Problems for Signals and Systems

Chapter 9. Laplace Transform

- **Definition and Properties of Laplace Transform**

1. Determine the Laplace transform and the associated region of convergence and pole-zero plot for each of the following functions of time (教材 9.21 题)

(a) $x(t)=e^{-2t}u(t)+e^{-3t}u(t)$ (b) $x(t)=e^{-4t}u(t)+e^{-5t}(\sin 5t)u(t)$

(c) $x(t)=e^{2t}u(-t)+e^{3t}u(-t)$ (d) $x(t)=te^{-2|t|}$

(e) $x(t)=|t|e^{-2|t|}$ (f) $x(t)=|t|e^{2t}u(-t)$

(g) $x(t)=\begin{cases}1, & 0\leqslant t\leqslant 1\\ 0, & \text{其余 } t\end{cases}$ (h) $x(t)=\begin{cases}t, & 0\leqslant t\leqslant 1\\ 2-t, & 1\leqslant t\leqslant 2\end{cases}$

(i) $x(t)=\delta(t)+u(t)$ (j) $x(t)=\delta(3t)+u(3t)$

2. Determine the Laplace transform for each of the following functions of time:

(a) $e^{-t}\sin 2t\, u(t)$;

(b) $te^{-(t-2)}u(t-1)$.

(c) $t[u(t-1)-u(t-2)]$.

(d) $\sin 2t\cdot u(t-1)$.

- **The Inverse Laplace Transform**

3. Determine the function of time x(t) for each of the following Laplace transforms and their associated regions of convergence (教材 9.22 题)

(a) $\frac{1}{s^2+9}$, $\mathcal{Re}\{s\}>0$ (b) $\frac{s}{s^2+9}$, $\mathcal{Re}\{s\}<0$

(c) $\frac{s+1}{(s+1)^2+9}$, $\mathcal{Re}\{s\}<-1$ (d) $\frac{s+2}{s^2+7s+12}$, $-4<\mathcal{Re}\{s\}<-3$

(e) $\frac{s+1}{s^2+5s+6}$, $-3<\mathcal{Re}\{s\}<-2$ (f) $\frac{(s+1)^2}{s^2-s+1}$, $\mathcal{Re}\{s\}>\frac{1}{2}$

(g) $\frac{s^2-s+1}{(s+1)^2}$, $\mathcal{Re}\{s\}>-1$

4. Determine the inverse Laplace transform for each of the following functions:

(a) $\frac{3s}{(s+4)(s+2)}, Re[s]>-2$;

(b) $\frac{s+3}{(s+1)^3(s+2)}, Re[s] > -1$;

(c) $\frac{e^{-s}}{4s(s^2+1)}, Re[s] > 0$.

5. Determine the inverse Laplace transform for each of the following functions:

(a) $\frac{1}{(s^2+3)^2}, Re[s] > 0$.

- **Solve Differential Equations Using Unilateral Laplace Transform**

6. Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = \frac{d}{dt}x(t) + 4x(t).$$

Determine the zero-input response and zero-state response under each of the following conditions.

(a) $x(t) = u(t)$, $y(0^-) = 0$, $y'(0^-) = 1$;

(b) $x(t) = e^{-2t}u(t)$, $y(0^-) = 1$, $y'(0^-) = 1$.