MTH 451/EGR 551 - Homework 3
Due Monday March 15, 2021 at 5 p.m.
Please submit all computer codes you developed to solve these problem.

1. An algorithm for the minimization of unconstrained functions that approximates the inverse of the Hessian is as below.

Algorithm 0.1 BFGS inverse Hessian approximation

Input: $x^{(0)}$, $G^{(0)} = I$ and a convergence tolerance $\epsilon_t > 0$.
Output: x such that $\|\nabla f(x)\| \leq \epsilon_t$.

for $k = 0, 1, \cdots$ **do**
  if $\|\nabla f(x^{(k)})\| \leq \epsilon_t$, $x = x^{(k)}$, break.
  $d^{(k)} = -G^{(k)} \nabla f(x^{(k)})$
  Compute α^* using the Armijo back-tracking line search
  Compute $x^{(k+1)} = x^{(k)} + \alpha^* d^{(k)}$
  $s = x^{(k+1)} - x^{(k)}$
  $y = \nabla f(x^{(k+1)}) - \nabla f(x^{(k)})$
  if $s^T y > 0$
    $$G^{(k+1)} = G^{(k)} + \left(1 + \frac{y^T G^{(k)} y}{s^T y}\right) \frac{s s^T}{s^T y} - \left(\frac{s y^T G^{(k)} + G^{(k)} y s^T}{s^T y}\right)$$
  else
    $G^{(k+1)} = G^{(k)}$
  end if
end for

(a) Implement this algorithm in Matlab/Octave.

(b) Use the implementations above to find a minimum of the following problems. In each case use $x^{(0)}$ as an initial estimate of the solution and print the optimal solution.

(a) $f(x) = \sum_{i=1}^{n-1} \left[(100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2\right]$, $x^{(0)} = [-1.2, 1, -1.2, 1, \cdots, -1.2, 1]^T$, $n = 10$.

(b) $f(x) = 100(x_1^2 - x_2)^2 + (x_1 - 1)^2 + (x_3 - 1)^2 + 90(x_3^2 - x_4)^2 + 10.1[(x_2 - 1)^2 + (x_4 - 1)^2] + 19.8(x_2 - 1)(x_4 - 1)$, $x^{(0)} = [-3, -1, -3, -1]^T$.

(c) $f(x) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4$, $x^{(0)} = [3, -1, 0, 1]^T$.

(d) $f(x) = 100(x_2 - x_1^3)^2 + (1 - x_1)^2$, $x^{(0)} = [-1.2, -1]^T$.

(e) $f(x) = \sum_{i=1}^{n} \left[n + i(1 - \cos x_i) - \sin x_i - \sum_{j=1}^{n} \cos x_j\right]^2$, $x^{(0)} = \left[\frac{1}{5n}, \cdots, \frac{1}{5n}\right]^T$, $n = 10$.