CP Workshop 2023

Wednesday, June 28th

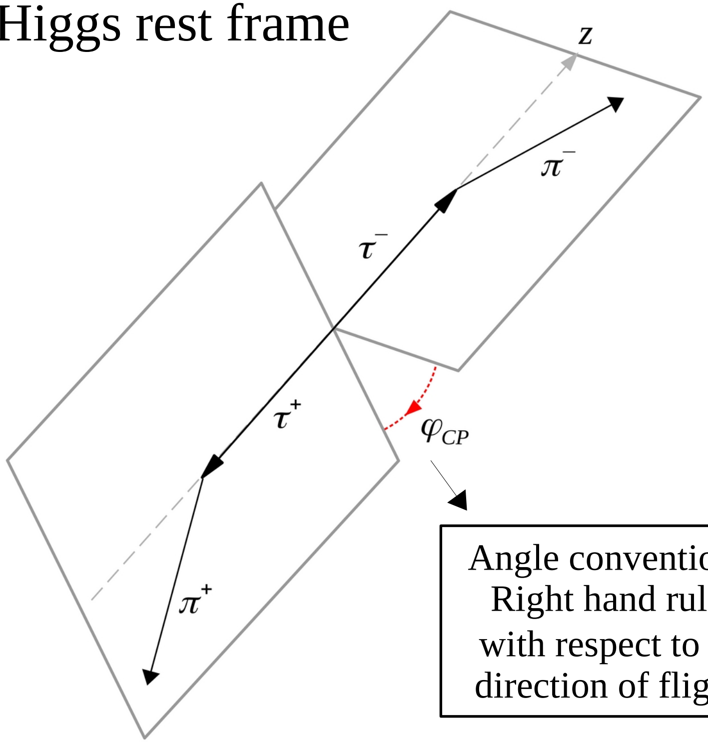
Methodologies to measure the CP structure of the Higgs y_τ coupling

Andrea Cardini

CP properties in H → ττ decays

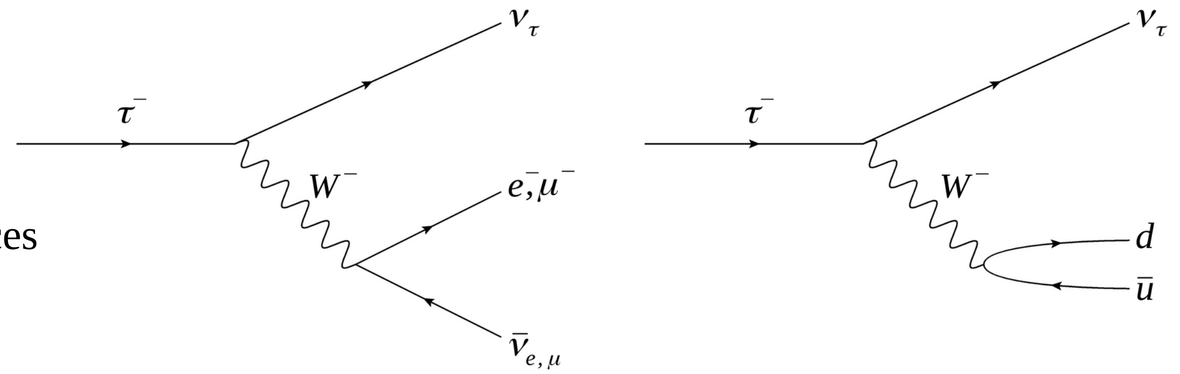
- H → ττ Yukawa interaction can be parametrized as follows:
  - $$\mathscr{L}_{Y,\tau} = -\frac{m_\tau}{v}\,\bar{\tau}(\kappa_\tau + i\gamma^5\tilde{\kappa}_\tau)H\tau$$
  - CP mixing encoded in $\Delta_{H\tau\tau}$:
    - $\kappa_\tau = \sqrt{\mu^H_{\tau\tau}}\cos\Delta_{H\tau\tau}$
    - $\tilde{\kappa}_\tau = \sqrt{\mu^H_{\tau\tau}}\sin\Delta_{H\tau\tau}$
- CP mixing alters transverse-spin correlation between τ leptons
  - $$\Gamma(H_{mix} \to \tau\tau) = \Gamma^{unpol}(1 - s^-_{//}s^+_{//} + s^-_{\perp}R(2\Delta_{H\tau\tau})s^+_{\perp})$$
  - The correlation carries over to the τ decay products

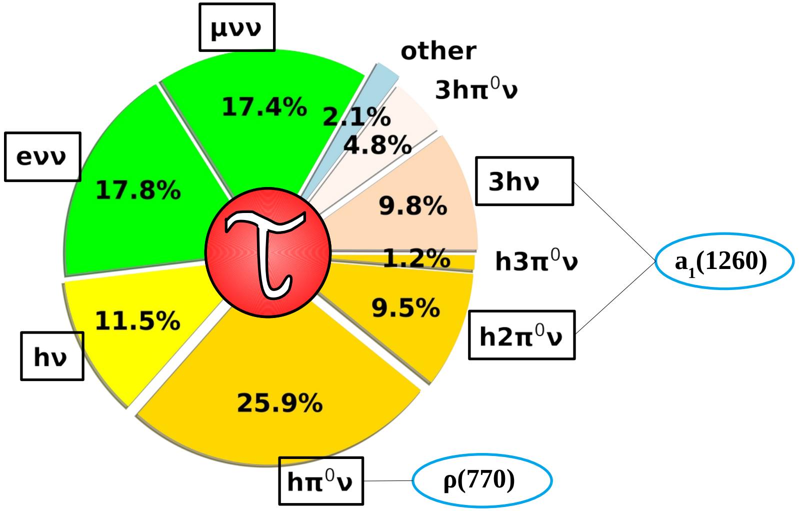
Higgs rest frame



Angle convention: Right hand rule with respect to τ⁻ direction of flight

- The tau is the heaviest lepton, and can only decay via electroweak interaction
- Its decay product *remember* the τ spin
  - Neutrinos have ~definite helicity
  - Hadronic decays are via mesonic resonances



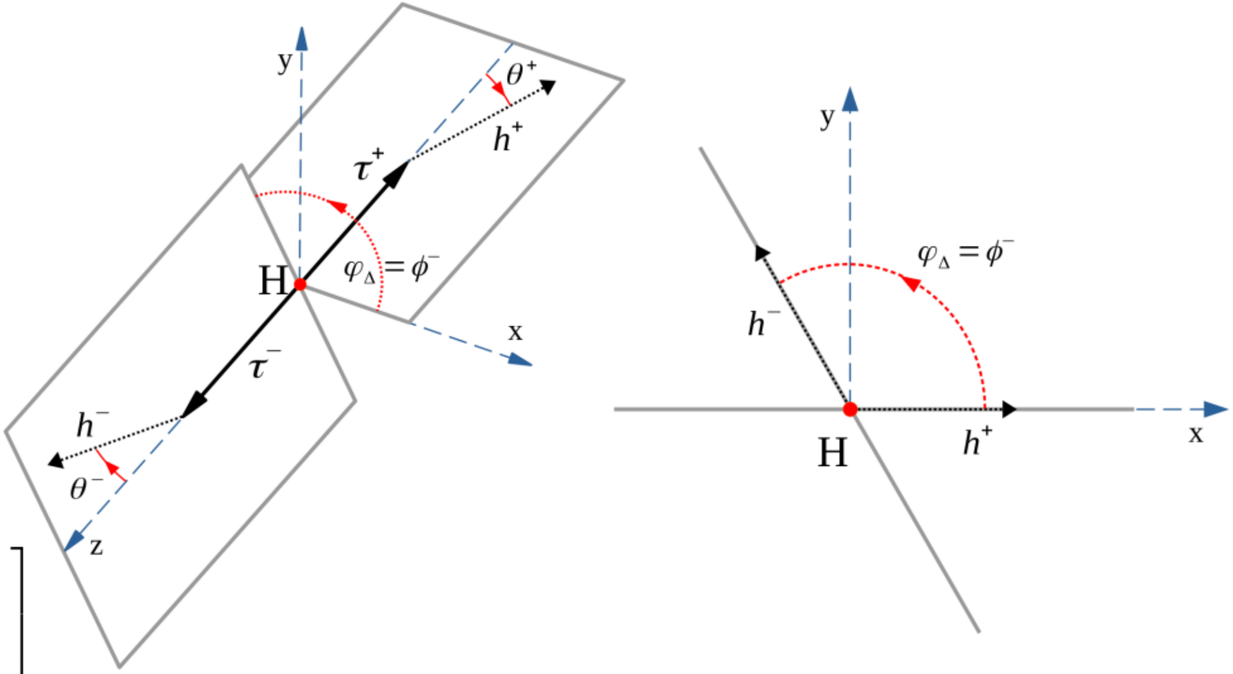


- The τ differential decay width can be written either encoding the spin correlation with the decay products in a polarimeter
  - $$\mathrm{d}\Gamma_{\tau\to X+\nu_\tau} = \frac{1}{2m_\tau}|\mathscr{M}|^2(1+h_\mu s^\mu)\mathrm{d}Lips$$
- Or as a function of a prong momenta
  - $$\mathrm{d}\Gamma_{\tau^-\to N^0 p^-} = A(x)[1 \pm B(x)(\vec{s}\cdot\hat{q})]\mathrm{d}x\frac{\mathrm{d}\Omega}{4\pi}$$

- Both tau decay width show a scalar product between the tau spin/polarization and a vector having the dimension of a momentum which is used to probe the tau spin

- In the Higgs rest frame one can align the z axis to the τ- d.o.f., x lies on the τ+-h+ plane, and y is orthogonal

- In this frame of reference the spin matrix R simplifies to



$$R(2\Delta_{H\tau\tau}) \simeq \begin{bmatrix} \cos(2\Delta_{H\tau\tau}) & \sin(2\Delta_{H\tau\tau}) & 0 \\ -\sin(2\Delta_{H\tau\tau}) & \cos(2\Delta_{H\tau\tau}) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\Gamma(H_{mix} \to \tau\tau) = \Gamma^{unpol}(1 - s^-_{//}s^+_{//} + s^-_{\perp}R(2\Delta_{H\tau\tau})s^+_{\perp})$$

The Z decays

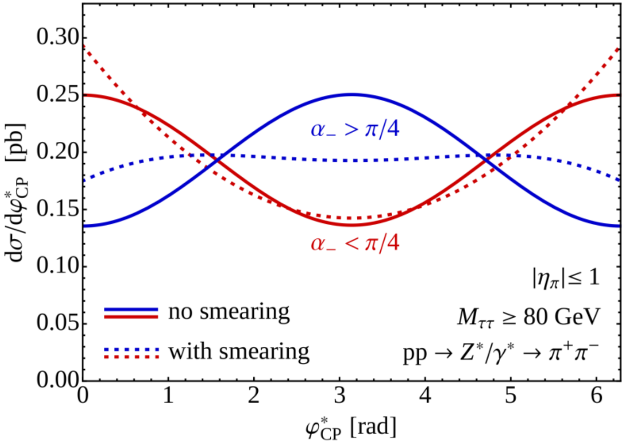
- The Z → ττ cross section includes terms related to the azymuthal difference between tau polarimeters
  - $$\mathrm{d}\sigma_{DY}/\mathrm{d}\cos(\theta^+)\mathrm{d}\cos(\theta^-)\mathrm{d}\cos(\phi^+)\mathrm{d}\varphi_\Delta \mathrm{d}E^+\mathrm{d}E^- \propto \sum_{B_1,B_2=Z,\gamma} a(B_1,B_2)$$
$$\times\Bigg\{V_\tau^{B_1}V_\tau^{B_2}\left[1-\left(\cos(\theta^+)\cos(\theta^-)\boxed{+\frac{1}{2}\sin(\theta^+)\sin(\theta^-)\cos(2\phi^+-\varphi_\Delta)}\right)\right]$$
$$+A_\tau^{B_1}A_\tau^{B_2}\left[1-\left(\cos(\theta^+)\cos(\theta^-)\boxed{-\frac{1}{2}\sin(\theta^+)\sin(\theta^-)\cos(2\phi^+-\varphi_\Delta)}\right)\right]$$
$$+(a^{B_1}V_\tau^{B_2}+V_\tau^{B_1}A_\tau^{B_2})(\cos(\theta^+)-\cos(\theta^-))\Bigg\},$$

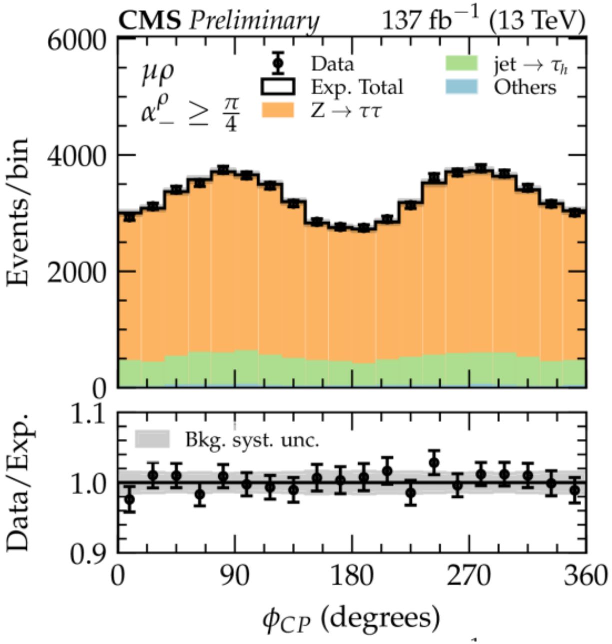
Integrating φ+ between 0 and 2π these terms go to 0

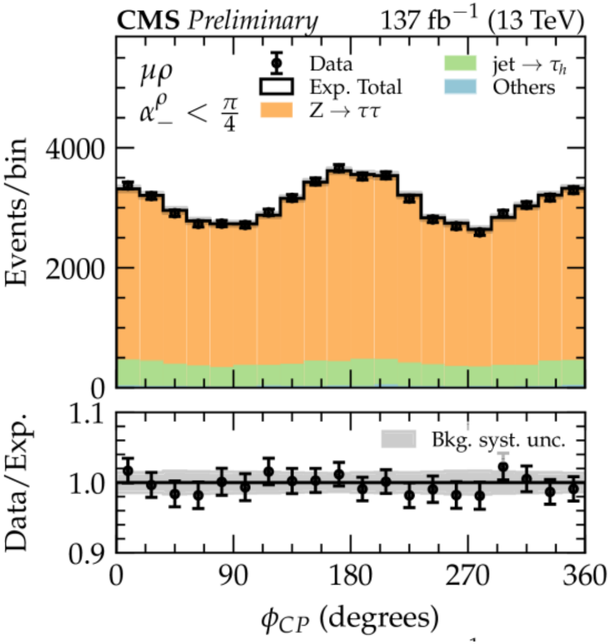
  - the dependence on the acoplanarity angle disappears integrating over the azymuthal angle of one of the tau polarimeters
- The dependence on φ_Δ can be recovered by integrating the cross section on smaller ranges in the azymuthal angle φ+

Acoplanarity in DY events

- By splitting events based on the τ- direction of flight with respect to the positive z direction in CMS (beam axis) the dependence on the acoplanarity angle is recovered
  - $\cos(\alpha_-) = \left| \frac{\hat{z} \times \vec{P}_-}{|\hat{z} \times \vec{P}_-|} \cdot \frac{\vec{R}_- \times \vec{P}_-}{|\vec{R}_- \times \vec{P}_-|} \right|$



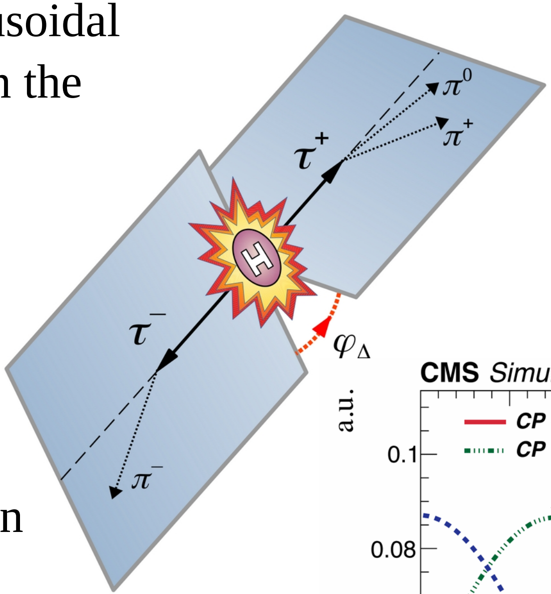




Defining our target

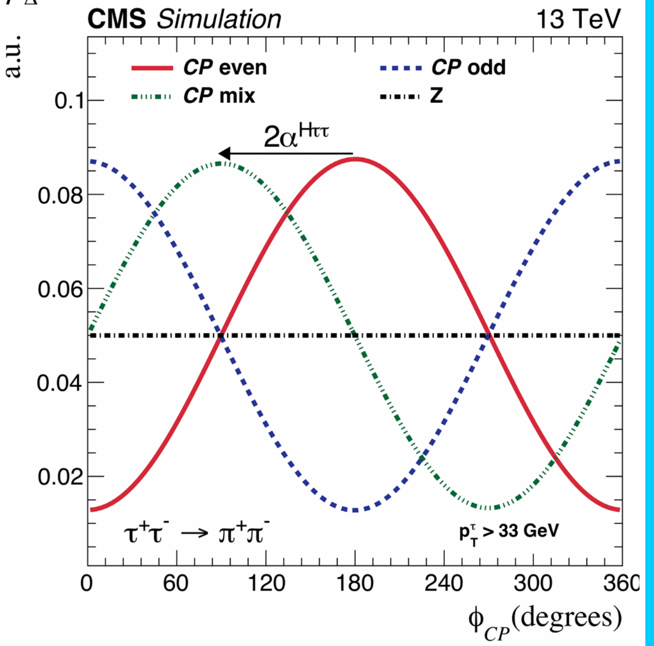
- The cross section of the H→ττ decay acquires a synusoidal dependence on the azymuthal angle spanned between the two τ polarimeters

$$\boxed{\frac{d\sigma}{d\varphi_\Delta} \propto const - \cos(\varphi_\Delta - 2\Delta_{H\tau\tau})}$$



- CP mixing appears as a **phase-shift** in the distribution

- If one describes the tau as decaying into a charged prong and a neutral system, the same relation is found using the charged prong momenta instead of the polarimeter



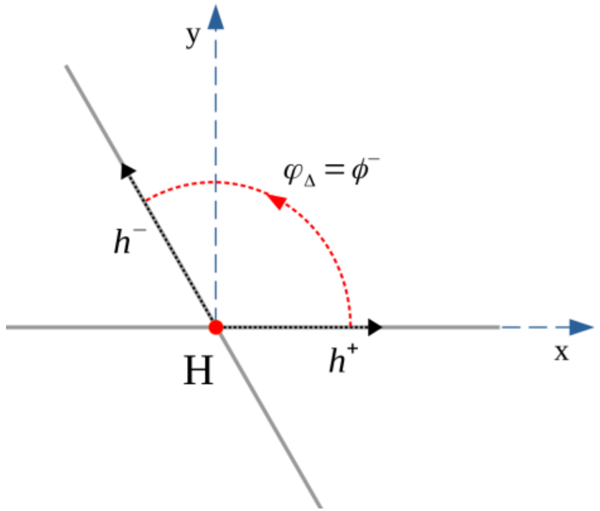
- This angle can be generalized to the angle between the τ decay planes, an **acoplanarity angle**

Acoplanarity calculation – polarimetric vectors

- In the Higgs rest frame, tau momenta are coaxial the calculation is as follows:

$$\varphi_\Delta = \begin{cases} \varphi^*, & \text{if } \mathcal{O} \geq 0 \\ 2\pi - \varphi^*, & \text{if } \mathcal{O} < 0 \end{cases},$$

2π angle definition



$$\varphi^* = \arccos\left(\frac{\vec{h}^- \times \vec{p}_\tau^-}{\left|\vec{h}^- \times \vec{p}_\tau^-\right|} \cdot \frac{\vec{h}^+ \times \vec{p}_\tau^+}{\left|\vec{h}^+ \times \vec{p}_\tau^+\right|}\right),$$

$$\mathcal{O} = \frac{(\vec{h}^+ \times \vec{h}^-) \cdot \vec{p}_{\tau^-}}{\left|(\vec{h}^+ \times \vec{h}^-) \cdot \vec{p}_{\tau^-}\right|}.$$

- Only the transverse components of the polarimeters contribute to the acoplanarity angle

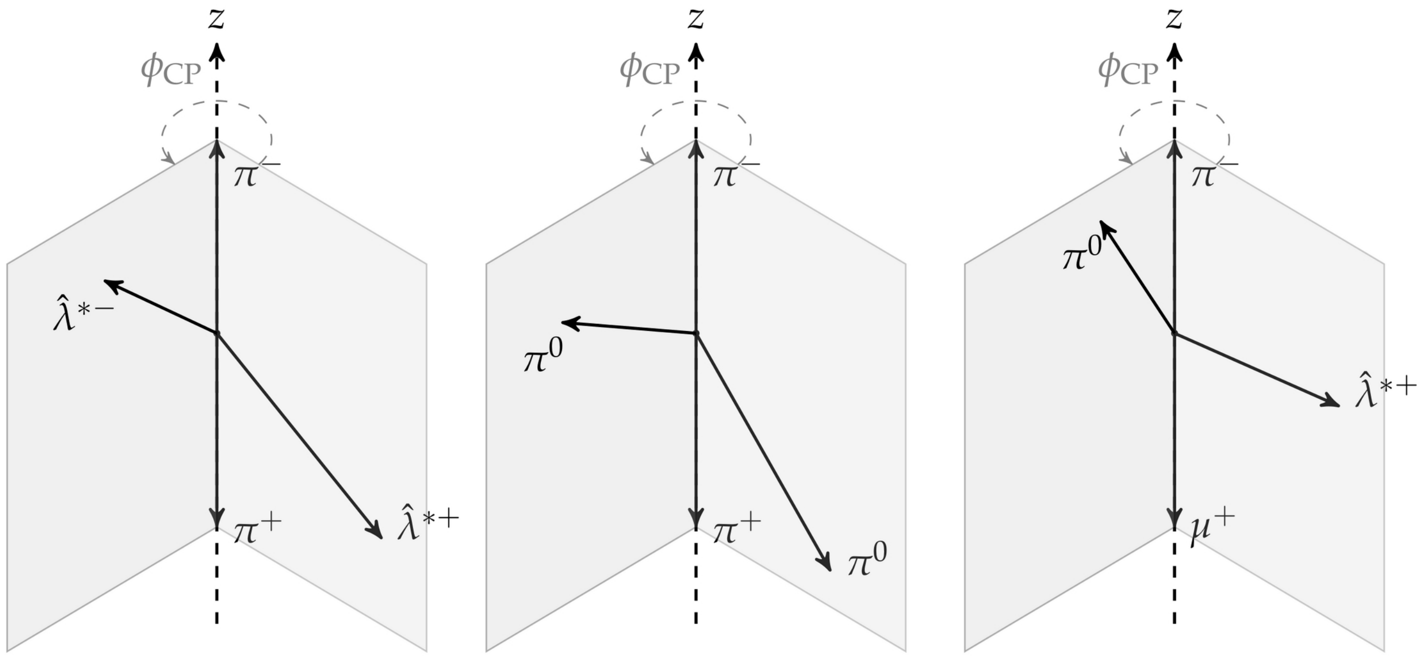
$$(\vec{h}_\perp^- \times \vec{p}_\tau^-) \cdot (\vec{h}_\perp^+ \times \vec{p}_\tau^+) = (\vec{h}_\perp^- \cdot \vec{h}_\perp^+)(\vec{p}_\tau^- \cdot \vec{p}_\tau^+) - \underbrace{(\vec{h}_\perp^- \cdot \vec{p}_\tau^+)}_{=0}\underbrace{(\vec{h}_\perp^+ \cdot \vec{p}_\tau^-)}_{=0}$$

$$= - |\vec{p}_\tau|^2 (\vec{h}_\perp^- \cdot \vec{h}_\perp^+) .$$

- The – sign is equivalent to a phase-shift of π in the acoplanarity angle calculation between the use of the double cross product, or the scalar product

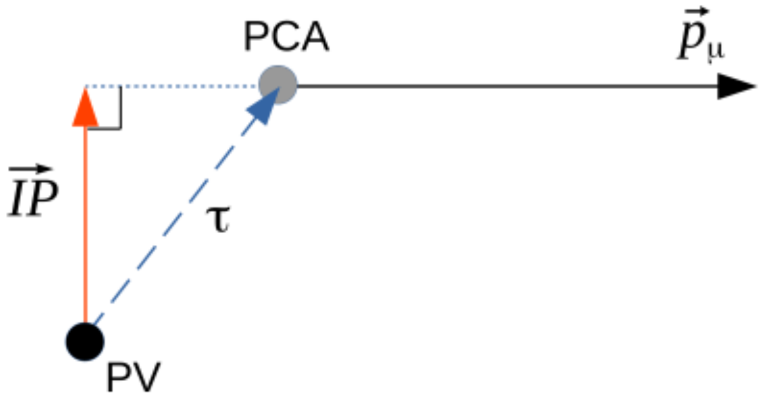
The “methods”

- In the Run 2 we used 3 methods to reconstruct the acoplanarity angle



IP method | Neutral pion | Mix method

IP method



$$\varphi_\Delta = \begin{cases} \varphi^*, & \text{if } \mathcal{O} \geq 0 \\ 2\pi - \varphi^*, & \text{if } \mathcal{O} < 0 \end{cases},$$

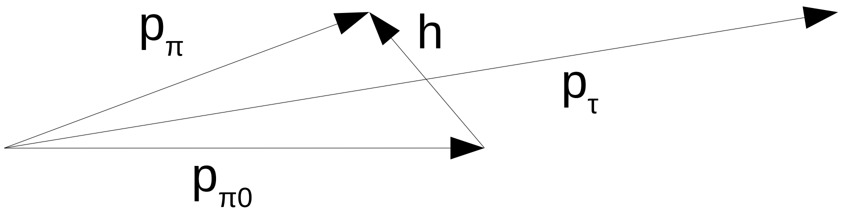
$$\varphi^* = \arccos\left(\overrightarrow{IP}_\perp^+ \cdot \overrightarrow{IP}_\perp^-\right),$$

$$\mathcal{O} = \left(\overrightarrow{IP}_\perp^+ \times \overrightarrow{IP}_\perp^-\right) \cdot \frac{\vec{p}_p^{\,-}}{|\vec{p}_p^{\,-}|}.$$

- For $\tau \rightarrow \pi\nu$ decays the polarimetric vector is defined as $h=p_\pi$
- The IP is an approximation of the tau direction of flight projected in the plane orthogonal to the pion momenta
- The IP x p_π vector product is // to h x p_τ → the geometrical operations done on the pair of vectors is the same
- Basically: it's the same as the polarimetric vector swapping h with IP and p_τ with $p_{\mu,\pi}$ in the calculations

- Caveat: there is a phase-shift of π with respect to the polarimetric vector method

- For $\tau \rightarrow \pi\pi0\nu$ decays the polarimetric vector is defined as
  - > $\vec{h} = \mathcal{N}(2(q\cdot N)\vec{q} - q^2\vec{N}) \simeq \mathcal{N}(2m_\tau(E_{\pi^\pm} - E_{\pi^0})\vec{q} - q^2\vec{N})$ with $q=p_\pi - p_{\pi0}$
- If one neglects the neutrino momenta component orthogonal to the $\pi - \pi0$ plane then the polarimetric vector is directed as $\sim p_\pi - p_{\pi0}$



- In this approximation, all vectors are coplanar, therefore
  - > $(\vec{p}_{\pi^\pm} \times \vec{p}_{\pi^0})\ //\ (\vec{h} \times \vec{p}_\tau)$
- Caveat: h changes sign with respect to the energy difference of the pions → a phase-shift of π is required based on the difference of pion energies

Calculating the acoplanarity angle

- All methods have at the core the same mathematical treatment
- They can be unified taking into account these common features
  - ZMF defined by two momenta ($\tau\tau$ for polarimetric vector method, $\pi\pi$ for the others)
    - They define a **frame of reference** where the two momenta are **co-axial**
  - Another vector per plane which is **not parallel** to the two momenta
  - The normal to these two vectors needs to be parallel to $(\vec{h} \times \vec{p}_\tau)$
    - The method precision depends on how close the normal to the plane is approximated:
      - IP method: PCA as approximation for the SV → IP as approximation for transverse component of the tau direction of flight
      - Neutral pion: neutrino momentum laying on the plane formed by the charged and neutral pion + energy difference between pions
      - Polarimetric vector: visible tau rest frame as Higgs rest frame + polarimeter reconstruction using only visible decay products

Unified notation

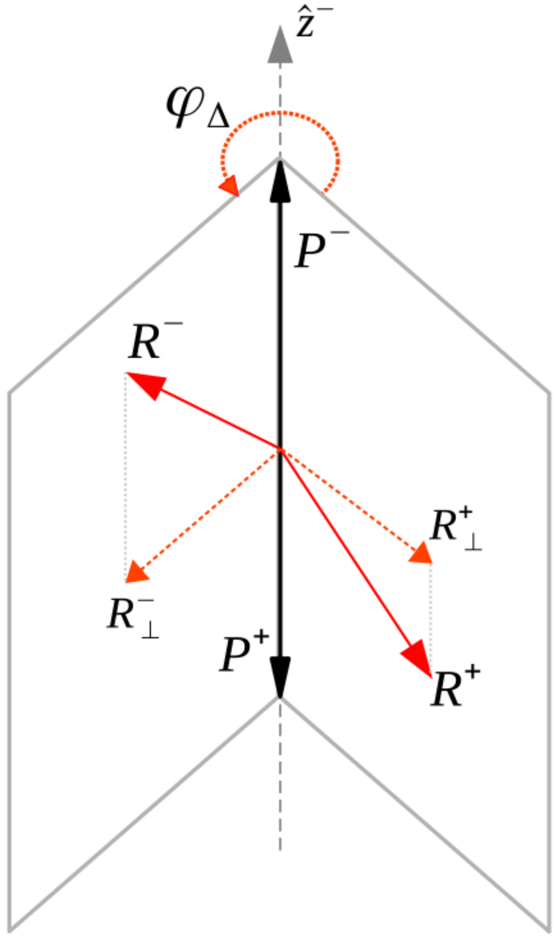
- Define each plane:
  - A momentum P
  - A *reference vector* R not parallel to P
- Boost all vectors in the ZMF of the two momenta
- Calculate the azymuthal difference between reference vectors
- Apply phase-shift of π if needed

$$\varphi_\Delta = \begin{cases} \varphi^*, & \text{if } \mathcal{O} \geq 0 \\ 2\pi - \varphi^*, & \text{if } \mathcal{O} < 0 \end{cases}$$

$$\varphi^* = \arccos\left(\frac{\vec{R}_\perp^-}{|\vec{R}_\perp^-|} \cdot \frac{\vec{R}_\perp^+}{|\vec{R}_\perp^+|} \right) ,$$

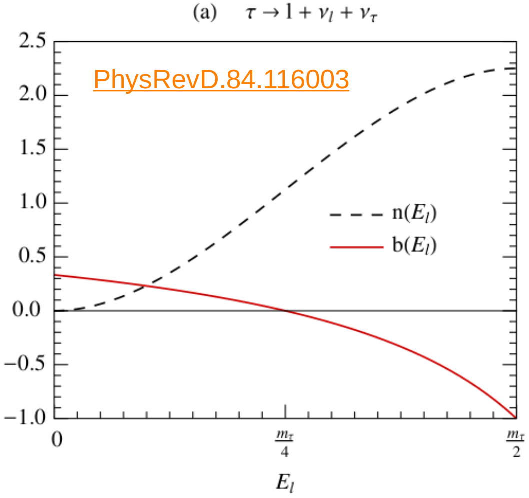
$$\mathcal{O} = (\vec{R}_\perp^+ \times \vec{R}_\perp^-) \cdot \vec{P}^- ,$$

$$\vec{R}_\perp^\pm = \vec{R}^\pm - (\vec{R}^\pm \cdot \vec{P}^\pm) \frac{\vec{P}^\pm}{|\vec{P}^\pm|} .$$



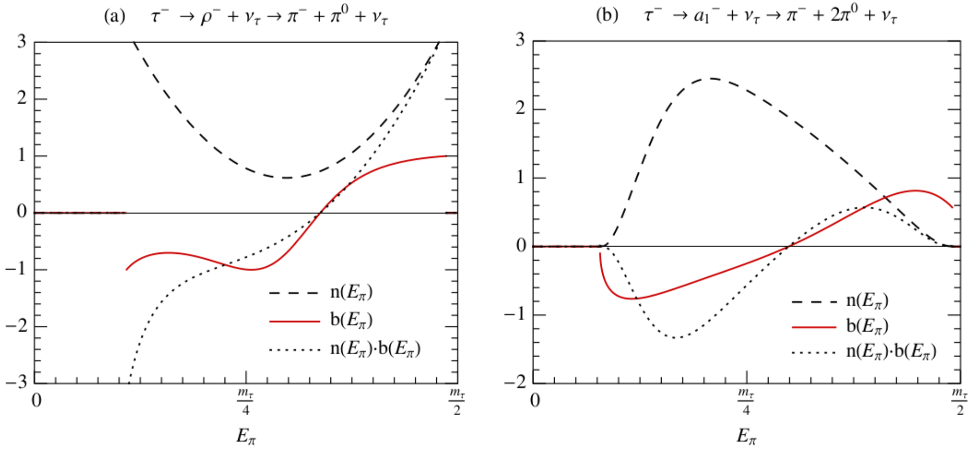
Phase-shift of π: how to

- A phase-shift of π is needed in the acoplanarity angle definition
  whenever the spin-correlation flips based on the decay product momenta
  - Basically, if the spectral function for $\tau \to$ p N^0 flips sign, then the polarimetric vector changes direction with respect to the prong momentum. For multi-prong decays more parameters are needed.
- Examples:
  - Spin correlation flips in leptonic decays if neutrinos carry most of the tau momentum
    - In this particular case we don’t particularly care since we cut at lep pT > 20 GeV
      → they carry most of the energy



PhysRevD.84.116003

  - Energy difference between charged and neutral pion(s)



Our methods in Run 2 under unified notation

Channel	P1	R1	P2	R2	π Phase-Shift
$\tau_{\mu,e,\pi} \times \tau_{\mu,e,\pi}$	$\vec{p}_{\mu,e,\pi}$	$\overrightarrow{IP}_{\mu,e,\pi}$	$\vec{p}_{\mu,e,\pi}$	$\overrightarrow{IP}_{\mu,e,\pi}$	-
$\tau_{\mu,e,\pi} \times \tau_{\rho}$	$\vec{p}_{\mu,e,\pi}$	$\overrightarrow{IP}_{\mu,e,\pi}$	$\vec{p}_{\pi^\pm}$	$\vec{p}_{\pi^0}$	$y_\rho < 0$
$\tau_{\mu,e,\pi} \times \tau_{a_1^{1\mathrm{Pr}}}$	$\vec{p}_{\mu,e,\pi}$	$\overrightarrow{IP}_{\mu,e,\pi}$	$\vec{p}_{\pi^\pm}$	$\vec{p}_{\pi^0\pi^0}$	$y_{a_1^{1\mathrm{Pr}}} < 0$
$\tau_{\mu,e,\pi} \times \tau^\pm_{a_1^{3\mathrm{Pr}}}$	$\vec{p}_{\mu,e,\pi}$	$\overrightarrow{IP}_{\mu,e,\pi}$	$\vec{p}_{\pi^\pm}$	$\vec{p}_{\pi^\mp}$	$y_{a_1^{3\mathrm{Pr}}} < 0$
$\tau_\rho \times \tau_\rho$	$\vec{p}_{\pi}$	$\vec{p}_{\pi^0}$	$\vec{p}_{\pi}$	$\vec{p}_{\pi^0}$	$y_\rho \cdot y_\rho < 0$
$\tau_\rho \times \tau_{a_1^{1\mathrm{Pr}}}$	$\vec{p}_{\pi}$	$\vec{p}_{\pi^0}$	$\vec{p}_{\pi}$	$\vec{p}_{\pi^0\pi^0}$	$y_\rho \cdot y_{a_1^{1\mathrm{Pr}}} < 0$
$\tau_\rho \times \tau^\pm_{a_1^{3\mathrm{Pr}}}$	$\vec{p}_{\pi}$	$\vec{p}_{\pi^0}$	$\vec{p}_{\pi^\pm}$	$\vec{p}_{\pi^\mp}$	$y_\rho \cdot y_{a_1^{3\mathrm{Pr}}} < 0$
$\tau_{a_1^{1\mathrm{Pr}}} \times \tau^\pm_{a_1^{3\mathrm{Pr}}}$	$\vec{p}_{\pi}$	$\vec{p}_{\pi^0\pi^0}$	$\vec{p}_{\pi^\pm}$	$\vec{p}_{\pi^\mp}$	$y_{a_1^{1\mathrm{Pr}}} \cdot y_{a_1^{3\mathrm{Pr}}} < 0$
$\tau_{a_1^{3\mathrm{Pr}}} \times \tau_{a_1^{3\mathrm{Pr}}}$	$\vec{p}_{\tau}$	$\vec{h}$	$\vec{p}_{\tau}$	$\vec{h}$	-

$$y_\rho = \frac{E_\pi - E_{\pi^0}}{E_\pi + E_{\pi^0}}, \quad y_{a_1^{1\mathrm{Pr}}} = \frac{E_\pi - E_{\pi^0\pi^0}}{E_\pi + E_{\pi^0\pi^0}}, \quad y_{a_1^{3\mathrm{Pr}}} = \frac{E_{\pi^\pm} - E_{\pi^\mp}}{E_{\pi^\pm} + E_{\pi^\mp}}$$

Where to improve

- Using this notation it is easier to compare / combine different methods, and identify quantities we could improve
  - The resolution of each method depends on the 4 vectors chosen and the boolean
  - IP resolution needs to be optimized in the direction orthogonal to the plane containing the track
  - Neutral pion direction of flight
  - Booleans for π phase-shift: finding better approximations based on decay chain
    - In DM 2 separating the two π0 allows the reconstruction of the charged ρ meson

$$y_{-}^{a_1} = \frac{E_{\rho^-} - E_{\pi^0}}{E_{\rho^-} + E_{\pi^0}} - \frac{m_{a_1}^2 - m_{\pi^0}^2 + m_{\rho^-}^2}{m_{a_1}^2}$$

Need to cross-check my old notes, I might have forgotten a factor 2

    - Same considerations can be applied to DM 10:

$$y_{a_1}^{\pm} = \frac{E^{\rho^0} - E^{\pi^\pm}}{E^{\rho^0} + E^{\pi^\pm}} - \frac{m_{a_1}^2 - m_{\pi^\pm}^2 + m_{\rho^0}^2}{2m_{a_1}^2}$$