一．问题与精确解

求解如下的对流方程

$u_t + u_x = 0, u(x,0) = \begin{cases} 1; x \geq 0 \\ 0; x < 0 \end{cases}$

二．数值格式

格式 1: $u_j^{n+1} = u_j^n - \lambda(u_{j+1}^n - u_j^n)$

格式 2: $u_j^{n+1} = u_j^n - \lambda(u_j^n - u_{j-1}^n)$

分别选取网格比$\lambda = 0.9$ 和 2。求 $T = 4$，$x \in [-5, 5]$的解。

三．数值解

方法 1：

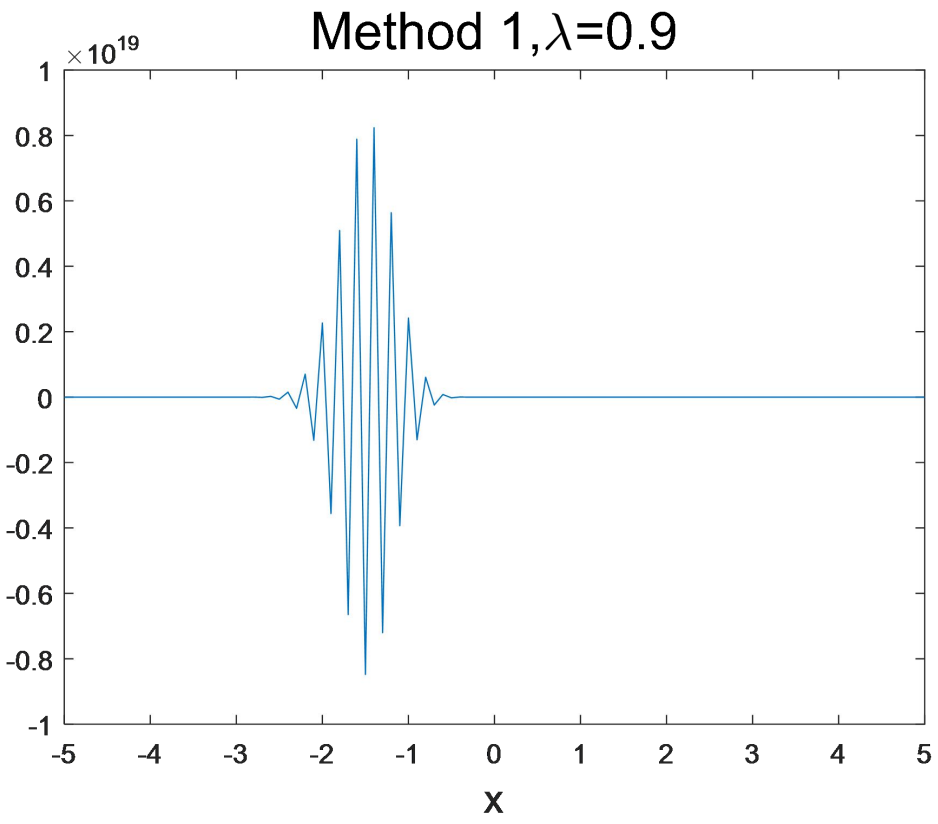

图 1 $\lambda = 0.9$,

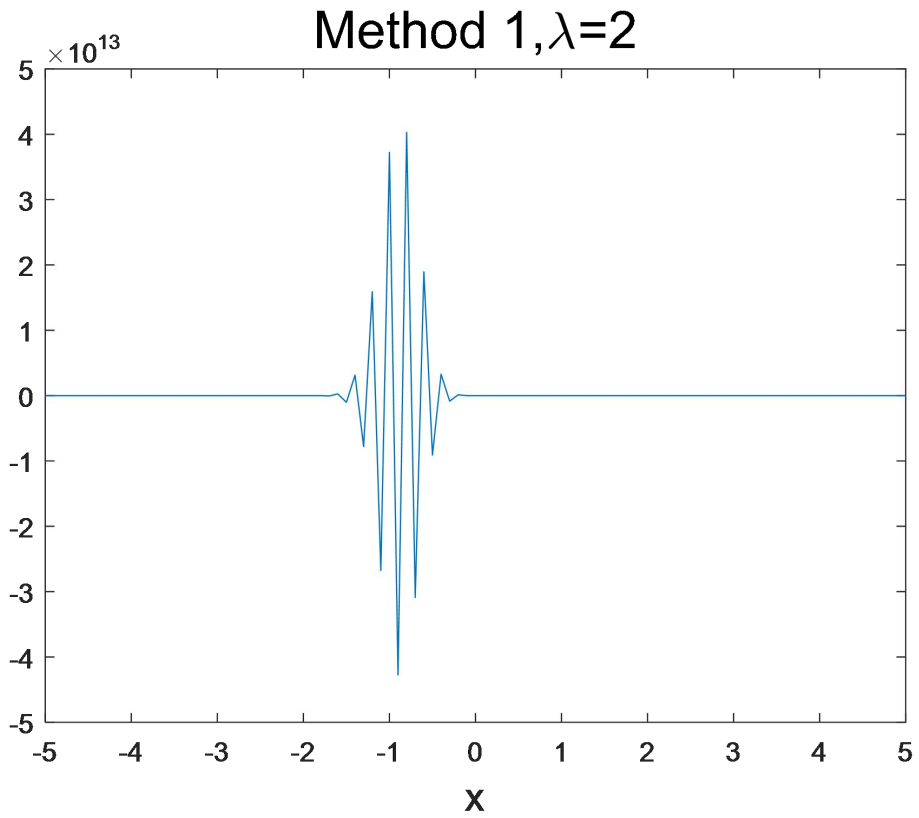
Method 1,λ=2
×10^13
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1 0 1 2 3 4 5
x

图 2 λ = 2

方法 2：

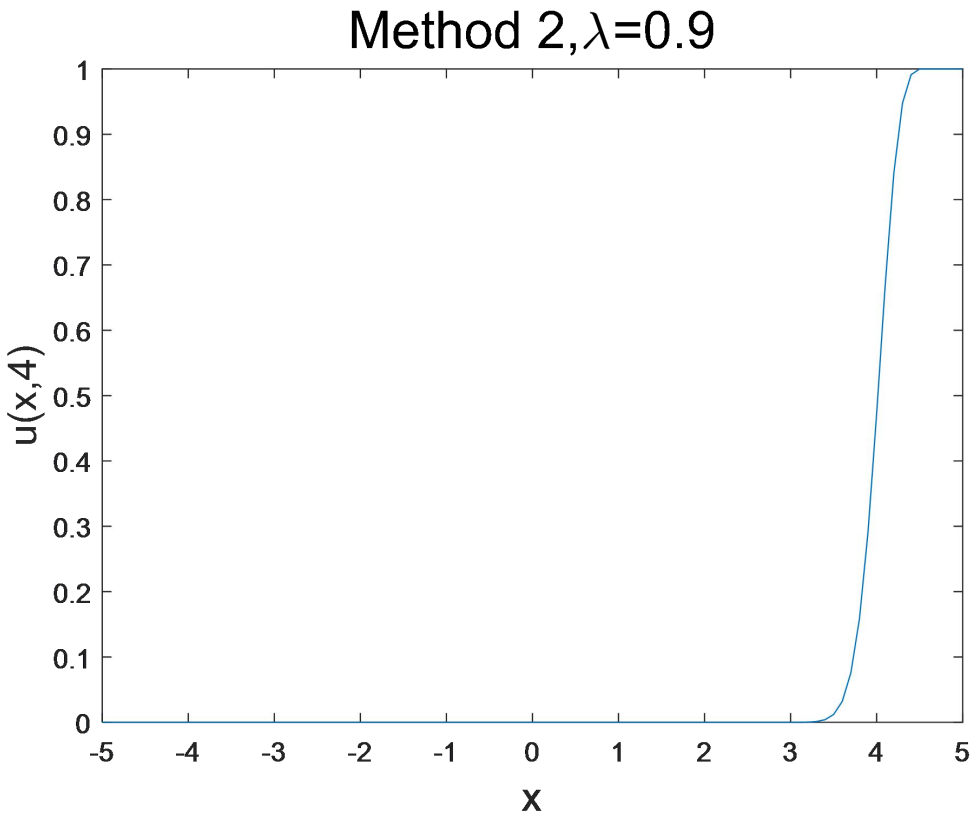
Method 2,λ=0.9
u(x,4)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-5 -4 -3 -2 -1 0 1 2 3 4 5
x

图 3 $\lambda = 0.9$

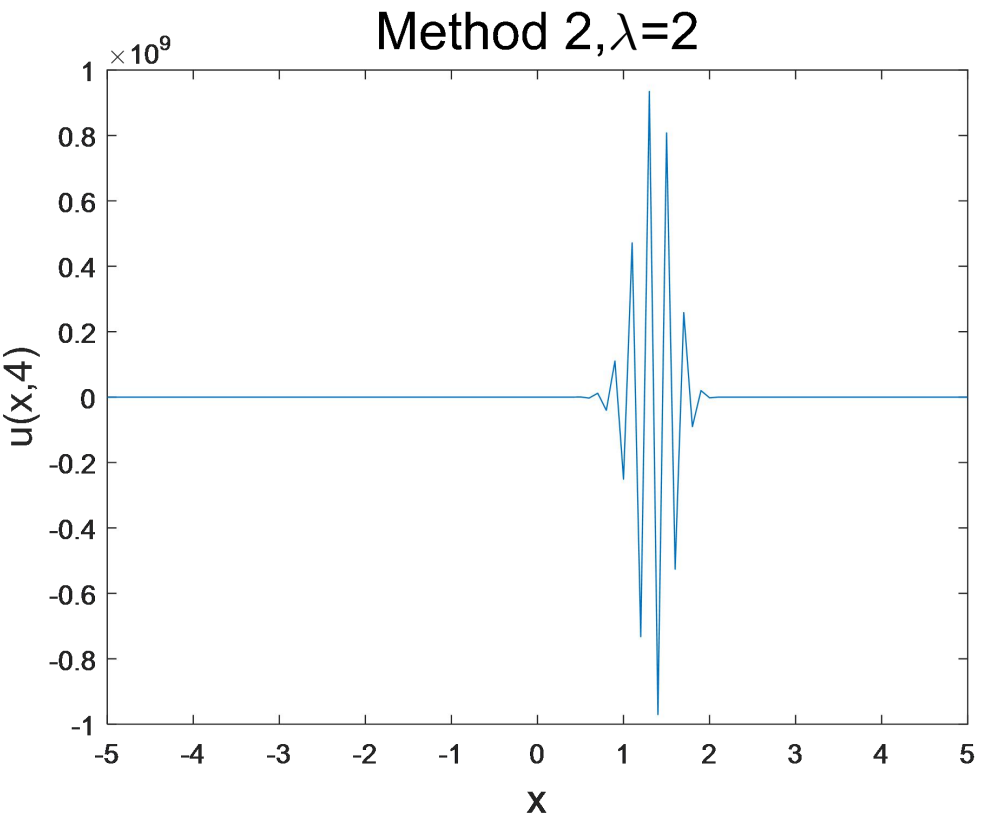

图 4 $\lambda = 2$

（3）求误差

$$T_1(x_j,t_n) = \frac{u(x_j,t_{n+1}) - u(x_j,t_n)}{\tau} - a\frac{u(x_{j+1},t_n) - u(x_j,t_n)}{h}$$

$$T_2(x_j,t_n) = \frac{u(x_j,t_{n+1}) - u(x_j,t_n)}{\tau} - a\frac{u(x_j,t_n) - u(x_{j-1},t_n)}{h}$$

运用泰勒展开

$$u(x_j,t_n+\tau) = u(x_j,t_n) + \tau u_t(x_j,t_n+\tau) + O(\tau^2)$$

$$u(x_j+h,t_n) = u(x_j,t_n) + hu_x(x_j,t_n+\tau) + O(h^2)$$

$$u(x_j-h,t_n) = u(x_j,t_n) - hu_x(x_j,t_n+\tau) + O(h^2)$$

可得

$$T_1(x_j,t_n) = u_t(x_j,t_n+\tau) + O(\tau) - au_x(x_j,t_n+\tau) + O(h) = O(\tau) + O(h)$$

$$T_2(x_j,t_n) = u_t(x_j,t_n+\tau) + O(\tau) - au_x(x_j,t_n+\tau) + O(h) = O(\tau) + O(h)$$

四讨论

关于迎风格式的稳定性，a= 1 > 0,所以向右偏心格式是无条件不稳定的，而当|λ| < 1 时，向左偏心格式稳定。

代码

```
clear,clc,close all
lamda=0.9;
tau=0.1;
x=[-5:tau:5];
t=[0:tau*lamda:4];
%³õÖµÌõ¼þ
for j=1:length(x)
    if x(j)>=0
        u(1,j)=1;
    else
        u(1,j)=0;
    end
end
u1=u;u2=u;
%
for i=1:length(t)
    %¸ñÊ½1
    for j=1:length(x)-1
        u1(i+1,j)=u1(i,j)-lamda*(u1(i,j+1)-u1(i,j));
        u1(i+1,end)=1;
    end
    %¸ñÊ½2
    for j=2:length(x)
        u2(i+1,j)=u2(i,j)-lamda*(u2(i,j)-u2(i,j-1));
        u2(i+1,1)=0;
    end
end
figure,plot(x,u1(end,:)),xlabel('u(x,4)','FontSize',1
5),xlabel('x','FontSize',15)
title('Method 1,\lambda=0.9','FontSize',20)
figure,plot(x,u2(end,:)),ylabel('u(x,4)','FontSize',1
5),xlabel('x','FontSize',15)
title('Method 2,\lambda=0.9','FontSize',20)
%½á¹ûÕ¹Ê¾
figure,subplot(1,2,1)
surf(u1);
title('Method 1','FontSize',20)
xlabel('x incretment','FontSize',15)
```

```
ylabel('t incretment','FontSize',15)
zlabel('u','FontSize',20)
subplot(1,2,2)
surf(u2)
title('Method 2','FontSize',20)
xlabel('x incretment','FontSize',15)
ylabel('t incretment','FontSize',15)
zlabel('u','FontSize',20)
```

满足离散最大模原理,具有最大模稳定性.