MATHEMATICS REASONING HEURISTIC (MRH): WRITING-TO-LEARN

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Mathematical problem solving requires a complex set of cognitive actions with many connections to cognitive structure and to the context of the situation. When solving problems, students practice cognitive activities that can enhance their mathematical thinking and reasoning by discussing different solutions for a mathematics problem. Scholars (Bereiter & Scardamalia, 1987) have argued that writing, as a tool for developing and communicating ideas, helps learners make the connection between existing knowledge and newly encountered information and experience in an organized manner because writers (learners) engage in a dialogue between their thoughts and their written statements. Writers must take into account multiple factors such as the topic, the audience to whom the text is written, the writing type used, and their knowledge about the topic (Bereiter & Scardamalia, 1987).

The mathematics reasoning heuristic (MRH) is a pedagogical tool for supporting mathematics classroom discourse by engaging students, through problem solving, in reasoning and communicating their ideas through dialogical interaction and writing tasks. Writing supported with public negotiation before production of a text encourages students to acknowledge the social and interpersonal dimensions of knowledge. There are two templates: teacher and student templates (see Table 1). The student template outlines a series of questions that students consider through problem solving; whereas, the teacher template guides teacher for the preparation of a unit and the implementation of the unit. The idea of the MRH teacher and student templates comes from the Science Writing Heuristic (SWH) (Hand & Keys, 1999).

The MRH refers to a conceptual framework that explains the relationship among students' knowledge of mathematics, teacher's knowledge of mathematics, interaction with students, negotiation of ideas, writing, and process of students' problem solving.

The teacher's knowledge of mathematics and students' knowledge of mathematics interact in the course of learning. However, prior to actual students' learning process, teacher has some initial understanding of students' mathematics to which he/she relates his/her own mathematics (Simon, 1995). According to the MRH, teacher defines big ideas and, at the same time, anticipates students' prior knowledge. This planning phase for learning goals and activities is crucial in implementing the MRH.

Method

The purpose of this study was to examine the effect of writing on students' understanding of mathematical content and their reasoning skills within these writing tasks for a "real numbers" unit. The main data sources were students' writing samples, videotapes, on-site observations, field notes, and students' pre- and post-test scores (for the statistical analysis). This study was conducted in a high school with an algebra teacher who taught three Algebra I classes divided into one control group (17 students) with no extra writing tasks and two treatment groups (24 and 25 students in each) who completed extra writing tasks. Students were asked to write a letter to a construction company about the area of a rectangular ranch style house.

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Teacher Template	Student Template
<u>Preparation</u>: - Identify the big ideas of the unit. - Make a concept map that relates sub-concepts to the big ideas. - Consider students' prior knowledge **<u>During the unit</u>:** **1. Students' knowledge of mathematics** - Give students opportunity to discuss their ideas - Have students put their ideas on the board for exploration **2. Teacher's knowledge of mathematics** - Use your knowledge to identify students' misconceptions - Guide students to the big ideas identified earlier during the preparation **3. Negotiation of ideas** - Create small groups and whole class discussion - Encourage students to reflect on each other's ideas **4. Writing** - Have students write about what they have learned in the unit to real audiences (teacher, parents, classmates, lower grades, etc.)	**1. What is my question (problem)?** - Specify what you are asked (What is the question asking?) - Outline the information/data given (What information is given?) **2. What can I claim about the solution?** - Use complete sentences how you will solve the problem - Tell what procedures you can follow **3. What did I do?** - What steps did I take to solve the problem? - Does my method make sense? **4. What are my reasons?** - Why did I choose the way I did? - How can I connect my findings to the information given in the problem? - How do I know that my method works? **5. What do others say?** - How do my ideas/solutions compare with others? a. My classmates b. Textbooks/Mathematicians **6. Reflection** – How have my ideas changed? - Am I convinced with my solution?

Table 1: MRH teacher and student templates

Results and Discussion

One-way analysis of variance (ANOVA) showed a non-significant result for the pre-test scores between control and treatment classrooms, ($F(1,63) = .43$, $p = .515$) and a significant difference between the control group and the treatment groups in favor of the treatment groups for the post-test ($F(1,63) = 4.82$, $p = .032$).

Students' reasoning varied from a trial-error process (e.g., *The best way to maximize area is to make it 40ft x 40 ft <u>a perfect square.</u> I found this by t<u>rial-error</u>*) to a mathematical demonstration (*It can have any sized length and any sized width <u>only if</u> it equals 160ft. It can be 10x70, 20x60, 30x50, and so on. My best opinion for the size of this ranch is 40x40 <u>because the rectangle ranch can be a square</u>…*). The analysis of students' written text suggested that the students appeared to solve the problem by intuition. Students' misconceptions also appeared in their writings: "*if we make it 40 feet on each side, it will be a square, not rectangle*." Some students recognized the condition for the problem (*any sized length and any sized width <u>only if</u> it equals 160ft.*) This is important for problem solving: stating which information can be used. Although this student did not provide evidence why the area would be the largest, she was aware that 'a square is a special case of rectangle' (i.e., *because the rectangle ranch <u>can be a square</u>*.)

The use of the writing task, while helping students on their test performance, also helped the teacher better understand students' understanding of the topic. While this is a pilot study, it does provide some encouragement to continue this line of research in the area of secondary schools.

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ROUTINE AND ADAPTIVE EXPERTS IN PROPORTIONAL REASONING

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Among the problems calling for proportional reasoning, those in which the task is a comparison of ratios can be classified according to several issues. One of the issues is the context; a possible classification according to it is in rate (extensive quantities), part-part-whole, and geometrical problems; in turn, part-part-whole problems can be mixture (e.g. the classical juice problem) or probability (e.g. double urn) problems. (Freudenthal, 1983; Tourniaire and Pulos, 1985; Lesh, Post and Behr, 1988; Lamon, 1993).

Another issue is the numerical structure. In a ratio or rate comparison there are four numbers in two "objects", each of which has an antecedent and a consequent. There is a classification of all possible such foursomes in 86 different situations that in turn can be grouped in three difficulty levels, L1, L2, and L3 (Alatorre, 2002; Alatorre and Figueras, 2003, 2004).

In the cited papers there is also a proposal for a classification of the strategies used by subjects in their answers to such problems; a brief description follows. Strategies can be simple (centrations or relations) or composed. Centrations can be on the totals, on the antecedents, or on the consequents. Relations can be order relations, subtractive relations, or proportionality relations RP; of the latter, five types are considered: three semi-formal (recognition of multiples, groupings, and equalizing), and two formal (quotient comparison and fractions properties). Composed strategies can take four forms of logical juxtapositions of two strategies. Strategies may be labeled as correct or incorrect, sometimes depending on the situation where they are used. Most strategies are incorrect; correct strategies are the proportionality relations and two kinds of informal strategies: some order relations RO, and some composed strategies that can be considered as theorems in action, TA (see e.g. Vergnaud, 1981).

The difficulty levels mentioned before refer to which correct strategies may be applied. L1 consists of all the situations where in addition to RP, RO and/or TA may be used. In L2 and L3 the only possible correct strategies are RP; L2 consists of situations of proportionality (both ratios or rates are the same), and L3 consists of situations of non-proportionality.

A case study was conducted in Mexico City with 23 subjects, aged from 9 to 65 and with schooling from 0 (illiterate adults) to 23 years (PhD). During the interviews, subjects were posed several questions in each of 8 sorts of problems, which were 4 Rate problems, 2 Mixture problems, and 2 Probability problems. Each of the problems was posed in different questions according to numerical structure. Each time, the subjects were asked to make a decision (object 1, object 2, or "it is the same") and to justify it. A total of 2152 answers was thus obtained, of which 80% were classified using the strategies system mentioned above.

In level L1 most subjects showed a good achievement. Here they used different correct strategies: almost never RP, almost always RO or TA, varying the strategy according to the context and the numerical structure, apparently searching each time the easiest way to solve the problem, in a manner that reminds of the adaptive experts described by Hatano (see e.g. Hatano and Oura, 2003). Thus, most of the subjects could be classified as locally (L1) adaptive experts.

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However, this kind of expertise is the only one that about half of the subjects could attain. Some of these subjects continued in L2 and L3 questions to use simple or composed centrations and incorrect order and subtractive relations, and thereby obtained incorrect answers. Only in scarce occasions of L2 and L3 questions did these subjects use some RP strategies.

If one considers that an expert in the kind of problems contemplated in this research is someone who solves a high percentage of questions using correct strategies, then the second half of the subjects can be considered as experts in at least one context. The easiest contexts were rate problems and the most difficult contexts were the probability problems. Only one of the subjects could be considered an expert in all contexts and difficulty levels.

The behavior of this last subject will be compared to the other ones. Given that he has a PhD in Chemistry, it would have been expected that Vicente (50 y.o.) was an adaptive expert, but he behaved as a routine expert: He used the same strategy almost in all the questions, monotonously calculating quotients. He did so even in most of the facile L1 questions.

Some of the other subjects had favorite strategies, but even if they did, they used at least two different kinds of RP strategies. The case of Dalia stands out as opposite to Vicente's. Dalia (25 y.o., with 3 years of schooling) can be considered an expert in all the contexts with the exception of the probability problems. In L1 questions she used mostly TA strategies, but also some RP strategies. In L2 and L3 she used an assortment of RP strategies. Dalia's behavior could be described as a constant search for the most comfortable strategy, taking into account the four numbers as well as the context. She prefers the use of informal or semi-formal strategies but uses formal calculations whenever she feels necessary. Dalia is an adaptive expert.

In conclusion, proportional reasoning can cover a range from informal strategies to formal calculations. In Vicente's case it seems that school favored a rigid use of formal strategies, while life has taught Dalia the ability to flexibly search for the most comfortable strategy covering the whole range of possibilities, although not in probability problems.

It has been argued (Hatano and Oura, 2003) that school is biased toward routine expertise even though adaptive expertise is more desirable. Dalia's case shows that in this sense life may be closer to the desired school than school itself, while Vicente's case is a confirmation of the routine expertise acquired at school, although this does certainly not prevent him from being a highly successful and creative teacher, lecturer, and scientist.

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CURRICULUM-SPECIFIC PROFESSIONAL DEVELOPMENT: A PHENOMENOGRAPHICAL STUDY OF TEACHERS' PERSPECTIVES

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To date, little research in the mathematics education literature addresses curriculum-specific professional development programs (i.e., professional development programs designed specifically for addressing the issues that arise when using a particular mathematics curriculum). As the use of reform curricula becomes more prevalent at all grade levels in the U. S., curriculum-specific professional development becomes critical. Consequently, understanding how teachers experience curriculum-specific professional development is vitally important; the more we know about how teachers experience professional development, the better we can support their learning as well as changes in classroom practice.

"Getting to the Core" was an NSF-funded professional development project that occurred from 2002-2004. During that time, 30 grades 8-12 teachers in one mid-western school district participated in approximately 200 hours of professional development focused on implementing *Contemporary Mathematics in Context* (Core-Plus) (Coxford et al., 1997, 1998, 1999, 2001). The professional development activities included: a) summer workshop-like sessions developed and implemented by a Core-Plus author, a university mathematics education faculty member, and the district 6-12 mathematics coordinator; and b) study groups throughout the two academic years.

In an attempt to focus on teachers' perspectives of the professional development, we utilized a phenomenographical approach in this study. "Phenomenography investigates the qualitatively *different* ways in which people experience or think about various phenomena" (Marton, 1996, p. 31, emphasis added). The goal of a researcher working within this approach is to "uncover all of the understandings people have of specific phenomena and to sort them into conceptual categories" (p. 32), and "these categorizations are the primary outcomes of phenomenographic research" (p. 33).

Methods of Inquiry/Data Sources

The primary data source in phenomenographical research is the interview. Interview transcripts are analyzed by first identifying and coding passages that relate to the research question. The marked passages are then "interpreted and classified in terms of the contexts from which they are taken" (Marton, 1996, p. 42). It is at this juncture that the unit of analysis shifts from individual subjects to interview passages or quotes. The quotes are then sorted into categories based on similarity. The resulting discrete categories are then used to describe the participants' different perspectives of a phenomenon.

We analyzed 29 semi-structured interviews from 21 teachers; 12 of these interviews occurred after the first year of the project, the other 17 occurred at the end of the project. Our analysis followed the sequence as described above (Marton, 1996). At the end of analysis, we had three discrete categories that described the participants' different perspectives of their participation in "Getting to the Core." Those categories comprise our findings.

Findings

The teachers experienced the professional development as an opportunity to *build knowledge* about the Core-Plus curriculum. They became more aware of strengths and weaknesses of Core-Plus; they learned more about how Core-Plus is different from traditional textbooks, both in philosophical underpinnings about learning and in structure and content. Most frequently, they spoke of knowing the curriculum itself much better after the professional development -- they understood better the articulation of the mathematical content within and across the four courses. Some teachers claimed that their new knowledge influenced their beliefs about the curriculum.

The teachers experienced the professional development as having an influence on their *classroom practice*. Many teachers spoke of attempts, successful and not, to transition to a more reform-oriented practice or philosophy.

Some teachers reported that they were more focused on their students than before the project, using student thinking to drive instructional decisions, and trying to understand what students understood about a topic. Many teachers attempted a new classroom physical layout – some of them grouping students for the first time. They spoke of explicitly establishing classroom norms with regard to teacher-student and student-student communication. The teachers found that they were trying new ways of managing the everyday work in the classroom – ways of collecting and distributing work and materials, as well as employing different methods for grouping students. Some teachers reported that they had been influenced in similar ways in their non-Core-Plus classes as well.

The teachers experienced the professional development as an opportunity to *collaborate* with other teachers in the district. They worked together, across the district, to share teaching ideas and plan lessons. A number of the teachers met in study groups to work through the Core-Plus curriculum in some manner. By collaborating to work through the curriculum, teachers told us that they deepened their understanding of mathematical content. Further, the teachers said that collaborating with other teachers as they worked through the curriculum allowed them to take on the role of the student.

Overwhelmingly the teachers talked about the opportunity to collaborate with different people over the course of the project. Along with teacher-to-teacher collaborations, the teachers also talked about the importance of collaborating with project staff. They perceived as particularly important the involvement of the Core-Plus author and experienced Core-Plus teachers, as well as local university faculty and project staff.

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INTEGRATING MATHEMATICS OF MEASUREMENT INTO ELEMENTARY TEACHERS' PEDAGOGY: COLLABORATIVE DESIGN AS A PROFESSIONAL DEVELOPMENT TOOL

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We report on a collaborative design project addressing the mathematics of measurement situated within elementary classrooms. We are working on a contextually-detailed account of teachers' growth in their pedagogical-content knowledge (PCK) of measurement topics, especially by selecting and implementing tasks (Simon & Tzur, 2004); we are beginning to sketch out a hypothetical learning trajectory on teaching measurement. (Empson & Turner, 2004; Hill, Schilling, & Ball, 2004).

Our understanding of the process of professional development of teachers is based on Piagetian concepts of *reflection* and *abstraction* (cf. Simon & Tzur, 2004). Ball et al. (2004)suggest that the work of teaching mathematics includes: giving and evaluating explanations, modeling operations as they link to concepts, judging representations, and interpreting students' mathematical ideas. We characterize teacher's developing abstractions along these critical aspects of teaching to elaborate on PCK of measurement. Children's understanding of measurement demands both mathematical and cognitive sophistication: for example, one must establish and extend units by clarifying the association between zero and one as locations along a line (Lehrer, 2003; Stephan & Clements, 2003). Lehrer and his colleagues (1998) observed a threefold improvement in teachers' classroom practices while engaged in a design experiment: (1) shifting from isolated task implementation to integrated sequences of tasks addressing themes, (2) shifting toward coordinated representations (3) shifting from gestures and brief verbal accounts of geometric ideas toward elaborate and specific verbal accounts of geometric concepts. Here, we examine the growth of a case-study teacher's thinking while engaged in a three-year professional development project (Thornton & Barrett, 2000)* centered on a reform curriculum.

How does a primary teacher's awareness of children's conceptualization of units and unit iteration contribute to changes in the teachers' explanations that fit children's strategies and ideas more closely, to improvements in task design, including the construction of representations that correspond to the mathematical foundations of measurement? We set up a teaching experiment using accounts of practice (Simon, 2000) to address this question; we examined the teacher's reflection while engaged in task selection and development, situating our analysis within the teachers' own classroom interaction patterns (D. Schifter, Bastable, & Russell, 2002; D. E. Schifter & O'Brien, 1997). We gathered three sources of data: (1) our own articulation of the learning trajectory throug written field notes, (2) the teachers' own reflective statements about lessons and (3) classroom videos.

To sketch some of our findings, we focus on the teachers' modeling of operations in relation to the concepts *unit* and *iteration* during the early period of the case study.

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Fall 2001, Day 1

Mark's explanations and discussions of measuring topics did not relate to mathematical notions of a unit or to an iterative pattern. He only used the word *unit* as a reference to standardization (cm or in).

Reflective Discussion after Day 1

The researcher (first author) proposed a task with non-standard rulers, labeled with number sequences beginning with an integer greater than one; the researcher sought to draw attention to students' understanding of units along the edge of a ruler within an iterated sequence.

Fall 2001, Day 2

Mark constructed such rulers and set up measuring activities based on these rulers alone. He found some of the students thought these novel rulers had different lengths depending on what number label they found marking the end of the ruler, even though they compared the rulers directly and found them to fit exactly alongside each other. Mark was puzzled when the students did not rely solely on direct visual comparison.

Post Lesson Discussion

The researcher suggested that such children may not have established a unit concept, nor had they iterated units to interpret the rulers. Mark suggested these students would understand if they were only shown how to re-number the inch marks along their rulers. He continued to model measure operations by verbally rehearsing his own actions but resisted suggestions that he talk to students about length units. He assessed students' knowledge by checking for a fit between students' statements and his own wording.

A Modification of Rule-Following During Fall 2002

Mark began to emulate the first author in using a tile-rolling procedure to help students use tiles to find perimeters for tile-based polygons; he centered his own instruction around a demonstration of an edge-rolling procedure, telling students to "go around the perimeter you are measuring and find the number of tile edges." While Mark was still asking students to learn his wording, he was now using words to identify successive iterations of a unit along a path to measure. We believe he was beginning to model operations that corresponded closely to conceptual aspects of measurement (unit-length edges of tiles), building a linkage between students' strategies and those conceptual aspects.

In this period, Mark transitioned from gestures and non-conceptual verbal accounts of geometric ideas toward elaborate and specific verbal accounts of units and iteration sequences, which he promoted as common procedures for measuring perimeter among his students. This is one of several changes in Marks' practices we attribute to collaborative participation in task design cycles. Across the entire teaching experiment, we observed a significant and positive development of PCK for measurement. Our findings suggest consistent reflection on task-effect relations directed at units, and unit iteration (Simon & Tzur, 2004) promote teacher growth.

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PERCEPTIONS OF THE MATHEMATICS ACHIEVEMENT GAP: A SURVEY OF THE NCTM MEMBERSHIP

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There is increasing concern among educators about the disparities that exist among ethnic groups in mathematics achievement. There is no simple explanation for the achievement gap. However, it is important to recognize that the achievement gap is not a result of membership in any group, but is instead a result of the conditions of education (Thompson & O'Quinn, 2001). Consequently, a variety of school, community, and home factors seem to underlie or contribute to the gap. For example, the lower mathematics achievement levels of minority students, particularly Black students, may be indicative of the curriculum and instruction these students receive (Lubienski, 2003). Other researchers have highlighted differences in teachers' expectations of students as a function of race, gender, and social class which influence achievement (Berry, 2004; Ferguson, 1998). However, there are few studies that have surveyed educators to explore their explanations of the achievement gap in mathematics.

Therefore, the purpose of this study was to survey the perceptions of members of the National Council of the Teachers of Mathematics (NCTM) on the achievement gap in mathematics education. For the purposes of this study the achievement gap was defined as an indictor of disparities between groups of students usually identified (accurately or not) by racial, ethnic, linguistic or socio economic class with regard to a variety of measures (attrition and enrollment rates, drug use, health, alienation for school and society attitude toward mathematics, as well as test scores). More specifically the following research questions were addressed:

(1) What do respondents perceive to be the most important contributors to the achievement gap in mathematics?
(2) Do these perceptions vary as a function of personal characteristics of the respondent (i.e., gender, ethnicity, or age)?
(3) Do these perceptions vary as a function of characteristics related to employment (i.e., position held, years of experience, or educational degree)?

Method

Data was collected via an online survey sent to a random sample of the NCTM membership. At the time of the survey, there were a total of 41,508 NCTM members in the population to draw the sample from. The random sample was composed of 5,000 non-student NCTM members. On March 9, 2004, the sampling of the NCTM membership received an email containing the URL link that opened the online survey. The online survey closed on March 29, 2004. Eight hundred seventy members from the random sample visited the website and 623 members completed the survey.

Data Sources

The data source was the questionnaire developed by the researchers. The first section contained items requesting information on demographic and employment characteristics. The next sections presented 23 rating scale items pertaining to factors contributing to the

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achievement gap. The items were organized into five sub-areas or scales and included (1) Background and Societal Influences, (2) Student Characteristics, (3) Curriculum and Instruction, (4) Politics and Policy, and (5) Language. Respondents were asked to rate the extent to which they agreed with the statement on 5-point Likert-type scale, ranging from "strongly disagree" (1) to "strongly agree" (5).

A factor analysis (principal components extraction method with varimax rotation) was conducted to empirically investigate the validity of the rating scale items. The results supported only 4 scales. The component matrix did not support the original scale called Background and Societal Influences. The final solution of four factors, all with eigen values greater than one, accounted for 52 percent of the variance. Reliability coefficients (Cronbach's alphas) for each of the scales ranged from a low of .61 to a high of .85. More detailed results and explanation of the factor analyses supporting the subscales appear in an article by Bol & Berry (2005).

Highlights of the Results

The results pertaining to the first research question highlight the complex nature of peoples' perceptions of the achievement gap. When looking at the items that were most strongly endorsed as contributors to the achievement gap, educators endorsed items related to student characteristics that focused on family support, student motivation, peer pressure, and intellectual ability. The mean ratings on these items were 4.00 or above. This is important because these factors can be perceived as primarily non-school factors that are more resistant to educational interventions. The mean ratings obtained on the other three scales were similar and somewhat lower, suggesting moderate levels of agreement.

The second research question addressed whether perceptions differed as function of personal characteristics of the respondents. Minority respondents were significantly more likely to agree that factors related to curriculum and instruction contributed to the achievement gap. A significant effect for gender was observed on the Language scale. Females were more likely to attribute the achievement gap to language differences or difficulties.

When examining the variation in factor scores by employment position, we found significant differences between mathematics supervisors and teachers across all grade levels on the Student Characteristic scale. This presents a dilemma because teachers who interact with students on a daily basis perceive that factors such as peer pressure, family support, motivation, intellect, and interest in mathematics are more contributory to the achievement gap than do mathematics supervisors. Perhaps, daily contact with students makes teachers more attuned to student characteristics as a contributory factor on student achievement.

Our findings illuminate mathematics educators' perceptions of the causes of the achievement gap. Additionally, they may inform future studies on interventions or strategies aimed at alleviating this gap.

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UNDERGRADUATE STUDENTS' INTERPRETATIONS OF MATHEMATICAL PROOF

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Objectives

Because of the central role proof plays in mathematics, scholars have called for the learning of proof to become a central goal of teaching mathematics, especially at the college level (e.g., RAND Mathematics Study Panel, 2002). However, in order to design instructional interventions that address this, we first need to understand what students know about proof. Research has begun to address issues such as students' difficulties with proof (e.g., Coe & Ruthven, 1994) and their cognitive proof schemes (Harel & Sowder, 1998). This study extends current work by exploring undergraduate students' competencies in evaluating mathematical proofs.

Methodology

Participants were 400 undergraduate students from six academically and geographically diverse US universities. Participation was voluntary and based on the criteria that (1) participants had not previously taken formal courses in mathematical proof, (2) had not completed beyond first semester calculus, and (3) were enrolled in a course for whom the instructor had agreed to administer data instruments during class instruction. While it was not possible to randomly select student participants, effort was made to use a variety of educational settings in order to have a sample that could be considered representative with respect to demographics and university type. Student self-reported statistics on factors such as gender and ethnicity indicate that the sample was representative of the overall US college student population.

An instrument consisting of a survey questionnaire and multiple-choice items was designed and administered during one classroom instructional period at the beginning of the Fall semester. The 45-question Likert-scale survey focused on students' proof construction and understanding, attitudes and beliefs about proof, and classroom experiences with proof. The 25-item multiple-choice test examined students' ability in evaluating simple proofs and elicited their personal views on the role of these proofs. Prior to the large-scale study, the instrument was pilot-tested and a task analysis (item difficulty, item discrimination and distractor analysis) was conducted in order to establish its reliability and validity. Using Cronbach's α, reliability was computed at 0.75 for the multiple-choice test and 0.77 for the survey questionnaire. Descriptive statistics based on frequency tables, simple correlations, and tests of significance were used to compare and interpret the data. This study reports findings from student responses to the multiple-choice portion of the instrument.

Results

The multiple choice test included 3 conjectures and 4 supporting arguments (empirical, narrative, visual, and deductive) from which students were asked to select the one they felt was (a) closest to the one they would construct, (b) the most rigorous, and (c) the one they would use to convince a peer. Results (see Table 1) suggest that students primarily chose an empirical

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approach for conjectures (1) and (3), but a deductive approach for (2) as the one closest to the argument they would construct. This discrepancy, which could be due to the statement of conjectures (1) and (3) in natural language and that of (2) in more symbolic form, or to potential learning effects of the instrument, is an area for further research. For all conjectures, students were also more likely to chose a narrative approach (over deductive) as the one closest to their own solution or to use to convince a peer.

For all conjectures, a significant majority of students (≥ 63%) chose the deductive argument as the most rigorous approach, while no more than 15% chose an empirical approach. In conjunction with this, responses to other test items indicated that students were aware of the limitations of an empirical approach, with at least 40% interpreting the empirical argument to be "true for only a few cases". This suggests that, while students may have difficulty in constructing deductive arguments, they can identify more mathematically rigorous arguments. More work is needed to determine what salient features of deductive arguments lead students who do not have formal training in writing proofs to select these as more rigorous.

Mathematical Conjecture	*(a) Solution closest to student approach*	*(b) Solution chosen as most rigorous*	*(c) Solution chosen to convince a peer*
(1) The sum of any two even numbers is even.	**empirical - 42%** narrative - 30% visual - 5% deductive - 23%	empirical - 15% narrative - 15% visual - 4% **deductive - 66%**	**empirical - 35%** narrative - 31% visual - 21% deductive - 13%
(2) For any integers a, b, and c, if a divides b with no remainder, then a divides bc with no remainder.	empirical - 31% narrative - 23% visual - 10% **deductive - 36%**	empirical - 15% narrative - 17% visual - 5% **deductive - 63%**	empirical - 25% **narrative - 29%** visual - 22% deductive - 24%
(3) The supplements of two congruent angles are congruent.	empirical - 12% narrative - 28% **visual - 42%** deductive - 18%	empirical - 7% narrative - 17% visual - 11% **deductive - 65%**	empirical - 15% narrative - 25% **visual - 52%** deductive - 8%

Table 1. Results of student selection of argument type.

Relationship to Goals of PME-NA

By offering insight into undergraduate students' interpretations of mathematical arguments, this study contributes to our understanding of the issues associated with the teaching and learning of proof, a domain central to students' mathematical development.

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THE ROLE OF EXAMPLE-GENERATION TASKS IN STUDENTS' UNDERSTANDING OF LINEAR ALGEBRA

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Background and Theoretical Framework

Examples play an important role in learning mathematics. Students are usually provided with examples by teachers or textbooks, and very rarely are asked to construct examples themselves, especially in postsecondary level courses. Research has shown that linear algebra is one of the postsecondary mathematics courses that students are having difficulty with (Dorier, 2000; Carlson et al, 1997). Part of the difficulty is due to the abstract nature of the subject. Dubinsky (1997) points out that the overall pedagogical approach in linear algebra is that of telling students about mathematics and showing how it works. There is a lack of pedagogical strategies that give students a chance to construct their own ideas about concepts in the subject.

As research shows (Hazzan & Zazkis, 1999; Watson & Mason, 2004), the construction of examples by students contributes to the development of understanding of the mathematical concepts. Simultaneously, learner-generated examples may highlight difficulties that students experience. The analysis and investigation of learner-generated examples has been guided by APOS (Action-Process-Object-Schema) theoretical framework for modeling mathematical mental constructions (Asiala, et al, 1996). This framework was developed for research and curriculum development in undergraduate mathematics education.

Methods or Modes of Inquiry / Data Sources or Evidence

Participants in this research were students enrolled in elementary linear algebra course. The data was collected through students' written responses to the example-generation tasks. These are non-standard questions that require understanding of the concept rather than merely demonstrating a learned algorithm or technique. The written questionnaires were administered to the participants during the course. The topics addressed in the questions included linear (in)dependence of vectors, matrix algebra, and linear transformations. Several examples of the tasks are listed below:

(Q1) Give an example of a 3x3 matrix A with nonzero real entries whose columns $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$ are linearly dependent. Now change as few entries of A as possible to produce a matrix B whose columns $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, $\boldsymbol{b}_3$ are linearly independent, explaining your reasoning. Interpret the span of columns of A geometrically.

(Q2) Give an example of a matrix for which the corresponding linear transformation maps the vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ *to* $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$.

Results

The study discussed students' difficulties with constructing examples, and also suggested possible correlations of students' understanding with the generated examples. Furthermore, it

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showed that the example-generation tasks reveal students' (mis)understanding of the mathematical concepts. In particular, generating examples for the mathematical statements require more than just procedural understanding of the topic.

Using the APOS theoretical framework for analyzing students' responses to (Q1), one can identify different levels of students' understanding of the linear dependence concept. When students construct examples using random guess-and-test strategy, they operate with an action conception of linear dependence. They have to perform row reduction on a matrix to find out if its columns are linearly dependent. Students that construct examples of matrices with the same rows or rows being multiples of each other, i.e. inverting the row reduction procedure mentally, understand linear dependence as a process. Students that emphasize relations between column vectors have encapsulated linear dependence as an object, and consequently will be able to construct any set of linearly dependent vectors.

This research provides a better understanding of the role of example-generation tasks in students' understanding of linear algebra. It analyzes students' difficulties involved in generating examples and how students' examples correlate with their understanding. It is a novel study on example-generation tasks as a research and pedagogical tool in postsecondary mathematics education. It provides a variety of tasks for implementation in Linear Algebra course, and opens opportunities for future research and development of pedagogy.

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INVESTIGATING DISTANCE PROFESSIONAL DEVELOPMENT: LESSONS LEARNED FROM RESEARCH

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Effective professional development (PD) is critical in retaining rural teachers (Storer & Crosswait, 1995). Although all school districts are charged with providing teachers access to high-quality PD (National Staff Development Council, NSDC, 2005), rural school districts increasingly face budgetary reductions for PD exacerbated by rising travel costs. Emerging technologies and alternative delivery methods, such as videoconferencing and web-based media, are enabling the implementation of novel and attractive distance professional development (D-PD) opportunities for rural teachers and districts.

The current study focused on a D-PD initiative in mathematics and science education and examined how face-to-face, web-based, and video conferencing technology delivery affects the learner-learner/learner-instructor environment, communication and interaction. The study sought to answer the questions: 1) how does D-PD via electronic modalities differ from traditional face-to-face instruction; 2) how can D-PD allow for these differences; 3) what affect does video conferencing and web-based technologies have on reducing isolation of rural teachers; and 4) in what ways are participant-participant and participant-instructor communication enabled and constrained by videoconferencing and web-based technologies?

Theoretical Framework

Within the construct of D-PD, PD providers in this project sought to provide rural teachers socially negotiated (Cobb, 1994) and authentically constructed (Brown et. al., 1993) learning environments. Researchers sought to facilitate and observe the construction of communities of practice (Wenger, 1998) and the interaction and communication within and among those communities. Participants were observed in communities of practice, working together towards a common goal of studying visualization technologies and their classroom applications.

The complexity of learning to pedagogically employ visualization technologies necessitated that participants continually collaborate and communicate. Participants simultaneously grappled with learning the underlying mathematics, to interpret the visualization in the context of the science, learning the epistemological and pedagogical use of the visualization technologies. Through various methods of instructional delivery, teachers in Illinois and North Carolina interacted with others (local and remote), and utilized various communication technologies.

Methodology

This initiative purposed to enhance the retention and renewal of rural mathematics and science teachers through community building by delivering content, providing mentoring, and creating virtual teams among teachers in different states. Through an environment which was designed to be rich in communication and interaction, teachers used visualization and immersive technologies to deepen their understanding of core mathematics and science topics. Data was collected on the effectiveness of individual instruction sessions, type of instruction, importance of communication, use of technology, and impact on the participants. Baseline questionnaires at the beginning, halfway through, and post institute questionnaires were administered. Changes in the distribution patters were measured using Wilcox on Signed Rank Tests.

Data Sources/Results

Teachers found videoconferencing effective as a D-PD delivery method, although participant satisfaction was higher for sessions where the presenter was live and local than when the presenter was remote and participants watched the electronic presentation. Although a class culture formed at each location through formal and informal interactions among professional developers and teachers, it did not translate well from onsite to the remote site. Therefore, if PD is limited to information dissemination, videoconferencing was effective; however, if PD connotes the creation of communities of practice, videoconferencing may yet be limited.

Typically, via electronic communication, natural, spontaneous, and voluntary communication between remote participants remained minimal. The most effective method of D-PD was the small group (by subject/grade) parallel session via videoconferencing. Participants were very likely to communicate and build communities of practice among teams of teachers from the same school and/or when they had common teaching interests (subjects/grades). Proximity seemed to be the decisive factor in whether a community of practice was formed.

Conclusions

Conducting high quality D-PD in mathematics and science using videoconferencing and web-based tools presents new challenges to providers. Particularly difficult is facilitating the social interactions between distance learning sites to create communities of practice among educators at remote locations. The researchers felt that a stronger emphasis on communication between participants at remote sites may have a greater impact on fostering communities of practice. These ideas have been implemented and are currently being studied under cohort II. As a methodology to enhance rural teacher renewal and retention, D-PD continues to be investigated through this project.

Relationship of paper to goals of PMENA

This research attempts to deepen understanding of D-PD using video-conferencing and web-based technologies and attempts to further understand how professional learning communities of rural teachers can be formed using D-PD to reduce teacher isolation in rural settings.

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TEACHER CANDIDATE EFFICACY IN MATHEMATICS: FACTORS THAT FACILITATE INCREASED EFFICACY

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The research objective of this study was to identify the specific experiences of preservice teachers that influence their efficacy in mathematics teaching. Research in the area of teacher efficacy (Gibson & Dembo, 1984; Bandura, 1997; Tschannen-Moran & Woolfolk Hoy, 2001; Goddard, Hoy & Woolfolk Hoy, 2004) has produced a solid body of literature that examines how teachers judge their own capability to bring about student learning. The teacher assesses his or her ability to perform a given task based on an analysis of what is required to accomplish the task, reflection on past similar situations, and assessment of resources available (Bandura, 1986). Those teachers who believe they are effective are more likely to set high goals and persist to meet those goals even when faced with obstacles. Those teachers are also willing to experiment in the classroom (Allinder, 1994) with instructional strategies and student-directed, activity based methods (Riggs & Enochs, 1990).

Elementary teachers are at a critical juncture where their sense of efficacy teaching mathematics has the potential to increase. Researchers in the field have called for studies that illustrate methods which promote the enhancement of teacher efficacy. The most powerful source of information on teacher efficacy is mastery experience. Self-efficacy generally rises with experience, particularly following practice teaching. However there have been few qualitative studies to examine the factors that facilitate increased efficacy, particularly in mathematics teaching. The case of mathematics teaching is complex because preservice teachers are placed in classrooms where there is a limited range of reform-based mathematics instructional practices being used (Ross, 1999). Yet the reform movement clearly indicates that student directed conceptual approaches and paradigms are more effective and impact positively on student achievement (Ross, McDougall & Hogaboam-Gray, 2002; Simon, Tzur, Heinz, Kinzel, 2000). Further, preservice teachers have reported that they experienced traditional programs as mathematics students (Bruce, 2004). Thus, preservice teachers are attempting practices they have not experienced as students or as observers of students in host classrooms. To understand the mechanisms through which preservice programs might influence teacher efficacy and math reform implementation, intensive case studies are required. Therefore, the research questions of this study were: (i) what is the nature of the learning trajectories of preservice teachers in a Bachelor of Education program; (ii) which methods contribute to preservice teacher development of efficacy related to mathematics reform based teaching; and, (iii) what are the implications for preservice programs?

The site of this study was a newly established Bachelor of Education program in Ontario, Canada. Participants in the study were 12 preservice teachers enrolled in an elementary mathematics methods course. Data sources included open-ended inventories, focus group and individual interviews, observations, the Teachers' Sense of Efficacy Scale (Tschannen-Moran & Woolfolk Hoy, 2001), and participant math log entries. A Constructivist Grounded Theory approach (Charmaz 2000, 2003), with a zig-zag method for data collection and analysis (Creswell, 2005) were used. Methods of open, active and axial coding were combined with

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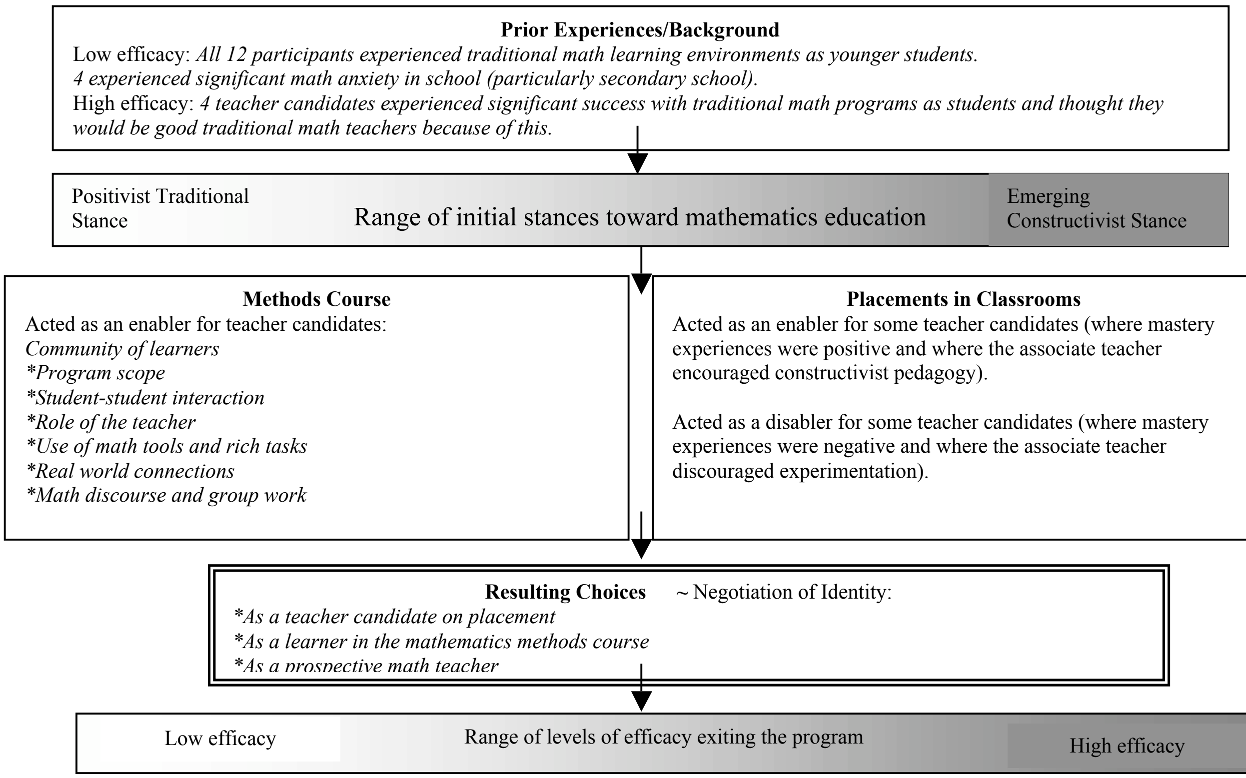
theory notes and visual maps to clarify and confirm understanding of the data. In order to fully mine the data, two case studies were completed to illustrate extreme trajectories: One extremely positive, the other very challenging. Other cases were used in a cross-case analysis to examine the range between the two extremes. For all participants, common stages in the trajectory were identified as were methods for enabling increased efficacy teaching mathematics.

The findings of all 12 participants are summarized in a diagram that illustrates the trajectories of preservice teachers, the influences on teacher efficacy in mathematics and the outcomes (see figure).



Factors that contributed to increases in teacher efficacy included features of both placements in schools and strategies used in the mathematics methods course. In most cases, opportunities to teach mathematics while on placement proved to be a tremendous confidence builder. However, in those cases where preservice teachers were strongly discouraged from experimenting with reform based methods of teaching math, teacher efficacy decreased. In the methods course, a combination of modelling reform based practices, developing a community of learners, encouraging student-student interaction, and guiding discourse were identified as enabling increased efficacy for all participants.

This study demonstrates that qualitative descriptions of shifts in teacher efficacy ratings experienced in a Bachelor of Education program are useful in identifying enablers of confidence teaching mathematics. Although challenging, mathematics methods courses can be structured and delivered to enhance preservice teacher efficacy. Further, teaching placements which support and encourage the use of reform based teaching strategies are strong influences on increased efficacy. The full findings of this study are important because the education community needs to establish and communicate researched effective strategies that enhance teacher efficacy at the preservice level in order to increase sustained implementation of mathematics reform.

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MODIFICATIONS GONE AWRY?: EXCLUDING FORMAL PROOFS TO ADDRESS EQUITY

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Mathematics education is widely viewed as an opportunity for students to develop skills and habits that will benefit them in the school mathematics curriculum, other subject areas, and in present and future life experiences. The National Council of Teachers of Mathematics (NCTM) (NCTM, 2000) describes not only the mathematical content that students should know but also ways through which content knowledge should be acquired and identifies reasoning and proof as two such skills that students should develop through their mathematical learning experiences. A high school geometry course *can* provide opportunities to develop systematic reasoning skills – a component of developing deductive reasoning and writing formal proofs. Students' justifications of mathematical arguments enhance their understanding through clarifying ideas and concepts. Thus, formal proofs are not to be viewed merely as an end product. Moreover, these skills are valued for their roles in preparing students for mathematics-intensive fields. Yet, students' overall performance in geometry, (Blank & Wilson, 2001), use of formal deduction (Burger & Shaughnessy, 1986), and ability to write formal proofs (Senk, 1985) are low.

Purposes

This research addresses two areas that are rarely considered concurrently – geometry and equity. Rather, at the secondary level algebra is viewed as a minimum requirement, and calculus is often viewed as an indicator of how successful one's mathematics education is. Although these perspectives are vital, an examination of the role of geometry in facilitating (or obstructing) equitable mathematics education is needed. The reasoning and logical thinking that accompany the construction of formal proofs in geometry are valuable skills that benefit students in future courses, real world applications, and access to advanced mathematics and mathematics-based majors and careers. Perhaps, greater concern is warranted for underrepresented students in that weakening the curriculum further impedes the realization of their mathematical potential.

I examine the influences and impact of the exclusion of formal proof from a high school geometry course disproportionately taken by students of color. This course, Modified Geometry, was designed to address inequities among students taking low level mathematics courses. I discuss how the course design addressed (1) the department's goals to increase the number of students who take more advanced mathematics courses and improve standardized test scores and (2) the department's quest to make mathematics education more equitable. While I assert that the exclusion of formal proofs negatively impacted students, I do not argue that the mere inclusion of formal proofs ensures desirable or equitable mathematics education.

Methods

This case study was conducted in a high school mathematics department in a school with approximately 1500 students. Approximately 50% of the students in the district received free or reduced lunch, and the racial demography was 65% White, 28% African American, 5% Hispanic, and 2% Asian/Pacific Islander. The graduation rate was 68% for the entire student

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body, 75% for Whites, 55% for African Americans, 41% for Hispanics, and 67% for Asian/Pacific Islanders.

Data were collected for eight months and included a focus survey, interviews, field notes from department meetings, and school documents. Twelve of the 13 members of the department including the chair and the district's curriculum coordinator participated. Their teaching experience averaged 10.8 years. Three key-informants participated in three interviews. The other teachers were interviewed once.

Data analysis consisted of triangulation of data and a search for disconfirming evidence. Transcripts of each interview and field notes were coded using an initial list of codes constructed from the research literature. I identified emergent themes after each iteration of data analysis and revised the list of codes, accordingly.

Findings

The department designed Modified Geometry for students who had completed Modified Algebra or who had passed the traditional algebra course with a C or below. Modified Geometry omitted formal proofs and was offered in addition to a traditional high school geometry course. A goal of Modified Geometry was to increase access to geometry topics to students who had previously stopped taking mathematics upon completing the four credit requirement. Equity is a multi-faceted construct that involves an examination of access/inputs, practices, and ends/outcomes. This analysis focuses on access, although access is not taken as a more important component. Greater access to geometric topics resulted. However, Secada (1989) outlined a second component to consider when examining how equitable mathematics education is – "is that which is being distributed worth having?" In this context, the exclusion of formal proofs and the accompanying deductive reasoning, yielded a more deprived course that impeded students' access to future mathematics courses and careers. Despite students' increased access to some geometry topics, the design modifications – which limited access beyond geometry, eliminated opportunities for developing critical skills (e.g. deductive reasoning), and perpetuated low expectations – had gone awry.

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HELPING PRESERVICE TEACHERS TO DEVELOP INSIGHT INTO CLASSROOM PRACTICE

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The use of videotaped classroom practice as an as an instrument of professional development has been well documented (for example: Davis, Maher & Martino, 1992; Fosnot & Dolk, 2002; Powell, Francisco, & Maher, 2003; Schmidt, McKnight & Raizen, 1996; Tarlow, 2004; Warner & Schorr, 2004;). However, in these studies, a fine-grained analysis of the data was utilized, which is something that may not be practical for classroom teachers or college students studying to become teachers. PRIVATE UNIVERSE (HSCFA, 2001), includes classroom clips to promote discussion. Although student artifacts and additional reading opportunities accompany this series, these supports are separate entities. The design of the project described here is distinctive in that supports are integrated directly into the actual software and can be viewed concurrently with the observation of the teaching and learning of mathematics.

The video vignettes that were examined for this study are part of a Virtual Learning Community (VLC) website. The VLC was created to support novice and mentor teachers as well as preservice teachers (Fraivillig, Wish & Bulgar, 2004). The question under study in this research is the following. How did the preservice teachers interpret the mathematical activity they observed through the use of supported videotaped vignettes and demonstrate their understanding of the interest and sustained engagement of the 2nd graders they observed?

The subjects of this study were full-time undergraduate students in two sections of a Mathematics Methods course at a small private university in NJ. They looked at vignettes from a 2nd grade elementary classroom as part of their regular university class experience. The data consist of undergraduate student work related to the project, including the answers to four questions about the vignettes. Undergraduate students viewed the vignettes prior to coming to class. In class, they were provided with laptop computers, working in groups of three or four to re-examine the vignettes, reflect upon their observations, discuss what they observed and answer four questions about the classroom mathematical activity. The following is an example of one of the questions: What is it about this task that makes it so engaging? The intent of this question was to focus attention on what led to student engagement in the hope that this would impact task design and selection for the future teachers.

The following excerpts from the undergraduate students' work products provide examples of the prospective teachers' responses.

Group 1: There are no limitations set for the way that they go about executing this task. The use of manipulatives kept the children engaged because it enabled them to see abstract ideas more [sic] concrete. The fact that they were able to talk to each other about the project also excited the children.

Group 2: They are extremely anxious to participate because of the atmosphere in the classroom and the interest in the problem.The problem is not traditional practice and drill exercise.

Group 3: …Students are given independence to complete the task using their ideas…Talking, using manipulatives, and conversation all allow the students to become engaged…

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Group 4: …They aren't concentrating on the math, but they are concentrating on the task at hand.

Group 5: …They each pick a strategy that works best for them.

The compiled responses indicate that the following criteria resulted in the young children's engagement: ownership of the task; authenticity; becoming decision-makers, empowerment; child-centered; enjoyment; teacher's encouragement and support; task relating to real-life; small group setting; collaborations; lends itself to the use of manipulatives; students have control over their ideas; students have control over their choices.

For future teachers to understand how to create and select mathematical tasks that will be engaging, they must first recognize the characteristics of engaging tasks. These pre-service teachers' responses indicated that viewing video vignettes with scaffolds embedded directly into the software provided them with an opportunity to identify criteria for engaging tasks.

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"MAKING MATHEMATICAL CONNECTIONS" IN THE TEACHING OF SCHOOL MATHEMATICS

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Problem Statement and Research Questions

The National Council of Mathematics Teachers' (NCTM) document, *Principles and Standards for School Mathematics* (2000) and its earlier versions in 1989 and 1991 establish a framework to guide improvement in the teaching and learning of mathematics in schools. The documents identify "mathematical connections" as one of the curriculum standards for all grades K to 12. In this framework, "… mathematics is not a set of isolated topics but rather a web of closely connected ideas" (NCTM, 2000, p. 200). Both the research literature and the pedagogical literature stress the value and importance of making mathematical connections, the rationale being that making connections will allow students to better understand, remember, appreciate and use mathematics. Teachers are exhorted to teach in ways that will encourage the making of useful mathematical connections by their students. Learners might make connections spontaneously, but "we cannot assume that the connection will be made without some intervention" (Weinberg, 2001, p.26). The implied role for teachers is to act in ways that will promote learners' making of mathematical connections (Thomas & Santiago, 2002). Studying teachers' pedagogical efforts to promote the making of mathematical connections necessitates considering the intersection of three frameworks – their own understanding of mathematics, their general pedagogical knowledge, and their specific pedagogical content knowledge (Shulman, 1986).

This report describes an exploratory study designed to identify emerging themes in teachers' thinking related to making connections by probing their own perceptions. The study addresses the questions:

- how do mathematics teachers conceptualize "making connections", and
- how do they see themselves attending to making mathematical connections in their teaching.

Research Setting

The participants are three secondary mathematics teachers with 5-8 years of teaching experience, all acknowledged to be excellent teachers. Each teacher participated in an audiotaped semi-structured interview lasting 30-60 minutes. Prepared questions focused on mathematical connections and how they are used in teaching practice, for example:

- What sorts of things come to mind for you when you hear the term "connections"?
- In addition to connections "to the real world", another interpretation of connections is "connections within mathematics". To what extent are these kinds of connections part of your teaching? Please give an example.
- What kinds of things do you do to show students connections within mathematics?

Follow-up questions were not pre-planned and probed teachers' individual initial responses Schram (2003). I transcribed the interviews and developed a coding scheme (Gall, Borg and Gall, 1996). Some categories of responses could be identified a priori because I had asked

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specific questions about certain topics – for example, mathematics background, beliefs about connections, teaching goals. Other general categories emerged from a reading of transcripts – for example, beliefs about the nature of mathematics, beliefs about learning and teaching, teaching strategies.

Data and Interpretations: Views of "Making Connections"

All three teachers reported similar views with respect to "making connections". The main issues that emerged in the interviews were:

- Making connections in mathematics means making connections to the "real world", specifically to finances, games and using mathematics as a tool in other subjects.
- Making connections can be useful in assisting students' memory and increasing their motivation. The teachers saw making connections as particularly important with younger students and students in non-academic math courses.
- None of the three teachers spontaneously included concept-to-concept connections within mathematics in their views. With further probing, they equated making connections between mathematical concepts to recalling previously learned ideas and procedures.
- The teachers found it difficult to present examples from their own teaching of specific events that they viewed as "making connections".

These teachers saw themselves as paying little attention to making connections in general, and even less to mathematical connections. A crucial next step in the research is to establish whether this belief is an accurate perception of their practice – do teachers *really* not attend to making connections? Or, might they be acting automatically and unaware of actions that others might identify as "making connections"? In either event, in the context of the NCTM's emphasis that understanding develops "only if students grasp the connections" (NCTM, 1989, p.147), this finding raises a crucial question about the nature of the connections that students make if the connections are not dealt with *explicitly* during their mathematics lessons.

Conclusion

This exploratory study identifies some emergent themes in the way that practicing teachers view "making mathematical connections" that appear to differ from the conceptualizations in the literature. The differences noted should be explored by a more in-depth study in order to verify these emerging themes with larger and broader samples of teachers, and using a range of methods, and then to build a model of the interaction of teachers' pedagogical beliefs, their own knowledge of mathematics, and their specific pedagogical content knowledge.

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THE EVOLUTION OF MATHEMATICAL EXPLORATIONS IN OPEN-ENDED PROBLEM SOLVING SITUATIONS

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The purpose of the study was to examine the problem solving processes that solvers use to solve open-ended mathematics problems. Subjects were interviewed as they solved a set of open-ended problems. Drawing from the episodes of two students solving a Number Array task, the analysis explains how the solvers' self-generated problem posing actions help to establish and then extend conceptual boundaries for their solution activity. Our on-going work in problem solving is to develop a model of the solvers' general exploration processes. The current study is part of this overall effort and is an extension of an earlier study (Cai & Cifarelli, in press).

Subjects and Analysis

The subjects were two secondary math education majors, Sarah and Gavin. The students were interviewed as they solved a Number Array task that required them to find relationships within a square array of numbers (Figure 1).

The data consisted of video protocols, written transcripts of the videos, the researchers' field notes, and the subjects' written work. We hypothesized that problem solving in open-ended tasks involves varying degrees of problem posing (that involves the solver's interpretations and how they give meaning to the tasks) and problem solving (in the sense that once goals are developed, the solver shifts his/her focus to the carrying out of action designed to achieve his/her goals). We believe this process to be recursive in nature, with each self-generated question indicating acts of problem posing that the solver initiates to frame and structure their subsequent actions.

Figure 1: Number Array Task

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Results

Working in the number array task provided the subjects with opportunities to explore and develop a variety of mathematical relationships; a subset of these is summarized in Table 1.

Table 1: Mathematical Relationships Generated by Solvers	Sarah	Gavin
Relationships about the arrangement of the numbers	4	4
Relationships about the sums of the numbers	7	2
Relationships about the products of the numbers	3	2
Relationships about number sequences	5	0
Total	19	8

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The results summarized in Table 1 indicate some compatibility in the relationships developed by the students. However, Sarah developed several more sophisticated relationships than did Gavin. These included an informal 'skipping' method to find the sums of the entries of all NXN blocks containing the square numbers on the diagonal.

> ***Sarah****: So, for a 1x1, I get a sum of 1. For a 2X2* ***(Points to 2X2 block [1,2:2,4][1])*** *I get a sum of 9 … but what happened to 4? It has been skipped!* ***(reflection)*** *Okay, let me try this, I will write down the sequence of squares of all numbers, all in a row* ***(She writes the sequence: 1, 4, 9, 16, 25, 36, 49, …, 2251).*** *The first number, 1, is the sum of the first matrix, a 1x1. And the first 2x2 has a sum of 9. So, I skipped over 4 to get the sum for the 2X2 in the upper-left* ***(crosses out the 4 in the sequence of squares)****, going from 1x1 to a 2x2, a sum of 9. The 4 is skipped! Interesting!*

Sarah was able to generalize her 'skip' method to all NXN blocks.

> ***Sarah:*** *So, going from the 2x2 to the 3x3 (****Points to 3X3 block [1,2,3:2,4,6:3,6,9]****), we go from 1, to 9, to 36 – so we skipped over the next two numbers, 16 and the 25* ***(crosses out 16 and 25 in the sequence)****, a skip of 2 in this sequence! Okay, then we will skip over the next 3 square numbers, and that should tell us the sum for a 4x4 should be 100* ***(crosses out the next 3 in the sequence after 25: 36, 49, 81)*** *– that is what I have over here !! Cool! So, for a 5X5, we skip over the next 4 numbers in the sequence, the number 121, 144, 169, 196 and get 225 – yes!*

Sarah had developed an in formal 'skipping' method for computing the sum of the entries of NXN matrices down the diagonal of the array. She generalized a more efficient algorithm that involved operations of the row and column numbers of each NXN.

> ***Sarah****: I wonder why this skipping works? Let's see, for the 6X6, we add the rows of the block, 21+42+…+126= 21(1+2+3+4+5+6)= 21x21=441. Do we get 441 by skipping the next 5 in this square sequence?* ***(She checks her original sequence and crosses out the corresponding 'skips', and gets 441 as the next number in the sequence!)*** *I notice that 21 over here* ***(she points to the factored form 21(1+2+3+4+5+6)*** *is the sum of the 6 numbers in that first row. Yes! So to find the sum of these NXN blocks, I bet you just need to look at the sum of 1 to N and then square the total to get the sum. Let's try 8x8 … it would be 1+2+…+8=36, and then I take* 36^2 *…1296. And does it check with my skipping sequence over here? So for 8x8 I first skip 6 over 21 to get* 28^2 *for 7x7, and then skip 7 more to get the one for 8x8, so 7 more is 35, and the next one is 36! So my algorithm works! It is efficient for large numbers – how about a 100x100 grid? – but the skipping relationship was pretty cool!*

Summary

The findings are consistent with research on open-ended problem solving that posit conceptual benefit when students solve open-ended problems (Becker and Shimada, 1997; Cai & Cifarelli, in press; Cifarelli & Cai, in press; Silver, 1994). Moreover, Sarah's evolution of her solution activity from informal methods to her invention of a sophisticated algorithm for NxN cases illustrates an important way that solvers stretch their conceptual boundaries when engaged in open-ended problem situations.

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[1] In order to refer to various blocks of numbers in the array, we use a notation that lists the top-to-bottom rows of the block. For example, the 3X3 block in the upper left position is denoted by [1,2,3:2,4,6:3,6,9].

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ANOTHER HIDDEN CURRICULUM: EAVESDROPPING ON STUDENT GROUPS

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Purpose

This paper reports the results of a pilot study in which I use ethnographic methods to demonstrate that even with reform-oriented, student-centered teaching methods, there exists another "hidden curriculum" in the math classroom. Historically, the hidden curriculum has focused on how teachers, administrators, and the institution of schooling perpetuate particular roles for students. As classrooms shift from being teacher directed to student centered, I show that there is *another hidden curriculum* – one where the responsible parties are the students themselves: as the classroom control is turned over to students, the hidden curriculum is being constituted by them. I highlight the ways female students position themselves within groups to be dually powerful and powerless.

Theoretical Framework

Critical multicultural mathematics education is "concerned with the social and political aspects of the learning of mathematics" (Skovsmose & Borba, 2004). The alienation of non-whites and women as a causal factor in their historical lack of success and participation in mathematics is indicated in numerous studies (Kincheloe & Steinberg, 2001). In the US, the National Council of Teachers of Mathematics *Standards* document supports a critical multicultural view of mathematics education through its mission to provide opportunities for "every child" to be successful in mathematics.

In this paper, I use a critical qualitative analysis to demonstrate the existence of *another* "hidden curriculum." This analyses benefits from a "Foucauldian gaze" which recognizes that power is present in all human relationships (Walshaw, 2001). Foucault argues that power is not solely owned by one person or one group, but rather exchanged and reformulated. In this paper, I focus on objectification and competing discourses in small group interactions to examine power relations. Objectification is defined as "a process whereby a powerful group establishes and maintains dominance over a less powerful group by teaching that the subordinate group is less than human or like an object" (Gamble, 1999, p. 286). Judith Baxter (2002) "shows the complexity of how girls are multiply positioned by competing classroom discourses as at times powerful and at other times powerless" (p.5).

Context of the Study and Methods

In this ethnographic pilot study, I observed a middle school mathematics classroom of 26 students with an equal number of males and females who, according to their teacher, Grace, represented "a mix of kids and abilities." I visited the classroom seven times during fall, 2004, to observe Grace, an award winning mathematics teacher, and her students. Field notes were taken and then coded to identify themes, of which the power and positioning of the female students in small groups became apparent. To illustrate the findings from this study, I present two situations from my data that occurred with the same students (working in a group of four) who had clear direction from their teacher to complete a certain task.

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Results

Objectification of the Female

In the first vignette, Janelle, described by Grace as a good student who cares about her grades, is trying to get her work done. While she is not the designated "quality control person," she becomes the group member who tries to focus the group to accomplish the task. While doing so, she is objectified by the boys in the group who say things like, "now she's acting motherly" and "now she's serious." Objectifying illustrates how men are socialized for gender dominance (Kincheloe & Steinberg, 2001). Although this interaction does not have explicit sexual overtones, the treatment of women as objects could be viewed as a source out of which grows masculine domination. This dominant vision of men, Bourdieu (1999) says, leads women to find their place in the social order and see it as normal. Carspecken (1996) contends that when subordinates accept their social status as natural or inevitable, oppression is reproduced.

Competing Discourses

In another situation, Janelle tries to focus the group, but after several attempts, she decides to do her own work and "ace the test." After making fun of one of her answers in the small group, one of the boys, Tim, uses her answer when called on later in a whole class discussion. When a male student takes a female student's answer, he is making a "claim to knowledge" (Kincheloe & Steinberg, 2001, p. 139). Male authority appropriation over what women have said is a power dimension that is illustrated daily on the individual level, e.g. during board meetings, union meetings, and teachers' meetings (Kincheloe & Steinberg, 2001, p.139).

When the group continues their off-task behavior, Janelle proceeds to work quietly on her own. When girls try to be nice, kind, and helpful – characteristics that teachers publicly hold up as good – they put themselves in psychic and social double-binds (Walkerdine, 1998). In this case, Janelle feels confident that she is capable of getting the work done by herself, but allows herself to be dismissed by Matt, saying "You **told** me to do my own [work]." Baxter (2002) demonstrates how and why girls can be silenced in classroom contexts by examining the *contradictory* positioning of the student. On one hand, Janelle is a capable, rule follower; on the other, her response is passive aggressive and later causes the teacher to reprimand her for not working with the group. This power struggle positions Janelle both as powerful (she'll "ace the test") and powerless (she was "told" what to do and gets in trouble with Grace).

Summary and Implications

Even an award-winning teacher has limited social control. She is practicing methods deemed useful in promoting equity, yet the gender roles are clearly apparent in the small group interactions. Kincheloe and Steinberg (2001) note that the "conditions under which knowledge is produced have changed dramatically over the past twenty years" resulting in new forms and guises of power and hegemony. Teachers interested in social justice need to be aware of another hidden curriculum—one that plays out in small groups-- and work to promote equity for historically subjugated groups.

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THE TEACHER AS A BROKER IN ESTABLISHING A CLASSROOM COMMUNITY OF PRACTICE

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Theoretical Framework

A classroom community of practice is used to describe the social setting of a classroom in which students come together to work towards a communal goal. It is within this classroom community of practice that the teacher acts as a broker (Wenger, 1999) to help guide the students to engage in activities in a way that is commensurate with the larger mathematical community. The term broker is used to recognize that the teacher was a member of both the classroom community and the larger community of mathematicians. Brokers thus have the role of aligning the communal goals of the classroom with those of the larger mathematical community. They encourage this alignment by initiating certain social norms which allow the sociomathematical norms (Yackel & Cobb, 1996) to emerge. They also aid this alignment by introducing tools and formal conventions accepted by the larger mathematical community.

Methods

This data was collected during a semester long teaching experiment (Cobb, 2000) in a mathematical structures course where students' transition from the computational mathematics of Algebra and Calculus to the more rigorous proof writing required for upper level mathematics. The students engaged in activities that encouraged them to use formal deductive arguments to justify their conclusions. Videos of the class sessions were analyzed and then transcribed. The transcripts were coded for areas in which the students and teacher were observed contributing to the norms of the classroom and points where the teacher's influence helped student learning.

To allow the sociomathematical norms to emerge, the teacher initiated the social norms of the classroom including that the students were to engage in classroom discussions, were to share their thinking about their solutions, were to try to make sense of other students' solutions, and were to challenge solutions they did not agree with. She did this by asking questions that encouraged students to participate, asking questions that encouraged students to check the validity of other students' solutions, and by inviting comments and critique from the students. Through these interactions the students became aware that she valued student participation.

Results and Discussion

The primary sociomathematical norm contributed to by the teacher and the students in this study was the criteria for what constituted a sufficient mathematical argument. The teacher served in her role as broker to encourage the students to create arguments that would be acceptable to the mathematical community while supporting students' emerging activity and arguments. The primary criterion for a sufficient mathematical argument became that the argument be deductive, meaning that it showed why a claim must be true.

At the beginning of the semester the teacher used the fact that students were explaining their solutions to the class and evaluating each other's solutions (social norms) to engage students in reflecting on what would count as a sufficient mathematical argument. She also supported this

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norm by pushing students to provide a more deductive or more complete argument when students did not at first provide one. Later in the semester students were more likely to provide such arguments or to require such arguments of others. Another important role of the teacher as broker was that she had to introduce the conventions and symbols that have been established by the mathematical community. Often students will have difficulty deriving these conventions or recognizing a need for them. It is the teacher's job to make sure the students use the same symbols as the mathematical community.

Finally the teacher as broker is responsible for helping students create the formal tools into the classroom community that are used by the community of mathematicians. While modeling the students thinking on a problem, the teacher created a grid that resembled the way they were discussing the problem. Noticing its similarity to a truth table, the teacher took the opportunity to introduce a formal truth table which is a tool of the mathematical community. The students were then able to use this tool in working on problems in order to make sense of them. In order to become a member of a community, whether it is the classroom community or the community of mathematicians, it is important for the individuals to be able to use the tools of that community.

In the examples above the teacher serves as a broker by helping to align the sociomathematical norms, tools and conventions of the class with that of the mathematical community. On the one hand the teacher has the role of engaging students in the classroom community of practice and working with in the boundaries of that situation. On the other hand the teacher has the responsibility as a representative of the mathematical community to work to align student activity with that of practicing mathematicians. These dual roles are illuminated by the use of Wenger's (1999) term broker and bring out the relationship between a classroom community and the mathematical community. This research is part of a larger study on the emergence of a classroom community of practice (Clark, 2005) and was supported in part by the National Science Foundation under Grant No. 0093494. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

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COMPARING THE MATHEMATICAL THINKING OF GRADES 2, 5 AND 8 STUDENTS ON IDENTICAL MATHEMATICAL TASKS

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Introduction

Some studies have found that as students progress through the grades, at least some of their mathematical thinking deteriorates (Burns, 1994; Kamii, 1989; Kamii, Lewis, & Livingston, 1993; Reid, 1995). Although students enter school as eager and capable mathematical thinkers, as they progress through the grades they appear to give up their own sense-making capacities and learn to rely on memorized procedures (McGowen & Davis, 2001a; 2001b; Romberg, 1992). This study explores grades 2, 5 and 8 students' mathematical thinking when completing identical mathematical tasks, in structured task-based interviews. The objective of the study is to develop a conceptualization of student mathematical thinking across these grades.

Methodology

Twenty students from each of grades 2, 5 and 8 in the same school were randomly selected from class lists. The interviews focused on the three tasks shown in Table 1. Task 1 was used in the study by Reid (1995). Task 2 is similar to questions used by Kamii (1989) to compare grades 2 and 4 student thinking. Task 3 offered students the opportunity to explore patterns in the context of making sense of division by zero.

Table 1. The mathematical tasks

TASK 1: Your parent gives your teacher \$6.25 to buy popsicles for your class. Popsicles cost 25 cents each. Will there be enough money to buy one for each student?
TASK 2: Think of the numbers 176 and 58 and 5. (*Written form: 176 + 58 + 5*) Please add these numbers in your head.
TASK 3: • What is 2 times 0? What is 2 divided by 0? • What is 2 divided by 2? Or in other words, how many twos are there in 2? • What is 2 divided by 1? How many ones are there in 2? • What is 2 divided by 1/2? How many halves are there in 2? How did you figure this out? • What is 2 divided by 1/4? How many quarters are there in 2? How did you figure this out? • What is 2 divided by 1/8? How many eighths are there in 2? How did you figure this out? • What would be the next question if we continue this pattern? What would be the answer? • What is happening to the numbers that we're dividing by? • What is happening to the answers to these division questions? • What do you think now about the question of 2 divided by 0?

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The interviews were tape-recorded and transcribed. Students were given positive feedback for their answers/contributions using phrases such as 'thank you' and 'that's very interesting', but they were not told whether they were right or wrong. Scaffolding was provided for grade 2 students for Task 3, as needed. A content analysis was then conducted of the transcribed interviews and field notes (Berg, 2004). The content analysis focused on the level and nature of students' mathematical thinking, using 4 levels of performance 'similar' to those found in the Ontario Mathematics curriculum documents (Ontario Ministry of Education, 1997).

Findings and Discussion

TASK 1. Unlike the study by Reid (1995), we did not find that grade 5 students performed less successfully than grade 2 students. The grade 5 students demonstrated the greatest variety of solutions. The grades 2 and 5 students were more likely than grade 8 students to rely on solutions that made use of graphical representations – all students who used such methods were successful in solving the problem.

TASK 2. Unlike the Kamii (1989) study, none of the grade 2 students were able to solve this problem. The grade 8 students performed less successfully than the grade 5 students. The grade 5 students were more likely to visualize the numbers in a column and apply the standard addition procedure. The grade 8 students used a greater variety of methods.

TASK 3. The level of mathematical engagement did not seem to vary among grades 2, 5 and 8 students. Although older students were typically more successful and needed less scaffolding in completing the questions involving operations with fractions, none of the students were able to engage with the task beyond the level of answering individual questions: They did not search for and did not identify patterns, and they did not see any connections to the final question of "What do you think now about the question of 2 divided by 0?"

In conclusion, unlike previous similar studies by Kamii (1989) and Reid (1995), older students in our study were generally more successful with problems involving number operations. At the same time, older students did not appear to have matured in their mathematical thinking, and could not see nor conjecture beyond small, individual tasks.

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EXAMINING PROSPECTIVE TEACHERS' GROWTH IN UNDERSTANDING SIMILARITY USING LESSON PLAN STUDY

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Focus of Study

The investigation of prospective teachers' (PT) knowledge of similarity was part of a 3-year study on PT change in understanding of the high school content, during Lesson Plan Study (LPS). This paper discusses the influences on PT preparation and the instructional teaching activities associated with the content of high school mathematics (Berenson, Cavey, Clark, & Staley, 2001).

Conceptual Framework

The growth of the PT knowledge of similarity was assessed within the Pirie-Kieren (1994) model of growth in understanding as adapted to teacher preparation by Berenson, Cavey, Clark and Staley (2001), while noting instances of folding back, and collecting (Pirie & Martin, 2000). The teacher preparation model is a framework for studying PT understanding of *what* and *how* to teach. The *what* of teaching includes representations, and the knowledge and understanding of the essential features of the concept. The *how* of teaching incorporates the ways of approaching the topic, the basic repertoire in teaching the topic, and the prospective teachers' knowledge about mathematics. Within the discussions and presentations of a lesson to introduce the topic of similarity, images and growth of PT knowledge of similarity were examined.

Methodology

Five participants (Alice, Anne, Ava, Rose, and Mary) were chosen for an analysis of their growth of understanding of similarity. They participated in the LPS on similarity, during their first of four mathematics education courses. The LPS contained four distinct stages occurring over a six-week period of time. The first stage was an individual interview in which a researcher got an initial understanding of what a PT knew about similarity and how they might teach it. The second stage was a group interview, the five participants together, were asked to construct a group presentation on similarity, and discuss their ideas. The third stage was the presentation of the group lesson to the methods class. In the last stage, the PT produced a reconstructed view of their individual lesson plans (Reference withheld).

Results

The PT images of *what* and *how* to teach similarity changed while involved in the LPS. At first, the PT related images of similarity with proportional sides and congruent angles in triangles. The group planning allowed the participants to make new images of *what* and *how* to teach by listening, discussing, and reflecting on their ideas. After the group planning stage, everyone collected and formalized Ava's images of the similarity postulates and Mary's images of modeling and indirect measurement. The following is a direct quote from Anne's final individual lesson plan:

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I will explain to the class that we have short cuts to find out if two triangles are similar or not. If it would take to long to see if all the angles are congruent or if the sides are proportional. What would we do if we were missing some information from the triangles? That is why we have conjectures.

Many of the PT made images of activities that allowed their students to derive mathematical similarity. Rose moved from a procedure-only approach of teaching to having an image of the importance of a conceptual activity.

Anne: What should be our hands-on activity?

Rose: I like her idea [*points to Ava, Anne nods*], I really do…

Anne: Yeah.

Alice: Me too.

Rose: Cause hers would take away my need to go, say okay well, "If you have two triangles and try and figure out if they are similar or not, and that actually teaches them.

While she did notice the importance of this activity, she never formalized this teaching strategy within her knowledge of how to teach and still believed that the best approach was a teacher-led lecture.

Towers (2001) stated that students' understanding is partly determined by teacher interventions. Interventions such as lesson-planning activities may allow PT to reflect on their own knowledge (Berenson, 2002; Davis, 2004; Davis & Staley, 2002; Staley & Davis, 2001). Through these reflections and discussions with colleagues and teachers, students could realize some of their own limitations which gave them the opportunity to improve upon their understandings.

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ON THE USES OF TREES AS REPRESENTATIONAL TOOLS IN ELEMENTARY PROBABILITY

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Difficulties of students in understanding and using tree diagrams are well-known of teachers, but researchers have extensively documented the problem. To cite one example, Green (1983) has provided extensive evidence that a great number of pupils who have been taught tree diagrams do not use them with success. Studies on use of tree diagrams in probability education have examined how to better teach these diagrams (Totohasina, 1994), and the effectiveness of trees as a pedagogical tool (Bernard, 2003; Dupuis & Rousset-Bert, 1996).

In this communication I carry out a discussion of the semantic transparency of tree diagrams by analyzing related literature and by adding to it observations from classroom experience.

What is So Sacred About Tree Diagrams?

Every representation has its history, and as with any idea, person or society, some may have a history of more conflicts, some may have had a brief existence, some may have been brought to existence rather recently and survived with no major difficulties due to favorable conditions and environment. It is not an accident that the Hindu-Arabic numeration system, for example, is such an effective notation: it has competed with dozens of other systems that were invented throughout a history spanning thousand of years and taking place in a vast geographical arena. But some notations do not have the "evolutionary pedigree" of the Hindu-Arabic numeration system (Cheng, 2003, p. 234).

What is the history of the tree diagrams? Hacking, who at the present is researching "the cultures and uses of tree-diagrams" (2005) but has not yet published on this topic, says that, while there are cognitive scientists who argue strongly that arranging hierarchies, taxonomies, or temporal processes in the form of tree-diagrams may be an innate tendency in humans, the use of tree diagrams seems very recent in history. At the moment, the earliest Western record of tree diagrams is from the 8th century; in the East, in Syriac, they go back to 5th century. Hacking doesn't have information on the earliest use of the diagrams to represent probabilities, though (Hacking, personal communication).

A history of the growth in the scope of use of tree diagrams may offer insight into what kinds of relations may be more naturally represented by trees (if we correspond "more natural" with that which was done earliest in history) and which ones took longer to be associated to trees. At least one study done in classroom seem to agree that some features may be more readily represented by tree diagrams than others: Pesci (1994) examined how tree graphs were used in eight third year Junior Middle School classes, and found that students immediately used tree graphs for solving a problem in which the various phases of the random experiment in the problem occurred successively in time. Tree graphs did not "come as naturally" in another problem proposed which had no explicit temporal sequence (p. 32).

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Alternatives to Tree Diagrams

Since representations naturally "compete for survival", falling in disuse when a better one is created, people should not refrain from designing alternative, or competitive, representations for the outcomes of multi-stage random experiments and their probabilities.

Konold has proposed a variation of the tree diagrams, which he has called *pipe diagrams*, with two major distinctions from the former: The branches, or pipes in his metaphor, are tagged with joint rather than conditional probabilities; and the values of these joint probabilities are graphically represented by pipe widths (1996).

Cheng (2003) designed "Probability Space" diagrams (PS diagrams) as an alternative to the current representations in probability theory, which he claims to combine the functions of Venn diagrams, tree diagrams, set theory notation, outcome tables and algebra.

Adding to Transparency

My experience suggests at least three reasons why tree diagrams do not have much semantic transparency:

Multiplicative Representation

One thing is to use tree diagrams to represent hierarchies, genealogies, or taxonomies, and quite another to represent the multiplicative principle. In a hierarchy tree or a genealogical tree, each item is represented in one and only one node. On the other hand, if you use a tree to denote possibilities, that changes. For example, in a family tree every node represents one and only one person: You can have kids make their family tree by pasting a picture of each relative on a branch. Now, take for example the task of making a tree for the different combinations of outfits we can have with two shirts, three skirts, and two pairs of sandals. Could we give students a picture or a template of the two different shirts, the three different skirts and the two different pairs of sandals and ask them to paste them on a tree? Would that amount of templates suffice? No, we would have to create, for each different shirt, templates for the 3 different skirts, etc. We would end up with two "copies" of the 3 different kinds of skirt and 6 copies of each kind of shoes (assuming we built the tree in the order "shirts, skirts, sandals"). This stems from the multiplicative principle, but students seem to have an awful hard time with that. The tree has "too many branches" for them. There inevitably is in every classroom of elementary education majors that I teach a great number of constructed trees that have too few branches – even after direct instruction on the issue.

Interestingly, the argument that tree diagrams "have too many branches" was heard from a student in a study by Figueiredo (2000), although she interpreted the assertion differently then I did above – she took that as evidence that we should avoid situations that would yield trees with a great number of ramifications.

We cannot rule out the possibility of these difficulties being due to an underlying weakness in combinatorial thinking, more than to the characteristics of the representation itself. Navarro-Pelayo, Batanero, & Godino (1996) have showed that combinatorial reasoning cannot be taken for granted, and have highlighted the importance of teaching that focuses on recursive thinking and systematic enumeration.

Where Is Each Outcome Represented?

Another problem I see my students (elementary education majors) with using trees to represent outcomes of multistage experiments is visualizing where each outcome is – in a path, not in a branch – especially not in the final branches, as students tend to think. I have found that asking student to highlight different paths in a tree and to write out the outcome that the path represents, as well as placing a tag next to every final branch on the tree where students are to list the outcome corresponding to following the path from the root of the tree to that final branch, has led to more successful uses of the trees.

Visualizing each Phase of the Experiment as a Level Across the Tree

If you make the tree from left to right, for example, each level or phase should be "read" vertically in columns. Vice-versa, a tree that is written vertically has a horizontal dimension. There is nothing particularly representing this on a tree, and students often have difficulty in adopting this convention. I have found that adding dashed lines to delineate the different levels of ramifications in a tree and labeling each of them, has made the representation more semantically transparent to students.

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COMMUNITY MATHEMATICS EDUCATION AS A FRAMEWORK FOR ELEMENTARY MATHEMATICS METHODS

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Purpose

In preparing elementary teachers to teach mathematics in urban schools, a tension exists between facilitating candidates' development of content and pedagogical content knowledge and helping candidates to be advocates for social justice in their schools and communities. Oakes, Franke, Quartz, & Rogers (2002) highlight this tension in their discussion of "high-quality urban teaching":

> An effective urban teacher cannot be skilled in the classroom but lack skills and commitment to equity, access, and democratic participation. Likewise, if one is to be a teacher, a deep caring and democratic commitment must be accompanied by highly developed subject matter and pedagogical skills. (Oakes et al., 2002, p. 229)

At the same time, those who are preparing to teach in high-poverty schools are often confronted by the lack of material resources- including curriculum materials, manipulatives, and technology- available for teaching mathematics, raising significant questions about equity and about the effectiveness of methods classes based on the use and availability of these resources.

The purpose of this paper is to describe a theoretical framework for an elementary mathematics methods course that seeks to address not only issues of equity, but also the tensions among content knowledge, pedagogical content knowledge, and a commitment to social justice. This framework builds on the work of Oakes et al. (2002) as well as a number of currently disconnected bodies of research highlighting the *human* and *social* resources available for the teaching of mathematics– including teacher candidates' mathematics identities, the mathematical thinking of K-5 students, and the mathematics "funds of knowledge" (Moll, 1992) and problem-solving opportunities available through parents, families, and communities. Furthermore, this framework provides an ecological perspective on mathematics instruction that prompts students, regardless of their teaching context, to explore and understand the mathematics resources available in any school or community and the value of connecting mathematics instruction to those resources.

Theoretical Framework

Teacher candidates often arrive in elementary methods classrooms aware of general feelings of like or dislike of mathematics, but with little idea how to use and build on specific mathematics pedagogies they have experienced or observed. Eliciting candidates' mathematics stories (LoPresto & Drake, 2004/2005) or autobiographies (Guillaume & Kirtman, 2005) can make these experiences more accessible as resources for teaching.

Similarly, teacher candidates are typically not aware of the variety of strategies and mathematical knowledge that students bring to the classroom. Explicit instruction and discussion about this variety and its usefulness as a pedagogical resource (e.g., Empson, 2002), as well as problem-solving interviews with children and the collection of student work around a common

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problem (e.g., Kazemi & Franke, 2004) illustrates the value of students' mathematical thinking as a resource for instruction.

Finally, teacher candidates are generally more aware of ways to use community and family resources in content areas such as literacy and social studies than in mathematics. Assignments asking students to design mathematics activities that could be completed during trips to various community sites, as well as interviews with parents eliciting the "funds of knowledge" (Moll, 1992) available within any group of parents help address this gap.

Pre-service teachers can better understand and utilize this framework not only by planning lessons that incorporate personal, student, and community resources, but also by observing examples of classroom practice and identifying the variety of ecological resources utilized in those lessons.

Implications and Future Research

To be clear, the four sources of human and social resources identified here are certainly not the only human and social resources available to urban teacher candidates. However, they were chosen because they are resources that are available in every context, whether urban or not, and because there is substantial research supporting the separate roles of each of these resources in elementary mathematics education. The focus on these four sources is not a replacement for discussion of curriculum, manipulatives, and technology as resources, but is instead presented as a complement to the use of these more traditional material resources.

Helping teacher candidates learn to utilize personal experiences, students' thinking, families, and communities as resources for mathematics instruction also highlights the importance, particularly in urban schools, of using mathematics and mathematical resources as *tools* for creating change within classrooms, schools, and communities. Much work remains to be done in the development and implementation of this framework. Nonetheless, the promise of this framework for teacher candidates, as well as the relationship of the framework to the PME-NA goal of more deeply understanding aspects of the teaching and learning of mathematics, is clear.

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BLOGGING PENTACUBES: ENHANCING CRITICAL READING AND WRITING SKILLS THROUGH COLLABORATIVE PROBLEM SOLVING WITH MATHEMATICS-BASED WEBLOGS

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Introduction

A number of educational studies have explored writing activities and their impact upon mathematical understanding of students. Researchers have investigated the use of journaling and "diaries" (Clarke, et al., 1993), submission of manuscripts to peer-reviewed journals (Brown, 1990), and mathematics "penpals" (Phillips & Crespo, 1996) to enhance student reflection on a variety of mathematical tasks. These and other studies suggest that mathematical writing has the potential to improve the communicative abilities and mathematical understandings of students.

Among the innovative approaches to writing in mathematics classroom, some, like diary-type journaling, require personal reflection on the mathematics at hand. Other approaches, such as peer-reviewed journals and math penpals, are based on critical peer interaction. Most of the epistemological work in the past two decades suggests that effective learning is inherently social (Ernest, 1998). Nevertheless, high-stakes testing pressures, saturated teaching schedules, and insufficient teacher training inhibit the realization of critical interactive writing in many classrooms. Despite NCTM recommendations, writing often takes a backseat to multiple choice assessments.

In this paper, we propose that weblogs (i.e. "blogs") are tools that may be used to address such limitations. Because blogs encourage academically-oriented interactions and mentoring among students and teachers *outside the traditional classroom*, use of the tools supports enhanced reading, writing, and mathematics skills without adding to time pressures that teachers typically face. In our study, pre-service teachers explore problems involving pentacubes as they compose initial drafts of problem solutions – then revise the drafts using a modified "writing workshop" model (Ray & Laminack, 2001) within an on-line blogging environment.

Framework for Understanding the Role of Blogging in the Study

In typical mathematics courses, students are assigned numerous homework problems of relatively low quality compared to those assigned to students in higher achieving countries. To help students develop conceptual understanding of mathematics rather than merely teaching them "how to obtain answers," we provide students with opportunities to submit multiple drafts of problem solutions. In peer-revision groups, students read and discuss mathematical work with others, provide encouragement and revision suggestions, and learn multiple solution strategies for a relatively small number of engaging problems. A model of this interaction is depicted in Figure 1.

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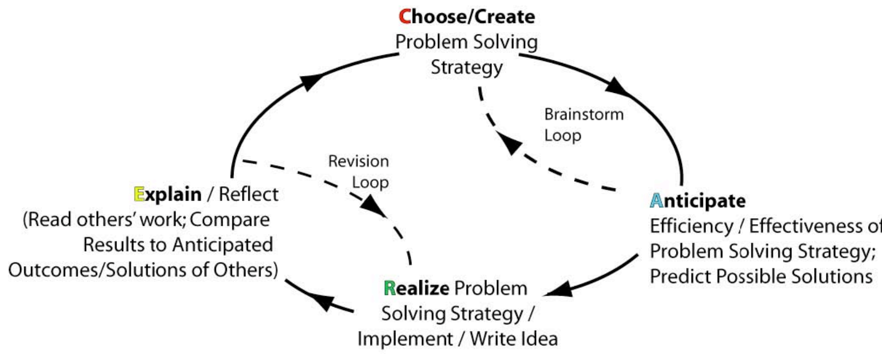

Figure 1: Collaborative Problem Solving Cycle (CARE)

In the cycle, brainstorming and revision mini-loops are implemented in traditional classroom and weblog settings. The model provides preservice teachers with the types of feedback and encouragement that supports quality teaching and learning.

Methodological Concerns

To informally measure the extent to which collaborative, "writing workshop"-style mathematics instruction impacts the reading and writing skills of preservice teachers, we collected and analyzed data from several sources: (1) Mathematics writing samples (draft and final responses to open-ended mathematics problems); (2) Short attitudinal questionnaires dealing with writing in mathematics classes; and (3) Writing samples taken from a class weblog. For all writing, structural and stylistic qualities of the samples were assessed using the "Holistic Rubric for the Ohio Graduation Test: Writing" (ODE,2005). The samples were analyzed both in terms of clarity and grammatical precision. Mathematics-specific work was assessed independently by two math educators to ensure inter-rater reliability.

Preliminary (Anticipated) Findings

Although the data analysis phase of the study is incomplete at the time of this writing, several interesting trends appear in the data. First and foremost, blogs appear to be useful tools for building student writing and reading skills in content areas outside of language arts. Assessment of student writing samples indicate a growth in students' writing during the study period -both stylistically and in terms of mathematical sophistication.

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A LOOK AT GENDER DIFFERENCES IN THIRD GRADERS' MATHEMATICAL PROBLEM SOLVING

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There has been much research done during the past twenty-five years on the differences in mathematics achievement of females and males (Hyde, Fennema & Lamon, 1990; Leder, 1992). Girls' achievement in mathematics, as measured by standardized tests, has generally been found to fall below that of boys by early adolescence, although there is evidence that this difference is becoming minimal (Cole, 1997; Hyde, Fennema & Lamon, 1990).

More recently, there has been a much smaller body of literature which presents compelling evidence that girls and boys are using different strategies to solve mathematical problems and computations in the early elementary grades (Carr & Davis, 2001; Fennema, Carpenter, Jacobs, Franke & Levi, 1998; Ricard, Paredes, Miller & Boerner, 1990; Zhang, Wilson & Manon, 1999). Additionally, Van den Heuvel-Panhuizen (2004) presents evidence that boys and girls perform differently depending on the mathematical nature of the tasks presented.

I will present the results from a study, the purpose of which is to identify and describe gender differences found in third grade students' work on a written mathematics assessment. In particular, comparisons will be made between the work of students in classrooms using *Investigations in Number, Data, and Space*, a reform-minded mathematics curriculum whose development has been funded by the National Science Foundation, and students in classrooms not using *Investigations*. A smaller sample of students will also be given task-based interviews.

The research questions that are the focus of this study are the following: (1) what strategies are third grade children using to solve specific mathematical tasks? (2) are there gender differences in the solutions or the strategies used or in the types of problems solved successfully? and (3) do these differences vary depending on whether or not the children are in *Investigations* classrooms?

This research for this study is being conducted as a piece of a much larger study. Diana Lambdin and Indiana University were awarded a subcontract to evaluate TERC's current revision of the *Investigations in Number, Data and Space* curriculum. The larger study looks students' mathematical growth and understanding of over a three-year period and compares the mathematical achievement of students in classrooms using the *Investigations* curriculum with students in classrooms that are not using *Investigations* ("comparison" classrooms).

The subjects for this study, who are participating in the larger *Investigations* evaluation study, are approximately 400 third grade students from a large, urban school district in the Midwestern United States. A smaller group of approximately 20-40 students will be asked to participate in task-based interviews.

The data is being collected using two methods: reviewing and analyzing the students' work on a written assessment instrument, and reviewing and analyzing children's responses to the interview questions and tasks. The written assessment was given to students in both the fall and the spring of the 2003-2004 school year, as a part of the larger *Investigations* study. The instrument includes six sections focusing on number and operations and two focusing on algebraic reasoning. The interview protocol will be used in the fall of 2005 with the smaller group of students to gather more in-depth data about the tasks and strategies used on the written assessment.

Coding for the data collected on the written assessment has been developed and defined by the I.U.-TERC research team as a part of the larger evaluation study. Students' answers have been coded as correct or incorrect, as well as by the types of strategies used to solve the problems. The results are being analyzed to look for growth in the students' achievement and differences in the work of children in *Investigations* and comparison classrooms. More detailed analyses will look at the strategies that girls and boys use to solve the problems and at the types of problems that girls and boys are able to answer successfully. Interview questions and tasks will be used with the smaller group of students to further investigate the similarities and differences found.

Preliminary results indicate that the gains made from fall to spring by *Investigations* classes were greater than those of the comparison classes. There also appear to be an indication of some differences between the percentages of boys and girls doing better on individual problems, depending on whether or not the children were in *Investigations* classrooms. Further analyses will be conducted throughout the spring and fall of 2005, and the results will be reported.

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TEACHER EDUCATORS' ACTIVITIES FOR TEACHING BASIC MATHEMATICAL CONCEPTS

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This study had two purposes: 1) to investigate pre-service elementary school teachers representational fluency and use of different kinds of manipulatives to solve problems in fraction equivalence, addition and subtraction and 2) to develop instructional activities that surround the Animated Fraction Addition and Subtraction Tool (A-FAST), that teacher educators can use to enable teachers have different kinds of representational and modeling experiences, so that they can use more modeling activities to introduce basic mathematical concepts. Seventy preservice elementary school teachers from three classes participated in this study. A pre-test was conducted which guided the researchers to design the instructional activities to be used with the preservice teachers. The methodology of "Multi-tiered Teaching Experiments" (Lesh & Kelly, 2000) with two levels (researchers and teacher educators, and preservice teachers) was used to conduct the study. At the end of the study the teachers' conceptual and pedagogical understanding was assessed using a post-test and lesson plans which they prepared in groups of three. Results from the study are presented below.

Participants

The participants for this study were seventy preservice elementary school teachers enrolled in a mathematics methods course at Indiana University School of education. These teachers have already completed their mathematics content courses offered by the mathematics department as a pre-requisite for this methods course. The assumption for this mathematics methods course is that after completing their mathematics content courses, the teachers have sufficient content knowledge to start thinking about how children learn mathematics and how to teach them for better understanding. The teachers have either done T104 (Teaching Elementary Mathematics via Problem Solving) or T101 (Mathematics for Elementary Teachers- I).

Method

At the beginning of the study the preservice teachers' were assessed using a paper-pencil pre-test and interviews regarding their knowledge of different representations for fractions, use of manipulative materials, and their reactions to common misconceptions of elementary school students. The results from these assessments led to the development of instructional activities around the software which were later used with the preservice teachers in two class periods of one hour and fifteen minutes each. After work with A-FAST and after the implementation of the instructional activities, the preservice teachers were asked to prepare lesson plans on "how to teach fractions" and write individual reflections on the thinking process they went through while working with the software and instructional activities, and while writing the lesson plans. A post-test was implemented with the same purpose as the pre-test.

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Analysis I

Pre-tests and interview questionnaires were used to collect the data for a preliminary analysis. Data analysis based on the preservice teachers' answers is presented: (1) Preservice teachers have a poor understanding of fractions. Concepts such as whole, part, whole-part, and part-part are not clear for them; (2) Preservice teachers aren't familiar with or able to use other manipulatives (square, Cuisenaire rods, or counters) except pies- and not in its totality; (3) Preservice teachers understand the importance of manipulatives in the classroom; (4) Preservice teachers don't feel prepared to teach fractions at schools-pedagogical ideas are their more important concerns; (5) Preservice teachers have a brief idea of how to add fractions where one of the denominators is a multiple of the other (e.g. 4 and 12) using manipulatives (pies most of the time) but they are not able to show this when the denominators aren't multiples (e.g. 4 and 7); (6) Preservice teachers ignore the concept of "common unit" while thinking about equivalent fractions and comparison of fractions.

Analysis II

The lesson plans, reflections, and post-tests were analyzed. The analysis is presented next:
(1) Preservice teachers support the view that models and manipulatives help in learning. Most of the teachers wished that if they had been exposed to fractions using models during their own elementary education or even during their more advance mathematics classes, they would have had a better understanding of fractions today; (2) Preservice teachers claim that their own knowledge of fractions has increased after they used the software and the instructional activities, and with this knowledge they feel better equipped to answer students' misconceptions about fractions; (3) Preservice teachers claim that their pedagogical knowledge of fractions has increased. The instructional activities put the preservice teachers in some real classroom situation that helped them to think about teaching fractions, and that helped them to come up with good teaching strategies; (4) Preservice teachers found the linear model most difficult to understand. The area models were the easiest for them to understand followed by the set model. This difficulty can be attributed to what research already showed- it is difficult for students to work with discrete objects (set model) than the region models like circles and squares (Post, 1981). Teachers' difficulties with the linear model can be explained by the design structure of the software and the lack of real manipulatives for simulating the actions of the software.

Results

Assertion 1: The instructional activities designed around the software influenced preservice teachers' beliefs about the use of different kinds of models and manipulatives for teaching fractions.

Assertion 2: The instructional activities designed around the software did have some positive influence on preservice teachers' representational fluency and fraction knowledge but not to the desired extent. *Assertion 3:* Although the preservice teachers claimed that will use different kinds of models and manipulatives, their own knowledge of using appropriate models for fractions and their representational fluency is still limited.

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EXAMINING PRESERVICE TEACHERS' KNOWLEDGE OF ALGEBRA

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Many researchers agree that subject matter knowledge is an important factor affecting classroom instruction. Ball (1990) analyzed preservice teachers' understandings of mathematics and concluded that subject matter knowledge should be a central focus of teacher education in order to teach mathematics effectively. However, understanding of subject matter is not sufficient condition to teach. Teachers should possess a representational repertoire, because teachers need to generate representations to facilitate students' learning (Wilson, Shulman & Richert, 1987). Their conceptions of knowledge may limit their ability to present subject matter in appropriate ways or give helpful explanations (Even & Tirosh, 1995).

In this study, false algebraic statements, taken from the study of Marquis (1988), were used. Our goal was to investigate whether or not preservice teachers would be able to determine why the statements were false. In addition, the researchers wanted to analyze preservice teachers' alternative solutions for common algebra mistakes.

Theoretical Framework

The data in this study was analyzed using the framework described in the study of Ball (1990). She stated that understanding mathematics for teaching requires both knowledge of mathematics (i.e., understanding of principles and meaning of underlying mathematical procedures) and knowledge about mathematics (i.e., understanding of the nature of knowledge in the discipline: where it comes from, how it changes, and how truth is established; the relative centrality of different ideas as well as what it is conventional or socially agreed upon in mathematics versus what is necessary or logical, p.6).

Methods

In this study, three audiotaped interviews were conducted with three preservice teachers. Harry was a sophomore and a middle school preservice teacher. Both Ashley and Jane were seniors and secondary preservice teachers. Each participant was given three algebraic statements with common mistakes and asked how they would respond to students who were making the mistakes.

Task 1: $\sqrt{x^2+y^2} = x+y$. Task 2: If $2(2-z) < 12$ then $z < -4$ Task 3: $\frac{xa+xb}{x+xd} = \frac{a+b}{d}$

Data Sources

When they were given Task 1, Harry easily showed that the statement was false. He first took square of both sides. Then, he said $(x + y)(x + y)$ was equal to $x^2 + 2xy + y^2$. However, he failed to explain why there should be 2xy. Ashley did not know how to solve the equation at all. Jane said students could not multiply $(x + y)$ and $(x + y)$ and explained why students made this mistake. Besides this approach she also suggested that students did not know the difference between the expressions $\sqrt{x^2}\ \sqrt{y^2}$, $\sqrt{x^2}+\sqrt{y^2}$, and $\sqrt{x^2+y^2}$ and demonstrated the difference among them.

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The participants solved the inequality for z correctly. However, they could not explain why they needed to reverse the inequality symbol, when both sides were divided by -2. Jane said, "Once, they divide by negative 2, you have to flip the inequality. It is probably because the reason I almost forgot. I myself do not have good understanding of why you have to flip it… You are hanging on the fact that you have to remember."

When Harry and Ashley were given Task 3, they said that the expression in parenthesis should be 1 + d, when x was factored out. However, they could not explain why it should be 1 + d. They suggested that writing one next to x might help students factor out the algebraic expression correctly. The following is an excerpt from Harry's interview.

Harry: You just have to put something in there. In this case is one… you can put the one in front of the x. See actually there is one in there.

After solving Task 3 correctly, Jane demonstrated an example that would help students understand why they need to place 1 when x is factored out. She said, "When students factor out x, they forgot they are supposed to leave 1 in there. If students did this, I would show them. When you factor out something, you should be able to reverse and get to the same thing. So if we take x+xd and x (d) and you reverse it. This is not the same thing (she wrote x + xd = x (1 + d) and x(d) = xd)."

Results

All the participants solved Task 2 correctly. However, they could not explain why the rule worked. For Task 3, Jane was the one participant who could provide an alternative approach by reversing the algebraic operation. She multiplied (1 + d) and (d) by x in order to show that they were not the same. She could not make any connection with other mathematical ideas (or properties). Ashley could not solve Task 1 at all. Harry solved it. However, he could not explain why the mistake occurred. After solving Task 1, Jane demonstrated three possible factors behind the mistake and showed the connections among them.

Conclusion

The results of this study revealed that the participants had difficulty solving the tasks as well as providing alternative solutions for common algebraic mistakes, because they had limited knowledge of mathematics. Their explanations were mainly based on algebraic procedures and arbitrary facts. They did not know the underlying meaning of algebraic procedures they used. They need to know more than describing steps and procedures. They should be able to give meaningful explanations and develop appropriate strategies for common algebraic mistakes.

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HOW ONE-TO-ONE TUTORING EFFECTS PROSPECTIVE MATHEMATICS TEACHERS' REFLECTIONS ON LEARNING MATHEMATICS

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While teachers do not necessarily have the time to work for extended periods one-on-one with students, pre-service programs can include these opportunities as part of the field-work. It has been shown that changes in teacher beliefs about learning may occur when given the opportunity to engage in one-on-one experiences (Pinnell, Lyons, Deford, Bryk, & Seltzer, 1994). Gipe & Richards (1992) found that reflective practices in pre-service education students could lead to improved performance in the field. With one-on-one teaching or learning situations considered being an effective method of instruction (e.g., Cohen, Kulik, & Kulik, 1982) tutoring of middle or high school students and their tutors (prospective mathematics teachers) could both benefit from the experience. Not only is the middle or high school student afforded the opportunity to get assistance with the mathematics s/he may be having trouble with but also the prospective teacher is provided an opportunity to develop the questioning and teaching skills needed in a classroom. Hedrick, McGee, & Mittag (1999) found that through tutoring sessions pre-service teachers experienced growth in understanding about the instructional cycle that included a consideration of cognitive, emotional and environmental factors.

The purpose of this study was to examine the qualitative nature of prospective mathematics teachers' analysis of and reflection on middle or high school students' mathematical thinking in a tutoring context. This research helps in the understanding of the process of how one comes to be a mathematics teacher and the effects of a specific "field-experience", namely, one-on-one tutoring, on the maturity of prospective mathematics teachers pedagogical and pedagogical content knowledge.

Methods

The mathematics education program at a large research institution now includes a year of tutoring as part of the field component for both middle grades and secondary mathematics education majors. This occurs in the junior year of preparation. Since fall 2001 a tutoring center for middle and high school students has been provided on the university campus as a service to the community. After each tutoring session prospective teachers are required to complete a reflections log to document what assistance was provided, what is recommended for subsequent visits, and their reflections on the mathematical thinking of the middle/high school tutee. Reflection logs from thirty-five tutors were collected. Interviews (either email or face-to-face) with ten prospective teachers constituted a second data source. Questions asked in the interview included "describe your typical tutoring session", "describe your interpretations of student difficulties", and "describe how you approach planning for teaching" (for those who were doing a field experience). The reflection logs and follow-up interviews constitute the data analyzed.

Data for two years (2002-03 and 03-04) were analyzed for evidence of the types of tutors' instructional strategies during the sessions; ways tutors overcame students' difficulties in understanding mathematics; and what was learned from the experience. Data were coded and

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categories for each area were developed. Using analytic induction assertions were made and data re-examined to determine if there was any evidence that would refute the assertion (Erickson, 1986).

Results

One-on-one tutoring experiences provided prospective mathematics teachers with an opportunity to work more closely with students who were not successful with mathematics and to develop a set of skills that could be used with this group of students.

As reported to one of the researchers during class, most of the prospective teachers had been in advanced or gifted mathematics classes in high school. Therefore, their perspective on teaching mathematics had been shaped by these classes. For the most part in their classes students were motivated to be successful regardless of the teaching strategies used by the teacher. Thus, as a prospective teacher each was challenged by interactions with middle or high school students who were not necessarily successful in mathematics or who were struggling. What might seem like a straightforward procedure or concept for the prospective teacher was a complicated and/or meaningless process or idea for the student. This was compounded by the less than enthusiastic attitudes held by the students for learning mathematics.

In reflecting on their tutoring sessions prospective teachers indicated areas they felt they struggled in, such as, finding appropriate problems, finding additional ways to explain a procedure (different from the book because it might not be helpful), and finding fruitful ways to ask questions rather than telling a student how to do something. These became critical teaching moments in the methods courses so that multiple strategies for dealing with these situations were explored.

Conclusions

The inclusion of year-long tutoring experience appears to have provided an environment for the prospective mathematics teachers through which they are able to develop skills for working with traditionally unsuccessful students in mathematics. Given that most of them have been successful and have a minimal awareness of difficulties someone might have in learning mathematics, the one-on-one opportunity provides real-life examples to situate what is being learned in the methods courses.

Further examination of the reflections shows how over the course of a year the nature of the reflections changes. Additional analyses revealed through the reflections the progression of the prospective teacher on the development of a more conceptual approach to teaching mathematics.

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DEVELOPING THE CRITICAL LENSES NECESSARY TO BECOME A LESSON STUDY COMMUNITY

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Conceptual Framework

Becoming a successful Lesson Study community requires developing three critical lenses: the researcher lens which requires teachers to design classroom experiences that explore questions they have about their practice; the curriculum developer lens which requires concern about how to organize, sequence and connect children's learning experiences; and, the student lens which requires the teacher to examine all aspects of the lesson through the eyes of the student. This is not a simple task. There are significant differences in the curricula and cultures of Japanese and U.S. teachers. This study raised the following question. Would U.S. teachers new to the Lesson Study process develop the critical lenses necessary to become a highly functioning Lesson Study group?

The School System

The CSD school system is a small urban system in the south with six elementary schools. Thirty-eight percent of the CSD students are on free or reduced lunch, 53% are African-American or other minority, and 47% are Caucasian. According to the mathematics coordinator, a teacher-directed model continues to be the primary mode of instruction in mathematics.

Participants

In an effort to find a self-sustaining model for change the system implemented a Lesson Study project during the 03-04 school year. Third grade teachers in the district were selected to participate. With only two teachers at that grade who were new to the system but not new to teaching, and a strong group of experienced teachers, third grade was perceived as a relatively stable grade level. Participation was voluntary and eight of the ten teachers opted to join the project, representing five of the six elementary schools. There were two African-American females, one Asian male, and five Caucasian females. Teachers ranged from 3 to 15 years experience in the elementary classroom.

Methods

In this study I looked for evidence that teachers were developing the three critical lenses necessary to become a functioning Lesson Study community. Videotapes were made of each planning and debriefing session. For this research, tapes from the initial and ending planning/debriefing sessions were selected for transcription. In consultation with the mathematics coordinator for the school system, a rubric was developed prior to coding. The rubric was used to describe possible language patterns or dialogue that we anticipated would be indicators of evidence of each of the three critical lenses. Each transcript was coded using R for the researcher lens, S for the student lens and CD for the curriculum developer lens, e.g., if teachers talked about what students said or did, we would code the event with an S. If teachers talked about how the mathematics in the lesson was presented or how it developed, we would

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code the event CD, or if the teachers talked about questions they had about how to best teach a particular topic we would code the event with an R. The clips were then sorted to determine if the sum of the clips in each category actually reflected the essence of the category. Finally, clips within each category were compared for qualitative differences in the conversation.

Results

Because of space limitations, I will give only two examples from the debriefing sessions; one at the beginning of the year and one at the end of the year. The clips are typical of teacher conversation with no probing or prompting by the facilitators.

Clip #1 ***[September- Third grade lesson on addition and subtraction word problems]***

T1: On the first one (problem) they didn't use the manipulatives at all . . .one strategy was used until you came over and asked "can you think of another way of doing this?" Then they suggested. . .let's draw a picture to answer the question and that's what they did. [S]

T2: They didn't really draw much of a picture [S]

T3: They used little models, little things to represent their thinking [S]

T1: and, each time they spoke they were very respectful of each other. They didn't say no, that's wrong. They were very calm in the group. [S]

Clip #2 ***[April – Lesson on identifying unshaded regions as fractional parts of a square]***

T1: I have down the word denominator in big letters because I really think that the concept of denominator is just hard. [CD] It seemed like my group could divide anything into one-eighth or one-sixth. They can do the ones but when you show them a larger region . . I think that was the big challenge. When they see a large region in their minds they see one-fourth, one-fourth, one-fourth instead of three-fourths. [S]

T2: I wonder about shading, if there was something about how she shaded. [CD]

T3: That makes it more challenging, not being shaded because they are use to that. [S]

T1: I guess it was two things (that made it hard), not shading it and giving them a larger region, so maybe one or the other would have been good to do. [CD]

Discussion

In the September clip, less than 10% of the comments were made from the curriculum developer's lens. Comments about the students [S] were primarily about what they did or said and about their behavior. In the second clip, the teachers are still talking about the students, but they are trying to take the perspective of the student (what was hard for them and what they were familiar with) and they were thinking about the organization of the learning [CD] indicating first that the concept of denominator is hard (although they don't indicate why) and that the instruction decision to give them a large region and one that was not shaded made the problem more difficult. Interesting, neither transcript provided evidence of the researcher lens.

There is no doubt that differences in the curricula and cultures of Japanese and U.S. teachers impacts successful adaptation of Lesson Study with U.S. teachers. Given the possible benefit for teacher learning through Lesson Study by gaining deeper insights into seeing the mathematics through the eyes of the student and thinking about the organization and presentation of the curriculum, further research in the area is clearly warranted

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A DISCURSIVE FRAMEWORK FOR EXAMINING THE POSITIONING OF A LEARNER IN A MATHEMATICS TEXTBOOK

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Personal Positioning in Relation to Mathematics

In *TMM*, first person pronouns[1] are entirely absent. Such an absence obscures the presence of human beings in a text. The second person pronoun ***you*** appears 263 times in *TMM*. Two forms are especially relevant: 1) ***you* + a verb** (165 times); and 2) **an inanimate object + an animate verb + *you* (as direct object**) (37 times). The most pervasive form, ***you* + a verb**, includes such phrases as *you find, you know,* and *you think*. In these statements, the authors tell the readers about themselves, defining and controlling the 'common knowledge' (Edwards & Mercer, 1987), and thus use such control to point out the mathematics they hope (or assume) the students are constructing. In *TMM*, the other common *you*-construction (**an inanimate object + an animate verb + *you* (as direct object**)) provides a striking example of obscured personal agency*:* inanimate objects perform activities that are typically associated with people – e.g., "The graph shows you…". In reality graphs cannot "show" you anything.

The modality of a text also points to the text's construction of the role of humans in relation to mathematics. The modality of the text includes "indications of the degree of likelihood, probability, weight or authority the speaker attaches to an utterance" (Hodge & Kress, 1993, p. 9). One set of modal forms, hedges*,* describe words that point at uncertainty. The most common hedge in *TMM* is *about* (12 instances), followed by *might* (7 instances) and *may* (5 instances). Modality also appears in the authors' verb choice: *would* (55 times), *can* and *will* (40 times each), *could* (13 times), and *should* (11 times). The frequency of these different modal verbs indicate an amplified voice of certainty because the verbs that express stronger conviction (*would, can*, and *will*) are much more common than those that communicate weaker conviction (*could* and *should*). The strong modal verbs, coupled with the lack of hedging, suggest that mathematical knowledge ought to be expressed with certainty, which could suggest that the knowledge is not contingent upon human relations.

Student Positioning in Relation to Peers and the Teacher

Pictures alongside verbal text can impact the reader's experience of the text. In *TMM*, for example, there are 24 pictorial images. Of these, only 7 are photographs. The textbook's preference for drawings, which are more generic than photographs, mirrors its linguistic obfuscation of particular people. Furthermore, only a quarter of the images show people, and among these we find only one image of a person doing mathematics – a drawing of a hand conducting a mathematical investigation. The disembodied, generic hand parallels the lost face of the mathematician in agency-masking sentences such as the ones discussed above.

Morgan (1996) asserts that imperatives (or commands) tacitly mark the reader as a capable member of the mathematics community. However, we suggest that such positioning is not clear

[1] Tools and concepts from discourse analysis are underlined here.

from the mere presence of imperatives. Rotman (1988) distinguishes between what he calls inclusive imperatives (e.g. *describe, explain, prove*), which ask the reader to be a thinker, and what he calls exclusive imperatives (e.g. *write, calculate, copy*), which ask the reader to be a scribbler. The thinker imperatives construct a reader whose actions are included in a community of people doing mathematics, whereas the scribbler imperatives construct one whose actions can be excluded from such a community. The student who 'scribbles' can work independent from other people (including her teacher and peers).

Student Positioning in Relation to the World

Most of the prompts in the analyzed textbook are referred to as 'real life', 'applications', and 'connections' (connections between mathematics and real life). Though the textbook consistently places its mathematics in 'real' <u>contexts</u> (with few exceptions), linguistic and other clues point to an inconsequential relationship between the student and her world. When we compare the instances of <u>low modality</u> (expressing low levels of certainty) with those of <u>high modality</u>, we begin to see what experiences the text foregrounds. The text refers with uncertainty to the student's experiences outside the classroom using hedging words like *probably* or *might*. However, the text expresses certainty about the student's abstract mathematical experiences, as in "In your earlier work, you saw that …" (p. 9). Because the authors know what the curriculum offers, they work under the assumption that the student has learned particular mathematical ideas. Yet, the authors cannot really know what their readers have seen. Students might be led to think that their everyday experiences matter less than their mathematical experiences?

Revisioning Mathematics Text

We were surprised by the results of our analysis of this textbook that we both appreciate for its constructivist approach to mathematics. The language forms and images suggest a different view of mathematics, one in which the student works independently from a pre-existent mathematics. How then does such a text become a tool for constructivist-informed education?

We see room for mathematics textbook writers to change the *form* of their writing to recognize the connections between readers and their world, which includes the people around them. Until such textbooks appear, we note that any textbook is mediated through a person (the teacher) in a conversation amongst many persons (students). In such a community, there is room to draw awareness to relationships between particular persons (historical or modern, professional or novice mathematicians) and the apparently abstract, static discipline of mathematics.

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LINGUISTIC INVENTION IN MATHEMATICAL COMMUNICATION AMONG PRACTICING ELEMENTARY SCHOOL TEACHERS

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Introduction

For many students, learning mathematics involves learning to communicate using conventional mathematics terminology (Moschkovich, 2003). However, as discussed by Brown (2001), students of mathematics should not only learn conventional methods of communication, but should also be able to verbally describe mathematical situations in relation to themselves. The practice of describing a mathematical situation in relation to oneself may be viewed as linguistic invention (Brown, 2001, p. 76). While conventional language identifies abstract mathematical concepts, linguistic invention draws on personal experience to provide those concepts with richer meaning.

For example, a student solving a story problem concerning a generic reservoir of water changes the context to that of a bathtub because she is more familiar with filling up bathtubs than filling up reservoirs. Meanwhile, another teacher communicates the conventionally stated idea of "the rate is increasing" by specifically saying that "the reservoir is filling up faster." In both cases, students of mathematics use linguistic invention, language that relates concepts to their personal experience, to describe unfamiliar or abstract mathematical concepts. The purpose of this study is to describe how practicing elementary school teachers optimally utilize linguistic invention to communicate mathematical ideas when working together to complete a mathematical task.

Method and Analysis

Twenty-four practicing elementary school teachers from one school district participated in a calculus course for elementary teachers as part of a larger research project in professional development. Participants worked together to determine how the volume of water in a reservoir changes when supplied with only a graph showing the rate of water entering the reservoir versus time (Connally et al., 1998, p 53).

Researcher field notes, participants' class notes and submitted solutions to the problem supported transcription and coding of approximately four hours of videotape showing classroom discourse related to the "Reservoir Task." Discourse turns taken by the participants were coded according to the types of language used in the turn and the effect of the turn on the conversation as observed in the reactions of the participants. Language codes ranged from conventional language ("the rate is increasing"), to uses of units and numerical values ("from one gallon a minute to two gallons a minute . . ."), to references to generic reservoirs ("the water's coming in faster"), to very personal references ("this is where you're actually turning the knob [on a bathtub faucet]"). Effect codes described whether each turn was (a) immediately verified, (b) skeptically questioned and/or clarified, (c) acknowledged in non-mathematical manners, (d) acknowledged with frustration, or (e) not acknowledged at all by the other participants.

Student work and transcripts were coded to identify personal linguistic invention that comprised five specific interpretations of the task. Interpretations that received the greatest

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amount of verification from the participants were viewed as optimal uses of linguistic invention. The developmental stages and public presentation of the various interpretations were studied to determine differences in language and effect code patterns between optimal and other instances of linguistic invention. Differences in the development and presentation of optimal and other instances of personal linguistic invention were analyzed to develop a theory of how linguistic invention was used in conjunction with other language forms to successfully create and communicate meaning for mathematical concepts.

Findings and Implications

Data analysis showed that optimal uses of linguistic invention involved five major elements. First, the linguistic invention was developed verbally by more than one participant. Second, overarching principles and assumptions which guided the interpretation of the graph given in the Reservoir Task were verbally identified and reiterated often. Third, during the development of linguistic invention, the personal situation was modified to fit the mathematical concepts represented on the graph in the Reservoir Task. Fourth, the linguistic invention was presented in conjunction with conventional language, which explicitly identified individual mathematical concepts common to both the given Reservoir Task and the personal situation. Finally, the participants seemed make the most progress with a simplest-case scenario involving a bathtub that involved the essential mathematical concepts present in the Reservoir graph yet eliminated the complicated details inherent in a reservoir situation.

In order to connect personal experience to mathematical situations in meaningful ways, students must identify key mathematical concepts common to both situations. A possible method for ensuring identification of key concepts is through the use of conventional language. As students describe what the given mathematics (in this study, a graph of rate versus time) and their personal experience (for our participants, filling up a bathtub) have in common, they further increase their understanding of the mathematical concepts involved (such as increasing, constant, and decreasing rate). If students are not familiar with conventional terms, their attempts to describe these mathematical concepts using personal language may provide a context for meaningful introduction of more conventional language.

Furthermore, teachers can use this information to monitor their students' linguistic invention. For example, if one is interpreting a graph of rate, linguistic invention that makes reference to volume rather than rate may not be as effective as a situation where rate is more prominent. Teachers can guide their students in identifying and differentiating between essential concepts and entertaining details in order to use linguistic invention more effectively in the classroom.

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THE DEVELOPMENT OF COMBINATORIAL THINKING IN UNDERGRADUATE STUDENTS

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Background and Objectives

This study examines the development of combinatorial reasoning and understanding among undergraduate liberal arts and social sciences students. My objective is to analyze students' difficulties in combinatorics and get a better insight into the development of understanding of elementary counting problems. Combinatorics is the study of ways to list and arrange elements of discrete sets according to specified rules (Cameron, 1994). Combinatorics is foundational in computer sciences, and it is used in many other fields of science such as chemistry and physics. In NCTM 2000 Principles and Standards, we see an explicit attention paid to the teaching and learning of combinatorics. Combinatorics is a growing field of mathematics and further attention has been paid to it in school curriculum in recent years. Unfortunately, mathematics education research has not yet caught up to this trend, and not much research has been done in this field.

On of the tasks in this study is students' generation of examples for particular combinatorial structures. These examples help us get a better understanding of students' difficulties with particular of the structures in elementary enumeration problems. Teachers use examples often in the mathematics classrooms to help students understand and explore different topics. When learners are invited by the teacher to construct their own examples, it helps them to think about the topic in a different way. According to Watson and Mason (2004) learner-generated examples promote reflection on the concept, encourage creative thought, and help learner to reason and communicate their understanding in more depth.

Theoretical Framework

Classification of problems and students' difficulties in each class of problems has been studied in detail for elementary arithmetic, and has been proven to be very useful for teaching and learning (Fennema et al., 1992). Batanero et al. (1997) and Rosen (2000) have presented a categorization of basic enumeration problems. I have modified their classification to design a new classification, which is more suitable for the purpose of this research:

1. Arrangement: Order of the elements within the configuration matters.
   - Unlimited repetition allowed: 'How many 4 digit passwords can you make?'
   - No repetition allowed (permutation): 'How many ways can 5 people sit in a row?'
   - Limited repetition allowed: 'How many 3-letter words can one make with the letters FINITELY?' (Note that there is no repetition except for the letter 'I', of which we have two.)
2. Selection: Selection of elements from a set such that the order of the elements within the configuration (selection) does not matter.
   - Unlimited repetition allowed: 'How many ways can you choose 3 roses if there are red and white roses available?'
   - No repetition allowed (combination): 'In how many ways can Kim choose 3 of her friends to invite for dinner if she has 10 friends?'

- Limited repetition allowed: 'How many different fruit baskets can one make with at least one fruit in it from 5 oranges, 3 peaches, 10 bananas?'

In this research, I have designed a variety of tasks, some of which requires the generation of examples, from different types of combinatorial structures to examine students' difficulties with different types of combinatorial configurations.

Modes of Enquiry and Data Source

The participants in this study are liberal arts students enrolled in an undergraduate mathematics course. The data is gathered from their written responses and a set of clinical interviews. Participants were presented with a variety of tasks in which they were asked (1) to solve given combinatorial problems (2) to generate an example of a combinatorial problem given the presented solution. The following is the example of (2).

Write three scenarios that can be modeled and solved using each of the following calculation:

a. 30*29*28*27*26
b. 30*29*28*27*26/5!
c. 30*29*28*27*26/4!

The goal of such questions is to invite students to think about two categories of enumeration problems (permutation in part a and combination in part b) and to explore their ability to generalize their examples to include a mixed configuration (as in part c). From the preliminary data, it appeared that students had most difficulty with tasks similar to the questions of part c. The data revealed students' difficulties strongly depended on the combinatorial structure of the task, particularly with the mixed configurations.

Conclusion

This study investigates students' development of combinatorial reasoning, and examines their difficulties in solving different types of combinatorial problems. I will suggest possible ways to help learners acquire understanding, by describing the source of their difficulties in this topic. Furthermore, I will categorize different enumeration problems. The systematic classification of different enumeration problems will help teachers to develop "a taxonomy of problem types" (Fennema et al., 1992). The distinction between the different types of problems will also reveal students' difficulties with particular problem types and assists teachers to help students overcome the obstacles particular to a certain problem type.

Relationship of the Paper with the Goals of PME-NA

Combinatorics is being taught in high schools and universities in North America, yet there are only a few studies dedicated to this growing field of mathematics. This study allows us to get a better understanding of students' difficulties with elementary enumeration problems, which are the basis of combinatorics. This research will also have pedagogical implication for teachers, in providing them with a deeper insight into challenges that their students face in this field and guiding how to design their teaching accordingly.

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