MTH 451/EGR 551 - Homework 4
Due Monday March 22, 2021 at 5 p.m.
Please submit all computer codes you developed to solve these problems.

1. An algorithm for finding the zeros (roots) of a system of nonlinear equations is shown below. Solve $F(x) = 0$ by minimizing $\phi(x)$. Here $J(x) = \frac{d}{dx}F(x)$ is the Jacobian of $F(x)$.

Input: x, $\sigma \in (0,1)$, $\beta \in (0,1)$, and a convergence tolerance $\epsilon_t > 0$
Output: x such that $\|F(x)\| \leq \epsilon_t$

Define $\phi(x) = \frac{1}{2}F(x)^T F(x)$
for $k = 0, 1, \cdots$ **do**
  if $\|F(x)\| \leq \epsilon_t$, break
  Solve $J(x)d = -F(x)$ for d
  for $j = 0, 1, \cdots$ **do**
    $\bar{x} = x + \beta^j d$
    if $\phi(\bar{x}) < (1 - \sigma\beta^j)\phi(x)$ break
  end for
  $x = \bar{x}$
end for

(a) Implement this algorithm in MATLAB/Octave. You can use the exact and/or approximate Jacobian.

(b) Use the implementations above to find a zero of the following problems. In each case print the solution and the number of iterations required for convergence. Use a convergence tolerance of $\epsilon_t = 10^{-6}$.

(a) Use (i) $x^{(0)} = [2,\ 3]^T$, (ii) $x^{(0)} = [-1,\ 1]^T$, and (iii) $x^{(0)} = [0,\ 0]^T$ (Dennis and Schnabel, 1983).

$$\begin{aligned} f_1 &= x_1^2 + x_2^2 - 2 = 0, \\ f_2 &= e^{x_1 - 1} + x_2^3 - 2 = 0, \end{aligned}$$

(b) Use (i) $x^{(0)} = [2,\ 2]^T$, and (ii) $x^{(0)} = [-2,\ -2]^T$ (Dennis and Schnabel, 1983).

$$\begin{aligned} f_1 &= 2(x_1 + x_2)^2 + (x_1 - x_2)^3 - 8 = 0, \\ f_2 &= 5x_1^2 + (x_2 - 3)^2 - 9 = 0, \end{aligned}$$

(c) Find a solution to the nonlinear equations (Stoer and Bulirsch, 2002),

$$\begin{aligned} f_1 &= = \exp(x_1^2 + x_2^2) - 3 = 0, \\ f_2 &= = x_1 + x_2 - \sin(3(x_1 + x_2)) = 0. \end{aligned}$$

(d) Solve the system of nonlinear equations given below.

$$F(x) = \begin{bmatrix} x_1 + 3\log|x_1| - x_2^2 \\ 2x_1^2 - x_1x_2 - 5x_1 + 1 \end{bmatrix} = 0.$$

Use $x^{(0)} = [2,\ 2]^T$ as the initial estimate of the solution (Conte and de Boor, 1980).