Quiz 3

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解：设 $v = v(x_j, t_n) =: v_j^n$ 精确满足 L-F 格式：

$$v_j^{n+1} = \frac{1}{2}\left(v_{j-1}^n + v_{j+1}^n\right) - \frac{a\tau}{2h}\left(v_{j+1}^n - v_{j-1}^n\right).$$

在 (x_j, t_n) 处 Taylor 展开 $\Rightarrow$

$$v + \tau v_t + \frac{\tau^2}{2} v_{tt} + O(\tau^3) = v + \frac{h^2}{2} v_{xx} + O(h^4) - a\tau (v_x) + O(\tau h^2).$$

$$\Rightarrow \quad v_t + a v_x = -\frac{\tau}{2} v_{tt} + \frac{h^2}{2\tau} v_{xx} + O\left(\tau^2 + h^2 + h^4/\tau\right).$$

当 $\lambda := \tau/h$ 固定时，$v_t + a v_x = -\frac{\tau}{2} v_{tt} + \frac{h^2}{2\tau} v_{xx} + O(\tau^2 + h^2)$.

$$\Rightarrow \quad v_{tt} = -a v_{tx} + O(\tau + h) = -a(-a v_{xx}) + O(\tau + h)$$

$$\Rightarrow \quad v_t + a v_x = -\frac{\tau}{2} a^2 v_{xx} + \frac{h^2}{2\tau} v_{xx} + O(\tau^2 + h^2 + \tau h)$$

$$= \underbrace{\frac{\tau}{2}\left(\frac{h^2}{\tau^2} - a^2\right) v_{xx}}_{\text{"Leading Error Term"}} + O(\tau^2 + h^2) \quad [\text{二阶修正方程}]$$

$\therefore$ 当 $\frac{h^2}{\tau^2} \geq a^2 \Rightarrow a\lambda \leq 1$, $(a>0)$，有数值 $\boxed{\text{耗散}}$

当 $a\lambda > 1$，误差项 $\hat{u}_t = -\varepsilon v_{xx}$, $\varepsilon > 0$ 不适定

$\Rightarrow$ L-F 不稳定.

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对 $\tilde{V}_t = \varepsilon \tilde{V}_{xx}$. 设 $\tilde{V} = e^{i(kx-\omega t)}$

$\Rightarrow \quad -i\omega = \varepsilon (ik)^2 \quad \Rightarrow \quad +i\omega = +\varepsilon k^2$

$\Rightarrow \quad v_p = \cancel{\frac{\omega}{k} =} -\frac{k}{i\varepsilon}$.

总之，误差项是耗散的. 期望在真实解间断处，

数值解峰值被"削平"、两边略有"凸起"，形似"扩散"作用.

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