

PROBLEM 3STEEPEST DESCENT, EXACT LINE SEARCHES

$$f(x) = \frac{1}{2} x^T Q x - b^T x$$

MATRIX DERIVATION FROM
APPENDIX A 3.7

$$f'(x) = \frac{1}{2} (Qx + Q^T x) - b$$

$$* Q \text{ is symmetric, } Q = Q^T *$$

$$f'(x) = \frac{1}{2} (Qx + Qx) - b$$

$$f'(x) = \frac{1}{2} (2Qx) - b$$

$$f'(x) = Qx - b$$

$$\nabla f(x) = Qx - b$$

$$(a). \quad d = -(Qx - b)$$

$$\hat{f}(\alpha) = f(x + \alpha d)$$

$$\hat{f}'(\alpha) = (Q(x + \alpha d) - b)^T d$$

$$\hat{f}'(\alpha) = (x^T Q + \alpha d^T Q - b^T)^T d$$

$$\hat{f}'(\alpha) = (x^T Q^T + \alpha d^T Q^T - b^T) d$$

$$* Q \text{ is symmetric, } Q = Q^T *$$

$$\hat{f}'(\alpha) = (x^T Q d + \alpha d^T Q d - b^T d)$$

$$0 = x^T Q d + \alpha^* d^T Q d - b^T d$$

$$-\alpha^* d^T Q d = x^T Q d - b^T d$$

$$\alpha = - \frac{x^T Q d - b^T d}{d^T Q d}$$

$$\alpha = - \frac{(x^T Q - b^T) d}{d^T Q d}$$

$$\alpha = - \frac{(Qx - b)^T d}{d^T Q d}$$

$$\alpha \cdot d = -(Qx - b)$$

$$(b) \quad \alpha = \frac{d^T d}{d^T Q d}$$