Combination of Rotation and Stratification

危国锐（硕士研究生）

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上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

水平−无地转：$\boldsymbol{V}_{\mathrm{h}} \cdot \nabla_{\mathrm{h}} \boldsymbol{V}_{\mathrm{h}} \sim -\frac{1}{\rho} \nabla_{\mathrm{h}} p \Rightarrow \frac{U^2}{L} = \frac{\Delta P}{\rho_0 L}.$

竖直-层化 & 静力平衡：

$$\left.\begin{aligned} p &= \rho g H \Rightarrow \Delta P = g H \Delta \rho \\ N^2 &= -\frac{g}{\rho}\frac{\partial \rho}{\partial z} \Rightarrow \Delta \rho = \frac{N^2 \rho_0 \Delta z}{g} \end{aligned}\right\} \Rightarrow \Delta P = N^2 H \rho_0 \Delta z.$$

水平−地转流：$\boldsymbol{V}_{\mathrm{h}} \sim \frac{\boldsymbol{k}}{\rho f} \times \nabla_{\mathrm{h}} p \Rightarrow \Omega U = \frac{\Delta P}{\rho_0 L}.$

$$U^2 = N^2 H \Delta z. \cdots (1)$$

$$W = \frac{\Delta z}{T} = \frac{\Delta z}{L/U} \Rightarrow \frac{W/H}{U/L} = \frac{\Delta z}{H}. \cdots (3)$$

$$U = \frac{N^2 H \Delta z}{\Omega L}. \cdots (2)$$

Case 1-**仅(竖直)层化、无地转**：

$$(1)\ \&\ (3) \Rightarrow \frac{W/H}{U/L} = \frac{\Delta z}{H} = \left(\frac{U}{NH}\right)^2 =: \mathrm{Fr}^2.$$

Case 2-**仅地转、无层化**：[1, pp.357-358]

$$\frac{W/H}{U/L} = \mathrm{Ro} := \frac{U}{\Omega L}.$$

Case 3-**(竖直)层化 + 地转**：

$$(2)\ \&\ (3) \Rightarrow \frac{W/H}{U/L} = \frac{\Delta z}{H} = \frac{\Omega L U}{N^2 H^2} = \frac{\mathrm{Fr}^2}{\mathrm{Ro}}.$$

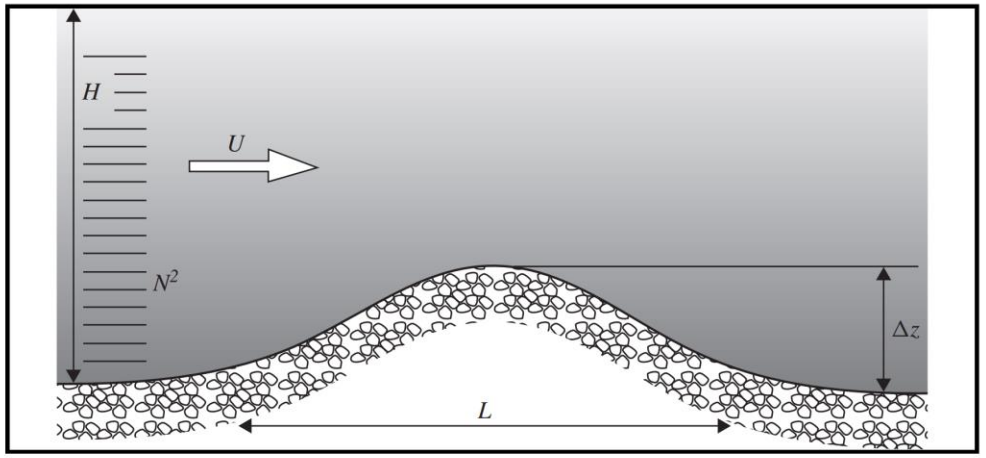
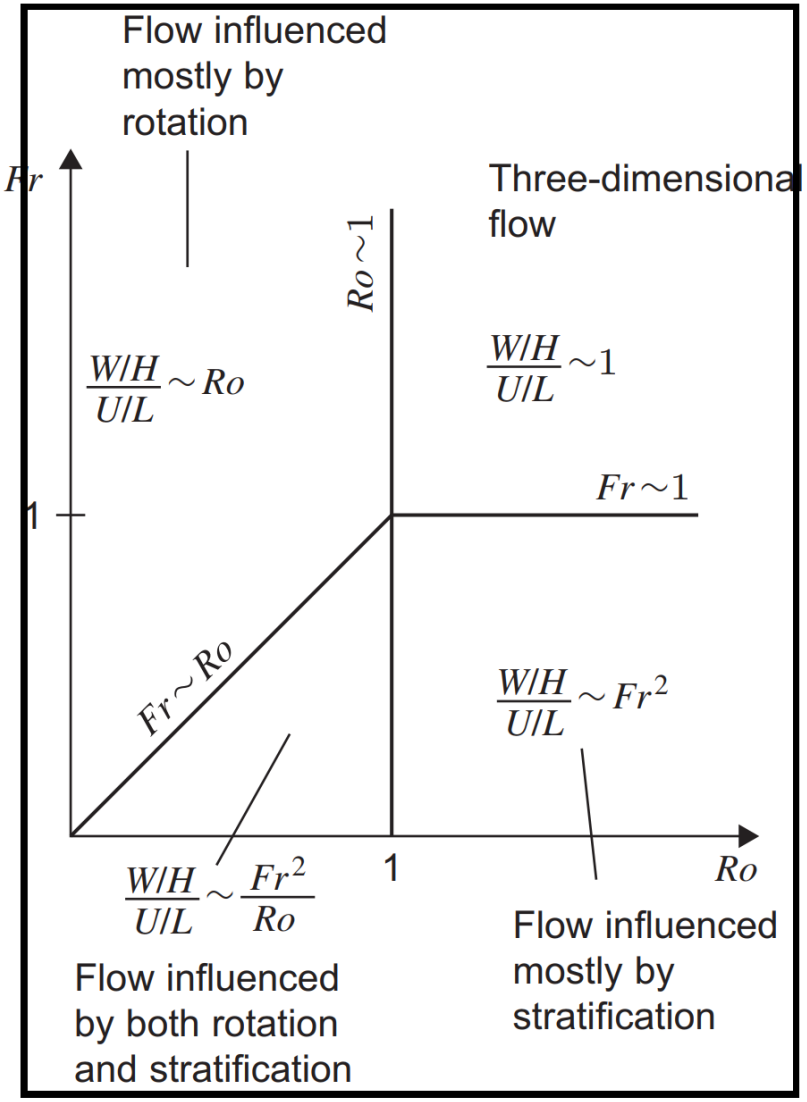


FIGURE 11.5 Situation in which a stratified flow encounters an obstacle, forcing some fluid parcels to move vertically against a buoyancy force.

上图来自：[1, p.356]

[1] Cushman-Roisin, B., & Beckers, J.-M. (2011). Chapter 11 - Stratification. In B. Cushman-Roisin & J.-M. Beckers (Eds.), *International Geophysics (Vol. 101, pp. 347-364). Academic Press.* https://doi.org/10.1016/B978-0-12-088759-0.00011-0



上图来自：[1, p.359]

竖直辐合(散) 与 水平辐散(合) 的量级之比

Case 1-**层化为主、地转可略** ($\mathrm{Fr} \lesssim 1, \mathrm{Ro} \gg 1$)

$$(1)\ \&\ (3) \Rightarrow \frac{W/H}{U/L} = \frac{\Delta z}{H} = \left(\frac{U}{NH}\right)^2 =: \mathrm{Fr}^2.$$

Case 2-**地转为主、层化可略** ($\mathrm{Fr} \gg 1, \mathrm{Ro} \lesssim 1$)

$$\frac{W/H}{U/L} = \mathrm{Ro} := \frac{U}{\Omega L}.$$

Case 3-**(竖直)层化 + 地转** ($\mathrm{Fr} \lesssim 1, \mathrm{Ro} \lesssim 1$)

$$(2)\ \&\ (3) \Rightarrow \frac{W/H}{U/L} = \frac{\Delta z}{H} = \frac{\Omega L U}{N^2 H^2} = \frac{\mathrm{Fr}^2}{\mathrm{Ro}}.$$

Burger number:

$$\mathrm{Bu} := \left(\frac{\mathrm{Ro}}{\mathrm{Fr}}\right)^2 = \left(\frac{NH}{\Omega L}\right)^2$$

当 $\mathrm{Fr}, \mathrm{Ro} \lesssim 1$，净浮力和科氏力比惯性力相当或更大，此时可用 Bu 衡量层化相对地转的重要性：

$\mathrm{Bu} \ll 1$：地转主导；
$\mathrm{Bu} \sim 1$：同等重要；
$\mathrm{Bu} \gg 1$：层化主导.

FIGURE 11.6 Recapitulation of the various scalings of the ratio of vertical convergence (divergence), W/H, to horizontal divergence (convergence), U/L, as a function of the Rossby number, $Ro = U/(\Omega L)$, and Froude number, $Fr = U/(NH)$.

[1] Cushman-Roisin, B., & Beckers, J.-M. (2011). Chapter 11 - Stratification. In B. Cushman-Roisin & J.-M. Beckers (Eds.), *International Geophysics (Vol. 101, pp. 347-364). Academic Press.* https://doi.org/10.1016/B978-0-12-088759-0.00011-0

谢谢！