Blow the wind southerly, southerly, southerly,
Blow the wind south o'er the bonny blue sea;
Blow the wind southerly, southerly, southerly,
Blow bonnie breeze, my lover to me.

Traditional English folk song, *Blow the Wind Southerly*, c. 1834.

CHAPTER 14

The Overturning Circulation: Hadley and Ferrel Cells

THE LARGE-SCALE CIRCULATION OF THE ATMOSPHERE is normally taken to mean the flow on scales of the weather — several hundred or a thousand kilometres, say — to the global scale. The *general circulation* is virtually synonymous with the large-scale circulation, although the former is sometimes taken to be the time- or ensemble-averaged flow. Our goal in this and the next few chapters is understand this circulation and other properties of the atmosphere that accompany it — the temperature and moisture fields, for example. We might hope to answer the simple question, why do the winds blow as they do? In this chapter we focus on the dynamics of the Hadley Cell and, rather descriptively, on the mid-latitude overturning cell or the Ferrel Cell, moving to a more dynamical view of the extratropical zonally averaged circulation in Chapter 15.

The atmosphere is a terribly complex system, and we cannot hope to fully explain its motion as the analytic solution to a small set of equations. Rather, a full understanding of the atmosphere requires describing it in a consistent way on many levels simultaneously. One of these levels involves simulating the flow by numerically solving the governing equations of motion as completely as possible by using a comprehensive General Circulation Model (GCM). Such a simulation brings problems sof its own, for example understanding the simulation itself and discerning whether it is a good representation of reality, and so we shall concentrate on simpler, more conceptual models and the basic theory of the circulation. We begin this chapter with a brief observational overview of some of the large-scale features of the atmosphere, concentrating on the zonally-averaged fields.[1]

14.1 BASIC FEATURES OF THE ATMOSPHERE

14.1.1 The Radiative Equilibrium Distribution

A gross but informative measure characterizing the atmosphere, and the effects that dynamics have on it, is the pole-to-equator temperature distribution. The *radiative equilibrium* temperature is the hypothetical, three-dimensional, temperature field that would obtain if there were no atmospheric or oceanic motion, given the composition and radiative properties of the atmosphere and surface. The field is a function of the incoming solar radiation and the atmospheric composition, and its determination entails a complicated calculation, especially as the radiative properties of the atmosphere depend heavily on the amount of water vapour and cloudiness it contains. (The distribution

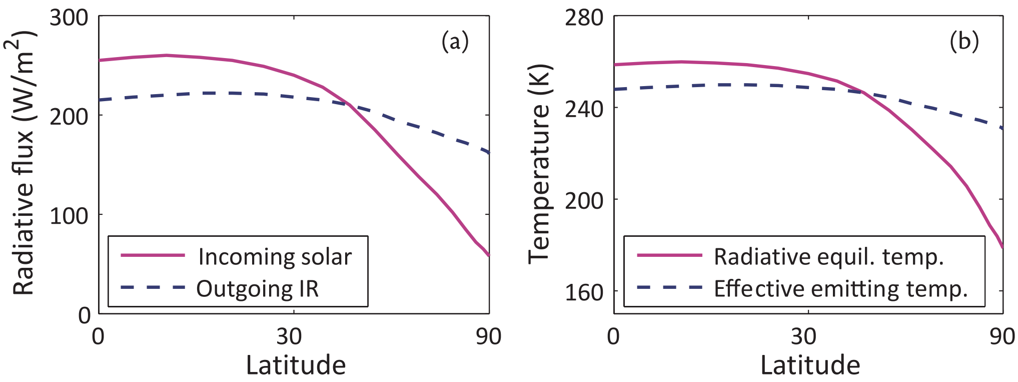

Fig. 14.1 (a) The (approximate) observed net average incoming solar radiation and outgoing infrared radiation at the top of the atmosphere, as a function of latitude (plotted on a sine scale). (b) The temperatures associated with these fluxes, calculated using $T = (R/\sigma)^{1/4}$, where R is the solar flux for the radiative equilibrium temperature and where R is the infrared flux for the effective emitting temperature. Thus, the solid line is an approximate radiative equilibrium temperature

of absorbers is usually taken to be that which obtains in the observed, moving, atmosphere, in order that the differences between the calculated radiative equilibrium temperature and the observed temperature are due to fluid motion.)

A much simpler calculation that illustrates the essence of the situation is to first note that at the top of the atmosphere the globally averaged incoming solar radiation is balanced by the outgoing infrared radiation. If there is no lateral transport of energy in the atmosphere or ocean then *at each latitude* the incoming solar radiation will be balanced by the outgoing infrared radiation, and if we parameterize the latter using a single latitudinally-dependent temperature we will obtain a crude radiative-equilibrium temperature (the 'radiative emitting temperature') for the atmospheric column at each latitude. Specifically, a black body subject to a net incoming radiation of S (watts per square metre) has a radiative-equilibrium temperature T_{rad} given by $\sigma T_{rad}^4 = S$, this being Stefan's law with Stefan–Boltzmann constant $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. Thus, for the Earth, we have, at each latitude,

$$\sigma T_{rad}^4 = S(\vartheta)(1 - \alpha), \tag{14.1}$$

where α is the albedo of the Earth and $S(\vartheta)$ is the incoming solar radiation at the top of the atmosphere, and its solution is shown in Fig. 14.1. The solid lines in the two panels show the net solar radiation and the solution to (14.1), T_{rad}; the dashed lines show the observed outgoing infrared radiative flux, I, and the effective emitting temperature associated with it, $(I/\sigma)^{1/4}$. The emitting temperature does not quantitatively characterize that temperature at the Earth's surface, nor at any single level in the atmosphere, because the atmosphere is not a black body and the outgoing radiation originates from multiple levels. Nevertheless, the qualitative point is evident: the radiative equilibrium temperature has a much stronger pole-to-equator gradient than does the effective emitting temperature, indicating that there is a poleward transport of heat in the atmosphere–ocean system. More detailed calculations indicate that the atmosphere is further from its radiative equilibrium in winter than summer, indicating a larger heat transport. The transport occurs because poleward moving air tends to have a higher static energy ($c_pT + gz$ for dry air; in addition there is some energy transport associated with water vapour evaporation and condensation) than the equatorward moving air, most of this movement being associated with the large-scale circulation. The radiative forcing thus seeks to maintain a pole-to-equator temperature gradient, and the ensuing circulation seeks to reduce this gradient.

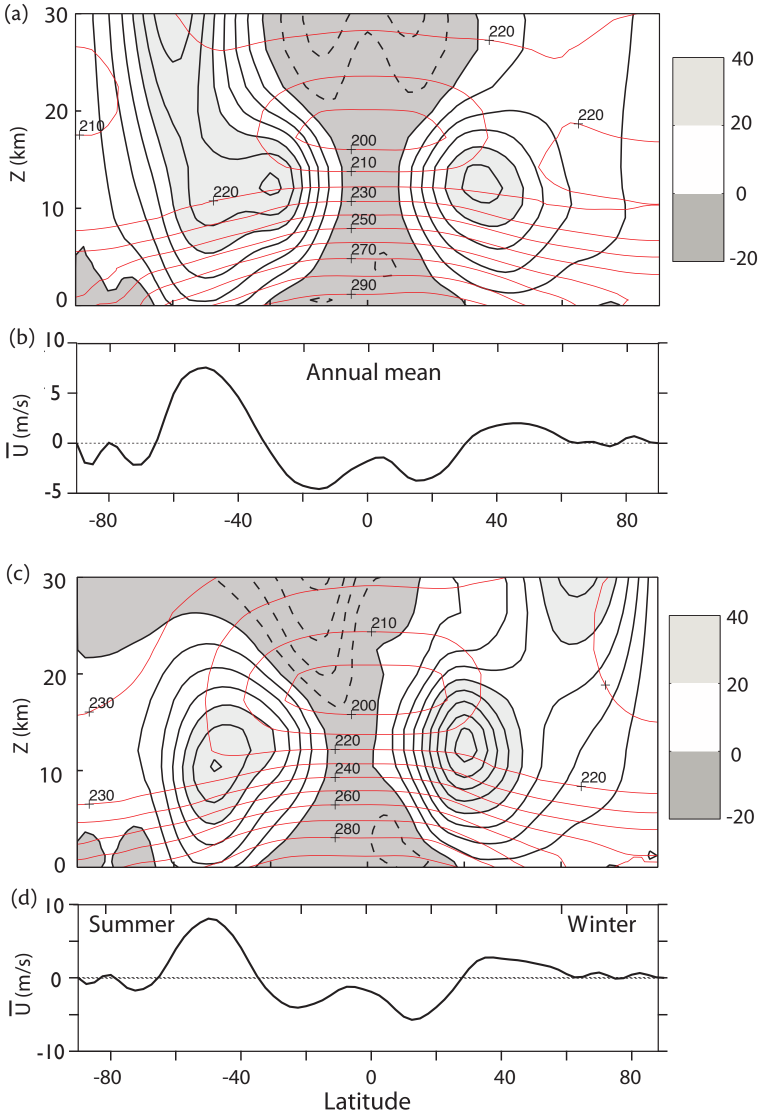

Fig. 14.2 (a) Annual mean, zonally-averaged zonal wind (heavy contours and shading) and the zonally-averaged temperature (red, thinner contours).

(b) Annual mean, zonally averaged zonal winds at the surface.

(c) and (d) Same as (a) and (b), except for northern hemisphere winter (December–January–February, or DJF).

The wind contours are at intervals of $5\,\mathrm{m\,s^{-1}}$ with shading for eastward winds above $20\,\mathrm{m\,s^{-1}}$ and for all westward winds, and the temperature contours are labelled. The ordinate of (a) and (c) is $Z = -H\log(p/p_R)$, where $H = 7.5$ km and $p_R = 100$ hPa.

14.1.2 Observed Wind and Temperature Fields

The observed zonally-averaged temperature and zonal wind fields are illustrated in Fig. 14.2. The vertical coordinate is log pressure, multiplied by a constant factor $H = RT_0/g = 7.5\,\mathrm{km}$, so that the ordinate is similar to height in kilometres. (In an isothermal hydrostatic atmosphere $(RT_0/g)\mathrm{d}\ln p = -\mathrm{d}z$, and the value of H chosen corresponds to $T_0 = 256\,\mathrm{K}$.) To a good approximation temperature and zonal wind are related by thermal wind balance, which in pressure coordinates is

$$f\frac{\partial u}{\partial p} = \frac{R}{p}\frac{\partial T}{\partial y}. \tag{14.2}$$

In the lowest several kilometres of the atmosphere temperature falls almost monotonically with latitude and height, and this region is called the *troposphere* (look ahead to Fig. 15.25). The temperature in the lower troposphere in fact varies more rapidly with latitude than does the effective emitting temperature, T_E, the latter being more characteristic of the temperature in the mid-to-

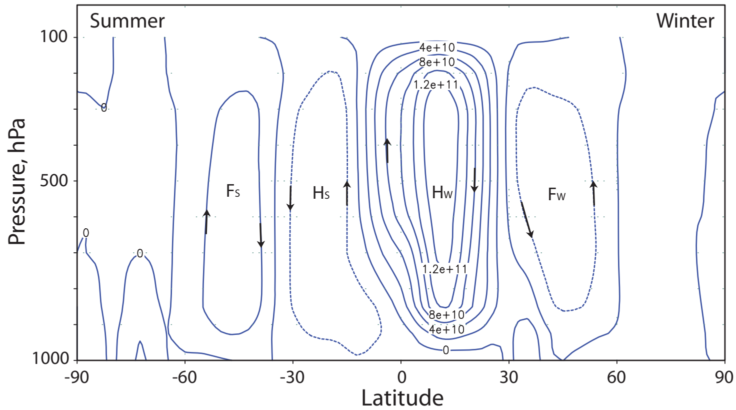

Fig. 14.3 The observed meridional overturning circulation (MOC) of the atmosphere ($\mathrm{kg\,s^{-1}}$) averaged over December–January–February. Note the direct *Hadley Cells,* particularly strong in winter (H_W and H_S, in winter and summer respectively) with rising motion near the equator, descending motion in the subtropics, and the weaker, indirect, *Ferrel Cells* (F_W and F_S) at mid-latitudes.

upper troposphere. The meridional temperature gradient is much larger in winter than summer, because in winter high latitudes receive virtually no direct heating from the Sun. The gradient is also strongest at the edge of the subtropics, and here it is associated with a zonal jet, particularly strong in winter. There is no need to 'drive' this wind with any kind of convergent momentum fluxes: given the temperature, the flow is a consequence of thermal wind balance, and to the extent that the upper troposphere is relatively frictionless there is no need to maintain it against dissipation. Of course just as the radiative-equilibrium temperature gradient is much larger than that observed, so the zonal wind shear associated with it is much larger than that observed. Thus, the overall effect of the atmospheric and oceanic circulation, and in particular of the turbulent circulation of the mid-latitude atmosphere, is to *reduce* the amplitude of the vertical shear of the eastward flow by way of a poleward heat transport. Observations indicate that about two-thirds of this transport is effected by the atmosphere, and about a third by the ocean, rather more in low latitudes.[2]

Above the troposphere is the *stratosphere,* and here temperature typically increases with height. The boundary between the two regions is called the *tropopause,* and this varies in height from about 16 km in the tropics to about 8 km in polar regions. We consider the maintenance of this stratification in Section 15.5.

The surface winds typically have, going from the equator to the pole, an E–W–E (easterly–westerly–easterly) pattern, although the polar easterlies are weak and barely present in the Northern Hemisphere. (Meteorologists use 'westerly' to denote winds from the west, that is eastward winds; similarly 'easterlies' are westward winds.) In a given hemisphere, the surface winds are stronger in winter than summer, and they are also consistently stronger in the Southern Hemisphere than in the Northern Hemisphere, because in the former the surface drag is weaker because of the relative lack of continental land masses and topography. The surface winds are *not* explained by thermal wind balance. Indeed, unlike the upper level winds, they must be maintained against the dissipating effects of friction, and this implies a momentum convergence into regions of surface westerlies and a divergence into regions of surface easterlies. Typically, the maxima in the eastward surface winds are in mid-latitudes and somewhat poleward of the subtropical maxima in the upper-level westerlies and at latitudes where the zonal flow is a little more constant with height. The mechanisms of the momentum transport in the mid-latitudes and the maintenance of the surface westerly winds are the topics of section 15.1.

14.1.3 Meridional Overturning Circulation

The observed (Eulerian) zonally-averaged meridional overturning circulation (MOC) is shown in Fig. 14.3. The figure shows a streamfunction, Ψ for the vertical and meridional velocities such that,

Some Features of the Large-scale Atmospheric Circulation

From Figures 14.1–14.3 we see or infer the following:

1. A pole–equator temperature gradient that is much smaller than the radiative equilibrium gradient.
2. A troposphere, in which temperature generally falls with height, above which lies the stratosphere, in which temperature increases with height. The two regions are separated by a tropopause, which varies in height from about 16 km at the equator to about 6 km at the pole.
3. A monotonically decreasing temperature from equator to pole in the troposphere, but a weakening and sometimes reversal of this above the tropopause.
4. A westerly (i.e., eastward) tropospheric jet. The time and zonally-averaged jet is a maximum at the edge or just poleward of the subtropics, where it is associated with a strong meridional temperature gradient. In mid-latitudes the jet has a stronger barotropic component.
5. An E–W–E (easterlies–westerlies–easterlies) surface wind distribution. The latitude of the maximum in the surface westerlies is in mid-latitudes, where the zonally-averaged flow is more barotropic. The surface easterlies at high latitudes are very weak and seasonal, barely showing on an annual average.

in the pressure coordinates used in the figure,

$$\frac{\partial \Psi}{\partial y} = -\overline{\omega}, \qquad \frac{\partial \Psi}{\partial p} = \overline{v}, \tag{14.3}$$

where the overbar indicates a zonal average. In each hemisphere there is rising motion near the equator and sinking in the subtropics, and this circulation is known as the *Hadley Cell.*[3] The Hadley Cell is a thermally direct cell (i.e., the warmer fluid rises, the colder fluid sinks), much stronger in the winter hemisphere, and extends to about 25–30°. In mid-latitudes the sense of the overturning circulation is apparently reversed, with rising motion in the high-mid-latitudes, at around 60° and sinking in the subtropics, and this is known as the *Ferrel Cell*. However, as with most pictures of averaged streamlines in unsteady flow, this gives a misleading impression as to the actual material flow of parcels of air because of the presence of eddying motion, and we discuss this in the next chapter. At low latitudes the circulation is more nearly zonally symmetric and the picture does give a qualitatively correct representation of the actual flow. At high latitudes there is again a thermally direct cell (although it is weak and not always present), and thus the atmosphere is often referred to as having a three-celled structure.

14.1.4 Summary

Some of the main features of the zonally-averaged circulation are summarized in the shaded box above. We emphasize that the zonally-averaged circulation is not synonymous with a zonally symmetric circulation, and the mid-latitude circulation is highly asymmetric. Any model of the mid-latitudes that did not take into account the zonal asymmetries in the circulation — of which the weather is the main manifestation — would be seriously in error. This was first explicitly realized in the 1920s, and taking into account such asymmetries is the main task of the dynamical

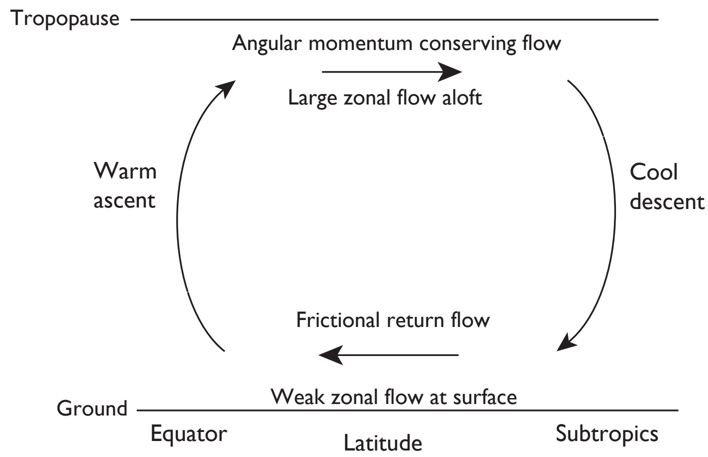

Fig. 14.4 A simple model of the Hadley Cell. Rising air near the equator moves poleward near the tropopause, descending in the subtropics and returning.

The poleward moving air conserves its angular momentum, leading to a shear of the zonal wind that increases away from the equator. By thermal wind the temperature of the air falls as it moves poleward, and to satisfy the thermodynamic budget it sinks in the subtropics.

meteorology of the mid-latitudes, and is the subject of the next chapter. The large-scale tropical circulation of the atmosphere is to a much larger degree zonally symmetric, and although monsoonal circulations and the Walker circulation (a cell with rising air in the Western Pacific and descending motion in the Eastern Pacific) are zonally asymmetric, they are relatively weaker than typical mid-latitude weather systems. Indeed the boundary between the tropics and mid-latitude may be usefully defined by the latitude at which such zonal asymmetries become dynamically important on the large scale and this boundary, at about 25°–30° on average, roughly coinciding with the edge of the Hadley Cell. We begin our dynamical description with a study of the low-latitude zonally symmetric atmospheric circulation.

14.2 A STEADY MODEL OF THE HADLEY CELL

Ceci n'est pas une pipe.
René Magritte. Title of painting, 1929.

14.2.1 Assumptions

Let us try to construct a zonally symmetric model of the Hadley Cell, recognizing that such a model is likely applicable mainly to the tropical atmosphere, this being more zonally symmetric than the mid-latitudes.[4] We suppose that heating is maximum at the equator, and our intuitive picture, drawing on the observed flow of Fig. 14.3, is of air rising at the equator and moving poleward at some height H, descending at some latitude ϑ_H, and returning equatorward near the surface. We will make three major assumptions:

(i) that the circulation is steady;

(ii) that the poleward moving air conserves its axial angular momentum, whereas the zonal flow associated with the near-surface, equatorward moving flow is frictionally retarded and weak;

(iii) that the circulation is in thermal wind balance.

We also assume the model is symmetric about the equator (an assumption we relax in Section 14.4). These are all reasonable assumptions, but they cannot be rigorously justified; in other words, we are constructing a *model* of the Hadley Cell, schematically illustrated in Fig. 14.4. The model defines a limiting case — steady, inviscid, zonally-symmetric flow — that cannot be expected to describe the atmosphere quantitatively, but that can be analysed fairly completely. Another limiting case, in which eddies play a significant role, is described in Section 14.5. The real atmosphere may defy such simple characterizations, but the two limiting cases provide useful benchmarks of understanding.

14.2.2 Dynamics

We now try to determine the strength and poleward extent of the Hadley circulation in our steady model. For simplicity we work with a Boussinesq atmosphere, but this is not an essential aspect. We first derive the conditions under which conservation of angular momentum will hold, and then determine the consequences of that.

The zonally-averaged zonal momentum equation may be easily derived from (2.50a) and/or (2.62) and in the absence of friction it is

$$\frac{\partial \overline{u}}{\partial t} - (f + \overline{\zeta})\overline{v} + \overline{w}\frac{\partial \overline{u}}{\partial z} = -\frac{1}{a\cos^2\vartheta}\frac{\partial}{\partial \vartheta}(\cos^2\vartheta\,\overline{u'v'}) - \frac{\partial \overline{u'w'}}{\partial z}, \tag{14.4}$$

where $\overline{\zeta} = -(a\cos\vartheta)^{-1}\partial_\vartheta(\overline{u}\cos\vartheta)$ and the overbars represent zonal averages. If we neglect the vertical advection and the eddy terms on the right-hand side, then a steady solution, if it exists, obeys

$$(f + \overline{\zeta})\overline{v} = 0. \tag{14.5}$$

Presuming that the meridional flow $\overline{v}$ is non-zero (an issue we address in Section 14.2.8) then $f + \overline{\zeta} = 0$, or equivalently

$$2\Omega\sin\vartheta = \frac{1}{a}\frac{\partial \overline{u}}{\partial \vartheta} - \frac{\overline{u}\tan\vartheta}{a}. \tag{14.6}$$

At the equator we shall assume that $\overline{u} = 0$, because here parcels have risen from the surface where, by assumption, the flow is weak. Equation (14.6) then has a solution of

$$\boxed{\overline{u} = \Omega a\frac{\sin^2\vartheta}{\cos\vartheta} \equiv U_M\,.} \tag{14.7}$$

This gives the zonal velocity of the poleward moving air in the upper branch of the (model) Hadley Cell, above the frictional boundary layer. We can derive (14.7) directly from the conservation of axial angular momentum, m, of a parcel of air at a latitude ϑ. In the shallow atmosphere approximation we have (cf. (2.64) and equations following)

$$\overline{m} = (\overline{u} + \Omega a\cos\vartheta)a\cos\vartheta, \tag{14.8}$$

and if $\overline{u} = 0$ at $\vartheta = 0$ and if $\overline{m}$ is conserved on a poleward moving parcel, then (14.8) leads to (14.7). It also may be directly checked that

$$f + \overline{\zeta} = -\frac{1}{a^2\cos\vartheta}\frac{\partial \overline{m}}{\partial \vartheta}. \tag{14.9}$$

We have thus shown that, if eddy fluxes and frictional effects are negligible, the poleward flow will conserve its angular momentum, the result of which, by (14.7), is that the magnitude of the zonal flow in the Earth's rotating frame will increase with latitude (see Fig. 14.5). (Also, given the absence of eddies our model is zonally symmetric and we shall drop the overbars over the variables.)

If (14.7) gives the zonal velocity in the upper branch of the Hadley Cell, and that in the lower branch is close to zero, then the thermal wind equation can be used to infer the vertically averaged temperature. Although the geostrophic wind relation is not valid at the equator (a more accurate balance is the gradient wind balance, $fu + u^2\tan\vartheta/a = -a^{-1}\partial\phi/\partial\vartheta$) the zonal wind is in fact geostrophically balanced until very close to the equator, and at the equator itself the horizontal temperature gradient in our model vanishes, because of the assumed interhemispheric symmetry. Thus, conventional thermal wind balance suffices for our purposes, and this is

$$2\Omega\sin\vartheta\frac{\partial u}{\partial z} = -\frac{1}{a}\frac{\partial b}{\partial \vartheta}, \tag{14.10}$$

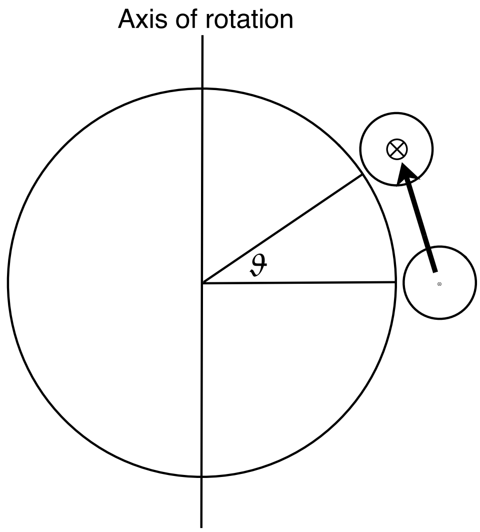

Fig. 14.5 If a ring of air at the equator moves poleward it moves closer to the axis of rotation. If the parcels in the ring conserve their angular momentum their zonal velocity must increase; thus, if $m = (\overline{u} + \Omega a \cos\vartheta) a \cos\vartheta$ is preserved and $\overline{u} = 0$ at $\vartheta = 0$ we recover (14.7).

where $b = g\,\delta\theta/\theta_0$ is the buoyancy and $\delta\theta$ is the deviation of potential temperature from a constant reference value θ_0. (Be reminded that θ is potential temperature, whereas ϑ is latitude.) Vertically integrating from the ground to the height H where the outflow occurs and substituting (14.7) for u yields

$$\frac{1}{a\theta_0}\frac{\partial\theta}{\partial\vartheta} = -\frac{2\Omega^2 a}{gH}\frac{\sin^3\vartheta}{\cos\vartheta}, \tag{14.11}$$

where $\theta = H^{-1}\int_0^H \delta\theta\,\mathrm{d}z$ is the vertically averaged potential temperature. If the latitudinal extent of the Hadley Cell is not too great we can make the small-angle approximation, and replace $\sin\vartheta$ by ϑ and $\cos\vartheta$ by one, then integrating (14.11) gives

$$\boxed{\theta = \theta(0) - \frac{\theta_0\Omega^2 y^4}{2gHa^2},} \tag{14.12}$$

where $y = a\vartheta$ and $\theta(0)$ is the potential temperature at the equator, as yet unknown. Away from the equator, the zonal velocity given by (14.7) increases rapidly poleward and the temperature correspondingly drops. How far poleward is this solution valid? And what determines the value of the integration constant $\theta(0)$? To answer these questions we turn to thermodynamics.

14.2.3 Thermodynamics

In the above discussion, the temperature field is slaved to the momentum field in that it seems to follow passively from the dynamics of the momentum equation. Nevertheless, the thermodynamic equation must still be satisfied. Let us assume that the thermodynamic forcing can be represented by a Newtonian cooling to some specified radiative equilibrium temperature, θ_E; this is a severe simplification, especially in equatorial regions where the release of heat by condensation is important. The thermodynamic equation is then

$$\frac{\mathrm{D}\theta}{\mathrm{D}t} = \frac{\theta_E - \theta}{\tau}, \tag{14.13}$$

where τ is a relaxation time scale, perhaps a few weeks. Let us suppose that θ_E falls monotonically from the equator to the pole, and that it increases linearly with height, and a simple representation of this is

$$\frac{\theta_E(\vartheta, z)}{\theta_0} = 1 - \frac{2}{3}\Delta_H P_2(\sin\vartheta) + \Delta_V\left(\frac{z}{H} - \frac{1}{2}\right), \tag{14.14}$$

where Δ_H and Δ_V are nondimensional constants that determine the fractional temperature difference between the equator and the pole, and the ground and the top of the fluid, respectively. P_2 is the second Legendre polynomial, and it is usually the leading term in the Taylor expansion of symmetric functions (symmetric around the equator) that decrease from pole to equator; it also integrates to zero over the sphere. $P_2(y) = (3y^2 - 1)/2$, so that in the small-angle approximation and at $z = H/2$, or for the vertically averaged field, we have

$$\frac{\theta_E}{\theta_0} = 1 + \frac{1}{3}\Delta_H - \Delta_H\left(\frac{y}{a}\right)^2 \qquad \text{or} \qquad \theta_E = \theta_{E0} - \Delta\theta\left(\frac{y}{a}\right)^2, \tag{14.15a,b}$$

where θ_{E0} is the equilibrium temperature at the equator, $\Delta\theta$ determines the equator–pole radiative-equilibrium temperature difference, and

$$\theta_{E0} = \theta_0(1 + \Delta_H/3), \qquad \Delta\theta = \theta_0\Delta_H. \tag{14.16}$$

Now, let us suppose that the solution (14.12) is valid between the equator and a latitude ϑ_H where $v = 0$, so that within this region the system is essentially closed. Conservation of potential temperature then requires that the solution (14.12) must satisfy

$$\int_0^{Y_H} \theta \,\mathrm{d}y = \int_0^{Y_H} \theta_E \,\mathrm{d}y, \tag{14.17}$$

where $Y_H = a\vartheta_H$ is as yet undetermined. Poleward of this, the solution is just $\theta = \theta_E$. Now, we may demand that the solution be continuous at $y = Y_H$ (without temperature continuity the thermal wind would be infinite) and so

$$\theta(Y_H) = \theta_E(Y_H). \tag{14.18}$$

The constraints (14.17) and (14.18) determine the values of the unknowns $\theta(0)$ and Y_H. A little algebra gives

$$Y_H = \left(\frac{5\Delta\theta g H}{3\Omega^2\theta_0}\right)^{1/2}, \tag{14.19}$$

and

$$\theta(0) = \theta_{E0} - \left(\frac{5\Delta\theta^2 g H}{18a^2\Omega^2\theta_0}\right). \tag{14.20}$$

A useful nondimensional number that parameterizes these solutions is

$$R \equiv \frac{gH\Delta\theta}{\theta_0\Omega^2 a^2} = \frac{gH\Delta_H}{\Omega^2 a^2}, \tag{14.21}$$

which is the square of the ratio of the speed of shallow water waves to the rotational velocity of the Earth, multiplied by the fractional temperature difference from equator to pole. Typical values for the Earth's atmosphere are a little less than 0.1. In terms of R we have

$$\boxed{Y_H = a\left(\frac{5}{3}R\right)^{1/2}, \qquad \theta(0) = \theta_{E0} - \left(\frac{5}{18}R\right)\Delta\theta.} \tag{14.22a,b}$$

The solution, (14.12) with $\theta(0)$ given by (14.22b) is plotted in Fig. 14.6. Perhaps the single most important aspect of the model is that it predicts that the Hadley Cell has a *finite* meridional extent, *even for an atmosphere that is completely zonally symmetric.* The baroclinic instability that does occur in mid-latitudes is not necessary for the Hadley Cell to terminate in the subtropics, although it may be an important factor, or even the determining factor, in the real world.

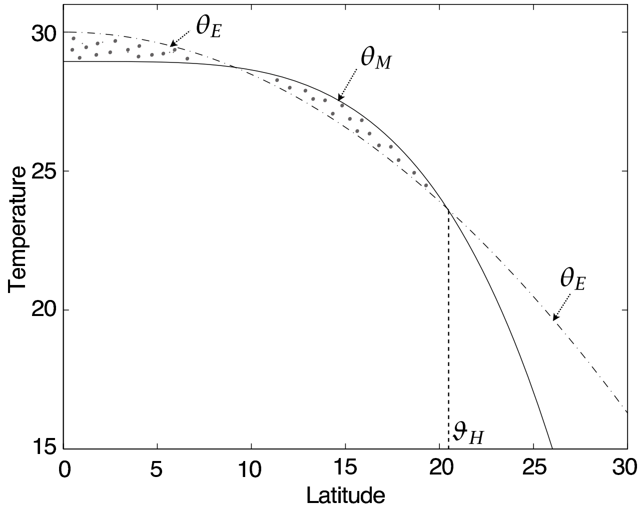

Fig. 14.6 The radiative equilibrium temperature (θ_E, dashed line) and the angular-momentum-conserving solution (θ_M, solid line) as a function of latitude. The two dotted regions have equal areas. The parameters are: $\theta_{EO} = 303\,\mathrm{K}$, $\Delta\theta = 50\,\mathrm{K}$, $\theta_0 = 300\,\mathrm{K}$, $\Omega = 7.272 \times 10^{-5}\,\mathrm{s}^{-1}$, $g = 9.81\,\mathrm{m\,s}^{-2}$, $H = 10\,\mathrm{km}$. These give $R = 0.076$ and $Y_H/a = 0.356$, corresponding to $\vartheta_H = 20.4°$.

14.2.4 Zonal Wind

The angular-momentum-conserving zonal wind is given by (14.7), which in the small-angle approximation becomes

$$U_M = \Omega \frac{y^2}{a}. \tag{14.23}$$

This relation holds for $y < Y_H$. The zonal wind corresponding to the radiative-equilibrium solution is given using thermal wind balance and (14.15b), which leads to

$$U_E = \Omega a R. \tag{14.24}$$

That the radiative-equilibrium zonal wind is a constant follows from our choice of the second Legendre function for the radiative equilibrium temperature and is not a fundamental result; nonetheless, for most reasonable choices of θ_E the corresponding zonal wind will vary much less than the angular-momentum-conserving wind (14.23). The winds are illustrated in Fig. 14.7. There is a discontinuity in the zonal wind at the edge of the Hadley Cell, and of the meridional temperature gradient, but not of the temperature itself.

14.2.5 Properties of the Solution

From (14.22) we can see that the model predicts that the latitudinal extent of the Hadley Cell is:

- proportional to the square root of the meridional radiative equilibrium temperature gradient: the stronger the gradient, the farther the circulation must extend to achieve thermodynamic balance via the equal-area construction in Fig. 14.6;
- proportional to the square root of the height of the outward flowing branch: the higher the outward flowing branch, the weaker the ensuing temperature gradient of the solution (via thermal wind balance), and so the further poleward the circulation must go;
- inversely proportional to the rotation rate Ω: the stronger the rotation rate, the stronger the angular-momentum-conserving wind, the stronger the ensuing temperature gradient and so the more compact the circulation.

These precise dependencies on particular powers of parameters are not especially significant in themselves, nor are they robust to changes in parameters. For example, were we to choose a meridional distribution of radiative equilibrium temperature different from (14.14) we might find different exponents in some of the solutions, although we would expect the same qualitative dependen-

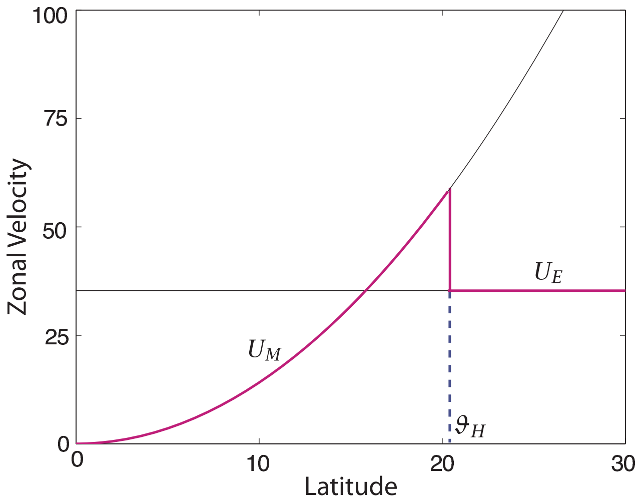

Fig. 14.7 The zonal wind corresponding to the radiative equilibrium temperature (U_E,) and the angular-momentum-conserving solution (U_M) as a function of latitude, given (14.23) and (14.24) respectively.

The parameters are the same as those of Fig. 14.6, and the radiative equilibrium wind, U_E is a constant, ΩaR. The actual zonal wind (in the model) follows the thick solid line: $u = U_m$ for $\vartheta < \vartheta_H$ ($y < Y_H$), and $u = U_E$ for $\vartheta > \vartheta_H$ ($y > Y_H$).

cies. However, the dependencies do provide predictions that may be tested with a numerical model. Also, as we have already noted, a key property of the model is that it predicts that the Hadley Cell has a finite meridional extent, even in the absence of mid-latitude baroclinic instability.

Another interesting property of the solutions is a discontinuity in the zonal wind. For tropical latitudes (i.e., $y < Y_H$), then $\overline{u} = U_M$ (the constant angular momentum solution), whereas for $y > Y_H$, $\overline{u} = U_E$ (the thermal wind associated with radiative equilibrium temperature θ_E). There is therefore a discontinuity of $\overline{u}$ at $y = Y_H$, because u is related to the meridional gradient of θ which changes discontinuously, even though θ itself is continuous. No such discontinuity is observed in the real world, although one may observe a baroclinic jet at the edge of the Hadley Cell.

14.2.6 Strength of the Circulation

We can make an estimate of the strength of the Hadley Cell by consideration of the thermodynamic equation at the equator, namely

$$w\frac{\partial\theta}{\partial z} \approx \frac{\theta_{E0} - \theta}{\tau}, \tag{14.25}$$

this being a balance between adiabatic cooling and radiative heating. If the static stability is determined largely by the forcing, and not by the meridional circulation itself, then $\theta_0^{-1}\partial\theta/\partial z \approx \Delta_V/H$, and (14.25) gives

$$w \approx \frac{H}{\theta_0 \Delta_V}\frac{\theta_{E0} - \theta}{\tau}. \tag{14.26}$$

Thus, the strength of the circulation is proportional to the distance of the solution from the radiative equilibrium temperature. The right-hand side of (14.25) can be evaluated from the solution itself, and from (14.22b) we have

$$\frac{\theta_{E0} - \theta}{\tau} = \frac{5R\Delta\theta}{18\tau}. \tag{14.27}$$

The vertical velocity is then given by

$$w \approx \frac{5R\Delta\theta H}{18\tau\Delta_V\theta_0} = \frac{5R\Delta_H H}{18\tau\Delta_V}. \tag{14.28}$$

Using mass continuity we can transform this into an estimate for the meridional velocity. Thus, if we let $(v/Y_H) \sim (w/H)$ and use (14.22), we obtain

$$v \sim \frac{R^{3/2} a \Delta_H}{\tau \Delta_V} \propto \frac{\Delta_H^{5/2}}{\Delta_V} \qquad \text{and} \qquad \Psi \sim vH \sim \frac{R^{3/2} a H \Delta_H}{\tau \Delta_V} \propto (\Delta\theta)^{5/2}, \tag{14.29}$$

where Ψ is the meridional overturning stream function Ψ, which evidently increases fairly rapidly as the gradient of the radiative equilibrium temperature increases. The characteristic overturning time of the circulation, τ_d is then

$$\tau_d = \frac{H}{w} \sim \frac{\tau \Delta_V}{R \Delta_H}. \tag{14.30}$$

We require $\tau_d/\tau \gg 1$ for the effects of the circulation on the static stability to be small and therefore $\Delta_V/(R\Delta_H) \gg 1$, or equivalently, using (14.16),

$$\theta_0 \Delta_V \gg R(\theta_{E0} - \theta_0). \tag{14.31}$$

If instead $\tau \gg \tau_d$, then the potential temperature would be nearly conserved as a parcel ascended in the rising branch of the Hadley Cell, and the static stability would be nearly neutral.

14.2.7 † Effects of Moisture

Suppose now that moisture is present, but that the Hadley Cell remains a self-contained system; that is, it neither imports nor exports moisture. We envision that water vapour joins the circulation by way of evaporation from a saturated surface into the equatorward, lower branch of the Hadley Cell, and that this water vapour then condenses in and near the upward branch of the cell. The latent heat released by condensation is exactly equal to the heat required to evaporate moisture from the surface, and no heat is lost or gained to the system. However, the heating *distribution* is changed from the dry case, becoming a strong function of the solution itself and likely to have a sharp maximum near the equator. Even if we were to try to parameterize the latent heat release by simply choosing a flow dependent radiative equilibrium temperature, the resulting problem would still be quite nonlinear and a general analytic solution seems out of our reach.[5]

Nevertheless, we may see quite easily the qualitative features of moisture, at least within the context of this model. The meridional distribution of temperature is still given by way of thermal wind balance with an angular-momentum-conserving zonal wind, and so is still given by (14.12). We may also assume that the meridional extent of the Hadley Cell is unaltered; that is, a solution exists with circulation confined to $\vartheta < \vartheta_H$ (although it may not be the unique solution). Then, if θ_E^* is the effective radiative equilibrium temperature of the moist solution, we have that $\theta_E^*(Y_H) = \theta_E(Y_H)$ and, in the small-angle approximation,

$$\int_0^{Y_H} \theta \, \mathrm{d}y = \int_0^{Y_H} \theta_E^* \, \mathrm{d}y = \int_0^{Y_H} \theta_E \, \mathrm{d}y, \tag{14.32}$$

where the first equality holds because it defines the solution, and the second equality holds because moisture provides no net energy source. Because condensation will occur mainly in the upward branch of the Hadley Cell, θ_E^* will be peaked near the equator, as sketched in Fig. 14.8. This construction makes it clear that the main difference between the dry and moist solutions is that the latter has a more intense overturning circulation, because, from (14.25), the circulation increases with the temperature difference between the solution and the forcing temperature. Concomitantly, our intuition suggests that the upward branch of the moist Hadley circulation will become much narrower and more intense than the downward branch because of the enhanced efficiency of moist convection, and these expectations are generally confirmed by numerical integrations of the moist equations of motion.

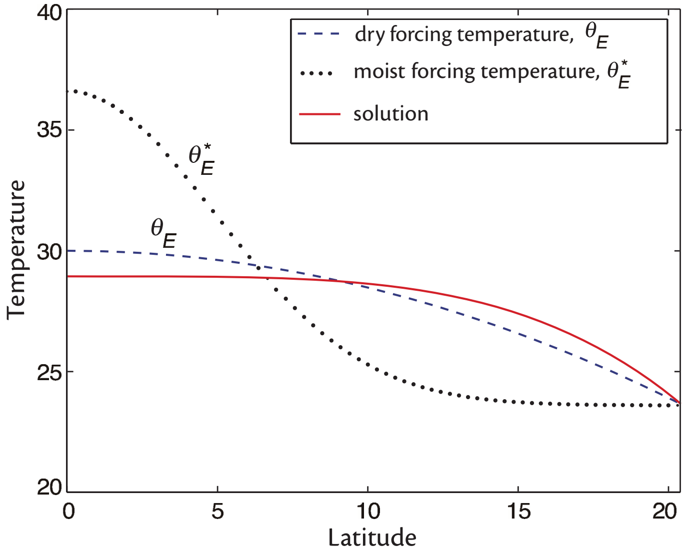

Fig. 14.8 Schematic of the effects of moisture on a model of the Hadley Cell. The temperature of the solution (solid line) is the same as that of a dry model, because this is determined from the angular-momentum-conserving wind. The heating distribution (as parameterized by a forcing temperature) is peaked near the equator in the moist case, leading to a more vigorous overturning circulation.

14.2.8 The Radiative Equilibrium Solution

Instead of a solution given by (14.12), could the temperature not simply be in radiative equilibrium everywhere? Such a state would have no meridional overturning circulation and the zonal velocity would be determined by thermal wind balance; that is,

$$v = 0, \qquad \theta = \theta_E, \qquad f\frac{u}{H} = -g\frac{\partial}{\partial y}\left(\frac{\theta_E}{\theta_0}\right). \tag{14.33}$$

To answer this question we consider the steady zonally symmetric zonal angular momentum equation with viscosity; that is, the zonally-averaged, viscous, steady, shallow atmosphere version of (2.68), namely

$$\frac{1}{a\cos\vartheta}\frac{\partial}{\partial\vartheta}(vm\cos\vartheta) + \frac{\partial(mw)}{\partial z} = \frac{\nu}{a\cos\vartheta}\frac{\partial}{\partial\vartheta}\left(\cos^2\vartheta\frac{\partial}{\partial\vartheta}\frac{u}{\cos\vartheta}\right) + \nu a\cos\vartheta\frac{\partial^2 u}{\partial z^2}, \tag{14.34}$$

where the variables vary only in the ϑ–z plane. The viscous term on the right-hand side arises from the expansion in spherical coordinates of the Laplacian. Note that it is angular *velocity*, not the angular momentum, that is diffused, because there is no diffusion of the angular momentum due to the Earth's rotation. However, to a very good approximation, the viscous term will be dominated by vertical derivatives and we may then write (14.34) as

$$\nabla_x \cdot (\boldsymbol{v}m) = \nu\frac{\partial^2 m}{\partial z^2}. \tag{14.35}$$

where $\nabla_x\cdot$ is the divergence in the meridional plane. The right-hand side now has a diffusive form, and in Section 13.5.1 we showed that variables obeying equations like this can have no extrema within the fluid. Thus, there can be no maximum or minimum of angular momentum in the interior of the fluid, a result known as Hide's theorem.[6] In effect, diffusion always acts to smooth away an isolated extremum, and this cannot be counterbalanced by advection. The result also implies that there can be no interior extrema in a statistically steady state if there is any zonally asymmetric eddy motion that transports angular momentum downgradient.

If the viscosity were so large that the viscous term was dominant in (14.34), and so with the horizontal term now important, then the fluid would evolve toward a state of solid body rotation, this being the fluid state with no internal stresses. In that case, there would be a maximum of angular momentum at the equator — a state of 'super-rotation'. (Related mechanisms have been proposed for the maintenance of super-rotation on Venus.[7])

Returning now to the question posed at the head of this section, suppose that the radiative equilibrium solution does hold. Then a radiative equilibrium temperature decreasing away from the equator more rapidly than the angular-momentum-conserving solution θ_M implies, using thermal wind balance, a maximum of m at the equator and above the surface, in violation of the no-extremum principle. Of course, we have derived the angular-momentum-conserving solution in the inviscid limit, in which the no-extrema principle does not apply. But any small viscosity will make the radiative equilibrium solution completely invalid, but potentially have only a small effect on the angular-momentum-conserving solution; that is, in the *limit* of small viscosity the angular-momentum-conserving solution can conceivably hold approximately, at least in the absence of boundary layers, whereas the radiative equilibrium solution cannot.

However, if the radiative equilibrium temperature varies more slowly with latitude than the temperature corresponding to the angular momentum conserving solution then a radiative equilibrium solution *can* obtain, without violating Hide's theorem. In particular, this is the case if $\theta_E \propto P_4(\sin\vartheta)$, where P_4 is the fourth Legendre polynomial, and so the possibility exists of two equilibrium solutions for the same forcing; however, P_4 is an unrealistically flat radiative equilibrium temperature for the Earth's atmosphere.

14.3 A SHALLOW WATER MODEL OF THE HADLEY CELL

Although expressed in the notation of the primitive equations, the model described above takes no account of any vertical structure in its stratification and is, *de facto,* a shallow water model. (We discuss how the primitive equations reduce to the shallow water equations in Sections 3.4 and 18.7.) Furthermore, the geometric aspects of sphericity play no essential role. Thus, we may transparently express the essence of the model by:

(i) explicitly using the shallow water equations instead of the stratified equations;

(ii) using the equatorial β-plane, with $f = f_0 + \beta y$ and $f_0 = 0$.

Let us therefore, if only as an exercise, construct a reduced-gravity model with an active upper layer overlying a stationary lower layer.

14.3.1 Momentum Balance

The inviscid zonal momentum equation of the upper layer is

$$\frac{\mathrm{D}u}{\mathrm{D}t} - \beta y v = 0 \tag{14.36}$$

or

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(u - \frac{\beta y^2}{2}\right) = 0, \tag{14.37}$$

which is the β-plane analogue of the conservation of axial angular momentum. (In this section, all variables are zonally averaged, but we omit any notation denoting that.) From (14.37) we obtain the zonal wind as a function of latitude,

$$u = \frac{1}{2}\beta y^2 + A, \tag{14.38}$$

where A is a constant, which is zero if $u = 0$ at the equator, $y = 0$. The flow given by (14.38) is then analogous to the angular momentum conserving flow in the spherical model, (14.7). Because the lower layer is stationary, the analogue of thermal wind balance in the stratified model is just geostrophic balance, namely

$$fu = -g' \frac{\partial h}{\partial y}, \tag{14.39}$$

where h is the thickness of the active upper layer. Using (14.39) and $f = \beta y$ we obtain

$$g' \frac{\partial h}{\partial y} = -\frac{1}{2}\beta^2 y^3, \qquad \text{whence} \qquad h = -\frac{1}{8g'}\beta^2 y^4 + h(0), \tag{14.40a,b}$$

where $h(0)$ is the value of h at $y = 0$.

14.3.2 Thermodynamic Balance

The thermodynamic equation in the shallow water equations is just the mass conservation equation, which we write as

$$\frac{\mathrm{D}h}{\mathrm{D}t} = -\frac{1}{\tau}(h - h^*), \tag{14.41}$$

where the right-hand side represents heating — h^* is the field to which the height relaxes on a time scale τ. For illustrative purposes we will choose

$$h^* = h_0(1 - \alpha|y|). \tag{14.42}$$

(If we chose the more realistic quadratic dependence on y, the model would be more similar to that of the previous section.) To be in thermodynamic equilibrium we require that the right-hand side integrates to zero over the Hadley Cell; that is

$$\int_0^Y (h - h^*)\,\mathrm{d}y = 0, \tag{14.43}$$

where Y is the latitude of the poleward extent of the Hadley Cell, thus far unknown. Poleward of this, the height field is simply in equilibrium with the forcing — there is no meridional motion and $h = h^*$. Since the height field must be continuous, we require that

$$h(Y) = h^*(Y). \tag{14.44}$$

The two constraints (14.43) and (14.44) provide values of the unknowns $h(0)$ and Y, and give

$$Y = \left(\frac{5h_0\alpha g'}{\beta^2}\right)^{1/3}, \tag{14.45}$$

which is analogous to (14.19), as well as an expression for $h(0)$ that we leave as a problem for the reader. The qualitative dependence on the parameters is similar to that of the full model, although the latitudinal extent of the Hadley Cell is proportional to the cube root of the meridional thickness gradient α.

14.4 † ASYMMETRY AROUND THE EQUATOR

The Sun is overhead at the equator but two days out of the year, and in this section we investigate the effects that asymmetric heating has on the Hadley circulation. Observations indicate that except for the brief periods around the equinoxes, the circulation is dominated by a single cell with rising motion centred in the summer hemisphere, but extending well into the winter hemisphere. That is, as seen in Fig. 14.3, the 'winter cell' is broader and stronger than the 'summer cell', and it behoves us to try to explain this. We will stay in the framework of the inviscid angular-momentum model

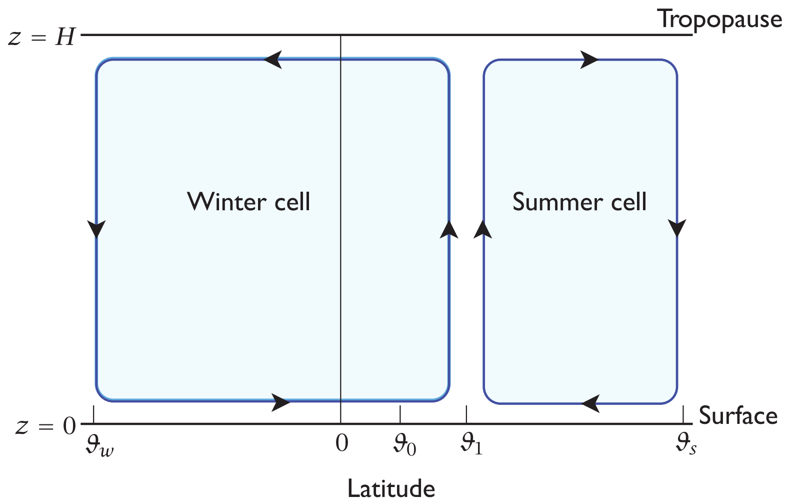

Fig. 14.9 A Hadley circulation model in which the heating is centred off the equator, at a latitude ϑ_0. The lower level convergence occurs at a latitude ϑ_1 that is not in general equal to ϑ_0. The resulting winter Hadley Cell is stronger and wider than the summer cell.

of Section 14.2, changing only the forcing field to represent the asymmetry and being a little more attentive to the details of spherical geometry.[8]

To represent an asymmetric heating we may choose a radiative equilibrium temperature of the form

$$\begin{aligned}\frac{\theta_E(\vartheta, z)}{\theta_0} &= 1 - \frac{2}{3}\Delta_H P_2(\sin\vartheta - \sin\vartheta_0) + \Delta_V\left(\frac{z}{H} - \frac{1}{2}\right)\\ &= 1 + \frac{\Delta_H}{3}\left[1 - 3(\sin\vartheta - \sin\vartheta_0)^2\right] + \Delta_V\left(\frac{z}{H} - \frac{1}{2}\right).\end{aligned} \tag{14.46}$$

This is similar to (14.14), but now the forcing temperature falls monotonically from a specified latitude ϑ_0. If $\vartheta_0 = 0$ the model is identical to the earlier one, but if not we envision a circulation as qualitatively sketched in Fig. 14.9, with rising motion off the equator at some latitude ϑ_1, extending into the winter hemisphere to a latitude ϑ_w, and into the summer hemisphere to ϑ_s. We will discover that, in general, $\vartheta_1 \neq \vartheta_0$ except when $\vartheta_0 = 0$. Following our procedure we used in the symmetric case as closely as possible, we then make the following assumptions:

(i) The flow is quasi-steady. That is, at any time of year the flow adjusts to a steady circulation on a time scale more rapid than that on which the solar zenith angle appreciably changes.

(ii) The flows in the upper branches conserve angular momentum, m. Further assuming that $u = 0$ at $\vartheta = \vartheta_1$ so that $m = \Omega a^2 \cos^2\vartheta_1$ we obtain

$$u(\vartheta) = \frac{\Omega a(\cos^2\vartheta_1 - \cos^2\vartheta)}{\cos\vartheta}. \tag{14.47}$$

Thus, we expect to see westward (negative) winds aloft at the equator. In the lower branches the zonal flow is assumed to be approximately zero, i.e., $u(0) \approx 0$.

(iii) The flow satisfies hydrostatic and gradient wind balance. The meridional momentum equation is then

$$fu + \frac{u^2\tan\vartheta}{a} = -\frac{1}{a}\frac{\partial\phi}{\partial\vartheta}, \tag{14.48}$$

and because the flow crosses the equator we cannot neglect the second term on the left-hand side. Combining this with hydrostatic balance ($\partial\phi/\partial z = g\theta/\theta_0$) leads to a generalized thermal wind balance, which may be written as

$$m\frac{\partial m}{\partial z} = -\frac{ga^2\cos^2\vartheta}{2\theta_0\tan\vartheta}\frac{\partial\theta}{\partial\vartheta}. \tag{14.49}$$

If the undifferentiated m is approximated by $\Omega a^2\cos^2\vartheta$, this reduces to conventional thermal wind balance, (14.10).

(iv) Potential temperature in each cell is conserved when integrated over the extent of the cell. Thus,

$$\int_{\vartheta_1}^{\vartheta_s}(\theta-\theta_E)\cos\vartheta\,\mathrm{d}\vartheta = 0, \qquad \int_{\vartheta_1}^{\vartheta_w}(\theta-\theta_E)\cos\vartheta\,\mathrm{d}\vartheta = 0, \tag{14.50}$$

for the summer and winter cells, respectively, where θ is the vertically averaged potential temperature.

(v) Potential temperature is continuous at the edge of each cell, so that

$$\theta(\vartheta_s) = \theta_E(\vartheta_s), \qquad \theta(\vartheta_w) = \theta_E(\vartheta_w), \tag{14.51}$$

and is also continuous at ϑ_1. This last condition must be explicitly imposed in the asymmetric model, whereas in the symmetric model it holds by symmetry. Now, recall from the symmetric model that the value of the temperature at the equator was determined by the integral constraint (14.17) and the continuity constraint (14.18). We have analogues of these in each hemisphere, namely (14.50) and (14.51), and thus, if ϑ_1 is set equal to ϑ_0 we cannot expect that they each would give the same temperature at ϑ_0. Thus, ϑ_1 must be a free parameter to be determined.

Given these assumptions, the solution may be calculated. Using thermal wind balance, (14.49), with $m(H) = \Omega a^2\cos^2\vartheta_1$ and $m(0) = \Omega a^2\cos^2\vartheta$ we find

$$-\frac{1}{\theta_0}\frac{\partial\theta}{\partial\vartheta} = \frac{\Omega^2a^2}{gH}\left(\frac{\sin\vartheta}{\cos^3\vartheta}\cos^4\vartheta_1 - \sin\vartheta\cos\vartheta\right), \tag{14.52}$$

which integrates to

$$\theta(\vartheta) - \theta(\vartheta_1) = -\frac{\theta_0\Omega^2a^2}{2gH}\frac{(\sin^2\vartheta - \sin^2\vartheta_1)^2}{\cos^2\vartheta}. \tag{14.53}$$

The value of ϑ_1, and the value of $\theta(\vartheta_1)$, are determined by the constraints (14.50) and (14.51). It is not in general possible to obtain a solution analytically, but one may be found numerically by an iterative procedure and one such is illustrated in Fig. 14.10. The zonal wind of the solution is always symmetric around the equator, because it is determined solely by angular momentum conservation. The temperature is therefore also symmetric, as (14.53) explicitly shows. However, the width of the solution in each hemisphere will, in general, be different.

Furthermore, because the strength of the circulation increases with difference between the temperature of the solution and the radiative equilibrium temperature, the circulation in the winter hemisphere will also be much stronger than that in the summer, a prediction that is consistent with the observations (see Fig. 14.3). More detailed calculations show that, because the strength of the model Hadley Cell increases nonlinearly with ϑ_0, the time-average strength of the Hadley Cell with seasonal forcing is stronger that that produced by annually averaged forcing. However, this does not appear to be a feature of either the observations or more complete numerical simulations, suggesting that an angular-momentum-conserving model has some deficiencies.[9]

The lack of consideration of zonal asymmetries and the lack of angular momentum conservation because of the effects of baroclinic eddies are issues that are shared with the steady model with

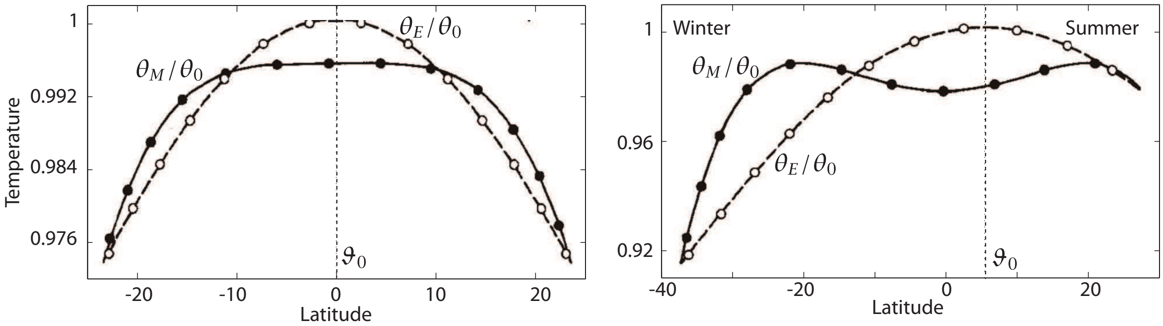

Fig. 14.10 Solutions of the Hadley Cell model with heating centred at the equator ($\vartheta_0 = 0°$, left) and off the equator ($\vartheta_0 = +6°$ N, right), with $\Delta_H = 1/6$. The dashed line is the radiative equilibrium temperature and the solid line is the angular-momentum-conserving solution. In the right-hand panel, $\vartheta_1 \approx +18°$, and the circulation is dominated by the cell extending from $+18°$ to $-36°$.[8]

hemispheric symmetry. A problem that is unique to the asymmetric model is the quasi-steady assumption, given the presence of a temporally progressing seasonal cycle. Because the latitude of the upward branch of the Hadley Cell varies with season, the value of the angular momentum entering the system also varies with time, and so a homogenized value of angular momentum is hard to achieve. Nonetheless, the overall picture that the model paints, with its qualitative explanation of the strengthened and extended winter Hadley Cell, is very useful, even if quantitatively flawed.

14.5 † EDDY EFFECTS ON THE HADLEY CELL

So far, we have ignored the effects of baroclinic eddies on the Hadley circulation although we have no reason to believe that their effects will be negligible. In fact, as the upper-level flow moves poleward the shear of the zonal wind increases, as described above, and at some point the flow will become baroclinically unstable. We first describe a simple model of this, before considering eddy fluxes more generally.[10]

14.5.1 A Hadley Cell Limited by Baroclinic Instability

Suppose that the flow moving poleward conserves its angular momentum, and for simplicity consider flow on a beta-plane. The flow is given by (14.7), which in the small angle approximation implies a shear, Λ_M, of

$$\Lambda_M \approx \frac{\Omega a \vartheta^2}{H} = \frac{\beta y^2}{2H}, \tag{14.54}$$

where H is the height of the outflow and the last two expressions hold in the small angle approximation. Now, in a quasi-geostrophic two-level model, the flow becomes unstable when the shear between upper and lower levels reaches a critical value, Λ_C, given by

$$\Lambda_C \equiv \frac{U_1 - U_2}{H/2} = \frac{1}{2H}\beta L_d^2, \tag{14.55}$$

where $L_d = NH/f$ is the baroclinic deformation radius, and on the sphere $\beta = 2\Omega\cos\phi/a$. Both β factor and the f hiding in L_d make Λ_C grow towards the equator. Equating (14.54) and (14.55) suggests that the angular-momentum conserving flow will become unstable at a latitude ϑ_C given by, in the small angle approximation,

$$\boxed{\vartheta_C \approx \left(\frac{NH}{2\Omega a}\right)^{1/2}.} \tag{14.56}$$

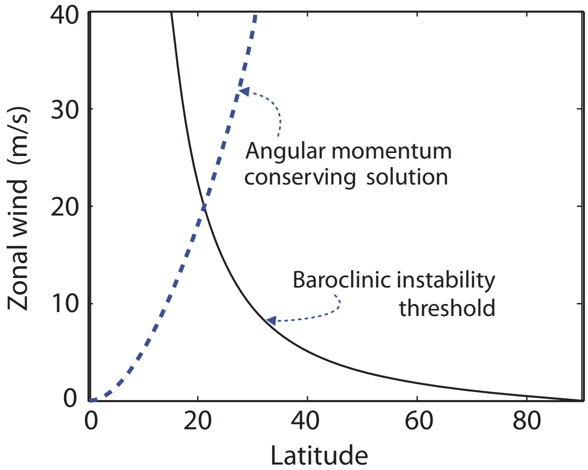

Fig. 14.11 Upper tropospheric zonal winds. The dashed curve shows the angular-momentum conserving wind, with $u = 0$ at the equator. The solid curve shows the threshold for baroclinic instability of the upper-level flow using a two-layer quasi-geostrophic calculation, (14.55), with f varying with latitude in the deformation radius.

The value of ϑ_C above should not be taken literally — the real atmosphere is not a two-level quasi-geostrophic model! But it does capture the essential truth that the angular momentum conserving solution will become baroclinically unstable at some latitude, as sketched in Fig. 14.11. It is a quantitative issue as to whether the Hadley flow becomes strongly unstable before it reaches its natural poleward extent. However, even if baroclinic instability itself is weak over much of the tropics, baroclinic instability further poleward will have an effect and lead to the non-conservation of angular momentum, as we now discuss.

14.5.2 Diagnostic Considerations

The zonally-averaged zonal momentum equation, (14.4), may be written as an equation for angular momentum, $\overline{m}$. Referring back to Section 2.2 if needs be, the equation may be written as

$$\begin{aligned}\frac{\partial \overline{m}}{\partial t} + \frac{1}{\cos\vartheta}\frac{\partial}{\partial y}(\overline{v}\,\overline{m}\cos\vartheta) + \frac{\partial}{\partial z}(\overline{w}\,\overline{m}) &= -\frac{1}{\cos\vartheta}\frac{\partial}{\partial y}(\overline{m'v'}\cos\vartheta) - \frac{\partial}{\partial z}(\overline{m'w'}) \\ &= -\frac{1}{\cos\vartheta}\frac{\partial}{\partial y}(\overline{u'v'}\,a\cos^2\vartheta) - \frac{\partial}{\partial z}(\overline{u'w'}\,a\cos\vartheta),\end{aligned} \tag{14.57}$$

where $\overline{m} = (\overline{u}+\Omega a\cos\vartheta)a\cos\vartheta$, $m' = u' a\cos\vartheta$, $y = a\vartheta$, and the vertical and meridional velocities are related by the mass continuity relation

$$\frac{1}{\cos\vartheta}\frac{\partial}{\partial y}(\overline{v}\cos\vartheta) + \frac{\partial\overline{w}}{\partial z} = 0. \tag{14.58}$$

In the angular-momentum-conserving model the eddy fluxes were neglected and (14.57) was approximated by the simple expression $\partial\overline{m}/\partial\vartheta = 0$, and by construction the Rossby number is $\mathcal{O}(1)$, because $\zeta = -f$.

The observed eddy heat and momentum fluxes are shown in Fig. 14.12. The eddy momentum flux is generally poleward, converging in the region of the mid-latitude surface westerlies. Its magnitude, and more particularly its meridional gradient, is as large or larger than the momentum flux associated with the mean flow. Neglecting vertical advection and vertical eddy fluxes, and using (14.58), (14.57) may be written as

$$\frac{\partial\overline{m}}{\partial t} + \overline{v}\frac{\partial\overline{m}}{\partial y} = -\frac{1}{\cos\vartheta}\frac{\partial}{\partial y}(\overline{u'v'}\,a\cos^2\vartheta). \tag{14.59}$$

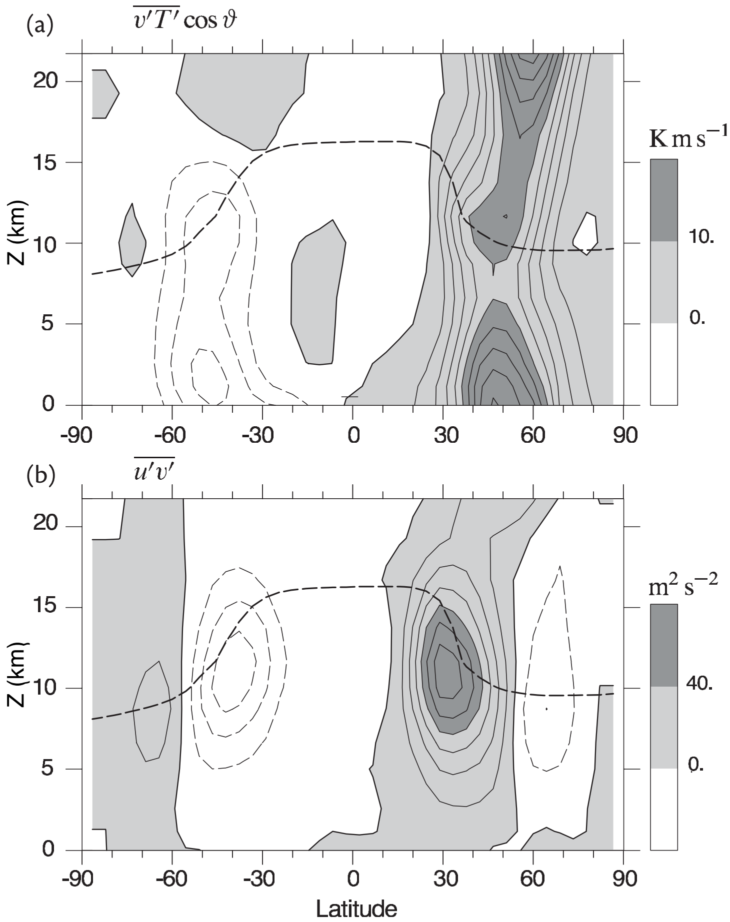

Fig. 14.12 (a) The average meridional eddy heat flux and (b) the eddy momentum flux in the northern hemisphere winter (DJF). The ordinate is log-pressure, with scale height $H = 7.5$ km. Positive fluxes are shaded, and the dashed line marks the thermal tropopause.

The eddy heat flux (contour interval $2\,\mathrm{K\,m\,s^{-1}}$) is largely poleward and downgradient in both hemispheres. The eddy momentum flux (contour interval $10\,\mathrm{m^2\,s^{-2}}$) is upgradient and *converges* in mid-latitudes in the region of the mean jet, leading to eastward surface winds.[11]

The quantity $\overline{u'v'}$ increases to a maximum at about 30°, so the right-hand side is negative in the tropics and lower subtropics. Thus, if $\overline{v} > 0$ (as in the upper branch of the Northern Hemisphere Hadley Cell) and the flow is steady, the observed eddy fluxes are such as to cause the angular momentum of the zonal flow to *decrease* as it moves poleward, and the zonal velocity is lower than it would be in the absence of eddies. (In the Southern Hemisphere the signs of v and the eddy momentum flux are reversed, but the dynamics are equivalent.)

The eddy flux of heat will also affect the Hadley Cell, although in a different fashion. We see from Fig. 14.12 that the eddy flux of temperature is predominantly poleward, and therefore that eddies export heat from the subtropics to higher latitudes. Now, the zonally-averaged thermodynamic equation may be written

$$\frac{\partial \overline{b}}{\partial t} + \frac{1}{\cos\vartheta}\frac{\partial}{\partial y}(\overline{v}\overline{b}\cos\vartheta) + \frac{\partial}{\partial z}(\overline{w}\overline{b}) = -\frac{1}{\cos\vartheta}\frac{\partial}{\partial y}(\overline{v'b'}\cos\vartheta) - \frac{\partial}{\partial z}(\overline{w'b'}) + Q_b, \qquad (14.60)$$

where Q_b represents the heating. After vertical averaging, the vertical advection terms vanish and the resulting equation is the thermodynamic equation implicitly used in the angular-momentum-conserving model, with the addition of the meridional eddy flux on the right-hand side. A diverging eddy heat flux in the subtropics (as in Fig. 14.12) is evidently equivalent to increasing the meridional gradient of the radiative equilibrium temperature, and therefore will increase the intensity of the overturning circulation.

14.5.3 An Idealized Eddy-driven Model

Consider now the extreme case of an 'eddy-driven' Hadley Cell. (The driving for the Hadley Cell, and the atmospheric circulation in general, ultimately comes from the differential heating between equator and pole. Recognizing this, 'eddy driving' is a convenient way to refer to the role of eddies in producing a zonally-averaged circulation. See also endnote 2 on page 858.) The model is over-simple, but revealing. Neglecting vertical derivatives the zonally-averaged zonal momentum equation (14.4) may be written

$$\frac{\partial \overline{u}}{\partial t} - (f + \overline{\zeta})\overline{v} = -\frac{\partial}{\partial y}\overline{u'v'}. \tag{14.61}$$

using Cartesian geometry for simplicity. If the Rossby number is sufficiently low this becomes

$$\frac{\partial \overline{u}}{\partial t} - f\overline{v} = M, \tag{14.62}$$

where $M = -\partial_y(\overline{u'v'})$, . This approximation is not quantitatively accurate but it will highlight the role of the eddies. Note the contrast between this model and the angular-momentum-conserving model. In the latter we assumed $f + \overline{\zeta} \approx 0$, and $Ro = \mathcal{O}(1)$; now we are neglecting $\overline{\zeta}$ and assuming the Rossby number is small. At a similar level of approximation let us write the thermodynamic equation, (14.60), as

$$\frac{\partial \overline{b}}{\partial t} + N^2 w = J, \tag{14.63}$$

where $J = Q_b - \partial_y(\overline{v'b'})$ represents the diabatic terms and eddy forcing. We are assuming, as in quasi-geostrophic theory, that the mean stratification, N^2 is fixed, and now $\overline{b}$ represents only the (zonally averaged) deviations from this. The mass continuity equation allows us to define a meridional streamfunction Ψ; that is

$$\frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0 \qquad \text{allows} \qquad \overline{w} = \frac{\partial \Psi}{\partial y}, \quad \overline{v} = -\frac{\partial \Psi}{\partial z}. \tag{14.64a,b}$$

We may then use the thermal wind relation, $f\partial\overline{u}/\partial z = -\partial\overline{b}/\partial y$, to eliminate time derivatives in (14.62) and (14.63), giving[12]

$$\boxed{f^2\frac{\partial^2 \Psi}{\partial z^2} + N^2\frac{\partial^2 \Psi}{\partial y^2} = f\frac{\partial M}{\partial z} + \frac{\partial J}{\partial y}.} \tag{14.65}$$

This is a linear equation for the overturning streamfunction, one that holds even if the flow is not in a steady state, and a positive value of Ψ, in the Northern Hemisphere, corresponds to rising at the equator. The equation is equally (or in fact more) valid in mid-latitudes as in the tropics. We see that the overturning circulation is forced by eddy fluxes of heat and momentum, as well as heating and other terms that might appear on the right-hand sides of (14.62) and (14.63). If we rescale the vertical coordinate by the Prandtl ratio (i.e., let $z = z' f/N$) then (14.65) is a Poisson equation for the streamfunction. A few other germane points are as follows:

- The horizontal gradient of the thermodynamic forcing partially drives the circulation, and both the heating term and the horizontal eddy flux divergence act in the same sense. A thermodynamically forced overturning circulation, with warm fluid rising and cold fluid sinking, is called a 'direct cell'.

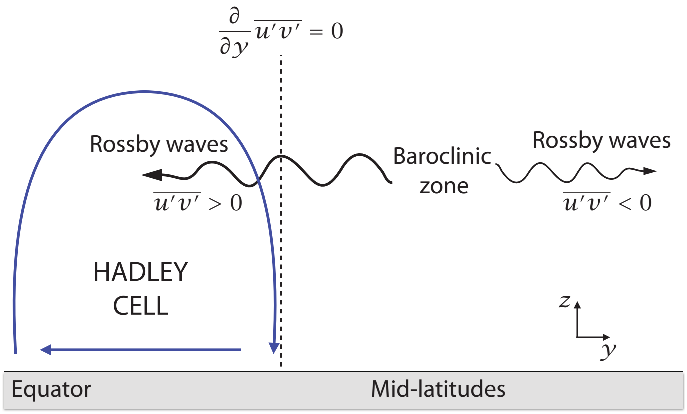

Fig. 14.13 Sketch of how eddy fluxes can affect the Hadley Cell even when the baroclinic zone is centred well poleward of the Cell.

Rossby waves are generated by baroclinic instability at mid-latitudes. Some propagate equatorward, and deposit westward momentum, $\partial(\overline{u'v'})/\partial y > 0$ inside the Hadley Cell. At some latitude the Rossby wave momentum flux is neither convergent nor divergent, $\partial(\overline{u'v'})/\partial y = 0$, corresponding to the edge of the Hadley cell.

- The vertical gradient of the horizontal eddy momentum divergence partially drives the circulation, and because $\partial_y(\overline{u'v'}) > 0$ over the tropics and subtropics (for the Northern Hemisphere, see Fig. 14.12) these fluxes intensify the circulation, weakening the zonal flow aloft terms and strengthening the overturning circulation. The Coriolis term fv is balanced by the eddy momentum flux convergence.
- If N is small, then the circulation becomes stronger if the other terms remain the same, because the air can circulate without transporting any heat.
- In winter, the increased strength of eddy momentum and buoyancy fluxes drives a stronger Hadley Cell. This constitutes a different mechanism from that given in Section 14.4 for the increased strength of the winter cell.

14.6 NON-LOCAL EDDY EFFECTS AND NUMERICAL RESULTS

14.6.1 † A Non-local Model

We saw above that eddy fluxes will tend to strengthen the Hadley Cell and weaken the zonal winds. These eddy fluxes can be important even if the main zone of baroclinic instability is well poleward of the Hadley Cell termination, because Rossby waves can propagate equatorward from the baroclinic zone into the subtropics (as sketched in Fig. 14.13). This propagation will be discussed more in Sections 15.1 and 16.2, but suffice it to say here that equatorward propagating Rossby waves produce a poleward momentum flux, $\overline{u'v'} > 0$ (we use Northern Hemisphere and Cartesian notation). Even if the main baroclinic activity occurs around, say, 45°, then the amplitude of the poleward eddy flux may reach its maximum value some distance equatorward of that latitude if the baroclinic zone itself extends further equatorward, and observations (Fig. 14.12) show that the flux is a maximum at about 30° (varying with season), diminishing equatorward of that, so that in the Hadley Cell $\partial_y(\overline{u'v'}) > 0$. The edge of the Hadley Cell is coincident with the latitude at which the poleward eddy flux divergence is zero, since the steady-state momentum equation is approximately

$$-(f + \overline{\zeta})\overline{v} = -\frac{\partial}{\partial y}\overline{u'v'}. \tag{14.66}$$

At the edge of the Hadley Cell we have $\overline{v} = 0$ and thus $\partial_y\left(\overline{u'v'}\right) = 0$. The model is a little simplistic because other terms in the momentum equation may then become important, but it is nevertheless instructive.

The latitude at which the right-hand side of (14.66) becomes small is not necessarily the same as the one where the poleward flow in the Hadley Cell becomes baroclinically unstable, although the two may be similar in practice. In fact, determining the latitudinal distribution of eddy fluxes is a difficult problem in wave–mean-flow interaction, since the eddy fluxes affect and are affected by the mean flow. As well as the generation of eddy fluxes by baroclinic instability, the absorption or dissipation of Rossby waves is an important factor, since in their absence the momentum flux would be approximately constant equatorward of the baroclinic zone, and the right-hand side of (14.66) would be zero over a range of latitudes. The absorption of Rossby waves is enhanced near critical latitudes (where the wave speed equals the mean fluid speed) and the wave activity and the momentum flux diminish equatorward of that, but the critical latitude itself need not correspond to the maximum of the eddy momentum fluxes. In the real world the critical latitude is not sharply defined and, although it is often equatorward of the edge of the Hadley Cell rather than coincident with it, Rossby waves may begin to dissipate well before reaching it, and so poleward of it.

The details of the dynamics determining the Hadley Cell edge are plainly rather complex, although a qualitative picture is simply described. The eddy momentum flux convergence, $\partial_y(\overline{u'v'})$, is negative in mid-latitudes and positive in low-latitudes, with a zero crossing ($\partial_y(\overline{u'v'}) = 0$) near the edge of the Hadley Cell, as in Fig. 14.13. Consistent with this, observations (e.g., Fig. 14.12) show that the latitude of the eddy momentum flux maximum coincides with a rapid change in tropopause height, which itself is generally coincident with the edge of the Hadley Cell.

Evidently, the effects of baroclinic instability, and not just the effects of a local instability, can greatly influence the Hadley Cell, and to round out this section we illustrate that fact with some numerical simulations.

14.6.2 Numerical Solutions

Some illustrative results from two idealized numerical experiments with a GCM are shown in Figs. 14.14 and 14.15. The GCM has no explicit representation of moisture, except that the lapse rate is adjusted to a value close to the moist adiabatic lapse rate if it exceeds that value. In one experiment the model is constrained to produce an axisymmetric solution (left-hand panels of the figures), and the zonal wind produced by the model in the Hadley Cell outflow is fairly close to being angular-momentum-conserving. In a three-dimensional version of the model, in which baroclinic eddies are allowed to form, the zonal wind is significantly reduced from its angular-momentum-conserving value, and correspondingly the overturning circulation is much stronger (right-hand panels). Indeed, the strength of the Hadley Cell increases roughly linearly with the strength of the eddies in a sequence of numerical integrations similar to those shown, as suggested by (14.65). Qualitatively similar results are found in a model with no convective parameterization. In this case, the lapse rate is closer to neutral, N^2 is small, and the overturning circulation is generally stronger, as also expected from (14.65). The results generally indicate very strong eddy effects on the strength of the Hadley Cell, and the value of the zonal wind within it, although a dry model may overemphasize the importance of eddy effects, because the circulation in a zonally symmetric dry model is weaker than a similar moist model, as discussed in Section 14.2.7.

14.6.3 Final Remarks

Is the real Hadley circulation 'eddy-driven', as in Sections 14.5 and above, or is it a largely zonally symmetric structure constrained by angular momentum conservation, as in Section 14.2? And how does this balance vary with season?

Observations of the overturning flow in summer and winter provide a guide. Figure 14.16 shows the thickness-weighted transport overturning circulation in isentropic coordinates, and (as discussed in Chapter 10) this circulation includes both the Eulerian mean transport and the transport due to eddies. The winter cell (the cross-equatorial cell with the upward branch in the summer hemisphere) is strong and self-contained, with considerable recirculation most of which comes from its zonally symmetric component. The winter cell is quite distinct from the mid-latitude circulation, suggesting the dominance of axisymmetric dynamics, for if it were solely a response to eddy heat and momentum fluxes one might expect it to join more smoothly with the mid-latitude

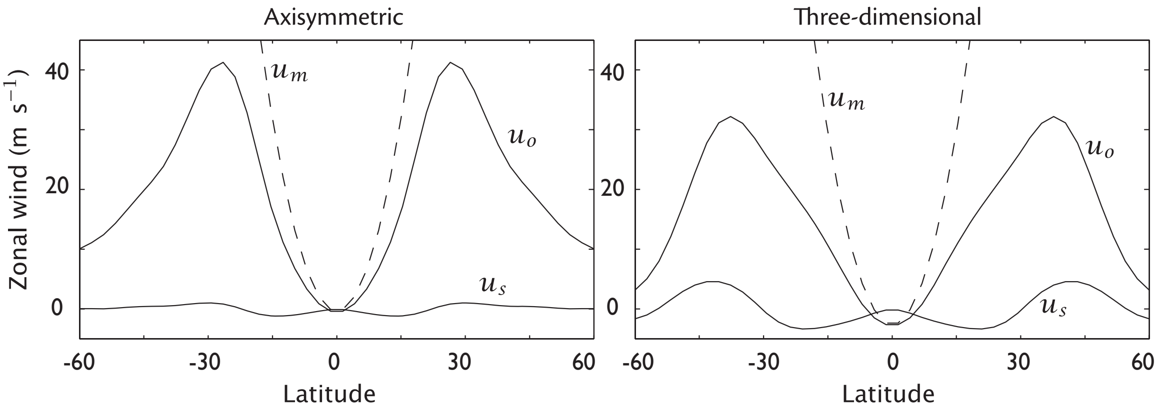

Fig. 14.14 The zonal wind in two numerical simulations. The right panel is from an idealized dry, three-dimensional atmospheric GCM, and the left panel is an axisymmetric version of the same model. Plotted are the zonal wind at the level of the Hadley Cell outflow, u_o; the surface wind, u_s; and the angular-momentum-conserving value, u_m.[13]

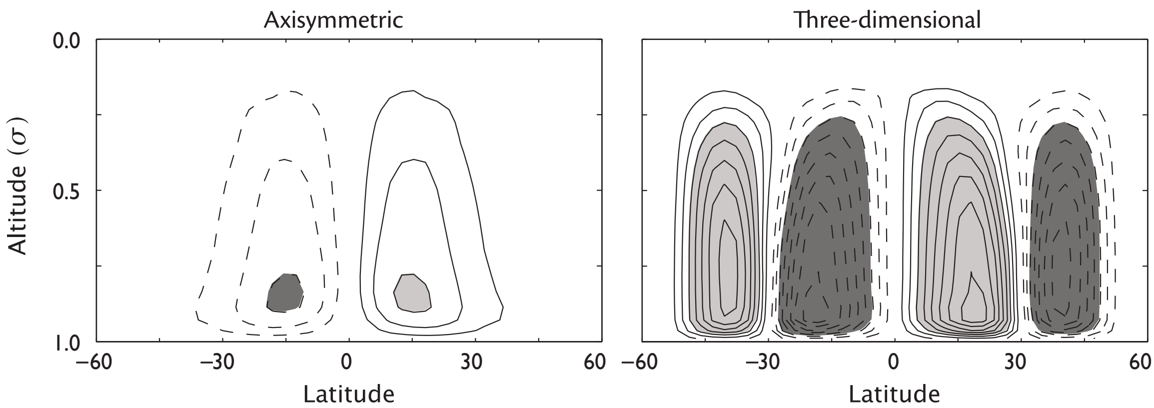

Fig. 14.15 As for Fig. 14.14, but now showing the streamfunction of the overturning circulation. 'Altitude' is $\sigma = p/p_s$, where p_s is surface pressure, and contour interval is 5 Sv (i.e., 5×10^9 kg s^{-1}). The effects of eddies may be exaggerated because the model is dry.

Ferrel Cell. The axisymmetric winter Hadley Cell is naturally stronger than its summer counterpart, even in a dry atmosphere, and the further effects of condensation and the concomitant concentration of the thermodynamic source may strengthen it further, giving the axisymmetric circulation a dominant role.

In summer, in contrast, there is virtually no recirculation within the Hadley Cell and it does not appear as a self-contained structure, suggestive of baroclinic eddy effects and/or a strong mid-latitude influence. And even without baroclinic eddies, zonally asymmetric circulations are important, for the Hadley Cell over India and South East Asia is intimately linked with monsoonal circulations. But tying the monsoon circulation into a theory of the Hadley Cell, and in particular into the transition from winter to summer dynamics, is, alas, a task for another day.

14.7 THE FERREL CELL

In this section we give a descriptive introduction to the Ferrel Cell, taking the eddy fluxes of heat and momentum to be given and viewing the circulation from a zonally averaged and Eulerian perspective. We investigate the associated dynamics in the next chapter.

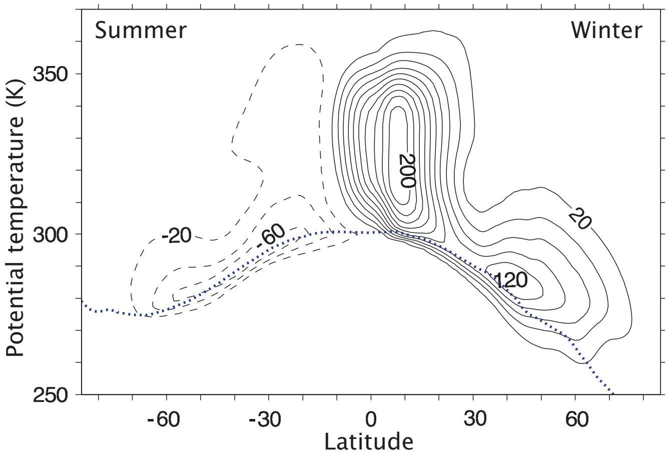

Fig. 14.16 The observed mass transport streamfunction in isentropic coordinates in northern hemisphere winter (DJF). The dotted line is the median surface temperature.

The return flow is nearly all in a layer near the surface, much of it at a lower temperature than the median surface temperature. Note the more vigorous circulation in the winter hemisphere.[14]

The Ferrel Cell is an indirect meridional overturning circulation in mid-latitudes (see Fig. 14.3) that is apparent in the zonally-averaged v and w fields, or the meridional overturning circulation defined by (14.3) or (14.64b). It is 'indirect' because cool air apparently rises in high latitudes, moves equatorward and sinks in the subtropics. Why should such a circulation exist? The answer, in short, is that it is there to balance the eddy momentum convergence of the mid-latitude eddies and it is effectively driven by those eddies. To see this, consider the zonally-averaged zonal momentum equation in mid-latitudes; at low Rossby number, and for steady flow this is just

$$-f\overline{v} = -\frac{1}{\cos^2\vartheta}\frac{\partial}{\partial\vartheta}(\cos^2\vartheta\,\overline{u'v'}) + \frac{1}{\rho}\frac{\partial\tau}{\partial z}. \tag{14.67}$$

This equation is a steady version of (14.62) with the addition of a frictional term $\partial\tau/\partial z$ on the right-hand side. At the surface we may approximate the stress by a drag, $\tau = r\overline{u}_s$, where r is a constant, with the stress falling away with height so that it is important only in the lowest kilometre or so of the atmosphere, in the atmospheric Ekman layer. Above this layer, the eddy momentum flux convergence is balanced by the Coriolis force on the meridional flow. In mid-latitudes (from about 30° to 70°) the eddy momentum flux divergence is negative in both hemispheres (Fig. 14.12) and therefore, from (14.67), the averaged meridional flow must be equatorward, as illustrated schematically in Fig. 14.17.

The flow cannot be equatorward everywhere, simply by mass continuity, and the return flow occurs largely in the Ekman layer, of depth d say. Here the eddy balance is between the Coriolis term and the frictional term, and integrating over this layer gives

$$-fV \approx -r\overline{u}_s, \tag{14.68}$$

where $V = \int_0^d \rho\overline{v}\,\mathrm{d}z$ is the meridional transport in the boundary layer, above which the stress vanishes. The return flow is poleward (i.e., $V > 0$ in the Northern Hemisphere) producing an eastward Coriolis force. This can be balanced by a westward frictional force provided that the surface flow has an eastward component. In this picture, then, the mid-latitude eastward zonal flow at the surface is a proximate consequence of the poleward flowing surface branch of the Ferrel Cell, this poleward flow being required by mass continuity given the equatorward flow in the upper branch of the cell. In this way, the Ferrel Cell is responsible for bringing the mid-latitude eddy momentum flux convergence to the surface where it may be balanced by friction (refer again to Fig. 14.17).

A more direct way to see that the surface flow must be eastward, given the eddy momentum flux convergence, is to vertically integrate (14.67) from the surface to the top of the atmosphere.

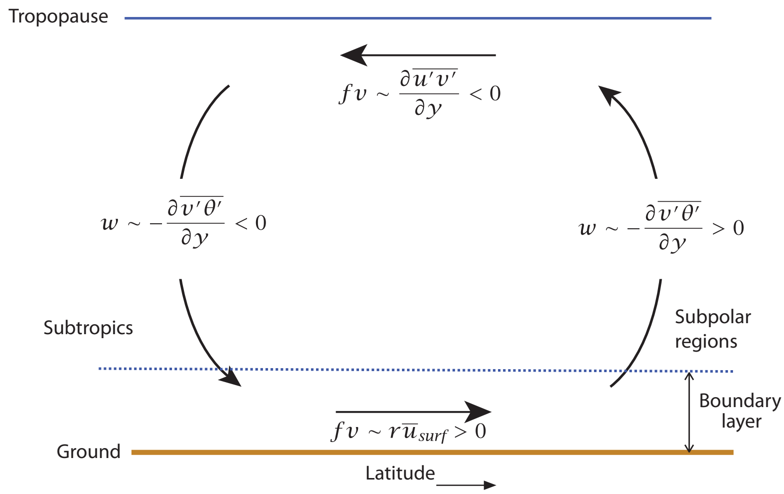

Fig. 14.17 The eddy-driven Ferrel Cell, from an Eulerian point of view.

Above the planetary boundary layer the mean flow is largely in balance with the eddy heat and momentum fluxes. The lower branch of the Ferrel Cell is largely confined to the boundary layer, where it is in a frictional–geostrophic balance.

By mass conservation, the Coriolis term vanishes (i.e., $\int_0^\infty f\rho\overline{v}\,dz = 0$) and we obtain

$$\int_0^\infty \frac{1}{\cos^2\vartheta}\frac{\partial}{\partial\vartheta}(\cos^2\vartheta\,\overline{u'v'})\rho\,dz = [\tau]_0^\infty = -r\overline{u}_s. \tag{14.69}$$

That is, the surface wind is proportional to the vertically integrated eddy momentum flux convergence. Because there *is* a momentum flux convergence, the left-hand side is negative and the surface winds are eastward.

The eddy heat flux also plays a role in the Ferrel Cell, for in a steady state we have, from (14.63)

$$w = \frac{1}{N^2}\left[Q_b - \frac{1}{\cos\vartheta}\frac{\partial(\overline{v'b'}\cos\vartheta)}{\partial y}\right], \tag{14.70}$$

and inspection of Fig. 14.12 shows that the observed eddy heat flux produces an overturning circulation in the same sense as the observed Ferrel Cell (again see Fig. 14.17).

Is the circulation produced by the heat fluxes *necessarily* the same as that produced by the momentum fluxes? In a non-steady state the effects of both heat and momentum fluxes on the Ferrel Cell are determined by (14.65) (an equation which applies more accurately at mid-latitudes than at low ones because of the low-Rossby number assumption), and there is no particular need for the heat and momentum fluxes to act in the same way. But in a steady state they must act to produce a consistent circulation. To see this, for simplicity let us take f and N^2 to be constant, let us suppose the fluid is incompressible and work in Cartesian coordinates. Take the y-derivative of (14.67) and the z-derivative of (14.70) and use the mass continuity equation. Noting that $\overline{v'\zeta'} = -\partial\overline{u'v'}/\partial y$ we obtain

$$\frac{\partial}{\partial y}\left(\overline{v'\zeta'} + \frac{f_0}{N^2}\frac{\partial\overline{v'b'}}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{f_0}{N^2}Q[b]\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho_0}\frac{\partial\tau}{\partial z}\right). \tag{14.71}$$

The expression on the left-hand side is the divergence of the eddy flux of quasi-geostrophic potential vorticity! That the heat and momentum fluxes act to produce a consistent overturning circulation is thus equivalent to requiring that the terms in the quasi-geostrophic potential vorticity equation are in a steady-state balance. The eddy fluxes of heat and momentum evidently play a huge role in the mid-latitude circulation, and in the next chapter we examine the fluid dynamics giving rise to these eddy fluxes.

Notes

1 Many of the observations presented here are so-called *reanalyses*, prepared by the National Centers for Environmental Prediction (NCEP) and the European Centre for Medium-Range Weather Forecasts (ECMWF), described in Kalnay (1996) and Dee *et al.* (2011). Reanalysis products are syntheses of observations and model results and so are not wholly accurate representations of the atmosphere. However, especially in data-sparse regions of the globe and for poorly measured fields, they are likely to be more accurate representations of the atmosphere than could be achieved using only the raw data. Of course, this in turn means they contain biases introduced by the models. A reanalysis is a particular form of *state estimate*, which is the more general name given to similar products and is the name used in oceanography.

2 Trenberth & Caron (2001).

3 George Hadley (1685–1768) was a British meteorologist who formulated the first dynamical theory for the trade winds, presented in a paper (Hadley 1735) entitled 'Concerning the cause of the general trade winds.' At that time, trade winds referred to any large-scale prevailing wind, and not just tropical winds. The name 'trade' may be associated with the commercial (i.e., trade) exploitation of the wind by mariners on long ocean journeys, but trade also means (or at least meant) customary, and trade winds customarily blow in one direction. Relatedly, in Middle English the word trade means path or track — hence the phrase 'the wind blows trade', meaning the wind is on track. Hadley realized that in order to account for the zonal winds, the Earth's rotation makes it necessary for there also to be a meridional circulation. His vision was of air heated at low latitudes, cooled at high latitudes, giving rise to a single meridional cell between the equator and each pole. Although he thought of the cell as essentially filling the hemisphere, and he did not account for the instability of such a flow, it was nevertheless a foundational contribution to meteorology. The thermally direct cell in low latitudes is now named after him.

A three-celled circulation was proposed by William Ferrel (1817–1891), an American school teacher and meteorologist, and the middle of these cells is now named for him. His explanation of the cell (Ferrel 1856a) was not correct, hardly surprising because the eddy motion that drives the Ferrel Cell was not understood for another 100 years or so. Ferrel's ideas evolved to something more akin to a two-celled picture (Ferrel 1859), similar to that proposed by J. Thomson in 1857. The history of these ideas is discussed by Thomson (1892). Ferrel did however give the first essentially correct description of the role of the Coriolis force and the geostrophic wind in the general circulation (Ferrel 1858, a paper with a quite modern style), a key development in the history of geophysical fluid dynamics. Ferrel also contributed to tidal theory (Ferrel 1856b) and to ocean dynamics. (See http://www.history.noaa.gov/giants/ferrel2.html).

Although Hadley's single-celled viewpoint was superseded by the three-celled and two-celled structures, the modern view of the overturning circulation is, ironically, that of a single cell of 'residual circulation', which, although having distinct tropical and extratropical components, in some ways qualitatively resembles Hadley's original picture.

4 Schneider (1977) proposed an axially-symmetric, angular momentum conserving model of the Hadley Cell. Held & Hou (1980) developed the model we follow here.

5 Fang & Tung (1996) do find some analytic solutions in the presence of moisture.

6 After Hide (1969).

7 Gierasch (1975).

8 Largely following Lindzen & Hou (1988). The solutions of Fig. 14.10 are taken from that paper.

9 See Dima & Wallace (2003) for some relevant observations. They noted that the asymmetry of the Hadley Cell is affected by monsoonal circulations (which are, of course, not accounted for in the model presented here). Fang & Tung (1999) investigated the effects of time dependence and note that quasi-steadiness is not well satisfied, although this alone was unable to limit the nonlinear amplification effect. In reality, the effects of baroclinic eddies are important in Hadley Cell dynamics, as we discuss in Section 14.5.

10 Although our presentation does not follow the historical order, that the effects of baroclinic eddies

are likely to be important in modifying the Hadley circulation is in fact the traditional view, emerging in the second half of the 20th century. Lorenz (1967), for example, discusses the fact that an ideal, axisymmetric, Hadley circulation is likely to be baroclinically unstable, although the Hadley circulation envisioned was one that filled the hemisphere. More recently Kim & Lee (2001), Walker & Schneider (2005), Frierson *et al.* (2007), and many others, have discussed the effects of baroclinic eddies on the Hadley Cell.

11 Figure courtesy of M. Juckes, using an ECMWF reanalysis.

12 A simple generalization of (14.65) is to replace (14.62) by (14.59) and then to use the thermal wind equation in the form

$$\frac{f}{a\cos\vartheta}\frac{\partial\overline{m}}{\partial z} = -\frac{\partial\overline{b}}{\partial y}. \tag{14.72}$$

An equation very similar to, but a little more general than, (14.65) may then be derived. A still more general, usually elliptic equation for the overturning circulation may be derived from the zonally-averaged primitive equations, assuming only that the zonally-averaged zonal wind is in gradient wind balance with the pressure field (Vallis 1982).

13 The simulations, kindly performed by C. Walker, are similar to those in Walker & Schneider (2005).

14 Figure courtesy of T. Schneider, using an ECMWF reanalysis.

Further Reading

Atmospheric dynamics and circulation

Andrews, D. G., Holton, J. R. & Leovy, C. B., 1987. *Middle Atmosphere Dynamics.*
Discusses the theory and observations of the middle atmosphere in some detail, including Rossby waves and wave–mean-flow interaction.

Green, J. S. A., 1999. *Atmospheric Dynamics.*
A personal view of the subject, with numerous insights about how the atmosphere works.

Lorenz, E. N., 1967. *The Nature and Theory of the General Circulation of the Atmosphere.*
A classic monograph on the atmospheric general circulation.

Marshall, J. & Plumb, R. A., 2008. *Atmosphere, Ocean and Climate Dynamics: An Introductory Text.*
Discusses the dynamics and circulation of both atmosphere and ocean at an advanced undergraduate, beginning graduate level.

Peixoto, J. P. & Oort, A. H., 1992. *Physics of Climate.*
A descriptive but physically-based discussion of the general circulation, emphasizing observations.

Randall, D. 2015. *An Introduction to the Global Circulation of the Atmosphere.*
Discusses both the observations and mechanisms of the large-scale atmospheric circulation.

Schneider, T. & Sobel, A., 2007. *The Global Circulation of the Atmosphere: Phenomena, Theory, Challenges.*
Contains several useful review articles on the large-scale atmospheric circulation.

Atmospheric physics and thermodynamics

Ambaum, M., 2010. *Thermal Physics of the Atmosphere.*
Distinguished by treating the subject as a branch of classical thermodynamics, which indeed it is.

Bohren, C. F., & Albrecht, B., 1998. *Atmospheric Thermodynamics.*
A well-known, and rather lively, introduction to the field.

Cabellero, R. 2014. *Physics of the Atmosphere.*
A short introduction to the subject of its title.

Emanuel, K., 1994. *Atmospheric Convection.*
A comprehensive discussion of convection, progressing from basic theory to an advanced level.

Pierrehumbert, R. T., 2010. *Principles of Planetary Climate.*
Pierrehumbert's book is an ideal complement to the one you are currently looking at.

Wallace, J. M. & Hobbs, P. V., 2006. *Atmospheric Science: An Introductory Survey.*
An introduction to a broad range of topics in atmospheric sciences, for scientists at all levels.