Problems for Signals and Systems

Chapter 6. Discrete-time Fourier Transform (DTFT)

- **Discrete-time Fourier Transform**

1. Compute the Fourier transform of each of the following discrete-time signals:

(a) $[a^n \sin \omega_0 n]u(n), \quad |a| < 1$

(b) $\cos(18\pi n/7) + \sin(2n)$;

(c) $\sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^n \delta(n-3k)$.

2. The following are the Fourier transforms of discrete-time signals. Determine the signal corresponding to each transform.

(a) $X(e^{j\omega}) = 1 - 2e^{-j3\omega} + 4e^{-j2\omega} + 3e^{-j6\omega}$;

(b) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(\omega - \frac{\pi k}{2})$;

(c) $X(e^{j\omega}) = cos^2\omega$.

3. Determine the signal corresponding to each of the following discrete-time Fourier transforms.

(a) $X(e^{j\omega}) = \frac{e^{-j\omega}}{1 + \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-j2\omega}}$;

(b) $X(e^{j\omega}) = \sum_{m=-\infty}^{\infty} [2\pi\delta(\omega - 2m\pi) + \pi\delta\left(\omega - \frac{\pi}{2} - 2m\pi\right) + \pi\delta\left(\omega + \frac{\pi}{2} - 2m\pi\right)]$;

(c) $X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}\left[\frac{\sin\frac{5\omega}{2}}{\sin\frac{\omega}{2}}\right] + 3\pi\delta(\omega), \quad |\omega| < \pi$.

- **Frequency Domain Analysis of Discrete-time System**

4. Consider a discrete-time LTI system with impulse response

$$h(n) = \left(\frac{1}{2}\right)^n u(n).$$

Using Fourier transform, determine the response of this system for each of the following input signals:

(a) $x(n) = (\frac{3}{4})^n u(n)$;

(b) $x(n) = (-1)^n$.

6. Consider a causal and discrete-time LTI system described by the difference equation

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{3}x(n-1)$$

Determine the frequency response of this system.