

10. A)

$$f'(x) = \underbrace{\frac{\bar{f}(x+h) - \bar{f}(x-h)}{2h}}_{\text{numerical derivative}} - \underbrace{\frac{h^2 B}{6}}_{\text{truncation error}} - \underbrace{\frac{E(x+h) - E(x-h)}{2h}}_{\text{roundoff error}}$$

$$f'(x) = \underbrace{\bar{f}'(x)}_{\text{numerical derivative}} - \underbrace{g(h)}_{\text{combined errors}}$$

→ \* minimize  $g(h)$  \* ←

$$g(h) = \frac{h^2 B}{6} + \frac{E(x+h) - E(x-h)}{2h}$$

$$g(h) = \left[ \frac{B}{6} \right] \left[ h^2 + \frac{3(E(x+h) - E(x-h))}{Bh} \right]$$

$$g(h) \leq \left[ \frac{B}{6} \right] \left[ h^2 + \frac{3\bar{E}}{h} \right]$$

$$g(h) \leq \left[ \frac{B}{6} \right] \left[ h^2 + \frac{3\epsilon_u \bar{E}}{h} \right] = \text{plu}(h)$$

$$g(h) \leq \left[ \frac{B}{6h} \right] \left[ h^3 + 3\epsilon_u \bar{E} \right]$$

$g(h)$  is minimized with  $h = 0(\sqrt[3]{\epsilon_u})$   
to accomodate where  $|x| \gg 1$

$$h = \sqrt[3]{\epsilon_m} \max(1, |x|)$$