第 6 次作业 (第 7 章)

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关键词：关键词 1，关键词 2

Homework 6 (Chapter 7)

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Keywords: keyword 1, keyword 2

目 录

1 Question 1

Suppose that a wind stress is applied to the ocean, taking the following simple form.

$$\tau_x = \begin{cases} A\cos\left(\frac{\pi y}{L}\right), & -L < y < L, \\ -A, & |y| \geq L. \end{cases}$$

Derive an equation for the vertical velocity at the bottom of the Ekman layer assuming a constant Coriolis parameter $f = f_0 = 2\ \Omega \sin 30^\circ$. Derive an equation for the integrated meridional transport V_I, using (7.14) with the f and β appropriate for 30°N latitude. Determine a numeric value for the maximum w_E and V_I using the following constants: A = 2 dyn cm^{-2} = 0.2 N m^{-2}, L = 1500 km, ρ_0 = 1025 kg m^{-3}. Plot τ_x, w_E, and V_I on the interval $-L < y < L$. Assuming that the ocean basin is 5000 km wide, calculate the water mass flux at 30°N associated with the interior flow. Compare this number with the estimate for the Gulf Stream mass flux given in Section 7.8. (Hartmann, 2016, p. 232)

1.1 Solution

Using Eq.(7.11) of (Hartmann, 2016), the vertical velocity at the bottom of the Ekman layer is

$$w_\mathrm{E} = \boldsymbol{k}\cdot\nabla\times\left(\frac{\boldsymbol{\tau}}{\rho_0 f}\right) = -\frac{\partial}{\partial y}\left(\frac{\tau_x}{\rho_0 f}\right) = \frac{A\pi}{\rho_0 f_0 L}\sin\left(\frac{\pi y}{L}\right), -L < y < L. \tag{1}$$

assuming that the vertical current vanishes at the surface.

Using Eq.(7.14) of (Hartmann, 2016), the integrated meridional transport V_I

$$V_\mathrm{I} = \frac{f}{\beta}\boldsymbol{k}\cdot\nabla\times\left(\frac{\boldsymbol{\tau}}{\rho_0 f}\right) = \frac{A}{\rho_0 f_0}\cos\left(\frac{\pi y}{L}\right) + \frac{A\pi}{\rho_0 \beta L}\sin\left(\frac{\pi y}{L}\right), -L < y < L, \tag{2}$$

where

$$\beta \coloneqq \frac{\partial f}{\partial y} = \frac{2\Omega}{R_\mathrm{E}}\cos\varphi\,, \tag{3}$$

R_E is the radius of the Earth, 6371 km.

Figure 1 shows a numeric value for the w_E and V_I using the following constants: A = 2 dyn cm^{-2} = 0.2 N m^{-2}, L = 1500 km, ρ_0 = 1025 kg m^{-3}.

Assuming that the ocean basin is 5000 km wide, the water mass flux at 30°N ($y = 0$) associated with the interior flow is

$$M_y\big|_{y=0} = \rho_0 W V_\mathrm{I}|_{y=0} = W\frac{A}{f_0} = 1.371\times 10^{10}\ \mathrm{kg/s}, \tag{4}$$

which is about 46% of the estimate for the Gulf Stream mass flux given in Section 7.8 (Hartmann, 2016, p. 229)

$$\begin{aligned}\text{Density}\times\text{width}\times\text{depth}\times\text{speed} &= 10^3\ \mathrm{kg\ m^{-3}}\times 100\ \mathrm{km}\times 600\ \mathrm{m}\times 0.5\ \mathrm{m\ s^{-1}} \\ &= 3\times 10^{10}\ \mathrm{kg\ s^{-1}}\end{aligned}$$

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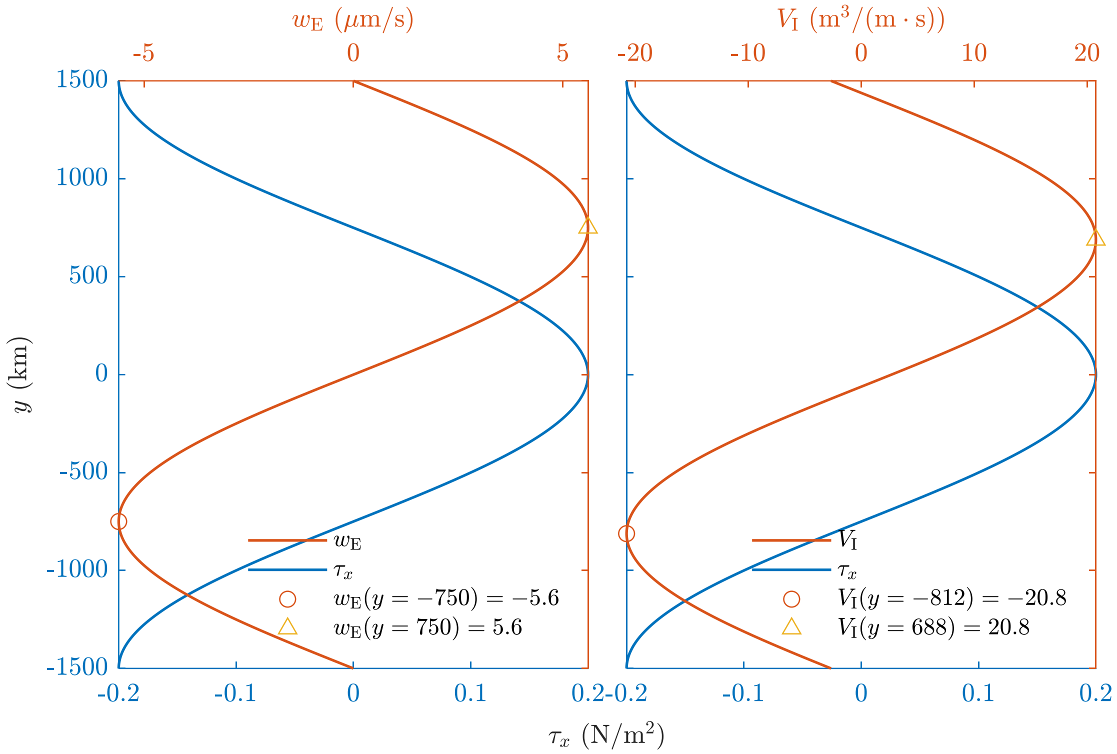

Figure 1 A numeric value for w_E and V_I using the following constants: A = 2 dyn cm^{-2} = 0.2 N m^{-2}, L = 1500 km, ρ_0 = 1025 kg m^{-3}.

References

Hartmann, D. L. (2016). Chapter 7 - The Ocean General Circulation and Climate. In D. L. Hartmann (Ed.), *Global Physical Climatology (Second Edition)* (pp. 195-232). Elsevier. https://doi.org/10.1016/B978-0-12-328531-7.00007-4

附录A 本文使用的 MATLAB 源代码

A.1 主程序

```
%% ch7_hw.m
% Description: MATLAB code for Homework 6 (Chapter 7) (MS3402, 2022 Spring)
% Author: Guorui Wei (危国锐) (313017602@qq.com; weiguorui@sjtu.edu.cn)
% Student ID: 516021910080
% Created: 2022-05-01
% Last modified: 2022-05-01
% References: [1] Hartmann, D. L. (2016). Chapter 7 - The Ocean General Circulation and Climate. In D. L. Hartmann (Ed.), Global Physical Climatology (Second Edition) (pp. 195-232). Elsevier. https://doi.org/10.1016/B978-0-12-328531-7.00007-4

%% Initialize project

clc; clear; close all
init_env();

%% Problem 7 of [1,p.232]

% params

A = .2;                 % [N/m^2]
L = 1500e3;             % [m]
rho_0 = 1025;           % [kg/m^3]
Omega = 7.292e-5;       % [rad/s] angular velocity of the Earth = 2*pi/(23*3600+56*60)
lat = pi/6;             % [rad E]
R_E = 6371e3;           % [m] radius of the Earth
W = 5000e3;             % [m] the ocean basin width

%
f_0 = 2*Omega*sin(lat);                     % [rad/s] Coriolis parameter
beta_0 = 2*Omega*cos(lat)/R_E;              % [rad/m/s]
tau_x = @(y) A*cos(pi*y/L);                 % [N/m^2] wind stress
w_E = @(y) A*pi/rho_0/f_0/L*sin(pi*y/L); % the vertical velocity at the bottom of the Ekman layer
V_I = @(y) A/rho_0/f_0*cos(pi*y/L) + A*pi/rho_0/beta_0/L*sin(pi*y/L); % [m^3/m/s] the integrated meridional transport (volume)
M_y = W*A/f_0; % [kg/s] the water mass flux at y = 0 associated with the interior flow
```

```
fprintf("M_y = %.3d (kg/s)\n",M_y);

%% solve

y = linspace(-L,L,2^13);
val_w_E = w_E(y)*1e6; % [um/s]
val_V_I = V_I(y);
[w_E_max,ind_w_E_max] = max(val_w_E);
[w_E_min,ind_w_E_min] = min(val_w_E);
[V_I_max,ind_V_I_max] = max(val_V_I);
[V_I_min,ind_V_I_min] = min(val_V_I);

%% plot
t_fig = figure("Name","Fig.1 Ch7 Q7");
t_TCL = tiledlayout(1,2,"TileSpacing","tight","Padding","tight");

%%% Fig.1(a) w_E
t_Axes_tau_x = nexttile(t_TCL,1);
t_plot_tau_x = plot(t_Axes_tau_x,tau_x(y),y/1000,'-','color','#0072BD',"DisplayName",'$\tau_x$','LineWidth',1);
set(t_Axes_tau_x,"YDir",'normal',"TickLabelInterpreter",'latex',"FontSize",10,'Box','off','XColor','#0072BD','YColor','#0072BD');
%
t_Axes_w_E = axes(t_TCL);
t_Axes_w_E.Layout.Tile = 1;
t_plot_w_E = plot(t_Axes_w_E,val_w_E,y/1000,'-','Color','#D95319',"DisplayName",'$w_{\rm{E}}$','LineWidth',1);
hold on
t_plot_w_E_min = plot(t_Axes_w_E,w_E_min,y(ind_w_E_min)/1000,'o',"DisplayName",sprintf("$w_{\\rm{E}}(y = %.0f) = \\rm{%.1f}$",y(ind_w_E_min)/1000,w_E_min));
t_plot_w_E_max = plot(t_Axes_w_E,w_E_max,y(ind_w_E_max)/1000,'^',"DisplayName",sprintf("$w_{\\rm{E}}(y = %.0f) = \\rm{%.1f}$",y(ind_w_E_max)/1000,w_E_max));
hold off
set(t_Axes_w_E,'YDir','normal','FontSize',10,'TickLabelInterpreter','latex','XAxisLocation','top','YAxisLocation','right','YTickLabel',{},'Box','off','Color','none','XColor','#D95319','YColor','#D95319','YLimitMethod','tight')
xlabel(t_Axes_w_E,"$w_{\rm{E}}$ $(\rm{\mu m}/\rm{s})$","FontSize",10,Interpreter="latex")
%
linkaxes([t_Axes_w_E,t_Axes_tau_x],'y');
legend([t_plot_w_E,t_plot_tau_x,t_plot_w_E_min,t_plot_w_E_max],"Location",'southeast','Interpreter','latex',"Box","off");
```

```matlab

%%% Fig.1(b) V_I
%
t_Axes_tau_x = nexttile(t_TCL,2);
t_plot_tau_x = plot(t_Axes_tau_x,tau_x(y),y/1000,'-',"color",'#0072BD',"DisplayName",'$\tau_x$','LineWidth',1);
set(t_Axes_tau_x,"YDir",'normal','YTickLabel',{},"TickLabelInterpreter",'latex',"FontSize",10,'Box','off','XColor','#0072BD','YColor','#0072BD');
%
t_Axes_V_I = axes(t_TCL);
t_Axes_V_I.Layout.Tile = 2;
t_plot_V_I = plot(t_Axes_V_I,val_V_I,y/1000,'-','Color','#D95319',"DisplayName",'$V_{\rm{I}}$','LineWidth',1);
hold on
t_plot_V_I_min = plot(t_Axes_V_I,V_I_min,y(ind_V_I_min)/1000,'o',"DisplayName",sprintf("$V_{\\rm{I}}(y = %.0f) = \\rm{%.1f}$",y(ind_V_I_min)/1000,V_I_min));
t_plot_V_I_max = plot(t_Axes_V_I,V_I_max,y(ind_V_I_max)/1000,'^',"DisplayName",sprintf("$V_{\\rm{I}}(y = %.0f) = \\rm{%.1f}$",y(ind_V_I_max)/1000,V_I_max));
hold off
set(t_Axes_V_I,'YDir','normal','FontSize',10,'TickLabelInterpreter','latex','XAxisLocation','top','YAxisLocation','right','YTickLabel',{},'Box','off','Color','none','XColor','#D95319','YColor','#D95319','YLimitMethod','tight')
xlabel(t_Axes_V_I,"$V_{\rm{I}}$ $(\rm{m}^3 / (\rm{m} \cdot \rm{s}))$","FontSize",10,Interpreter="latex")
%
linkaxes([t_Axes_V_I,t_Axes_tau_x],'y');
legend([t_plot_V_I,t_plot_tau_x,t_plot_V_I_min,t_plot_V_I_max],"Location",'southeast','Interpreter','latex',"Box","off");

%%%
xlabel(t_TCL,"$\tau_x$ $(\rm{N}/\rm{m}^2)$","FontSize",10,Interpreter="latex")
ylabel(t_TCL,"$y$ (km)","FontSize",10,Interpreter="latex")
[t_title_t,t_title_s] = title(t_TCL,"\bf 2022 Spring MS3402 Hw6 (Chapter 7) Pblm 7","Guorui Wei 516021910080","Interpreter",'latex');
set(t_title_s,'FontSize',8)
%
exportgraphics(t_TCL,"..\\doc\\fig\\ch7_P7.emf",'Resolution',800,'ContentType','auto','BackgroundColor','none','Colorspace','rgb')
exportgraphics(t_TCL,"..\\doc\\fig\\ch7_P7.png",'Resolution',800,'ContentType','auto','BackgroundColor','none','Colorspace','rgb')

```

```
%% local functions

%% Initialize environment
function [] = init_env()
    % set up project directory
    if ~isfolder("../doc/fig/")
        mkdir ../doc/fig/
    end
    % configure searching path
    mfile_fullpath = mfilename('fullpath'); % the full path and name of the file in which the call occurs, not including the filename extension.
    mfile_fullpath_without_fname = mfile_fullpath(1:end-strlength(mfilename));
    addpath(genpath(mfile_fullpath_without_fname + "../data"), ...
            genpath(mfile_fullpath_without_fname + "../inc")); % adds the specified folders to the top of the search path for the current MATLAB® session.
end
```

A.2 子程序