

## Double-diffusive convection

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In this paper we present a rather personal view of the important developments in double-diffusive convection, a subject whose evolution has been the result of a close interaction between theoreticians, laboratory experimenters and sea-going oceanographers. More recently, applications in astrophysics, engineering and geology have become apparent. In the final section we attempt to draw some general conclusions and suggest that further progress will again depend on a close collaboration between fluid dynamicists and other scientists.

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### 1. Introduction

The subject to be reviewed in this article had its beginnings in the same year as the first issue of the *Journal of Fluid Mechanics* in 1956. With hindsight, one can find earlier suggestions in the literature of phenomena which would now be described as double-diffusive processes (Jevons 1857, Mendenhall & Mason 1923), but a correct description of the phenomenon in physical terms was not presented then, nor was any generality of the ideas perceived by these previous workers.

Even the seminal paper by Stommel, Arons, & Blanchard (1956) did not seem a very promising start to what was to become a new field of fluid mechanics. Its title was ‘An oceanographical curiosity: the perpetual salt fountain’ and there was little indication then that in 25 years the phenomenon which it described, and related processes, would play an important role in oceanography and more recently in astrophysics, geology and engineering. The development of the subject has perhaps not been as rapid as that of other areas of fluid mechanics. A contributing factor has been the relatively small size of the oceanographic community and the lack of immediate practical incentives which speeded the development of other branches of fluid dynamics, such as the aircraft industry, ship hydrodynamics or meteorology.

The field has been reviewed several times before (see, for example, Turner 1974, 1979, 1981), but the present survey is less didactic and more personal. The development of the subject has been largely due to a rather close interaction between a small number of people: theoreticians, laboratory experimenters and sea-going oceanographers. On the observational side, the field has been strongly influenced by the appearance in the mid 1960s of new instruments capable of measuring fluctuations of temperature and salinity as a continuous function of depth, hence permitting investigations on much smaller scales than had previously been possible. Among the first surprising results were indications that vertical temperature and salinity

profiles contained thin regions of rapid, almost discontinuous change. These and other observations of finestructure and microstructure in the deep ocean have led to theoretical and experimental work aimed at explaining them, and the laboratory experiments have in turn stimulated new observations. Recently, the ideas which were first conceived with the oceans in mind have begun to have an influence in other fields and some of the latest developments in geology, of particular interest to the authors, will be described in a later section.

Convective motions produced in a fluid when only one property affects the density – for example, temperature differences in a fluid layer heated from below – are now very well understood. When gradients of more than one diffusing property are important a whole new range of phenomena can arise, and intuition based on simple thermal convection can be misleading. For example, in many systems of interest, instabilities can develop even when the net density decreases upwards, and thus the system would be judged hydrostatically stable in a single-component fluid. Diffusion, which is a stabilising influence in a fluid containing a single solute, can in the double or multiple component case act to release the potential energy in the component that is heaviest at the top. The two requirements for the occurrence of double-diffusive convection are that the fluid contain two or more components with different molecular diffusivities and that these components make opposing contributions to the vertical density gradient.

Consider now explicitly the phenomenon which Stommel *et al.* (1956) described. They imagined a heat-conducting pipe suspended vertically in an ocean with its upper end in warm salty water and its lower end in denser, cooler and fresher water (a temperature and salinity distribution which is common in the sub-tropical ocean). If the pipe is primed by pumping water up from the lower end, the relatively fresh water in the pipe will come to the same temperature as that outside at each level. The water in the pipe will remain fresher and therefore less dense than the water at the same depth outside because salt cannot diffuse through the walls. The difference in head causes the water to flow up the pipe, and the flow will continue after the pumping is stopped. A similar argument shows that motion would also continue if the initial pumping was downwards. These flows are of course not ‘perpetual motion’ in the usual sense, since they draw on the potential energy supply in the salinity distribution and will continue only so long as a salinity gradient remains. The serendipitous discovery of this phenomenon is humorously described by Arons (1980) in a résumé of Stommel’s work. A feasibility study was carried out by Groves (1959), who calculated that under typical conditions the flow rate would be of the order of  $5000 \text{ cm}^3 \text{ s}^{-1}$ , an impressive rate in itself, but not really large enough to bring up sufficient nutrients to run an ocean farm, as suggested by Stommel *et al.* To our knowledge there has been only one field attempt to investigate the feasibility of the idea in the ocean. In 1971 L. N. Howard and H. Stommel lowered some 1000 m of flexible plastic tube into the Caribbean off Martinique. They observed a variable flow of water up the pipe, but the experiment cannot be regarded as a success since they were unconvinced that the flow was not pumped by the wave-induced motion of the float securing the top of the pipe.

The next step in the development was taken by Stern (1960) who pointed out that the walls of the pipe are not essential to the above argument, since salt diffuses so much more slowly than heat. Similar motions, alternating upwards and downwards,

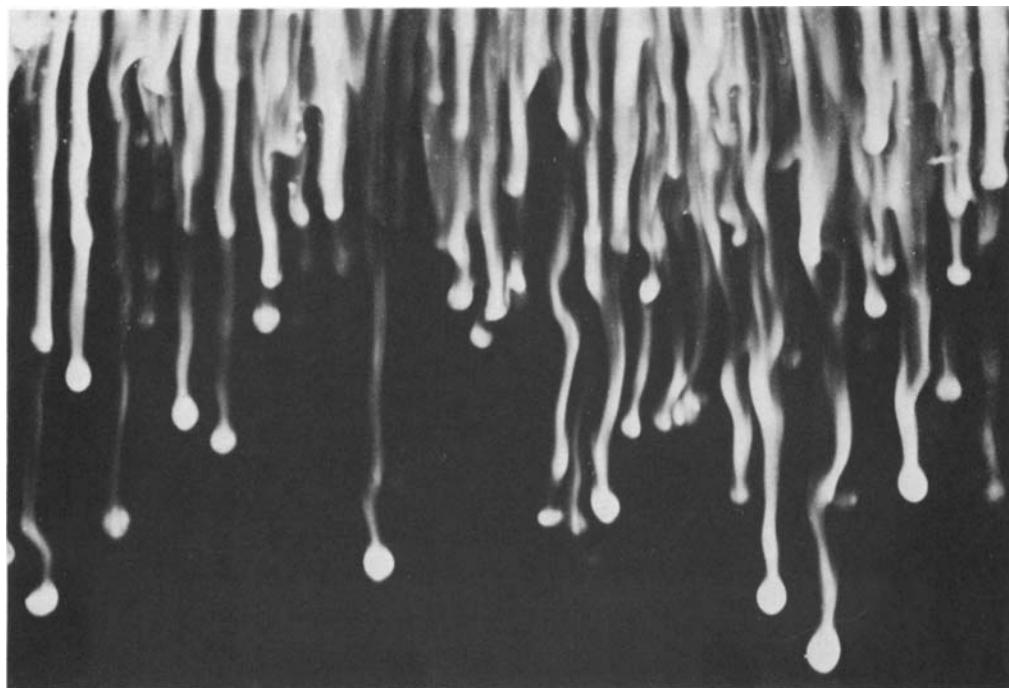


FIGURE 1. A field of salt fingers formed by setting up a stable temperature gradient and pouring a little salt solution on top. The downward-moving fingers were made visible by adding fluorescein to the salt and lighting through a slit from below.

can be set up in the interior of a fluid containing gradients of temperature and salinity (hotter and saltier at the top) because of this lower transfer of salt horizontally relative to heat. Long narrow convection cells, now called salt fingers, readily form, and their scale was predicted by Stern using a linear stability calculation. A report of a laboratory demonstration of the phenomenon by A. J. Faller and H. Stommel was included in Stern's paper. Figure 1 presents a photograph of a field of salt fingers made visible by adding fluorescein dye.

The opposite situation, with warmer, saltier water underneath colder, fresher and less dense water, was first considered by Stern (1960) in a much overlooked footnote. He stated there the correct result that the linear mode of instability is oscillatory, which allows the potential energy stored in the thermal field to be released. The subsequent behaviour was first studied a little later by Turner & Stommel (1964) following a thought experiment by Stommel (1962). After the initial oscillatory instability, the thermal boundary layer breaks down to form a shallow convecting layer, which grows by incorporating fluid from the gradient region above it. When the thermal boundary layer ahead of the convecting region reaches a critical Rayleigh number, it too becomes unstable, and a second layer forms above the first. Convection is sustained by a more rapid vertical diffusive transport of heat relative to salt, and eventually many such layers form. An interface which separates hot salty water beneath colder fresher water has subsequently been called a 'diffusive' interface because heat and salt are transported through the interface solely by molecular diffusion. Figure 2 shows a typical series of diffusive interfaces photographed during

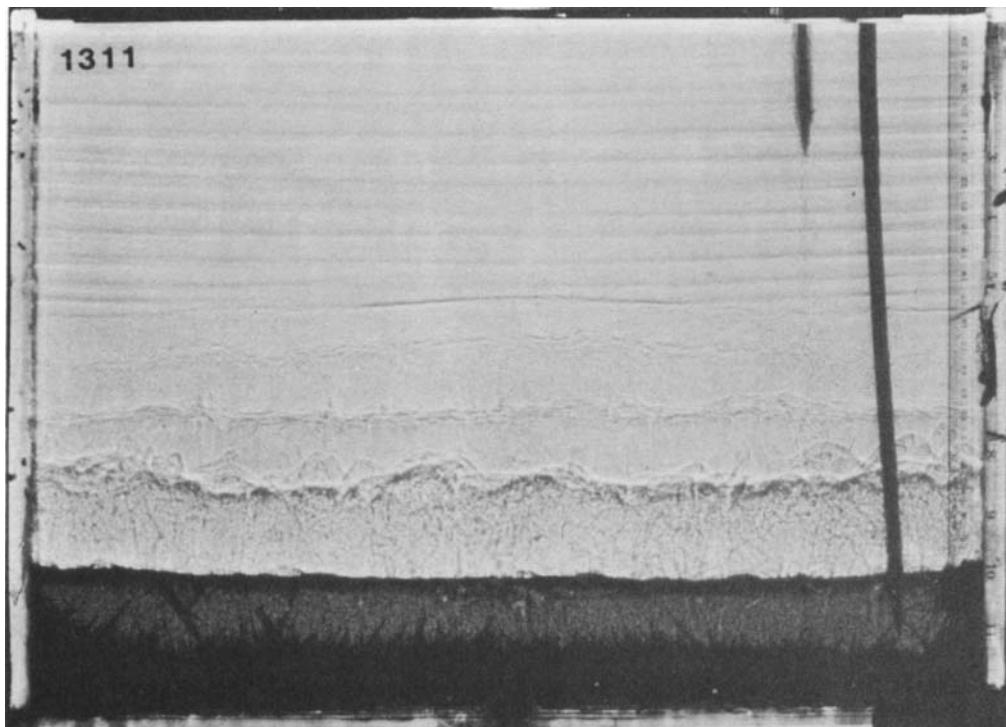


FIGURE 2. A series of convecting layers and 'diffusive' interfaces, formed by heating a gradient of  $K_2CO_3$  solution from below. In this experiment the heating was provided by a hot layer of dyed  $KNO_3$  solution inserted below the gradient region, to simulate the influx of new magma into the base of a magma chamber. Note the growth of crystals in the lowest layer due to convective cooling from above, as described in § 6. (The thinner layers at the top were formed during the filling of the tank with a gradient of cold solution and are due to side-wall heating.)

a laboratory experiment modelling a geological phenomenon which will be described in § 6.

In the next section we will trace the development of the laboratory work which followed these early experiments. In § 3 the corresponding theoretical work will be described, though it will be seen that there has not been a very direct connection between experiment and theory, except for some mechanistic models which have been developed specifically to interpret experiments. In § 4 some examples of the oceanographic measurements will be given, and the interactions with the laboratory and theoretical work explored further. In § 5 other suggested and potential applications will be referred to more briefly, while § 6 makes particular reference to geological applications which are currently engaging our attention. In the final section we try to make some general deductions from this brief history of the subject, point out the essential nature of collaboration between scientists of different disciplines and speculate on how the subject might best be developed in the future.

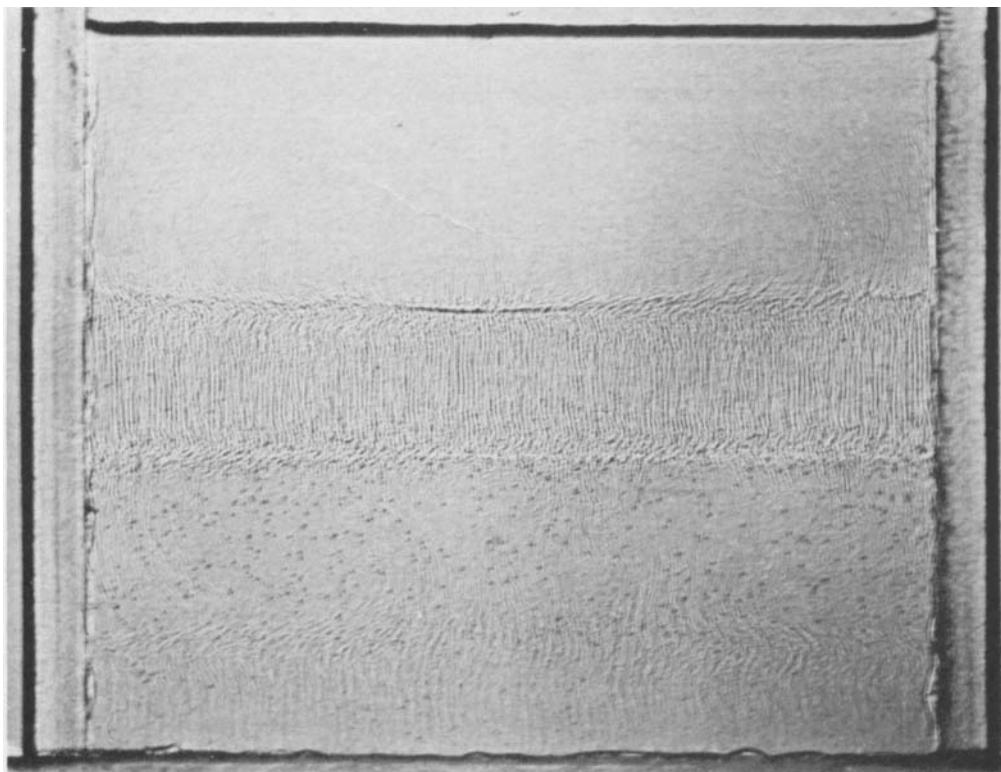
## 2. Laboratory experiments

The first quantitative laboratory experiments were carried out before there were any field observations with which to compare them, though Stommel's arguments were so persuasive that their oceanographic relevance even then seemed in little doubt. The first case considered quantitatively was a single diffusive interface, which was chosen as the simplest problem which could be built up to more complicated situations. Turner (1965) measured the relative rates of transport of heat and salt through such an interface and showed that both the heat transfer and  $\mathcal{R}$ , the ratio of the rate of turbulent transfer of salt to that of heat, depend systematically on the density ratio  $R_\rho = \beta\Delta S/\alpha\Delta T$ . The most intriguing result was that, to within experimental accuracy, for  $R_\rho > 2$ ,  $\mathcal{R}$  was a constant, 0.15. No really satisfactory explanation of this result has yet been given though there are mechanistic arguments which make it plausible. Shortly after the experimental results were reported, observations of interfaces which could be identified with this process were observed in Antarctic lakes (Hoare 1966) and at the bottom of the Red Sea (Munns, Stanley & Densmore 1967). Preliminary observations of this latter case by Swallow (1965), who reported sharp interfaces recorded on echo sounders, were particularly important in gaining acceptance of this idea among oceanographers.

The formation of layers from a linear salinity gradient due to heating from below was studied by Turner (1968). He concentrated on the formation of the first layer and made satisfactory predictions of its scale in relation to the salinity gradient and heating rate. It was not until Huppert & Linden (1979) took up the problem again, however, that the formation of multiple layers and the transports between them were properly understood. From their combined numerical and experimental study, they were able to determine expressions for the rate of advance of the layers, their mean thickness and the effective thermal conductivity for the series of layers.

In the steady state, there must be a balance at a diffusive interface between the diffusive fluxes of heat and salt, which tend to spread the interface out, and the convective stirring driven by the net unstable buoyancy flux into the convecting layers on each side, which tends to sharpen the interface. In the salt-finger situation, the first measurements of the relative fluxes were conducted by Turner (1967) in the expectation that there should also be a balance between the interfacial and convective processes in this case, with only the mechanism of transport, which is now due to the salt fingers, being different.

It was nevertheless surprising to see the first oceanic profiles showing steps in the 'finger' situation, with warmer, saltier water above colder, fresher water underneath the Mediterranean outflow (Tait & Howe 1968). The early laboratory experiments had tended to draw the distinction between the diffusive case, in which horizontal layering was produced, and the finger situation, which led to the formation of long narrow convection cells extending through the depth of the fluid. The observations of Tait & Howe implied that the latter system should be unstable and that a deep layer of fingers should break down to form a series of convecting layers with fingers confined to interfaces. This was demonstrated explicitly by Stern & Turner (1969) using the analogue system of sugar and salinity gradients which has been used for much subsequent work. The more slowly diffusing sugar plays the role of salt in the



**FIGURE 3.** Shadowgraph picture of sugar-salt fingers at an interface about  $2\frac{1}{2}$  cm deep, separating convecting layers of sugar solution (from Turner 1979).

salt-heat system, and even though the ratio of diffusivities is about  $1/3$  instead of  $10^{-2}$ , vigorous convecting layers separated by finger interfaces can be produced. A shadowgraph picture of an interface between sugar and salt layers is shown in figure 3.

The acceptance of the idea of salt fingers in the ocean was not so whole-hearted. Objections were raised that the motions were so easily disrupted by imposed turbulence that they would not survive in the ocean. On the other hand, it was also argued that the process, if it worked, was so efficient that all the salt would drop out of the oceans! The experiments of Linden (1971, 1973) were conducted partly with these arguments in mind. He showed that a steady shear across a finger interface has little effect on the vertical fluxes, though it changes the fingers into two-dimensional sheets aligned down shear. Unsteady shears, such as those produced by stirring on both sides of the interface, can, however, rapidly disrupt the interface and actually decrease the vertical salt flux. In fact, as discussed in § 4, salt fingers interact with the internal wave field and may be intermittent.

These early experiments, which obtained measurements of the coupled vertical fluxes across existing interfaces, concentrated thinking on one-dimensional phenomena, including the processes of layer formation. There also tended to be an identification between the layer scales observed and those found in the laboratory due to heating a gradient from below, which was the only prediction of a layer scale available

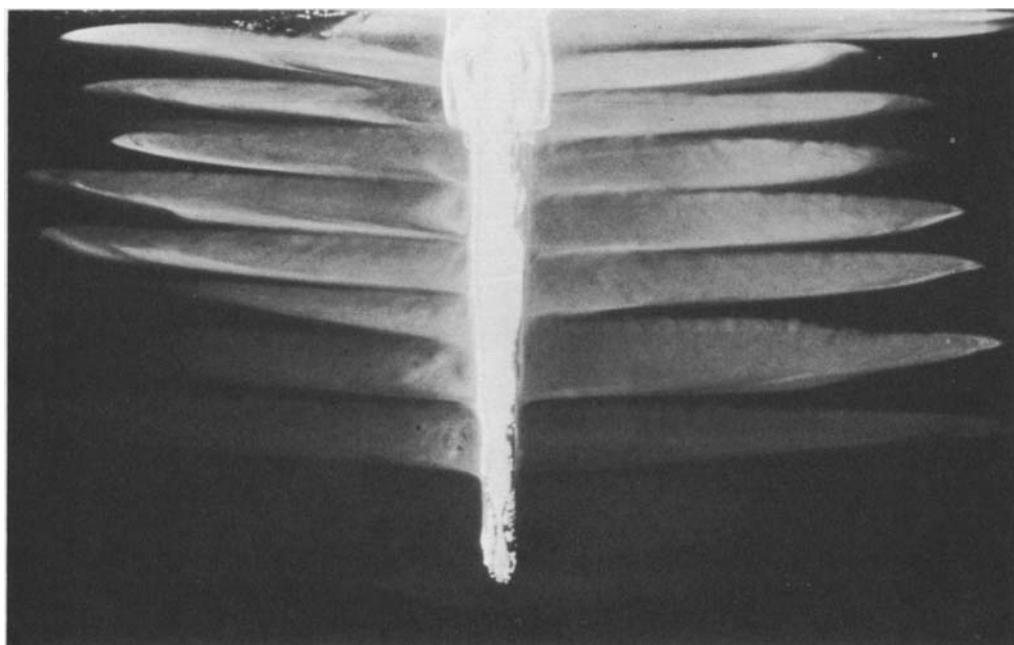


FIGURE 4. A series of layers formed by the melting of an ice block into a salinity gradient. Fluorescein dye, originally frozen into the ice, has been illuminated from the side to show the final distribution of the fresh water (from Huppert & Turner 1980).

at that time. Only more recently has it become clear that the formation of layers is more likely to be dominated by horizontal variations of properties and consequent intrusive motions, though once layers have formed the vertical flux relations probably still apply.

It was inadvertently found in early laboratory experiments that double-diffusive layers readily form when a salt gradient is heated from the side. For example, the heat generated by a light shone, for photographic purposes, onto a salt-stratified system is sufficient to produce layers. This effect was studied systematically by Thorpe, Hutt & Soulsby (1969) and Chen, Briggs & Wirtz (1971), but it was deliberately excluded from most experiments conducted with the ocean in mind, and the main advantage of the sugar-salt technique seemed to be that it eliminated these unwanted side-wall heating and cooling effects. However, Turner & Chen (1974) conducted a series of exploratory experiments to investigate two-dimensional effects, and this is one of the main thrusts of current experimental work. For example, Huppert & Turner (1978, 1980) and Huppert & Josberger (1980) have carried out an extensive set of experiments to understand the process of icebergs melting into a salinity gradient. They have shown, as depicted in figure 4, that the melt water spreads out into a series of slightly inclined convecting layers all along the depth of a vertical wall of melting ice, rather than convecting right to the surface and collecting in a pool on the top as it would were the ambient fluid unstratified. Thus, the collection of fresh melt water from icebergs is likely to be much more difficult technically than was originally suggested. The predictions have already received some confirmation from recent observations off the Erebus Glacier tongue in the Antarctic (Jacobs *et al.* 1981). They observed a step structure in the temperature and salinity

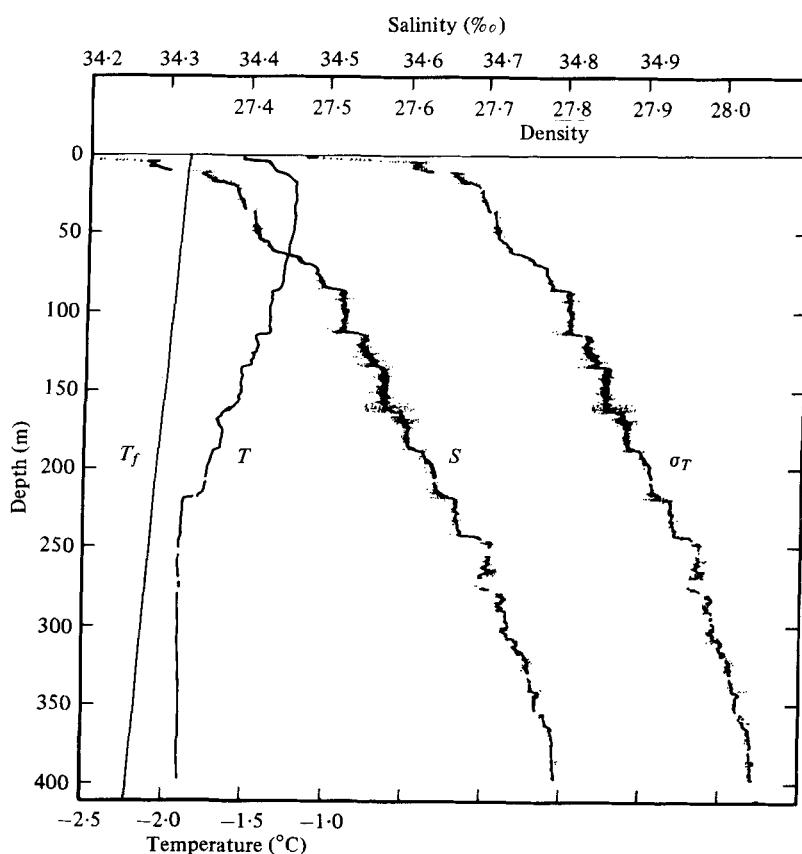


FIGURE 5. Profiles of freezing point temperature, temperature, salinity and density measured off the Erebus Glacier ice tongue. Note the steps produced by side-wall cooling and melting, even though the overall distributions of properties are both stable.

field just off the tip of the Erebus tongue with a mean step size of approximately 20 m, as shown in figure 5, in agreement with the theoretical relationships previously determined. It is also worth pointing out here that the mechanism of introducing cold melt water laterally allowed double-diffusive steps to occur even in the presence of profiles of temperature which decreased and of salinity which increased with depth. These distributions by themselves are not subject to double-diffusive instabilities as they are in the situation to be shown in figure 8.

Turner (1978) has investigated from a different point of view the intrusion of fluid with one set of temperature and salinity properties into another having different properties. He used sources of sugar and salt solutions released into gradients of these properties and showed that when different solutes are used the behaviour is very different from that of a single intrusion of salt water into a gradient of salinity at its neutral buoyancy level – there is strong vertical convection near the source followed by several noses spreading more rapidly at levels above and below the source. The total volume of fluid affected by mixing is many times that of the input, showing that the intrusions are overtaking and incorporating the environment rather than just displacing it. A further step in this series of experiments has been taken by Ruddick

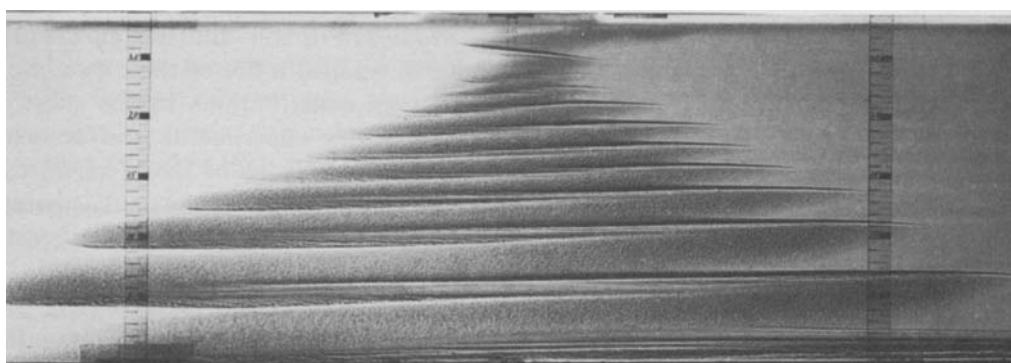


FIGURE 6. A system of interleaving layers produced by removing a barrier separating sugar solution (left) and salt solution (right) which have identical linear vertical density distributions (from Ruddick & Turner 1979).

& Turner (1979), who investigated the motions produced at a front, separating fluids (i.e. salt and sugar solutions) having the same vertical density gradient but different molecular diffusivities. In this case a series of regular interleaving layers develops, as shown in figure 6, with the vertical spacing and speed of advance proportional to the horizontal property differences. Apart from small wave-like adjustments, none of these motions would occur without double-diffusive processes.

Many of the developments described in this section have close parallels in the oceanic measurements, only some of which have been described. The current state of the observational work, and likely future developments will be taken up later. Implications of another group of laboratory experiments in which the fluxes of several components across diffusive interfaces have been measured (Turner, Shirtcliffe & Brewer 1970; Griffiths 1979) will also be discussed then. A different laboratory situation in which double-diffusive effects can occur in a surprising way is provided by the mechanism of the Soret effect, whereby a composition gradient arises from the diffusion induced by a temperature gradient. Hurle & Jakeman (1969, 1971) were the first to show that the small compositional gradient needed to set up an oscillatory instability even in a nominally pure fluid heated from below could be provided by the Soret effect. They found that predictions of linear stability calculations incorporating the Soret effect compared well with experiments using water-methanol mixtures. In their later paper they question whether the Soret effect has perhaps unknowingly played a role in previous experimental studies of purely thermal convection. Nonlinear calculations incorporating the Soret effect and the related Dufour effect (diffusion induced temperature gradient due to a composition gradient) have been carried out by Prigogine and his co-workers (see, for example, Prigogine & Rice 1975).

### 3. Theory

The theoretical prediction of double-diffusive phenomena has not always been as closely linked to laboratory results as either theoreticians or experimentalists would wish. This has been mainly due to the fact that many laboratory investigations have been of essentially turbulent situations, which are difficult to handle theoretically. Nevertheless, some of the theory has influenced and guided the experimental studies

and, as we shall see in this section, theoretical studies of double-diffusive effects are being actively pursued at the moment and have developed a life of their own. For the purposes of this review many of the theoretical contributions to the subject are described more naturally alongside those laboratory experiments and/or field observations to which they are related. This is particularly so of the mechanistic models which have been developed. In this section we shall concentrate on discussing the more fundamental studies, which form the theoretical framework of the subject, and defer discussion of further extensions to subsequent sections.

The first theoretical calculation was that of Stern (1960), who realised that the different molecular diffusivities of heat and salt naturally supplied the essentials of the vertical pipe impervious to salt that had been previously considered by Stommel *et al.* (1956). Stern considered temperature and salinity profiles both of which increased with height between two horizontal boundaries separated by a distance  $H$ . He wrote down the partial differential equation governing the stability to infinitesimal perturbations of the temperature gradient  $T_z$  and the salinity gradient  $S_z$ . Asserting that the instability would be initiated as a time-independent mode, Stern showed that the equations for marginal stability were of exactly the same form as those for a one-component fluid. He thus proved that instability would set in when an appropriate Rayleigh number, say  $Ra$ , was of order  $10^3$ , with

$$Ra = \frac{gH^4}{\nu} \left( \frac{\alpha}{\kappa_T} T_z + \frac{\beta}{\kappa_S} S_z \right), \quad (3.1)$$

where  $g$  is the acceleration due to gravity,  $\nu$  the coefficient of kinematic viscosity,  $\alpha$  the coefficient of thermal expansion,  $\beta$  the corresponding coefficient for salt and  $\kappa_T$  and  $\kappa_S$  are the molecular diffusivities of heat and salt respectively. It also follows that the horizontal dimensions of the subsequent motion are comparable to  $H$ . However, an experiment of Stommel and Faller, cited in Stern's paper, indicated that the cells were long and thin. Stern hence calculated the width of the fastest growing mode based on linear theory, which yielded a result in good agreement with the experiment. This width, under the condition that  $\kappa_T \gg \kappa_S$ , is given by  $[\alpha g T_z / \kappa_T \nu]^{-\frac{1}{4}}$  and represents a balance between the destabilizing diffusive effects and dissipative viscous effects. If the fingers are too wide the heat cannot be diffused efficiently and if they are too narrow viscous effects restrain the motion. This is also the appropriate balance when the motion is nonlinear, as shown by Huppert & Manins (1973), thus explaining why the calculation of the fastest growing linear mode yields a result in agreement with experiment.

With the realisation that in both laboratory experiments and in the ocean a salt-finger field broke down into a series of layers and interfaces, Stern (1969) initiated a study of the stability of salt fingers to internal gravity waves whose wavelengths are very much larger than the salt-finger width. He showed that perturbations to the velocity in the fingers could interact with the shear in the internal waves and cause fingers beyond a critical length to break up. Neglecting the diffusion of salt and assuming that the fluxes of heat,  $F_T$ , and of salt  $F_S$ , are unaltered by the rotation induced by the internal waves, Stern calculated that all (stable) salt fingers must satisfy

$$\frac{\beta F_S - \alpha F_T}{\rho_0 \nu (\alpha T_z - \beta S_z)} < 1, \quad (3.2)$$

where  $T_z$  and  $S_z$  are the vertical temperature and salinity gradients in the fingers and  $\rho_0$  is the mean density. A more rigorous calculation, which analyses the non-linear interactions between the small-scale fingers and the larger-scale internal waves using formal expansion techniques, has been carried out by Holyer (1981) and yields  $1/3$  rather than  $1$  on the right-hand side of (3.2). More detailed experiments are needed to examine the validity of Holyer's version of (3.2) critically, but the results obtained by Stern & Turner (1969) are in reasonable agreement with it.

It was immediately realized that the diffusive layers and interfaces first studied by Turner & Stommel (1964) could only be explained theoretically by a nonlinear theory. To this end, Veronis (1965, 1968) commenced the study of the nonlinear behaviour of the two-dimensional motion of a fluid confined between two long horizontal planes heated and salted from below. The most extensive numerical calculations of this extended Rayleigh-Bénard problem have been undertaken by Huppert (1976) and Huppert & Moore (1976). Solving the three partial differential equations representing conservation of momentum, heat and salt, they found that there were two different solution branches, a steady branch and an oscillatory branch, and mapped out each branch as a function of the thermal Rayleigh number,  $R_T$ , representing the magnitude of the destabilizing temperature field. For  $\tau = \kappa_S/\kappa_T = 0.1$ , the smallest value for which their numerical scheme remained stable, they found that it was possible for the minimum value of  $R_T$  at which nonlinear steady convection occurred to be less than the linear oscillatory value.

The next step has been taken by Proctor (1981), who was able to describe the steady solutions in the vicinity of the minimum value of  $R_T$  by a boundary-layer analysis valid as  $\tau \rightarrow 0$ . The essential idea is that, for sufficiently small  $\tau$ , salt can be swept to the boundaries of the convection cells, where the only salinity gradients appear. The salt field is thus uniform in the interior and plays a minor role in the determination of the thermal field. The conclusion of Proctor's analysis is that, in the limit of  $\tau = 0$ , a value of  $R_T$  sufficient to initiate purely thermal convection is sufficient to drive a steady motion in the presence of a stabilizing salt field no matter how strong it is initially. This explains why Turner & Stommel were able to generate convective motions so easily and why considerable care is required to observe the oscillatory behaviour predicted by linear theory.

Over the last few years, there has been much interest in the generic morphology of solutions to ordinary differential and difference equations (see, for example, Marzec & Spiegel 1980; May 1976). Indeed Huppert & Moore calculated aperiodic solutions to their system of three partial differential equations which may be likened to the solutions of a strange attractor. Da Costa, Knobloch & Weiss (1981) developed the analogy further by determining numerical solutions to a fifth order ordinary differential system, which is the severest nonlinear truncation to a Fourier representation of the complete system studied by Huppert & Moore. Their results mimic many of those obtained from the partial differential system and the relative simplicity of the system allows longer and more detailed calculations to be made, in particular in parameter ranges not investigated for the complete system. The agreement between the results obtained by these two different approaches is somewhat surprising in view of the *disagreement* obtained by pursuing the same line for purely thermal convection, where the solutions of the Lorenz model, which involves three nonlinear ordinary differential equations, soon depart from those obtained by the complete system.

The nonlinear behaviour of two-dimensional salt-fingers extending between two horizontal boundaries has been investigated by Straus (1972). Considering the limit  $\nu \gg \kappa_T \gg \kappa_S$ , Straus used a Fourier series representation to solve the governing equations. He carried out calculations for a range of horizontal wavelengths and determined that wavelength which would maximize the salt-flux, pointing to the analogous criterion for wavelength selection initially proposed by Malkus (1954). Straus found that as the (destabilizing) salinity gradient across the horizontal boundaries increased, the preferred wavelength decreased, so that the fingers looked geometrically more like the long slender fingers observed in experiments. Straus also carried out a three-dimensional linear stability analysis of the nonlinear two-dimensional solutions and determined the wavelength of the ‘most stable’ solution, being that for which the growth rate was least. The wavelengths selected by the two different criteria were rather similar. The form of the predicted salt flux as a function of the maintained vertical salinity difference was in qualitative agreement with the laboratory measurements of Turner (1967) across an interface, but quantitatively it was an order of magnitude smaller. This disagreement is not too surprising given that the calculation considered laminar fingers extending between horizontal boundaries while in the experiments the salt fingers were localised at an interface, on either side of which there was three-dimensional turbulent convection.

A step towards a more realistic calculation has been taken by Piacsek & Toomre (1980), who obtained numerical solutions of the equations governing the two-dimensional growth of salt fingers across an initially sharp interface. As the fingers grew, the ends developed blobs which could proceed ahead of the fingers (cf. figure 1). In the numerical experiments these blobs hit the containing boundaries, which were too close to allow for a convecting layer to develop independently of the fingers. In the opposite case, Elder (1969) carried out some preliminary calculations using the ‘mean-field’ approximations due to Herring (1963) to investigate the development of an initially sharp diffusive interface. In the calculations, the interface first increased in thickness by diffusion and after a time instabilities on either side of the interface limited its growth. Eventually, for reasons not yet understood, the interface broke down and a one-layer fluid remained. It appears that further numerical simulations, using more sophisticated approaches, need to be carried out before we have models of a finger and of a diffusive interface whose predictions are in agreement with laboratory measurements.

A different and imaginative approach has been outlined by Stern (1976). Rather than solving equations, Stern determines the conditions which maximise the buoyancy flux given a series of argued constraints governing the nature of the salt fingers at the interface. He determines a relationship for the corresponding heat and salt flux which are in reasonable agreement with experimental results. Clarification of the constraints suggested by Stern and of their implications is required before one can be confident of the validity of the results obtained and of the usefulness of the general approach.

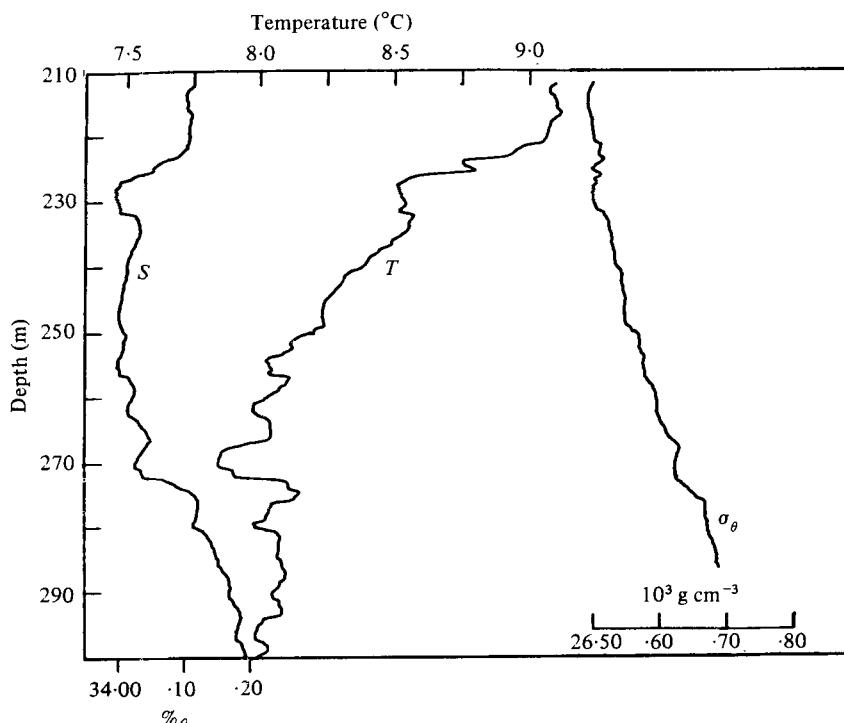


FIGURE 7. Profiles of temperature, salinity and density measured by Gregg & Cox (1972) using a freely falling microprofiler.

#### 4. Observations in the ocean

Until as recently as twenty years ago, observational oceanographers studying the interior of the oceans concentrated on the larger scales of motion, and it seemed clear that molecular processes should be entirely negligible. The transports of heat, salt and momentum were described by eddy coefficients which were orders of magnitude larger than molecular values, and this led to the idea that turbulent mixing must dominate in the deep ocean. Only the development of new instruments has made it possible for the accepted picture of the ocean to change so rapidly and drastically, from the previous ideas based on smoothly varying profiles drawn through the widely spaced points obtained from water bottle samples. The newly-developed rapidly-responding probes used to measure vertical distributions of temperature and salinity soon showed that the profiles are not always smooth, but frequently consist of a series of layers in which the gradients are weak, separated by interfaces in which the gradients are much larger. Typical profiles of temperature, salinity and density measured by Gregg & Cox (1972) off California are shown in figure 7. Furthermore, small scale fluctuations are very weak through most of the volume of the deep ocean because of the stratification, and active turbulence tends to occur in thin but horizontally elongated patches. Undoubtedly the need to understand the implications of these new observations for oceanic mixing processes provided the major stimulation for a field which might otherwise have remained an exercise in pure fluid dynamics.

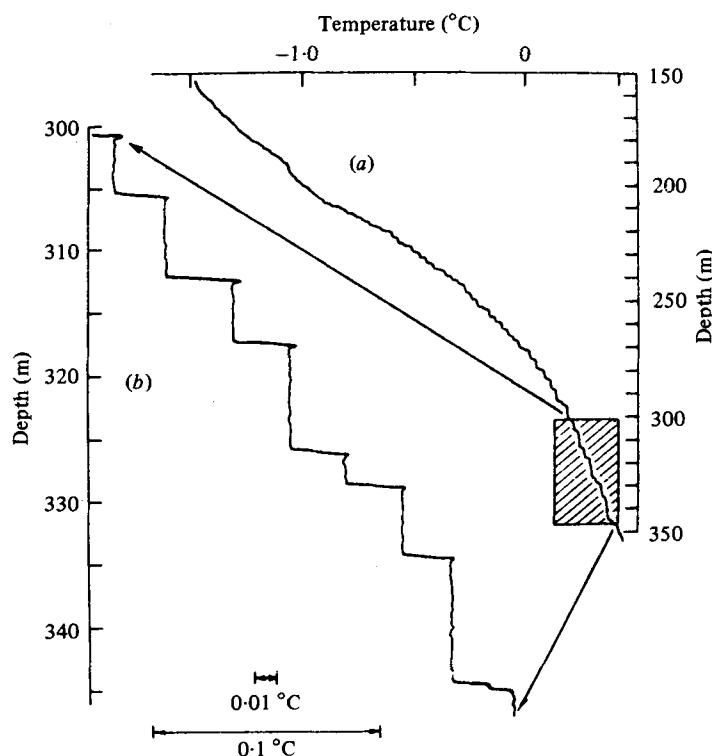


FIGURE 8. A temperature profile obtained under an Arctic ice island by Neal, Neshyba & Denner (1969) showing steps formed by the double-diffusive mechanism. (a) Typical temperature profile section. (b) Section of profile recorded at high gain.

Although there were some prior indications of layering in measurements made by Cooper (1967), who correctly inferred their presence from closely-spaced water bottle samples pushed to the limit of their precision, and in bathythermograph records from many parts of the ocean (many of which revealed sharp steps and inversions which were discounted and attributed to malfunctioning of the instrument), the real start was made when continuous recorders of salinity and temperature against depth (STD recorders) became widely available. An early detailed study, which has since been widely quoted, was carried out by Stommel & Fedorov (1967). Their measurements showed many small inversions, extrema, and indications of homogeneous regions separated by sharp gradients, and they showed that some features were horizontally coherent over many kilometres. They made estimates of the probable timescale and dissipation rates of the laminae, and related them to the understanding of vertical mixing processes. They pointed explicitly to the importance of coupled temperature and salinity changes in which temperatures and salinity increased or decreased together, and the ways in which interleaving of various kinds could produce laminae with inverted salinity or temperature gradients. They suggested 'that this may be direct evidence of the more exotic types of two-diffusivity convection' and related it to the laboratory work of Turner & Stommel (1964).

At about the same time as temperature steps were observed in Lake Vanda and at the bottom of the Red Sea, a fine example of diffusive layers in the deep ocean was

reported by Neal, Neshyba & Denner (1969) and analysed by Neshyba, Neal & Denner (1971). As shown in figure 8, they monitored layers about 5 m thick (with temperature steps of a few hundredths of a degree) in the Arctic Ocean where colder, fresher melt water overlies warm salty water underneath a drifting ice island. It was relatively straightforward to identify the layers and interfaces observed with the temperature and salinity distributions in the 'diffusive' sense, helped by the corresponding laboratory phenomenon which showed directly how layers could form from smooth gradients. Although good examples of layering consistent with the finger distributions of temperature and salinity were also observed from 1967 onwards (Tait & Howe, 1968, 1971; Cooper & Stommel, 1968), the association of this with salt fingering was not immediately accepted, in spite of the laboratory work which showed how fingers can form at the interfaces between convecting layers.

The direct detection of salt fingers in the ocean by Williams (1974) and their clear association with the strongest gradients of temperature and salinity (i.e. the interfaces predicted by the laboratory work) was an important step in the development of the subject. Williams, an ocean engineer with a background in optics, became interested in the detection of salt fingers in the ocean following conversations with Stommel and Turner during the summer of 1969. After developing the appropriate instrumentation he succeeded in photographing salt fingers below the Mediterranean outflow in July 1973, using a sophisticated variant of the shadowgraph technique. He also obtained photographic evidence of salt fingers in the Tyrrhenian Sea (Molcard & Williams 1975). Direct evidence of conductivity fluctuations of the right scale to be identified with salt fingers was also obtained by Magnell, a Ph.D. student of Stommel's, at about the same time, using a towed instrument. Magnell (1976) obtained spectra of the temperature and salinity fluctuations and compared them with a theoretical and experimental study of the spectra conducted by Huppert & Linden (1976).

Conditions are especially favourable for double-diffusive convection to occur when a layer with compensating temperature and salinity differences intrudes at its own density level into an environment with different properties. This is true of the Mediterranean outflow into the Atlantic, below which the strongest layering in the finger sense has been observed, and it was clear from the observations of Stommel & Fedorov (1967) and later Pingree (1971) that horizontal gradients and advection often play a significant role in the formation of layers. In view of this, it may be surprising that the preoccupation with one-dimensional processes persisted so long, both in the ocean and in the associated laboratory and theoretical work. Fedorov (1970) attempted to use one-dimensional laboratory experiments to explain layer scales in some of these observations, but the same author's recent monograph (1976, English translation 1978) has brought together the considerable body of evidence which shows that much of the fine structure, or horizontally extensive layering in the ocean, is associated with intrusive motions. Even when the mean distributions would not in themselves give rise to double-diffusive transports (e.g. in much of the Pacific Ocean where both the temperature and salinity distributions are 'stabilizing'), horizontal intrusions can so reorganise the vertical gradients that double-diffusive processes can operate, as for example in the situation depicted in figure 5. It is now recognized that microstructure measurements must be made with a full knowledge of the fine structure environment, i.e. of the layer and interface structure of the water column, and there are now many measurements (see, for example, Gregg

& Briscoe 1979) which show that the regions of most intense activity are the upper and lower boundaries of intrusions produced by the interleaving of two water masses.

In general, the strongest layering is found near boundaries between water masses of different origin. Layers are most prominent when there are large horizontal contrasts in temperature and salinity, but a small net density difference. For example, various measurements at the Antarctic Polar front (Gordon, Georgi & Taylor 1977; Joyce, Zenk & Toole 1978) have revealed inversions that decrease in strength with increasing distance away from the front. A definite slope of the intrusions relative to density surfaces has been documented, supporting an explanation in terms of double-diffusive processes. Gregg (1980) has recently reported a very detailed study of an intrusion just below the base of the seasonal thermocline. A cold, fresher tongue of water about 6 km long, 2 km wide and 10 m thick was shown to slope downwards by about  $1^\circ$  relative to isopycnal surfaces, away from an origin at a front; the feature was almost undetectable in the density profiles. The sense of this slope, and the scale of the intrusion, are in good agreement with the predictions made on the basis of the laboratory model of Ruddick & Turner (1979), and support double diffusion as the most probable driving mechanism.

It is worth pausing to point to a remarkable implication of these measurements of intrusions across fronts. They imply that molecular effects can not only influence motions on the layer scale, i.e. tens of metres in the vertical, but they could play a significant role in the large-scale mixing between water masses, and thus they could affect the salt balance in the Southern Ocean. This is not yet fully accepted by the oceanographic community, and there is little direct evidence to support it, but it seems important to devise experiments to distinguish carefully between intrusions which are actively driven across fronts by double-diffusive processes and those which represent a more passive response to a distortion of isotherms near a front.

While one-dimensional processes of formation of interfaces were unduly stressed when interpreting the early observations in the ocean, the fluxes through the interfaces between the layers once they have formed can probably be adequately treated using one-dimensional models. Although the analogue experiments using sugar and salt solutions have been helpful in obtaining a qualitative understanding of many new processes and instabilities, it is only the salt-heat experiments which have been directly helpful in estimating vertical fluxes in the ocean and their effect on the structure.

In this context, we will consider the finger case first. It is predicted on the basis of the stability arguments that salt fingers at an interface can readily form even when the ratio  $\alpha\Delta T/\beta\Delta S$  is as large as  $10^3$  (Huppert & Manins 1973). Thus they should be extremely common below any region where warm salty water is formed at the surface by solar heating and evaporation. Regular layered structures are only observed, however, when this density ratio is less than about 1.7. Their presence implies that locally the vertical 'transport coefficient'  $K_S$  for salt, defined as the vertical flux divided by the mean gradient, is greater than that of the corresponding coefficient for heat, and in the strongly layered situation underneath the Mediterranean outflow Turner (1967) deduced that  $K_S$  is equal to  $5 \text{ cm}^2 \text{ s}^{-1}$ . In many cases, they are not vigorous enough to produce layering, and they cannot remain steady in the presence of other mixing mechanisms. Schmitt & Evans (1978) have considered

the behaviour of salt fingers in the presence of an internal wave field. When the temperature and salinity gradients are nearly equal, salt fingers grow rapidly enough to become active on the high gradient regions produced by internal wave straining. Even though this salt fingering is intermittent, it can produce mean salt fluxes (calculated using laboratory data and the measured temperature and salinity profiles) which are comparable to the surface input of salt due to evaporation. Thus they deduce that fingers can account for all the vertical flux in the ocean. Lambert & Sturges (1977) reached similar conclusions about the importance of the finger fluxes through a system of layers and interfaces observed below the core of a warm saline intrusion in the Caribbean Sea. Schmitt (1981) has suggested further that the *T-S* relationships of the central water masses of the oceans are better fitted by a model in which temperature and salinity are mixed at different rates by salt fingering rather than by turbulent mixing processes.

In the diffusive case, where both salinity and temperature increase with depth, some estimates of the fluxes through sharp interfaces have also been made, but much more needs to be done. Huppert & Turner (1972) used the Lake Vanda data to show that the one-dimensional flux relation obtained in the laboratory can be applied quantitatively to comparable large-scale motions. Steady-state situations are uncommon, however, and Huppert (1971) showed theoretically that a stationary, diffusive interface will only be stable if  $R_\rho = \beta\Delta S/\alpha\Delta T$  is greater than about 2 and unstable otherwise. Observations of stable layers and interfaces in the ocean are generally consistent with this criterion, but more attention should be paid in the future to the study of the time history of a series of diffusive layers. Further, an interface can break down in two ways, either remaining stationary, while the density difference across its decreases to zero, or by migrating vertically to the adjacent interface. McDougall (1981) has recently conducted a laboratory experiment in which this latter process is prominent. He has incorporated nonlinear density effects which can lead to a 'cabbeling' instability and has shown that the interface migration can be interpreted as being solely due to double-diffusive convection modified by the nonlinear equation of state. He finds that it is not possible to explain the observations in the Weddell Sea, for example, by neglecting double-diffusion, and considering the conventional cabbeling instability alone.

It is now accepted that when vertical transports are produced by double-diffusive processes the 'transport coefficients'  $K_i$  must be different for the two properties, and that a single eddy diffusivity for both of them is not an appropriate concept. It is not so widely appreciated that the individual fluxes of several dissolved species can depend strongly on molecular diffusivity. Griffiths (1979) has suggested that  $K_1/K_2$ , for two components, both of which are driven across an interface by convective heating in a 'diffusive' situation, should be proportional to  $\tau^{\frac{1}{2}} = (\kappa_1/\kappa_2)^{\frac{1}{2}}$  (where  $\kappa_1$  and  $\kappa_2$  are the molecular diffusivities) at low total solute/heat density ratios,  $R_\rho$ , and to  $\tau$  at higher  $R_\rho$ . Experiments conducted with interfacial density ratios  $R_\rho$  between 2 and 4 are consistent with  $K_1/K_2 = \tau$ , but there is greater separation of components at higher  $R_\rho$ . The experimental results have potentially far-reaching implications for geochemical studies in the ocean which have not yet been thoroughly investigated. When a tracer is used to mark a water mass, it is assumed that its dilution is a measure of the mixing rate for the water mass as a whole. But if, for example, a tracer was put into the lowest brine layer in the Red Sea, its transport

through the diffusive interfaces above would not necessarily be a good indication of the flux of a major component, much less of heat. The prevalence of double-diffusive phenomena in finestructure observations certainly suggests that a single eddy diffusivity for all properties must be used with great caution in the deep ocean.

### 5. Other applications of double-diffusive convection

Though the oceanographic applications have been emphasized in this review because of the important part they have played in the development of the subject as a whole, applications of the ideas have also been made in diverse fields. In fact the phenomena described are widespread and probably occur in contexts which have not yet been recognized – transfer of this understanding to other fields will depend mainly on the sometimes accidental communication between scientists whose interests appear to be at first sight very different.

There have been some suggestions that double-diffusive processes might be important in the atmosphere, but because the diffusivities of heat and water vapour in air are so close (the latter being about 20% larger than the former) it is harder to produce clear-cut observational evidence for the existence of such effects. But the possibility certainly exists, for example in the case where a cold moist layer of air underlies warmer dryer air at a cold front. The interface is often very sharp, and since moist air is lighter than dry air at the same temperature, there is potential energy in the moisture distribution, and double-diffusive convection could help to maintain the sharpness of such an interface. Radar observations of ascending moist parcels of air show that the tops are much sharper when the parcels are colder than when they are warmer than the environment, and this too can be explained in similar terms. Schaefer (1975) and Merceret (1977) have put forward specific examples of atmospheric phenomena which they have interpreted in terms of double-diffusive convection, but further work along these lines is needed to establish its relevance to the atmosphere.

A direct analogue of heat/salt diffusive convection has been invoked to explain the properties of large stars with a helium rich core. Spiegel (1971, 1972), an astrophysicist who first heard of double-diffusive convection from the geophysical fluid dynamics community at Woods Hole, has shown that gradients of chemical composition can produce a density gradient, which is heated from below and therefore convecting. Whether the convection is in layers is still a matter for debate. Outside the core, lighter hydrogen predominates, but helium is transported upwards by double-diffusive convection. In this context the name semi-convection has been given to the process, which allows the motion outside the core to regulate the transport and remain nearer the condition of convective neutrality. In this state the potential temperature will be nearly uniform, since this is the driving component for the motion, but there can still be a stabilizing potential density gradient. Another instability of astrophysical interest, which is akin to salt fingers, may arise in the radiative zone of a star and has been discussed by Goldreich & Schubert (1967). Due to the differential rotation in a star, an angular momentum distribution which decreases radially outward and is hence potentially unstable can be superimposed on a stable temperature gradient. Since in a star the molecular diffusivity of heat greatly exceeds that of momentum, an instability drawing on the energy in the

differential rotation can develop. Other examples of double-diffusive instabilities dependent upon unstable angular momentum distributions have been investigated by McIntyre (1970a, b) and Baker (1971).

There are many fields of engineering in which previously puzzling phenomena have now been explained in terms of double-diffusive processes. The particular examples given here are among those which have come to our attention, often through a direct contact with someone working in the relevant field.

The first examples to be given are closely related to the oceanographic systems already described. The 'solar pond' is an artificially stratified lake which relies on opposing density gradients due to salinity and temperature for its operation (Tabor 1979). It has been developed for use in sunny climates to store radiant energy, using the salinity gradient to protect the heated bottom layers of the pond from losses to the air above. The aim here is to prevent convection altogether, by maintaining a large enough salinity gradient. If two or more convecting layers, separated by interfaces, were allowed to form, then the efficiency would be reduced because of the increased rate of vertical transport of heat. A commercially successful pond of this type is already operational in Israel.

Salt fingering near an intrusion can also be of importance to engineers. Fischer (1971) has shown that this process can be relevant in the context of sewage disposal in the sea. Effluent, which can be regarded as nearly fresh (though polluted) water, is commonly ejected from a pipe laid along the bottom. The effluent rises as a plume, and if the sea is strongly stratified in temperature, but of almost uniform salinity (as occurs off California), the effluent, diluted with cold sea water, will spread out in a layer below the thermocline. The layer will however remain colder and fresher than the water above it, so that the salt finger mechanism can cause it to thicken vertically and even extend to the surface. Another related case in which the environmental effects could be even more serious arises in the disposal of effluent from a desalination plant. If the brine from which water has been evaporated and the heated water from the cooling plant are mixed together, this hot salty effluent will have about the same density as the original sea water. Precisely what happens to it will depend on whether it is slightly heavier or slightly lighter. If it is slightly more dense and flows out along the bottom a diffusive interface will be formed, and the coupled transport will tend to increase the density difference and keep the layer intact. If it is put in at the surface, or at an intermediate level in a gradient, there will be a more rapid vertical mixing, perhaps with the production of layers. Only one thing is certain; double-diffusive effects cannot be neglected if one is to make a realistic assessment of the mixing.

Double-diffusive processes can be important in other systems besides aqueous solutions, and two applications arise in the context of storage and transport of liquid natural gas (LNG). If a storage tank is refilled from below with relatively warm LNG containing heavier fractions, two distinct layers separated by a diffusive interface can form. Gas is drawn off at the top at a rate which is adjusted to the conditions at the liquid surface. As the lighter fractions boil off, the upper fluid may increase in density more rapidly than the lower can by cooling through the interface. If the densities become equal, a sudden overturning or 'roll-over' takes place, bringing warmer LNG to the surface; the pressure can build up rapidly and as discussed by Sarsten (1972) there can be a sudden uncontrollable increase in the rate of release of

gas. A second potentially dangerous situation occurs if LNG or some other liquefied gas spills on to the sea surface (see Fay & MacKenzie 1972). The liquid quickly evaporates to form a layer of cold gas, predominantly methane, which would be lighter than the air above it except that it is much colder. Since both methane and the water vapour picked up from the sea surface have larger diffusivities than heat in air, double-diffusive effects can again be important in this gaseous system. The driving energy comes from the distribution of methane and water vapour, so the interface is 'diffusive'. Observations show that the top of such a layer is very sharp, and its vertical rate of spread small. It stays coherent and close to the sea surface for a long distance downwind, which is consistent with a self-stabilising double-diffusive transport across the interface.

Another application, with cross connections to the melting of icebergs considered previously and to some of the geological problems to be discussed in the next section, concerns the solidification of crystals from a surrounding melt. Even neglecting convective effects, a spatial variation in the deposition of new material can develop, and the stability of such a solid-liquid interface has been extensively examined over the last decade. Only recently have the additional effects of fluid motion begun to be investigated. Coriell *et al.* (1980) have analysed the linear stability of a surface translating uniformly upwards due to the solidification of a single-phase binary alloy. A temperature and compositional gradient occurs ahead of the surface in the 'finger sense': the compositional gradient is destabilizing. Including the effects of an applied magnetic field, they derive a specific stability criterion for the solidification of lead containing tin and stress that the criterion differs substantially from that of a net neutral density gradient. Direct evidence of finger motions in such solidifying castings is provided by 'freckling', or long narrow vertical regions in which impurities segregate (Copley *et al.* 1970).

Undoubtedly many more effects remain to be discovered; it is likely that related phenomena have already been recognized, though they may not have been considered explicitly in these terms. It is clear that when a fluid contains two or more components with comparable influence on the density, the rate of mixing cannot be reliably predicted knowing the net density distribution alone. Not only will the mixing rate be wrong, but a whole range of qualitatively different phenomena can be missed if the double-diffusive nature of the fluid is ignored.

## 6. Some geological applications

We will consider in more detail a recent field of application of double-diffusive convection, both because it is of current interest to us, and because it illustrates a more general point about the spread of information from one field to another.

Five years ago, one of us (J.S.T.) moved from a group of fluid dynamicists interested in problems of the ocean and atmosphere to a Research School interested mainly in problems of the solid earth. Discussions with new colleagues revealed a phenomenon of geological interest which was clearly related to a problem already studied in an oceanographic context – the flow of hot saline solutions from vents in the sea floor. This is important in the genesis of massive sulphide and other ore deposits, but the fluid dynamics of the phenomenon had not been treated in any detail by geologists. Turner & Gustafson (1978) therefore wrote a review of various



FIGURE 9. Layers in the Upper Zone of the Skaergaard igneous intrusion, made visible by contrasting light and dark colours of different minerals (from Mc Birney & Noyes 1979).

flow phenomena involving fluids with thermal and compositional variations. When the work was begun, we had in mind the formation of ore deposits on ancient sea floors, but there has been an increased incentive to study the process since hot plumes at up to 350 °C have been observed directly at the bottom of the ocean.

Of particular interest is the case where the effluent is both very salty and hot, so that the two properties have opposing effects on the density difference. This can lead first to a very nonlinear density behaviour during mixing, which makes it possible for initially light fluid to become heavier than sea water and for an oscillating flow to develop. Even more important, and previously totally unknown in the geological context, are the double-diffusive effects, which can cause an outflow to separate into two parts, a hot, less concentrated plume which rises and a warm concentrated flow which spreads as a bottom current away from the source, maintaining a sharp boundary with the overlying sea water as it does so. Further double-diffusive processes which could act on this outflow were also described, and a preliminary study was made of two additional processes of potential geological importance; precipitation due to a chemical reaction between the outflow and the environment, and crystallization due to supersaturation associated with the cooling.

These latter observations were the prelude to the more systematic crystallization experiments which we now describe. It was suggested by Turner & Gustafson that double-diffusive processes might be important in the formation of layered igneous complexes (i.e. in liquid magmas as well as in aqueous solutions). There are many examples of igneous intrusions exhibiting prominent layers, in some cases with horizontal extents of many kilometres (see for example the descriptions in one of

geology's classics Wager & Brown, 1967). An example given by Mc Birney & Noyes (1979) is reproduced in figure 9. Common explanations have invoked the settling of crystals and the intermittent formation of gravity currents, but the regularity of the layers suggest that they are more likely to have formed in the fluid state by some double-diffusive mechanism, and that the growth of crystals is subsequently affected by these pre-existing layers.

This idea was pursued by Chen & Turner (1980), who added crystallization to double-diffusive systems previously studied in the laboratory. Again, however, the model system used was an aqueous solution, initially of sodium carbonate, cooled in various configurations. This salt was chosen because of the strong dependence of the saturation concentration on temperature. The first experiments started with a gradient of  $\text{Na}_2\text{CO}_3$  concentration, because of our previous experience with layering in stratified systems, but later experiments showed how stratification could develop from an initially homogeneous fluid. When the gradient of sodium carbonate was cooled from the top, a series of convecting fluid layers was first produced, separated by sharp interfaces – this is physically equivalent to heating a salinity gradient from below. When the top boundary was held below the eutectic temperature, crystals of  $\text{Na}_2\text{CO}_3$  grew and adhered to the boundary. Their subsequent growth, and that of the eutectic, was clearly influenced by the position of the interfaces in the fluid. Distinct banding was also observed in the eutectic layer, at levels corresponding to the fluid interfaces. The advancing front of the growing crystals remained nearly horizontal, again being strongly influenced by the position of the interfaces.

The second geometry used was a stable concentration gradient cooled through one plane side wall, above the eutectic temperature. As in the rather different experiments with melting ice walls (Huppert & Turner 1978), the main effect on the interior fluid was due to the imposed lateral temperature gradient. A series of nearly horizontal convecting layers was generated, the sense of circulation in each layer being towards the cooling wall at the top, down along the wall and away from it at the bottom of the layer. When crystallization began, it was most pronounced where the fluid was coldest, just as it left the cooled wall.

Another process is very important, however. Crystallization leaves behind a boundary layer of fluid depleted in  $\text{Na}_2\text{CO}_3$  that is lighter than its surroundings (in spite of being colder). The continuing upflow in the boundary layer causes an accumulation of fluid at the top of the region, thus displacing all the underlying interfaces and layers downwards. As a result, the locations of more rapid crystal growth do not remain in the same relation to the interfaces, and any persistent layer structure in the crystals tends to be obliterated. In other experiments, where the interfaces did stay at a fixed level, the growth of the crystals was clearly affected by the existence of the fluid layers and interfaces.

The phenomenon of upflow in a boundary layer described above, due to the crystallization of a heavier component at a vertical wall, has an important application to the understanding of structures observed in large igneous intrusions. Consider what will happen when a chamber filled with *homogeneous*  $\text{Na}_2\text{CO}_3$  solution is cooled at a vertical wall, so that crystallization occurs. The lightest, most depleted fluid is continually fed to the top of the chamber, so building up a stable density gradient, by the 'filling box' mechanism described by Baines & Turner (1968). Thus even if the region was not stratified to begin with, the flow produced by crystallization

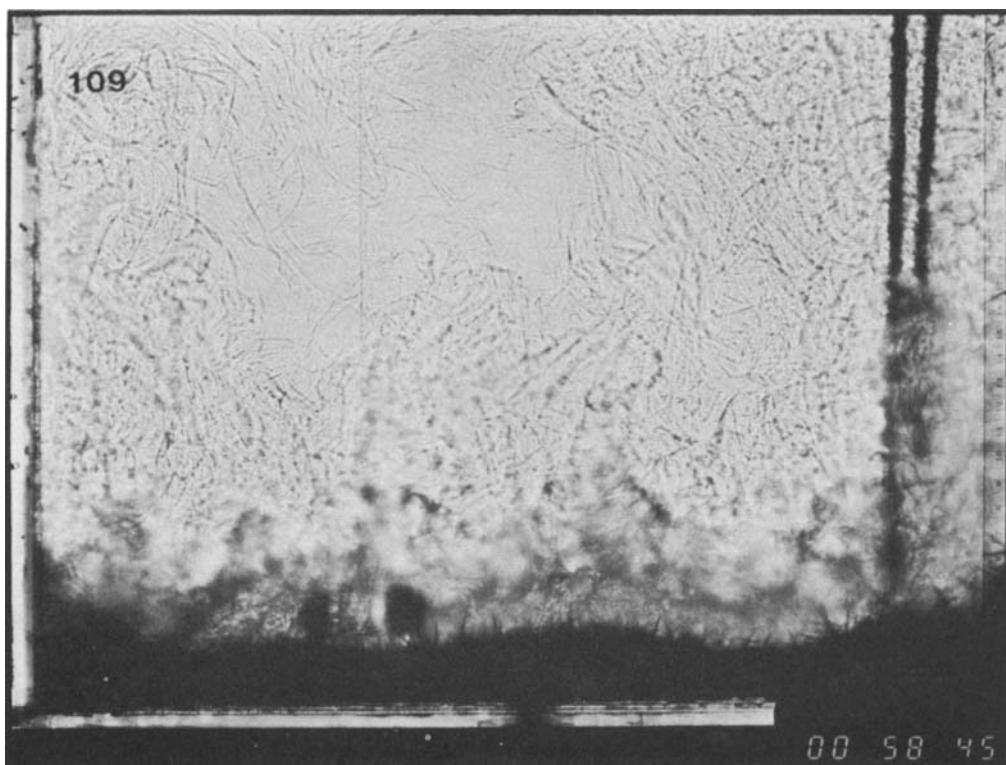


FIGURE 10. A crystal column produced by cooling an initially homogeneous solution of  $\text{Na}_2\text{CO}_3$  at a central pipe. Upflow in the boundary layer due to crystallization has produced stratification and layering and associated 'scallops' on the crystal surface. The left of the picture is a shadowgraph and the right shows a dye streak against diffuse back lighting, with the distortions indicating the motion in the layers.

makes it so. The combination of cooling from above and from the side produces a series of layers which moves downwards in time until the whole region is stratified and layered. An example of layering produced by cooling at a central rod is shown in figure 10.

If a second solute is added to the initial fluid, the process of differentiation in a magma chamber can be modelled explicitly. Turner (1980) used a homogeneous mixed solution of  $\text{Na}_2\text{CO}_3$  and  $\text{K}_2\text{CO}_3$ , with cooling at a central vertical metal rod. Again pure hydrated sodium carbonate crystallized out over the range of temperatures and concentrations used. Thus as well as being depleted in sodium, the upward flowing boundary layer was relatively enriched in potassium. Measurements of the ratio of sodium to potassium demonstrated directly that 'differentiation' had been produced in the experimental tank. In a continuing series of experiments, Turner & Gustafson (1981) are also investigating the case where more than one component crystallizes out.

The laboratory studies described above have concentrated on explaining how vertical and compositional stratification can arise from crystallization processes, and the accompanying boundary layer flows, set up by side wall cooling in a magma chamber which is initially homogeneous. Huppert & Sparks (1980, 1981), a fluid dynamicist and a geologist, have recently suggested a theoretical model of the



**FIGURE 11.** A laboratory model of a magma chamber, using a layer of hot  $\text{KNO}_3$  below cold  $\text{NaNO}_3$ . At the time shown, the densities of the two layers have become equal, and the interface is breaking down.

evolution of some basaltic magma chambers subject to quite different initial conditions. They suppose that periodically there is an influx of new magma into the base of the chamber, which due to compositional differences is heavier than the resident magma, even though it is hotter. The new magma forms a layer at the base of the chamber and cools because of the heat transfer across the relatively thin interface above. The layer cools with negligibly small compositional transfer across the interface and crystals form within it. Initially the crystals of olivine are kept in suspension by the convective motion, and not until the two layers have almost reached thermal equilibrium do the crystals fall out. The density of the residual liquid decreases with decreasing temperature, and once the crystals have separated, the remaining liquid can mix rapidly with the upper layer. The model explains how a magma chamber can act as a buffer between the new magma, containing 18%  $\text{MgO}$ , and the magma erupted from the chamber, which is observed to contain 10%  $\text{MgO}$ .

We (Huppert & Turner 1981) have modelled this process in the laboratory, apart from the actual suspension of the crystals in the lower layer, which is not an essential aspect of the process. The lower layer used was hot  $\text{KNO}_3$  solution, for which saturated solutions become less dense as the temperature decreases. With a cold, deeper layer of less dense  $\text{NaNO}_3$  above, there was a convective transfer of heat through the interface, at a rate which could be predicted by drawing on previous studies carried out in an oceanographic context. Crystals grew on the bottom, and the

density of the residual liquid in the lower layer decreased till finally it became equal to that of the upper layer. Then, as depicted in figure 11, the interface broke down and the layers mixed thoroughly together, leaving a layer of  $\text{KNO}_3$  crystals at the base. This is quite different from what was observed when the hot input fluid was forced to mix initially with the cold solution already in the chamber; no crystallization at all occurred in that case. Related experiments have also been conducted with a gradient of  $\text{K}_2\text{CO}_3$  above the hot  $\text{KNO}_3$ , which is the case used for illustration in figure 2, or alternatively using a continuous input of the lower fluid rather than a sudden injection.

We have so far concentrated on areas close to our own interests, but there have been other examples in the recent literature where double-diffusive processes have been invoked in geological contexts, though they have not always been identified by this term. McBirney & Noyes (1979) have written a penetrating paper questioning the validity of previously proposed explanations for the rhythmical layering of the Skaergaard intrusion in Greenland. They put forward a model based on the chemical oscillations which arise due to the different rates of thermal and chemical diffusion as crystallization proceeds at a solid boundary. The qualitative features of the model appears to agree better with the observational evidence than previous ideas based on successive deposition by gravity currents.

In another paper, McBirney (1980) has suggested that the compositional differences observed in some large-scale calc-alkaline suites may be explained by fractionation and the resultant collection of light liquid at the top of a reservoir. He examined this process in a laboratory experiment, which can be viewed as an extension of the experiments conducted by Chen & Turner (1980). McBirney again used  $\text{Na}_2\text{CO}_3$  solution, but his experimental tank was fitted with an arched top through which cold water was circulated. Buoyant liquid, released as the  $\text{Na}_2\text{CO}_3$  cooled and crystallized, accumulated under the arch and an interface separating it from the underlying denser solution developed and with time slowly migrated downward. Above this interface McBirney measured a strong stable density gradient and inferred that there was a correspondingly strong compositional gradient. This compositional variation resembles that believed to exist in the magma chamber beneath the Long Valley Caldera in California. As another example, the variation of composition of lavas discharged during the nine-year eruption of the Paricutin volcano suggest that a compositional gradient had been built up in the magma chamber from which they came.

On a much larger scale, there has recently appeared a number of studies investigating the form of global convection in the Earth's mantle. It is believed by most geophysicists that convection must be the major driving mechanism involved in plate tectonics. The convection might be constrained to the upper mantle, with a penetration to only approximately 650 km, where there is believed to be a phase change transition and also possibly a compositional discontinuity. The convection might on the other hand occur throughout the mantle to a depth of 2900 km. An interesting possibility, which combines aspects of both of the previous ideas and seems consistent with much of the geochemical and seismological data, is that convection takes place in two or more distinct layers: one layer penetrating to 650 km, beneath which there is another layer (or possibly several) down to the core-mantle boundary.

A two-layer convective system for the Earth was discussed by Stevenson & Turner

(1979), who drew analogies with the fundamental properties of double-diffusive layering reviewed in the earlier sections of this paper. Richter (1979) suggested that layering in the mantle may have originated by the process of heating a chemical gradient from below, using the same principles involved in heating a salinity gradient. The precise details of the layering produced would require a knowledge of how the Earth accreted and its initial thermal conditions. A step towards determining this has been undertaken in a study of the Earth's thermal history by McKenzie & Richter (1981). Assuming that the layers are Newtonian fluids, each with constant viscosity, they showed that the response time of the Earth's heat loss to the decay of radioactive elements within it depends strongly upon whether the convection extends to the core-mantle boundary or not. If the convection occurs in two layers they conclude that the present surface heat flux is determined mainly by the initial thermal conditions of the Earth.

It seems clear that double-diffusive convection will be important in understanding a large range of geological problems. While some of the fundamental principles are at hand, no doubt there remain many others to be discovered and investigated. Further, there are many quantitative investigations needed to determine relationships for the relevant geological parameter values. For example, the ratio of diffusivities for the much considered system of salt and sugar is  $1/3$  and for heat and salt it is  $10^{-2}$ . How do relationships established for these two systems carry over to ratios typical for magmas,  $10^{-5}$ , and for the mantle,  $10^{-12}$ ? The flow patterns observed for any of these systems may to a large extent be similar, but the quantitative variations in such parameters as the rate of heat transfer may be rather different. In general, it is only by obtaining quantitative predictions that models can be tested against field data and either suitably altered, or perhaps rejected.

## 7. Conclusions

The justification for attempting a review of this kind, with a very personal bias and with some of the material quite recent and undigested, is the hope that more general deductions can be drawn from the way the field has developed. There is no doubt that cross-links with the rapidly developing field of observational oceanography have been a vital element in the growth of the subject, but what about the connections with other fields?

In the geological context in which we are currently working, it is clear that much more interaction with petrologists and geochemists will be essential before the most important problems can be defined, and certainly before the fluid-dynamical ideas can become widely accepted among geologists. Our own personal connections with geological colleagues have provided the incentive for our current work, and it has become clear that both a detailed knowledge of the fluid dynamical processes and of the large-scale phenomena to which we wish to apply them are essential elements in the solution of such multi-disciplinary problems. It is too much to expect that one individual can become an instant expert in a new field, particularly when the training in the two disciplines is almost mutually exclusive. For example, geology as it is presently taught, is a mainly descriptive subject, with the more mathematically-minded students becoming geophysicists rather than petrologists, and it would seem

more feasible at present to encourage appropriate interactions rather than expect a single person to do it all.

Can such interactions be systematically fostered, or must they always be fortuitous? It seems likely from the short history we have sketched that double-diffusive convection is a fundamental phenomenon which is likely to be acting in other contexts of which we are unaware – there are certainly chemical effects which we have not attempted to review, and some recent suggestions that related processes are important in biological systems (see, for example, Preston *et al.* 1980). How can or should such ideas be transferred to other fields more efficiently, and do those working on the fundamental aspects of a problem have any special responsibility to make it more widely known, in particular to scientists working in more applied areas?

There are very few scientists who are willing to take the considerable time needed to discuss their new work with others in very different fields and to probe deeply for any, as yet unknown, connections. To add to any natural reluctance, such scientists are sometimes accused of unfairly pushing their own work, or of becoming ‘popularizers’. Reluctantly, we conclude that there must be a large element of chance in the spread of ideas across disciplines. Desirable interactions do not arise if one of the parties is merely consulted on a particular problem – for example, the solution of a boundary-layer equation, the physical context of which is not made clear. Progress is also restricted if the background from which the problem arose is not understood and the consultant is unaware which aspects of the problem are important and which may be neglected. The most exciting interactions are those in which the several parties can both contribute to and gain from the solution to a problem of mutual interest. Many fields could benefit from more interaction with fluid dynamicists, and we should be alert to the potential application of our particular specialty to other fields. Realistically, however, only a fraction of the possible collaborative projects are likely to bear fruit, especially in fields where there is no obvious technological incentive to produce rapid solutions to particular problems.

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