第 5 次作业

危国锐 516021910080
（上海交通大学电子信息与电气工程学院，上海 200240）

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Homework 5

Guorui Wei 516021910080
(*School of Electronic Information and Electrical Engineering*,
Shanghai Jiao Tong University, *Shanghai* 200240, *China*)

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目 录

1 Question 1

Draw Fig. 6.7 for the Southern Hemisphere using the same coordinate axes. Make sure that what you draw transports heat and momentum toward the South Pole. (Hartmann, 2016, p. 192)

1.1 Solution

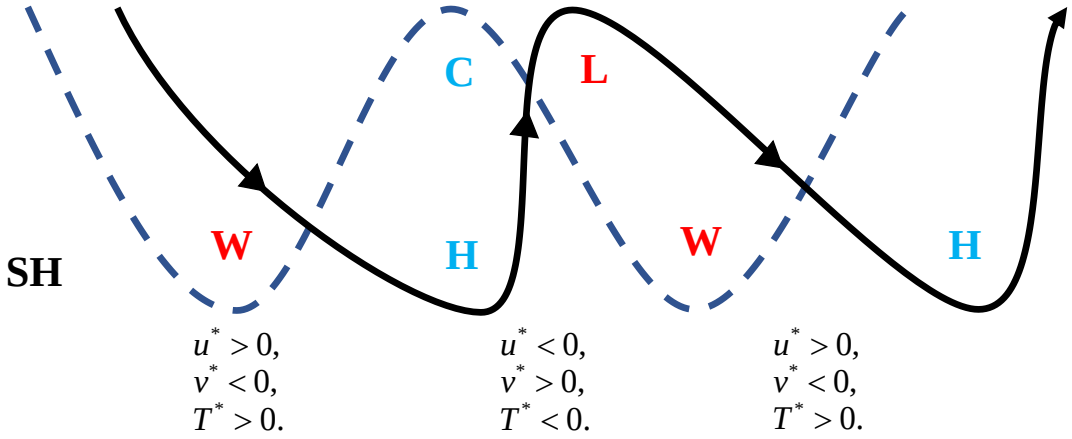

Figure 1.1 Schematic of the streamlines (black with arrows) and isotherms (colors) associated with a large-scale atmospheric disturbance in mid-latitudes of the Southern Hemisphere. Arrows along the streamline contour indicate the direction of wind velocity. The streamlines correspond approximately to lines of constant pressure, since the winds are nearly geostrophic. The signs of the deviations of the wind components and temperature from their zonal average values are shown with asterisks to illustrate that the NW–SE tilt of the streamlines indicates a southward zonal momentum transport, and the westward phase shift of the temperature wave relative to the pressure wave gives a southward heat transport. (Hartmann, 2016, p. 169)

2 Question 2

Derive (6.19) from (6.15) and (6.13). (Hartmann, 2016, p. 192)

2.1 Definitions and Basic Properties of Time- and Zonal Average

时-空场 x 的（有限）**时间平均**定义为

$$\overline{x} \coloneqq \frac{1}{\Delta t}\int_0^{\Delta t} x\,\mathrm{d}t, \tag{2.1}$$

故当 Δt 取定时，时间平均算子 $\overline{(\cdot)}$ 是线性算子. 定义 x 对其时间平均的偏移量

$$x' \coloneqq x - \overline{x}, \tag{2.2}$$

则有

$$\overline{x'} = \overline{x - \overline{x}} = \overline{x} - \overline{x} = 0. \tag{2.3}$$

本来，定义式 (2.1) 不能保证 $\overline{x}$ 不依赖于时间，但若物理量 x 存在某个稳态值，或存在周期，则适当选取 Δt 可能使得 $\overline{x}$ 与时间无关. 故在本题中，我擅自假定 $\overline{x}$ 不依赖于时间，从而有

$$\frac{\partial \overline{x}}{\partial t} \equiv 0 \Rightarrow \overline{\overline{x}} = \overline{x}. \tag{2.4}$$

由 (2.2) 至 (2.4) 得

$$\overline{xy} = \overline{(\overline{x} + x')(\overline{y} + y')} = \overline{x}\,\overline{y} + \overline{x'y'}. \tag{2.5}$$

时-空场 x 的**纬向平均**定义为

$$[x] \coloneqq \frac{1}{2\pi}\int_0^{2\pi} x\,\mathrm{d}\lambda, \tag{2.6}$$

故纬向平均算子 $[\cdot]$ 是线性算子. 定义 x 对其纬向平均的偏移量

$$x^* \coloneqq x - [x], \tag{2.7}$$

则有

$$[x^*] = \big[x - [x]\big] = [x] - [x] = 0. \tag{2.8}$$

与有限时间平均的情况不同，纬向平均的定义式 (2.6) 保证了 $[x]$ 不依赖于经度 λ，从而

$$[xy] = [([x] + x^*)([y] + y^*)] = [x][y] + [x^*y^*]. \tag{2.9}$$

假定时间平均算子与纬向平均算子可交换次序，即

$$[\overline{x}] = \overline{[x]}, \tag{2.10}$$

则有

$$\overline{x^*} \coloneqq \overline{x - [x]} = \overline{x} - \overline{[x]} = \overline{x} - [\overline{x}] =: \overline{x}^*, \tag{2.11}$$

$$[x'] \coloneqq [x - \overline{x}] = [x] - [\overline{x}] = [x] - \overline{[x]} =: [x]', \tag{2.12}$$

即 $(\cdot)^*$ 与 $\overline{(\cdot)}$ 可交换次序，$(\cdot)'$ 与 $[\cdot]$ 可交换次序.

2.2 Solution

由上一节的结果，有

$$[\overline{xy}] = \big[\overline{x}\,\overline{y} + \overline{x'y'}\big] = [\overline{x}\,\overline{y}] + \big[\overline{x'y'}\big] = [\overline{x}][\overline{y}] + [x^*y^*] + \big[\overline{x'y'}\big]. \tag{2.13}$$

取 $x = v$, $y = M \coloneqq (\Omega a\cos\varphi + u)a\cos\varphi$，则 $y' = u'a\cos\varphi$，$y^* = u^*a\cos\varphi$，代入 (2.13) 立得

$$\big[\overline{vM}\big] = [\overline{v}](\Omega a\cos\varphi + [\overline{u}])a\cos\varphi + \big([v^*u^*] + \big[\overline{v'u'}\big]\big)a\cos\varphi\,. \tag{2.14}$$

References

Hartmann, D. L. (2016). Chapter 6 - Atmospheric General Circulation and Climate. In D. L. Hartmann (Ed.), *Global Physical Climatology (Second Edition)* (pp. 159-193). Elsevier. https://doi.org/10.1016/B978-0-12-328531-7.00006-2