Evolutionary Algorithms:
Constrained Optimization, Parallel EAs

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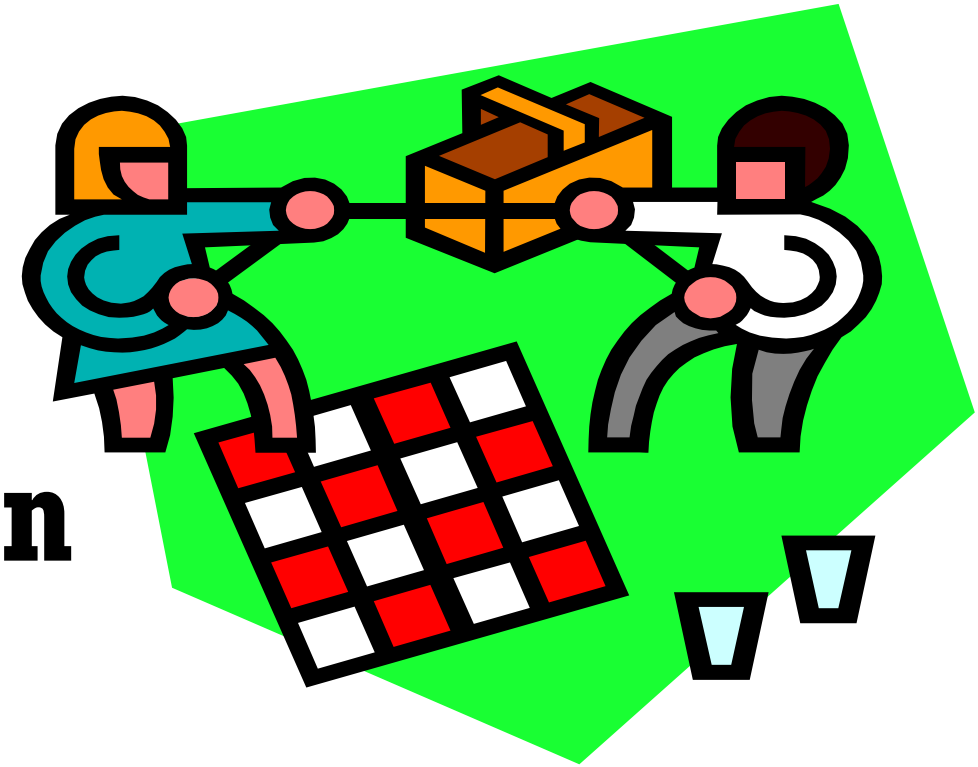
Constrained Evolutionary Optimization



Constrained Optimization (1)

SEAL.

❖ What is the Constrained Optimization Problems (COPs) ?

- ➢ A class of **Optimization Problems** that have **Constraint Conditions**
  - ✓ The aim is **to find the best solution(s)** while ***satisfying the Constraint Conditions***

For a search space Ω

There is a function $f : \Omega \mapsto \Re$

Unconstrained Optimization

The task is to find $x^* = \arg\max_{x \in \Omega} f$

Here, x is a vector of decision variables, and f is the objective function

Constrained Optimization

The task is to find $x^* = \arg\max_{x \in \Omega} f$

subject to $g_i(x) \leq 0, \ i = 1, \cdots, m$

$h_i(x) = 0, \ j = 1, \cdots, p$

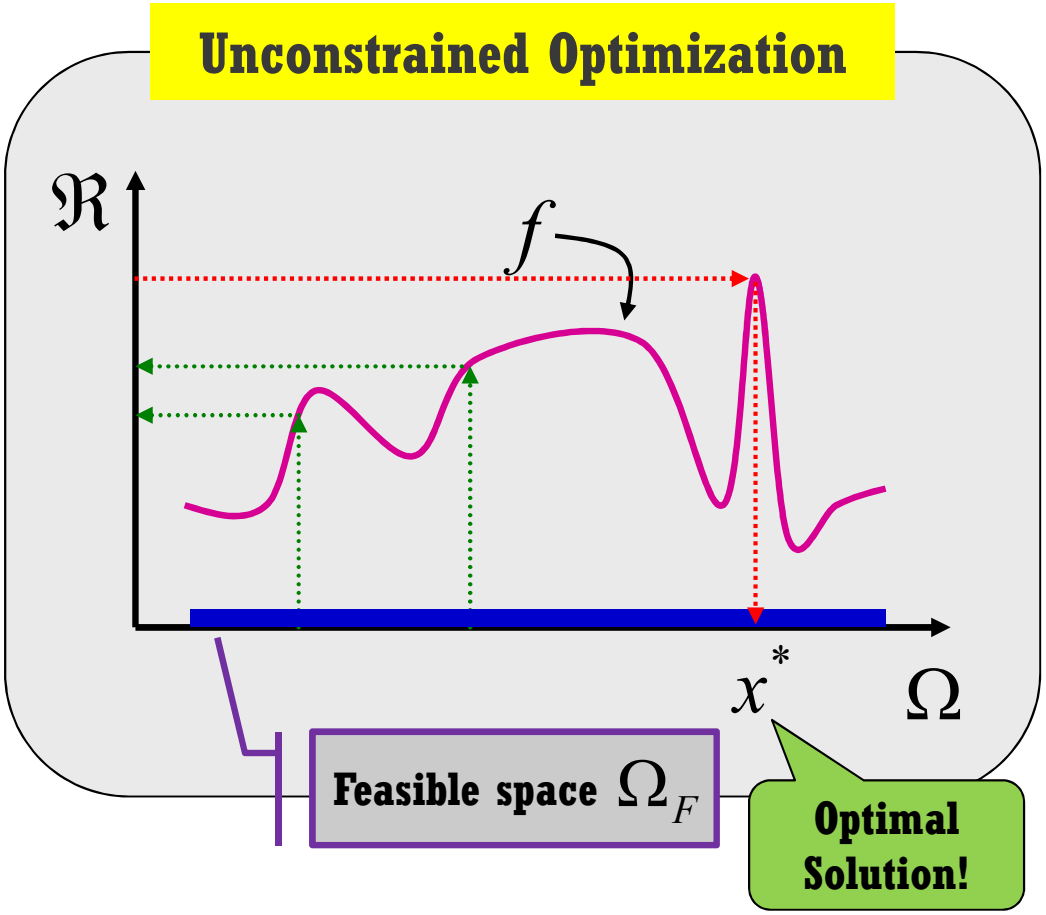
Inequality Constraints

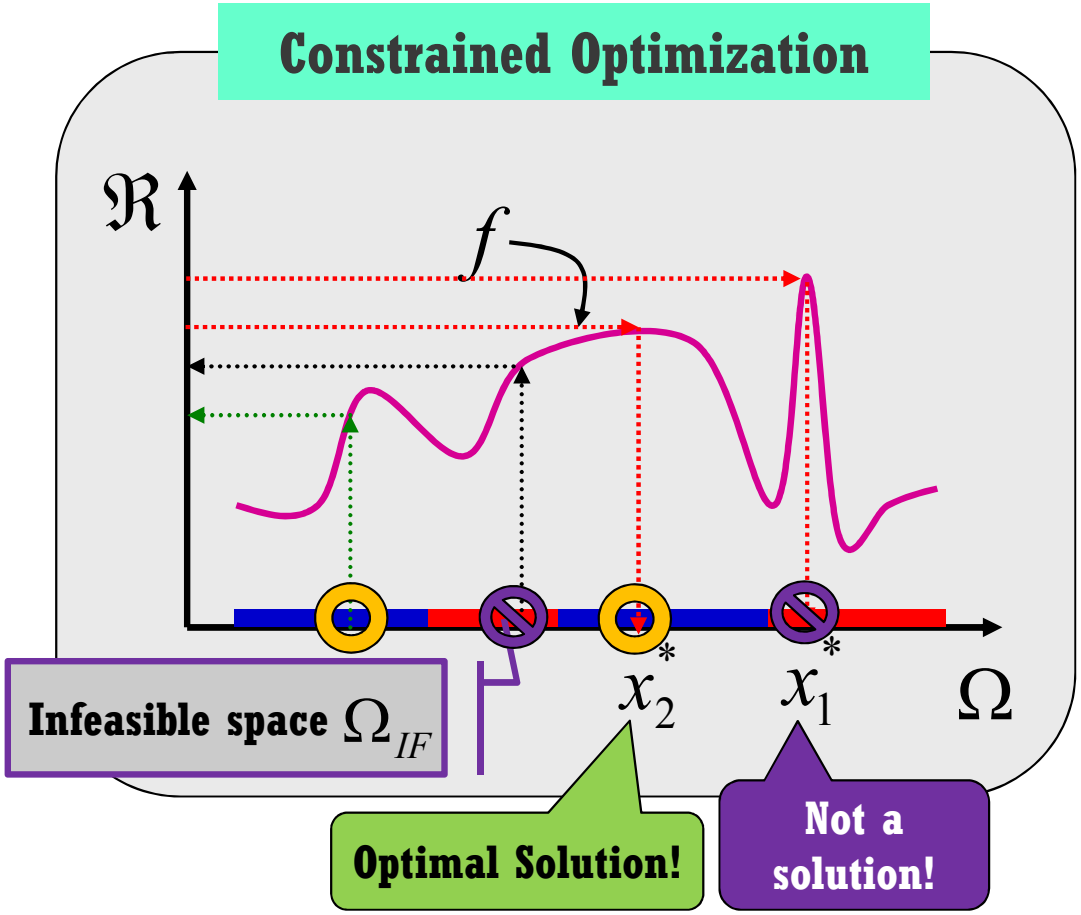
Equality Constraints

Constrained Optimization (2)

❖ Regarding the Search Space

- ➢ **Feasible search space of COPs is a key issue!**
  - ✓ **It is relatively very narrow! ➔ Thus, hard to discover the optimal solution!**



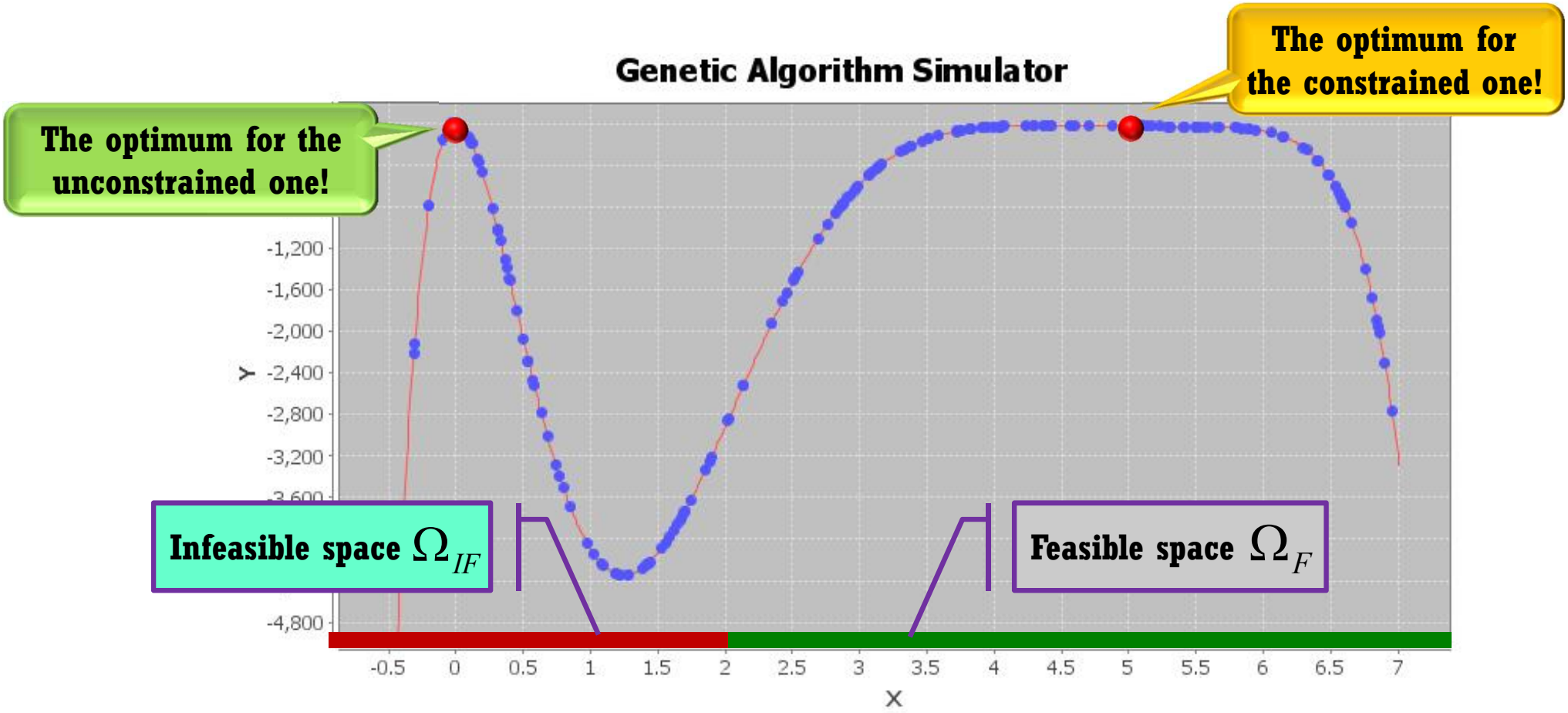


Constrained Optimization (3)

❖ Example

➢ **Find the optimal solutions w.r.t the maximization of** $f(x) = -x^2\{1+(x-5)^6\}$

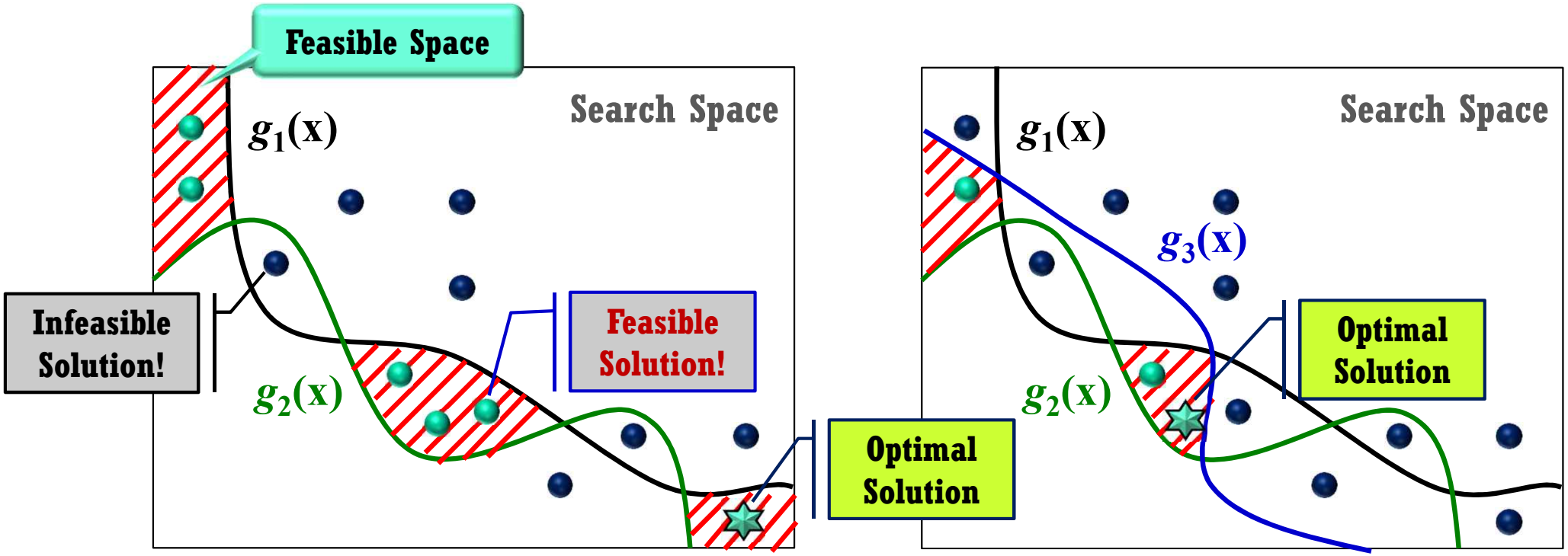
- ✓ **If there is no constraint, then x=0 is the optimum!**
- ✓ **If a constraint is u(x-2) ≥ 0, then x=5 is the optimum!**



Constrained Optimization (4)

❖ How Difficult the Constraints are?

- ➢ **Constraints significantly reduce the amount of feasible search space.**
  - ✓ In real-world cases, the feasible spaces are very narrow! (e.g., less than 0.001%)
  - ✓ To make matters worse, it is very hard to handle the constraints mathematically!



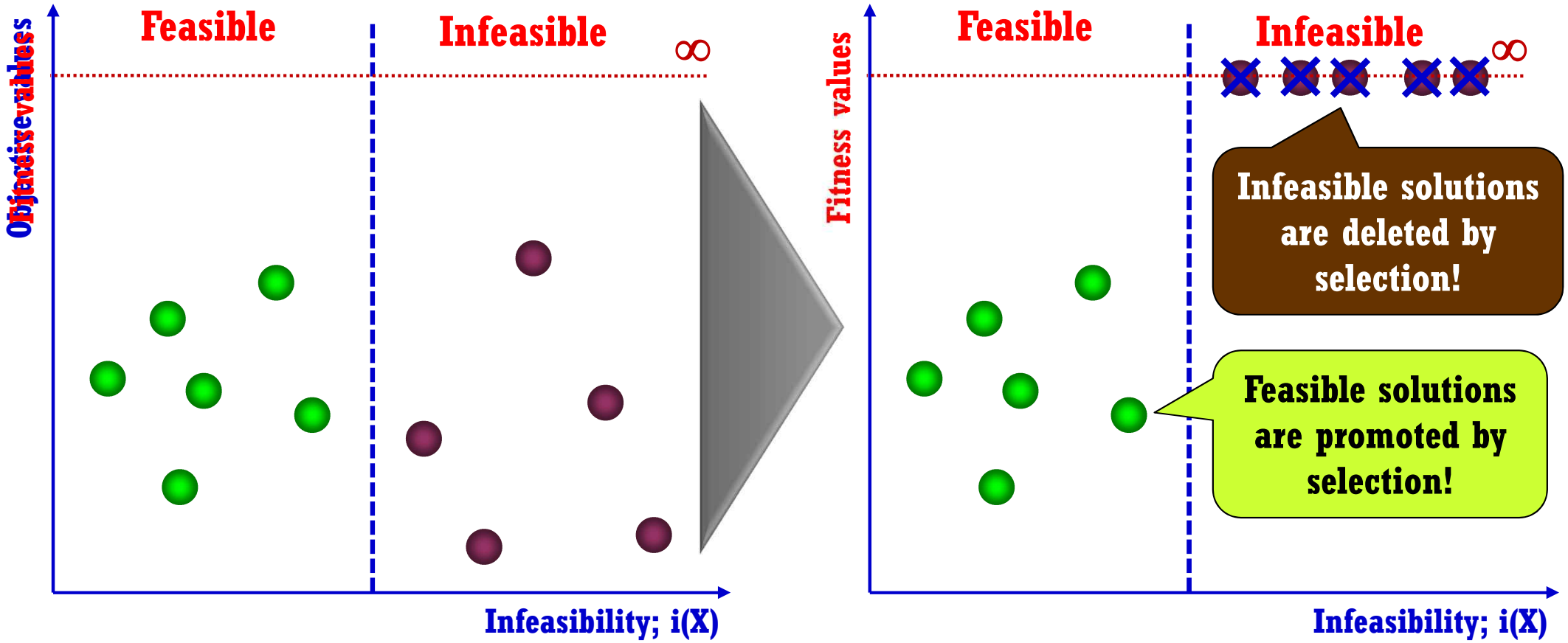
Case I. Number of Constraints is 2

Case II. Number of Constraints is 3

Constrained Evolutionary Opt. (1)

❖ How to Handle the Constraints in terms of GAs?

- ➢ The Simplistic Method: Not consider the infeasible solutions!
  - ✓ EAs solve the optimization problems by referring to the fitness values
  - ✓ In the minimization case, the infinity value is assigned to the infeasible solutions



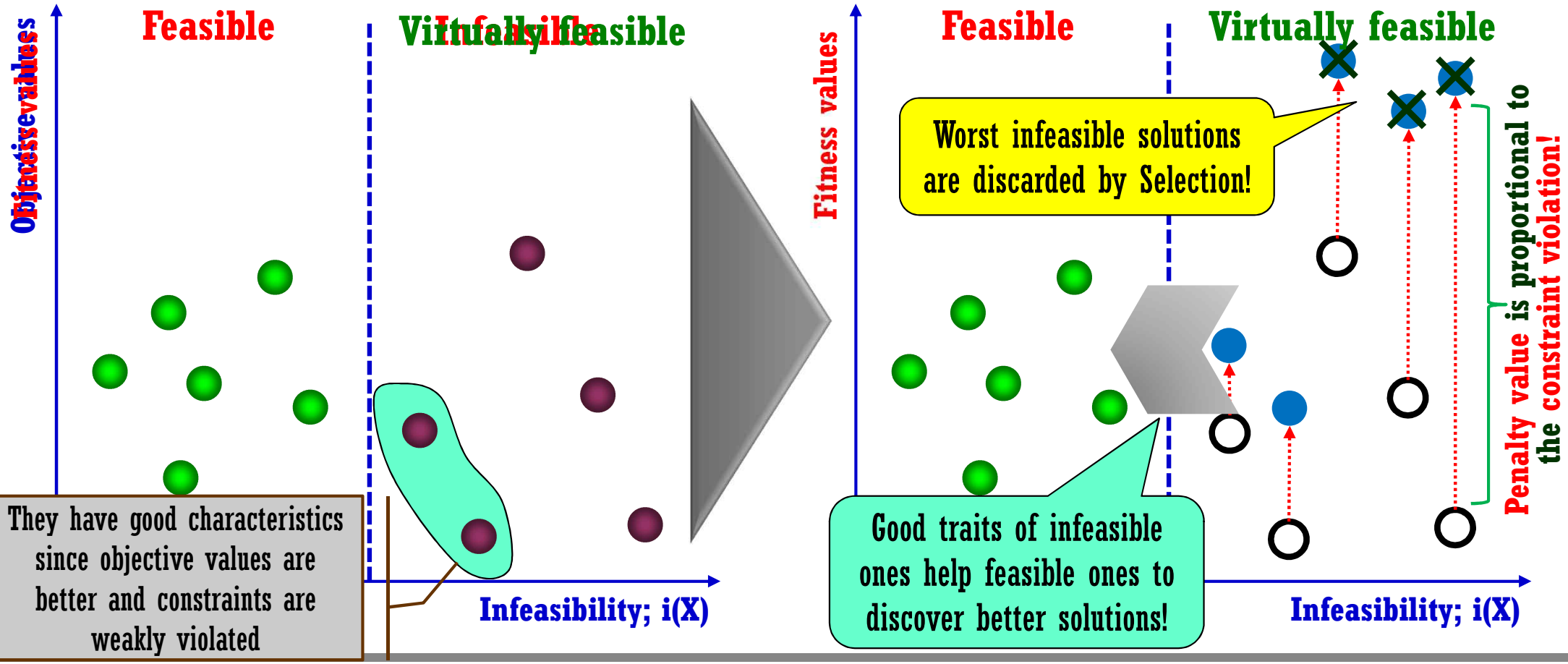
Constrained Evolutionary Opt. (2)

❖ More Efficient Method?

➢ **The Penalty Method: Participate the infeasible solutions as well in evolution!**

✓ Transform a COP into an unconstrained one by adding/subtracting a certain value.

✓ In the minimization case, the value is based on the amount of constraint violation.



Constrained Evolutionary Opt. (3)

❖ Example: Knapsack Problem

- A set of n items is available to packed into a knapsack with capacity C units.
- Item i has a value w_i (e.g., $) and uses up c_i units (e.g., kg) of capacity
- The aim is to *maximize* the amount of values while keeping the overall capacity

Problem Formulation

Objective function $f(x)$

$$\max \sum_{i=1}^{n} w_i x_i \quad \text{subject to} \quad \sum_{i=1}^{n} c_i x_i \le C$$

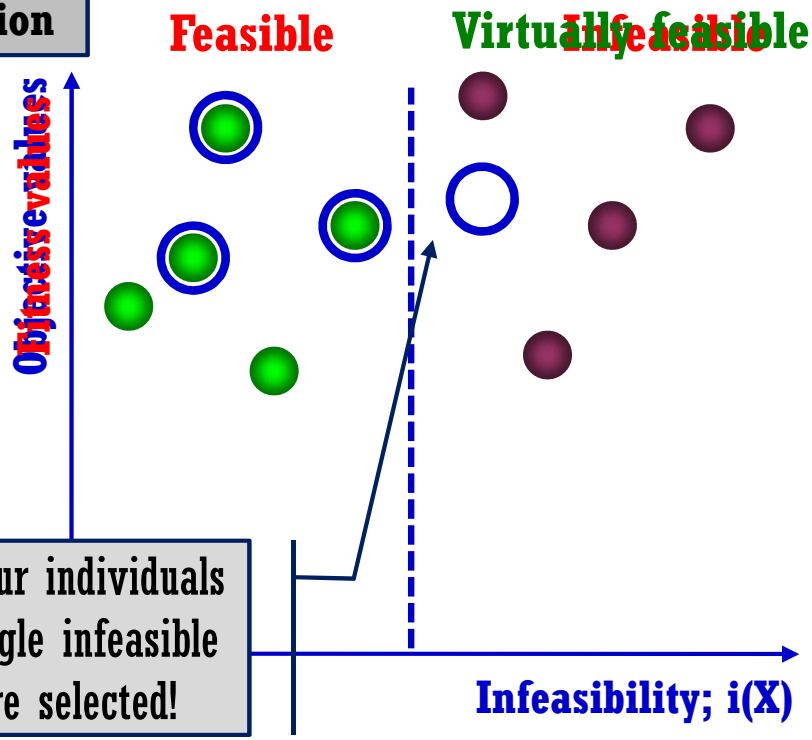
Constraint condition

Penalty function $p(x)$

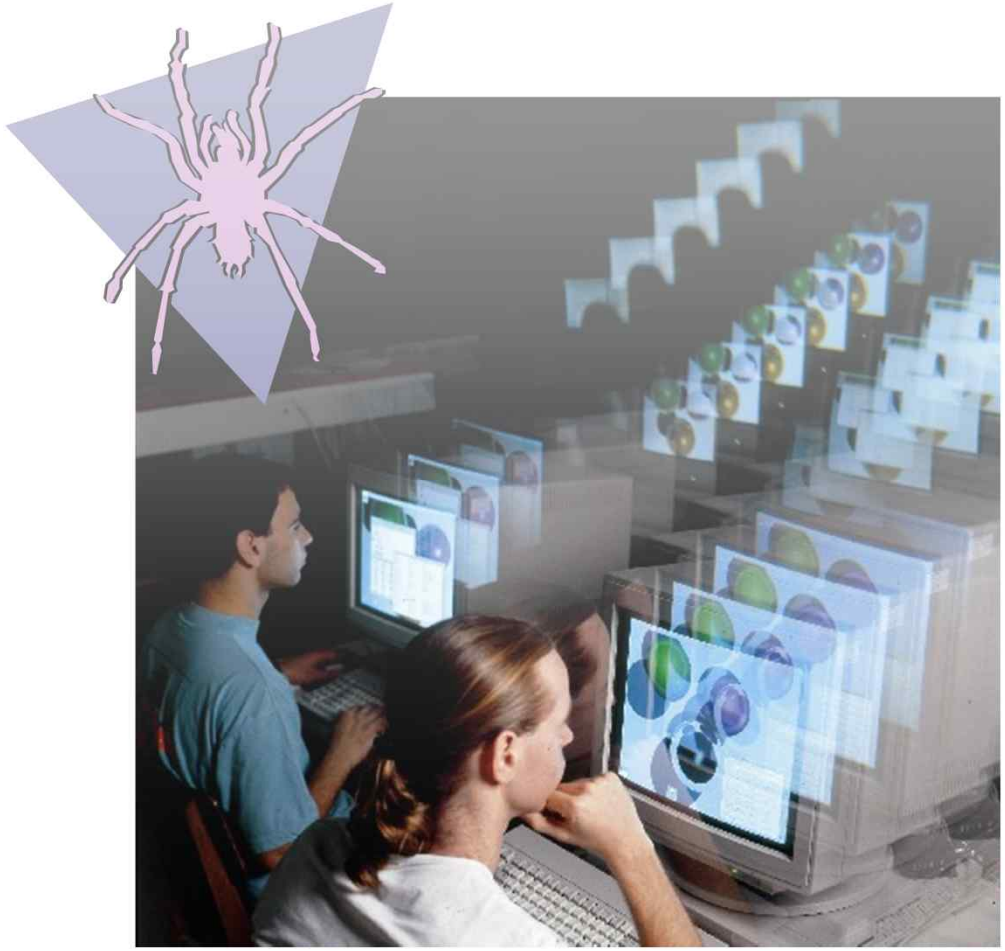
$$p(x) = \begin{cases} 0, & \text{if feasible solution} \\ \alpha(\sum c_i x_i - C)^2, & \text{otherwise} \end{cases}$$

Fitness function = Objective function – Penalty function

$$F(x) = f(x) - p(x) = \sum_{i=1}^{n} w_i x_i - p(x)$$

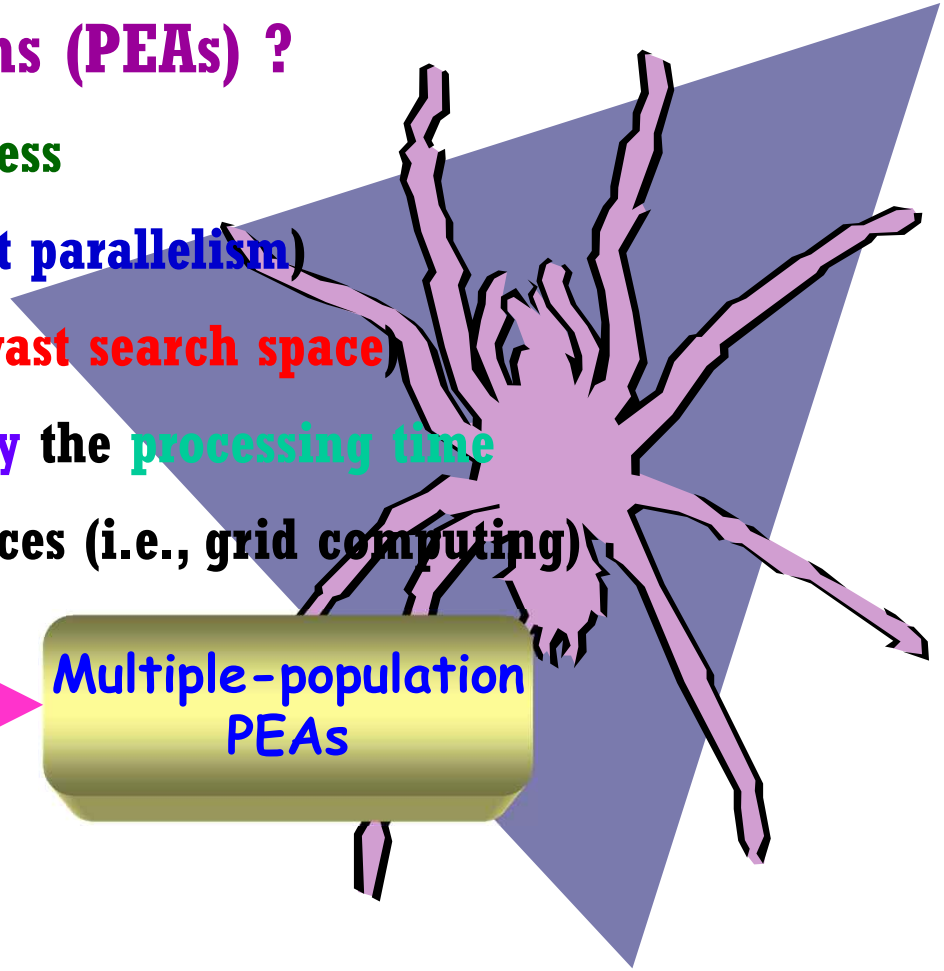


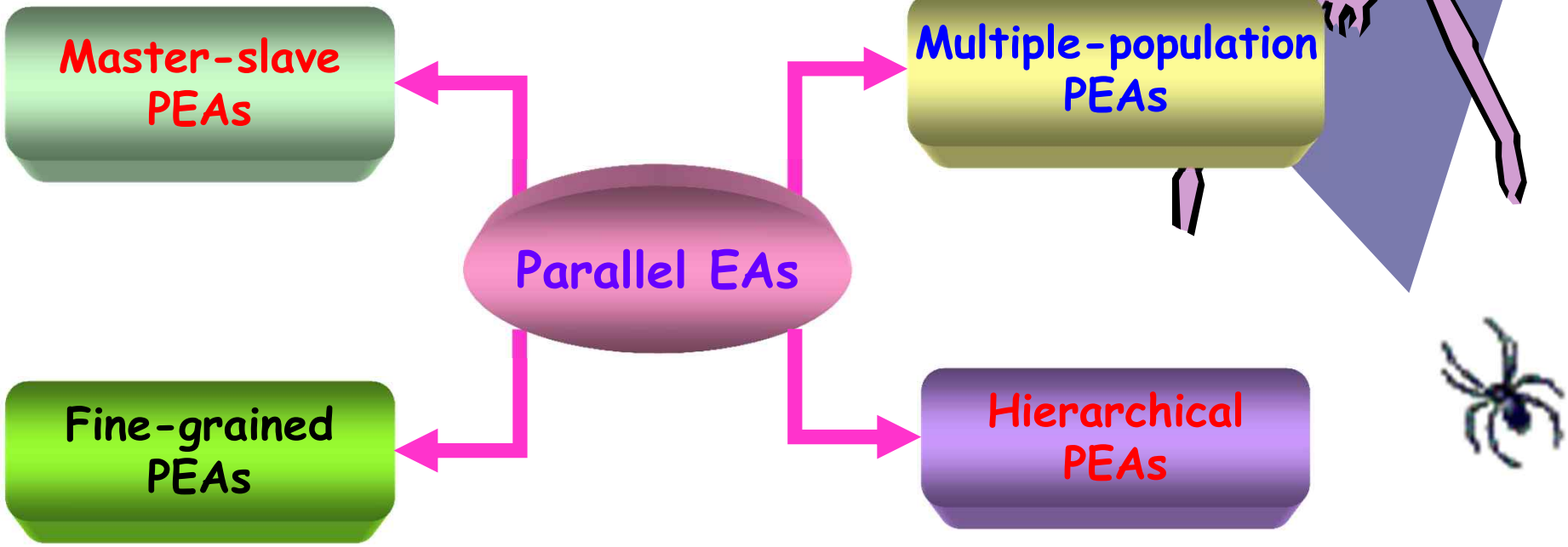
Parallel Evolutionary Algorithms



Parallel EAs (1/3)

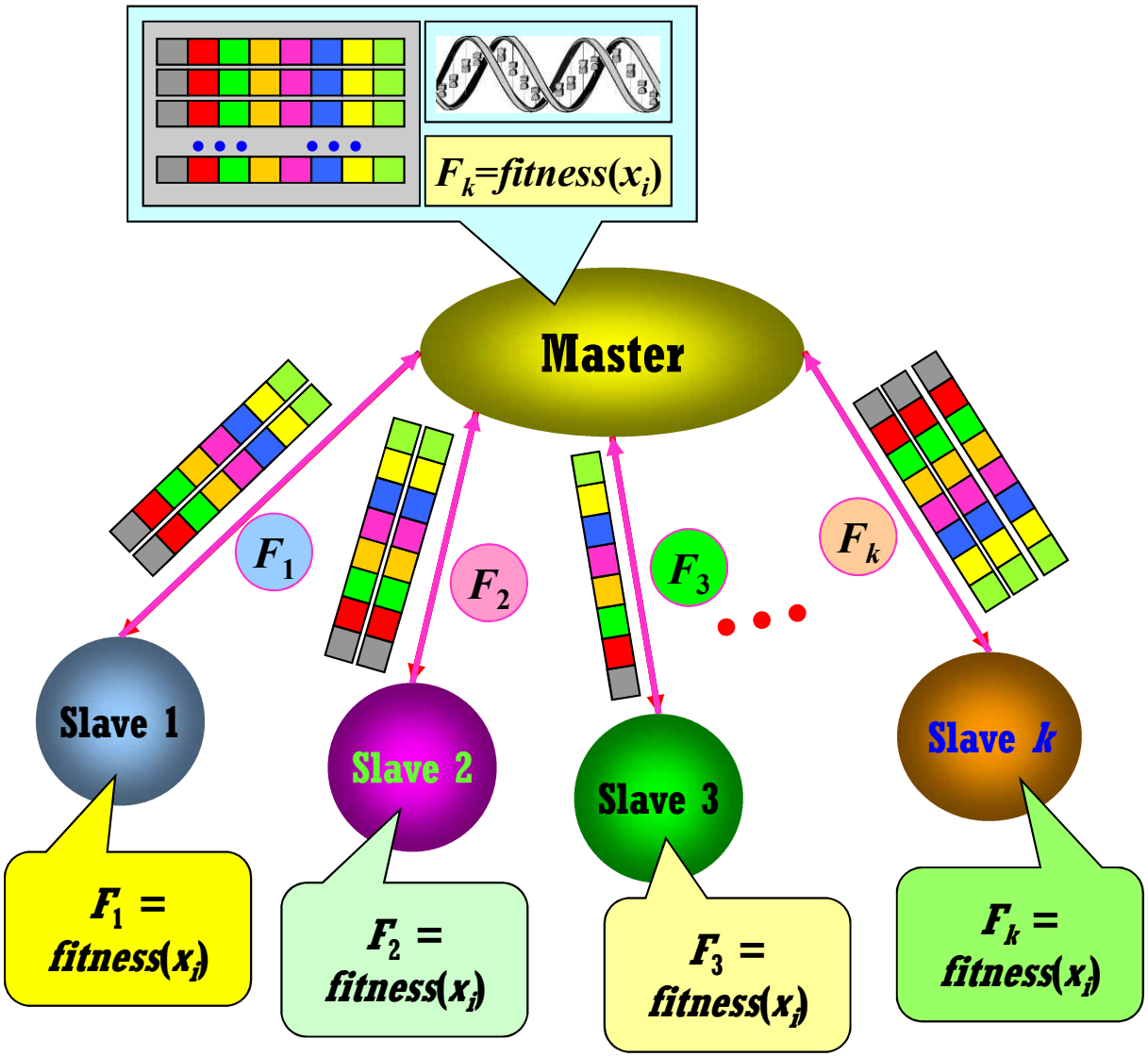
- **Why Parallel Evolutionary Algorithms (PEAs) ?**
  - Evolution itself is a highly parallel process
  - EAs are easy to parallelize (i.e., inherent parallelism)
  - PEAs can solve difficult problems (i.e., vast search space)
  - Parallel mechanism reduces considerably the processing time
  - There are tremendous computing resources (i.e., grid computing)



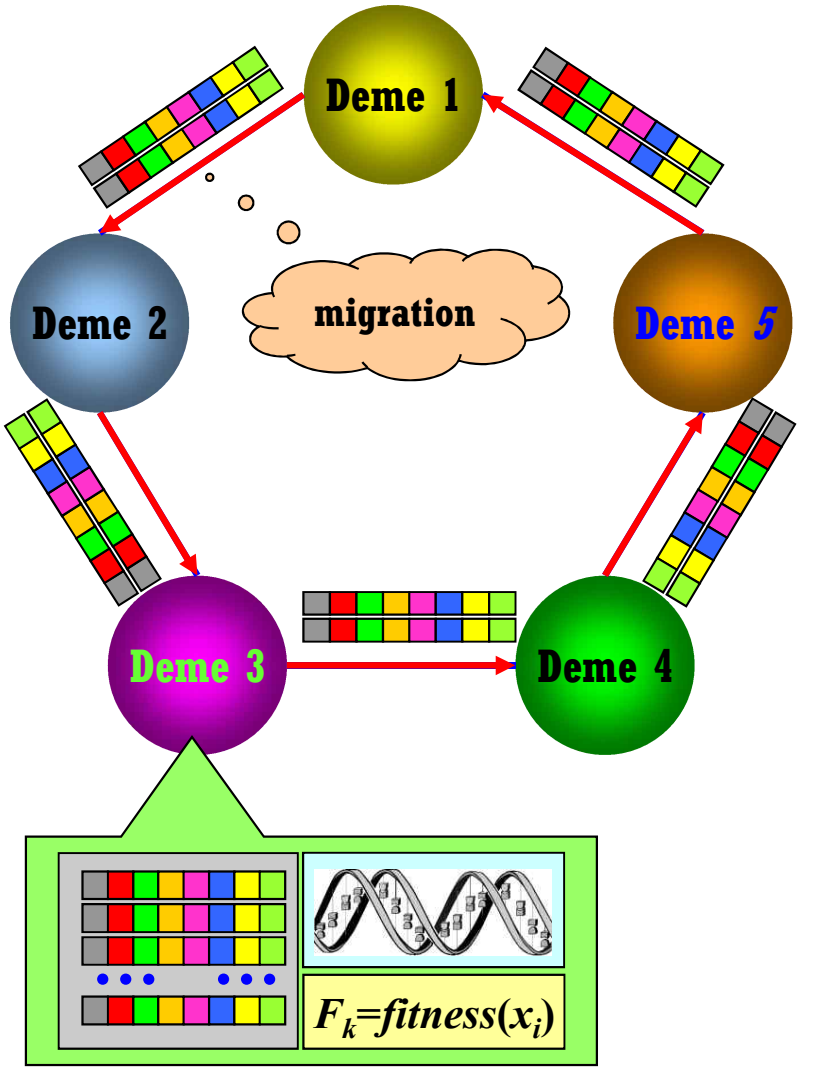


Parallel EAs (2/3)

Master-slave PEAs

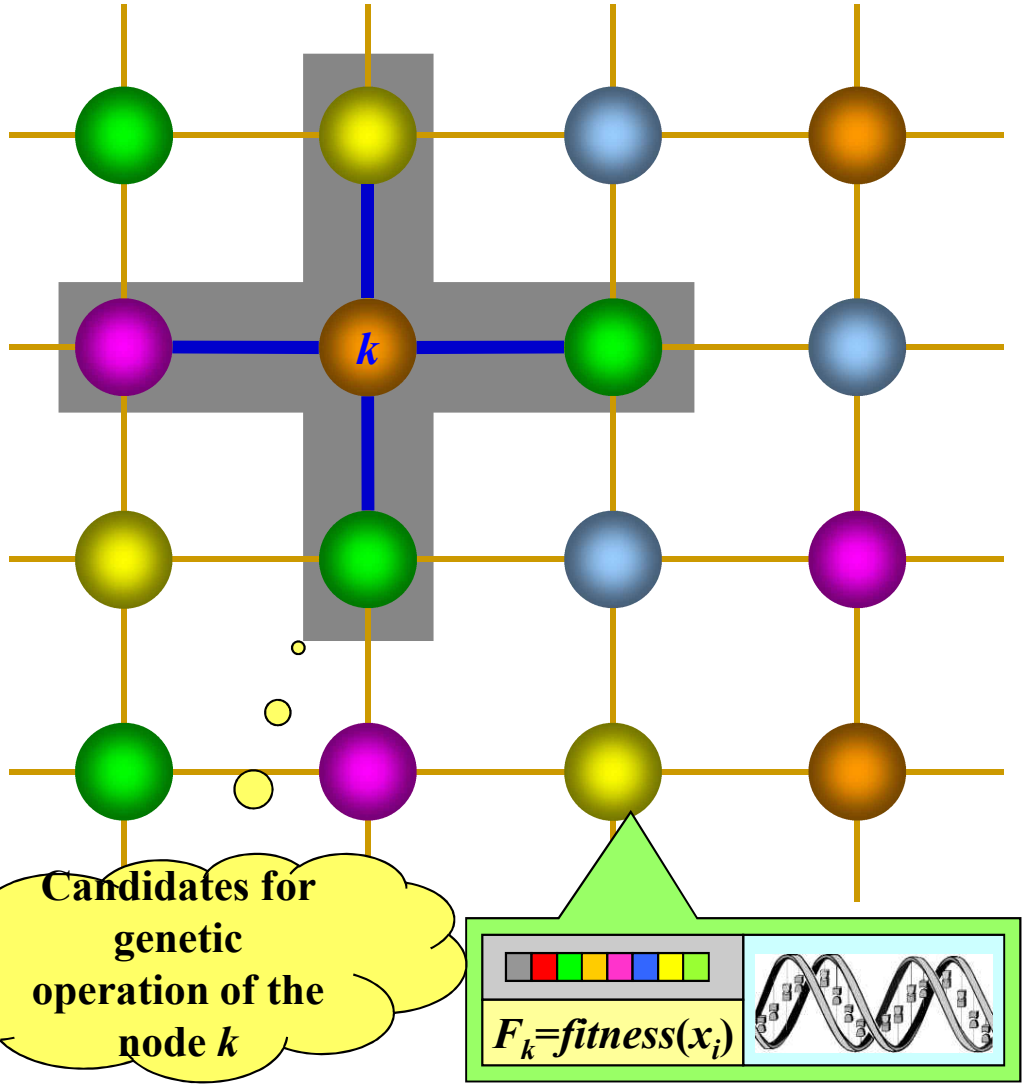


Multiple-population PEAs

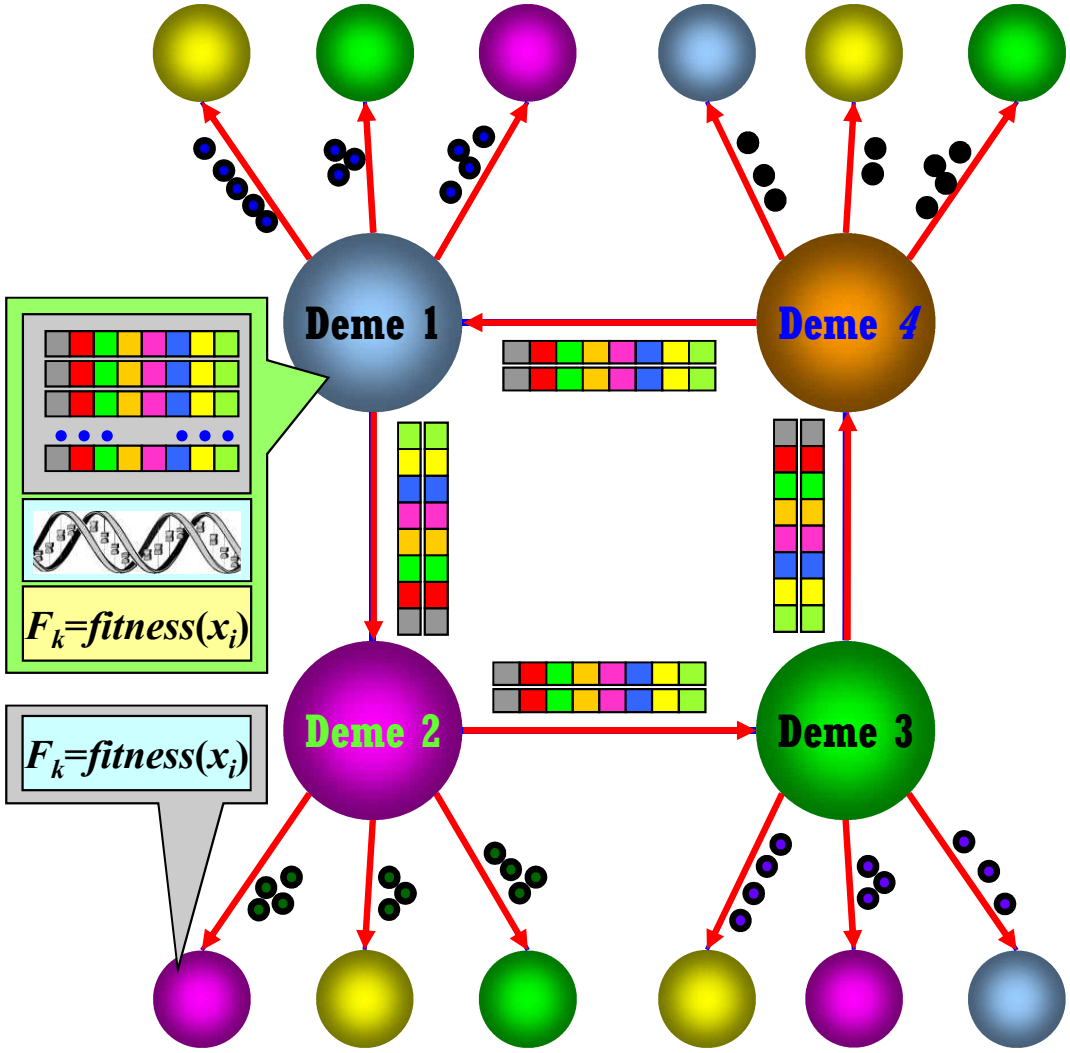


Parallel EAs (3/3)

Fine-grained PEAs



Hierarchical PEAs



Summary

❖ Constrained Evolutionary Algorithms

- ➢ Constraints make optimization problems difficult to be solved
- ➢ Feasible space significantly reduces as the number of constraints increase
- ➢ Penalty value is imposed on the objective value by the constraint violation
- ➔ The design of penalty function is a key issue in COPs!

❖ Parallel Evolutionary Algorithms (PEAs)

- ➢ It is natural to make EAs parallel due to their Inherent Parallelism
- ➢ Generally, parallel EAs outperform sequential EAs
- ➔ The topology and the migration of PEAs are important topics in PEAs!