MTH 451/EGR 551 - Homework 6
Due Monday April 5, 2021 at 5 p.m.
Please submit all computer codes you developed to solve these problems.

1. A Gauss-Newton continuation method for solving a parameter dependent system of nonlinear equations is as follow.

Input: y_0, such that $H(y_0) = 0$, τ_0, such that $\|\tau_0\| = 1$, h, $h_{\min} < h_{\max}$, $N_{\text{opt}} > 0$, $\eta > 0$, and $\epsilon_t > 0$.
Output: y_k such that $\|H(y_k)\| \le \epsilon_t$, $k = 1, 2, \cdots$.

for $k = 0, 1, \cdots$ **do**
    `% Predictor`
    $y = y_k + h\tau_k$
    `% Corrector`
    for $j = 0, 1, \cdots$ **do**
        if $\|H(y)\| \le \epsilon_t$, $y_{k+1} = y$ **break**
        Solve $G(y)\Delta y = -H(y)$
        $y = y + \Delta y$
    end for
    if $\lambda_{k+1} \ge \eta$ **break**
    `% Tangent computation`
    Solve $G(y_{k+1})v = G(y_{k+1})\tau_k$
    $\tau = \tau_k - v$
    $\tau_{k+1} = \tau / \|\tau\|$
    `% Step size adjustment`
    $h = h\sqrt{N_{\text{opt}}/(j+1)}$
    $h = \max(h_{\min}, \min(h_{\max}, h))$
end for

(a) Implement this algorithm in MATLAB/Octave using a damped Newton's method to solve the equations $H(x, \lambda)$ for some given continuation parameter λ. Your solver interface should have the following format.

```
X = solver_name(H, y0, h, h_min, h_max, tau0, eta, tol)
```

Here, `H` is the function that computes $H(y)$ given the vector y. (The last element of y is the continuation parameter λ.) The initial estimate is `y0`. The initial step size is `h`. The minimum step size is `h_min`. The maximum step size is `h_max`. The vector `tau0` is the initial tangent vector. The terminal value for λ is η. The convergence tolerance `tol`. The matrix `X` stores the solution of the equations along the path.

(b) Use the Gauss-Newton continuation method to solve the following problem.

$$
\begin{aligned}
-3.933x_1 + 0.107x_2 + 0.126x_3 - 9.99x_5 \qquad & \\
-45.83\lambda - 0.727x_2x_3 + 8.39x_3x_4 - 684.4x_4x_5 + 63.5x_4\lambda &= 0, \\
-0.987x_2 - 22.95x_4 - 28.37u + 0.949x_1x_3 + 0.173x_1x_5 &= 0, \\
0.002x_1 - 0.235x_3 + 5.67x_5 \qquad &
\end{aligned}
$$

$$
\begin{aligned}
-0.921\lambda - 0.713x_1x_2 - 1.578x_1x_4 + 1.132x_4\lambda &= 0, \\
x_2 - x_4 - 0.168u - x_1x_5 &= 0, \\
-x_3 - 0.196x_5 - 0.0071\lambda + x_1x_4 &= 0
\end{aligned}
$$

(a) Set $u = 0$. Starting at $x_i = 0,\ i = 1, 2, \ldots, 5$, and $\lambda = 0$, compute the solution in the interval $0 \leq \lambda \leq 1$. Plot x_i versus λ, $i = 1, 2, \cdots, 5$.

(b) Set $u = -0.008$, $\lambda = 0$ and determine the values of x_i, $i = 1, 2, \ldots, 5$ that solve the nonlinear equations. Then, compute the solution of the nonlinear equations in the interval $0 \leq \lambda \leq 1$. Plot x_i versus λ, $i = 1, 2, \cdots, 5$.