# Giuseppe Savaré - CURRICULUM

#### PERSONAL INFORMATION

Savaré Giuseppe ORCID: 0000-0002-0104-4158 MathSciNet: 336952

ResearcherID: H-3651-2014 Scopus: 6602154611 Google Scholar: w7qH1YYAAAAJ

Nationality: Italian Born in Pavia, 25/08/1966

**EDUCATION** 

1984–1988: Laurea (cum laude) in Mathematics, University of Pavia, Italy

#### **CURRENT POSITION**

2020-: Full Professor, Mathematical Analysis. Department of Decision Sciences, Bocconi University

#### **PREVIOUS POSITIONS**

2000–2020: Full Professor, Mathematical Analysis Department of Mathematics, University of Pavia

1998–2000: Associate professor, University of Pavia

1990-1998: Researcher, IMATI-CNR, Pavia

### FELLOWSHIPS AND AWARDS

2019-2023: Hans Fischer senior fellow, Institute for Advanced Study, Technical University of Munich

2019: Plenary Speaker, XXI Congresso Unione Matematica Italiana

2016: Invited Speaker, 7th European Congress of Mathematics, Berlin: section "Analysis and PDEs"

2015: John von Neumann Visiting Professor, Technical University of Munich

2014: the results on Gradient Flows has been presented in the *Exposé Bourbaki 1065: Flots de gradient dans les espaces métriques et leurs applications d'aprè Ambrosio-Gigli-Savaré*, Asterisque 361.

2012: The paper Chemical reactions as  $\Gamma$ -limit of diffusion SIAM Rev. 54 (2012) (with M. Peletier,

M. Veneroni) has been published as SIAM Review's SIGEST award.

2011: Ennio De Giorgi Prize, awarded by the Italian Mathematical Union

1994: Gioachino Japichino Prize, awarded by the Accademia Nazionale dei Lincei

#### SUPERVISION OF GRADUATE STUDENTS

Bocconi University:

current: Alessandro Pinzi

IAS-TUM, Munich:

2022: Giacomo Sodini (co-advised with M. Fornasier)

*University of Pavia*: 2019: Nicolò De Ponti 2017: Luca Minotti

2015: Giovanni Bonaschi (co-advised with M. Peletier, Eindhoven)

2014: Dario Mazzoleni (co-advised with A. Pratelli, Erlangen)

2011: Luca Natile 2007: Marco Veneroni 2006: Stefano Lisini 2005: Riccarda Rossi

1998: Simona Sanfelici (co-advised with P. Colli Franzone)

I also mention the supervision of the Master thesis (subsequently published in international mathematical journals) of Federico Bassetti, Laura Spinolo, Sara Daneri, Giulia Luise

# POSTDOCTORAL FELLOWS

2021-2023: Luca Tamanini 2017-2019: Giulia Cavagnari 2016-2017: Dario Mazzoleni

2015-2016: Carlo Orrieri, Matteo Muratori

2009-2011: Edoardo Mainini 2007-2008: Daniel Matthes 2006-2008: Antonio Marigonda

### **TEACHING ACTIVITIES**

University of Pavia:

1998-2020: courses of *Mathematical Methods* (Engineering Faculty) and *Mathematical Analysis* (degree in Mathematics), typically two main courses every year,~140 hours/year

2000-2020: several Ph.D. courses (PDEs, Semigroups, Calculus of Variations, Optimal transport) *Bocconi University*:

2020-2022: Mathematical Analysis (Bachelor), Introduction to Real Analysis (Ph.D.)

Several invited courses to international advanced schools (as CIME, HIM, SNS, EVEQ)

### **ORGANISATION OF SCIENTIFIC MEETINGS**

2024: Workshop on *Mathematics for Artificial Intelligence and Machine Learning*, Università Bocconi, Milano (Co-organizers: B. Morini, E. De Vito, S. Pieraccini)

2023: Workshop on *Variational and geometric structures for evolution*, CIRM, Trento (Co-organizers: D. Knees, R. Rossi, M. Thomas)

2023: Workshop on *Workshop on Optimal Transport, Mean-Field Models, and Machine Learning,* IAS-TUM, Munich (Co-organizers: M. Burger, M. Fornasier, G. Peyré)

2022, 2018, 2016, 2014, 2012, 2010, 2008: *Workshops on Optimal Transportation and Applications*, Pisa (Co-organizers: L. Ambrosio, G. Buttazzo, N. Gigli)

2022: *Contemporary Trends in Kinetic Theory and PDEs*, Pavia (Co-organizers: J.A. Carrillo, A. Pulvirenti, M. Zanella)

2018: School on *Optimal transport: numerical methods and applications*, Lake Como School of Advanced Studies (Co-organizer: F. Santambrogio).

2018: Workshop *Optimal Control and Mean Field Games*, Pavia (Co-organizers: G. Cavagnari, S. Lisini, C. Orrieri)

2016: Bimester on *Nonlinear Flows*, Research Centre ESI, Vienna (Co-organizers: E. Feireisl, A. Juengel, A. Mielke, U. Stefanelli)

2014 and 2011: MFO Workshop *Variational Methods for Evolution*, Oberwolfach (Co-organizers: L. Ambrosio, A. Mielke, M. Peletier, F. Otto, U. Stefanelli)

2011: Conference Analysis and Numerics of PDEs. In memory of Enrico Magenes, Pavia

2010: BIRS Workshop: Rate-independent systems: Modeling, Analysis, and Computations, Banff (Coorganizer: U. Stefanelli)

2008: CIME Course: Nonlinear Partial Differential Equations and Applications, Cetraro (Co-organizer: L. Ambrosio)

#### **INSTITUTIONAL RESPONSIBILITIES**

2019: Director of the Advanced School of Ph.D. Higher Education (SAFD), University of Pavia

2014–2016: University Assessment Commission, University of Pavia

2001-2008: Director of the Ph.D. program in Mathematics and Statistics, University of Pavia

1998-present: IMATI-CNR, research associate

## **REVIEWING ACTIVITIES**

2023-: ESAIM: Control, Optimisation and Calculus of Variations, member of the editorial board

2018-: Unione Matematica Italiana, member of the Scientific Advisory Board

2014–: C.I.M.E. Foundation, member of the *Scientific Advisory Board* 

2016-: Applied Mathematics and Optimization, member of the editorial board

2013-: Potential Analysis, member of the editorial board

# MEMBERSHIPS OF SCIENTIFIC SOCIETIES

2009-: Membro Corrispondente, Istituto Lombardo, Accademia di Scienze e Lettere, Milano

# **Publications**

The MSC database attributes to me 102 publications, 5500 citations by 3100 authors, H-index 35. Seven of the papers listed below belong to the group of Highly cited papers according to WOS. The most relevant contributions of my research activity over the past ten years concern:

♦ Foundation of the theory of metric-measure spaces with *Riemannian Ricci curvature bounded* from below (the so-called RCD(K, N) condition):

*Calculus and heat flow in metric measure spaces and applications to spaces with Ricci bounds from below* (with L. Ambrosio, N. Gigli). Invent. Math. 195 (2014), 289–391. (\*)

[Heat flow and Cheeger energy in metric–measure spaces:  $L^2$  and Optimal Transport theory]

*Metric measure spaces with Riemannian Ricci curvature bounded from below* (with L. Ambrosio, N. Gigli). Duke Math. J., 163 (2014):1405–1490. (\*).

[The first analysis and characterization of Riemannian RCD( $K,\infty$ ) spaces with quadratic Cheeger energy]

*Bakry-Émery curvature-dimension condition and Riemannian Ricci curvature bounds* (with L. Ambrosio, N. Gigli). Annals of Probability, 43 (2015): 339–404. (\*)

[The full identification between the Bakry-Émery condition and RCD( $K,\infty$ ) spaces]

Self-improvement of the Bakry-Émery condition and Wasserstein contraction of the heat flow in RCD(K, ∞) metric measure spaces Discrete Contin. Dyn. Syst. 34 (2014): 1641–1661. (\*)

[Second order calculus, measure-valued  $\Gamma_2$ -tensor, and improved Bakry-Émery condition]

Convergence of pointed non-compact metric measure spaces and stability of Ricci curvature bounds and heat flows (with N. Gigli, A. Mondino) Proc. Lond. Math. Soc. 111 (2015): 1071–1129. (\*)

Nonlinear diffusion equations and curvature conditions in metric measure spaces (with L. Ambrosio, A. Mondino) Mem. Amer. Math. Soc. 262 (2019), no. 1270, v+121 pp.

[The full identification between the Bakry-Émery condition and the RCD(K, N) spaces]

- Metric-Sobolev spaces: identification of the construction by Lipschitz functions and Cheeger energy with the Newtonian approach
  - Density of Lipschitz functions and equivalence of weak gradients in metric measure spaces (with L. Ambrosio, N. Gigli) Rev. Mat. Iberoam. 29 (2013): 969–996. (\*)
- ♦ Entropy-Transport formulation for unbalanced optimal transport and the characterization of the new Hellinger-Kantorovich distance
  - *Optimal entropy-transport problems and a new Hellinger-Kantorovich distance between positive measures* (with M. Liero, A. Mielke). Invent. Math. 211 (2018): 969–1117. (\*)
- ♦ Foundation of Balanced Viscosity and Visco-Energetic solutions to rate-independent processes in infinite-dimensional spaces

Balanced viscosity (BV) solutions to infinite-dimensional rate-independent systems (with A. Mielke, R. Rossi). J. Eur. Math. Soc. (JEMS) 18 (2016): 2107–)2165. [The first contribution to existence, characterization, and properties of Balanced Viscosity solutions in infinite dimension.]

Viscous corrections of the time incremental minimization scheme and visco-energetic solutions to rate-independent evolution problems. (with L. Minotti) Arch. Ration. Mech. Anal. 227 (2018), no. 2, 477-543.

♦ The mean-field formulation of spatially inhomogeneous Evolutionary Games

Spatially Inhomogeneous Evolutionary Games (with L. Ambrosio, M. Fornasier, M. Morandotti) Comm. Pure Appl. Math. 74 (2021): 1353—1402

## Monographs and contributions to volumes

Sobolev Spaces in Extended Metric-Measure Spaces in New Trends on Analysis and Geometry in Metric Spaces. Lecture Notes in Mathematics, Springer 2022, 117–276.

[The refined theory of metric Sobolev space generated by sub-algebra of Lipschitz functions in extended metric-measure spaces.]

<sup>\*</sup>An Highly cited paper according to WOS

- *Gradient flows in metric spaces and in the space of probability measures* (with L. Ambrosio, N. Gigli) Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 2005 (second edition 2008). The two editions have received more than 1500 citations, according to MSC.
- Computational electrocardiology: mathematical and numerical modeling (with P. Colli Franzone, L. F. Pavarino; contribution). In Complex systems in biomedicine, pages 187–241. Springer Italia, Milan, 2006.

# Invited presentations to conferences (selection)

- ICMS Workshop: Optimal Transport and the Calculus of Variations, Edinburgh, 2023, Geodesic convexity of entropy functionals in Hellinger-Kantorovich metric
- Workshop on The Mathematics of Subjective Probability, Milano, 2023, Sobolev spaces on the Wasserstein space of probability measures
- Workshop Calculus of Variations & Geometric Measure Theory, Pisa, 2023, Evolution of probability measures: beyond gradient flows
- BIRS Workshop: Nonlinear Diffusion and nonlocal Interaction Models Entropies, Complexity, and Multi-Scale Structures, Granada, 2023, A Lagrangian approach to dissipative evolutions of probability measures
- Workshop Frontiers of Numerical PDEs, Maryland, 2023, Evolution equations in spaces of probability measures
- Workshop Interpolation of Measures, Paris, 2023, Fine properties of geodesics and geodesic  $\lambda$ -convexity for the Hellinger-Kantorovich distance
- MFO workshop: Heat Kernels, Stochastic Processes and Functional Inequalities, Oberwolfach, 2022, Capacitary modulus and Newtonian-Sobolev capacity in metric measure spaces
- HCM Conference: From Dirichlet Forms to Wasserstein Geometry, Bonn, 2022, Density of subalgebras of Lipschitz functions in metric Sobolev spaces and applications to measured Wasserstein spaces
- Workshop on Frontiers in Nonlocal Nonlinear PDEs, Anacapri, 2022, Dissipative evolutions of probability measures
- Workshop 20 years of Summer Schools on CalcVar in Rome, Roma, 2022, Evolution of probability measures
- Workshop Geometric Measure Theory and applications, Cortona, 2021, Lipschitz approximation, Capacity and Capacitary Modulus in metric Newtonian-Sobolev spaces
- MFO workshop: Applications of Optimal Transportation in the Natural Sciences (online), Oberwolfach, 2021, Dissipative evolution of measures
- Workshop: Calculus of Variations and Applications, SISSA, Trieste, 2020, Singular perturbation of gradient flows and rate-independent evolution
- XXI Congresso Unione Matematica Italiana, 2019. Plenary Speaker
- Workshop: Optimal transport and Geometric Analysis, Venice, 2019.
- ICMS Workshop: Gradient flows: challenges and new directions, Edinburgh, 2018.
- BIRS workshop: Topics in the Calculus of Variations: Recent Advances and New Trends, Banff, 2018.
- BIRS workshop: Entropies, the Geometry of Nonlinear Flows, and their Applications, Banff, 2018.
- MFO workshop: Variational Methods for Evolution Oberwolfach, 2017
- MFO workshop: Applications of Optimal Transportation in the Natural Sciences Oberwolfach, 2017
- 7<sup>th</sup> European Congress of Mathematics, Berlin: section "Analysis and PDEs". Invited speaker.
- MFO workshop: Heat Kernels, Stochastic Processes, and Functional Inequalities, Oberwolfach, 2016.
- Conference on New trends in Optimal Transport, HIM, Bonn, 2015.
- BIRS workshop: Entropy Methods, PDEs, Functional Inequalities, and Applications, Banff, 2014.
- International Conference on Fractal Geometry and Stochastics V, Tabarz, 2014. Plenary speaker.
- Workshop on Infinite-Dimensional Geometry, MSRI, Berkeley, 2013.

- EQUADIFF 2013, Prague. Plenary speaker.
- BIRS Workshop: Optimal Transportation and Differential Geometry Banff, 2012.
- MFO workshop: Interplay of Analysis and Probability in Physics, Oberwolfach, 2012.
- MFO mini-workshop: Manifolds with Lower Curvature Bounds, Oberwolfach, 2012.
- RISM meeting: Multiphase and Multiphysics problems, Verbania (IT), 2011.
- BIRS workshop: Nonlinear Diffusions and Entropy Dissipation: From Geometry to Biology, 2010.
- CIRM-HCM Meeting: Stochastic Analysis, SPDEs, Particle Systems, Optimal Transport 2010.
- Wokshop on Particle systems, nonlinear diffusions, and equilibration, HCM, Bonn, 2007.
- Workshop on Optimal Transportation, and Applications to Geophysics and Geometry, Edinburgh, 2007.
- Workshop on Optimal transport: theory and application, Centro De Giorgi, Pisa, 2006.
- ICMS Workshop: Optimal Transportation, Transport Equations and Hydrodynamics, Edinburgh, 2005.
- 10th Conference on Free Boundary Problems, Coimbra, June 7-12, 2005. Plenary speaker.

# Invited courses to international advanced schools

- CIME course on New Trends on Analysis and Geometry in Metric Spaces, Levico Terme, Italy 2017:
  Sobolev Spaces in Extended Metric-Measure Spaces
- *Gradient flows and entropy methods, HIM, Bonn, 2015:* The Weighted Energy-Dissipation (WED) principle for gradient flows.
- Analysis and Geometry on Singular Spaces, Scuola Normale Superiore, Pisa, 2014: Metric measure spaces with Riemannian Ricci curvature bounded from below.
- Seventh Summer School in Analysis and Applied Mathematics, Roma, 2013: Gradient flows and rate-independent evolutions: a variational approach.
- CNA Summer School on "New Vistas in Image Processing and PDEs" Carnegie Mellon University, Pittsburgh, 2010: Applications of optimal transport to evolutionary PDEs.
- School on "Optimal transport: Theory and applications" Institut Fourier, Grenoble, 2009: Gradient flows and optimal transport.
- EVEQ2008, Prague, 2008: A variational approach to gradient flows and rate-independent problems.
- School in Nonlinear Analysis and Calculus of Variations Scuola Normale Superiore, Pisa, 2006: Gradient flows: a variational approach.

Milano, January 24, 2024

Giuseppe Savaré

# List of publications

- [1] Giulia Cavagnari, Giuseppe Savaré, and Giacomo Enrico Sodini. "Dissipative probability vector fields and generation of evolution semigroups in Wasserstein spaces". In: *Probab. Theory Related Fields* 185.3-4 (2023), pp. 1087–1182. ISSN: 0178-8051. URL: https://doi.org/10.1007/s00440-022-01148-7.
- [2] Massimo Fornasier, Giuseppe Savaré, and Giacomo Enrico Sodini. "Density of subalgebras of Lipschitz functions in metric Sobolev spaces and applications to Wasserstein Sobolev spaces". In: *J. Funct. Anal.* 285.11 (2023), Paper No. 110153, 76. ISSN: 0022-1236,1096-0783. URL: https://doi.org/10.1016/j.jfa.2023.110153.
- [3] Matthias Liero, Alexander Mielke, and Giuseppe Savaré. "Fine properties of geodesics and geodesic *λ*-convexity for the Hellinger–Kantorovich distance". In: *Arch. Ration. Mech. Anal.* 247.6 (2023), Paper No. 112, 73. ISSN: 0003-9527,1432-0673. URL: https://doi.org/10.1007/s00205-023-01941-1.
- [4] Dario Mazzoleni and Giuseppe Savaré. "L²-gradient flows of spectral functionals". In: Discrete Contin. Dyn. Syst. 43.3-4 (2023), pp. 1560–1594. ISSN: 1078-0947. URL: https://doi.org/10.3934/dcds.2022123.
- [5] Giulia Cavagnari et al. "Lagrangian, Eulerian and Kantorovich formulations of multi-agent optimal control problems: equivalence and gamma-convergence". In: *J. Differential Equations* 322 (2022), pp. 268–364. ISSN: 0022-0396. URL: https://doi.org/10.1016/j.jde.2022.03.019.
- [6] Mark A. Peletier et al. "Jump processes as generalized gradient flows". In: Calc. Var. Partial Differential Equations 61.1 (2022), Paper No. 33, 85. ISSN: 0944-2669. URL: https://doi.org/10.1007/s00526-021-02130-2.
- [7] Giuseppe Savaré. "Sobolev spaces in extended metric-measure spaces". In: *New trends on analysis and geometry in metric spaces*. Vol. 2296. Lecture Notes in Math. Springer, Cham, 2022, pp. 117–276. URL: https://doi.org/10.1007/978-3-030-84141-6\_4.
- [8] Giuseppe Savaré and Giacomo E. Sodini. "A simple relaxation approach to duality for optimal transport problems in completely regular spaces". In: *J. Convex Anal.* 29.1 (2022), pp. 1–12. ISSN: 0944-6532.
- [9] Luigi Ambrosio and Giuseppe Savaré. "Duality properties of metric Sobolev spaces and capacity". In: Math. Eng. 3.1 (2021), Paper No. 1, 31. url: https://doi.org/10.3934/mine. 2021001.
- [10] Luigi Ambrosio et al. "Spatially inhomogeneous evolutionary games". In: Comm. Pure Appl. Math. 74.7 (2021), pp. 1353–1402. ISSN: 0010-3640. URL: https://doi.org/10.1002/cpa.21995.
- [11] Giulia Luise and Giuseppe Savaré. "Contraction and regularizing properties of heat flows in metric measure spaces". In: *Discrete Contin. Dyn. Syst. Ser. S* 14.1 (2021), pp. 273–297. ISSN: 1937-1632. URL: https://doi.org/10.3934/dcdss.2020327.
- [12] Emanuele Naldi and Giuseppe Savaré. "Weak topology and Opial property in Wasserstein spaces, with applications to gradient flows and proximal point algorithms of geodesically convex functionals". In: *Atti Accad. Naz. Lincei Rend. Lincei Mat. Appl.* 32.4 (2021), pp. 725–750. ISSN: 1120-6330. URL: https://doi.org/10.4171/rlm/955.
- [13] Florentine Fleissner and Giuseppe Savaré. "Reverse approximation of gradient flows as minimizing movements: a conjecture by De Giorgi". In: *Ann. Sc. Norm. Super. Pisa Cl. Sci.* (5) 20.2 (2020), pp. 677–720. ISSN: 0391-173X. URL: https://doi.org/10.2422/2036-2145.201711\_008.
- [14] Matteo Muratori and Giuseppe Savaré. "Gradient flows and evolution variational inequalities in metric spaces. I: Structural properties". In: *J. Funct. Anal.* 278.4 (2020), pp. 108347, 67. ISSN: 0022-1236. URL: https://doi.org/10.1016/j.jfa.2019.108347.

- [15] Luigi Ambrosio, Andrea Mondino, and Giuseppe Savaré. "Nonlinear diffusion equations and curvature conditions in metric measure spaces". In: *Mem. Amer. Math. Soc.* 262.1270 (2019), pp. v+121. ISSN: 0065-9266. URL: https://doi.org/10.1090/memo/1270.
- [16] M. Fornasier et al. "Mean-field optimal control as gamma-limit of finite agent controls". In: European J. Appl. Math. 30.6 (2019), pp. 1153–1186. ISSN: 0956-7925. URL: https://doi.org/10.1017/s0956792519000044.
- [17] Carlo Orrieri, Alessio Porretta, and Giuseppe Savaré. "A variational approach to the mean field planning problem". In: *J. Funct. Anal.* 277.6 (2019), pp. 1868–1957. ISSN: 0022-1236. URL: https://doi.org/10.1016/j.jfa.2019.04.011.
- [18] Riccarda Rossi et al. "Weighted energy-dissipation principle for gradient flows in metric spaces". In: *J. Math. Pures Appl.* (9) 127 (2019), pp. 1–66. ISSN: 0021-7824. URL: https://doi.org/10.1016/j.matpur.2018.06.022.
- [19] Matthias Liero, Alexander Mielke, and Giuseppe Savaré. "Optimal entropy-transport problems and a new Hellinger-Kantorovich distance between positive measures". In: *Invent. Math.* 211.3 (2018), pp. 969–1117. ISSN: 0020-9910. URL: https://doi.org/10.1007/s00222-017-0759-8.
- [20] Alexander Mielke, Riccarda Rossi, and Giuseppe Savaré. "Global existence results for viscoplasticity at finite strain". In: *Arch. Ration. Mech. Anal.* 227.1 (2018), pp. 423–475. ISSN: 0003-9527. URL: https://doi.org/10.1007/s00205-017-1164-6.
- [21] Luca Minotti and Giuseppe Savaré. "Viscous corrections of the time incremental minimization scheme and visco-energetic solutions to rate-independent evolution problems". In: *Arch. Ration. Mech. Anal.* 227.2 (2018), pp. 477–543. ISSN: 0003-9527. URL: https://doi.org/10.1007/s00205-017-1165-5.
- [22] Giuseppe Savaré. "Diffusion, optimal transport and Ricci curvature". In: *European Congress of Mathematics*. Eur. Math. Soc., Zürich, 2018, pp. 311–331.
- [23] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. "Diffusion, optimal transport and Ricci curvature for metric measure spaces". In: *Eur. Math. Soc. Newsl.* 103 (2017), pp. 19–28. ISSN: 1027-488X. URL: https://doi.org/10.4171/news/103/4.
- [24] Riccarda Rossi and Giuseppe Savaré. "From visco-energetic to energetic and balanced viscosity solutions of rate-independent systems". In: *Solvability, regularity, and optimal control of boundary value problems for PDEs.* Vol. 22. Springer INdAM Ser. Springer, Cham, 2017, pp. 489–531.
- [25] Luigi Ambrosio, Matthias Erbar, and Giuseppe Savaré. "Optimal transport, Cheeger energies and contractivity of dynamic transport distances in extended spaces". In: *Nonlinear Anal.* 137 (2016), pp. 77–134. ISSN: 0362-546X. URL: https://doi.org/10.1016/j.na.2015.12.006.
- [26] Luigi Ambrosio, Andrea Mondino, and Giuseppe Savaré. "On the Bakry-Émery condition, the gradient estimates and the local-to-global property of  $RCD^*(K,N)$  metric measure spaces". In: *J. Geom. Anal.* 26.1 (2016), pp. 24–56. ISSN: 1050-6926. URL: https://doi.org/10.1007/s12220-014-9537-7.
- [27] Matthias Liero, Alexander Mielke, and Giuseppe Savaré. "Optimal transport in competition with reaction: the Hellinger-Kantorovich distance and geodesic curves". In: *SIAM J. Math. Anal.* 48.4 (2016), pp. 2869–2911. ISSN: 0036-1410. URL: https://doi.org/10.1137/15M1041420.
- [28] Alexander Mielke, Riccarda Rossi, and Giuseppe Savaré. "Balanced viscosity (BV) solutions to infinite-dimensional rate-independent systems". In: *J. Eur. Math. Soc. (JEMS)* 18.9 (2016), pp. 2107–2165. ISSN: 1435-9855. URL: https://doi.org/10.4171/JEMS/639.
- [29] Alexander Mielke, Riccarda Rossi, and Giuseppe Savaré. "Balanced-viscosity solutions for multi-rate systems". In: *J. Phys. Conf. Ser.* 727 (2016), pp. 012010, 26. ISSN: 1742-6588. URL: https://doi.org/10.1088/1742-6596/727/1/012010.

- [30] Virginia Agostiniani, Riccarda Rossi, and Giuseppe Savaré. "On the transversality conditions and their genericity". In: *Rend. Circ. Mat. Palermo* (2) 64.1 (2015), pp. 101–116. ISSN: 0009-725X. URL: https://doi.org/10.1007/s12215-014-0184-4.
- [31] Luigi Ambrosio, Simone Di Marino, and Giuseppe Savaré. "On the duality between *p*-modulus and probability measures". In: *J. Eur. Math. Soc. (JEMS)* 17.8 (2015), pp. 1817–1853. ISSN: 1435-9855. URL: https://doi.org/10.4171/JEMS/546.
- [32] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. "Bakry-Émery curvature-dimension condition and Riemannian Ricci curvature bounds". In: *Ann. Probab.* 43.1 (2015), pp. 339–404. ISSN: 0091-1798. URL: https://doi.org/10.1214/14-AOP907.
- [33] Nicola Gigli, Andrea Mondino, and Giuseppe Savaré. "Convergence of pointed non-compact metric measure spaces and stability of Ricci curvature bounds and heat flows". In: *Proc. Lond. Math. Soc.* (3) 111.5 (2015), pp. 1071–1129. ISSN: 0024-6115. URL: https://doi.org/10.1112/plms/pdv047.
- [34] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. "Calculus and heat flow in metric measure spaces and applications to spaces with Ricci bounds from below". In: *Invent. Math.* 195.2 (2014), pp. 289–391. ISSN: 0020-9910. URL: https://doi.org/10.1007/s00222-013-0456-1.
- [35] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. "Metric measure spaces with Riemannian Ricci curvature bounded from below". In: *Duke Math. J.* 163.7 (2014), pp. 1405–1490. ISSN: 0012-7094. URL: https://doi.org/10.1215/00127094-2681605.
- [36] Sara Daneri and Giuseppe Savaré. "Lecture notes on gradient flows and optimal transport". In: *Optimal transportation*. Vol. 413. London Math. Soc. Lecture Note Ser. Cambridge Univ. Press, Cambridge, 2014, pp. 100–144.
- [37] Giuseppe Savaré. "Self-improvement of the Bakry-Emery condition and Wasserstein contraction of the heat flow in RCD(*K*, ∞) metric measure spaces". In: *Discrete Contin. Dyn. Syst.* 34.4 (2014), pp. 1641–1661. ISSN: 1078-0947. URL: https://doi.org/10.3934/dcds.2014.34.1641.
- [38] Giuseppe Savaré and Giuseppe Toscani. "The concavity of Rényi entropy power". In: *IEEE Trans. Inform. Theory* 60.5 (2014), pp. 2687–2693. ISSN: 0018-9448. URL: https://doi.org/10.1109/TIT.2014.2309341.
- [39] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. "Density of Lipschitz functions and equivalence of weak gradients in metric measure spaces". In: *Rev. Mat. Iberoam.* 29.3 (2013), pp. 969–996. ISSN: 0213-2230. URL: https://doi.org/10.4171/RMI/746.
- [40] Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré. "Heat flow and calculus on metric measure spaces with Ricci curvature bounded below—the compact case". In: *Analysis and numerics of partial differential equations*. Vol. 4. Springer INdAM Ser. Springer, Milan, 2013, pp. 63–115. URL: https://doi.org/10.1007/978-88-470-2592-9\_8.
- [41] Y. Brenier et al. "Sticky particle dynamics with interactions". In: *J. Math. Pures Appl.* (9) 99.5 (2013), pp. 577–617. ISSN: 0021-7824. URL: https://doi.org/10.1016/j.matpur.2012.09.013.
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