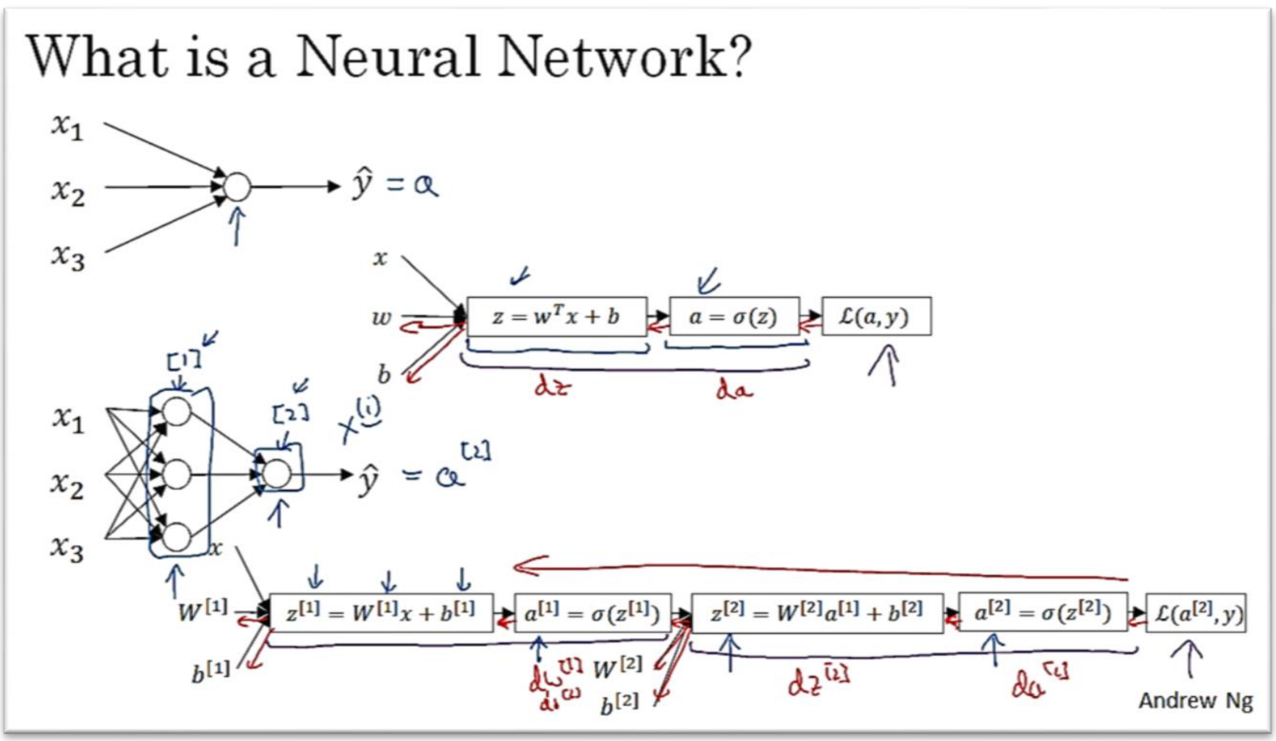
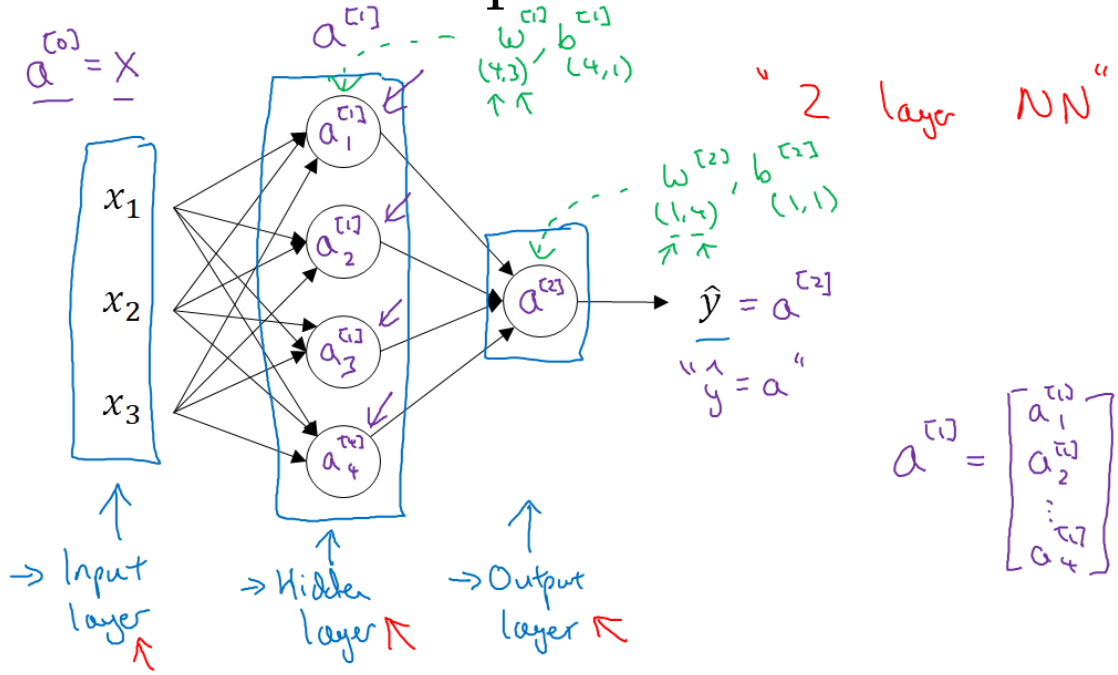
deeplearning.ai

One hidden layer Neural Network

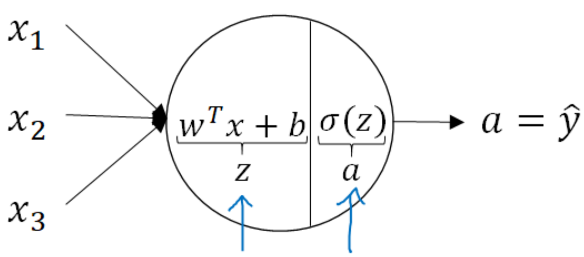
Neural Networks Overview

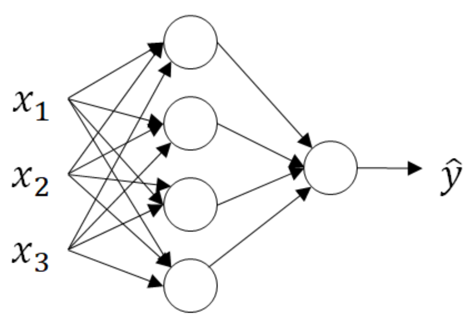


Neural Network Representation



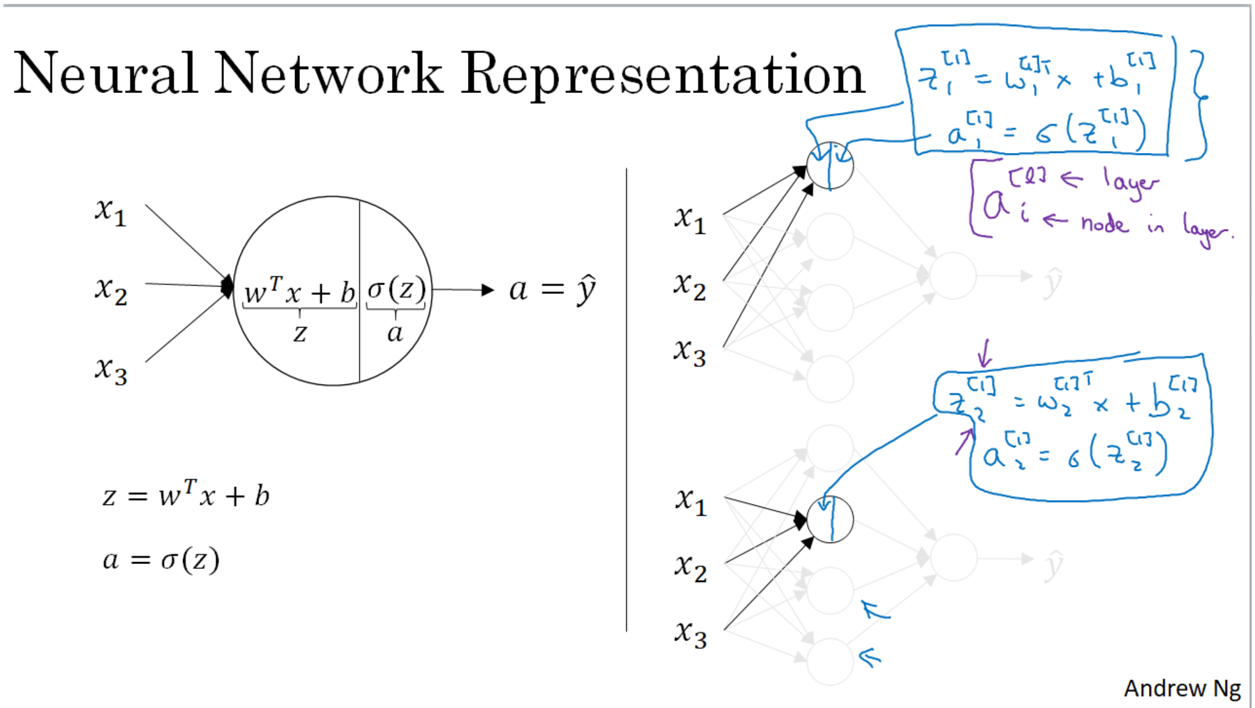
Neural Network Representation





$z = w^T x + b$

$a = \sigma(z)$


Neural Network Representation
x_1
x_2
x_3
$w^T x + b$
$\sigma(z)$
$a = \hat{y}$
z
a
$z = w^T x + b$
$a = \sigma(z)$
$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}$
$a_1^{[1]} = \sigma(z_1^{[1]})$
$a_i^{[l]}$ ← layer
← node in layer.
$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}$
$a_2^{[1]} = \sigma(z_2^{[1]})$
$\hat{y}$
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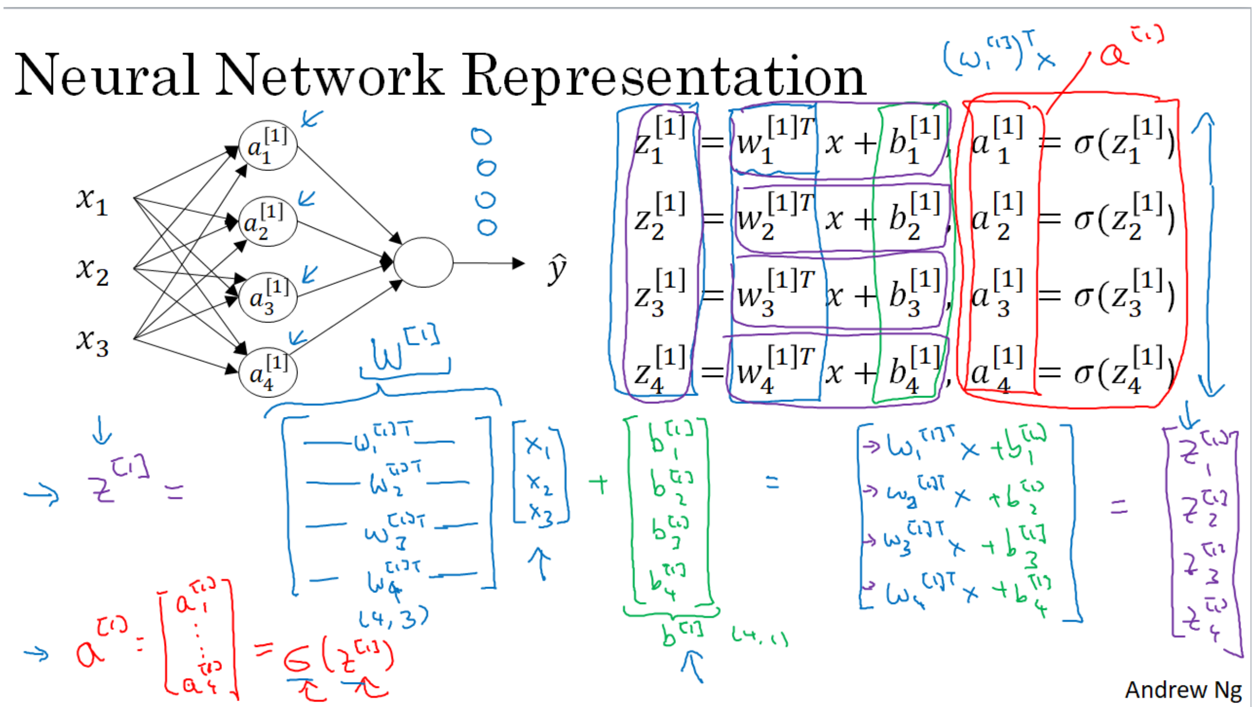
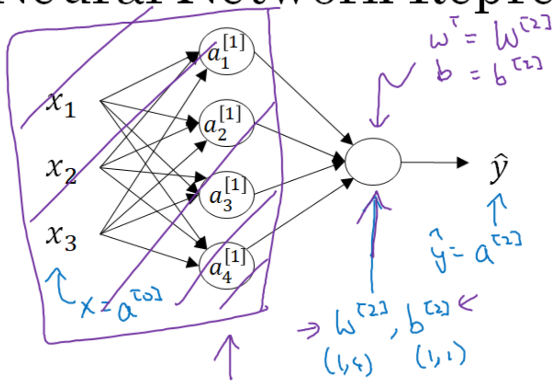

Neural Network Representation
$a_1^{[1]}$
$a_2^{[1]}$
$a_3^{[1]}$
$a_4^{[1]}$
$\hat{y}$
$(w_1^{[1]})^T x$
$a^{[1]}$
$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}, \quad a_1^{[1]} = \sigma(z_1^{[1]})$
$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}, \quad a_2^{[1]} = \sigma(z_2^{[1]})$
$z_3^{[1]} = w_3^{[1]T} x + b_3^{[1]}, \quad a_3^{[1]} = \sigma(z_3^{[1]})$
$z_4^{[1]} = w_4^{[1]T} x + b_4^{[1]}, \quad a_4^{[1]} = \sigma(z_4^{[1]})$
$W^{[1]}$
(4, 3)
$b^{[1]}$ (4, 1)
$z^{[1]} = \begin{bmatrix} - w_1^{[1]T} - \\ - w_2^{[1]T} - \\ - w_3^{[1]T} - \\ - w_4^{[1]T} - \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix} = \begin{bmatrix} w_1^{[1]T} x + b_1^{[1]} \\ w_2^{[1]T} x + b_2^{[1]} \\ w_3^{[1]T} x + b_3^{[1]} \\ w_4^{[1]T} x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}$
$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})$
Andrew Ng

Neural Network Representation learning



$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

Given input x:

$$\underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}} \underset{(3,1)}{\cancel{x}\,a^{[0]}} + \underset{(4,1)}{b^{[1]}}$$

$$\underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}} \underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

$$\underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

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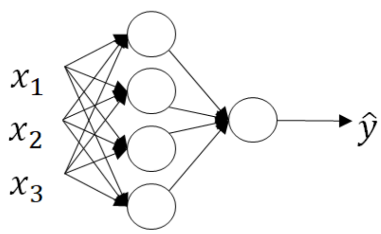
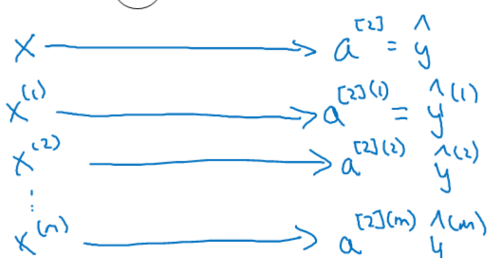
One hidden layer
Neural Network

Vectorizing across
multiple examples

Vectorizing across multiple examples





$a^{[2](i)}$ ← example i ← layer 2

$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$

for $i = 1$ to m,

$$z^{[1](i)} = W^{[1]} x^{(i)} + b^{[1]}$$
$$a^{[1](i)} = \sigma(z^{[1](i)})$$
$$z^{[2](i)} = W^{[2]} a^{[1](i)} + b^{[2]}$$
$$a^{[2](i)} = \sigma(z^{[2](i)})$$

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Vectorizing across multiple examples

for i = 1 to m:

$$\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$$
$$\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$$
$$\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$$
$$\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$$

$$\boxed{\begin{aligned} Z^{[1]} &= W^{[1]}X + b^{[1]} \\ A^{[1]} &= \sigma(Z^{[1]}) \\ Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\ A^{[2]} &= \sigma(Z^{[2]}) \end{aligned}}$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \quad (n_x, m)$$

training examples

hidden units

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

hidden units

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One hidden layer Neural Network

Explanation for vectorized implementation

Justification for vectorized implementation

$$z^{[1](1)} = W^{[1]}x^{(1)} + \cancel{b^{[1]}}, \quad z^{[1](2)} = W^{[1]}x^{(2)} + \cancel{b^{[1]}}, \quad z^{[1](3)} = W^{[1]}x^{(3)} + \cancel{b^{[1]}}$$

0 0 0

$$W^{[1]} = \begin{bmatrix} \text{———} \\ \text{———} \\ \text{———} \\ \text{———} \end{bmatrix} \quad W^{[1]}x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \quad W^{[1]}x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \quad W^{[1]}x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \end{bmatrix} = Z^{[1]}$$

$+b^{[1]}$ $+b^{[1]}$ $+b^{[1]}$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$W^{[1]}x^{(i)} = z^{[1](i)}$$

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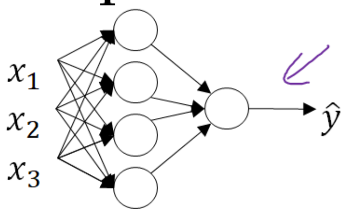
Recap of vectorizing across multiple examples



for i = 1 to m

$$\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$$
$$a^{[1](i)} = \sigma(z^{[1](i)})$$
$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$
$$a^{[2](i)} = \sigma(z^{[2](i)})$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$A^{[0]}$ $\quad X = a^{[0]}$ $\quad x^{(i)} = a^{[0](i)}$

$$Z^{[1]} = W^{[1]}X + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$$
$$A^{[1]} = \sigma(Z^{[1]})$$
$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = \sigma(Z^{[2]})$$

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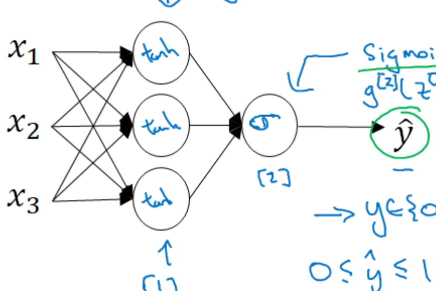
Activation functions

$g^{[1]}(z^{[1]}) = \tanh(z^{[1]})$

Sigmoid $\quad g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$



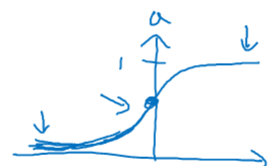
$\rightarrow y \in \{0,1\}$

$0 \leq \hat{y} \leq 1$

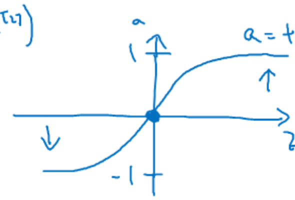
$-1 \quad 1$

Given x:

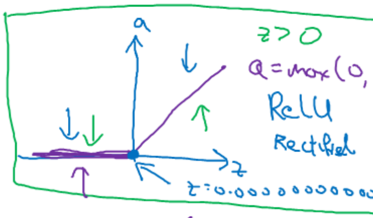
$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$\rightarrow a^{[1]} = \cancel{\sigma(z^{[1]})}\ g^{[1]}(z^{[1]})$$
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$\rightarrow a^{[2]} = \cancel{\sigma(z^{[2]})}\ g^{[2]}(z^{[2]})$$



$a = \frac{1}{1+e^{-z}}$



$a = \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$



$z > 0$

$a = \max(0, z)$

ReLU

Rectified Linear Unit

leaky ReLU

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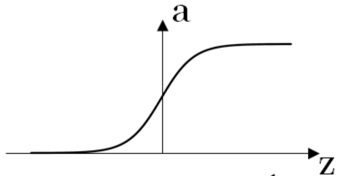
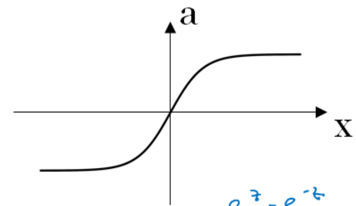
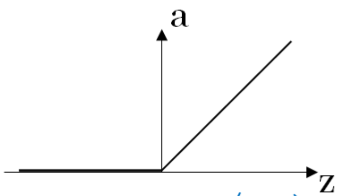
Pros and cons of activation functions



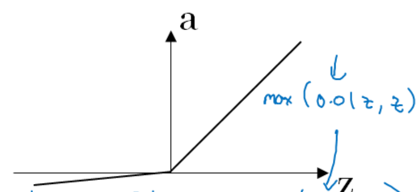
sigmoid: $a = \dfrac{1}{1+e^{-z}}$



tanh: $a = \dfrac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$



ReLU $a = \max(0, z)$



$\max(0.01z, z)$

Leaky ReLU $a = \max(0.01z, z)$

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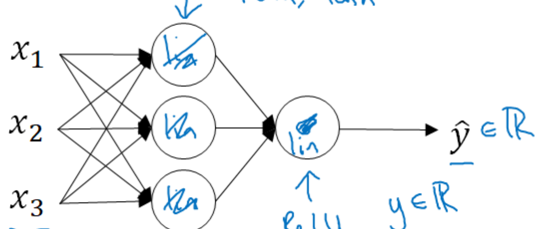
One hidden layer
Neural Network

Why do you
need non-linear
activation functions?

Activation function

ReLU, tanh



$y \in \mathbb{R}$

$0 ..... $1,000,000

$\hat{y} \geq 0$.

Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = \cancel{g^{[1]}(z^{[1]})}\; z^{[1]}$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = \cancel{g^{[2]}(z^{[2]})}\; z^{[2]}$$

$g(z) = z$

"linear activation function"

$$a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = W^{[2]}\underbrace{(W^{[1]}x + b^{[1]})}_{a^{[1]}} + b^{[2]}$$

$$= \underbrace{(W^{[2]}W^{[1]})}_{W'}x + \underbrace{(W^{[2]}b^{[1]} + b^{[2]})}_{b'}$$

$$= W'x + b'$$

$g(z) = z$

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One hidden layer
Neural Network

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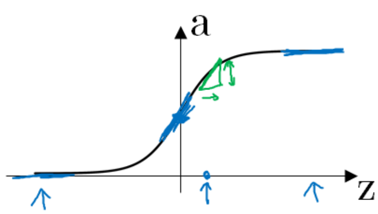
Derivatives of
activation functions

Sigmoid activation function



$$g(z) = \frac{1}{1+e^{-z}}$$

$a = g(z) = \frac{1}{1+e^{-z}}$

$\boxed{g'(z)} = \boxed{\frac{d}{dz} g(z)}$ = slope of $g(x)$ at z

$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$

$= \underline{g(z)\left(1 - g(z)\right)}$

$= \boxed{a(1-a)}$

$g'(z) = a(1-a)$

$z = 10$. $g(z) \approx 1$

$\frac{d}{dz} g(z) \approx 1(1-1) \approx 0$

$z = -10$ $g(z) \approx 0$

$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$

$z = 0$ $g(z) = \frac{1}{2}$

$\frac{d}{dz} g(z) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4}$

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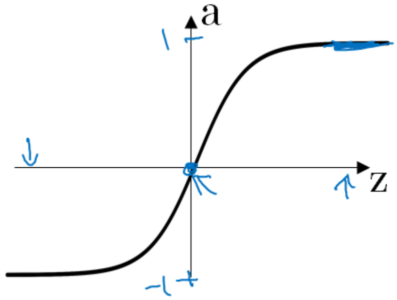
Tanh activation function



$g(z) = \tanh(z)$

$= \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$

$z = 10 \quad \tanh(z) \approx 1$

$g'(z) \approx 0$

$z = -10 \quad \tanh(z) \approx -1$

$g'(z) \approx 0$

$z = 0 \quad \tanh(z) = 0$

$g'(z) = 1$

$g'(z) = \frac{d}{dz} g(z)$ = slope of $g(z)$ at z

$= 1 - (\tanh(z))^2$

$a = g(z), \quad g'(z) = 1 - a^2$

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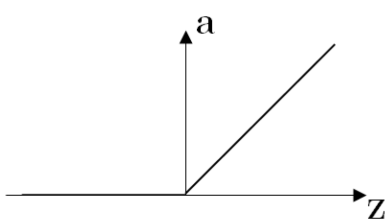
ReLU and Leaky ReLU

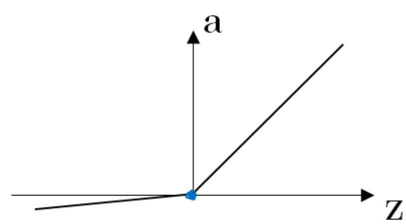


ReLU

$g(z) = \max(0, z)$

$$g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \\ \cancel{\text{undef}} & \cancel{\text{if } z = 0} \end{cases}$$

$z = 0.00000\ldots0$



Leaky ReLU

$g(z) = \max(0.01z, z)$

$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

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One hidden layer
Neural Network

Gradient descent for
neural networks

Gradient descent for neural networks

Parameters: $\underset{(n^{[1]},n^{[0]})}{W^{[1]}}, \underset{(n^{[1]},1)}{b^{[1]}}, \underset{(n^{[2]},n^{[1]})}{W^{[2]}}, \underset{(n^{[2]},1)}{b^{[2]}}$ $n_x = n^{[0]}, n^{[1]}, n^{[2]} = 1$

Cost function: $J(W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$ ← $a^{[2]}$

Gradient descent:

→ Repeat {

→ Compute predicts $(\hat{y}^{(i)}, i=1 \ldots, m)$

$dW^{[1]} = \frac{\partial J}{\partial W^{[1]}}$, $db^{[1]} = \frac{\partial J}{\partial b^{[1]}}$, ...

$W^{[1]} := W^{[1]} - \alpha\, dW^{[1]}$

$b^{[1]} := b^{[1]} - \alpha\, db^{[1]}$

$W^{[2]} := \ldots$ $b^{[2]} := \ldots$

}

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Formulas for computing derivatives

Forward propagation:

$Z^{[1]} = W^{[1]}X + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$ ←

$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$

$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$

Back propagation:

$dZ^{[2]} = A^{[2]} - Y$ ← $Y = [y^{(1)}\ y^{(2)} \ldots, y^{(m)}]$

$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$ $(n^{[2]},)$ ← $(n^{[2]}, 1)$ ←

$db^{[2]} = \frac{1}{m}$ np.sum($dZ^{[2]}$, axis=1, keepdims=True)

$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]},m)} * \underbrace{g^{[1]'}(Z^{[1]})}_{(n^{[1]},m)}$ ← element-wise product

$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$

$db^{[1]} = \frac{1}{m}$ np.sum($dZ^{[1]}$, axis=1, keepdims=True)

$(n^{[1]},1)$ $(n^{[1]},)$ reshape ↑

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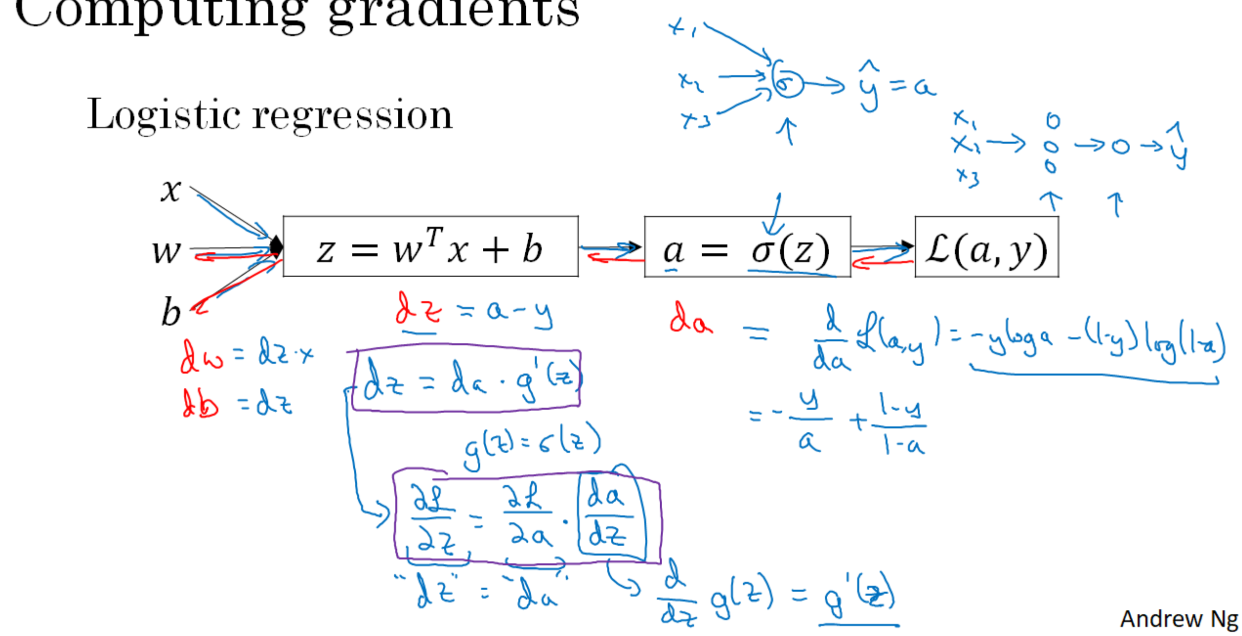
One hidden layer Neural Network

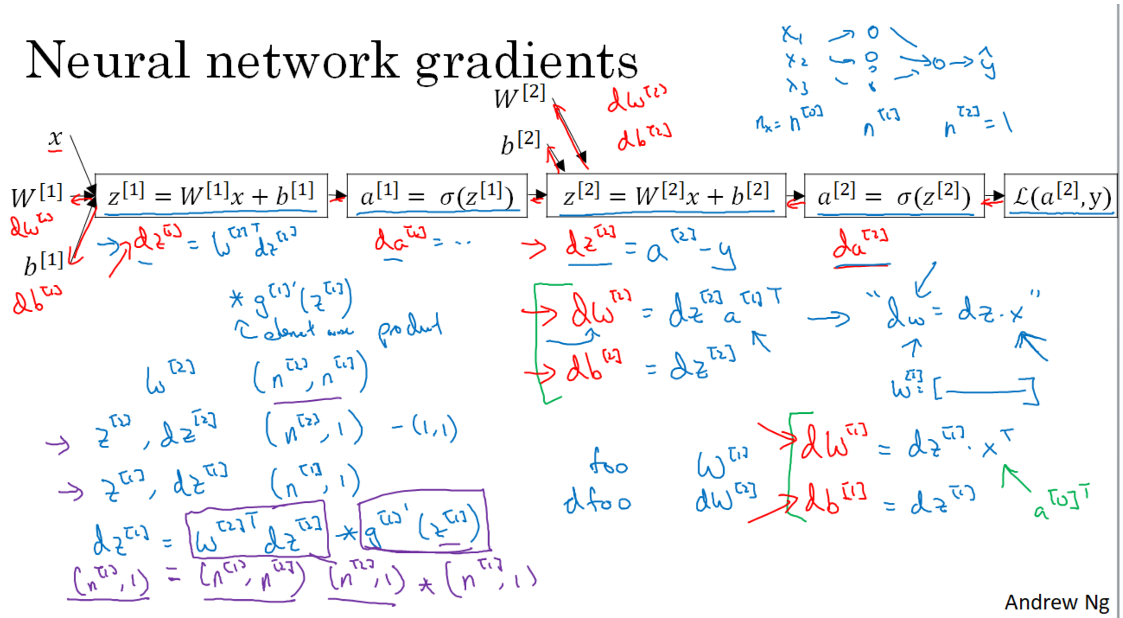
Backpropagation intuition (Optional)

Computing gradients

Logistic regression



Neural network gradients



Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

Vectorized Implementation:

$$z^{[1]} = W^{[1]} x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

Summary of gradient descent

$\underline{dz^{[2]}} = \underline{a^{[2]}} - \underline{y}$	$\underline{dZ^{[2]} = A^{[2]} - Y}$
$dW^{[2]} = dz^{[2]} a^{[1]^T}$	$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$
$db^{[2]} = dz^{[2]}$	$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True)$
$\underset{(n^{[1]},1)}{dz^{[1]}} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$	$\underset{(n^{[1]},m)}{dZ^{[1]}} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]},m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]},m)}$
$dW^{[1]} = dz^{[1]} x^T$	$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$
$db^{[1]} = dz^{[1]}$	$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)$

$$J(\ldots) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$$

elementwise product

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One hidden layer
Neural Network

Random Initialization

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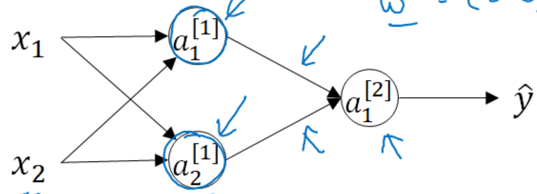
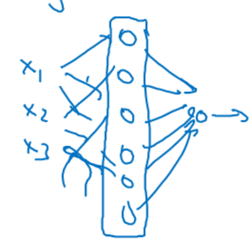
What happens if you initialize weights to zero?

$W^{[2]} = [0\ 0]$



$n^{[0]} = 2$ $n^{[1]} = 2$

Symmetric



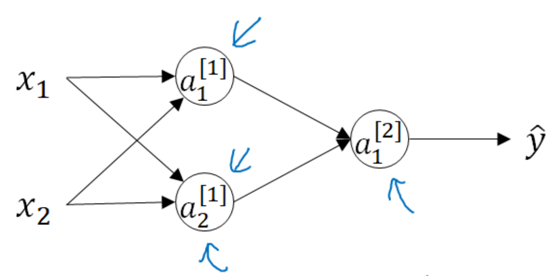
$W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$W^{[1]} = \begin{bmatrix} \cdots\cdots \\ \cdots\cdots \end{bmatrix}$

$a_1^{[1]} = a_2^{[1]}$ $dz_1^{[1]} = dz_2^{[1]}$

$dW = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$ $W^{[1]} = W^{[1]} - \alpha dW$

Random initialization



$\rightarrow$ $W^{[1]} =$ np.random.randn((2,2)) $* 0.01$ $\quad \frac{}{100?}$

$b^{[1]} =$ np.zeros((2,1))

$W^{[2]} =$ np.random.randn((1,2)) $* 0.01$

$b^{[2]} = 0$

$\rightarrow z^{[1]} = W^{[1]}x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$