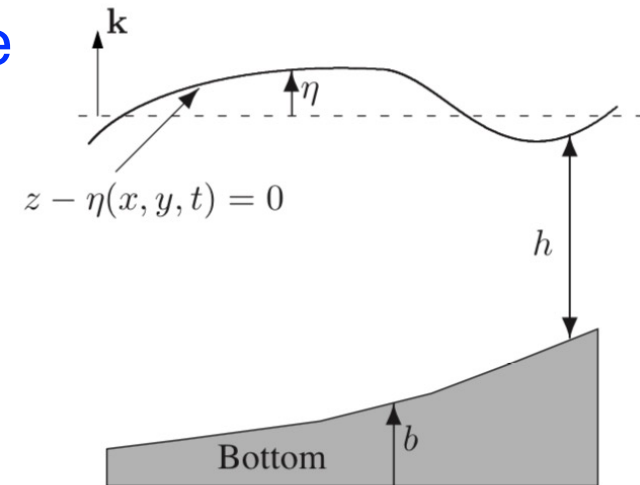


Dynamic boundary conditions — force

Pressure:

$$p_{\text{atm}} = p_{\text{sea}} \quad \text{at air-sea interface.}$$

$$p_{\text{sea}}(z = 0) = p_{\text{atm at sea level}} + \rho_0 g \eta$$



Surface stress:

Stress must be continuous along moving boundaries (sea surface)

$$\rho_0 \nu_E \left(\frac{\partial u}{\partial z} \right) \Big|_{\text{at surface}} = \tau^x, \quad \rho_0 \nu_E \left(\frac{\partial v}{\partial z} \right) \Big|_{\text{at surface}} = \tau^y$$

The stress must be equal to the wind stress that is parameterized by:

$$U_{10} = \sqrt{u_{10}^2 + v_{10}^2} \quad \tau^x = C_d \rho_{\text{air}} U_{10} u_{10}, \quad \tau^y = C_d \rho_{\text{air}} U_{10} v_{10}$$

Bottom stress:

linear parameterization: $\tau = r_1 \mathbf{u}$

quadratic parameterization: $\tau = r_2 |\mathbf{u}| \mathbf{u}$

logarithmic parameterization: $\tau = \frac{K^2}{\ln^2(z/z_0)} |\mathbf{u}| \mathbf{u}$

K : von Kármán's constant
 z_0 : roughness length

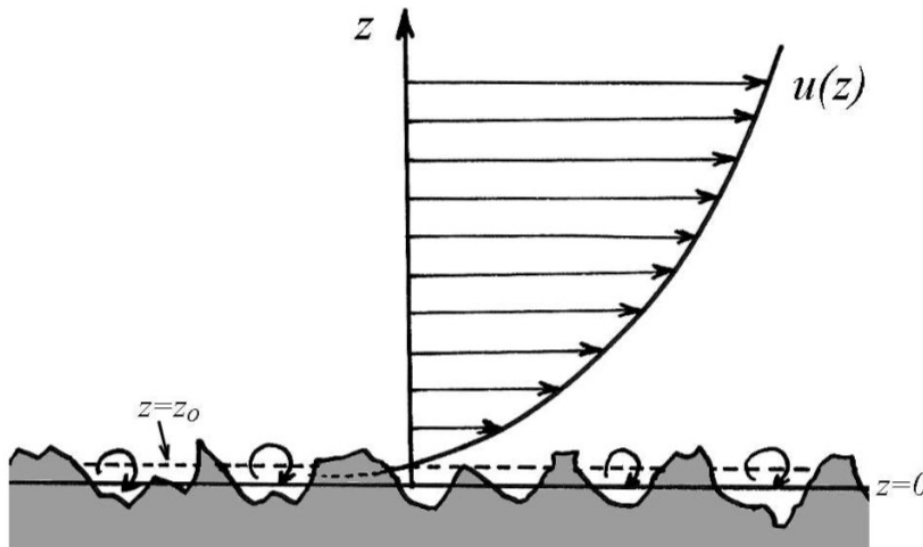


Figure 8-2 Velocity profile in the vicinity of a rough wall. The roughness height z_0 is smaller than the averaged height of the surface asperities. So, the velocity u falls to zero somewhere within the asperities, where local flow degenerates into small vortices between the peaks, and the negative values predicted by the logarithmic profile are not physically realized.

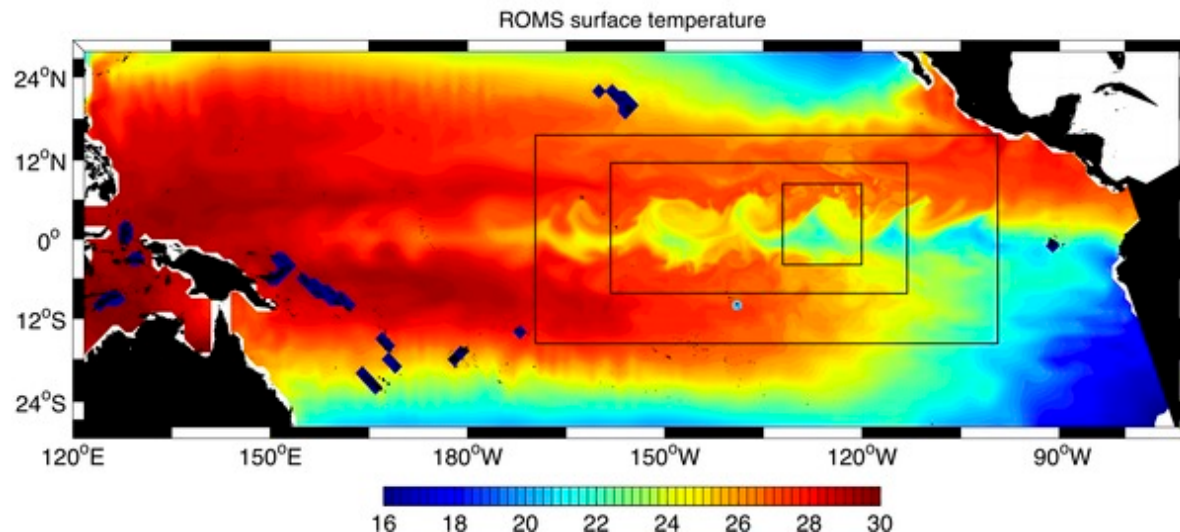
Open boundary conditions

Dirichlet condition: Prescribing the value of the variable at the boundary ($\phi = \phi_0$)

Newman condition: setting the gradient to impose the diffusive flux of a quantity ($\kappa \frac{\partial \phi}{\partial n}$)

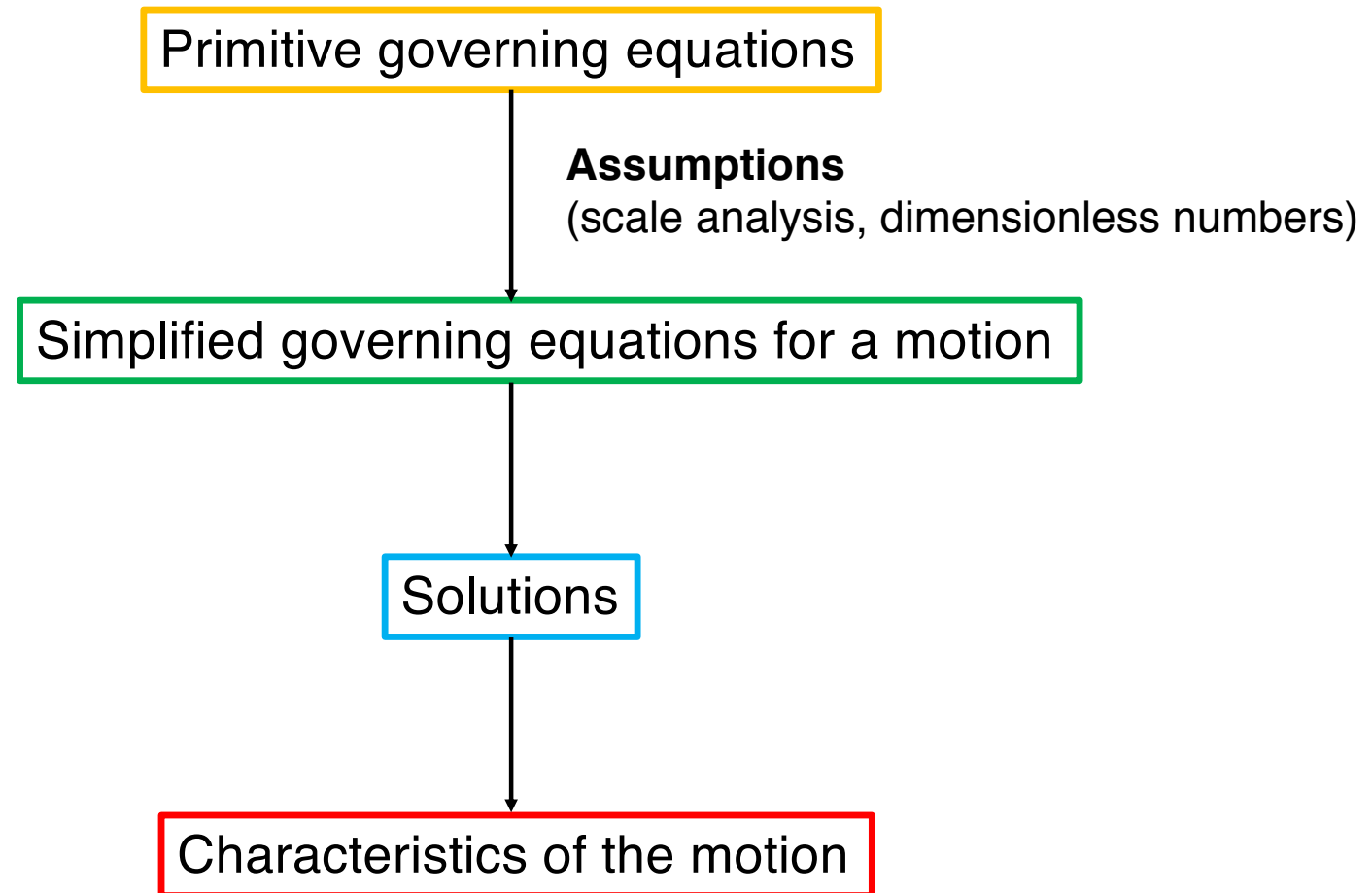
Cauchy condition: Prescribing a total, advective plus diffusive, flux ($u\phi - \kappa \frac{\partial \phi}{\partial x}$)

Data source for open boundary conditions: **observations, modelling results**



nested models

Part III. Different types of motions



Free motions – Inertial Oscillations

Not subject to real force:

$$\frac{du}{dt} - fv = 0$$

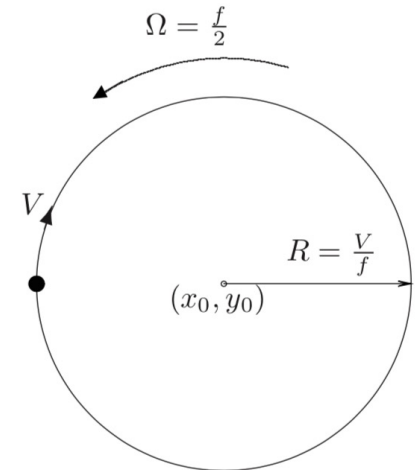
$$\frac{dv}{dt} + fu = 0$$

$$V = \sqrt{u^2 + v^2} \quad \frac{dx}{dt} \quad u = V \sin(ft + \phi) \quad \frac{dy}{dt} \quad v = V \cos(ft + \phi)$$

$$x = x_0 - \frac{V}{f} \cos(ft + \phi)$$

$$y = y_0 + \frac{V}{f} \sin(ft + \phi)$$

$$(x - x_0)^2 + (y - y_0)^2 = \left(\frac{V}{f}\right)^2$$



f : inertial frequency

$$T = \frac{2\pi}{f}$$

Table 9.1 Inertial Oscillations

Latitude (φ)	T_i (hr)	D (km)
	for $V = 20$ cm/s	
90°	11.97	2.7
35°	20.87	4.8
10°	68.93	15.8

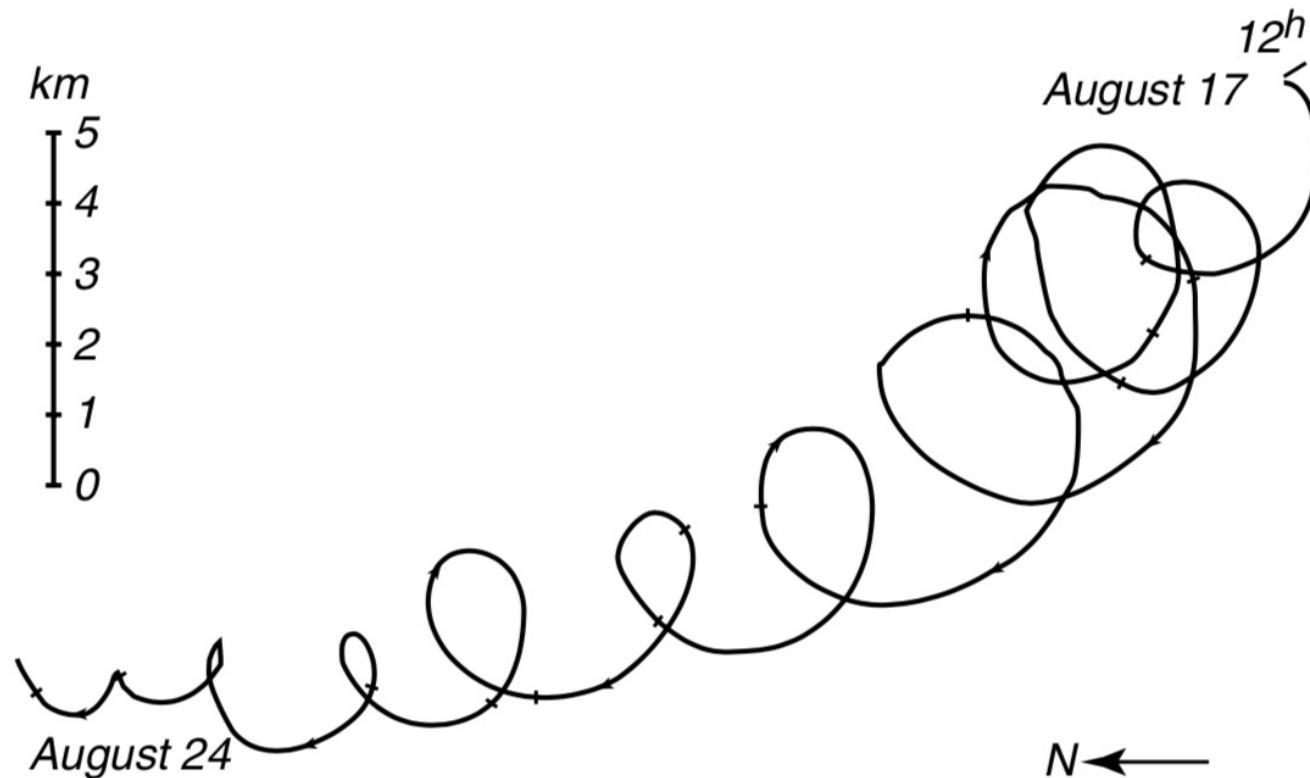
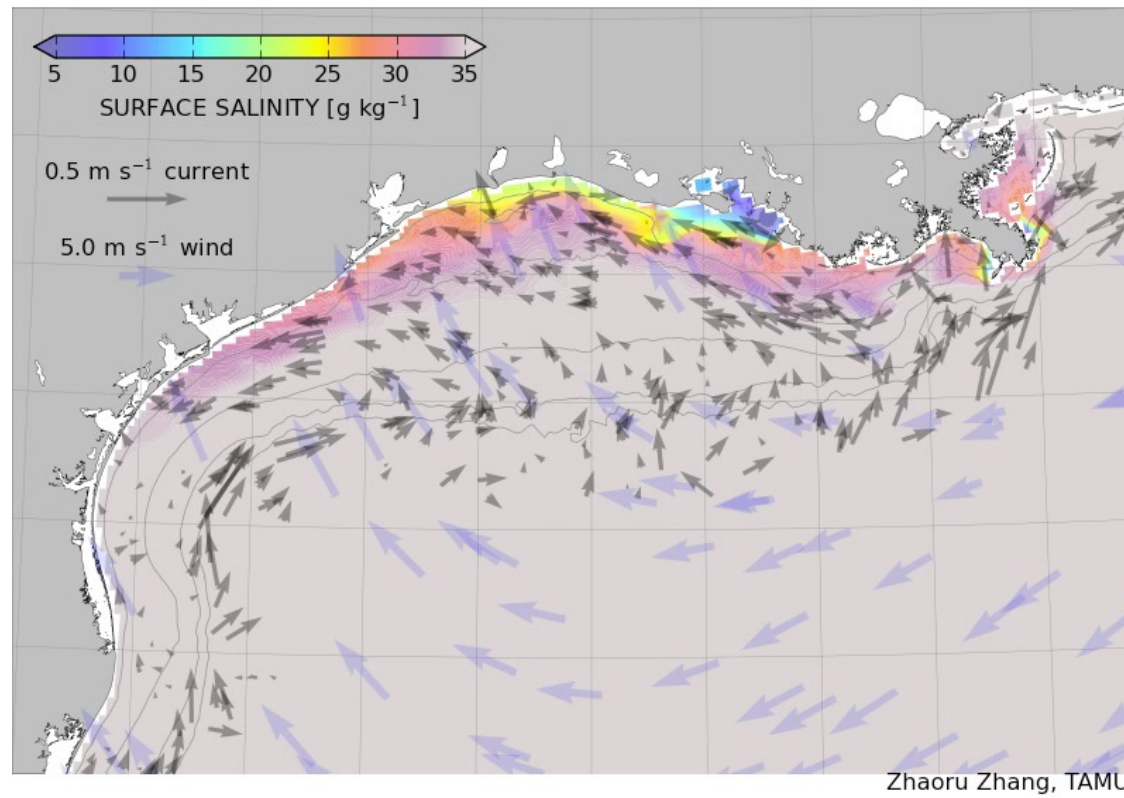
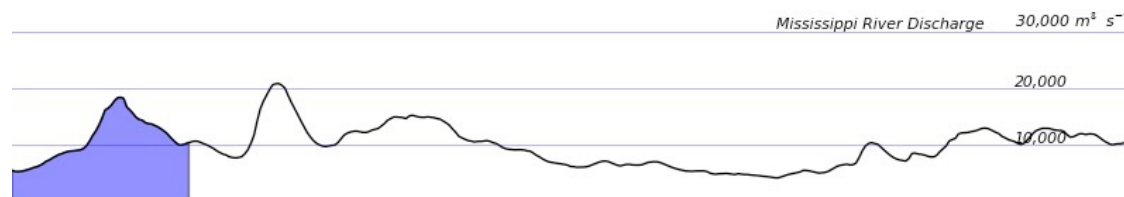


Figure 9.1 Trajectory of a water parcel calculated from current measured from August 17 to August 24, 1933 at $57^{\circ}49'N$ and $17^{\circ}49'E$ west of Gotland (From Sverdrup, Johnson, and Fleming, 1942).



2006 Feb 28 00:00 GMT



Geostrophic flows

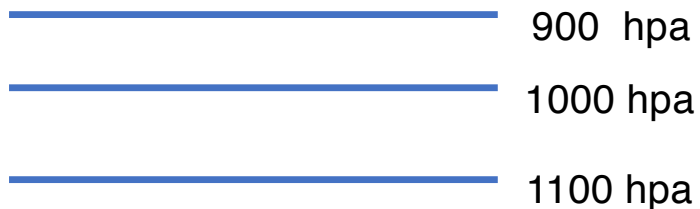
Assumptions:

$$R_o \ll 1 \text{ and } E_k \ll 1 \text{ (ocean interior)}$$

Inertial acceleration, nonlinear advection and viscosity terms are neglected

Governing equations

低压



高压

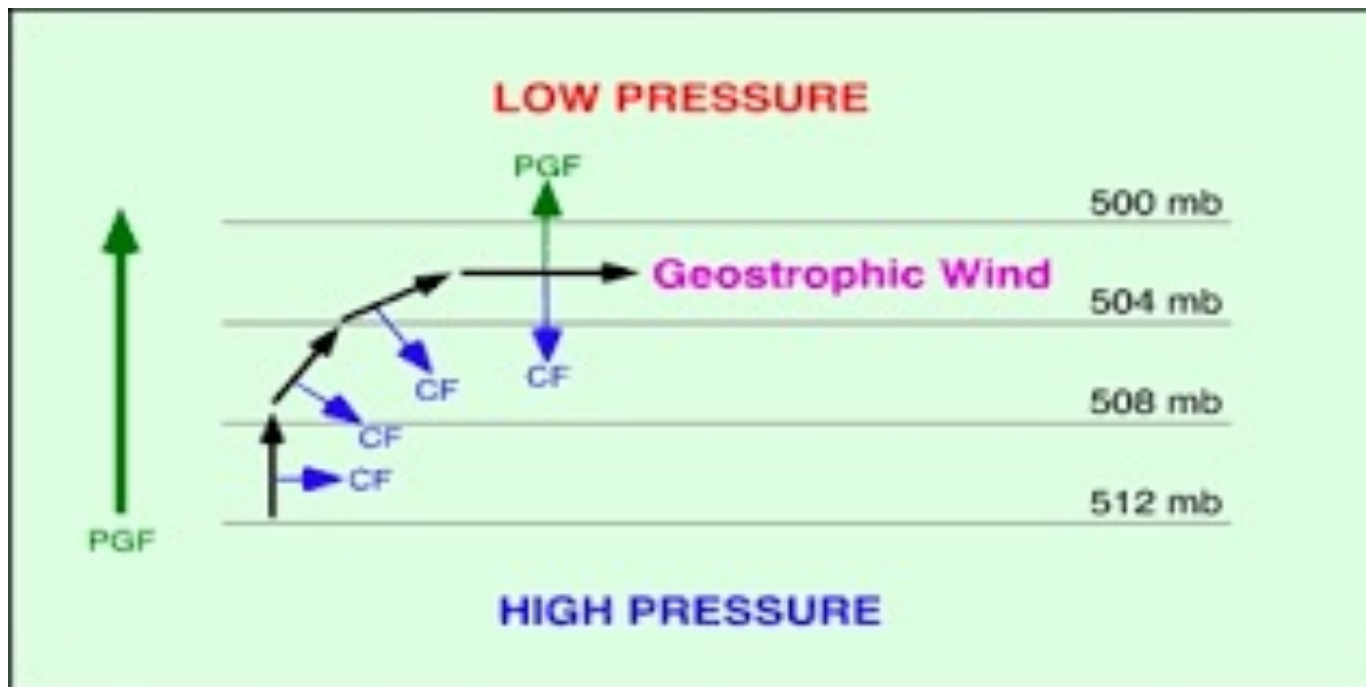
$$\begin{aligned} -fv &= -\frac{1}{\rho_0} \frac{\partial p}{\partial x} \\ +fu &= -\frac{1}{\rho_0} \frac{\partial p}{\partial y} \end{aligned}$$

Geostrophic balance

balance between the pressure gradient term and the Coriolis term

$$\mathbf{u} \cdot \nabla p = 0$$

Geostrophic flow is perpendicular to the pressure gradient (force).



High pressure is to the right (left) of the geostrophic flow in the northern (southern) hemisphere

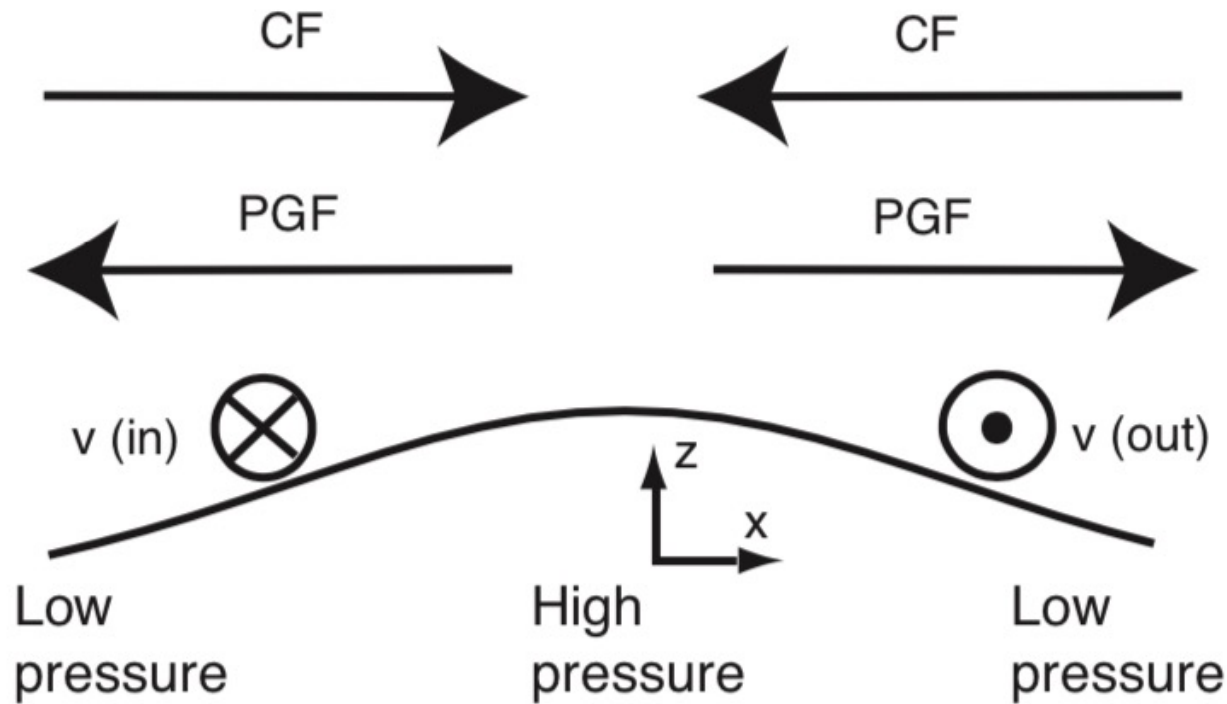
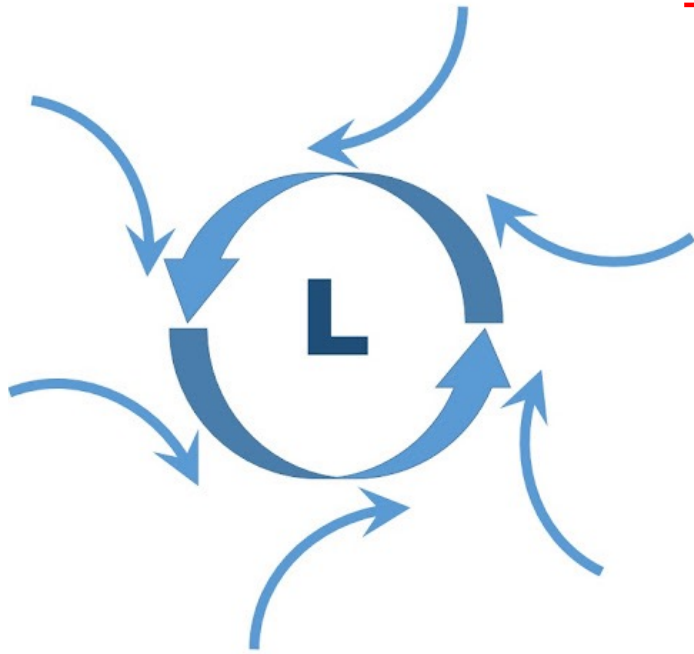
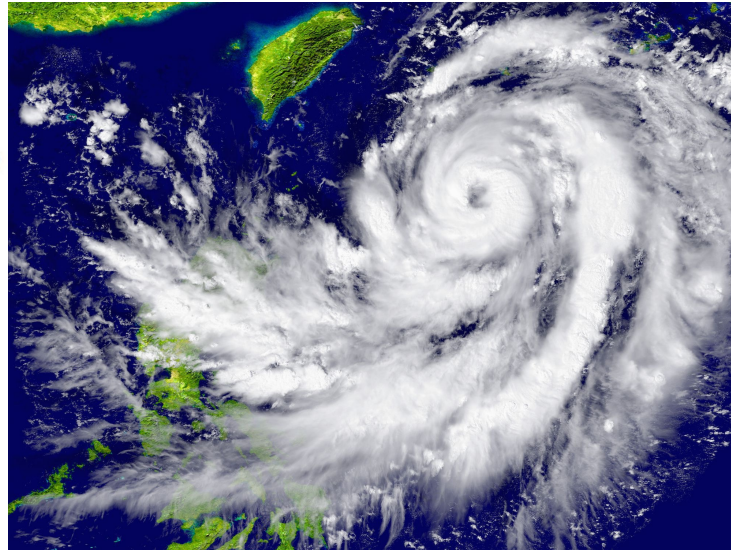


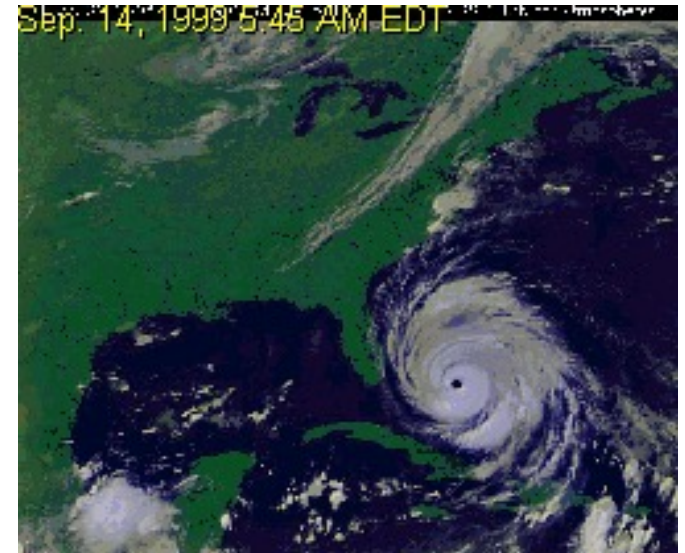
FIGURE 7.9 Geostrophic balance: horizontal forces and velocity. PGF = pressure gradient force. CF = Coriolis force. v = velocity (into and out of page). See also Figure S7.17.



Typhoon: low pressure center



Hurricane



In the northern hemisphere, **low-pressure** center induces **anticlockwise** flow (cyclone)

Cyclones and anticyclones

Northern Hemisphere upper-air pattern



Anticyclonic geostrophic
clockwise flow

Cyclonic geostrophic
counterclockwise flow

Southern Hemisphere upper-air pattern

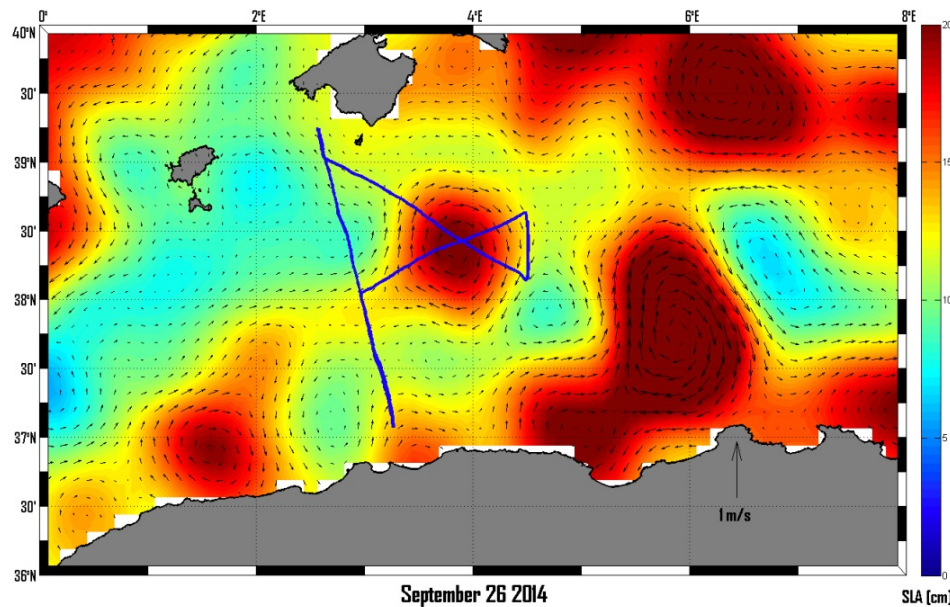


Anticyclonic geostrophic
counterclockwise flow

Cyclonic geostrophic
clockwise flow

Cyclonic and anticyclonic eddies

northern hemisphere



southern hemisphere

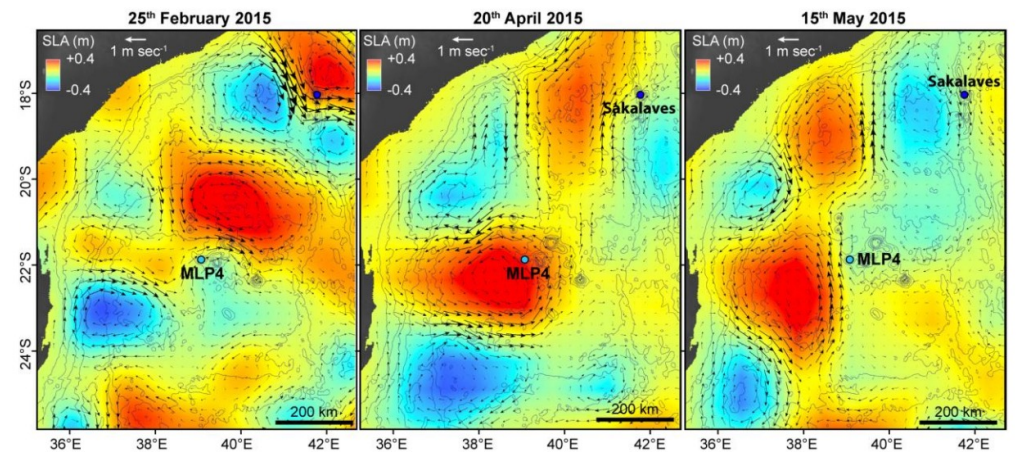


Fig. 16. Sea level anomaly (SLA) and geostrophic velocity anomalies (black arrows) in the Mozambique Channel showing the interaction of large anticyclonic eddies (positive anomalies in red colour) near the seamounts. Bathymetric contours are represented every 500 m.

Figure 1: Sea level anomaly map (color scale) and associated geostrophic velocity anomalies (black arrows) from AVISO data on 26 September 2014. Blue line shows the glider track from 15 September to 20 October 2014.

Miramontes et al. (2018)

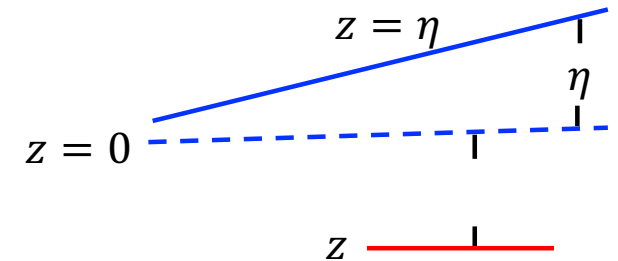
Cotroneo et al. (2015)

Hydrostatic balance $\frac{\partial p}{\partial z} + \rho g = 0$

If $\rho = \rho_0$ everywhere, vertically integrating the equation from z to the surface η :

$$\int_z^\eta \frac{\partial p}{\partial z} dz + \rho_0 g (\eta - z) = 0$$

$$p|_z = \overset{P_o}{p|_\eta} + \rho_0 g (\eta - z)$$



The the horizontal momentum equations become:

$$-fv = -\frac{1}{\rho_0} \frac{\partial}{\partial x} (P_o + \rho_0 g \eta - \rho_0 g z) = -g \frac{\partial \eta}{\partial x}$$

$$fu = -\frac{1}{\rho_0} \frac{\partial}{\partial y} (P_o + \rho_0 g \eta - \rho_0 g z) = -g \frac{\partial \eta}{\partial y}$$



So for $\rho=\rho_0$:

$$-fv = -g \frac{\partial \eta}{\partial x}$$

$$fu = -g \frac{\partial \eta}{\partial y}$$

$$\frac{\partial u}{\partial z} = 0$$

$$\frac{\partial v}{\partial z} = 0$$



The flow has no vertical shear and the fluid moves like a slab – **Taylor column**

$$\begin{aligned} -fv &= -\frac{1}{\rho_0} \frac{\partial p}{\partial x} \\ +fu &= -\frac{1}{\rho_0} \frac{\partial p}{\partial y} \end{aligned}$$

For f-plane:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial}{\partial x} \left(\frac{1}{\rho_0 f} \frac{\partial p}{\partial y} \right) + \frac{\partial}{\partial y} \left(\frac{1}{\rho_0 f} \frac{\partial p}{\partial x} \right) = 0.$$

Geostrophic flows are horizontally non-divergent

From the continuity equation: $\frac{\partial w}{\partial z} = 0$ For flat surface or bottom, $w = 0$ through the water column

Streamfunction ψ

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

Geostrophic flow over irregular bottom

Boundary conditions at the surface and bottom:

$$\boxed{\frac{\partial w}{\partial z} = 0}$$

$$w|_{z=\eta} = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} + v \frac{\partial \eta}{\partial y}$$

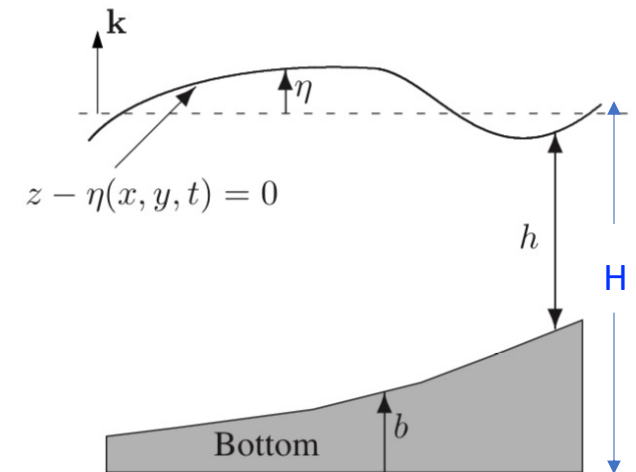
$$w|_{z=b} = u \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y}$$

$$\frac{\partial \eta}{\partial t} + u \frac{\partial (\eta - b)}{\partial x} + v \frac{\partial (\eta - b)}{\partial y} = 0$$

For steady motion

$$\mathbf{u} \cdot \nabla h = 0$$

Geostrophic flows must follow constant h



$$\boxed{\eta = h + b - H}$$

Geostrophic currents tend to flow along the isobaths

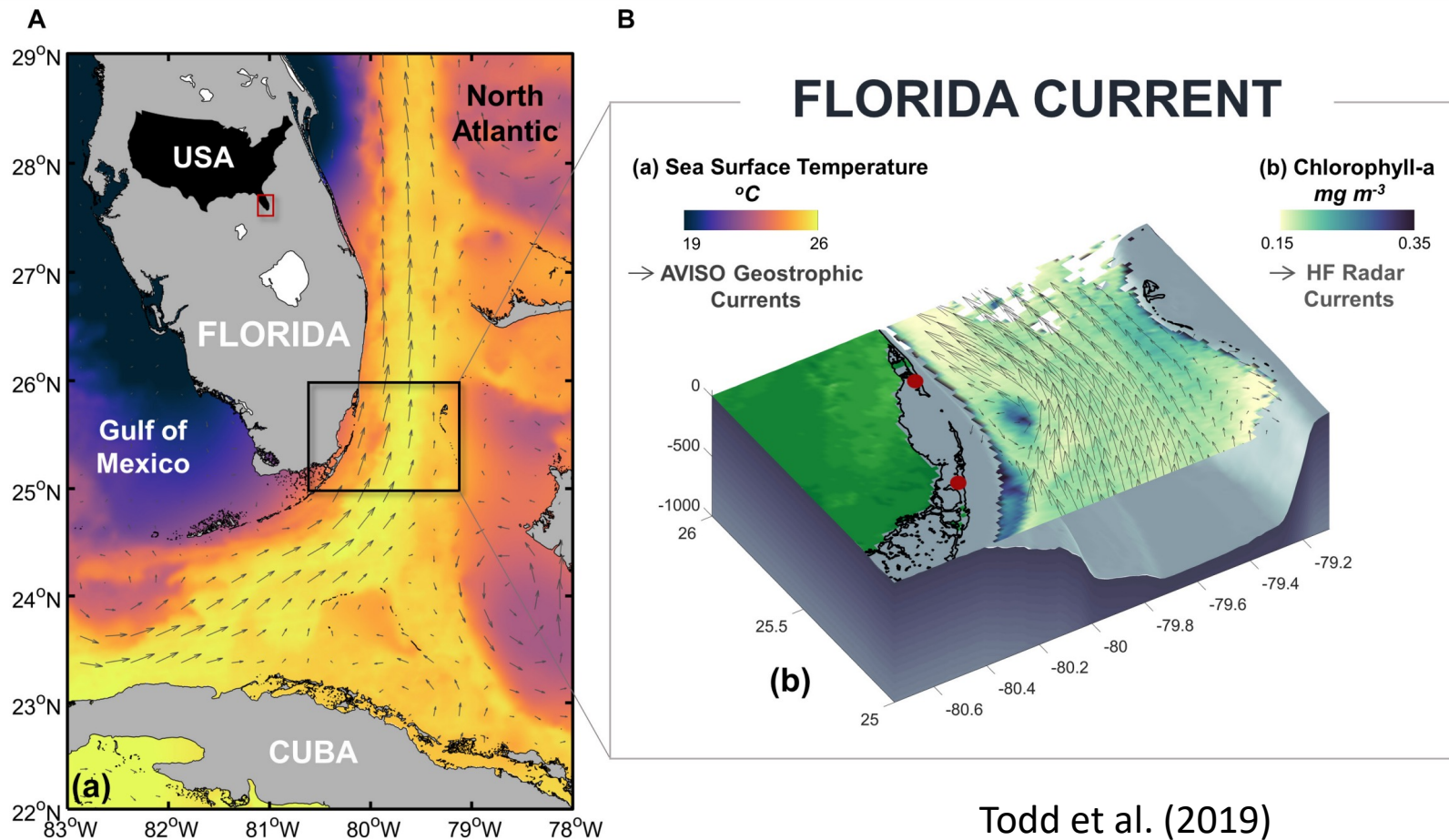


FIGURE 4 | Example of combined satellite- and land-based remote sensing of the Florida Current. **(A)** SST from GHRSSST and surface geostrophic currents from AVISO. **(B)** Chlorophyll from MODIS AQUA and surface currents from HF radars (HF radar data from Archer et al., 2017b).

Thermal wind balance

$$\begin{aligned} -fv &= -\frac{1}{\rho_0} \frac{\partial p}{\partial x} \\ +fu &= -\frac{1}{\rho_0} \frac{\partial p}{\partial y} \end{aligned}$$

Hydrostatic balance:

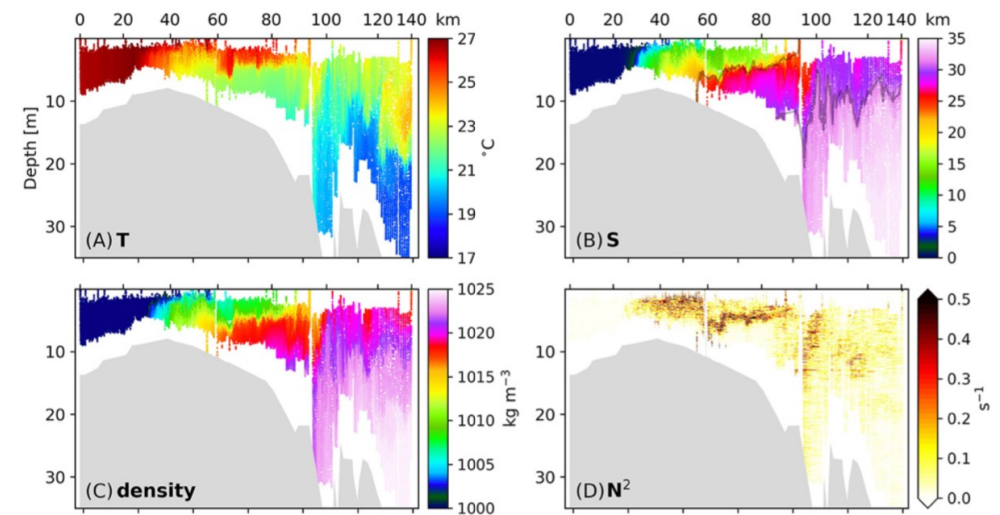
$$\frac{\partial p}{\partial z} + \rho g = 0$$

$$\frac{\partial v}{\partial z} = -\frac{g}{\rho_0 f} \frac{\partial \rho}{\partial x}$$

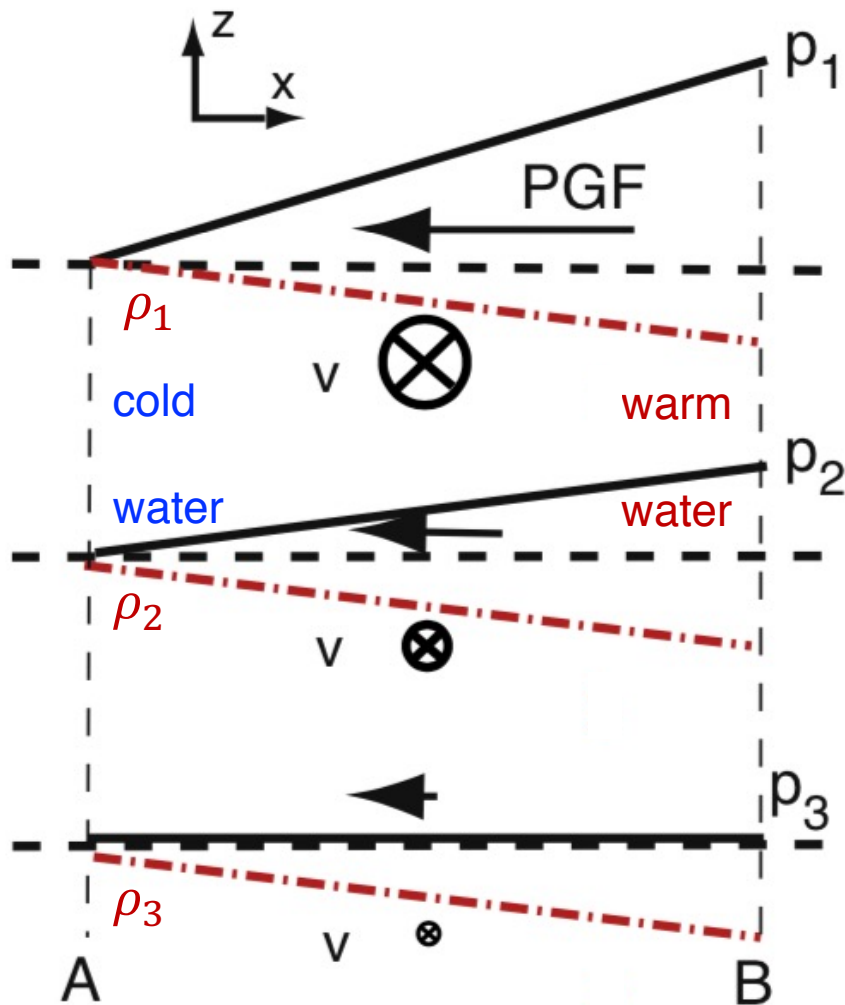
$$\frac{\partial u}{\partial z} = +\frac{g}{\rho_0 f} \frac{\partial \rho}{\partial y}$$



oceanic front



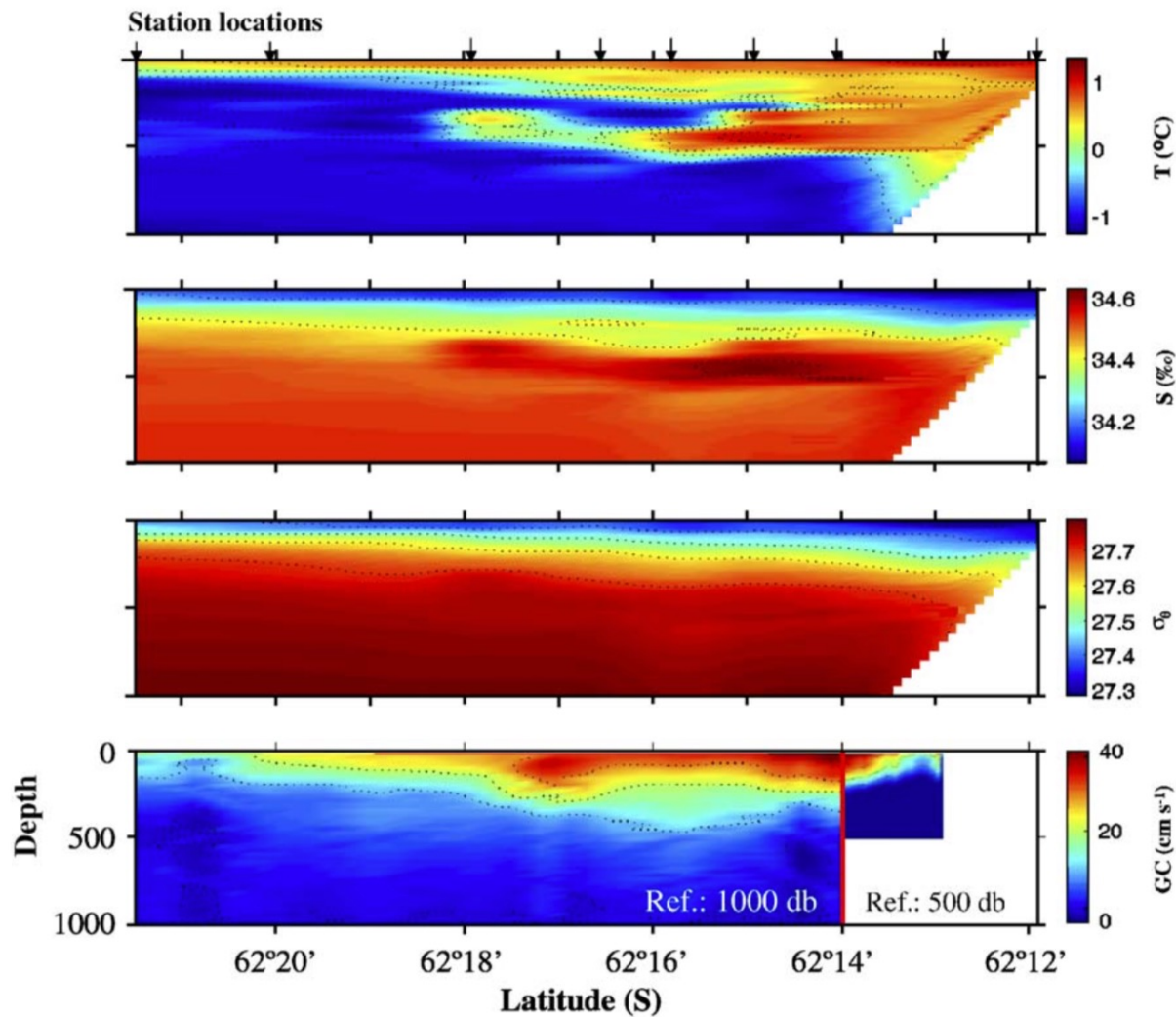
Zhang et al. (2020)



$$-fv = -\frac{1}{\rho_0} \frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial z} = -\frac{g}{\rho_0 f} \frac{\partial \rho}{\partial x}$$

FIGURE 7.10 Geostrophic flow and thermal wind balance: schematic of change in pressure gradient force (PGF) with depth. The horizontal geostrophic velocity v is into the page for this direction of PGF and is strongest at the top, weakening with depth, as indicated by the circle sizes. Density (dash-dot) increases with depth, and isopycnals are tilted. With the sea surface at B higher than at A, the PGF at the sea surface (h_1) is to the left. The PGF decreases with increasing depth, as indicated by the flattening of the isobars p_2 and p_3 .



Zhou et al. (2006)

$$\frac{\partial u}{\partial z} = + \frac{g}{\rho_0 f} \frac{\partial \rho}{\partial y}$$

$$\int_{ref}^z \frac{\partial u}{\partial z} dz = \frac{g}{\rho_0 f} \int_{ref}^z \frac{\partial \rho}{\partial y} dz$$

$$u|_z = u|_{ref} + \frac{g}{\rho_0 f} \int_{ref}^z \frac{\partial \rho}{\partial y} dz$$

$= 0$ > 0
 $f < 0$

Assume at $z=1000\text{m}$ (reference depth), $u = 0$

$$\frac{\partial \rho}{\partial y} < 0$$

$u|_z > 0$ (eastward current)

$$\frac{\partial u}{\partial z} > 0$$