第 5 章作业

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摘　要： 摘要。

关键词： 词 1，词 2

Homework (Chapter 5)

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Abstract: Abstract.

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6. 解：LTI 系统:因果 $\Longleftrightarrow h[n]=0\ (\forall n<0)$,

LTI 系统:稳定 $\Longleftrightarrow \sum_{n=-\infty}^{+\infty}|h[n]| < +\infty$.

(a) ~~$h[-1]\neq 0$~~ $\Rightarrow$ 显然 因果； $\sum_n |h[n]| = \sum_{n=0}^{+\infty}\left(\frac{1}{2}\right)^n = \frac{1}{1-1/2} < \infty \Rightarrow$ 稳定；

(b) ~~$h[n]=0\ (\forall n$~~ 非因果； $\sum_n |h[n]| = \sum_{n=-\infty}^{0}\left(\frac{1}{2}\right)^n = \sum_{n=0}^{+\infty} 2^n \to +\infty$
$\Rightarrow$ 不稳定.

(c) $h[-1]\neq 0 \Rightarrow$ 非因果； $\sum_n |h[n]| = \sum_{n=-\infty}^{2} 3^n = \sum_{n=-2}^{+\infty}\left(\frac{1}{3}\right)^n < \infty \Rightarrow$ 稳定；

(d) $h[n]=0\ (\forall n<0) \Rightarrow$ 因果； $\sum_n |h[n]| = \sum_{n=0}^{+\infty} 3 \to +\infty \Rightarrow$ 不稳定.

7. 解：$h[n] = h_1[n] * (h_2[n] - h_3[n])$

$= u[n] * (\delta[n] - \delta[n-N]) = u[n] - u[n-N]$.

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$x[n] = x[n+N], \forall n\in\mathbb{Z}$. ($\exists N\in\mathbb{N}$,) (希望) $x[n] = \sum_k a_k e^{j\xi_k n}$, $e^{j\xi_k n}$ 具有 N 周期,

$\Longleftrightarrow \xi_k N = 2k\pi \Longleftrightarrow \xi_k = k\frac{2\pi}{N}$. $\therefore x[n] = \sum_k^{(N)} a_k e^{jk\frac{2\pi}{N}n} \Rightarrow a_k = \dfrac{\langle x, e^{jk\frac{2\pi}{N}n}\rangle}{\langle e^{jk\frac{2\pi}{N}n}, e^{jk\frac{2\pi}{N}n}\rangle}$

(最小二乘逼近, 法方程) $\Rightarrow a_k = \frac{1}{N}\sum_n^{(N)} x[n] e^{-jk\frac{2\pi}{N}n}$.

定义 $x_N[n] = \begin{cases} x[n], & n\in \text{主周期, e.g. } [-N/2, N/2] \text{ 取 } N \text{ 个}. \\ x_N[n+N], & \forall n. \end{cases}$ 则 $\forall n: \lim_{N\to+\infty} x_N[n] = x[n]$,

而 $x_N[n] = \sum_k^{(N)} e^{jk\frac{2\pi}{N}n} \frac{1}{N}\sum_m^{(N)} x[m] e^{-jk\frac{2\pi}{N}m}\, \frac{2\pi}{N}\cdot\frac{N}{2\pi} \xrightarrow{N\to+\infty} \int_{2\pi}\frac{1}{2\pi}\cdot e^{j\omega n}\, d\omega \sum_m^{(N)} x[m] e^{-j\omega m}$ $=x[n]$

定义 $X(\omega) := \mathcal{F}\boxed{\sum_n x[n] e^{-j\omega n} =: \mathcal{F}[x[n]]}$, 则 $\boxed{x[n] = \frac{1}{2\pi}\int_{2\pi} X(\omega) e^{j\omega n}\, d\omega} =: \mathcal{F}^{-1}[X(\omega)]$.

连续FT：$X(\omega) := \mathcal{F}[x(t)] := \int_{\mathbb{R}} x(t) e^{-j\omega t}\, dt$, $x(t) = \mathcal{F}^{-1}[X(\omega)] := \frac{1}{2\pi}\int_{\mathbb{R}} X(\omega) e^{j\omega t}\, d\omega$.

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1. 解：(a) $a^n u[n] \leftrightarrow \sum_{n=0}^{+\infty} a^n e^{-j\omega n} \overset{|a|<1}{=} \frac{1}{1-ae^{-j\omega}}$.

$\sin(\omega_0 n) = \frac{1}{2j}\cdot\left(e^{j\omega_0 n} - e^{-j\omega_0 n}\right)$. $\quad x[n]e^{j\omega_0 n} \leftrightarrow = \frac{1}{2\pi}\int_{2\pi} \underset{\omega'-\omega_0}{\underbrace{X(\omega)}} e^{j\overbrace{(\omega+\omega_0)}^{\omega'}n} \, d\underbrace{(\omega+\omega_0)}_{\omega'} \leftrightarrow X(\omega-\omega_0)$.

$\therefore\ a^n u[n]\cdot\sin(\omega_0 n) = a^n u[n]\cdot\frac{1}{2j}\left(e^{j\omega_0 n} - e^{-j\omega_0 n}\right) \leftrightarrow \frac{1}{2j}\left[\frac{1}{1-ae^{-j(\omega-\omega_0)}} - \frac{1}{1-ae^{-j(\omega+\omega_0)}}\right]$.

(b) $\delta[n] := \begin{cases} 1, & n=0, \\ 0, & \sim \end{cases} \leftrightarrow \sum_n \delta[n] e^{-j\omega n} = 1$.

$\sum_l \delta(\omega-2\pi l) \leftrightarrow \frac{1}{2\pi}\int_{2\pi} \sum_l \delta(\omega-2\pi l) e^{j\omega n} d\omega = 1/2\pi \Rightarrow \mathcal{F}[1] = 2\pi\sum_l \delta(\omega-2\pi l)$, i.e.

$\boxed{\sum_{n=-\infty}^{+\infty} e^{-j\omega n} = 2\pi\sum_l \delta(\omega-2\pi l)}$. 类比 CT: $\mathcal{F}[1] = \boxed{2\pi\delta(\omega) = \int_{\mathbb{R}} e^{-j\omega t} dt}$.

$\therefore\ \cos(\frac{18}{7}\pi n) \cancel{\leftrightarrow} = \left(e^{j\frac{18}{7}\pi n} + e^{-j\frac{18}{7}\pi n}\right)/2 \leftrightarrow \pi\sum_l \left[\delta(\omega-\frac{4}{7}\pi+2\pi l) + \delta(\omega+\frac{4}{7}\pi+2\pi l)\right]$

$\sin(2n) = \frac{1}{2j}\left(e^{j2n} - e^{-j2n}\right) \leftrightarrow \frac{\pi}{j}\sum_l \left[\delta(\omega-2+2\pi l) \mp \delta(\omega+2+2\pi l)\right]$.

$\therefore\ x[n] \leftrightarrow \pi\sum_l \left[\delta(\omega+\frac{4}{7}\pi+2\pi l) + \delta(\omega-\frac{4}{7}\pi+2\pi l) + j\delta(\omega+2+2\pi l) - j\delta(\omega-2+2\pi l)\right]$.

(c) $\sum_{k=0}^{+\infty} (\frac{1}{4})^n \delta[n-3k] = \sum_{k\geq 0} \frac{1}{4} \sim \leftrightarrow \sum_n e^{-j\omega n} \sum_{k=0}^{+\infty} (\frac{1}{4})^n \delta[n-3k]$

原式 $= \sum_{k=0}^{+\infty} (\frac{1}{4})^{3k} \sum_n e^{-j\omega n}\delta[n-3k] = \sum_{k=0}^{+\infty}\left(\frac{1}{4}e^{-j\omega}\right)^{3k} = \frac{1}{1-\frac{1}{64}e^{-3j\omega}}$.

(c) 另解：$x[n] := \sum_{k=0}^{+\infty} (\frac{1}{4})^n \delta[n-3k] \overset{\cancel{m=3k}}{=\!=} \begin{cases} (\frac{1}{4})^n, & 3|n,\ n\geq 0 \\ 0, & 其他. \end{cases}$

$\Rightarrow y[n] := x[3n] = (\frac{1}{4})^{3n} u[n]$, $\Rightarrow x[n] = \begin{cases} y[n/3], & n\in 3\mathbb{Z}, \\ 0, & 其他. \end{cases}$

$Y(\omega) = 1/[1-\frac{1}{64}e^{-j\omega}]$.

$\Rightarrow X(\omega) := \sum_n x[n]e^{-j\omega n} = \sum_{n\in 3\mathbb{Z}} y[n/3]e^{-j\omega n} \overset{n=3k}{=\!=} \sum_k y[n]e^{-j\omega 3k} = Y(3\omega)$

$= 1/[1-\frac{1}{64}e^{-j3\omega}]$.

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2. 解: (a) $x[n-n_0] = \frac{1}{2\pi}\int_{2\pi} X(\omega) e^{j\omega(n-n_0)} d\omega \leftrightarrow X(\omega)e^{-j\omega n_0}$.

$1 \leftrightarrow \mathcal{F}[1] = \sum_n e^{-j\omega n} = 2\pi\sum_l \delta(\omega-2\pi l)$, $\delta[n] \leftrightarrow \sum_n \delta[n]e^{-j\omega n} = 1$.

$\therefore x[n] = \mathcal{F}^{-1}[X(\omega)] = \delta[n] - 2\delta[n-3] + 4\delta[n-2] + 3\delta[n-6]$.

(b) $x[n] = \frac{1}{2\pi}\int_{0-0}^{2\pi-0} X(\omega)e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{0-0}^{2\pi-0} e^{-j\omega n}d\omega \cdot \sum_k (-1)^k \delta(\omega-\frac{\pi}{2}k)$

$= \frac{1}{2\pi}\int_{0-0}^{2\pi-0} d\omega \cdot \sum_{k=0}^{3}(-1)^k e^{-jk\frac{\pi}{2}n}\delta(\omega-\frac{\pi}{2}k) = \frac{1}{2\pi}\left[\sum_{k=0}^{3} (-1)^k e^{-jk\frac{\pi}{2}n}\right.$

$= \frac{1}{2\pi}[1-(-j)^n + (-1)^n - (j)^n]$.

(c) $\cos^2\omega = \frac{1}{2} + \frac{1}{2}\cos(2\omega) = \frac{1}{2}+\frac{1}{4}(e^{j2\omega}+e^{-j2\omega})$.

$\leftrightarrow \frac{1}{2}\delta[n] + \frac{1}{4}(\delta[n+2]+\delta[n-2])$.

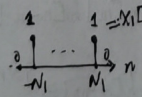
3. 解: (a) $X(\omega) \xlongequal{u=e^{-j\omega}} \frac{-6u}{u^2-u-6} = \frac{-18/5}{u-3} + \frac{12/(-5)}{u+2} = \frac{6}{5}\frac{1}{1-\frac{1}{3}e^{-j\omega}} - \frac{6}{5}\frac{1}{1+\frac{1}{2}e^{-j\omega}}$

$\leftrightarrow x[n] := \frac{6}{5} u[n]\left((\frac{1}{3})^n - (-\frac{1}{2})^n\right)$.

$[\ \alpha^n u[n] \leftrightarrow \sum_{n=0}^{+\infty}(\alpha e^{-j\omega})^n = \frac{1}{1-\alpha e^{-j\omega}},\ |\alpha|<1\ ]$

(b) $1 \leftrightarrow 2\pi\sum_m \delta(\omega-2\pi m)$, $e^{j\omega_0 n}x[n] \leftrightarrow \sum_n x[n]e^{-j(\omega-\omega_0)n} = X(\omega-\omega_0)$.

$\therefore x[n] \cancel{\leftrightarrow} = 1+\frac{1}{2}\left(e^{j\frac{\pi}{2}n}+e^{-j\frac{\pi}{2}n}\right) = 1+\cos(\frac{\pi}{2}n)$.

(c) 

$\leftrightarrow X_1(\omega) = \sum_{-N_1}^{N_1} e^{-j\omega n} = \frac{e^{j\omega N_1}(1-e^{-j\omega(2N_1+1)})}{1-e^{-j\omega}}$

$= \frac{2j\sin[\omega(N_1+\frac{1}{2})]}{2j\sin(\omega/2)} = \begin{cases} \frac{\sin[\omega(N_1+\frac{1}{2})]}{\sin(\omega/2)}, & \omega\neq 0, \\ \sum_n x[n] = 2N_1+1, & \omega = 0. \end{cases}$

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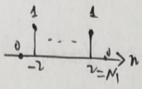
$\Rightarrow \quad x_1[n; N_1=2] =$  $\leftrightarrow X_1(\omega; N_1=2) = \begin{cases} \dfrac{\sin(\frac{5}{2}\omega)}{\sin(\omega/2)}, & \omega \neq 0, \\ \sum_n x_1[n] = 5, & \omega = 0. \end{cases}$

$$x_2[n] := \sum_{m=-\infty}^{n} x_1[n; N_1=2] \leftrightarrow X_2(\omega).$$

$$\Rightarrow \quad x_2[n] - x_2[n-1] = x_1[n; N_1=2] \leftrightarrow (1-e^{-j\omega}) X_2(\omega) = X_1(\omega; N_1=2)$$

$$\Rightarrow \quad X_2(\omega) = \frac{X_1(\omega; N_1=2)}{1-e^{-j\omega}} + \overline{x_2} \cdot 2\pi \sum_l \delta(\omega - 2\pi l),$$

$$\overline{x_2} = \lim_{N\to+\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \sum_{m=-\infty}^{+\infty} x_1[m; N_1=2]\, u[n-m] = \lim_{N\to+\infty} \sum_m \frac{N-m+1}{2N+1} X_1(\omega)\Big|_{\omega=0} = \frac{1}{2} X(\omega)$$

$$\Rightarrow \quad X_2(\omega) = \frac{X_1(\omega; N_1=2)}{1-e^{-j\omega}} + \pi X_1(0)\, \delta(\omega), \quad |\omega| < \pi$$

$$= \frac{1}{1-e^{-j\omega}} \frac{\sin\frac{5\omega}{2}}{\sin\frac{\omega}{2}} + 5\pi \cdot \delta(\omega), \quad |\omega| < \pi.$$

$$\therefore \quad X(\omega) = X_2(\omega) - 2\pi\delta(\omega) \overset{|\omega|<\pi}{\leftrightarrow} x_2[n] - 1 = \sum_{m=-\infty}^{n} x_1[m; N_1=2] - 1,$$

$$x_2[n] := \sum_{m=-\infty}^{n} x_1[n; N_1=2] = \sum_{m=-\infty}^{n} \left[\delta[m+2] + \dots + \delta[m-2] \right]$$

$$= u[n+2] + \dots + u[n-2]. \qquad \left[\sum_{m=-\infty}^{n} \delta[m] = u[n] \right]$$

$$\therefore \quad x[n] = u[n+2] + \dots + u[n-2] - 1.$$

4. 解：$h[n] = \left(\frac{1}{2}\right)^n u[n] \leftrightarrow H(\omega) := \sum_n h[n] e^{-j\omega n} = \sum_{n=0}^{+\infty} \left(\frac{1}{2} e^{-j\omega}\right)^n = 1/\left[1 - \frac{1}{2} e^{-j\omega}\right].$

$$x[n] = x[n] * \delta[n] = \sum_m x[m]\,\delta[n-m] \xrightarrow{LSI} y_{zs}[n] = \sum_m x[m]\, h[n-m] = x[n] * h[n].$$

$$\therefore \quad Y(\omega) := \mathcal{F}[y_{zs}[n]] = \sum_n e^{-j\omega n} \sum_m x[m]\, h[n-m] = \sum_m x[m] e^{-j\omega m} \sum_n h[n-m] e^{-j\omega(n-m)} = X(\omega) H(\omega)$$

4(a) $X(\omega) = 1/[1-\frac{3}{4}e^{-j\omega}] \Rightarrow Y(\omega) = H(\omega)X(\omega) = \dfrac{1}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{3}{4}e^{-j\omega})}$

$= \dfrac{1/(-\frac{1}{2})}{1-\frac{1}{2}e^{-j\omega}} + \dfrac{1/(1-2/3)}{1-\frac{3}{4}e^{-j\omega}} \Leftrightarrow y_{zs}[n] = \left(-2\cdot(\frac{1}{2})^n + 3(\frac{3}{4})^n\right)u[n].$

(b) $x[n] = (-1)^n = e^{j\pi n} \Leftrightarrow X(\omega) = 2\pi\sum_m \delta(\omega-\pi+2\pi m).$

$\Rightarrow Y(\omega) = H(\omega)X(\omega) = \dfrac{2\pi\sum_m \delta(\omega-\pi+2\pi m)}{1-\frac{1}{2}e^{-j\omega}} = 2\pi\sum_m \dfrac{\delta(\omega-\pi+2\pi m)}{1-\frac{1}{2}e^{j\pi}}$

$= \frac{2}{3}\cdot 2\pi\sum_m \delta(\omega-\pi+2\pi m) \Longleftrightarrow y_{zs}[n] = \frac{2}{3}e^{j\pi n} = \frac{2}{3}(-1)^n.$

5. 另解：(a) $y_{zs}[n] = x[n]*h[n] = \sum_m (\frac{3}{4})^m u[m](\frac{1}{2})^{n-m}u[n-m]$

$= (\frac{1}{2})^n\sum_{m=0}^{n}(\frac{3}{2})^m u[n] = (\frac{1}{2})^n\cdot\dfrac{1-(\frac{3}{2})^{n+1}}{1-\frac{3}{2}}u[n] = u[n]\cdot\left[-2(\frac{1}{2})^n + 3(\frac{3}{4})^n\right]$

(b) $y_{zs}[n] = x[n]*h[n] = \sum_m (-1)^{n-m}(\frac{1}{2})^m u[m] = (-1)^n\sum_{m=0}^{+\infty}(-\frac{1}{2})^m$

$= (-1)^n\dfrac{1}{1-(-1/2)} = \frac{2}{3}(-1)^n.$

6. 解：$\sum_k a_k y[n-k] = \sum_k b_k x[n-k] \xRightarrow{\mathcal{F}} Y(\omega)\sum_k a_k e^{-jk\omega} = X(\omega)\sum_k b_k e^{-jk\omega}$

$\Rightarrow H(\omega) = \dfrac{Y(\omega)}{X(\omega)} = \dfrac{\sum b_k e^{-jk\omega}}{\sum a_k e^{-jk\omega}} = \dfrac{1+\frac{1}{3}e^{-j\omega}}{1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-j2\omega}}$ 1/2 1/4 × −1 −1

$= \cancel{1-e}\ \dfrac{(1+\frac{2}{3})/(1-\frac{1}{2})}{1-\frac{1}{2}e^{-j\omega}} + \dfrac{(1+\frac{4}{3})/(1-2)}{1-\frac{1}{4}e^{-j\omega}}.$

References