ctype Tree where
    Lf : Int → Tree
    Br : Int → Tree → Tree → Tree

Induction

There is a base case

True for x ⇒ True for x+1

$$\forall n:\mathbb{N}\; P(n)$$

$$\left|\left| \begin{array}{l} P(0) \\ \wedge\; \forall n\; P(n) \Rightarrow P(n+1) \end{array} \right.\right.$$

etype $[a]$ where

$[\,]$: $[a]$

$(::)$: $a \to [a] \to [a]$

Induction principle $P(l)$ $\forall l : [a]$

Base $P([\,])$

Step $\big(P(xs) \Rightarrow P(x :: xs)\big)$

$\forall x, xs$

++ is associative

$$P: \quad a ++ (b ++ c) \qquad (a ++ b) ++ c$$

$$[\,] ++ ys = ys \qquad E1$$

$$(x :: xs) ++ ys = x :: (xs ++ ys) \qquad E2$$

$P[\,]$

$$[\,] ++ (b ++ c) = ([\,] ++ b) ++ c$$

$$(b ++ c) \qquad (b) ++ c$$

$$a ++ (b ++ c) = (a ++ b) ++ c \qquad IH$$

TPT $(x :: a) ++ (b ++ c) = ((x :: a) ++ b) ++ c$

$$\begin{aligned}
LHS &= (x :: a) ++ (b ++ c) \\
&= x :: (a ++ (b ++ c)) \\
&= x :: ((a ++ b) ++ c) \\
&\quad (x :: (a ++ b)) ++ c \\
&\quad ((x :: a) ++ b) ++ c
\end{aligned}$$

$$\text{map}.g.\left(\text{map}.f.[x_1 \cdots x_n]\right)$$

$$[f.x_1 \cdots f\,x_n]$$

$$[g.(f.x_1) \cdots g.(f.x_n)]$$

$$f;g$$

$$\left(\text{map}\,f;\ \text{map}.g\right).l = \left(\text{map}.(f;g)\right).l$$

$$\forall x\ f.x = g.x \Leftrightarrow f = g$$