APPENDIX

# A

# Calculation of Insolation Under Current Conditions

## A.1 SOLAR ZENITH ANGLE

Consider a unit circle representing the Earth, as pictured in Fig. A.1. We are interested in calculating the solar zenith angle $\theta$s and the solar azimuth angle ξ at the point X on the surface of the sphere, located at latitude $\phi$. We draw a radial from the center of the sphere through the point X to define the local zenith direction, Z. Another radial from the center of Earth to the Sun crosses through the surface of the sphere at the subsolar point, ss. The point ss occurs at a latitude of δ, which is equal to the declination angle. At the point X we draw another line to the Sun. Since the Sun is many Earth radii from Earth, the lines drawn to the Sun from the center of the Earth and point X are parallel. The arc length on the unit sphere between X and ss is equal to the desired solar zenith angle, $\theta$s. We can now apply the law of cosines to the oblique spherical triangle defined by the points at the pole, P, the subsolar point, ss, and the point, X, where we want the solar zenith angle. For this triangle, we know the arc length of two sides and one interior angle, and we wish to know the length of the third side. The law of cosines requires that,

$$\cos\theta_s = \cos(90-\phi)\cos(90-\delta) + \sin(90-\phi)\sin(90-\delta)\cos h \qquad (A.1)$$

or

$$\cos\theta_s = \sin\phi\sin\delta + \cos\phi\cos\delta\cos h \qquad (A.2)$$

The solar azimuth angle ξ, the angle between the subsolar point and due south (or equatorward) can be obtained from the law of sines for an oblique spherical triangle.

$$\frac{\sin(180-\xi)}{\sin(90-\delta)} = \frac{\sin h}{\sin\theta_s} \qquad (A.3)$$

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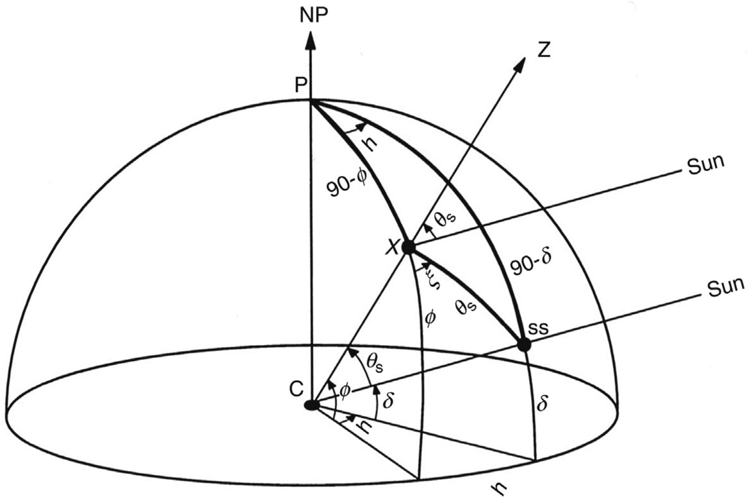


**FIGURE A.1** Spherical geometry for solar zenith angle calculation.

or

$$\sin \xi = \frac{\cos \delta \sin h}{\sin \theta_s} \tag{A.4}$$

## A.2 DECLINATION ANGLE

The annual variation of the declination angle for current conditions can be defined to a good approximation in terms of a truncated Fourier series in the time of year. If the time of year is expressed in radians according to the following formula, where $d_n$ is the day number, which ranges from 0 on January 1 to 364 on December 31, then

$$\theta_d = \frac{2\pi d_n}{365}. \tag{A.5}$$

The declination angle is given to a good approximation by the Fourier series,

$$\delta = \sum_{n=0}^{3} a_n \cos(n\theta_d) + b_n \sin(n\theta_d) \tag{A.6}$$

where the coefficients are as given in the following table.

| $n$ | $\mathbf{a}_n$ | $\mathbf{b}_n$ |
|---|---|---|
| 0 | 0.006918 | |
| 1 | −0.399912 | 0.070257 |
| 2 | −0.006758 | 0.000907 |
| 3 | −0.002697 | 0.001480 |

Because of the eccentricity of the Earth's orbit, the Earth–Sun distance varies according to the time of year.

A Fourier series formula for the squared ratio of the mean Earth–Sun distance to the actual distance has also been derived by Spencer (1971).

$$\left(\frac{\bar{d}}{d}\right)^2 = \sum_{n=0}^{2} a_n \cos(n\theta_\mathrm{d}) + b_n \sin(n\theta_\mathrm{d}) \tag{A.7}$$

where the coefficients are given by

| $n$ | $\mathbf{a}_n$ | $\mathbf{b}_n$ |
|---|---|---|
| 0 | 1.000110 | |
| 1 | 0.034221 | 0.001280 |
| 2 | 0.000719 | 0.000077 |