Vortex Doublet in the presence of a wall

Abstract

Interaction of two counter rotating vortex patches in the presence of wall is being studied as a part of this assignment. Here, we use image method and discrete vortex method to simulate the motion of two vortex patches in the presence of the wall. Each vortex patch is approximated by a uniform distribution of vortex blobs. The no normal flow condition on the wall is satisfied by using image vortices.

Description of computational methodology

The initial distribution of vortices was generated using a c code named *InitialDistribution.c*. This code generates the position data of the vortex blobs such that they will be uniformly distributed in the circular patch. The code makes sure that the number of vortex blobs at a radial location is proportional to the distance from center. This is to ensure uniform distribution of vortex blobs. If N_r is the number of radial locations at which blobs will be placed, and N is the total number of blobs to be placed, then it will be convenient if $N \propto N_r(N_r + 1)$. This will ensure that there is symmetric placement of blobs in the circumferential direction. The number of blobs on the circumference of the circle at a radius r_i is N_θ. This N_θ should be proportional to the radius r_i. The proportionality constant has been chosen appropriately to finally get N number of blobs.

$$r_i = \frac{i}{N_r} \tag{1}$$

$$N_\theta = \frac{2Ni}{N_r(N_r+1)} \tag{2}$$

$$\theta_j = \frac{2\pi j}{N_\theta} \tag{3}$$

Finally, initial positions generated are outputted to a text file in a fashion which is expected by the python code for the simulation. The python code $ME14B027 - DiscreteVortexMethod\_Wall + Doublet.py$ defines a class of vortex blobs and uses Euler scheme of integration for computing the time evolution. The class of vortex blobs named *vortexParticle* contains function named *vel* which computes the velocity induced by the vortex blob on any point in space. Using this, we compute the instantaneous velocity of every vortex blob. We include image vortices to compute velocities of all the real vortices.Using Euler scheme we march in time and compute the new positions of the blobs. The new positions of image vortices are simply reflection of the real vortices about the wall i.e., $y = 0$. The instantaneous positions of the blobs are plotted at regular intervals and saved as figures in the working directory. Also, the plots positions of the center of mass $\bar{\vec{x}}$ of the two patches were made. To quantify merging or splitting of patches, a quantity σ analogous to correlation has been defined. This quantity will be 1 for completely mixed patches and is greater than 1 for unmixed patches. If two patches are splitting apart, we can see that the corresponding σ will become very high.

$$\bar{\vec{x}}_1 = \frac{\sum \vec{x}_1}{n_1} \tag{4}$$

$$\bar{\vec{x}}_2 = \frac{\sum \vec{x}_2}{n_2} \tag{5}$$

$$\bar{\vec{x}} = \frac{\sum \vec{x}}{n} \tag{6}$$

$$r_1 = \sqrt{\frac{\sum((x_i - \bar{x}_1)^2 + (y_i - \bar{y}_1)^2)}{n_1}} \tag{7}$$

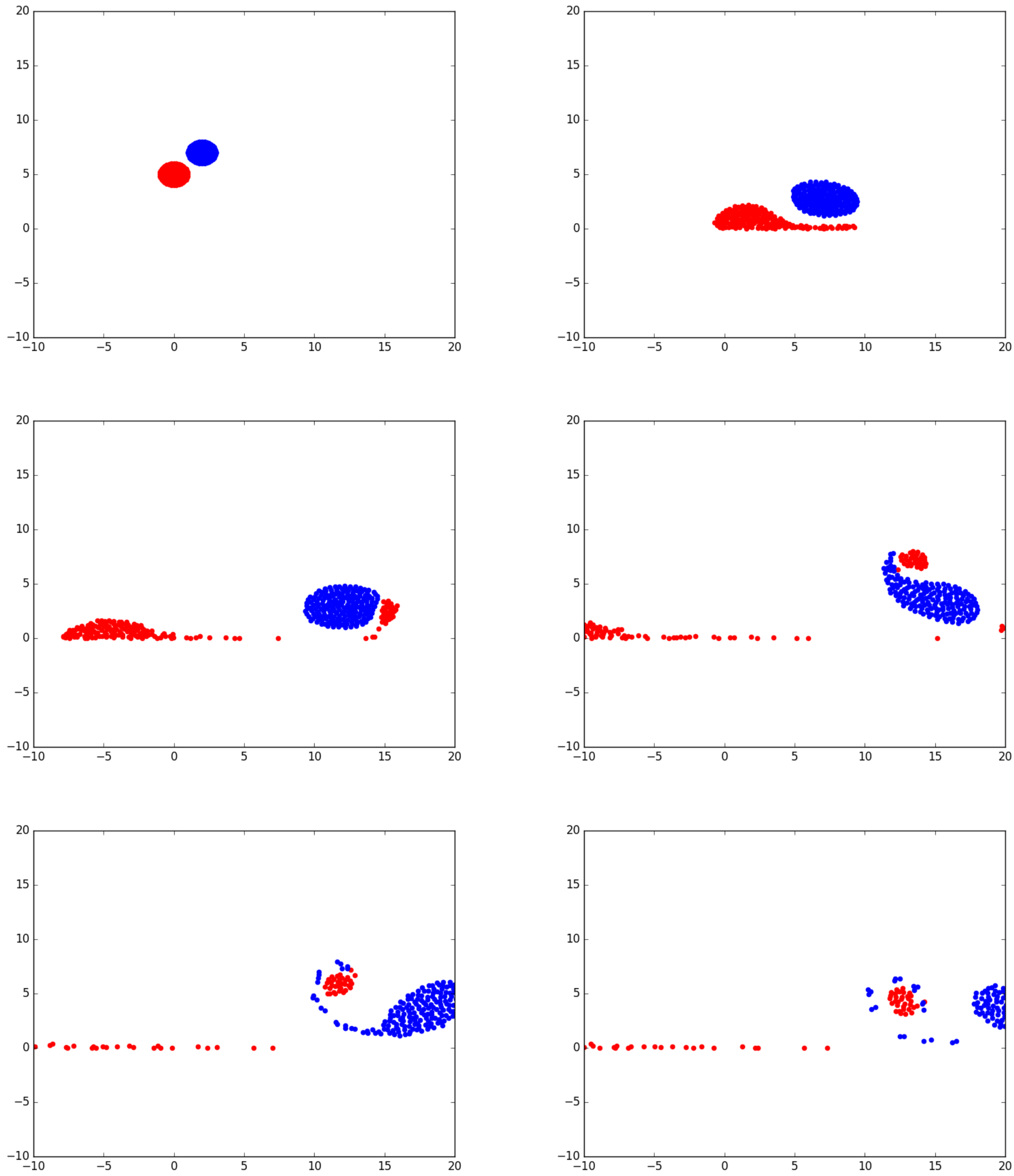
$$r_2 = \sqrt{\frac{\sum((x_i - \bar{x}_2)^2 + (y_i - \bar{y}_2)^2)}{n_2}} \tag{8}$$

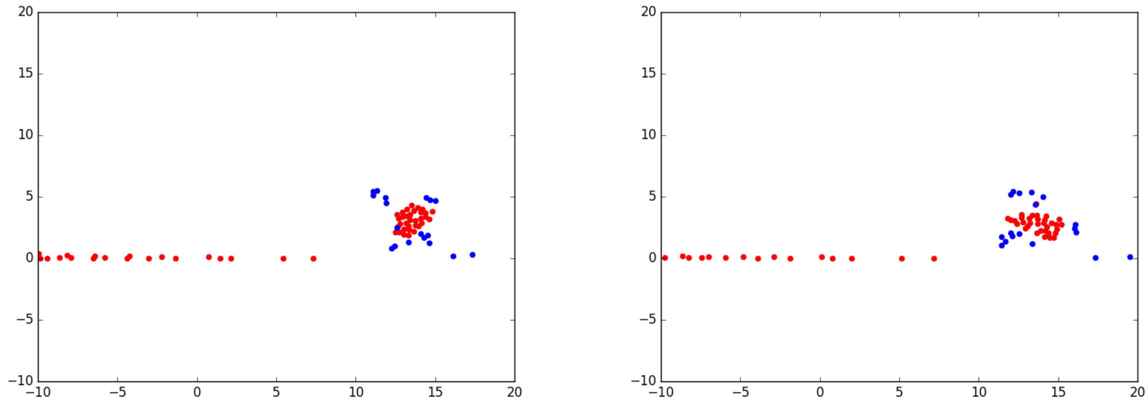
$$r = \sqrt{\frac{\sum((x_i - \bar{x})^2 + (y_i - \bar{y})^2)}{n}} \tag{9}$$

$$\sigma = \frac{r^2}{r_1 \times r_2} \tag{10}$$

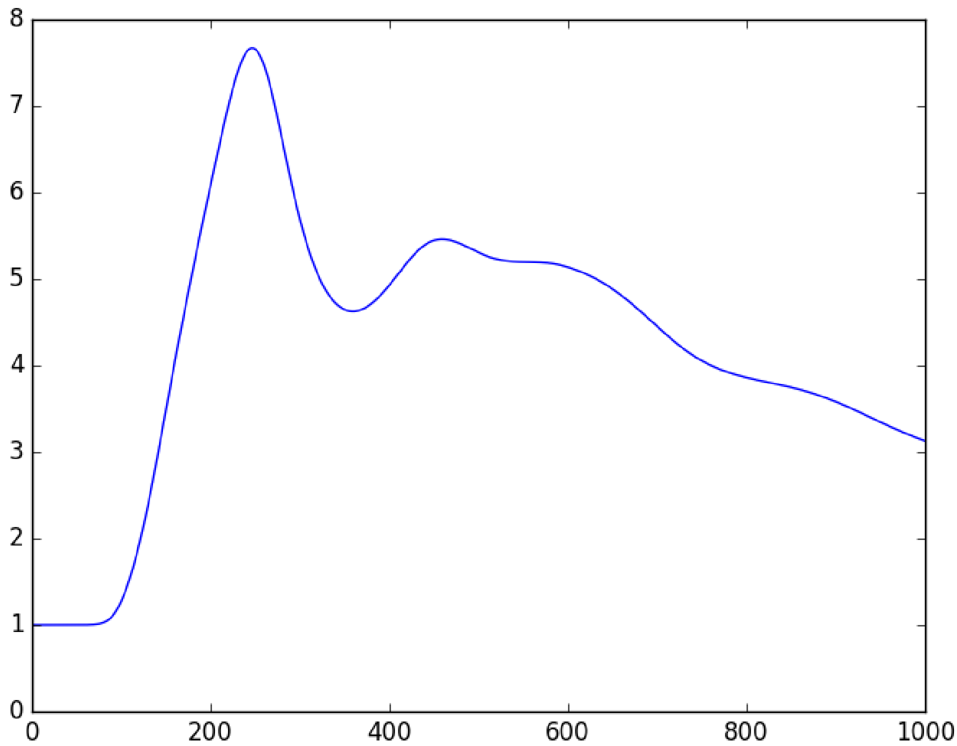
Results: $N = 1000$

The following are the results obtained by simulating two vortex patches of radii 1, number of blobs 180 each and positions as shown in the figures. There is a wall at $y = 0$. Each of the image is at a real time difference of 0.1 from the previos image

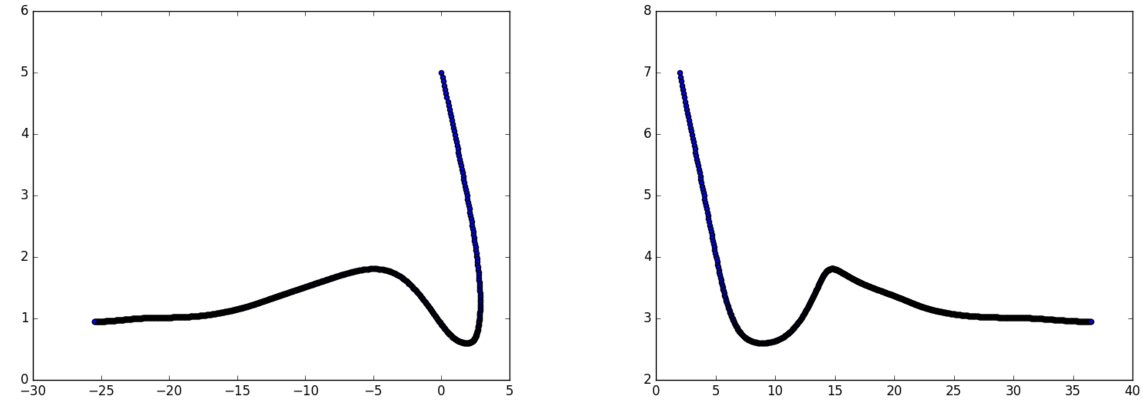




We can clearly see that there is a vortex sheet formation at $y = 0$ by the vortex patch coloured red. Also, a part of it gets detached and starts rotating at he same location while also warping the other patch. The correlation as a fuction of time shows a plot shown below. σ starts increasing after 100 steps which is exactly when the patches start splitting.

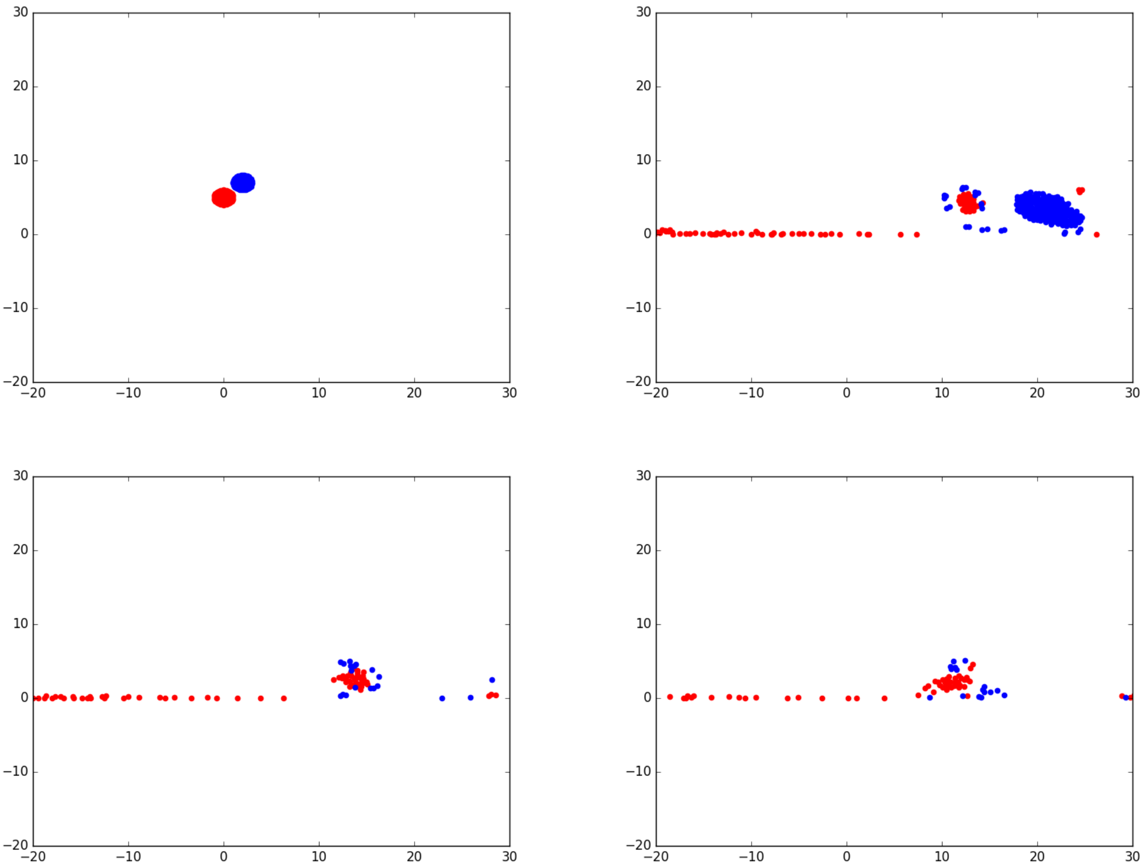


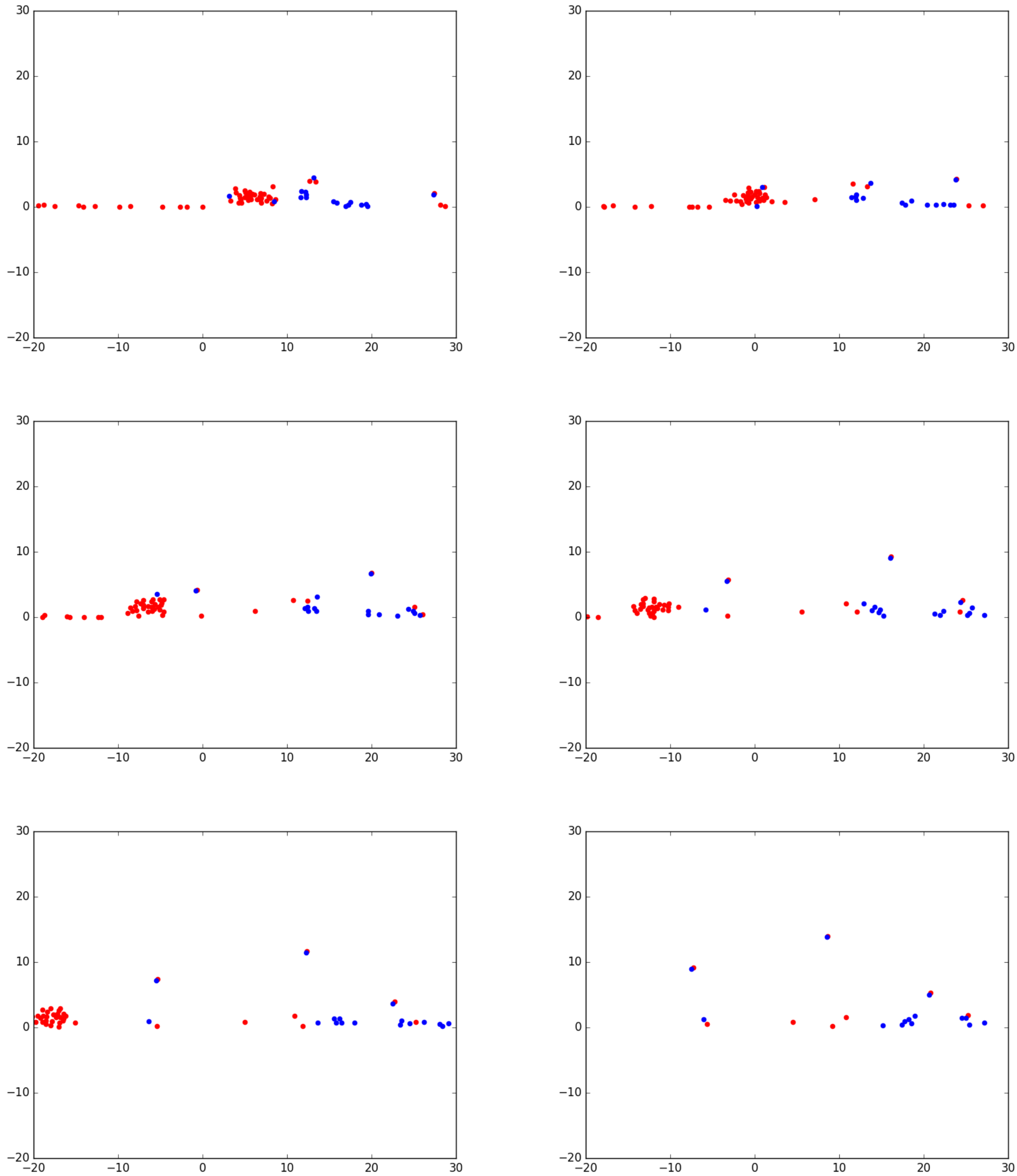
The plots of the center of mass of both the patches are shown below.



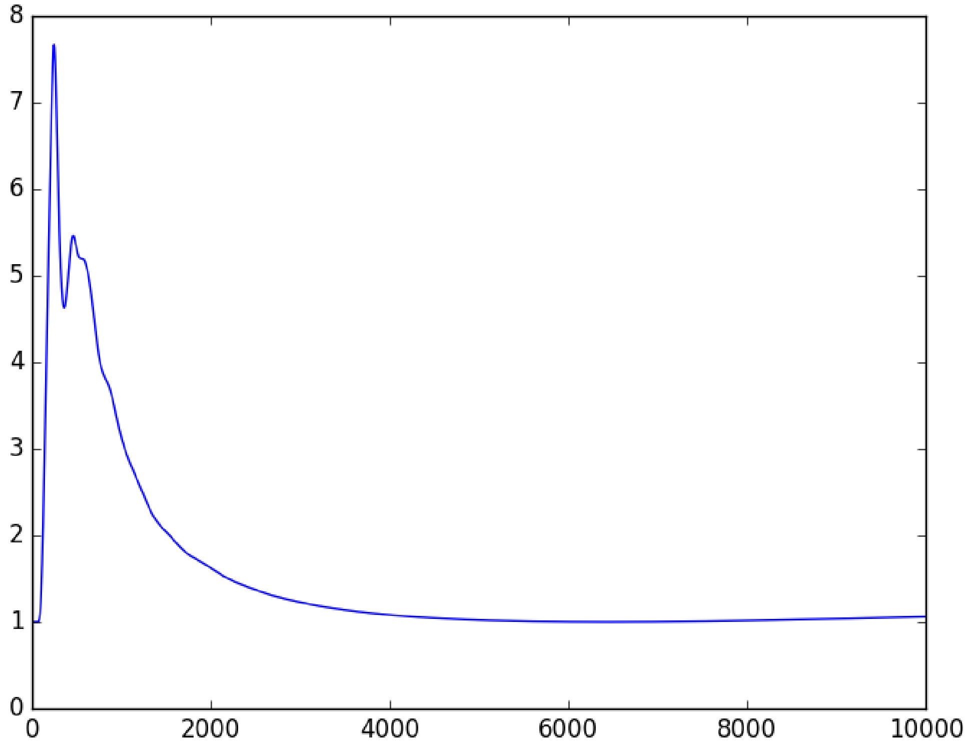
Results: $N = 10000$

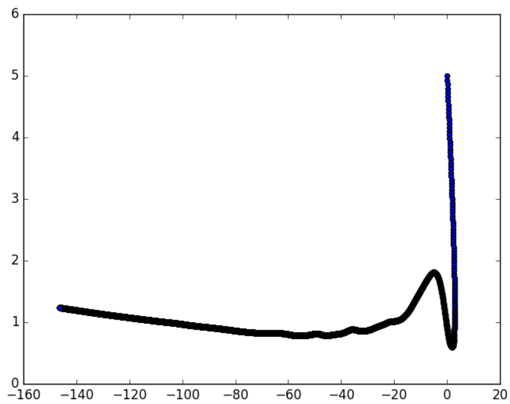
The following are the results obtained by simulating two vortex patches of radii 1, number of blobs 180 each and positions as shown in the figures. There is a wall at $y = 0$. Each of the image is at a real time difference of 0.5 from the previos image

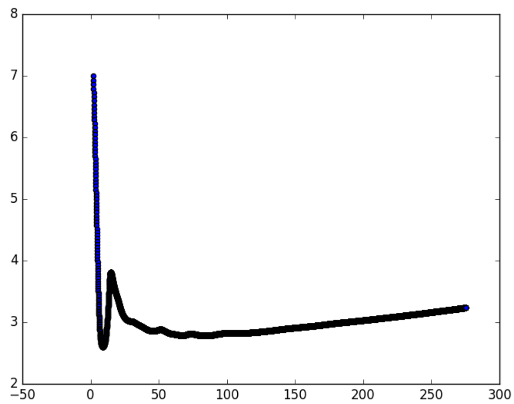




In this simulation, it is very clear that there is a vortex sheet formation at $y = 0$ by the vortex patches. The σ plot and the center of mass plots are as follows.







On close inspection of the gifs created, we can see that there might be another vortex sheet forming at exactly 45^0 to the wall i.e., the initial angle of propogation of the vortices. We can ascertain this by increasing the number of blobs.