Chapter 5 The Discrete-Time Fourier Transform

5.1 THE DISCRETE-TIME FORURIER TRANSFORM

- **Development**
- **Convergence Issues**
- **Examples**

5.2 PROPERTIES OF THE DISCRETE-TIME FORURIER TRANSFORM

5.3 THE CONVOLUTION PROPERTY AND FEQUENCY RESPONSE

5.4 LINEAR CONSTANT-COEFFICIENT DIFFERENCE EQUATION

5.5 DISCRETE-TIME PROCESSING OF CONTINUOUS-TIME SIGNALS

5.1 THE DISCRETE-TIME FORURIER TRANSFORM

- Development
- Convergence Issues
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$$x[n]=\frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$X(e^{j\omega})=\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

基本信号$e^{j\omega n}$关于ω以2π为周期

$X(e^{j\omega})$ **is periodic with period** 2π

- $X[e^{j(\omega+2\pi k)}]=X(e^{j\omega})$

注：低频出现在 π 的偶数倍，高频出现在 π 的奇数倍

- $$x[n]=\frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

注：上式积分区间为2π，例：（$-\pi$，π）或（0，2π）

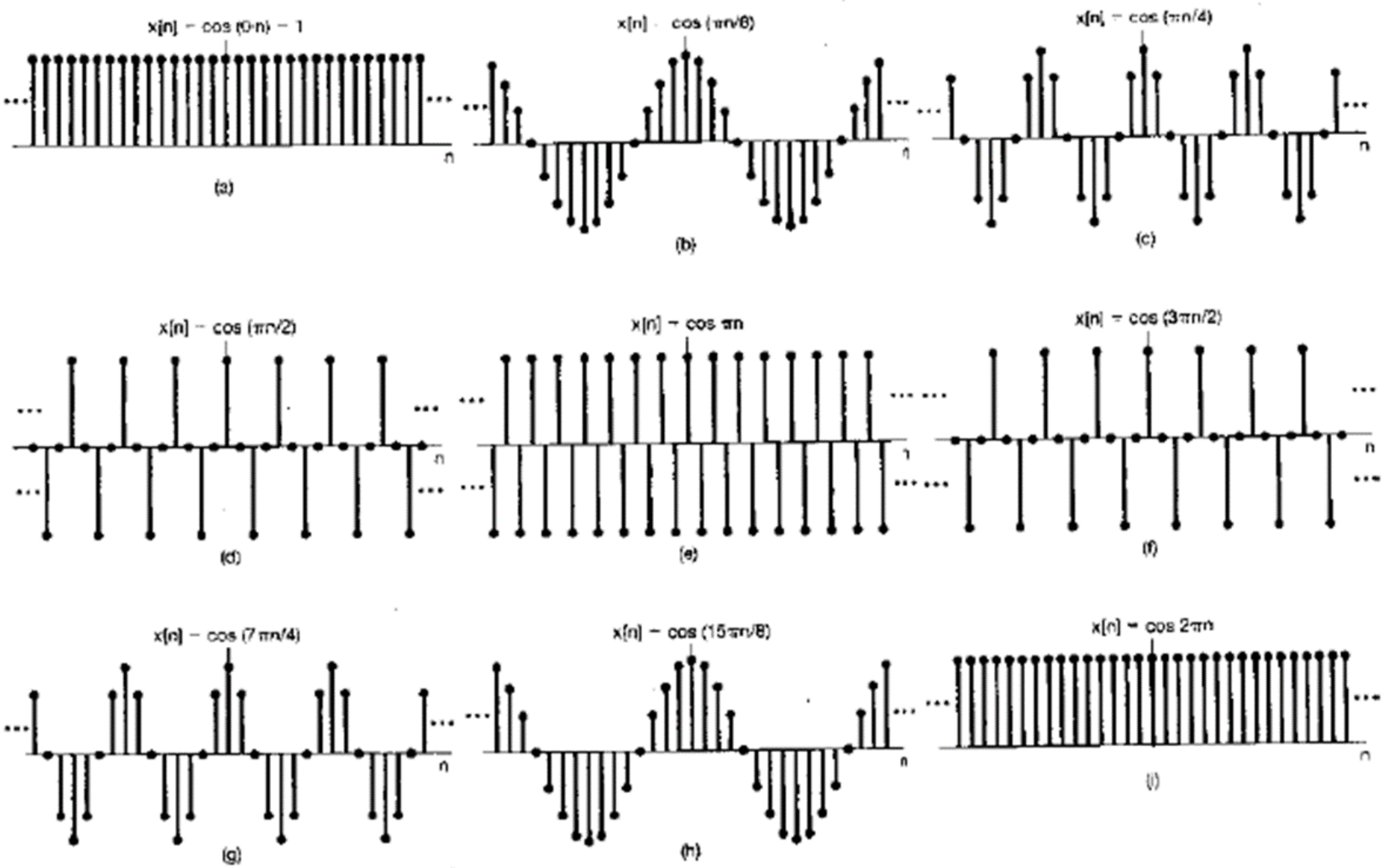
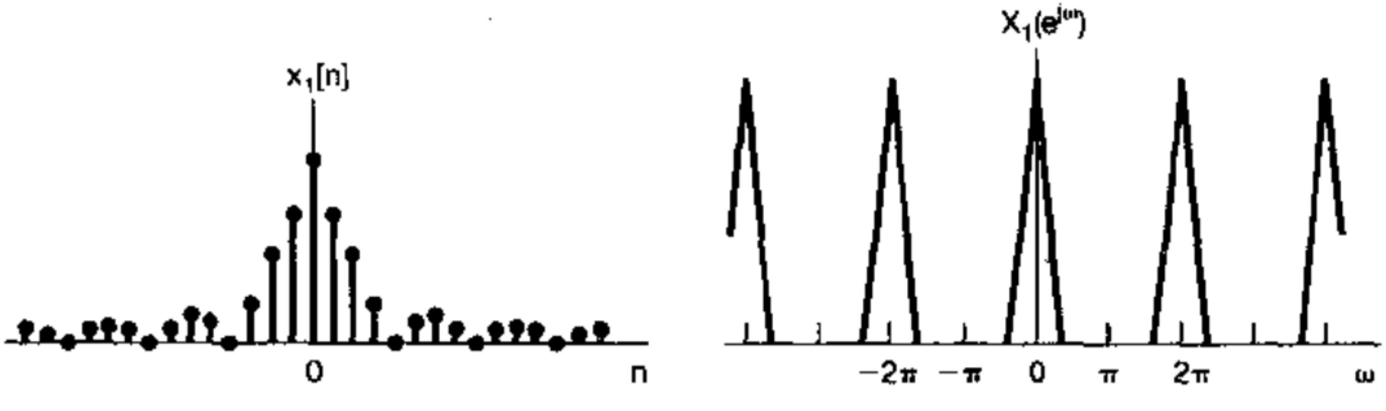
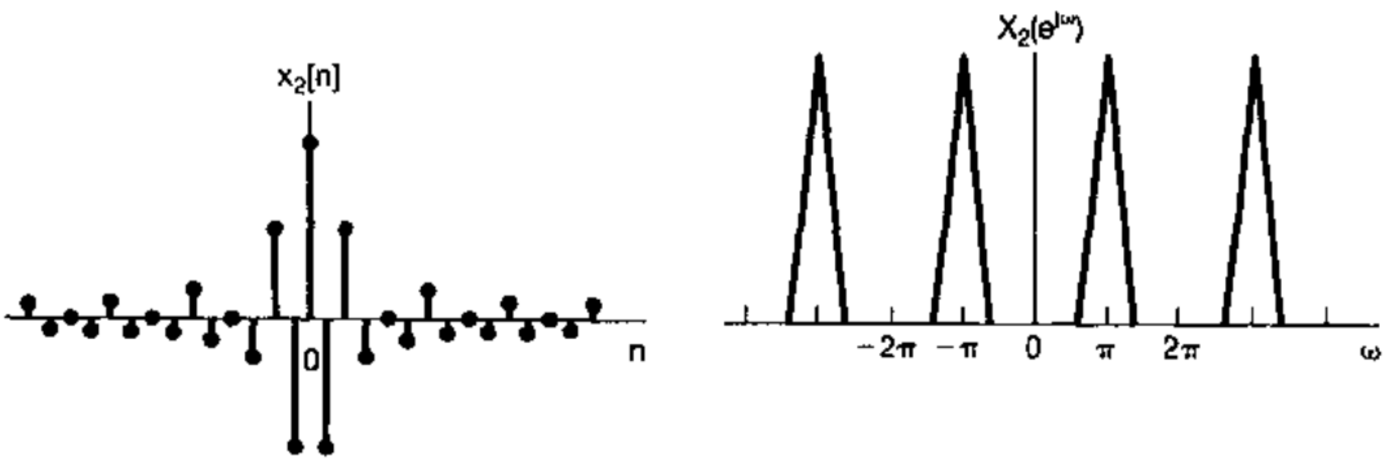


图 1.27　对应于几个不同频率时的离散时间正弦序列


$x_1[n]$
0
n
$X_1(e^{j\omega})$
-2π
$-\pi$
0
π
2π
ω

(a)

(b)


$x_2[n]$
0
n
$X_2(e^{j\omega})$
-2π
$-\pi$
0
π
2π
ω

(c)

(d)

5.1 THE DISCRETE-TIME FORURIER TRANSFORM

- Development
- Convergence Issues
- Examples

$\sum_n x[n]e^{-j\omega n}$ will converge either if x[n] is absolutely summable or has finite energy

- 能量有限，即

$$\sum_n |x[n]|^2 < \infty$$

- 绝对可和，即

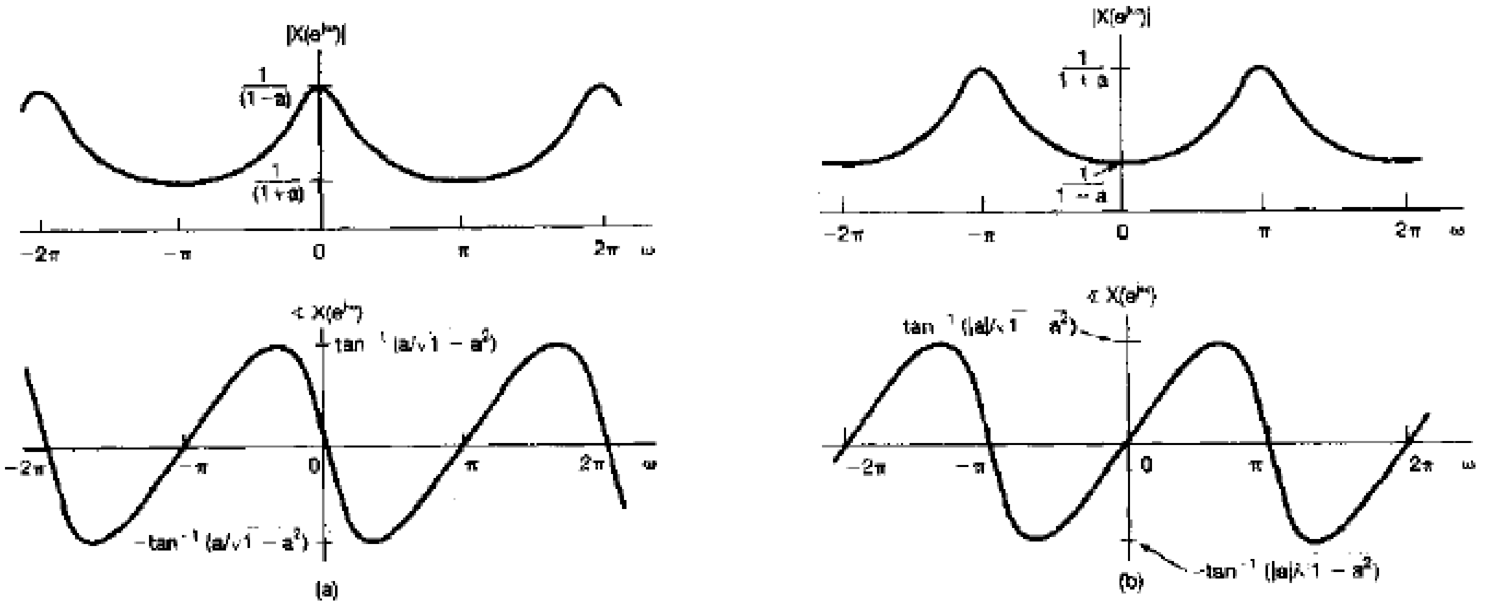
$$\sum_n |x[n]| < \infty$$

5.1 THE DISCRETE-TIME FORURIER TRANSFORM

- Development
- Convergence Issues
- Examples

①**Exponential Signal**

$$x[n]=\alpha^n u[n] \leftrightarrow X(e^{j\omega})=\frac{1}{1-\alpha e^{-j\omega}} \qquad |\alpha|<1$$



(a) α > 0 (b) α < 0

②Unit Impulse

$$x[n]=\delta[n]\leftrightarrow X(e^{j\omega})=1$$

③Dc Signal

$$x[n]=1\leftrightarrow X(e^{j\omega})=2\pi\sum_{l=-\infty}^{\infty}\delta[\omega-2\pi l]$$

Proof:

$$\frac{1}{2\pi}\int_{2\pi}X(e^{j\omega})e^{j\omega n}d\omega=\frac{1}{2\pi}\int_{-\pi}^{\pi}2\pi\sum_{l}\delta(\omega-2\pi l)e^{j\omega n}d\omega$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi}2\pi\delta(\omega)e^{j\omega n}d\omega=1$$

④Rectangular Pulse

$$x[n]=\begin{cases}1, & |n|\le N_1\\ 0, & |n|>N_1\end{cases}\leftrightarrow\frac{\sin\omega(N_1+\frac{1}{2})}{\sin(\omega/2)}$$

⑤ $$\frac{\sin Wn}{\pi n}\leftrightarrow X(e^{j\omega})=\begin{cases}1, & 0\le|\omega|\le W\\ 0, & W<|\omega|\le\pi\end{cases}$$ 注: $W<\pi$

Proof:

$$x[n] = 1, |n| \le N_1 \leftrightarrow X(e^{j\omega}) = \sum_{n=-N_1}^{N_1} e^{-j\omega n}$$

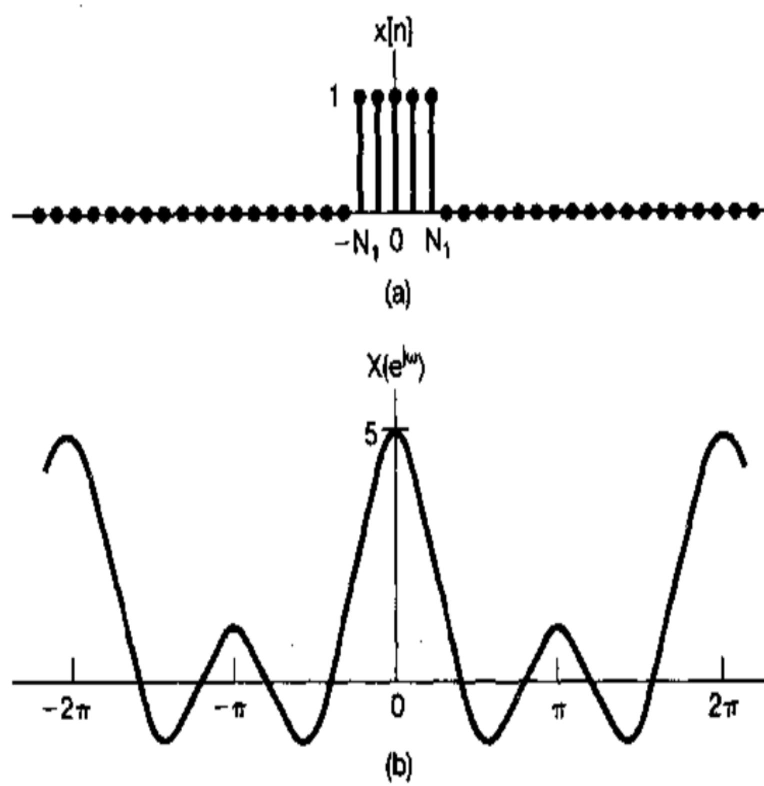
设m=N_1+n

$$X(e^{j\omega}) = \sum_{m=0}^{2N_1+1} e^{-j\omega(m-N_1)} = \sum_{m=0}^{2N_1+1} e^{-j\omega m}$$

$$= e^{j\omega N_1} \frac{1 - e^{-j\omega(2N_1+1)}}{1 - e^{-j\omega}}$$

$$= \frac{e^{j\omega N_1} - e^{-j\omega N_1} \cdot e^{-j\omega}}{1 - e^{-j\omega}} = \frac{(e^{j\omega N_1} e^{j\frac{\omega}{2}} - e^{-j\omega N_1} e^{-j\frac{\omega}{2}})}{e^{j\frac{\omega}{2}} - e^{j\frac{\omega}{2}}} =$$

$$\frac{\sin(\omega(N_1 + \frac{1}{2}))}{\sin(\frac{\omega}{2})}$$



5.2 PROPERTIES OF THE DISCRETE-TIME FORURIER TRANSFORM

- **Periodicity**
- **Linearity**
- **Time Shifting and Frequency Shifting**
- **Time Expansion**
- **……**

•Periodicity

$$\boxed{X(e^{j(\omega+2\pi)}) = X(e^{j\omega})}$$

Example:

Suppose: $X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} (-1)^k \delta(\omega - \frac{\pi k}{2})$

Determine: $x[n]$

提示：在$(-\pi$，$\pi]$内，$X(e^{j\omega}) = -\delta(\omega + \frac{\pi}{2}) + \delta(\omega) - \delta(\omega - \frac{\pi}{2}) + \delta(\omega - \pi)$

- **Linearity**

$$x_1[n] \leftrightarrow X_1(e^{j\omega}) \quad x_2[n] \leftrightarrow X_2(e^{j\omega})$$

$$\boxed{ax_1[n] + bx_2[n] \leftrightarrow aX_1(e^{j\omega}) + bX_2(e^{j\omega})}$$

- **Time Shifting and Frequency Shifting**

$$x[n] \leftrightarrow X(e^{j\omega})$$

$$\boxed{\begin{aligned} x[n-n_0] &\leftrightarrow e^{-j\omega n_0} X(e^{j\omega}) \\ e^{j\omega_0 n} x[n] &\leftrightarrow X[e^{j(\omega-\omega_0)}] \end{aligned}}$$

Example: $x[n]=\alpha^n \cos\omega_0 n\cdot u[n], \quad |a|<1$

$$x[n]=\alpha^n u[n]\cdot\frac{e^{j\omega_0 n}+e^{-j\omega_0 n}}{2}$$

$$\because \alpha^n u[n]\leftrightarrow\frac{1}{1-\alpha e^{-j\omega}}$$

$$\therefore x[n]\leftrightarrow X(e^{j\omega})=\frac{1}{2}[\frac{1}{1-\alpha e^{-j(\omega-\omega_0)}}+\frac{1}{1-\alpha e^{-j(\omega+\omega_0)}}]$$

$$=\frac{1-\alpha\cos\omega_0 e^{-j\omega}}{1-2\alpha\cos\omega_0 e^{-j\omega}+\alpha^2 e^{-j2\omega}}, \quad |a|<1$$

Example: $$X(e^{j\omega}) = \sin^2 3\omega$$

$$X(e^{j\omega}) = \sin^2 3\omega$$

$$= \frac{1-\cos 6\omega}{2} = \frac{1}{2} - \frac{e^{j6\omega} + e^{-j6\omega}}{4}$$

$$\leftrightarrow \frac{1}{2}\delta[n] - \frac{1}{4}(\delta[n+6] + \delta[n-6])$$

About Time Expansion:

- Continuous-time signal

$$x(t) \leftrightarrow X(j\omega)$$

$$x(at) \leftrightarrow \frac{1}{|a|} X(j\frac{\omega}{a})$$

- Discrete-time signal

$$x[n] \leftrightarrow X(e^{j\omega}) \qquad x[an] \leftrightarrow X_a(e^{j\omega})$$

$X_a(e^{j\omega})$与$X(e^{j\omega})$，则不一定有如上确定关系 !

- **Time Expansion** $x[n] \leftrightarrow X(e^{j\omega})$

Defination:

$$x_{(k)}[n] = \begin{cases} x[n/k], & 当n为k的整数倍 \\ 0, & 当n不为k的整数倍 \end{cases} \leftrightarrow X_{(k)}(e^{j\omega})$$

$$\boxed{X_{(k)}(e^{j\omega}) = X(e^{jk\omega})}$$

注：$x_{(k)}[n]$ 是在x[n]的相邻样本点插入(k-1)个零，相对于x[n]来说是“时域展宽”

$X_{(k)}(e^{j\omega})$ 的重复周期变为 $2\pi/k$ ，相对于 $X(e^{j\omega})$ 是“频域压缩”

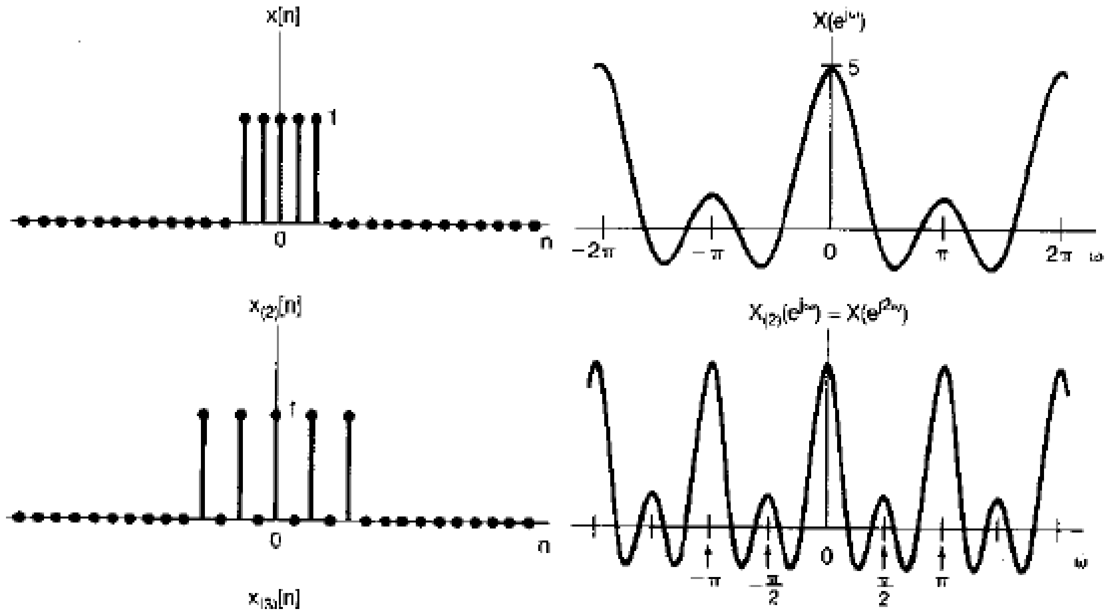
Proof:

$$x_k[n] \leftrightarrow \sum_n x_k[n] e^{-j\omega n}$$

$$\overset{n/k=r}{=} \sum_r x_k[kr] e^{-j\omega kr}$$

$$\overset{x_k[kr]=x[r]}{=} \sum_r x[r] e^{-j(\omega k)r} = X(e^{jk\omega})$$

Example：x[n]与x[n/2]



- **Time Reversal**

$$x[-n] \leftrightarrow X(e^{-j\omega})$$

•Conjugation and Conjugate Symmetry

$$x[n] \leftrightarrow X(e^{j\omega})$$

$$\boxed{x^*[n] \leftrightarrow X^*(e^{-j\omega})}$$

Proof:

$$x^*[n] \leftrightarrow \sum_n x^*[n] e^{-j\omega n}$$

$$= \{\sum_n x[n] e^{j\omega n}\}^*$$

$$= X^*(e^{-j\omega})$$

If x[n] is real valued

$$X(e^{j\omega}) = X^*(e^{-j\omega})$$ **—Conjugate Symmetry**

① $$X(e^{j\omega}) = \mathrm{Re}[X(e^{j\omega})] + jI_m[X(e^{j\omega})]$$

$$\mathrm{Re}[X(e^{j\omega})] = \mathrm{Re}[X(e^{-j\omega})]$$

$$I_m[X(e^{j\omega})] = -I_m[X(e^{-j\omega})]$$

② $$X(e^{j\omega}) = |X(e^{j\omega})|\, e^{j\measuredangle X(e^{j\omega})}$$

$$\left|X(e^{j\omega})\right| = \left|X(e^{-j\omega})\right|$$

$$\angle X(e^{j\omega}) = -\angle X(e^{-j\omega})$$

③ $x[n]$ real and even $\leftrightarrow X(e^{j\omega})$ real and even

$x[n]$ real and odd $\leftrightarrow X(e^{j\omega})$ purely imaginary and odd

$$x_e[n] = \frac{1}{2}[x[n] + x[-n]] \leftrightarrow \mathrm{Re}[X(e^{j\omega})]$$

$$x_0[n] = \frac{1}{2}[x[n] - x[-n]] \leftrightarrow jI_m[X(e^{j\omega})]$$

• **Differencing and Accumulation**

$$x[n]-x[n-1]\leftrightarrow(1-e^{-j\omega})X(e^{j\omega})$$

$$\sum_{m=-\infty}^{n}x[m]\leftrightarrow\frac{1}{1-e^{-j\omega}}X(e^{j\omega})+\pi X(e^{j0})\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k)$$

Example : $\because u[n]=\sum_{m=-\infty}^{n}\delta[m]$

$$\delta[n]\leftrightarrow 1$$

$$\therefore u[n]\leftrightarrow\frac{1}{1-e^{-j\omega}}+\pi\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k)$$

•Differentiation in Frequency

$$\boxed{nx[n] \leftrightarrow j\frac{dX(e^{j\omega})}{d\omega}}$$

Proof：

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

两边求微分

$$\frac{d}{d\omega}X(e^{j\omega}) = -j\sum_{n=-\infty}^{\infty} nx[n]e^{-j\omega n}$$

即

$$j\frac{d}{d\omega}X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \{nx[n]\}e^{-j\omega n}$$

Example: 已知$x[n]=n(\frac{1}{2})^{|n|}$，求其傅里叶变换$X(e^{j\omega})$

$$x_1[n]=(\frac{1}{2})^{|n|}$$

$$\leftrightarrow X_1(e^{j\omega})=\sum_{n=-\infty}^{-1}(\frac{1}{2})^{-n}e^{-j\omega n}+\sum_{n=0}^{\infty}(\frac{1}{2})^{n}e^{-j\omega n}$$

$$=\frac{\frac{1}{2}e^{j\omega}}{1-\frac{1}{2}e^{j\omega}}+\frac{1}{1-\frac{1}{2}e^{-j\omega}}=\frac{\frac{3}{4}}{(\frac{5}{4}-\cos\omega)}$$

$$X(e^{j\omega})=j\frac{dX_1(e^{j\omega})}{d\omega}=-j\frac{\frac{3}{4}\sin\omega}{(\frac{5}{4}-\cos\omega)^2}$$

•Parseval's Relation

$$\sum_{n=-\infty}^{\infty}|x[n]|^2=\frac{1}{2\pi}\int_{2\pi}|X(e^{j\omega})|^2\,d\omega$$

其中，$\left|X(e^{j\omega})\right|^2$ 称为能谱密度

Proof:

$$\sum_{n=-\infty}^{\infty}|x[n]|^2=\sum_{n=-\infty}^{\infty}x[n]x^*[n]=\sum_{n=-\infty}^{\infty}x[n]\left[\frac{1}{2\pi}\int_{2\pi}X(e^{j\omega})\,e^{j\omega n}d\omega\right]^*$$

$$=\sum_{n=-\infty}^{\infty}x[n]\left[\frac{1}{2\pi}\int_{2\pi}X^*(e^{j\omega})\,e^{-j\omega n}d\omega\right]=\frac{1}{2\pi}\int_{2\pi}X^*(e^{j\omega})\left[\sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}\right]d\omega$$

$$=\frac{1}{2\pi}\int_{2\pi}X^*(e^{j\omega})X(e^{j\omega})\,d\omega=\frac{1}{2\pi}\int_{2\pi}\left|X(e^{j\omega})\right|^2d\omega$$

5.3 THE CONVOLUTION PROPERTY AND FEQUENCY RESPONSE

- **Convolution Property**
- **Frequency Response**

$$x_1[n] \leftrightarrow X_1(e^{j\omega}) \quad x_2[n] \leftrightarrow X_2(e^{j\omega})$$

$$\boxed{x_1[n] * x_2[n] \leftrightarrow X_1(e^{j\omega}) \cdot X_2(e^{j\omega})}$$

Example: $$\sum_{m=-\infty}^{n} x[m] = x[n] * u[n] \leftrightarrow X(e^{j\omega}) \cdot [\frac{1}{1-e^{j\omega}} + \pi \sum_k \delta(\omega - 2\pi k)]$$

$$= \frac{X(e^{j\omega})}{1-e^{j\omega}} + \pi \sum_k X(0)\delta(\omega - 2\pi k)$$

Exercise: $x_1[n] = \alpha^n u[n] \quad x_2[n] = \beta^n u[n]$

$$Determine\ y[n] = x_1[n] * x_2[n]$$

5.3 THE CONVOLUTION PROPERTY AND FEQUENCY RESPONSE

- **Convolution Property**
- **Frequency Response**

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = x[n] * h[n]$$

设 $x[n] \leftrightarrow X(e^{j\omega})$ $y[n] \leftrightarrow Y(e^{j\omega})$

$$\boxed{H(e^{j\omega}) = \sum_n h[n] e^{-j\omega n}}$$ —frequency response

$$Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega}) \qquad H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

注：H($e^{j\omega}$)存在的条件 $\sum_n |h[n]| < \infty$ 此时描述的LTI系统是稳定的

1) $H(e^{j\omega})$可完全描述一个（稳定的）LTI系统

2) $H(e^{j\omega})$一般为复数

$$H(e^{j\omega}) = |H(e^{j\omega})| e^{j\measuredangle H(e^{j\omega})}$$

物理意义：给出任意频率分量经过LTI系统后幅度和相位的改变量

注：

- 频谱的周期性
- 低频/高频分量

Example: $x[n]=e^{j\omega_0 n}$

$$y[n]=h[n]*x[n]=\sum_k h[k]e^{j\omega_0(n-k)}$$

$$=e^{j\omega_0 n}\sum_k h[k]e^{-j\omega_0 k}=H(e^{j\omega_0})e^{j\omega_0 n}$$

Example: Determine the Response of an LTI system with impulse response $h[n]=a^n u[n]$, to the input $x[n]=\cos(\omega_0 n)$, where $|a|<1$

Solution1: $H\left(e^{j\omega}\right)=\frac{1}{1-ae^{-j\omega}}$ $\qquad x[n]=\frac{1}{2}e^{j\omega_0 n}+\frac{1}{2}e^{-j\omega_0 n}$

$$y[n]=\frac{1}{2}H\left(e^{j\omega_0}\right)e^{j\omega_0 n}+\frac{1}{2}H\left(e^{-j\omega_0}\right)e^{-j\omega_0 n}=\mathcal{R}e\left\{H\left(e^{j\omega_0}\right)e^{j\omega_0 n}\right\}$$

$$=\frac{1}{\sqrt{1-2a\cos\omega_0+a^2}}\cos\left(\omega_0 n-\arctan\frac{a\sin\omega_0}{1-a\cos\omega_0}\right)$$

Solution2:

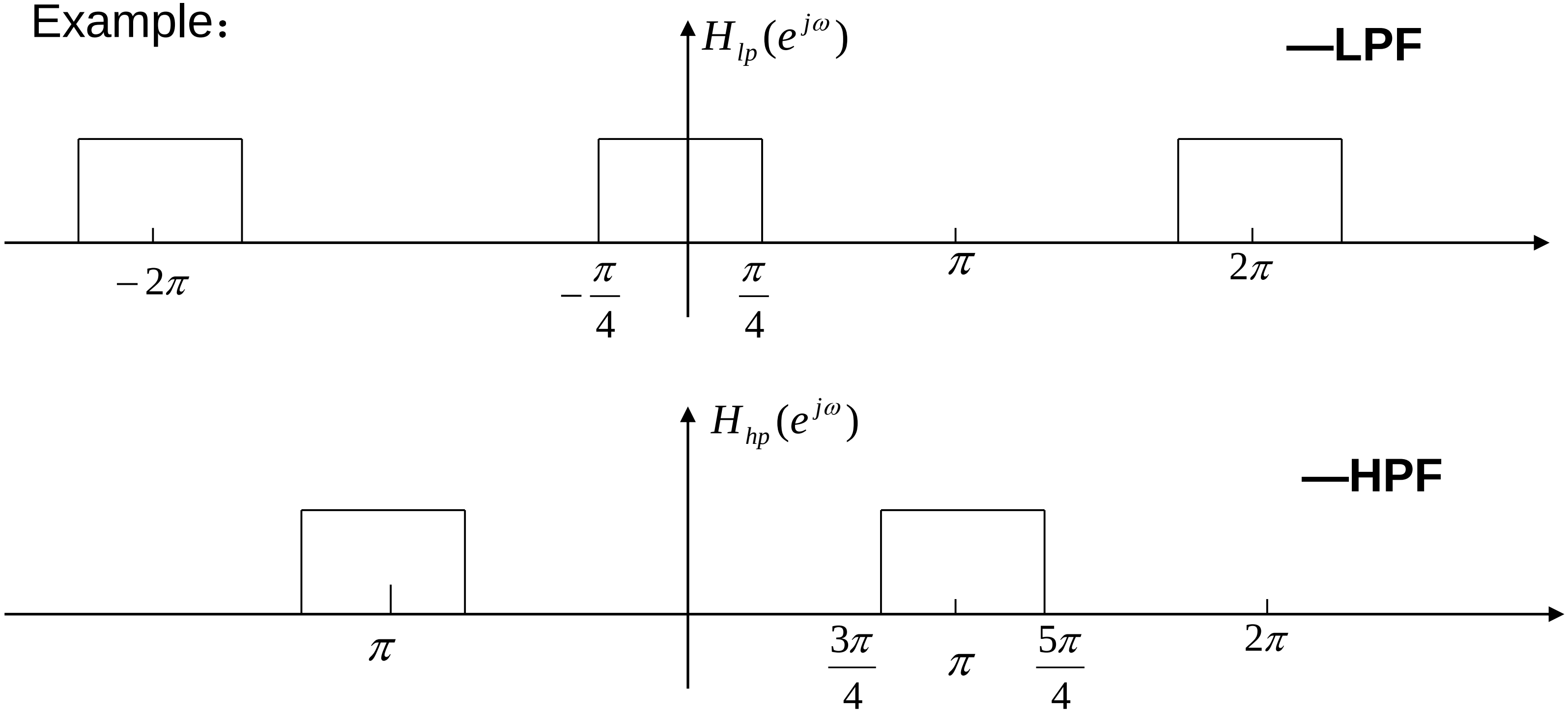
$$x[n] = \frac{1}{2}e^{j\omega_0 n} + \frac{1}{2}e^{-j\omega_0 n}$$

$$\leftrightarrow X\left(e^{j\omega}\right) = \sum_{k=-\infty}^{\infty} [\pi\delta(\omega - \omega_0 + 2k\pi) + \pi\delta(\omega + \omega_0 + 2k\pi)]$$

$$Y\left(e^{j\omega}\right) = X\left(e^{j\omega}\right)H\left(e^{j\omega}\right) = \sum_{k=-\infty}^{\infty} \pi H\left(e^{j(\omega_0 - 2k\pi)}\right)\delta(\omega - \omega_0 + 2k\pi) + \sum_{k=-\infty}^{\infty} \pi H\left(e^{-j(\omega_0 + 2k\pi)}\right)\delta(\omega + \omega_0 + 2k\pi)$$

$$\leftrightarrow y[n] = \frac{1}{2}H\left(e^{j\omega_0}\right)e^{j\omega_0 n} + \frac{1}{2}H\left(e^{-j\omega_0}\right)e^{-j\omega_0 n}$$

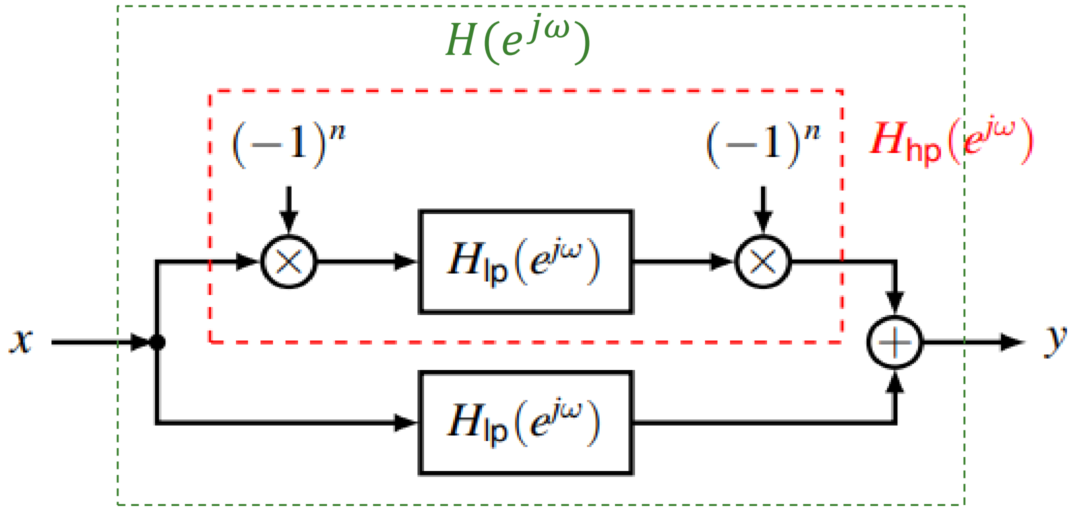
Example：

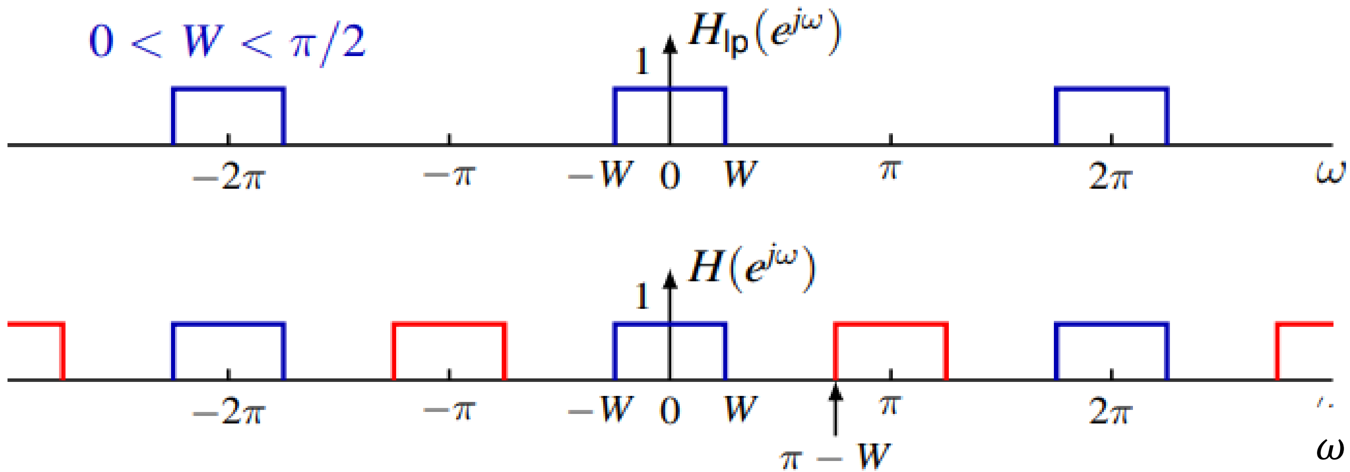


$$H_{hp}(e^{j\omega}) = H_{lp}(e^{j(\omega-\pi)})$$

$$h_{hp}[n] = e^{jn\pi} h_{lp}[n] = (-1)^n h_{lp}[n]$$

Example:





—ideal bandstop

附：**Multiplicaton Property**

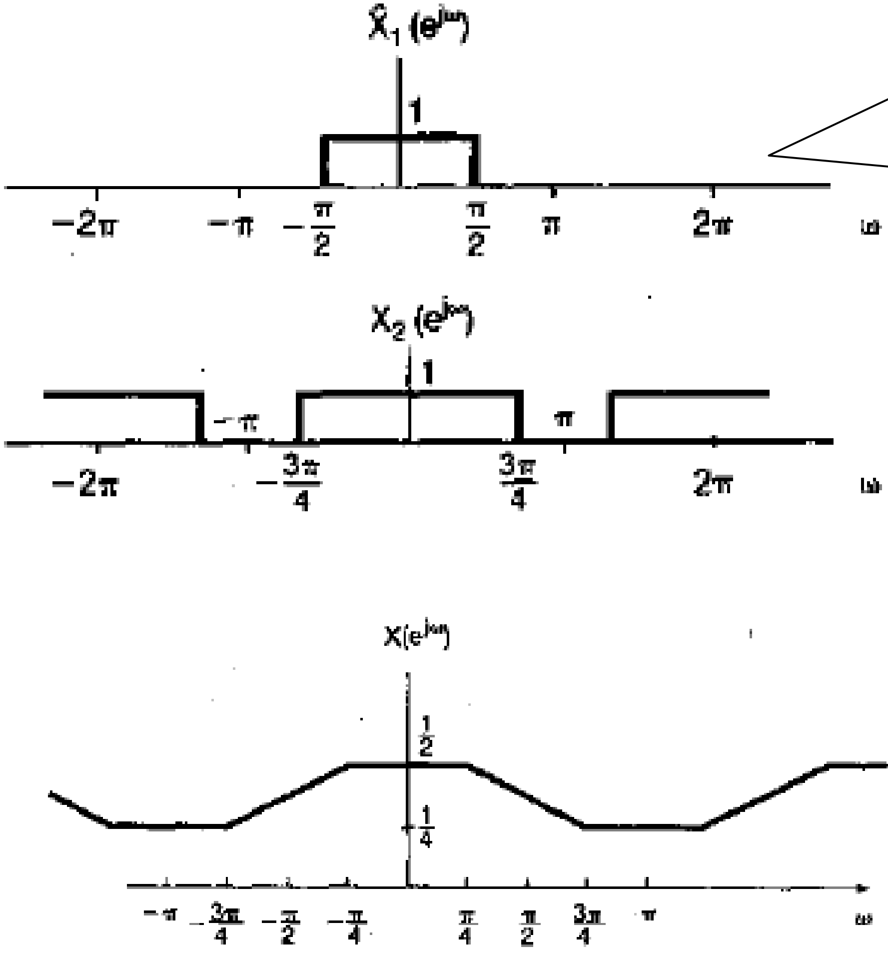
周期卷积！

$$x_1[n] \leftrightarrow X_1(e^{j\omega}) \quad x_2[n] \leftrightarrow X_2(e^{j\omega})$$

$$\boxed{x_1[n]\cdot x_2[n] \leftrightarrow \frac{1}{2\pi}\int_{2\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta}$$

Example: $x_1[n] = \dfrac{\sin(3\pi n/4)}{\pi n} \quad x_2[n] = \dfrac{\sin(\pi n/2)}{\pi n}$

$$x_1[n]\cdot x_2[n] \leftrightarrow ?$$



取$X_1(e^{j\omega})$的主周期

将周期卷积转换为非周期卷积

5.4 LINEAR CONSTANT-COEFFICENT DIFFERENCE EQUATION

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$\Downarrow$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}}$$

Example:
$$y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=2x[n]$$

$$H(j\omega)=\frac{2}{1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-j2\omega}}$$

$$=\frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}$$

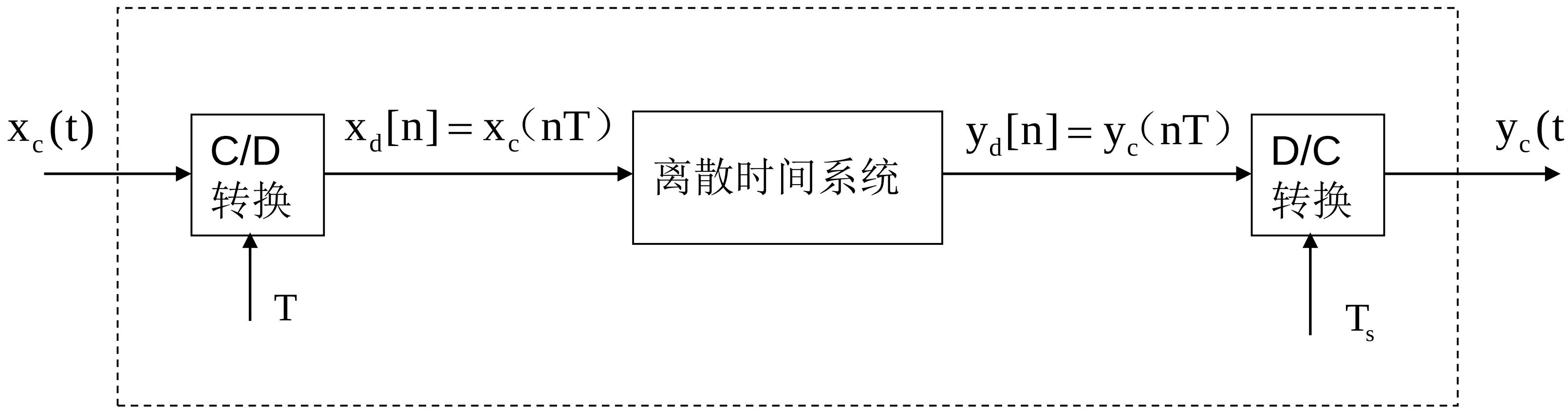
$$H(j\omega)=\frac{4}{1-\frac{1}{2}e^{-j\omega}}-\frac{2}{1-\frac{1}{4}e^{-j\omega}}$$

Expanded by the method of partial fractions

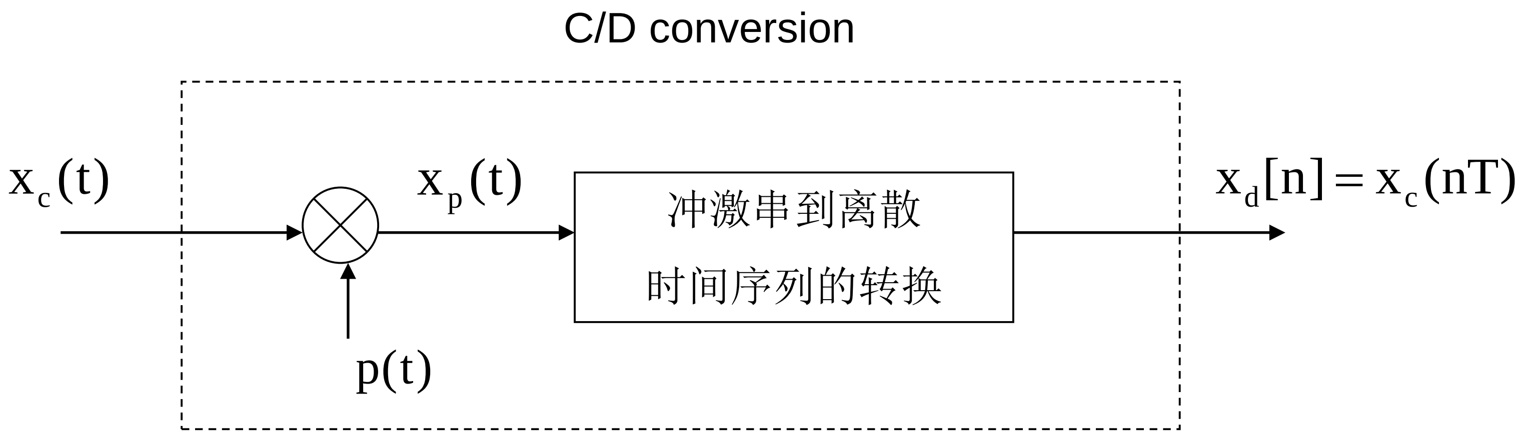
$$h[n]=4(\frac{1}{2})^n u[n]-2(\frac{1}{4})^n u[n]$$

5.5 DISCRETE-TIME PROCESSING OF CONTINUOUS-TIME SIGNALS

——将连续时间信号转换成离散时间信号，经离散时间系统处理后再转为连续时间信号。



- C/D转换



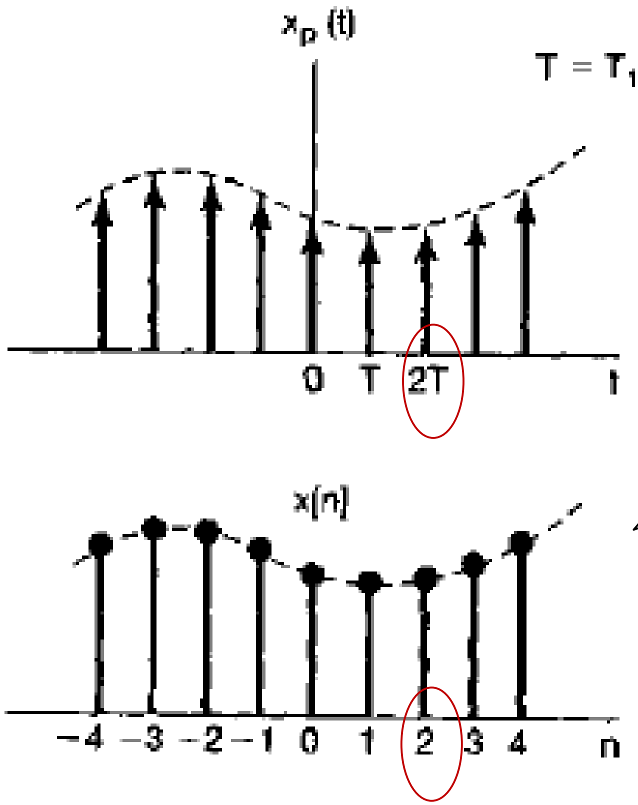
1）取样—须满足取样定理，不产生频谱混叠

2）冲激串到离散时间序列的转换

注：用于实现C/D转换的器件——模拟数字（A/D）转换器

✓ 关于冲激串(impulse train)到离散时间序列(D-T sequence)的转换

- 时域：$x_d[n]=x_c[nT]$



时域的归一化过程

- 频域：

连续时间信号的频率为ω，离散时间信号的频率为Ω

$$\because x_p(t)=\sum_{n=-\infty}^{\infty}x_c(nT)\delta(t-nT)$$

$$\therefore X_p(j\omega)=\int\sum_{n=-\infty}^{\infty}x_c(nT)\delta(t-nT)\cdot e^{-j\omega t}dt=\sum_{n=-\infty}^{\infty}x_c(nT)e^{-j\omega nT}$$

$$\text{又}\ X_d(e^{j\Omega})=\sum_{n=-\infty}^{\infty}x_d[n]e^{-j\Omega n}=\sum_{n=-\infty}^{\infty}x_c[nT]e^{-j\Omega n}$$

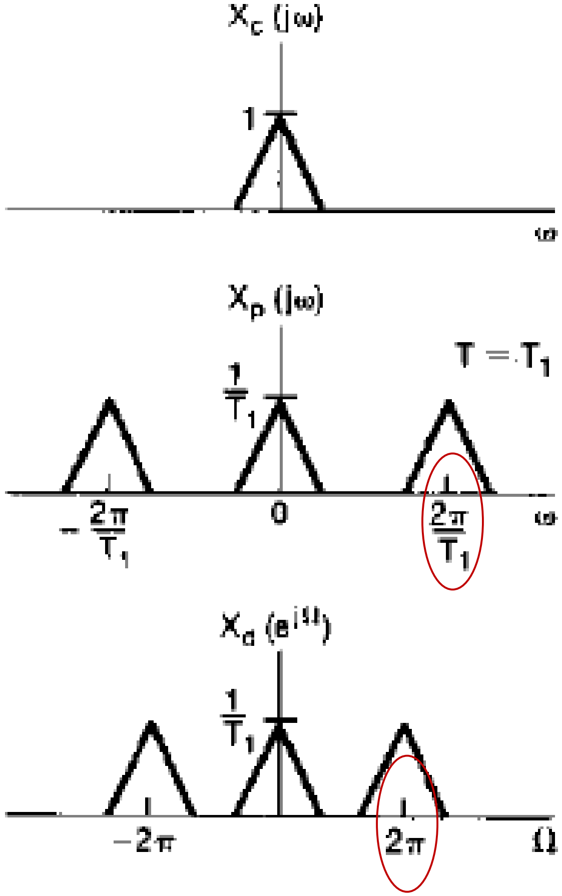
当 $\Omega=\omega T$ 时

$$X_d(e^{j\Omega})=X_p(j\omega)$$

注：时域除以T，频域乘以T——时域和频域的对偶特性！

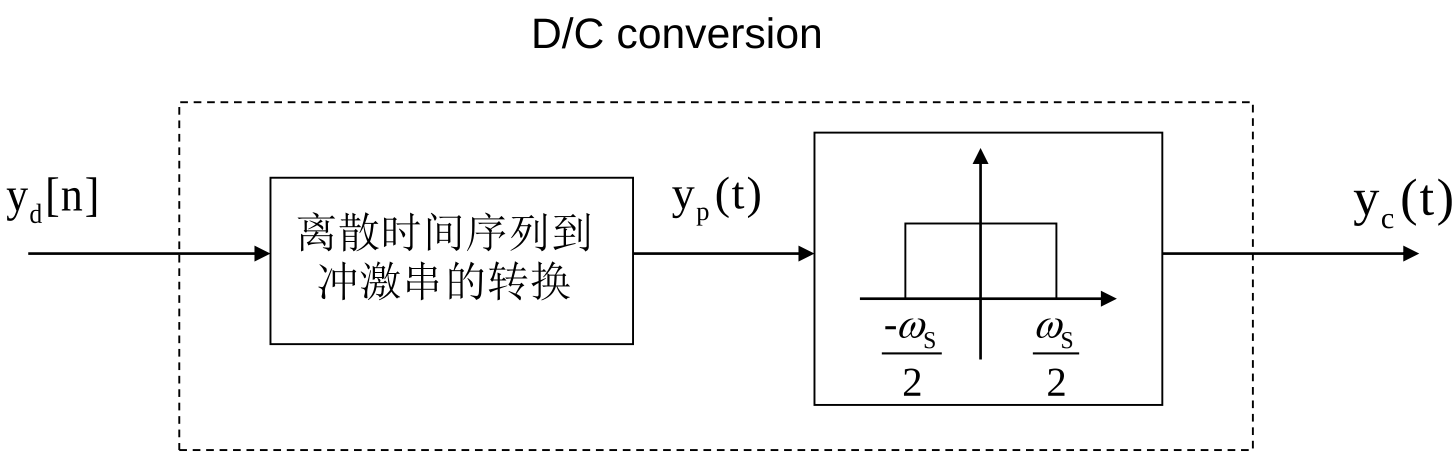
$$\because X_p(j\omega) = \frac{1}{T}\sum X_c(j(\omega - k\omega_s)) \quad \omega_s = \frac{2\pi}{T}$$

$$\therefore X_d(e^{j\Omega}) = \frac{1}{T}\sum X_c(j(\Omega - k2\pi)/T)$$

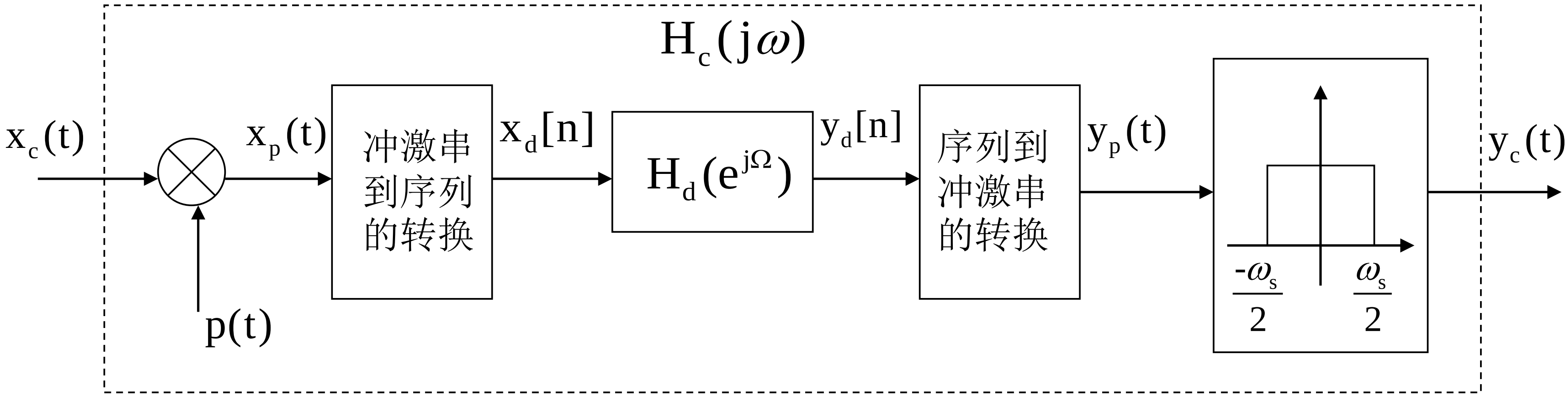


频域的归一归一化过程

- D/C转换


D/C conversion
$y_d[n]$
离散时间序列到冲激串的转换
$y_p(t)$
$-\frac{\omega_s}{2}$
$\frac{\omega_s}{2}$
$y_c(t)$

■ 冲激响应不变法



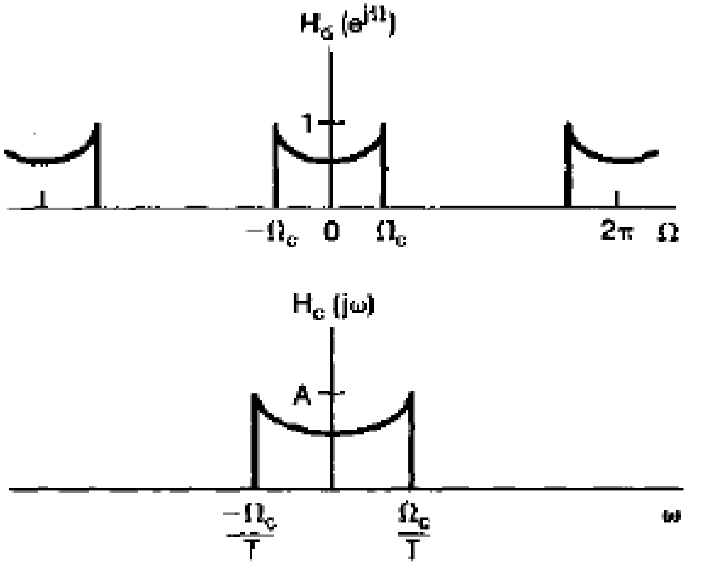
$H_c(j\omega)$
$x_c(t)$
$x_p(t)$
$p(t)$
冲激串到序列的转换
$x_d[n]$
$H_d(e^{j\Omega})$
$y_d[n]$
序列到冲激串的转换
$y_p(t)$
$\frac{-\omega_s}{2}$
$\frac{\omega_s}{2}$
$y_c(t)$

若， 1）离散时间系统是线性和时不变的

2）输入信号是带限的，且采样频率足够高

则，上述整个系统就等效为一个频率响应为 $H_c(j\omega)$ 的连续时间系统

$$H_c(j\omega)=\begin{cases} H_d(e^{j\omega T}), & |\omega|<\dfrac{\omega_s}{2} \\ 0, & |\omega|>\dfrac{\omega_s}{2} \end{cases}$$



✓ 关于冲激响应不变法

$$\because H_d(e^{j\Omega}) = H_c(j\omega), \quad |\omega| < \frac{\omega_s}{2}$$

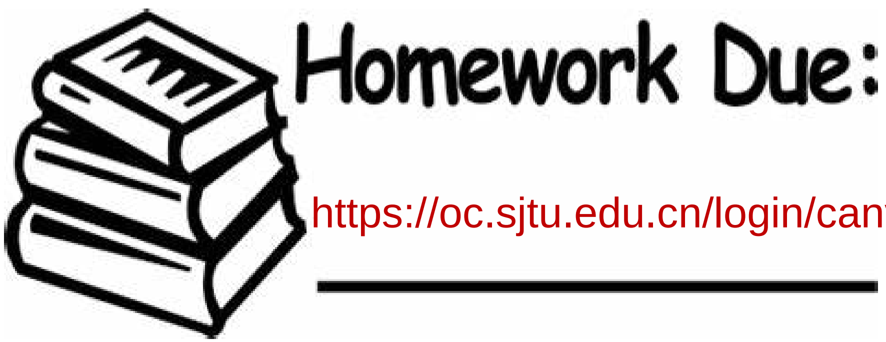
$$H_d(e^{j\Omega}) \leftrightarrow h_d[n] \quad H_c(j\omega) \leftrightarrow h_c(t)$$

若设$h_d[n]$是$h_c(t)$采样获得

考虑到$x_d[n] = x_c(nT)$时，有 $X_d(e^{j\Omega}) = \frac{1}{T}\sum X_c[j(\omega - k\omega_s)]$

$$\therefore \quad \boxed{h_d[n] = Th_c(nT)}$$

Q & A



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