Evolutionary Algorithms:
More Investigation – Crossover

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Prof. Chang Wook Ahn

Sungkyunkwan Evolutionary Algorithms Lab.
School of Info. & Comm. Eng.
Sungkyunkwan University

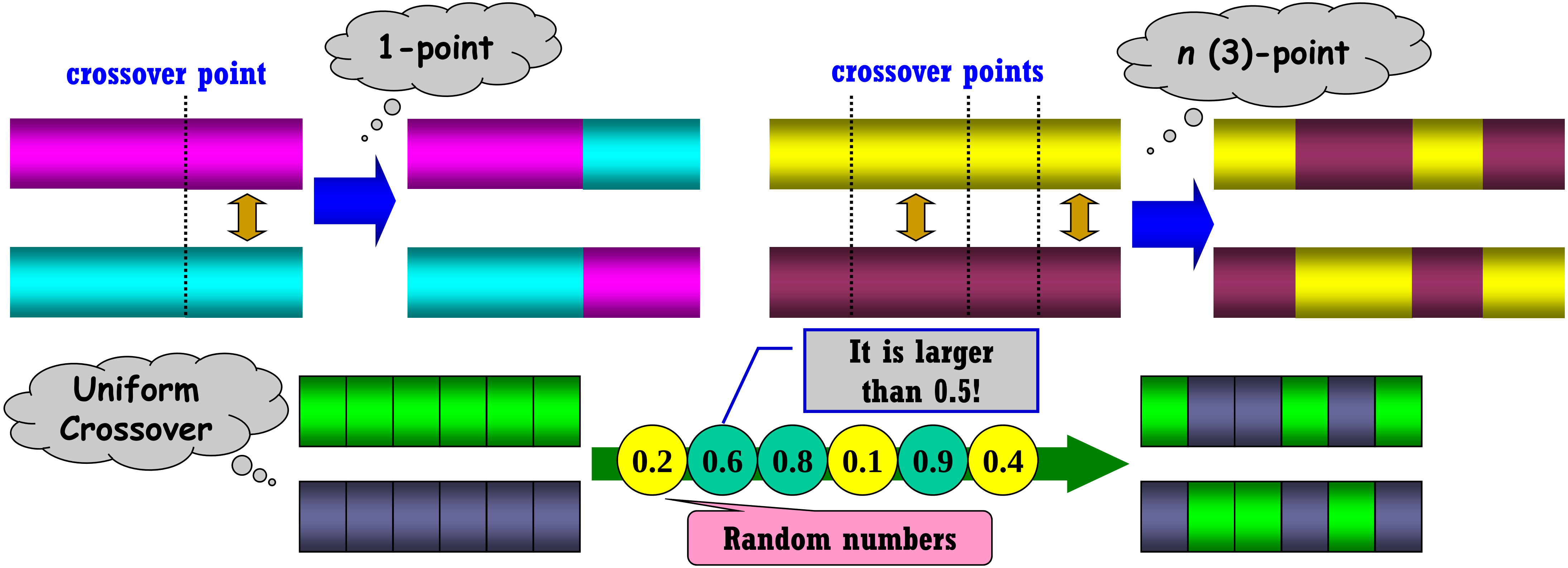
Further
Studies on
Crossover

Crossover (1)

- **Crossover (Recombination)**
  - Imitating the genetic inheritance
    - ✓ by recombining segments belonging to the individuals corresponding to parents
  - Ensuring the exploration of search space
  - One-point crossover, *n*-point crossover, Uniform crossover, etc.

Which one is the Most Promising Crossover?

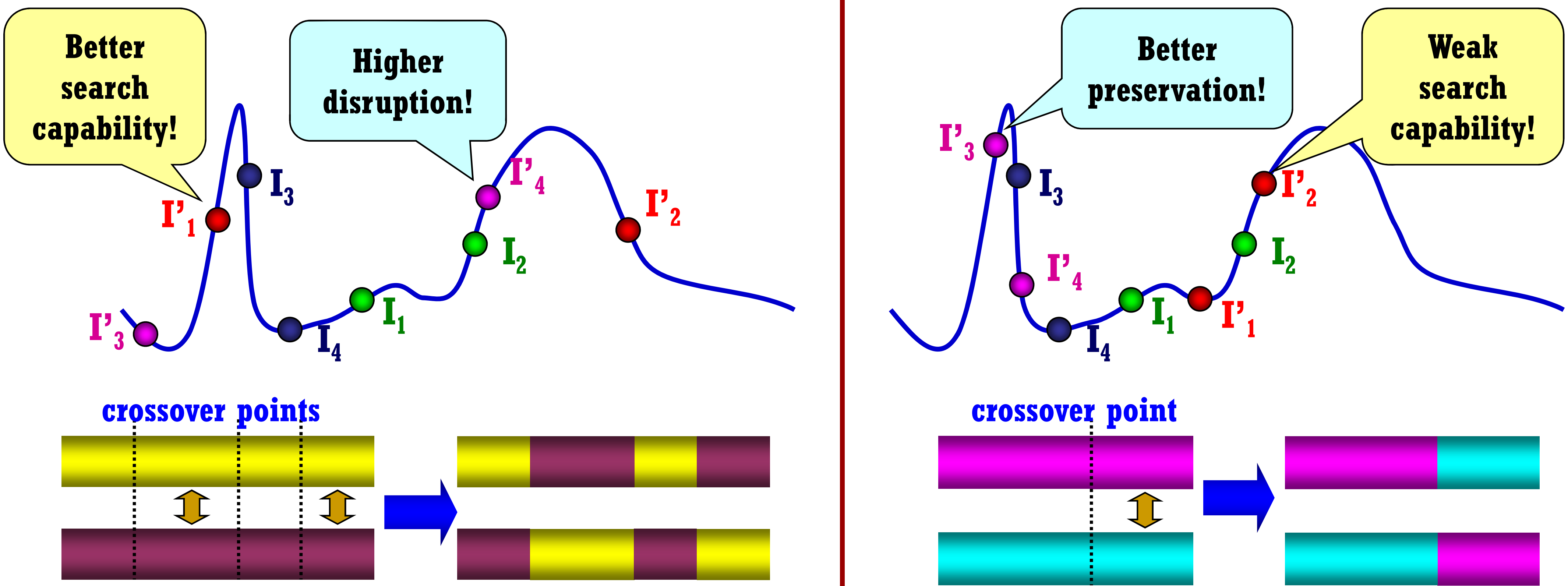
➔ If n increases, the n-point crossover becomes the uniform crossover



Crossover (2)

❖ Dynamics of Crossover

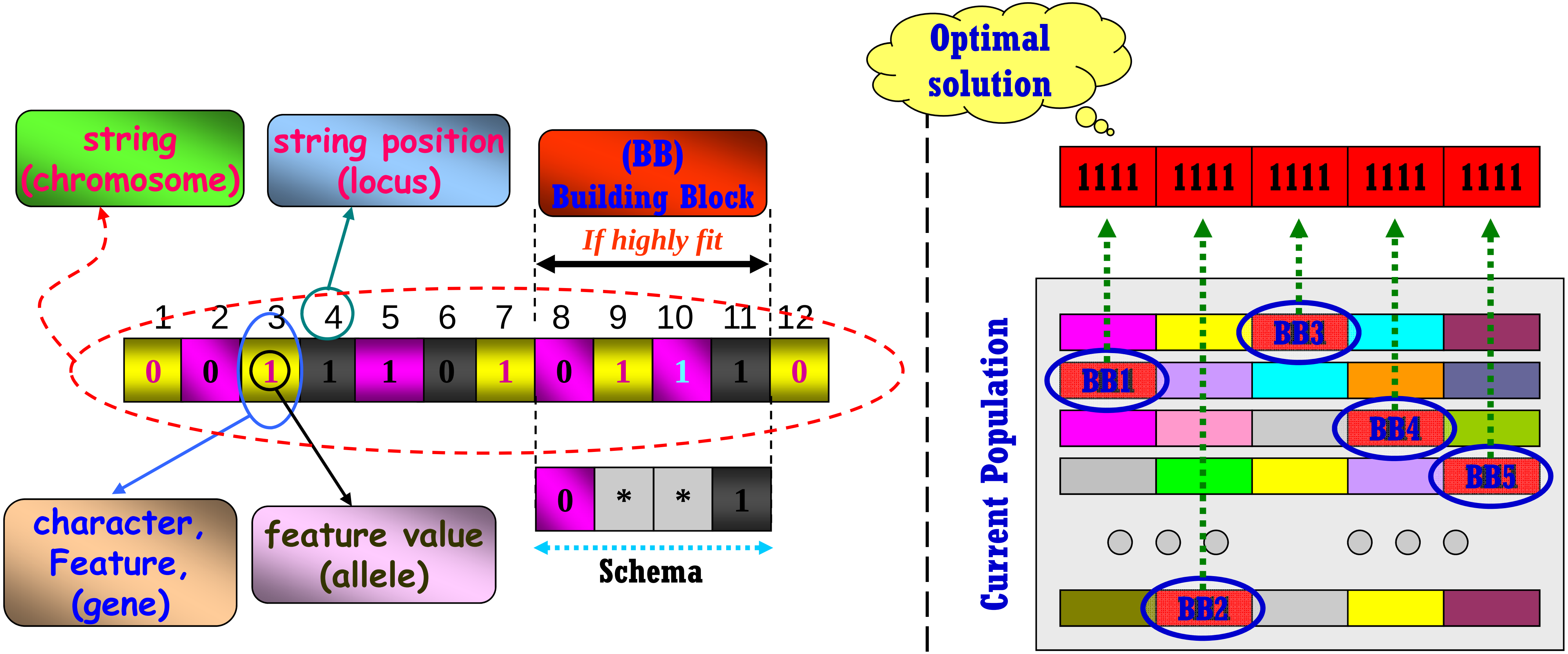
- As the number of crossover points increases,
  - ✓ Exploratory power is increased
  - ✓ Genes of each parent are more likely scrambled/disrupted
- As the number of crossover points decreases,
  - ✓ Exploratory power is decreased, but Genes of each parent are more likely preserved



Building-Blocks: BBs (1)

❖ Building-Blocks (BBs)

- ➢ They are partial solutions whose contributions to fitness are very high
- ➢ For instance, the global optimum is formed by combining a set of subsolutions: These subsolutions are defined as Building-Blocks (BBs).
- ➢ Thus, BBs must be preserved and bred for reliably discovering the optimum



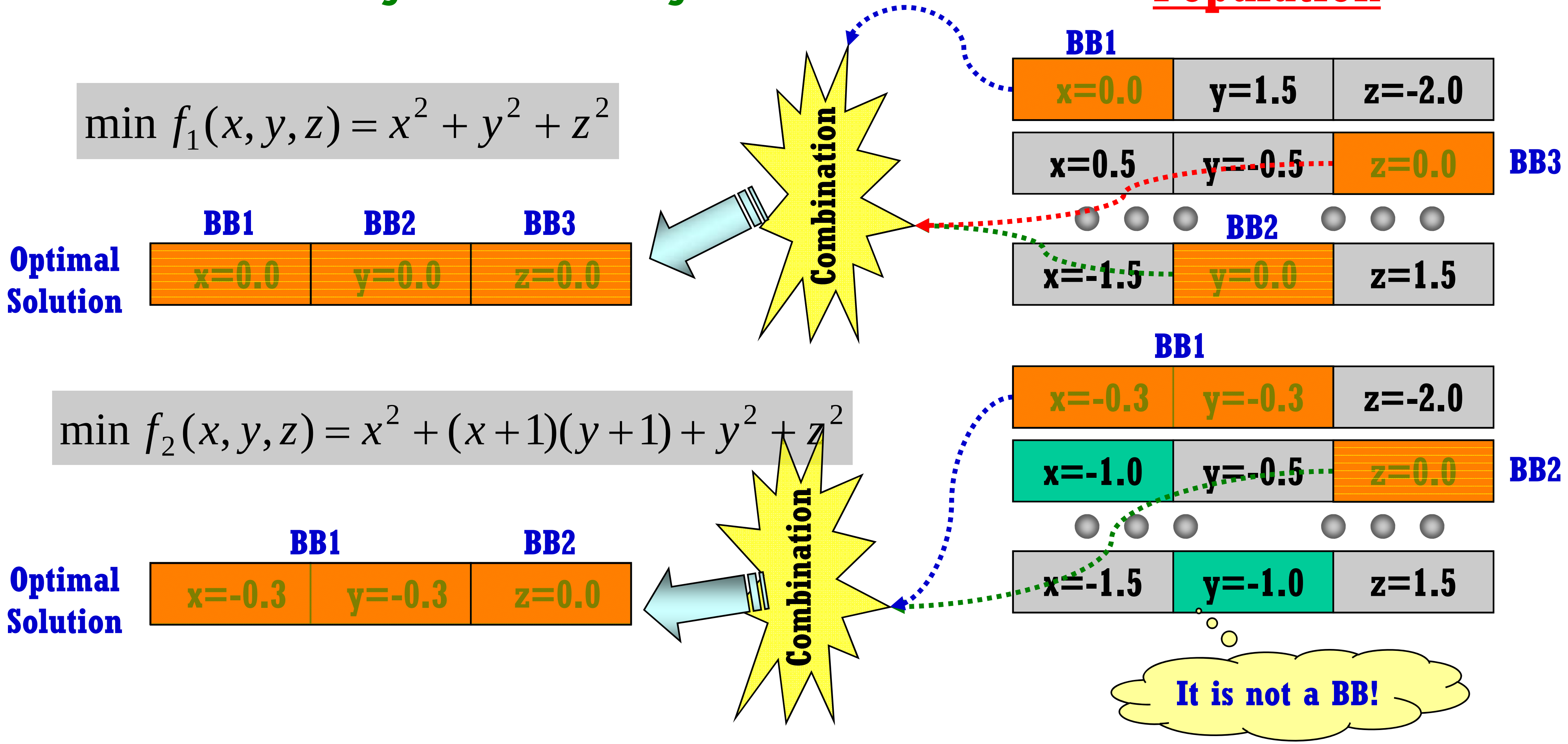
Building-Blocks: BBs (2)

❖ Building-Blocks (BBs): Example

- We can build up the optimal solution by **carefully assembling BBs**
- How to achieve this goal?
  - ➔ **Preserving BBs** & **Breeding the BBs**

$$\min f_1(x,y,z) = x^2 + y^2 + z^2$$

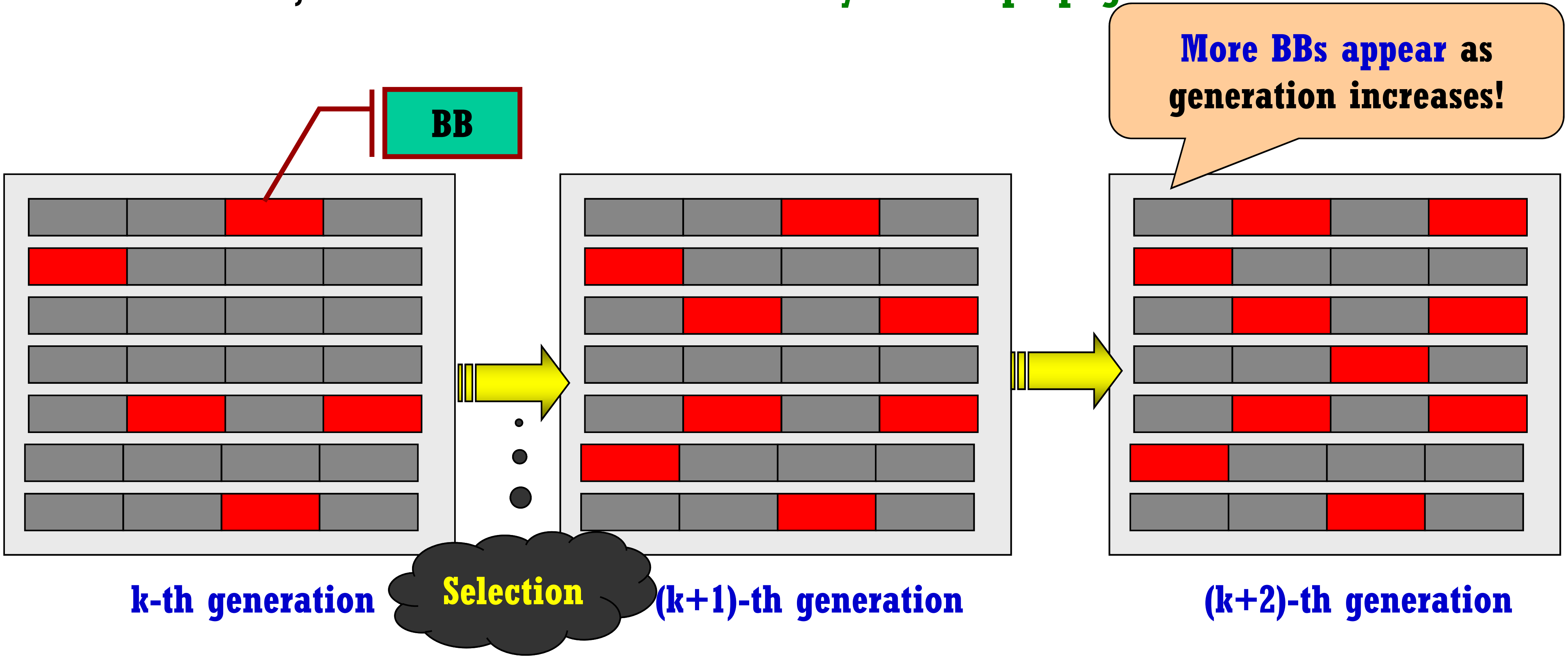
$$\min f_2(x,y,z) = x^2 + (x+1)(y+1) + y^2 + z^2$$



Building-Blocks: BBs (3)

❖ Effect of Selection

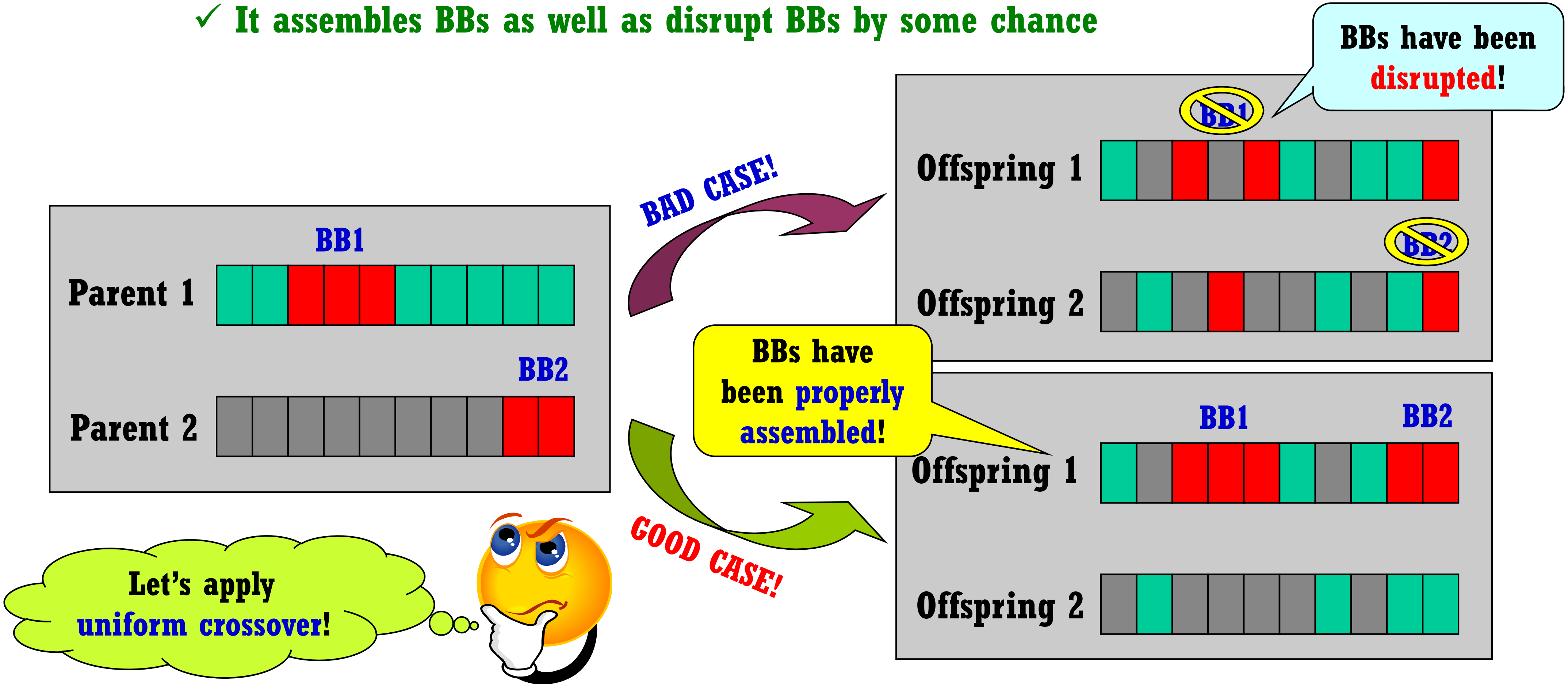
- The selection plays an important role in preserving & breeding BBs
  - The individuals that contain BBs have higher fitness:
  - Such individuals will survive from the selection
  - Thus, BBs also survive and then they can be propagated!



Building-Blocks: BBs (4)

❖ Effect of Crossover

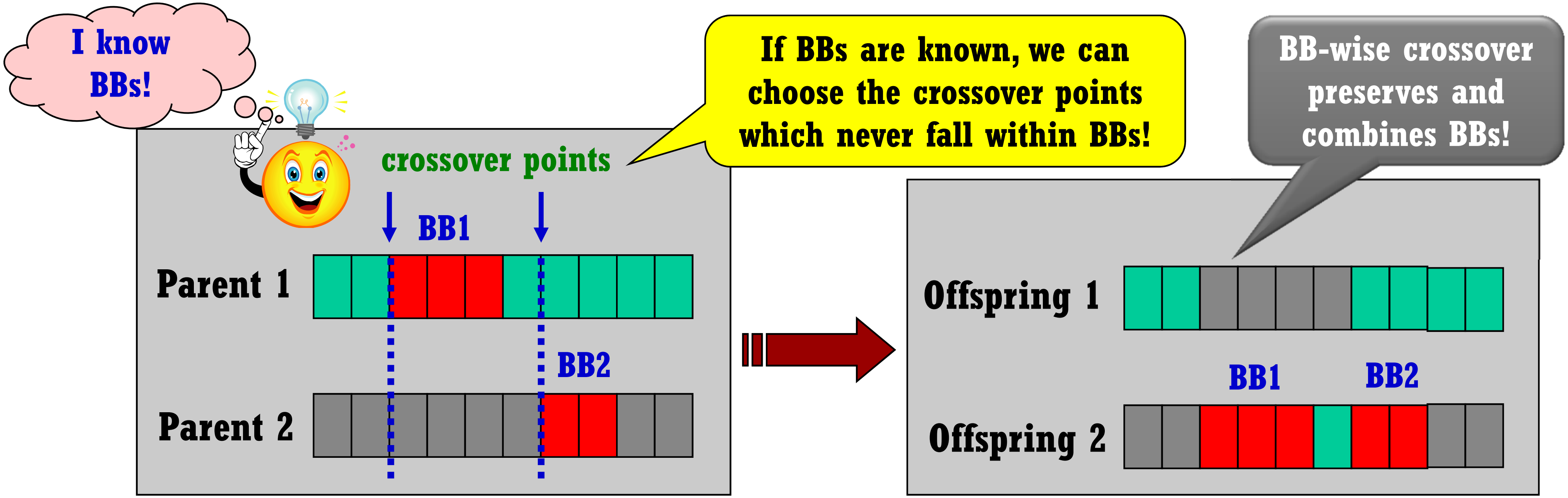
- The crossover **exchanges** some genes of parents
- To find the optimum, **BBs** must be **preserved** and **combined** within an individual
- Crossover looks like Janus's two faces!
  - ✓ It assembles BBs as well as disrupt BBs by some chance



Building-Blocks: BBs (5)

❖ BB-wise Crossover

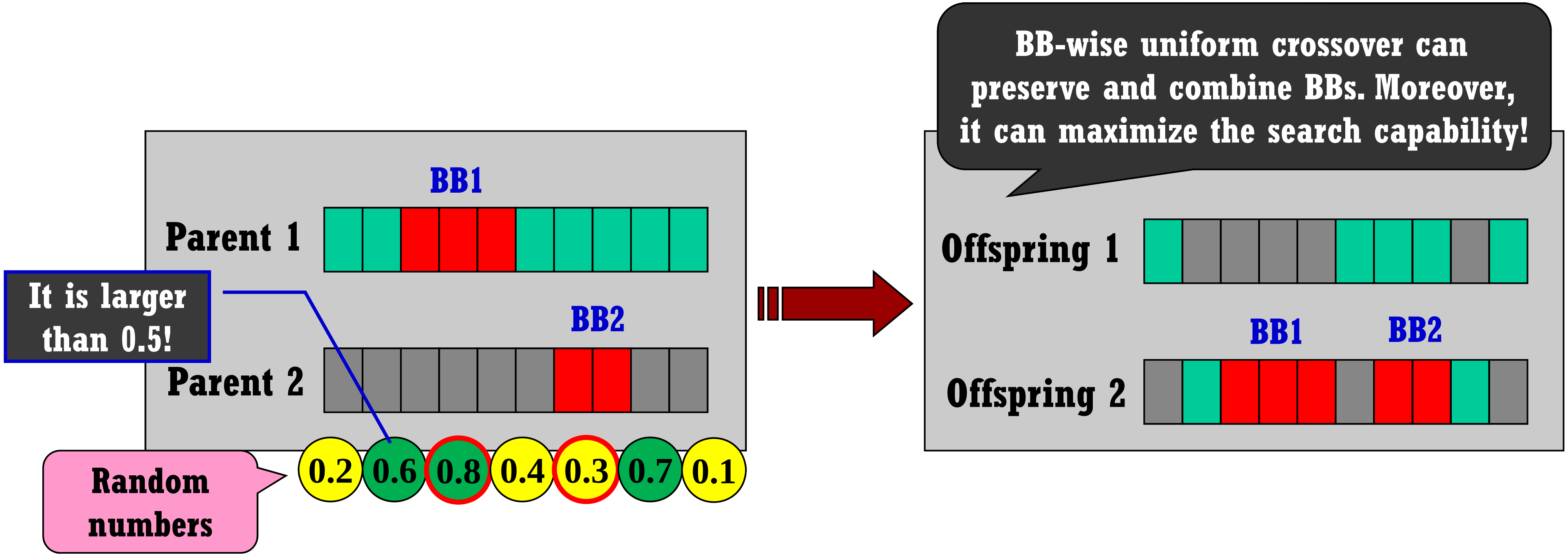
- ➢ To increase search capability, the number of crossover points must be increased
  - ➔ But increasing points brings forth higher possibility of BB disruption!
- ➢ To preserve BBs, the number of crossover points must be decreased
  - ➔ But decreasing points brings about weaker search capability!
- ➢ If BBs are known, we can perform a crossover at the level of BBs
  - ➔ It is referred to as "BB-wise Crossover".



Building-Blocks: BBs (6)

❖ BB-wise Uniform Crossover

- Uniform crossover **maximizes** the mixing rate of genes (**search capability**)
  - ➔ But it also maximize **the disruption rate of BBs**!
- If BBs are known, we can apply the **uniform crossover at the level of BBs**.
  - ➔ It is so-called "**BB-wise Uniform Crossover**"
- It can **maximize the exploratory power without destroying BBs**
  - ➔ Thus, the population converges to the optimum **quickly** and **reliably**!

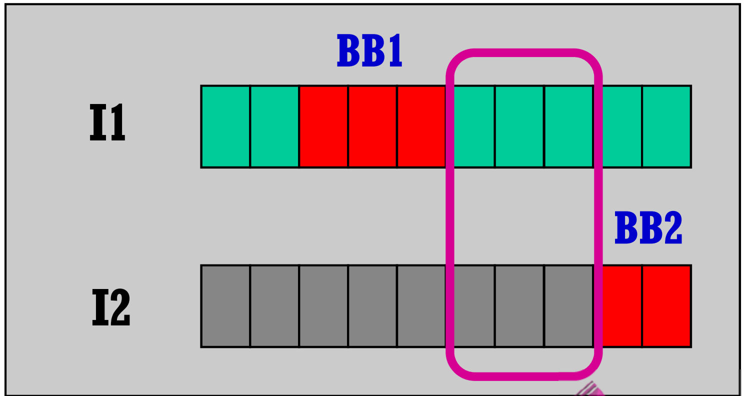


Building-Blocks: BBs (7)

❖ Why 1- or 2-point Crossover?

- BB information is not available in most problems.
- High-order crossover would be very harmful without the knowledge of BBs
- BB-preservation is more important than the increase of search capability
- As such, a natural choice must preserve BBs under the minimal search capability

➔ If BBs are not known, 1- or 2-point crossover is the best choice!



Individual Description

The individual length = 10
The number of possible crossover points = 9
The size of BB1 = 3
The number of possible crossover points in BB1 = 2;
The size of BB2 = 2
The number of possible crossover points in BB2 = 1;

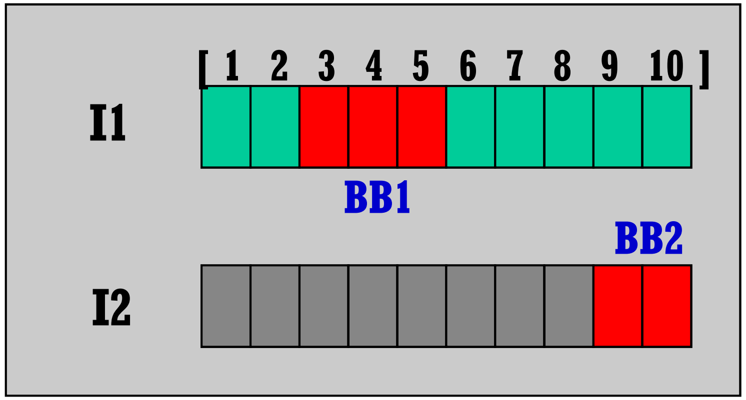
One-point Crossover

P[BB1 is preserved] = P[Crossover point does not fall at any position of BB1] = 1–(2/9) = 7/9

P[BB2 is preserved] = P[Crossover point does not fall at any position of BB2] = 1–(1/9) = 8/9

P[BB1 & BB2 appear after crossover] = P[Crossover point falls at the position between BB1 and BB2] = $\boxed{4/9}$

Building-Blocks: BBs (8)



Notation

P[BB1] = Prob. that BB1 is preserved.
P[BB2] = Prob. that BB2 is preserved.
P[BB1 in I1] = Prob. that BB1 exists in I1
P[BB1 in I2] = Prob. that BB1 exists in I2
P[BB2 in I1] = Prob. that BB2 exists in I1
P[BB2 in I2] = Prob. that BB2 exists in I2

Uniform Crossover

P[BB1] = P[Alter I1[3] & I1[4] & I1[5]] + P[Not alter I1[3] & I1[4] & I1[5]] = $(1/2)^3+(1/2)^3$ = 1/4
P[BB2] = P[Alter I1[9] & I1[10]] + P[Not alter I1[9] & I1[10]] = $(1/2)^2+(1/2)^2$ = 1/2

P[BB1 in I1 | BB1] = P[BB1 in I1 $\cap$ BB1]/P[BB1] = P[BB1 in I1]/P[BB1] = 1/2
P[BB2 in I1 | BB2] = P[BB2 in I1 $\cap$ BB2]/P[BB2] = P[BB2 in I1]/P[BB2] = 1/2
P[BB1 in I2 | BB1] = P[BB1 in I2 $\cap$ BB1]/P[BB1] = P[BB1 in I2]/P[BB1] = 1/2
P[BB2 in I2 | BB2] = P[BB2 in I2 $\cap$ BB2]/P[BB2] = P[BB2 in I2]/P[BB2] = 1/2

P[BB1 & BB2 appear after crossover] = P[BB1 & BB2 appear in I1] + P[BB1 & BB2 appear in I2] =
P[BB1 appears in I1] * P[BB2 appears in I1] + P[BB1 appears in I2] * P[BB2 appears in I2] =
P[BB1 in I1 | BB1]P[BB1] * P[BB2 in I1 | BB2]P[BB2] + P[BB1 in I2 | BB1]P[BB1] * P[BB2 in I2 | BB2]P[BB2] =
(1/2)(1/4) * (1/2)(1/2) + (1/2)(1/4) * (1/2)(1/2) = $\boxed{1/16}$

Summary

- There are Many Crossover Schemes!
  - One-, two-, n-point, and uniform crossover
    (n-point crossover becomes uniform crossover as n increases)
  - Search capability grows as n increases (but the disruption of BB increases)
- To Quickly Find the Optimum, BBs must be Preserved and Bred.
  - Crossover must be performed at the level of BBs:
    that is, BB-wise (n-point) crossover, BB-wise uniform crossover
- BBs are not Known in Most Real-world Problems
  - Under this circumstance, the primary importance is the BB preservation
  - To do this, the search capability must be offered at the minimum level
  - Thus, one- or two-point crossover is the most promising choice!

If BBs are known, we can maximize the performance by using the BB-wise uniform crossover. But BB information is not available in general.
Thus, knowing BB information is a very important issue of GAs. How?