# Numerical methods for PDEs: HW5

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## 1 Theory

(6 pts)

1. (路金甫，第三版) 第四章习题 2，6.

2. Consider the Allen-Cahn equation in 1D

$$u_t = u_{xx} + u(1-u^2).$$

If we do not use the time-splitting method, one may use the implicit-explicit (IMEX) method for such equations. Explain why one wants to use implicit method for $u_{xx}$ and explicit method for $u(1-u^2)$.

## 2 Numerics

(14 pts)

1. (路金甫，第三版) 第四章习题 10 （只用前三个格式，不需要求解Du Fort-Frankel）

2. Consider the viscous Burger's equation

$$u_t + uu_x = \nu u_{xx}$$

Use a time splitting approach to solve this Burger's equation for initial value

$$u_0(x) = \begin{cases} 1 & x \in [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

Take $\nu = 1$ and $\nu = 10^{-3}$ to do the simulation. Plot the solution at different times.

For the $u_t + uu_x = 0$ part, you may use some classical finite volume scheme, for example, the Engquist-Osher scheme flux. In particular,

$$\frac{u_j^{n+1} - u_j^n}{\tau} + \frac{f_{j+1/2} - f_{j-1/2}}{h} = 0,$$

where $f_{j+1/2} = F(u_j, u_{j+1})$, with

$$F(U_l, U_r) = f(U_r) - \int_{U_l}^{U_r} (f'(u))^+ du = f(U_r) - \int_{U_l}^{U_r} u^+ du,$$

where $a^+ = \max(a, 0)$. In particular,

$$F(U_l, U_r) = \frac{1}{2}(U_l(U_l + |U_l|) + U_r(U_r - |U_r|)).$$

The Engquist-Osher flux is a simplification to the Godunov flux.

3. Consider the reaction-diffusion equation

$$u_t = u_{xx} + u_{yy} + \frac{1}{\epsilon}u(1 - u^2), (x, y) \in \Omega = [0, 1] \times [0, 1]$$
$$u|_{\partial\Omega} = 1$$

Here, $\epsilon = 0.01$. Choose the initial data that is $-1$ inside $9(x - 0.5)^2 + 25(y - 0.5)^2 = 1$ and 1 otherwise. Solve this equation using a splitting method. Describe the dynamics of the zero level set: $u = 0$, which is a curve.

*Comment: The zero-level set of $u$ follows the so-called flow by mean curvature as $\epsilon \to 0$.*