EE6412 Project: Minimum time cornering of a race car

Team 8
Srikanth sarma - ME14B027
Rangeesh - ME14B055

May 7, 2017

Abstract

This project investigates the minimum time maneuvering of cars using optimal control techniques. In this project, we would be refering to the paper [1]. This paper [1] discusses the vehicle model equations and formulates the optimal control problem as a time minimal optimal control problem. Constaints have been provided to the rate of contol of the control parameters like throttle and steer to account for the limitations of real drivers. Also, constraints have been provided for the vehicle trajectory to stay within the road boundaries. We take these model equaitons as motivation and simpl,itfy them into simple models and perform simulations of a car taking a 180^0 turn.

Introduction

The optimal control problem in hand is a time optimal control problem which is solved using direct simultaneous approach. We start with a simple model of a car and add modules to the car as we progress. These models, even though don't depict the exact behaviour of cars, serve as a good approximation when it comes to finding optimal driver controls. A similar approach has been used in [2].

Numerical scheme

This section discussed the numerical scheme and the conventions followed throughout the project. The main executable code will be named *main.m*, the constraints will be given through a function named *confun_.m* and the objetive function is named *objfun_.m*. The optimizer used all through the project is *fmincon*. The following flow chart describes the overall flow of information among the functions.

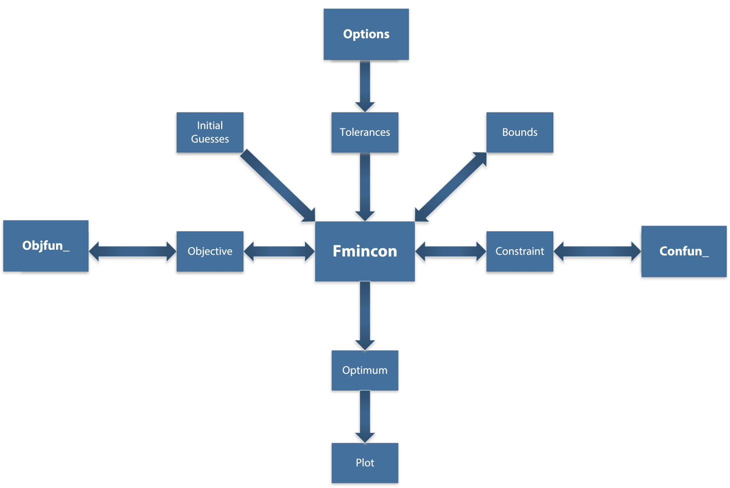

Figure 1: Flow chart of the structure of the code

All through the project, discretization has been done using Euler forward stepping scheme. The basic idea of this is that if we have a system of differental equations governed by

$$\dot{\vec{x}}(t) = \vec{F}(\vec{x}(t), \vec{u}(t)) \tag{1}$$

the discretized equations for each step will be:

$$\vec{x}(i+1) - \vec{x}(i) = \vec{F}(\vec{x}(i), \vec{u}(i))\Delta t \tag{2}$$

1 Model 1

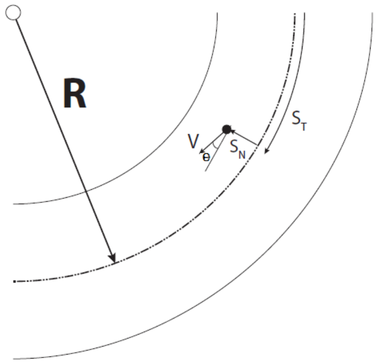

Figure 2: Curvilinear coordinates

In this model, the dimensions of the car are neglected and the car is assumed to be concentrated in a point. The car moves with a fixed speed V. The only control is the steering angle θ. The state space of the car comprises of the curvilinear coordinates s_t and s_n. The curvilinear coordinates adopted here is depicted in figure 2. The model equations are as follows:

$$\dot{s_n}(t) = V sin(\theta(t)) \tag{3}$$

$$\dot{s_t}(t) = \frac{V cos(\theta(t))}{1 - \frac{s_n(t)}{R}} \tag{4}$$

As a case for simulation, we consider a car taking a 180^0 turn. The car starts at the center of the track and is free to leave from anywhere in the track at the exit. The objective is to reach the exit in the shortest time possible. Direct simultaneous method has been adopted for solving this optimal control problem. The problem has been recast as a mayer problem and the resulting state equations have been discretized using Euler forward scheme as discussed in equations (1) and (2). This results in a Nonlinear programming problem which has been solved using the *fmincon* function in *MATLAB*. The following discusses the numerical procedure is briefly discussed below:

$$\vec{x} = \begin{pmatrix} s_n(1:N) \\ s_t(1:N) \\ \Delta t \\ \theta(1:N) \end{pmatrix} \tag{5}$$

$$J[\vec{x}] = x(2N+1) \tag{6}$$

$$x(i+1) - x(i) - V sin(x(2N+1+i))\, x(2N+1) = 0 \qquad \forall\, i \in [1,\, N-1] \tag{7}$$

$$x(i+1) - x(i) - \frac{V cos(x(N+1+i))}{1 - \frac{x(i-N)}{R}}\, x(2N+1) = 0 \qquad \forall\, i \in [N+1,\, 2N-1] \tag{8}$$

On top of these, we have added constraints on the steeing rates i.e., the rate at with the steering angle changes. This is a constraint which takes into the ability of a real driver. There's a lot of litrature which talks about the ability of an average driver. We adopt [3] and set this limit as $0.3\, rads/s$. These types of constraints are called bandwidth limitations. Along with these, constraint on s_n was given such that the car doesn't go out of the road.

$$|\dot{\theta}(t)| \leq 0.3 \tag{9}$$

$$|\frac{x(i+1) - x(i)}{x(2N+1)}| \leq 0.3 \qquad \forall\, i \in [2N+2,\, 3N] \tag{10}$$

$$|s_n(t)| \leq \frac{w}{2} \tag{11}$$

Results

The following are the results obtained after simulations of this model for different cases. The simulations have been performed for various different values radius of curvature of the turn (R)), width of the road (w), speed of the car (V) and number of stages of discretization (N). Two of these results have been included below for discussion.

1.1 $R = 50m,\ w = 10m,\ V = 10\,m/s,\ N = 80$

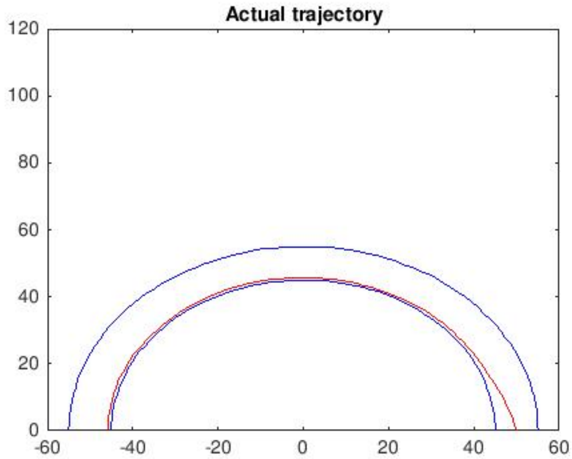

Figure 3: Trajectory

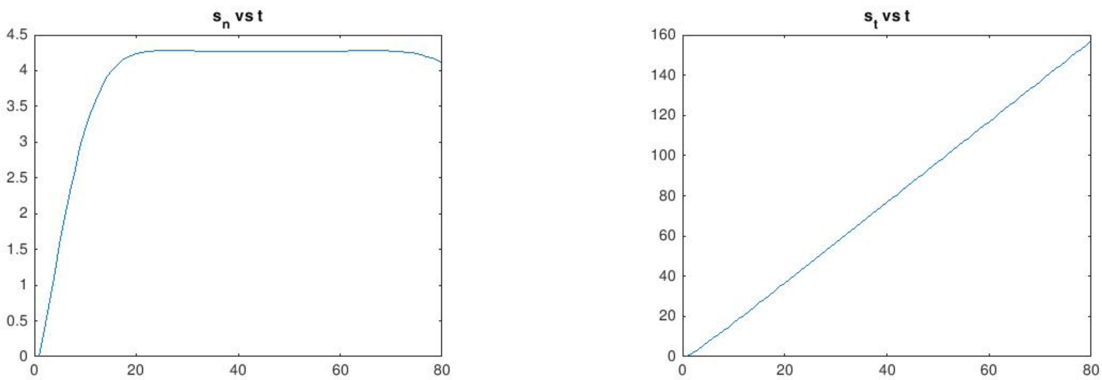

Figure 4: States

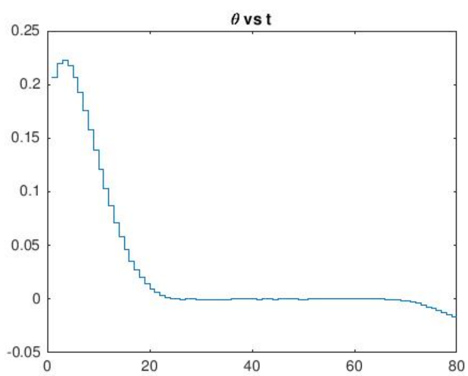

Figure 5: controls

In this case, as is intutivelty expected, the car goes towards the inner limit of the track and goes along it to minimize the distance covered and since speed is constant, also the total time. The total time for this manoeuvre was $T = 14.5116\,s$ while the time to go along the center line is $T_0 = 15.708\,s$. $T < T_0$ proves that the solution obtained is sane at least.

1.2 $R = 50m,\ w = 10m,\ V = 15\,m/s,\ N = 80$

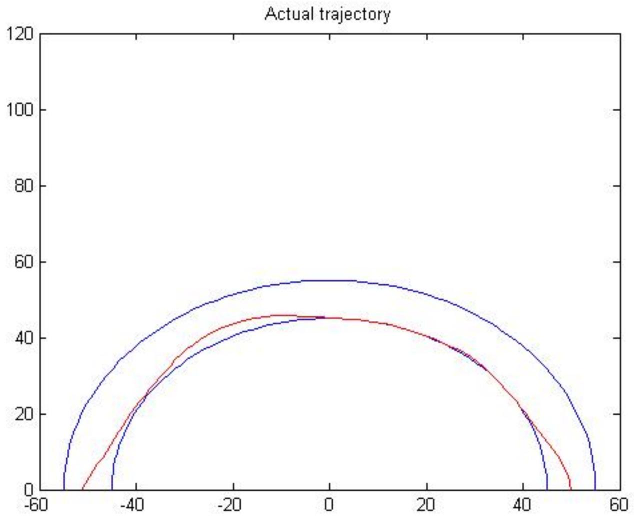

Figure 6: Trajectory

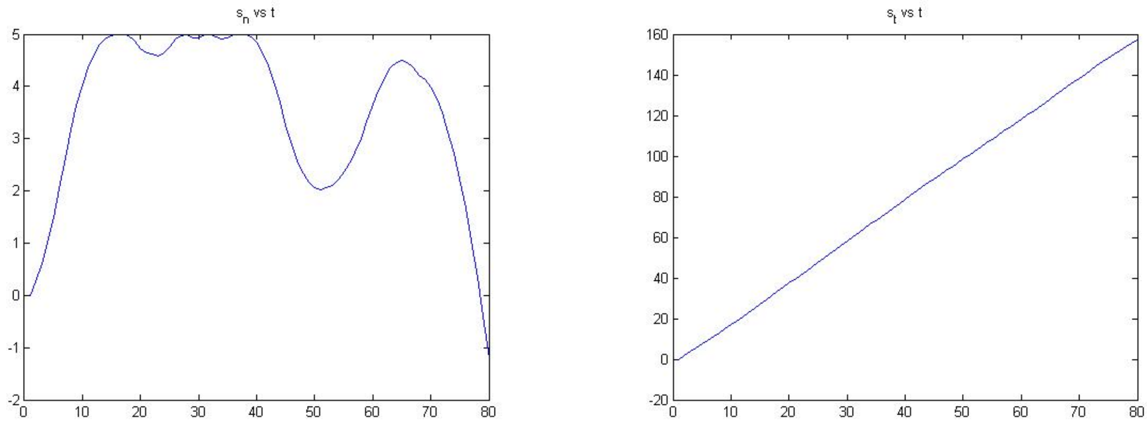

Figure 7: States

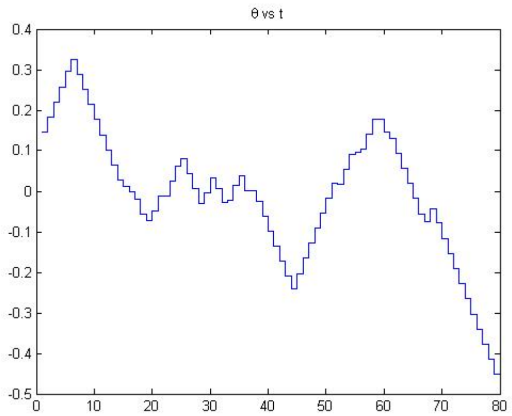

Figure 8: controls

In this case, as is observed in the previous case, the car goes towards the inner limit of the track. But, since the speed is higher and the steering angle rate constraint doesnt allow the steering angle to change by more than $0.3\,rads/s$, the car can't sustain in the inner track and hence will go outward and come in again showing a wobbly trajectory as shown in figure 6. The manoeuvre takes a total time of $T = 9.8326\,s$. This is lower than the time to go along the center line i.e., $T_0 = 10.4720\,s$. This proves the sanity of the solution.

2 Model 2

This model is same as the Model 1 except that here, the speed of the vehicle is a variable. And, here we have a new control variable which is acceleration (*a(t)*). The model equations are as follows:

$$\dot{s_n}(t) = V(t)sin(\theta(t)) \tag{12}$$

$$\dot{s_t}(t) = \frac{V(t)cos(\theta(t))}{1 - \frac{s_n(t)}{R}} \tag{13}$$

$$\dot{V}(t) = a(t) \tag{14}$$

For simulations, similar cases as consider in Model 1 will be considered i.e., a 180^0 turn. The car starts at the center of the track and is free to leave from anywhere at the exit. Same methodology as discussed in Model 1 is for solving the minimal time control problem. The discretized equations are:

$$\vec{x} = \begin{pmatrix} s_n(1:N) \\ s_t(1:N) \\ \Delta t \\ \theta(1:N) \\ V(1:N) \\ a(1:N) \end{pmatrix} \tag{15}$$

$$J[\vec{x}] = x(2N+1) \tag{16}$$

$$x(i+1) - x(i) - x(3N+1+i)sin(x(2N+1+i))\,x(2N+1) = 0 \qquad \forall\, i \in [1,\, N-1] \tag{17}$$

$$x(i+1) - x(i) - \frac{x(2N+1+i)cos(x(N+1+i))}{1 - \frac{x(i-N)}{R}}\,x(2N+1) = 0 \qquad \forall\, i \in [N+1,\, 2N-1] \tag{18}$$

$$x(i+1) - x(i) - x(N+i)\,x(2N+1) = 0 \qquad \forall\, i \in [3N+2,\, 4N+1] \tag{19}$$

Similar to Model 1, we apply bandwidth limitations over both seering rates and acceleration rate. Following the arguments in [3] we have decided upon a acceleration rate constraint at $5g\ m/s^2$.

$$|\dot{\theta}(t)| \leq 0.3 \tag{20}$$

$$|\dot{a}(t)| \leq 5g \tag{21}$$

$$|\frac{x(i+1) - x(i)}{x(2N+1)}| \leq 0.3 \qquad \forall\, i \in [2N+2,\, 3N] \tag{22}$$

$$|\frac{x(i+1) - x(i)}{x(2N+1)}| \leq 5g \qquad \forall\, i \in [4N+2,\, 5N] \tag{23}$$

Also, bounds were given to all the state variables. This is especially very important for the state variable s_n, it ensures that the car stays within the track.

$$|s_n(t)| \leq \frac{w}{2} \tag{24}$$

Results

Simulations have been performed for various different values radius of curvature of the turn (R), width of the road (w), initial speed of the car (V_0) and number of stages of discretization (N). Two of these results have been included below for discussion.

2.1 $R = 50m,\ w = 10m,\ V = 15\,m/s,\ N = 80$

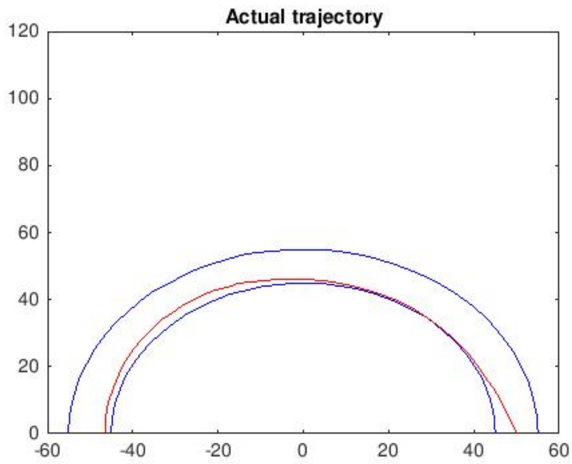

Figure 9: Trajectory

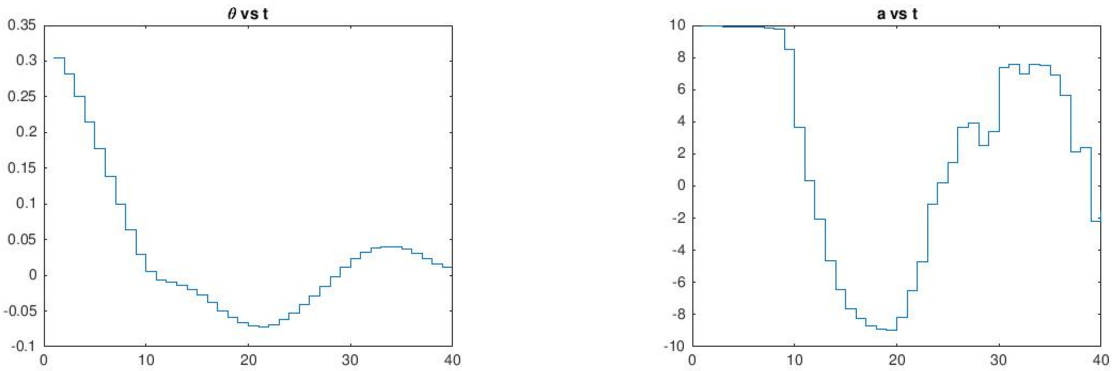

Figure 10: controls

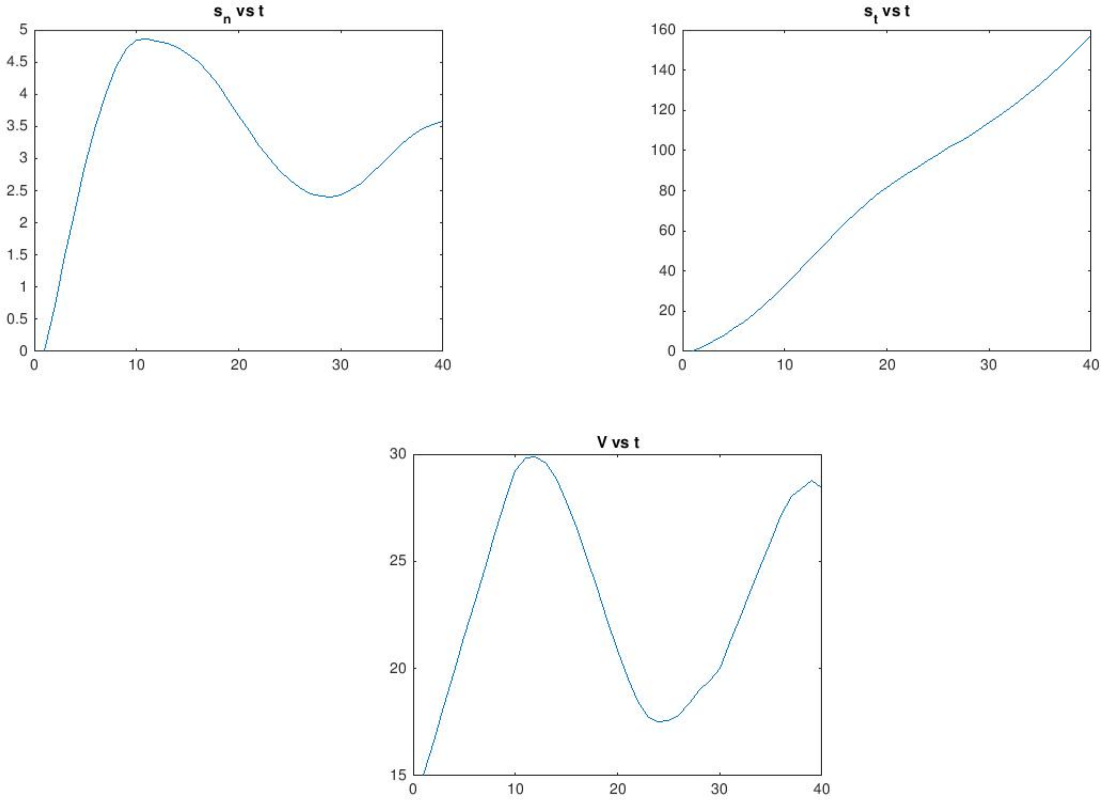

Figure 11: States

In this case, the car as it has two controls θ and V, faces a competition between speed and distance. It can either cover a small distance at a lower speed or a larger distance at a higher speed. As we can see from figures 9, 10 and 11, the car initially accelerates and moves inward, as it approaches the turn it starts decelerating and then touches the innercircle at a point popularly called as the apex and then starts accelerating again. The total manouevre time was $T = 6.3404\,s$. This is less than that for the solution obtained with no acceleration in section 1.2 i.e., $T_1 = 9.8326\,s$ as expected.

2.2 $R = 50m,\ w = 10m,\ V = 20\,m/s,\ N = 80$

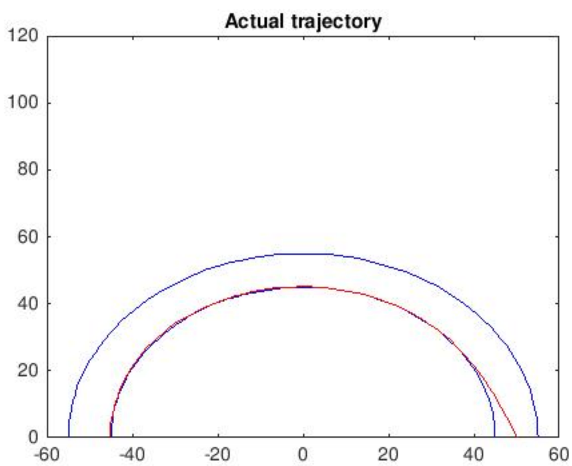

Figure 12: Trajectory

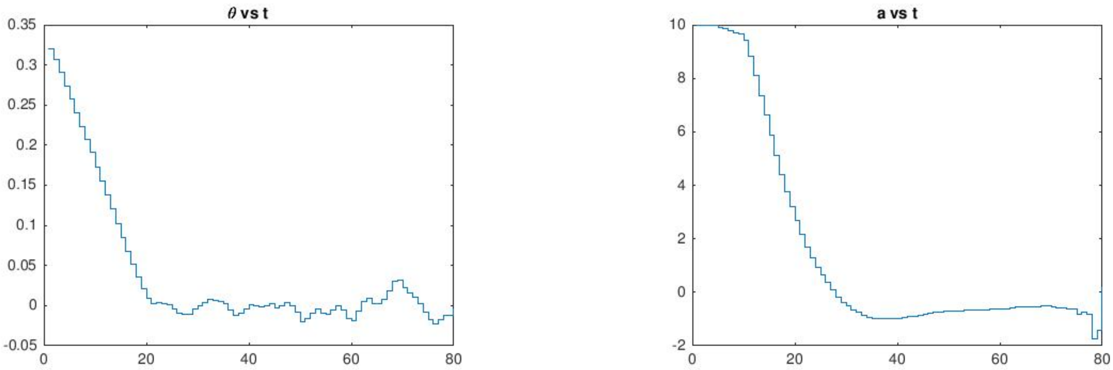

Figure 13: controls

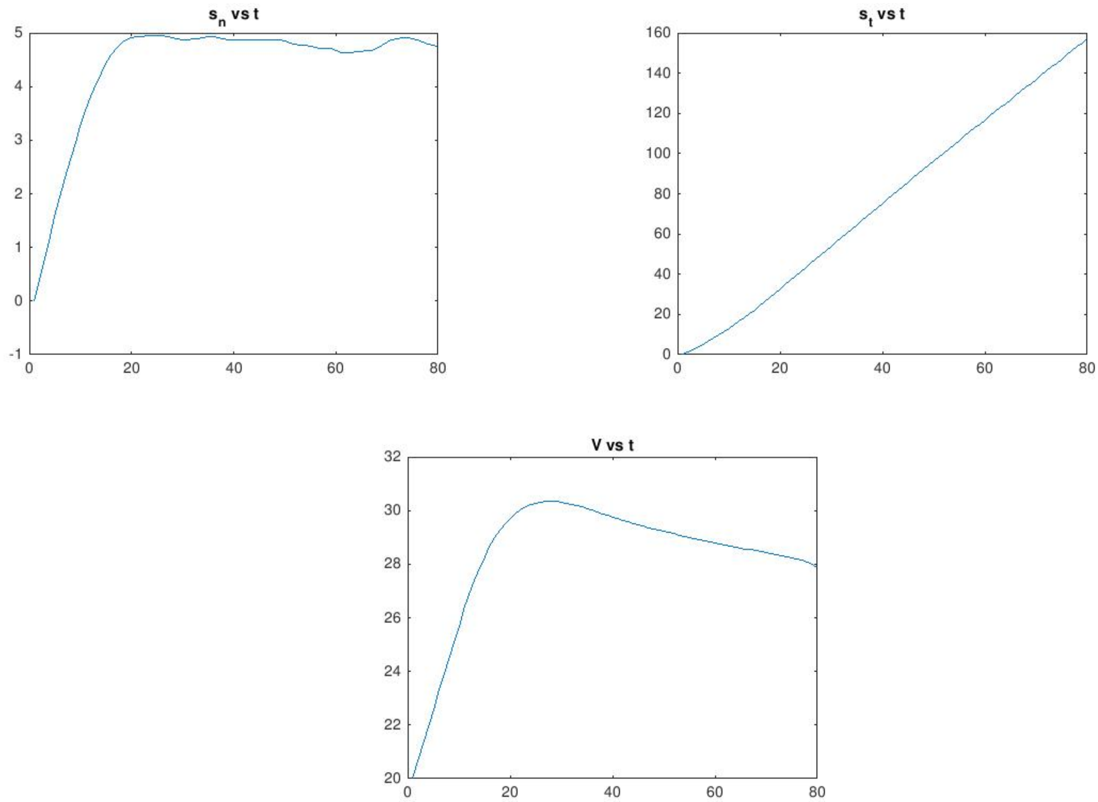

Figure 14: States

The time taken for this manoeuvre was $T = 5.0645\,s$ while time for the car to move along the center line with constant initial velocity is $7.8540\,s$. We can clearly see a reduction in maoeuvre time is decreasing with increasing initial speed.

3 Model 3

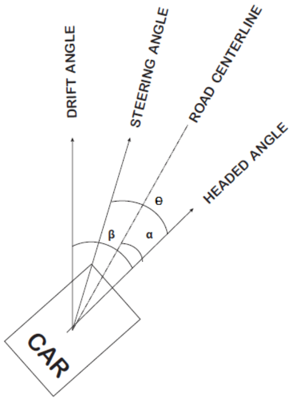

Figure 15: Angle naming convention

All of the previously discussed models are kinematic models. There was no dynamics involved in the model equations. This model brigs in simple dynamics to the car model. We consider a car with equivalently oonly one tyre and neglect the dimensions of the car i.e., approximate it as a point mass. This simlifies the state equations given in [1]. Also, we neglect the suspension dynamics to simplify the problem even further. The simplified state equations are as follows:

$$\dot{s_n}(t) = V(t)sin(\alpha(t) + \beta(t)) \tag{25}$$

$$\dot{s_t}(t) = \frac{V(t)cos(\alpha(t) + \beta(t))}{1 - \frac{s_n(t)}{R}} \tag{26}$$

$$\dot{V}(t) = g\big(S(t)\, cos(\theta(t) - \beta(t)) - f(t)\, sin(\theta(t) - \beta(t))\big) - C_d V^2 cos^3(\beta(t)) \tag{27}$$

$$\dot{\omega}(t) = \frac{Mg(\tau(t) - S(t)r)}{I} \tag{28}$$

$$\dot{\beta}(t) = g\big(S(t)\, sin(\theta(t) - \beta(t)) + f(t)\, cos(\theta(t) - \beta(t))\big) - C_d V\, cos^2(\beta(t))\, sin(\beta(t)) \tag{29}$$

$$\dot{\alpha}(t) = \frac{V(t)cos(\alpha(t) + \beta(t))}{R - s_n(t)} \tag{30}$$

Where, $S(t)$ and $f(t)$ are the traction force and the lateral forceper unit car weight respectively. These are given by the tyre equations. In [1], Pacjeka's magic equation has been used for tyre model. $S(t)$ and $f(t)$ are given by:

$$S(t) = \mu_s = \frac{\sigma_x}{\sigma} D\, sin\big(C\, tan^{-1}(B\sigma - E(B\sigma - tan^{-1}(B\sigma)))\big) \tag{31}$$

$$f(t) = \mu_f = \frac{\sigma_y}{\sigma} D\, sin\big(C\, tan^{-1}(B\sigma - E(B\sigma - tan^{-1}(B\sigma)))\big) \tag{32}$$

Where, σ_x, σ_y and σ are logitudinal slip, lateral slip and equivalent slip respectively. They are given by,

$$\sigma_x = \frac{\kappa}{1 + \kappa} \tag{33}$$

$$\sigma_y = \frac{tan(\lambda)}{1 + \kappa} \tag{34}$$

$$\sigma = \sqrt{\sigma_x^2 + \sigma_y^2} \tag{35}$$

Where,

$$\kappa = \frac{\omega r - V\, cos(\beta - \theta)}{V\, cos(\beta - \theta)} \tag{36}$$

$$\lambda = \beta - \theta \tag{37}$$

This model has 6 state variables and also the equations are highly nonlinear.Providing the initial guess is a very non-trivial task in this model. Simulations performed with this model had mostly converged at an infeasible point. So, one must think of a smart way to provide initial guess for this problem. In the previous cases, initial guesses were given as the car moving along the center line. But, the difficulty here is that the reverse calculation of states for the car moving along the center line is not straight forward. Nevertheless, if one can provide a good initial guess, the code written by us should work well. Making a few assumptions would reduce the complexity of the problem. Assuming that the moment of inertia of the wheel is negligible, the equation (28) reduces to:

$$S(t) = \tau(t)r \tag{38}$$

And, assuming $\mu_f = 0$, we end up with a simplified problem. The following is the solution obtained for a simulation of this problem.

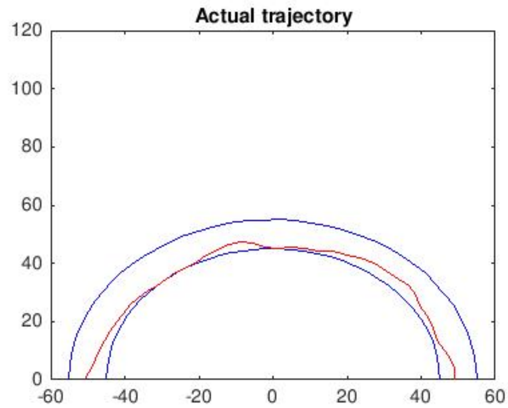

Figure 16: Trajectory

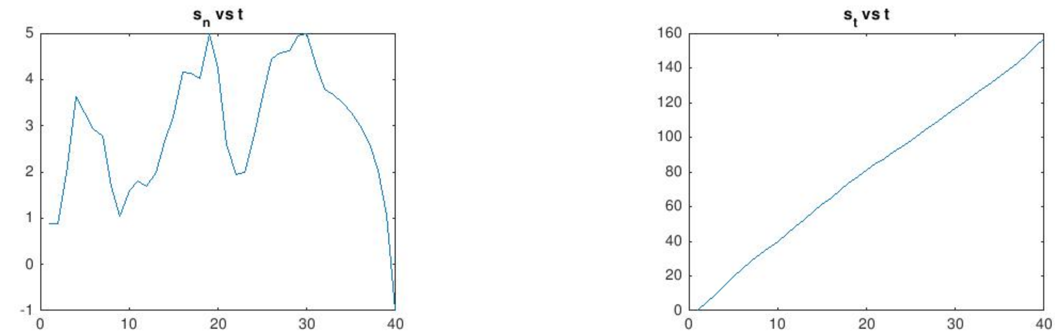

Figure 17: States

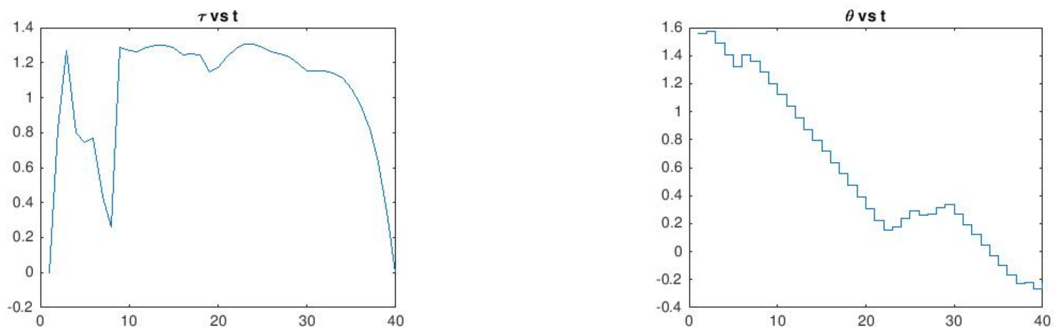

Figure 18: controls

These were results of prematurely terminated simulations. These simulations ran over more than an hour and would converge if they were run for a longer time.

References

[1] D. Tavernini, M. Massaro, E. Velenis, D. I. Katzourakis, and R. Lot, “Minimum time cornering: the effect of road surface and car transmission layout,” *Vehicle System Dynamics*, vol. 51, no. 10, pp. 1533–1547, 2013.

[2] A. Saccon, “Minimum time maneuver for a nonholonomic car with acceleration constraints: Preliminary results,” in *Proceedings of the 2005 IEEE International Symposium on, Mediterrean Conference on Control and Automation Intelligent Control, 2005.*, pp. 1337–1342, June 2005.

[3] M. Buehler, K. Iagnemma, and S. Singh, *The DARPA Urban Challenge: Autonomous Vehicles in City Traffic*. Springer Publishing Company, Incorporated, 1st ed., 2009.