988), Schmitt (1988), and Hamilton et al (1989, 1993) point out that the equivalent relation for salt fingers is

$$K_S = \frac{[R_\rho - 1]\varepsilon}{[1-\gamma]N^2} = \Gamma^f \frac{\varepsilon}{N^2}. \tag{8}$$

This expression can give a diffusivity over ten times the Osborn formula for the same dissipation rate due to the high efficiency of salt fingers in converting haline to thermal potential energy. Turbulence, however, dissipates most of its kinetic energy, converting only a small fraction to potential energy. As Schmitt (1988) points out, this relation is very sensitive to the value of the flux ratio, leading to some uncertainty in the effective diffusivity expected in salt finger regions.

Oakey (1985) has used ocean turbulence measurements to estimate Γ as the scaled ratio of χ to ε:

$$\Gamma = \frac{\chi N^2}{2\varepsilon \bar{T}_z^2}. \tag{9}$$

He found that Γ is around 0.17–0.26 in non-double diffusive regimes in the ocean. Hamilton et al (1989) and McDougall & Ruddick (1992) show how the measured Γ should differ for turbulence and salt fingers. Hamilton et al (1993) report that Γ is indeed above the value expected for turbulence, and consistent with salt finger theory, in a finger-favorable ocean staircase. McDougall & Ruddick (1992) address the problem of interpreting such measurements in environments where both turbulence and salt fingers may be active. The sensitivity to flux ratio is somewhat problematic; the asymptotic theory of Stern (1975) differs from the complete solution of Schmitt (1979a) for both high and low values of R_ρ. In addition to differences based on theoretical models, Γ is a particularly difficult variable to estimate, because of (*a*) questions about the isotropy of microstructure, (*b*) the fact that both χ and ε have non-Gaussian distributions, and (*c*) the variability of ocean hydrodynamic regimes over vertical scales of a few meters. Even so, the consistency of the Hamilton et al (1993) data with salt finger theory is encouraging. We can expect to see further advances with this approach, as new instrumentation capable of defining the background Richardson numbers as well as R_ρ, χ, and ε (Schmitt et al 1988), is deployed in a variety of ocean environments.

DIFFUSIVE CONVECTION

As mentioned above, Stern (1960) noted the possibility of an oscillatory instability when a stable solute distribution compensates for an unstable temperature profile. That is, when warm, salty water (sitting beneath cold, fresh) is displaced upward, the loss of heat causes an enhanced restoring force (an overstability) and a growing oscillation. The experiments of Turner & Stommel (1964) showed that a steady convection is realized in such a system when thin interfaces separate well-mixed layers. Thermal diffusion across the interfaces drives the convection; weaker salt diffusion acts as a brake on the system. When a stable salt gradient is heated from below, a series of mixed layers and interfaces forms a staircase in temperature and salinity profiles analogous to the salt finger case. Walin (1964) studied the stability properties of different thermohaline stratifications. The diffusive convection system is more profitably analyzed in terms of classic Rayleigh-Bénard convection than salt fingers, since the limiting effect of diffusive fluxes through the boundaries is an accurate analog, and the convection cell size is of the order of the layer depth. Veronis (1965, 1968) examined the finite amplitude behavior of diffusive convection using numerical integration of a truncated spectral model. Relative to the non-double-diffusive problem, the presence of the solute delays the onset of convection. However, at sufficiently high Rayleigh number, fluxes are nearly as large. Also of interest is the salt/heat buoyancy flux ratio, which Veronis showed tends toward $\tau^{-1/2}$. Nield (1967) and Baines & Gill (1969) have also examined the stability of such systems for various boundary conditions.

The low value of the flux ratio compared to salt fingers is of interest and can be readily understood, as it reflects the differing roles of the two components in the two systems. That is, energy in the salt field is released in salt fingers by diffusion of heat, and advection is the primary vertical flux agent. In the diffusive convection case it is the thermal stratification that contains the energy, and the vertical flux is diffusive at the interface. As in Linden & Shirtcliffe (1978), we suppose that the diffusion of heat and salt across a diffusive interface yields penetration distances into the adjacent mixed layer proportional to the one half power of the diffusivity. Since heat penetrates further, an unstable region forms which convects away from the interface when sufficiently thick. The relative portions of heat and salt carried away will reflect their different penetration scales, giving a salt/heat buoyancy flux ratio of $\tau^{-1/2}$.

Shirtcliffe (1967) found that an oscillatory instability does indeed first appear when a stable salt gradient is heated from below. Continued heating drives overturning cells at finite amplitude, which generate well-mixed layers in the previously stratified fluid (Turner 1968). The initial bottom layer grows to a height given by scaling arguments, then additional layers form in succession. This transition to a series of high gradient interfaces separating well-mixed layers enables the fluid to support much higher fluxes. In two-layer experiments Turner (1965), Marmorino & Caldwell (1976), and Linden (1974b) have shown that the flux ratio is indeed low for high density ratios far from one, though modestly above the $\tau^{-1/2}$ expected from theory. As the density ratio approaches one, however, the flux ratio approaches one due to disruption of the interface by mixed layer turbulence. That is, turbulent fluid parcels will penetrate and disrupt the interface so that direct fluid transfer is achieved. The laboratory experiments provide a "4/3" flux law, based on the temperature difference across the interface. Recently, Kelley (1990) has questioned the validity of the 4/3 power law, suggesting that an exponent between 4/3 and 5/4 (depending on Rayleigh number) may be more accurate.

Whereas salt fingers tend to be found in the low to mid-latitude evaporative regions of the ocean, conditions favorable to diffusive convection are more frequently found in high-latitude precipitation zones. The penetration of warmer, saltier water from low latitudes beneath cold, fresh surface waters, sets up diffusive convection sites over vast regions of the Arctic Ocean and various sites around Antarctica. Salt stratified lakes with geothermal or solar heating also provide diffusively favorable sites (Hoare 1966), as do hot, salty brines in ocean deeps at spreading centers (Swallow & Crease 1965). Steps under the Arctic ice were first reported by Neal et al (1969) and Neshyba et al (1971). More recent observations have been provided by Padman & Dillon (1987, 1988, 1989) in other parts of the Arctic. A diffusive thermohaline staircase between about 200 and 400 m depth appears to be a ubiquitous feature under most of the Arctic ice field away from boundaries. There is also a weak internal wave field there (Levine et al 1987) perhaps due to the rigid lid, but also possibly due to enhanced wave decay in the convectively mixed staircase (Padman 1991). Areas near topography associated with stronger wave energy and turbulence show less propensity to form steps, probably due to the down-gradient buoyancy flux (an up-gradient buoyancy flux is required to maintain a staircase).

The scales of layers in diffusive convective staircases have been addressed by Huppert & Linden (1979), Kelley (1984, 1988), and Federov (1988). All suggest that the layer thickness H must scale as $H \sim (\kappa_T/N)^{1/2}$. Kelley (1984) finds that the proportionality constant is a function of R_ρ. His analysis indicates that both laboratory and oceanic staircase heights can be incorporated into the same scaling—a rather remarkable result. In the salt finger case there was great discrepancy between lab and ocean data in the application of similar scaling laws by Stern & Turner (1969). Kelley (1988) maintains that the R_ρ dependence of the scaling can be determined from a model of the competing effects of layer splitting and merging. We also note that Fernando (1987) has proposed a different scaling for the layer thickness. Kelley (1988) provides an estimate of the dependence of the effective vertical diffusivity on R_ρ, a parameterization which should prove useful for understanding the larger scale effects of diffusive convection. The eddy diffusivities are in the range of 10^{-2}–10^{-1} cm^2/s. In general, vertical diffusive fluxes in the Arctic may be small compared to lateral advection, and seem unable to account for the evolution of the warm, salty Atlantic water within the Arctic basin (Padman & Dillon 1987), which may be dominated by lateral processes.

Staircase layering in the diffusive sense is also a prominent feature of the ocean stratification around Antarctica (Foster & Carmack 1976, Middleton & Foster 1980, Muench et al 1990). Muench et al find staircases to be a common feature over much of the Weddell Sea. The layers were much thicker (10–100 m) than commonly found in the Arctic, and may support a larger flux. They estimate upward heat flux using laboratory flux laws to be about 15 W/m^2 in open waters. This is sufficient to be important in the upper ocean heat budget and may help to maintain ice-free conditions in the summer.

INTRUSIONS

Whereas salt fingers and diffusive convection have distinct geographical regimes in terms of the large-scale mean stratification of the ocean (low to mid-latitude versus high latitude), there are widespread opportunities for the transient occurence of both types of instability on smaller scales. This arises from the horizontal contrast in temperature and salinity along density surfaces, and the propensity for interleaving of water masses in such conditions. Finescale horizontal intrusions, in which temperature inversions are stabilized by a density compensating salinity profile, are found within water mass fronts at all latitudes (Figure 5). The vertical scales range from 5–100 m; a typical horizontal scale is of order 10 km. The distinct role of double diffusion in mixing, and probably driving such intrusions, is now well recognized.

The potential for double-diffusive generation of horizontal intrusions was first recognized by Stern (1967). In the presence of isopycnal gradicnts of temperature and salinity, lateral displacements can set up sites for enhanced double-diffusive mixing which generate pressure gradients that serve to reinforce the motion. The greater transfer of salt than heat by fingers means that warm, salty intrusions should rise across isopycnals as

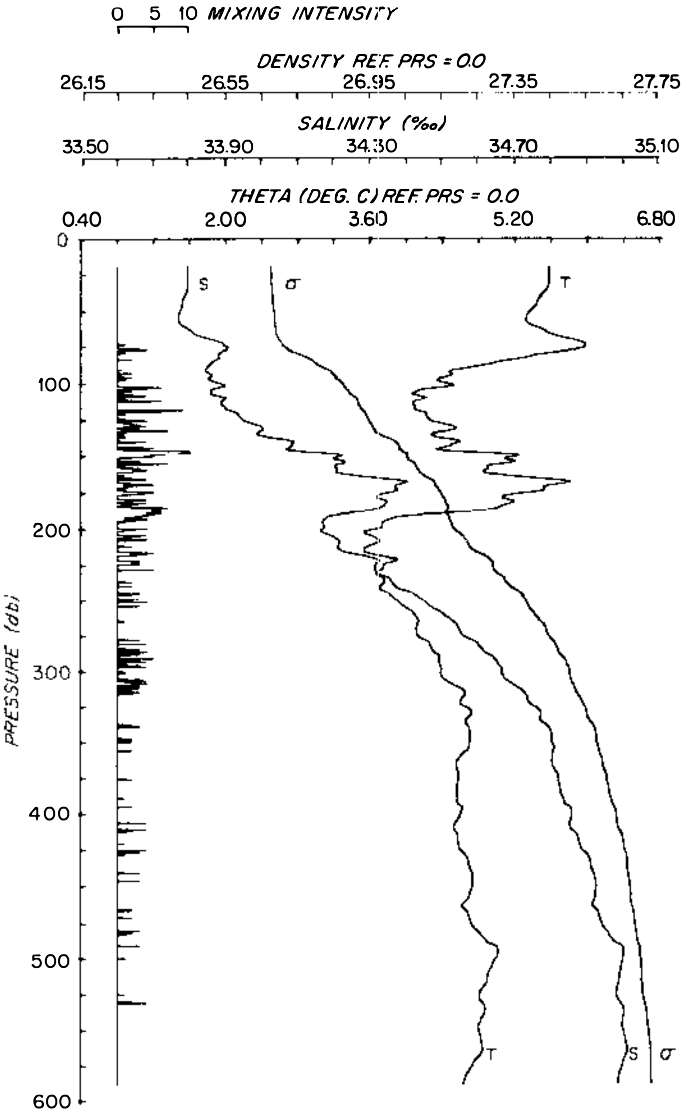

Figure 5 Temperature, salinity, and potential density profiles from the North Atlantic Current east of Newfoundland, from the surface to about 600 m depth (1 db in pressure is approximately 1 m in depth). The prominent intrusions found in this region arise because of the strong horizontal gradients in temperature and salinity along density surfaces. The profile of mixing intensity (*left*), derived from an optical shadowgraph, indicates that the boundaries of the intrusions have the most microstructure. These are regions where R_ρ is near 1, and double-diffusive mixing should be strong. (From Schmitt & Georgi 1983.)

they cool and freshen; cold, fresh intrusions should sink. Stern suggested that this mechanism could help maintain the tightness of the temperature-salinity relationship. That is, intrusive anomalies would be the sites of enhanced double diffusion which would cause the intrusions to migrate across density surfaces until their *T-S* properties matched those of the surrounding fluid. In Stern's model, small vertical scales were preferred for thc instability, because the enhanced flux convergence could more effectively accelerate the motions. Toole & Georgi (1981) extended the model to include viscosity, and found that the fastest growing features had an intermediate scale. Turner (1978) found that finger-dominated intrusions in the laboratory did indeed cross density surfaces as Stern predicted, and those dominated by diffusive interfaces were inclined in the opposite sense. Ruddick & Turner (1979) further demonstrated the intrusion phenomena in a beautiful set of experiments. They suggested that the vertical scale of the intrusions was set by the potential energy available across the initial sharp front of the experiments, giving a linear dependence on lateral salt contrast. The model of Toole & Georgi predicts an inverse square root dependence of vertical scale on horizontal salt gradient, giving a vertical intrusion scale of order 10s of meters.

Further advances in the theory of double-diffusive intrusions have been made by: Posmentier & Hibbard (1982), who examined the role of tilt on intrusion growth; Ruddick (1984), who studied the evolution of an intrusive layer subject to both finger and diffusive fluxes; McDougall (1985a,b), who developed linear-stability and finite-amplitude intrusion models; and Niino (1986), who studied the stability of a finite width front. In addition, Richards (1991) extended the linear stability problem to the equatorial beta plane, Posmentier & Kirwan (1985) suggested that intrusions could have dynamical effects on mesoscale eddies, McDougall (1986) pointed out the limitations of the Ruddick & Turner (1979) intrusion model, and Yoshida et al (1989) presented a generalized model that quantified the applicability of the Ruddick & Turner or Toole & Georgi scalings for the intrusion thickness. Attempts have been made to estimate the effects of intrusions on larger scales by Joyce (1977), who suggested a formalism to infer the effective lateral diffusivity due to intrusive interleaving, and Garrett (1982), who parameterized the diapycnal fluxes due to the migration of intrusions across density surfaces.

Oceanographers have reported on the existence of finescale thermohaline interleaving at a variety of ocean fronts. Joyce et al (1978) and Toole (1981a) reported on intrusions in the Antarctic polar front. They found that intrusions did slope as predicted by Stern. Gregg & McKenzie (1979) and Gregg (1980) found similar behavior in mid-latitude intrusions. Toole (1981a) reported reasonable agreement with the length scale of the

Toole and Georgi model, though eddy diffusivities and horizontal gradients are difficult to specify. Toole (1981b) described different intrusions in the equatorial Pacific, dominated by temperature variations, for which the vertical scale was more accurately given by the Ruddick & Turner expression.

Williams (1981) used his shadowgraph device to find evidence for active double diffusion in a Gulf Stream intrusion. Schmitt & Georgi (1982) also used the Williams shadowgraph in strong intrusions in the North Atlantic Current east of Newfoundland, and reported evidence for salt finger activity (Figure 5). They found plausible agreement of the intrusion scale with the Toole & Georgi model and also applied the Joyce scheme for estimating the effective lateral diffusivity of intrusions on larger scales. They suggest that intrusions may give lateral diffusivities two orders of magnitude larger than predicted from the shear dispersion of internal waves by Young et al (1982). Schmitt et al (1986) found intrusions around a warm core ring of the Gulf Stream which appeared to be dominated by diffusive-convection based on intrusion slopes and the relative intensity of fingering and diffusive microstructure. They also point out that the advective velocities and relative shears found around strong eddies significantly complicate our capacity to map intrusions and may contribute sufficient turbulence to mask the finescale effects of double diffusion.

Finally, Ruddick (1992) has analyzed data from a lens of warm, salty Mediterranean water in the eastern Atlantic (a "Meddy"). He finds diffusively-dominated intrusions at the upper edges of the eddy with a typical vertical scale of 13 m, and finger-dominated intrusions at the lower edges with vertical scales near 25 m. The slopes in the respective cases are as expected for the double-diffusive driving mechanisms and clearly inconsistent with the McIntyre (1970) process of differential diffusion of momentum and mass. This data set comprises the best evidence to date that double-diffusive intrusions play a significant role in ocean mixing. Hebert (1989) and Hebert et al (1990) have estimated that the intrusions were the most important dissipation mechanism acting on the Meddy, which was observed to decay over a two-year period.

Double-diffusively driven intrusions could turn out to be a primary horizontal mixing mechanism of the ocean. While "isopycnal" displacements do not require a change in potential energy, and thus should be readily accomplished, it appears that the internal wave climate provides rather modest lateral mixing rates (Young et al 1982). Though large-scale baroclinic instability is an active "stirring" mechanism, serving to increase mesoscale lateral gradients, it does not actually cause mixing (the destruction of gradients). Intrusions provide a key link in conveying heat and salt variance from the mesoscale to the microscale. The double-diffusive driving can be strong because the density ratio approaches one for both the finger and diffusive boundaries of the inversion. This releases energy in the isopycnal heat and salt gradients and appears to accomplish substantial lateral mixing. While we know rather little about lateral mixing processes, preliminary indications are that other finescale mechanisms are of lesser strength (Schmitt & Georgi 1982). Thus, there is reason to believe that double-diffusive intrusions are one of the most important lateral mixing agents in those regions with isopycnal gradients of temperature and salinity.

LARGE-SCALE IMPLICATIONS

Conditions favorable to salt fingering are very common in the main thermocline of the subtropical gyres. Ingham (1966) reports that in 90% of the Atlantic Ocean the main thermocline has a density ratio less than 2.3. At 24° N in the Atlantic, Schmitt (1990) finds that 95% of the upper kilometer is fingering favorable, with over 65% having a density ratio between 1.5 and 2.5. The Pacific is less conducive to salt fingering, but the Indian Ocean also harbors vast regions of low density ratio [65% has a thermocline $R_\rho < 4.5$, according to Ingham (1966)]. Given the widespread nature of fingering-favorable conditions in the world ocean, it is important to evaluate the effects of double diffusion on larger scale thermohaline structure.

As already discussed, evidence for active salt fingering in the ocean can be seen with a variety of measurement techniques. The agreement of spectra with theory, the increased occurrence of microstructure at low density ratio, the performance of spectral slope and kurtosis discriminators, the scaled ratio of χ to ε, the persistence and lateral property variations of thermohaline staircases, the finger-scale microstructure in the interfaces and the plume-like structure in the mixed layers, the occurrence of steps at low density ratio, and the behavior and microstructure of thermohaline intrusions, all indicate that salt fingering is an active ocean mixing process.

But is it an important process, given the failure of present models in describing the structural details of finger interfaces (Fleury & Lueck 1992) and the inadequacy of laboratory flux laws (Kunze 1987)? I would argue that it is, based on the structure of the large-scale temperature-salinity relation in the ocean. As noted by Ingham (1966) and Schmitt (1981), the form of the temperature-salinity relationship in most of the Central Waters of the various oceans is better fit by a curve of constant density ratio than a straight line (Figure 6*a*). Greengrove & Rennie (1991) find similar results for the South Atlantic. Schmitt (1981) showed how the dependence of salt finger mixing intensity on R_ρ and the greater transport of salt than heat could cause such structure. In this mechanism, variations away from a constant R_ρ profile are sites of salt flux convergence or divergence that tend to remove the R_ρ perturbations. A linearized analysis showed that R_ρ could "diffuse" vertically with an effective diffusivity well above that for heat or salt. Numerical examples of the nonlinear one-dimensional mechanism showed how salt fingers can rotate the slope of the T-S relation and induce curvature toward constant R_ρ, in distinct contrast to the straight line T-S relation given by ordinary turbulence, with equal mixing rates for T and S (Figure 6*b*). The suggested decrease in finger mixing rate as R_ρ exceeds 2 provides a rationale for the large portions of the Central Waters that have R_ρ close to 2. As an alternative, Stommel (1993) has proposed a lateral mixing mechanism for the surface mixed layer that also produces $R_\rho = 2$. However, the lateral gradients he requires would be strongly unstable to double-diffusive intrusions. Also, examination of density ratio distributions shows that it is generally less likely to have a value of 2 near the surface than deeper in the water column (Schmitt 1990). This suggests that the internal mixing and advection processes are most likely the dominant determinants of the density ratio.

In retrospect, the mixing rates chosen for the Schmitt (1981) model are too high, based on what was learned from C-SALT. However, our understanding of internal wave induced mixing has also led to a lowering of expected turbulence rates in the thermocline over the past decade. Gregg (1989) finds low levels of turbulent vertical mixing under typical internal wave conditions, giving $K_\rho < 0.1$ cm^2/s. The Schmitt (1981) mechanism will still function so long as the transfer of salt exceeds the transfer of heat and the mixing rate is a function of R_ρ. The model of Kunze (1990) shows a variation of mixing rate with R_ρ similar to what I proposed, though at a lower overall level. Progress in further understanding the role of salt fingers in large-scale balances may come from an analysis technique developed by Schmitt (1990). There, a relation is derived that expresses the potential for vertical shear to act on isopycnal gradients of temperature and salinity and thus modify the density ratio. The tendency of shear to rotate the slope of the T-S relation can be balanced by the differential vertical transport of heat and salt by fingers, but not by turbulence. The technique shows promise for the "constant R_ρ" regions of the subtropical gyres, where the absence of R_ρ gradients makes the R_ρ balance insensitive to the absolute velocity, and the vertical shear can be estimated from the thermal-wind relation.

Another approach has been taken by Gargett & Holloway (1992). They explored the influence of different diffusion rates for T and S in a numerical model of the ocean circulation. Dramatic modifications of the large-scale

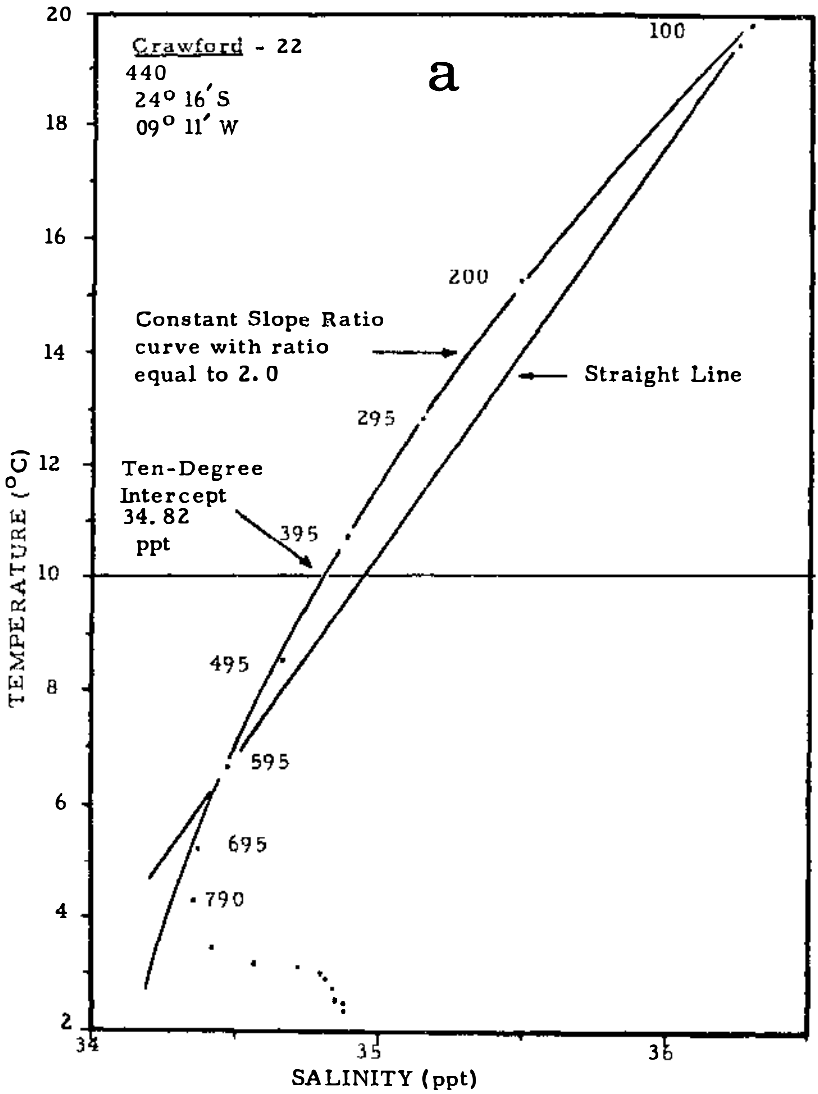

Figure 6 (*a*) Temperature-salinity diagram from the main thermocline of the South Atlantic, from Ingham (1966). The curve of constant $R_\rho = 2.0$ (his "slope ratio") is a better description of the data than a straight line fit for depths from 100 to 800 m.

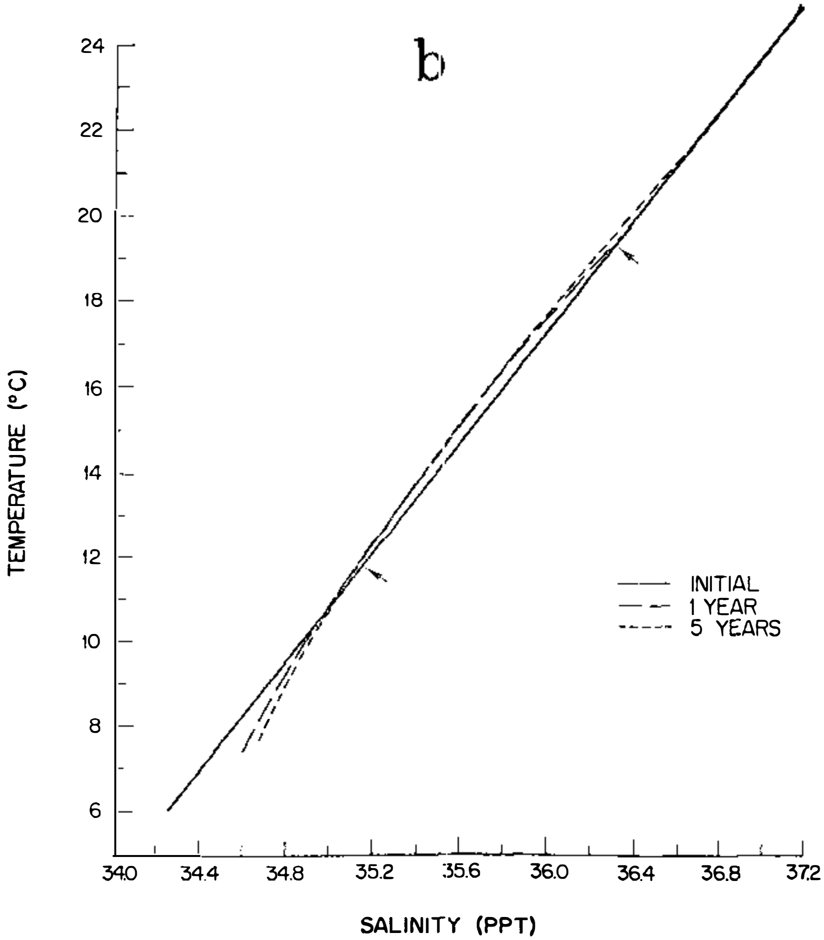

Figure 6 (*b*) Temperature-salinity diagram from the salt finger model of Schmitt (1981). The induction of curvature into an initially linear T-S relation over 1–5 years is caused by enhanced vertical transport of salt relative to heat in low R_ρ portions of the water column. This predicted effect of salt fingers eventually yields a constant R_ρ T-S curve, rather than the straight line expected from ordinary turbulent mixing. (Note: Constant R_ρ corresponds to a curve on a T-S diagram because of the nonlinear equation of state for seawater, primarily due to the increase in thermal expansion coefficient with temperature.)

meridional transports resulted because the vertical buoyancy-flux changes sign in the double-diffusive case relative to the constant diffusivity assumption, requiring different advective regimes to achieve a local density balance. Gargett & Holloway suggest that the ratio of the effective heat and salt diffusivities is more important than the specific value of the mixing rate. This is an encouraging result, since ratios are often more easily determined from the data (e.g. the horizontal density ratio observed in C-SALT) than are absolute fluxes. Much remains to be done with such modeling, including the use of diffusivities with the expected R_ρ dependence (Schmitt 1981, Kelley 1990), rather than a simple on/off function.

SUMMARY

Double diffusion has long graduated from the status of an "oceanographic curiosity." There is overwhelming evidence from fine- and microstructure studies that both salt fingers and diffusive convection are active ocean mixing processes. Much of the main thermocline of the mid- and low-latitude ocean is unstable to salt fingers, and double-diffusive intrusions are likely an important lateral mixing agent in some areas. There is also suggestive evidence from the character of the large-scale temperature-salinity relation in the Central Waters that fingers cause unequal mixing rates for heat and salt. Model results indicate that the presence of double-diffusive mixing has profound implications for the thermohaline circulation (and thus ocean climate as well). Further advancement of our understanding of the role of double diffusion in the ocean should have high priority. Improving observational capabilities will help to accomplish this goal, though field studies must be augmented by continuing improvement in laboratory, numerical, and theoretical models.

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