AS5435 - Waves in Fluids - Assignment 1
Out: Feb 6, 2018. Due: Feb 16, 2018.

1. **Acoustic Waves with Gravity**: Derive the dispersion relation for acoustic waves in the presence of constant gravity **g** (vector). Unlike in the class, express the final wave equation in terms of the pressure perturbation p. Assume the background state to be quiescent fluid of uniform density ρ_0 and hydrostatic pressure distribution. Equation of State: Assume the density perturbation to be directly proportional to the pressure perturbation via the proportionality constant $\rho\kappa_s$, where ρ is the net density and κ_s the isentropic compressibility of the fluid.

2. Plot (on a software like Matlab) the Gaussian spectrum $\hat{\phi}(k) = \hat{\phi}_0 \exp(-\sigma^2(k-k_0)^2)$ with k/k_0 on the $x-$axis and $\hat{\phi}/\hat{\phi}_0$ on the $y-$axis. Also, plot the corresponding wave form $\phi(x, t = 0)$ on the graph of $k_0 x$ on the $x-$axis and ϕ/ϕ_0 on the $y-$axis. Do the plots for three different values of σ: (i) $\sigma^{-1} = 0.01k_0$, (ii) $\sigma^{-1} = 0.1k_0$, (iii) $\sigma^{-1} = 1k_0$. Comment on the trends you see. Derive the wave packet shape $\phi(x, t)$ analytically by keeping up to the second order terms in the Taylor series expansion of $\omega(k)$. Estimate the times below which the wave packet can be assumed to propagate without significant dispersion. Plot the wave packet (from your analytical expression) for $\sigma^{-1} = 0.1k_0$ and $\omega_0'' = 4c_g/k_0$ and (i) t = 0, and (ii) $c_g t = 100/k_0$.

Please keep the number of graphs to a minimum, and plot only what you have been asked to. Label your axes clearly, and ensure that the font size on the axes are sufficiently large. Include your codes also along with your assignment submission. Some of you may be unfamiliar with Matlab or any other plotting software - so, use this assignment as an opportunity to learn Matlab or any other equivalent software. Your final submission should be one pdf file (including your solutions, graphs and codes) sent by email to: m.s.manikandan@gmail.com