Equation of State (EOS)

Specific volume:

$$\alpha = 1/\rho$$

$$\alpha = \alpha(\mathrm{T}, \mathrm{S}, \mathrm{P})$$

For small variations of α around a reference value:

$$d\alpha = \frac{\partial \alpha}{\partial T} dT + \frac{\partial \alpha}{\partial S} dS + \frac{\partial \alpha}{\partial p} dp = \alpha(\beta_T dT - \beta_S dS - \beta_p dp)$$

β_T:thermal expansion coefficient

β_s: salt contraction coefficient

β_p: compressibility coefficient

If we treat these coefficents as constants:

$$\alpha = \alpha_0 \left[1 + \beta_T (T - T_0) - \beta_S (S - S_0) - \beta_p (p - p_0)\right]$$

$$\rho = \rho_0 \left[1 - \beta_T (T - T_0) + \beta_S (S - S_0) + \beta_p (p - p_0)\right]$$

linear EOS

In reality the coefficients are not constant:

nonlinear nonlinear

$$\alpha = \alpha_0 \left[1 + \beta_T (1 + \gamma^* p)(T - T_0) + \frac{\beta_T^*}{2}(T - T_0)^2 - \beta_S (S - S_0) - \beta_p (p - p_0) \right]$$

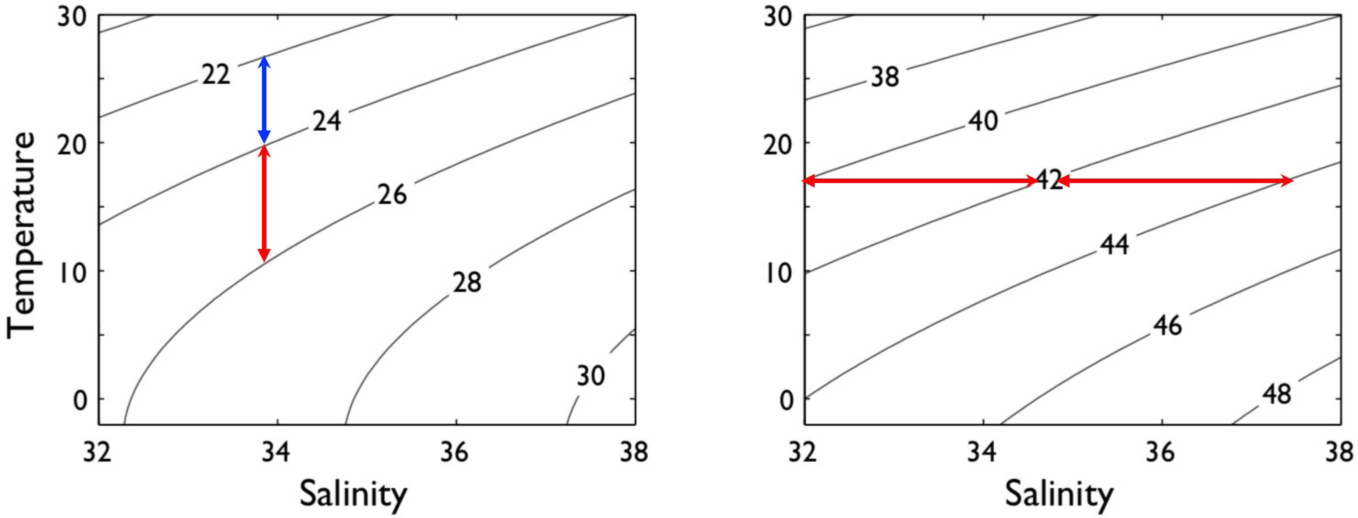

Fig. 1.3 A temperature–salinity diagram for seawater, calculated using an accurate empirical equation of state. Contours are (density-1000) kg m^{-3}, and the temperature is potential temperature, which in the deep ocean may be less than *in situ* temperature by a degree or so (see Fig. 1.4). Left panel: at sea-level ($p = 10^5$ Pa $= 1000$ mb). Right panel: at $p = 4 \times 10^7$ Pa, a depth of about 4 km. Note that in both cases the contours are slightly convex.

TEOS-10

Thermodynamic Equation Of Seawater - 2010

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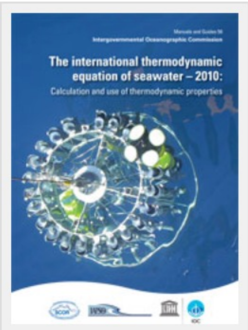

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HOME

This site is the official source of information about the Thermodynamic Equation Of Seawater - 2010 (TEOS-10), and the way in which it should be used.

TEOS-10 is based on a Gibbs function formulation from which all thermodynamic properties of seawater (density, enthalpy, entropy sound speed, etc.) can be derived in a thermodynamically consistent manner. TEOS-10 was adopted by the Intergovernmental Oceanographic Commission at its 25th Assembly in June 2009 to replace EOS-80 as the official description of seawater and ice properties in marine science.

A significant change compared with past practice is that TEOS-10 uses Absolute Salinity S_A (mass fraction of salt in seawater) as opposed to Practical Salinity S_P (which is essentially a measure of the conductivity of seawater) to describe the salt content of seawater. Ocean salinities now have units of g/kg.

Absolute Salinity (g/kg) is an SI unit of concentration. The thermodynamic properties of seawater, such as density and enthalpy, are now correctly expressed as functions of Absolute Salinity rather than being functions of the conductivity of seawater. Spatial variations of the composition of seawater mean that Absolute Salinity is not simply proportional to Practical Salinity; TEOS-10 contains procedures to correct for these effects.

github.com/TEOS-10/GSW-Python

README.md

gsw Python package

build failing | Install with conda | DOI 10.5281/zenodo.5214122

This Python implementation of the Thermodynamic Equation of Seawater 2010 (TEOS-10) is based primarily on numpy ufunc wrappers of the GSW-C implementation. We expect it to replace the original python-gsw pure-python implementation after a brief overlap period. The primary reasons for this change are that by building on the C implementation we reduce code duplication and we gain an immediate update to the 75-term equation. Additional benefits include a major increase in speed, a reduction in memory usage, and the inclusion of more functions. The penalty is that a C (or MSVC C++ for Windows) compiler is required to build the package from source.

Warning: this is for Python >=3.5 only.

Documentation is provided at https://teos-10.github.io/GSW-Python/.

For the core functionality, we use an auto-generated C extension module to wrap the C functions as numpy ufuncs, and then use an autogenerated Python module to add docstrings and handle masked arrays. 165 scalar C functions with only double-precision arguments and return values are wrapped as ufuncs, and 158 of these are exposed in the `gsw` namespace with an additional wrapper in Python.

A hand-written wrapper is used for one C function, and others are re-implemented directly in Python instead of being wrapped. Additional functions present in GSW-Matlab but not in GSW-C may be re-implemented in Python, but there is no expectation that all such functions will be provided.

The package can be installed from a clone of the repo using `pip install .`. It is neither necessary nor recommended to run the code generators, and no instructions are provided for them; their output is included in the repo. You will need a suitable compiler: gcc or clang for unix-like systems, or the MSVC compiler set used for Python itself on Windows. For Windows, some of the source code has been modified to C++ because the MSVC C compiler does not support the C99 complex data type used in original GSW-C.

Summary of the governing equations

Momentum equation:

$$\frac{d\boldsymbol{v}}{dt} + f\boldsymbol{k}\times\boldsymbol{v} = -\frac{1}{\rho}\nabla p + \nu \quad \nabla^2\boldsymbol{v} + \boldsymbol{g}$$

Continuity equation:

$$\frac{d\rho}{dt} + \rho\nabla \cdot \boldsymbol{v} = 0$$

Tracer equations:

$$\frac{dT}{dt} = K\nabla^2 T$$

$$\frac{dS}{dt} = K\nabla^2 S$$

Equation of state:

$$\rho = \rho(\mathrm{T}, \mathrm{S}, \mathrm{P})$$

Part II. Equation approximation and simplifications

Key points:

1. What are the **Boussinesq approximation and Reynolds average**? How do the primitive equations change after these operations?

2. **Scale analysis** is a powerful tool to simplify the equations (dimensionless numbers).

Boussinesq approximation

$$\rho = \rho_0 + \Delta\rho(x, y, z, t)$$
$$= \boldsymbol{\rho_0} + \boldsymbol{\overline{\rho}(z)} + \boldsymbol{\rho'(x, y, z, t)}$$
$$= \tilde{\rho}\,(z) + \rho'(x, y, z, t)$$

$\boxed{\bar{\rho}, \quad \rho' << \rho_0}$

$\boldsymbol{\rho_0}$: mean (reference) density (~1025 kg m^{-3})

$\boldsymbol{\overline{\rho}(z)}$: density variation due to stratification

$\boldsymbol{\rho'}$: density variation due to perturbation

For pressure:

dynamic pressure

$$p = \tilde{p}(z) + p'(x, y, z, t)$$

hydrostatic pressure

$$\tilde{p}(z) = P_0 - \rho_0 g z$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = -\frac{1}{\rho}\left(\frac{d\tilde{p}}{dz} + \frac{\partial p'}{\partial z}\right) = \frac{\rho_0}{\rho} g - \frac{1}{\rho}\frac{\partial p'}{\partial z}$$

Then the z-momentum equation becomes:

$b = -\Delta\rho g$: buoyancy force

$$\frac{\mathrm{dw}}{\mathrm{dt}} + f_* u = -\frac{1}{\rho}\frac{\partial p'}{\partial z} - \frac{\Delta\rho}{\rho} g + \nu\nabla^2 \mathrm{w}$$

ρ_0 ρ_0

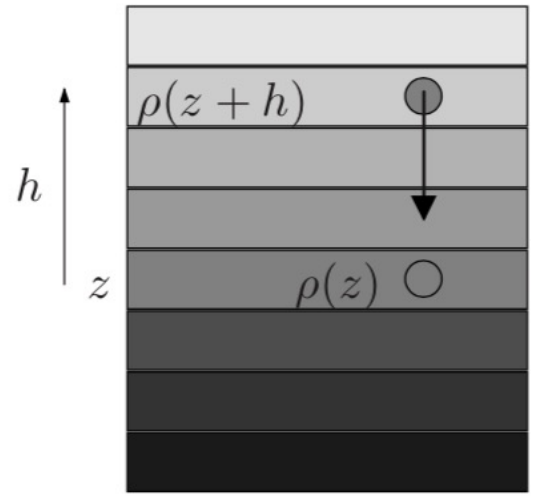
Boussinesq approximation: ρ can be replaced by ρ_0 (or $\Delta\rho$ can be neglected) everywhere except in the gravity term

Horizontal momentum equations

$$\frac{du}{dt} + f_* w - f v = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu \nabla^2 u$$

$$\frac{dv}{dt} + f u = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \nu \nabla^2 v.$$

$$\frac{\mathrm{dw}}{\mathrm{dt}} + f_* u = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - \frac{\rho}{\rho_o} g + \nu\nabla^2 \mathrm{w}$$



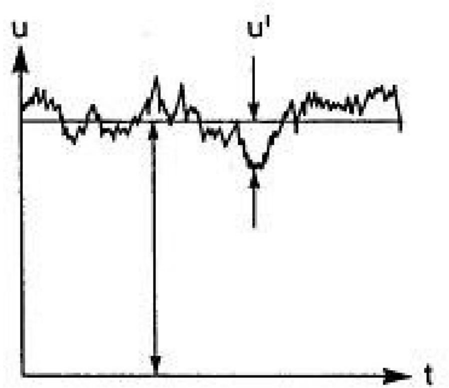
Reynolds-averaged (time-average) equations

The velocity can be decomposed into (time) mean velocity and perturbation (turbulent) velocity

$$u=\bar{u}+u', \quad v=\bar{v}+v', \quad w=\bar{w}+w', \quad p=\bar{p}+p'$$

$$\overline{u'}=0, \ \overline{v'}=0, \ \overline{w'}=0, \ \overline{p'}=0$$

$$\overline{uv}=\overline{(\bar{u}+u')(\bar{v}+v')}=\bar{u}\bar{v}+\overline{u'v'}$$



The advection term in the x momentum equation can be written as:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{\partial(u^2)}{\partial x}+\frac{\partial(uv)}{\partial y}+\frac{\partial(uw)}{\partial z}-u\underbrace{\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)}_{=0}$$

$$=\frac{\partial(u^2)}{\partial x}+\frac{\partial(uv)}{\partial y}+\frac{\partial(uw)}{\partial z}$$

Take the time average of the x-direction momentum equation:

$$\overline{\frac{\partial u}{\partial t}}+\overline{\frac{\partial uu}{\partial x}}+\overline{\frac{\partial uv}{\partial y}}+\overline{\frac{\partial uw}{\partial z}}-\overline{fv}+\overline{f_* w}=\overline{-\frac{1}{\rho_0}\frac{\partial p}{\partial x}}+\overline{\nu\nabla^2 u}$$

$$F^x=\frac{\partial\tau^{xx}}{\partial x}+\frac{\partial\tau^{yx}}{\partial y}+\frac{\partial\tau^{zx}}{\partial z}$$
$$=\frac{1}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{1}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\frac{1}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)$$
$$=\mu\,\nabla^2 u$$

$$\frac{\partial\bar{u}}{\partial t}+\frac{\partial\overline{uu}}{\partial x}+\frac{\partial\overline{uv}}{\partial y}+\frac{\partial\overline{uw}}{\partial z}-f\bar{v}+f_*\bar{w}=-\frac{1}{\rho_0}\frac{\partial\bar{p}}{\partial x}+\nu\nabla^2\bar{u}$$

$$\frac{\partial\bar{u}}{\partial t}+\frac{\partial}{\partial x}(\bar{u}\bar{u}+\overline{u'u'})+\frac{\partial}{\partial y}(\bar{u}\bar{v}+\overline{u'v'})+\frac{\partial}{\partial z}(\bar{u}\bar{w}+\overline{u'w'})-f\bar{v}+f_*\bar{u}=-\frac{1}{\rho_0}\frac{\partial\bar{p}}{\partial x}+\nu\nabla^2\bar{u}$$

$$\frac{\partial\bar{u}}{\partial t}+\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\bar{u}}{\partial y}+\bar{w}\frac{\partial\bar{u}}{\partial z}-f\bar{v}+f_*\bar{u}=-\frac{1}{\rho_0}\frac{\partial\bar{p}}{\partial x}+\nu\nabla^2\bar{u}-\frac{\partial\,\overline{u'u'}}{\partial x}-\frac{\partial\,\overline{u'v'}}{\partial y}-\frac{\partial\,\overline{u'w'}}{\partial z}$$

$$-\overline{u'u'}=\upsilon_E\frac{\partial\bar{u}}{\partial x}\qquad\overline{-u'v'}=\upsilon_E\frac{\partial\bar{u}}{\partial y}\qquad-\overline{u'w'}=\upsilon_E\frac{\partial\bar{u}}{\partial z}$$

υ_E: eddy (turbulent) viscosity coefficient

Reynolds stress: stress induced by turbulence acting on mean flow

$$\frac{\partial\bar{u}}{\partial t}+\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\bar{u}}{\partial y}+\bar{w}\frac{\partial\bar{u}}{\partial z}-f\bar{v}+f_*\bar{u}=-\frac{1}{\rho_0}\frac{\partial\bar{p}}{\partial x}+(\nu+\upsilon_E)\nabla^2\bar{u}$$

molecular viscosity coefficient

	μ (kg m^{-1} s^{-1})	ν (m^2 s^{-1})
Air	1.8×10^{-5}	1.5×10^{-5}
Water	1.1×10^{-3}	1.1×10^{-6}
Mercury	1.6×10^{-3}	1.2×10^{-7}

eddy viscosity can be orders of magnitude larger than the molecular viscosity

$$\frac{\partial \bar{u}}{\partial \mathrm{t}} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} - f\bar{v} + f_*\bar{u} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x} + \upsilon_E \nabla^2 \bar{u}$$

From now on, for simplicity we will remove the bar symbol, but note that the variables in the equation actually denote time-mean variables.

We normally distinguish between the horizonal and vertical eddy viscosity, so the equation become:

$$\text{x:}\ \frac{\partial u}{\partial t} + \mathrm{u}\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial v}{\partial y} + w\frac{\partial u}{\partial z} - fv + f_*w = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right)$$

Reynolds-averaged equations

Momentum

$$\text{x:}\ \frac{du}{dt} - fv + f_* w = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(A_H \frac{\partial \mathrm{u}}{\partial \mathrm{x}}\right) + \frac{\partial}{\partial y}\left(A_H \frac{\partial \mathrm{u}}{\partial \mathrm{y}}\right) + \frac{\partial}{\partial \mathrm{z}}\left(A_V \frac{\partial \mathrm{u}}{\partial \mathrm{z}}\right)$$

$$\text{y:}\frac{dv}{dt} + fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(A_H \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_H \frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_V \frac{\partial v}{\partial z}\right)$$

$$\text{z:}\ \frac{dw}{dt} + f_* u = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\left(A_H \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_H \frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_V \frac{\partial w}{\partial z}\right) - \frac{\rho}{\rho_0} g$$

Tracers

$$\frac{dT}{dt} = \frac{\partial}{\partial x}\left(K_H \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_H \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_V \frac{\partial T}{\partial z}\right)$$

$$\frac{dS}{dt} = \frac{\partial}{\partial x}\left(K_H \frac{\partial S}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_H \frac{\partial S}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_V \frac{\partial S}{\partial z}\right)$$

Scale analysis

Characteristic scale: a typical scale for a variable

Table 4.1 TYPICAL SCALES OF ATMOSPHERIC AND OCEANIC FLOWS

Variable	Scale	Unit	Atmospheric value	Oceanic value	
x, y	L	m	100 km = 10^5 m	10 km = 10^4 m	10^4~10^6 km
z	H	m	1 km = 10^3 m	100 m = 10^2 m	10^1~10^3 km
t	T	s	$\geq \frac{1}{2}$ day $\simeq 4 \times 10^4$ s	≥ 1 day $\simeq 9 \times 10^4$ s	
u, v	U	m/s	10 m/s	0.1 m/s	
w	W	m/s			
p	P	$\mathrm{kg\,m^{-1}\,s^{-2}}$	variable		
ρ	$\Delta\rho$	kg/m^3			

The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial x} = \frac{\Delta u}{\Delta x} \sim \frac{U}{L}$$

scale $\qquad \frac{U}{L} \qquad \frac{U}{L} \qquad \frac{W}{H}$

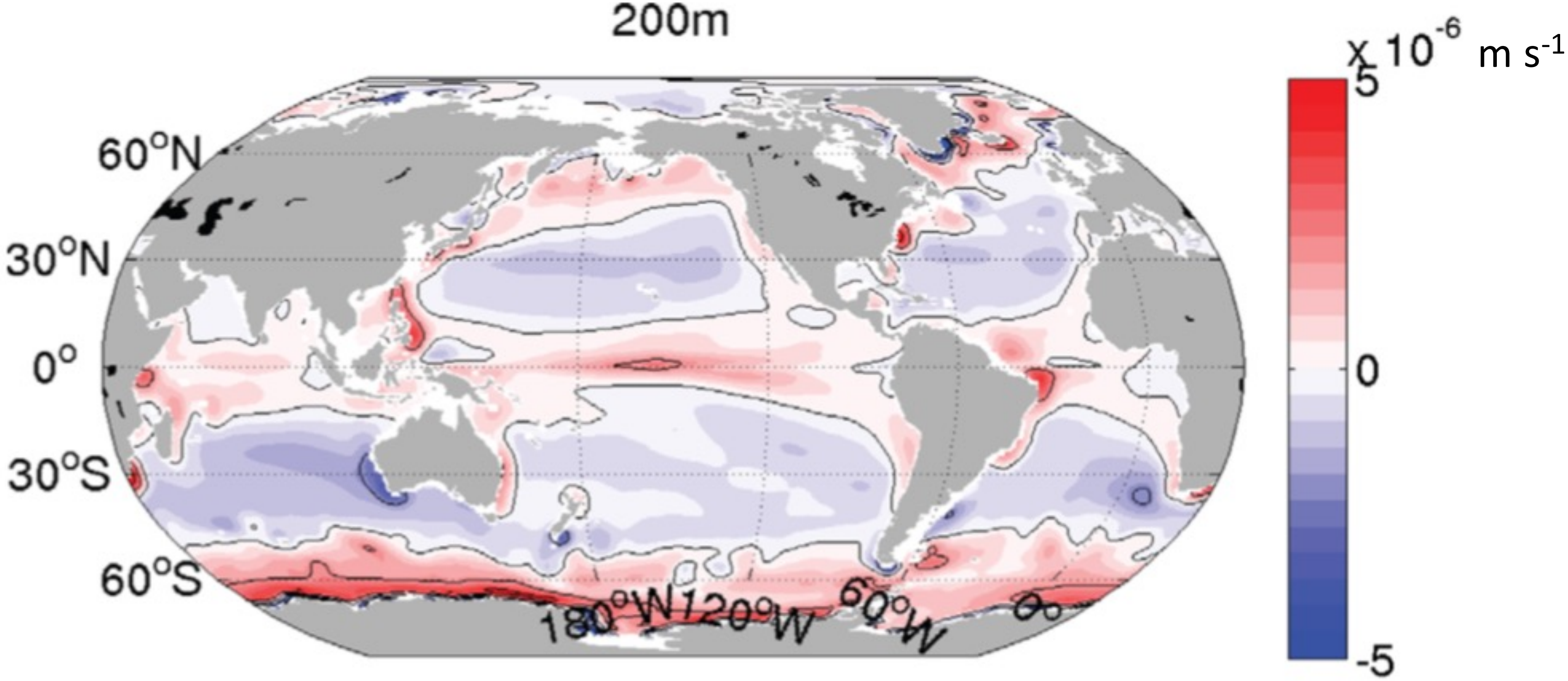
$$\frac{U}{L} \sim \frac{W}{H}$$

$$W \sim U\frac{H}{L} \sim U\delta$$

$\delta = \frac{H}{L}$: aspect ratio ($10^{-4} \sim 10^{-2}$)

$$W \ll U$$

Vertical velocity in the global ocean



Liang et al. (2017, JGR)

The momentum equation

x direction: $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv + \cancel{f_* w} =$$

$$\frac{U}{T} \qquad \frac{U^2}{L} \qquad \frac{U^2}{L} \qquad W\frac{U}{H} \qquad fU \qquad \cancel{fW}$$

$W \ll U$

$$-\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_V\frac{\partial u}{\partial z}\right)$$

$$\frac{P}{\rho_0 L} \qquad A_H\frac{U}{L^2} \qquad A_H\frac{U}{L^2} \qquad A_V\frac{U}{H^2}$$

Non-dimensional numbers

Rossby number (R_o) = scale of nonlinear advection term / scale of Coriolis term

$$= \frac{U^2}{L} / \mathrm{f}\mathrm{U} = U/fL$$

measure of the role of Earth's rotation in motions

$R_o \ll 1$: rotation is important for the motion

Ekman number (E_k) = scale of vertical viscosity term / scale of Coriolis term

$$= A_V \frac{U}{H^2} / fU = \frac{A_V}{fH^2}$$

measure of the importance of frictional force

$E_k << 1$: friction can be neglected

Reynolds number (Re) = scale of the nonlinear term / scale of viscous term

$$= \frac{U^2}{L} / \nu \frac{U}{L^2} = \frac{UL}{\nu}$$

small Re: viscous flow

large Re: inviscous, turbulent flow