PROBLEM 4

GRADIENT OF $V(\theta)$

1ST TERM

$$V_1(\theta) = \sum_{i=1}^{N-1} m_i g h_i$$

$$V_1(\theta) = \sum_{i=1}^{N-1} \left(\frac{1}{N-1}\right) g \left(h_{i-1} + L_i \sin \theta_i\right)$$

$$V_1(\theta) = \frac{g}{N-1} \sum h_{i-1} + L_i \sin \theta$$

$$V_1'(\theta) = \frac{g}{N-1} \sum L_i \cos \theta$$

$$\longrightarrow V_1'(\theta) = \frac{g}{N-1} \left(\frac{L}{N}\right) \sum_{i=1}^{N-1} \cos \theta$$

2ND TERM

$$V_2(\theta) = \frac{\rho}{2} \left(\mu L - \sum_{i=1}^{N} L_i \cos \theta\right)^2$$

$$V_2'(\theta) = 2 \cdot \frac{\rho}{2} \left(\mu L - \sum_{i=1}^{N} L_i \cos \theta\right)\left(\sum_{i=1}^{N} L_i \sin \theta\right)$$

$$\longrightarrow V_2'(\theta) = \rho \left(\mu L - \left(\frac{L}{N}\right) \sum_{i=1}^{N} \cos \theta\right)\left(\left(\frac{L}{N}\right) \sum_{i=1}^{N} \sin \theta\right)$$

3RD TERM

$$V_3(\theta) = \frac{\rho}{2} \left(\sum L_i \sin \theta\right)^2$$

$$V_3'(\theta) = 2 \left(\frac{\rho}{2}\right)\left(\sum L_i \sin \theta\right)\left(\sum L_i \cos \theta\right)$$

$$\longrightarrow V_3'(\theta) = \rho \left(\frac{L}{N}\right)^2 \left(\sum_{i=1}^{N} \sin \theta\right)\left(\sum_{i=1}^{N} \cos \theta\right)$$