About This Graph

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1 Background on Smooth Numbers

Think of the common quantities that you can buy packs of beer in: 6, 12, 15, 18, 24, 30, 36, etc. These values are chosen for a few reasons, some of which you can probably come up with on your own. They're great values for splitting amongst different numbers of friends, and they're also great for stacking in different ways. These same numbers show up for other items you might buy large quantities of, and you probably see common values for even larger amounts (48, 60, 72, ...).

One way we can try to describe such numbers is defining how *smooth* they are. Roughly speaking, a smoother number has smaller prime divisors than one that is less smooth. We don't classify numbers as smooth or non-smooth, the way we do with odds and evens or primes and composites. Instead, we define them based off a level of smoothness. Formally speaking, we say that a number n is y-smooth (or y-friable) if all of its prime factors p satisfy $p \leq y$. For example, 14 is 7-smooth but not 5-smooth, and all of the numbers above are 5-smooth. It's often convenient to find the maximum value y for which n is smooth, and this is simply the largest prime divisor of n, which is commonly denoted $P^+(n)$. The first graph below displays the different values of $P^+(n)$ for the first 100 positive integers on a logarithmic scale. Out of convention, we take $P^+(1) = \infty$, and we'll ignore this for the pretty pictures. The second graph is a the same graph except for the first 10000 positive integers, rescaled (logarithmically), and the third is the first million. Maybe take a second to try to find some of the cool patterns in each of them.

Smoother numbers have certain divisibility properties that are useful in certain applications. This isn't the only way to describe these properties. There are a few ways to generalize these ideas, and this is what some of my research involves, so stay tuned for more fun pictures and explanations!

Figure 1: Graph of $P^+(n)$ for $n \in [1, 100]$

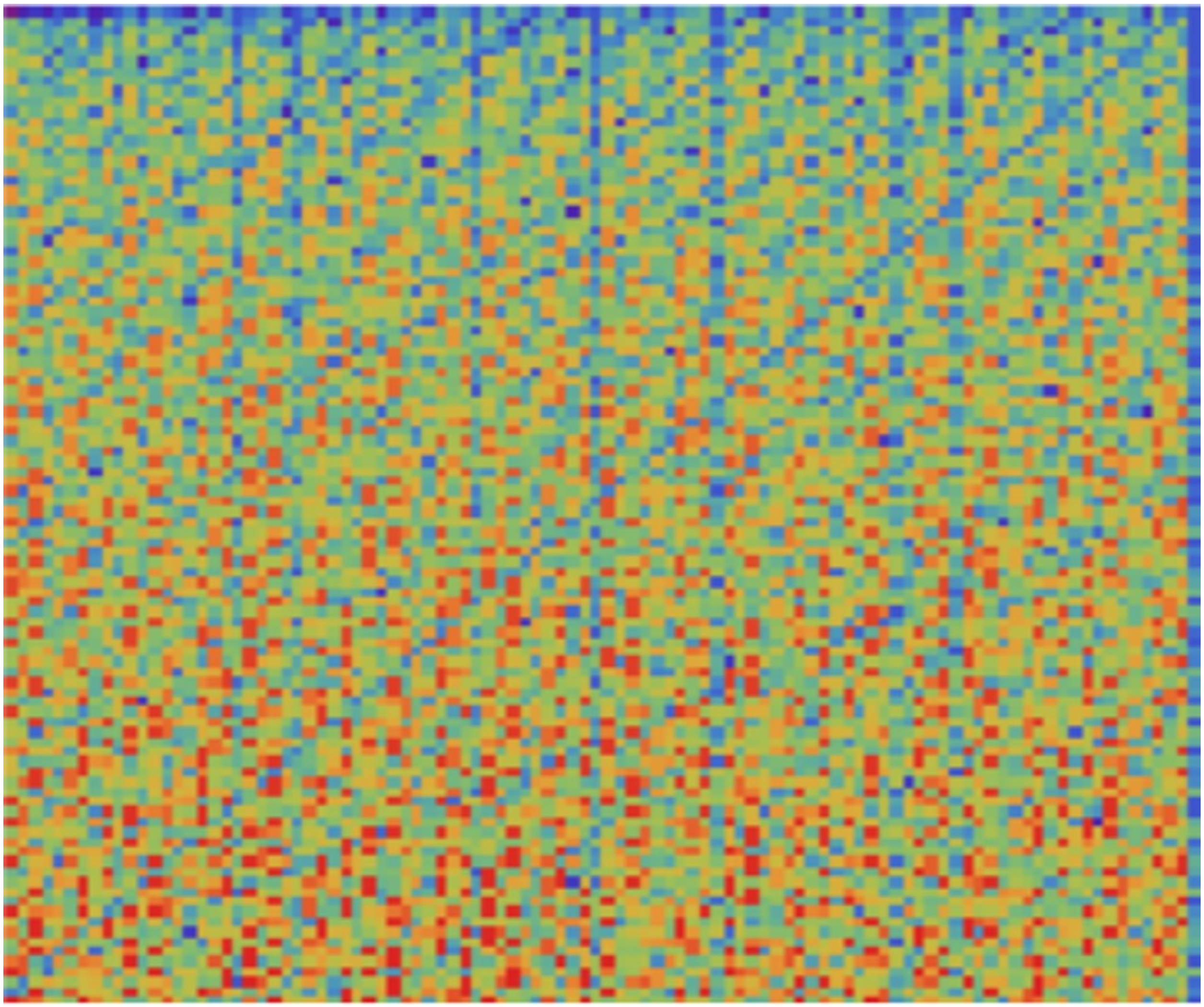

Figure 2: Graph of $P^+(n)$ for $n \in [1, 10000]$

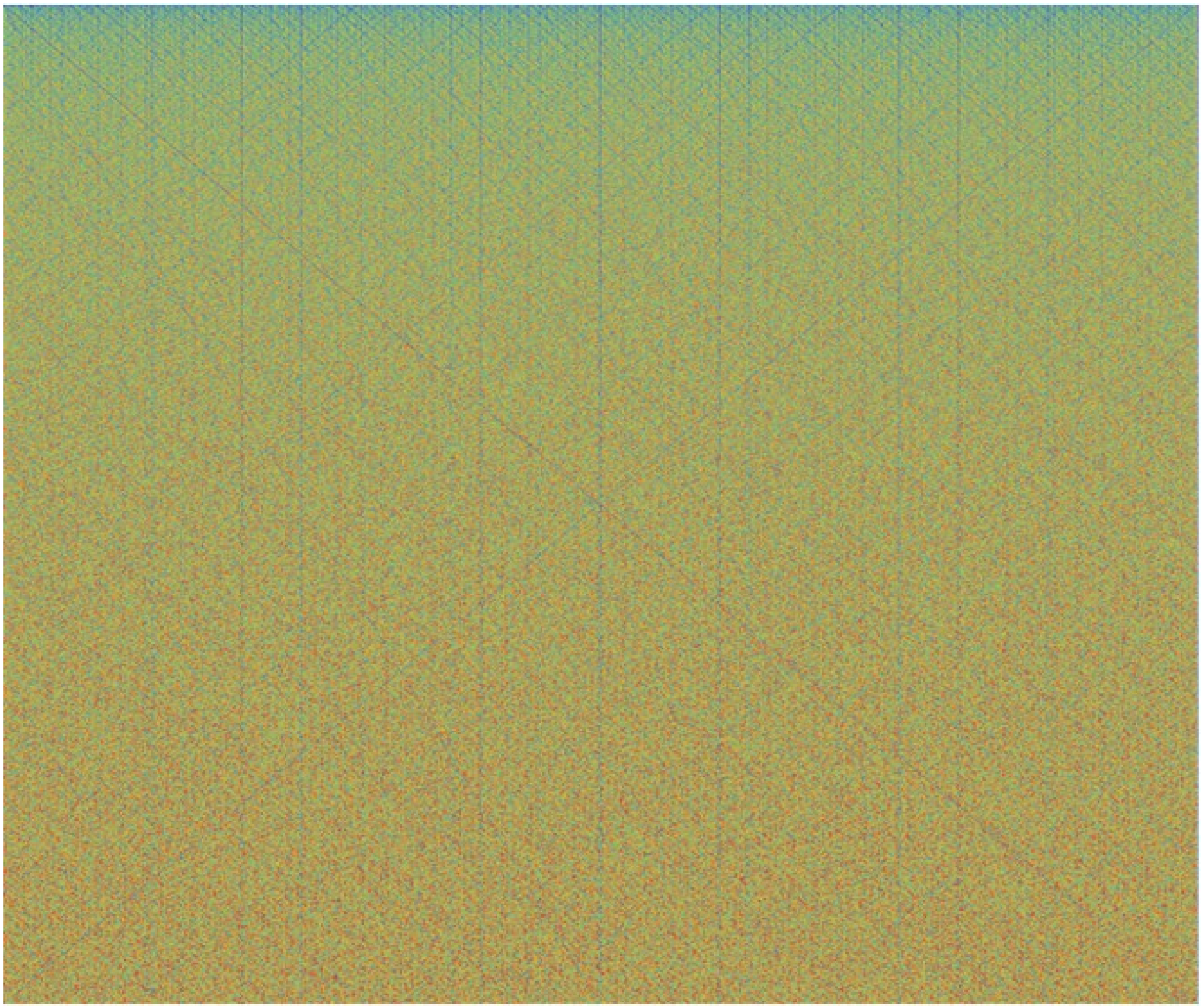

Figure 3: Graph of $P^+(n)$ for $n \in [1, 1000000]$