Genetic and Evolutionary Algorithms:
Schema Theorem

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Theoretical Basis of Trust in the Potential of Genetic Algorithms!

Schema Theorem

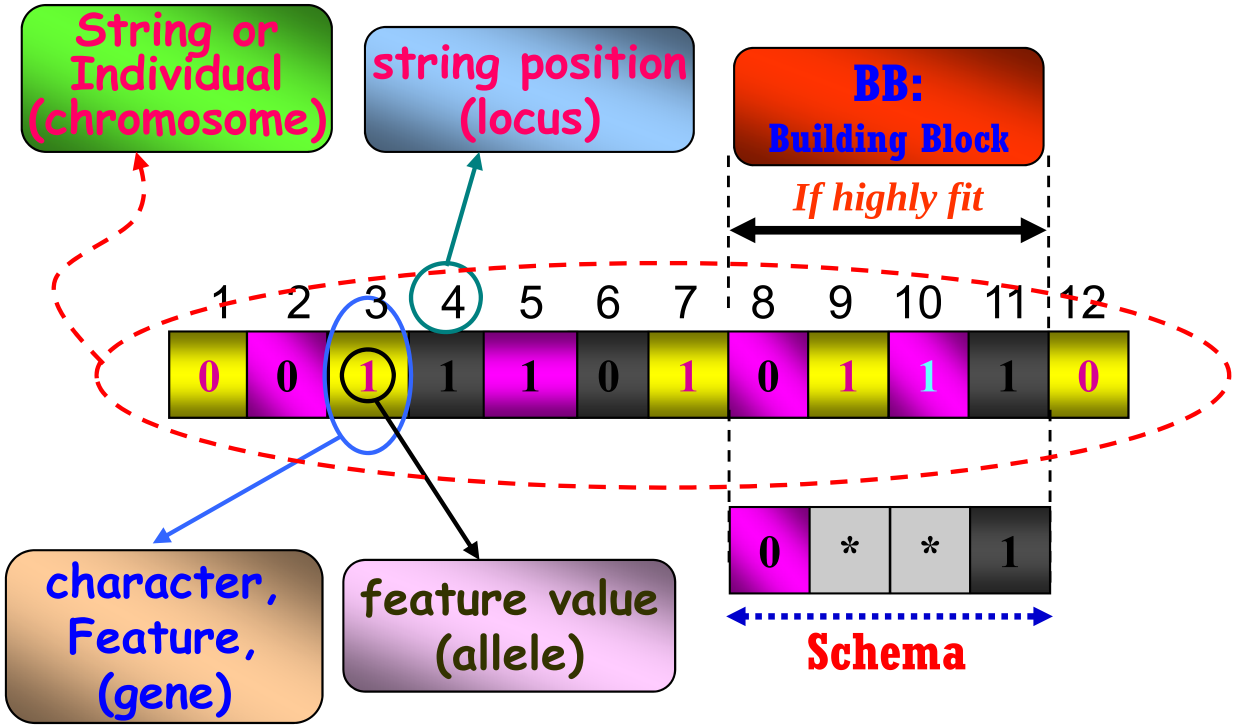
Schema Theorem (1)

❖ Schema Theorem

- It guarantees the success of GAs by just applying genetic operations
  - ✓ Concept: the good schemata increase their market share in the population

Cf) What is Schema?

- ✓ A subset of the space in which all the strings share a particular defined set



Schema Theorem (2)

- **Order of a schema *H*:** $o(H)$

  - Number of fixed positions present in the template

011*1**	$o(011*0**)=4$	**0********	$o(0******)=1$

- **Length of a schema *H*:** $\delta(H)$

  - The distance between the first and last specific string position

011*1**	$\delta(H_1)=5-1=4$	**0********	$\delta(H_2)=1-1=0$

- **How many expected number of schema will be inherited to the next generation?**

  - The survival rate under simple reproduction: $\frac{f(H)}{\overline{f}}$ **Proportional selection**

where $f(H): average\ fitness\ of\ the\ strings\ representing\ schema\ H$

$\overline{f}: average\ fitness\ of\ the\ entire\ population\ (=\sum f_i / n)$

Schema Theorem (3)

- **Consider the following situation on Crossover:**

$A =$ 0111000 → $A =$ 011|1000

$H_1 =$ *1****0 → $H_1 =$ *1*|***0

$H_2 =$ ***10** → $H_2 =$ ***|10**

The schema H_1 will be destroyed by the one-point crossover!

- If the crossover site is selected uniformly among $l - 1 = 7 - 1 = 6$, then the prob. that schema H_1 is destroyed is given as

$$p_d = \delta(H_1)/(l-1) = 5/6$$

: ***Just a conditional prob.***

- The survival prob. under simple crossover is given by

$$p_s \geq 1 - p_c \cdot \frac{\delta(H)}{l-1}$$

where p_c *is a crossover prob.*

Schema Theorem (4)

- **Mutation** **is the** ***random alteration*** **of a** **single position** **with the prob.** p_m
- **Single allele** **survives** **with the prob.** $(1-p_m)$
- **The** **schema survival prob.** **is obtained by**

$$(1-p_m)^{o(H)} \approx 1-o(H)\cdot p_m$$

- **Particular** **schema** ***H*** **receives** **an expected number of copies** **in the** **next generation**:

$$\boxed{m(H,t+1)} = m(H,t)\cdot P[H \text{ is survival under reproduction} \cap \text{crossover} \cap \text{mutation}]$$

$$= m(H,t)\cdot P[H \text{ is survival under reproduction}]\cdot P[H \text{ is survival under crossover}]$$

$$\cdot P[H \text{ is survival under mutation}]$$

$$\geq m(H,t)\cdot \frac{f(H)}{\overline{f}}\cdot\left[1-p_c\cdot\frac{\delta(H)}{l-1}\right]\cdot\left[1-o(H)\cdot p_m\right] \boxed{\geq m(H,t)\cdot \frac{f(H)}{\overline{f}}\cdot\left[1-p_c\cdot\frac{\delta(H)}{l-1}-o(H)\cdot p_m\right]}$$

Short, Low-order schemata receive exponentially increasing trials in subsequent generation!