第6次作业 (第7章)

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关键词：关键词1，关键词2

Homework 6 (Chapter 7)

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**Keywords:** keyword 1, keyword 2

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# Question 1

Suppose that a wind stress is applied to the ocean, taking the following simple form.

Derive an equation for the vertical velocity at the bottom of the [Ekman layer](https://www.sciencedirect.com/topics/earth-and-planetary-sciences/ekman-layer) assuming a constant Coriolis parameter *f* = *f*0 = 2 Ω sin 30o. Derive an equation for the integrated meridional transport *V*I, using [(7.14)](https://www.sciencedirect.com/science/article/pii/B9780123285317000074?via%3Dihub#e0065) with the *f* and *β* appropriate for 30°N latitude. Determine a numeric value for the maximum *w*E and *V*I using the following constants: A = 2 dyn cm–2 = 0.2 N m–2, *L* = 1500 km, *ρ*0 = 1025 kg m–3. Plot *τx*, *w*E, and *V*I on the interval –*L* < *y* < *L*. Assuming that the ocean basin is 5000 km wide, calculate the water mass flux at 30°N associated with the interior flow. Compare this number with the estimate for the Gulf Stream mass flux given in [Section 7.8](https://www.sciencedirect.com/science/article/pii/B9780123285317000074?via%3Dihub#s0065). ([Hartmann, 2016, p. 232](#_ENREF_1))

## Solution

Using Eq.[(7.11)](https://www.sciencedirect.com/science/article/pii/B9780123285317000074?via%3Dihub" \l "e0060) of ([Hartmann, 2016](#_ENREF_1)), the vertical velocity at the bottom of the [Ekman layer](https://www.sciencedirect.com/topics/earth-and-planetary-sciences/ekman-layer) is

assuming that the vertical current vanishes at the surface.

Using Eq.[(7.14)](https://www.sciencedirect.com/science/article/pii/B9780123285317000074?via%3Dihub#e0075) of ([Hartmann, 2016](#_ENREF_1)), the integrated meridional transport *V*I

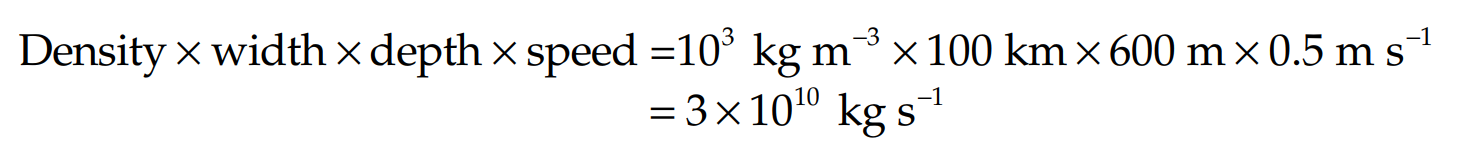
where

is the radius of the Earth, 6371 km.

**Figure 1** shows a numeric value for the *w*E and *V*I using the following constants: *A* = 2 dyn cm–2 = 0.2 N m–2, *L* = 1500 km, *ρ*0 = 1025 kg m–3.

Assuming that the ocean basin is 5000 km wide, the water mass flux at 30°N (*y* = 0) associated with the interior flow is

which is about 46% of the estimate for the Gulf Stream mass flux given in [Section 7.8](https://www.sciencedirect.com/science/article/pii/B9780123285317000074?via%3Dihub#s0065) ([Hartmann, 2016, p. 229](#_ENREF_1))





**Figure 1** A numeric value for *w*E and *V*I using the following constants: *A* = 2 dyn cm–2 = 0.2 N m–2, *L* = 1500 km, *ρ*0 = 1025 kg m–3.

References

Hartmann, D. L. (2016). Chapter 7 - The Ocean General Circulation and Climate. In D. L. Hartmann (Ed.), *Global Physical Climatology (Second Edition)* (pp. 195-232). Elsevier. <https://doi.org/10.1016/B978-0-12-328531-7.00007-4>

1. 本文使用的MATLAB源代码
   1. 主程序

|  |  |
| --- | --- |
| 1 | %% ch7\_hw.m |
| 2 | % Description: MATLAB code for Homework 6 (Chapter 7) (MS3402, 2022 Spring) |
| 3 | % Author: Guorui Wei (危国锐) (313017602@qq.com; weiguorui@sjtu.edu.cn) |
| 4 | % Student ID: 516021910080 |
| 5 | % Created: 2022-05-01 |
| 6 | % Last modified: 2022-05-01 |
| 7 | % References: [1] Hartmann, D. L. (2016). Chapter 7 - The Ocean General Circulation and Climate. In D. L. Hartmann (Ed.), Global Physical Climatology (Second Edition) (pp. 195-232). Elsevier. https://doi.org/10.1016/B978-0-12-328531-7.00007-4 |
| 8 |  |
| 9 | %% Initialize project |
| 10 |  |
| 11 | clc; clear; close all |
| 12 | init\_env(); |
| 13 |  |
| 14 | %% Problem 7 of [1,p.232] |
| 15 |  |
| 16 | % params |
| 17 |  |
| 18 | A = .2;             % [N/m^2] |
| 19 | L = 1500e3;         % [m] |
| 20 | rho\_0 = 1025;       % [kg/m^3] |
| 21 | Omega = 7.292e-5;   % [rad/s] angular velocity of the Earth = 2\*pi/(23\*3600+56\*60) |
| 22 | lat = pi/6;         % [rad E] |
| 23 | R\_E = 6371e3;       % [m] radius of the Earth |
| 24 | W = 5000e3;         % [m] the ocean basin width |
| 25 |  |
| 26 | % |
| 27 | f\_0 = 2\*Omega\*sin(lat);                  % [rad/s] Coriolis parameter |
| 28 | beta\_0 = 2\*Omega\*cos(lat)/R\_E;           % [rad/m/s] |
| 29 | tau\_x = @(y) A\*cos(pi\*y/L);              % [N/m^2] wind stress |
| 30 | w\_E = @(y) A\*pi/rho\_0/f\_0/L\*sin(pi\*y/L); % the vertical velocity at the bottom of the Ekman layer |
| 31 | V\_I = @(y) A/rho\_0/f\_0\*cos(pi\*y/L) + A\*pi/rho\_0/beta\_0/L\*sin(pi\*y/L); % [m^3/m/s] the integrated meridional transport (volume) |
| 32 | M\_y = W\*A/f\_0; % [kg/s] the water mass flux at y = 0 associated with the interior flow |
| 33 | fprintf("M\_y = %.3d (kg/s)\n",M\_y); |
| 34 |  |
| 35 | %% solve |
| 36 |  |
| 37 | y = linspace(-L,L,2^13); |
| 38 | val\_w\_E = w\_E(y)\*1e6; % [um/s] |
| 39 | val\_V\_I = V\_I(y); |
| 40 | [w\_E\_max,ind\_w\_E\_max] = max(val\_w\_E); |
| 41 | [w\_E\_min,ind\_w\_E\_min] = min(val\_w\_E); |
| 42 | [V\_I\_max,ind\_V\_I\_max] = max(val\_V\_I); |
| 43 | [V\_I\_min,ind\_V\_I\_min] = min(val\_V\_I); |
| 44 |  |
| 45 | %% plot |
| 46 | t\_fig = figure("Name","Fig.1 Ch7 Q7"); |
| 47 | t\_TCL = tiledlayout(1,2,"TileSpacing","tight","Padding","tight"); |
| 48 |  |
| 49 | %%% Fig.1(a) w\_E |
| 50 | t\_Axes\_tau\_x = nexttile(t\_TCL,1); |
| 51 | t\_plot\_tau\_x = plot(t\_Axes\_tau\_x,tau\_x(y),y/1000,'-',"color",'#0072BD',"DisplayName",'$\tau\_x$','LineWidth',1); |
| 52 | set(t\_Axes\_tau\_x,"YDir",'normal',"TickLabelInterpreter",'latex',"FontSize",10,'Box','off','XColor','#0072BD','YColor','#0072BD'); |
| 53 | % |
| 54 | t\_Axes\_w\_E = axes(t\_TCL); |
| 55 | t\_Axes\_w\_E.Layout.Tile = 1; |
| 56 | t\_plot\_w\_E = plot(t\_Axes\_w\_E,val\_w\_E,y/1000,'-','Color','#D95319',"DisplayName",'$w\_{\rm{E}}$','LineWidth',1); |
| 57 | hold on |
| 58 | t\_plot\_w\_E\_min = plot(t\_Axes\_w\_E,w\_E\_min,y(ind\_w\_E\_min)/1000,'o',"DisplayName",sprintf("$w\_{\\rm{E}}(y = %.0f) = \\rm{%.1f}$",y(ind\_w\_E\_min)/1000,w\_E\_min)); |
| 59 | t\_plot\_w\_E\_max = plot(t\_Axes\_w\_E,w\_E\_max,y(ind\_w\_E\_max)/1000,'^',"DisplayName",sprintf("$w\_{\\rm{E}}(y = %.0f) = \\rm{%.1f}$",y(ind\_w\_E\_max)/1000,w\_E\_max)); |
| 60 | hold off |
| 61 | set(t\_Axes\_w\_E,'YDir','normal','FontSize',10,'TickLabelInterpreter','latex','XAxisLocation','top','YAxisLocation','right','YTickLabel',{},'Box','off','Color','none','XColor','#D95319','YColor','#D95319','YLimitMethod','tight') |
| 62 | xlabel(t\_Axes\_w\_E,"$w\_{\rm{E}}$ $(\rm{\mu m}/\rm{s})$","FontSize",10,Interpreter="latex") |
| 63 | % |
| 64 | linkaxes([t\_Axes\_w\_E,t\_Axes\_tau\_x],'y'); |
| 65 | legend([t\_plot\_w\_E,t\_plot\_tau\_x,t\_plot\_w\_E\_min,t\_plot\_w\_E\_max],"Location",'southeast','Interpreter','latex',"Box","off"); |
| 66 |  |
| 67 | %%% Fig.1(b) V\_I |
| 68 | % |
| 69 | t\_Axes\_tau\_x = nexttile(t\_TCL,2); |
| 70 | t\_plot\_tau\_x = plot(t\_Axes\_tau\_x,tau\_x(y),y/1000,'-',"color",'#0072BD',"DisplayName",'$\tau\_x$','LineWidth',1); |
| 71 | set(t\_Axes\_tau\_x,"YDir",'normal','YTickLabel',{},"TickLabelInterpreter",'latex',"FontSize",10,'Box','off','XColor','#0072BD','YColor','#0072BD'); |
| 72 | % |
| 73 | t\_Axes\_V\_I = axes(t\_TCL); |
| 74 | t\_Axes\_V\_I.Layout.Tile = 2; |
| 75 | t\_plot\_V\_I = plot(t\_Axes\_V\_I,val\_V\_I,y/1000,'-','Color','#D95319',"DisplayName",'$V\_{\rm{I}}$','LineWidth',1); |
| 76 | hold on |
| 77 | t\_plot\_V\_I\_min = plot(t\_Axes\_V\_I,V\_I\_min,y(ind\_V\_I\_min)/1000,'o',"DisplayName",sprintf("$V\_{\\rm{I}}(y = %.0f) = \\rm{%.1f}$",y(ind\_V\_I\_min)/1000,V\_I\_min)); |
| 78 | t\_plot\_V\_I\_max = plot(t\_Axes\_V\_I,V\_I\_max,y(ind\_V\_I\_max)/1000,'^',"DisplayName",sprintf("$V\_{\\rm{I}}(y = %.0f) = \\rm{%.1f}$",y(ind\_V\_I\_max)/1000,V\_I\_max)); |
| 79 | hold off |
| 80 | set(t\_Axes\_V\_I,'YDir','normal','FontSize',10,'TickLabelInterpreter','latex','XAxisLocation','top','YAxisLocation','right','YTickLabel',{},'Box','off','Color','none','XColor','#D95319','YColor','#D95319','YLimitMethod','tight') |
| 81 | xlabel(t\_Axes\_V\_I,"$V\_{\rm{I}}$ $(\rm{m}^3 / (\rm{m} \cdot \rm{s}))$","FontSize",10,Interpreter="latex") |
| 82 | % |
| 83 | linkaxes([t\_Axes\_V\_I,t\_Axes\_tau\_x],'y'); |
| 84 | legend([t\_plot\_V\_I,t\_plot\_tau\_x,t\_plot\_V\_I\_min,t\_plot\_V\_I\_max],"Location",'southeast','Interpreter','latex',"Box","off"); |
| 85 |  |
| 86 | %%% |
| 87 | xlabel(t\_TCL,"$\tau\_x$ $(\rm{N}/\rm{m}^2)$","FontSize",10,Interpreter="latex") |
| 88 | ylabel(t\_TCL,"$y$ (km)","FontSize",10,Interpreter="latex") |
| 89 | [t\_title\_t,t\_title\_s] = title(t\_TCL,"\bf 2022 Spring MS3402 Hw6 (Chapter 7) Pblm 7","Guorui Wei 516021910080","Interpreter",'latex'); |
| 90 | set(t\_title\_s,'FontSize',8) |
| 91 | % |
| 92 | exportgraphics(t\_TCL,"..\\doc\\fig\\ch7\_P7.emf",'Resolution',800,'ContentType','auto','BackgroundColor','none','Colorspace','rgb') |
| 93 | exportgraphics(t\_TCL,"..\\doc\\fig\\ch7\_P7.png",'Resolution',800,'ContentType','auto','BackgroundColor','none','Colorspace','rgb') |
| 94 |  |
| 95 | %% local functions |
| 96 |  |
| 97 | %% Initialize environment |
| 98 | function [] = init\_env() |
| 99 | % set up project directory |
| 100 | if ~isfolder("../doc/fig/") |
| 101 | mkdir ../doc/fig/ |
| 102 | end |
| 103 | % configure searching path |
| 104 | mfile\_fullpath = mfilename('fullpath'); % the full path and name of the file in which the call occurs, not including the filename extension. |
| 105 | mfile\_fullpath\_without\_fname = mfile\_fullpath(1:end-strlength(mfilename)); |
| 106 | addpath(genpath(mfile\_fullpath\_without\_fname + "../data"), ... |
| 107 | genpath(mfile\_fullpath\_without\_fname + "../inc")); % adds the specified folders to the top of the search path for the current MATLAB® session. |
| 108 | end |
| 109 |  |

* 1. 子程序