Causal.jl:
A Modeling and Simulation Framework for Causal Models

Answers to the Questions/Comments

Zekeriya Sarı Serkan Günel

We would like to thank Chad Scherrer for contributing to our study with his valuable comments. The text was reviewed accordingly. Below are the specific answers to the questions/comments.

1. (On page 3): *This section is very wordy. and seems to be at a very low level of detail given the length of the article. Possibly more helpful would be*

   - *Comparison to eg. channels or Dagger.jl*
   - *Is this pull based of push based.*

   The paragraph explaining the details of how the data can flow through the connections is removed from the text. Instead, a comparison between the approaches adopted in Causal.jl and Dagger.jl is included.

2. *(On page 3) "outputs are directly dependent on their inputs" is this correct? Aren't outputs always directly dependent on input?*

   The outputs may not *directly* be dependent on the inputs. For example, consider the following simple dynamical system with right-hand-side and readout functions

   $$\dot{x} = f(x, u, t) = -x + u \tag{1}$$
   $$y = g(x, u, t) = x \tag{2}$$

   where x, u, y are the state, input and output at time t, respectively. Note that the output y depends on just the state x, but not the input y.

3. *(On page 4) Typo*

   The typo is corrected.

4. *(On page 4) These quotes are very unusual, especially, the second one. Maybe turn it around a monospace font.*

   The quotes are removed and a monospace font is used, instead.

5. *(On page 4) Very nice plots but there is quite a lot of overplottting. Making the curves semi-transparent would probably help here.*

   Because the chaotic attractors of the dynamical systems given in equations (1) and (2) are very dense naturally, the simulation is performed for a time duration of 15 seconds instead of 100 seconds to make the plots semi-transparent. Also, the line widths of plots are decreased.

6. *(On page 4) "topological" Is this a standard user of this term? I could see it being confused for a topology on the observation space. Not a requirement, but to me "dependency structure" would be more clear.*

   The term "topological structure of a model" is replaced by "dependency structure of a model"

7. *(On page 5) Could there be a better way to visualize this network? Some possibilities:*

   - *Graph layout, eg. GraphViz*
   - *Make nodes smaller ,or maybe even remove.*
   - *Another option - maybe instead visualize ϵ_{ij} matrix directly.*

   The network is re-drawn using GraphPlot.jl. The layout of the network is changed from random-layout to circular-layout make the network be visualized better. Also, the node size is decreased.

8. *(On page 5) A dot is missing in the caption of the Figure 9.*

   The dot is placed.

9. *(On page 5) Is this standard notation? $\boldsymbol{P}$ is often a projection matrix.*

   The notation is not standard. So, the notation $\boldsymbol{P} = diag(p_1, p_2, \ldots, p_d)$ is replaced by $\boldsymbol{\Gamma} = diag(\gamma_1, \gamma_2, \ldots, \gamma_d)$.

10. *How can a diagonal do this? Wouldn't it instead determine the strength of the interaction?*

    The diagonal matrix $P = diag(p_1, p_2, \ldots, p_n)$ determines by which state variable the nodes are connected. For example, consider a very simple network consisting of $n = 2$ nodes each of which is of degree $d = 3$. Consider also that these two nodes are connected to each other with a coupling strength of ϵ, which implies $\epsilon_{11} = -\epsilon_{12} = -\epsilon_{21} = \epsilon_{22} = \epsilon$ From (5), the input to the first node is

    $$\boldsymbol{u}_1 = \epsilon \boldsymbol{P} \boldsymbol{x}_1 - \epsilon \boldsymbol{P} \boldsymbol{x}_2 = \epsilon \boldsymbol{P}(\boldsymbol{x}_1 - \boldsymbol{x}_2) \tag{3}$$

    where $\boldsymbol{x}_i = [x_{i,1}, x_{i,2}, x_{i,3}], i = 1, 2$ and $\boldsymbol{u}_1 = [u_{1,1}, u_{1,2}, u_{1,3}]$. If $\boldsymbol{P} = diag(1, 1, 1)$, than we have,

    $$u_{1,1} = \epsilon(x_{1,1} - x_{2,1}) \tag{4}$$
    $$u_{1,2} = \epsilon(x_{1,2} - x_{2,3}) \tag{5}$$
    $$u_{1,3} = \epsilon(x_{1,3} - x_{2,3}) \tag{6}$$
    $$\tag{7}$$

    which implies the nodes are coupled through their all three state variables $x_{i,j},\ i = 1, 2,\ j = 1, 2, 3$. If $\boldsymbol{P} = diag(1, 0, 1)$, then the input to the first node is

    $$u_{1,1} = \epsilon(x_{1,1} - x_{2,1}) \tag{8}$$
    $$u_{1,2} = 0 \tag{9}$$
    $$u_{1,3} = \epsilon(x_{1,3} - x_{2,3}) \tag{10}$$
    $$\tag{11}$$

which implies that the nodes are coupled through their first and third states $x_{i,j},\ i=1,2,\ j=1,3.$. There is no interaction through the second state variables $x_{1,2}$ and $x_{2,2}$.

Therefore, the diagonal matrix $\boldsymbol{P} = diag(p_1, p_2, \ldots, p_d)$ determines through which state variables the nodes are connected. There exists a connection through the state variables $x_{i,j},\ i=1,2,\ldots,n,\ j=1,2,\ldots,d$ if $p_j \neq 0,\ j=1,2,\ldots,d$.