Problems for Signals and Systems

Chapter 10. Z Transform

- **Definition and Properties of Z Transform**

1. Determine the Z transform for each of the following sequences. Sketch the pole-zero plot and indicate the region of convergence. Indicate whether or not the Fourier transform of the sequence exists.

(a) $(\frac{1}{2})^n u(-n)$;

(b) $(\frac{1}{2})^{n-1} u(n-1)$.

(c) $(\frac{1}{2})^{|n|}$.

- **Inverse Z Transform**

3. Following are several Z transforms $X(z)$. For each one, determine the inverse Z transform $x(n)$.

(a) $X(z) = \frac{10}{(1-0.5z^{-1})(1-0.25z^{-1})}$, $|z| > 0.5$;

(b) $X(z) = \frac{10z^2}{(z-1)(z+1)}$, $|z| > 1$;

(c) $X(z) = \frac{z^{-1}}{(1-6z^{-1})^2}$, $|z| > 6$;

(d) $X(z) = \frac{1+z^{-1}}{1-2z^{-1}\cos\omega+z^{-2}}$, $|z| > 1$;

(e) $X(z) = \frac{z^{-2}}{1+z^{-2}}$, $|z| > 1$.

4. Plot the pole-zero pattern of $X(z) = \frac{-3z^{-1}}{2-5z^{-1}+2z^{-2}}$.

Determine $x(n)$ corresponding to each of the three ROCs, and indicate whether or not $x(n)$ is right-sided sequence, left-sided sequence, or two-sided sequence.

(a) $|z| > 2$;

(b) $|z| < 0.5$;

(c) $0.5 < |z| < 2$.

- **Solve Difference Equation Using Unilateral Z Transform**

5. Solve the following difference equations using unilateral Z transform:

(a) $y(n) + 0.1y(n-1) - 0.02y(n-2) = 10u(n)$, $y(-1) = 4$, $y(-2) = 6$;

(b) $y(n) - 0.9y(n-1) = 0.05u(n)$, $y(-1) = 1$;

- **Z-Domain Analysis**

6. Consider an LTI system with input $x(n)$ and output $y(n)$ for which

$$y(n-1) - \frac{5}{2}y(n) + y(n+1) = x(n).$$

The system may or may not be stable or causal.

(a) Determine the system function, and plot its pole-zero pattern.

(b) Determine three possible choices for the unit sample response $h(n)$ of the system.

(c) For each choice of $h(n)$, is this system stable? Is this system causal?

7. The following is known about a discrete-time LTI system with input $x(n)$ and output $y(n)$:

(1) If $x(n) = (-2)^n$ for all n, then $y(n) = 0$ for all n.

(2) If $x(n) = (\frac{1}{2})^n u(n)$ for all n, then $y(n)$ for all n is of the form

$$y(n) = \delta(n) + a(\frac{1}{4})^n u(n),$$

where a is a constant.

(a) Determine the value of the constant a.

(b) Determine the response $y(n)$ if the input $x(n)$ is

$$x(n) = 1, \quad \text{for all } n.$$

8. Consider an LTI system with input $x(n)$ and output $y(n)$ for which

$$y(n) - \frac{5}{2}y(n-1) + y(n-2) = 6x(n) - 7x(n-1) + 5x(n-2).$$

Realize the system using the direct-form II structure, and plot the block-diagram representation of this system.