Problems for Signals and Systems

Chapter 2 Time Domain Analysis of Continuous Time Systems

- **Solve Differential Equations**

1. Consider the corresponding homogeneous equations of system differential equations

$$\text{(a)}\ \frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+3y(t)=0$$

$$\text{(b)}\ \frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+y(t)=0$$

For each of the above two cases, if the initial state is $y(0^-)=0,\ y'(0^-)=2$, determine the zero-input response for each case.

2. Consider

$$\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}+5y(t)=\frac{d^2x(t)}{dt^2}-2\frac{dx(t)}{dt}+x(t)$$

$$x(t)=2e^{-2t}u(t),\ \ y(0^+)=-4,\ \ y'(0^+)=6,$$

Determine the complete response $y(t)$.

- **Convolution**

3. Compute the convolution $f_1(t)*f_2(t)$ of the following functions:

(a) $f_1(t)=f_2(t)=u(t)$;

(b) $f_1(t)=f_2(t)=u(t+\tau)-u(t-\tau)$;

(c) $f_1(t)=f_2(t)=u(t)-u(t-\tau)$;

(d) $f_1(t)=u(t+\tau)-u(t-\tau), f_2(t)=u(t+2\tau)-u(t-2\tau)$;

(g) $f_1(t)=u(t)-u(t-4),\ f_2(t)=\sin\pi t\cdot u(t)$。

4. Let $f_1(t)=u(t+1)-u(t-1),\ \ f_2(t)=\delta(t+5)+\delta(t-5),\ \ f_3(t)=\delta\left(t+\frac{1}{2}\right)+\delta\left(t-\frac{1}{2}\right)$, plot the waveforms of the following convolutions :

(a) $s_1(t) = f_1(t) * f_2(t)$;

(b) $s_2(t) = f_1(t) * f_2(t) * f_2(t)$;

(c) $s_3(t) = \{[f_1(t) * f_2(t)][u(t+5) - u(t-5)]\} * \ f_2(t)$;

(d) $s_4(t) = f_1(t) * f_3(t)$。

- **Properties of Systems**

5. (a) Consider an LTI system with input $x(t)$ and output $y(t)$ related through the equation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2) d\tau$$

What is the impulse response $h(t)$ for this system?

(b) Determine the response of the system when the input $x(t)$ is as shown in Figure 2.1.

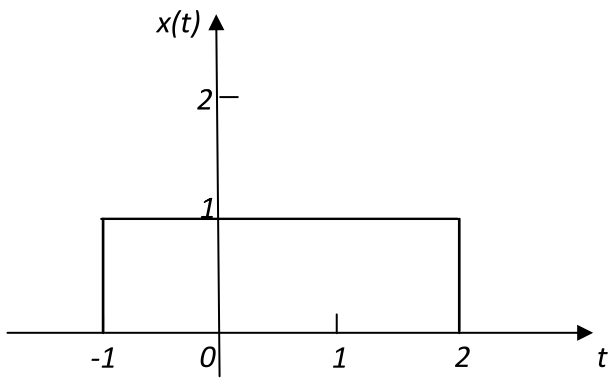

Figure 2.1

6. The system shown in Figure 2.2 is constituted by several subsystems, whose impulse responses are $h_a(t) = \delta(t-1)$, $h_b(t) = u(t) - u(t-3)$ respectively. Determine the impulse response $h(t)$ of the overall system.

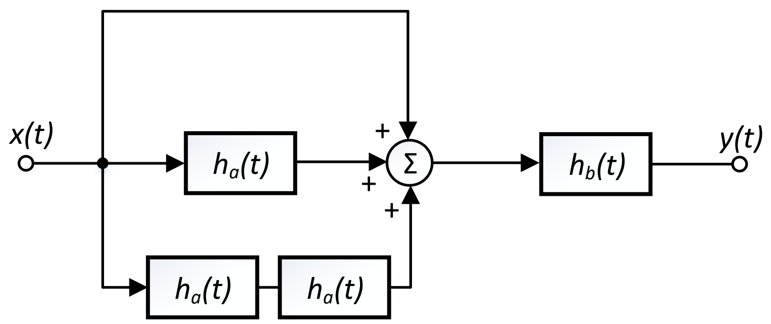

Figure 2.2

7. A LTI system's zero-state response $y_{zs}(t)$ to the excitation signal $\sin t \cdot u(t)$

is $y_{zs}(t) = \begin{cases} 1-|t-1|, & 0 \le t \le 2 \\ 0, & otherwise \end{cases}$, as shown in Figure 2.3

Determine the system's impulse response.

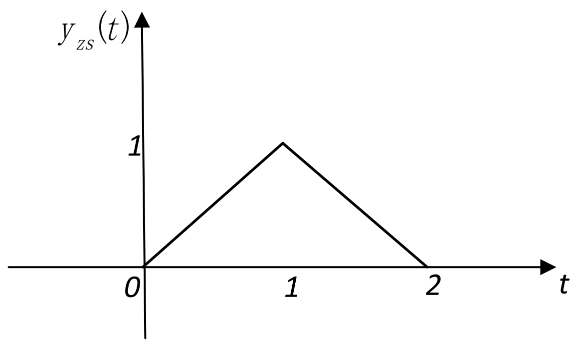

Figure 2.3