FRC 2014

Shooter physics analysis

for Team 8

Hayden Brownell   Jan 11, 2014

The purpose of this analysis is to estimate quantitatively what is required for each type of shooter and a means for quick comparative assessment. A rough theoretical assessment can save you weeks of experimentation in eliminating options that clearly won't fit within the FRC size and weight limits. But this model is just theory. Experimental results are the only way to prove your design is accurate enough.

The following is a typical physics approach considering energy and momentum. The concepts here should be familiar to a first year physics student though I have tried to present them so that everyone can follow the reasoning. The ball is assumed to be perfectly elastic, springs are assumed to be ideal and friction is neglected in order to reach a conclusion in reasonable time. These assumptions are likely close enough to the actual case (if good bearings are chosen) that the analytical results will yield a good approximation to actual performance. Air friction is likely to be the largest contributor to the expected error. Effects such as knuckling may be an issue depending on the ball velocity if the ball is launched with no spin.

This description is pretty dense. If the details make your eyes glaze over, you can still estimate the desired launch speed (s_0) from Equations 4 and 3 and then compute the minimum work required from Equation 10 and determine which push/kick mechanisms can supply that much work or determine the roller length and torque needed by Equation 9. Remember to use consistent units or else the results are useless.

Shooter requirements

The shooter must launch the ball so that it passes through the goal opening. Given the ball is spherical (when in the air), it is convenient to define its location as the center of the ball. The range of the center position that will allow the ball to pass through the opening is then the opening width as seen from the direction that the ball approaches the goal minus the ball diameter. For example, given the upper goal width $G = 37''$ and the ball diameter $D = 25''$, the center position range is $12''$ when approaching head on. If the ball approaches at $45°$ from normal (i.e. perpendicular) to the goal, then the range is only $(37'' \cos[45] - 25'') = 1''$; in other words very unlikely to get through. Therefore, the ball trajectory should be within roughly 20 degrees of normal to have a reasonable chance of success. The lower goals have similar considerations.

After the ball is launched by the shooter, it moves only under the influence of gravity. The acceleration of the ball is a constant $g = 10$ meters/second^2 vertically downward, independent of its mass. (Let x indicate the horizontal position and y the vertical.) The horizontal acceleration is zero so the horizontal speed of the center of the ball is constant (assuming the ball is spherical when in the air): $s_x = s_0 \cos[\theta]$ where s_0 is the initial speed and θ is the inclination angle of the shooter. The horizontal center position is then

$$x[t] = x_0 + s_0 \cos[\theta]\, t \tag{1}$$

where t is the elapsed time from launch and x_0 is the shooter horizontal position. Gravity constantly changes the vertical speed in the downward direction (assumed to be the negative y direction): $s_y[t] = s_0 \sin[\theta] - gt$. Adding the incremental change in position at at each increment in time $\Delta y[t] = y[t] + s_y[t]\Delta t$ yields the vertical center position

$$y[t] = y_0 + s_0 \sin[\theta]\, t - \frac{1}{2}gt^2, \tag{2}$$

where y_0 is the vertical position of the shooter. Note that if the ball is spinning when launched, the ball keeps spinning without affecting the center position trajectory (unless air friction is significant, in which case you get a curve ball in the direction that the front of the ball is spinning).

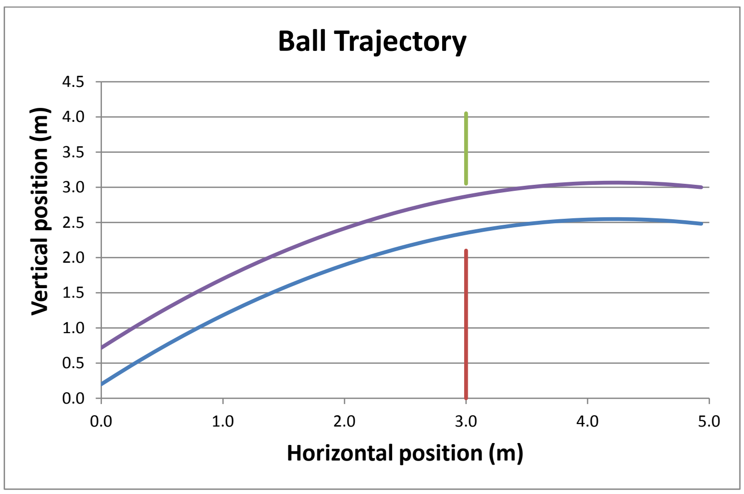
This trajectory is an inverted parabola. The ball reaches a maximum height when the vertical speed is zero: $t_{\max} = s_0 \sin[\theta]/g$. The peak height is then

$$y_{\max} = y[t_{\max}] = y_0 + s_0^2 \sin^2[\theta]/2g \tag{3}$$

which occurs at a horizontal position of

$$x_{\max} = x[t_{\max}] = x_0 + s_0^2 \sin[\theta] \cos[\theta]/g. \tag{4}$$

Try plotting $y[t]$ versus $x[t]$ in Excel or any other plotting application and vary the input parameters to see what combinations work. Remember to keep all units consistent otherwise your results are nonsense! The most common units system is "MKS" with lengths in meters, masses in kilograms and time in seconds. Then force is measured in Newtons (N $\equiv$ kg m / s^2), pressure in Pascals (Pa $\equiv$ N / m^2) and energy in Joules (J $\equiv$ kg m^2 / s^2). Figure shows an example of shooting at the high goal.



Now estimating the likely horizontal distance from shooter to goal and the height of the goal, the initial speed s_0 and inclination angle can be estimated that gives the best chance of scoring. You should factor in that the conditions may not be as modeled here. The "best chance of scoring" means a solution where the ball still passes through the goal even if the initial speed, inclination angle and/or distance to the goal are varied by the greatest extent around their chosen values.

Bonus points: An idealized drag effect due to air friction can be taken into account by including a drag force with gravity in Newton's equation ($F = Ma$), where M is the ball mass. For a sphere traveling through air at moderate speed, the drag force can approximated as $F_{\text{drag}} = (1/4)(\text{density of air})(\pi D^2/4)(\text{speed}^2)$ opposing the direction of motion. The acceleration at any given time is then $a_x[t] = (F_{\text{drag}}[t]/M) \cos[\phi[t]]$ and $a_y[t] = g + (F_{\text{drag}}[t]/M) \sin[\phi[t]]$ in the horizontal and vertical directions respectively. The direction of travel is defined by the instantaneous inclination angle $\phi[t] = \tan^{-1}[y[t]/x[t]]$. Because the drag force is proportional to the square of the speed, there is no solution in terms of common functions. You need to sum the incremental change in speed and position implied by the acceleration at each time increment to get the trajectory.

Shooter design:

The shooter mechanisms considered so far fall into three categories: Push, kick and roller. Push and kick can be analyzed similarly while rollers involve different issues. The common requirement is the ball is launched with the desired initial speed (s_0) found above. A push mechanism accelerates the ball from rest. A kick mechanism collides with the ball producing impulse acceleration. In a roller shooter, the ball is accelerated by contact with moving belts or wheels. A guiding structure steers the ball in the desired direction.

Push and Kick mechanisms:

In either case, the force can be applied through gravity, springs, elastic bands, pneumatic piston, etc., whether directly or indirectly through levers or gears. In every case, the driving mechanism initially has potential energy that is converted into kinetic energy (motion) of the ball. (Note that the ball is highly elastic. There is some initial deformation of the ball by the actuator. Any deformation of the ball acts like a spring, storing some potential energy which is then converted to kinetic energy when the deformed surface recoils against the actuator.) The amount of energy exchanged is called the work done (W). The task is to determine what conditions allow for the most work done so that the mechanism can store the least potential energy. Generally, a mechanism is larger and more difficult to use the greater its energy capacity. The mechanism that has the greatest energy density (i.e. capacity per volume) will usually be the easier choice to implement.

The work done can be calculated as the product of the applied force times the distance moved (designated here by Δz): $\Delta W = F\Delta z$. If the force changes during the motion, then you need to add the incremental work done for increments in distance moved over which the force is roughly constant: $W = \sum_i F[z_i]\Delta z_i$. (This approach leads to an integral formula $W = \int F[z]dz$ when the increments are very small.)

For push mechanisms, the actuator applies a force on the ball from when the ball is at rest through to when the actuator loses contact with the ball.

The force applied and work done by various mechanisms are:

- Spring: $F_{\text{spring}} = k(z - \bar{z})$ where k is the spring constant and $\bar{z}$ is the relaxed length of the spring. You can measure k by hanging a known weight from one end and measuring the change in length. The total work done by a spring $W_{\text{spring}} = k(z_0 - \bar{z})^2/2$ where z_0 is the initial stretched or compressed length.
- Surgical tubing acts like a spring.
- Pneumatics: Given the mechanism is a gas reservoir of volume V_0 filled initially to pressure P_0 pressing on a piston actuator with bore diameter (B). The bore area is then $A = \pi B^2/4$. As the actuator extends, the volume that the gas occupies increases and the pressure decreases inversely. Therefore, the pressure for a given actuator position is $P[z] = P_0V_0/(zA)$ if it is understood that the initial value of z is $z_0 = V_0/A$. The force applied by the actuator is the pressure times the bore area: $F_p[z] = P[z]A = P_0V_0/z$. The work done by a pneumatic piston is, after integration, $W_p = P_0V_0 \ln[z_{\text{final}}/z_0]$ where $\ln[u]$ is the natural logarithm and z_{final} is the actuator position when the actuator loses contact with the ball.
- Gravity: The gravitational force on mass m is $F_g = mg$ in the vertical direction. Therefore a lever or gear system must be used to convert vertical motion into motion at the shooter inclination angle. Note that a pendulum is essentially a lever; a vertical drop followed by a small lever has the same outcome as a pendulum. The work done by gravity is proportional to the vertical drop of the mass: $W_g = mg(z_0 - z_{\text{final}})$ where $(z_0 - z_{\text{final}})$ is the total vertical drop.

Note that if, for example, a short, stiff spring is more convenient for other reasons, you can extend the distance force is applied to the ball by inserting a lever or gear pair. This distance is changed by the ratio of the lever arms or gear diameter ratio but the force is also changed inversely proportionally so that the total work done remains the same: $\Delta W = (F/\text{ratio})(\text{ratio}\,\Delta z) = F\Delta z$.

The work done by the push mechanism equals the total energy of the ball plus any residual energy in the mechanism. The ball energy immediately after launch is practically all kinetic energy: $E_{\text{ball}} = Ms_0^2/2$. The residual energy of the mechanism is primarily the kinetic energy of the moving parts of the mechanism (i.e piston, lever, spring, etc.) which is proportional to their mass. The aim is to maximize the ball energy and minimize the residual energy, which can be achieved by minimizing the actuator mass. If the residual energy is small enough that neglecting it causes an insignificant error, then the initial ball speed is $s_0 = \sqrt{2MW}$. You can now judge which mechanisms have sufficient stored energy in a small enough space appropriate for your robot design.

Kick mechanisms act essentially the same as push in the sense that an intermediate weight (i.e. the actuator with possibly additional weight attached) is pushed until it contacts the ball, at which point the intermediate mass, rather than the actuating force, exerts a short impulse on the ball. In this elastic collision between intermediate mass and ball, both total momentum and energy remain constant. Before the collision, ball is at rest and so has zero momentum while the intermediate mass (m) has speed ($s_{\text{m,before}}$) calculated as in the push case, so the total momentum is $p_{\text{before}} = ms_{\text{m,before}}$. After the collision, the momentum is $p_{\text{after}} = ms_{\text{m,after}} + Ms_0$. The energy just before and just after (after the ball has recovered its spherical shape) is entirely kinetic so that the total energy before and after is: $E_{\text{before}} = ms^2_{\text{m,before}}/2$ and $E_{\text{after}} = ms^2_{\text{m,after}}/2 + Ms_0^2/2$. The conservation of momentum and energy can then be expressed by these equations:

$$\begin{aligned} ms_{\text{m,before}} &= ms_{\text{m,after}} + Ms_0 && (5) \\ \frac{1}{2}ms^2_{\text{m,before}} &= \frac{1}{2}ms^2_{\text{m,after}} + \frac{1}{2}Ms_0^2. && (6) \end{aligned}$$

Nature obeys both conditions simultaneously. Solving these two equations for the initial ball speed yields: $s_0 = (2/(\sqrt{m/M} + \sqrt{M/m}))s_m$. The coefficient has a maximum value of one at $m = M$ and is smaller for either $m < M$ or $m > M$. This is the case where the intermediate mass is stopped by the collision and therefore has no residual energy after the collision, consistent with the reasoning for push mechanisms. Therefore, the intermediate mass should equal the ball mass to get the largest transfer of energy.

Note that if the shooter causes the ball to spin, independent of the mechanism, then some of the energy is consumed by the kinetic energy of the spin ($E_{\text{spin}} = (2/5)M(D/2)^2\omega^2/2$ where omega is the angular speed of spin) and therefore reduces the initial speed s_0 of the center of the ball. One case where this can be quantified concretely is when friction against the guiding rails does not allow the ball to slip. Then the spin rate is simply related to the center speed: $\omega = s_0/(D/2)$. The spin energy is then $E_{\text{spin}} = (1/5)Ms_0^2$, which is added to the ball energy:

Push case with spinning ball: $E_{\text{ball}} = \frac{7}{10}Ms_0^2$;

Kick case with spinning ball: $E_{\text{after}} = \frac{1}{2}ms^2_{\text{m,after}} + \frac{7}{10}Ms_0^2$.

Roller mechanisms

A roller mechanism accelerates the ball through friction between the ball and roller. The friction should be sufficient to assure no slipping, which would reduce the acceleration. The ball will be deformed by contact but then the ball as a whole will accelerate when it recovers its spherical shape. Rollers might be implemented as a series of individual wheels or a single belt on a single rail. (A sequence of multiple belts is feasible too and may be preferable in some cases.) Multiple rails might be implemented for guiding the ball. Motors that drive the rollers are usually rated by torque so the required torque is derived here.

In the case of a sequence of wheels, each wheel would boost the ball speed so that the desired launch speed (s_0) is achieved after the final wheel. Therefore, each wheel must be spinning to match the speed of the ball at that point in the sequence. For example, if there are four wheels, the wheel rims should be moving at $s_0/4, s_0/2, 3s_0/4, s_0$ respectively. The spin rate equals the rim speed divided by the wheel radius. Separate drive systems are then required to achieve the correct individual spin rates. (You could achieve some simplification by applying a single ramping voltage to all of the motors.) The torque required to drive the wheels is the force divided by the wheel radius. The force in this can be estimated as the change in the momentum of the ball divided by the time the wheel is in contact with the ball. The change in momentum is $\Delta p = M\Delta s$. The duration of contact can be estimated by the angular range of contact divided by the spin rate. For an N wheel sequence $\Delta s = s_0/N$. Let the angular range of contact (a measured quantity) be α. The spin rate of the nth wheel in the sequence should be equal to $\Omega = n\, s_0/NR$, where R is the wheel radius. Then the required torque would be

$$\tau_{\rm wheel} = \frac{M\Delta s}{\alpha/\Omega} \tag{7}$$

$$= \frac{n}{N^2}\frac{Ms_0^2}{\alpha R^2} \tag{8}$$

split Therefore, you want many, large wheels to reduce the required torque. To launch the ball, the wheels could be started with the ball in contact or the ball could be pushed onto the already moving wheels.

The advantage of a belt is the duration of contact is much longer and so the necessary torque is lower if the belt is started with the stationary ball in contact. (This is similar to the push case.) The belt speed can be ramped up from zero to the desired launch speed in time to coincide with ball exiting the shooter. This scheme requires only a single motor to drive the belt and the launch speed is adjustable in real time. To analyze this case, the ball accelerates from zero to s_0 over the length of the belt (L). If you choose a constant ramp in speed, the acceleration is constant. Then analogous to Equation 3, the acceleration must be $a_{\rm belt} = s_0^2/2L$ to achieve a launch speed of s_0 over the belt length L. The duration of contact is then $\Delta t_{\rm belt} = s_0/a_{\rm belt}$ and the applied force is $F_{\rm belt} = Ma_{\rm belt}$. Assuming that the pulley driving the belt has a radius of R, the required torque is

$$\tau_{\rm belt} = \frac{Ms_0^2}{2LR}. \tag{9}$$

The belt requires lower torque than a wheel sequence if $2L > \alpha NR$ Because αNR is the total length the ball is in contact with the wheels, which cannot be greater than roughly $L/3$, then the belt scheme requires less than one sixth the torque of the wheel scheme.

Conclusion

Several theoretical conclusions can be drawn from this analysis. First, the desired speed and direction provided by the the shooter can be estimated from equations 1 and 2. Second, the mechanism must have enough initially stored energy to give to the ball:

$$E_{mechanism} > W = E_{\rm ball} = Ms_0^2/2. \tag{10}$$

Third, the active mass of the shooter should be as small as possible for push mechanisms or equal to the ball mass for kick mechanisms. Fourth, roller mechanism analysis indicates a clear advantage both in simplicity and required torque in favor of belt systems.