Problems for Signals and Systems

Chapter 4-3. Frequency Domain Analysis of Continuous Time System

- **Frequency Response**

1. For the system as shown in Figure 4.9, $H_1(\omega)$ has the property of ideal low-pass filter

$$H_1(\omega)=\begin{cases} e^{-j\omega t_0}, & |\omega|\le 1 \\ 0, & |\omega|>1 \end{cases}$$

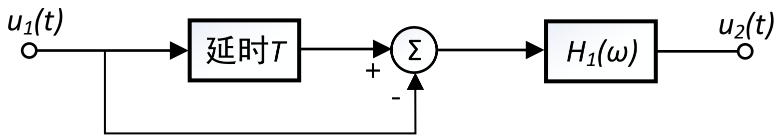


Figure 4.9

(a) If $u_1(t)$ is the unite step signal $u(t)$, write down the expression of $u_2(t)$;

(b) If $u_1(t)=\frac{2\sin(t/2)}{t}$, write down the expression of $u_2(t)$.

2. Consider a causal LTI system with frequency response $H(j\omega)=-2j\omega$. Calculate the system's zero-state responses $y_{zs}(t)$ to the following input signals.

(a) $x(t)=e^{jt}$;

(b) $x(t)=\sin\omega_0 t\cdot u(t)$

3. If a LTI system's frequency response $H(j\omega)$ is the same as that in Question 2, given the following frequency spectra of input signals, determine the system's zero-state response $y_{zs}(t)$:

(a) $X(j\omega)=\frac{1}{j\omega(6+j\omega)}$;

(b) $X(j\omega)=\frac{1}{2+j\omega}$.

- **Filter**

4. Consider a continuous time system with the frequency response $H(j\omega) = \frac{j\omega}{3\pi}, \quad -3\pi < \omega < 3\pi$, it is called low pass differentiator. For each of the following input signals $x(t)$, determine the system's output $y(t)$:

(a) $x(t) = \cos(2\pi t + \theta)$;

(b) $x(t) = \cos(4\pi t + \theta)$.

5. The amplitude property and phase property of an ideal band-pass filter are shown in Figure 4.10.

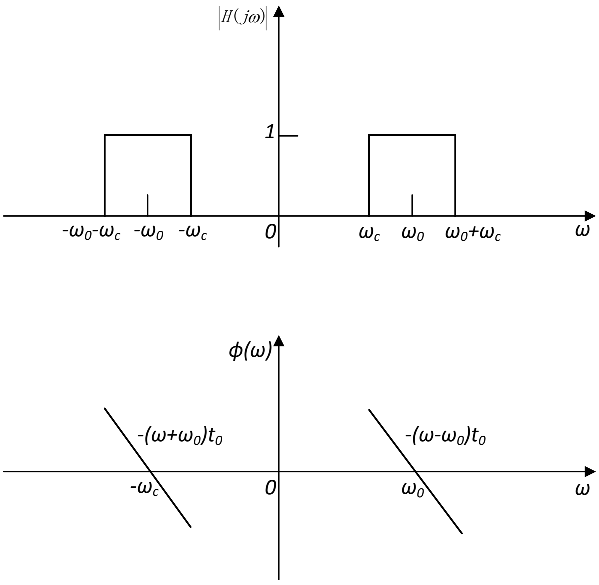


Figure 4.10

(a) Determine the system's unit impulse response. Plot its waveform, and discuss whether or not the system is physically attainable;

(b) If $\omega_0 = 2\omega_c$, when the excitation is $x(t) = \mathrm{S}a^2(\frac{\omega_c t}{2})\cos\omega_0 t$, determine the response $y(t)$ of the filter.

- **Modulation**

6. In Figure 4.11, an synchronous demodulation system is shown with input signal $f(t) = g(t)\cos(\omega_c t)$. When carrier phase θ_c is random, prove that $w(t)$ can be denoted as $w(t) = \frac{1}{2} g(t)\cos\theta_c + \frac{1}{2} g(t)\cos(2\omega_c t + 2\theta_c)$ .Does g(t) can be recovered from this demodulation system now?

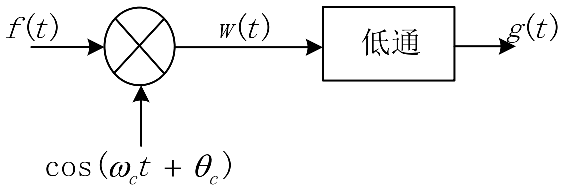


Figure 4.11

7. Figure 4.12 shows an amplitude modulation system, which consists of two parts: the sum of modulation signal and carrier get squared firstly, then the modulated signal is obtained through a band-pass filter. If $g(t)$ is band-limited, i.e., $G(\omega) = 0,\ |\omega| > \omega_m$. Determine the band-pass filter's parameters A, ω_L and ω_H that make $f(t) = g(t)\cos\omega_c t$, and clearly state the constraints on ω_c and ω_m.

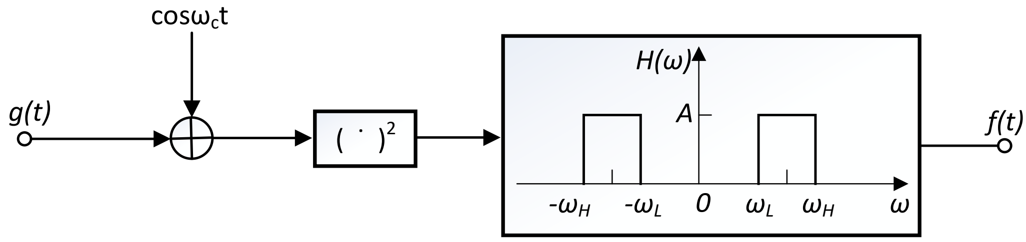
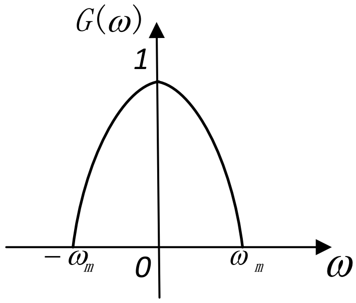
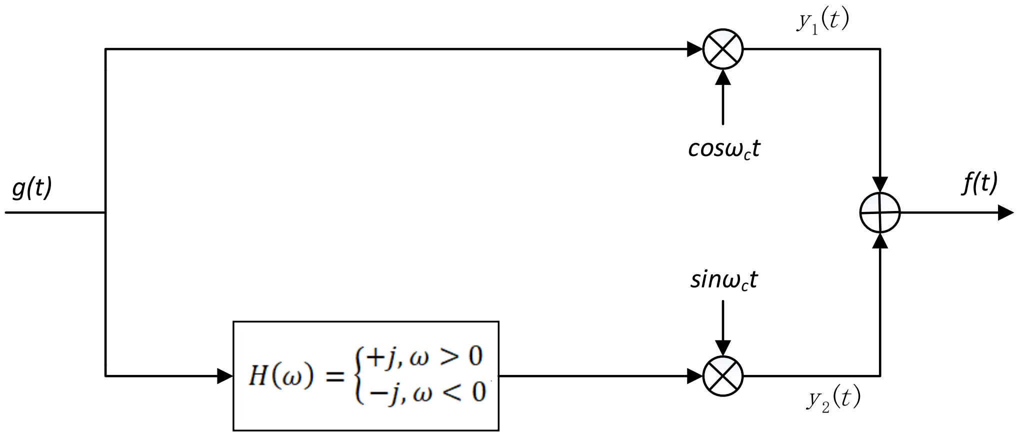


Figure 4.12

8. Figure 4.13(b) shows a sinusoidal carrier modulation system. The frequency spectrum $G(\omega)$ of the modulation signal $g(t)$ is shown in Figure 4.13(a).



(a)



(b)

Figure 4.13

(a) Plot frequency spectra $Y_1(\omega)$, $Y_2(\omega)$ and $F(\omega)$ for $y_1(t)$, $y_2(t)$ and $f(t)$ respectively.

(b) Explain that the system is a single side band (SSB) modulation system and point out which side band is going to be retained between the upper and lower side band.

- **Sampling**

9. Identify the minimum sampling frequency and Nyquist period for the following signals:

(1) $1 + \cos(2000\pi t) + \sin(4000\pi t)$;

(2) $(Sa)^2(100t)$.

10. Signal $f(t)$ is convoluted from two band-limited signals $f_1(t)$ and $f_2(t)$, i.e., $y(t) = f_1(t) * f_2(t)$, and

$F_1(\omega) = \mathfrak{F}[f_1(t)] = 0,\ \ |\omega| > 1000\pi$; $F_2(\omega) = \mathfrak{F}[f_2(t)] = 0,\ \ |\omega| > 2000\pi$.

Make impulse-train sampling on $y(t)$ and obtain

$$y_s(t) = \sum_{n=-\infty}^{\infty} y(nT)\delta(t - nT)$$

Determine the range of sampling period T that ensures $y(t)$ can be recovered from $y_s(t)$.

- **Differential Equation and Frequency Response**

11. If a LTI system's zero-state response to the excitation $x(t) = [e^{-t} + e^{-3t}]u(t)$ is $y(t) = [2e^{-t} - 2e^{-4t}]u(t)$

(a) Determine the frequency response of the system;

(b) Determine the impulse response of the system;

(c) Determine the differential equation of the system.

12. The output $y(t)$ and input $x(t)$ of a causal LTI system is combined by the following equation:

$$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau - x(t)$$

in which $z(t) = e^{-t}u(t) + 3\delta(t)$.

(a) Determine the system's frequency response $H(j\omega) = Y(j\omega)/X(j\omega)$. Plot the amplitude and phase response of $H(j\omega)$.

(b) Determine the system's impulse response.