MTH 451/EGR 551 - Homework 5
Due Monday March 29, 2021 at 5 p.m.
Please submit all computer codes you developed to solve these problems.

1. A simple continuation method for solving a parameter dependent system of nonlinear equations is as follow.

Input: $H(x, \lambda)$, x_0, $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_N]$, $\epsilon_t > 0$
Output: (x_k, λ_k) such that $H(x_k, \lambda_k) = 0$, $k = 1, 2, \ldots, N$

1: **for** $k = 1, 2, \ldots, N$ **do**
2: $\quad \lambda = \Lambda_k$
3: $\quad$ Set $x_k^0 = x_{k-1}$ as an initial estimate, solve $H(x, \lambda) = 0$ to get x_k using a damped Newton's method
4: **end for**

(a) Implement this algorithm in MATLAB/Octave using a damped Newton's method to solve the equations $H(x, \lambda)$ for some given continuation parameter λ. Your solver interface should have the following format.

```
X = solver_name(H, x0, Lambda, tol)
```

Here, `H` is the function that computes $H(x, \lambda)$ given the vector x and the continuation parameter λ. The initial estimate is `x0`. The vector `Lambda` contains the values of continuation parameters where the problem will be solved. The convergence tolerance `tol`. The matrix `X` stores the solution of the equations for each element of `Lambda`. The i-th row of `X` has the solution to the equations for $\lambda =$ `Lambda[i]`.

(b) Use the implementation above to find a solution to the following nonlinear equations.

$$\begin{aligned} f_1 &= a_1 \cos\theta_1 + a_2 \cos\theta_2 + a_3 \cos\theta_3 - 4.5 = 0, \\ f_2 &= a_1 \sin\theta_1 + a_2 \sin\theta_2 + a_3 \sin\theta_3 + 3.0 = 0, \end{aligned}$$

where $a_1 = 1.5$, $a_2 = 5.0$ and $a_3 = 3.0$.
The equations f_1 and f_2 represent the x-axis and y-axis components of the loop closure equation for a four bar mechanism. Here, a_1 is the crank of the mechanism with the ground pivot at $(0, 0)$; a_2 is the connecting rod; and a_3 is the follower link with ground pivot at $(4.5, -3)$.
Solve these equations for $\theta_1 = 0, 0.02\pi, 0.04\pi, \ldots, 2\pi$. (That is, $\theta_1 =$ `linspace(0, 2*pi, 101);`.) Assume that θ_2 and θ_3 are unknowns, and θ_1 is the continuation parameter. Using the results obtained plot the points

$$\begin{aligned} x_p &= a_1 \cos\theta_1 + 10.5 \cos\theta_2 + 0.5 \cos(\theta_2 - \pi/2), \\ y_p &= a_1 \sin\theta_1 + 10.5 \sin\theta_2 + 0.5 \sin(\theta_2 - \pi/2). \end{aligned}$$

Also plot (θ_1, θ_2) and (θ_1, θ_3).