Evolutionary Algorithms:
Constrained Evolutionary Optimization

Nov. 11, 2019

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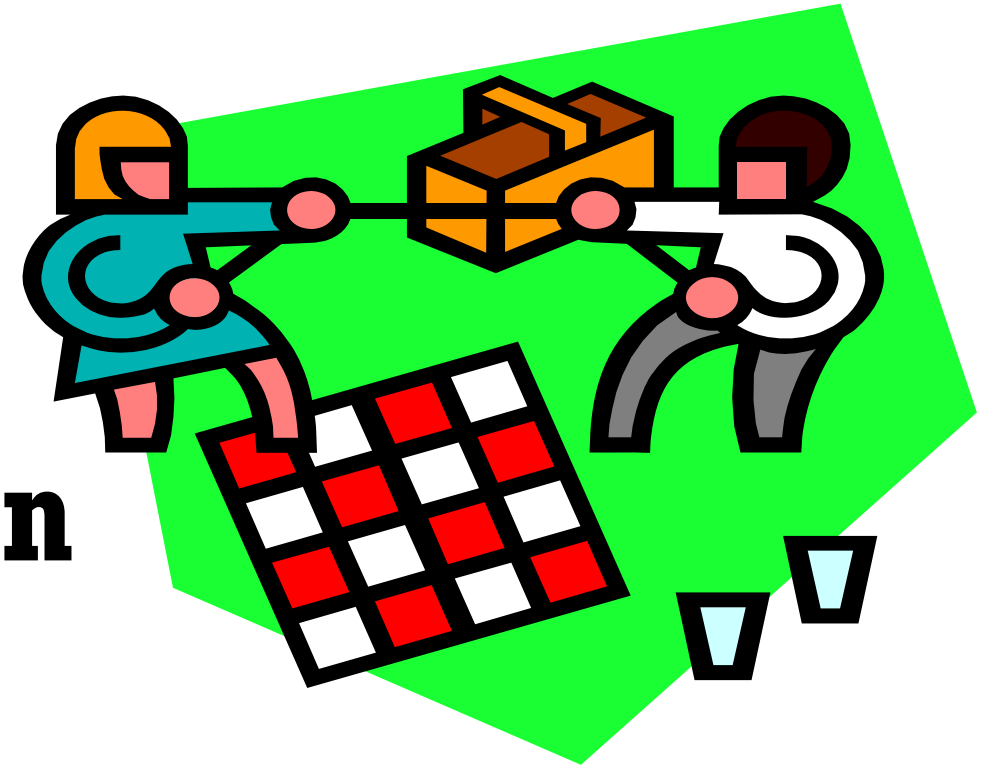
Constrained Evolutionary Optimization



Constrained Optimization (1)

MEMI.

❖ What are the Constrained Optimization Problems (COPs) ?

- ➢ A class of **Optimization Problems** that have **Constraint Conditions**
  - ✓ The aim is **to find the best solution(s)** while ***satisfying the Constraint Conditions***

For a search space Ω

There is a function $f : \Omega \mapsto \Re$

Unconstrained Optimization

The task is to find $x^* = \arg\max_{x \in \Omega} f$

Here, x is a vector of decision variables, and f is the objective function

Constrained Optimization

The task is to find $x^* = \arg\max_{x \in \Omega} f$

subject to $g_i(x) \le 0, \; i = 1, \cdots, m$

$h_i(x) = 0, \; j = 1, \cdots, p$

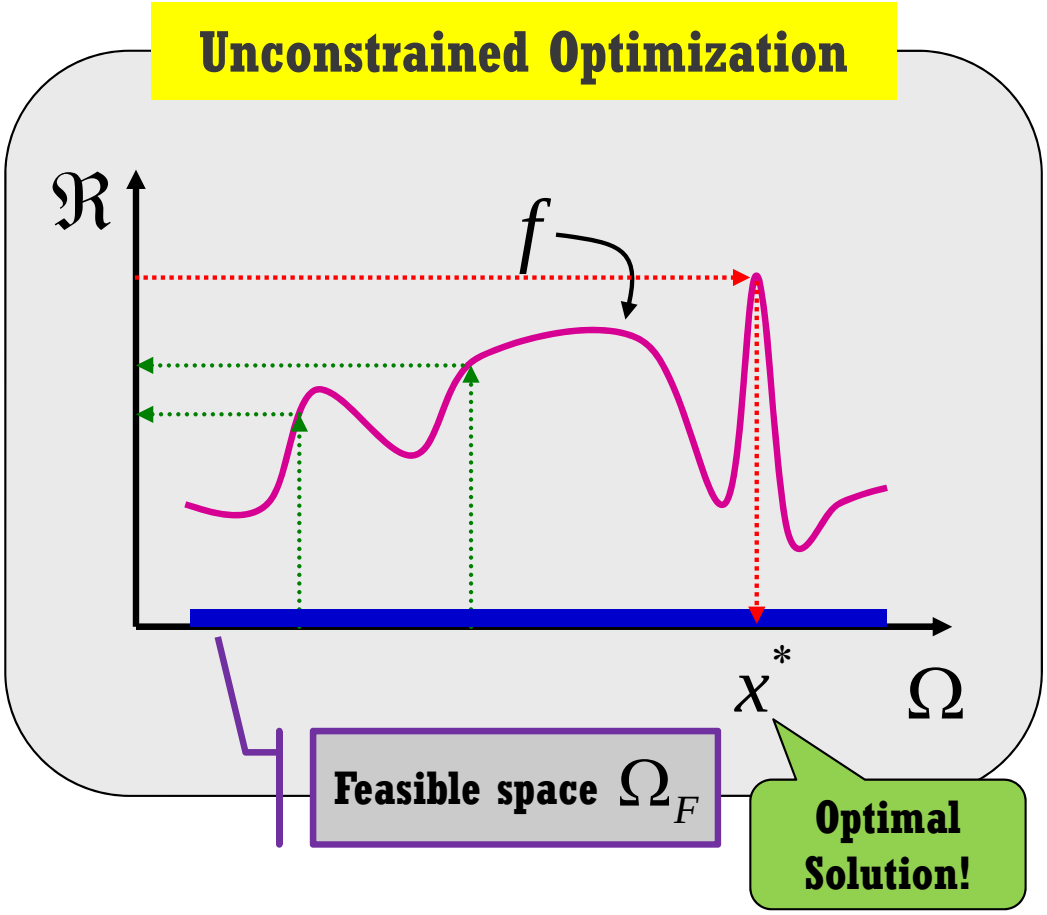
Inequality Constraints

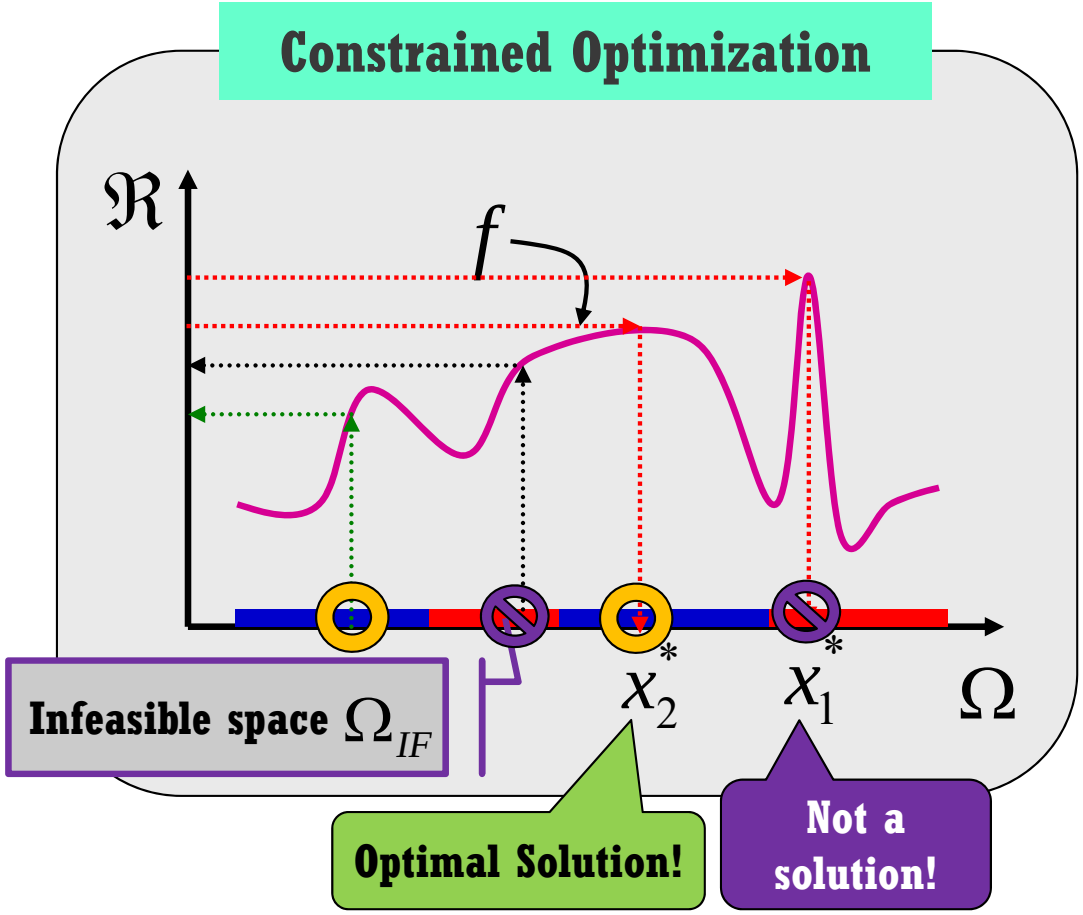
Equality Constraints

Constrained Optimization (2)

❖ Regarding the Search Space

- ➢ **Feasible search space of COPs is a key issue!**
  - ✓ **It is relatively very narrow! ➔ Thus, hard to discover the optimal solution!**



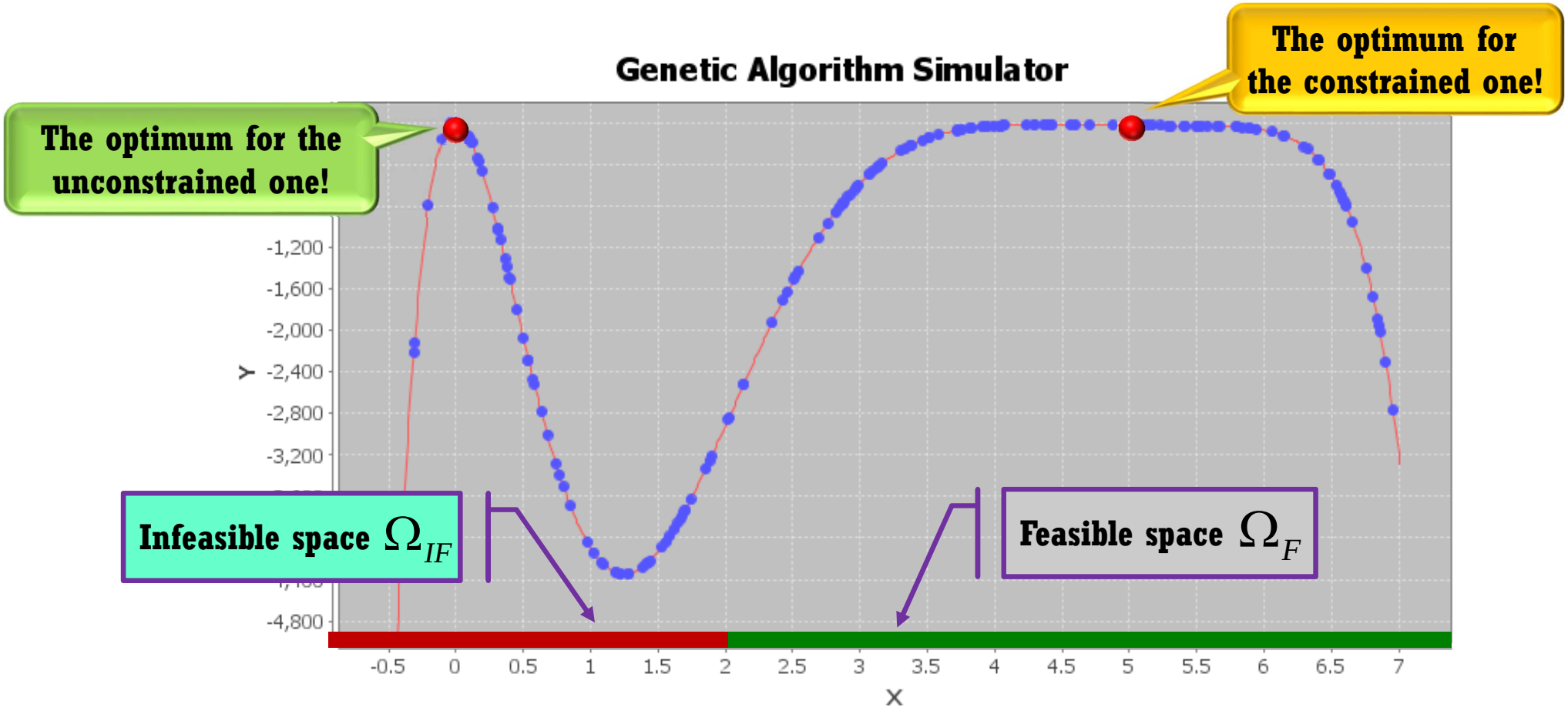


Constrained Optimization (3)

❖ Example

➢ **Find the optimal solutions w.r.t the maximization of** $f(x) = -x^2\{1+(x-5)^6\}$

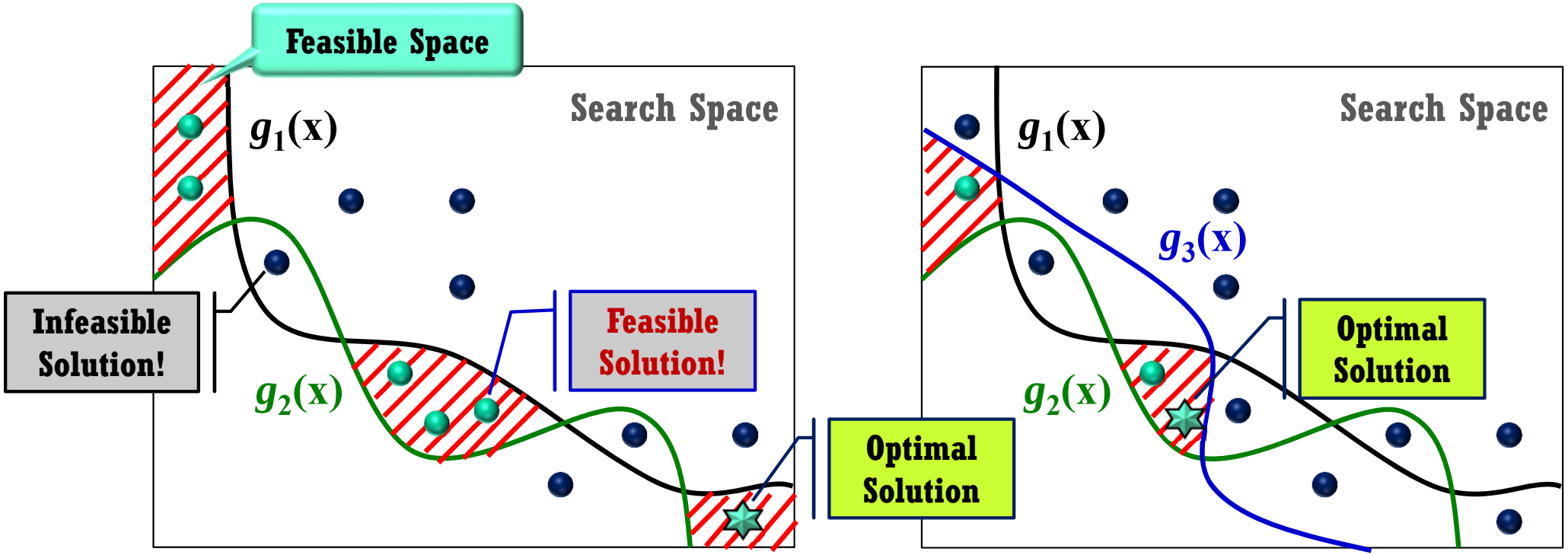
- ✓ **If there is no constraint, then x=0 is the optimum!**
- ✓ **If a constraint is u(x-2) ≥ 0, then x=5 is the optimum!**



Constrained Optimization (4)

❖ How Difficult the Constraints are?

- ➢ **Constraints significantly reduce the amount of feasible search space.**
  - ✓ In real-world cases, the feasible spaces are very narrow! (e.g., less than 0.001%)
  - ✓ To make matters worse, it is very hard to handle the constraints mathematically!



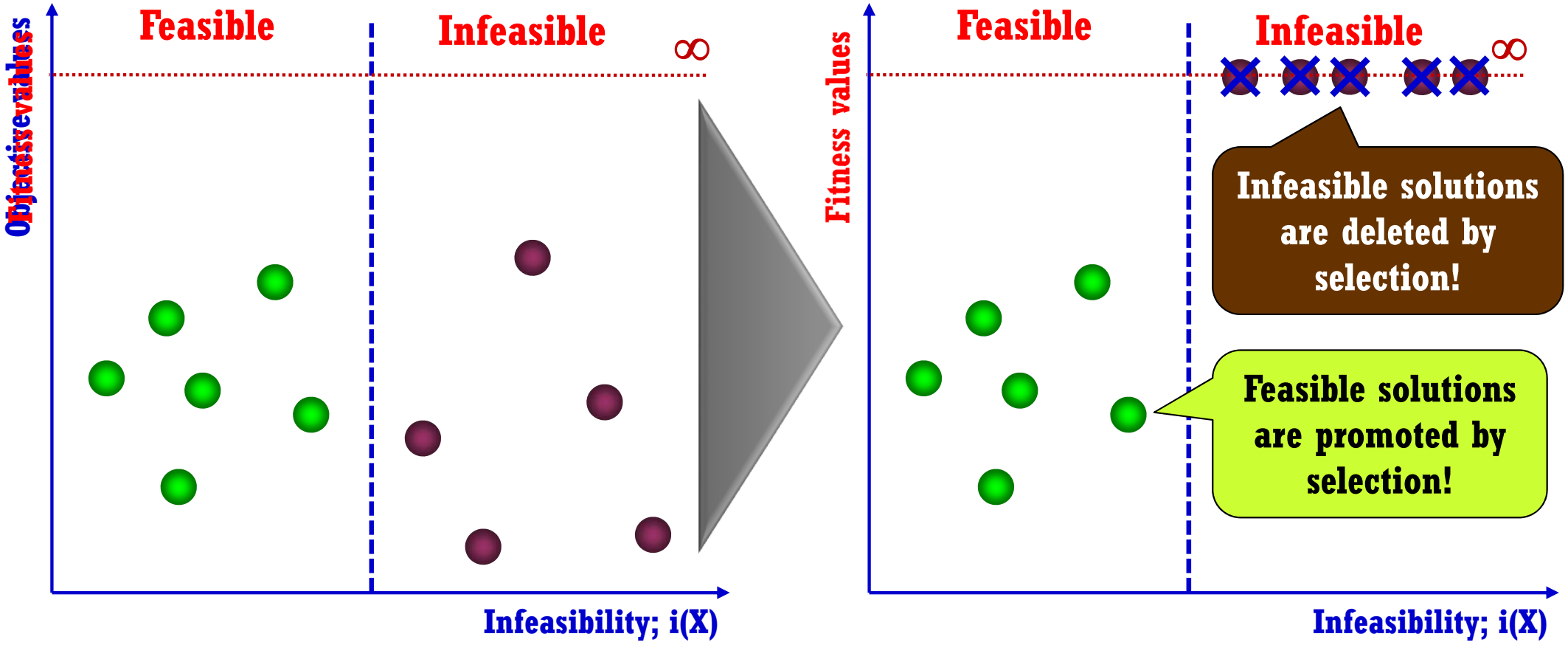
Case I. Number of Constraints is 2

Case II. Number of Constraints is 3

Constrained Evolutionary Opt. (1)

❖ How to Handle the Constraints in terms of GAs?

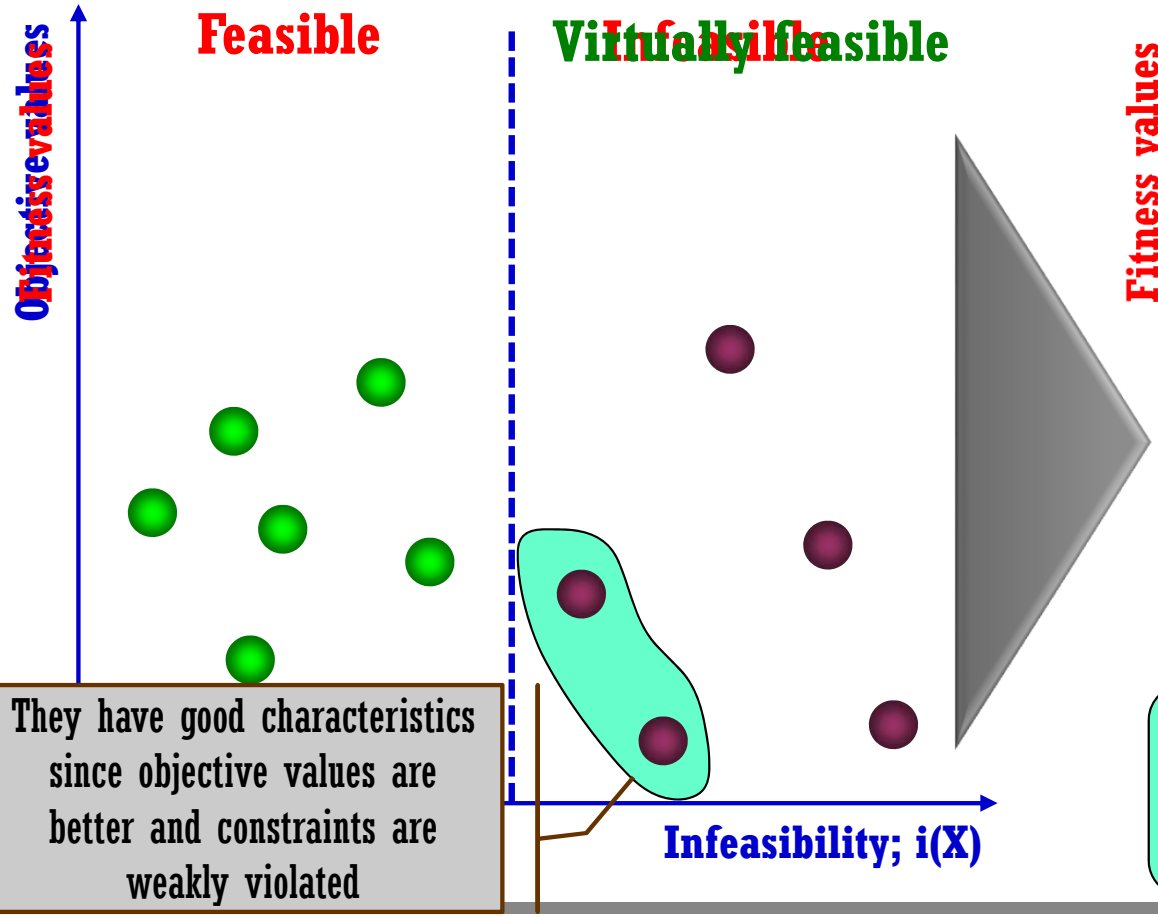
- ➢ The Simplistic Method: Not consider the infeasible solutions!
  - ✓ EAs solve the optimization problems by referring to the fitness values
  - ✓ In the minimization case, the infinity value is assigned to the infeasible solutions

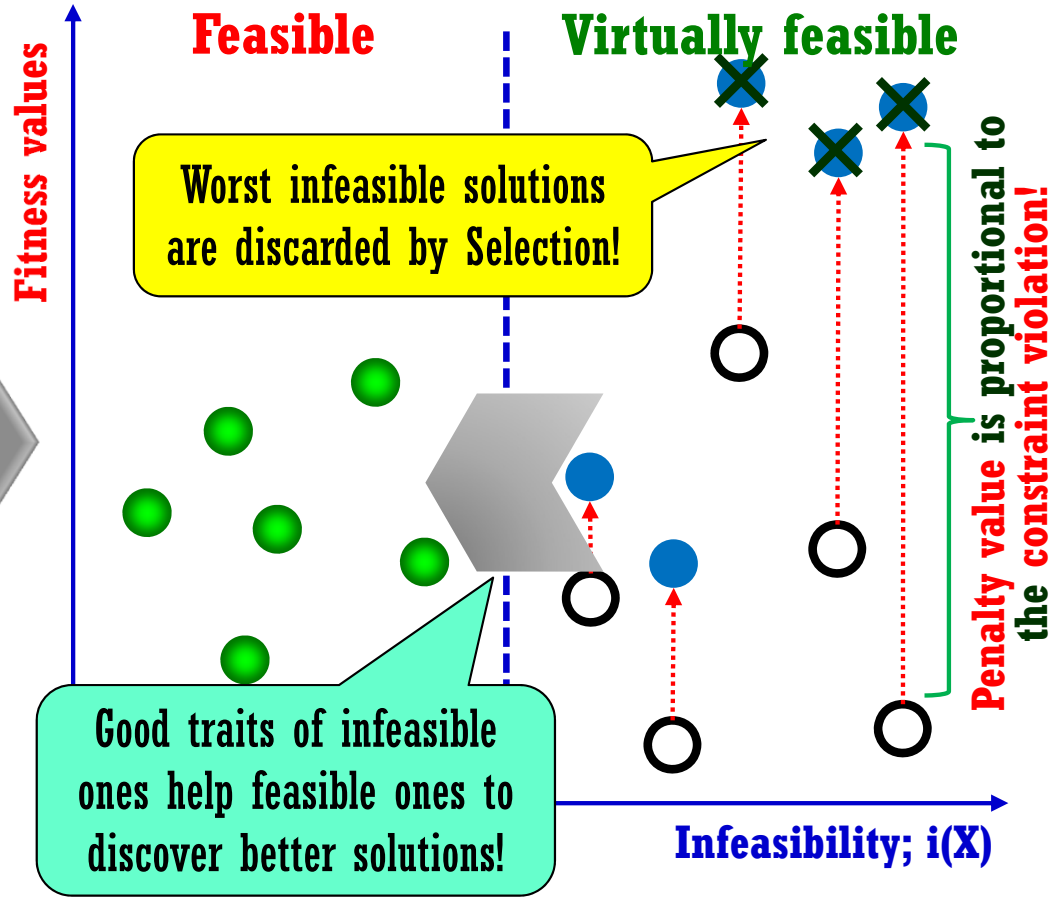


Constrained Evolutionary Opt. (2)

❖ More Efficient Method?

- ➢ **The Penalty Method: Participate the infeasible solutions as well in evolution!**
  - ✓ Transform a COP into an unconstrained one by adding/subtracting a certain value.
  - ✓ In the minimization case, the value is based on the amount of constraint violation.





Constrained Evolutionary Opt. (3)

❖ Example: Knapsack Problem

- A set of n items is available to packed into a knapsack with capacity C units.
- Item i has a value w_i (e.g., $) and uses up c_i units (e.g., kg) of capacity
- The aim is to *maximize* the amount of values while keeping the overall capacity

Problem Formulation

Objective function $f(x)$

$$\max \sum_{i=1}^{n} w_i x_i \quad \text{subject to} \quad \sum_{i=1}^{n} c_i x_i \le C$$

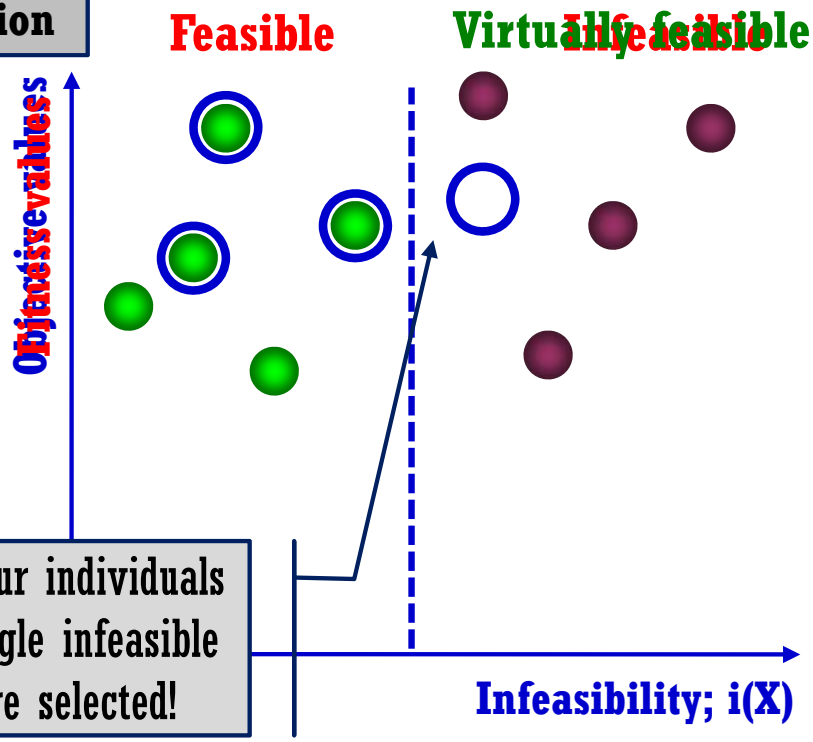
Constraint condition

Penalty function $p(x)$

$$p(x) = \begin{cases} 0, & \text{if feasible solution} \\ \alpha(\sum c_i x_i - C)^2, & \text{otherwise} \end{cases}$$

Fitness function = Objective function – Penalty function

$$F(x) = f(x) - p(x) = \sum_{i=1}^{n} w_i x_i - p(x)$$



Summary

❖ Constrained Evolutionary Algorithms

- ➢ **Constraints** make optimization problems **difficult to be solved**
- ➢ **Feasible space significantly reduces** as the number of **constraints increases**
- ➢ **Penalty value** is imposed on **the objective value** by the **constraint violation**
- ➔ **The design of penalty function is a key issue in COPs!**