greisen

MTH 451 EXAM 1

PROBLEM # 1

TAYLOR SERIES EXPANSION (APPENDIX A 2.9) @ n=2

$$f(x) = f(\bar{x}) + [x-\bar{x}]f'(\bar{x}) + \frac{1}{2}[x-\bar{x}]^2 f''(\bar{x}) + R_3(x)$$

$$f(x) = f(\bar{x}) + [x-\bar{x}]f'(x) + \frac{1}{2}[x-\bar{x}]^2 f''(x) + \frac{1}{6}[x-\bar{x}]f'''(\xi)$$

* where $\xi = x + \Theta(x-\bar{x})$ for $\Theta \in (0,1)$ *

* substitute η for ξ, $\eta \in (x, \bar{x})$ *

$$f(x) = f(\bar{x}) + [x-\bar{x}]f'(\bar{x}) + \frac{1}{2}[x-\bar{x}]^2 f''(\bar{x}) + \frac{1}{6}[x-\bar{x}]^3 f'''(\eta)$$

⟶ SUB (x+h) for x ⟵

$$f(x+h) = f(x) + [h]f'(x) + \frac{1}{2}[h]^2 f''(x) + \frac{1}{6}[h]^3 f'''(\eta)$$

⟶ SUB (x+2h) for x ⟵

$$f(x+2h) = f(x) + [2h]f'(x) + \frac{1}{2}[2h]^2 f''(x) + \frac{1}{6}[2h]^3 f'''(\eta)$$

$$f(x+2h) = f(x) + 2hf'(x) + 2h^2 f''(x) + \frac{4}{3}h^3 f'''(\eta)$$

⟶ LINEAR COMBINATION, $f(x+2h) - 4f(x+h)$ ⟵

$$f(x+2h) - 4f(x+h) = -3f(x) - 2hf'(x) + \frac{2}{3}h^3 f'''(\eta)$$

$$-2hf'(x) = 3f(x) - 4f(x+h) + f(x+2h) - \frac{2}{3}h^3 f'''(\eta)$$

$$f'(x) = \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h} + \underbrace{\frac{1}{3}h^2 f'''(\eta)}_{\text{TRUNCATION ERROR} = O(h^2)}$$

(a).

$$f'(x) = \underbrace{\frac{-3f(x)+4f(x+h)-f(x+2h)}{2h}}_{\text{NUMERICAL DERIVATIVE}} + \underbrace{\frac{h^2}{3}f'''(\eta)}_{\text{TRUNCATION ERROR}} + \underbrace{\frac{-3E(x)+4E(x+h)-E(x+2h)}{2h}}_{\text{ROUNDOFF ERROR}}$$

$$f'(x) = \underbrace{\bar{f}'(x)}_{\text{NUMERICAL DERIVATIVE}} - \underbrace{g(h)}_{\text{COMBINED ERROR}}$$

$$g(h) = \frac{h^2}{3}f'''(\eta) + \frac{-3E(x)+4E(x+h)-E(x+2h)}{2h}$$

* SUB $f'''(\eta) = B$ *

* SUB $3E(x)+4E(x+h)-E(x+2h) = \epsilon_u \bar{E}$ *

$$g(h) = \frac{h^2 B}{3} + \frac{\epsilon_u \bar{E}}{2h}$$

$$g(h) = \frac{B}{3}\left[h^2 + \frac{3}{2}\frac{\epsilon_u \bar{E}}{hB}\right]$$

* SUB $\epsilon_u \bar{E}/B = A\epsilon_m$ *

$$g(h) = \frac{B}{3}\underbrace{\left[h^2 + \frac{3A\epsilon_m}{2h}\right]}_{q(h)}$$

$$q(h) = h^2 + \frac{3}{2}A\epsilon_m h^{-1}$$

$$q'(h) = 2h - \frac{3}{2} A \epsilon_m h^{-2}$$

$$0 = 2h - \frac{3}{2} A \epsilon_m h^{-2}$$

$$0 = 2h^3 - \frac{3}{2} A \epsilon_m$$

$$2h^3 = \frac{3}{2} A \epsilon_m$$

$$h = \sqrt[3]{\frac{3}{4} A \epsilon_m}$$

(b.) $h = O\left(\sqrt[3]{\epsilon_m}\right)$