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Basics of Neural Network Programming

Binary Classification

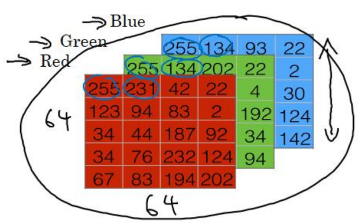
Binary Classification

64

64

$\longrightarrow$ 1 (cat) vs 0 (non cat)

y



$$x = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ 255 \\ 134 \\ \vdots \end{bmatrix}$$

$64 \times 64 \times 3 = 12288$

$n = n_x = 12288$

$x \longrightarrow y$

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Notation

$(x, y) \qquad x \in \mathbb{R}^{n_x}, \; y \in \{0, 1\}$

m training examples : $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$

$M = M_{train} \qquad M_{test} = \#\text{test examples.}$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \; n_x$$

$\longleftarrow m \longrightarrow$

$\cancel{\begin{bmatrix} - x^{(1)T} - \\ \vdots \\ - x^{(m)T} - \end{bmatrix}}$

$X \in \mathbb{R}^{n_x \times m} \qquad X.\text{shape} = (n_x, m)$

$Y = [y^{(1)} \; y^{(2)} \; \ldots, y^{(m)}]$

$Y \in \mathbb{R}^{1 \times m}$

$Y.\text{shape} = (1, m)$

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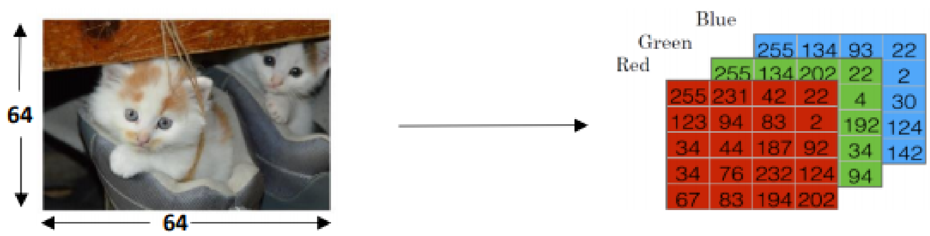
Binary Classification

In a binary classification problem, the result is a discrete value output.

For example - account hacked (1) or compromised (0)
- a tumor malign (1) or benign (0)

Example: Cat vs Non-Cat

The goal is to train a classifier that the input is an image represented by a feature vector, x, and predicts whether the corresponding label y is 1 or 0. In this case, whether this is a cat image (1) or a non-cat image (0).



An image is store in the computer in three separate matrices corresponding to the Red, Green, and Blue color channels of the image. The three matrices have the same size as the image, for example, the resolution of the cat image is 64 pixels X 64 pixels, the three matrices (RGB) are 64 X 64 each.

The value in a cell represents the pixel intensity which will be used to create a feature vector of n-dimension. In pattern recognition and machine learning, a feature vector represents an object, in this case, a cat or no cat.

To create a feature vector, x, the pixel intensity values will be “unroll” or “reshape” for each color. The dimension of the input feature vector x is $n_x = 64\ x\ 64\ x\ 3 = 12\ 288$.

$$x = \begin{bmatrix} 255 \\ 231 \\ 42 \\ \vdots \\ 255 \\ 134 \\ 202 \\ \vdots \\ 255 \\ 134 \\ 93 \\ \vdots \end{bmatrix} \begin{array}{l} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right] \text{red} \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right] \text{green} \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right] \text{blue} \end{array}$$

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Basics of Neural Network Programming

Logistic Regression

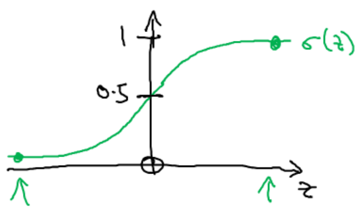
Logistic Regression

Given x, want $\hat{y} = P(y=1 \mid x)$

$x \in \mathbb{R}^{n_x}$ $\qquad 0 \le \hat{y} \le 1$

Parameters: $\boxed{w} \in \mathbb{R}^{n_x}$, $\boxed{b} \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$x_0 = 1, \quad x \in \mathbb{R}^{n_x+1}$

$\hat{y} = \sigma(\theta^T x)$

$$\theta = \underbrace{\begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n_x} \end{bmatrix}}$$

$\theta_0 \leftarrow b$; $\theta_1, \dots, \theta_{n_x} \leftarrow w$

$\sigma(z) = \frac{1}{1 + e^{-z}}$

If z large $\sigma(z) \approx \frac{1}{1+0} = 1$

If z large negative number

$\sigma(z) = \frac{1}{1+e^{-z}} \approx \frac{1}{1 + \text{Bignum}} \approx 0$

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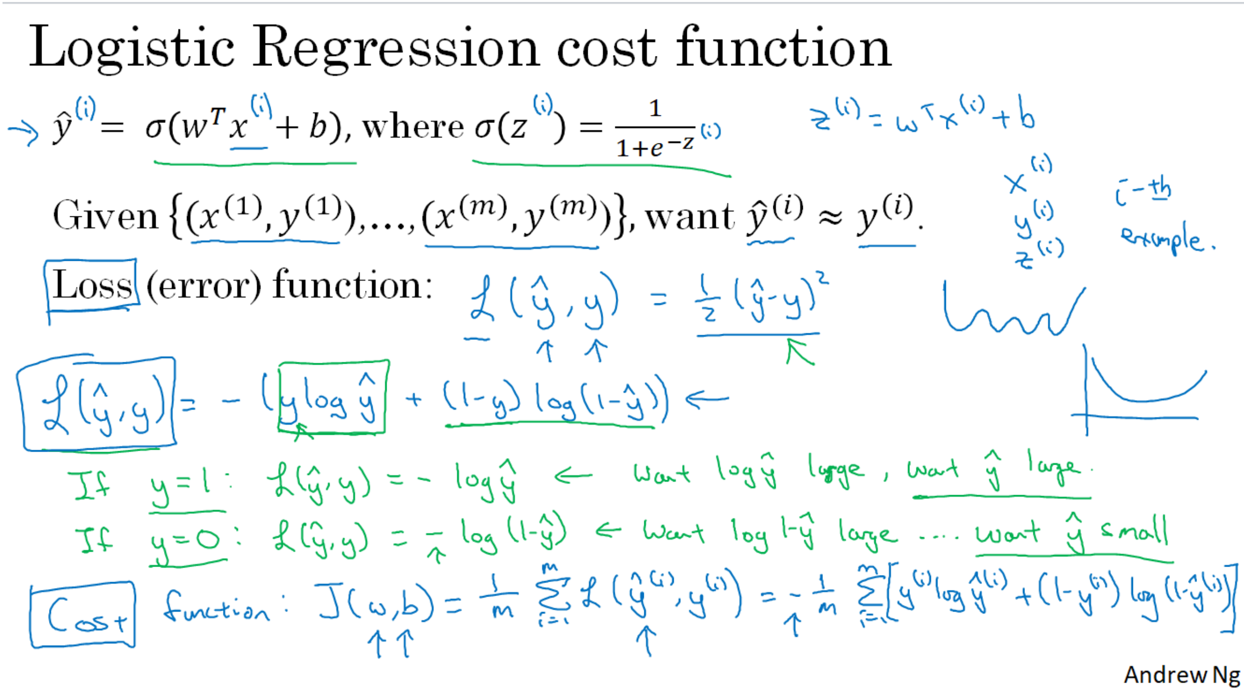
Basics of Neural Network Programming

Logistic Regression cost function



Logistic Regression: Cost Function

To train the parameters w and b, we need to define a cost function.

Recap:

$\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

$x^{(i)}$ the i-th training example

Given $\{(x^{(1)}, y^{(1)}), \cdots, (x^{(m)}, y^{(m)})\}$, *we want* $\hat{y}^{(i)} \approx y^{(i)}$

Loss (error) function:

The loss function measures the discrepancy between the prediction ($\hat{y}^{(i)}$) and the desired output ($y^{(i)}$). In other words, the loss function computes the error for a single training example.

$$L(\hat{y}^{(i)}, y^{(i)}) = \frac{1}{2}(\hat{y}^{(i)} - y^{(i)})^2$$

$$L(\hat{y}^{(i)}, y^{(i)}) = -(y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)})\log(1 - \hat{y}^{(i)})$$

- If $y^{(i)} = 1$: $L(\hat{y}^{(i)}, y^{(i)}) = -\log(\hat{y}^{(i)})$ where $\log(\hat{y}^{(i)})$ and $\hat{y}^{(i)}$ should be close to 1
- If $y^{(i)} = 0$: $L(\hat{y}^{(i)}, y^{(i)}) = -\log(1 - \hat{y}^{(i)})$ where $\log(1 - \hat{y}^{(i)})$ and $\hat{y}^{(i)}$ should be close to 0

Cost function

The cost function is the average of the loss function of the entire training set. We are going to find the parameters w *and* b that minimize the overall cost function.

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} L(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}[(y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)})\log(1 - \hat{y}^{(i)})]$$

Basics of Neural Network Programming

Gradient Descent

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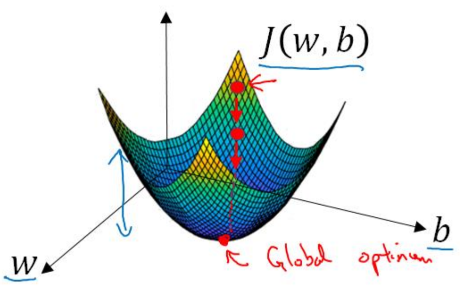
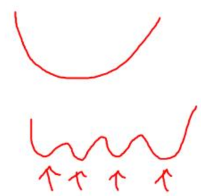
Gradient Descent

Recap: $\hat{y} = \sigma(w^T x + b)$, $\sigma(z) = \frac{1}{1+e^{-z}}$

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\boxed{\hat{y}^{(i)}}, \boxed{y^{(i)}}) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

Want to find w, b that minimize $J(w,b)$





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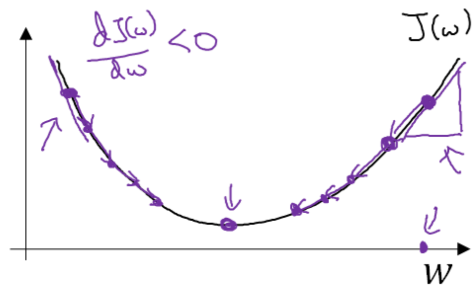
Gradient Descent



learning rate

Repeat {

$$w := w - \alpha \boxed{\frac{dJ(w)}{dw}}$$

}

"dw"

$w := w - \alpha\, dw$

$\frac{dJ(w)}{dw} = ?$

$J(w,b)$

$$w := w - \alpha \boxed{\frac{dJ(w,b)}{dw}} \quad \boxed{\frac{\partial J(w,b)}{\partial w}} \rightarrow dw$$

$$b := b - \alpha \boxed{\frac{dJ(w,b)}{db}} \quad \boxed{\frac{\partial J(w,b)}{\partial b}} \rightarrow db$$

∂ "partial derivative"

d J

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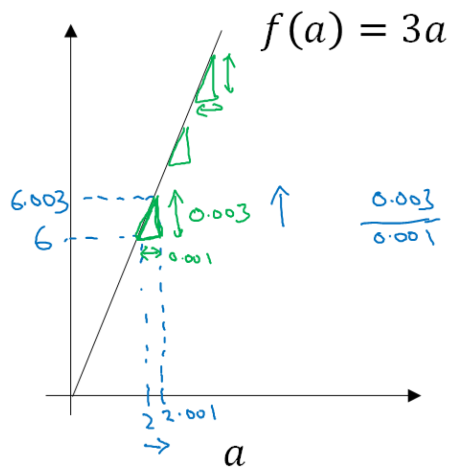
Derivatives

Intuition about derivatives



$\frac{0.003}{0.001}$ $\frac{\text{height}}{\text{width}}$

→ $a = 2$ $f(a) = 6$

$a = 2.\underline{001}$ $f(a) = 6.\underline{003}$

→ slope (derivative) of $f(a)$ at $a = 2$ is 3

→ $a = 5$ $f(a) = 15$

$a = 5.\underline{001}$ $f(a) = 15.\underline{003}$

slope at $a = 5$ is also 3

$$\frac{d f(a)}{da} = 3 = \frac{d}{da} f(a)$$

0.001
0.000000001
0.00000000001

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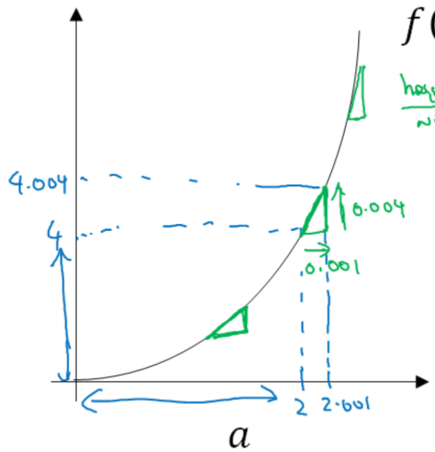
More derivatives examples

Intuition about derivatives

0.001
0.000000.....01

$f(a) = a^2$



$\frac{d}{da} a^2 = 2a$

0.001

$(2a) \times 0.001$

$a = 2$ $\quad f(a) = 4$

$a = 2.001$ $\quad f(a) \approx 4.004$

(4.004001)

slope (derivative) of f(a) at $a = 2$ is 4.

$\boxed{\frac{d}{da} f(a) = 4}$ when $\boxed{a = 2}$.

$a = 5$ $\quad f(a) = 25$

$a = 5.001$ $\quad f(a) \approx 25.010$

$\boxed{\frac{d}{da} f(a) = 10}$ when $\boxed{a = 5}$

$\frac{d}{da} f(a) = \frac{d}{da} a^2 = \boxed{2a}$

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More derivative examples

$f(a) = a^2$ $\qquad \frac{d}{da} f(a) = \underbrace{2a}_{4}$ $\qquad a = 2$ $\quad f(a) = 4$

$a = 2.001$ $\quad f(a) \approx 4.004$

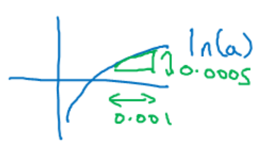
$f(a) = a^3$ $\qquad \frac{d}{da} f(a) = \underbrace{3a^2}_{3 \times 2^2 = 12}$ $\qquad a = 2$ $\quad f(a) = 8$

$a = 2.001$ $\quad f(a) \approx 8.012$

$f(a) = \log_e(a)$ $\qquad \frac{d}{da} f(a) = \frac{1}{a}$ $\qquad a = 2$ $\quad f(a) \approx 0.69315$

$\ln(a)$

$a = 2.001$ $\quad f(a) \approx 0.69365$



$\frac{d}{da} f(a) = \boxed{\frac{1}{2}}$

0.0005

0.0005

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Basics of Neural Network Programming

Computation Graph

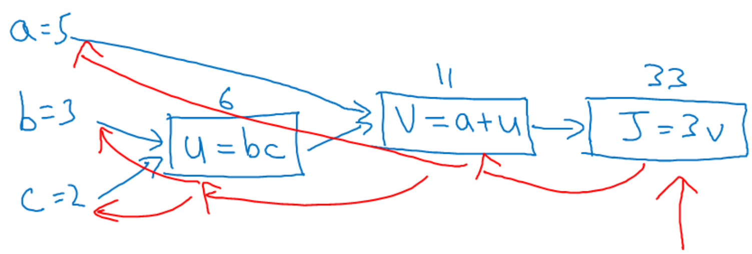
Computation Graph

$$J(a,b,c) = \underbrace{3(\underbrace{a+\underbrace{bc}_{u}}_{v})}_{J} = 3(5+3\times 2) = 33$$

$u = bc$

$v = a + u$

$J = 3v$



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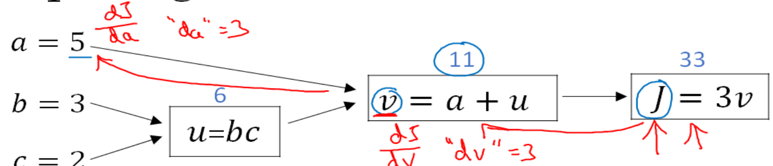
Basics of Neural Network Programming

Derivatives with a Computation Graph

Computing derivatives



$\frac{dJ}{da}$ "da" = 3

$\frac{dJ}{dv}$ "dv" = 3

$J = 3v$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$\boxed{\frac{dJ}{dv} = ? = 3}$

$a \rightarrow v \rightarrow J$

$\frac{dJ}{da} = 3 = \frac{dJ}{dv}\,\frac{dv}{da}$

3×1

$\boxed{\frac{dv}{da} = 1}$

$a = 5 \rightarrow 5.001$

$\rightarrow v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$\boxed{\frac{d\,\text{FinalOutputVar}}{d\,\text{var}}}$

dJdvar

"dvar"

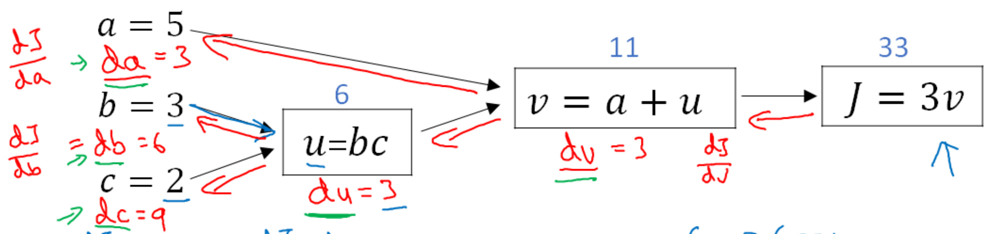
$\boxed{f(a) = 3a}$

$\frac{df(a)}{da} = \frac{df}{da} = 3$

$\boxed{J = 3v}$

$\frac{dJ}{dv} = 3$

Computing derivatives



$$\frac{dJ}{du} = 3 = \underbrace{\frac{dJ}{dv}}_{3} \cdot \underbrace{\frac{dv}{du}}_{1}$$

$$\frac{dJ}{db} = \underbrace{\boxed{\frac{dJ}{du}}}_{\to 3} \cdot \underbrace{\frac{du}{db}}_{=2} = 6$$

$$\frac{dJ}{da} = \boxed{\frac{dJ}{du}} \cdot \frac{du}{da} = 9$$

$u = 6 \rightarrow 6.001$
$v = 11 \rightarrow 11.001$
$J = 33 \rightarrow 33.003$

$b = 3 \rightarrow 3.001$
$u = b \cdot c = 6 \rightarrow 6.002$ $\quad c = 2$
$J = 33.006$ $\quad .006$

$v = 11.002$
$J = 3v$

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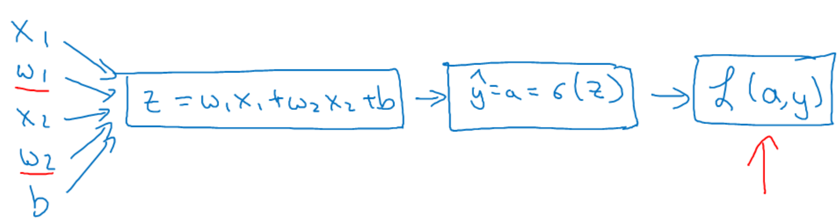
Basics of Neural Network Programming

Logistic Regression
Gradient descent

Logistic regression recap

$$\rightarrow z = w^T x + b$$
$$\rightarrow \hat{y} = a = \sigma(z)$$
$$\rightarrow \mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$



Logistic regression derivatives

$$x_1,\ w_1,\ x_2,\ w_2,\ b \longrightarrow \boxed{z = w_1x_1 + w_2x_2 + b} \longrightarrow \boxed{a = \sigma(z)} \longrightarrow \boxed{\mathcal{L}(a, y)}$$

$$dz = \frac{d\mathcal{L}}{dz} = \frac{d\mathcal{L}(a,y)}{dz}$$

$$= a - y$$

$$= \boxed{\frac{d\mathcal{L}}{da}} \cdot \boxed{\frac{da}{dz}} \qquad a(1-a)$$

$$\text{"}da\text{"} = \frac{d\mathcal{L}(a,y)}{da} = \boxed{-\frac{y}{a} + \frac{1-y}{1-a}}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = \text{"}dw_1\text{"} = x_1 \cdot dz. \qquad dw_2 = x_2 \cdot dz. \qquad db = dz.$$

$$w_1 := w_1 - \alpha\, dw_1$$
$$w_2 := w_2 - \alpha\, dw_2$$
$$b := b - \alpha\, db.$$

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Basics of Neural Network Programming

Gradient descent on m examples

Logistic regression on m examples

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)}) \qquad (x^{(i)}, y^{(i)})$$

$$\to a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b) \qquad dw_1^{(i)},\ dw_2^{(i)},\ db^{(i)}$$

$$\frac{\partial}{\partial w_1} J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \underbrace{\frac{\partial}{\partial w_1}\mathcal{L}(a^{(i)}, y^{(i)})}_{dw_1^{(i)}\ -\ (x^{(i)},\, y^{(i)})}$$

Logistic regression on m examples

$J=0;\ dw_1=0;\ dw_2=0;\ db=0$

For $i=1$ to m

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J += -[y^{(i)} \log a^{(i)} + (1-y^{(i)}) \log(1-a^{(i)})]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$

$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$

dw_3

$\vdots$

dw_n

$db \mathrel{+}= dz^{(i)}$

$n=2$

$J \mathrel{/}= m$

$dw_1 \mathrel{/}= m;\ dw_2 \mathrel{/}= m;\ db \mathrel{/}= m.$

$$dw_1 = \frac{\partial J}{\partial w_1}$$

$w_1 := w_1 - \alpha\, dw_1$

$w_2 := w_2 - \alpha\, dw_2$

$b := b - \alpha\, db$

Vectorization

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