Part 1.

$$z = f(x,y) = ax + by + c$$

$$\nabla z = \frac{\delta f}{\delta x}\hat{i} + \frac{\delta}{\delta y} f \hat{j}$$

$$\frac{\delta}{\delta x} f = a$$

$$\frac{\delta}{\delta y} f = b$$

$$\nabla z = a\hat{i} + b\hat{j}$$

$$z = f(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} a(x_i - b_i) + S = a x_1 + a_2 x_2 \ldots + a_N x_N + d$$

$$\nabla z = \frac{\delta}{\delta x_1} f \hat{i}_1 + \frac{\delta}{\delta x_2} f \hat{i}_2 \ldots + \frac{\delta}{\delta x_N} f \hat{i}_N$$

$$= \sum_{i=1}^{N} \frac{\delta f}{\delta x_i} \hat{i}_i = \sum_{i=1}^{N} a_i \hat{i}_i$$

$$z = f(x,y) = A(x-x_0)^2 + B(y-y_0)^2 + C$$

$$\frac{\delta}{\delta x} f = \frac{\delta}{\delta x}\left(A(x-x_0)^2 + \cancel{B(y-y_0)^2 + C}\right)$$

$$= \frac{\delta}{\delta x}\left(A(x^2 - 2xx_0 + \cancel{x_0^2})\right)$$

$$= 2Ax - 2A$$

$$\frac{\delta}{\delta y} f = \frac{\delta}{\delta y}\left(\cancel{A(x-x_0)^2} + B(y-y_0)^2 + \cancel{C}\right)$$

$$= \frac{\delta}{\delta y}\left(B(y^2 - 2yy_0 + y_0^2)\right)$$

$$= 2By - 2B$$

$$X = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \quad y = \begin{pmatrix} 2 & 5 & 1 \end{pmatrix} \quad A = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 1 & 5 \\ 6 & 4 & 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 3 & 5 \\ 5 & 2 \\ 1 & 4 \end{pmatrix}$$

$$X^T = \begin{pmatrix} 3 & 4 & 1 \end{pmatrix}$$

$$y^T = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$$

$$X \cdot X = 3 \cdot 3 + 4 \cdot 4 + 1 \cdot 1 = 26$$

$$X \cdot y^T = 3 \cdot 2 + 4 \cdot 5 + 1 \cdot 1 = 27$$

$$X \times y = \begin{pmatrix} 3 \cdot 2 & 3 \cdot 5 & 3 \cdot 1 \\ 4 \cdot 2 & 4 \cdot 5 & 4 \cdot 1 \\ 1 \cdot 2 & 1 \cdot 5 & 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 6 & 15 & 3 \\ 8 & 20 & 4 \\ 2 & 5 & 1 \end{pmatrix}$$

$$y \times X = (y \cdot x) = (27)$$

$$A \times X = \begin{pmatrix} 4 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 \\ 3 \cdot 3 + 4 \cdot 1 + 1 \cdot 5 \\ 3 \cdot 6 + 4 \cdot 4 + 1 \cdot 3 \end{pmatrix} = \begin{pmatrix} 12 + 20 + 2 \\ 9 + 4 + 5 \\ 18 + 16 + 3 \end{pmatrix} = \begin{pmatrix} 34 \\ 18 \\ 37 \end{pmatrix}$$

$$A \times B = \begin{pmatrix} 3 \cdot 4 + 5 \cdot 5 + 1 \cdot 2 & 5 \cdot 4 + 2 \cdot 5 + 4 \cdot 2 \\ 3 \cdot 3 + 5 \cdot 1 + 1 \cdot 5 & 5 \cdot 3 + 2 \cdot 1 + 4 \cdot 5 \\ 3 \cdot 6 + 5 \cdot 4 + 1 \cdot 3 & 5 \cdot 6 + 2 \cdot 4 + 4 \cdot 3 \end{pmatrix} = \begin{pmatrix} 40 & 38 \\ 19 & 37 \\ 41 & 50 \end{pmatrix}$$

$$B.\text{reshape}(1,6) = \begin{pmatrix} 3 & 5 & 5 & 2 & 1 & 4 \end{pmatrix}$$

$$y = M(x|p) = mx + b$$

$$Loss = \sum_{i=1}^{N} (y_i - (mx_i + b))^2 = 0$$

$$\Rightarrow \sum_{i=1}^{N} (y_i - b - mx_i)^2 = 0$$

$$\Rightarrow \sum_{i=1}^{N} (y_i - \bar{y} + m\bar{x} - mx_i) = 0$$

$$\Rightarrow \sum_{i=1}^{N} (y_i - \bar{y}) + \sum_{i=1}^{N} (m\bar{x} - mx_i) = 0$$

$$\sum_{i=1}^{N} (y_i - \bar{y}) = \sum_{i=1}^{N} (mx_i - m\bar{x})$$

$$\frac{\sum (x_i - \bar{x})}{\sum (x_i - \bar{x})} \cdot \frac{\sum_{i=1}^{N} (y_i - \bar{y})}{\sum_{i=1}^{N} (x_i - \bar{x})} = m$$

$$= \frac{Cov(x,y)}{Var(x)} = m$$

$$y_i = mx_i + b$$

$$\bar{y} = \frac{\sum_{i=1}^{N} y_i}{N} \qquad \bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

$$\bar{y} = \frac{1}{N} \sum_{i=1}^{N} (mx_i + b)$$

$$= \frac{m \sum_{i=1}^{N} x_i + b}{N}$$

$$\bar{y} = m\bar{x} + b$$

$$b = \bar{y} - m\bar{x}$$

$$b = \bar{y} - m\bar{x}$$

$$b = \bar{y} - \frac{Cov(x,y)}{Var(x)} \cdot \bar{x}$$