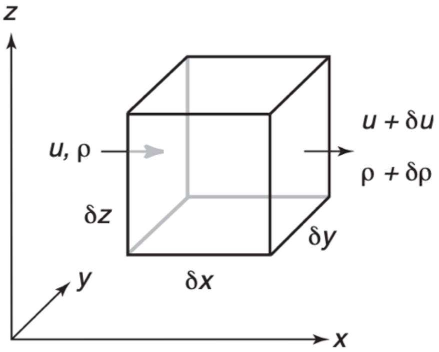
Homework 1

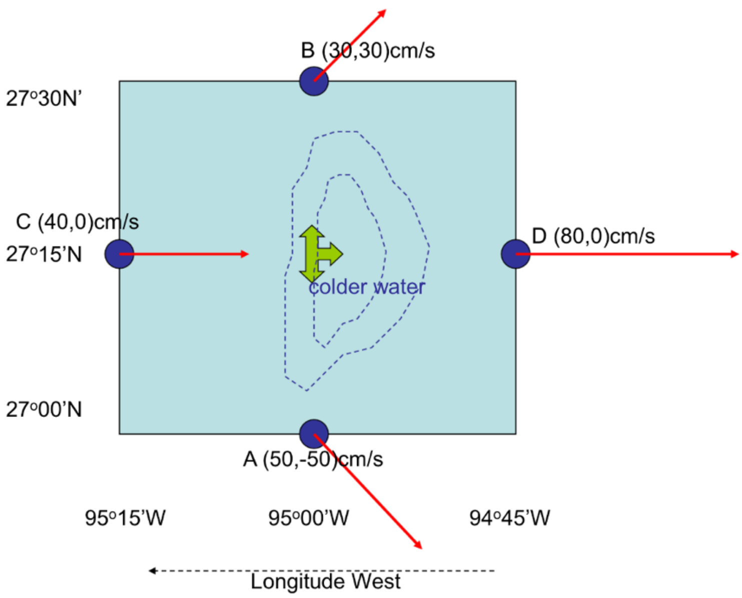
1. In class, we derived the continuity equation using the vector form. Now consider a small box of fluid as shown below, and derive the continuity equation with the scalar form (Hint: mass flow into the box – mass flow out of the box = rate of change of mass, and given that $\delta x, \delta y, \delta z \to 0$).



2. There are two sites in the ocean, site A and site B. The distributions of temperature (T), salinity (S) and density (ρ) with depth at A and B are shown in the following table. 1) Plot the vertical profiles of T, S and ρ (y-axis is depth and x-axis is the variable, and sea surface is at y=0); 2) Compute the Brunt–Väisälä frequency N using density in the depth range of 20-100 m; 3) Based on the calculation of N, at which site the water column is more stable?

	Site A				Site B		
Depth (m)	S1 (psu)	T1 (°C)	ρ1 (kg/m3)		S2 (psu)	T2 (°C)	ρ2 (kg/m3)
0	35.1	28.5	1022.308		33.5	2.5	1026.73
20	34.99	28.45	1022.327		33.5	3.74	1026.712
40	34.88	28.35	1022.363		34.25	4.02	1027.374
60	34.78	24.55	1023.572		34.55	4.1	1027.697
80	34.68	22.75	1024.111		34.65	4.15	1027.864
100	34.6	20.55	1024.748		34.74	4.2	1028.023

3. In an oceanic area shown below, horizontal velocity at four points A, B, C and D at the boundaries are provided, and the dimensions in the longitudinal and latitudinal directions are given (you can use $1° \approx 100$km in both directions). 1) Judge if the flow is horizontally divergent or convergent (hint: compute $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ and see its sign); 2) Assuming that the vertical velocity at the sea surface is zero, compute the vertical velocity at 50 m; 3) With such vertical velocity, how long does it take for a particle to move from 50 m to the sea surface?


B (30,30)cm/s
27°30N'
C (40,0)cm/s
27°15'N
D (80,0)cm/s
colder water
27°00'N
A (50,-50)cm/s
95°15'W
95°00'W
94°45'W
Longitude West