Mech 422/522 **Project II** **March 23, 2021**

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Flow around a Bluff-Body in a Channel Flow

Modify the program described in class to solve for the flow through the channel below Assume

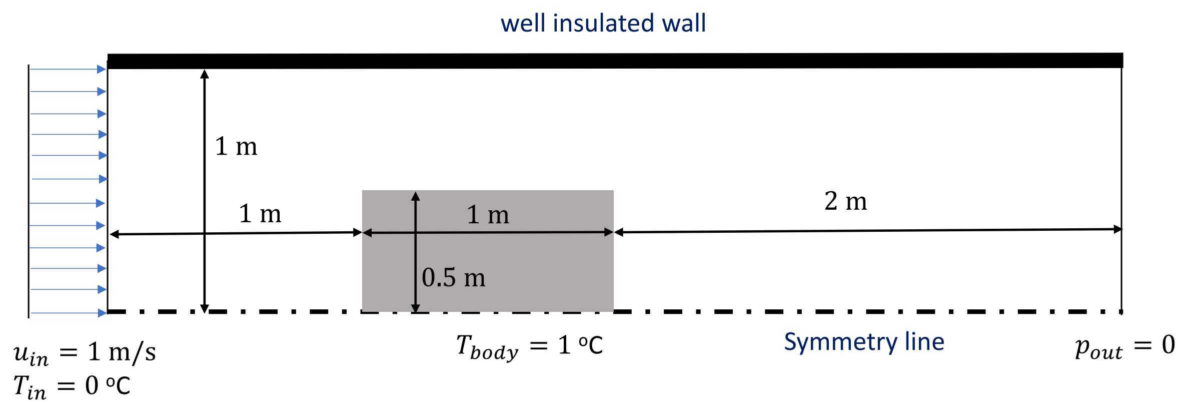

Figure 1: Sketch for the two-dimensional flow around a bluff-body in a channel.

uniform inflow and outflow, with $u_{in} = 1$ m/s. The fluid density is assumed to be constant at $\rho = 1$ kg/m^3. Take a viscosity value so that the Reynolds number $Re = 20$, based on the height of the channel ($H = 2$ m) and inlet velocity. Notice that the top wall is a no-slip wall but the boundary condition for the bottom symmetry boundary can be taken as a full-slip wall.

1. Find the force on the object versus time. (*Hint:* Use the formulation discussed in lecture 8 on slide 100 titled "Computing Total Force and Heat Transfer".)

2. Add a code to solve the energy equation

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \alpha \nabla^2 T, \tag{1}$$

where the velocity comes from a solution of the Navier-Stokes equations. The inlet temperature is 0, the temperature at the surface of the object is 1, and the normal gradient of the temperature at the outlet, the top and the bottom are all zero. Show the temperature flux from the object versus time. Take $\alpha = \mu/\rho$, i.e., Prandtl number is unity.

Report your results for the force as a drag coefficient defined as $C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A}$ where $U = u_{in}$ is the average velocity, A is the cross-sectional area of the block and F_D is the drag force acting on the block, and the heat transfer in terms of the average Nusselt number, defined by:

$$Nu = \frac{hL}{k} = \frac{L}{\Delta T}\frac{1}{A}\iint_A \left(-\frac{\partial T}{\partial n}|_{wall}\right) dA, \tag{2}$$

where h is the convective heat transfer coefficient of the fluid, L is the characteristic length, k is the thermal conductivity of the fluid, $\Delta T = T_{body} - T_{in}$ is the temperature difference and A is the surface area of the block.

You should hand in a discussion of what you have done and the tests that you conducted to demonstrate the accuracy of your solution. Your report should include a printout of your code and plots of the steady-state solution (velocity vectors, pressure contours, and temperature) for at least two different resolutions. You can also compute the streamfunction and the vorticity.

Hint: You can start with the following code. You need to modify it.

```
nx=49;ny=49;dt=0.001;nstep=200;mu=0.1;maxit=100;beta=1.2;h=1/nx;
u=zeros(nx+1,ny+2);v=zeros(nx+2,ny+1);p=zeros(nx+2,ny+2);
ut=zeros(nx+1,ny+2);vt=zeros(nx+2,ny+1);c=zeros(nx+2,ny+2)+0.25;
uu=zeros(nx+1,ny+1);vv=zeros(nx+1,ny+1);w=zeros(nx+1,ny+1);
c(2,3:ny)=1/3;c(nx+1,3:ny)=1/3;c(3:nx,2)=1/3;c(3:nx,ny+1)=1/3;
c(2,2)=1/2;c(2,ny+1)=1/2;c(nx+1,2)=1/2;c(nx+1,ny+1)=1/2;
un=1;us=0;ve=0;vw=0;time=0.0;
for is=1:nstep
u(1:nx+1,1)=2*us-u(1:nx+1,2);u(1:nx+1,ny+2)=2*un-u(1:nx+1,ny+1);
v(1,1:ny+1)=2*vw-v(2,1:ny+1);v(nx+2,1:ny+1)=2*ve-v(nx+1,1:ny+1);
for i=2:nx, for j=2:ny+1 % temporary u-velocity
ut(i,j)=u(i,j)+dt*(-(0.25/h)*((u(i+1,j)+u(i,j))^2-(u(i,j)+...
u(i-1,j))^2+(u(i,j+1)+u(i,j))*(v(i+1,j)+...
v(i,j))-(u(i,j)+u(i,j-1))*(v(i+1,j-1)+v(i,j-1)))+...
(mu/h^2)*(u(i+1,j)+u(i-1,j)+u(i,j+1)+u(i,j-1)-4*u(i,j)));
end, end
for i=2:nx+1, for j=2:ny % temporary v-velocity
vt(i,j)=v(i,j)+dt*(-(0.25/h)*((u(i,j+1)+u(i,j))*(v(i+1,j)+...
v(i,j))-(u(i-1,j+1)+u(i-1,j))*(v(i,j)+v(i-1,j))+...
(v(i,j+1)+v(i,j))^2-(v(i,j)+v(i,j-1))^2)+...
(mu/h^2)*(v(i+1,j)+v(i-1,j)+v(i,j+1)+v(i,j-1)-4*v(i,j)));
end, end
for it=1:maxit % solve for pressure
```

```
for i=2:nx+1, for j=2:ny+1
p(i,j)=beta*c(i,j)*(p(i+1,j)+p(i-1,j)+p(i,j+1)+p(i,j-1)-...
(h/dt)*(ut(i,j)-ut(i-1,j)+vt(i,j)-vt(i,j-1)))+(1-beta)*p(i,j);
end, end
end
% correct the velocity
u(2:nx,2:ny+1)=...
ut(2:nx,2:ny+1)-(dt/h)*(p(3:nx+1,2:ny+1)-p(2:nx,2:ny+1));
v(2:nx+1,2:ny)=...
vt(2:nx+1,2:ny)-(dt/h)*(p(2:nx+1,3:ny+1)-p(2:nx+1,2:ny));
time=time+dt
% plot results
uu(1:nx+1,1:ny+1)=0.5*(u(1:nx+1,2:ny+2)+u(1:nx+1,1:ny+1));
vv(1:nx+1,1:ny+1)=0.5*(v(2:nx+2,1:ny+1)+v(1:nx+1,1:ny+1));
w(1:nx+1,1:ny+1)=(u(1:nx+1,2:ny+2)-u(1:nx+1,1:ny+1)-...
v(2:nx+2,1:ny+1)+v(1:nx+1,1:ny+1))/(2*h);
hold off,quiver(uu',vv','r');
hold on;contour(w',20),axis equal,pause(0.01)
end
```

The staggered grid arrangement is shown in Fig. 2.

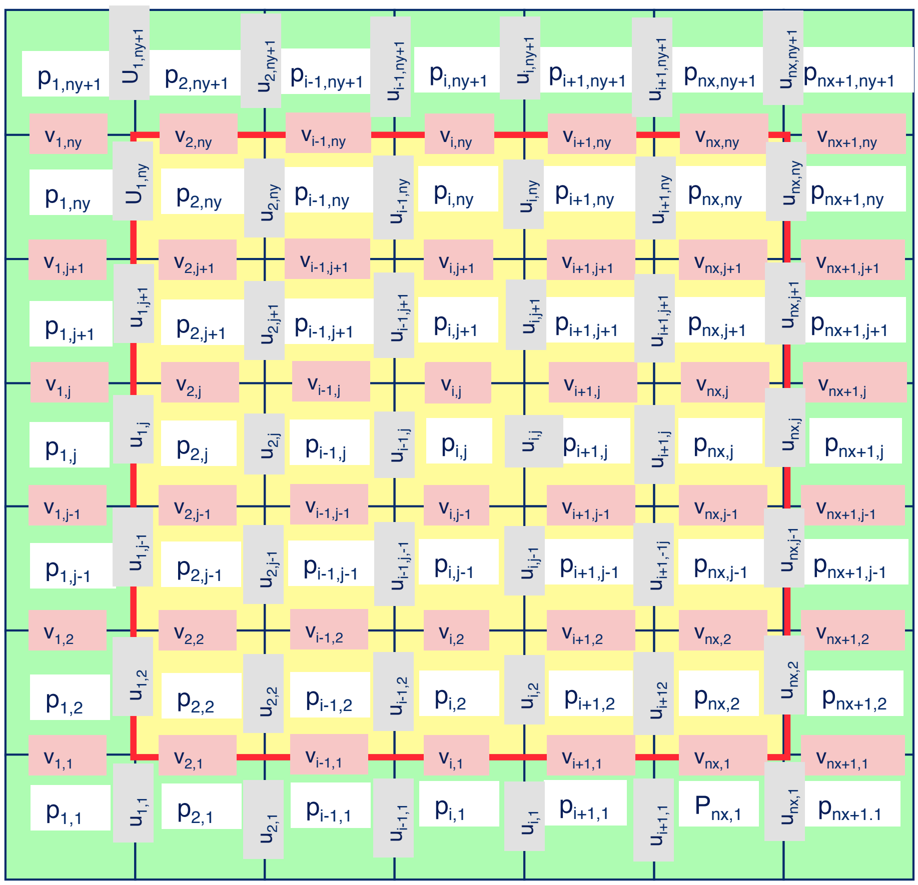

Figure 2: Staggered grid arrangement