第 1 次作业

危国锐　120034910021

（上海交通大学海洋学院，上海 200030）

摘　要：移动控制体是一种不必追随流体运动的区域. 关于移动控制体的 Reynolds 输运定理（RTT）是关于含参变量积分的 Leibniz 定理在高维情形下的推广. 包括随体导数公式在内的一些流体力学基本结果可视作 RTT 的直接推论. 由 RTT 可以统一对描述流体运动的两种方法——Lagrangian 描述和 Eulerian 描述的理解. 不可压缩流体的连续方程是流速的散度为 0. 海水的状态方程是非线性的. TEOS-10 基于 Gibbs 函数公式，海水的所有热力学特性（密度、焓、熵声速等）都可以热力学一致的方式推导出来. 使用 TEOS-10 计算位温、密度等海水状态参量，首先要遵循 TEOS-10 规范将海水现场温度和以实用盐标定义的盐度转换为“绝对盐度”（*Absolute Salinity*）和“保守温度”（*Conservative Temperature*）. 本文使用的程序和文档发布于 https://grwei.github.io/SJTU_2021-2022-2-MS8402/.

关键词：雷诺输运定理，流体运动学，连续方程，状态方程，TEOS-10

Homework 1

Guorui Wei　120034910021

(*School of Oceanography*, *Shanghai Jiao Tong University*, *Shanghai* 200030, *China*)

Abstract: A moving control volume is a region that does not have to follow the motion of the fluid. The Reynolds transport theorem (RTT) for a moving control volume is an extension of Leibniz's theorem for integrals with parameters in high-dimensional cases. The basic results of fluid mechanics including the volume derivative formula. RTT can unify the understanding of the two methods of describing fluid motion - Lagrangian description and Eulerian description. The continuity equation of an incompressible fluid is that the divergence of the flow velocity is 0. The state of seawater The equation is nonlinear. TEOS-10 is based on the Gibbs function formula, and all thermodynamic properties of seawater (density, enthalpy, entropy sound velocity, etc.) can be derived in a thermodynamically consistent way. Using TEOS-10 to calculate seawater state parameters such as potential temperature and density, First, the field temperature of seawater and the salinity defined by the practical salt scale are converted into "*Absolute Salinity*" and "*Conservative Temperature*" according to the TEOS-10 specification. The programs and documents used in this article are published at https://grwei.github.io/SJTU_2021-2022-2-MS8402/.

Keywords: Reynolds Transport Theorem, Fluid Kinematics, Continuity Equation, Equation of State, TEOS-10

目 录

1　流体运动学基础

本节简要回顾流体运动学中一些基本且重要的结果.

描述流体运动的两种方法——Lagrangian 描述（流体质点的观点）和 Eulerian 描述（场的观点）的联系被定义为（Kundu et al., 2016, p. 82）

$$F[\mathbf{r}(t;\mathbf{r}_o,t_o),t] = F(\mathbf{x},t) \text{ when } \mathbf{x} = \mathbf{r}(t;\mathbf{r}_o,t_o) \tag{3.2}$$

其中 F 是任意标量，向量或张量. 由此定义随体导数（*material*, *substantial*, or *particle derivative*），它是 Lagrangian 描述下 F 对时间的全导数，并可导出其 Eulerian 描述下的表达式（p. 83）:

$$\frac{DF}{Dt} \equiv \frac{\partial F}{\partial t} + \mathbf{u}\cdot\nabla F, \text{or } \frac{DF}{Dt} \equiv \frac{\partial F}{\partial t} + u_i\frac{\partial F}{\partial x_i}. \tag{3.5}$$

上式给出了两种描述方法的联系.

Kundu et al. (2016) 引入移动**控制体**（Control Volume, CV）的概念，推导了雷诺输运定理（Reynolds transport theorem, RTT）. 由此可统一对两种描述方法的理解.

RTT 描述的是移动控制体（不必追随流体质点）所包含的物理量对时间的变化率. 从数学上，就是建立了被积函数和积分区域都含参变量的积分对这参变量的导数. 事实上，我们在数学分析课程（陈纪修 et al., 2019, p. 315）中已证过一维 RTT，那就是下面的**积分号下求导定理**

定理 15. 1. 3（积分号下求导定理）　设 $f(x,y)$, $f_y(x,y)$ 都在闭矩形 $[a,b]\times[c,d]$ 上连续，则 $I(y)=\int_a^b f(x,y)\,\mathrm{d}x$ 在 $[c,d]$ 上可导，并且在 $[c,d]$ 上成立

$$\frac{\mathrm{d}I(y)}{\mathrm{d}y} = \int_a^b f_y(x,y)\,\mathrm{d}x.$$

和 Leibniz **定理**

定理 15. 1. 4　设 $f(x,y)$, $f_y(x,y)$ 都是闭矩形 $[a,b]\times[c,d]$ 上的连续函数，又设 $a(y)$, $b(y)$ 是在 $[c,d]$ 上的可导函数，满足 $a\leqslant a(y)\leqslant b$, $a\leqslant b(y)\leqslant b$，则函数

$$F(y) = \int_{a(y)}^{b(y)} f(x,y)\,\mathrm{d}x$$

在 $[c,d]$ 上可导，并且在 $[c,d]$ 上成立

$$F'(y) = \int_{a(y)}^{b(y)} f_y(x,y)\,\mathrm{d}x + f(b(y),y)b'(y) - f(a(y),y)a'(y).$$

Leibniz 定理在二维、三维情形下的推广，便是著名的**雷诺输运定理**（Reynolds transport theorem, RTT）. Kundu et al. (2016) 从数学上不很严格地给出了推导（pp. 99-103），其结果是

$$\frac{d}{dt}\int_{V^*(t)} F(\mathbf{x},t)dV = \int_{V^*(t)} \frac{\partial F(\mathbf{x},t)}{\partial t}dV + \int_{A^*(t)} F(\mathbf{x},t)\mathbf{b}\cdot\mathbf{n}dA. \tag{3.35}$$

式中各字母的含义示于下图（Fig 3.20）. 这样，Lagrangian 描述和 Eulerian 描述，就分别对应于（3.35）中 $\boldsymbol{b}=\boldsymbol{u},\mathbf{0}$ 的情形. 如此，两种描述方法不仅是联系的，而且是统一的.

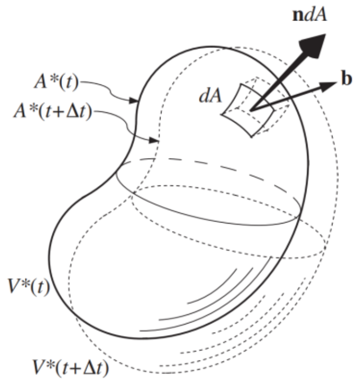


FIGURE 3.20 Geometrical depiction of a control volume $V^*(t)$ having a surface $A^*(t)$ that moves at a nonuniform velocity **b** during a small time increment Δt. When Δt is small enough, the volume increment $\Delta V = V^*(t+\Delta t) - V^*(t)$ will lie very near $A^*(t)$, so the volume-increment element adjacent to dA will be $(\mathbf{b}\Delta t)\cdot\mathbf{n}dA$ where **n** is the outward normal on $A^*(t)$.

从 RTT (3.35) 出发，可获得一些重要推论. 例如，在 (3.35) 中取 $F \equiv 1, |V^*| = \delta V \to 0^+$，就得到

$$\lim_{\delta V \to 0^+} \frac{1}{\delta V}\frac{\mathrm{d}}{\mathrm{d}t}\delta V = \nabla\cdot\boldsymbol{b}. \tag{1}$$

利用（1），在（3.35）中取 $|V^*| = \delta V \to 0^+$，就得到 RTT 的微分形式：

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\partial F}{\partial t} + (\boldsymbol{b}\cdot\nabla)F. \tag{2}$$

在（1）中取 $\boldsymbol{b} = \boldsymbol{u}$，便得到人们熟悉的速度散度的物理意义（相对体积膨胀率）. 在（2）中取 $\boldsymbol{b} = \boldsymbol{u}$，便得到**随体导数公式**，它是“the *Eulerian representation of the Lagrangian derivative as applied to a field*.”（Vallis, 2017, p. 5）

另外，吴望一（1982）从定义出发，推导了线、面、体元的随体导数（1982a, pp. 135-138）和体、面、线积分的随体导数（1982b, pp. 479-484）. 这些结果可视为一维 RTT 及其微分形式在曲线曲面情形下的推广，正如 Green, Gauss, Stokes 公式是 Newton-Leibniz 公式在高维情形下的推广.

关于 RTT (3.35)，受 Vallis (2017, pp. 3-7) 和吴望一（1982a, pp. 135-138）的启发，现重新推导如下：

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\int_{V^*(t)} F\,\mathrm{d}V &= \int_{V^*(t)}\frac{\mathrm{d}}{\mathrm{d}t}(F\,\delta V) = \int_{V^*(t)}\left(\frac{\mathrm{d}F}{\mathrm{d}t} + F\frac{1}{\delta V}\frac{\mathrm{d}\delta V}{\mathrm{d}t}\right)\delta V \\
&= \int_{V^*(t)}\left(\frac{\mathrm{d}F}{\mathrm{d}t} + F\nabla\cdot\boldsymbol{b}\right)\mathrm{d}V = \int_{V^*(t)}\left(\frac{\partial F}{\partial t} + \nabla\cdot(\boldsymbol{b}F)\right)\mathrm{d}V \\
&= \int_{V^*(t)}\frac{\partial F}{\partial t}\mathrm{d}V + \int_{A^*(t)} F\boldsymbol{b}\cdot\boldsymbol{n}\,\mathrm{d}S.
\end{aligned}$$

在推导中利用了（1）（2）. 这种推导方法在数学上不严格，然而物理意义较清晰，便于记忆. 应该指出，这种方法不是我的原创.

2 第 1 题

2.1 问题描述

1. For a fluid volume, show that $\frac{d}{dt}\int_v \rho\, dV = \int_v \frac{\partial\rho}{\partial t} dV$.

2.2 解决方案

在 RTT（3.35）中取 $\boldsymbol{b} \equiv \mathbf{0}$ 立得

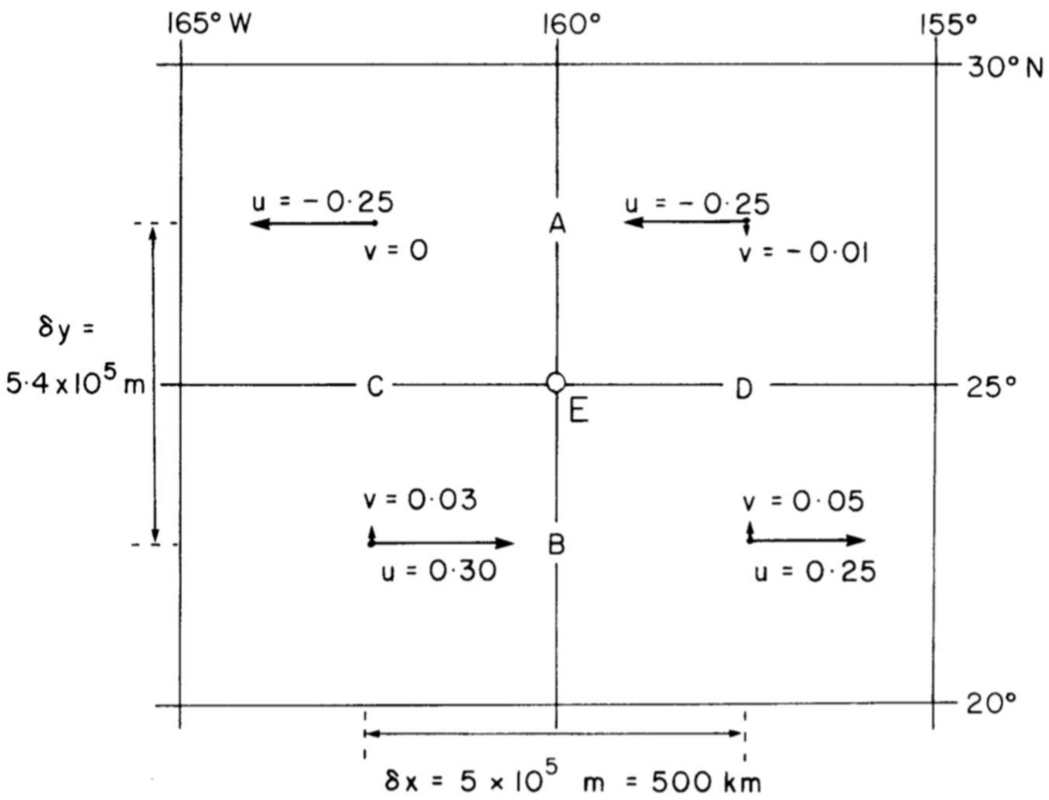
$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_0} F\,\mathrm{d}V = \int_{V_0} \frac{\partial F}{\partial t}\mathrm{d}V. \tag{3}$$

此时，积分区域 V_0 是空间位置固定的**控制体** (Control Volume, CV). 事实上，上式就是积分号下求导定理在三维情形下的推广.

3 第 2 题

3.1 问题描述

2. Use the following configuration for a domain in the ocean, derive the vertical velocity *w* at 50 m (assuming incompressible fluids and *w=0* at surface) based on the continuity equation. Hint: you can obtain the horizontal velocity at points A, B, C and D first, and then use these values to compute the horizonal divergence at point E.



3.2 解决方案

从左上象限起，按逆时针方向，为四个数据点依次编号 1 至 4.

首先，用线性插值估计 A 至 D 点的水平流速梯度：

$$\frac{\partial}{\partial x}(u,v)\bigg|_{\mathrm{A}} \approx \frac{(u,v)_2-(u,v)_1}{\delta x}, \qquad \frac{\partial}{\partial x}(u,v)\bigg|_{\mathrm{B}} \approx \frac{(u,v)_3-(u,v)_4}{\delta x},$$
$$\frac{\partial}{\partial y}(u,v)\bigg|_{\mathrm{C}} \approx \frac{(u,v)_1-(u,v)_4}{\delta y}, \qquad \frac{\partial}{\partial y}(u,v)\bigg|_{\mathrm{D}} \approx \frac{(u,v)_2-(u,v)_3}{\delta y}.$$

然后，用线性插值估计 E 点的水平流速梯度：

$$\frac{\partial}{\partial x}(u,v)\bigg|_{\mathrm{E}} \approx \frac{1}{2}\left(\frac{\partial}{\partial x}(u,v)\bigg|_{\mathrm{A}}+\frac{\partial}{\partial x}(u,v)\bigg|_{\mathrm{B}}\right),$$
$$\frac{\partial}{\partial y}(u,v)\bigg|_{\mathrm{E}} \approx \frac{1}{2}\left(\frac{\partial}{\partial y}(u,v)\bigg|_{\mathrm{C}}+\frac{\partial}{\partial y}(u,v)\bigg|_{\mathrm{D}}\right).$$

假定：1）流体为不可压缩的，从而有连续方程

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\equiv 0;$$

2）水平流速在 z 方向上不变，即

$$\frac{\partial}{\partial z}(u,v)\equiv 0.$$

从而，E 点上方 $z=z$ 垂直流速

$$w_{\mathrm{E}}(z)=w_{\mathrm{E}}(0)+\int_0^z \frac{\partial w}{\partial z}\bigg|_{\mathrm{E}}\mathrm{d}z = w_{\mathrm{E}}(0)-\int_0^z \left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\bigg|_{\mathrm{E}}\mathrm{d}z$$
$$= w_{\mathrm{E}}(0)-\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\bigg|_{\mathrm{E}} z.$$

代入数据，得 E 点 50 米深处垂直流速 $w_{\mathrm{E}}(-50)=-6.67\times 10^{-6}$ (m/s).

4 第 3 题

“TEOS-10 is based on a Gibbs function formulation from which all thermodynamic properties of seawater (density, enthalpy, entropy sound speed, etc.) can be derived in a thermodynamically consistent manner. TEOS-10 was adopted by the Intergovernmental Oceanographic Commission at its 25th Assembly in June 2009 to replace EOS-80 as the official description of seawater and ice properties in marine science.” (http://www.teos-10.org/index.htm)

4.1 问题描述

3. There are two sites in the ocean, A and B. The distributions of temperature (C°) and salinity (‰) with pressure (P, dbar) at these sites are shown in the following table.

	Site-A		Site-B	
P	S	T	S	T
0	35.10	28.50	33.50	2.50
20	34.99	28.45	33.50	3.74
40	34.88	28.35	34.25	4.02
60	34.78	24.55	34.55	4.10
80	34.68	22.75	34.65	4.15
100	34.60	20.55	34.74	4.20
200	34.45	15.50	34.90	4.30
250	34.35	13.00	35.10	4.35
500	34.25	6.58	35.23	4.25
1000	34.53	4.20	35.40	3.75

1) Calculate density with the linear Equation of State (EOS) as shown below, and plot the density profiles separately for A and B.

$$\rho = \rho_0\left[1 - \beta_T(T - T_0) + \beta_S(S - S_0) + \beta_p(p - p_0)\right]$$

where $\rho_0 = 1027$ kg m^{-3}, β_T=0.15 kg m^{-3} C°$^{-1}$, β_S=0.78 kg m^{-3} ‰$^{-1}$, β_P=4.5 kg m^{-3} dbar^{-1}, and p_0=0 dbar。

2) Use the Thermodynamic Equation of Seawater – 2010 (TEOS2010) (matlab or python packages are available at Github, named as "Gibbs Sea Water (GSW)") to compute the density again, and plot the profiles on the figure drawn in 1) to make a comparison between the density distributions obtained from linear and nonlinear EOS.

4.2 解决方案

题目未指明原始数据中“温度”和“盐度”两词的定义. 若“温度”是指现场（in-situ）温度，而“盐度”的单位是实用盐标（psu），则当使用 TEOS-10 (McDougall et al., 2011) 时，需要先转换为 *Absolute Salinity* (TEOS-10) 和 *Conservative Temperature* (TEOS-10)，这需要使用 A, B 两地的位置，然而题目并未提供. 鉴此，我暂且将原始数据的“温度”和“盐度”理解为 *Absolute Salinity* (TEOS-10) 和 *Conservative Temperature* (TEOS-10).

题中给出的线性 EOS 公式似乎有小笔误. 我擅自将其修改为

$$\rho = \rho_0 - \beta_T(T - T_0) + \beta_S(S - S_0) + \beta_p(p - p_0),$$

其中 ρ_0 选为由 TEOS-10 在 $p = 0$ 处计算得到的密度值，β_p 的数值修改为在原来的基础上除以 1000，$\beta_T, \beta_S, \beta_p$ 的单位亦作相应修改.

图 4.1 展现了使用线性与非线性 EOS 的计算结果对比. 可见线性 EOS 公式在 B 处的性能与 TEOS-10 的相近，而在 A 处比 TEOS-10 的密度计算结果偏高（偏高量对深度递增）.

提示线性 EOS 的参数选取可能依赖具体海区. 若参数选取得当，线性 EOS 也可能取得可接受的效果.

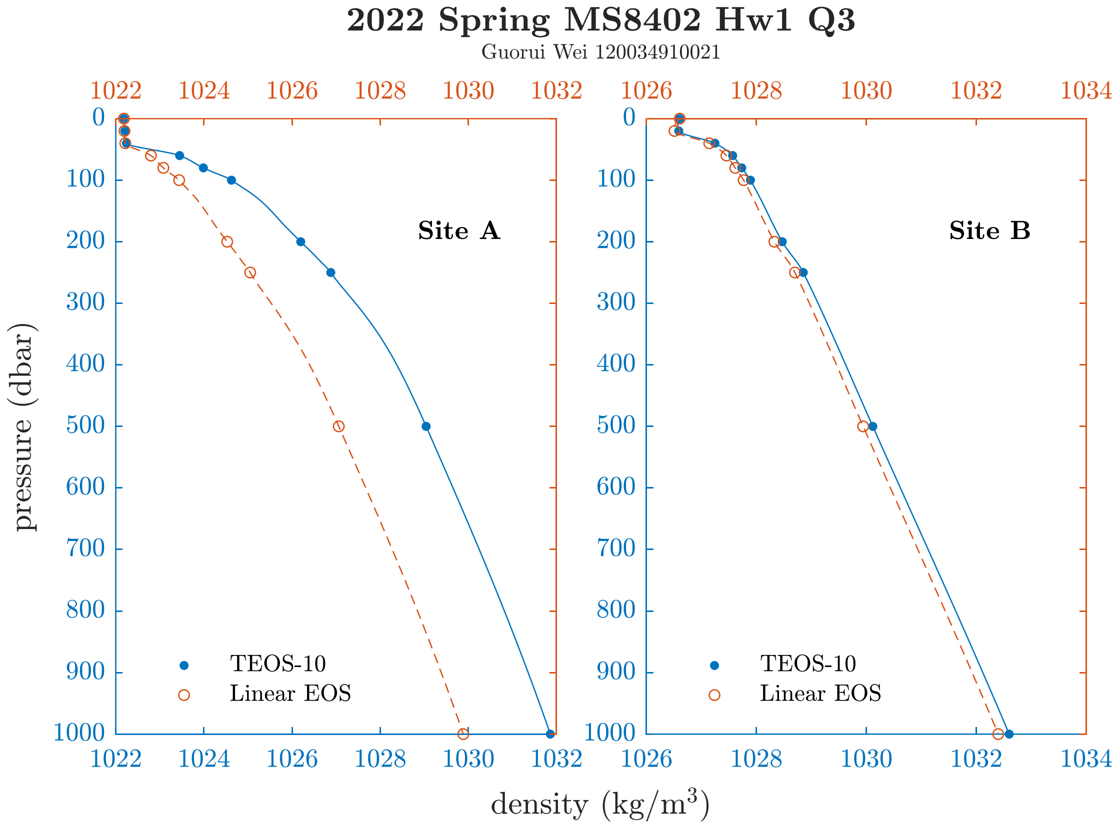


图 4.1 Site A 和 Site B 处的海水密度剖面. 蓝点、橙圈分别是 TEOS-10、Linear EOS 的计算结果，蓝实线、橙虚线分别是 TEOS-10、线性 EOS 数据的插值结果. 可见线性 EOS 公式在 B 处的性能与 TEOS-10 的相近，而在 A 处比 TEOS-10 的密度计算结果偏高（偏高量对深度递增）. 该图可用于说明使用线性与非线性 EOS 的密度计算结果的差异.

References

http://www.teos-10.org/index.htm. Retrieved 2022-02-28 from http://www.teos-10.org/index.htm

Kundu, P. K., Cohen, I. M., & Dowling, D. R. (2016). Chapter 3 - Kinematics. In P. K. Kundu, I. M. Cohen, & D. R. Dowling (Eds.), *Fluid Mechanics (Sixth Edition)* (pp. 77-108). Academic Press. https://doi.org/10.1016/B978-0-12-405935-1.00003-4

McDougall, Barker, T. J., & M, P. (2011). Getting started with TEOS-10 and the Gibbs Seawater (GSW) oceanographic toolbox. *Scor/Iapso WG*, *127*, 1-28.

Vallis, G. K. (2017). *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation* (2 ed.). Cambridge University Press. https://doi.org/10.1017/9781107588417

陈纪修, 於崇华, & 金路. (2019). *数学分析(下册)* (3 ed.). 高等教育出版社.

吴望一. (1982a). *流体力学(上册)*. 北京大学出版社.

吴望一. (1982b). *流体力学(下册)*. 北京大学出版社.

附录A 本作业使用的 MATLAB 程序源代码

A.1 主程序

```matlab
%% hw1.m
% Description: MATLAB code used in homework 1 (MS8402, 2022 Spring)
% Author: Guorui Wei (危国锐) (313017602@qq.com; weiguorui@sjtu.edu.cn)
% Student ID: 120034910021
% Created: 2022-02-27
% Last modified: 2022-03-06

%% Initialize project

clc; clear; close all
init_env();

%% Question 2

%
u = [-.25,-.25,.25,.30]; % [m/s]
v = [0,-.01,.05,.03]; % [m/s]
delta_x = 5e5; % [m]
delta_y = 5.4e5; % [m]
%
u_wrt_x = (u(2)-u(1)+u(3)-u(4))/(2*delta_x); % [1/s]
v_wrt_y = (v(1)-v(4)+v(2)-v(3))/(2*delta_y); % [1/s]
w = @(z) -(u_wrt_x+v_wrt_y)*z; % [m/s]
fprintf("w(-50) = %.2d cm/s\n",w(-50)*100);

%% Question 3

%
site_A.p = [0,20,40,60,80,100,200,250,500,1000].'; % [dbar]
site_A.SA =
[35.10,34.99,34.88,34.78,34.68,34.60,34.45,34.35,34.25,34.53].'; % [g/kg]
site_A.CT = [28.50,28.45,28.35,24.55,22.75,20.55,15.50,13.00,6.58,4.20].'; %
[deg C]
site_B.p = site_A.p; % [dbar]
site_B.SA =
[33.50,33.50,34.25,34.55,34.65,34.74,34.90,35.10,35.23,35.40].'; % [g/kg]
site_B.CT = [2.50,3.74,4.02,4.10,4.15,4.20,4.30,4.35,4.25,3.75].'; % [deg C]
p_i = (site_A.p(1):1:site_A.p(end)).'; % specific query points at which the
interpolated SA_i and CT_i are required [ dbar ]
```

```matlab

%% Figure.

t_TCL = tiledlayout(1,2,"TileSpacing","tight","Padding","tight");
%
site_A = hw1_3(site_A,p_i,t_TCL,1,"\bf Site A");
site_B = hw1_3(site_B,p_i,t_TCL,2,"\bf Site B");
%
xlabel(t_TCL,"density (kg/$\rm{m}^3$)","Interpreter",'latex');
ylabel(t_TCL,"pressure (dbar)","Interpreter",'latex');
[t_title_t,t_title_s] = title(t_TCL,"\bf 2022 Spring MS8402 Hw1 Q3","Guorui Wei 120034910021","Interpreter",'latex');
set(t_title_s,'FontSize',8)
%
exportgraphics(t_TCL,"..\\doc\\fig\\hw1_Q3.emf",'Resolution',800,'ContentType','auto','BackgroundColor','none','Colorspace','rgb')
exportgraphics(t_TCL,"..\\doc\\fig\\hw1_Q3.png",'Resolution',800,'ContentType','auto','BackgroundColor','none','Colorspace','rgb')

%% local functions

%% Initialize environment

function [] = init_env()
    % set up project directory
    if ~isfolder("../doc/fig/")
        mkdir ../doc/fig/
    end
    % configure searching path
    mfile_fullpath = mfilename('fullpath'); % the full path and name of the file in which the call occurs, not including the filename extension.
    mfile_fullpath_without_fname = mfile_fullpath(1:end-strlength(mfilename));
    addpath(genpath(mfile_fullpath_without_fname + "../data"), ...
            genpath(mfile_fullpath_without_fname + "../inc")); % adds the specified folders to the top of the search path for the current MATLAB® session.
end

%% Question 3

function [site_struct] = hw1_3(site_struct,p_i,t_TCL,num_Tile,textbox_string)
%% hw1_3
```

```
% Description.
arguments
    site_struct
    p_i
    t_TCL
    num_Tile = 1
    textbox_string = "\bf Site"
end

site_struct.p_i = p_i; % specific query points at which the interpolated SA_i and CT_i are required [ dbar ]
[site_struct.SA_i,site_struct.CT_i] = gsw_SA_CT_interp(site_struct.SA,site_struct.CT,site_struct.p,p_i); % SA and CT interpolation to p_i on a cast

%%% 1. Use TEOS-10 to compute the density.
% NOTE: Since the location of site A & B is not provided, I have to let SA, CT
% (TEOS-10) be SP, t, and therefore the results here should not be correct.

site_struct.rho_TEOS_10 = gsw_rho_CT_exact(site_struct.SA,site_struct.CT,site_struct.p); % Calculates in-situ density from Absolute Salinity and Conservative Temperature.
site_struct.rho_i_TEOS_10 = gsw_rho_CT_exact(site_struct.SA_i,site_struct.CT_i,site_struct.p_i);

%%% 2. Calculate density with the linear Equations of State (EOS).

rho_0 = site_struct.rho_TEOS_10(1); % [kg/m^3]
beta_T = 0.15; % [kg/m^3/(deg C)]
beta_S = 0.78; % [kg/m^3/(g/kg)]
beta_p = 4.5e-3;  % [kg/m^3/dbar]

func_rho_linear = @(T,S,p) rho_0 - beta_T*(T-T(1)) + beta_S*(S-S(1)) + beta_p*(p-p(1));
site_struct.rho_linear = func_rho_linear(site_struct.CT, site_struct.SA, site_struct.p);
site_struct.rho_i_linear = func_rho_linear(site_struct.CT_i,site_struct.SA_i,site_struct.p_i);

%%% plot

%
t_Axes_TEOS = nexttile(t_TCL,num_Tile);
```

```
t_plot_1 = plot(t_Axes_TEOS,site_struct.rho_i_TEOS_10,site_struct.p_i,'-',"color",'#0072BD',"DisplayName",'TEOS-10');
hold on
t_plot_2 = plot(site_struct.rho_TEOS_10,site_struct.p,'.',"color",'#0072BD',"MarkerSize",10,"DisplayName",'');
set(t_Axes_TEOS,"YDir",'reverse',"TickLabelInterpreter",'latex',"FontSize",10,'Box','off',"XColor",'#0072BD',"YColor",'#0072BD');
%
t_Axes_linear = axes(t_TCL);
t_Axes_linear.Layout.Tile = num_Tile;
t_plot_3 = plot(t_Axes_linear,site_struct.rho_i_linear,site_struct.p_i,'--','Color','#D95319',"DisplayName",'LEOS');
hold on
t_plot_4 = plot(site_struct.rho_linear,site_struct.p,'o',"MarkerSize",4,'Color','#D95319',"DisplayName",'');
set(t_Axes_linear,'YDir','reverse','FontSize',10,'TickLabelInterpreter','latex','XAxisLocation','top','YAxisLocation','right','YTickLabel',{},'Box','off','Color','none','XColor','#D95319','YColor','#D95319','YLimitMethod','tight')
%
linkaxes([t_Axes_TEOS,t_Axes_linear],'xy');
legend([t_plot_2,t_plot_4],["TEOS-10","Linear EOS"],"Location",'southwest','Interpreter','latex',"Box","off");
annotation('textbox',[.36+0.48*(num_Tile-1) .68 .10 .06],'String',textbox_string,'LineStyle','none','FontWeight','bold','Interpreter','latex');

end

```

A.2 子程序

发布于 https://github.com/grwei/SJTU_2021-2022-2-MS8402.