Elec 405 - Homework 1

In [5]:
```
using LinearAlgebra
using Plots
```

8. Affine Approximation

$$a(\mathbf{x}) = f(\mathbf{x}_0) + \nabla f(\mathbf{x}_0)^T(\mathbf{x} - \mathbf{x}_0) \tag{7}$$

is an affine approximation of $f(\mathbf{x})$ around $\mathbf{x}_0$.
Note that this affine function defines an hyperplane in $\Re^{n+1}$ in the standard form

$$\begin{bmatrix} \nabla f(\mathbf{x}_0) \\ -1 \end{bmatrix}^T \left(\begin{bmatrix} \mathbf{x} \\ a(\mathbf{x}) \end{bmatrix} - \begin{bmatrix} \mathbf{x}_0 \\ f(\mathbf{x}_0) \end{bmatrix} \right) = 0. \tag{8}$$

Now consider the function

$$f(\mathbf{x}) = \mathbf{x}^T\mathbf{A}\mathbf{x} \tag{9}$$

defined over $\Re^2$, where

$$\mathbf{A} = \begin{bmatrix} 0.01 & 0.001 \\ 0.001 & 0.01 \end{bmatrix}. \tag{10}$$

In [6]:
```
A = [0.01 0.001; 0.001 0.01]
A
```

Out[6]:
```
2×2 Array{Float64,2}:
 0.01   0.001
 0.001  0.01
```

In [7]:
```
function draw_hyperplane_3D_modified(a, b, camera=(60, 45))

#     pyplot()
    x=range(-50, stop=50, length=100)
    y=range(-50, stop=50, length=100)
    g(x, y) = (dot(a, b) - dot(a[1: 2], [x; y]))/a[3]
    plot!(x, y, g,
        label="$(a)'(x-$(b))=0",
        xlabel="x_1", ylabel="x_2", zlabel="x_3",
        st=:surface, camera=camera)

    # # plot the normal vector, since b is on the plane plot the vector b to b+a
    a_x1 = range(b[1], stop=(b[1]+15*a[1]), length=100)
    a_x2 = range(b[2], stop=(b[2]+15*a[2]), length=100)
    a_x3 = range(b[3], stop=(b[3]+15*a[3]), length=100)
    plot!(a_x1, a_x2, a_x3, label="a")
end

function draw_hyperplane_3D(a, b, p, camera=(60, 45))
    pyplot()
    x=range(-2, stop=2, length=100)
    y=range(-2, stop=2, length=100)
    g(x, y) = (dot(a, b) - dot(a[1: 2], [x; y]))/a[3]
    p= plot(x, y, g,
        title="Hyperplane in 3D",
        label="$(a)'(x-$(b))=0",
        xlabel="x_1", ylabel="x_2", zlabel="x_3",
        xlims= (-2, 2), ylims=(-2, 2), zlims=(-2, 2),
        st=:surface, camera=camera)

    # plot the normal vector, since b is on the plane plot the vector b to b+a
    a_x1 = range(b[1], stop=(b[1]+a[1]), length=100)
    a_x2 = range(b[2], stop=(b[2]+a[2]), length=100)
    a_x3 = range(b[3], stop=(b[3]+a[3]), length=100)
    plot!(a_x1, a_x2, a_x3, label="a")
end
```

Out[7]: draw_hyperplane_3D (generic function with 2 methods)

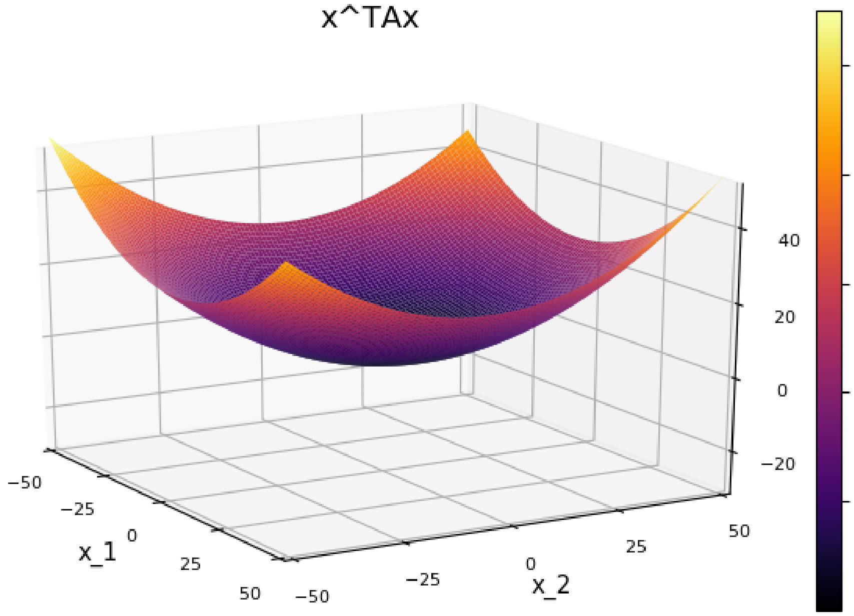
In [8]:
```
function affine_approx(x, f, f_der, x_0)
    return f(x_0) + dot(f_der(x_0), (x-x_0))
end

function f(x)
    return x' * A * x
end

function nabla_f(x)
    return (A+A')x
end
```

Out[8]: nabla_f (generic function with 1 method)

In [11]:

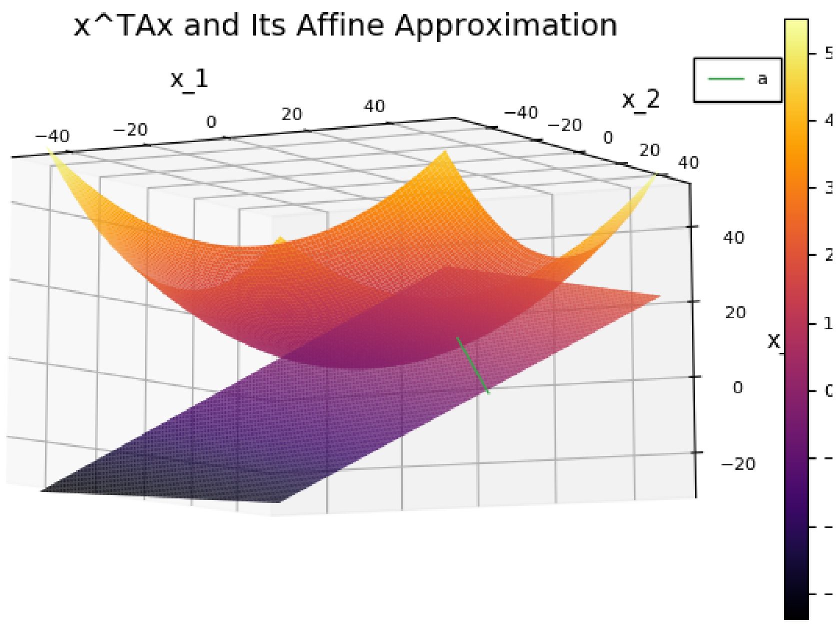
```
camera=(60, 20)
# draw_hyperplane_3D()
pyplot()
x=range(-50, stop=50, length=100)
y=range(-50, stop=50, length=100)
g(x, y) = ([x y] * A * [x; y])[1]
p= plot(x, y, g,
    title="x^TAx",
    label="x^TAx",
    xlabel="x_1", ylabel="x_2",
    zlims=(-30,50),
    st=:surface, camera=camera)
```

Out[11]:



In [12]:
```
camera=(30, -10)


pyplot()
x=range(-50, stop=50, length=100)
y=range(-50, stop=50, length=100)
g(x, y) = ([x y] * A * [x; y])[1]
p= plot(x, y, g,
    title="x^TAx and Its Affine Approximation",
    label="x^TAx",
    xlabel="x_1", ylabel="x_2",
    zlims=(-30,50),
    st=:surface, camera=camera)

x_0 = [25; 0]
a = [nabla_f(x_0); -1]
b = [x_0; f(x_0)]

draw_hyperplane_3D_modified(a, b, camera)
```

Out[12]:



In [ ]: