Chapter 1

Introduction

1.1 DYNAMIC METEOROLOGY

Dynamic meteorology is the study of air motion in the Earth's atmosphere that is associated with weather and climate. These motions organize into coherent circulation features that affect human activity primarily through wind, temperature, clouds, and precipitation patterns. Short-lived features, lasting from a few minutes to a few days, are related to weather, and some familiar weather examples that we will examine in this book include tropical and extratropical cyclones, organized thunderstorms, and local wind patterns such as those that occur near mountains. Figure 1.1 illustrates the mixing effect of larger weather patterns in the atmosphere, from large areas of convective cloud in the tropics to extratropical cyclones in the higher latitudes of the Northern and Southern Hemispheres. These weather elements occur in the *troposphere*, which is the portion of the atmosphere in contact with the surface. The troposphere normally exhibits a drop in temperature with elevation and contains most of the water vapor, clouds, and precipitation found in the atmosphere. On average, the troposphere extends vertically about 10 kilometers, where the *tropopause* is located. Above the tropopause is the stratosphere, where the temperature increases with elevation due to heating of the air by absorption of ultraviolet radiation by ozone. Most of the topics addressed in this book concern the dynamics of the troposphere and stratosphere.

Over longer time periods, the realm of climate, circulation features may persist from seasons to years over large regions of Earth. Examples of climate variability include shifts in the locations where storms occur, oscillations in large-scale pressure patterns, and planetary patterns of variability associated with the El Niño Southern Oscillation (ENSO) phenomenon of the tropical Pacific Ocean. ENSO reminds us that although dynamic meterology involves the study of air motion in the atmosphere, this motion links to other parts of Earth's system, including the oceans, biosphere, and cryosphere, and plays an active role in the transport of chemical species. Moreover, many of the ideas we present here also apply to the atmospheres of other planets.

Before we set off to explore the landscape of dynamic meteorology, we devote this first chapter to introducing fundamental concepts that will guide the

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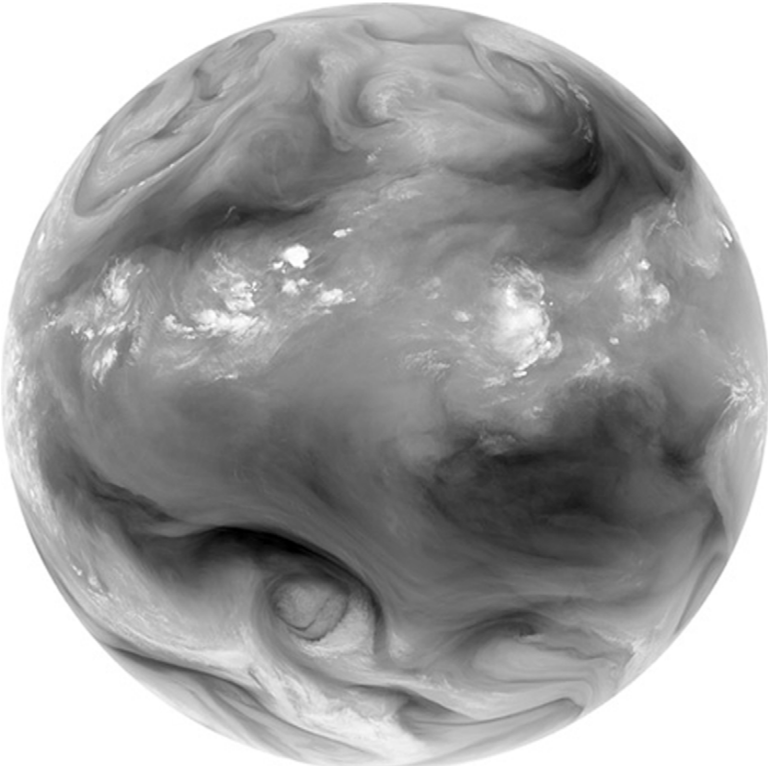

FIGURE 1.1 Infrared satellite image near a wavelength of 6.7 μm, which is known as the "water vapor" channel since it captures the distribution of that field in a layer roughly 5 to 10 km above Earth's surface. Because water vapor is continuously distributed, in contrast to clouds, atmospheric motion is especially well captured. Here we see the convective clouds in the tropics and the mixing effects of eddies at higher latitude. (*Source: NASA.*)

journey. First, note that the laws that govern atmospheric motion satisfy the principle of *dimensional homogeneity*, which means that all terms in the equations expressing these laws must have the same physical dimensions. These dimensions can be expressed in terms of multiples and ratios of four dimensionally independent properties: length, time, mass, and thermodynamic temperature. To measure and compare the scales of terms in the laws of motion, a set of units of measure must be defined for these four fundamental properties. In this text the international system of units (SI) will be used almost exclusively. The four fundamental properties are measured in terms of the SI *base units* shown in Table 1.1. All other properties are measured in terms of SI *derived units*, which are units formed from products or ratios of the base units. For example, velocity has the derived units of meter per second ($\mathrm{m\ s^{-1}}$).

TABLE 1.1 SI Base Units

Property	Name	Symbol
Length	Meter (meter)	m
Mass	Kilogram	kg
Time	Second	s
Temperature	Kelvin	K

TABLE 1.2 SI Derived Units with Special Names

Property	Name	Symbol
Frequency	Hertz	$\mathrm{Hz(s^{-1})}$
Force	Newton	$\mathrm{N(kg\ m\ s^{-2})}$
Pressure	Pascal	$\mathrm{Pa(N\ m^{-2})}$
Energy	Joule	$\mathrm{J\ (N\ m)}$
Power	Watt	$\mathrm{W(J\ s^{-1})}$

A number of important derived units have special names and symbols. Those that are commonly used in dynamic meteorology are indicated in Table 1.2. In addition, the supplementary unit designating a plane angle, the radian (rad), is required for expressing angular velocity (rad s^{-1}) in the SI system.[1] Finally, Table 1.3 lists the symbols frequently used in this book for some of the basic physical quantities. Note that the full three-dimensional velocity vector, $\mathbf{U}$, is related to the horizontal velocity vector, $\mathbf{V}$, by $\mathbf{U} = (\mathbf{V}, w)$ and $\mathbf{U} = (\mathbf{V}, \omega)$ in height and pressure vertical coordinates, respectively. We shall use the term "zonal" to refer to the East–West direction and "meridional" to refer to the North–South direction.

Dynamic meteorology applies the conservation laws of classical physics for momentum (Newton's laws of motion), mass, and energy (First law of thermodynamics) to the atmosphere. An essential aspect of this application involves the *continuum* approximation, whereby the properties of discrete molecules are ignored in favor of a continuous representation involving a local average over a blob of molecules. This approximation is common to all fluids, including liquids and gases, and allows atmospheric properties (e.g., pressure, density, temperature), or "field variables," to be represented as smooth functions taking on unique values

[1] Note that *Hertz* measures frequency in *cycles* per second, not in radians per second.

TABLE 1.3 Symbols and Units of Basic Physical Quantities

Quantity	Symbol	Units
Three-dimensional velocity vector	$\mathbf{U}$	m s^{-1}
Horizontal velocity vector	$\mathbf{V}$	m s^{-1}
Eastward component of velocity	u	m s^{-1}
Northward component of velocity	v	m s^{-1}
Upward component of velocity	w (ω)	m s^{-1} (Pa s^{-1})
Pressure	P	N m^{-2}
Density	ρ	kg m^{-3}
Temperature	T	K (or °C)

in the independent variables of space and time. A "point" in the continuum is regarded as a volume element that is very small compared with the volume of atmosphere under consideration, but that still contains a large number of molecules. The expressions *air parcel* and *air particle* are both commonly used to refer to such a point. In Chapter 2, the fundamental conservation laws are applied to a small volume element of the atmosphere subject to the continuum approximation in order to obtain the governing equations. Our goal here is to provide a survey of the main forces that influence atmospheric motions.

1.2 CONSERVATION OF MOMENTUM

Newton's first law states that an object at rest or moving with a constant velocity remains so unless acted upon by an external unbalanced force. Once the forces are identified, Newton's second law states that the temporal change of momentum (i.e., acceleration) is a vector having direction given by the net force (the sum over all forces) and magnitude given by the size of the net force divided by the object's mass. These forces can be classified as either *body forces* or *surface forces*. Body forces act on the center of mass of a fluid parcel and have magnitudes proportional to the mass of the parcel; gravity is an example of a body force. Surface forces act across the boundary surface separating a fluid parcel from its surroundings and have magnitudes independent of the mass of the parcel; the pressure force is an example.

For atmospheric motions of meteorological interest, the forces that are of primary concern are the pressure gradient force, the gravitational force, and the frictional force. These *fundamental* forces determine acceleration as measured relative to coordinates fixed in space. If, as is the usual case, the motion is referred to a coordinate system rotating with Earth, Newton's second law may still be applied provided that certain *apparent* forces, the centrifugal force and the Coriolis force, are included. The fundamental forces are discussed subsequently, and the apparent forces are introduced in Section 1.3.

1.2.1 Pressure Gradient Force

Pressure is defined as the force per unit area acting normal to a surface. In a gas such as the atmosphere, pressure at a point acts equally in all directions due to random molecular motion. Therefore, the magnitude of the net force due to molecules colliding with a surface is independent of the orientation of the surface; note that the *direction* of the net force changes with the orientation of the surface, but the magnitude does not. Placing a wall in a gas, with the pressure on one side different from that on the other, yields a net force that will accelerate the wall toward the side having lower pressure; this net force associated with pressure differences is the essence of the pressure *gradient* force.

Consider now an infinitesimal volume element of air, $\delta V = \delta x \delta y \delta z$, centered at the point x_0, y_0, z_0, as illustrated in Figure 1.2. Due to random molecular motions, momentum is continually imparted to the walls of the volume element by the surrounding air. This momentum transfer per unit time per unit area is just the pressure exerted on the walls of the volume element by the surrounding air. If the pressure at the center of the volume element is designated by p_0, then the pressure on the wall labeled A in Figure 1.2 can be expressed in a Taylor series expansion as

$$p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2} + \text{higher-order terms}$$

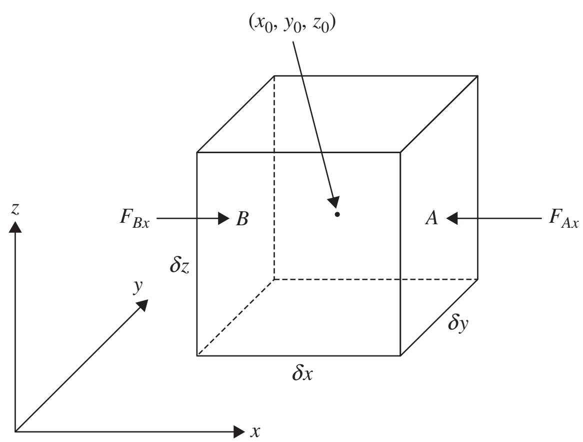

FIGURE 1.2 The x component of the pressure gradient force acting on a fluid element.

Neglecting the higher-order terms in this expansion, the pressure force acting on the volume element at wall A is

$$F_{Ax} = -\left(p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y\,\delta z$$

where $\delta y \delta z$ is the area of wall A. Similarly, the pressure force acting on the volume element at wall B is just

$$F_{Bx} = +\left(p_0 - \frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y\,\delta z$$

Therefore, the net x component of this force acting on the volume is

$$F_x = F_{Ax} + F_{Bx} = -\frac{\partial p}{\partial x}\,\delta x\,\delta y\,\delta z$$

Because the net force is proportional to the derivative of pressure, it is referred to as the *pressure gradient force*. The mass m of the differential volume element is simply the density ρ times the volume: $m = \rho\delta x\delta y\delta z$. Thus, the x component of the pressure gradient force per unit mass is

$$\frac{F_x}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

Similarly, it can easily be shown that the y and z components of the pressure gradient force per unit mass are

$$\frac{F_y}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \text{and} \qquad \frac{F_z}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

so that the total pressure gradient force per unit mass is the vector given by

$$\frac{\mathbf{F}}{m} = -\frac{1}{\rho}\nabla p \tag{1.1}$$

The gradient operator $\nabla = \left(\mathbf{i}\frac{\partial}{\partial x}, \mathbf{j}\frac{\partial}{\partial y}, \mathbf{k}\frac{\partial}{\partial z}\right)$ acts on functions to its right to yield vectors that point toward higher values of the function. It is important to note that (1) the pressure gradient points from low to high pressure, but the pressure gradient *force* points from high to low pressure, and (2) the pressure gradient force is proportional to the *gradient* of the pressure field, not to the pressure itself.

1.2.2 Viscous Force

Any real fluid is subject to internal friction, called viscosity, which causes it to resist the tendency to flow. Viscosity arises when the fluid velocity varies spatially so that random molecular motion accomplishes a net transport of momentum from molecules in faster-moving air parcels to molecules in nearby slower-moving air parcels. This momentum exchange between parcels may be expressed as a viscous force, F, acting *along* the face of the air parcel, which produces a *shear* stress, τ, on the parcel per area, A,

$$\tau = \frac{F}{A} \tag{1.2}$$

Therefore, the viscous force is given by $F = \tau A$. For a Newtonian fluid, we assume that the shear stress depends linearly on the fluid speed, which is a very good approximation for air. In the vertical direction, for example, variation in the x component of the wind, u, produces the stress

$$\tau \approx \mu \frac{\partial u}{\partial z} \tag{1.3}$$

where μ is the viscosity coeficient, which depends on the fluid. As in the pressure gradient force discussion, we need the net force acting on the air parcel from viscous effects. Following a similar Taylor-approximation approach as for the pressure gradient derivation, but noting that the force is directed *along* rather than *normal* to the face of the parcel volume, we find that the viscous force per unit mass due to vertical shear of the component of motion in the x direction is

$$\frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right) \tag{1.4}$$

For constant μ, the right side may be simplified to $\nu \partial^2 u/\partial z^2$, where $\nu = \mu/\rho$ is the *kinematic viscosity coefficient*. For standard atmosphere[2] conditions at sea level, $\nu = 1.46 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$. Note that (1.4) represents only the contribution from x momentum shear stress in the z direction, and the net force in the x direction, F_{rx}, also includes contributions from the x and y directions. The net frictional force components per unit mass in the three Cartesian coordinate directions are

$$\begin{aligned}
F_{rx} &= \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \\
F_{ry} &= \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] \\
F_{rz} &= \nu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right]
\end{aligned} \tag{1.5}$$

Each component frictional force represents diffusion of momentum in that coordinate direction, since, for example, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \nabla \cdot \nabla u = \nabla^2 u$. For any

[2]The U.S. standard atmosphere is a specified vertical profile of atmospheric structure.

vector **A**, $\nabla \cdot \mathbf{A}$ is a scalar quantity called the *divergence* of **A**, since it is positive when vectors diverge away from a point; negative divergence is called *convergence.* At a local maximum in u, ∇u points toward (converges on) the maximum, and therefore $\nabla^2 u < 0$, resulting in a loss of momentum from the maximum value to the surrounding region. This process is called *downgradient diffusion*, since momentum diffuses down the gradient, from large to small values.

For the atmosphere below 100 km, ν is so small that molecular viscosity is negligible except in a thin layer within a few centimeters of Earth's surface, where the vertical shear is very large. Away from this surface molecular boundary layer, momentum is transferred primarily by turbulent eddy motions, which are discussed in Chapter 8.

1.2.3 Gravitational Force

The sole body force on atmospheric air parcels is due to gravity. Newton's law of universal gravitation states that any two elements of mass in the universe attract each other with a force proportional to their masses and inversely proportional to the square of the distance separating them. Thus, if two mass elements M and m are separated by a distance $r \equiv |\mathbf{r}|$ (with the vector $\mathbf{r}$ directed toward m as shown in Figure 1.3), then the force exerted by mass M on mass m due to gravitation is

$$\mathbf{F}_g = -\frac{GMm}{r^2}\left(\frac{\mathbf{r}}{r}\right) \tag{1.6}$$

where G is a universal constant called the gravitational constant. The law of gravitation as expressed in (1.6) actually applies only to hypothetical "point" masses, since for objects of finite extent $\mathbf{r}$ will vary from one part of the object to another. However, for finite bodies, (1.6) may still be applied if $|\mathbf{r}|$ is interpreted

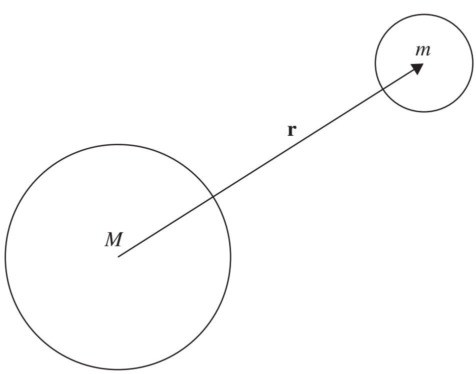

FIGURE 1.3 Two spherical masses whose centers are separated by a distance r.

as the distance between the centers of mass of the bodies. Thus, if Earth is designated as mass M, and m is a mass element of the atmosphere, then the force per unit mass exerted on the atmosphere by the gravitational attraction of Earth is

$$\frac{\mathbf{F}_g}{m} \equiv \mathbf{g}^* = -\frac{GM}{r^2}\left(\frac{\mathbf{r}}{r}\right) \tag{1.7}$$

In dynamic meteorology it is customary to use the height above mean sea level as a vertical coordinate. If the mean radius of Earth is designated by a and the distance above mean sea level is designated by z, then neglecting the small departure of the shape of Earth from sphericity, $r = a + z$. Therefore, Eq. (1.7) can be rewritten as

$$\mathbf{g}^* = \frac{\mathbf{g}_0^*}{(1 + z/a)^2} \tag{1.8}$$

where $\mathbf{g}_0^* = -(GM/a^2)(\mathbf{r}/r)$ is the gravitational force at mean sea level. For meteorological applications, $z \ll a$, so that with negligible error we can let $\mathbf{g}^* = \mathbf{g}_0^*$ and simply treat the gravitational force as a constant. Note that this treatment of the gravitational force will be modified in Section 1.3.2 to account for centrifugal forces due to Earth's rotation.

1.3 NONINERTIAL REFERENCE FRAMES AND "APPARENT" FORCES

In formulating the laws of atmospheric dynamics, it is natural to use a *geocentric* reference frame—that is, a frame of reference at rest with respect to rotating Earth. Newton's first law of motion states that a mass in uniform motion relative to a coordinate system fixed in space will remain in uniform motion in the absence of any forces. Such motion is referred to as *inertial motion*, and the fixed reference frame is an inertial, or absolute, frame of reference. It is clear, however, that an object at rest or in uniform motion with respect to rotating Earth is not at rest or in uniform motion relative to a coordinate system fixed in space.

Therefore, motion that appears to be inertial motion to an observer in a geocentric reference frame is really accelerated motion. Hence, a geocentric reference frame is a *noninertial* reference frame. Newton's laws of motion can only be applied in such a frame if the acceleration of the coordinates is taken into account. The most satisfactory way of including the effects of coordinate acceleration is to introduce "apparent" forces in the statement of Newton's second law. These apparent forces are the inertial reaction terms that arise because of the coordinate acceleration. For a coordinate system in uniform rotation, two such apparent forces are required: the centrifugal force and the Coriolis force.

1.3.1 Centripetal Acceleration and Centrifugal Force

To illustrate the essential aspects of noninertial frames, we consider a ball of mass m attached to a string and whirled through a circle of radius r at a constant angular velocity ω. From the point of view of an observer in inertial space the speed of the ball is constant, but its direction of travel is continuously changing so that its velocity is not constant. To compute the acceleration we consider the change in velocity $\delta \mathbf{V}$ that occurs for a time increment δt during which the ball rotates through an angle $\delta\theta$ as shown in Figure 1.4. Because $\delta\theta$ is also the angle between the vectors $\mathbf{V}$ and $\mathbf{V} + \delta\mathbf{V}$, the magnitude of $\delta\mathbf{V}$ is just $|\delta\mathbf{V}| = |\mathbf{V}|\,\delta\theta$. If we divide by δt and note that in the limit $\delta t \to 0$, $\delta\mathbf{V}$ is directed toward the axis of rotation, we obtain

$$\frac{D\mathbf{V}}{Dt} = |\mathbf{V}|\frac{D\theta}{Dt}\left(-\frac{\mathbf{r}}{r}\right)$$

However, $|\mathbf{V}| = \omega r$ and $D\theta/Dt = \omega$, so that

$$\frac{D\mathbf{V}}{Dt} = -\omega^2\mathbf{r} \tag{1.9}$$

Therefore, viewed from fixed coordinates, the motion is one of uniform acceleration directed toward the axis of rotation at a rate equal to the square of the angular velocity times the distance from the axis of rotation. This acceleration is called *centripetal acceleration*. It is caused by the force of the string pulling the ball.

Now suppose that we observe the motion in a coordinate system rotating with the ball. In this rotating system the ball is stationary, but there is still a force acting on the ball—namely, the pull of the string. Therefore, in order to apply Newton's second law to describe the motion relative to this rotating

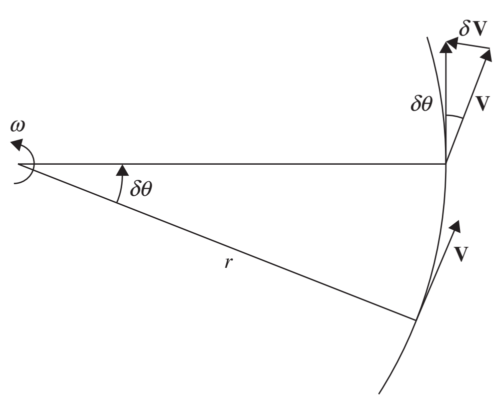

FIGURE 1.4 Centripetal acceleration is given by the rate of change of the direction of the velocity vector, which is directed toward the axis of rotation, as illustrated here by $\delta\mathbf{V}$.

coordinate system, we must include an additional apparent force, the *centrifugal force*, which just balances the force of the string on the ball. Thus, the centrifugal force is equivalent to the inertial reaction of the ball on the string and just equal and opposite to the centripetal acceleration.

To summarize, observed from a fixed system, the rotating ball undergoes a uniform centripetal acceleration in response to the force exerted by the string. Observed from a system rotating along with it, the ball is stationary and the force exerted by the string is balanced by a centrifugal force.

1.3.2 Gravity Revisited

An object at rest on the surface of Earth is not at rest or in uniform motion relative to an inertial reference frame except at the poles. Rather, an object of unit mass at rest on the surface of Earth is subject to a centripetal acceleration directed toward the axis of rotation of Earth given by $-\Omega^2\mathbf{R}$, where $\mathbf{R}$ is the position vector from the axis of rotation to the object and $\Omega = 7.292 \times 10^{-5}\,\mathrm{rad\,s^{-1}}$ is the angular speed of rotation of Earth.[3] Since, except at the equator and poles, the centripetal acceleration has a component directed poleward along the horizontal surface of Earth (i.e., along a surface of constant *geopotential*), there must be a net horizontal force directed poleward along the horizontal to sustain the horizontal component of the centripetal acceleration.

This force arises because rotating Earth is not a sphere but has assumed the shape of an oblate spheroid in which there is a poleward component of gravitation along a constant geopotential surface just sufficient to account for the poleward component of the centripetal acceleration at each latitude for an object at rest on the surface of Earth. In other words, from the point of view of an observer in an inertial reference frame, geopotential surfaces slope upward toward the equator (Figure 1.5). As a consequence, the equatorial radius of Earth is about 21 km larger than the polar radius.

Viewed from a frame of reference rotating with Earth, however, a geopotential surface is everywhere normal to the sum of the true force of gravity, $\mathbf{g}^*$, and the centrifugal force $\Omega^2\mathbf{R}$ (which is just the reaction force of the centripetal acceleration). A geopotential surface is thus experienced as a level surface by an object at rest on rotating Earth. Except at the poles, the weight of an object of mass m at rest on such a surface, which is just the reaction force of Earth on the object, will be slightly less than the gravitational force $m\mathbf{g}^*$ because, as illustrated in Figure 1.5, the centrifugal force partly balances the gravitational force. It is, therefore, convenient to combine the effects of the gravitational force and centrifugal force by defining *gravity* $\mathbf{g}$ such that

$$\mathbf{g} \equiv -g\mathbf{k} \equiv \mathbf{g}^* + \Omega^2\mathbf{R} \tag{1.10}$$

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[3]Earth revolves around its axis once every *sidereal* day, which is equal to 23 h 56 min 4 s (86,164 s). Thus, $\Omega = 2\pi/(86{,}164\,\mathrm{s}) = 7.292 \times 10^{-5}\,\mathrm{rad\,s^{-1}}$.

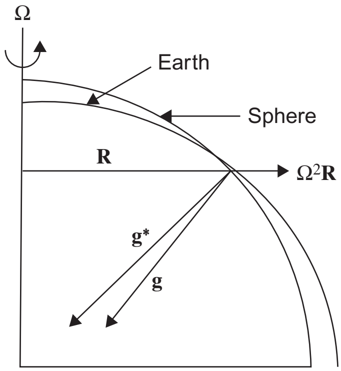

FIGURE 1.5 Relationship between the true gravitation vector $\mathbf{g}^*$ and gravity $\mathbf{g}$. For an idealized homogeneous spherical Earth, $\mathbf{g}^*$ would be directed toward the center of Earth. In reality, $\mathbf{g}^*$ does not point exactly to the center except at the equator and the poles. Gravity, $\mathbf{g}$, is the vector sum of $\mathbf{g}^*$ and the centrifugal force and is perpendicular to the level surface of Earth, which approximates an oblate spheroid.

where $\mathbf{k}$ designates a unit vector parallel to the local vertical. Gravity, g, sometimes referred to as "apparent gravity," will here be taken as a constant ($g = 9.81\,\mathrm{m\,s^{-2}}$). Except at the poles and the equator, $\mathbf{g}$ is not directed toward the center of Earth, but is perpendicular to a geopotential surface as indicated by Figure 1.5. True gravity $\mathbf{g}^*$, however, is not perpendicular to a geopotential surface, but has a horizontal component just large enough to balance the horizontal component of $\Omega^2\mathbf{R}$.

Gravity can be represented in terms of the gradient of a potential function Φ, which is just the geopotential referred to before:

$$\nabla\Phi = -\mathbf{g}$$

However, because $\mathbf{g} = -g\mathbf{k}$, where $g \equiv |\mathbf{g}|$, it is clear that $\Phi = \Phi(z)$ and $d\Phi/dz = g$. Thus, horizontal surfaces on Earth are surfaces of constant geopotential. If the value of dz should be dz', where dz' is a dummy variable of integration, geopotential is set to zero at mean sea level, the geopotential $\Phi(z)$ at height z is just the work required to raise a unit mass to height z from mean sea level:

$$\Phi = \int_0^z g\,dz \tag{1.11}$$

Despite the fact that the surface of Earth bulges toward the equator, an object at rest on the surface of rotating Earth does not slide "downhill" toward the pole because, as indicated previously, the poleward component of gravitation is balanced by the equatorward component of the centrifugal force. However, if the object is put into motion relative to Earth, this balance will be disrupted. Consider a frictionless object located initially at the North Pole. Such an object has zero angular momentum about the axis of Earth. If it is displaced away from the pole in the absence of a zonal torque, it will not acquire rotation and thus will feel a restoring force due to the horizontal component of true gravity, which, as indicated before, is equal and opposite to the horizontal component of the centrifugal force for an object at rest on the surface of Earth. Letting R be the distance from the pole, the horizontal restoring force for a small displacement is thus $-\Omega^2 R$, and the object's acceleration viewed in the inertial coordinate system satisfies the equation for a simple harmonic oscillator:

$$\frac{d^2R}{dt^2} + \Omega^2 R = 0 \tag{1.12}$$

The object will undergo an oscillation of period $2\pi/\Omega$ along a path that will appear as a straight line passing through the pole to an observer in a fixed coordinate system, but will appear as a closed circle traversed in 1/2 day to an observer rotating with Earth (Figure 1.6). From the point of view of an Earth-bound observer, there is an apparent deflection force that causes the object to deviate to the right of its direction of motion at a fixed rate.

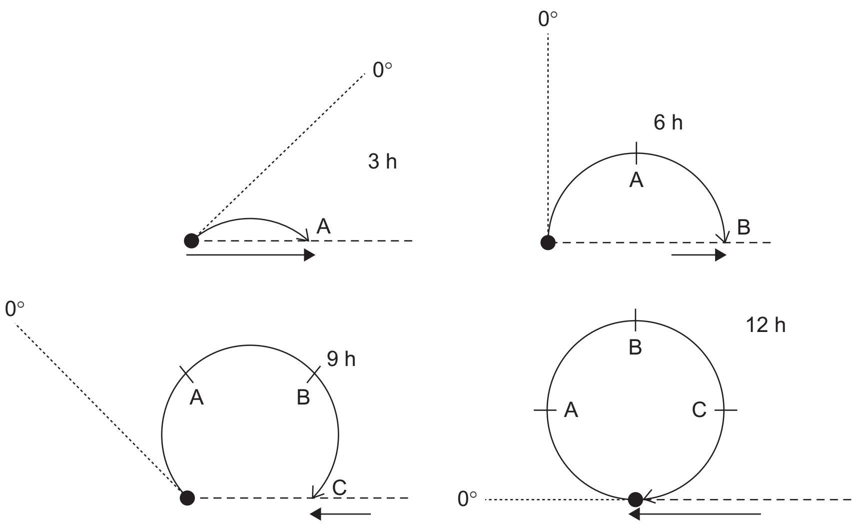

FIGURE 1.6 Motion of a frictionless object launched from the North Pole along the 0° longitude meridian at $t = 0$, as viewed in fixed and rotating reference frames at 3, 6, 9, and 12 h after launch. The *horizontal dashed line* marks the position that the 0° longitude meridian had at $t = 0$, and the short dashed lines show its position in the fixed reference frame at subsequent 3-h intervals. The *horizontal arrows* show 3-h displacement vectors as seen by an observer in the fixed reference frame. *Heavy curved arrows* show the trajectory of the object as viewed by an observer in the rotating system. Labels A, B, and C show the position of the object relative to the rotating coordinates at 3-h intervals. In the fixed coordinate frame, the object oscillates back and forth along a straight line under the influence of the restoring force provided by the horizontal component of gravitation. The period for a complete oscillation is 24 h (only 1/2 period is shown). To an observer in rotating coordinates, however, the motion appears to be at constant speed and describes a complete circle in a clockwise direction in 12 h.

1.3.3 The Coriolis Force and the Curvature Effect

Newton's second law of motion expressed in coordinates rotating with Earth can be used to describe the force balance for an object at rest on the surface of Earth, provided that an apparent force, the centrifugal force, is included among the forces acting on the object. If, however, the object is in motion along the surface of Earth, additional apparent forces are required in the statement of Newton's second law. The Coriolis force will be given a more formal mathematical treatment in Chapter 2, and our purpose here is to deduce the effect by building upon the centrifugal force discussion of the previous section.

Angular momentum, $\mathbf{m} = \mathbf{r} \times \mathbf{p}$, provides a measures of the rotation traced by the linear momentum vector $\mathbf{p}$ with respect to a set of coordinates, the origin of which defines the position vector $\mathbf{r}$. We note that the dynamically important piece of the angular momentum vector lies parallel to Earth's rotational axis, $m = \mathbf{m} \cos\phi$. For now, assume that the linear momentum vector points eastward, having contributions from eastward air motion, u, and from the planetary rotation, $R\Omega$, where $R = r\cos\phi$ is the distance of the air parcel from the axis of rotation (Figure 1.7). If there is no torque in the east–west direction (i.e., no pressure gradient or viscous forces), then m is conserved following the motion,

$$\frac{Dm}{Dt} = \frac{DR}{Dt}(2R\Omega + u) + R\frac{Du}{Dt} = 0 \tag{1.13}$$

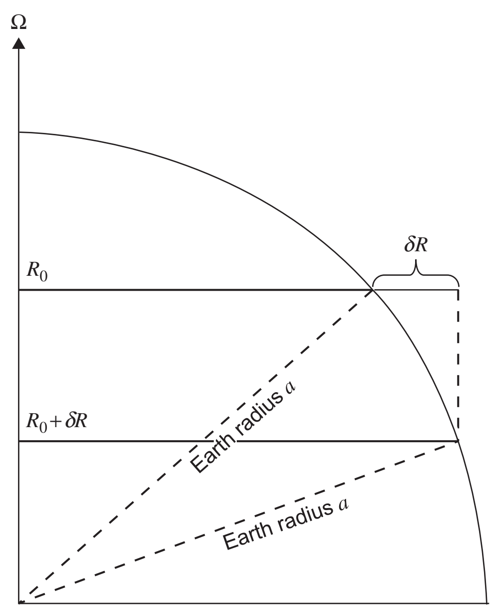

FIGURE 1.7 For poleward motion, air parcels move closer to the axis of rotation and, through angular momentum conservation, the zonal wind accelerates.

so that

$$\frac{Du}{Dt} = -\frac{(2\Omega R + u)}{R}\frac{DR}{Dt} \tag{1.14}$$

Figure 1.7 shows that moving an air parcel closer to the axis of rotation, $\frac{DR}{Dt} < 0$, while conserving angular momentum, increases the westerly linear momentum, analagous to an ice skater spinning faster as the person's arms are drawn inward.

We expand the right side of (1.14) by first noting that

$$\frac{DR}{Dt} = \frac{Dr}{Dt}\cos\phi + r\frac{D}{Dt}\cos\phi = w\cos\phi - v\sin\phi \tag{1.15}$$

where v and w are the northward and upward velocity components, respectively. With this relation, (1.14) becomes

$$\frac{Du}{Dt} = (2\Omega\sin\phi)v - (2\Omega\cos\phi)w - \frac{uw}{r} + \frac{uv}{r}\tan\phi \tag{1.16}$$

The first two terms on the right in (1.16) are the zonal component of the *Coriolis force* due to meridional and vertical motion, respectively. The last two terms on the right are referred to as *metric terms* or *curvature effects* because they arise from the curvature of Earth's surface; since r is large, these terms are negligibly small except near large u.

Suppose now that the object is set in motion in the eastward direction by an impulsive force. Axial angular momentum is not conserved in this case, but considering again the centrifugal force will help expose the meridional component of the Coriolis force. Because the object is now rotating faster than Earth, the centrifugal force on the object will be increased. The excess of the centrifugal force over that for an object at rest is

$$\left(\Omega + \frac{u}{R}\right)^2 \mathbf{R} - \Omega^2\mathbf{R} = \frac{2\Omega u\mathbf{R}}{R} + \frac{u^2\mathbf{R}}{R^2}$$

The terms on the right represent *deflecting* forces, which act outward along the vector $\mathbf{R}$ (i.e., perpendicular to the axis of rotation). The meridional and vertical components of these forces are obtained by taking meridional and vertical components of $\mathbf{R}$, as shown in Figure 1.8, to yield

$$\frac{Dv}{Dt} = -2\Omega u\sin\phi - \frac{u^2}{a}\tan\phi \tag{1.17}$$

$$\frac{Dw}{Dt} = 2\Omega u\cos\phi + \frac{u^2}{a} \tag{1.18}$$

The first terms on the right are the meridional and vertical components, respectively, of the Coriolis forces for zonal motion; the second terms on the right are curvature effects.

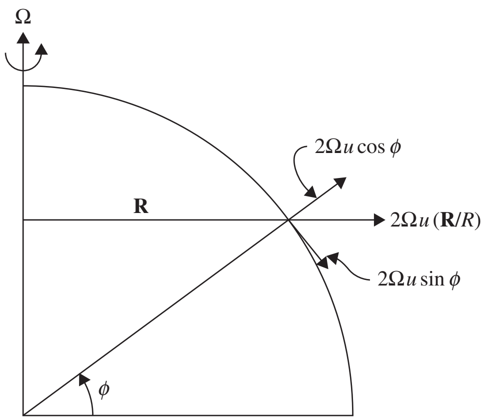

FIGURE 1.8 Components of the Coriolis force due to relative motion along a latitude circle.

For larger-scale motions, the curvature terms can be neglected as an approximation. Therefore, relative horizontal motion produces a horizontal acceleration perpendicular to the direction of motion given by

$$\frac{Du}{Dt} = 2\Omega v \sin\phi = fv \tag{1.19}$$

$$\frac{Dv}{Dt} = -2\Omega u \sin\phi = -fu \tag{1.20}$$

where $f \equiv 2\Omega \sin\phi$ is the *Coriolis parameter*.

Thus, for example, an object moving eastward in the horizontal is deflected equatorward by the Coriolis force, whereas a westward moving object is deflected poleward. In either case the deflection is to the right of the direction of motion in the Northern Hemisphere and to the left in the Southern Hemisphere. The vertical component of the Coriolis force in (1.18) is ordinarily much smaller than the gravitational force so that its only effect is to cause a very minor change in the apparent weight of an object depending on whether the object is moving eastward or westward.

The Coriolis force is negligible for motions with time scales that are very short compared to the period of Earth's rotation (a point that is illustrated by several problems at the end of the chapter). Thus, the Coriolis force is not important for the dynamics of individual cumulus clouds but is essential for an understanding of longer time scale phenomena such as synoptic scale systems. The Coriolis force must also be taken into account when computing long-range missile or artillery trajectories.

As an example, suppose that a ballistic missile is fired due eastward at 43°N latitude ($f = 10^{-4}\,\mathrm{s}^{-1}$ at 43°N). If the missile travels 1000 km at a horizontal speed $u_0 = 1000\,\mathrm{m\,s}^{-1}$, by how much is the missile deflected from its eastward path by the Coriolis force? Integrating (1.20) with respect to time, we find that

$$v = -fu_0 t \tag{1.21}$$

where it is assumed that the deflection is sufficiently small so that we may let f and u_0 be constants. To find the total displacement, we must integrate (1.13) with respect to time:

$$\int_0^t v dt = \int_{y_0}^{y_0+\delta y} dy = -fu_0 \int_0^t t dt$$

Thus, the total displacement is

$$\delta y = -fu_0 t^2/2 = -50\,\text{km}$$

Therefore, the missile is deflected southward by 50 km due to the Coriolis effect. Further examples of the deflection of objects by the Coriolis force are given in some of the problems at the end of the chapter.

The x and y components given in (1.19) and (1.20) can be combined in vector form as

$$\left(\frac{D\mathbf{V}}{Dt}\right)_{Co} = -f\mathbf{k} \times \mathbf{V} \tag{1.22}$$

where $\mathbf{V} \equiv (u, v)$ is the horizontal velocity, $\mathbf{k}$ is a vertical unit vector, and the subscript Co indicates that the acceleration is due solely to the Coriolis force. Since $-\mathbf{k} \times \mathbf{V}$ is a vector rotated 90° to the right of $\mathbf{V}$, (1.22) clearly shows the deflection character of the Coriolis force. The Coriolis force can only change the direction of motion, not the speed.

1.3.4 Constant Angular Momentum Oscillations

Suppose an object initially at rest on Earth at the point (x_0, y_0) is impulsively propelled along the x axis with a speed V at time $t = 0$. Then, from (1.19) and (1.20), the time evolution of the velocity is given by $u = V\cos ft$ and $v = -V\sin ft$. However, because $u = Dx/Dt$ and $v = Dy/Dt$, integration with respect to time gives the position of the object at time t as

$$x - x_0 = \frac{V}{f}\sin ft \quad \text{and} \quad y - y_0 = \frac{V}{f}(\cos ft - 1) \tag{1.23}$$

where the variation of f with latitude is neglected. Equations (1.23) show that in the Northern Hemisphere, where f is positive, the object orbits clockwise (anticyclonically) in a circle of radius $R = V/f$ about the point $(x_0, y_0 - V/f)$ with a period given by

$$\tau = 2\pi R/V = 2\pi/f = \pi/(\Omega\sin\phi) \tag{1.24}$$

Thus, an object displaced horizontally from its equilibrium position on the surface of Earth under the influence of the force of gravity will oscillate about its equilibrium position with a period that depends on latitude and is equal to one sidereal day at 30° latitude and 1/2 sidereal day at the pole. Constant angular momentum oscillations (often referred to as "inertial oscillations") are commonly observed in the oceans, but are apparently not of importance in the atmosphere.

1.4 STRUCTURE OF THE STATIC ATMOSPHERE

The thermodynamic state of the atmosphere at any point is determined by the values of pressure, temperature, and density (or specific volume) at that point. These field variables are related to one an other by the equation of state for an ideal gas. Letting p, T, ρ, and $\alpha(\equiv \rho^{-1})$ denote pressure, temperature, density, and specific volume, respectively, we can express the equation of state for dry air as

$$p\alpha = RT \quad \text{or} \quad p = \rho RT \tag{1.25}$$

where R is the gas constant for dry air ($R = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$).

1.4.1 The Hydrostatic Equation

In the absence of atmospheric motions, the gravity force must be exactly balanced by the vertical component of the pressure gradient force. Thus, as illustrated in Figure 1.9,

$$dp/dz = -\rho g \tag{1.26}$$

This condition of *hydrostatic balance* provides an excellent approximation for the vertical dependence of the pressure field in the real atmosphere. Only for intense small-scale systems, such as squall lines and tornadoes, is it necessary to consider departures from hydrostatic balance. Integrating (1.26) from a height z to the top of the atmosphere, we find that

$$p(z) = \int_z^\infty \rho g dz \tag{1.27}$$

so that the pressure at any point is simply equal to the weight of the unit cross-section column of air overlying the point. Thus, mean sea level pressure $p(0) = 1013.25\,\mathrm{hPa}$ is simply the average weight per square meter of the total atmospheric column.[4] It is often useful to express the hydrostatic equation in

[4]For computational convenience, the mean surface pressure is often assumed to equal 1000 hPa.

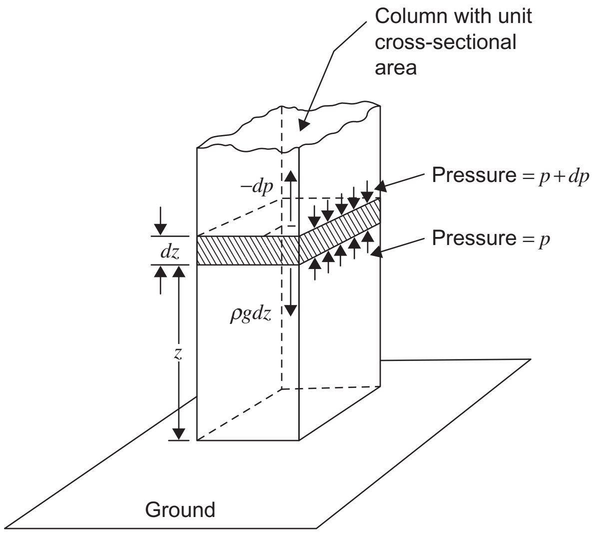

FIGURE 1.9 Balance of forces for hydrostatic equilibrium. *Small arrows* show the upward and downward forces exerted by air pressure on the air mass in the shaded block. The downward force exerted by gravity on the air in the block is given by $\rho g dz$, whereas the net pressure force given by the difference between the upward force across the lower surface and the downward force across the upper surface is $-dp$. Note that dp is negative, as pressure decreases with height. (*After Wallace and Hobbs, 2006*.)

terms of the geopotential rather than the geometric height. Noting from (1.11) that $d\Phi = gdz$ and from (1.25) that $\alpha = RT/p$, we can express the hydrostatic equation in the form

$$gdz = d\Phi = -(RT/p)dp = -RTd\ln p \tag{1.28}$$

Thus, the variation in geopotential with respect to pressure depends only on temperature. Integration of (1.28) in the vertical yields a form of the *hypsometric equation*:

$$\Phi(z_2) - \Phi(z_1) = g_0(Z_2 - Z_1) = R\int_{p_2}^{p_1} Td\ln p \tag{1.29}$$

Here $Z \equiv \Phi(z)/g_0$ is the *geopotential height*, where $g_0 \equiv 9.80665\,\mathrm{m\,s^{-2}}$ is the global average of gravity at mean sea level. Thus, in the troposphere and lower stratosphere, Z is numerically almost identical to the geometric height z. In terms of Z the hypsometric equation becomes

$$Z_T \equiv Z_2 - Z_1 = \frac{R}{g_0}\int_{p_2}^{p_1} Td\ln p \tag{1.30}$$

where Z_T is the *thickness* of the atmospheric layer between the pressure surfaces p_2 and p_1. Defining a layer mean temperature

$$\langle T \rangle = \left[\int_{p_2}^{p_1} d \ln p \right]^{-1} \int_{p_2}^{p_1} T d \ln p$$

and a layer mean scale height $H \equiv R\langle T \rangle / g_0$, we have from Eq. (1.30)

$$Z_T = H \ln(p_1/p_2) \tag{1.31}$$

Thus, the thickness of a layer bounded by isobaric surfaces is proportional to the mean temperature of the layer. Pressure decreases more rapidly with height in a cold layer than in a warm layer. It also follows immediately from (1.31) that in an isothermal atmosphere of temperature T, the geopotential height is proportional to the natural logarithm of pressure normalized by the surface pressure clearer:

$$Z = H \ln(p_0/p) \tag{1.32}$$

where p_0 is the pressure at $Z = 0$. Thus, in an isothermal atmosphere the pressure decreases exponentially with geopotential height by a factor of e^{-1} per scale height:

$$p(Z) = p(0)e^{-Z/H}$$

1.4.2 Pressure as a Vertical Coordinate

From the hydrostatic equation (1.26), it is clear that a single-valued monotonic relationship exists between pressure and height in each vertical column of the atmosphere. Thus, we may use pressure as the independent vertical coordinate and height (or geopotential) as a dependent variable. The thermodynamic state of the atmosphere is then specified by the fields of $\Phi(x, y, p, t)$ and $T(x, y, p, t)$.

Now the horizontal components of the pressure gradient force given by Eq. (1.1) are evaluated by partial differentiation holding z constant. However, when pressure is used as the vertical coordinate, horizontal partial derivatives must be evaluated holding p constant. Transformation of the horizontal pressure gradient force from height to pressure coordinates may be carried out with the aid of Figure 1.10. Considering only the x, z plane, we see from Figure 1.10 that

$$\left[\frac{(p_0 + \delta p) - p_0}{\delta x} \right]_z = \left[\frac{(p_0 + \delta p) - p_0}{\delta z} \right]_x \left(\frac{\delta z}{\delta x} \right)_p$$

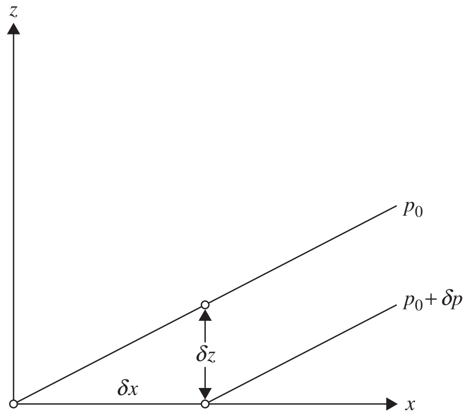

FIGURE 1.10 Slope of pressure surfaces in the x, z plane.

where subscripts indicate variables that remain constant in evaluating the differentials. For example, in the limit $\delta z \to 0$

$$\left[\frac{(p_0 + \delta p) - p_0}{\delta z}\right]_x \to \left(-\frac{\partial p}{\partial z}\right)_x$$

where the minus sign is included because $\delta z < 0$ for $\delta p > 0$.

Taking the limits $\delta x, \delta z \to 0$, we obtain[5]

$$\left(\frac{\partial p}{\partial x}\right)_z = -\left(\frac{\partial p}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_p$$

which after substitution from the hydrostatic equation (1.26) yields

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = -g\left(\frac{\partial z}{\partial x}\right)_p = -\left(\frac{\partial \Phi}{\partial x}\right)_p \qquad (1.33)$$

Similarly, it is easy to show that

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right)_z = -\left(\frac{\partial \Phi}{\partial y}\right)_p \qquad (1.34)$$

Thus, in the *isobaric* coordinate system the horizontal pressure gradient force is measured by the gradient of geopotential at constant pressure. Density no longer appears explicitly in the pressure gradient force; this is a distinct advantage of the isobaric system.

[5]It is important to note the minus sign on the right in this expression!

1.4.3 A Generalized Vertical Coordinate

Any single-valued monotonic function of pressure or height may be used as the independent vertical coordinate. For example, in many numerical weather prediction models, pressure normalized by the pressure at the ground, $\sigma \equiv p(x, y, z, t)/p_s(x, y, t)$, is used as a vertical coordinate. This choice guarantees that the ground is a coordinate surface ($\sigma \equiv 1$) even in the presence of spatial and temporal surface pressure variations. Thus, this so-called σ coordinate system is particularly useful in regions of strong topographic variations.

We now obtain a general expression for the horizontal pressure gradient, which is applicable to any vertical coordinate $s = s(x, y, z, t)$ that is a single-valued monotonic function of height. Referring to Figure 1.11 we see that for a horizontal distance δx, the pressure difference evaluated along a surface of constant s is related to that evaluated at constant z by the relationship

$$\frac{p_C - p_A}{\delta x} = \frac{p_C - p_B}{\delta z}\frac{\delta z}{\delta x} + \frac{p_B - p_A}{\delta x}$$

Taking the limits as $\delta x, \delta z \to 0$, we obtain

$$\left(\frac{\partial p}{\partial x}\right)_s = \frac{\partial p}{\partial z}\left(\frac{\partial z}{\partial x}\right)_s + \left(\frac{\partial p}{\partial x}\right)_z \tag{1.35}$$

Using the identity $\partial p/\partial z = (\partial s/\partial z)(\partial p/\partial s)$, we can express (1.35) in the alternate form

$$\left(\frac{\partial p}{\partial x}\right)_s = \left(\frac{\partial p}{\partial x}\right)_z + \frac{\partial s}{\partial z}\left(\frac{\partial z}{\partial x}\right)_s\left(\frac{\partial p}{\partial s}\right) \tag{1.36}$$

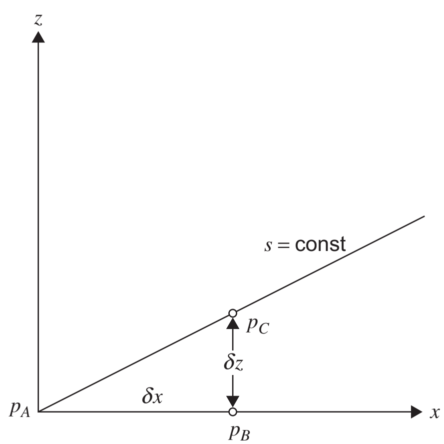

FIGURE 1.11 Transformation of the pressure gradient force to s coordinates.

In later chapters we will apply (1.35) or (1.36) and similar expressions for other fields to transform the dynamical equations to several different vertical coordinate systems.

1.5 KINEMATICS

Kinematics involves the analysis of motion without reference to forces that change the motion in time. It provides a diagnosis of motion at a particular instant in time, which may in turn prove useful for understanding the dynamics of the flow as it evolves in time. There are many aspects of kinematics, but usually one is interested in the structure of the flow, and here we will limit attention to the horizontal flow. One way to quantify flow structure is to examine linear variations in the flow near an arbitrary point (x_0, y_0). A leading-order Taylor approximation to the wind near the point is

$$u(x_0 + dx, y_0 + dy) \approx u(x_0, y_0) + \left.\frac{\partial u}{\partial x}\right|_{(x_0,y_0)} dx + \left.\frac{\partial u}{\partial y}\right|_{(x_0,y_0)} dy \tag{1.37a}$$

$$v(x_0 + dx, y_0 + dy) \approx v(x_0, y_0) + \left.\frac{\partial v}{\partial x}\right|_{(x_0,y_0)} dx + \left.\frac{\partial v}{\partial y}\right|_{(x_0,y_0)} dy \tag{1.37b}$$

Making the following definitions

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \delta \tag{1.38a}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta \tag{1.38b}$$

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = d_1 \tag{1.38c}$$

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = d_2 \tag{1.38d}$$

allows the derivatives in (1.37a,b) to be replaced in favor of the elemental quanties δ, ζ, d_1, and d_2:

$$u(x_0 + dx, y_0 + dy) \approx u(x_0, y_0) + \frac{1}{2}(\delta + d_1)dx + \frac{1}{2}(d_2 - \zeta)dy \tag{1.39a}$$

$$v(x_0 + dx, y_0 + dy) \approx v(x_0, y_0) + \frac{1}{2}(\zeta + d_2)dx + \frac{1}{2}(\delta - d_1)dy \tag{1.39b}$$

The advantage of this manipulation is that we can now think about the wind near a point as a linear combination of the elemental fluid properties. The vorticity, ζ, represents pure rotation (about the vertical direction); δ represents pure divergence; and $\delta < 0$ is called convergence. Pure deformation is represented by d_1

and d_2, where the wind field contracts, or is *confluent* in one direction, called the axis of contraction, and the wind field stretches in the normal direction, called the axis of dilatation. d_2 represents a 45° rotation of d_1, and therefore they are not independent patterns. The leading, constant, terms in (1.39a,b) represent uniform translation.

By setting all elemental quantities in (1.39a,b) except one to zero, we visualize the spatial distribution of the horizontal wind associated with one elemental quantity (Figure 1.12). Note that the pure deformation pattern represents d_1 only and that, although the vectors appear to converge and diverge, the divergence is exactly zero in this field. This is an important example of the fact that confluence (difluence) in vector fields is not the same as convergence (divergence). In the lower right panel of Figure 1.12, we see a linear combination of vorticity

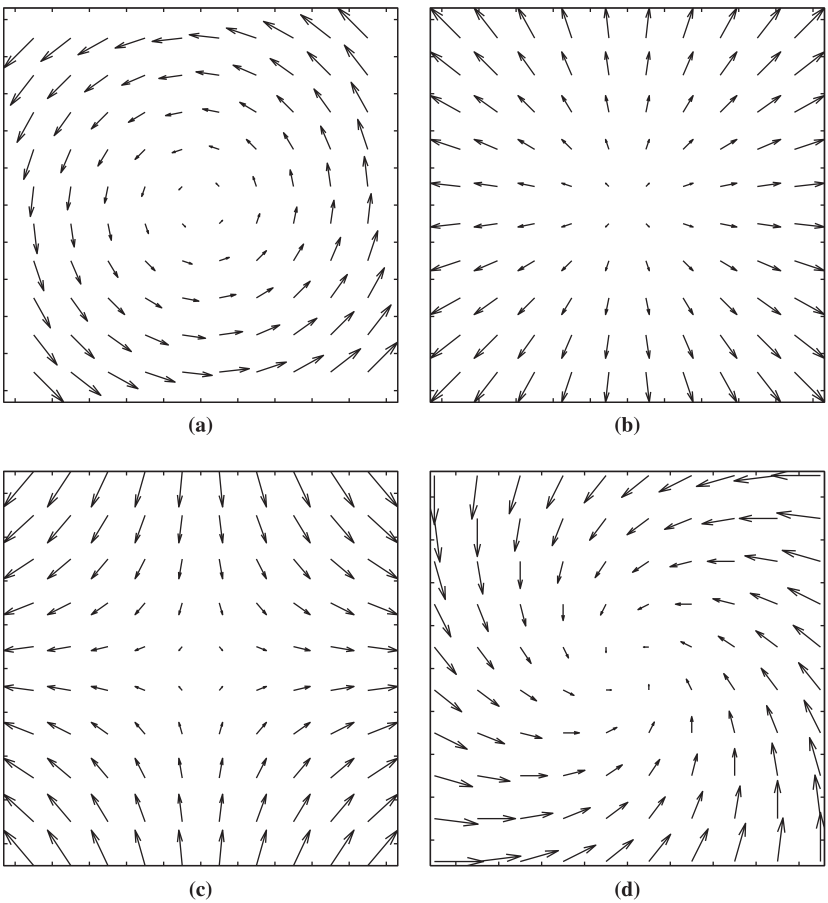

FIGURE 1.12 Velocity fields associated with pure vorticity (a), pure divergence (b), pure deformation (c), and a mixture of vorticity and convergence (d).

and divergence, illustrating a more complicated pattern than the elemental fields in isolation. By computing the elemental variables at a point, one can visualize the linear variation of the flow near that point through (1.39a,b).

This taste of kinematics highlights the importance of certain properties of the wind field that will be explored in greater depth in future chapters. Divergence is connected to vertical motion by mass conservation, as will be discussed in Chapter 2. Vorticity is fundamental to understanding dynamic meteorology and will be explored in detail in Chapter 3. Deformation is important for creating and destroying boundaries in fluids, such as horizontal temperature contrasts known as *frontal zones*, which will be explored in Chapter 9.

1.6 SCALE ANALYSIS

Scale analysis, or scaling, is a convenient technique for estimating the magnitudes of various terms in the governing equations for a particular type of motion. In scaling, typical expected values of the following quantities are specified:

1. Magnitudes of the field variables
2. Amplitudes of fluctuations in the field variables
3. Characteristic length, depth, and time scales on which the fluctuations occur

These typical values are then used to compare the magnitudes of various terms in the governing equations. For example, in a typical midlatitude synoptic[6] cyclone, the surface pressure might fluctuate by 10 hPa over a horizontal distance of 1000 km. Designating the amplitude of the horizontal pressure fluctuation by δp, the horizontal coordinates by x and y, and the horizontal scale by L, the magnitude of the horizontal pressure gradient may be estimated by dividing δp by the length L to get

$$\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}\right) \sim \frac{\delta p}{L} = 10\,\text{hpa}/10^3\ \text{km}\ \left(10^{-3}\ \text{Pa m}^{-1}\right)$$

Pressure fluctuations of similar magnitudes occur in other motion systems of vastly different scale such as tornadoes, squall lines, and hurricanes. Thus, the horizontal pressure gradient can range over several orders of magnitude for systems of meteorological interest. Similar considerations are also valid for derivative terms involving other field variables. Therefore, the nature of the dominant terms in the governing equations is crucially dependent on the horizontal scale of the motions. In particular, motions with horizontal scales of only a few kilometers tend to have short time scales so that terms involving the rotation of Earth are negligible, while for larger scales they become very

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[6] The term *synoptic* designates the branch of meteorology that deals with the analysis of observations taken over a wide area at or near the same time. This term is commonly used (as here) to designate the characteristic scale of the disturbances that are depicted on weather maps.

TABLE 1.4 Scales of Atmospheric Motions

Type of Motion	Horizontal Scale (m)
Molecular mean free path	10^{-7}
Minute turbulent eddies	10^{-2} to 10^{-1}
Small eddies	10^{-1} to 1
Dust devils	1 to 10
Gusts	10 to 10^2
Tornadoes	10^2
Cumulonimbus clouds	10^3
Fronts, squall lines	10^4 to 10^5
Hurricanes	10^5
Synoptic cyclones	10^6
Planetary waves	10^7

important. Because the character of atmospheric motions depends so strongly on the horizontal scale, this scale provides a convenient method for the classification of motion systems. Table 1.4 classifies examples of various types of motions by horizontal scale for the spectral region from 10^{-7} to 10^7 m. In the following chapters, scaling arguments are used extensively in developing simplifications of the governing equations for use in modeling various types of motion systems.

SUGGESTED REFERENCES

Complete reference information is provided in the Bibliography at the end of the book.

Curry and Webster, *Thermodynamics of Atmospheres and Oceans*, contains a more advanced discussion of atmospheric statistics.

Durran (1993) discusses the constant angular momentum oscillation in detail.

Wallace and Hobbs, *Atmospheric Science: An Introductory Survey*, discuss much of the material in this chapter at an elementary level.

PROBLEMS

1.1. Neglecting the latitudinal variation in the radius of Earth, calculate the angle between the gravitational force and gravity vectors at the surface of Earth as a function of latitude. What is the maximum value of this angle?

1.2. Calculate the altitude at which an artificial satellite orbiting in the equatorial plane can be a synchronous satellite (i.e., can remain above the same spot on the surface of Earth).

1.3. An artificial satellite is placed in a natural synchronous orbit above the equator and is attached to Earth below by a wire. A second satellite is attached to the first by a wire of the same length and is placed in orbit directly above the first at the same angular velocity. Assuming that the wires have zero mass, calculate the tension in the wires per unit mass of satellite. Could this tension be used to lift objects into orbit with no additional expenditure of energy?

1.4. A train is running smoothly along a curved track at the rate of 50 $\mathrm{m\,s^{-1}}$. A passenger standing on a set of scales observes that his weight is 10% greater than when the train is at rest. The track is banked so that the force acting on the passenger is normal to the floor of the train. What is the radius of curvature of the track?

1.5. If a baseball player throws a ball a horizontal distance of 100 m at 30° latitude in 4 s, by how much is the ball deflected laterally as a result of the rotation of Earth?

1.6. Two balls 4 cm in diameter are placed 100 m apart on a frictionless horizontal plane at 43°N. If the balls are impulsively propelled directly at each other with equal speeds, at what speed must they travel so that they just miss each other?

1.7. A locomotive of mass 2×10^5 kg travels 50 $\mathrm{m\,s^{-1}}$ along a straight horizontal track at 43°N. What lateral force is exerted on the rails? Compare the magnitudes of the upward reaction force exerted by the rails for cases where the locomotive is traveling eastward and westward, respectively.

1.8. Find the horizontal displacement of a body dropped from a fixed platform at a height h at the equator, neglecting the effects of air resistance. What is the numerical value of the displacement for $h = 5$ km?

1.9. A bullet is fired directly upward with initial speed w_0 at latitude ϕ. Neglecting air resistance, by what distance will it be displaced horizontally when it returns to the ground? (Neglect $2\Omega u \cos\phi$ compared to g in the vertical momentum equation.)

1.10. A block of mass $M = 1$ kg is suspended from the end of a weightless string. The other end of the string is passed through a small hole in a horizontal platform and a ball of mass $m = 10$ kg is attached. At what angular velocity must the ball rotate on the horizontal platform to balance the weight of the block if the horizontal distance of the ball from the hole is 1 m? While the ball is rotating, the block is pulled down 10 cm. What is the new angular velocity of the ball? How much work is done in pulling down the block?

1.11. A particle is free to slide on a horizontal frictionless plane located at a latitude ϕ on Earth. Find the equation governing the path of the particle if it is given an impulsive northward velocity $v = V_0$ at $t = 0$. Give the solution for the position of the particle as a function of time. (Assume that the latitudinal excursion is sufficiently small that f is constant.)

1.12. Calculate the 1000 to 500 hPa thickness for isothermal conditions with temperatures of 273 and 250 K, respectively.

1.13. Isolines of 1000 to 500 hPa thickness are drawn on a weather map using a contour interval of 60 m. What is the corresponding layer mean temperature interval?

1.14. Show that a homogeneous atmosphere (density independent of height) has a finite height that depends only on the temperature at the lower boundary. Compute the height of a homogeneous atmosphere with surface temperature $T_0 = 273$ K and surface pressure 1000 hPa. (Use the ideal gas law and hydrostatic balance.)

1.15. For the conditions of the previous problem, compute the variation of the temperature with respect to height.

1.16. Show that in an atmosphere with uniform *lapse rate* γ (where $\gamma \equiv -dT/dz$) the geopotential height at pressure level p_1 is given by

$$Z = \frac{T_0}{\gamma}\left[1 - \left(\frac{p_0}{p_1}\right)^{-R\gamma/g}\right]$$

where T_0 and p_0 are the sea-level temperature and pressure, respectively.

1.17. Calculate the 1000 to 500 hPa thickness for a constant lapse rate atmosphere with $\gamma = 6.5\,\mathrm{K\,km^{-1}}$ and $T_0 = 273\,\mathrm{K}$. Compare your results with the results in Problem 1.12.

1.18. Derive an expression for the variation in density with respect to height in a constant lapse rate atmosphere.

1.19. Derive an expression for the altitude variation in the pressure change δp that occurs when an atmosphere with a constant lapse rate is subjected to a height-independent temperature change δT while the surface pressure remains constant. At what height is the magnitude of the pressure change a maximum if the lapse rate is 6.5 K km^{-1}, $T_0 = 300$, and $\delta T = 2\,\mathrm{K}$?

MATLAB Exercises

M1.1. This exercise investigates the role of the curvature terms for high-latitude constant angular momentum trajectories.

(a) Run the **coriolis.m** script with the following initial conditions: initial latitude 60°, initial velocity $u = 0$, $v = 40\,\mathrm{m\,s^{-1}}$, run time = 5 days. Compare the appearance of the trajectories for the case with the curvature terms included and the case with the curvature terms neglected. Qualitatively explain the difference that you observe. Why is the trajectory not a closed circle as described in Eq. (1.15) of the text? [*Hint*: Consider the separate effects of the term proportional to tan ϕ and of the spherical geometry.]

(b) Run **coriolis.m** with latitude 60°, $u = 0$, $v = 80$ m/s. What is different from case (a)? By varying the run time, see if you can determine how long it takes for the particle to make a full circuit in each case, and compare this to the time given in Eq. (1.24) for $\phi = 60°$.

M1.2. Using the MATLAB script from Problem M1.1, compare the magnitudes of the lateral deflection for ballistic missiles fired eastward and westward at 43° latitude. Each missile is launched at a velocity of 1000 m s^{-1} and travels 1000 km. Explain your results. Can the curvature term be neglected in these cases?

M1.3. This exercise examines the strange behavior of constant angular momentum trajectories near the equator. Run the **coriolis.m** script for the following contrasting cases: (a) latitude $0.5°$, $u = 20\ \mathrm{m\,s^{-1}}$, $v = 0$, run time = 20 days and (b) latitude $0.5°$, $u = -20\ \mathrm{m\,s^{-1}}$, $v = 0$, run time = 20 days. Obviously, eastward and westward motion near the equator leads to very different behavior. Briefly explain why the trajectories are so different in these two cases. By running different time intervals, determine the approximate period of oscillation in each case (i.e., the time to return to the original latitude).

M1.4. More strange behavior near the equator. Run the script **const_ang_mom_traj1.m** by specifying initial conditions of latitude = 0, $u = 0$, $v = 50\ \mathrm{m\,s^{-1}}$, and a time of about 5 or 10 days. Notice that the motion is symmetric about the equator and that there is a net eastward drift. Why does providing a parcel with an initial poleward velocity at the equator lead to an eastward average displacement? By trying different initial meridional velocities in the range of 50 to 250 $\mathrm{m\,s^{-1}}$, determine the approximate dependence of the maximum latitude reached by the ball on the initial meridional velocity. Also determine how the net eastward displacement depends on the initial meridional velocity. Show your results in a table, or plot them using MATLAB.