第 9 周作业

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摘　要：主教材：[1]. 截止日期：2022-04-20.

关键词：词 1，词 2

Homework (Week 9)

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Abstract: Textbook: [1]. Due date: 2022-04-20.

Keywords: keyword 1, keyword 2

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$\Rightarrow \pi$ 是同态. $\therefore \pi$ 是 $\mathbb{Z}_m \oplus \mathbb{Z}_n \to \mathbb{Z}_{mn}$ 的同构.

"$\Rightarrow$". $\mathbb{Z}_m \oplus \mathbb{Z}_n \cong \mathbb{Z}_{mn} \Rightarrow \exists\, (\bar{s},\bar{t}) \in \mathbb{Z}_m \oplus \mathbb{Z}_n$:

$\mathbb{Z}_m \oplus \mathbb{Z}_n = \langle (\bar{s},\bar{t}) \rangle$, $o(\bar{s},\bar{t}) = |\mathbb{Z}_{mn}| = mn$.

$\therefore \mathbb{Z}_m = \langle \bar{s} \rangle$, $\mathbb{Z}_n = \langle \bar{t} \rangle$. $\Rightarrow o(\bar{s}) = |\mathbb{Z}_m| = m$,

$o(\bar{t}) = |\mathbb{Z}_n| = n$. 又已证 $o(\bar{s},\bar{t}) = [o(\bar{s}), o(\bar{t})]$ (注),

$\Rightarrow mn = [m,n] \Rightarrow (m,n) = \dfrac{mn}{[m,n]} = 1$. #

注: 对 "$o(\bar{s},\bar{t}) = [o(\bar{s}), o(\bar{t})]$" 的证明.

由 $(\bar{s},\bar{t})^{[o(\bar{s}),o(\bar{t})]} = (0,0)$, $\Rightarrow o(\bar{s},\bar{t}) \mid [o(\bar{s}),o(\bar{t})]$.

由 $(\bar{s},\bar{t})^{o(\bar{s},\bar{t})} = \left(\bar{s}^{o(\bar{s},\bar{t})}, \bar{t}^{o(\bar{s},\bar{t})}\right) = (0,0) \Rightarrow o(\bar{s}) \mid o(\bar{s},\bar{t})$ 且 $o(\bar{t}) \mid o(\bar{s},\bar{t})$

$\Rightarrow [o(\bar{s}),o(\bar{t})] \mid o(\bar{s},\bar{t})$. $\therefore o(\bar{s},\bar{t}) = [o(\bar{s}), o(\bar{t})]$.

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(p.73) 2. 解: $392 = 2^3 \times 7^2 \Rightarrow$ 共有 $P(3)P(2) = 3 \times 2 = 6$ 个互不同构的 392 阶 Abel 群, 它们是: $H_1 = \mathbb{Z}_{2^3} \oplus \mathbb{Z}_{7^2}$, $H_2 = \mathbb{Z}_{2^2} \oplus \mathbb{Z}_{2^1} \oplus \mathbb{Z}_{7^2}$,

$H_3 = \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_{7^2}$, $H_4 = \mathbb{Z}_{2^3} \oplus \mathbb{Z}_7 \oplus \mathbb{Z}_7$,

$H_5 = \mathbb{Z}_{2^2} \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_7 \oplus \mathbb{Z}_7$, $H_6 = \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_7 \oplus \mathbb{Z}_7$.

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(p.80) 1. 解: $G_1 = \{(1),(34)\} = G_2$, $G_3 = \{(1),(12)\} = G_4$.

2. 解: 设 $G \times G/H \to G/H$ 的映射"$\times$"满足:

(i) $e \times gH \overset{def}{=} (eg)H = gH$, $\forall gH \in G/H$;

(ii) $t_1 \times (t_2 \times gH) \overset{def}{=} t_1 \times (t_2 g)H \overset{def}{=} (t_1(t_2 g))H = ((t_1 t_2) g)H \overset{def}{=}$

$\overset{def}{=} t_1 t_2 \times gH$, $\forall t_1, t_2 \in G$, $\forall gH \in G/H$.

∴ G/H 是一个(左)G-集.

$\forall gH \in G/H$, 记其稳定子群为 $\cancel{G/}$ $G_{gH} := \{ t \in G \mid t \times gH = gH \}$.

有 $t \in G_{gH} \overset{def}{\iff} t \times gH = gH \overset{def}{\iff} (tg)H = gH \iff g^{-1}tg \in H \iff t \in gHg^{-1}$,

∴ $G_{gH} = gHg^{-1} \Rightarrow |G_{gH}| = |gHg^{-1}| = |H|$. ∴ 轨道公式成为 [$G \cdot gH = G/H$, $\forall g \in G$](可迁)

$$|G/H| = \underbrace{|G \cdot gH|}_{[gH\text{的}G\text{-轨道}]} = [G : G_{gH}] \overset{Lagrange}{=\!=} \frac{|G|}{|G_{gH}|} = |G|/|H|, \forall g \in G.$$

5. 证: $\forall m_1, m_2 \in M$. 由轨道公式, $|Nm_i| = \frac{|N|}{|N_{m_i}|}$. 故要证 $|Nm_1| = |Nm_2|$, 只需证稳定子群满足 $|N_{m_1}| = |N_{m_2}|$, $\forall m_1, m_2 \in M$. 下证之.

因 G 在 M 上的作用是可迁的, 故 $\exists\, g \in G$ s.t. $m_2 = gm_1$. 考察 N_{m_2}. 有 $\forall n_2 \in N_{m_2}$ 则

$\overset{def}{\iff} n_2 m_2 = m_2 \Rightarrow n_2 g m_1 = g m_1 \iff \underbrace{g^{-1} n_2 g}_{\in N \lhd G} m_1 = m_1 \overset{def}{\iff} g^{-1} n_2 g \in N_{m_1} \iff n_2 \in g N_{m_1} g^{-1}$.

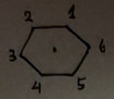
∴ $N_{m_2} \subseteq g N_{m_1} g^{-1}$. 反之, $\forall g n_1 g^{-1} \in g N_{m_1} g^{-1}$ ($n_1 \in N_{m_1}$), 有 $\underbrace{g n_1 g^{-1}}_{\in N \lhd G} m_2 = g n_1 g^{-1} g m_1$

$= g n_1 m_1 = g m_1 = m_2 \Rightarrow g n_1 g^{-1} \in N_{m_2}. \Rightarrow g N_{m_1} g^{-1} \subseteq N_{m_2}$.

∴ $N_{m_2} = g N_{m_1} g^{-1}$. $\Rightarrow |N_{m_2}| = |g N_{m_1} g^{-1}| = |N_{m_1}|$. $\Rightarrow |Nm_1| = \frac{|N|}{|N_{m_1}|}$

$= \frac{|N|}{|N_{m_2}|} = |Nm_2|$, $\forall m_1, m_2 \in M$. 即 M 的每条 N-轨道等长. ∎

(p.80) 6. 解: $D_6 = \{ (1), (123456), (135)(246), (14)(25)(36), (153)(264), (654321),$
$(26)(35), (12)(36)(45), (13)(46), (23)(14)(56), (15)(34), (16)(25)(34) \}.$



按型分类. $1^6 : \{(1)\}$, $6^1 : \{ (123456), (654321) \}$,

$3^2 : \{ (135)(246), (153)(264) \}$, $2^3 : \{ (14)(25)(36), (12)(36)(45), (23)(14)(56), (16)(25)(34) \}$,

$2^2 1^2 : \{ (26)(35), (13)(46), (15)(34) \}$.

References

[1] 刘绍学, 章璞. 近世代数导引 [M]. 1 ed. 北京: 高等教育出版社, 2011.