第 3 次作业

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关键词：关键词 1，关键词 2

Homework 3

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目 录

1 Question 2

"Derive (3.23) using (3.22), (3.19) and the ideal gas law."(Hartmann, 2016, p. 93)

1.1 Solution

除特别说明外，本节各符号的含义同 Hartmann (2016)中的.

1.1.1 Brief Review: *Lambert–Bouguer–Beer Law of Extinction*

由 *Lambert–Bouguer–Beer* **消光定律**（*law of extinction*）

$$\mathrm{d}F = -F\rho M_\mathrm{a} k_\mathrm{abs}\,\mathrm{d}s, \qquad \mathrm{d}s = -\sec\theta\,\mathrm{d}z \tag{1.1}$$

得

$$\frac{\mathrm{d}F}{F} = \rho M_\mathrm{a} k_\mathrm{abs} \sec\theta\,\mathrm{d}z. \tag{1.2}$$

对 z 从高度 $z = z$ 到大气上界 $z = +\infty$ 积分，得

$$F = F_\infty \exp(-\tau\sec\theta), \tag{1.3}$$

其中 F_∞, θ 是常数，

$$\tau := \int_z^{+\infty} \rho M_\mathrm{a} k_\mathrm{abs}\,\mathrm{d}z \tag{1.4}$$

被称为沿传输路径的**光学厚度**（*optical depth*）. 由（1.3）得

$$\mathrm{d}F = -F\sec\theta\,\mathrm{d}\tau, \tag{1.5}$$

从而**吸收率**（*Absorption rate*）

$$\frac{\mathrm{d}F}{\mathrm{d}z} = -F_\infty \exp(-\tau\sec\theta)\sec\theta\frac{\mathrm{d}\tau}{\mathrm{d}z}. \tag{1.6}$$

下面将本节的结果应用于静力平衡状态下的平面平行等温大气.

1.1.2 A Special Case: Isothermal Plane-parallel Atmosphere in Hydrostatic Balance

将理想气体状态方程

$$p = \rho RT \tag{1.7}$$

代入静力平衡方程

$$\mathrm{d}p = -\rho g\,\mathrm{d}z \tag{1.8}$$

得

$$\frac{\mathrm{d}p}{p} = \frac{\mathrm{d}\rho}{\rho} = -\frac{\mathrm{d}z}{H}, \tag{1.9}$$

其中

$$H := RT/g \tag{1.10}$$

为**大气标高**（scale height）. 对（1.10）从下表面（$z = 0$）到高度 $z = z$ 积分，得

$$p = p_\mathrm{s}\exp(-z/H), \tag{1.11}$$

$$\rho = \rho_\mathrm{s}\exp(-z/H). \tag{1.12}$$

将（1.12）代入（1.4），假定 $M_\mathrm{a}, k_\mathrm{abs}$ 为常数，得

$$\tau = k_\mathrm{abs} M_\mathrm{a} \rho_\mathrm{s} H \exp(-z/H), \tag{1.13}$$

从而

$$\mathrm{d}\tau = -\tau\,\mathrm{d}z/H\,. \tag{1.14}$$

将（1.14）代入（1.6），得

$$\frac{\mathrm{d}F}{\mathrm{d}z} = \frac{F_\infty}{H}\tau\exp(-\tau\sec\theta)\sec\theta\,. \tag{1.15}$$

易知吸收率 $\mathrm{d}F/\mathrm{d}z$（作为 τ 的函数）有最大值，下求之.

令

$$0 = \frac{\mathrm{d}}{\mathrm{d}\tau}\ln\frac{\mathrm{d}F}{\mathrm{d}z} = \frac{1}{\tau} - \sec\theta\,, \tag{1.16}$$

得 $\ln(\mathrm{d}F/\mathrm{d}z)$ 的唯一驻点

$$\tau = \tau_{\max\mathrm{abs}} \coloneqq \cos\theta\,, \tag{1.17}$$

故吸收率 $\mathrm{d}F/\mathrm{d}z$ 在 $\tau = \cos\theta$ 处取最大值. 将（1.17）代入（1.13）得该最大值点位于高度

$$z = z_{\max\mathrm{abs}} \coloneqq H\ln(k_{\mathrm{abs}}M_{\mathrm{a}}\rho_{\mathrm{s}}H\sec\theta) \tag{1.18}$$

处，再由（1.11）得该处的压强为

$$p = p_{\max\mathrm{abs}} \coloneqq p_{\mathrm{s}}\frac{\cos\theta}{Hk_{\mathrm{abs}}\rho_{\mathrm{as}}}, \tag{1.19}$$

其中

$$\rho_{\mathrm{as}} = M_{\mathrm{a}}\rho_{\mathrm{s}} \tag{1.20}$$

"is the density of the absorber at the surface." (Hartmann, 2016, p. 64)

2 Question 1

"Suppose a gas that absorbs solar radiation has a uniform mixing ratio of 1 g kg^{-1} and an absorption cross-section of 5 m^2 kg^{-1}. At what altitude will the maximum rate of energy absorption per unit volume occur? Assume an isothermal atmosphere with $T = 260$ K, and a surface pressure of 1.025×10^5 Pa, and that the Sun is directly overhead."(Hartmann, 2016, p. 93)

2.1 Solution

将（1.7）（1.10）代入（1.18）得

$$z = z_{\max\mathrm{abs}} \coloneqq \frac{RT}{g}\ln\left(k_{\mathrm{abs}}M_{\mathrm{a}}\frac{p_{\mathrm{s}}}{g}\sec\theta\right), \tag{2.1}$$

代入 $\theta = 0$ 和其他数据得

$$z_{\max\mathrm{abs}} = 30091\ (\mathrm{m}). \tag{2.2}$$

References

Hartmann, D. L. (2016). Chapter 3 - Atmospheric Radiative Transfer and Climate. In D. L. Hartmann (Ed.), *Global Physical Climatology (Second Edition)* (pp. 49-94). Elsevier. https://doi.org/10.1016/B978-0-12-328531-7.00003-7