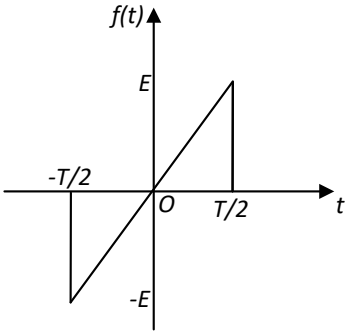
Problems for Signals and Systems

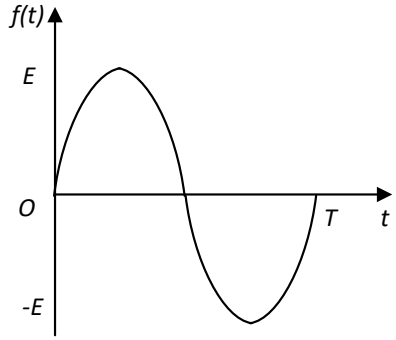
Chapter 4-2. Fourier Transform of Continuous Time Signal

- **Definition of Fourier Transform**

1. Determine the Fourier transforms of the sawtooth pulse and sinusoidal pulse shown in Figure 4.4.



(a)



(b)

Figure 4.4

- **Properties of Fourier Transform**

2. Determine the Fourier transforms of the following signals.

(1) $e^{2+t}u(-t)+1$;

(2) $e^{-3|t|}\sin 2t$;

(3) $[e^{-\alpha t}\cos\omega_0 t]\,u(t)$;

(4) $[te^{-2t}\sin 4t]u(t)$.

3. Determine the inverse Fourier transforms of $F(\omega)$ shown in Figure 4.5.

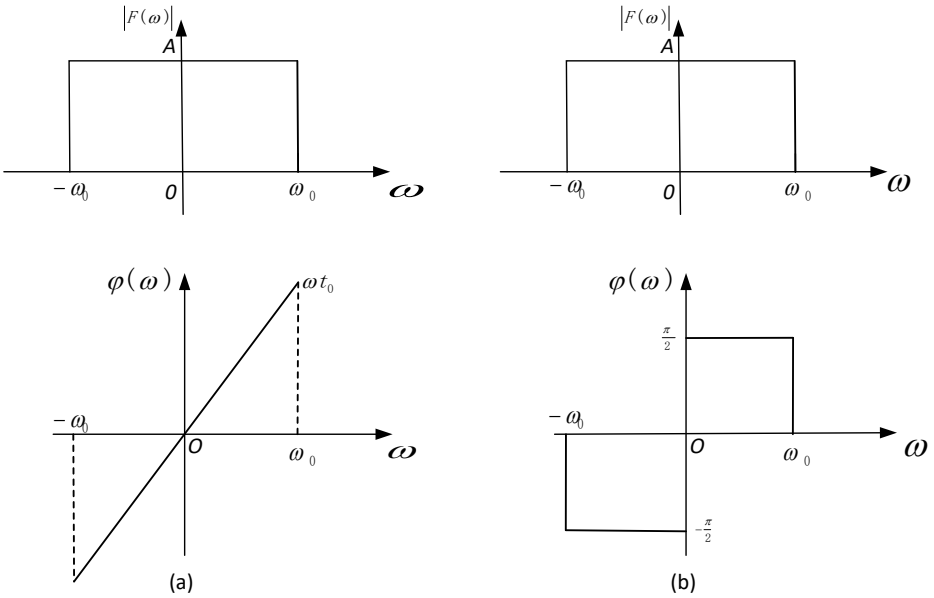


Figure 4.5

4. Given $F(\omega)$ is the frequency spectrum of $f(t)$, determine the frequency spectra of the following signals.

(a) $tf(2t)$;

(b) $t\frac{df(t)}{dt}$;

(c) $(t-2)f(-2t)$;

(d) $(1-t)f(1-t)$;

(e) $f(6-2t)$.

5. Determine the frequency spectrum of the triangular amplitude modulation(AM) signal shown in Figure 4.6.

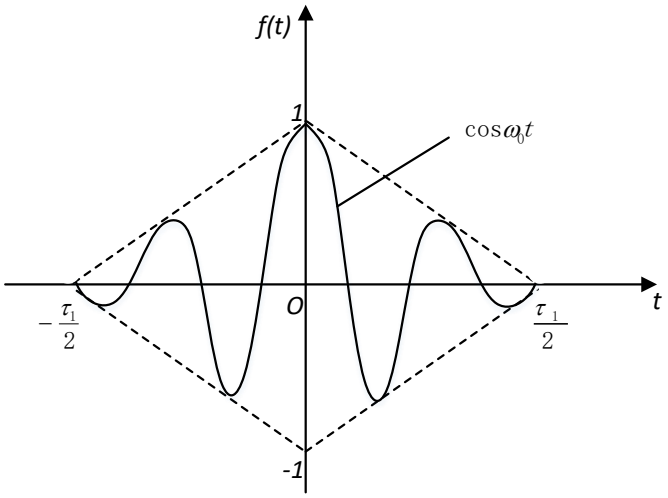


Figure 4.6

6. Determine the Fourier transform of the following signal.

$$\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$$

7. Determine the continuous time signal corresponding to each of the following transform.

(a) $F(\omega) = \frac{2\sin[3(\omega-2\pi)]}{(\omega-2\pi)}$;

(b) $F(\omega) = \cos(4\omega + \pi/3)$;

(c) $F(\omega) = 2[\delta(\omega-1) - \delta(\omega+1)] + 3[\delta(\omega-2\pi) + \delta(\omega+2\pi)]$.

8. Let $F(\omega)$ denote the Fourier transform of the signal $f(t)$ depicted in Figure 4.7,

(a) Find $\arg F(\omega)$;

(b) Find $\mathrm{Re}\{F(\omega)\}$;

(c) Find $F(0)$;

(d) Evaluate $\int_{-\infty}^{\infty} F(\omega)d\omega$;

(e) Evaluate $\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$;

(f) Evaluate $\int_{-\infty}^{\infty} F(\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}d\omega$

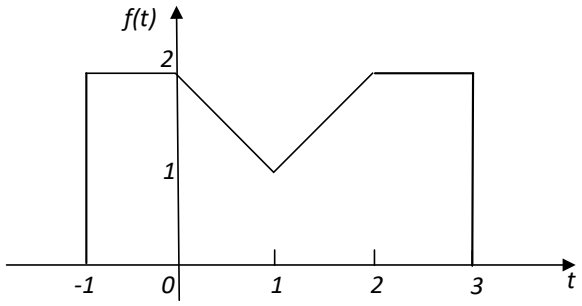


Figure 4.7

9. Use the energy equation

$$\int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

to calculate the following integral.

(a) $\int_{-\infty}^{\infty}(\frac{\sin t}{t})^2 dt$;

(b) $\int_{-\infty}^{\infty} \frac{dt}{(1+t^2)^2}$.

10. Determine the inverse transform of $F(\omega)$ shown in Figure 4.8.

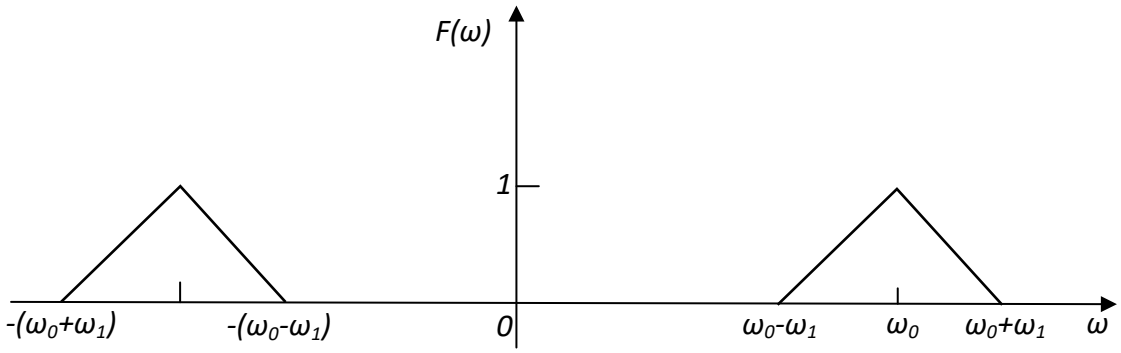


Figure 4.8