Problems for Signals and Systems

Chapter 9. Laplace Transform

- **S-Domain Analysis**

7. The signal

$$y(t) = e^{-2t}\ u(t)$$

is the output of a causal all-pass system for which the system function is

$$H(s) = \frac{s-1}{s+1}.$$

(a) Find and sketch at least two possible inputs $x(t)$ that could produce $y(t)$.

(b) What is the input $x(t)$ if it is known that

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty\ ?$$

(c) What is the input $x(t)$ if it is known that a stable (but not necessarily causal) system exists that will have $x(t)$ as an output if $y(t)$ is the input? Find the impulse response $h(t)$ of this system.

8. The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}.$$

Determine and sketch the response $y(t)$ when the input is

$$x(t) = e^{-|t|}, \qquad -\infty < t < \infty.$$

9. Determine the system functions $H(s) = \frac{U_2(s)}{U_1(s)}$ of the circuits shown in Figure 9.1

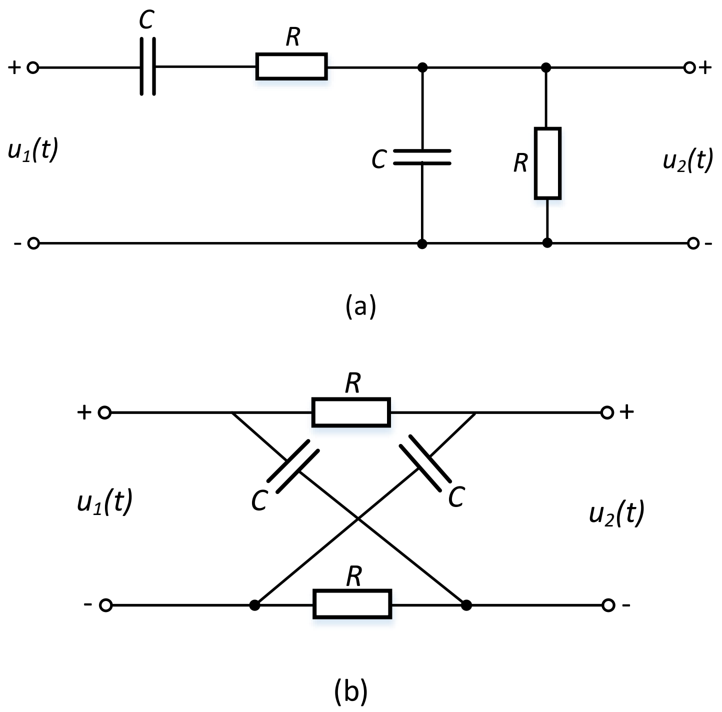

Figure 9.1

10. Consider the following driving-point impedance functions, and plot the corresponding circuits.

(a) $s+\frac{1}{s}$;

(b) $\frac{s}{s^2+s+1}$.

11. An LTI system has the same initial state in the following conditions. When the excitation is $x_1(t)=\delta(t)$, the total response is $y_1(t)=\delta(t)+e^{-t}u(t)$; when the excitation is $x_2(t)=u(t)$, its total response is $y_2(t)=3e^{-t}u(t)$.

(a) Determine the total response of this system for the excitation $x_3(t)=e^{-2t}u(t)$.

(b) Determine the total response of this system for the excitation $x_4(t)=2u(t-1)$.

12. Consider the LTI system shown in Figure 9.2 for which we are given the following information:

$$X(s) = \frac{s+2}{s-2},$$

$$x(t) = 0, \quad t > 0,$$

and

$$y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t).$$

(a) Determine $H(s)$ and its region of convergence.

(b) Determine $h(t)$.

(c) Using the system function $H(s)$ found in part (a), determine the output $y(t)$ if the input is

$$x(t) = e^{3t}, \qquad -\infty < t < \infty.$$

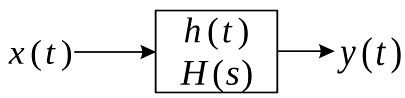

Figure 9.2

13. Given that the system function is

$$H(s) = \frac{s}{s^2+3s+2}.$$

(a) Determine the response of this system for the input $x(t) = 10 \cdot u(t)$, and point out the natural response component and the forced response component respectively.

(b) Determine the response of this system for the input $x(t) = 10\sin t \cdot u(t)$, and point out the natural response component and the forced response component respectively.

14. Suppose that we are given the following information about an LTI system:

(1) The system function $H(s)$ has a zero at $z = 0$, and a pair of conjugate poles at $p_1 = -1 + j\frac{\sqrt{3}}{2}$, $p_2 = -1 - j\frac{\sqrt{3}}{2}$;

(2) The value of the impulse response at $t=0^{+}$ is 2.

Determine the steady-state response of this system for the input

$$x(t) = \sin\frac{\sqrt{3}}{2}t \cdot u(t).$$

15. The system function $H(s)$ has the pole-zero pattern shown in Figure 9.3.

Plot the magnitude and phase of the frequency response.

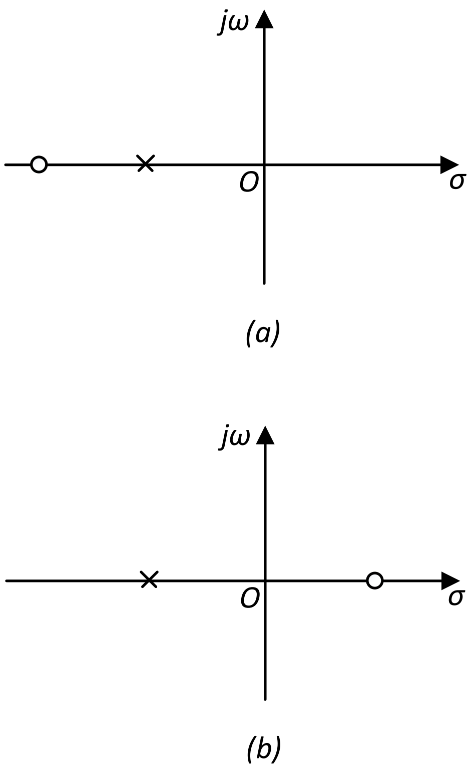

Figure 9.3

16. Suppose a system is characterized by the differential equation

$$\frac{d^3}{dt^3}y(t) + 7\frac{d^2}{dt^2}y(t) + 10\frac{d}{dt}y(t) = 5\frac{d}{dt}x(t) + 5x(t).$$

Using three different forms, plot the flow graph representation of the system.

17. Given that the system function is

$$H = \frac{Y}{X} = \frac{H_5[1-(G_1+G_2H_3+G_3)+G_1G_3]+H_1H_2(1-G_3)+H_1H_3H_4}{1-(G_1+G_2H_3+G_3)+G_1G_3}.$$

Plot the flow graph representation of this system.