# Introduction to Programming
# with
# Scientific Applications

**Course evaluation**

*”Den første forelæsning var meget* ***skræmmende*** *og overvældende”*

# Course description – kursuskatalog.au.dk/en/course/111388/

## Introduction to Programming with Scientific Applications

**Description of qualifications**

After the course the participants will have knowledge of principles and techniques for systematic **construction** of **programs**.

At the end of the course, the participants will be able to:

- apply constructions of a common programming language,
- develop **well-structured** programs and perform **testing** and **debugging** of these,
- explain fundamental programming concepts and basic algorithmic techniques,
- apply standard **tools for scientific applications**,
- use the documentation for a programming language and available software packages.

**Contents**

The course gives an introduction to programming with scientific applications. Programming concepts and techniques are introduced using the **Python** programming language. The programming concepts are **illustrated in other programming languages**. The following content is included.

*Basic programming constructs*: Data types, operators, variables, flow of control, conditionals, loops, functions, recursion, scope, exceptions. *Object orientation*: Abstract data types, classes, inheritance, encapsulation. *Basic algorithmic techniques*: Sorting, binary search, dynamic programming. *Systematic development of programs*: Testing and debugging. File-based input/output, numerical analysis, functional programming. Scientific computing using standard packages for Python.

**ECTS** 10

**Hours - weeks - periods**
Lectures 2 x 2 hours/week
TA sessions 1 x 3 hours/week
Study café 3 x 1 hour/week

**Language of instruction**
Danish

**Instructor**
Gerth Stølting Brodal

**Academic prerequisites**
(Some) Linear algebra

**Exam**
**5 hour programming**
Aid: Computer and Internet, headphones,
7-point grading scale

**Prerequisites for examination participation**
Submission and approval of 10 mandatory assignments and submission of **1 implementation project**

**Notes** Grade reflects an overall assessment of implementation project and written examination.

# Lecturer

| | |
|---|---|
| Name | Gerth Stølting Brodal |
| Research | Algorithms and Data Structures (Computer Science) |
| Teaching | |
| 2018 - | BSc course on Introduction to Programming with Scientific Applications |
| 2003 - | BSc course on Introduction to Algorithms and Data Structures |
| 1999 - 17 | MSc courses on Computational Geometry, Algorithm Engineering, Advanced Data Structures, External Memory Algorithms and Data Structures |
| Python | Advanced Beginner |

# Question – Primary Education?

a) Mathematics
b) Mathematics-Economics
c) Data Science
d) Chemestry
e) Physics
f) Other Science-Technology
g) Other

# Question – Programming languages you know?

+750 listed on en.wikipedia.org/wiki/List_of_programming_languages

# Question – Programming experience?

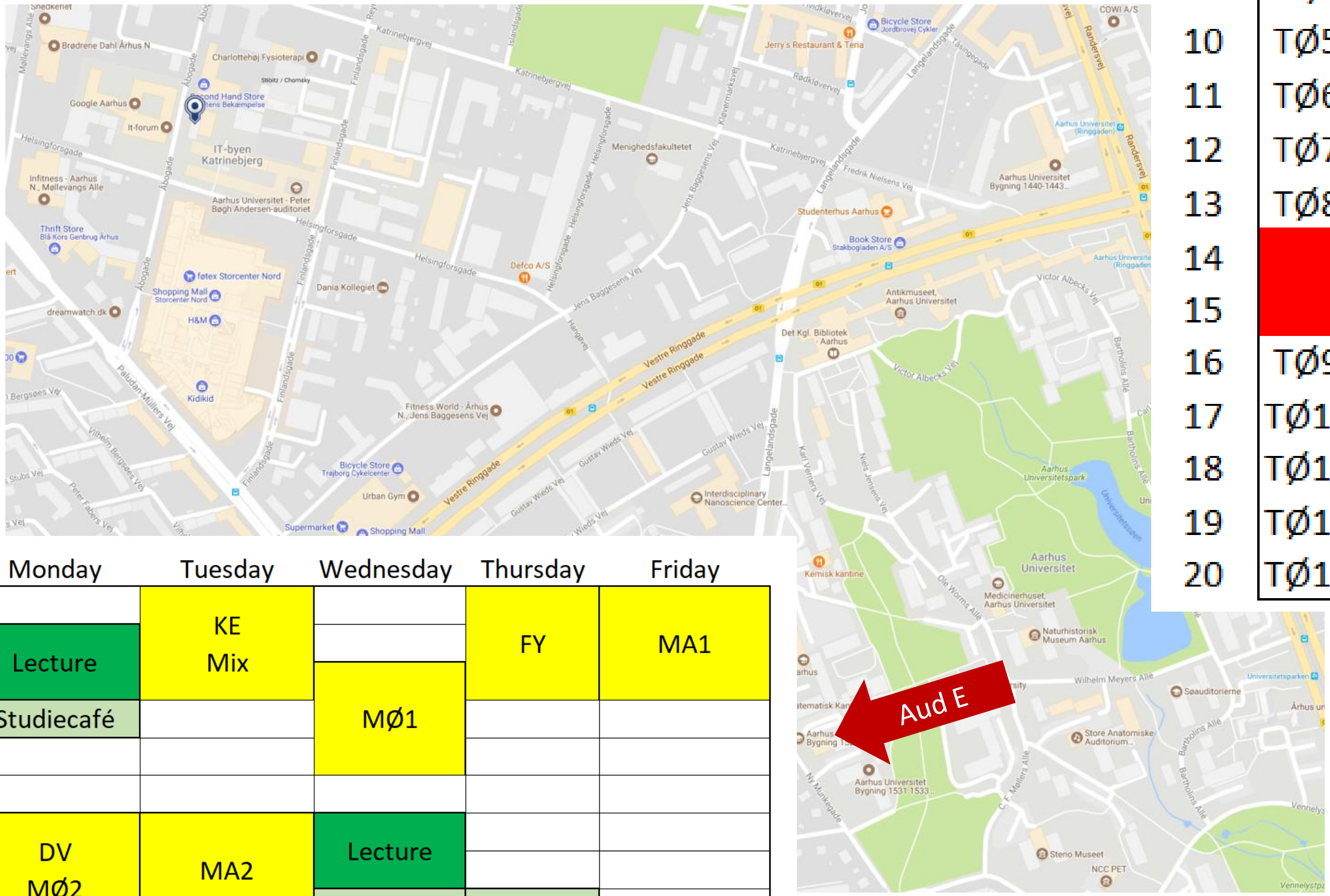
For the programming language you know best (if any) please state you proficiency level within the language.

a) None
b) Fundamental awareness (basic knowledge)
c) Novice (limited experience)
d) Intermediate (practical application)
e) Advanced (applied theory)
f) Expert (recognized authority)

# Some course practicalities

## Primary lecture material = slides

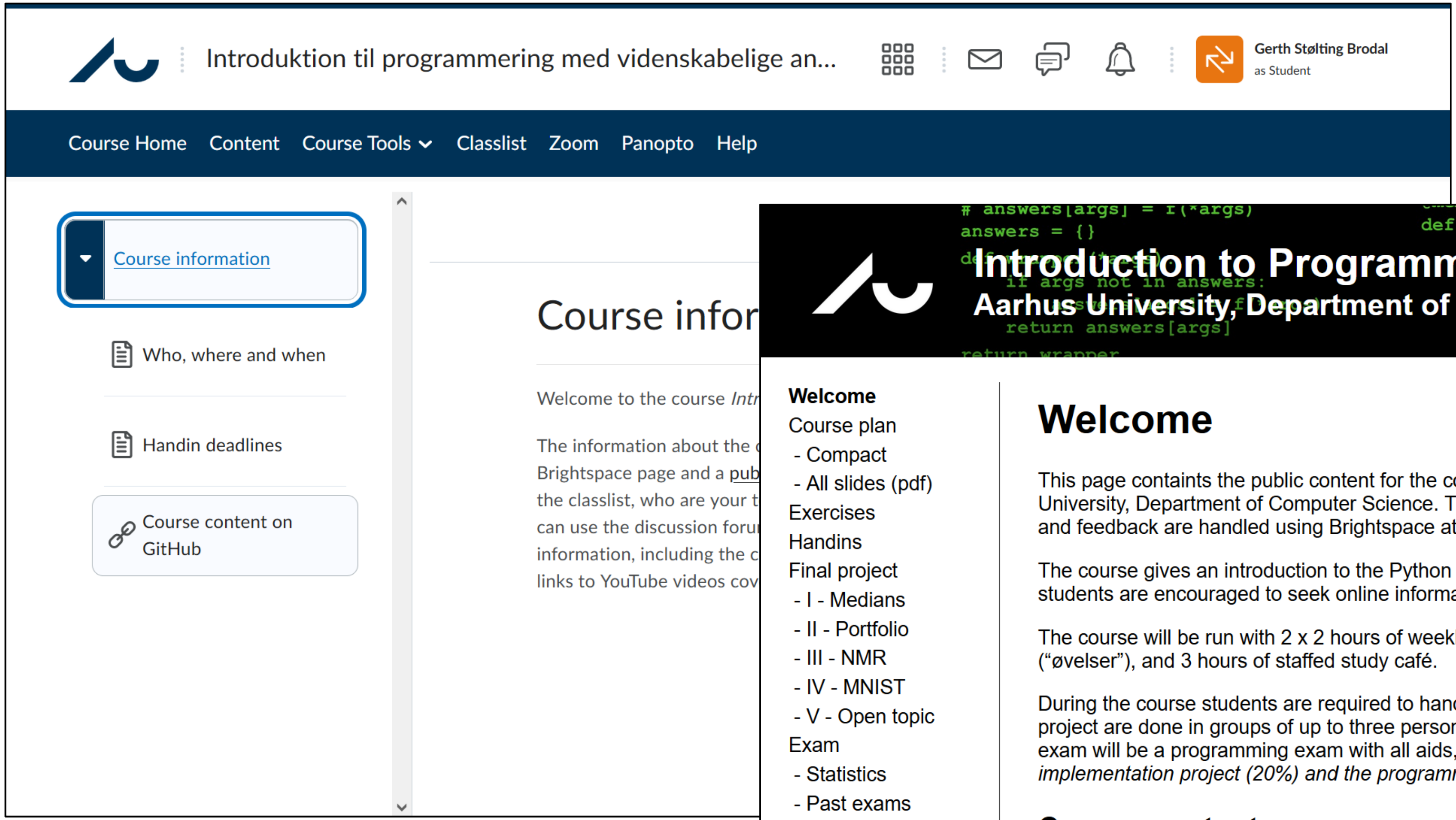
| Week | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| 5 | - / F1 | no TA clas: | - / F2 | | TØ1 |
| 6 | TØ1 / F3 | TØ1 | TØ1 / F4 | TØ1 | TØ2 |
| 7 | TØ2 / F5 | TØ2 | TØ2 / F6 | TØ2 | TØ3 |
| 8 | TØ3 / F7 | TØ3 | TØ3 / F8 | TØ3 | TØ4 |
| 9 | TØ4 / F9 | TØ4 | TØ4 / F10 | TØ4 | TØ5 |
| 10 | TØ5 / F11 | TØ5 | TØ5 / F12 | TØ5 | TØ6 |
| 11 | TØ6 / F13 | TØ6 | TØ6 / F14 | TØ6 | TØ7 |
| 12 | TØ7 / F15 | TØ7 | TØ7 / F16 | TØ7 | TØ8 |
| 13 | TØ8 / F17 | TØ8 | TØ8 / F18 | TØ8 | TØ9 |
| 14 | Easter break | | | | |
| 15 | | - | - / F21 | TØ9 | TØ10 |
| 16 | TØ9 / F19 | TØ9 | TØ9 / F20 | TØ10 | TØ11 |
| 17 | TØ10 / F22 | TØ10 | TØ10 / F23 | TØ11 | Kapsejlads? |
| 18 | TØ11 / F24 | TØ11 | TØ11 / F25 | TØ12 | Prayer Day |
| 19 | TØ12 / F26 | TØ12 | TØ12 / - | TØ13 | TØ12 |
| 20 | TØ13 / F27 | TØ13 | TØ13 / - | Ascesion Day | TØ13 |

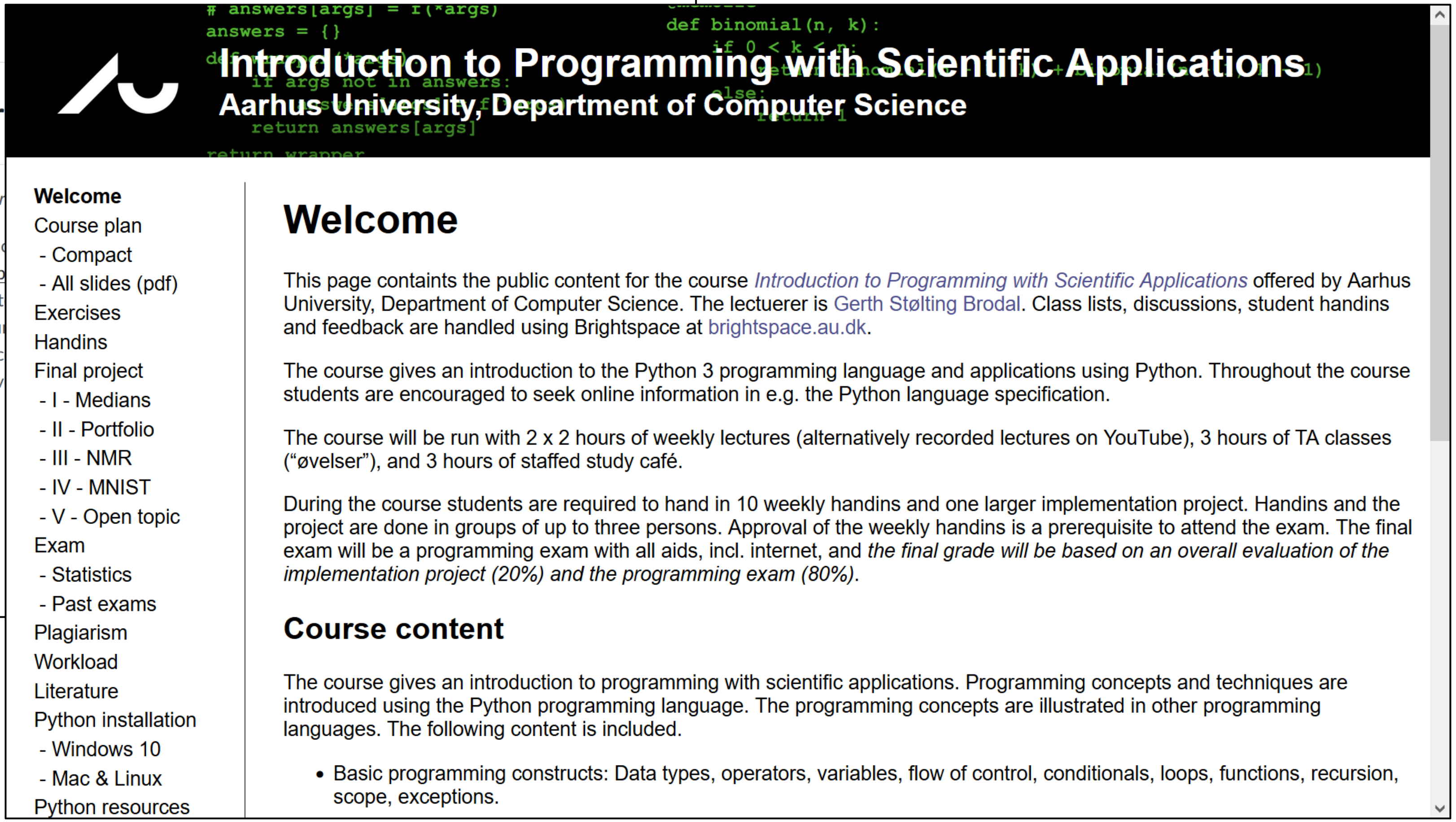



| | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| 8:15-9:00 | | KE Mix | | FY | MA1 |
| 9:15-10:00 | Lecture | | | | |
| 10:15-11:00 | | | MØ1 | | |
| 11:15-12:00 | Studiecafé | | | | |
| 12:15-13:00 | | | | | |
| 13:15-14:00 | | | | | |
| 14:15-15:00 | DV MØ2 | MA2 | Lecture | | |
| 15:15-16:00 | | | | | |
| 16:15-17:00 | | | Studiecafé | Studiecafé | |
| 17:15-18:00 | | | | | |

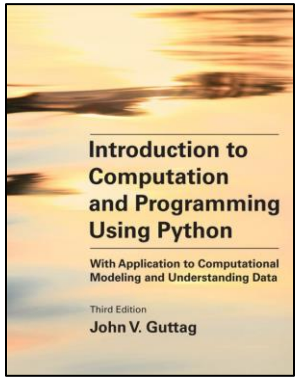
# Course page on Brightspace and GitHub

gsbrodal.github.io/ipsa



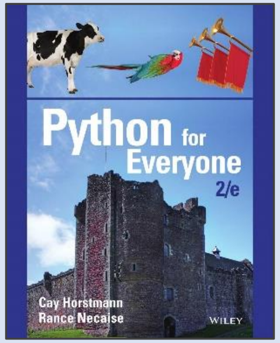



# Course text book – optional




John V. Guttag: **Introduction to Computation and Programming Using Python, Third Edition With Application to Computational Modeling and Understanding Data**. Third Edition. 664 pages. MIT Press, 2021.

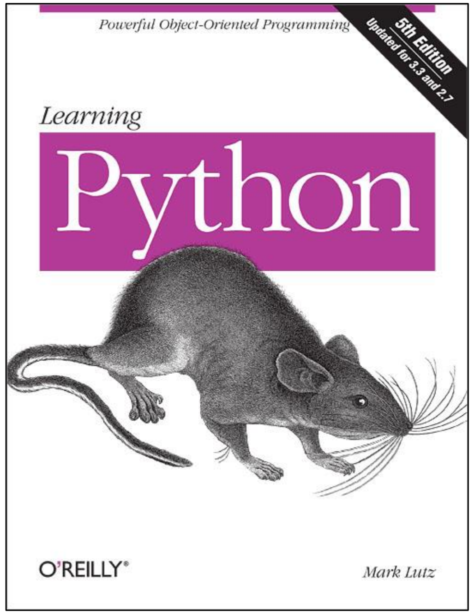
- *[Guttag, 2nd Ed., page 8] "The reader should be forewarned that this book is by no means a comprehensive introduction to Python". 3rd Ed. added about 80 pages on introduction to Python.*
- *Covers all basic features of Python enabling you to deal with data in Chapters 1-10 (212 pages) - remaining chapters are applications*
- *Other resources: Google, stack**overflow**, Python.org, YouTube, ...*



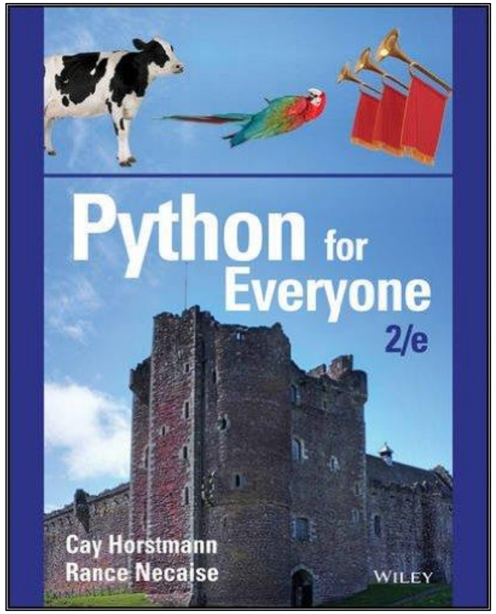

Comparison to a standard text book on the *programming language* Python by Cay Horstmann and Rance Necaise:

Topic **recursion** is covered by Guttag on page 123 (2nd edition on page 50), Horstmann and Necaise do it on page 611

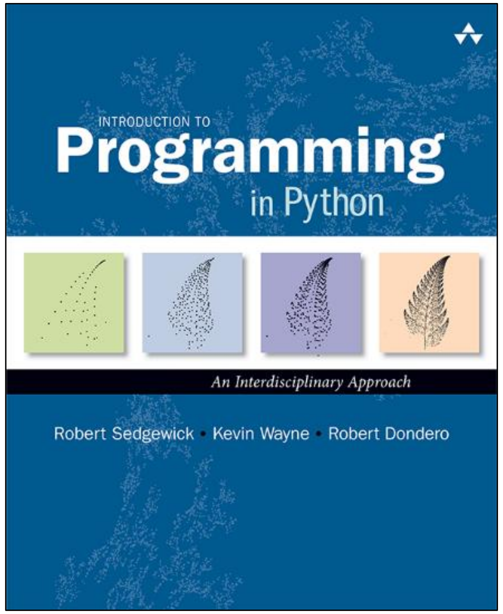
# Some other books on Python



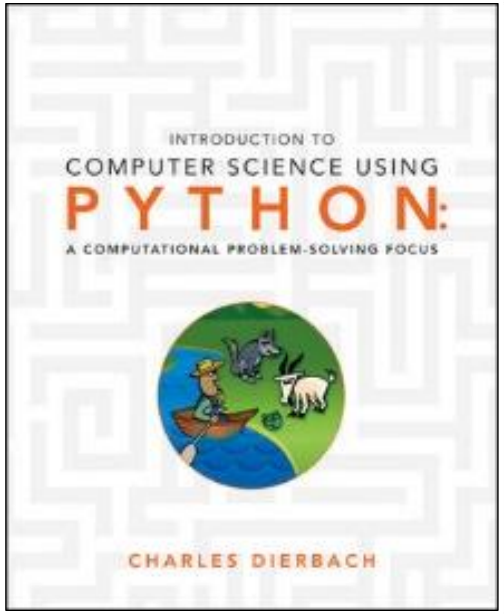

O'Reilly, 2013
1684 pages



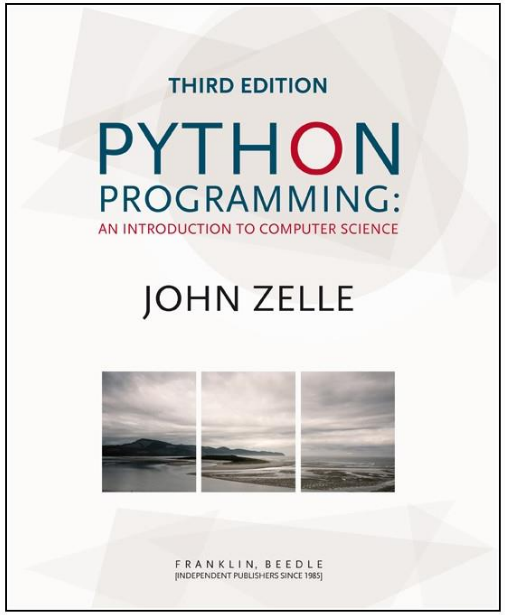

Wiley, 2016
752 pages




Addison-Wesley, 2015
794 pages




Wiley, 2013
580 pages




Franklin & Beedle, 2016
552 pages

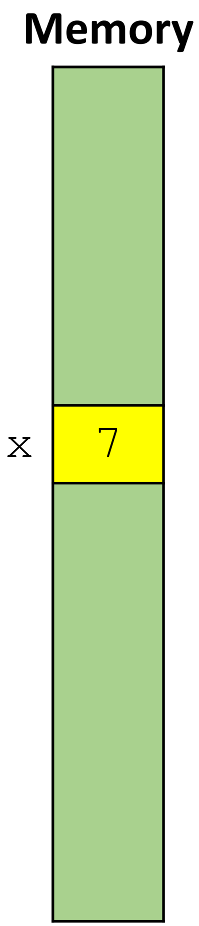
... numerous online introduction texts/courses/videos on Python

# Two Python programs

# A Python program

**Python shell**

```
> x = 7
> print(x * x)
| 49
```



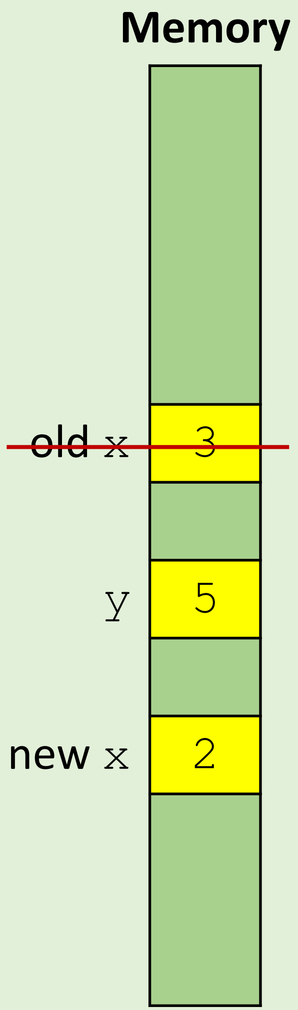

- `7` is an *integer literal* – in Python denoted an “`int`”
- `x` is the name of a *variable* that can hold some value
- `=` is assigning a value to a variable
- `*` denotes multiplication
- `print` is the name of a built-in *function*,
  here we call `print` to print the result of 7*7
- A program consists of a sequence of *statements*, executed sequentially

# Question – What  is the result of this program?

**Python shell**

```
> x = 3
> y = 5
> x = 2
> print(x * y)
```

x assigned new value

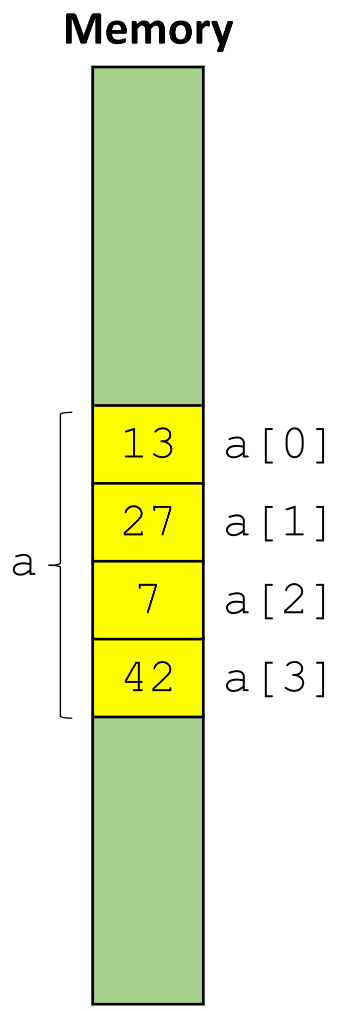



a) 10
b) 15
c) 25
d) [15, 10]
e) Error
f) Don’t know

# Another Python program using lists

**Python shell**

```
> a = [13, 27, 7, 42]
> print(a)
| [13, 27, 7, 42]
> print(a[2])
| 7
```




- `[13, 27, 7, 42]` is a *list* containing four integers
- `a[2]` refers to the entry in the list with *index* 2
  (the first element has index 0, i.e. `a[2]` is the 3rd element of the list)
- Note that `print` also can print a list

# Question – What is the result of this program?

**Python shell**

```
> a = [3, 5, 7]
> print(a[1] + a[2])
```

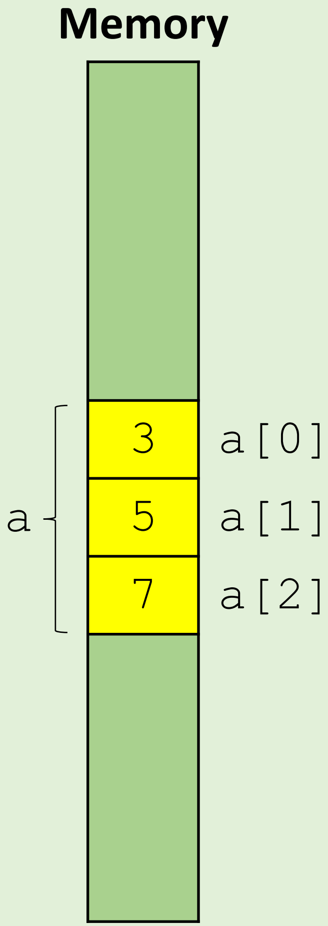
a) 8

b) 10

c) 12

d) 15

e) Don’t know



# Why Python ?

the next slides will be technical

# TIOBE Index January 2023

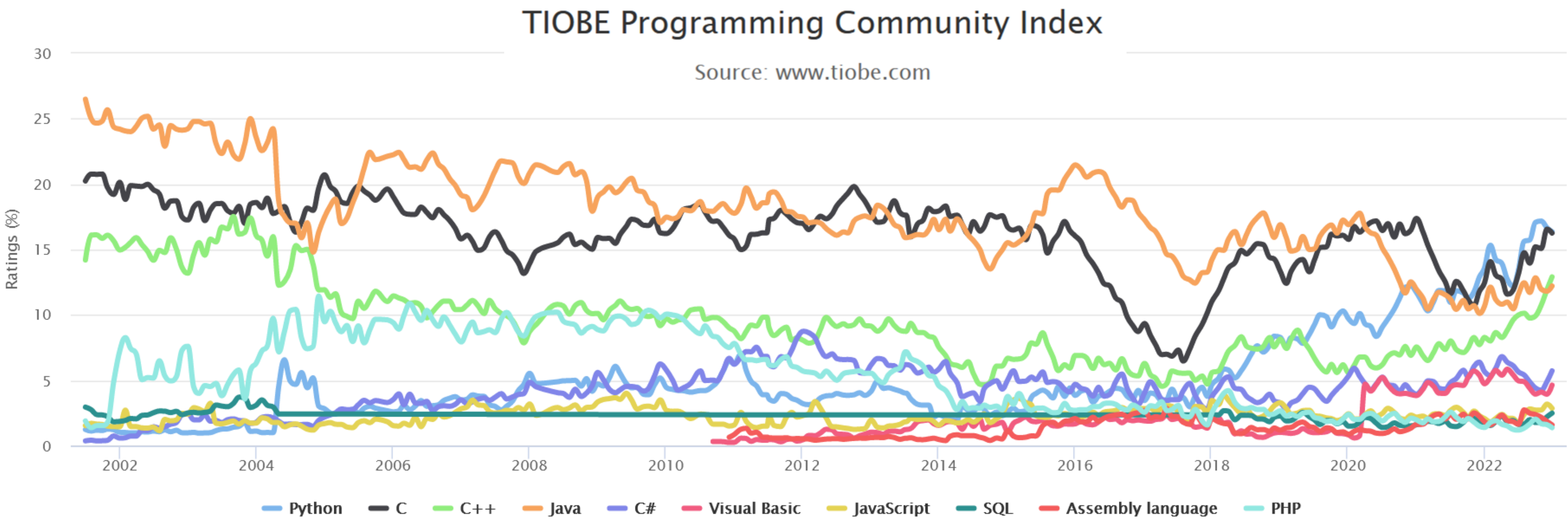
Since November 2021 Python #1

| Jan 2023 | Jan 2022 | Change | Programming Language | Ratings | Change |
|---|---|---|---|---|---|
| 1 | 1 | | Python | 16.36% | +2.78% |
| 2 | 2 | | C | 16.26% | +3.82% |
| 3 | 4 | ^ | C++ | 12.91% | +4.62% |
| 4 | 3 | v | Java | 12.21% | +1.55% |
| 5 | 5 | | C# | 5.73% | +0.05% |
| 6 | 6 | | Visual Basic | 4.64% | -0.10% |
| 7 | 7 | | JavaScript | 2.87% | +0.78% |
| 8 | 9 | ^ | SQL | 2.50% | +0.70% |
| 9 | 8 | v | Assembly language | 1.60% | -0.25% |
| 10 | 11 | ^ | PHP | 1.39% | -0.00% |

The TIOBE Programming Community index is an indicator of the *popularity of programming languages*. The index is updated once a month. The ratings are based on the number of skilled engineers world-wide, courses and third party vendors. Popular search engines such as Google, Bing, Yahoo!, Wikipedia, Amazon, YouTube and Baidu are used to calculate the ratings. It is important to note that the TIOBE index is not about the *best* programming language or the language in which *most lines of code* have been written.

www.tiobe.com

# Popularity of programming languages




Most Popular Programming Languages 1965 – 2022 (YouTube)

# “Hello World”

- In Java, C, C++ a lot of “{“, “}” and “;” are needed
- Java tends to have a lot of “public...” details that need to be spelled out
- Python is concise

**Java**

```java
public class HelloWorld {
    public static void main( String[] args ) {
        System.out.println( "Hello World!" );
        System.exit( 0 );
    }
}
```

**C**

```c
#include <stdio.h>

int main(int argc, char **argv) {
    printf("Hello World");
    return 0;
}
```

**C++**

```cpp
#include <iostream>
using namespace std;

int main(int argc, char** argv) {
    cout << "Hello, World!";
    return 0;
}
```

**Python 2**

```python
print "Hello world"
```

**Python 3**

```python
print("Hello world")
```

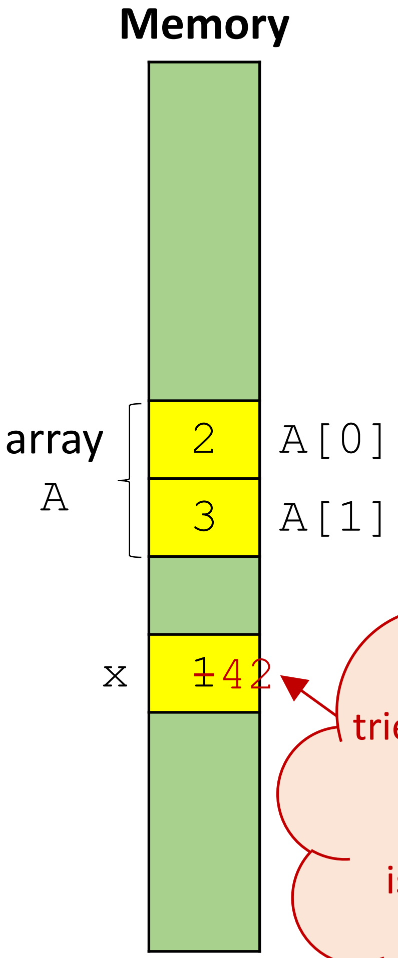
# Why Python ?

- **Short concise code**

(C developed by Dennis Ritchie 1969-73)

# C index out of bounds

**Debugging** is the process of finding and resolving defects or problems within a computer program that prevent correct operation of computer software or a system.
*en.wikipedia.org/wiki/Debugging*




"A" only has size 2, but tries to update the 4th entry. No warning is giving. Something unexpected is overridden in memory. **Have fun debugging!**

**indexing.c**

```c
#include <stdio.h>
int main() {
  int x = 1;
  int A[2] = {2, 3}; // A[0] = 2, A[1] = 3
  printf("x = %d, A = {%d, %d}\n", x, A[0], A[1]);
  A[3] = 42; // index A[3] out of bounds
  printf("x = %d, A = {%d, %d}\n", x, A[0], A[1]);
  return 0;
}
```

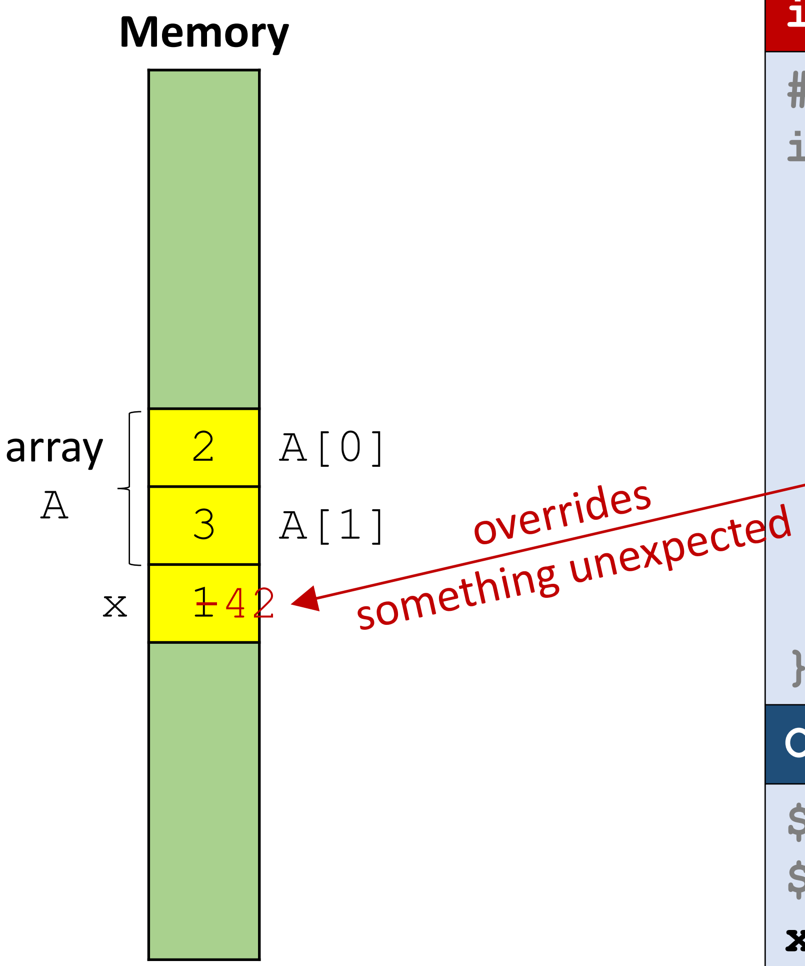
**Output**

```
$ gcc indexing.c
$ ./a.exe
x = 1, A = {2, 3}
x = 42, A = {2, 3}
```

Skipping checking for invalid indexing makes programs faster, but also requires disciplined programming

(C++ was developed by Bjarne Stroustrup 1985)

# … and C++ index out of bounds



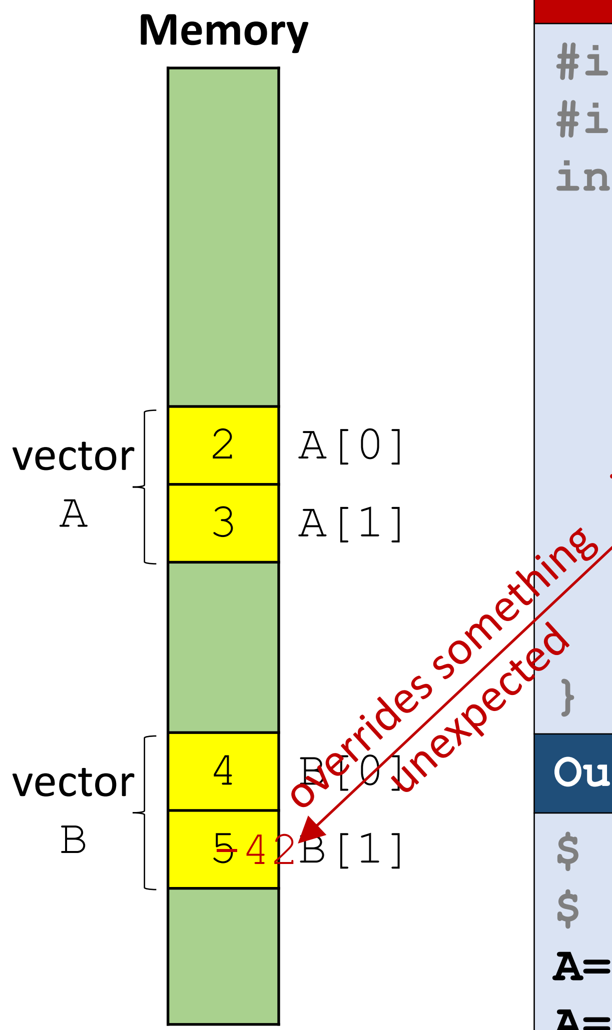

**indexing.cpp**

```
#include <iostream>
int main() {
  int x = 1;
  int A[2] = {2, 3}; // A[0] = 2, A[1] = 3
  std::cout << "x = " << x << ", A = {"
            << A[0] << ", " << A[1] << "}" << std::endl;
  A[2] = 42; // index A[2] out of bounds
  std::cout << "x = " << x << ", A = {"
            << A[0] << ", " << A[1] << "}" << std::endl;
  return 0;
}
```

**Output**

```
$ g++ indexing.cpp
$ ./a.exe
x = 1, A = {2, 3}
x = 42, A = {2, 3}
```

# … and C++ `vector` index out of bounds




**indexing.cpp**

```
#include <iostream>
#include <vector>
int main() {
   std::vector<int> A = {2, 3};  // A[0] = 2, A[1] = 3
   std::vector<int> B = {4, 5};  // B[0] = 4, B[1] = 5
   std::cout << "A={" << A[0] << ", " << A[1] << "}, ";
   std::cout << "B={" << B[0] << ", " << B[1] << "}" << std::endl;
   A[9]=42; // index A[9] out of bounds
   std::cout << "A={" << A[0] << ", " << A[1] << "}, ";
   std::cout << "B={" << B[0] << ", " << B[1] << "}" << std::endl;
   return 0;
}
```

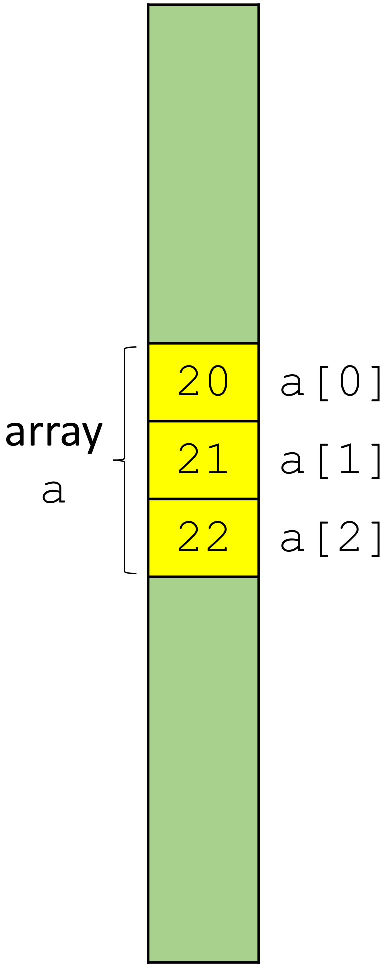
**Output**

```
$ g++ -std=c++11 indexing-vector.cpp
$ ./a.exe
A={2, 3}, B={4, 5}
A={2, 3}, B={4, 42}
```

(Java was developed by James Gosling 1995)

# … and Java index out of bounds exception

**Memory**




**indexing.java**

```
class IndexingTest{
    public static void main(String args[]){
        int a[] = {20, 21, 22};
        a[5] = 42; // index a[5] out of bounds
    }
}
```

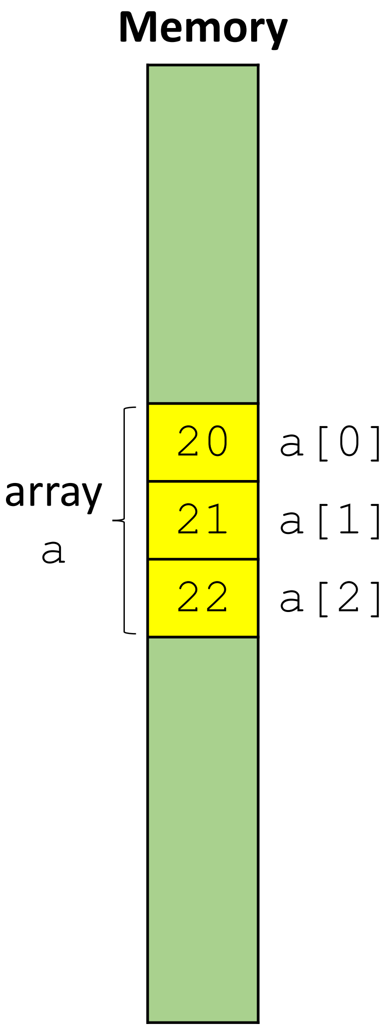
**Output**

```
$ javac indexing.java
$ java IndexingTest
Exception in thread "main"
java.lang.ArrayIndexOutOfBoundsException: 5
        at IndexingTest.main(indexing.java:5)
```

Java provides error message when running the program

(Python first release by Guido van Rossum 1991)

# … and Python index out of bounds exception




**indexing.py**

```
a = [20, 21, 22]
a[5] = 42   # index a[5] out of bounds
```

**Output**

```
$ python indexing.py
Traceback (most recent call last):
  File "indexing.py", line 3, in <module>
    a[5] = 42
IndexError: list assignment index out of range
```

Python provides error message when running the program

# Why Python ?

- Short concise code
- **Index out-of-range exceptions**

# C++ different ways to print a vector

**vector-iterator.cpp**

```
#include <iostream>
#include <vector>
int main() {
  // Vector is part of STL (Standard Template Library)
  std::vector<int> A = {20, 23, 26};
  // "C" indexing - since C++98
  for (int i = 0; i < A.size(); i++)
    std::cout << A[i] << std::endl;
  // iterator - since C++98
  for (std::vector<int>::iterator it = A.begin(); it != A.end(); ++it)
    std::cout << *it << std:: endl;
  // "auto" iterator - since C++11
  for (auto it = A.begin(); it != A.end(); ++it)
    std::cout << *it << std:: endl;
  // Range-based for-loop - since C++11
  for (auto e : A)
    std::cout << e << std:: endl;
}
```

elegant

# Java - different ways to print a vector

**vector-iterator.java**

```
import java.util.Vector;
import java.util.Iterator;

class IteratorTest{
   public static void main(String[] args) {
       Vector<Integer> a = new Vector<Integer>();
       a.add(7);
       a.add(42);
       // "C" for-loop & get method
       for (int i = 0; i < a.size(); i++)
          System.out.println(a.get(i));
       // iterator
       for (Iterator it = a.iterator(); it.hasNext(); )
          System.out.println(it.next());
       // for-each loop - since Java 5
       for (Integer e : a)
          System.out.println(e);
   }
}
```

elegant

# The Python way to print a list

**print-list.py**

```
a = [20, 23, 26]

for e in a:
    print(e)
```

**Output**

```
$ python print-list.py
20
23
26
```

# Why Python ?

- Short concise code
- Index out of range exceptions
- **Elegant for-each loop**

# C++ how not to print a vector

```
$ g++ -std=c++11 print-vector.cpp
cpp-error-message.cpp: In function 'int main()':
cpp-error-message.cpp:7:13: error: no match for 'operator<<' (operand types are 'std::ostream {aka std::basic_ostream<char>}' and 'std::vector<int>')
   std::cout << A << std::endl;
             ^
In file included from /usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/iostream:39:0,
                 from cpp-error-message.cpp:1:
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:628:5: note: candidate: std::basic_ostream<_CharT, _Traits>& std::operator<<(std::basic_ostream<_CharT, _Traits>&&, const _Tp&) [with _CharT = char; _Traits = std::char_traits<char>; _Tp = std::vector<int>] <near match>
     operator<<(basic_ostream<_CharT, _Traits>&& __os, const _Tp& __x)
     ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:628:5: note:   conversion of argument 1 would be ill-formed:
cpp-error-message.cpp:7:16: error: cannot bind 'std::ostream {aka std::basic_ostream<char>}' lvalue to 'std::basic_ostream<char>&&'
   std::cout << A << std::endl;
                ^
In file included from /usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/iostream:39:0,
                 from cpp-error-message.cpp:1:
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:108:7: note: candidate: ...
std::basic_ostream<_CharT, _Traits>::__ostream_type = std::basic_ostream<char>]
       operator<<(__ostream_type& (*__pf)(__ostream_type&))
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:108:7: note:   no known conversion for argument 1 from 'std::vector<int>' to 'std::basic_ostream<char>::__ostream_type& (*)(std::basic_ostream<char>::__ostream_type&) {aka std::basic_ostream<char>& (*)(std::basic_ostream<char>&)}'
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:117:7: note: candidate: std::basic_ostream<_CharT, _Traits>::__ostream_type& std::basic_ostream<_CharT, _Traits>::operator<<(std::basic_ostream<_CharT, _Traits>::__ios_type& (*)(std::basic_ostream<_CharT, _Traits>::__ios_type&)) [with _CharT = char; _Traits = st
_Traits>::__ostream_type = std::basic_ostream<char>; std::basic_ostream<_CharT, _Traits>::__ios_type = std::basic_ios<char>]
       operator<<(__ios_type& (*__pf)(__ios_type&))
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:117:7: note:   no known conversion for argument 1 from 'std::vector<int>' to 'std::basic_ostream<char>::__ios_type& (*)(std::basic_ostream<char>::__ios_type&) {aka std::basic_ios<char>& (*)(std::basic_ios<char>&)}'
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:127:7: note: candidate: std::basic_ostream<_CharT, _Traits>::__ostream_type& std::basic_ostream<_CharT, _Traits>::operator<<(std::ios_base& (*)(std::ios_base&)) [with _CharT = char; _Traits = std::char_traits<char>; std::basic_ostream<_CharT, _Traits>::__ostrean
       operator<<(ios_base& (*__pf) (ios_base&))
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:127:7: note:   no known conversion for argument 1 from 'std::vector<int>' to 'std::ios_base& (*)(std::ios_base&)'
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:166:7: note: candidate: std::basic_ostream ...
       operator<<(long __n)
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:166:7: note:   no known conversion for arg ...
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:170:7: note: candidate: std::basic_ostream ...
       operator<<(unsigned long __n)
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:170:7: note:   no known conversion for arg ...
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:174:7: note: candidate: std::basic_ostream ...
       operator<<(bool __n)
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:174:7: note:   no known conversion for arg ...
In file included from /usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:638:0,
                 from /usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/iostream:39,
                 from cpp-error-message.cpp:1:
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/bits/ostream.tcc:91:5: note: candidate: std::basic ...
     basic_ostream<_CharT, _Traits>::
     ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/bits/ostream.tcc:91:5: note:   no known conversion ...
In file included from /usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/iostream:39:0,
                 from cpp-error-message.cpp:1:
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:181:7: note: candidate: std::basic_ostream ...
       operator<<(unsigned short __n)
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:181:7: note:   no known conversion for arg ...
In file included from /usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:638:0,
                 from /usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/iostream:39,
                 from cpp-error-message.cpp:1:
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/bits/ostream.tcc:105:5: note: candidate: std::basi ...
     basic_ostream<_CharT, _Traits>::
     ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/bits/ostream.tcc:105:5: note:   no known conversio ...
In file included from /usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/iostream:39:0,
                 from cpp-error-message.cpp:1:
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:192:7: note: candidate: std::basic_ostream ...
       operator<<(unsigned int __n)
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:192:7: note:   no known conversion for argument 1 from 'std::vector<int>' to 'unsigned ...
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:201:7: note: candidate: std::basic_ostream<_CharT, _Traits>::__ostream_type& std::basic ...
       operator<<(long long __n)
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:201:7: note:   no known conversion for argument 1 from 'std::vector<int>' to 'long long ...
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:205:7: note: candidate: std::basic_ostream<_CharT, _Traits>::__ostream_type& std::basic ...
       operator<<(unsigned long long __n)
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:205:7: note:   no known conversion for argument 1 from 'std::vector<int>' to 'long long unsigned int'
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:220:7: note: candidate: std::basic_ostream<_CharT, _Traits>::__ostream_type& std::basic_ostream<_CharT, _Traits>::operator<<(double) [with _CharT = char; _Traits = std::char_traits<char>; std::basic_ostream<_CharT, _Traits>::__ostream_type = std::basic_ostream<
       operator<<(double __f)
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:220:7: note:   no known conversion for argument 1 from 'std::vector<int>' to 'double'
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:224:7: note: candidate: std::basic_ostream<_CharT, _Traits>::__ostream_type& std::basic_ostream<_CharT, _Traits>::operator<<(float) [with _CharT = char; _Traits = std::char_traits<char>; std::basic_ostream<_CharT, _Traits>::__ostream_type = std::basic_ostream<c
       operator<<(float __f)
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:224:7: note:   no known conversion for argument 1 from 'std::vector<int>' to 'float'
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:232:7: note: candidate: std::basic_ostream<_CharT, _Traits>::__ostream_type& std::basic_ostream<_CharT, _Traits>::operator<<(long double) [with _CharT = char; _Traits = std::char_traits<char>; std::basic_ostream<_CharT, _Traits>::__ostream_type = std::basic_ostr
       operator<<(long double __f)
       ^
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:232:7: note:   no known conversion for argument 1 from 'std::vector<int>' to 'long double'
/usr/lib/gcc/x86_64-pc-cygwin/5.4.0/include/c++/ostream:245:7: note: candidate: std::basic_ostream<_CharT, _Traits>::__ostream_type& std::basic_ostream<_CharT, _Traits>::operator<<(const void*) [with _CharT = char; _Traits = std::char_traits<char>; std::basic_ostream<_CharT, _Traits>::__ostream_type = std::basic_ost
```

**print-vector.cpp**

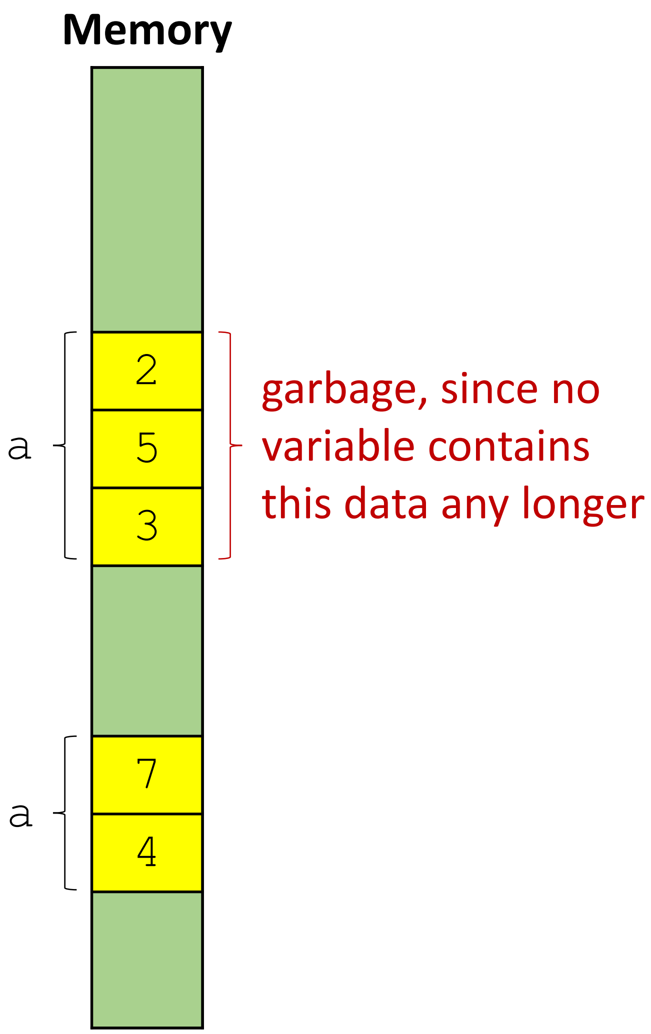
```cpp
#include <iostream>
#include <vector>

int main() {
  std::vector<int> A = {2, 3};
  std::cout << A << std::endl;
  return 0;
}
```

C++ vectors cannot be printed directly – mistake results in +200 lines of error messages

# Why Python ?

- Short concise code
- Index out of range exceptions
- Elegant for-each loop
- **Python hopefully better error messages than C++**

# Python and garbage collection




**garbage.py**

```
a = [2, 5, 3]
a = [7,4]
```

`a` gets new value

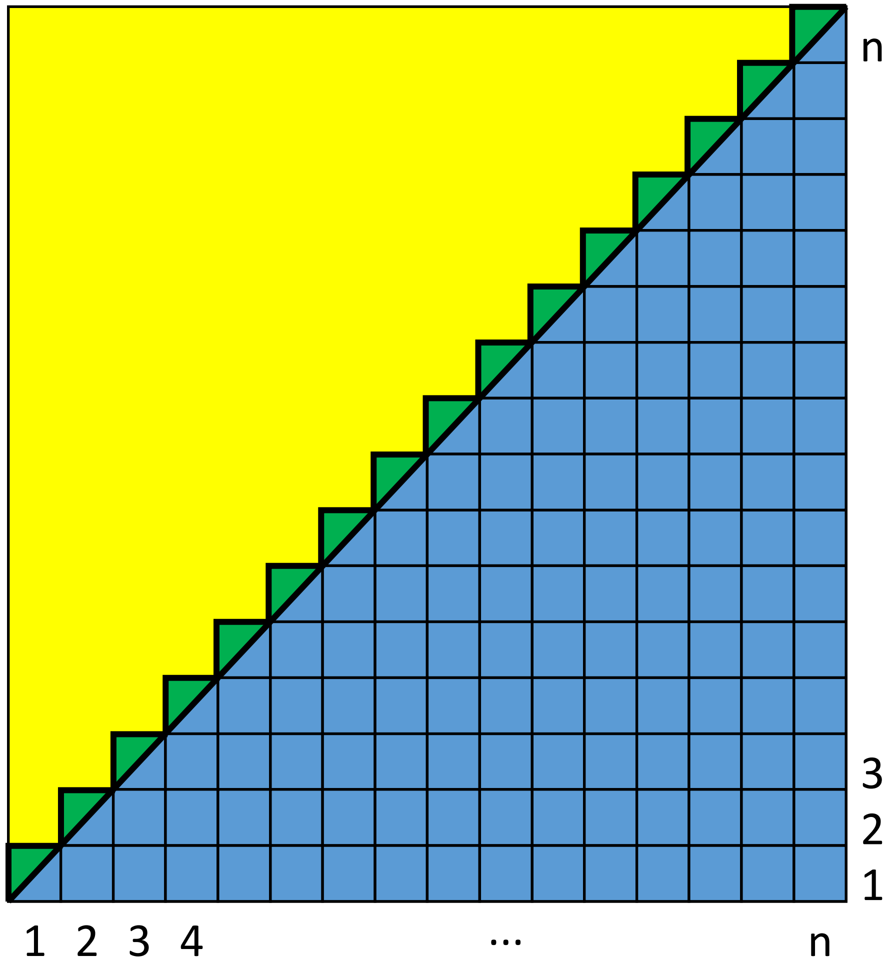
- Python and e.g. Java, C# and JavaScript have a **garbage collector** to automatically recycle garbage
- C and C++ garbage collection must be done explicitly by the program; forgetting to **free** memory again results in **memory leaks** – which can be really hard to find. **Have fun debugging!**

# Why Python ?

- Short concise code
- Index out of range exceptions
- Elegant for-each loop
- Python hopefully better error messages than C++
- **Garbage collection is done automatically**

# Python performance vs C, C++ and Java

Compute sum $1 + 2 + 3 + \cdots + n = n^2/2 + n/2$



# $1 + 2 + \cdots + n$

**add.py**

```python
import sys

n = int(sys.argv[1])
sum = 0
for i in range(1, n + 1):
    sum += i
print("Sum = %d" % sum)
```

**add.c**

```c
#include <stdio.h>
#include <stdlib.h>

int main(int argc, char *argv[]) {
  int n = atoi(argv[1]);
  int sum = 0;
  for (int i = 1; i <= n; i++)
    sum += i;
  printf("Sum = %d\n", sum);
}
```

**add.cpp**

```cpp
#include <iostream>
#include <cstdlib>
using namespace std;

int main(int argc, char *argv[]) {
  int n = atoi(argv[1]);
  int sum = 0;
  for (int i = 1; i <= n; i++)
    sum += i;
  cout << "Sum = " << sum << endl;
}
```

**add.java**

```java
class Add{
    public static void main(String args[]){
        int n = Integer.parseInt(args[0]);
        int sum = 0;
        for (int i = 1; i <= n; i++)
            sum += i;
        System.out.println("Sum = " + sum);
    }
}
```

# Timing results

| n | C (gcc 9.2) | C++, int (g++ 9.2 ) | C++, long (g++ 9.2 ) | Java (12.0) | CPython (3.8.1) | PyPy (7.3.0) | Numba, int64 |
|---|---|---|---|---|---|---|---|
| | | | | | Python | | |
| $10^7$ | 0.001 sec* | 0.001 sec* | 0.003 sec | 0.006 sec* | 1.5 sec | 0.27 sec | 0.002 sec |
| $10^9$ | 0.10 sec** | 0.10 sec** | 0.30 sec | 0.40 sec** | 145 sec | 27 sec | 0.2 sec |

**Wrong output (overflow)**

* -2004260032 instead of 50000005000000

** -243309312 instead of 500000000500000000

- since C, C++, and Java only uses 32 bits to represent integers (and 64 bits for ”long” integers)

Have fun debugging!

```
Bit           6666666666555555555544444444443333333333222222222211111111110000000000
position      9876543210987654321098765432109876543210987654321098765432109876543210

bin(10**9)                                            111011100110101100101000000000

bin(50000005000000)                     1011010111100110001000100010010110101101000000
bin(-2004260032+2**32)                                10001000100010010110101101000000

bin(500000000500000000)    11011110000010110110101100111110001011111110110010100000000
bin(-243309312+2**32)                                 11110001011111110110010100000000
```

Try Google: civilization gandhi overflow

# Timing results

| n | C (gcc 9.2) | C++, int (g++ 9.2 ) | C++, long (g++ 9.2 ) | Java (12.0) | Python (3.8.1) | PyPy (7.3.0) | Numba, int64 |
|---|---|---|---|---|---|---|---|
| $10^7$ | 0.001 sec* | 0.001 sec* | 0.003 sec | 0.006 sec* | 1.5 sec | 0.27 sec | 0.002 sec |
| $10^9$ | 0.10 sec** | 0.10 sec** | 0.30 sec | 0.40 sec** | 145 sec | 27 sec | 0.2 sec |

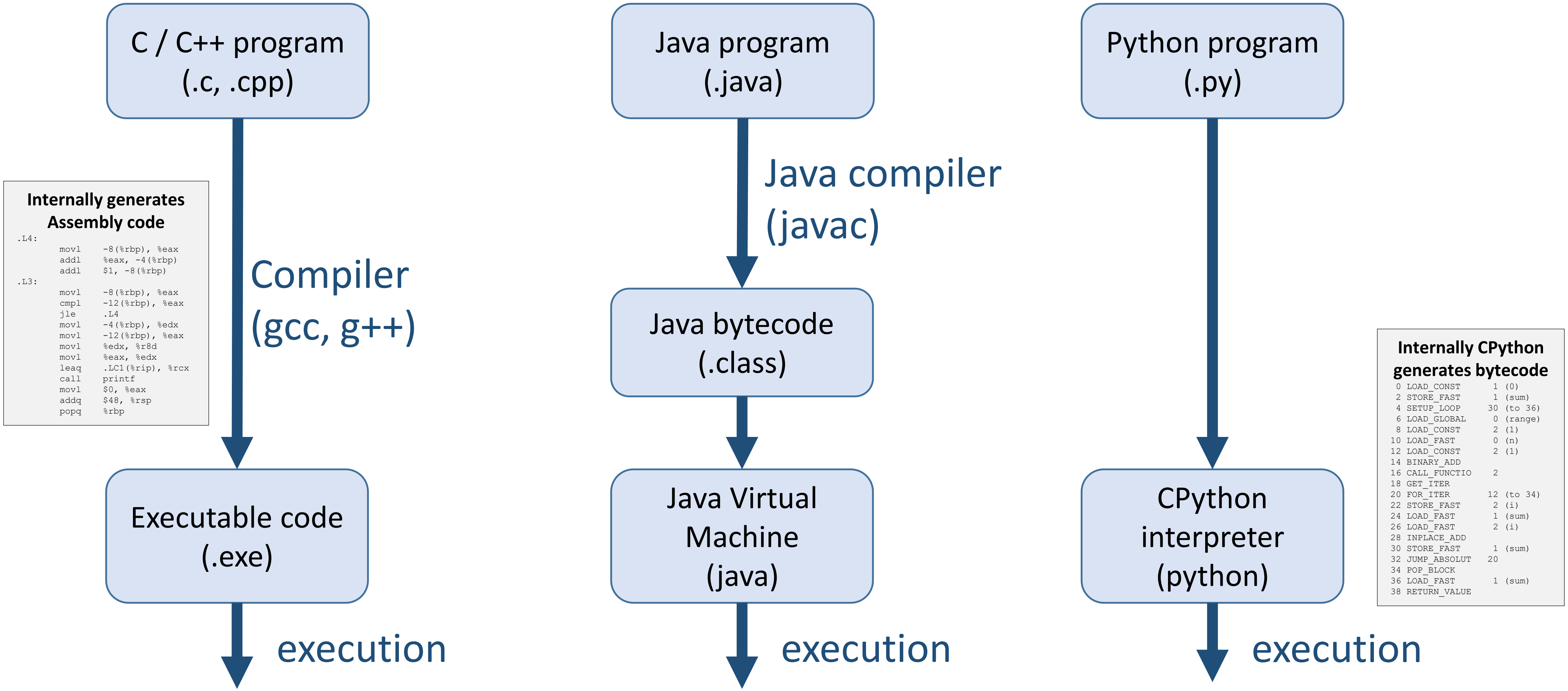
Python

- Relative speed

**C ≈ C++ > Java >> Python**

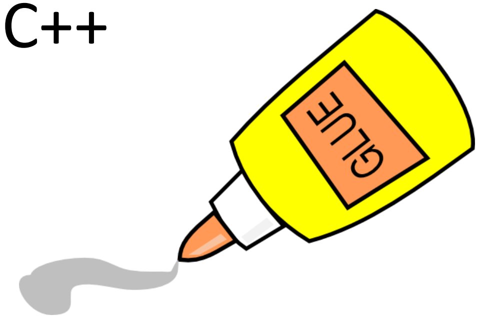
- C, C++, Java need to care about integer overflows – select integer representation carefully with sufficient number of bits (8, 16, 32, 64, 128)
- Python natively works with arbitrary long integers (as memory on your machine allows). Also possible in Java using the class java.math.BigInteger
- Python programs can (sometimes) run faster using PyPy
- Number crunching in **Python** should be delegated to **specialized modules (e.g. Numpy, CPLEX, Numba)** – often written in C or C++ and requires selecting right integer representation

# Interpreter vs Compiler

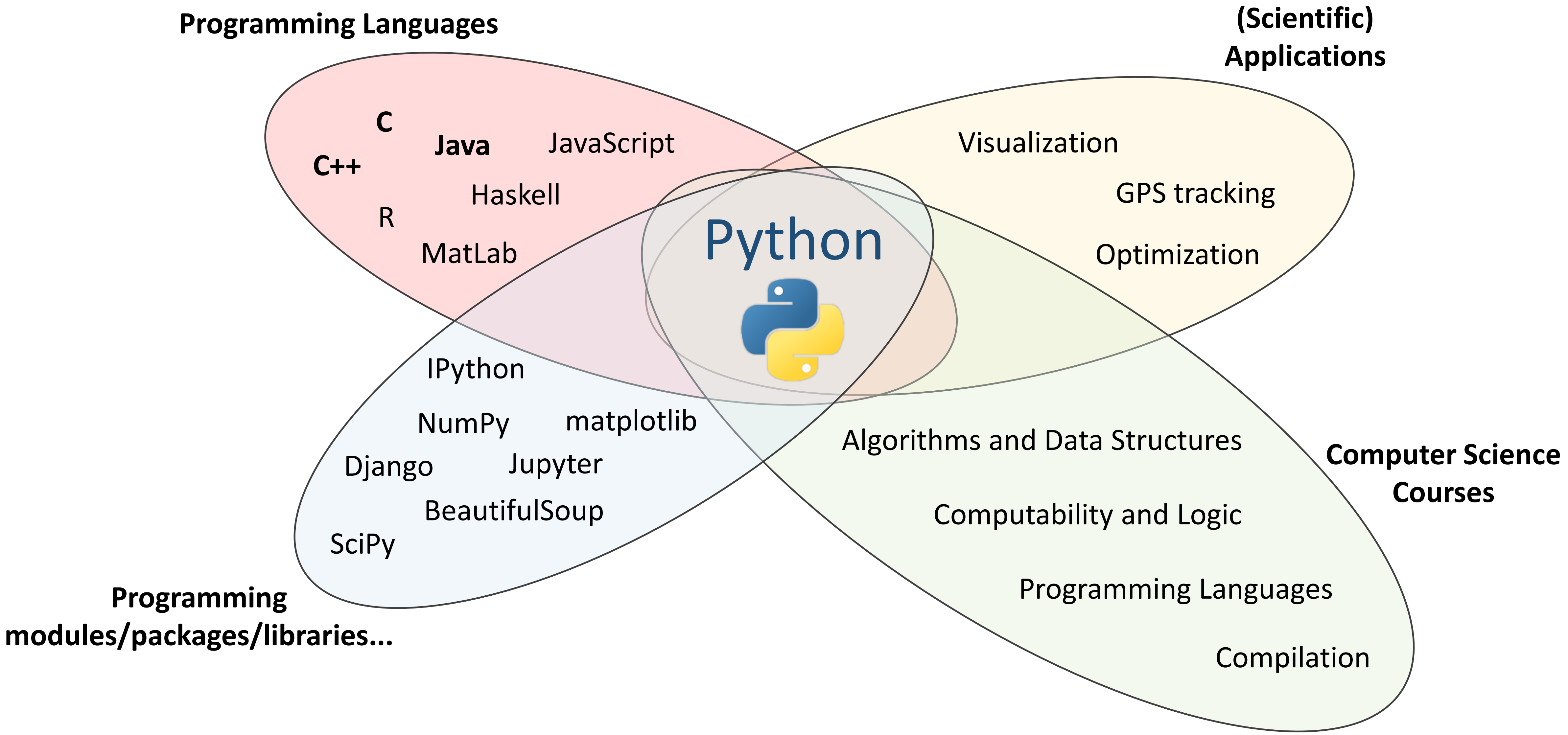


# Why Python ?

- Short concise code
- Index out of range exceptions
- Elegant for-each loop
- Python hopefully better error messages than C++
- Garbage collection is done automatically
- **Exact integer arithmetic (no overflows)**
- **Can delegate number crunching to C, C++, ...**



# This course


Programming Languages
C
C++
Java
JavaScript
Haskell
R
MatLab
(Scientific) Applications
Visualization
GPS tracking
Optimization
Python
IPython
NumPy
matplotlib
Django
Jupyter
BeautifulSoup
SciPy
Programming modules/packages/libraries...
Algorithms and Data Structures
Computability and Logic
Programming Languages
Compilation
Computer Science Courses

# Course overview

Basic programming
Advanced / specific python
Libraries & applications

| | | |
|---|---|---|
| 1. Introduction to Python | 10. Functions as objects | 19. Linear programming |
| 2. Python basics / if | 11. Object oriented programming | 20. Generators, iterators, with |
| 3. Basic operations | 12. Class hierarchies | 21. Modules and packages |
| 4. Lists / while / for | 13. Exceptions and files | 22. Working with text |
| 5. Tuples / comprehensions | 14. Doc, testing, debugging | 23. Relational data |
| 6. Dictionaries and sets | 15. Decorators | 24. Clustering |
| 7. Functions | 16. Dynamic programming | 25. Graphical user interfaces (GUI) |
| 8. Recursion | 17. Visualization and optimization | 26. Java vs Python |
| 9. Recursion and Iteration | 18. Multi-dimensional data | 27. Final lecture |

**10 handins**
**1 final project (last 1 month)**

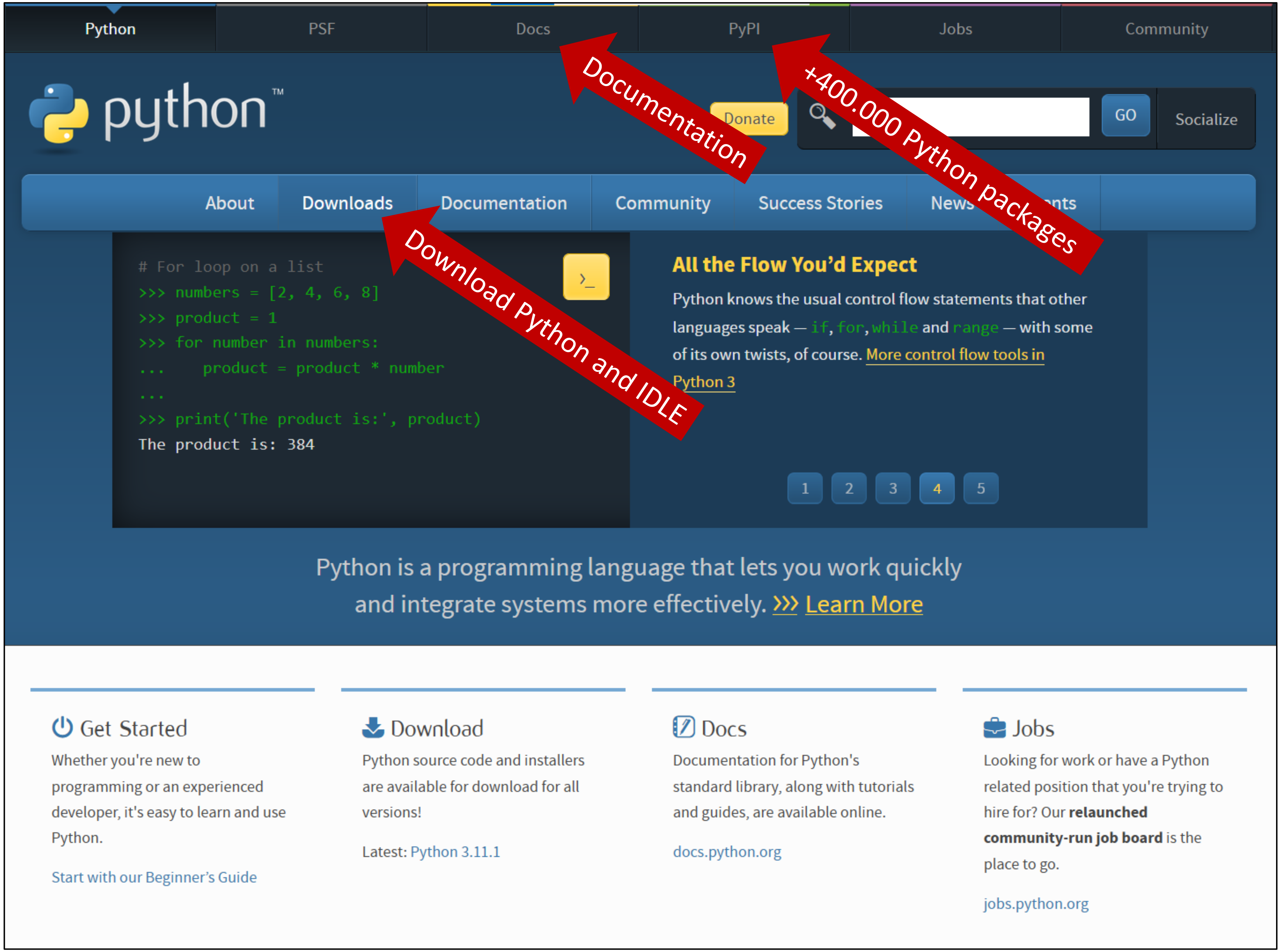
# History of Python development

- Python created by Guido van Rossum in 1989, first release 0.9.0 1991
- Python 2 → Python 3 (clean up of Python 2 language)
  - Python 2 – version 2.0 released 2000, final version 2.7 released mid-2010
  - Python 3 – released 2008, current release 3.11.1
- Python 3 is *not* backward compatible, libraries incompatible

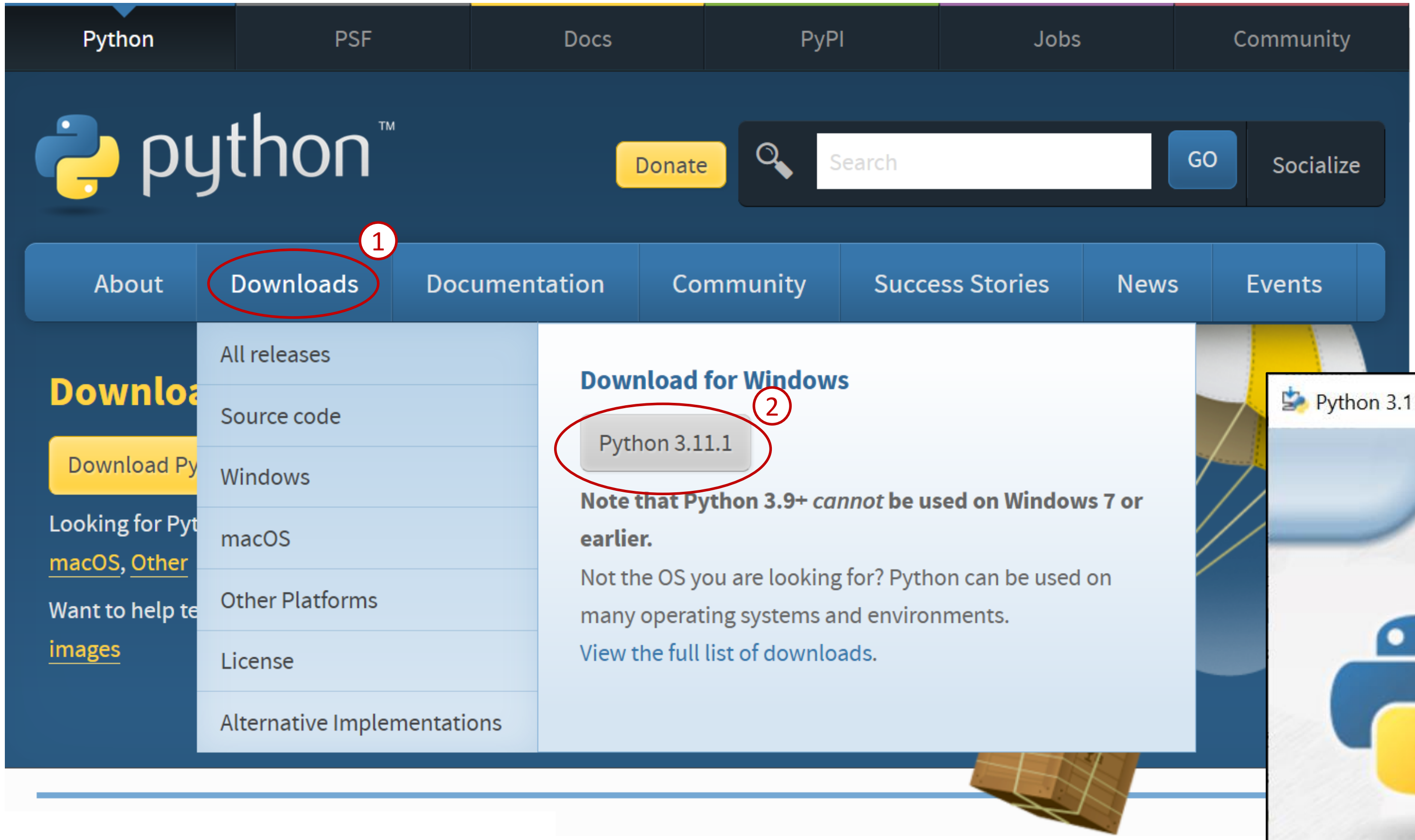
| **Python 2** | **Python 3** |
|---|---|
| print 42 | print(42) |
| int = C long (32 bits) | int = arbitrary number of digits (= named “long” in Python 2) |
| 7/3 → 2   returns “int” | 7/3 → 2.333...   returns “float” |
| range()   returns list (memory intensive) | range()   returns iterator (memory efficient; xrange in Python 2) |

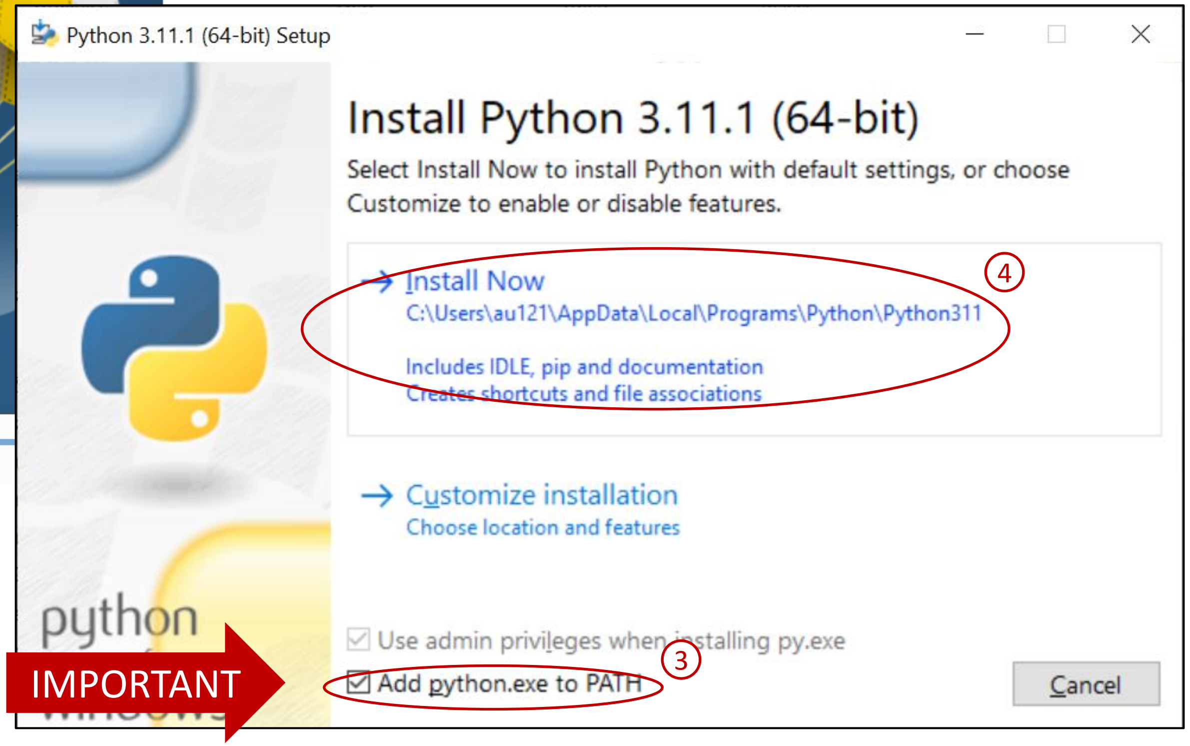
100th episode of Talk Python To Me: **Python past, present, and future with Guido van Rossum**

# Python.org


Python
PSF
Docs
PyPI
Jobs
Community
python™
Donate
GO
Socialize
About
Downloads
Documentation
Community
Success Stories
News
Documentation
+400.000 Python packages
Download Python and IDLE
# For loop on a list
>>> numbers = [2, 4, 6, 8]
>>> product = 1
>>> for number in numbers:
...     product = product * number
...
>>> print('The product is:', product)
The product is: 384
All the Flow You'd Expect
Python knows the usual control flow statements that other languages speak — if, for, while and range — with some of its own twists, of course. More control flow tools in Python 3
1 2 3 4 5
Python is a programming language that lets you work quickly and integrate systems more effectively. >>> Learn More
Get Started
Whether you're new to programming or an experienced developer, it's easy to learn and use Python.
Start with our Beginner's Guide
Download
Python source code and installers are available for download for all versions!
Latest: Python 3.11.1
Docs
Documentation for Python's standard library, along with tutorials and guides, are available online.
docs.python.org
Jobs
Looking for work or have a Python related position that you're trying to hire for? Our relaunched community-run job board is the place to go.
jobs.python.org

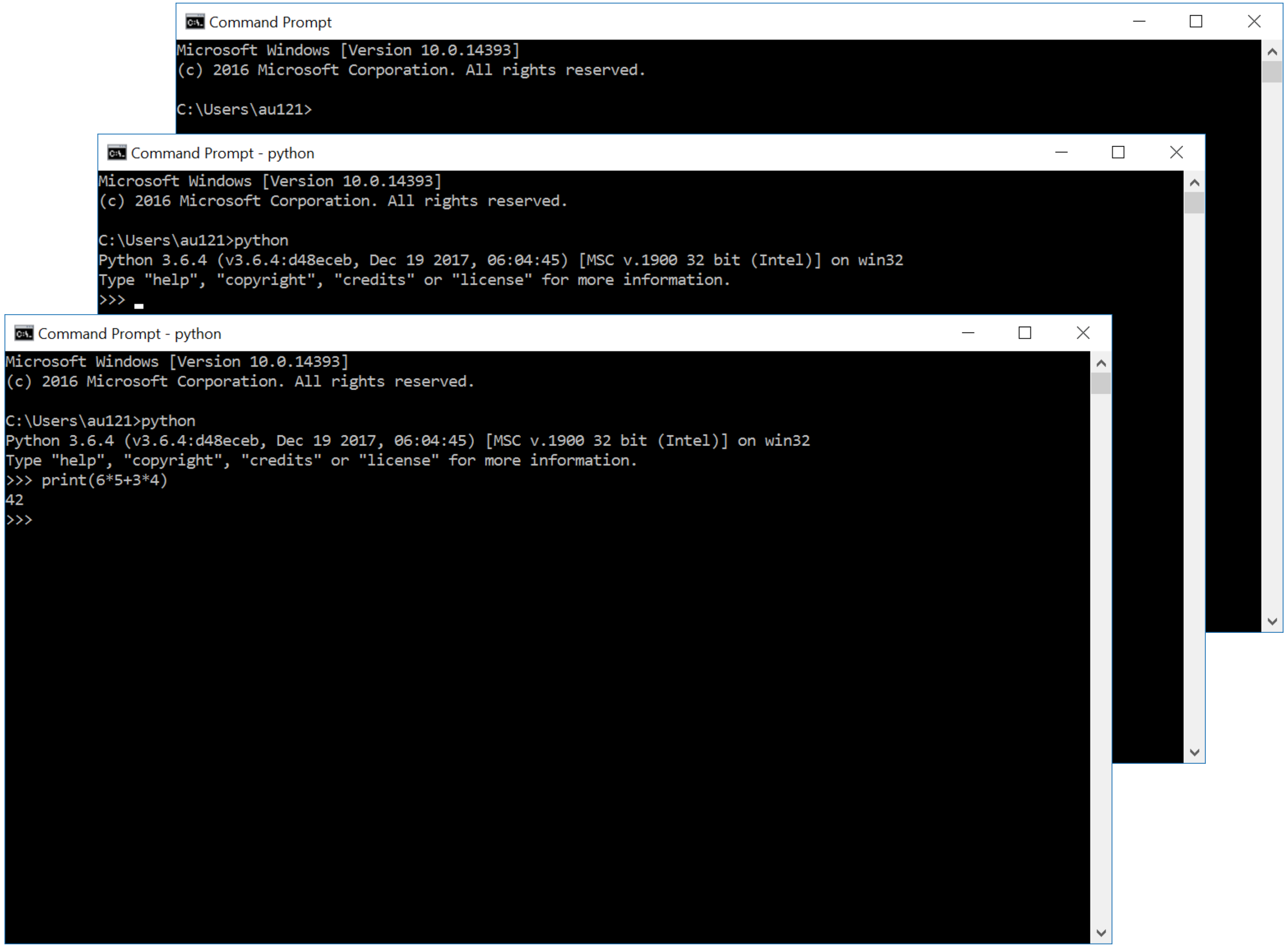
# Installing Python



# Running the Python Interpreter from a terminal

- Open Command Prompt (Windows-key + cmd)
- Type “python” + return
- Start executing Python statements




- To exit shell:
  Ctrl-Z + return *or*
  exit() + return

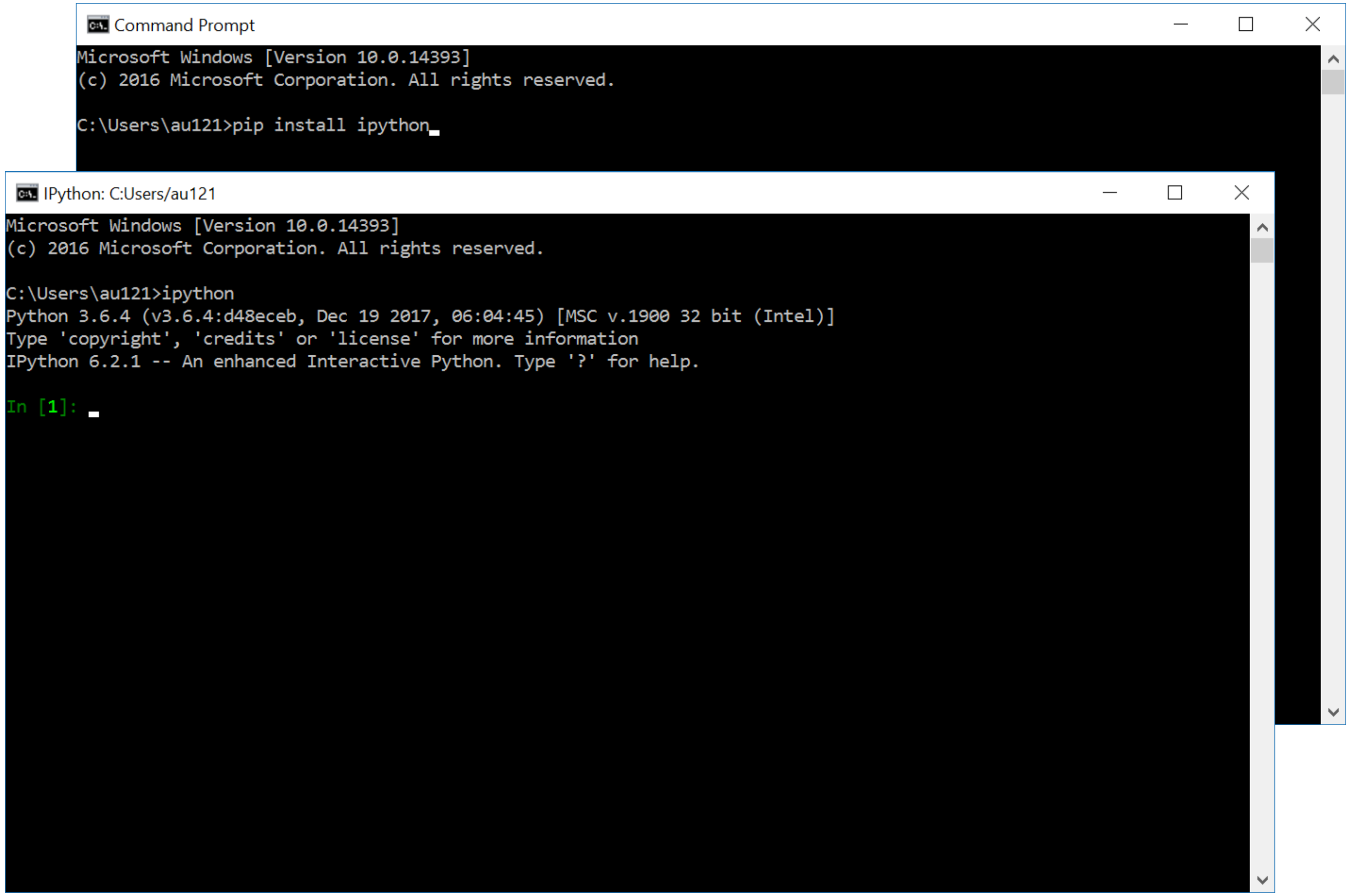
# Installing IPython – A more powerful interactive Python shell

- Open Command Prompt
- Execute:

  ```
  pip install ipython
  ```

- Start ipython

  ```
  ipython
  ```



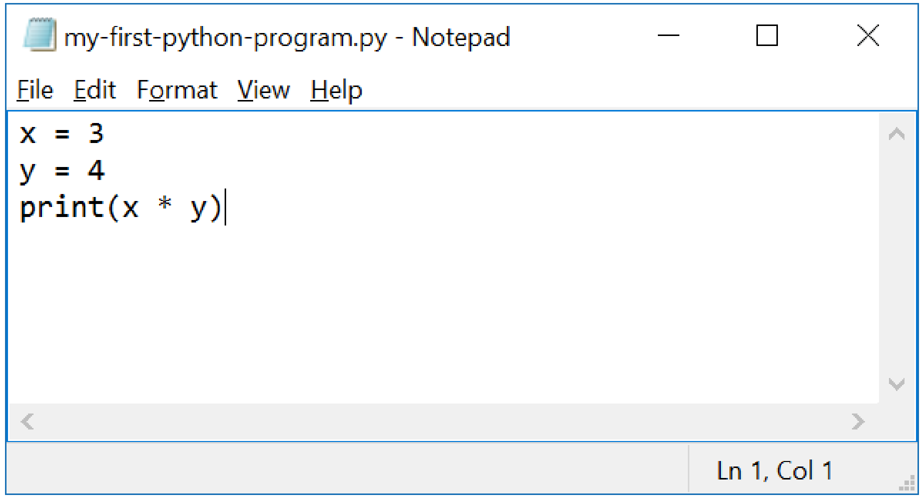

pip = the Python package manager

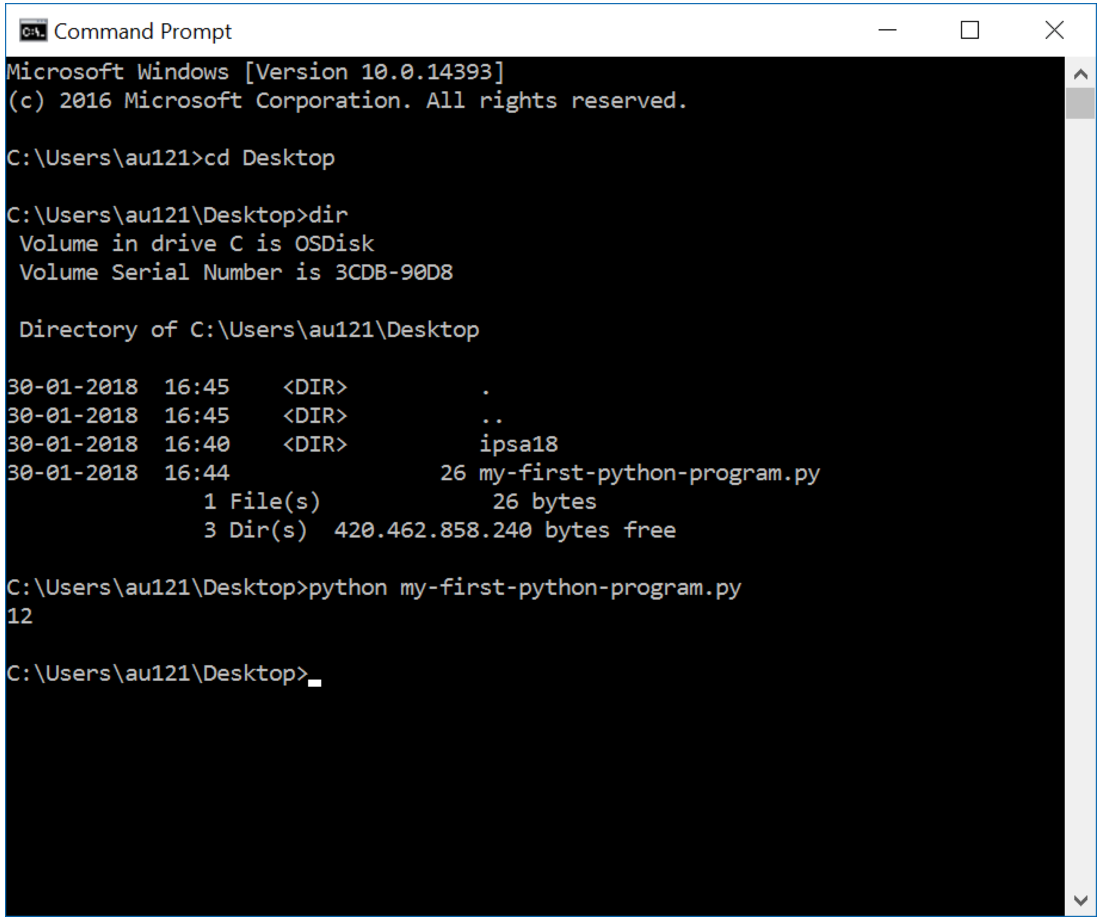
# Some other usefull packages

- Try installing some more Python packages:

| | |
|---|---|
| `pip install numpy` | linear algebra support (N-dimensional arrays) |
| `pip install scipy` | numerical integration and optimization |
| `pip install matplotlib` | 2D plotting library |
| `pip install pylint` | Python source code analyzer enforcing a coding standard |

# Creating a Python program the very basic way

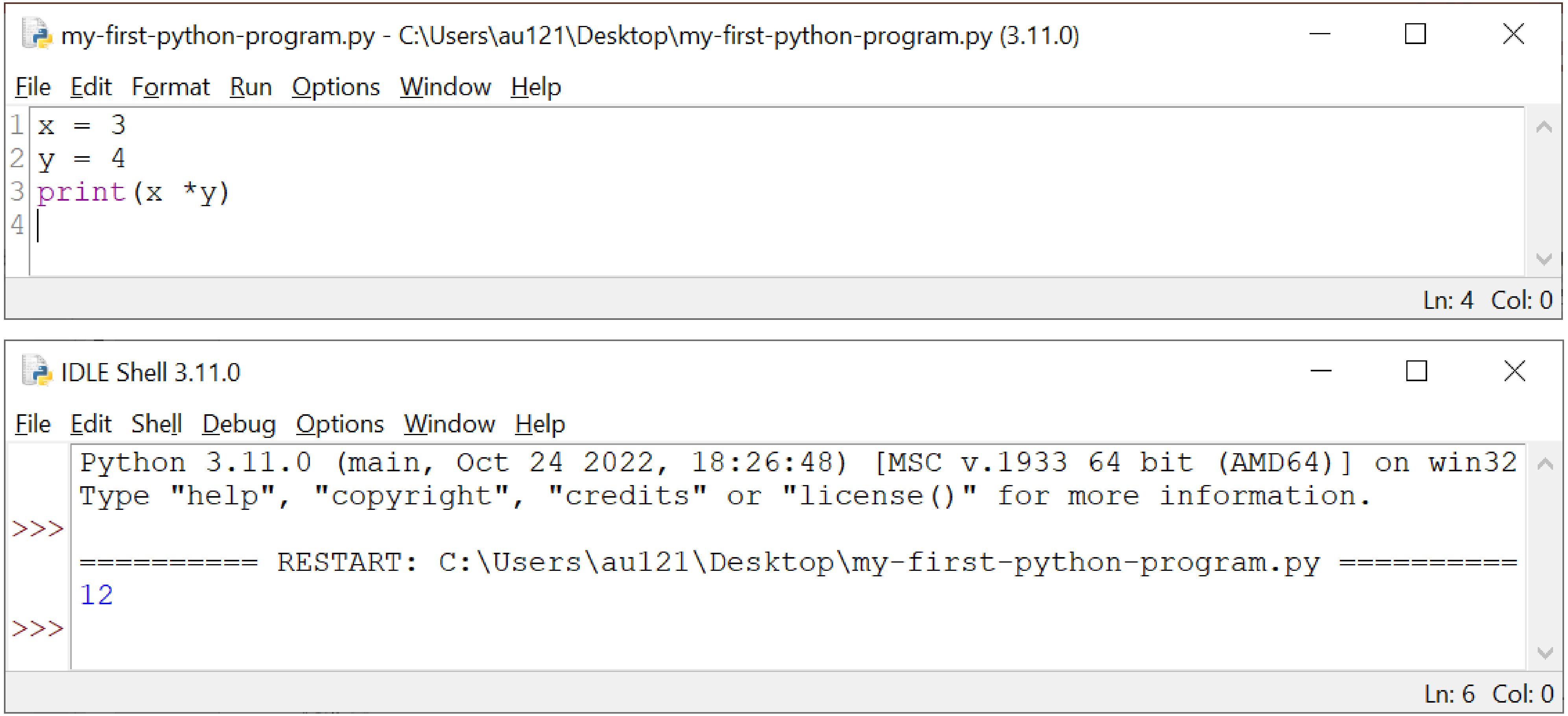



- Open Notepad (or TextEdit on Mac)
  - write a simple Python program
  - save it
- Open a command prompt
  - go to folder (using cd)
  - run the program using

```
python <program name>.py
```

# … or open IDLE and run program with F5




enable
line numbers
under options

- IDLE ships with Python from python.org
- Good beginner IDE (Integrated Development Environment)

# The Python Ecosystem

- **Interpreters/compiler**
  - CPython – reference C implementation from python.org
  - PyPy – written in RPython (a subset of Python) – faster than Cpython
  - Jython – written in Java and compiles to Java bytecode, runs on the JVM
  - IronPython – written in C#, compiles to Microsoft's Common Language Runtime (CLR) bytecode
  - Cython – project translating Python-ish code to C
- **Shells (IPython, IDLE, Jupyter)**
- **Libraries/modules/packages**
  - pypi.python.org/pypi (PyPI - the Python Package Index, +400.000 packages)
- **IDEs (Integrated development environment)**
  - IDLE comes with Python (docs.python.org/3/library/idle.html)
  - Anaconda w. Spyder, IPython (www.anaconda.com/download)
  - Canopy (enthought.com/product/canopy)
  - Visual Studio Code (code.visualstudio.com)
  - Python tools for Visual Studio (github.com/Microsoft/PTVS)
  - PyCharm (www.jetbrains.com/pycharm/)
  - Emacs (Python mode and ElPy mode)
  - Notepad++
- **Python Style guide (PEP8)**
  - pylint, pep8, flake8
- **Python online**
  - Google colab (colab.research.google.com), repl.it, sagemath.org, …

Try to google ”best ide python”

# Python basics

- Comments
- ”;”
- Variable names
- int, float, str
- type conversion
- assignment (=)
- print(), help(), type()

# Python comments

A ‘#’ indicates the beginning of a comment. From ‘#’ until of end of line is ignored by Python.

```
x = 42  # and here goes the comment
```

Comments useful to describe what a piece of code is supposed to do, what kind of input is expected, what is the output, side effects...

# The ";" in Python

- Normally statements follow in consecutive lines with identical indentation

```
x = 1
y = 1
```

- but Python also allows multiple statements on one line, separated by ";"

```
x = 1; y = 1
```

Command Prompt

```
Microsoft Windows [Version 10.0.14393]
(c) 2016 Microsoft Corporation. All rights reserved.

C:\Users\au121>cd Desktop

C:\Users\au121\Desktop>pylint semicolon.py
No config file found, using default configuration
************* Module semicolon
C:  1, 0: Missing module docstring (missing-docstring)
C:  1, 0: Constant name "x" doesn't conform to UPPER_CASE naming style (invalid-name)
C:  1, 7: More than one statement on a single line (multiple-statements)
C:  1, 7: Constant name "y" doesn't conform to UPPER_CASE naming style (invalid-name)

-------------------------------------
Your code has been rated at -10.00/10


C:\Users\au121\Desktop>flake8 semicolon.py
semicolon.py:1:6: E702 multiple statements on one line (semicolon)

C:\Users\au121\Desktop>
```

neither **pylint** or **flake8** like ";"

- General Python PEP 8 guideline: **avoid using ";"**
- Other languages like C, C++ and Java require ";" to end/separate statements

# Variable names

- Variable name = sequence of **letters** ‘a’-’z’, ‘A’-’Z’, **digits** ‘0’-’9’, and **underscore** ‘_’

```
v, volume, height_of_box, WidthOfBox, x0, _v12_34B, _
```

(snake_case) (CamelCase)

  - a name cannot start with a digit
  - names are case sensitive (`AB`, `Ab`, `aB` and `ab` are different variables)

- Variable names are references to objects in memory
- **Use meaningful variables names**
- **Python 3 reserved keywords:** `and`, `as`, `assert`, `break`, `class`, `continue`, `def`, `del`, `elif`, `else`, `except`, `False`, `finally`, `for`, `from`, `global`, `if`, `import`, `in`, `is`, `lambda`, `nonlocal`, `None`, `not`, `or`, `pass`, `raise`, `return`, `True`, `try`, `while`, `with`, `yield`

# Question – Not a valid Pyton variable name?

a) `print`

b) `for` Python reserved keyword

c) `_100`

d) `x`

e) `_`

f) `python_for_ever`

g) Don’t know

**Python shell**

```
> print = 7
> print(42)
| Traceback (most recent call last):
|   File "<stdin>", line 1, in <module>
| TypeError: 'int' object is not callable
```

`print` is a valid variable name, with default value a builtin *function* to print output to a shell – assigning a new value to `print` is very likely a bad idea (like many others `sum`, `int`, `str`, ...)

# Integer literals

- …. `-4`, `-3`, `-2`, `-1`, `0`, `1`, `2`, `3`, `4` ….
- Python integers can have an arbitrary number of digits
(only limited by machine memory)
- Can be preceded by a plus (`+`) or minus (`-`)
- For readability underscores (_) can be added between digits,

`2_147_483_647`

(for more, see PEP 515 - Underscores in Numeric Literals)

# Question – What statement will not fail?

a) `x = _42`
b) `_10 = -1_1`
c) `x = 1___0`
d) `x = +1_0_`
e) Don't know

# Float literals

- Decimal numbers are represented using **float** – contain "." or "e"
- Examples
  - `3.1415`
  - `-.00134`
  - `124e3` $= 124 \cdot 10^3$
  - `-2.345e2 = -234.5`
  - `12.3e-4 = 0.00123`
- Floats are often only approximations, e.g. `0.1` is *not* 1/10
- Extreme values (CPython)
  - max = 1.7976931348623157e+308
  - min = 2.2250738585072014e-308
- NB: Use module `fractions` for exact fractions/rational numbers.

**Python shell**

```
> 0.1 + 0.2 + 0.3
| 0.6000000000000001
> (0.1 + 0.2) + 0.3
| 0.6000000000000001
> 0.1 + (0.2 + 0.3)
| 0.6
> type(0.1)
| <class 'float'>
> 1e200 * 1e300
| inf
> 0.1+(0.2+0.3) == (0.1+0.2)+0.3
| False
> x = 0.1 + 0.2
> y = 0.3
> x == y
| False
> print(f'{x:.30f}')   # 30 decimals
| 0.300000000000000044408920985006
> print(f'{y:.30f}')   # 30 decimals
| 0.299999999999999988897769753748
> import sys
> sys.float_info.min
| 2.2250738585072014e-308
> sys.float_info.max
| 1.7976931348623157e+308
```

Associativity rule does not apply to floats

# Question – What addition order is ”best”?

a) `1e10 + 1e-10 + -5e-12 + -1e10`

b) `1e10 + -1e10 + 1e-10 + -5e-12`

c) `1e-10 + 1e10 + -1e10 + -5e-12`

d) `-5e-12 + -1e10 + 1e10 + 1e-10`

e) Any order is equally good

f) Don’t know

```
 1e10  =  10000000000
-1e10  = -10000000000
 1e-10 =             0.0000000001
-5e-12 =            -0.000000000005
```

**Python shell**

```
> 1e10 + 1e-10 + -5e-12 + -1e10
| 0.0
> 1e10 + -1e10 + 1e-10 + -5e-12
| 9.500000000000001e-11
> 1e-10 + 1e10 + -1e10 + -5e-12
|  -5e-12
> -5e-12 + -1e10 + 1e10 + 1e-10
|  1e-10
```

a) - d) give four different outputs

# Approximating
# π = 3.14159265359...

$$\frac{\pi^2}{6} = \sum_{k=1}^{+\infty} \frac{1}{k^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$

$$= 1.6449340668\ldots$$

Riemann zeta function ζ(2)

**pi_approximation_riemann.py**

```python
apx = 0.0
k = 0.0
while True:
    k = k + 1.0
    apx = apx + 1.0 / (k * k)
    print(k, apx)
```

**Output**

```
...
94906261.0 1.6449340578345741
94906262.0 1.6449340578345744
94906263.0 1.6449340578345746
94906264.0 1.6449340578345748
94906265.0 1.644934057834575
94906266.0 1.644934057834575
94906267.0 1.644934057834575
94906268.0 1.644934057834575
94906269.0 1.644934057834575
94906270.0 1.644934057834575
...
```

⚠ This is not a course in numeric computations – but now you are warned....

# Python float ≡ IEEE-754 double precision*

- A binary number is a number in base 2 with digits/bits from {0,1}

$$10110_2 = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 16 + 4 + 2 = 22_{10}$$

- IEEE-754 64-bit double

| sign $s$ 1 bit | exponent $e$ 11 bits | coefficient $c$ 52 bits |
|---|---|---|

$s = 0$ $c$

+1 0011101101011100111010101010101010101010101011101100 1001010101100.0100011110010101111001110101010111110101100110101

$\underbrace{\phantom{0000000000}}_{e\,-\,1023 \text{ bits}}$ $\uparrow 2^{-6}$

| Float value | Case |
|---|---|
| $(-1)^s \cdot (1 + c \cdot 2^{-52}) \cdot 2^{e\,-\,1023}$ | $0 < e < 2047$ |
| $(-1)^s \cdot c \cdot 2^{-1074}$ | $e = 0, c \neq 0$ |
| +0 and -0 | $e = 0, c = 0$ |
| +∞ and -∞ | $e = 2047, c = 0$ |
| NaN ("not a number") | $s = 0, e = 2047, c \neq 0$ |

**Python shell**

```
> 1e200 * 1e200
| inf
> -1e200 * 1e200
| -inf
> 1e-200 * 1e-200
| 0.0
> -1e-200 * 1e-200
| -0.0
> 1e200 * 1e200 * 0.0
| nan
```

(*most often, but there is no guarantee given in the Python language specification that floats are represented using IEEE-754)

# String literals (type `str`)

- Sequence of characters enclosed by single ( `'` ) or double ( `"` ) quotes

```
"a 'quoted' word"    "Hello World"    'abc'
'a "quoted" word'    '_"_\'_"_'
```

- Escape characters

| | |
|---|---|
| `\n` | newline |
| `\t` | tab |
| `\\` | backslash |
| `\'` | single quote |
| `\"` | double quote |

- A backslash ( `\` ) a the end of line, will continue line/string on next line
- Use triple single or double quotes ( `'''` or `"""` ) for enclosing strings spanning more lines

  (in particular for Python Dosctrings, see PEP 257)

**string-test.py**

```
print("abc")
print('de\'f')
print("'ghi'")
print("'jk\nl'\"")
print("mn\
o")
print("p\\q\tr")
```

**Output**

```
$ python string-test.py
abc
de'f
'ghi'
'jk
l'"
mno
p\q	r
```

# Question – What does the following print ?

```
print("\\\\"\\n\n'")
```

a) `\\\"\\n\n'`

b) `\"\nn'`

c)
```
\"\n
'
```

d) `"nn'`

e)
```
\"

'
```

f) Don't know

# Long string literals

- Long string literals often need to be split over multiple lines
- In Python two (or more) string literals following each other will be treated as a single string literal (they can use different quotes)
- Putting parenthesis around multiple literals allows line breaks
- Advantages:
  - avoids the backslash at the end of line
  - can use indentation to increase readability
  - allows comments between literals

**long-string-literals.py**

```
s1 = 'abc' "def"  # two string literals
print(s1)
s2 = '"' "'" '"'  # avoid escaping quotes
print(s2)
s3 = 'this is a really, really, really, \
really, really, long string'
print(s3)
s4 = ('this is a really, really, '
      'really, really, really, '
      'long string')
print(s4)
very_very_long_variable_name = (
    'this is a really, really, '   # line 1
    'really, really, really, '     # line 2
    "long string"                  # line 3
)
print(very_very_long_variable_name)
```

**Python shell**

```
| abcdef
| "'"
| this is a really, really, really, really,
  really, long string
| this is a really, really, really, really,
  really, long string
| this is a really, really, really, realy,
  really, long string
```

# Raw string literals

- By prefixing a string literal with an `r`, the string literal will be considered a raw string and backslashes become literal characters
- Useful in cases where you actually need backslashes in your strings, e.g. when working with Python's regular expression module `re`

**Python shell**

```
> print('\let\epsilon\varepsilon')      # \v = vertical tab
| \let\epsilon
  arepsilon
> print('\\let\\epsilon\\varepsilon')   # many backslashs
| \let\epsilon\varepsilon
> print(r'\let\epsilon\varepsilon')     # more readable
| \let\epsilon\varepsilon
```

# print(...)

- `print` can print zero, one, or more values
- default behavior
  - print a space between values
  - print a line break after printing all values
- default behavior can be changed by **keyword arguments** “`sep`“ and “`end`“

**Python shell**

```
> print()
|
> print(7)
| 7
> print(2, 'Hello')
| 2 Hello
> print(3, 'a', 4)
| 3 a 4
> print(3, 'a', 4, sep=':')
| 3:a:4
> print(5); print(6)
| 5
| 6
> print(5, end=', '); print(6)
| 5, 6
```

# print(...) and help(...)

**Python shell**

```
> help(print)
| Help on built-in function print in module builtins:
|
| print(...)
|     print(value, ..., sep=' ', end='\n', file=sys.stdout, flush=False)
|
|     Prints the values to a stream, or to sys.stdout by default.
|     Optional keyword arguments:
|     file:  a file-like object (stream); defaults to the current sys.stdout.
|     sep:   string inserted between values, default a space.
|     end:   string appended after the last value, default a newline.
|     flush: whether to forcibly flush the stream.
```

# Assignments

- *variable = expression*

```
x = 42
```

- Multiple assignments – right hand side evaluated before assignment

```
x, y, z = 2, 5, 7
```

- Useful for swapping

```
x, y = y, x
```

- Assigning multiple variables same value in left-to-right

```
x = y = z = 7
```

**Warning**

```
i = 1
i = v[i] = 3  # v[3] is assigned value 3
```

In languages like C and C++ instead v[1] is assigned 3

# Python is dynamically typed, type(...)

- The current type of a value can be inspected using the **type()** function (that returns a type object)
- In Python the values contained in a variable over time can be of different type
- In languages like C, C++ and Java variables are declared with a given type, e.g.

```
int x = 42;
```

and the different values stored in this variable must remain of this type

**Python shell**

```
> x = 1
> type(x)
| <class 'int'>
> x = 'Hello'
> type(x)
| <class 'str'>
> type(42)
| <class 'int'>
> type(type(42))
| <class 'type'>
```

x new type

# Type conversion

- Convert a value to another type:

  *new-type*(*value*)

- Sometimes done automatically:

`1.0+7=1.0+float(7)=8.0`

**Python shell**

string concatenation

```
> float(42)
| 42.0
> int(7.8)
| 7
> x = 7
> print("x = " + x)
| Traceback (most recent call last):
|   File "<stdin>", line 1, in <module>
| TypeError: must be str, not int
> print("x = " + str(x))
| x = 7
> print("x = " + str(float(x)))
| x = 7.0
> int("7.3")
| Traceback (most recent call last):
|   File "<stdin>", line 1, in <module>
| ValueError: invalid literal for int() with base 10: '7.3'
> int(float("7.3"))
| 7
```

# Questions – `str(float(int(float("7.5"))))` ?

a) 7
b) 7.0
c) 7.5
d) "7"
e) "7.0"
f) "7.5"
g) Don't know

# Control structures

- input()
- if-elif-else
- while-break-continue

# input

- The builtin function `input(`*message*`)` prints *message*, and waits for the user provides a line of input and presses return. The line of input is returned as a `str`
- If you e.g. expect input to be an `int`, then remember to convert the input using `int()`

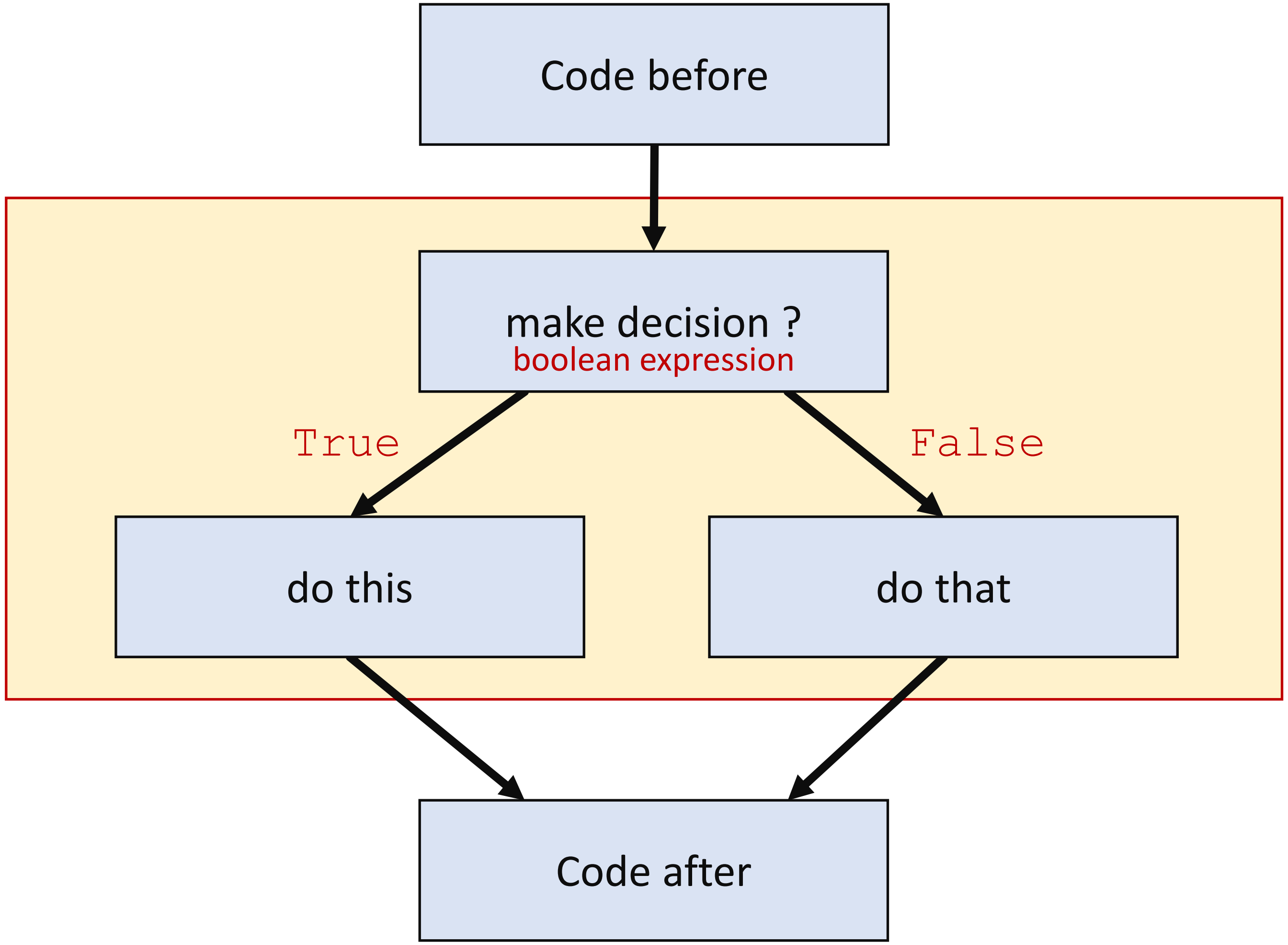
**name-age.py**

```
name = input('Name: ')
age = int(input('Age: '))
print(name, 'is', age, 'years old')
```

**Python shell**

```
> Name: Donald Duck
> Age: 84
| Donald Duck is 84 years old
```

# Branching – do either this or that ?



# Basic if-else

`if` *boolean expression*:

identical indentation
: *code*
  *code*
  *code*

`else:`

identical indentation
: *code*
  *code*
  *code*

**if-else.py**

```
if x % 2 == 0:
    print('even')
else:
    print('odd')
```

Identical indentation for a sequence of lines = the same spaces/tabs should precede code

# pass

- `pass` is a Python statement doing nothing. Can be used where a statement is required but you want to skip (e.g. code will be writen later)

- Example (bad example, since `else` could just be omitted):

**if-else.py**

```
if x % 2 == 0:
    print('even')
else:
    pass
```

# if-elif-else

```
if condition:
        code
elif condition:      # zero or more “elfi“ ≡ “else if”
        code
else:      # optional
        code
```

```
if (condition) {
  code
} else if (condition) {
  code
} else {
  code
}
```

Java, C, C++ syntax

Other languages using indentation for blocking: ABC (1976), occam (1983), Miranda (1985)

**if.py**

```python
if x == 0:
    print('zero')
```

**if-else.py**

```python
if x % 2 == 0:
    print('even')
else:
    print('odd')
```

**elif.py**

```python
if x < 0:
    print('negative')
elif x == 0:
    print('zero')
elif x == 1:
    print('one')
else:
    print('>= 2')
```

# Questions – What value is printed?

```
x = 1
if x == 2:
    x = x + 1
else:
    x = x + 1
    x = x + 1
x = x + 1
print(x)
```

a) 1
b) 2
c) 3
d) 4
e) 5
f) Don’t know

# Nested if-statements

**nested-if.py**

```
if x < 0:
    print('negative')
elif x % 2 == 0:
    if x == 0:
        print('zero')
    elif x == 2:
        print('even prime number')
    else:
        print('even composite number')
else:
    if x == 1:
        print('one')
    else:
        print('some odd number')
```

# Common mistake

**if-if.py**

```python
x = int(input())
if x == 0:
    print('zero')
if x % 2 == 0:
    print('even')
```

**Python shell**

```
> 0
| zero
| even
```

**if-elif.py**

```python
x = int(input())
if x == 0:
    print('zero')
elif x % 2 == 0:
    print('even')
```

**Python shell**

```
> 0
| zero
```

# if-else *expressions*

- A very common computation is

```
if test:
    x = true-expression
else:
    x = false-expression
```

- In Python there is a shorthand for this:

```
x = true-expression if test else false-expression
```

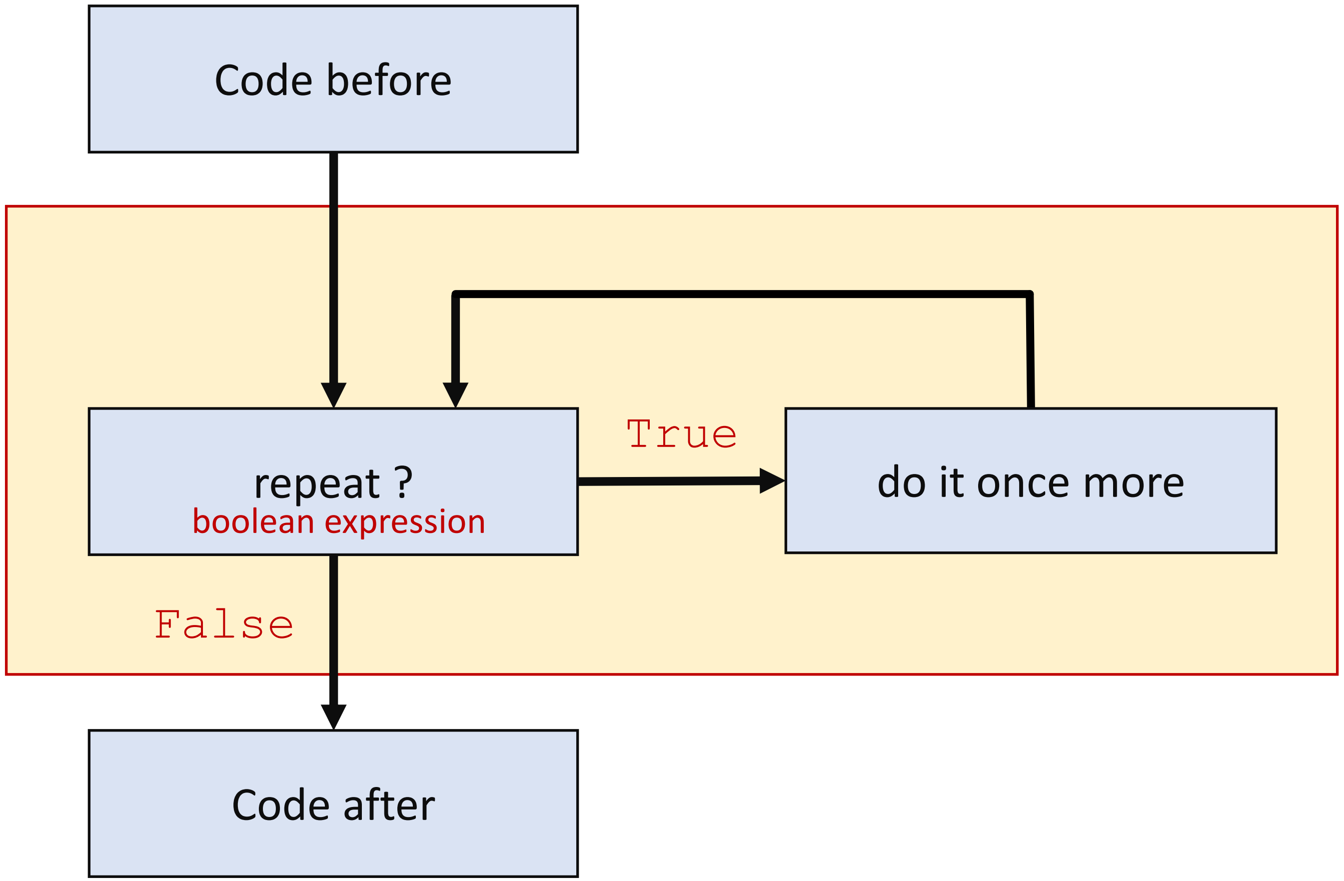
think of this as the "common case" and the "exceptional case"

(see What's New in Python 2.5 - **PEP 308: Conditional Expressions**)

- In C, C++ and Java the equivalent notation is (note the different order)

```
x = test ? true-expression : false-expression
```

# Repeat until done



# while-statement

```
while condition:
    code
    ...
    break  # jump to code after while loop
    ...
    continue #  jump to condition at the
    ...      #  beginning of while loop
```

The function `randint(a, b)` from module `random` returns a random integer from {a, a + 1,..., b – 1, b}

**random-pair.py**

```
from random import randint
while True:
    x = randint(1, 10)
    y = randint(1, 10)
    if abs(x - y) >= 2:
        break
    print('too close', x, y)
print(x, y)
```

**Python shell**

```
| too close 4 4
| too close 10 9
| 8 5
```

**count.py**

```
x = 1
while x <= 5:
    print(x, end=' ')
    x = x + 1
print('and', x)
```

**Python shell**

```
| 1 2 3 4 5 and 6
```

```
while (condition) {
  code
}
```

Java, C, C++ syntax

# Computing $\lfloor\sqrt{x}\rfloor$ using binary search

An exercise asks to simplify the code

int-sqrt.py

```
x = 20
low = 0
high = x + 1
while True:   # low <= sqrt(x) < high
    if low + 1 == high:
        break
    mid = (high + low) // 2
    if mid * mid <= x:
        low = mid
        continue
    high = mid
print(low)   # low = floor(sqrt(x))
```

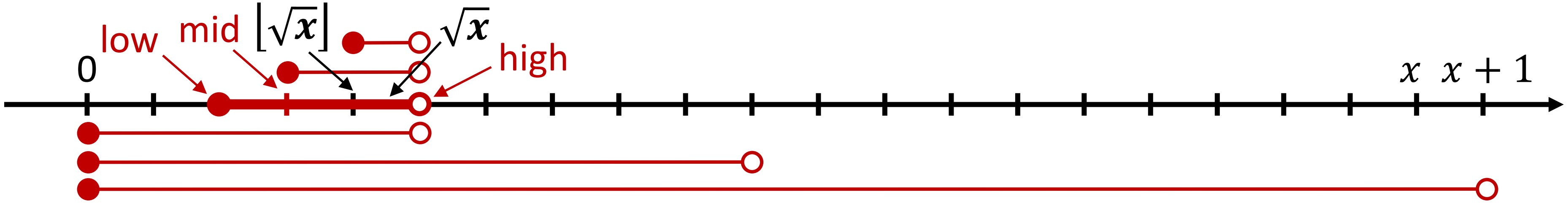
Integer division

$$\left\lfloor \frac{\mathtt{high}+\mathtt{low}}{2} \right\rfloor$$

$$\mathtt{mid} \leq \sqrt{x}$$

$$\Updownarrow$$

$$\mathtt{mid}^2 \leq x$$

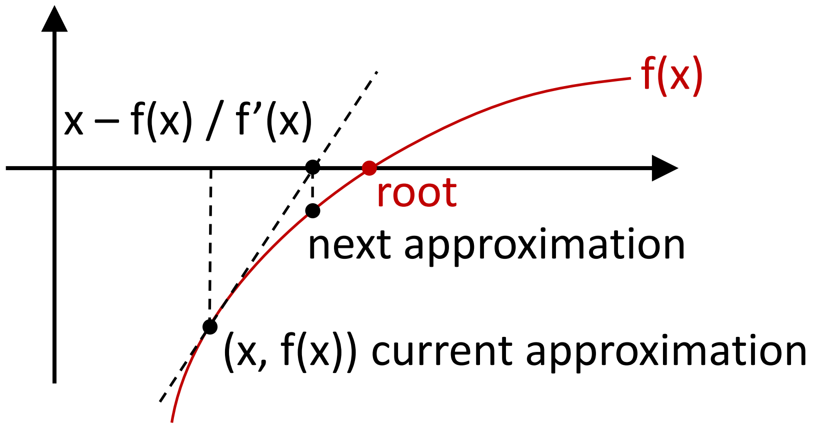


# Division using the Newton-Raphson method

- **Goal:** Compute 1 / n only using +, -, and *
- x = 1 / n $\iff$ f(x) = n – 1 / x = 0
- Problem reduces to finding root of f
- Newton-Raphson:

$$x := x - f(x)/f'(x) = x-(n-1/x)/(1/x^2) = (2-n\cdot x)\cdot x$$

since $f'(x) = 1 / x^2$ for $f(x) = n - 1 / x$




**division.py**

```python
n = 0.75  # n in [0.5, 1.0]
x = 1.0
last = 0.0
while last < x:
    print(x)
    last = x
    x = (2 - n * x) * x
print('Apx of 1.0 /', n, '=', x)
print('Python 1.0 /', n, '=', 1.0 / n)
```

**Python shell**

```
| 1.0
| 1.25
| 1.328125
| 1.3331298828125
| 1.333333330228925
| 1.3333333333333333
| Apx of 1.0 / 0.75 = 1.3333333333333333
| Python 1.0 / 0.75 = 1.3333333333333333
```

en.wikipedia.org/wiki/Newton's_method

# Operations

- None, bool
- basic operations
- strings
- += and friends

# NoneType

- The type None has only one value: `None`
- Used when context requires a value, but none is really available
- **Example**: All functions must return a value. The function `print` has the *side-effect* of printing something to the standard output, but returns `None`
- **Example**: Initialize a variable with no value, e.g. list entries `mylist = [None, None, None]`

**Python shell**

```
> x = print(42)
| 42
> print(x)
| None
```

# Type bool

- The type `bool` only has two values: `True` and `False`

- Logic truth tables:

| *x* **or** *y* | True | False |
|---|---|---|
| True | True | True |
| False | True | False |

| *x* **and** *y* | True | False |
|---|---|---|
| True | True | False |
| False | False | False |

| *x* | **not** *x* |
|---|---|
| True | False |
| False | True |

# Scalar vs Non-scalar Types

- **Scalar types** (atomic/indivisible): `int`, `float`, `bool`, `None`
- **Non-scalar**: Examples `strings` and `lists`

```
"string"[3] = "i"
[2, 5, 6, 7][2] = 6
```

# Questions – What is `[7,3,5][[1,2,3][1]]` ?

a) 1
b) 2
c) 3

d) 5
e) 7
f) Don’t know

# Operations on int and float

**Result is float if and only if at least one argument is float, except ** with negative exponent always gives a float**

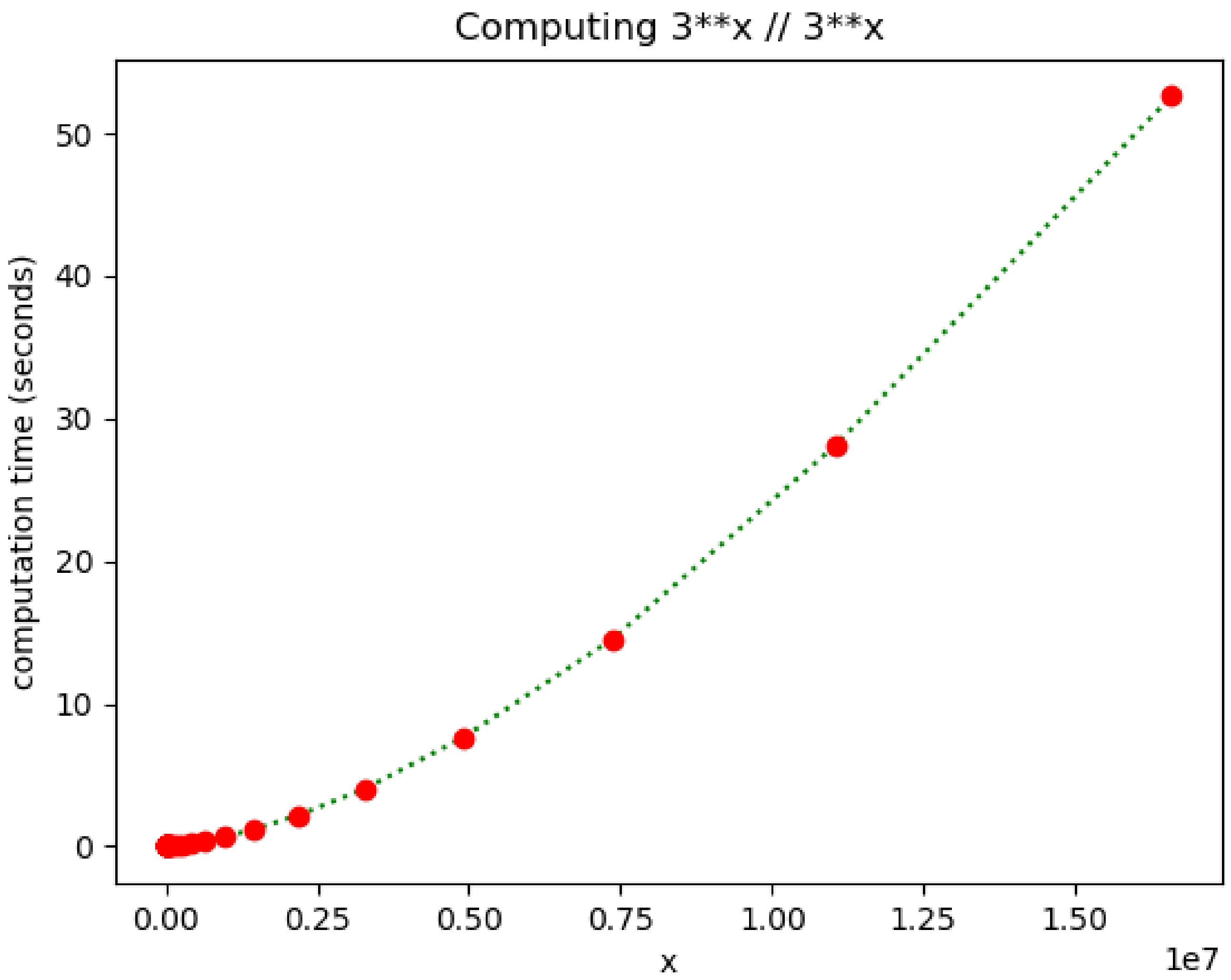
- `+`, `-`, `*` addition multiplication, e.g. `3.0*2` = `6.0`
- `**` and `pow(x, y)` power, e.g. `2**3=pow(2,3)=8`, `2**-2=0.25`
- `//` integer division = ⌊x / y⌋
  e.g. `15.0//4= 3.0`. Note: `-8//3=-3`
- `/` division returns float, `6/3=2.0`
- `abs(x)` absolute value
- `%` integer division remainder (modulo)
  `11%3 =2`
  `4.7%0.6=0.50000000000000003`

**Python shell**

```
> 0.4 // 0.1
| 4.0
> 0.4 / 0.1
| 4.0
> 0.3 // 0.1
| 2.0
> 0.3 / 0.1
| 2.9999999999999996
> 10**1000 / 2
| OverflowError: integer division
  result too large for a float
```

# Running time for 3 ** x // 3 ** x

Working with larger integers takes slightly more than linear time in the number of digits



integer-division-timing.py

```python
from time import time
import matplotlib.pyplot as plt

bits, compute_time = [], []

for i in range(42):
    x = 3 ** i // 2 ** i
    start = time()
    result = 3 ** x // 3 ** x     # the computation we time
    end = time()
    t = end - start
    print('i =', i, 'x =', x, 'Result =', result, 'time(sec) =', t)
    bits.append(x)
    compute_time.append(t)

plt.title('Computing 3**x // 3**x')
plt.xlabel('x')
plt.ylabel('computation time (seconds)')
plt.plot(bits, compute_time, 'g:')
plt.plot(bits, compute_time, 'ro')
plt.show()
```

# module math

Many standard mathematical functions are available in the Python module "`math`", e.g.

sqrt, sin, cos, tan, asin, acos, atan, log(natural), log10, exp, ceil, floor, ...

- To use all the functions from the math module use `import math`
  Functions are now available as e.g. `math.sqrt(10)` and `math.ceil(7.2)`
- To import selected functions you instead write `from math import sqrt, ceil`
- The library also contains some constants, e.g.
  `math.pi` = 3.141592... and `math.e` = 2.718281...
- Note: `x ** 0.5` significantly faster than `sqrt(x)`

**Python shell**

```
> (0.1 + 0.2) * 10
| 3.0000000000000004
> math.ceil((0.1 + 0.2) * 10)
| 4
```

docs.python.org/3/library/math.html

module `statistics` contains basic functions like mean and variance, see docs.python.org/3/library/statistics.html

**Python shell**

```
> import math
> math.sqrt(8)
| 2.8284271247461903
> from math import pi, sqrt
> pi
| 3.141592653589793
> sqrt(5)
| 2.23606797749979
> from math import sqrt as kvadratrod
> kvadratrod(3)
| 1.7320508075688772

> import timeit
> timeit.timeit("1e10**0.5")
| 0.021124736888936863
> timeit.timeit("sqrt(1e10)", "from math import sqrt")
| 0.1366314052865789
> timeit.timeit("math.sqrt(1e10)", "import math")
| 0.1946660841634582
```

docs.python.org/3.6/library/timeit.html

# Rounding up integer fractions

- Python: ⌈`x/y`⌉ = `-(-x//y)`

| -(-13/3) | | |
|---|---|---|
| **Python** | **Java** | **C** |
| -(-13//3) = 5 | -(-13/3) = 4 | -(-13/3) = 4 |

The intermediate result `x/y` in

`math.ceil(x/y)`

is a float with limited precision

- Alternative computation:

⌈`x/y`⌉ = `(x+(y-1))//y`

**Python shell**

```
> from math import ceil
> from timeit import timeit
> 13 / 3
| 4.333333333333333
> 13 // 3
| 4
> -13 // 3
| -5
> -(-13 // 3)
| 5
> ceil(13 / 3)
| 5
> -(-22222222222222222223 // 2)
| 11111111111111111112
> ceil(22222222222222222223 / 2)
| 11111111111111110656
> timeit('ceil(13 / 3)', 'from math import ceil')
| 0.27746671276099973
> timeit('-(-13 // 3)')   # negation trick is fast
| 0.05231945830200857
```

# floats : Overflow, inf, -inf, nan

- There exists special float values
  `inf, -inf, nan`
  representing “+infinity”, “-infinity” and “not a number”
- Can be created using e.g.
  `float('inf')`
  or imported from the `math` module
- Some overflow operations generate an `OverflowError`, other return `inf` and allow calculations to continue !
- Read the IEEE 754 standard if you want to know more details...

**Python shell**

```
> 1e250 ** 2
| OverflowError:
  (34, 'Result too large')
> 1e250 * 1e250
| inf
> -1e250 * 1e250
| -inf
> import math
> math.inf
| inf
> type(math.inf)
| <class 'float'>
> math.inf / math.inf
| nan
> type(math.nan)
| <class 'float'>
> math.nan == math.nan
| False
> float('inf') - float('inf')
| nan
```

# Operations on bool

- The operations `and`, `or`, and `not` behave as expected when the arguments are `False`/`True`.
- The three operators also accept other types, where the following values are considered *false*:

  `False, None, 0, 0.0, "", [],…`

  (see The Python Standard Library > 4.1. True Value Testing for more *false* values)
- *Short-circuit evaluation*: The rightmost argument of `and` and `or` is only evaluated if the result cannot be determined from the leftmost argument alone. The result is either the leftmost or rightmost argument (see truth tables), i.e. the result is not necessarily `False`/`True`.

  `True or 7/0` is completely valid since 7/0 will never be evaluated (which otherwise would throw a ZeroDivisionError exception)

| `x` | `x or y` |
|---|---|
| *false* | `y` |
| otherwise | `x` |

| `x` | `x and y` |
|---|---|
| *false* | `x` |
| otherwise | `y` |

| `x` | `not x` |
|---|---|
| *false* | `True` |
| otherwise | `False` |

# Questions – What is `"abc" and 42` ?

a) `False`
b) `True`
c) `"abc"`
d) `42`
e) TypeError
f) Don’t know

# Comparison operators (e.g. int, float, str)

`==` test if two objects are equal, returns bool

not to be confused with the assignment operator (=)

`!=` not equal

`>`

`>=`

`<`

`<=`

**Python shell**

```
> 3 == 7
| False
> 3 == 3.0
| True
> "-1" != -1
| True
> "abc" == "ab" + "c"
| True
> 2 <= 5
| True
> -5 > 5
| False
> 1 == 1.0
| True
> 1 == 1.0000000000000001
| True
> 1 == 1.000000000000001
| False
```

# Chained comparisons

- A recurring condition is often

```
x < y and y < z
```

- If `y` is a more complex expression, we would like to avoid computing `y` twice, i.e. we often would write

```
tmp = complex expression
x < tmp and tmp < z
```

- In Python this can be written as a chained comparisons (which is shorthand for the above)

```
x < y < z
```

- Note: Chained comparisons do not exist in C, C++, Java, ...

# Questions – What is `1 < 0 < 6/0` ?

a) `True`
b) `False`
c) `0`
d) `1`
e) `6`
f) ZeroDivisionError
g) Don't know

# Binary numbers and operations

- Binary number = integer written in base 2: $101010_2 = 42_{10}$
- Python constant prefix 0b: `0b101010` → `42`
- `bin(x)` converts integer to string: `bin(49)` → `"0b110001"`
- `int(x,2)` converts binary string value to integer: `int("0b110001",2)` → `49`
- Bitwise operations
  - `|` Bitwise OR
  - `&` Bitwise AND
  - `~` Bitwise NOT (`~ x` equals to `-x - 1`)
  - `^` Bitwise XOR
- Example: `bin(0b1010 | 0b1100)` → `"0b1110"`
- Hexadecimal = base 16, Python prefix 0x: `0x30` → `48`, `0xA0` → `160`, `0xFF` → `255`
- `<<` and `>>` integer bit shifting left and right, e.g. `12 >> 2` → `3`, and `1 << 4` → `16`

# Operations on strings

- `len(`*str*`)` returns length of *str*
- *str*`[`*index*`]` returns *index*+1'th symbol in *str*
- $str_1$ `+` $str_2$ returns concatenation of two strings
- *int* `*` *str* concatenates *str* with itself *int* times
- Formatting: % operator or .format() function

  old Python 2 way    since Python 3.0

  or formatted string literals (f-strings) with prefix

  since Python 3.6

  letter `f` and Python expressions in `{}`

  (see pyformat.info for an introduction)

**From "What's New In Python 3.0", 2009:** *A new system for built-in string formatting operations replaces the % string formatting operator. (However, the % operator is still supported; it will be deprecated in Python 3.1 and removed from the language at some later time.) Read PEP 3101 for the full scoop.*

% formatting (inherited from C's sprint() function) was supposed to be on the way out - but is still going strong in Python 3.10

**Python shell**

```
> len("abcde")
| 5
> "abcde"[2]
| 'c'
> x = 2; y = 3
> "x = %s, y = %s" % (x, y)
| 'x = 2, y = 3'
> "x = {}, y = {}".format(x, y)
| 'x = 2, y = 3'
> f'x + y = {x + y}'
| 'x + y = 5'
> f'{x + y = }'   # >= Python 3.8
| 'x + y = 5'
> f'{x} / {y} = {x / y:.3}'
| '2 / 3 = 0.667'
> "abc" + "def"
| 'abcdef'
> 3 * "x--"
| 'x--x--x--'
> 0 * "abc"
| ''
```

# … more string functions

- *str*`[`−*index*`]` returns the symbol i positions from the right, the rightmost *str*`[-1]`
- *str*`[`*from*`:`*to*`]` substring starting at index *from* and ending at index *to*-1
- *str*`[`*from*`:`−*to*`]` substring starting at *form* and last at index `len(`*str*`)` − *to* `-1`
- *str*`[`*from*`:`*to*`:`*step*`]` only take every *step*'th symbol in *str*[*from*:*to*]
  - *from* or/and *to* can be omitted and defaults to the beginning/end of string
- `chr(`*x*`)` returns a string of length 1 containing the *x*'th Unicode character
- `ord(`*str*`)` for a string of length 1, returns the Unicode number of the symbol
- *str*`.lower()` returns string in lower case
- *str*`.split()` split string into list of words, e.g.

```
"we love python".split() = ['we', 'love', 'python']
```

docs.python.org/3/library/string.html

# Questions – What is `s[2:42:3]`?

```
     0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44
s = 'abwwdexy   lwtopavghevt xypxxyattx hxwoadnxxx'
```

a) `'wwdexy___lwtopavghevt_xypxxyattx_hxwoadn'`

b) `'we_love_python'`

c) `'we_love_java'`

d) Don’t know

# Strings are immutable

- Strings are non-scalar, i.e. for `s = "abcdef"`, `s[3]` will return `"d"`
- Strings are immutable and cannot be changed once created. I.e. the following natural update is not possible (but is e.g. allowed in C)

```
s[3] = "x"
```

- To replace the `"d"` with `"x"` in `s`, instead do the following update

```
s = s[:3] + "x" + s[4:]
```

# Operators
# Precedence rules & Associativity

Example: * has higher precedence than +

2 + 3 * 4 ≡ 2 + (3 * 4) → 14 and (2 + 3) * 4 → 20

All operators in same group are evaluated left-to-right

2 + 3 - 4 - 5 ≡ ((2 + 3) - 4) - 5 → -4

except for **, that is evaluated right-to-left

2**2**3 ≡ 2**(2**3) → 256

Rule: Use parenthesis whenever in doubt of precedence!

docs.python.org/3/reference/expressions.html

| Precedence (low to high) |
|---|
| `or` |
| `and` |
| `not x` |
| `in` `not in` `is` `is not` `==` `<` `<=` `!=` `>` `>=` |
| `\|` |
| `^` |
| `&` |
| `<<` `>>` |
| `+` `-` |
| `*` `@` `/` `//` `%` |
| `+x` `-x` `~x` |
| `**` |

# Long expressions

- Long expressions can be broken over several lines by putting parenthesis around it
- The PEP8 guidelines recommend to limit **all** lines to a maximum of 79 characters

**Python shell**

```
> (1
    + 2 +
        3)
| 6
```

https://www.python.org/dev/peps/pep-0008/#maximum-line-length

# += and friends

- Recurring statement is

```
x = x + value
```

- In Python (and many other languages) this can be written as

```
x += value
```

- This also applies to other operators like

```
+=   -=   *=   /=   //=   **=
|=   &=   ^=   <<=   >>=
```

**Python shell**

```
> x = 5
> x *= 3
> x
| 15
> a = 'abc'
> a *= 3
> a
| 'abcabcabc'
```

# := assignment expressions (the “Walrus Operator”)

New in Python 3.8

- Syntax

  `name := expression`

- Evaluates to the value of `expression`, with the side effect of assigning result to `name`
- Useful for naming intermediate results/repeating subexpressions for later reusage
- See PEP 572 for further details and restrictions of usage

**Python shell**

```
> (x := 2 * 3) + 2 * x
| 18
> print(1 + (x := 2 * 3), 2 + x)
| 7 8
> x := 7
| SyntaxError
> (x := 7)  # valid, but not recommended
> while line := input():
      print(line.upper())
> abc
| ABC
```

- In some languages, e.g. Java, C and C++, “`=`“ also plays the role of “`:=`“, implying “`if (x=y)`” and “`if (x==y)`” mean quite different things (common typo)

# Lists

- List syntax
- List operations
- copy.deepcopy
- range
- while-else
- for
- for-break-continue-else

# List operations

- List syntax `[`*value*$_0$`, ` *value*$_1$`, ..., ` *value*$_{k\text{-}1}$`]`
- List indexing `L[`*index*`]`, `L[-`*index*`]`
- List slices `L[`*from*`:`*to*`]`, `L[`*from*`:`*to*`:`*step*`]` or `L[slice(`*from*`,`*to*`,`*step*`)]`
- Creating a copy of a list `L[:]` or `L.copy()`
- List concatenation (creates new list) `X + Y`
- List repetition (repeated concatenation with itself) `42 * L`
- Length of list `len(L)`
- Check if element is in list `e in L`
- Index of first occurrence of element in list `L.index(e)`
- Number of occurrences of element in list `L.count(e)`
- Check if element is not in list `e not in L`
- `sum(L)` `min(L)` `max(L)`

docs.python.org/3/library/stdtypes.html#sequence-types-list-tuple-range

# List modifiers (lists are mutable)

- Extend list with elements (`X` is modified) `X.extend(Y)`
- Append an element to a list (`L` is modified) `L.append(42)`
- Replace sublist by another list (length can differ) `X[i:j] = Y`
- Delete elements from list `del L[i:j:k]`
- Remove & return element at position `L.pop(i)`
- Remove first occurrence of element `L.remove(e)`
- Reverse list `L.reverse()`
- `L *= 42`
- `L.insert(i, x)` same as `L[i:i] = [x]`

**Python shell**

```
> x = [1, 2, 3, 4, 5]
> x[2:4] = [10, 11, 12]
> x
| [1, 2, 10, 11, 12, 5]
> x = [1, 2, 11, 5, 8]
> x[1:4:2] = ['a', 'b']
| [1, 'a', 11, 'b', 8]
```

docs.python.org/3/library/stdtypes.html#sequence-types-list-tuple-range

# Questions – What is `x` ?

```
x = [1,2,3,4,5,6,7,8,9,10]
x[2:8:3] = ['a', 'b']
```

a) `[1,2,'a','b',5,6,7,8,9,10]`

b) `[1,'a',3,4,5,6,7,'b',9,10]`

c) `[1,2,3,4,5,6,7,'a','b']`

d) `[1,2,'a',4,5,'b',7,8,9,10]`

e) ValueError

f) Don’t know

# Questions – What is `y` ?

```
y = [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19]
y = y[3:15:3][1:4:2]
```

a) `[3,6,9,12,15]`

b) `[7,13]`

c) `[1,9]`

d) `[4,7,10,13,2,4]`

e) TypeError

f) Don’t know

# Nested lists (multi-dimensional lists)

- Lists can contain lists as elements, that can contain lists as elements, that ...
- Can e.g. be used to store multi-dimensional data (list lengths can be non-uniform)

**Note**: For dealing with matrices the NumPy module is a better choice

**multidimensional-lists.py**

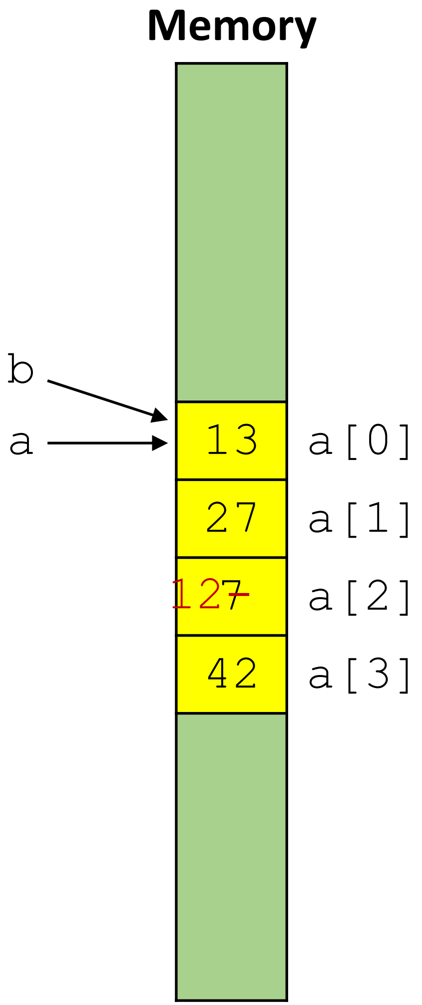
```
list1d = [1, 3, 5, 2]
list2d = [[1, 2, 3, 4],
          [5, 6, 7, 9],
          [0, 8, 2, 3]]
list3d = [[[5,6], [4,2], [1,7], [2,4]],
          [[1,2], [6,3], [2,5], [7,5]],
          [[3,8], [1,5], [4,3], [2,4]]]

print(list1d[2])
print(list2d[1][2])
print(list3d[2][0][1])
```

**Python shell**

```
| 5
| 7
| 8
```

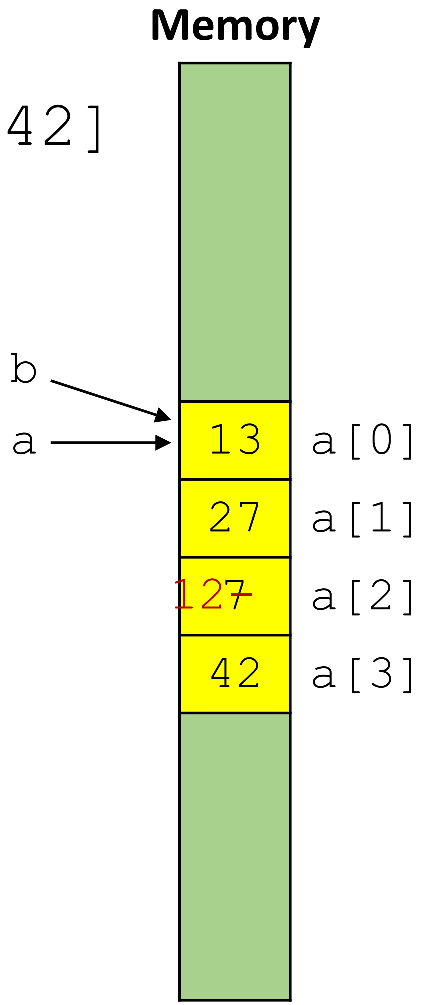
# aliasing

```
a = [13, 27, 7, 42]
b = a
a[2] = 12
```

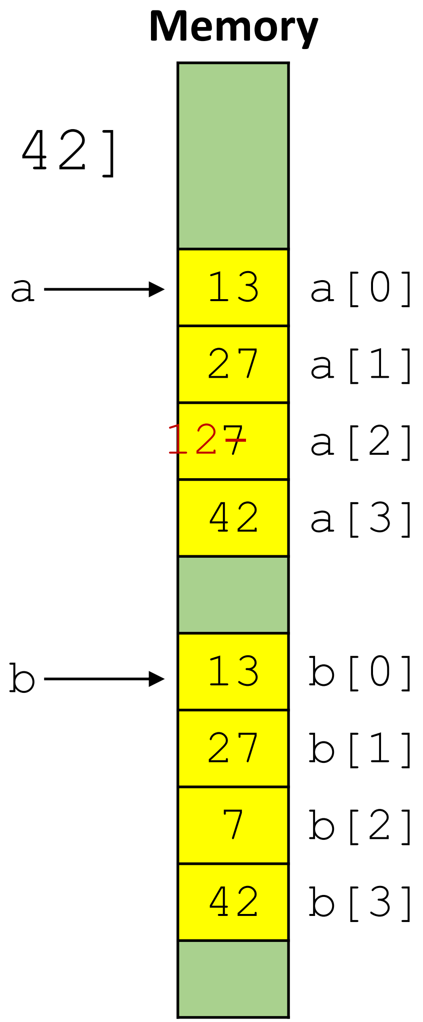


# y = x VS y = x[:]

```
a = [13, 27, 7, 42]
b = a
a[2] = 12
```

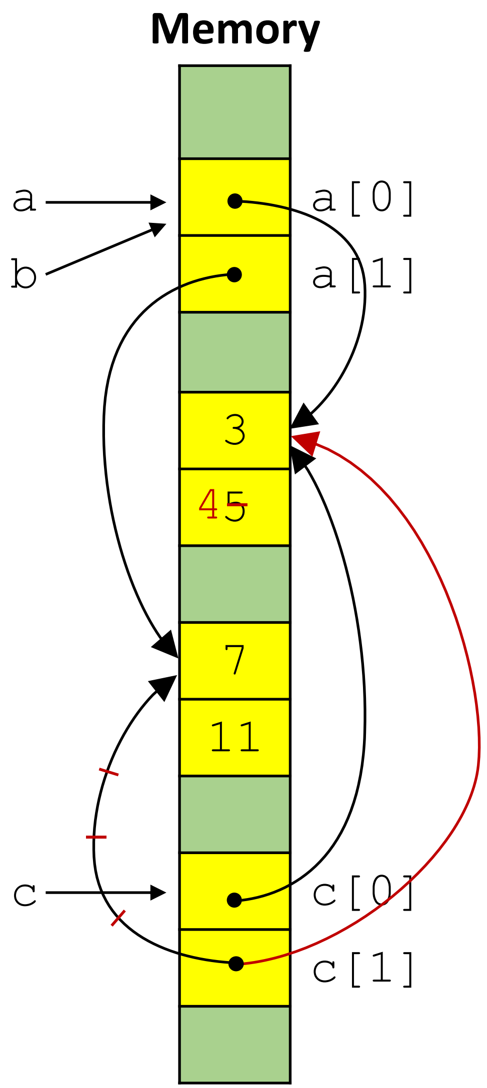



```
a = [13, 27, 7, 42]
b = a[:]
a[2] = 12
```



# `x[:]` vs nested structures

```
a = [[3,5],[7,11]]
b = a
c = a[:]
a[0][1] = 4
c[1] = b[0]
```



# Question – what is `c` ?

a) `[[3,5],[7,11]]`

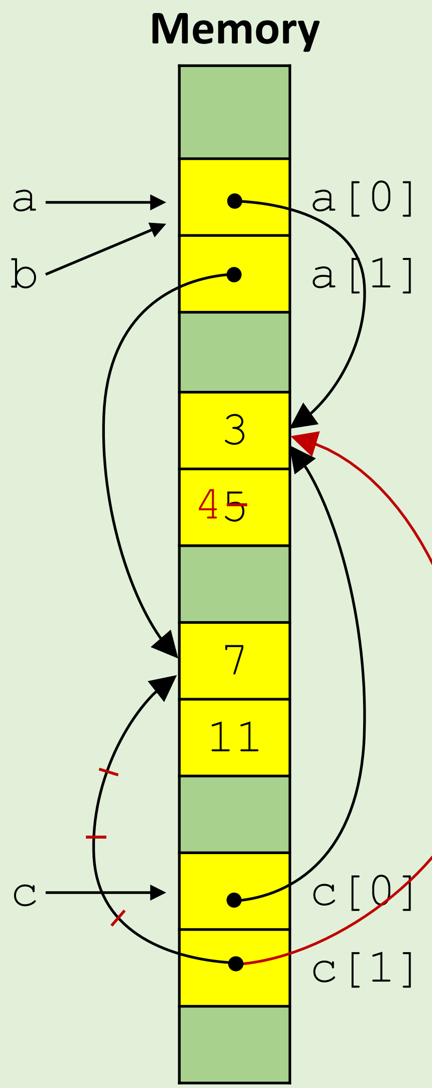
b) `[[3,5],[3,5]]`

c) `[[3,4],[3,5]]`

d) `[[3,4],[3,4]]`

e) Don’t know

```
a = [[3,5],[7,11]]
b = a
c = a[:]
a[0][1] = 4
c[1] = b[0]
```

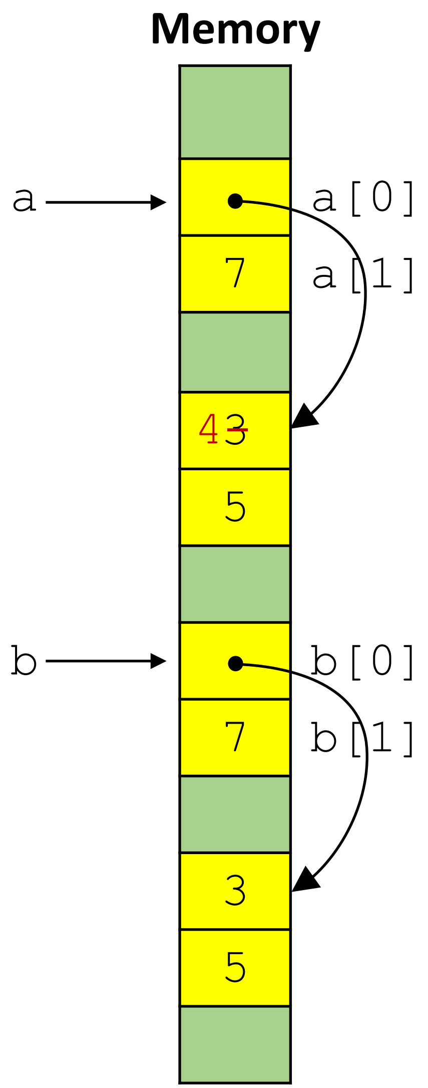


# copy.deepcopy

- To make a copy of all parts of a composite value use the function deepcopy from module copy

**Python shell**

```
> from copy import deepcopy
> a = [[3, 5], 7]
> b = deepcopy(a)
> a[0][0] = 4
> a
| [[4,5],7]
> b
| [[3,5],7]
```



# Initializing a 2-dimensional list

**Python shell**

```
> x = [1] * 3
> x
| [1, 1, 1]
> y = [[1] * 3] * 4
> y
| [[1, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]]
> y[0][0] = 0
> y
| [[0, 1, 1], [0, 1, 1], [0, 1, 1], [0, 1, 1]]
```

**Python shell**

```
> y = []
> for _ in range(4): y.append([1] * 3)
> y[0][0] = 0
> y
| [[0, 1, 1], [1, 1, 1], [1, 1, 1], [1, 1, 1]]
```

# range(*from*, *to*, *step*)

In Python 2, `range` generates the explicit list, i.e. always use memory proportional to the length; `xrange` in Python 2 corresponds to `range` in Python 3; Python 3 is more memory friendly

- `range(`*from*`,` *to*`,` *else*`)` generates the sequence of numbers starting with *from*, with increments of *step*, and smaller/greater than *to* if *step* positive/negative

| | | |
|---|---|---|
| `range(5)` | `: 0, 1, 2, 3, 4` | (default *from* = 0, *step* = 1) |
| `range(3, 8)` | `: 3, 4, 5, 6, 7` | (default *step* = 1) |
| `range(-2, 7, 3)` | `: -2, 1, 4` | (*from* and *to* can be any integer) |
| `range(2, -5, -2)` | `: 2, 0, -2, -4` | (decreasing sequence if *step* negative) |

- Ranges are immutable, can be indexed like a list, sliced, and compared (i.e. generate the same numbers)
- `list(range(...))` generates the explicit list of numbers

**Python shell**

```
> range(1, 10000000, 3)[2]
| 7
> range(1, 10000000, 3)[100:120:4]
| range(301, 361, 12)
> range(1, 10000000, 3)[100:120:4][2:3]
| range(325, 337, 12)
> list(range(5, 14, 3))
| [5, 8, 11]
```

# Question – What is `range(3,20,4)[2:4][1]` ?

a) 3
b) 7
c) 11
d) 15
e) 19
f) Don’t know

# for - loop

- For every element in a sequence execute a block of code:

  `for` *var* `in` *sequence*:
  
      *block*

- Sequences can e.g. be lists, strings, ranges
- `break` and `continue` can be used like in a while-loop to break out of the for-loop or continue with the next element in the sequence

**Python shell**

```
> for x in [1, "abc", [2, 3], 5.0]:
>     print(x)
| 1
| abc
| [2, 3]
| 5.0
> for x in "abc":
>     print(x)
| a
| b
| c
> for x in range(5, 15, 3):
>     print(x)
| 5
| 8
| 11
| 14
```

# Question – What is printed ?

**Python shell**

```
> for i in range(1, 4):
>     for j in range(i, 4):
>         print(i, j, sep=':', end=' ')
```

a) 1:1 1:2 1:3 2:1 2:2 2:3 3:1 3:2 3:3

b) 1:1 1:2 1:3 2:2 2:3 3:3

c) 1:1 2:1 3:1 1:2 2:2 3:2 1:3 2:3 3:3

d) 1:1 2:1 3:1 2:2 3:2 3:3

e) Don’t know

# Question – `break`, what is printed ?

**Python shell**

```
> for i in range(1, 4):
>     for j in range(1, 4):
>         print(i, j, sep=':', end=' ')
>         if j >= i:
>             break
```

a) *nothing*

b) `1:1`

c) `1:1 2:1 2:2 3:1 3:2 3:3`

d) `1:1 2:2 3:3`

e) Don't know

In nested `for`- and `while`-loops, `break` only breaks the innermost loop

```
    **** COMMODORE 64 BASIC V2 ****

 64K RAM SYSTEM  38911 BASIC BYTES FREE

READY.
10 FOR I=1 TO 10
20 PRINT I
30 NEXT
RUN
 1
 2
 3
 4
 5
 6
 7
 8
 9
 10

READY.
```

```
    **** COMMODORE 64 BASIC V2 ****

 64K RAM SYSTEM  38911 BASIC BYTES FREE

READY.
10 FOR I=1 TO 3
20   FOR J=I TO 3
30     PRINT I,J
40   NEXT
50 NEXT
RUN
 1         1
 1         2
 1         3
 2         2
 2         3
 3         3

READY.
```

# Palindromic substrings

- Find all palindromic substrings of length ≥ 2, i.e. substrings spelled identically forward and backwards:

  **abracadrabratrallalla**

  i j i j

- **Algorithm:** Test all possible substrings (brute force/exhaustive search)
- **Note**: the slice `t[::-1]` is `t` reversed

**palindrom.py**

```python
s = "abracadrabratrallalla"

for i in range(len(s)):
    for j in range(i + 2, len(s) + 1):
        t = s[i:j]
        if t == t[::-1]:
            print(t)
```

**Python shell**

```
| aca
| alla
| allalla
| ll
| llall
| lal
| alla
| ll
```

# Sieve of Eratosthenes

- Find all prime numbers ≤ n
- Algorithm:

2 3 4 5 6 7 8 9 10 11 12 13 14 ...
2 3 4 5 6 7 8 9 10 11 12 13 14 ...
2 3 4 5 6 7 8 9 10 11 12 13 14 ...
2 3 4 5 6 7 8 9 10 11 12 13 14 ...
2 3 4 5 6 7 8 9 10 11 12 13 14 ...
2 3 4 5 6 7 8 9 10 11 12 13 14 ...

**eratosthenes.py**

```python
n = 100
prime = [True] * (n + 1)

for i in range(2, n):
    for j in range(2 * i, n + 1, i):
        prime[j] = False

for i in range(2, n + 1):
    if prime[i]:
        print(i, end=' ')
```

**Python shell**

```
| 2 3 5 7 11 13 17 19 23 29 31 37 41
  43 47 53 59 61 67 71 73 79 83 89
  97
```

en.wikipedia.org/wiki/Sieve_of_Eratosthenes

# while-else and for-else loops

- Both for- and while-loops can have an optional “else”:

```
for var in sequence:
        block
else:
        block

while condition:
        block
else:
        block
```

- The “else” block is only executed if no `break` is performed in the loop
- The “else” construction for loops is specific to Python,
and does not exist in e.g. C, C++ and Java

# Linear search

linear-search-while.py

```
L = [7, 3, 6, 4, 12, 'a', 8, 13]
x = 4

i = 0
while i < len(L):
    if L[i] == x:
        print(x, "at position", i, "in", L)
        break
    i = i + 1

if i >= len(L):
    print(x, "not in", L)
```

linear-search-while-else.py

```
i = 0
while i < len(L):
    if L[i] == x:
        print(x, "at position", i, "in", L)
        break
    i = i + 1
else:
    print(x, "not in", L)
```

linear-search-for.py

```
found = False
for i in range(len(L)):
    if L[i] == x:
        print(x, "at position", i, "in", L)
        found = True
        break

if not found:
    print(x, "not in", L)
```

linear-search-for-else.py

```
for i in range(len(L)):
    if L[i] == x:
        print(x, "at position", i, "in", L)
        break
else:
    print(x, "not in", L)
```

linear-search-builtin.py

```
if x in L:
    print(x, "at position", L.index(x), "in", L)
else:
    print(x, "not in", L)
```

# Some performance considerations

# String concatenation

- To concatenate two (or few) strings use

  $str_1$ + $str_2$
  *var* += *str*

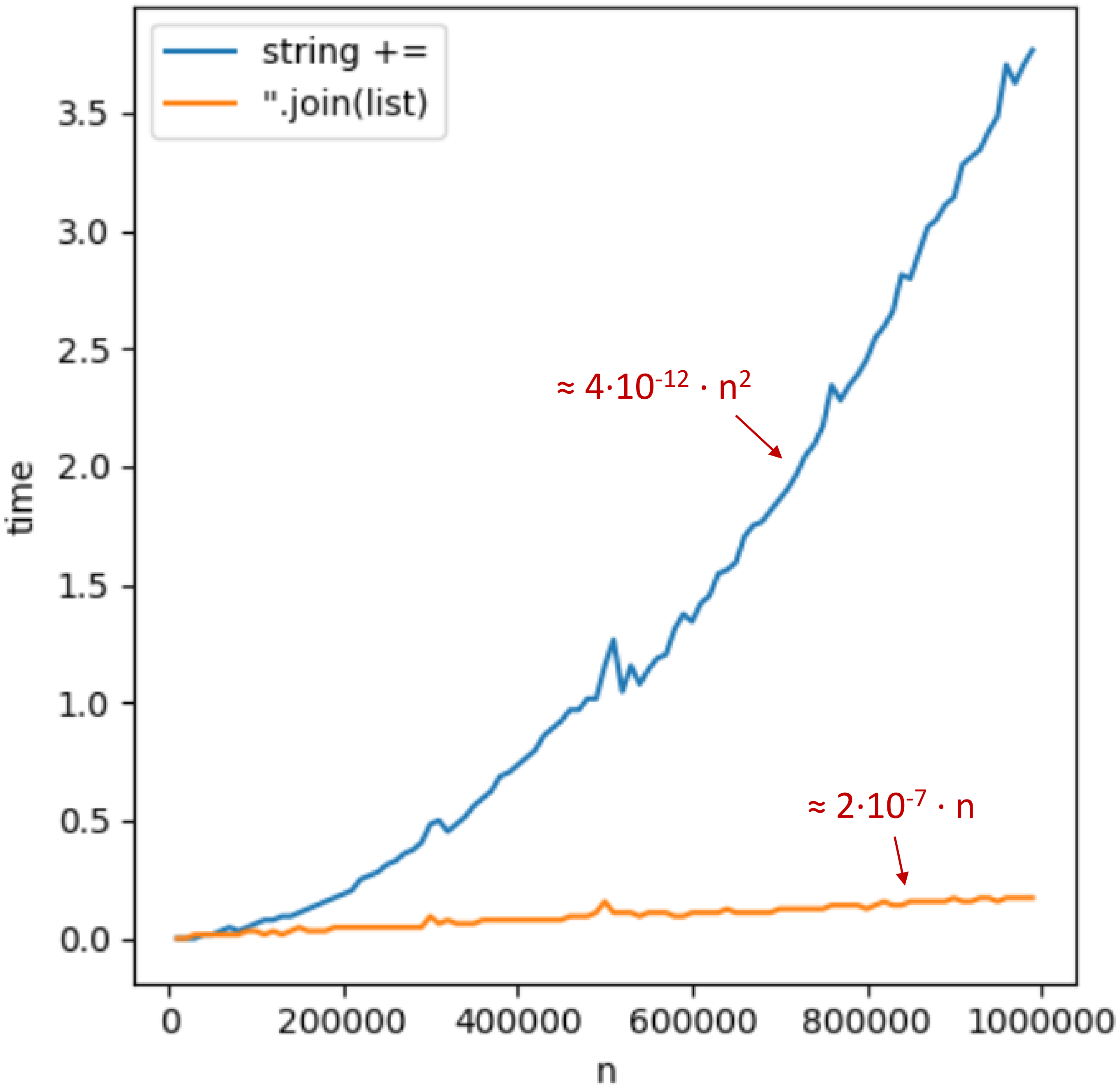
- To concatenate several/many strings use

  `''.join(`[$str_1$, $str_2$, $str_3$, … , $str_n$]`)`

- Concatenating several strings by repeated use of + generates explicitly the longer-and-longer intermediate results; using `join` avoids this slowdown

**Python shell**

```
> s = 'A' + 'B' + 'C'
> s
| 'ABC'
> 'x'.join(['A', 'B', 'C'])
| 'AxBxC'
> s = ''
> s += 'A'
> s += 'B'
> s += 'C'
> s
| 'ABC'
> L = []
> L.append('A')
> L.append('B')
> L.append('C')
> L
| ['A', 'B', 'C']
> s = ''.join(L)
> s
| 'ABC'
```

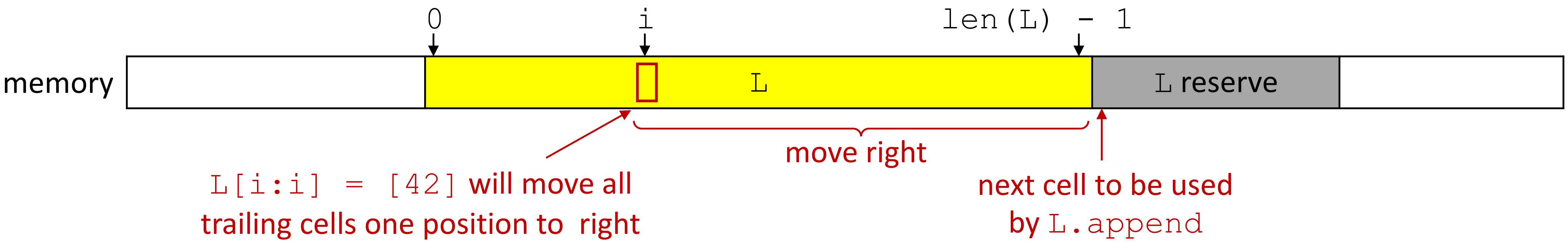



**string-concatenation.py**

```
from time import time
from matplotlib import pyplot as plt

ns = range(10_000, 1_000_000, 10_000)
time_string = []
time_list = []
for n in ns:
    start = time()
    s = ''
    for _ in range(n):
        s += 'abcdefgh'   # slow
    end = time()
    time_string.append(end - start)

    start = time()
    substrings = []
    for _ in range(n):
        substrings.append('abcdefgh');
    s = ''.join(substrings);
    end = time()
    time_list.append(end - start)

plt.plot(ns, time_string, label='string +=')
plt.plot(ns, time_list, label="''.join(list)")
plt.xlabel('n')
plt.ylabel('time')
plt.legend()
plt.show()
```

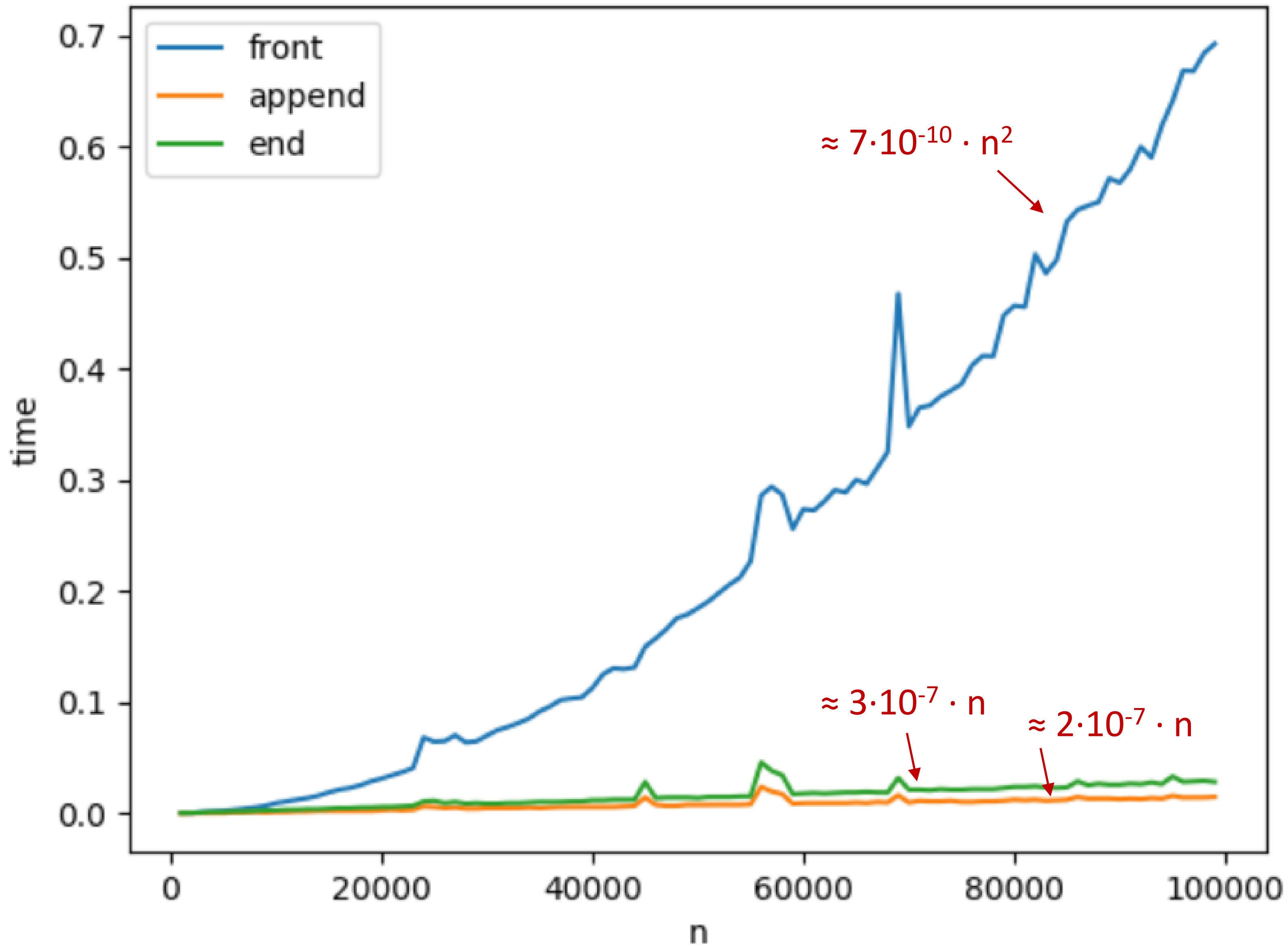
# The internal implementation of Python lists

- Accessing and updating list positions take the same time independently of position
- Creating new / deleting entries in a list depends on position, Python optimizes towards updates at the end
- Try to organize your usage of lists to insert / delete elements at the end `L.append(`*element*`)` and `L.pop()`.
- Python lists internally have space for adding ≈ 12.5 % additional entries at the end; when the reserved extra space is exhausted the list is moved to a new chunk of memory with ≈ 12.5 % extra space




github.com/python/cpython/blob/master/Objects/listobject.c

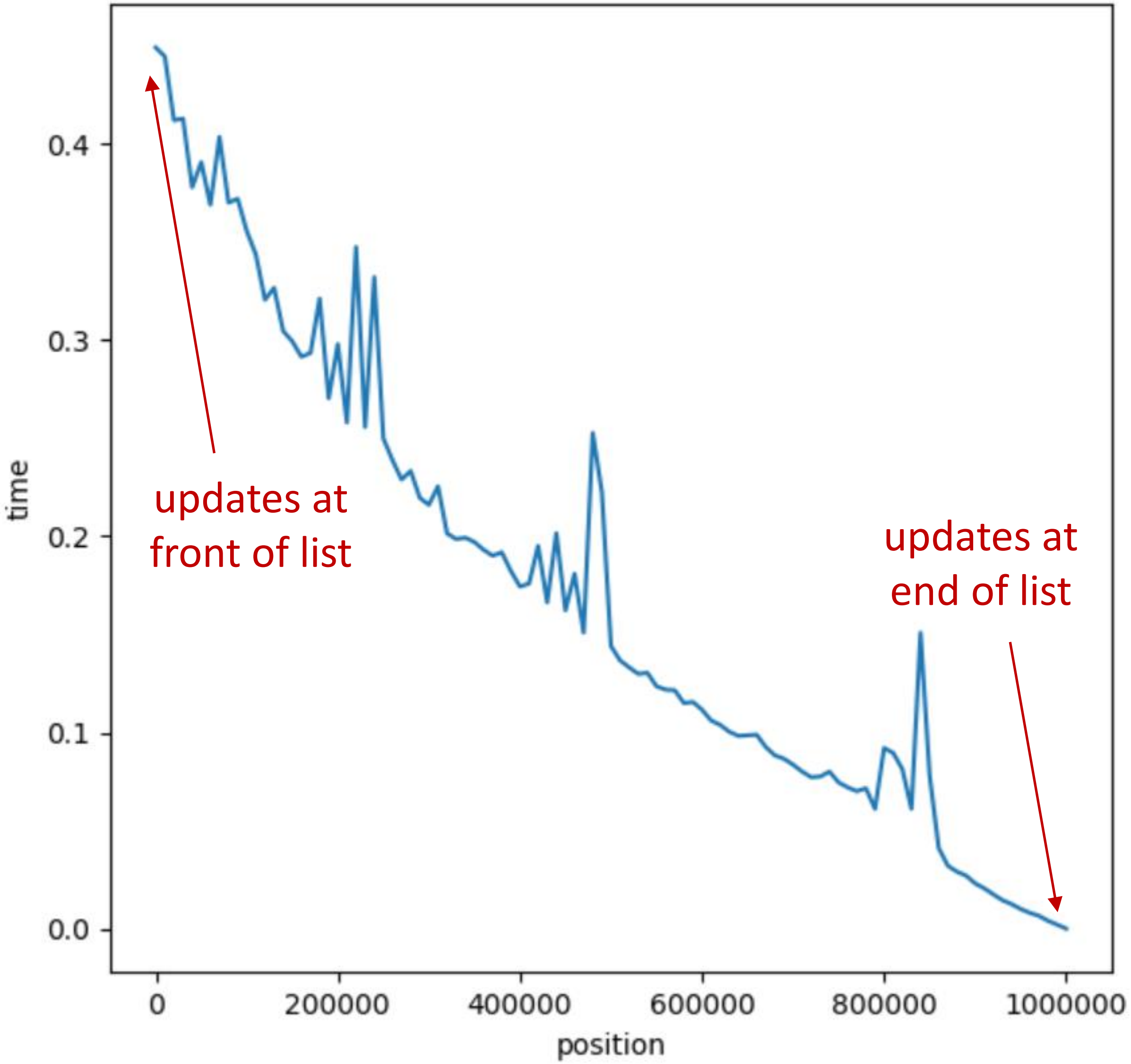
# List insertions at begin vs end




list-insertions.py

```python
from time import time
from matplotlib import pyplot as plt

ns = range(1000, 100_000, 1000)
time_end = []
time_append = []
time_front = []
for n in ns:
    start = time()
    L = []
    for i in range(n):
        L[i:i] = [i]   # insert after list
    end = time()
    time_end.append(end - start)

    start = time()
    L = []
    for i in range(n):
        L.append(i)   # append to list
    end = time()
    time_append.append(end - start)

    start = time()
    L = []
    for i in range(n):
        L[0:0] = [i]   # insert at front
    end = time()
    time_front.append(end - start)

plt.plot(ns, time_front, label='front')
plt.plot(ns, time_append, label='append')
plt.plot(ns, time_end, label='end')
plt.xlabel('n')
plt.ylabel('time')
plt.legend()
plt.show()
```

# Updates (insertions + deletions) in the middle of a list




list-updates.py

```
from time import time
from matplotlib import pyplot as plt

ns = range(0, 1_000_001, 10_000)
time_pos = []
L = list(range(1_000_000))  # L = [0, ..., 999_999]
for i in ns:
    start = time()
    for _ in range(1000):
        L[i:i] = [42]  # insert element before L[i]
        del L[i]  # remove L[i] from L
    end = time()
    time_pos.append(end - start)

plt.plot(ns, time_pos)
plt.xlabel('position')
plt.ylabel('time')
plt.show()
```

# Tuples and lists

- tuples
- lists
- mutability
- list comprehension
- for-if, for-for
- list()
- any(), all()
- enumerate(), zip()

# Tuples

$$(\text{value}_0,\ \ \text{value}_1,\ \ \ldots\ \ ,\ \ \text{value}_{k\text{-}1})$$

- Tuples can contain a sequence of zero or more elements, enclosed by ”(” and ”)”
- Tuples are immutable
- Tuple of length 0: `()`
- Tuple of length 1: `(value,)`

  Note the comma to make a tuple of length one distinctive from an expression in parenthesis
- In many contexts a tuple with ≥ 1 elements can be written without parenthesis
- Accessors to lists also apply to tuples, slices, ...

**Python shell**

```
> (1, 2, 3)
| (1, 2, 3)
> ()
| ()
> (42)
| 42
> (42,)
| (42,)
> 1, 2
| (1, 2)
> 42,
| (42,)
> x = (3, 7)
> x
| (3, 7)
> x = 4, 6
> x
| (4, 6)
> x[1] = 42
| TypeError: 'tuple' object does
  not support item assignment
```

# Question – What value is `((42,))` ?

a) `42`

b) `(42)`

c) `(42,)`

d) `((42,),)`

e) Don't know

# Question – What is `x` ?

```
x = [1, [2, 3], (4, 5)]
x[2][0] = 42
```

a) `[1, [42, 3], (4, 5)]`

b) `[1, [2, 3], (42, 5)]`

c) `[1, [2, 3], 42]`

d) TypeError

e) Don't know

# Question – What tree is `('A',(('B','C'),'D'))` ?

```
          /\
         /  \
        /    \
       /      \
      /\      'D'
     /  \
    /    \
   /\    'C'
  /  \
'A'  'B'
```

a)

```
          /\
         /  \
        /    \
       /      \
      /        \
     /\        'D'
    /  \
   /    \
 'A'    /\
       /  \
     'B'  'C'
```

b)

```
         /\
        /  \
       /    \
      /      \
     /        \
    /\        /\
   /  \      /  \
 'A'  'B'  'C'  'D'
```

c)

```
          /\
         /  \
        /    \
       /      \
      /        \
    'A'        /\
              /  \
             /    \
            /\    'D'
           /  \
         'B'  'C'
```

d)

```
        /\
       /  \
      /    \
     /      \
   'A'      /\
           /  \
          /    \
        'B'    /\
              /  \
            'C'  'D'
```

e)

f) Don’t know

# Tuple assignment

**Python shell**

```
> point = (10, 25)
> x, y = point
> x
| 10
> y
| 25
```

- Parallel assignments

  `x, y, z = a, b, c`

  is a short hand for a tuple assignment (right side is a single tuple)

  `(x, y, z) = (a, b, c)`

- First the right-hand side is evaluated completely, and then the individual values of the tuple are assigned to `x`, `y`, `z` left-to-right (length must be equal on both sides)

# Nested tuple/lists assignments

- Let hand side can be nested
  (great for unpacking data)

```
(x, (y, (a[0], w)), a[1])
    = 1, (2, (3, 4)), 5
```

- [...] and (...) on left side matches both lists and tuples of equal length
  (but likely you would like to be consistent with type of parenthesis)

**Python shell**

```
> two_points = [(10, 25), (30, 40)]
> (x1, y1, x2, y2) = two_points
| ValueError: not enough values to
  unpack (expected 4, got 2)
> ((x1, y1), (x2, y2)) = two_points
> a = [None, None]
> v = ((2, (3, 4)), 5)
> ((y, (a[0], w)), a[1]) = v
> a
| [3, 5]
> [x, y, z] = (3, 5, 7)
> (x, y, z) = [3, 5, 7]
> [x, (y, z), w] = (1, [2, 3], 4)
> [x, (y, z), w] = (1, [2, (5, 6)], 4)
> z
| (5, 6)
```

# Tuples vs lists: `a += b`

- Lists
  Extends existing list, i.e. same as `a.extend(b)`
- Tuples
  Must create a new tuple `a + b` and assign to `a`
  (since tuples are immutable)

**Python shell**

```
> (1, 2) + (3, 4)
| (1, 2, 3, 4)
> x = [1, 2]
> y = x
> y += [3, 4]
> y
| [1, 2, 3, 4]
> x
| [1, 2, 3, 4]

> x = (1, 2)
> y = x
> y += (3, 4)
> y
| (1, 2, 3, 4)
> x
| (1, 2)
```

# *variable assignment

- For a tuple of variable length a single **variable name* on the left side will be assigned a list of the remaining elements not matched by variables preceding/following *
- *Example*

```
a, *b, c = t
```

  is equivalent to

```
a = t[0]
b = t[1:-1]
c = t[-1]
```

- There can be a single * in a left-hand-side tuple (but one new * in each nested tuple)

**Python shell**

```
> (a, *b, c, d) = (1, 2, 3, 4, 5, 6)
> b
| [2, 3, 4]
> (a, *b, c, d) = (1, 2, 3)
> b
| []
> (a, *b, c, d) = (1, 2)
| ValueError: not enough values to
  unpack (expected at least 3, got 2)
> v = ((1,2,3),4,5,6,(7,8,9,10))
> ((a, *b), *c, (d, *e)) = v
> b
| [2, 3]
> c
| [4, 5, 6]
> e
| [8, 9, 10]
> head, *tail = [1, 2, 3, 4]
> head
| 1
> tail
| [2, 3, 4]
```

# Question – What is `b` ?

```
(*a,(b,),c) = ((1,2),((3,4)),((5,)),(6))
```

a) `(1,2)`
b) `(3,4)`
c) `5`
d) `(5,)`
e) `(6)`
f) Don't know

**Python shell**

```
> (*a,(b,),c) = ((1,2),((3,4)),((5,)),(6))
> a
| [(1, 2), (3, 4)]
> b
| 5
> c
| 6
```

# * in list and tuple construction

- When constructing a list or tuple you can insert zero or more elements from another list/tuple/sequence by inserting **expression*
- There can be an arbitrary number of * expressions in a tuple or list construction

**Python shell**

```
> A = (1, 2, 3)
> B = ['B', 'C']
> L = [A, B, 4, 5]
> L
| [(1, 2, 3), ['B', 'C'], 4, 5]
> len(L)
| 4
> L = [*A, *B, 4, 5]
> L
| [1, 2, 3, 'B', 'C', 4, 5]
> len(L)
| 7
> (*A, *B, 4, 5)
| (1, 2, 3, 'B', 'C', 4, 5)
```

**Python shell**

```
> from timeit import timeit
> timeit('A + A + A + A + A + A + A + A', setup='A = [1,2,3,4,5,6,7,8,9,10]')
| 0.665172699955292  # repeated concatenation can be slow
> timeit('[*A, *A, *A, *A, *A, *A, *A, *A]', setup='A = [1,2,3,4,5,6,7,8,9,10]')
| 0.3259985999027103  # * notation can be faster for multiple concatenation
```

# List comprehension (cool stuff)

- Example:

```
[x*x for x in [1, 2, 3]]
```

returns

```
[1, 4, 9]
```

- General

  [*expression* for *variable* in *sequence*]

  returns a list, where *expression* is computed for each element in *sequence* assigned to *variable*

**Python shell**

```
> [2*x for x in [1,2,3]]
| [2, 4, 6]
> [2*x for x in (1,2,3)]
| [2, 4, 6]
> [2*x for x in range(10,15)]
| [20, 22, 24, 26, 28]
> [2*x for x in 'abc']
| ['aa', 'bb', 'cc']
> [(None, None) for _ in range(2)]
| [(None, None), (None, None)]
```

# List comprehension (it's just syntactic sugar...)

**Python shell**

```
> [x*2 for x in [1, 2, 3]]
| [2, 4, 6]

> L = []
> for x in [1, 2, 3]:
>     L.append(x*2)
> L
| [2, 4, 6]
```

# List comprehension (more cool stuff)

- Similarly to the left-hand-side in assignments, the variable part can be a (nested) tuple of variables for unpacking elements:

[*expression* `for` *tuple of variables* `in` *sequence*]

**Python shell**

```
> points = [(3, 4), (2, 5), (4, 7)]
> [(x, y, x*y) for (x, y) in points]
| [(3, 4, 12), (2, 5, 10), (4, 7, 28)]
> [(x, y, x*y) for x, y in points]
| [(3, 4, 12), (2, 5, 10), (4, 7, 28)]
> [x, y, x*y for (x, y) in points]
| SyntexError: invalid syntax
```

⚠ parenthesis required for the constructed tuples

# List comprehension – for-if and multiple for

- List comprehensions can have nested for-loops

  $$[\mathit{expression}\ \texttt{for}\ v_1\ \texttt{in}\ s_1\ \texttt{for}\ v_2\ \texttt{in}\ s_2\ \texttt{for}\ v_3\ \texttt{in}\ s_3]$$

- Can select a subset of the elements by adding an if-condition

  $$[\mathit{expression}\ \texttt{for}\ v_1\ \texttt{in}\ s_1\ \texttt{if}\ \mathit{condition}]$$

- and be combined...

**Python shell**

```
> [(x, y) for x in range(1, 3) for y in range(4, 6)]
| [(1, 4), (1, 5), (2, 4), (2, 5)]
> [x for x in (1, 2) for x in (4, 5)]
| [4, 5, 4, 5]
> [x for x in range(1, 101) if x % 7 == 1 and x % 5 == 2]
| [22, 57, 92]
> [(x, y, x*y) for x in range(1, 11) if 6 <= x <= 7 for y in range(x, 11) if 6 <= y <= 7 and not x == y]
| [(6, 7, 42)]
```

# Question – What will print the same?

```
points = [(3,7), (4,10),(12,3), (9,11), (7,5)]
print([(x, y) for x, y in points if x < y])
```

a) `print([x, y for x, y in points if x < y])`

b) `print([(x, y) for p in points if p[0] < p[1]])`


c) `print([p for p in points if p[0] < p[1]])`

d) `print([[x, y] for x, y in points if x < y])`

e) Don't know

# any, all

- `any(L)` checks if at least one element in the sequence `L` is true (list, tuple, ranges, sequence, strings, ...)

  `any([False, True, False])`

- `all(L)` checks if all elements in the sequence `L` are true

  `all([False, False, True])`

- `any` and `all` return `True` or `False`

**Python shell**

```
> any((False, True, False))
| True
> any([False, False, False])
| False
> any([])
| False
> all([False, False, True])
| False
> all((True, True, True))
| True
> all(())
| True
> L = (7, 42, 13)
> any([x == 42 for x in L])
| True
> all([x == 42 for x in L])
| False
```

# Example – computing primes

**Python shell**

```
> [x for x in range(2, 50) if all([x % f for f in range(2, x)])]
| [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47]

> [10 % f for f in range(2, 10)]
| [0, 1, 2, 0, 4, 3, 2, 1]
> all([10 % f for f in range(2, 10)])   #  == 0 is considered False
| False
> [13 % f for f in range(2, 13)]
| [1, 1, 1, 3, 1, 6, 5, 4, 3, 2, 1]
> all([13 % f for f in range(2, 13)])
| True
```

# enumerate

`list(enumerate(L))`

returns

`[(0, L[0]), (1, L[1]), ..., (len(L) - 1, L[-1])]`

**Python shell**

```
> points = [(1, 2), (3, 4), (5, 6)]
> [(idx, x * y) for idx, (x, y) in enumerate(points)]
| [(0, 2), (1, 12), (2,30)]
> L = ('a', 'b', 'c')
> list(enumerate(L))
| [(0, 'a'), (1, 'b'), (2, 'c')]
> L_ = []
> for idx in range(len(L)):
>       L_.append((idx, L[idx]))
> print(L_)
| [(0, 'a'), (1, 'b'), (2, 'c')]
> list(enumerate(['a', 'b', 'c'], start=7))
| [(7, 'a'), (8, 'b'), (9, 'c')]
```

# zip

$\texttt{list(zip(}L_1\texttt{,}L_2\texttt{,...,}L_k\texttt{))} = \texttt{[(}L_1[0],L_2[0],\ldots,L_k[0]\texttt{),...,(}L_1[n\text{-}1],L_2[n\text{-}1],\ldots,L_k[n\text{-}1]\texttt{)]}$

where $n = \min(\mathrm{len}(L_1), \mathrm{len}(L_2),\ldots, \mathrm{len}(L_k))$

- Example ("matrix transpose"):

```
list(zip([1,2,3],
         [4,5,6],
         [7,8,9]))
```

returns

```
[(1, 4, 7),
 (2, 5, 8),
 (3, 6, 9)]
```

**Python shell**

```
> x = [1, 2, 3]
> y = [4, 5, 6]
| zip(x, y)
> <zip at 0xb02b530>
> points = list(zip(x, y))
> print(points)
| [(1, 4), (2, 5), (3, 6)]
```

**Python shell**

```
> first = ['Donald', 'Mickey', 'Scrooge']
> last = ['Duck', 'Mouse', 'McDuck']

> for i, (a, b) in enumerate(zip(first, last), start=1):
>     print(i, a, b)

| 1 Donald Duck
| 2 Mickey Mouse
| 3 Scrooge McDuck
```

# (Simple) functions

- You can define your own functions using:

  `def` *function-name* `(`*var*$_1$, ..., *var*$_k$`):`

  body code

- If the body code executes

  `return` *expression*

  the result of *expression* will be returned by the function. If expression is omitted or the body code terminates without performing `return`, then `None` is returned.

- When *calling* a function *name* `(`*value*$_1$,..., *value*$_k$`)` body code is executed with *var*$_i$=*value*$_i$

**Python shell**

```
> def sum3(x, y, z):
      return x + y + z

> sum3(1, 2, 3)
| 6
> sum3(5, 7, 9)
| 21

> def powers(L, power):
      P = [x**power for x in L]
      return P

> powers([2, 3, 4], 3)
| [8, 27, 64]
```

# Question – What tuple is printed ?

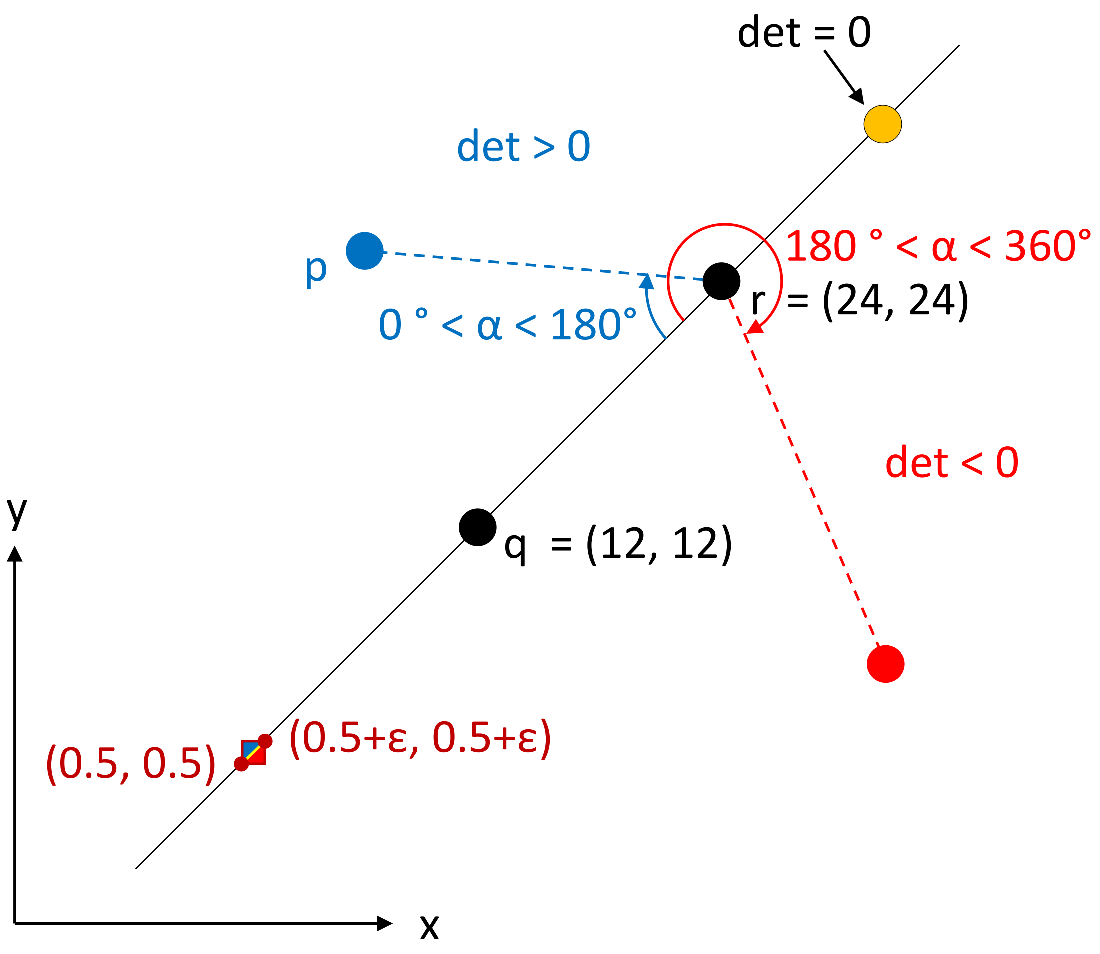
```
def even(x):
    if x % 2 == 0:
        return True
    else:
        return False

print((even(7), even(6)))
```

a) (False, False)
b) (False, True)
c) (True, False)
d) (True, True)
e) Don't know

# Geometric orientation test

**Purpose of example**

- illustrate tuples
- list comprehension
- matplotlib.pyplot
- floats are strange ⚠



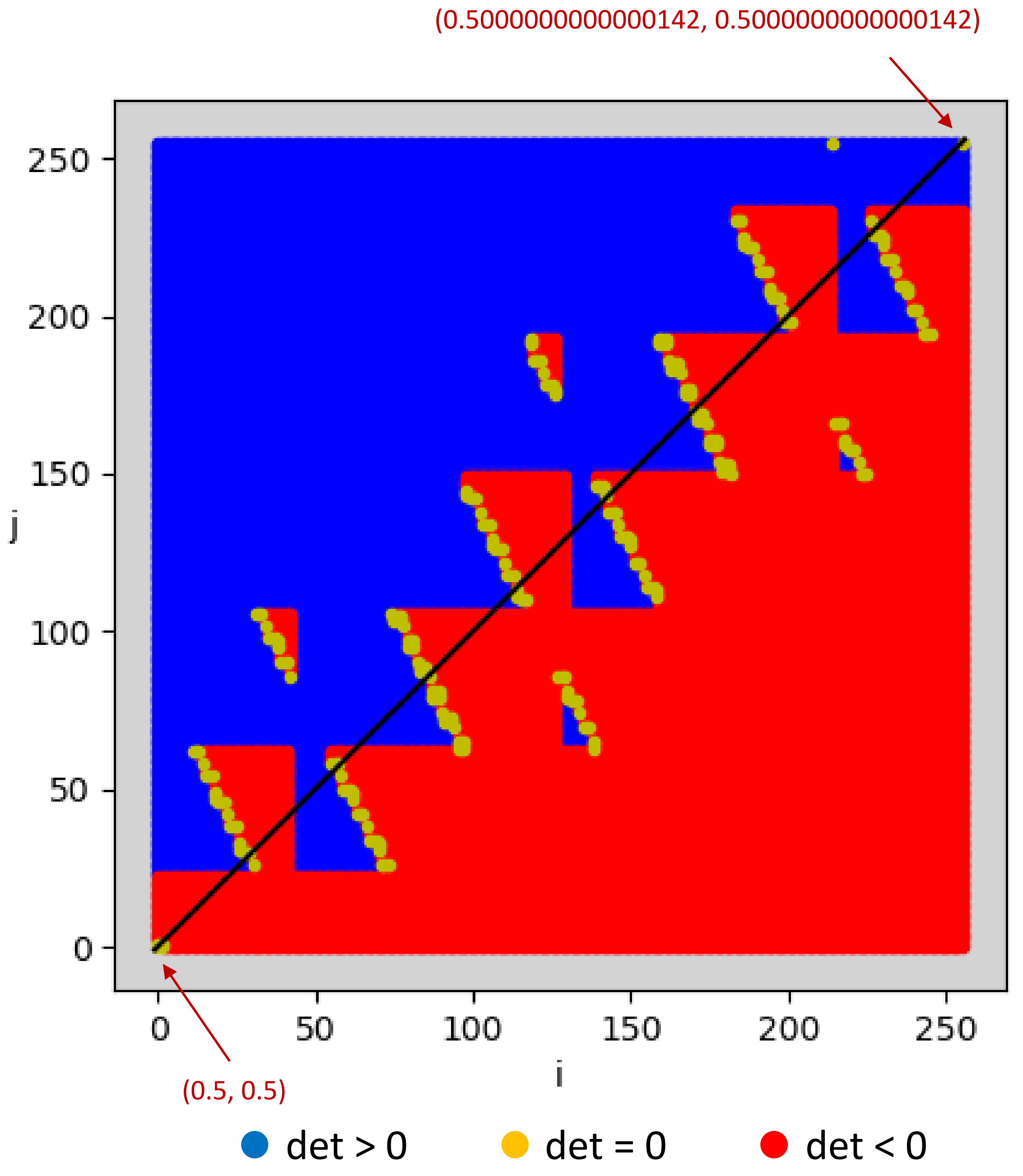

$$\det = \begin{vmatrix} 1 & q_x & q_y \\ 1 & r_x & r_y \\ 1 & p_x & p_y \end{vmatrix} = \underbrace{r_x p_y - p_x r_y - q_x p_y + p_x q_y + q_x r_y - r_x q_y}_{6\,!\,=\,720 \text{ different orders to add}}$$

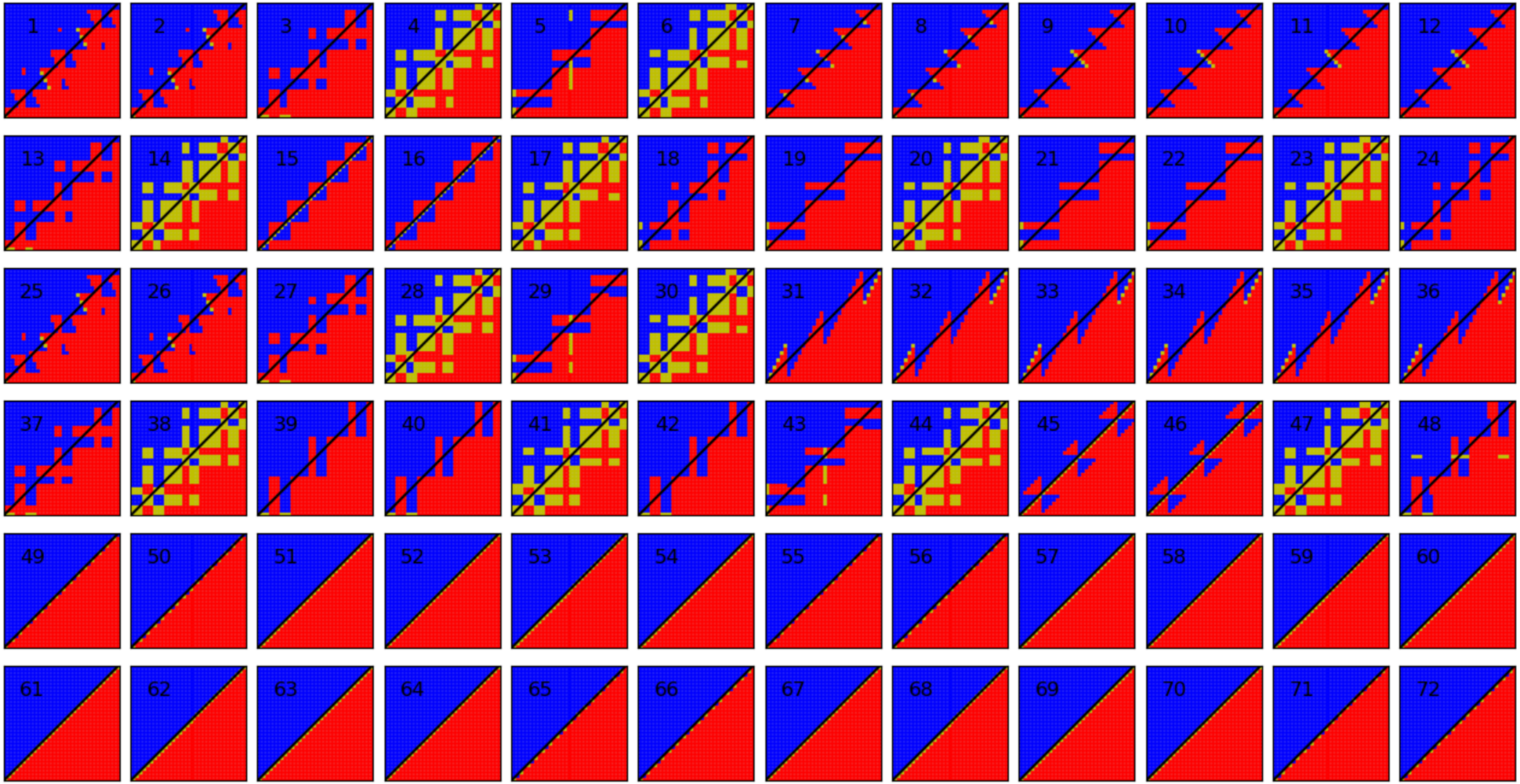
6 ! = 720 different orders to add ⚠

Kettner, Mehlhorn, Pion, Schirra, Yap:
Classroom Examples of Robustness Problems in Geometric Computations

**sign-plot.py**

```
import matplotlib.pyplot as plt

N = 256
delta = 1 / 2**54
q = (12, 12)
r = (24, 24)
P = []  # points (i, j, det)

for i in range(N):
    for j in range(N):
        p = (1/2 + i * delta, 1/2 + j * delta)
        det = (q[0]*r[1] + r[0]*p[1] + p[0]*q[1]
               - r[0]*q[1] - p[0]*r[1] - q[0]*p[1])
        P.append((i, j, det))

pos = [(i, j) for i, j, det in P if det > 0]
neg = [(i, j) for i, j, det in P if det < 0]
zero = [(i, j) for i, j, det in P if det == 0]

plt.subplot(facecolor='lightgrey', aspect='equal')
plt.xlabel('i')
plt.ylabel('j', rotation=0)

for points, color in [(pos, "b"), (neg, "r"), (zero, "y")]:
    X = [x for x, y in points]
    Y = [y for x, y in points]
    plt.plot(X, Y, color + ".")

plt.plot([-1, N], [-1, N], "k-")
plt.show()
```

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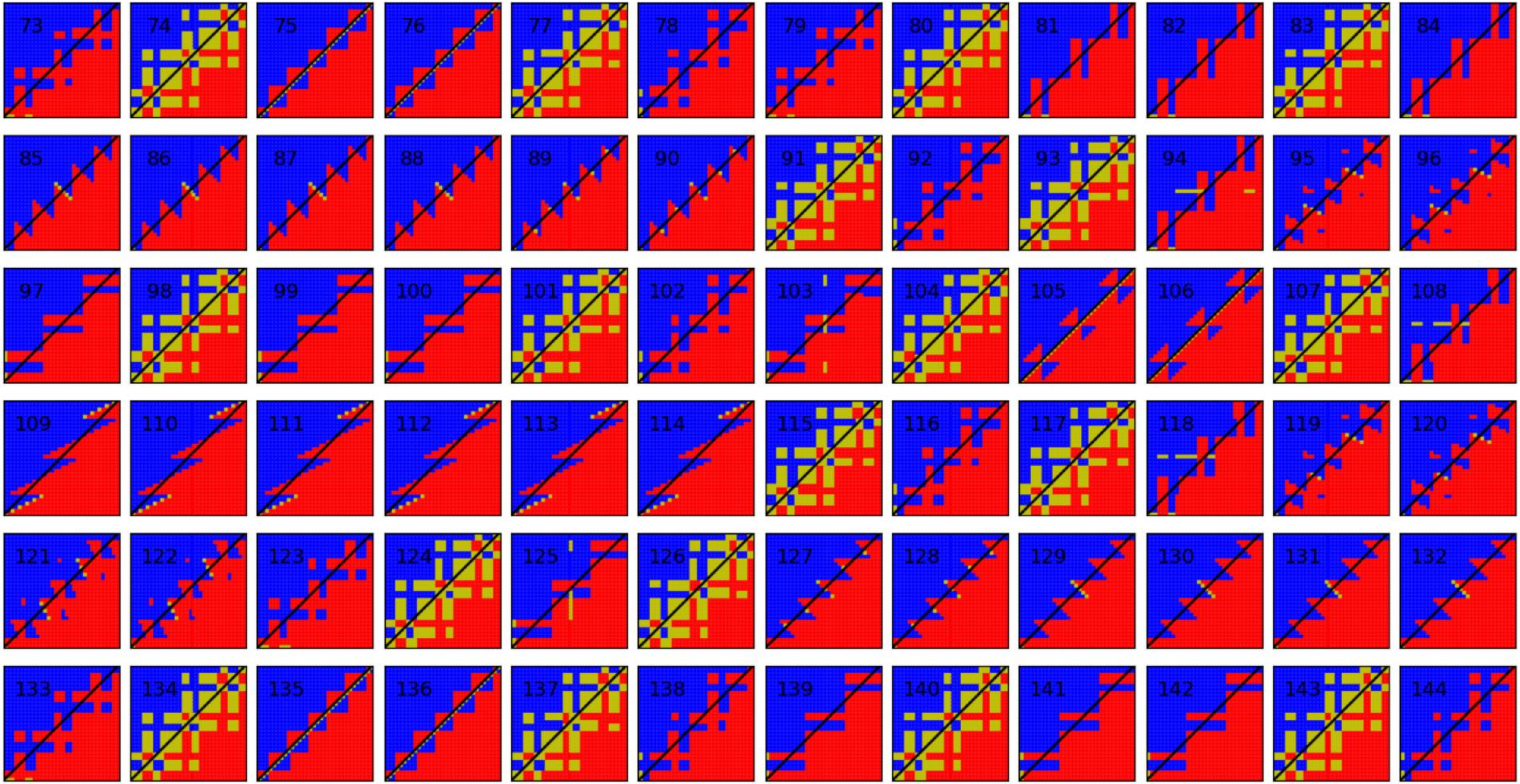
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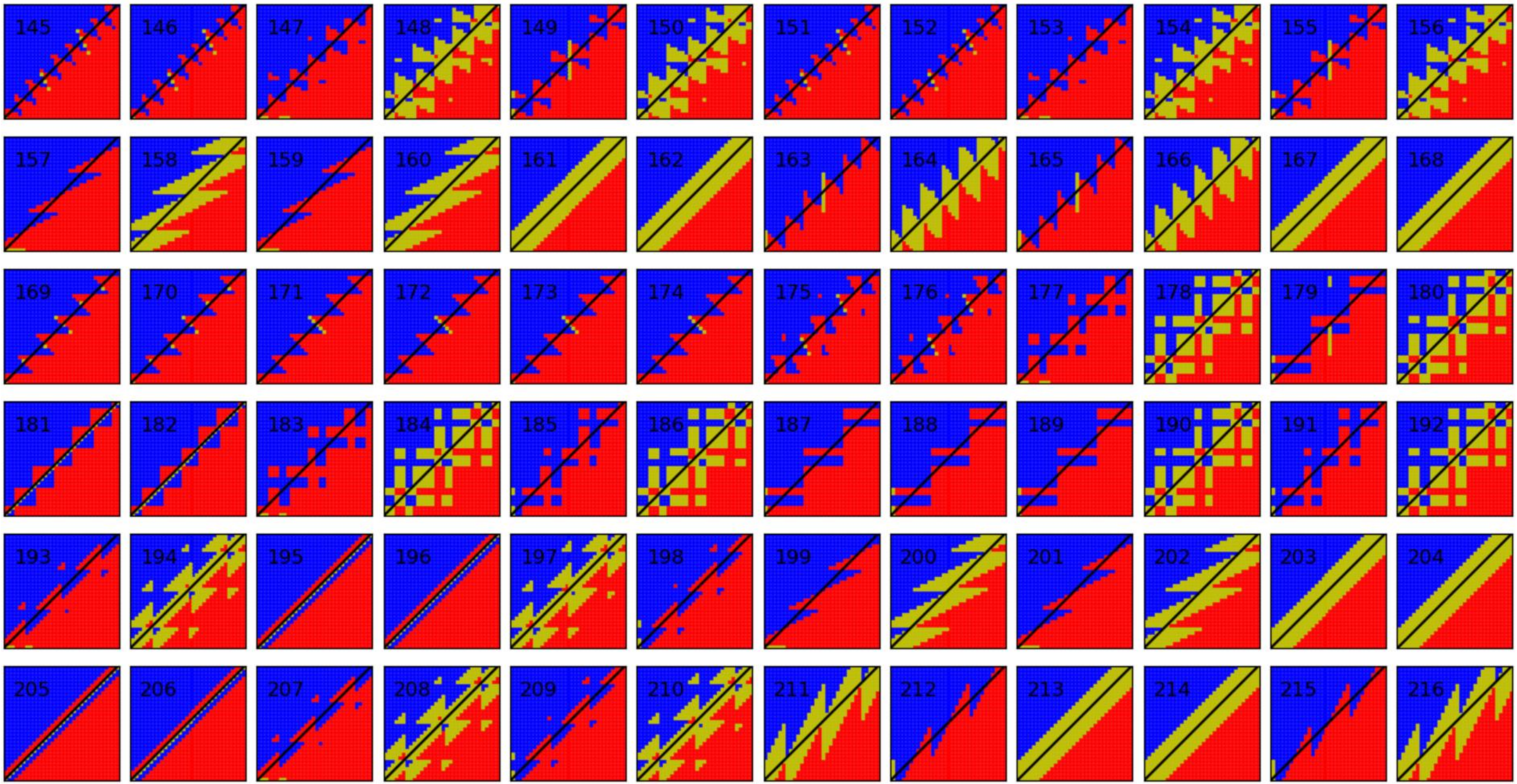
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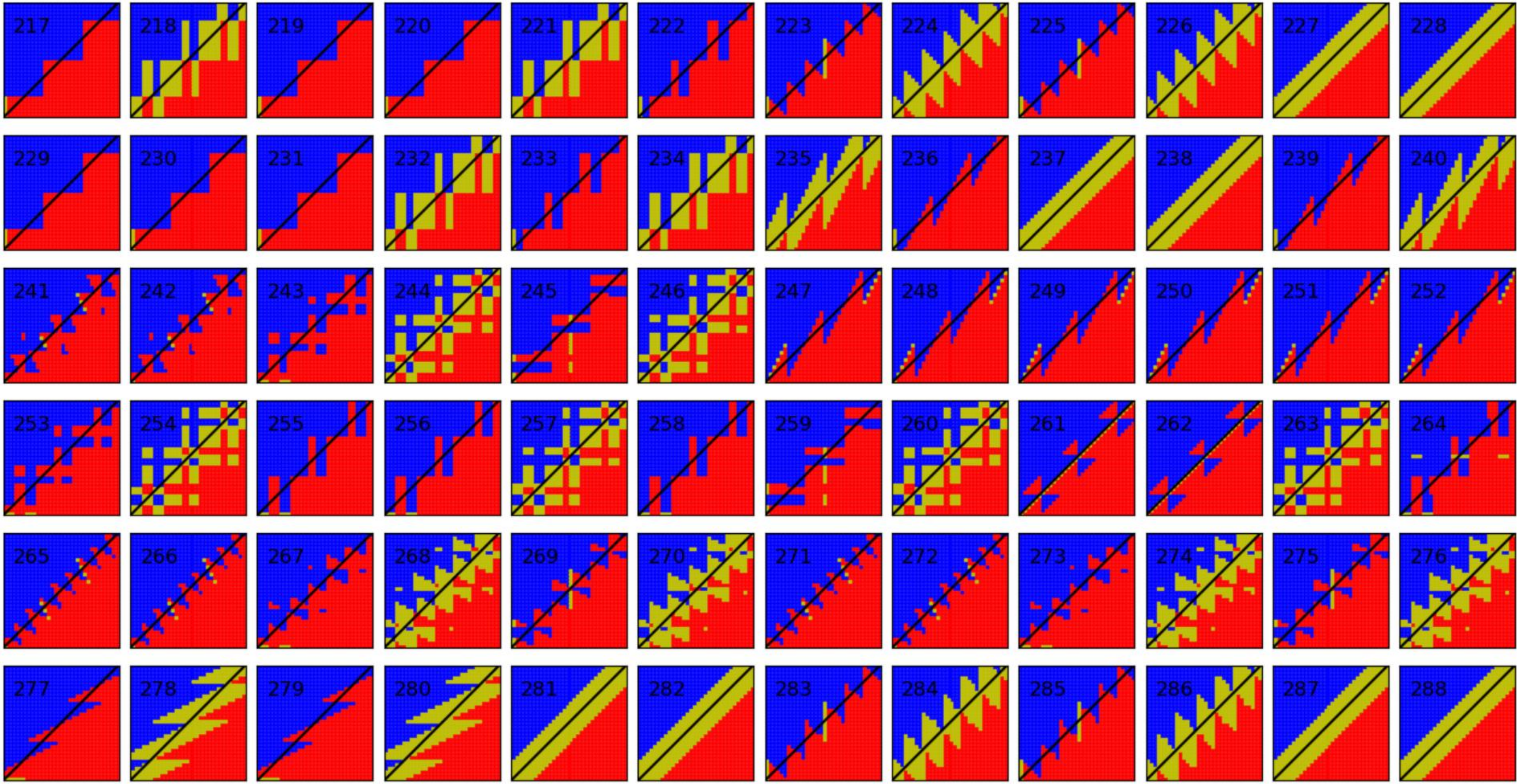
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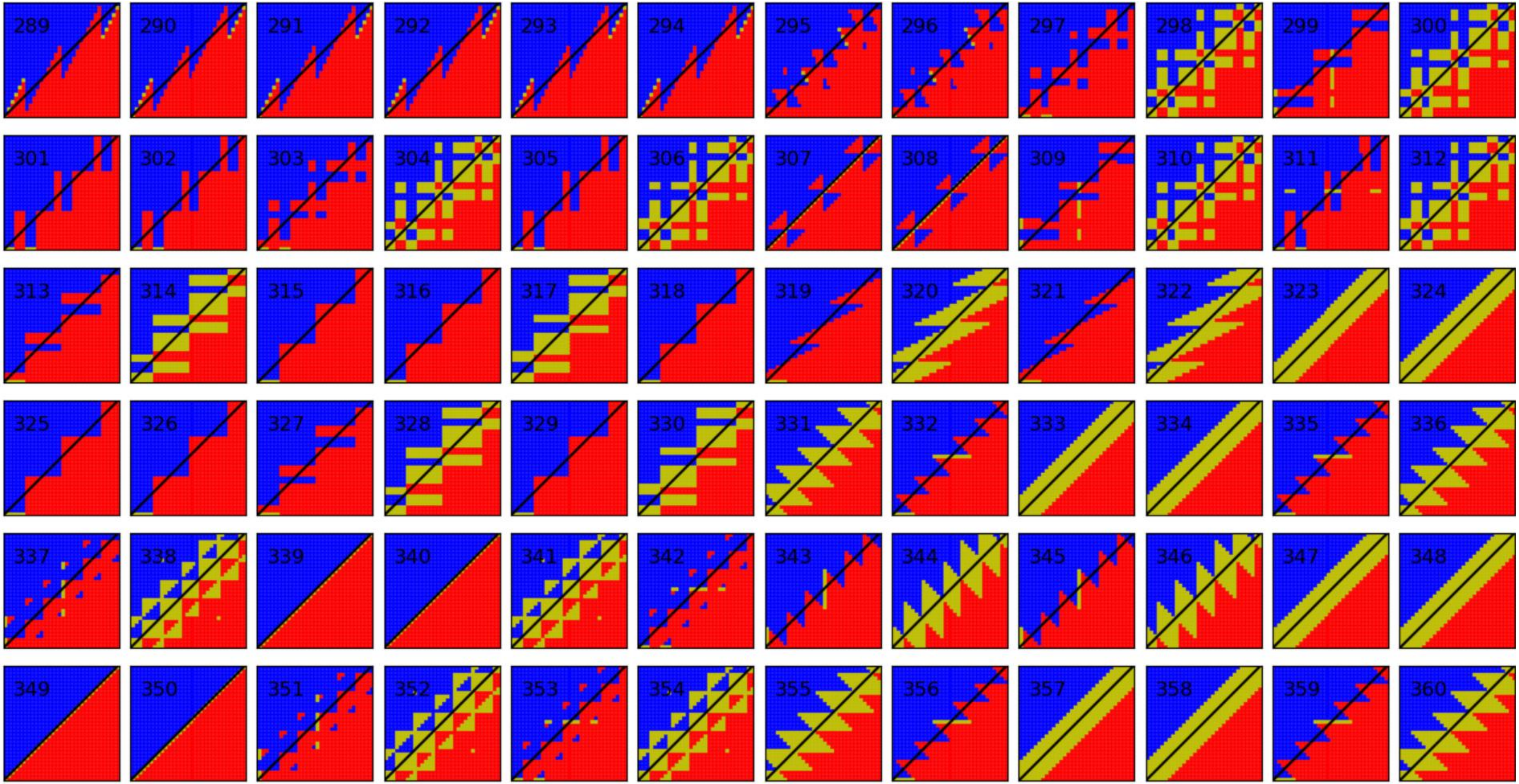
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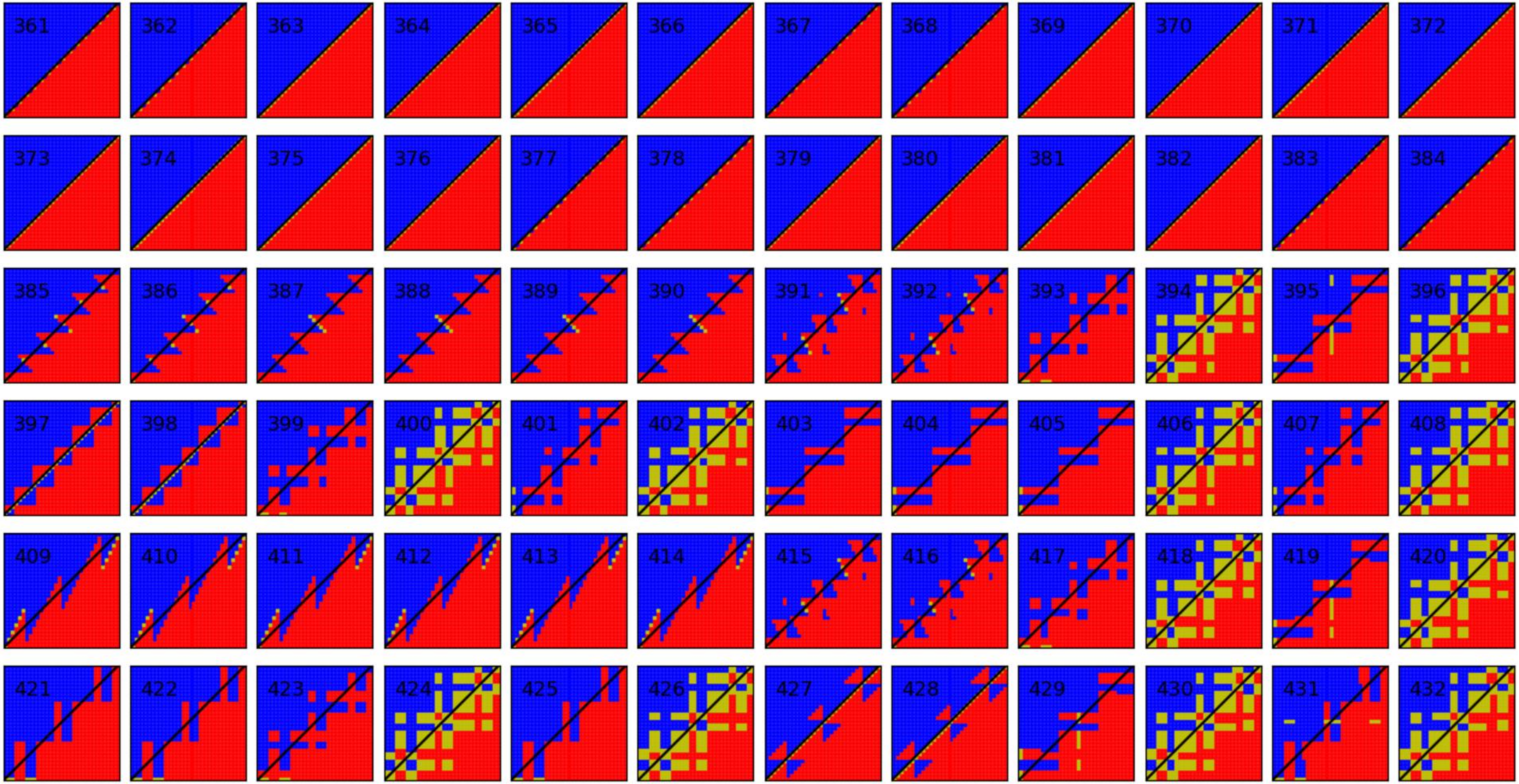
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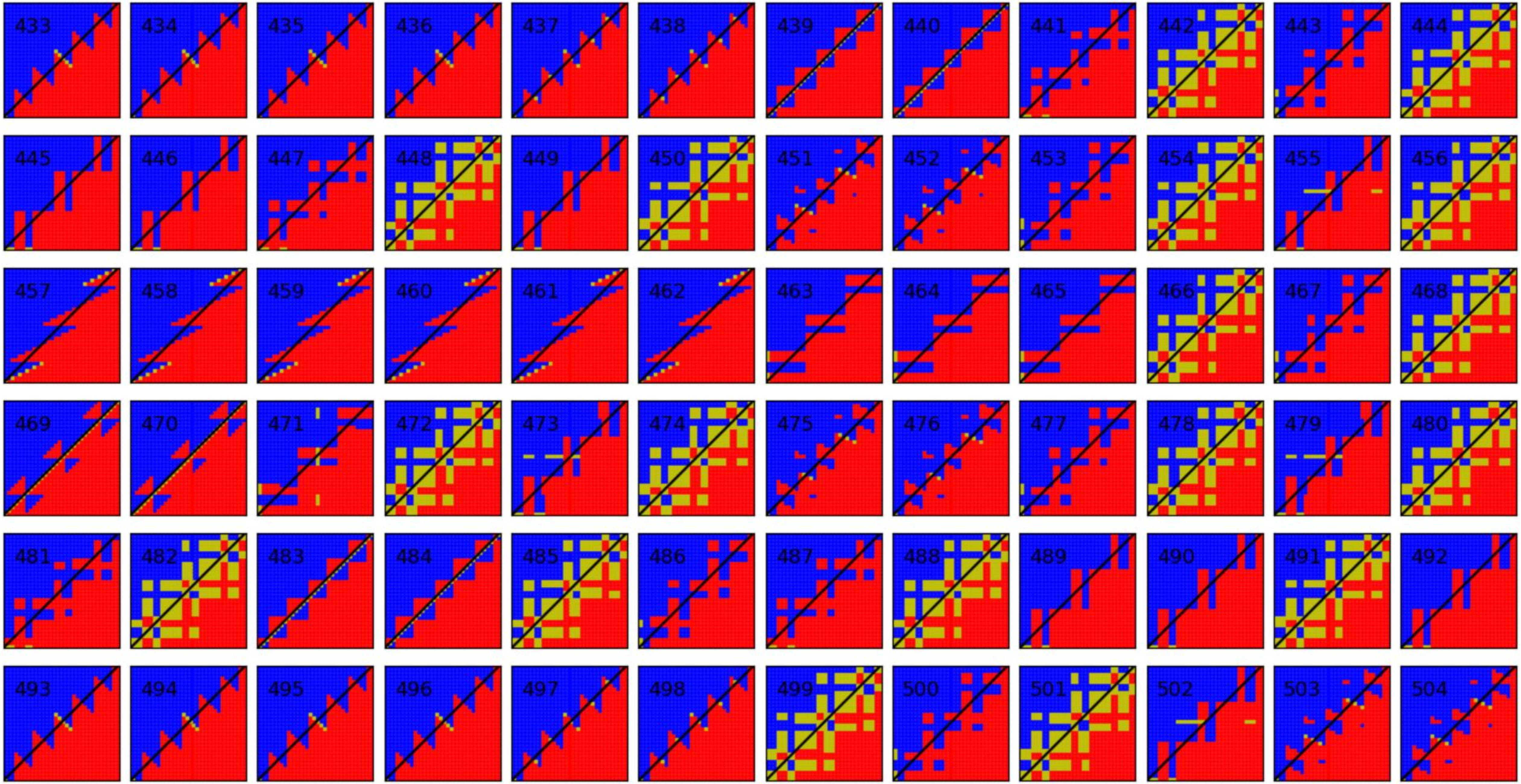
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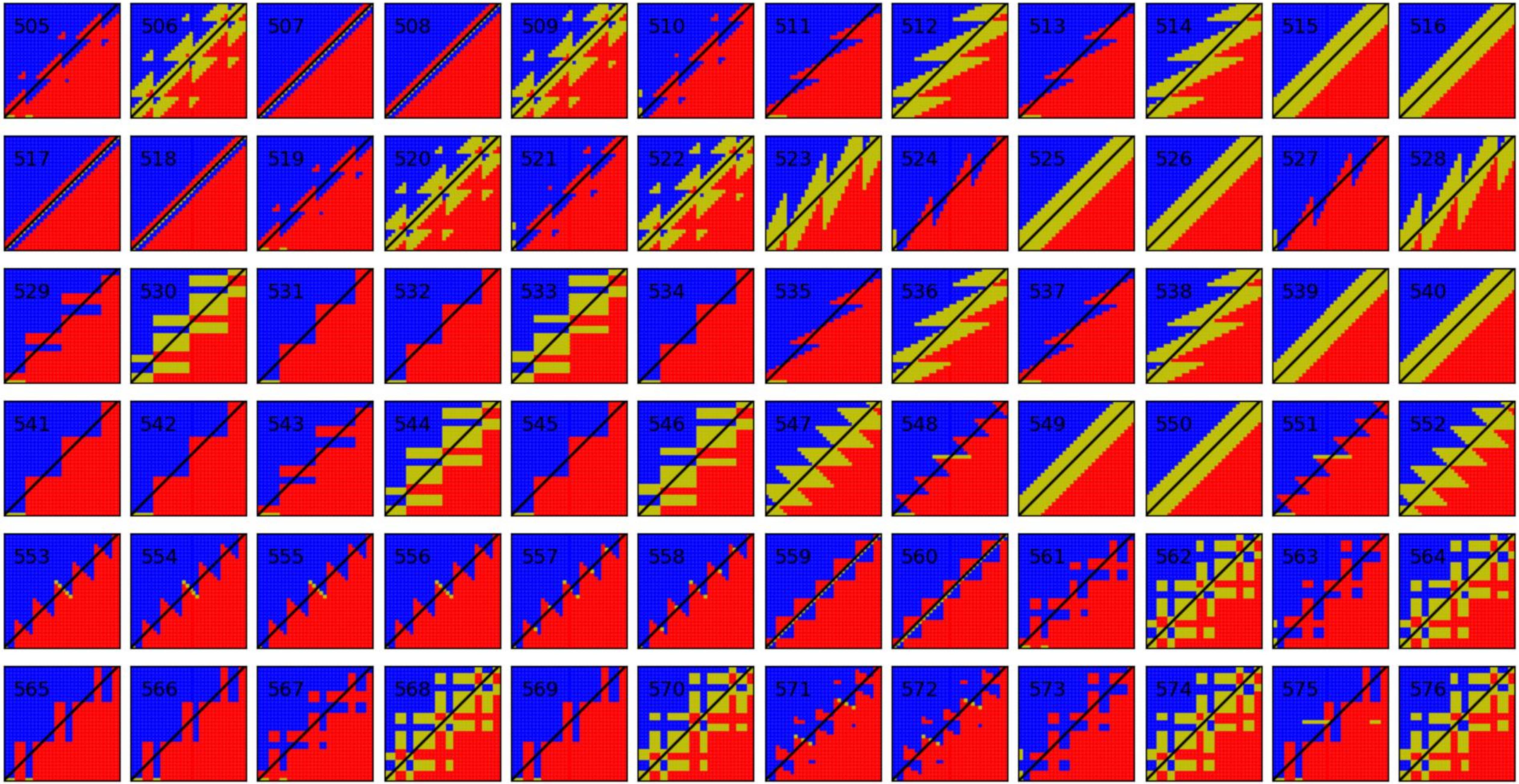
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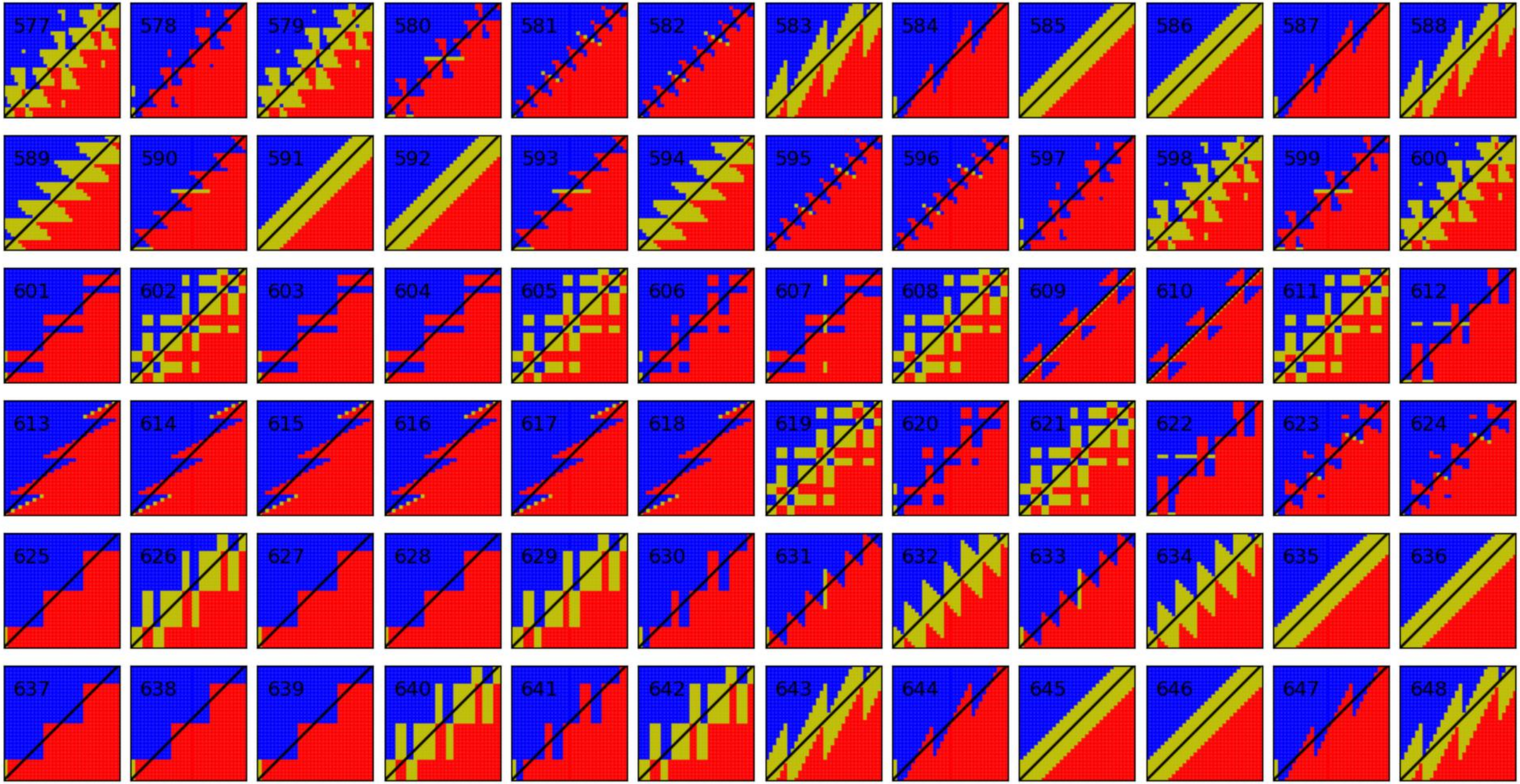
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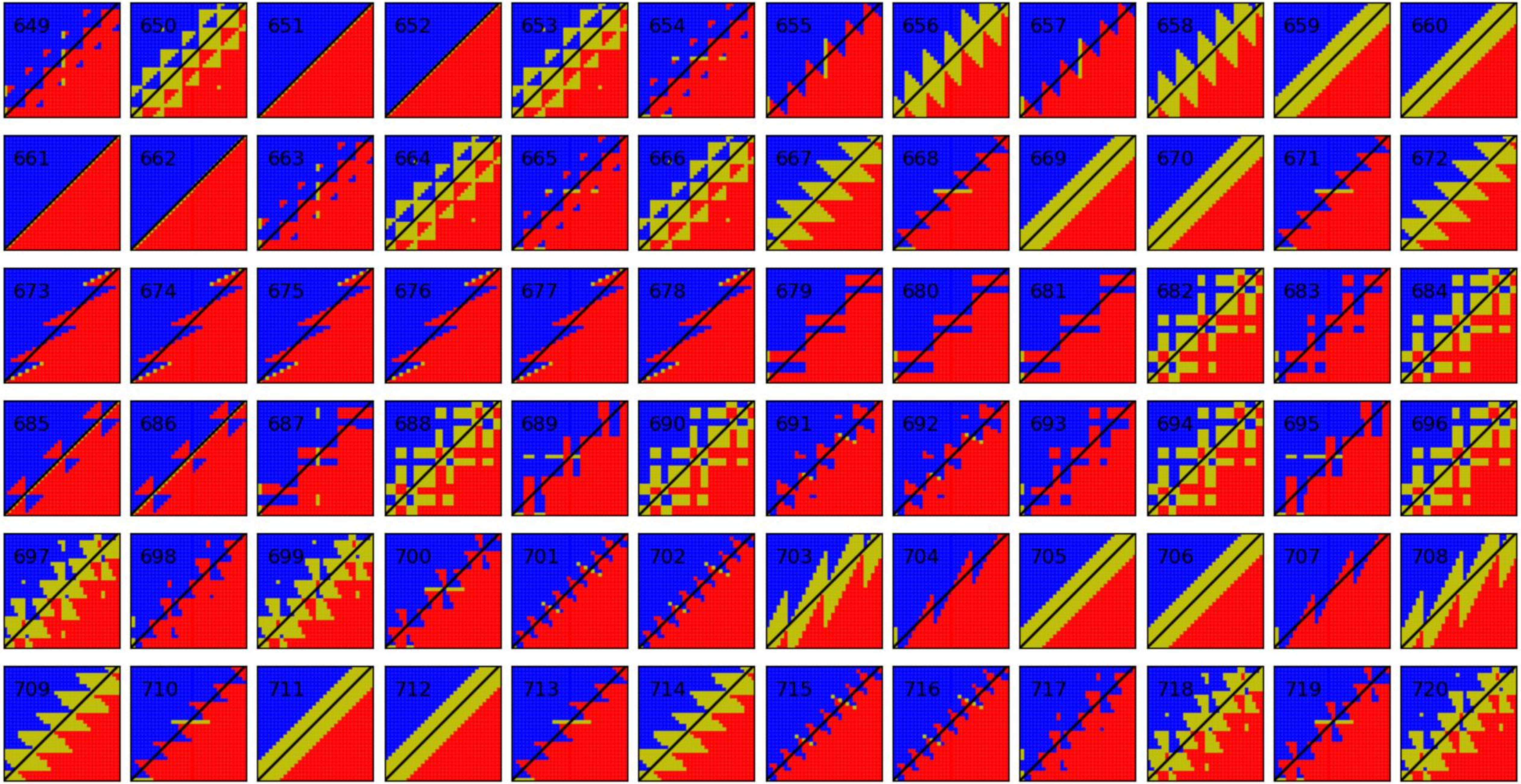
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# Dictionaries and Sets

- dict
- set
- frozenset
- set/dict comprehensions

# Dictionaries (type dict)

$$\{\, key_1 :\ \ value_1 ,\ \ \ldots ,\ \ key_k :\ \ value_k \,\}$$

- Stores a mutable set of (key, value) pairs, denoted *items*, with distinct keys, i.e. *maps* keys to values
- Constructing empty dictionary: `dict()` or `{}`
- *dict* [ *key* ] lookup for key in dictionary, and returns associated value. Key must be present otherwise a KeyError is raised.
- *dict* [ *key* ] = *value* assigns value to *key*, overriding exising value if present.

| key | value |
|---|---|
| 'a' | 7 |
| 'foo' | '42nd' |
| 5 | 29 |
| '5' | 44 |
| 5.5 | False |
| False | True |
| (3, 4) | 'abc' |

distinct keys,
i.e. not "=="

https://docs.python.org/3/tutorial/datastructures.html#dictionaries

# Dictionaries (type dict)

**Python shell**

```
> d = {'a': 42, 'b': 57}
> d
| {'a': 42, 'b': 57}

> d.keys()
| dict_keys(['a', 'b'])

> list(d.keys())
| ['a', 'b']

> d.items()
| dict_items([('a', 42), ('b', 57)])

> list(d.items())
| [('a', 42), ('b', 57)]
```

```
> for key in d:
        print(key)
| a
| b

> for key, val in d.items():
        print("Key", key, "has value", val)
| Key a has value 42
| Key b has value 57

> {5: 'a', 5.0: 'b'}
| {5: 'b'}
```

**Python shell**

```
> surname = dict(zip(['Donald', 'Mickey', 'Scrooge'], ['Duck', 'Mouse', 'McDuck']))
> surname['Mickey']
| 'Mouse'
```

# Dictionaries (type dict)

**Python shell**

```
> gradings = [('A', 7), ('B', 4), ('A', 12), ('C', 10), ('A', 7)]
> grades = {}  # empty dictionary
> for student, grade in gradings:
      if student not in grades:  # is key in dictionary
          grades[student] = []
      grades[student].append(grade)
> grades
| {'A': [7, 12, 7], 'B': [4], 'C': [10]}
> print(grades['A'])
| [7, 12, 7]
> print(grades['E'])  # can only lookup keys in dictionary
| KeyError: 'E'
> print(grades.get('E'))  # .get returns None if key not in dictionary
| None
> print(grades.get('E', []))  # change default return value
| []
> print(grades.get('A', []))
| [7, 12, 7]
```

# Dictionary initialization

```
Python shell
> d1 = {'A': 7, 'B': 42}
> d2 = dict([('A', 7), ('B', 42)])  # list of (key, value) pairs
> d3 = dict(A=7, B=42)              # keyword arguments to constructor
> d1 == d2 == d3
| True
```

- **Note**: *keyword initialization* only works if keys are strings which are valid keyword arguments to a function – but saves writing a lot of quotes

| and |=

new in Python 3.9 PEP 584

```
Python shell
> d1 = dict(A=1, B=2)
> d2 = dict(B=3, C=4)
> d1 | d2                          # merge dictionaries
| {'A': 1, 'B': 3, 'C': 4}         # rightmost value for 'B' wins
> {**d1, **d2, 'D': 5}             # ** inserts dictionary content
| {'A': 1, 'B': 3, 'C': 4, 'D': 5} # rightmost value for 'B' wins
> d1 |= d2                         # same as d1.update(d2)
> d1
| {'A': 1, 'B': 3, 'C': 4}
```

| Dictionary operation | Description |
|---|---|
| len(d) | Items in dictionary |
| d[key] | Lookup key |
| d[key] = value | Update value of key |
| del d[key] | Delete an existing key |
| key in d | Key membership |
| key not in d | Key non-membership |
| clear() | Remove all items |
| copy() | Shallow copy |
| get(key), get(key, default) | d[key] if key in dictionary, otherwise `None` or default |
| items() | *View* of the dictionaries items |
| keys() | *View* of the dictionaries keys |
| values() | *View* of the dictionaries values |
| pop(key) | Remove key and return previous value |
| popitem() | Remove and return an arbitrary item |
| update() | Update key/value pairs from another dictionary |

https://docs.python.org/3/library/stdtypes.html#mapping-types-dict

# Tuples as dictionary keys

- A tuple can be used as a dictionary key, and parenthesis can be omitted

**Python shell -**

```
> d = {('a', 1): 7, ('b', 2): 42}
> d[('b', 2)]
| 42
> d['b', 2]   # same as above, parenthesis omitted
| 42
> T = [[None, None], [42, None]]  # 2D table as lists-of-lists
> T[1][0]
| 42
> T[1, 0]  # wrong, T is a list (of lists)
| TypeError: list indices must be integers or slices, not tuple
> T = {(1, 0): 42}  # 2D table as dictionary
> T[1, 0]  # dictionary lookup with tuple (1, 0) as key
| 42
> T[1][0]  # wrong, T has only one key = the tuple (1, 0)
| KeyError: 1
```

# Order returned by `list(d.keys())` ?

**The Python Standard Library**
**Mapping Types — dict**

"Dictionaries preserve insertion order. Note that updating a key does not affect the order. Keys added after deletion are inserted at the end." (since Python 3.7)

docs.python.org/3/library/stdtypes.html


Raymond Hettinger @ Twitter

**Python shell**

```
> d = {'d': 1, 'c': 2, 'b': 3, 'a':4}
> d['x'] = 5    # new key at end
> d['c'] = 6    # overwrite value
> del d['b']    # remove key 'b'
> d['b'] = 7    # reinsert key 'b' at end
> d
| {'d': 1, 'c': 6, 'a': 4, 'x': 5, 'b': 7}
```

**See also Raymond's talk @ PyCon 2017**
Modern Python Dictionaries
A confluence of a dozen great ideas

# Dictionary comprehension

- Similarly to creating a list using list comprehension, one can create a set of key-value pairs:

{*key* : *value* `for` *variable* `in` *list*}

**Python shell**

```
> names = ['Mickey', 'Donald', 'Scrooge']
> list(enumerate(names, start=1))
| [(1, 'Mickey'), (2, 'Donald'), (3, 'Scrooge')]
> dict(enumerate(names, start=1))
| {1: 'Mickey', 2: 'Donald', 3: 'Scrooge'}
> {name: idx for idx, name in enumerate(names, start=1)}
| {'Mickey': 1, 'Donald': 2, 'Scrooge': 3}
```

# Sets (set and frozenset)

$$\{\, value_1,\ \ldots,\ value_k \,\}$$

- Values of type `set` represent mutable sets, where "==" elements only appear once
- Do **not** support: indexing, slicing
- `frozenset` is an immutable version of `set`

**Python shell**

```
> S = {2, 5, 'a', 'c'}
> T = {3, 4, 5, 'c'}
> S | T
| {2, 3, 4, 5, 'a', 'c'}
> S & T
| {5, 'c'}
```

```
> S ^ T
| {2, 3, 4, 'a'}
> S - T
| {2, 'a'}
> {4, 5, 5.0, 5.1}
| {4, 5, 5.1}
```

⚠

| Operation | Description |
|---|---|
| S \| T | Set union |
| S & T | Set intersection |
| S – T | Set difference |
| S ^ T | Symmetric difference |
| set() | Empty set<br>( {} = empty dictionary ⚠ ) |
| set(L) | Create set from list |
| x in S | Membership |
| x not in S | Non-membership |
| S.isdisjoint(T) | Disjoint sets |
| S <= T | Subset |
| S < T | Proper subset |
| S >= T | Superset |
| S > T | Proper superset |
| len(S) | Size of S |
| S.add(x) | Add x to S (not frozenset) |

https://docs.python.org/3/tutorial/datastructures.html#sets
https://docs.python.org/3/library/stdtypes.html#set-types-set-frozenset

# Question – What value has the expression ?

```
sorted( { 5, 5.5, 5.0, '5' } )
```

a) `{'5', 5, 5.0, 5.5}`

b) `{5, 5.5}`

c) `['5', 5.0, 5.5]`

d) `['5', 5, 5.5]`

e) TypeError

f) Don’t know

# Sets of (frozen) sets

- Sets are mutable, i.e. cannot be used as dictionary keys or elements in sets
- Frozen sets can

**Python shell**

```
> S = {{'a'}, {'a', 'b'}, {'a', 'c'}}
| TypeError: unhashable type: 'set'
> S = {frozenset({'a'}), frozenset({'a', 'b'}), frozenset({'a', 'c'})}
> frozenset({'a', 'b'}) in S
| True
> {'a', 'b'} in S  # automatically converts unhashable set to frozenset
| True
> {'a', 'b'} == frozenset(['a', 'b'])  # frozenset from list
| True
> D = {frozenset({'a', 'b'}): 42}  # dictionary
> frozenset({'a', 'b'}) in D
| True
> {'a', 'b'} in D  # dictionaries are not that friendly as sets
| TypeError: unhashable type: 'set'
```

# Set comprehension

- Similarly to creating a list using list comprehension, one can create a set of values (also using nested for- and if-statements):

  `{`*value* `for` *variable* `in` *list*`}`

- A value occurring multiple times as *value* will only be included once

**primes_set.py**

```
n = 101
not_primes = {m for f in range(2, n) for m in range(2 * f, n, f)}
primes = set(range(2, n)) - not_primes
```

**Python shell**

```
> L = ['a', 'b', 'c']
> {(x,(y,z)) for x in L for y in L for z in L if x != y and y != z and z != x}
| {('a',('b','c')),('a',('c','b')),('b',('a','c')),...,('c',('b','a'))}
> L = {'a', 'b', 'c'}
> {(x,(y,z)) for x in L for y in L - {x} for z in L - {x, y}}
| {('c',('b','a')),('c',('a','b')),('a',('c','b')),...,('b',('a','c'))}
```

# Hash, equality, and immutability

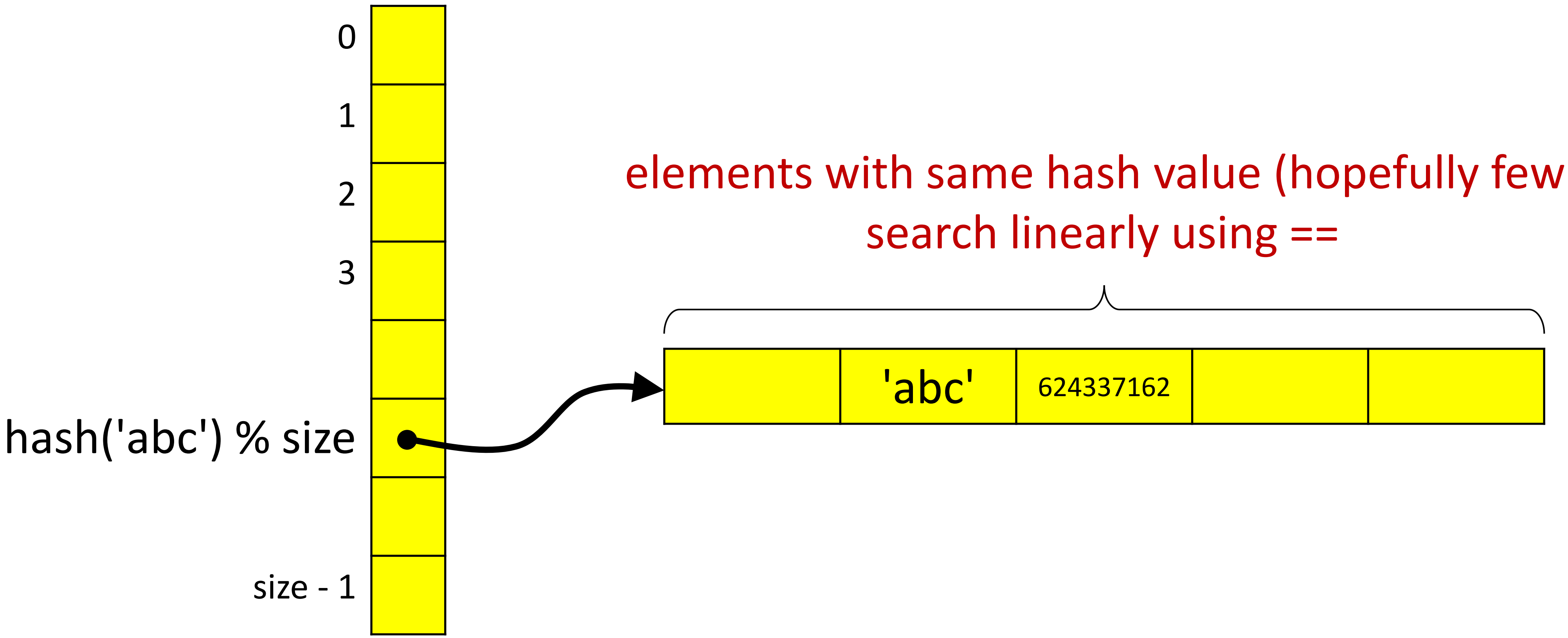
- Keys for dictionaries and sets must be *hashable*, i.e. have a __hash__() method returning an integer that does not change over their lifetime and an __eq__() method to check for equality with "==".

  `'abc'.__hash__()` could e.g. return `624337162`

  `(624337162).__hash__()` would also return `624337162`

- All built-in immutable types are hashable. In particular tuples of immutable values are hashable. I.e. trees represented by nested tuples like `((('a'),'b'),('c',('d','e')))` can be used as dictionary keys or stored in a set.

# Sketch of internal set implementation




Raymond Hettinger,
*Modern Python Dictionaries*
*- A confluence of a dozen great ideas*

# Module `collections` (container datatypes)

- Python builtin containers for data: `list`, `tuple`, `dict`, and `set`.
- The module **`collections`** provides further alternatives
  (but these are not part of the language like the builtin containers)

| | |
|---|---|
| `deque` | double ended queue |
| `namedtuple` | tuples allowing access to fields by name |
| `Counter` | special dictionary to count occurrences of elements |
| … | |

https://docs.python.org/3/library/collections.html

# deque – double ended queues

- Extends lists with efficient updates at the front
- ⚠ Inserting at the front of a standard Python list takes linear time in the size of the list – very slow for long lists

**Python shell**

```
> L = list()
> L.append(1)
> L.append(2)
> L.insert(0, 0)   # insert at the front
> L.insert(0, -1)  # slow for long lists
> L.insert(0, -2)
> L
| [-2, -1, 0, 1, 2]
```

```
> from collections import deque
> d = deque()   # create empty deque
> d.append(1)
> d.append(2)
> d.appendleft(0)   # efficient
> d.appendleft(-1)
> d.appendleft(-2)
> d
| deque([2, 1, 0, -1, -2])
> for e in d: print(e, end=', ')
| -2, -1, 0, 1, 2,
```

# namedtuple – tuples with field names

- Compromise between `tuple` and `dict`, can increase code readability

**Python shell**

```
> person = ('Donald Duck', 1934, '3 feet')  # as tuple
> person[1]  # not clear what is accessed
| 1934
> person = {'name': 'Donald Duck', 'appeared': 1934, 'height': '3 feet'}  # as dict
> person['appeared']  # more clear what is accessed, but ['...'] overhead
| 1934
> from collections import namedtuple
> Person = namedtuple('Person', ['name', 'appeared', 'height'])  # create new type
> person = Person('Donald Duck', 1934, '3 feet')  # as namedtuple
> person
| Person(name='Donald Duck', appeared=1934, height='3 feet')
> person.appeared   # short and clear
| 1934
> person[1] # still possible
| 1934
```

# Counter – dictionaries for counting

**Python shell**

```
> from collections import Counter
> fq = Counter('abracadabra')  # create new counter from a sequence
> fq
| Counter({'a': 5, 'b': 2, 'r': 2, 'c': 1, 'd': 1})  # frequencies of the letters
> fq['a']
| 5
> fq.most_common(3)
| [('a', 5), ('b', 2), ('r', 2)]
> fq['x'] += 5  # increase count of 'x', also valid if 'x' not in Counter yet
> Counter('aaabbbcc') - Counter('aabdd')  # counters can be subtracted
| Counter({'b': 2, 'c': 2, 'a': 1})
> Counter([1, 2, 1, 3, 4, 5]) + Counter([3, 3, 3])  # counters can be added
| Counter({3: 4, 1: 2, 2: 1, 4: 1, 5: 1})

> T = 'AfD adsf dsa f dsaf daf dsaf DSA fda f SA dsa f dsa fdsa f dsAf sAf f dsaf'
> Counter(T.lower().split()).most_common(3)
| [('f', 5), ('dsa', 4), ('dsaf', 4)]
```

# Handin 3 & 4 – Triplet distance (Dobson, 1975)

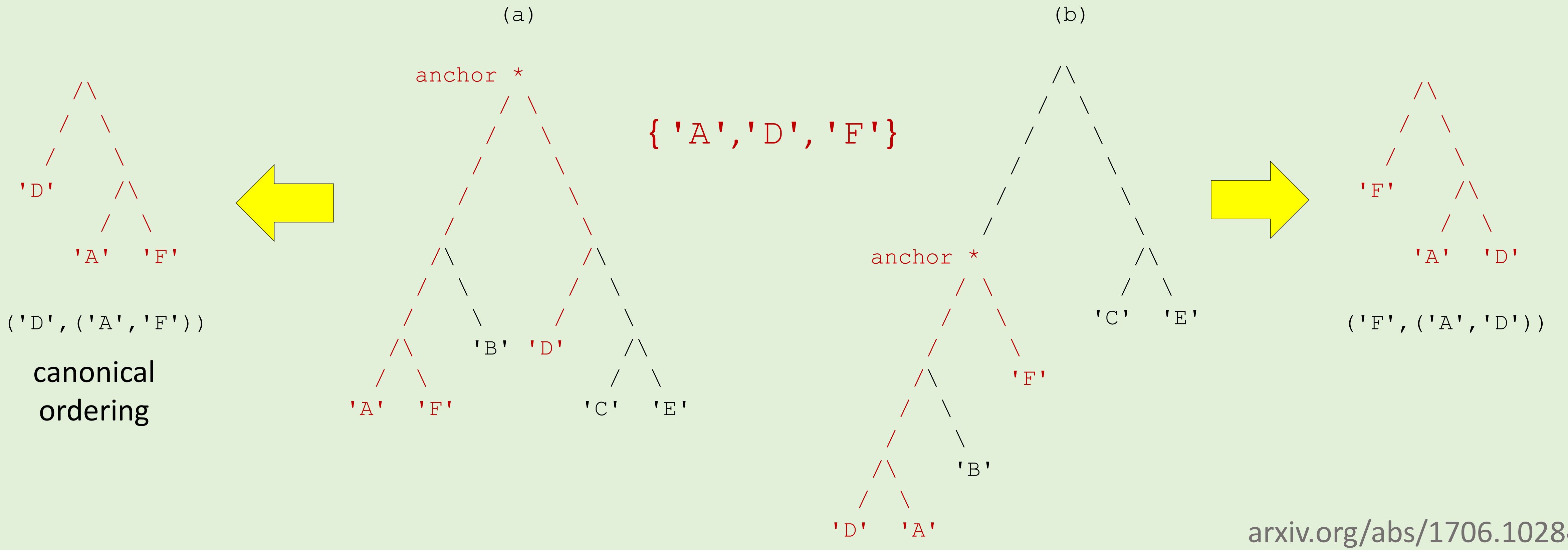
```
((('A','F'),'B'),('D',('C','E')))   (((('D','A'),'B'),'F'),('C','E'))

               (a)                                (b)

               /\                                 /\
              /  \                               /  \
             /    \                             /    \
            /      \                           /      \
           /        \                         /        \
          /          \                       /          \
         /\          /\                     /\          /\
        /  \        /  \                   /  \        /  \
       /    \      /    \                 /    \     'C'  'E'
      /\    'B'  'D'    /\               /      \
     /  \              /  \             /\      'F'
   'A'  'F'          'C'  'E'          /  \
                                      /    \
                                     /\    'B'
                                    /  \
                                  'D'  'A'
```

How similar are the trees ?

arxiv.org/abs/1706.10284

# Handin 3 & 4 – Triplet distance (Dobson, 1975)

Consider all $\binom{n}{3}$ subsets of size three, and count how many do not have identical substructure (topology) in the two trees.




arxiv.org/abs/1706.10284

# Functions

- functions
- return
- scoping
- arguments
- keyword arguments
- *, **
- global variables

# (Simple) functions

- You can define your own functions using:

  **def** ***function-name*** **(*$var_1$, ..., $var_k$*):**
  **body code**

  $var_1$, ..., $var_k$ are the *formal parameters*

- If the body code executes

  `return` *expression*

  the result of *expression* will be returned by the function. If expression is omitted or the body code terminates without performing `return`, then `None` is returned.

- When *calling* a function ***name*** **(*$value_1$,..., $value_k$*)** body code is executed with $var_i$=$value_i$

**Python shell**

```
> def sum3(x, y, z):
      return x+y+z

> sum3(1, 2, 3)
| 6
> sum3(5, 7, 9)
| 21

> def powers(L, power):
      P = [x**power for x in L]
      return P

> powers([2,3,4], 3)
| [8, 27, 64]
```

# Questions – `poly(3, "10", '3')` ?

```
def poly(z, x, y):
    return z * x + y
```

a) 33
b) 1010103
c) '33'
d) '1010103'
e) TypeError
f) Don't know

# Why functions ?

**Some other Python language features helping structuring programs**

- Object orientation
- Modules
- Decorators
- Context managers
- Exceptions
- Doc strings
- doctest

- Avoid writting the same code multiple times, *re-usability*
- Be able to *name a functionality*
- Clearly state the functionality of a piece of code, *abstraction*: *Input* = arguments, *output* = return value (and/or side effects)
- *Encapsulate* code with clear interface to the dependency to the outside world/code
- Share functionality in modules/libraries/packages with other users, *code sharing*
- Increase *readability* of code, smaller independent blocks of code
- Easier systematically *testing* of code
- ...

# Local variables in functions

- The formal arguments and variables assigned to in the body of a function are created as temporary *local variables*

| Global variables | | Local variables | |
|---|---|---|---|
| sum3 | <function> | x | 4 |
| a | 3 | y | 5 |
| y | 42 | z | 6 |
| | | a | 9 |
| | | b | 15 |

state just before `return b`

**Python shell**

```
> def sum3(x, y, z):
      a = x + y
      b = a + z
      return b

> a = 3
> y = 42
> w = sum3(4, 5, 6)
> w
| 15
> a
| 3
> b
| NameError: name 'b' is not defined
> x
| NameError: name 'x' is not defined
> y
| 42
> sum3
| <function sum3 at 0x0356DA98>
```

# Global variables

- Variables in function bodies that are only read, are considered access to *global variables*

**Python shell**

```
> prefix = "The value is"
> def nice_print(x):
      print(prefix, x)
> nice_print(7)
| The value is 7
> prefix = "Value ="
> nice_print(42)
| Value = 42
```

| Global variables | | Local variables | |
|---|---|---|---|
| nice_print | <function> | x | 42 |
| prefix | "Value =" | | |

state just before returning from 2nd `nice_print`

# Global vs local variables

- If a function contains an assignment to a variable, the variable is local throughout the function – also before the first assignment

**Python shell**

```
> x = 42
> def f():
      print(x)   # refers to local variable
      x = 7   # x declared local variable
> f()
| UnboundLocalError: local variable 'x' referenced before assignment
```

# global

- Global variables that should be updated in the function body must be declared global in the body:

  `global` *variable*, *variable*, ...

- Note: If you only need to read a global variable, it is not required to be declared global (but would be polite to the readers of your code)

Since `counter` assigned in body, `counter` will be considered to be a local variable

**Python shell**

```
> counter = 1

> def counted_print(x):
      global counter
      print("(%d)" % counter, x)
      counter += 1

> counted_print(7)
| (1) 7
> counted_print(42)
| (2) 42

> def counted_print(x):
      print("(%d)" % counter, x)
      counter += 1

> counted_print(7)
| UnboundLocalError: local variable
  'counter' referenced before
  assignment
```

# Question – What value is printed ?

```
x = 1

def f(a):
    global x
    x = x + 1
    return a + x

print(f(2) + f(4))
```

a) 6

b) 7

c) 8

d) 9

e) 10

f) 11

g) 12

h) Don't know

# Arbitrary number of arguments

- If you would like your function to be able to take a variable number of additional arguments in addition to the required, add a **variable* as the last argument.
- In a function call *variable* will be assigned a tuple with all the additional arguments.

**Python shell**

```
> def my_print(x, y, *L):
      print("x =", x)
      print("y =", y)
      print("L =", L)

> my_print(2, 3, 4, 5, 6, 7)
| x = 2
| y = 3
| L = (4, 5, 6, 7)
> my_print(42)
| TypeError: my_print() missing 1
  required positional argument: 'y'
```

# Unpacking a list of arguments in a function call

- If you have list `L` (or tuple) containing the arguments to a *function call*, you can unpack them in the function call using `*L`

  ```
  L = [x, y, z]
  f(*L)
  ```

  is equivalent to calling

  ```
  f(L[0], L[1], L[2])
  ```

  i.e.

  ```
  f(x, y, z)
  ```

- Note that `f(L)` would pass a single argument to `f`, namely a list
- In a function call several `*` expressions can appear, e.g. `f(*L1, x, *L2, *L3)`

**Python shell**

```
> import math
> def norm(x, y):
      return math.sqrt(x * x + y * y)
> norm(3, 5)
| 5.830951894845301
> point = (3, 4)
> print(*point, sep=':')
| 3:4
> norm(point)
| TypeError: norm() missing 1 required positional argument: 'y'
> norm(*point)
| 5.0
> def dist(x0, y0, x1, y1):
      return math.sqrt((x1 - x0) ** 2 + (y1 - y0) ** 2)
> p = 3, 7
> q = 7, 4
> dist(p, q)
| TypeError: dist() missing 2 required positional arguments: 'x1' and 'y1'
> dist(*p, *q)
| 5.0
```

# Question – How many arguments should `f` take ?

```
a = [1, 2, 3]
b = [4, 5]
c = (6, 7, 8)
d = (9, 10)
f(*a, b, c, *d)
```

a) 4
b) 5
c) 6
d) 7
e) 8
f) 9
g) 10
h) Don’t know

# Question – What is `list(zip(*zip(*L)))` ?

```
L = [[1, 2, 3], [4, 5], [6, 7, 8]]
```

a) `[(([1, 2, 3],),), (([4, 5],),), (([6, 7, 8],),)]`
b) `[((1, 4, 6),), ((2, 5, 7),)]`
c) `[(1, 2), (4, 5), (6, 7)]`
d) `[(1, 2, 3), (4, 5), (6, 7, 8)]`
e) `[([1, 2, 3], [4, 5], [6, 7, 8])]`
f) Don't know

**Python shell**

```
> list(zip((1, 2, 3), (4, 5, 6)))
| [(1, 4), (2, 5), (3, 6)]
```

# Keyword arguments

- Previously we have seen the following (strange) function calls

```
print(7, 14, 15, sep=":", end="")
enumerate(my_list, start=1)
```

- *name* = refers to one of the formal arguments, known as a keyword argument. A *name* can appear at most once in a function call.
- In function calls, keyword arguments must follow positional arguments.
- Can e.g. be useful if there are many arguments, and the order is not obvious, i.e. improves readability of code.

```
complicated_function(
    name="Mickey",
    city="Duckburg",
    state="Calisota",
    occupation="Detective",
    gender="Male"
)
```

**Python shell**

```
> def sub(x, y):
      return x - y

> sub(9, 4)
| 5
> sub(y=9, x=4)
| -5
```

# Keyword arguments, default values

- When calling a function arguments can be ommitted if the corresponding arguments in the function definition have default values *argument = value*.

**Python shell**

```
> def my_print(a, b, c=5, d=7):
      print("a=%s, b=%s, c=%s, d=%s" % (a, b, c, d))

> my_print(2, d=3, b=4)
| a=2, b=4, c=5, d=3
```

# Question – What is `f(6, z=2)` ?

```
def f(x, y=3, z=7):
    return x + y + z
```

a) `10`

b) `11`

c) `16`

d) `TypeError: f() missing 1 required positional argument: 'y'`

e) Don’t know

# Keyword arguments, mutable default values ⚠

- Be carefull: Default value will be shared among calls (which can be usefull)

**The Python Language Reference**
**8.7 Function definitions**

”**Default parameter values are evaluated from left to right when the function definition is executed**. This means that the expression is evaluated once, when the function is defined, and that the same “pre-computed” value is used for each call. This is especially important to understand when a default parameter is a mutable object, such as a list or a dictionary: if the function modifies the object (e.g. by appending an item to a list), the default value is in effect modified. This is generally not what was intended. A way around this is to use None as the default”

**Python shell**

```
> def list_append(e, L=[]):
        L.append(e)
        return L
> list_append('x', ['y', 'z'])
| ['y', 'z', 'x']
> list_append("a")
| ['a']
> list_append("b")
| ['a', 'b']
> list_append("c")
| ['a', 'b', 'c']
```

**Python shell**

```
> def list_append(e, L=None):
      if L == None:
          L = []
      L.append(e)
      return L
> list_append('x', ['y', 'z'])
| ['y', 'z', 'x']
> list_append("a")
| ['a']
> list_append("b")
| ['b']
> list_append("c")
| ['c']
```

# Function call, dictionary of keyword arguments

- If you happen to have a *dictionary* containing the keyword arguments you want to pass to function, you can give all dictionary items as arguments using the single argument ***dictionary*

**Python shell**

```
> print(3, 4, 5, sep=":", end='#\n')
| 3:4:5#
> print_kwarg = {'sep': ':', 'end': '#\n'}
> print(3, 4, 5, **print_kwarg)
| 3:4:5#
```

# Function definition, arbitrary keyword arguments

- If you want a function to accept arbitrary keyword arguments, add an argument *****argument*** to the function definition.
- When the function is called *argument* will be assigned a dictionary containing the excess keyword arguments.

**Python shell**

```
> def my_print(a, b=3, **c):
        print("a =", a)
        print("b =", b)
        print("c =", c)

> my_print(x=27, y=42, a=7)
| a = 7
| b = 3
| c = {'x': 27, 'y': 42}
```

# Example

Python shell

```
> L1 = [1, 'a']
> L2 = ['b', 2, 3]
> D1 = {'y':4, 's':10}
> D2 = {'t':11, 'z':5.0}

> def f(a, b, c, d, e, *f, q=0, x=1, y=2, z=3, **kw):
      print("a=%s, b=%s, c=%s, d=%s, e=%s, " % (a, b, c, d, e),
            "f=%s\n" % str(f),
            "q=%s, x=%s, y=%s, z=%s, " % (q, x, y, z),
            "kw=%s" % kw,
            sep="")

> f(7, *L1, 9, *L2, x=7, **D1, w=42, **D2)
| a=7, b=1, c=a, d=9, e=b, f=(2, 3)
| q=0, x=7, y=4, z=5.0, kw={'w': 42, 's': 10, 't': 11}
```

non-keyword arguments must appear before keyword arguments

all arguments must have names

# A confusing example

**Python shell**

```
> def f(*a, **kw):
      print(f'{a=} {kw=}')
> f(a=42)   # no positional arguments
| a=() kw={'a': 42}
```

# Forwarding function arguments

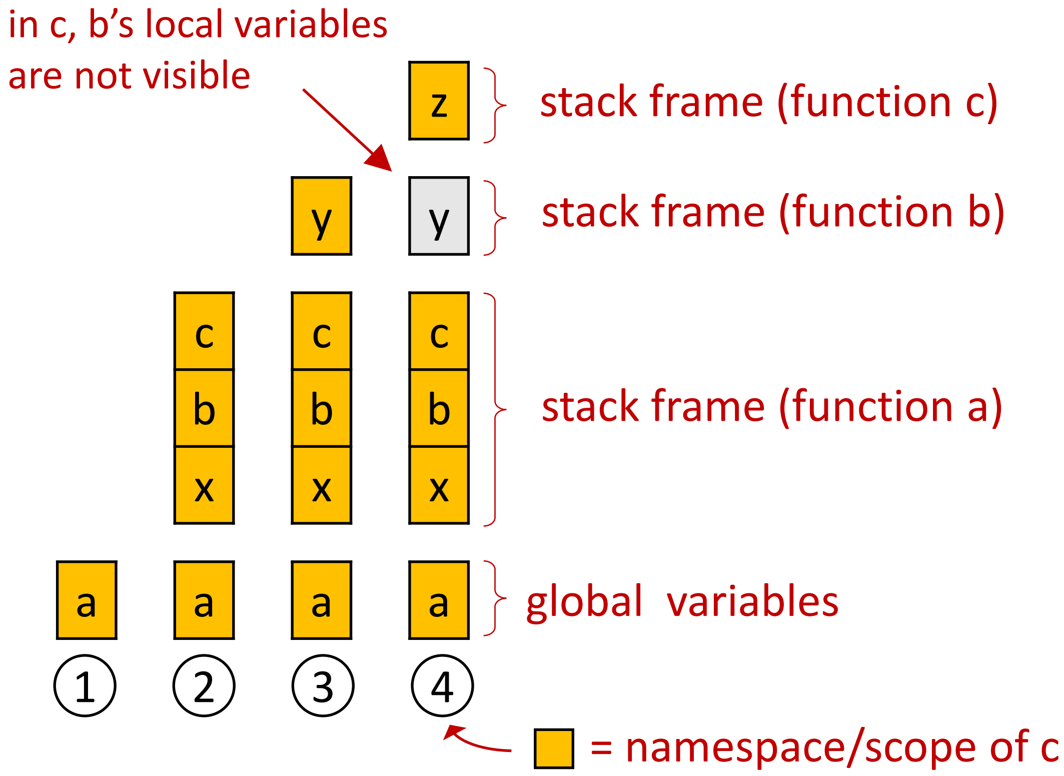
- * and ** can e.g. be used to forward (unknown) arguments to other function calls

**Python shell**

```
> def my_print(*positional_arguments, sep=":", **keyword_arguments):
      print(*positional_arguments, sep=sep, **keyword_arguments)
> my_print(7, 42)
| 7:42
> my_print("x", "y", end="<")
| x:y<
> my_print("x", "y", sep="_")
| x_y
```

# Local function definitions and namespaces

- Function definitions can contain (nested) local function definitions, only accessable inside the function
- *static/lexical scoping*, i.e. can see from the code which variables are in scope




**Python shell**

```
> def a(x):
      def b(y):
          print("b: y=%s x=%s" % (y, x))
          c(y + 1)
      def c(z):
          print("c: z=%s x=%s" % (z, x))
      print("a: x=%s" % x)
      b(x + 1)
> a(42)
| a: x=42
| b: y=43 x=42
| c: z=44 x=42
```

# Example – nested function definitions

**Python shell**

```
> def a(x):
      def b(y):
          print("Enter b (y=%s, x=%s)" % (y, x))
          c(y + 1)
          print("leaving b")
      def c(x):  # x hides argument of function a
          def d(z):
              print("Enter d (z=%s, x=%s)" % (z, x))
              print("leaving d")
          print("Enter c (x=%s)" % x)
          d(x + 1)
          print("leaving c")
      print("Enter a (x=%s)" % x)
      b(x + 1)
      print("leaving a")
```

```
> a(5)
| Enter a (x=5)
| Enter b (y=6, x=5)
| Enter c (x=7)
| Enter d (z=8, x=7)
| leaving d
| leaving c
| leaving b
| leaving a
```

# Example – nested functions and default values

**Python shell**

```
> def init_none(var_name):
>     print('initializing', var_name)
>     return None   # redundant line
> def f(a=init_none('a')):
>     def g(b=init_none('b')):
>         print('b =', b)
>     print("a =", a)
>     g(a + 1)
| initializing a
> f(10)
| initializing b
| a = 10
| b = 11
```

# nonlocal

- The `nonlocal` statement causes the listed identifiers to refer to previously bound variables in the nearest *enclosing scope excluding globals*.
- `nonlocal` *variable*, *variable*, ...

**Python shell**

```
> x = 0

> def f():
      y = 1
      def f_helper(z):
          global x
          nonlocal y
          print("(%s:%s) %s" % (x, y, z))
          y += 1
          x += 3

      f_helper(7)
      f_helper(42)

> f()
| (0:1) 7
| (3:2) 42
> f()
| (6:1) 7
| (9:2) 42
```

docs.python.org/3/reference/simple_stmts.html#the-nonlocal-statement

# Positional and keyword only arguments

- A function definition can contain / and * as arguments.
  Arguments before / must be provided as positional arguments in a call, and arguments after * cannot be positional arguments

**Python shell**

```
> def f(a, /, b, *, c):
      print(a, b, c)

> f(a=1, b=2, c=3)
| TypeError: f() got some positional-only arguments passed as keyword arguments: 'a'
> f(1, b=2, c=3)
| 1 2 3
> f(1, 2, c=3)
| 1 2 3
> f(1, 2, 3)
| TypeError: f() takes 2 positional arguments but 3 were given
```

# A note on Python and functions

- Similarities between Python and other languages:
  - functions are widely supported (sometimes called methods and procedures)
  - scoping rules is present in many languages (but details differ)
- Python specific (but nice):
  - how to handle global, local and nonlocal variables
  - keyword arguments
  - *, **

# Recursion

- symbol table
- stack frames

# Recursion

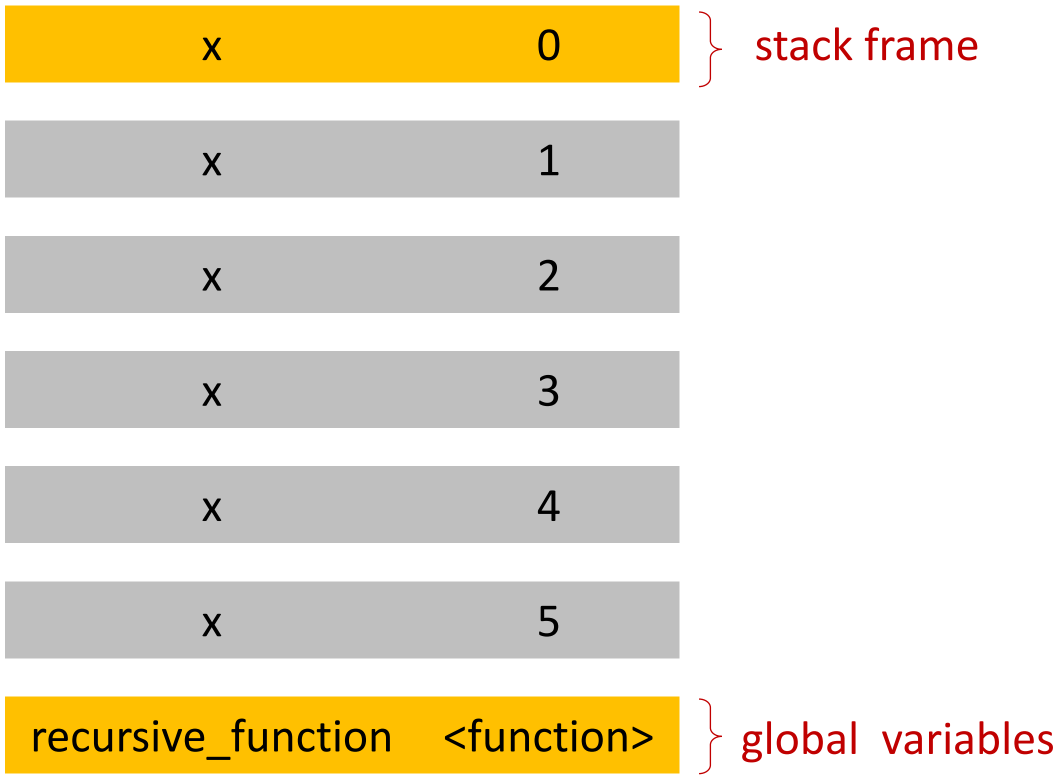
Recursive function
≡
”function that calls itself”

| | | |
|---|---|---|
| start 5 | `recursive_function(5)` | end 5 |
| start 4 | `recursive_function(4)` | end 4 |
| start 3 | `recursive_function(3)` | end 3 |
| start 2 | `recursive_function(2)` | end 2 |
| start 1 | `recursive_function(1)` | end 1 |
| | `recursive_function(0)` | |
| | done | |

**Python shell**

```
> def recursive_function(x):
      if x > 0:
          print("start", x)
          recursive_function(x - 1)
          print("end", x)
      else:
          print("done")

> recursive_function(5)
| start 5
| start 4
| start 3
| start 2
| start 1
| done
| end 1
| end 2
| end 3
| end 4
| end 5
```

# Recursion




Recursions stack when x = 0 is reached

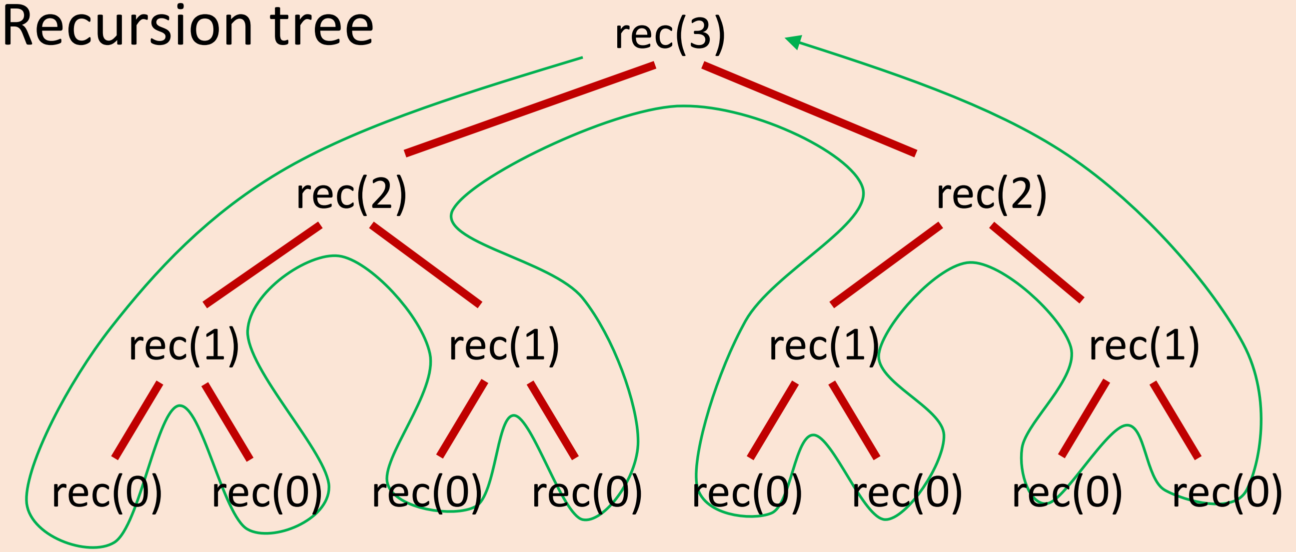
**Python shell**

```
> def recursive_function(x):
      if x > 0:
          print("start", x)
          recursive_function(x - 1)
          print("end", x)
      else:
          print("done")

> recursive_function(5)
| start 5
| start 4
| start 3
| start 2
| start 1
| done
| end 1
| end 2
| end 3
| end 4
| end 5
```

**Python shell**

```
> def rec(x):
      if x > 0:
          print("start", x)
          rec(x - 1)
          rec(x - 1)
          print("end", x)
      else:
          print("done")
```

Recursion tree




**Python shell**

```
> rec(3)
| start 3
| start 2
| start 1
| done
| done
| end 1
| start 1
| done
| done
| end 1
| end 2
| start 2
| start 1
| done
| done
| end 1
| start 1
| done
| done
| end 1
| end 2
| end 3
```

# Question – How many times does `rec(5)` print "`done`"?

**Python shell**

```
> def rec(x):
      if x > 0:
          print("start", x)
          rec(x - 1)
          rec(x - 1)
          rec(x - 1)
          print("end", x)
      else:
          print("done")
```

a) 3

b) 5

c) 15

d) 81

e) 125

f) $243 = 3^5$

g) Don't know

# Factorial

$$n\,! = n \cdot \underbrace{(n - 1) \cdot (n - 2) \cdots 3 \cdot 2 \cdot 1}_{(n-1)\,!}$$

**Observation**
(recursive definition)

$$1\,! = 1$$

$$n\,! = n \cdot (n - 1)\,!$$

factorial.py

```
def factorial(n):
    if n <= 1:
        return 1
    return n * factorial(n - 1)
```

factorial_iterative.py

```
def factorial(n):
    result = 1
    for i in range(2, n + 1):
        result *= i
    return result
```

factorial.py

```
def factorial(n):
    return n * factorial(n - 1) if n > 1 else 1
```

# Binomial coefficient $\binom{n}{k}$

- $\binom{n}{k}$ = number of ways to pick k elements from a set of size n
- $$\binom{n}{k} = \begin{cases} 1 & \text{if } k = 0 \text{ or } k = n \\ \binom{n-1}{k} + \binom{n-1}{k-1} & \text{otherwise} \end{cases}$$

**binomial_recursive.py**

```
def binomial(n, k):
    if k == 0 or k == n:
        return 1
    return binomial(n - 1, k) + binomial(n - 1, k - 1)
```

- Unfolding computation shows $\binom{n}{k}$ 1's are added → slow

# Tracing the recursion

- At beginning of function call, print arguments
- Before returning, print return value
- Keep track of recursion depth in a argument to print indentation

binomial_trace.py

```
def binomial(n, k, indent=0):
    print('   ' * indent + f'binomial({n}, {k})')
    if k == 0 or k == n:
        result = 1
    else:
        result = binomial(n - 1, k, indent=indent + 1) + \
                 binomial(n - 1, k - 1, indent=indent + 1)
    print('   ' * indent + f'return {result}')
    return result
```

Python shell

```
> binomial(4, 2)
| binomial(4, 2)
|    binomial(3, 2)
|       binomial(2, 2)
|       return 1
|       binomial(2, 1)
|          binomial(1, 1)
|          return 1
|          binomial(1, 0)
|          return 1
|       return 2
|    return 3
|    binomial(3, 1)
|       binomial(2, 1)
|          binomial(1, 1)
|          return 1
|          binomial(1, 0)
|          return 1
|       return 2
|       binomial(2, 0)
|       return 1
|    return 3
| return 6
| 6
```

# Binomial coefficient $\binom{n}{k}$

Observation $\binom{n}{k} = \dfrac{n\,!}{(n-k)\,!\cdot k\,!}$

**binomial_factorial.py**

```
def binomial(n, k):
    return factorial(n) // factorial(k) // factorial(n - k)
```

- Unfolding computation shows 2n - 2 multiplications and 2 divisions → fast
- Intermediate value n ! can have significantly more digits than result (bad)

# Binomial coefficient $\binom{n}{k}$

Observation $\binom{n}{k} = \dfrac{n \cdot (n-1) \cdot (n-2) \cdots (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdots 1} = \binom{n-1}{k-1} \cdot \dfrac{n}{k}$

**binomial_recursive_product.py**

```
def binomial(n, k):
    if k == 0:
        return 1
    else:
        return binomial(n - 1, k - 1) * n // k
```

- Unfolding computation shows k multiplications and divisions → fast
- Multiplication with fractions ≥ 1 → intermediate numbers limited size

# Questions – Which correctly computes $\binom{n}{k}$ ?

Observation $\binom{n}{k} = \dfrac{n \cdot (n-1) \cdot (n-2) \cdots (n-k+1)}{k \cdot (k-1) \cdot (k-2) \cdots 1}$

a) binomial_A
b) binomial_B
c) both
d) none
e) Don’t know

**binomial_iterative.py**

```
def binomial_A(n, k):
    result = 1
    for i in range(k):
        result = result * (n - i) // (k - i)
    return result

def binomial_B(n, k):
    result = 1
    for i in range(k)[::-1]:
        result = result * (n - i) // (k - i)
    return result
```

**Python shell**

```
> binomial_A(5, 2)
| 8
> binomial_B(5, 2)
| 10
```

# Recursively print all leaves of a tree

- Assume a recursively nested tuple represents a tree with strings as leaves

**Python shell**

```
> def print_leaves(tree):
      if isinstance(tree, str):
          print("Leaf:", tree)
      else:
          for child in tree:
              print_leaves(child)

> print_leaves(('a',('b','c')))
| Leaf: a
| Leaf: b
| Leaf: c
```

```
      /\
     /  \
   'a'  /\
       /  \
     'b'  'c'
```

# Question – How many times is `print_leaves` function called in the example?

```
   /\
  /  \
'a'  /\
    /  \
  'b'  'c'
```

**Python shell**

```
> def print_leaves(tree):
      if isinstance(tree, str):
          print("Leaf:", tree)
      else:
          for child in tree:
              print_leaves(child)

> print_leaves(('a',('b','c')))
| Leaf: a
| Leaf: b
| Leaf: c
```

a) 3
b) 4
c) 5
d) 6
e) Don’t know

**Python shell**

```
> def collect_leaves_wrong(tree, leaves = set()):
      if isinstance(tree, str):
          leaves.add(tree)
      else:
          for child in tree:
              collect_leaves_wrong(child, leaves)
      return leaves

> def collect_leaves_right(tree, leaves = None):
      if leaves == None:
          leaves = set()
      if isinstance(tree, str):
          leaves.add(tree)
      else:
          for child in tree:
              collect_leaves_right(child, leaves)
      return leaves
```

```
> collect_leaves_wrong(('a',('b','c')))
| {'a', 'c', 'b'}
> collect_leaves_wrong(('d',('e','f')))
| {'b', 'e', 'a', 'f', 'c', 'd'}

> collect_leaves_right(('a',('b','c')))
| {'b', 'a', 'c'}
> collect_leaves_right(('d',('e','f')))
| {'f', 'd', 'e'}
```

**Python shell**

```
> def collect_leaves(tree):
      leaves = set()

      def traverse(tree):
          nonlocal leaves  # can be omitted
          if isinstance(tree, str):
              leaves.add(tree)
          else:
              for child in tree:
                  traverse(child)

      traverse(tree)
      return leaves

> collect_leaves(('a', ('b', 'c')))
| {'b', 'a', 'c'}
> collect_leaves(('d', ('e', 'f')))
| {'f', 'd', 'e'}
```

# Maximum recursion depth ?

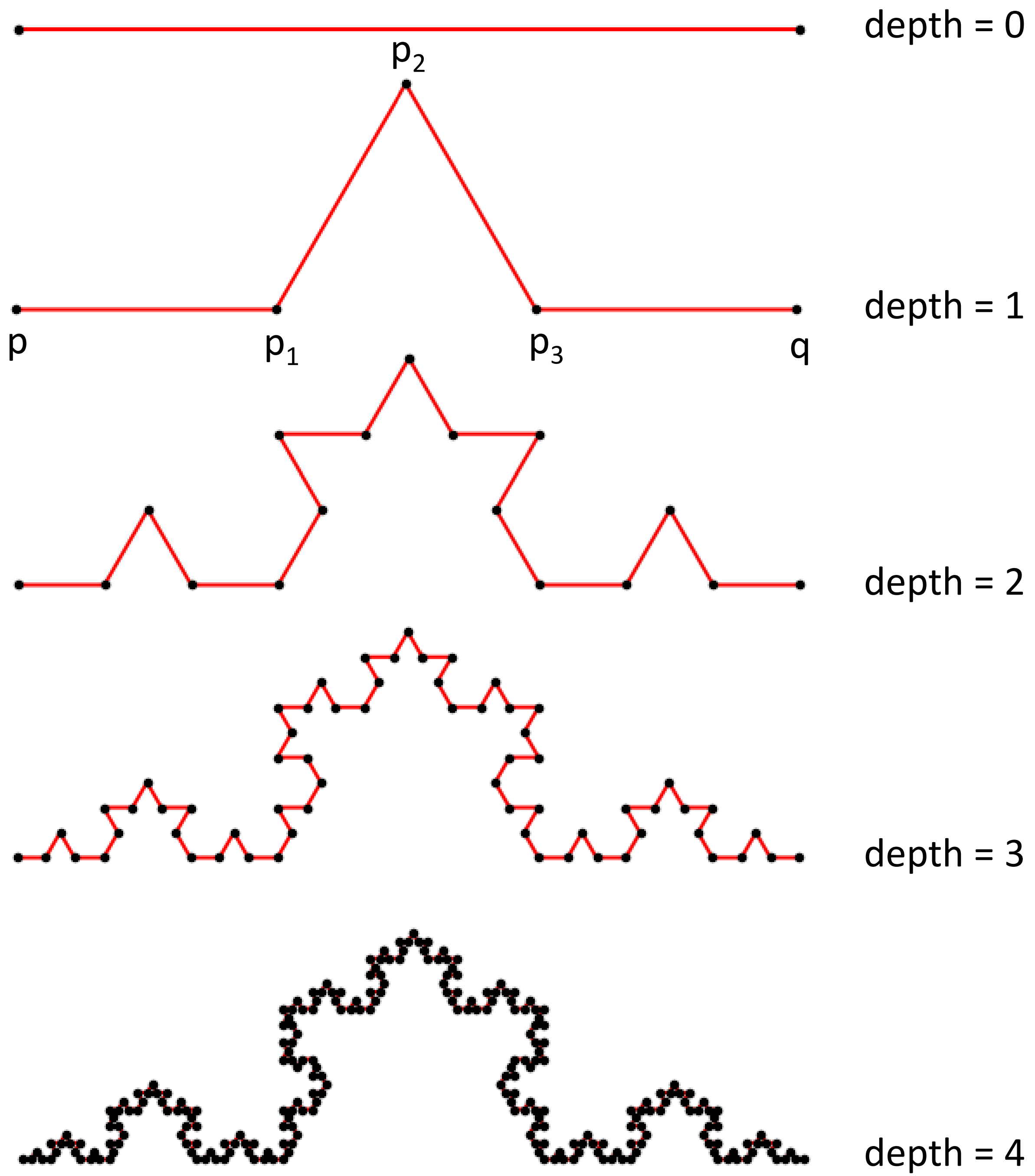
- Pythons maximum allowed recursion depth can be increased by

```
import sys
sys.setrecursionlimit(1500)
```

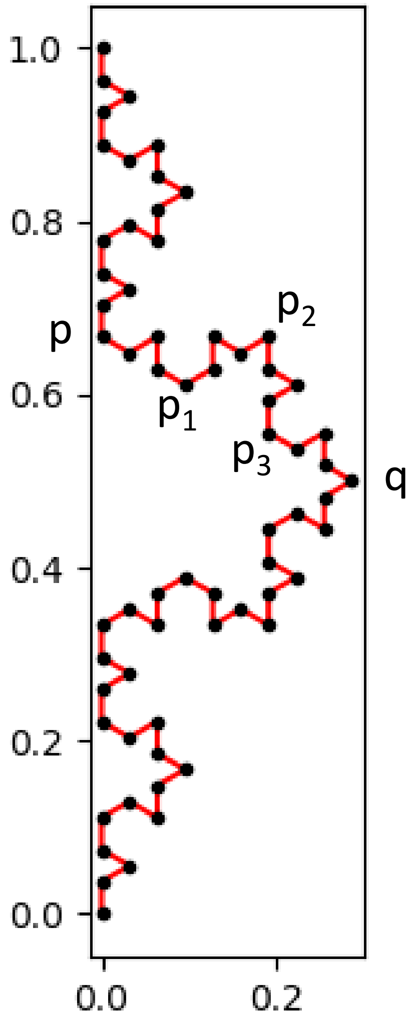
**Python shell**

```
> def f(x):
      print("#", x)
      f(x + 1)

> f(1)
| # 1
| # 2
| # 3
| ...
| # 975
| # 976
| # 977
| # 978
| RecursionError: maximum
  recursion depth exceeded
  while pickling an object
```

# Koch Curves



# Koch Curves




**koch_curve.py**

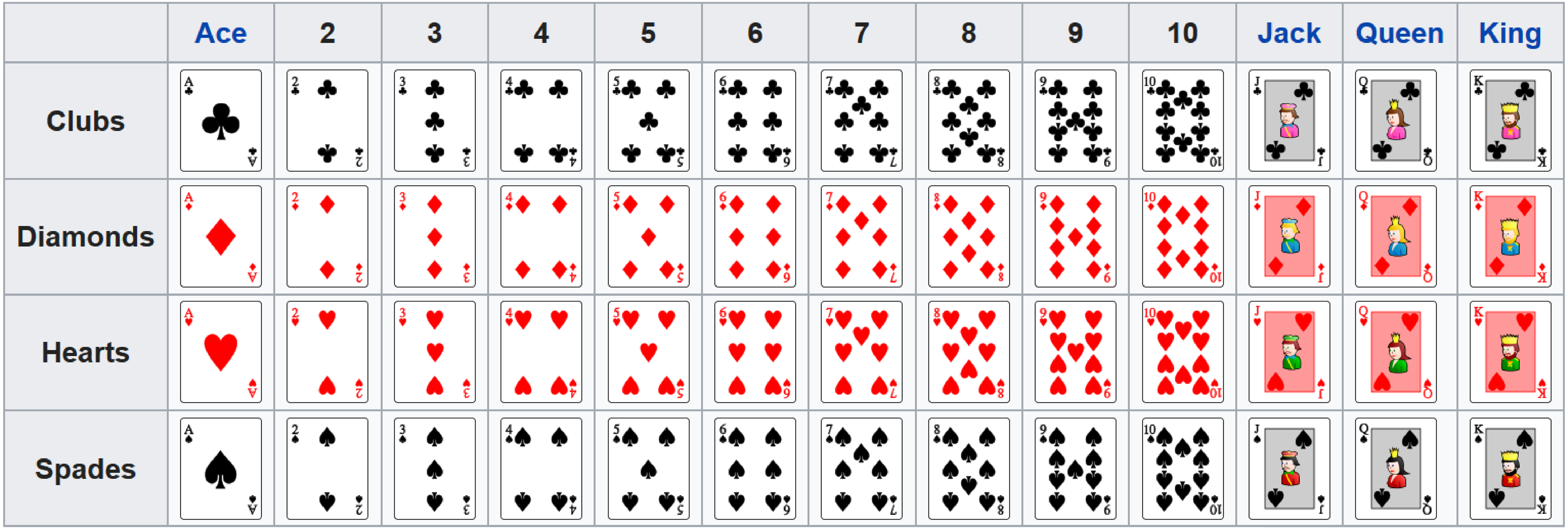
```
import matplotlib.pyplot as plt
from math import sqrt

def koch(p, q, depth=3):
    if depth == 0:
        return [p, q]

    dx, dy = q[0] - p[0], q[1] - p[1]
    h = 1 / sqrt(12)
    p1 = p[0] + dx / 3, p[1] + dy / 3
    p2 = p[0] + dx / 2 - h * dy, p[1] + dy / 2 + h * dx
    p3 = p[0] + dx * 2 / 3, p[1] + dy * 2 / 3
    return (koch(p,  p1, depth - 1)[:-1]
          + koch(p1, p2, depth - 1)[:-1]
          + koch(p2, p3, depth - 1)[:-1]
          + koch(p3, q,  depth - 1))

points = koch((0, 1), (0, 0), depth=3)
X, Y = zip(*points)

plt.subplot(aspect='equal')
plt.plot(X, Y, 'r-')
plt.plot(X, Y, 'k.')
plt.show()
```

remove last point (equal to first point in next recursive call)

# Recursion and iteration

- algorithm examples

# Standard 52-card deck

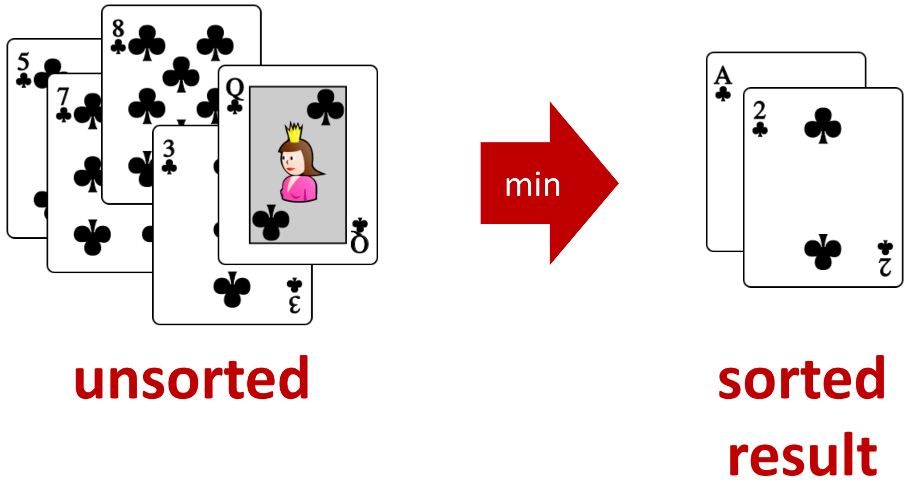



en.wikipedia.org/wiki/Standard_52-card_deck

# Selection sort

selection_sort.py

```python
def selection_sort(L):
    unsorted = L[:]
    result = []

    while unsorted:
        e = min(unsorted)
        unsorted.remove(e)
        result.append(e)

    return result
```

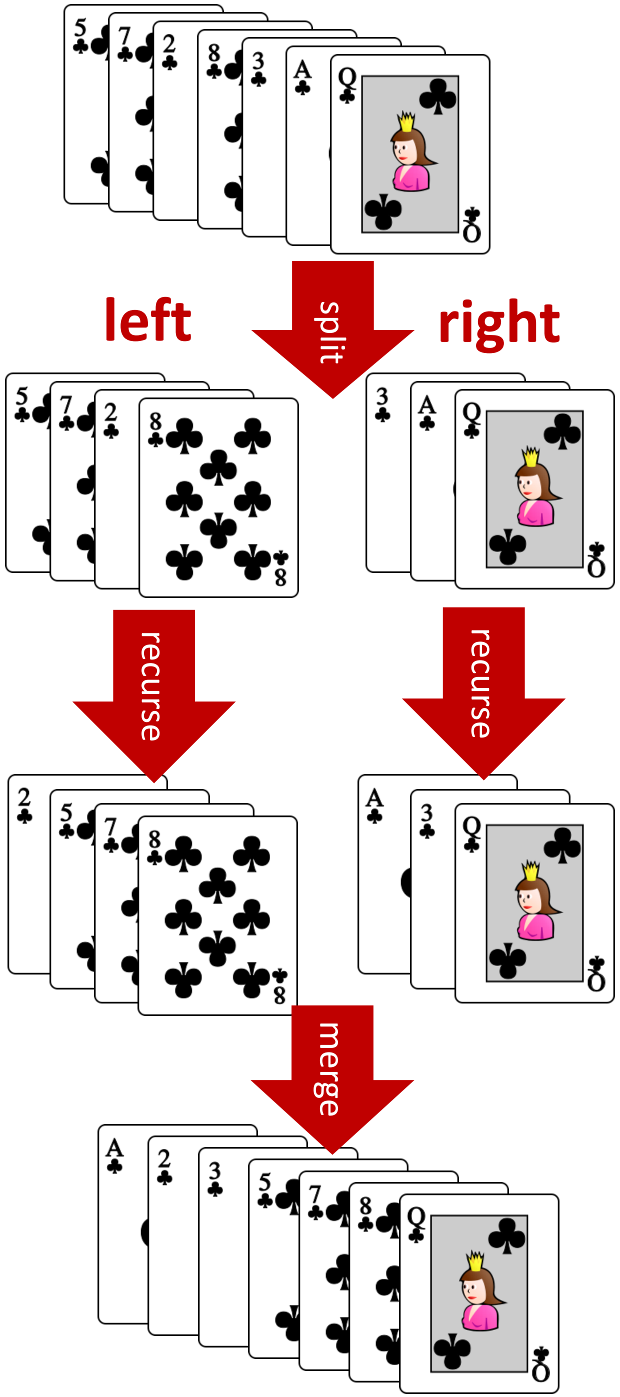



- `min` and `.remove` scan the remaining `unsorted` list for each element moved to `result`
- order $|L|^2$ comparisons

# Sorting a pile of cards (Merge sort)

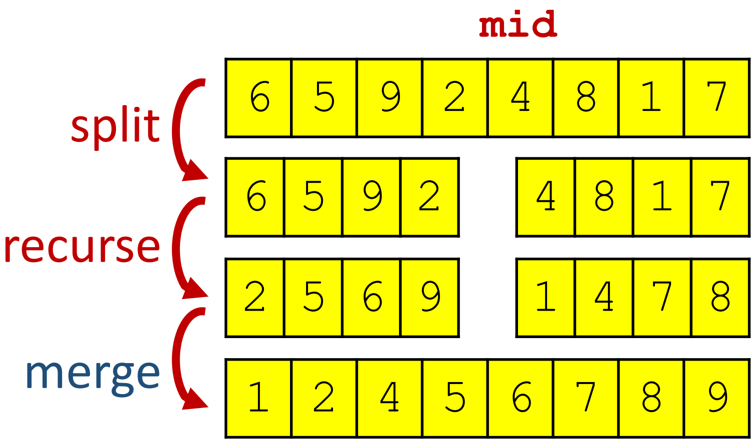
- If one card in pile, i.e. pile is sorted
- Otherwise

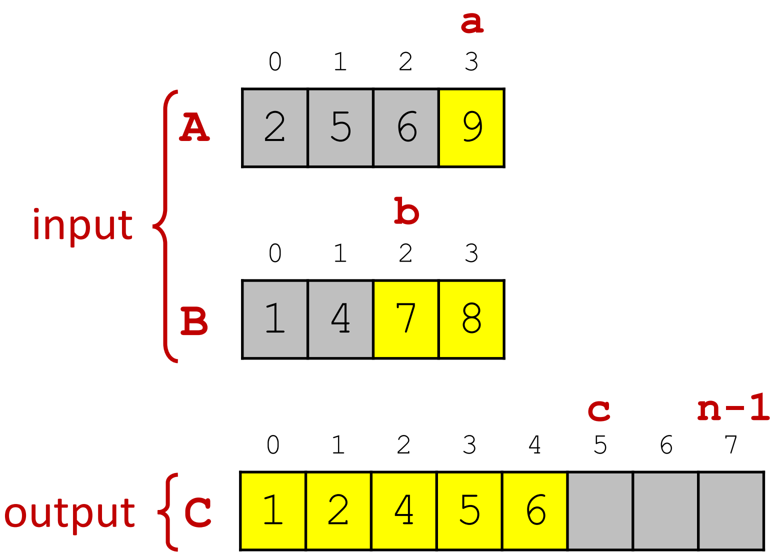
  1) Split pile into two piles, **left** and **right**, of approximately same size
  2) Sort **left** and **right** recursively (independently)
  3) Merge **left** and **right** (which are sorted)



**merge_sort.py**

```python
def merge_sort(L):
    n = len(L)
    if n <= 1:
        return L[:]
    mid = n // 2
    left, right = L[:mid], L[mid:]
    return merge(merge_sort(left), merge_sort(right))

def merge(A, B):
    n = len(A) + len(B)
    C = n * [None]
    a, b = 0, 0
    for c in range(n):
        if a < len(A) and (b == len(B) or A[a] < B[b]):
            C[c] = A[a]
            a = a + 1
        else:
            C[c] = B[b]
            b = b + 1
    return C
```



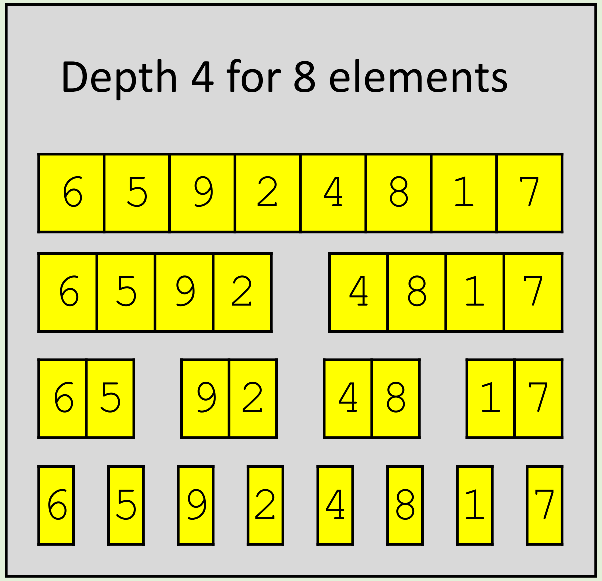



# Question – Depth of recursion for 52 elements

a) 1
b) 2
c) 3
d) 4
e) 5
f) 6
g) 7
h) 8
i) 9
j) 10
k) Don't know

Max recursive subproblem size

52 → 26 → 13 → 7 → 4 → 2 → 1



# Question – Order of comparisons by Merge sort ?

a) $\sim n$

b) $\sim n\sqrt{n}$

c) $\sim n \log_2 n$

d) $\sim n^2$

e) $\sim n^3$

f) Don't know

merge_sort.py

```
def merge_sort(L):
    n = len(L)
    if n <= 1:
        return L[:]
    else:
        mid = n // 2
        left, right = L[:mid], L[mid:]
        return merge(merge_sort(left), merge_sort(right))

def merge(A, B):
    n = len(A) + len(B)
    C = n * [None]
    a, b = 0, 0
    for c in range(n):
        if a < len(A) and (b == len(B) or A[a] < B[b]):
            C[c] = A[a]
            a = a + 1
        else:
            C[c] = B[b]
            b = b + 1
    return C
```

# Merge sort without recursion

- Start with piles of size one
- Repeatedly merge two smallest piles

**merge_sort.py**

```python
def merge_sort_iterative(L):
    Q = [[x] for x in L]
    while len(Q) > 1:
        Q.insert(0, merge(Q.pop(), Q.pop()))
    return Q[0]

from collections import deque

def merge_sort_deque(L):
    Q = deque([[x] for x in L])
    while len(Q) > 1:
        Q.appendleft(merge(Q.pop(), Q.pop()))
    return Q[0]
```

insert at front of list inefficient

deques are a generalization of lists with efficient updates at both ends

```
merge_sort_iterative([7,1,9,3,-2,5])
```

**Values of Q in while-loop**

```
[[7], [1], [9], [3], [-2], [5]]
[[-2, 5], [7], [1], [9], [3]]
[[3, 9], [-2, 5], [7], [1]]
[[1, 7], [3, 9], [-2, 5]]
[[-2, 3, 5, 9], [1, 7]]
[[-2, 1, 3, 5, 7, 9]]
```

**Note**: Lists in Q appear in non-increasing length order, where longest ≤ 2· shortest

# Question – Number of iterations of while-loop ?

```
merge_sort_iterative([7, 1, 9, 3, -2, 5])
```

a) 1
b) 2
c) 3
d) 4
e) 5
f) 6
g) 7
h) Don't know

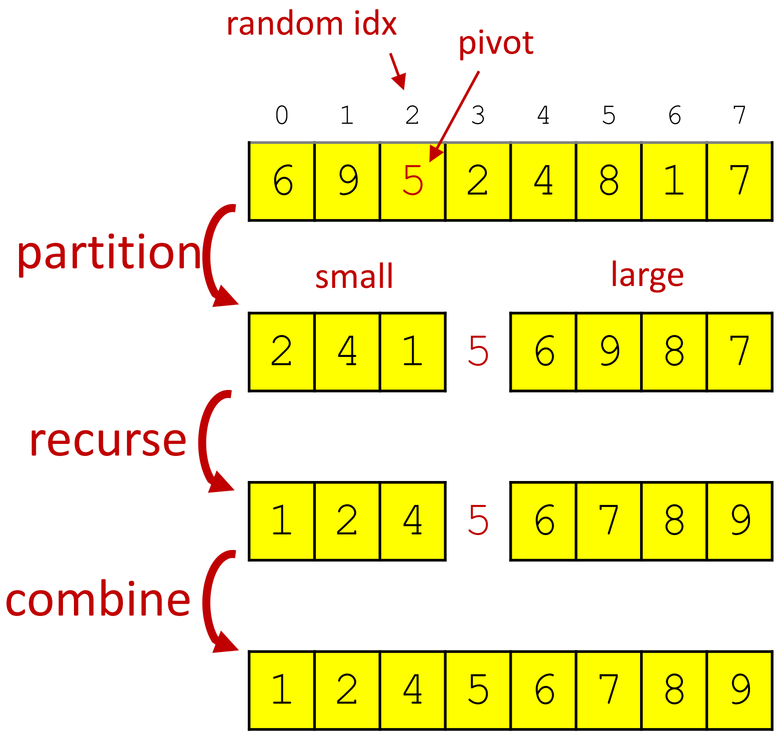
**merge_sort.py**

```
def merge_sort_iterative(L):
    Q = [[x] for x in L]
    while len(Q) > 1:
        Q.insert(0, merge(Q.pop(), Q.pop()))
    return Q[0]
```

# Quicksort (randomized)

**quicksort.py**

```
import random

def quicksort(L):
    if len(L) <= 1:
        return L[:]

    idx = random.randint(0, len(L) - 1)
    pivot = L[idx]
    other = L[:idx] + L[idx + 1:]

    small = [e for e in other if e < pivot]
    large = [e for e in other if e >= pivot]

    return quicksort(small) + [pivot] + quicksort(large)
```



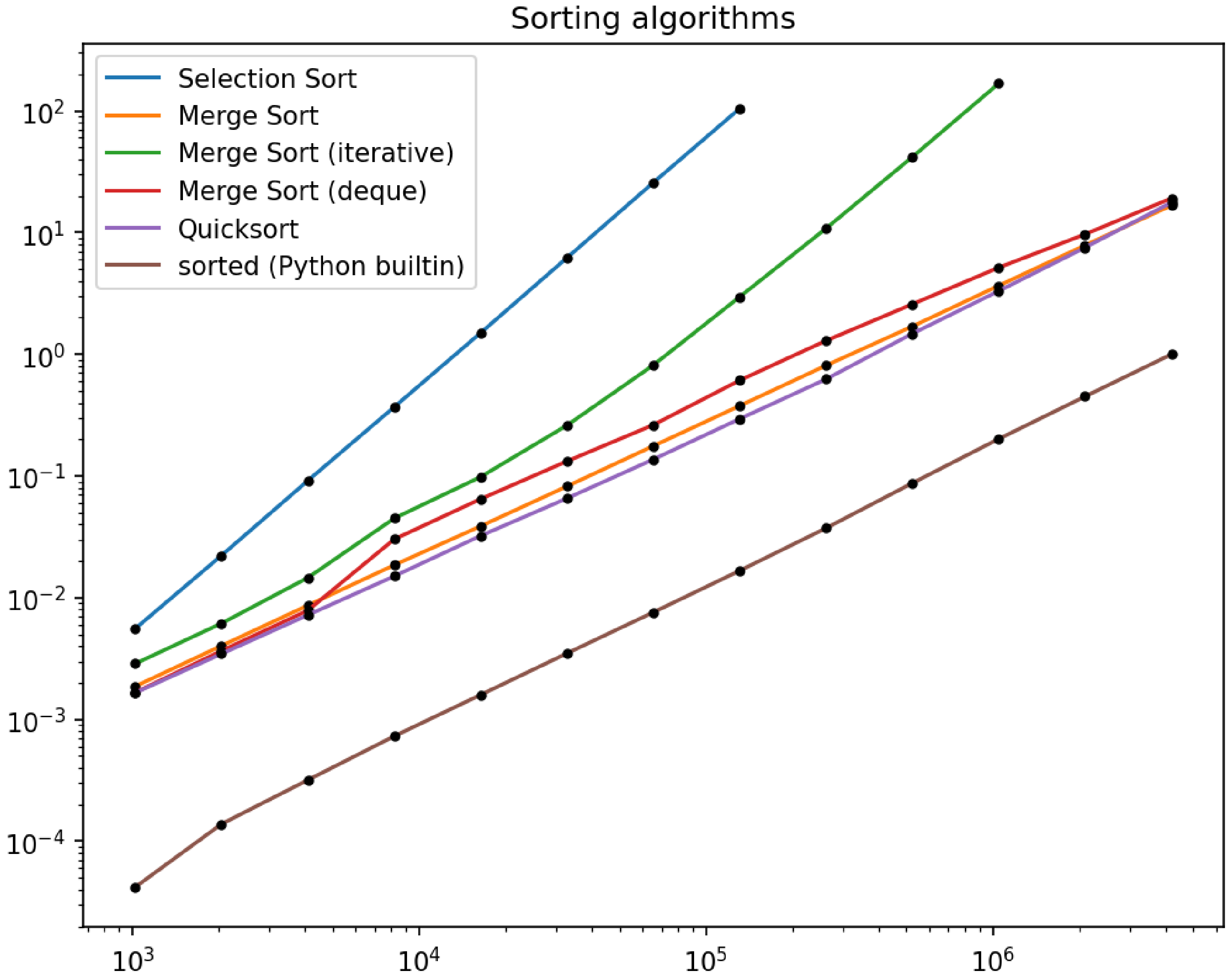

order $|L| \cdot \log_2 |L|$ comparisons, expected

# Sorting comparison (single run)

tuned merge-sort (Tim-sort)
implementation in C

| $\|L\|$ | Selection sort | Merge sort Recursive | Merge sort Iterative | Merge sort Deque | Quicksort | sorted (Python builtin) |
|---|---|---|---|---|---|---|
| $2^{10}$ | 0.006 | 0.002 | 0.003 | 0.002 | 0.002 | 0.00004 |
| $2^{11}$ | 0.02 | 0.004 | 0.006 | 0.000 | 0.003 | 0.0001 |
| $2^{12}$ | 0.09 | 0.008 | 0.01 | 0.008 | 0.007 | 0.0003 |
| $2^{13}$ | 0.37 | 0.02 | 0.04 | 0.03 | 0.02 | 0.0007 |
| $2^{14}$ | 1.50 | 0.04 | 0.10 | 0.06 | 0.03 | 0.002 |
| $2^{15}$ | 6.19 | 0.08 | 0.26 | 0.13 | 0.07 | 0.003 |
| $2^{16}$ | 25.67 | 0.18 | 0.81 | 0.26 | 0.14 | 0.008 |
| $2^{17}$ | 104.20 x 4 | 0.38 | 2.96 | 0.61 | 0.29 | 0.02 |
| $2^{18}$ | | 0.81 | 10.78 | 1.29 | 0.62 | 0.04 |
| $2^{19}$ | | 1.69 | 41.71 | 2.58 | 1.48 | 0.09 |
| $2^{20}$ | | 3.65 | 167.31 x 4 | 5.15 | 3.30 | 0.20 |
| $2^{21}$ | | 7.85 | | 9.68 | 7.53 | 0.45 |
| $2^{22}$ | | 16.69 x 2 | | 19.09 x 2 | 17.6 x 2 | 1.00 x 2 |

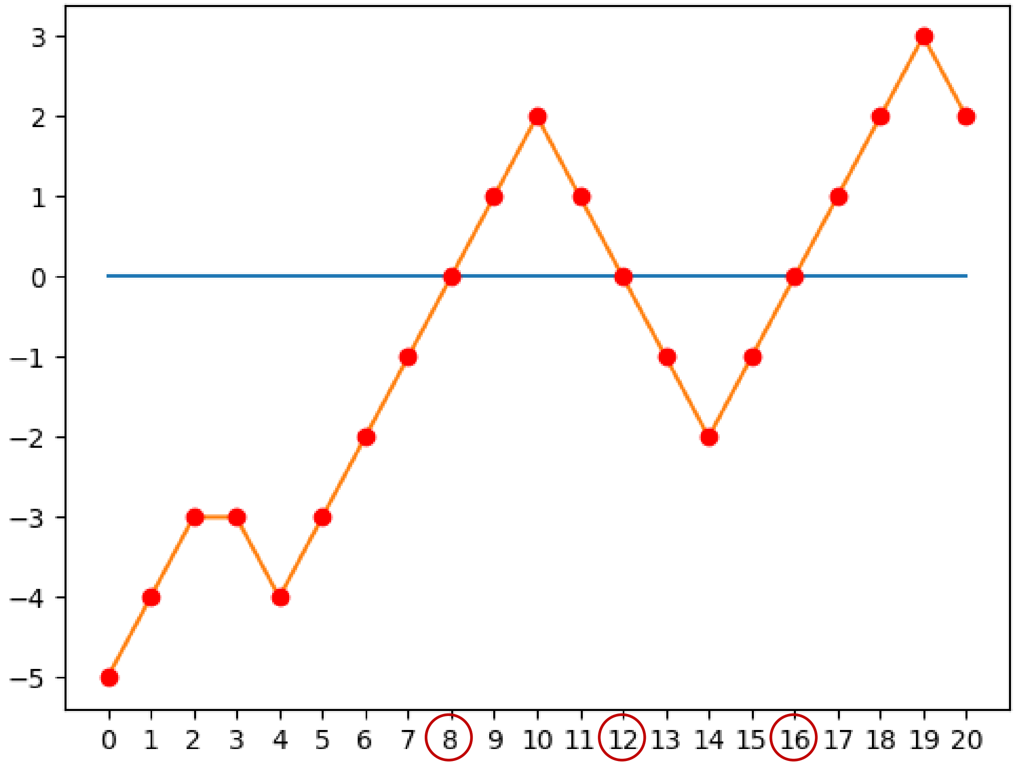
# Sorting comparison



# Find zero

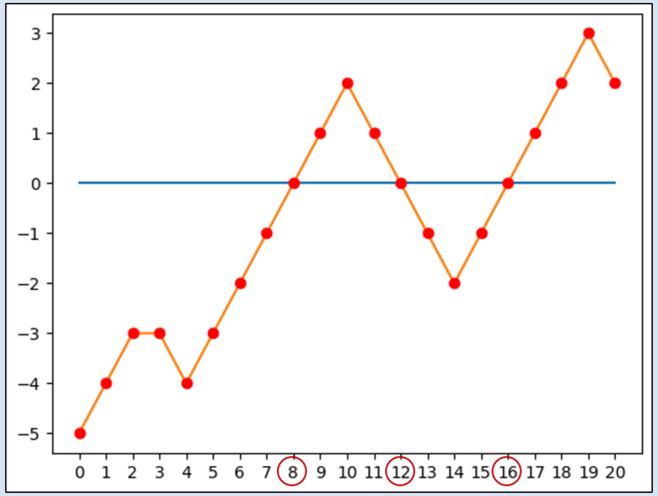
- Given a list L of integers starting with a negative and ending with a positive integer, and where |L[i+1] - L[i]| ≤ 1, find the position of a zero in L.

L = [-5, -4, -3, -3, -4, -3, -2, -1, 0, 1, 2, 1, 0, -1, -2, -1, 0 , 1, 2, 3, 2]



**find_zero.py**

```python
def find_zero_loop(L):
    i = 0
    while L[i] != 0:
        i += 1
    return i

def find_zero_enumerate(L):
    for i, e in enumerate(L):
        if e == 0:
            return i

def find_zero_index(L):
    return L.index(0)
```




```python
def find_zero_binary_search(L):
    low = 0
    high = len(L) - 1
    while True:  # L[low] < 0 < L[high]
        mid = (low + high) // 2
        if L[mid] == 0:
            return mid
        elif L[mid] < 0:
            low = mid
        else:
            high = mid

def find_zero_recursive(L):
    def search(low, high):
        mid = (low + high) // 2
        if L[mid] == 0:
            return mid
        elif L[mid] < 0:
            return search(mid, high)
        else:
            return search(low, mid)

    return search(0, len(L) - 1)
```

**find_zero.py**

```python
def find_zero_loop(L):
    i = 0
    while L[i] != 0:
        i += 1
    return i

def find_zero_enumerate(L):
    for i, e in enumerate(L):
        if e == 0:
            return i

def find_zero_index(L):
    return L.index(0)
```

| Function ($\|L\| = 10^6$) | Time, sec |
|---|---|
| find_zero_loop | 0.13 |
| find_zero_enumerate | 0.10 |
| find_zero_index | 0.015 |
| find_zero_binary_search | 0.000015 |
| find_zero_recursive | 0.000088 |

```python
def find_zero_binary_search(L):
    low = 0
    high = len(L) - 1
    while True:  # L[low] < 0 < L[high]
        mid = (low + high) // 2
        if L[mid] == 0:
            return mid
        elif L[mid] < 0:
            low = mid
        else:
            high = mid

def find_zero_recursive(L):
    def search(low, high):
        mid = (low + high) // 2
        if L[mid] == 0:
            return mid
        elif L[mid] < 0:
            return search(mid, high)
        else:
            return search(low, mid)

    return search(0, len(L) - 1)
```

# Greatest Common Divisor (GCD)

**Notation** $x \uparrow y$ denotes y is divisible by x, e.g. $3 \uparrow 12$
i.e. $y = a \cdot x$ for some integer a

**Definition** $\mathrm{gcd}(m, n) = \max \{ x \mid x \uparrow m \text{ and } x \uparrow n \}$

**Fact** if $x \uparrow y$ and $x \uparrow z$ then $x \uparrow (y + z)$ and $x \uparrow (y - z)$

**Observation**
(recursive definition)

$$\mathrm{gcd}(m, n) = \begin{cases} m & \text{if } m = n \\ \mathrm{gcd}(m, n - m) & \text{if } m < n \\ \mathrm{gcd}(m - n, n) & \text{if } m > n \end{cases}$$

**gcd(90, 24)**

| m | n |
|---|---|
| 90 | 24 |
| 66 | 24 |
| 42 | 24 |
| 18 | 24 |
| 18 | 6 |
| 12 | 6 |
| 6 | 6 |

# Greatest Common Divisor (GCD)

gcd_slow.py

```python
def gcd(m, n):
    while m != n:
        if n > m:
            n = n - m
        else
            m = m - n
    return m
```

gcd.py

```python
def gcd(m, n):
    while n != 0:
        m, n = n, m % n
    return m
```

gcd_slow_recursive.py

```python
def gcd(m, n):
    if m == n:
        return m
    elif m > n:
        return gcd(m - n, n)
    else:
        return gcd(m, n - m)
```

gcd_recursive.py

```python
def gcd(m, n):
    if n == 0:
        return m
    else:
        return gcd(n, m % n)
```

gcd_recursive_one_line.py

```python
def gcd(m, n):
    return m if n == 0 else gcd(n, m % n)
```

# Permutations

- Generate a list L of all permutations of a tuple

**Python shell**

```
> permutations(('a', 'b', 'c'))
| [('a', 'b', 'c'), ('b', 'a', 'c'), ('b', 'c', 'a'),
   ('a', 'c', 'b'), ('c', 'a', 'b'), ('c', 'b', 'a')]
```

**permutations.py**

```python
def permutations(L):
    if len(L) == 0:
        return [L[:]]  # empty tuple (ensures same type as L)
    else:
        P = permutations(L[1:])
        return [p[:i] + L[:1] + p[i:] for p in P for i in range(len(L))]
```

- An implementation of "permutations" exists in the "itertools" library

# Maze solver

## Input

- First line #rows and #columns
- Following #rows lines contain strings containing #column characters
- There are exactly one 'A' and one 'B'
- '.' are free cells and '#' are blocked cells

## Output

- Print whether there is a path from 'A' to 'B' or not

**maze input**

```
11 19
#######A###########
#.......#.....#...#
#.###.###...#.#.#.#
#...#.....#.#...#.#
#.#.###.#.#.#.###.#
#.#.....#...#.#...#
#.###########.#.#.#
#.#.#.....#...#.#.#
#.#.#####.#####.#.#
#.........#.....#.#
###############B###
```

# Maze solver (recursive)

**maze_solver.py**

```python
def explore(i, j):
    global solution, visited

    if (0 <= i < n and 0 <= j < m and
        maze[i][j] != "#" and not visited[i][j]):

        visited[i][j] = True

        if maze[i][j] == 'B':
            solution = True

        explore(i - 1, j)
        explore(i + 1, j)
        explore(i, j - 1)
        explore(i, j + 1)
```

```python
def find(symbol):
    for i in range(n):
        j = maze[i].find(symbol)
        if j >= 0:
            return (i, j)

n, m = [int(x) for x in input().split()]
maze = [input() for i in range(n)]

solution = False
visited = [m * [False] for i in range(n)]

explore(*find('A'))

if solution:
    print("path from A to B exists")
else:
    print("no path")
```

**maze input**

```
11 19
#######A###########
#.......#.....#...#
#.###.###...#.#.#.#
#...#.....#.#...#.#
#.#.###.#.#.#.###.#
#.#.....#...#.#...#
#.###########.#.#.#
#.#.#.....#...#.#.#
#.#.#####.#####.#.#
#.........#.....#.#
###############B###
```

# Maze solver (iterative)

**maze_solver_iterative.py**

```
def explore(i, j):
    global solution, visited

    Q = [(i, j)]  # cells to visit

    while Q:
        i, j = Q.pop()
        if (0 <= i < n and 0 <= j < m and
            maze[i][j] != '#' and not visited[i][j]):

            visited[i][j] = True

            if maze[i][j] == 'B':
                solution = True

            Q.append((i - 1, j))
            Q.append((i + 1, j))
            Q.append((i, j - 1))
            Q.append((i, j + 1))
```

```
def find(symbol):
    for i in range(n):
        j = maze[i].find(symbol)
        if j >= 0:
            return (i, j)

n, m = [int(x) for x in input().split()]
maze = [input() for i in range(n)]

solution = False
visited = [m*[False] for i in range(n)]

explore(*find('A'))

if solution:
    print("path from A to B exists")
else:
    print("no path")
```

# Question – How difficult is the triplet project on a scale 1 – 10 ?

a) 1 (I’m offended by how trivial the project was)
b) 2 (very easy)
c) 3 (a quite standard review exercise)
d) 4 (not too complicated, got some known concepts repeated)
e) 5 (good exercise to repeat standard programming techniques)
f) 6 (had to use more advanced techniques in a familiar way)
g) 7 (quite complicated, but manageable)
h) 8 (very abstract exercise, using complicated language constructs)
i) 9 (very complicated – barely manageable spending all my time)
j) 10 (this is a research project – could be an MSc thesis/PhD project)
k) 25 (this is wayyy too complicated for a university course)

# Functions as objects

- lambda
- higher-order functions
- map, filter, reduce

# Aliasing functions – both user defined and builtin

**Python shell**

```
> def square(x):
      return x * x
> square
| <function square at 0x0329A390>
> square(8)
| 64
> kvadrat = square
> kvadrat(5)
| 25
> len
| <built-in function len>
> length = len
> length([1, 2, 3])
| 3
```

# Functions as values

**square_or_double.py**

```
def square(x):
      return x * x

def double(x):
      return 2 * x

while True:
    answer = input("square or double ? ")
    if answer == "square":
        f = square
        break
    if answer == "double":
        f = double
        break

answer = input("numbers: ")
L_in = [int(x) for x in answer.split()]
L_out = [f(x) for x in L_in]
print(L_out)
```

`f` will refer to one of the functions `square` and `double` refer to

call the function `f` is referring to with argument `x`

**Python shell**

```
| square or double ? square
| numbers: 3 6 7 9
| [9, 36, 49, 81]

| square or double ? double
| numbers: 2 3 4 7 9
| [4, 6, 8, 14, 18]
```

# Functions as values and namespaces

**say.py**

```python
def what_says(name):
    def say(message):
        print(name, "says:", message)
    return say

alice = what_says("Alice")
peter = what_says("Peter")

alice("Where is Peter?")
peter("I am here")
```

**Python shell**

```
| Alice says: Where is Peter?
| Peter says: I am here
```

- `what_says` is a function returning a function (`say`)
- Each call to `what_says` with a single string as its argument **creates a new say** function with the current `name` argument in its namespace
- In each call to a an instance of a `say` function, `name` refers to the string in the namespace when the function was created, and `message` is the string given as an argument in the call

# Question – What list is printed ?

```
def f(x):
    def g(y):
        nonlocal x
        x = x + 1
        return x + y
    return g
a = f(3)
b = f(6)
print([a(3), b(2), a(4)])
```

a) [7, 7, 10]
b) [7, 9, 8]
c) [7, 9, 9]
d) [7, 9, 12]
e) [7, 10, 10]
f) Don’t know

# map

- `map(function, list)` applies the function to each element of the sequence `list`
- `map(function, list`$_1$`, …, list`$_k$`)` requires `function` to take k arguments, and creates a sequence with the i'th element being `function(list`$_1$`[i], …, list`$_k$`[i])`

**Python shell**

```
> def square(x):
      return x*x
> list(map(square, [1,2,3,4,5]))
| [1, 4, 9, 16, 25]
> def triple_sum(x, y, z):
      return x + y + z
> list(map(triple_sum, [1,2,3], [4,5,6], [7,8,9]))
| [12, 15, 18]
> list(map(triple_sum, *zip(*[(1,4,7), (2,5,8), (3,6,9)])))
| [12, 15, 18]
```

# sorted

- A list `L` can be sorted using `sorted(L)`
- A user defined order on the elements can be defined by providing a function using the keyword argument `key`, that maps elements to values with some default ordering

**Python shell**

```
> def length_square(p):
      x, y = p
      return x**2 + y**2  # no sqrt

> L = [(5, 3), (2, 5), (1, 9), (2, 2), (3, 4)]
> list(map(length_square, L))
| [34, 29, 82, 8, 25]
> sorted(L)  # default lexicographical order
| [(1, 9), (2, 2), (2, 5), (3, 4), (5, 3)]
> sorted(L, key=length_square)  # order by length
| [(2, 2), (3, 4), (2, 5), (5, 3), (1, 9)]
```

https://docs.python.org/3/library/functions.html#sorted

# Question – What list does sorted produce ?

```
sorted([2, 3, -1, 5, -4, 0, 8, -6], key=abs)
```

**key** 2 3 1 5 4 0 8 6

a) `[-6, -4, -1, 0, 2, 3, 5, 8]`

b) `[0, 2, 3, 5, 8, -1, -4, -6]`

c) `[0, -1, 2, 3, -4, 5, -6, 8]`

d) `[8, 5, 3, 2, 0, -1, -4, -6]`

e) `[0, 1, 2, 3, 4, 5, 6, 8]`

f) Don’t know

**Python shell**

```
> abs(7)
| 7
> abs(-42)
| 42
```

# filter

- `filter(function, list)` returns the subsequenece of `list` where `function` evaluates to true
- Essentially the same as

  `[x for x in list if function(x)]`

**Python shell**

```
> def odd(x):
      return x % 2 == 1

> filter(odd, range(10))
| <filter object at 0x03970FD0>
> list(filter(odd, range(10)))
| [1, 3, 5, 7, 9]
```

# reduce (in module functools)

- Python's "reduce" function is in other languages often denoted "foldl"

$$\texttt{reduce(f, [}x_1\texttt{, }x_2\texttt{, }x_3\texttt{, ..., }x_k\texttt{])} = \texttt{f(}\cdots\texttt{f(f(}x_1\texttt{, }x_2\texttt{), }x_3\texttt{)}\cdots\texttt{, }x_k\texttt{)}$$

**Python shell**

```
> from functools import reduce
> def power(x, y):
      return x ** y
> reduce(power, [2, 2, 2, 2, 2])
| 65536
```

# lambda (anonymous functions)

- If you need to define a *short* function, that *returns a value*, and the function is only *used once* in your program, then a lambda function might be appropriate:

```
lambda arguments: expression
```

- Creates a function with no name that takes zero or more arguments, and returns the value of the single expression

**Python shell**

```
> f = lambda x, y : x + y
> f(2, 3)
| 5
> list(filter(lambda x: x % 2, range(10)))
| [1, 3, 5, 7, 9]
```

# Examples: sorted using lambda

**Python shell**

```
> L = [ 'AHA', 'Oasis', 'ABBA', 'Beatles', 'AC/DC', 'B. B. King', 'Bangles', 'Alan Parsons']

> # Sort by length, secondary after input position (default, known as stable)
> sorted(L, key=len)
| ['AHA', 'ABBA', 'Oasis', 'AC/DC', 'Beatles', 'Bangles', 'B. B. King', 'Alan Parsons']

> # Sort by length, secondary alphabetically
> sorted(L, key=lambda s: (len(s), s))
| ['AHA', 'ABBA', 'AC/DC', 'Oasis', 'Bangles', 'Beatles', 'B. B. King', 'Alan Parsons']

> # Sort by most 'a's, if equal by number of 'b's, etc.
> sorted(L, key=lambda s: sorted([a.lower() for a in s if a.isalpha()] + ['~']))
| ['Alan Parsons', 'ABBA', 'AHA', 'Beatles', 'Bangles', 'AC/DC', 'Oasis', 'B. B. King']

> sorted([a.lower() for a in 'Beatles' if a.isalpha()] + ['~'])
| ['a', 'b', 'e', 'e', 'l', 's', 't', '~']
```

# min and max

- Similarly to `sorted`, the functions `min` and `max` take a keyword argument `key`, to map elements to values with some default ordering

**Python shell**

```
> max(['w', 'xyz', 'abcd', 'uv'])
| 'xyz'
> max(['w', 'xyz', 'abcd', 'uv'], key=len)
| 'abcd'
> sorted([210, 13, 1010, 30, 27, 103], key=lambda x: str(x)[::-1])
| [1010, 210, 30, 103, 13, 27]
> min([210, 13, 1010, 30, 27, 103], key=lambda x: str(x)[::-1])
| 1010
```

docs.python.org/3/library/functions.html#min

# defaultdict (from module collections)

- An extension of the built-in dict that automatically initializes undefined items on access by calling a function (factory) to produce a default value to be inserted

`defaultdict`(*function to create default value*, *normal dict arguments*)

**Python shell**

```
> scores = {'Mickey': [2, 3, 1], 'Goofy': [1, 0, 2]}
> scores['Gladstone']  # access to undefined key in a standard dictionary
| KeyError: 'Gladstone'
> from collections import defaultdict
> scores = defaultdict(lambda : [], {'Mickey': [2, 3, 1], 'Goofy': [1, 0, 2]})
> scores
| defaultdict(<function <lambda> at 0x0000026460F0BBE0>, {'Mickey': [2, 3, 1], 'Goofy': [1, 0, 2]})
> scores['Gladstone']  # calls lambda without arguments to initialize scores['Gladstone']
| []
> scores
| defaultdict(<function <lambda> at ...>, {'Mickey': [2, 3, 1], 'Goofy': [1, 0, 2], 'Gladstone': []})
> scores = defaultdict(list, Mickey=[2, 3, 1], Goofy=[1, 0, 2])
> scores['daisy'].append(7)  # calls list() to initialize scores['daisy']
> scores
| defaultdict(<class 'list'>, {'Mickey': [2, 3, 1], 'Goofy': [1, 0, 2], 'daisy': [7]})
```

docs.python.org/3/library/collections.html#collections.defaultdict

# History of lambda in programming languages

- lambda calculus invented by Alonzo Church in 1930s
- Lisp has had lambdas since 1958
- C++ got lambdas in C++11 in 2011
- Java first got lambdas in Java 8 in 2014
- Python has had lambdas since Version 1.0 in 1994

| polynomial.py | Python shell |
|---|---|
| | |

```python
def linear_function(a, b):
    return lambda x: a * x + b

def degree_two_polynomial(a, b, c):
    def evaluate(x):
        return a * x**2 + b * x + c
    return evaluate

def polynomial(coefficients):
    return lambda x: sum([c * x**p for p, c in enumerate(coefficients)])

def combine(f, g):
    def evaluate(*args, **kwargs):
        return f(g(*args, **kwargs))
    return evaluate

f = linear_function(2, 3)
for x in [0, 1, 2]:
    print("f(%s) = %s" % (x, f(x)))
p = degree_two_polynomial(1, 2, 3)
for x in [0, 1, 2]:
    print("p(%s) = %s" % (x, p(x)))
print("polynomial([3, 2, 1])(2) =", polynomial([3, 2, 1])(2))
h = combine(abs, lambda x, y: x - y)
print("h(3, 5) =", h(3, 5))
```

```
| f(0) = 3
| f(1) = 5
| f(2) = 7
| p(0) = 3
| p(1) = 6
| p(2) = 11
| polynomial([3, 2, 1])(2) = 11
| h(3, 5) = 2
```

# Question – What value is h(1) ?

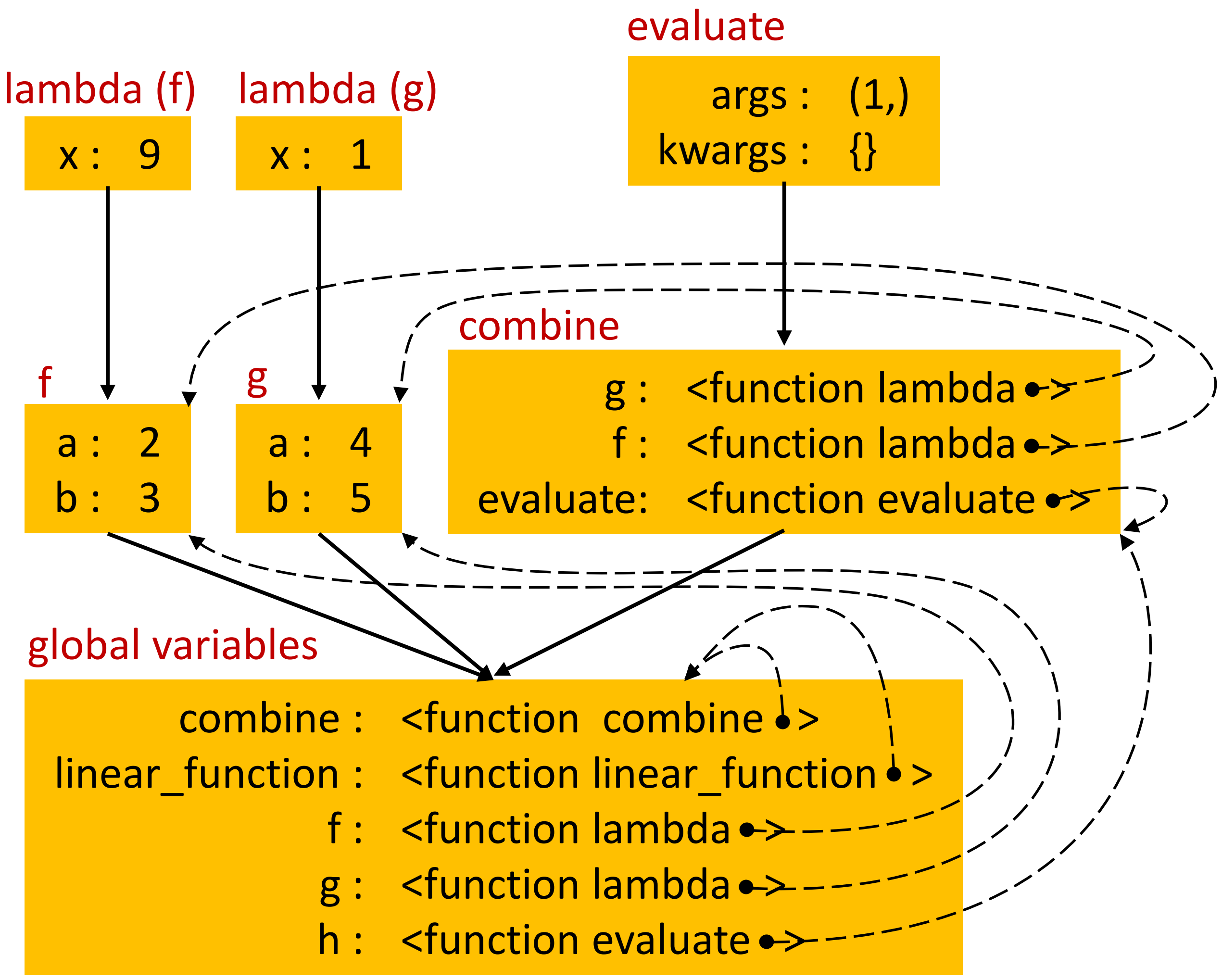
linear_combine.py

```
def combine(f, g):
    def evaluate(*args, **kwargs):
        return f(g(*args, **kwargs))

    return evaluate

def linear_function(a, b):
    return lambda x: a * x + b

f = linear_function(2, 3)
g = linear_function(4, 5)

h = combine(f, g)

print(h(1))
```

a) 5
b) 9
c) 16

d) 21
e) 25
f) Don’t know

# Namespace example




**linear_combine.py**

```python
def combine(f, g):
    def evaluate(*args, **kwargs):
        return f(g(*args, **kwargs))

    return evaluate

def linear_function(a, b):
    return lambda x: a * x + b

f = linear_function(2, 3)
g = linear_function(4, 5)

h = combine(f, g)

print(h(1))
```

# partial (trace of computation)

**partial_trace.py**

```python
def partial(fn, *args):
    def new_f(*a):
        print(f'new_f: fn={fn.__name__}, args={args}, a={a})')
        answer = fn(*args, *a)
        print(f'answer={answer}')
        return answer

    return new_f

def f(x, y, z):
    print(f'f({x},{y},{z})')
    return x + 2 * y + 3 * z

g = partial(f, 7)
h = partial(f, 2, 1 )
k = partial(g, 1, 2)

print(f'{g(2, 1)=}\n')   # 7 + 2 * 2 + 3 * 1 = 14
print(f'{h(3)=}\n')      # 2 + 2 * 1 + 3 * 3 = 13
print(f'{k()=}\n')       # 7 + 2 * 1 + 3 * 2 = 15
```

**Python shell**

```
| new_f: fn=f, args=(7,), a=(2, 1))
| f(7,2,1)
| answer=14
| g(2, 1)=14
|
| new_f: fn=f, args=(2, 1), a=(3,))
| f(2,1,3)
| answer=13
| h(3)=13
|
| new_f: fn=new_f, args=(1, 2), a=())
| new_f: fn=f, args=(7,), a=(1, 2))
| f(7,1,2)
| answer=15
| answer=15
| k()=15
```

**Python shell**

```
> def f(x): return x
> g = lambda x: x
> f.__name__
| 'f'
> g.__name__
| '<lambda>'
```

functools.partial

# Object oriented programming

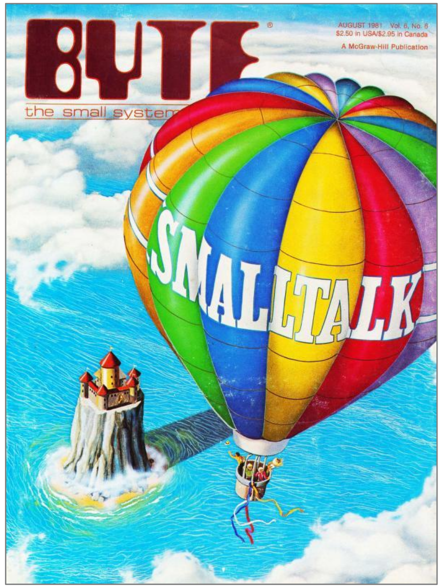
- classes, objects
- self
- construction
- encapsulation

# Object Oriented Programming

- **Programming paradigm**, other paradigms are e.g.
  - *functional programming* where the focus is on functions, lambda's and higher order functions, and
  - *imperative programming* focusing on sequences of statements changing the state of the program
- Core concepts are **objects**, **methods** and **classes**,
  - allowing one to construct *abstract data types*, i.e. user *defined types*
  - objects have states
  - methods manipulate objects, defining the interface of the object to the rest of the program
- OO supported by many programming languages, including Python

# Object Oriented Programming - History
## (selected programming languages)

| | |
|---|---|
| **Mid 1960's** | **Simula 67** |
| | (Ole-Johan Dahl and Kristen Nygaard, Norsk Regnesentral Oslo)<br>Introduced classes, objects, virtual procedures |
| **1970's** | **Smalltalk** (Alan Kay, Dan Ingalls, Adele Goldberg, Xerox PARC) |
| | Object-oriented programming, fully dynamic system<br>(opposed to the static nature of Simula 67 ) |
| **1985** | **Eiffel** (Bertrand Meyer, Eiffel Software) |
| | Focus on software quality, capturing the full software cycle |
| **1985** | **C++** (Bjarne Stroustrup [MSc Aarhus 1975], AT&T Bell Labs) |
| **1995** | **Java** (James Gosling, Sun) |
| **2000** | **C#** (Anders Hejlsberg (studied at DTU) et al., Microsoft) |
| **1991** | **Python** (Guido van Rossum) |
| | Multi-paradigm programming language, fully dynamic system |

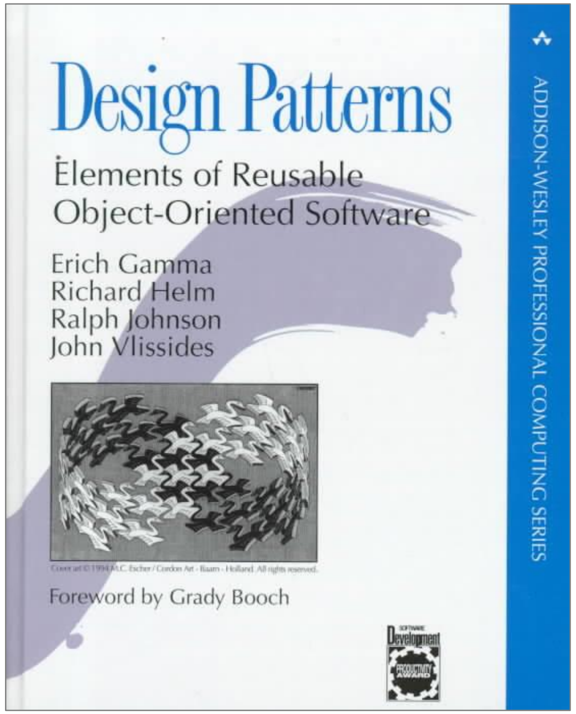


Byte Magazine, August 1981

**Note**: Java, C++, Python, C# are among Top 5 on TIOBE January 2020 index of popular languages (only non OO language among Top 5 was C)

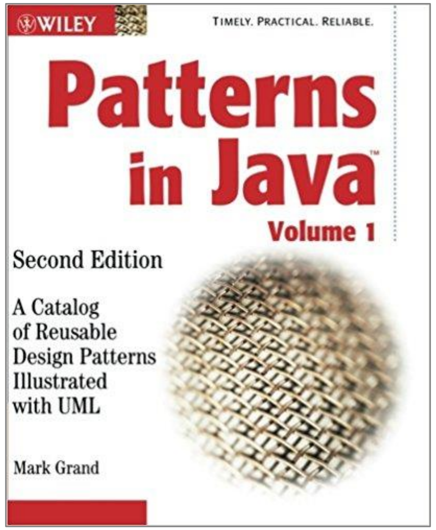
# Design Patterns (not part of this course)

*reoccuring patterns in software design*

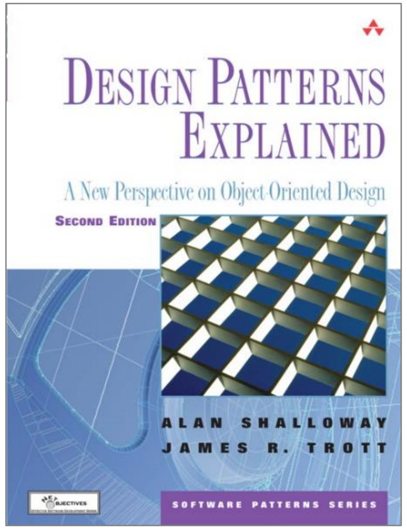



**The Classic book 1994**
(C++ cookbook)

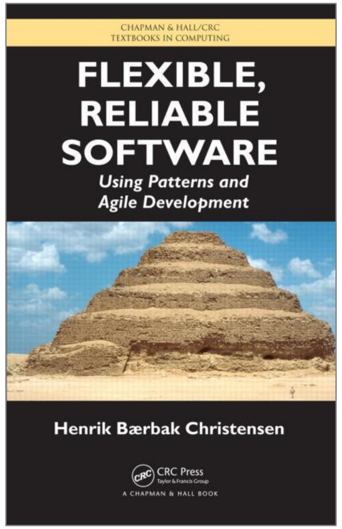

A very alternative book 2004
(Java, very visual)




Java cookbook 2003




Java textbook 2004




Java textbook 2010

...and many more books on the topic of Design Patterns, also with Python

# Some known classes, objects, and methods

| Type / class | Objects | Methods (examples) |
|---|---|---|
| int | 0 -7 42 1234567 | .__add__(x), .__eq__(x), .__str__() |
| str | "" 'abc' '12_ a' | .isdigit(), .lower(), .__len__() |
| list | [] [1,2,3] ['a', 'b', 'c'] | .append(x), .clear(), .__mul__(x) |
| dict | {'foo' : 42, 'bar' : 5} | .keys(), .get(), .__getitem__(x) |
| NoneType | None | .__str__() |

**Example**:

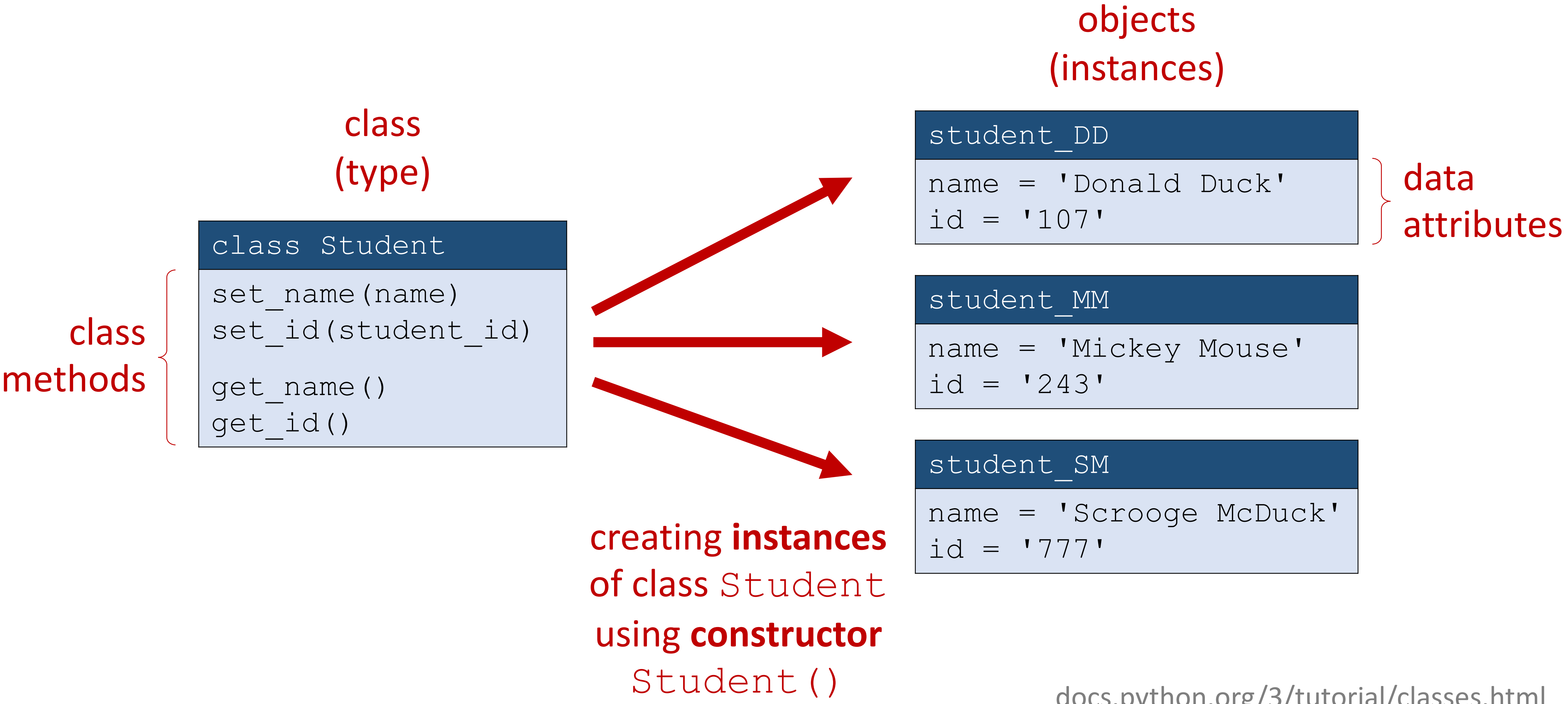
The function `str(obj)` calls the methods `obj.__str__()` or `obj.__repr__()`, if `obj.__str__` does not exist.

`print` calls `str`.

**Python shell**

```
> 5 + 7  # + calls .__add__(7)
| 12
> (5).__add__(7)  # eq. to 5 + 7
| 12
> (7).__eq__(7)  # eq. to 7 == 7
| True
> 'aBCd'.lower()
| 'abcd'
> 'abcde'.__len__()
  # .__len__() called by len(...)
| 5
> ['x', 'y'].__mul__(2)
| ['x', 'y', 'x', 'y']
> {'foo' : 42}.__getitem__('foo')
  # eq. to {'foo' : 42}['foo']
| 42
> None.__str__()  # used by str(...)
| 'None'
> 'abc'.__str__(), 'abc'.__repr__()
| ('abc', "'abc'")
```

# Classes and Objects




docs.python.org/3/tutorial/classes.html

# Using the `Student` class

**student.py**

```python
student_DD = Student()
student_MM = Student()
student_SM = Student()

student_DD.set_name('Donald Duck')
student_DD.set_id('107')

student_MM.set_name('Mickey Mouse')
student_MM.set_id('243')

student_SM.set_name('Scrooge McDuck')
student_SM.set_id('777')

students = [student_DD, student_MM, student_SM]

for student in students:
    print(student.get_name(),
          "has id",
          student.get_id())
```

**Python shell**

```
| Donald Duck has id 107
| Mickey Mouse has id 243
| Scrooge McDuck has id 777
```

Call **constructor** for class `Student`. Each call returns a new `Student` object.

Call class methods to set data attributes

Call class methods to read data attributes

# class Student

name of class

docstring containing documentation for class

class definitions start with the keyword **class**

**student.py**

```
class Student:
    '''Documentation of class'''

    def set_name(self, name):
        self.name = name

    def set_id(self, student_id):
        self.id = student_id

    def get_name(self):
        return self.name

    def get_id(self):
        return self.id
```

the first argument to all class methods is a reference to the object called upon, and by convention the first argument should be named **self**.

use **self.** to access an attribute of an object or class method (attribute reference)

often called **mutator methods**, since they change the state of an object

often called **accessor methods**, since they only read the state of an object

class method definitions start with keyword **def** (like normal function definitions)

**Note** In other OO programming languages the explicit reference to **self** is not required (in Java and C++ **self** is the keyword **this**)

# When are object attributes initialized ?

**Python shell**

```
> x = Student()
> x.set_name("Gladstone Gander")
> x.get_name()
| 'Gladstone Gander'
> x.get_id()
| AttributeError: 'Student' object has no attribute 'id'
```

- Default behaviour of a class is that instances are created with no attributes defined, but has access to the attributes / methods of the class
- In the previous class `Student` both the `name` and `id` attributes were first created when set by `set_name` and `set_id`, respectively

# Class construction and `__init__`

- When an object is created using `class_name()` it's initializer metod `__init__` is called.
- To initialize objects to contain default values, (re)define this function.

**student.py**

```
class Student:
    def __init__(self):
        self.name = None
        self.id = None

    ... previous method definitions ...
```

# Question – What is printed ?

**Python shell**

```
> class C:
      def __init__(self):
          self.v = 0
      def f(self):
          self.v = self.v + 1
          return self.v
> x = C()
> print(x.f() + x.f())
```

a) 1
b) 2
c) 3
d) 4
e) 5
f) Don't know

# `__init__` with arguments

- When creating objects using *`class_name(args)`* the initializer method is called as `__init__(args)`
- To initialize objects to contain default values, (re)define this function to do the appropriate initialization

**student.py**

```python
class Student:
    def __init__(self, name=None, student_id=None):
        self.name = name
        self.id = student_id

    ... previous method definitions ...
```

**Python shell**

```
> p = Student("Pluto")
> print(p.get_name())
| Pluto
> print(p.get_id())
| None
```

# Are accessor and mutator methods necessary ?

No - but good programming style

**Python shell**

```
> p = Pair(3, 5)
> p.sum()
| 8
> p.set_a(4)
> p.sum()
| 9
> p.a       # access object attribute
| 4
> p.b = 0 # update object attribute
> p.sum()
| 9         # the_sum not updated
```

**pair.py**

```
class Pair:
    """ invariant: the_sum = a + b """
    def __init__(self, a, b):
        self.a = a
        self.b = b
        self.the_sum = self.a + self.b
    def set_a(self, a):
        self.a = a
        self.the_sum = self.a + self.b
    def set_b(self, b):
        self.b = b
        self.the_sum = self.a + self.b
    def sum(self):
        return self.the_sum
```

constructor

mutator

accessor

# Defining order on instances of a class (sorting)

- To define an order on objects, define the “<“ operator by defining `__lt__`
- When ”<” is defined a list `L` of students can be sorted using `sorted(L)` and `L.sort()`

**student.py**

```
class Student:
    def __lt__(self, other):
        return self.id < other.id

    ... previous method definitions ...
```

**Python shell**

```
> student_DD < student_MM
| True
> [x.id for x in students]
| ['243', '107', '777']
> [x.id for x in sorted(students)]
| ['107', '243', '777']
```

# Converting objects to `str`

- To be able to convert an object to a string using `str(`*`object`*`)`, define the method `__str__`
- `__str__` is e.g. used by `print`

**student.py**

```
class Student:
    def __str__(self):
        return ("Student['%s', '%s']"
                % (self.name, self.id))

    ... previous method definitions ...
```

**Python shell**

```
> print(student_DD)   # without __str__
| <__main__.Student object at 0x03AB6B90>
> print(student_DD)   # with __str__
| Student['Donald Duck', '107']
```

# Nothing is private in Python

- Python does not support **hiding information** inside objects
- Recommendation is to start attributes with underscore, if these should be used only locally inside a class, i.e. be considered ”private”
- **PEP8:** “Use one leading underscore only for non-public methods and instance variables“

**private_attributes.py**

```python
class My_Class:
    def set_xy(self, a, b):
        self._x = a
        self._y = b

    def get_sum(self):
        return self._x + self._y

obj = My_Class()
obj.set_xy(3, 5)

print("Sum =", obj.get_sum())
print("_x =", obj._x)
```

**Python shell**

```
| Sum = 8
| _x = 3
```

# C++ private, public

**C++ vs Python**

1. argument types
2. return types
3. `void` = `NoneType`
4. `private` / `public` access specifier
5. types of data attributes
6. data attributes must be defined in class
7. object creation
8. no `self` in class methods

**private_attributes.cpp**

```
#include <iostream>
using namespace std;
class My_Class {
private:④
 ⑤int x, y;⑥
public:④      ⑧①       ①
②③void set_xy(int a, int b) {
     x = a;
     y = b
   };            ⑧
 ② int get_sum() {
     return x + y;
   };
};
main() {
⑦ My_Class obj;
   obj.set_xy(3, 5);
   cout << "Sum = " << obj.get_sum() << endl;
   cout << "x = " << obj.x << endl;
}
```

invalid reference

# Java private, public

**Java vs Python**

1. argument types
2. return types
3. `void` = `NoneType`
4. `private` / `public` access specifier
5. types of data attributes
6. data attributes must be defined in class
7. object creation
8. no `self` in class methods

**private_attributes.java**

```
class My_Class {
④ private ⑤int x, y; ⑥
④ public ②③void set_xy⑧①(int a, ①int b) {
    x = a; y = b;
  }
④ public ②int get_sum⑧() { return x + y; };
};
class private_attributes {
  public static void main(String args[]){
  ⑦ My_Class obj = new My_Class();
    obj.set_xy(3, 5);
    System.out.println("Sum = " + obj.get_sum());
    System.out.println("x = " + obj.x);
  }
}
```

invalid reference

# Name mangling (partial privacy)

- Python handles references to class attributes inside a class definition with *at least two leading underscores and at most one trailing underscore* in a special way: `__attribute` is textually replaced by `_classname__attribute`

- Note that [Guttag, p. 126] states ”that attribute is not visible outside the class” – which only is partially correct (see example)

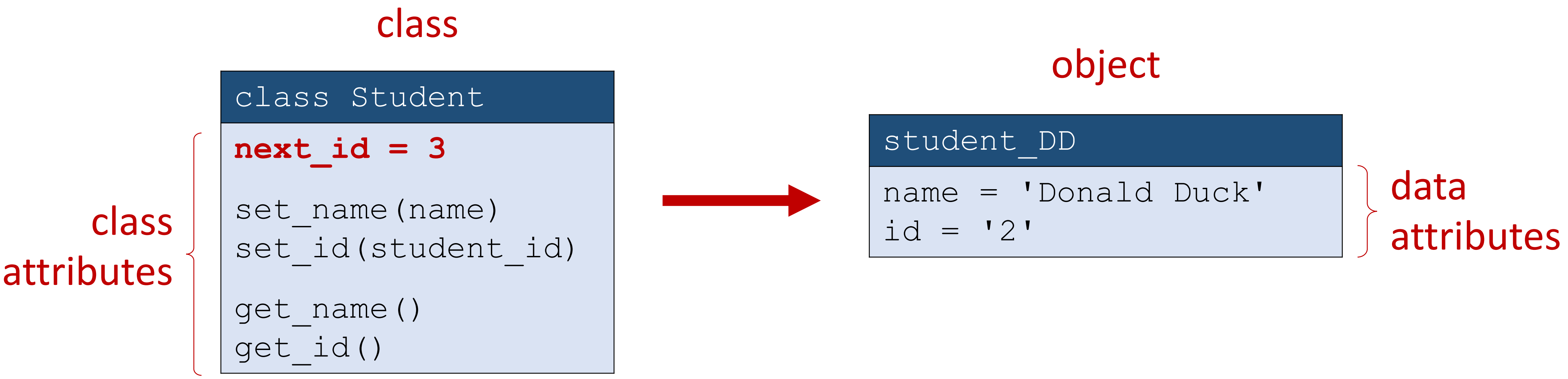
**name_mangeling.py**

```python
class MySecretBox:
    def __init__(self, secret):
        self.__secret = secret
```

**Python shell**

```
> x = MySecretBox(42)
> print(x.__secret)
| AttributeError: 'MySecretBox'
  object has no attribute
  '__secret'
> print(x._MySecretBox__secret)
| 42
```

# Class attributes




- *obj.attribute* first searches the objects attributes to find a match, if no match, continuous to search the attributes of the class
- Assignments to *obj.attribute* are always to the objects attribute (possibly creating the attribute)
- Class attributes can be accesed directly as *class.attribute* (or *obj.__class__.attribute*)

# Class data attribute

- **`next_id`** is a class attribute
- Accessed using **`Student.next_id`**
- The lookup ① can be replaced with **`self.next_id`**, since only the class has this attribute, looking up in the object will be propagated to a lookup in the class attributes
- In the update ② it is crucial that we update the class attribute, since otherwise the incremented value will be assigned as an object attribute

  (What will the result be?)

**student_auto_id.py**

```
class Student:
    next_id = 1  # class attribute
    def __init__(self, name):
        self.name = name         ①
        self.id = str(Student.next_id)
      ② Student.next_id += 1
    def get_name(self):
        return self.name
    def get_id(self):
        return self.id

students = [Student('Scrooge McDuck'),
            Student('Donald Duck'),
            Student('Mickey Mouse')]
for student in students:
    print(student.get_name(),
          "has student id",
          student.get_id())
```

**Python shell**

```
| Scrooge McDuck has student id 1
| Donald Duck has student id 2
| Mickey Mouse has student id 3
```

# Question – What does `obj.get()` return ?

**Python shell**

```
> class MyClass:
      x = 2

      def get(self):
          self.x = self.x + 1
          return MyClass.x + self.x

> obj = MyClass()
> print(obj.get())
| ?
```

a) 4

b) 5

c) 6

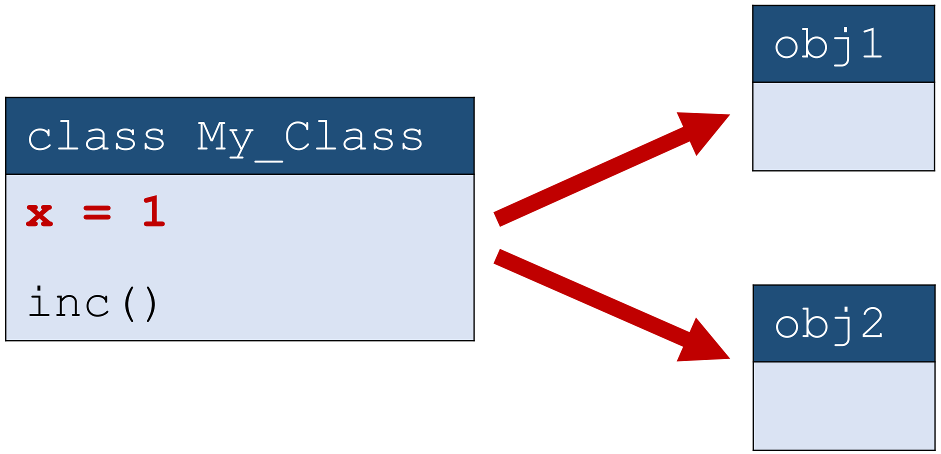
d) UnboundLocalError

e) Don't know

| class MyClass |
| --- |
| **x = 2** |
| get() |

| obj |
| --- |
| **x = 3** |

# Class data attribute example (in Python)




**class_attributes.py**

```python
class My_Class:
    x = 1  # class attribute
    def inc(self):
        My_Class.x = self.x + 1
obj1 = My_Class()
obj2 = My_Class()
obj1.inc()
obj2.inc()
print(obj1.x, obj2.x)
```

**Python shell**

```
| 3 3
```

- Note that `My_Class.x` and `self.x` refer to the same class attribute (since `self.x` has never been assigned a value)

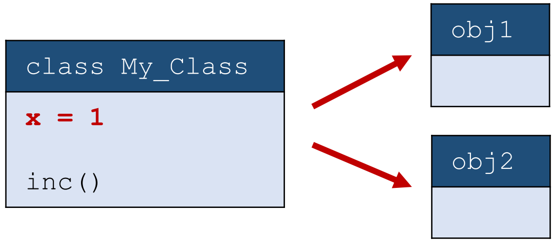
# __dict__, __name__ and __class__

Python shell

```
> MM = Student('Mickey Mouse')
> MM.__dict__                          # objects attributes
| {'name': 'Mickey Mouse', 'id': '1'}
> MM.__class__                         # objects class (reference to object of type class)
| <class '__main__.Student'>
> Student.__name__                     # class name (string)
| 'Student'
> Student.__dict__                     # class attributes
| mappingproxy({
  '__module__': '__main__',                                           # module where class defined
  'next_id': 2,                                                       # class data attriute
  '__init__': <function Student.__init__ at 0x000002831344CD30>,     # class method
  'get_name': <function Student.get_name at 0x000002831344CE50>,     # class method
  'get_id': <function Student.get_id at 0x000002831344CEE0>,         # class method
  '__dict__': <attribute '__dict__' of 'Student' objects>,           # attributes of class
  '__weakref__': <attribute '__weakref__' of 'Student' objects>,     # (for garbage collecting)
  '__doc__': None                                                     # docstring
  })
```

# Java `static`

- In Java *class attributes*, i.e. attribute values shared by all instances, are labeled **`static`**
- Python allows both class and instance attributes with the same name – in Java at most one of them can exist




**static_attributes.java**

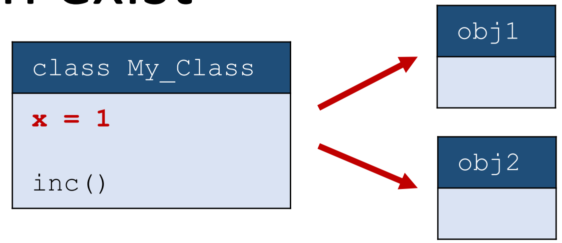
```java
class My_Class {
    public static int x = 1;
    public void inc() { x += 1; };
}

class static_attributes {
    public static void main(String args[]){
        My_Class obj1 = new My_Class();
        My_Class obj2 = new My_Class();
        obj1.inc();
        obj2.inc();
        System.out.println(obj1.x);
        System.out.println(obj2.x);
    }
}
```

**Java output**

```
3
3
```

# C++ static

- In C++ *class attributes*, i.e. attribute values shared by all instances, are labeled **static**
- ISO C++ forbids in-class initialization of non-const static member
- Python allows both class and instance attributes with the same name – in C++ at most one of them can exist




**static_attributes.cpp**

```
#include <iostream>
using namespace std;

class My_Class {
public:
   static int x; // "= 1" is not allowed
   void inc() { x += 1; };
};

int My_Class::x = 1; // class initialization

int main(){
   My_Class obj1;
   My_Class obj2;
   obj1.inc();
   obj2.inc();
   cout << obj1.x << endl;
   cout << obj2.x << endl;
}
```

**C++ output**

```
| 3
| 3
```

# Constants

- A simple usage of class data attributes is to store a set of constants (but there is nothing preventing anyone to chance these values)

**Python shell**

```
> class Color:
      RED   = "ff0000"
      GREEN = "00ff00"
      BLUE  = "0000ff"
> Color.RED
| 'ff0000'
```

# PEP8 Style Guide for Python Code (some quotes)

- Class names should normally use the CapWords convention.
- Always use self for the first argument to instance methods.
- Use one leading underscore only for non-public methods and instance variables.
- For simple public data attributes, it is best to expose just the attribute name, without complicated accessor/mutator methods.
- Always decide whether a class's methods and instance variables (collectively: "attributes") should be public or non-public. If in doubt, choose non-public; it's easier to make it public later than to make a public attribute non-public.

www.python.org/dev/peps/pep-0008/

# Some methods many class have

| Method | Description |
|---|---|
| `__eq__(self, other)` | Used to test if two elements are equal<br>Two elements where __eq__ is true must have equal __hash__ |
| `__str__(self)` | Used by `str` and `print` |
| `__repr__(self)` | Used by `repr`, e.g. for printing to the IDE shell<br>(usually something that is a valid Python expression for `eval()`) |
| `__len__(self)` | Length (integer) of object, e.g. lists, strings, tuples, sets, dictionaries |
| `__doc__(self)` | The docstring of the class |
| `__hash__(self)` | Returns hash value (integer) of object<br>Dictionary keys and set values must have a __hash__ method |
| `__lt__(self, other)` | Comparison (less than, <) used by `sorted` and `sort()` |
| `__init__(self,...)` | Class initializer |

Overloading operators: [www.python-course.eu/python3_magic_methods.php](www.python-course.eu/python3_magic_methods.php)

# Class hierarcies

- inheritance
- method overriding
- super
- multiple inheritance

# Calling methods of a class

- If an object *obj* of class *C* has a method `method`, then usually you call *obj*`.`*method*`()`

- It is possible to call the method in the class directly using *C*`.`*method*, where the object is the first argument

  *C*`.`*method*`(`*obj*`)`

**X.py**

```python
class X:
    def set_x(self, x):
        self.x = x

    def get_x(self):
        return self.x

obj = X()

obj.set_x(42)

print("obj.get_x() =", obj.get_x())
print("obj.x =", obj.x)
print("X.get_x(obj) =", X.get_x(obj))
```

**Python shell**

```
| obj.get_x() = 42
| obj.x = 42
| X.get_x(obj) = 42
```

# Classes and Objects

**Observation:** students and employees are persons with additional attributes

```
class Person
set_name(name)
get_name()

set_address(address)
get_address()
```

instance

```
Person object
name = 'Mickey Mouse'
address = 'Mouse Street 42, Duckburg'
```

```
class Student
set_name(name)
get_name()

set_address(address)
get_address()

set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

instance

```
Student object
name = 'Donald Duck'
address = 'Duck Steet 13, Duckburg'
id = '1094'
grades = {'programming' : 'A' }
```

```
Employee object
name = 'Goofy'
address = 'Clumsy Road 7, Duckburg'
employer = 'Yarvard University'
```

# Classes and Objects

```
class Person
set_name(name)
get_name()

set_address(address)
get_address()
```

```
class Student
set_name(name)
get_name()

set_address(address)
get_address()

set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

person attributes

**Goal** – avoid redefining the 4 methods below from `person` class again in `student` class

**person.py**

```
class Person:
    def set_name(self, name):
        self.name = name
    def get_name(self):
        return self.name
    def set_address(self, address):
        self.address = address
    def get_address(self):
        return self.address
```

# Classes inheritance

| class Person |
| --- |
| set_name(name)<br>get_name()<br><br>set_address(address)<br>get_address() |

| class Student |
| --- |
| set_name(name)<br>get_name()<br><br>set_address(address)<br>get_address()<br>person attributes |
| set_id(student_id)<br>get_id()<br><br>set_grade(course, grade)<br>get_grades() |

class `Student` **inherits** from class `Person`

class `Person` is the **base class** of `Student`

**person.py**

```
class Student(Person):
    def set_id(self, student_id):
        self.id = student_id
    def get_id(self):
        return self.id
    def set_grade(self, course, grade):
        self.grades[course] = grade
    def get_grades(self):
        return self.grades
```

# Classes constructors

```
class Person
set_name(name)
get_name()

set_address(address)
get_address()
```

```
class Student
set_name(name)
get_name()

set_address(address)
get_address()

set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

person attributes

**person.py**

```python
class Person:
    def __init__(self):
        self.name = None
        self.address = None
    ...

class Student(Person):
    def __init__(self):
        self.id = None
        self.grades = {}
        Person.__init__(self)
    ...
```

constructor for `Person` class

constructor for `Student` class

**Notes**

1) If `Student.__init__` is not defined, then `Person.__init__` will be called
2) `Student.__init__` must call `Person.__init__` to initialize the `name` and `address` attributes

# super()

```
class Person
set_name(name)
get_name()

set_address(address)
get_address()
```

```
class Student
set_name(name)                 person
get_name()                     attributes

set_address(address)
get_address()

set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

**person.py**

```
class Person:
    def __init__(self):
        self.name = None
        self.address = None
    ...

class Student(Person):
    def __init__(self):
        self.id = None
        self.grades = {}
        Person.__init__(self)
        super().__init__()
    ...
```

alternative constructor

**Notes**

1) Function `super()` searches for attributes in base class
2) `super` is often a keyword in other OO languages, like Java and C++
3) Note `super().__init__()` does not need `self` as argument

# Method search order

```
class Person
set_name(name)
get_name()

set_address(address)
get_address()
```

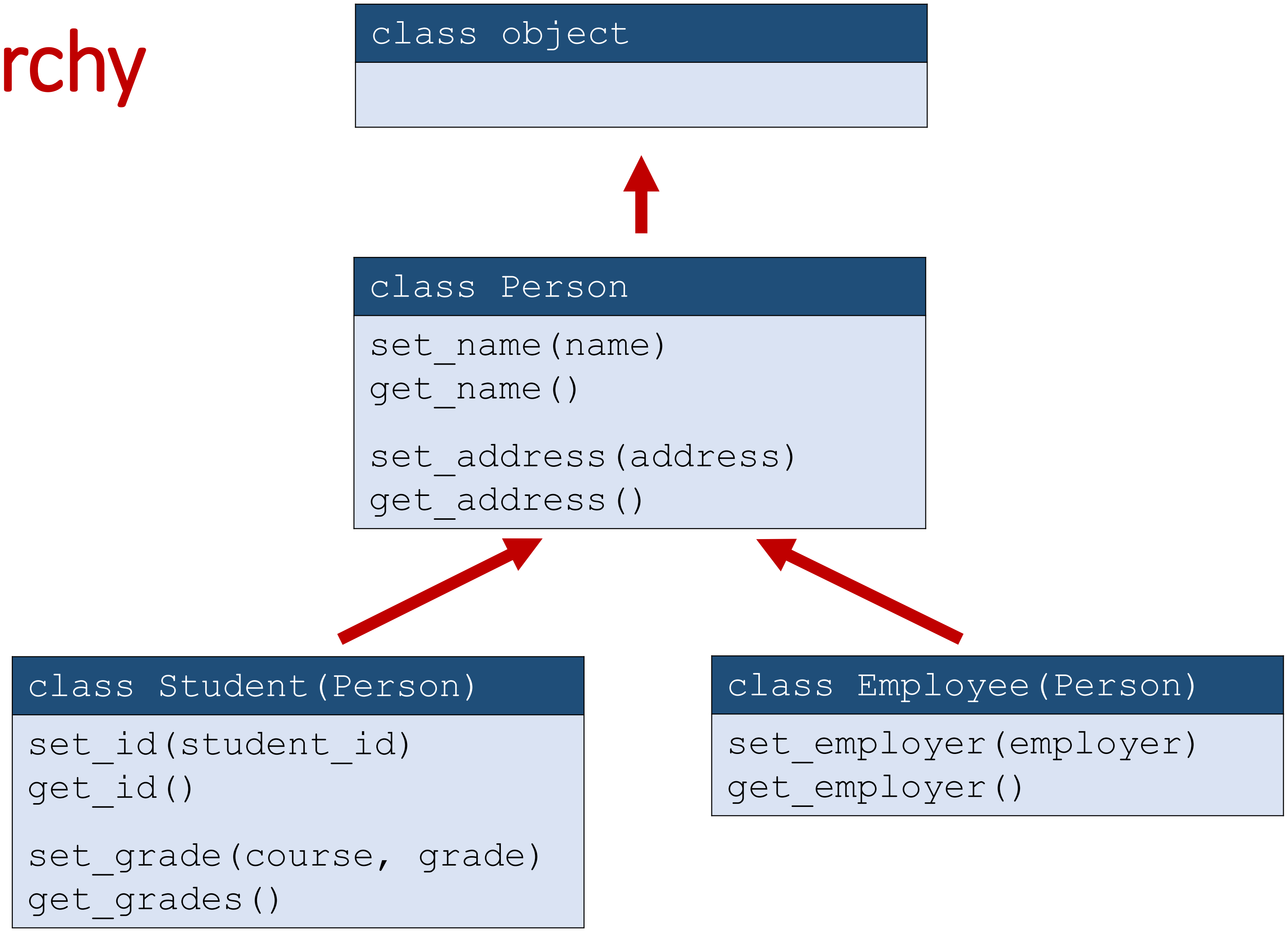
**parent class**

```
class Student(Person)
set_id(student_id)
get_id()

set_grade(course, grade)
get_grades()
```

**instance of**

```
Student object
name = 'Donald Duck'
address = 'Duck Steet 13, Duckburg'
id = '1094'
grades = {'programming' : 'A' }
```

# Class hierarchy



# Method overriding

**overloading.py**

```
class A:
    def say(self):
        print("A says hello")

class B(A):  # B is a subclass of A
    def say(self):
        print("B says hello")
        super().say()
```

**Python shell**

```
> B().say()
| B says hello
| A says hello
```

In Java one can use the keyword "`finally`" to prevent any subclass to override a method

# Question – What does `b.f()` print ?

**Python shell**

```
> class A():
      def f(self):
          print("Af")
          self.g()
      def g(self):
          print("Ag")

> class B(A):
      def g(self):
          print("Bg")

> b = B()
> b.f()
| ?
```

a) `AttributeError`

b) `Af Ag`

c) `Af Bg`

d) Don’t know

# Undefind methods in superclass ?

**Python shell**

```
> class A():
      def f(self):
          print("Af")
          self.g()

      def g(self):
          print("Ag")
> class B(A):
      def g(self):
          print("Bg")
> b = B()
> b.f()
| Af
| Bg
> a = A()
> a.f()
| Af
| Ag
```

**Python shell**

```
> class A():
      def f(self):
          print("Af")
          self.g()
> class B(A):
      def g(self):
          print("Bg")
> b = B()
> b.f()
| Af
| Bg
> a = A()
> a.f()
| Af
| AttributeError: 'A' object has no attribute 'g'
```

method `g` undefined in class `A`; subclasses must implement `g` to be able to call `f`

in Java, `A` would have been required to be declared an *abstract class*

can create instance of `A`

fails since `g` is not defined in class `A`

# Name mangling and inheritance ⚠

**Python shell**

```
> class A():
      def f(self):
          print("Af")
          self.__g()
      def __g(self):
          print("Ag")
> class B(A):
      def __g(self):
          print("Bg")
> b = B()
> b.f()
| Af
| Ag
```

- The call to `A.__g` in `A.f` forces a call to `__g` to stay within `A`
- Recall that due to name mangling, `__g` is accessible as `A._A__g`

# Multiple inheritance

- A class can inherit attributes from multiple classes (in example two)
- When calling a method defined in several ancestor classes, Python executes only one of the these (in the example `say_hello`)
- Which one is determined by the so called ”C3 Method Resolution Order” (originating from the Dylan language)

Raymond Hettinger, *Super considered super!* Conference talk at PyCon 2015

**multiple_inheritance.py**

```python
class Alice:
    def say_hello(self):
        print("Alice says hello")
    def say_good_night(self):
        print("Alice says good night")
class Bob:
    def say_hello(self):
        print("Bob says hello")
    def say_good_morning(self):
        print("Bob says good morning")
class X(Alice, Bob):  # Multiple inheritance
    def say(self):
        self.say_good_morning()
        self.say_hello()          # C3 resolution
        Alice.say_hello(self)     # from Alice
        Bob.say_hello(self)       # from Bob
        self.say_good_night()
```

**Python shell**

```
> X().say()
| Bob says good morning
| Alice says hello
| Alice says hello
| Bob says hello
| Alice says good night
```

since `Alice` before `Bob` in list of super classes

# C3 Method resolution order

- Use `help(class)` to determine the resolution order for the class
- or access the `__mro__` attribute of the class

**Python shell**

```
> X.__mro__
| (<class '__main__.X'>, <class '__main__.Alice'>,
  <class '__main__.Bob'>, <class 'object'>)
> help(X)
| Help on class X in module __main__:
| class X(Alice, Bob)
|   |  Method resolution order:
|   |      X
|   |      Alice
|   |      Bob
|   |      builtins.object
|   |  Methods defined here:
|   |  say(self)
|   |  ----------------------------------
|   |  Methods inherited from Alice:
|   |  say_good_night(self)
|   |  say_hello(self)
|   |  ----------------------------------
|   |  ...
|   |  ----------------------------------
|   |  Methods inherited from Bob:
|   |  say_good_morning(self)
```

# Question – Who says hello ? Bob says good morning

**inheritance.py**

```
class Alice:
    def say_hello(self):
        print("Alice says hello")
class Bob:
    def say_hello(self):
        print("Bob says hello")
    def say_good_morning(self):
        self.say_hello()
        print("Bob says good morning")
class X(Alice, Bob):  # Multiple inheritance
    pass

X().say_good_morning()
```

a) Alice

b) Bob

c) Dont’ know

…example of code injection using multiple inheritance and where body of new class is empty

# Comparing objects and classes

- `id(obj)` returns a unique identifyer for an object (in CPython the memory address)
- `obj1 is obj2` tests if `id(obj1)==id(obj2)`
- `type(obj)` and `obj.__class__` return the class of an object
- `isinstance(object, class)` checks if an object is of a particular class, *or a derived subclass*
- `issubclass(class1, class2)` checks if `class1` is a subclass of `class2`

# `is` is not for integers, strings, ... and `is` is not ==

Python shell

```
> 500 + 500 is 1000
| True
> x = 500
> x + x is 1000
| False
> x + x == 1000   # int.__eq__(...)
| True
> for x in range(0, 1000):
      if x - 1 + 1 is not x:
          print(x)
          break
| 257
> for x in range(0, -1000, -1):
      if x + 1 - 1 is not x:
          print(x)
          break
| -6
```

Python shell

```
> "abc" is "abc"
| True
> "abc" is "xabc"[1:]
| False
> x, y = "abc", "xabc"[1:]
> x, y
| ('abc', 'abc')
> x is y
| False
> x == y  # x.__eq__(y)
| True
```

- Only use `is` on objects !
- Even though `isinstance(42, object)` and `isinstance("abc", object)` are true, do not use `is` on integers and strings !

# Comparison of OO in Python, Java and C++

- private, public, .... – in Python everything in an object is public
- class inheritance – core concept in OO programming
  - Python and C++ support multiple inheritance
  - Java only allows single inheritance, but Java ”interfaces” allow for something like multiple inheritance
- Python and C++ allow overloading standard operators (+, *, ...). In Java it is not possible.
- Overloading methods
  - Python extremely dynamic (hard to say anything about the behaviour of a program in general)
  - Java and C++’s type systems allow several methods with same name in a class, where they are distinguished by the type of the arguments, whereas Python allows only one method that can have * and ** arguments

# C++ example

- Multiple methods with identical name (`print`)
- The types distinguish the different methods

**printing.py**

```python
class MyClass:
    def print(self, value):
        if isinstance(value, int):
            print('An integer', value)
        elif isinstance(value, str):
            print('A string', value)

C = MyClass()
C.print(42)
C.print("abc")
```

**printing.cpp**

```cpp
#include <iostream>
using namespace std;

class MyClass {
public:
    void print(int x) {
        cout << "An integer " << x << endl;
    };

    void print(string s) {
        cout << "A string " << s << endl;
    };
};

main() {
    MyClass C;

    C.print(42);
    C.print("abc");
}
```

**Shell**

```
| An integer 42
| A string abc
```

# Exceptions and file input/output

- try-raise-except-finally
- Exception
- control flow
- match - case
- file open/read/write
- sys.stdin, sys.stdout, sys.stderr

# Exceptions – Error handling and control flow

- Exceptions is a widespread technique to handle run-time **errors** / abnormal behaviour (e.g. in Python, Java, C++, JavaScript, C#)

- Exceptions can also be used as an **advanced control flow mechanism** (e.g. in Python, Java, JavaScript)

  - *Problem: How to perform a "break" in a recursive function ?*

# Built-in exceptions (class hierarchy)

```
BaseException
 +-- SystemExit
 +-- KeyboardInterrupt
 +-- GeneratorExit
 +-- Exception
      +-- StopIteration
      +-- StopAsyncIteration
      +-- ArithmeticError
      |    +-- FloatingPointError
      |    +-- OverflowError
      |    +-- ZeroDivisionError
      +-- AssertionError
      +-- AttributeError
      +-- BufferError
      +-- EOFError
      +-- ImportError
      |    +-- ModuleNotFoundError
      +-- LookupError
      |    +-- IndexError
      |    +-- KeyError
      +-- MemoryError
      +-- NameError
      |    +-- UnboundLocalError
      +-- TypeError
      +-- ValueError
      |    +-- UnicodeError
      |         +-- UnicodeDecodeError
      |         +-- UnicodeEncodeError
      |         +-- UnicodeTranslateError
      |
      +-- OSError
      |    +-- BlockingIOError
      |    +-- ChildProcessError
      |    +-- ConnectionError
      |    |    +-- BrokenPipeError
      |    |    +-- ConnectionAbortedError
      |    |    +-- ConnectionRefusedError
      |    |    +-- ConnectionResetError
      |    +-- FileExistsError
      |    +-- FileNotFoundError
      |    +-- InterruptedError
      |    +-- IsADirectoryError
      |    +-- NotADirectoryError
      |    +-- PermissionError
      |    +-- ProcessLookupError
      |    +-- TimeoutError
      +-- ReferenceError
      +-- RuntimeError
      |    +-- NotImplementedError
      |    +-- RecursionError
      +-- SyntaxError
      |    +-- IndentationError
      |         +-- TabError
      +-- SystemError
      +-- Warning
           +-- DeprecationWarning
           +-- PendingDeprecationWarning
           +-- RuntimeWarning
           +-- SyntaxWarning
           +-- UserWarning
           +-- FutureWarning
           +-- ImportWarning
           +-- UnicodeWarning
           +-- BytesWarning
           +-- ResourceWarning
```

docs.python.org/3/library/exceptions.html

# Typical built-in exceptions

## and unhandled behaviour

**Python shell**

```
> 7 / 0
| ZeroDivisionError: division by zero
> int('42x')
| ValueError: invalid literal for int() with base 10: '42x'
> x = y
| NameError: name 'y' is not defined
> L = list(range(1000000000))
| MemoryError
> 2.5 ** 1000
| OverflowError: (34, 'Result too large')
> t = (3, 4)
> t[0] = 7
| TypeError: 'tuple' object does not support item assignment
> t[3]
| IndexError: tuple index out of range
> t.x
| AttributeError: 'tuple' object has no attribute 'x'
> x = {}
> x['foo']
| KeyError: 'foo'
> def f(x): f(x + 1)
> f(0)
| RecursionError: maximum recursion depth exceeded
> def f(): x = x + 1
> f()
| UnboundLocalError: local variable 'x' referenced before assignment
```

# Catching exceptions – Fractions (I)

fraction1.py

```
while True:
    numerator = int(input('Numerator = '))
    denominator = int(input('Denominator = '))
    result = numerator / denominator
    print('%s / %s = %s' % (numerator, denominator, result))
```

Python shell

```
| Numerator = 10
| Denominator = 3
| 10 / 3 = 3.3333333333333335
| Numerator = 20
| Denominator = 0
| ZeroDivisionError: division by zero
```

# Catching exceptions – Fractions (II)

catch exception

**fraction2.py**

```
while True:
    numerator = int(input('Numerator = '))
    denominator = int(input('Denominator = '))
    try:
        result = numerator / denominator
    except ZeroDivisionError:
        print('cannot divide by zero')
        continue
    print('%s / %s = %s' % (numerator, denominator, result))
```

**Python shell**

```
| Numerator = 10
| Denominator = 0
| cannot divide by zero
| Numerator = 20
| Denominator = 3
| 20 / 3 = 6.666666666666667
| Numerator = 42x
| ValueError: invalid literal for int() with base 10: '42x'
```

# Catching exceptions – Fractions (III)

**fraction3.py**

```
while True:
    try:
        numerator = int(input('Numerator = '))
        denominator = int(input('Denominator = '))
    except ValueError:
        print('input not a valid integer')
        continue
    try:
        result = numerator / denominator
    except ZeroDivisionError:
        print('cannot divide by zero')
        continue
    print('%s / %s = %s' % (numerator, denominator, result))
```

catch exception

catch exception

**Python shell**

```
| Numerator = 5
| Denominator = 2x
| input not a valid integer
| Numerator = 5
| Denominator = 2
| 5 / 2 = 2.5
```

**fraction3.py**

```python
while True:
    try:
        numerator = int(input('Numerator = '))
        denominator = int(input('Denominator = '))
    except ValueError:
        print('input not a valid integer')
        continue
    try:
        result = numerator / denominator
        print('%s / %s = %s' % (numerator, denominator, result))
    except ZeroDivisionError:
        print('cannot divide by zero')
```

**Python shell**

```
| Numerator = 1000000000000000000000000000000000000000000000000000000000
  0000000000000000000000000000000000000000000000000000000000000000000000
  0000000000000000000000000000000000000000000000000000000000000000000000
  0000000000000000000000000000000000000000000000000000000000000000000000
  000000000000000000000000000000000000000000
| Denominator = 1
| OverflowError: integer division result too large for a float
```

exception not caught

# Catching exceptions – Fractions (IV)

catch exceptions

**fraction4.py**

```
while True:
    try:
        numerator = int(input('Numerator = '))
        denominator = int(input('Denominator = '))
        result = numerator / denominator
        print('%s / %s = %s' % (numerator, denominator, result))
    except ValueError:
        print('input not a valid integer')
    except ZeroDivisionError:
        print('cannot divide by zero')
```

**Python shell**

```
| Numerator = 3
| Denominator = 0
| cannot divide by zero
| Numerator = 3x
| input not a valid integer
| Numerator = 4
| Denominator = 2
| 4 / 2 = 2.0
```

# Keyboard interrupt (Ctrl-c)

- throws **`KeyboardInterrupt`** exception

**infinite-loop1.py**

```
print('starting infinite loop')

x = 0
while True:
    x = x + 1

print('done (x = %s)' % x)
input('type enter to exit')
```

**Python shell**

```
| starting infinite loop
| Traceback (most recent call last):
|   File 'infinite-loop1.py', line 4, in <module>
|     x = x + 1
| KeyboardInterrupt
```

**infinite-loop2.py**

```
print('starting infinite loop')

try:
    x = 0
    while True:
        x = x + 1
except KeyboardInterrupt:
    pass

print('done (x = %s)' % x)
input('type enter to exit')
```

**Python shell**

```
| starting infinite loop
| done (x = 23890363)   # Ctrl-c
| type enter to exit
```

# Keyboard interrupt (Ctrl-c)

- Be aware that you likely would like to leave the Ctrl-c generated **`KeyboardInterrupt`** exception unhandled, except when stated explicitly

**read-int1.py**

```
while True:
    try:
        x = int(input('An integer: '))
        break
    except ValueError:  # only ValueError
        continue

print('The value is:', x)
```

**Python shell**

```
| An integer:          Ctrl-c
| KeyboardInterrupt
```

**read-int2.py**

```
while True:
    try:
        x = int(input('An integer: '))
        break
    except:  # all exceptions
        continue

print('The value is:', x)
```

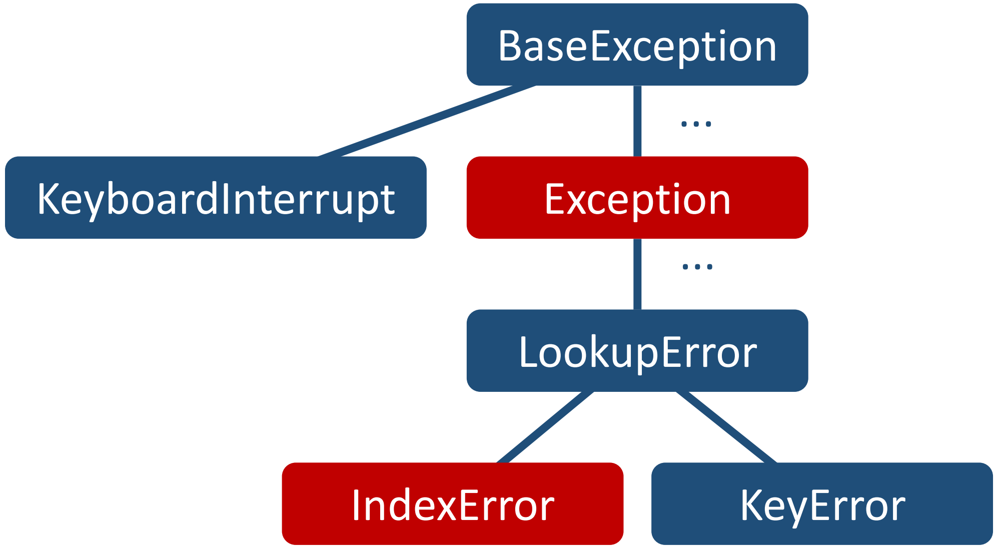
catches KeyboardInterrupt

**Python shell**

```
| An integer:          Ctrl-c
| An integer:          Ctrl-c
| An integer:
```

- (left) `KeyboardInterrupt` is unhandled (right) it is handled (intentionally?)

# Exception class hierarchy




**except-twice1.py**

```
try:
    L[4]
except IndexError:  # must be before Exception
    print('IndexError')
except Exception:
    print('Fall back exception handler')
```

**except-twice2.py**

```
try:
    L[4]
except Exception:  # and subclasses of Exception
    print('Fall back exception handler')
except IndexError:
    print('IndexError')   # unreachable
```

# `try` statement syntax

arbitrary number of except cases

```
try:
    code
except ExceptionType1:
    code   # executed if raised exception instanceof
           # ExceptionType1 (or subclass of ExceptionType1)
except ExceptionType2:
    code   # executed if exception type matches and none of
           # the previous except statements matched
...
else:
    code   # only executed if no exception was raised
finally:
    code   # always executed independent of exceptions
           # typically used to clean up (like closing files)
```

# except variations

```
except:                    # catch all exceptions

except ExceptionType:      # only catch exceptions of class ExceptionType
                           # or subclasses of ExceptionType

except (ExceptionType_1, ExceptionType_2, ..., ExceptionType_k):
                           # catch any of k classes (and subclasses)

except ExceptionType as e:
                           # catch exception and assign exception object to e
                           # e.args contains arguments to the raised exception
```

# Raising exceptions

- An exception is raised (or trown) using one of the following (the first being an alias for the second):

  **`raise ExceptionType`**

  **`raise ExceptionType()`**

  **`raise ExceptionType(args)`**

**abstract.py**

```python
class A():
    def f(self):
        print('f')
        self.g()
    def g(self):
        raise NotImplementedError
class B(A):
    def g(self):
        print('g')
```

**Python shell**

```
> B().f()
| f
| g
> A().f()
| f
| NotImplementedError
```

# User exceptions

- New exception types are created using class inheritance from an existing exception type (possibly defining `__init__`)

**tree-search.py**

```python
class SolutionFound(Exception):  # new exception
    pass

def recursive_tree_search(x, t):
    if isinstance(t, tuple):
        for child in t:
            recursive_tree_search(x, child)
    elif x == t:
        raise SolutionFound  # found x in t

def tree_search(x, t):
    try:
        recursive_tree_search(x, t)
    except SolutionFound:
        print('found', x)
    else:
        print('search for', x, 'unsuccessful')
```

**Python shell**

```
> tree_search(8, ((3,2),5,(7,(4,6))))
| search for 8 unsuccessful
> tree_search(7, ((3,2),5,(7,(4,6))))
| found 7
```

# match – case (since Python 3.10)

- Assume we want to do different things depending on the value of an expression (different *cases*)
- Can be done using `if`, but also using `match` – `case`, that is also evaluated top-down

**match-case.py**

```
x = 7
if x == 1:
    print('x is one')
elif x == 2:
    print('x is two')
elif x == 3 or x == 4 or x == 5:
    print('x is three, four or five')
else:
    print(x, 'is not in the range 1-5')
```

**Python shell**

```
| 7 is not in the range 1-5
```

**match-case.py**

```
x = 7
match x:  # match expression
    case 1:
        print('x is one')
    case 2:
        print('x is two')
    case 3 | 4 | 5:  # match any of the cases
        print('x is three, four or five')
    case value:  # else, value = variable name
        print(value, 'is not in the range 1-5')
```

**Python shell**

```
| 7 is not in the range 1-5
```

# match – case

Can match...

- simple values
- named variable values
- guards (`if`)
- sequences of values
- dictionaries
- builtin types
- user defined classes
- nested structures of the above

**match-case.py**

```
class Color:
    RED   = 'ff0000'
    GREEN = '00ff00'
    BLUE  = '0000ff'

class Point:
    __match_args__ = ('x', 'y')

    def __init__(self, x, y):
        self.x = x
        self.y = y

    def __repr__(self):
        return f'Point({self.x}, {self.y})'
```

**match-case.py**

```
match x:
    case 42:
        return 'the integer 42'
    case 1 | 2 | 3 | 4 | 5:
        return 'integer in range(1, 6)'
    case (1, 2):
        return 'sequence containing the elements 1 and 2'
    case [x, 2]:
        return 'sequence of length 2, last=2, first=' + str(x)
    case (x, y) if x + y == 7:  # guard
        return 'sequence with two values with sum 7'
    case [0, 1, *x]:  # x is list of remaining elements in sequence
        return 'sequence starting with 1 and 2, and tail ' + str(x)
    case {'a': 7, 'b': x}:
        return 'dictionary "a" -> 7, "b" -> ' + str(x)
    case (('a' | 'b'), ('c' | 'd')):
        return 'tuple length 2, first "a" or "b", last "c" or "d"'
    case (('x' | 'y') as fst, ('x' | 'y') as snd):
        return '(fst, snd), where fst=' + str(fst) + ', snd=' + str(snd)
    case float(value):  # test on builtin type
        return 'a float ' + str(value)
    case Color.RED:  # class or object attribute
        return 'the color red'
    case Point(x=7, y=value):  # Point object with attributes x and y
        return 'a Point object with x=7, and y=' + str(value)
    case Point(x, y):  # requires __match_args__ in class Point
        return 'a point Point(' + str(x) + ', ' + str(y) + ')'
    case e:  # ⚠ using the wildcard _ would not bind to a variable
        return 'cannot match ' + repr(e)
```

**Python shell**

```
> for x in [42, 1, [1, 2], [7, 2], range(3, 5), (3, (5, 7)), (0, 1, 2, 3, 4, 5), {'a':7, 'b':42, 'c':1},
  ('b', 'c'), ('y', 'x'), 3.14, 'ff0000', Point(7, 42), Point(3, 5), 'abc']:
      print('f(' + repr(x) + ') = ' + repr(f(x)))
| f(42) = 'the integer 42'
| f(1) = 'integer in range(1, 6)'
| f([1, 2]) = 'sequence containing the elements 1 and 2'
| f([7, 2]) = 'sequence of length 2, last=2, first=7'
| f(range(3, 5)) = 'sequence with two values with sum 7'
| f((3, (5, 7))) = 'a triplet (3, (5, 7))'
| f((0, 1, 2, 3, 4, 5)) = 'sequence starting with 1 and 2, and tail [2, 3, 4, 5]'
| f({'a': 7, 'b': 42, 'c': 1}) = 'dictionary "a" -> 7, "b" -> 42'
| f(('b', 'c')) = 'tuple length 2, first "a" or "b", last "c" or "d"'
| f(('y', 'x')) = '(fst, snd), where fst=y, snd=x'
| f(3.14) = 'a float 3.14'
| f('ff0000') = 'the color red'
| f(Point(7, 42)) = 'a Point object with x=7, and y=42'
| f(Point(3, 5)) = 'a point Point(3, 5)'
| f('abc') = "cannot match 'abc'"
```

# 3 ways to read lines from a file

**Steps**

1. Open file using `open`
2. Read lines from file using
   a) *`filehandler`*`.readline`
   b) *`filehandler`*`.readlines`
   c) `for` *`line`* `in` *`filehandler`*`:`
3. Close file using `close`

`open ('filename.txt')` assumes the file to be in the same folder as your Python program, but you can also provide a full path `open('c:/Users/gerth/Documents/filename.txt')`

try to open file for reading

filename

filehandle

iterate over lines in file

close file when done

**reading-file1.py**

```
f = open('reading-file1.py')
for line in f:
    print('>', line[:-1])
f.close()
```

read all lines into a list of strings

**reading-file2.py**

```
f = open('reading-file2.py')
lines = f.readlines()
f.close()
for line in lines:
    print('>', line[:-1])
```

read single line (terminated by '\n')

**reading-file3.py**

```
f = open('reading-file3.py')
line = f.readline()
while line != '':
    print('>', line[:-1])
    line = f.readline()
f.close()
```

# 3 ways to write lines to a file

- Opening file:
  `open(filename, mode)`
  where *mode* is a string, either `'w'` for opening  a new (or truncating an existing file) and `'a'` for appending to an existing file
- Write single string:
  `filehandle.write(string)`
  Returns the number of characters written
- Write list of strings strings:
  `filehandle.writeline(list)`
- Newlines (`'\n'`) must be written explicitly
- `print` can take an optional `file` argument

try to open file for writing

write mode

write single string to file

write list of strings to file

append to existing file

**write-file.py**

```
f = open('output-file.txt', 'w')
f.write('Text 1\n')
f.writelines(['Text 2\n', 'Text 3 '])
f.close()

g = open('output-file.txt', 'a')
print('Text 4', file=g)
g.writelines(['Text 5 ', 'Text 6'])
g.close()
```

**output-file.txt**

```
Text 1
Text 2
Text 3 Text 4
Text 5 Text 6
```

# Exceptions while dealing with files

- When dealing with files one should be prepared to handle errors / raised exceptions, e.g. `FileNotFoundError`

**reading-file4.py**

```
try:
    f = open('reading-file4.py')
except FileNotFoundError:
    print('Could not open file')
else:
    try:
        for line in f:
            print('> ', line[:-1])
    finally:
        f.close()
```

# Opening files using `with`

- The Python keyword **`with`** allows to create a *context manager* for handling files
- *Filehandle will automatically be closed, also when exceptions occur*
- Under the hood: filehandles returned by `open()` support `__enter__()` and `__exit__()` methods

f = result of calling `__enter__()` on result of `open` expression, which is the file handle

**reading-file5.py**

```
with open('reading-file5.py') as f:
    for line in f:
        print('> ', line[:-1])
```

# Does a file exist?

- Module `os.path` contains a method `isfile` to check if a file exists

**checking-files.py**

```
import os.path

filename = input('Filename: ')
if os.path.isfile(filename):
    print('file exists')
else:
    print('file does not exist')
```

docs.python.org/3/library/os.path.html

# module `sys`

- Module `sys` contains the three standard file handles

  `sys.stdin` (used by the `input` function)

  `sys.stdout` (used by the `print` function)

  `sys.stderr` (error output from the Python interpreter)

**sys-test.py**

```
import sys
sys.stdout.write('Input an integer: ')
x = int(sys.stdin.readline())
sys.stdout.write('%s square is %s' % (x, x**2))
```

**Python shell**

```
| Input an integer: 10
| 10 square is 100
```

docs.python.org/3/library/sys.html

# print(..., file=*output file*)

sys-print-file.py

```
import sys

def complicated_function(file):
    print('Hello world', file=file)  # print to file or STDOUT

while True:
    file_name = input('Output file (empty for STDOUT): ')

    if file_name == '':
        file = sys.stdout
        break
    else:
        try:
            file = open(file_name, 'w')
            break
        except Exception:
            pass

complicated_function(file)

if file != sys.stdout:
    file.close()
```

# PEP8 on exceptions

- For all try/except clauses, limit the try clause to the absolute minimum amount of code necessary.
- The class naming convention applies (CapWords)
- Use the suffix "Error" on your exception names (if the exception actually is an error)
- A bare **`except:`** clause will catch **`SystemExit`** and **`KeyboardInterrupt`** exceptions, making it harder to interrupt a program with Control-C, and can disguise other problems.
  If you want to catch all exceptions that signal program errors, use **`except Exception:`**

www.python.org/dev/peps/pep-0008/

# Performance of scanning a file

- Python can efficiently scan through quite big files

| File | Size | Time |
|---|---|---|
| Atom_chem_shift.csv | ≈ 750 MB | ≈ 8 sec |
| cano.txt | ≈ 3.7 MB | ≈ 0.1 sec |

The first search finds all lines related to ThrB12-DKP-insulin (Entry ID 6203) in a chemical database available from www.bmrb.wisc.edu

The second search finds all occurrences of “Germany” in Conan Doyle's complete Sherlock Holmes available at sherlock-holm.es

**file-scanning.py**

```python
from time import time

for filename, query in [
        ('Atom_chem_shift.csv', ',6203,'),
        ('cano.txt', 'Germany')
    ]:
    count = 0
    matches = []
    start = time()
    with open(filename) as f:
        for i, line in enumerate(f, start=1):
            count += 1
            if query in line:
                matches.append((i, line))
    end = time()

    for i, line in matches:
        print(i, ':', line, end='')
    print('Duration:', end - start)
    print(len(matches), 'of', count, 'lines match')
```

**Python shell**

```
...
3057752 : 195,,2,2,30,30,THR,HB,H,1,4.22,0.02,,1,,,,,,,,,,,,,,,,,,,,,228896,6203,2
3057753 : 196,,2,2,30,30,THR,HG21,H,1,1.18,0.02,,1,,,,,,,,,,,,,,,,,,,,,228896,6203,2
3057754 : 197,,2,2,30,30,THR,HG22,H,1,1.18,0.02,,1,,,,,,,,,,,,,,,,,,,,,228896,6203,2
3057755 : 198,,2,2,30,30,THR,HG23,H,1,1.18,0.02,,1,,,,,,,,,,,,,,,,,,,,,228896,6203,2
Duration: 7.760039329528809
329 of 9758361 lines match
57557 :       "Well, then, to the West, or to England, or to Germany, where father
66515 :       kind master. He wanted me to go with his wife to Germany yesterday,
66642 :       of business in Germany in the past and my name is probably familiar
73273 :       associates with Germany. This he placed in his instrument cupboard.
Duration: 0.07700657844543457
4 of 76764 lines match
```

**sudoku.py**

```python
class Sudoku:
  def __init__(self, puzzle):
    self.puzzle = puzzle

  def solve(self):
    def find_free():
      for i in range(9):
        for j in range(9):
          if self.puzzle[i][j] == 0:
            return (i, j)
      return None

    def unused(i, j):
      i_, j_ = i // 3 * 3, j // 3 * 3
      cells = {(i, k) for k in range(9)}
      cells |= {(k, j) for k in range(9)}
      cells |= {(i, j) for i in range(i_, i_ + 3)
                       for j in range(j_, j_ + 3)}
      return set(range(1, 10)) - {self.puzzle[i][j] for i, j in cells}

    class SolutionFound(Exception):
      pass

    def recursive_solve():
      cell = find_free()
      if not cell:
        raise SolutionFound
      i, j = cell
      for value in unused(i, j):
        self.puzzle[i][j] = value
        recursive_solve()
      self.puzzle[i][j] = 0

    try:
      recursive_solve()
    except SolutionFound:
      pass
```

**sudoku.py (*continued*)**

```python
  def print(self):
    for i, row in enumerate(self.puzzle):
      cells = [' %s ' % c if c else ' . ' for c in row]
      print('|'.join([''.join(cells[j:j+3]) for j in (0,3,6)]))
      if i in (2,5):
        print('---------+---------+---------')

with open('sudoku.txt') as f:
  A = Sudoku([[int(x) for x in line.strip()] for line in f])

A.solve()
A.print()
```

**sudoku.txt**

```
517600034
289004000
346205090
602000010
038006047
000000000
090000078
703400560
000000000
```

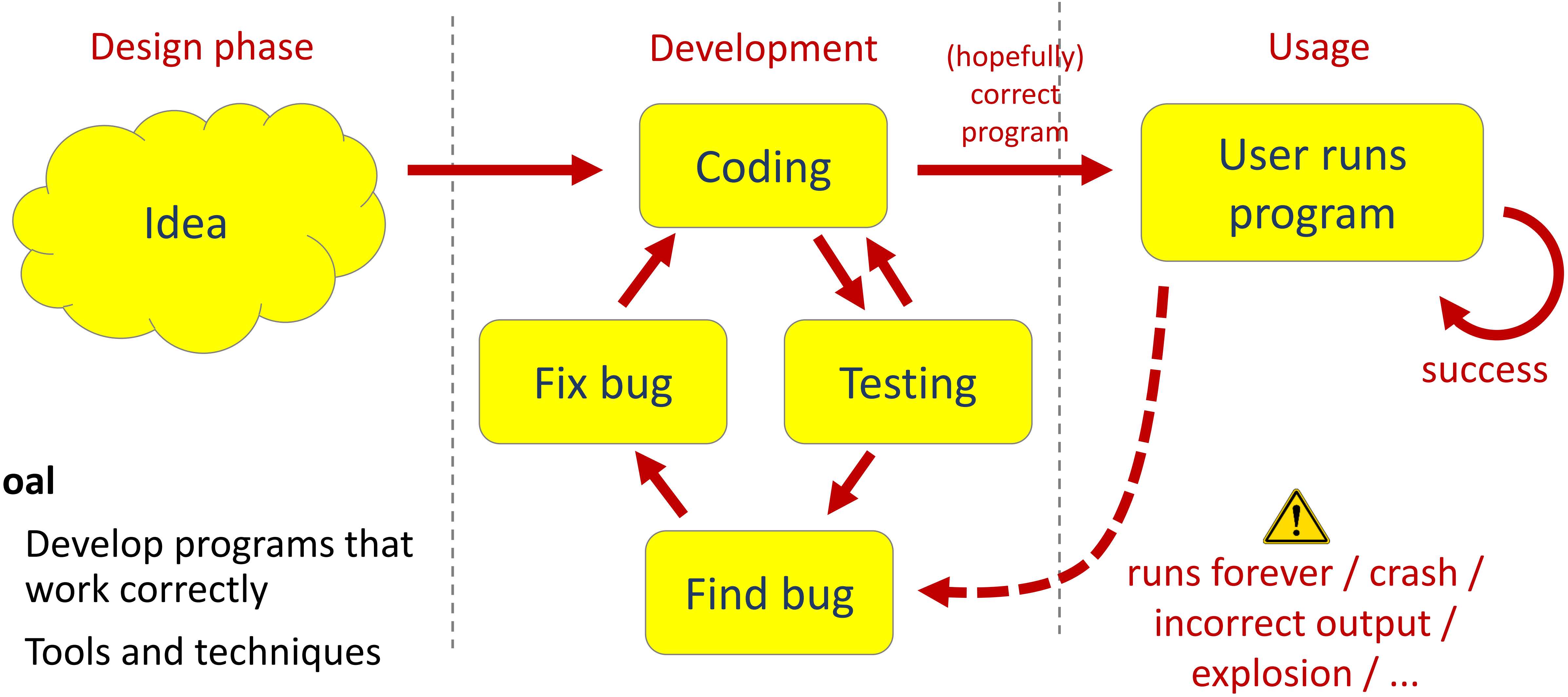
**Python shell**

```
| 5  1  7 | 6  9  8 | 2  3  4
| 2  8  9 | 1  3  4 | 7  5  6
| 3  4  6 | 2  7  5 | 8  9  1
| ---------+---------+---------
| 6  7  2 | 8  4  9 | 3  1  5
| 1  3  8 | 5  2  6 | 9  4  7
| 9  5  4 | 7  1  3 | 6  8  2
| ---------+---------+---------
| 4  9  5 | 3  6  2 | 1  7  8
| 7  2  3 | 4  8  1 | 5  6  9
| 8  6  1 | 9  5  7 | 4  2  3
```

# Documentation, testing and debugging

- docstring
- defensive programming
- assert
- test driven developement
- assertions
- testing
- unittest
- debugger
- coverage
- static type checking (mypy)

# Ensuring good quality code ?




**Goal**

- Develop programs that work correctly
- Tools and techniques

# What is good code ?

- Readability
  - well-structured
  - documentation
  - comments
  - follow some standard structure (easy to recognize, follow PEP8 Style Guide)

- Correctness
  - outputs the correct answer on valid input
  - eventually stops with an answer on valid input (should not go in infinite loop)

- Reusable...

# Why ?

## Documentation

- *specification of functionality*
- docstring
  - *for users of the code*
  - modules
  - methods
  - classes
- comments
  - *for readers of the code*

## Testing

- Correct implementation ?
- Try to predict behavior on unknown input ?
- Performance guarantees ?

## Debugging

- *Where is the #!¤$ bug ?*

”Program testing can be used to show the presence of bugs, but never to show their absence” – Edsger W. Dijkstra

# Built-in exceptions (class hierarchy)

```
BaseException
 +-- SystemExit
 +-- KeyboardInterrupt
 +-- GeneratorExit
 +-- Exception
      +-- StopIteration
      +-- StopAsyncIteration
      +-- ArithmeticError
      |    +-- FloatingPointError
      |    +-- OverflowError
      |    +-- ZeroDivisionError
      +-- AssertionError
      +-- AttributeError
      +-- BufferError
      +-- EOFError
      +-- ImportError
      |    +-- ModuleNotFoundError
      +-- LookupError
      |    +-- IndexError
      |    +-- KeyError
      +-- MemoryError
      +-- NameError
      |    +-- UnboundLocalError
      +-- TypeError
      +-- ValueError
      |    +-- UnicodeError
      |         +-- UnicodeDecodeError
      |         +-- UnicodeEncodeError
      |         +-- UnicodeTranslateError
      |
      +-- OSError
      |    +-- BlockingIOError
      |    +-- ChildProcessError
      |    +-- ConnectionError
      |    |    +-- BrokenPipeError
      |    |    +-- ConnectionAbortedError
      |    |    +-- ConnectionRefusedError
      |    |    +-- ConnectionResetError
      |    +-- FileExistsError
      |    +-- FileNotFoundError
      |    +-- InterruptedError
      |    +-- IsADirectoryError
      |    +-- NotADirectoryError
      |    +-- PermissionError
      |    +-- ProcessLookupError
      |    +-- TimeoutError
      +-- ReferenceError
      +-- RuntimeError
      |    +-- NotImplementedError
      |    +-- RecursionError
      +-- SyntaxError
      |    +-- IndentationError
      |         +-- TabError
      +-- SystemError
      +-- Warning
           +-- DeprecationWarning
           +-- PendingDeprecationWarning
           +-- RuntimeWarning
           +-- SyntaxWarning
           +-- UserWarning
           +-- FutureWarning
           +-- ImportWarning
           +-- UnicodeWarning
           +-- BytesWarning
           +-- ResourceWarning
```

docs.python.org/3/library/exceptions.html

# Testing for unexpected behaviour ?

**infinite-recursion1.py**

```
def f(depth):
    f(depth + 1)   # infinite recursion

f(0)
```

**Python shell**

```
| RecursionError: maximum recursion depth exceeded
```

**infinite-recursion2.py**

```
def f(depth):
    if depth > 100:
        print("runaway recursion???")
        raise SystemExit   # raise built-in exception
    f(depth + 1)

f(0)
```

**Python shell**

```
| runaway recursion???
```

**infinite-recursion3.py**

```
import sys

def f(depth):
    if depth > 100:
        print("runaway recursion???")
        sys.exit()   # system function
    f(depth + 1)

f(0)
```

raises SystemExit

**Python shell**

```
| runaway recursion???
```

- let the program eventually fail
- check and raise exceptions
- check and call `sys.exit`

# Catching unexpected behaviour – `assert`

**infinite-recursion4.py**

```python
def f(depth):
    assert depth <= 100  # raise exception if False
    f(depth + 1)

f(0)
```

**Python shell**

```
| File "...\infinite-recursion4.py", line 2, in f
|     assert depth <= 100
| AssertionError
```

**infinite-recursion5.py**

```python
def f(depth):
    assert depth <= 100, "runaway recursion???"
    f(depth + 1)

f(0)
```

**Python shell**

```
| File "...\infinite-recursion5.py", line 2, in f
|     assert depth <= 100, "runaway recursion???"
| AssertionError: runaway recursion???
```

- keyword `assert` checks if boolean expression is true, if not, raises exception AssertionError
- optional second parameter passed to the constructor of the exception

**infinite-recursion6.py**

```python
def f(depth):
    if not depth <= 100:
        raise AssertionError("runaway recursion???")
    f(depth + 1)

f(0)
```

**Python shell**

```
| File "...\infinite-recursion6.py", line 3, in f
|     raise AssertionError("runaway recursion???")
| AssertionError: runaway recursion???
```

# Disabling `assert` statements

Command Prompt

```
C:\Users\au121\Desktop>python -O infinite-recursion5.py
Traceback (most recent call last):
  File "infinite-recursion5.py", line 5, in <module>
    f(0)
  File "infinite-recursion5.py", line 3, in f
    f(depth + 1)
  File "infinite-recursion5.py", line 3, in f
    f(depth + 1)
  File "infinite-recursion5.py", line 3, in f
    f(depth + 1)
  [Previous line repeated 995 more times]
RecursionError: maximum recursion depth exceeded

C:\Users\au121\Desktop>
```

- `assert` statements are good to help check correctness of program – but can slow down program
- invoking Python with option `-O` disables all assertions (by setting `__debug__` to `False`)

docs.python.org/3/reference/simple_stmts.html#assert

Example

$$\left\lfloor \sqrt{x} \right\rfloor$$

# First try... (seriously, the bugs were not on purpose)

**intsqrt_buggy.py**

```python
def int_sqrt(x):
    low = 0
    high = x
    while low < high - 1:
        mid = (low + high) / 2
        if mid ** 2 <= x:
            low = mid
        else:
            high = mid
    return low
```

**Python shell**

```
> int_sqrt(10)
| 3.125  # 3.125 ** 2 = 9.765625
> int_sqrt(-10)
| 0  # what should the answer be ?
```

# Let us add a specification...

**intsqrt.py**

```
def int_sqrt(x):
    """Compute the integer square root of an integer x.

    Assumes that x is an integer and x >= 0
    Returns integer floor(sqrt(x))"""

    ...
```

docstring

input requirements

output guarantees

**Python shell**

```
> help(int_sqrt)
| Help on function int_sqrt in module __main__:
|
| int_sqrt(x)
|     Compute the integer square root of an integer x.
|     Assumes that x is an integer and x >= 0
|     Returns integer floor(sqrt(x))
```

- all methods, classes, and modules can have a **`docstring`** (ideally have) as a **specification**
- for methods: summarize purpose in first line, followed by input requirements and ouput guarantees
- the docstring is assigned to the object's `__doc__` attribute

**PEP 257 -- Docstring Conventions**
www.python.org/dev/peps/pep-0257/

# Let us check input requirements...

intsqrt.py

```
def int_sqrt(x):
    """Compute the integer square root of an integer x.

    Assumes that x is an integer and x >= 0
    Returns integer floor(sqrt(x))"""

    assert isinstance(x, int)    check input
    assert 0 <= x                requirements
    ...
```

Python shell

```
> int_sqrt(-10)
|  File "...\int_sqrt.py", line 7, in int_sqrt
|     assert 0 <= x
| AssertionError
```

- doing explicit checks for valid input arguments is part of **defensive programming** and helps spotting errors early

  (instead of continuing using likely wrong values... resulting in a final meaningless error)

# Let us check if output correct...

**intsqrt.py**

```
def int_sqrt(x):
    """Compute the integer square root of an integer x.

    Assumes that x is an integer and x >= 0
    Returns integer floor(sqrt(x))"""

    assert isinstance(x, int)
    assert 0 <= x
    ...
    assert isinstance(result, int)              } check
    assert result ** 2 <= x < (result + 1) ** 2 } output
    return result
```

**Python shell**

```
> int_sqrt(10)
|   File "...\int_sqrt.py", line 20, in int_sqrt
|     assert isinstance(result, int)
| AssertionError
```

- output check identifies the error

  `mid = (low + high) / 2`

- should have been

  `mid = (low + high) // 2`

- The output check helps us to ensure that function specifications are satisfied in applications

# Let us test some input values...

**intsqrt.py**

```
def int_sqrt(x):
    ...

assert int_sqrt(0) == 0
assert int_sqrt(1) == 1
assert int_sqrt(2) == 1
assert int_sqrt(3) == 1
assert int_sqrt(4) == 2
assert int_sqrt(5) == 2
assert int_sqrt(200) == 14
```

**Python shell**

```
| Traceback (most recent call last):
|   File "...\int_sqrt.py", line 28, in <module>
|     assert int_sqrt(1) == 1
|   File "...\int_sqrt.py", line 21, in int_sqrt
|     assert result ** 2 <= x < (result + 1) ** 2
| AssertionError
```

- test identifies wrong output for x = 1

# Let us check progress of algorithm...

**intsqrt.py**

```
...
low, high = 0, x
while low < high - 1:  # low <= floor(sqrt(x)) < high
    assert low ** 2 <= x < high ** 2   } check invariant
    mid = (low + high) // 2            } for loop
    if mid ** 2 <= x:
        low = mid
    else:
        high = mid
result = low
...
```

**Python shell**

```
| Traceback (most recent call last):
|   File "...\int_sqrt.py", line 28, in <module>
|     assert int_sqrt(1) == 1
|   File "...\int_sqrt.py", line 21, in int_sqrt
|     assert result ** 2 <= x < (result + 1) ** 2
| AssertionError
```

- test identifies
  wrong output for x = 1
- but invariant apparently correct ???
- problem

  `low == result == 0`
  `high == 1`

  implies loop never entered
- output check identifies the error

  `high = x`
- should have been

  `high = x + 1`

# Final program

**We have used assertions to:**

- Test if input arguments / usage is valid (defensive programming)
- Test if computed result is correct
- Test if an internal invariant in the computation is satisfied
- Perform a final test for a set of test cases (should be run whenever we change anything in the implementation)

intsqrt.py

```python
def int_sqrt(x):
    """Compute the integer square root of an integer x.

    Assumes that x is an integer and x >= 0
    Returns the integer floor(sqrt(x))"""

    assert isinstance(x, int)
    assert 0 <= x

    low, high = 0, x + 1
    while low < high - 1:  # low <= floor(sqrt(x)) < high
        assert low ** 2 <= x < high ** 2
        mid = (low + high) // 2
        if mid ** 2 <= x:
            low = mid
        else:
            high = mid
    result = low

    assert isinstance(result, int)
    assert result ** 2 <= x < (result + 1) ** 2

    return result

assert int_sqrt(0) == 0
assert int_sqrt(1) == 1
assert int_sqrt(2) == 1
assert int_sqrt(3) == 1
assert int_sqrt(4) == 2
assert int_sqrt(5) == 2
assert int_sqrt(200) == 14
```

# Which checks would you add to the below code?

binary-search.py

```
def binary_search(x, L):
    """Binary search for x in sorted list L

    Assumes x is an integer, and L a non-decreasing list of integers

    Returns index i, -1 <= i < len(L), where L[i] <= x < L[i+1],
    assuming L[-1] = -infty and L[len(L)] = +infty"""

    low, high = -1, len(L)
    while low + 1 < high:
        mid = (low + high) // 2
        if x < L[mid]:
            high = mid
        else:
            low = mid
    result = low

    return result
```

**binary-search-assertions.py**

```python
def binary_search(x, L):
    """Binary search for x in sorted list L

    Assumes x is an integer, and L a non-decreasing list of integers

    Returns index i, -1 <= i < len(L), where L[i] <= x < L[i+1],
    assuming L[-1] = -infty and L[len(L)] = +infty"""
    assert isinstance(x, int)
    assert isinstance(L, list)
    assert all([isinstance(e, int) for e in L])
    assert all([L[i] <= L[i + 1] for i in range(len(L) - 1)])
    low, high = -1, len(L)
    while low + 1 < high:  # L[low] <= x < L[high]
        assert (low == -1 or L[low] <= x) and (high == len(L) or x < L[high])
        mid = (low + high) // 2
        assert isinstance(L[mid], int)
        assert (low == -1 or L[low] <= L[mid]) and (high == len(L) or L[mid] <= L[high])
        if x < L[mid]:
            high = mid
        else:
            low = mid
    result = low
    assert (isinstance(result, int) and -1 <= result < len(L) and
            ((result == -1 and (len(L) == 0 or x < L[0])) or
             (result == len(L) - 1 and x >= L[-1]) or
             (0 <= result < len(L) - 1 and L[result] <= x < L[result + 1])))
    return result

assert binary_search(42, []) == -1
assert binary_search(42, [7]) == 0
assert binary_search(7, [42]) == -1
assert binary_search(7, [42,42,42]) == -1
assert binary_search(42, [7,7,7]) == 2
assert binary_search(42, [7,7,7,56,81]) == 2
assert binary_search(8, [1,3,5,7,9]) == 3
```

input

loop

input

output

① inefficient

②

test cases

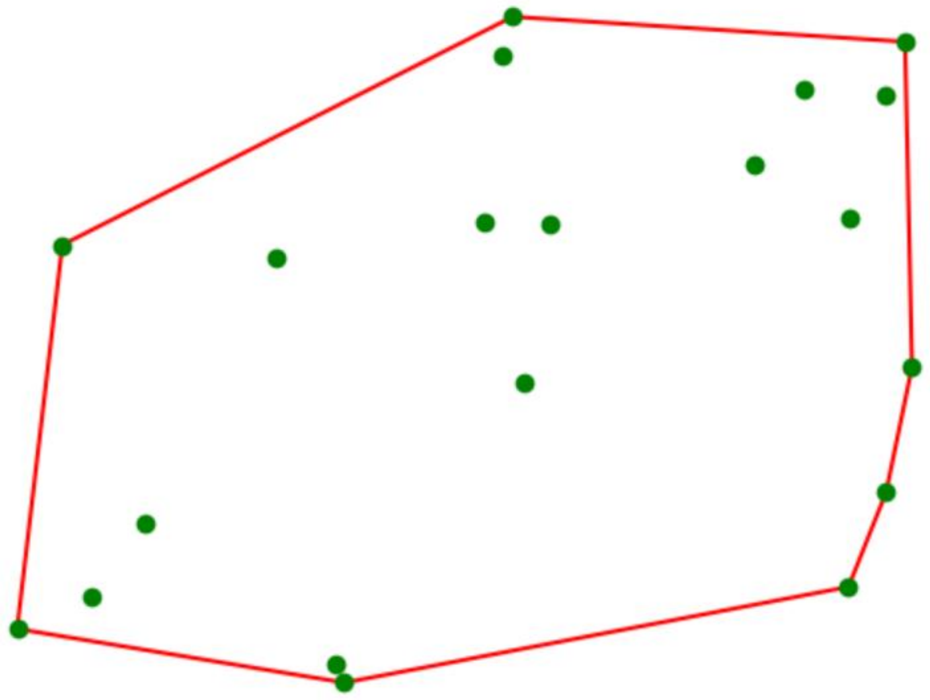
① Verifying if L is a sorted list of integers can slow down the program significantly

② Alternative is to only verify if the part of L visited is a sorted subsequence

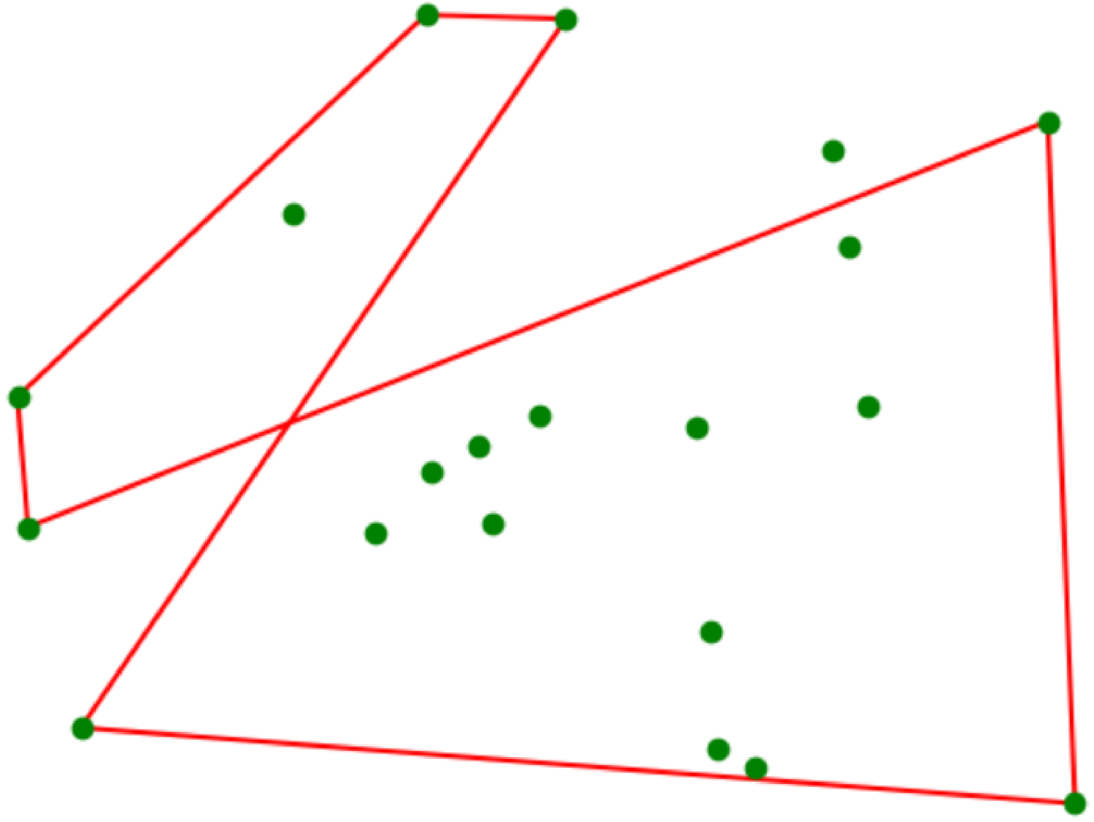
# Testing – how ?

- Run set of test cases
  - test all cases in input/output specification **(black box testing)**
  - test all special cases **(black box testing)**
  - set of tests should force all lines of code to be tested **(glass box testing)**
- Visual test
- Automatic testing
  - Systematically / randomly generate input instances
  - Create function to **validate** if output is correct
    (hopefully easier than finding the solution)
- Formal verification
  - Use computer programs to do formal proofs of correctness, like using Coq

# Visual testing – Convex hull computation



**Correct**



**Bug !**
**(not convex)**

# doctest

- Python module
- Test instances (pairs of input and corresponding output) are written in the doc strings, formatted as in an interactive Python session

**binary-search-doctest.py**

```python
def binary_search(x, L):
    """Binary search for x in sorted list L

    Examples:
    >>> binary_search(42, [])
    -1
    >>> binary_search(42, [7])
    0
    >>> binary_search(42, [7,7,7,56,81])
    2
    >>> binary_search(8, [1,3,5,7,9])
    3
    """

    low, high = -1, len(L)
    while low + 1 < high:
        mid = (low + high) // 2
        if x < L[mid]:
            high = mid
        else:
            low = mid
    return low

import doctest
doctest.testmod(verbose=True)
```

**Python shell**

```
| Trying:
|     binary_search(42, [])
| Expecting:
|     -1
| ok
| Trying:
|     binary_search(42, [7])
| Expecting:
|     0
| ok
| Trying:
|     binary_search(42, [7,7,7,56,81])
| Expecting:
|     2
| ok
| Trying:
|     binary_search(8, [1,3,5,7,9])
| Expecting:
|     3
| ok
| 1 items had no tests:
|     __main__
| 1 items passed all tests:
|    4 tests in __main__.binary_search
| 4 tests in 2 items.
| 4 passed and 0 failed.
| Test passed.
```

docs.python.org/3/library/doctest.html

# pytest

- Run all tests stored in functions prefixed by `test` or `test` prefixed test methods inside `Test` prefixed test classes
- `pip install pytest`
- Run the `pytest` program from a shell

pytest.org

**binary-search-pytest.py**

```python
def binary_search(x, L):
    """Binary search for x in sorted list L"""
    low, high = -1, len(L)
    while low + 1 < high:
        mid = (low + high) // 2
        if x < L[mid]:
            high = mid
        else:
            low = mid
    return low

def test_binary_search():
    assert binary_search(42, []) == -1
    assert binary_search(42, [7]) == 0
    assert binary_search(42, [7,7,7,56,81]) == 2
    assert binary_search(8, [1,3,5,7,9]) == 3

import pytest

def test_types():
    with pytest.raises(TypeError):
        _ = binary_search(5, ['a', 'b', 'c'])
```

**Shell**

```
> pytest binary-search-pytest.py
| ============= test session starts =============
| platform win32 -- Python 3.7.2, pytest-4.3.1, ... 0.9.0
| collected 2 items
| binary-search-pytest.py .. [100%]
| ============= 2 passed in 0.06 seconds =============
```

# unittest

- Python module
- A comprehensive object-oriented test framework, inspired by the corresponding JUnit test framework for Java

binary-search-unittest.py

```python
def binary_search(x, L):
    """Binary search for x in sorted list L"""

    low, high = -1, len(L)
    while low + 1 < high:
        mid = (low + high) // 2
        if x < L[mid]:
            high = mid
        else:
            low = mid
    return low

import unittest

class TestBinarySearch(unittest.TestCase):
    def test_search(self):
        self.assertEqual(binary_search(42, []), -1)
        self.assertEqual(binary_search(42, [7]), 0)
        self.assertEqual(binary_search(42, [7,7,7,56,81]), 2)
        self.assertEqual(binary_search(8, [1,3,5,7,9]), 3)

    def test_types(self):
        self.assertRaises(TypeError, binary_search, 5, ['a', 'b', 'c'])

unittest.main(verbosity=2)
```

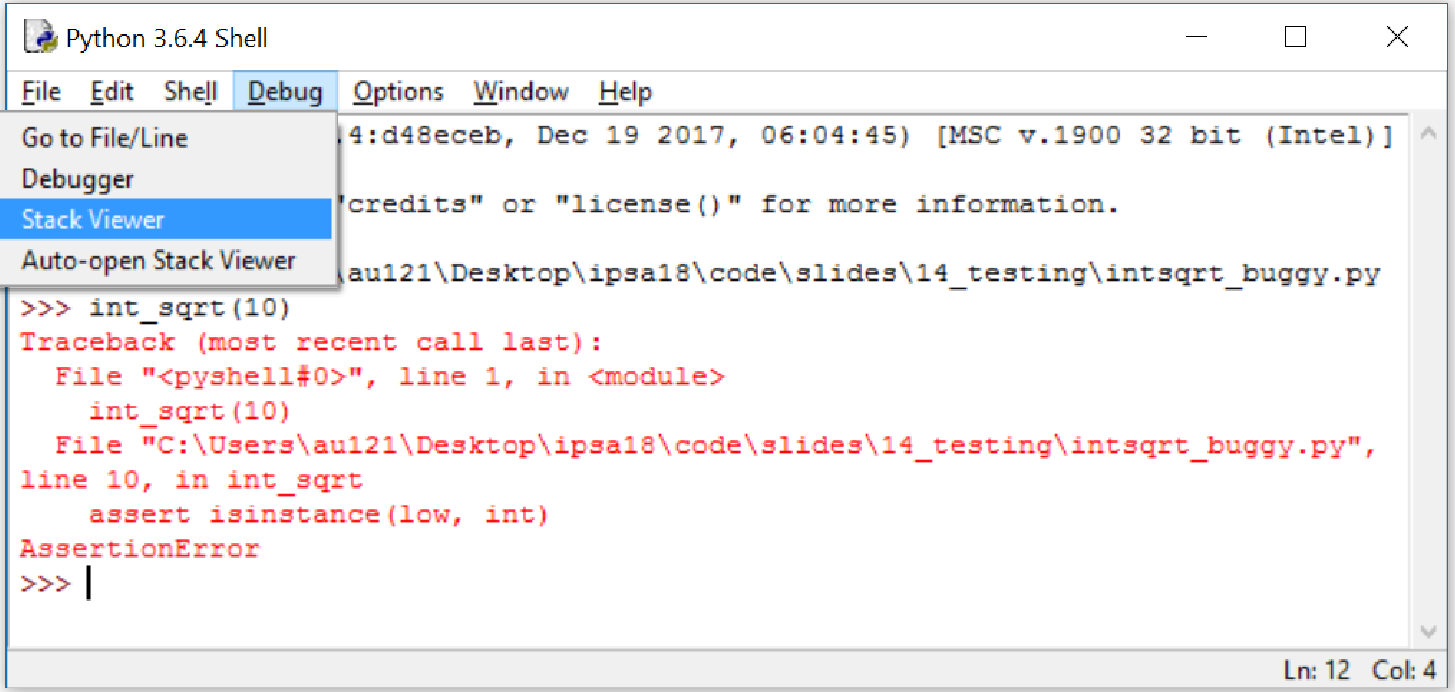
Python shell

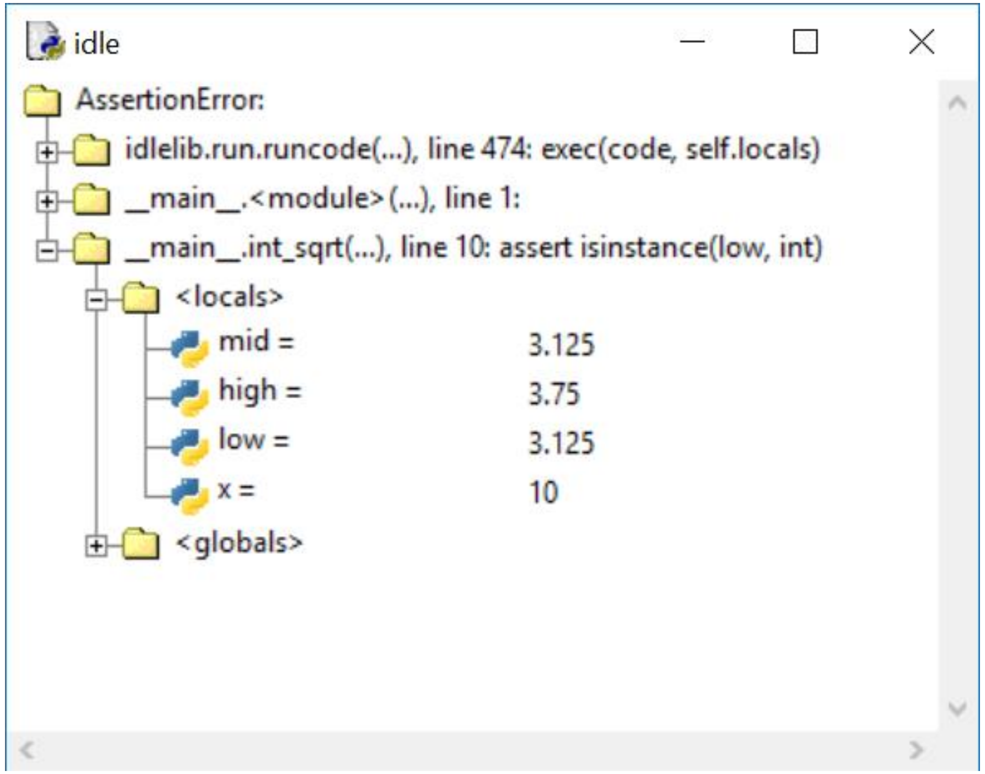
```
| test_search (__main__.TestBinarySearch) ... ok
| test_types (__main__.TestBinarySearch) ... ok
| ----------------------------------------------------------------------
| Ran 2 tests in 0.051s
| OK
```

docs.python.org/3/library/unittest.html

# Debugger (IDLE)

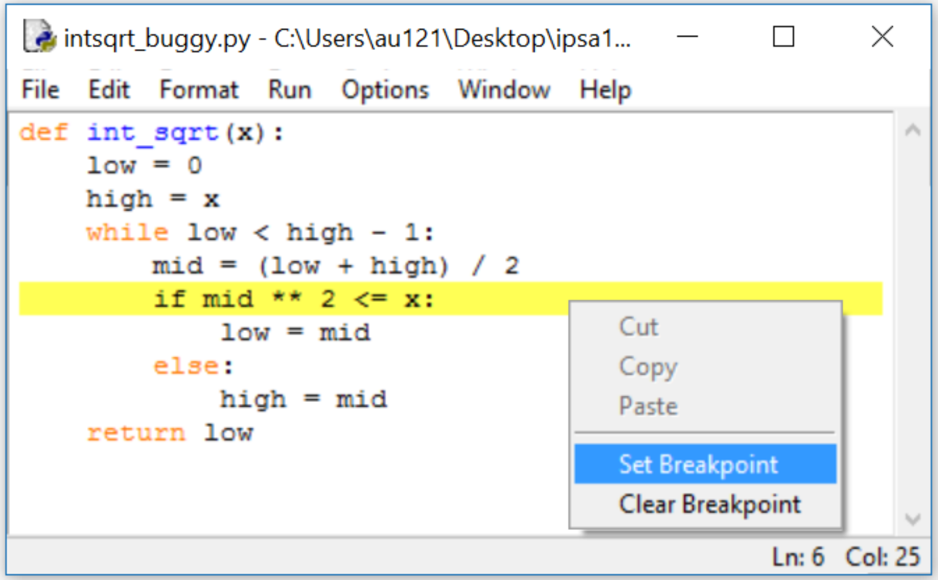
- When an exception has stopped the program, you can examine the state of the variables using **Debug > Stack Viewer** in the Python shell

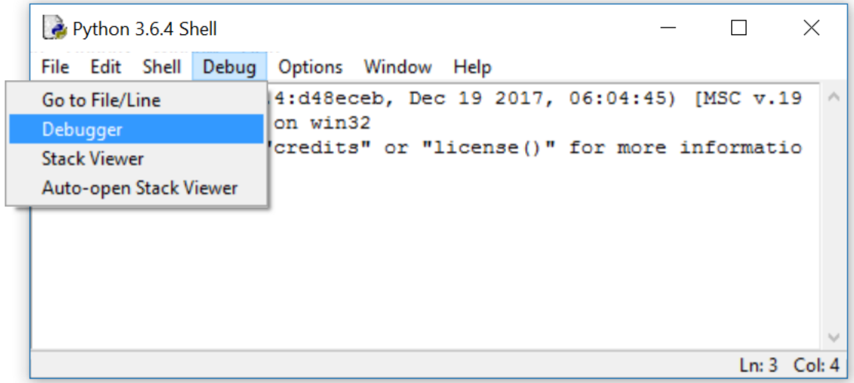


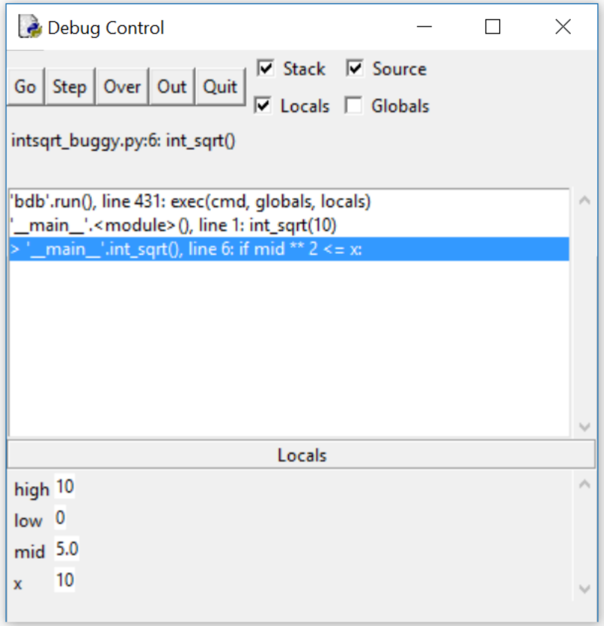



# Stepping through a program (IDLE debugger)

- **Debug > Debugger** in the Python shell opens Debug Control window
- **Right click** on a code line in editor to set a "breakpoint" in your code
- **Debug Control:** Go → run until next breakpoint is encountered; Step → execute one line of code; Over → run function call without details; Out → finish current function call; Quit → Stop program;



# Coverage

- Ensure that your tests cover the whole code and all possible branches are taken
- The module `coverage` can monitor running your code and report which lines and branches were not executed
- `pip install coverage`
- **Note** 100% coverage does not guarantee that there are no errors... just fewer

**goldbach.py**

```python
 1 def odd(x):
 2   return x % 2 == 1
 3 def sum_of_three_primes(n):
 4   assert odd(n) and n > 5
 5   primes = (set(range(2, n + 1)) -
 6             set(x for f in range(2, n + 1)
 7                   for x in range(2 * f, n + 1, f)))
 8   for x in primes:
 9     for y in primes:
10       for z in primes:
11         if n == x + y + z:
12           print(n, 'is the sum of three primes', x, y, z)
13           return
14   print(n, 'is not the sum of three primes')
15 for n in range(7, 1000, 2):
16   sum_of_three_primes(n)
```

**Shell**

```
> coverage run --branch goldbach.py
| 7 is the sum of three primes 2 2 3
| 9 is the sum of three primes 2 2 5
| ...
| 999 is the sum of three primes 3 5 991
> coverage report -m goldbach.py
| Name          Stmts   Miss Branch BrPart  Cover   Missing
| ---------------------------------------------------------
| goldbach.py      14      1     12      1    92%   14
| ---------------------------------------------------------
| TOTAL            14      1     12      1    92%
```

pypi.org/project/coverage

en.wikipedia.org/wiki/Goldbach's_weak_conjecture

# coverage html

Coverage for **goldbach.py**: 92%

14 statements | 13 run | 1 missing | 0 excluded | 1 partial

« prev   ^ index   » next   *coverage.py v6.5.0, created at 2022-10-05 17:23 +0200*

```
1  def odd(x):
2      return x % 2 == 1
3  def sum_of_three_primes(n):
4      assert odd(n) and n > 5
5      primes = (set(range(2, n + 1)) -
6                set(x for f in range(2, n + 1)
7                      for x in range(2 * f, n + 1, f)))
8      for x in primes:                                          8 ↛ 14
9          for y in primes:
10             for z in primes:
11                 if n == x + y + z:
12                     print(n, 'is the sum of three primes', x, y, z)
13                     return
14     print(n, 'is not the sum of three primes')
15 for n in range(7, 1001, 2):
16     sum_of_three_primes(n)
```

line 8 didn't jump to line 14, because the loop on line 8 didn't complete

# Concluding remarks

- Simple debugging: add print statements
- **Test driven development** → Strategy for code development, where tests are written before the code
- **Defensive programming** → add tests (assertions) to check if input/arguments are valid according to specification
- When designing tests, ensure **coverage** (the set of test cases should make sure all code lines get executed)
- **Python testing frameworks: doctest, unittest, pytest, ...**

# Mypy – a static type checker for Python

Experimental ⚠

- **Static type checking** tries to analyze a program for potential type errors without executing the program
- Installing:

```
pip install mypy
```

- Running Python will cause an error during execution, whereas using mypy the error will be found without executing the program
- Standard (and required) in statically typed languages like Java, C, C++

**mypy-simple.py**

```python
print("start")
print(42 + "abc")   # error
print("end")
```

**Shell**

```
> python mypy-simple.py
| start
| TypeError: unsupported operand type(s)
  for +: 'int' and 'str'
> mypy mypy-simple.py
| mypy-simple.py:2: error: Unsupported
  operand types for + ("int" and "str")
```

mypy-lang.org

PEP 484 - Type Hints

# Type hints (PEP 484)

- Python allows type hints in programs
- They are ignored at run-time by Python, but useful for static type analysis (mypy)
- Syntax

  *variable : type*

  *variable : type = value*

**mypy-basic-types.py**

```
x : int   # type hint
x = 42
x = "abc"   # type error
y : int = 42   # type hint
y = "abc"   # type error
z = 42
z = "abc"   # type changed from int to str
print(x, y, z)
```

**Shell**

```
> python mypy-basic-types.py
| abc abc abc
> mypy mypy-basic-types.py
| mypy-basic-types.py:3: error: Incompatible
  types in assignment (expression has type
  "str", variable has type "int")
| mypy-basic-types.py:5: error: ...
| mypy-basic-types.py:7: error: ...
```

# Type hints – functions

`def name(variable : type, ...) -> return type:`

| mypy-function.py | Shell |
|---|---|
| <pre>def f(x: int, units: str) -> str:<br>    return str(x) + " " + units<br><br>def g(x, units: str) -> str:<br>    return str(x) + " " + units<br><br>print(f(3, "cm"))<br>print(f("one", "meter"))<br><br>print(g(3, "cm"))<br>print(g("one", "meter"))<br><br>print(f.__annotations__)</pre> | <pre>> python mypy-function.py<br>\| 3 cm<br>\| one meter<br>\| 3 cm<br>\| one meter<br>\| {'x': <class 'int'>, 'units': <class 'str'>,<br>  'return': <class 'str'>}<br>> mypy mypy-function.py<br>\| mypy-function.py:8: error: Argument 1 to "f"<br>  has incompatible type "str"; expected "int"</pre> |

- Note: for functions and methods *function*`.__annotations__` is a dictionary with the annotation

# Type hints – objects

**mypy-classes.py**

```python
class A:
    pass
class B(A):
    pass
class C:
    pass
a : A
b : B
c : C
```

```python
a = A()
a = B()  # valid, B subclass of A
a = C()  # error
b = A()  # error
b = B()
b = C()  # error
c = A()  # error
c = B()  # error
c = C()
```

**Shell**

```
> mypy mypy-classes.py
| mypy-classes.py:15: error: Incompatible types in assignment (expression has type "C", variable has type "A")
| mypy-classes.py:16: error: Incompatible types in assignment (expression has type "A", variable has type "B")
| mypy-classes.py:18: error: Incompatible types in assignment (expression has type "C", variable has type "B")
| mypy-classes.py:19: error: Incompatible types in assignment (expression has type "A", variable has type "C")
| mypy-classes.py:20: error: Incompatible types in assignment (expression has type "B", variable has type "C")
```

# More type hints... see PEP 484 for even more...

**mypy-typing.py**

```
from typing import Mapping, Set, List, Tuple, Union, Optional

S : Set = {}                                     # error {} dictionary
S2 : Set[int] = {1, 2, "abc"}                    # error "abc" is not int
D : Mapping[int, int] = {1: 42, 'a': 1}          # error 'a' is not int
T : Tuple[int, str] = (42, 7)                    # error 7 is not str
L : List[Union[int, str]] = [42, 'a', None]      # list can only contain int and str
L2 : List[Optional[str]] = ['abc', None, 42]     # list can only contain str og None
```

**Shell**

```
> mypy mypy-typing.py
| mypy-typing.py:3: error: Incompatible types in assignment (expression has type "Dict[<nothing>,
  <nothing>]", variable has type "Set[Any]")
| mypy-typing.py:4: error: Argument 3 to <set> has incompatible type "str"; expected "int"
| mypy-typing.py:5: error: Dict entry 1 has incompatible type "str": "int"; expected "int": "int"
| mypy-typing.py:6: error: Incompatible types in assignment (expression has type "Tuple[int, int]",
  variable has type "Tuple[int, str]")
| mypy-typing.py:7: error: List item 2 has incompatible type "None"; expected "Union[int, str]"
| mypy-typing.py:8: error: List item 2 has incompatible type "int"; expected "Optional[str]"
```

PEP 484 - Type Hints

# … the same in Python 3.10

**mypy-typing-new.py**

```python
# deprecated: from typing import Mapping, Set, List, Tuple, Union, Optional

S : set = {}                                           # error {} dictionary
S2 : set[int] = {1, 2, "abc"}                          # error "abc" is not int
D : dict[int, int] = {1: 42, 'a': 1}                   # error 'a' is not int
T : tuple[int, str] = (42, 7)                          # error 7 is not str
L : list[int | str] = [42, 'a', None]                  # list can only contain int and str
L2 : list[str | None] = ['abc', None, 42]              # list can only contain str og None
```

**Shell**

```
> mypy mypy-typing-new.py
| mypy-typing-new.py:3: error: Incompatible types in assignment (expression has type "Dict[<nothing>,
  <nothing>]", variable has type "Set[Any]")
| mypy-typing-new.py:4: error: Argument 3 to <set> has incompatible type "str"; expected "int"
| mypy-typing-new.py:5: error: Dict entry 1 has incompatible type "str": "int"; expected "int": "int"
| mypy-typing-new.py:6: error: Incompatible types in assignment (expression has type "Tuple[int, int]",
  variable has type "Tuple[int, str]")
| mypy-typing-new.py:7: error: List item 2 has incompatible type "None"; expected "Union[int, str]"
| mypy-typing-new.py:8: error: List item 2 has incompatible type "int"; expected "Optional[str]"
```

PEP 585 - Type Hinting Generics In Standard Collections (Python 3.9)
PEP 604 – Allow writing union types as X | Y (Python 3.10)

# Decorators

- @

www.python.org/dev/peps/pep-0318/

# Course overview

Basic programming
Advanced / specific python
Libraries & applications

| | | |
|---|---|---|
| 1. Introduction to Python | 10. Functions as objects | 19. Linear programming |
| 2. Python basics / if | 11. Object oriented programming | 20. Generators, iterators, with |
| 3. Basic operations | 12. Class hierarchies | 21. Modules and packages |
| 4. Lists / while / for | 13. Exceptions and files | 22. Working with text |
| 5. Tuples / comprehensions | 14. Doc, testing, debugging | 23. Relational data |
| 6. Dictionaries and sets | 15. Decorators | 24. Clustering |
| 7. Functions | 16. Dynamic programming | 25. Graphical user interfaces (GUI) |
| 8. Recursion | 17. Visualization and optimization | 26. Java vs Python |
| 9. Recursion and Iteration | 18. Multi-dimensional data | 27. Final lecture |

**10 handins**
**1 final project (last 1 month)**

# Python decorators are just syntatic sugar

```
Python
@dec2
@dec1
def func(arg1, arg2, ...):
    pass
```

≡

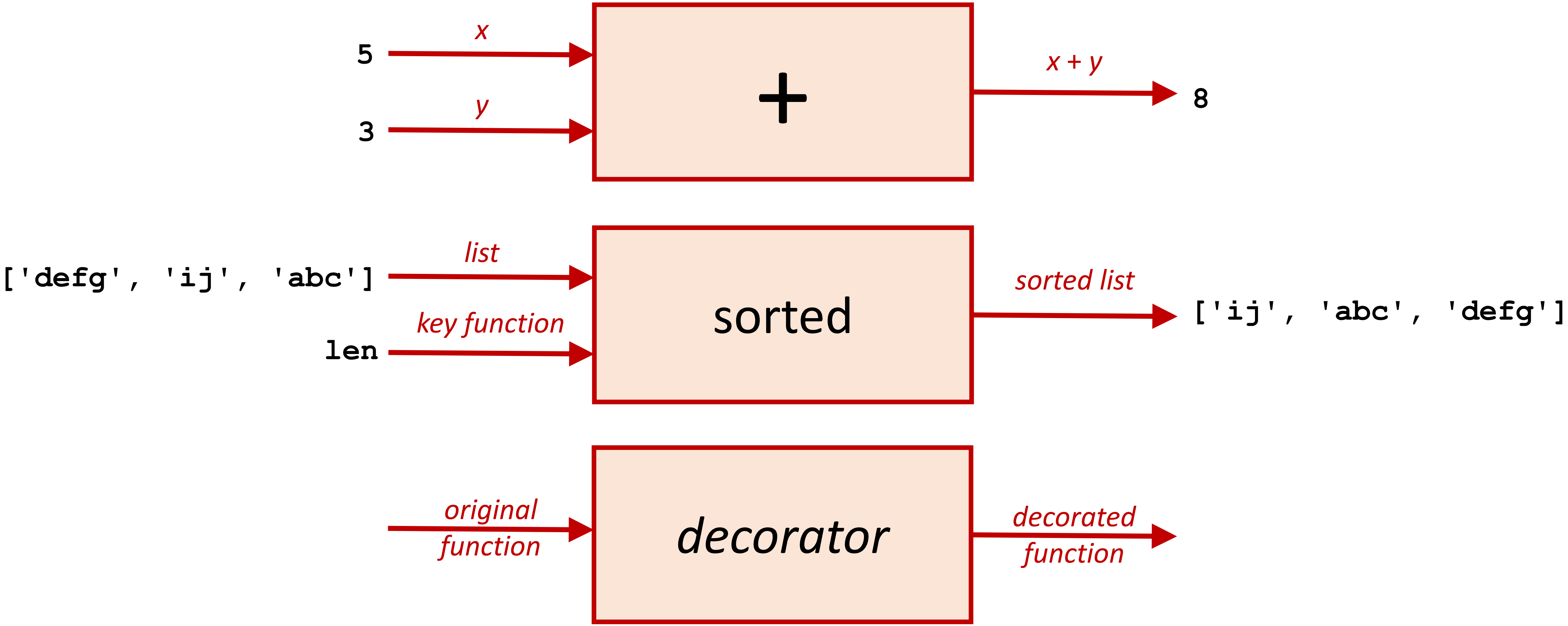
```
Python
def func(arg1, arg2, ...):
    pass

func = dec2(dec1(func))
```

'pie-decorator' syntax

dec1, dec2, ... are functions (decorators) taking a *function as an argument* and *returning a new function*

Note: decorators are listed bottom up in order of execution

# Recap functions



# Contrived example : Plus one (I-II)

**plus_one1.py**

```
def plus_one(x):
    return x + 1

def square(x):
    return x ** 2

def cube(x):
    return x ** 3

print(plus_one(square(5)))
print(plus_one(cube(5)))
```

**Python shell**

```
| 26
| 126
```

Assume we *always* need to call `plus_one` on the result of `square` and `cube` (don't ask why!)

**plus_one2.py**

```
def plus_one(x):
    return x + 1

def square(x):
    return plus_one(x ** 2)

def cube(x):
    return plus_one(x ** 3)

print(square(5))
print(cube(5))
```

**Python shell**

```
| 26
| 126
```

We could call `plus_one` inside functions (but could be more `return` statements in functions)

# Contrived example : Plus one (III-IV)

**plus_one3.py**

```
def plus_one(x):
    return x + 1
def square(x):
    return x ** 2
def cube(x):
    return x ** 3
square_original = square
cube_original = cube
square = lambda x: plus_one(square_original(x))
cube = lambda x: plus_one(cube_original(x))
print(square(5))
print(cube(5))
```

**Python shell**

```
| 26
| 126
```

Overwrite `square` and `cube` with decorated versions

**plus_one4.py**

```
def plus_one(x):
    return x + 1
def plus_one_decorator(f):
    return lambda x: plus_one(f(x))
def square(x):
    return x ** 2
def cube(x):
    return x ** 3
square = plus_one_decorator(square)
cube = plus_one_decorator(cube)
print(square(5))
print(cube(5))
```

**Python shell**

```
| 26
| 126
```

Create a decorator function `plus_one_decorator`

# Contrived example : Plus one (V-VI)

**plus_one5.py**

```python
def plus_one(x):
    return x + 1

def plus_one_decorator(f):
    return lambda x: plus_one(f(x))

@plus_one_decorator
def square(x):
    return x ** 2

@plus_one_decorator
def cube(x):
    return x ** 3

print(square(5))
print(cube(5))
```

**Python shell**

```
| 26
| 126
```

Use Python decorator syntax

**plus_one6.py**

```python
def plus_one_decorator(f):
    def plus_one(x):
        return f(x) + 1

    return plus_one

@plus_one_decorator
def square(x):
    return x ** 2

@plus_one_decorator
def cube(x):
    return x ** 3

print(square(5))
print(cube(5))
```

**Python shell**

```
| 26
| 126
```

Create local function instead of using `lambda`

# Contrived example : Plus one (VII)

plus_one7.py

```
def plus_one_decorator(f):
    def plus_one(x):
        return f(x) + 1
    return plus_one

@plus_one_decorator
@plus_one_decorator
def square(x):
    return x ** 2

@plus_one_decorator
@plus_one_decorator
@plus_one_decorator
def cube(x):
    return x ** 3

print(square(5))
print(cube(5))
```

Python shell

```
| 27
| 128
```

- A function can have an arbitrary number of decorators (also the same repeated)
- Decorators are listed bottom up in order of execution

# Handling arguments

**run_twice1.py**

```python
def run_twice(f):
    def wrapper():
        f()
        f()
    return wrapper
@run_twice
def hello_world():
    print("Hello world")
hello_world()
```

**Python shell**

```
| Hello world
| Hello world
```

”`wrapper`” is a common name for the function returned by a decorator

**run_twice2.py**

```python
def run_twice(f):
    def wrapper(*args):
        f(*args)
        f(*args)
    return wrapper
@run_twice
def hello_world():
    print("Hello world")
@run_twice
def hello(txt):
    print("Hello", txt)
hello_world()
hello("Mars")
```

**Python shell**

```
| Hello world
| Hello world
| Hello Mars
| Hello Mars
```

`args` holds the arguments in a tuple given to the function to be decorated

# Question – What does the decorated program print ?

**decorator_quizz.py**

```python
def double(f):
    def wrapper(*args):
        return 2 * f(*args)
    return wrapper

def add_three(f):
    def wrapper(*args):
        return 3 + f(*args)
    return wrapper

@double
@add_three
def seven():
    return 7

print(seven())
```

- 7
- 10
- 14
- 17
- 20
- Don’t know

# Example: Enforcing argument types

- Defining decorators can be (slightly) complicated
- Using decorators is easy

integer_sum1.py

```python
def integer_sum(*args):
    assert all([isinstance(x, int) for x in args]),\
           "all arguments most be int"
    return sum(args)
```

Python shell

```
> integer_sum(1, 2, 3, 4)
| 10
> integer_sum(1, 2, 3.2, 4)
| AssertionError: all arguments most be int
```

integer_sum2.py

```python
def enforce_integer(f):    # decorator function
    def wrapper(*args):
        assert all([isinstance(x, int) for x in args]),\
               "all arguments most be int"
        return f(*args)
    return wrapper

@enforce_integer
def integer_sum(*args):
    return sum(args)
```

Python shell

```
> integer_sum(1, 2, 3, 4)
| 10
> integer_sum(1, 2, 3.2, 4)
| AssertionError: all arguments most be int
```

# Decorators can take arguments

Python

```python
@dec(argA, argB, ...)
def func(arg1, arg2, ...):
    pass
```

≡

Python

```python
def func(arg1, arg2, ...):
    pass
func = dec(argA, argB, ...)(func)
```

`dec` is a function (decorator) that takes a *list of arguments* and *returns a function* (to decorate `func`) that takes a *function as an argument* and *returns a new function*

# Example: Generic type enforcing

print_repeated.py

```python
def enforce_types(*decorator_args):
    def decorator(f):
        def wrapper(*args):
            assert len(args) == len(decorator_args),\
                   ("got %s arguments, expected %s" % (len(args), len(decorator_args)))
            assert all([isinstance(x, t) for x, t in zip(args, decorator_args)]),\
                   "unexpected types"
            return f(*args)
        return wrapper
    return decorator

@enforce_types(str, int)  # decorator with arguments
def print_repeated(txt, n):
    print(txt * n)

print_repeated("Hello ", 3)
print_repeated("Hello ", "world")
```

Python

```python
@dec(argA, argB, ...)
def func(arg1, arg2, ...):
    pass
```

≡

Python

```python
def func(arg1, arg2, ...):
    pass
func = dec(argA, argB, ...)(func)
```

Python shell

```
| Hello Hello Hello
| AssertionError: unexpected types
```

# Example: A timer decorator

**time_it.py**

```
import time

def time_it(f):
    def wrapper(*args, **kwargs):
        t_start = time.time()
        result = f(*args, **kwargs)
        t_end = time.time()
        t = t_end - t_start
        print("%s took %.2f sec" % (f.__name__, t))
        return result

    return wrapper

@time_it
def slow_function(n):
    sum_ = 0
    for x in range(n):
        sum_ += x
    print("The sum is:", sum_)

for i in range(6):
    slow_function(1_000_000 * 2 ** i)
```

**Python shell**

```
| The sum is: 499999500000
| slow_function took 0.27 sec
| The sum is: 1999999000000
| slow_function took 0.23 sec
| The sum is: 7999998000000
| slow_function took 0.41 sec
| The sum is: 31999996000000
| slow_function took 0.81 sec
| The sum is: 127999992000000
| slow_function took 1.52 sec
| The sum is: 511999984000000
| slow_function took 3.12 sec
```

# Built-in `@property`

- decorator specific for class methods
- allows accessing `x.attribute()` as `x.attribute`, convenient if `attribute` does not take any arguments (also readonly)

**rectangle1.py**

```
class Rectangle:
    def __init__(self, width, height):
        self.width = width
        self.height = height

#   @property
    def area(self):
        return self.width * self.height
```

**Python shell**

```
> r = Rectangle(3, 4)
> print(r.area())
| 12
```

**rectangle2.py**

```
class Rectangle:
    def __init__(self, width, height):
        self.width = width
        self.height = height

    @property
    def area(self):
        return self.width * self.height
```

**Python shell**

```
> r = Rectangle(3, 4)
> print(r.area)
| 12
```

# Class decorators

Python

```python
@dec2
@dec1
class A:
  pass
```

≡

Python

```python
class A:
  pass

A = dec2(dec1(A))
```

# Module `dataclasses` (Since Python 3.7)

- New (and more configurable) alternative to `namedtuple`

**Python shell**

```
> from dataclasses import dataclass
> @dataclass  # uses a decorator to add methods to the class
  class Person:
      name: str  # uses type annotation to define fields
      appeared: int
      height: str = 'unknown height'  # field with default value
> person = Person('Donald Duck', 1934, '3 feet')
> person
| Person(name='Donald Duck', appeared=1934, height='3 feet')
> person.name
| 'Donald Duck'
> Person('Mickey Mouse', 1928)
| Person(name='Mickey Mouse', appeared=1928, height='unknown height')
```

docs.python.org/3/library/dataclasses.html#module-dataclasses

Raymond Hettinger - Dataclasses: The code generator to end all code generators - PyCon 2018

# @functools.total_ordering (class decorator)

**student.py**

```python
import functools

@functools.total_ordering
class Student():
    def __init__(self,name, student_id):
        self.name = name
        self.id = student_id
    def __eq__(self, other):
        return (self.name == other.name
                and self.id == other.id)
    def __lt__(self, other):
        my_name = ', '.join(reversed(self.name.split()))
        other_name = ', '.join(reversed(other.name.split()))
        return (my_name < other_name
                or (my_name == other_name and self.id < other.id))
donald = Student('Donald Duck', 7)
gladstone = Student('Gladstone Gander', 42)
grandma = Student('Grandma Duck', 1)
```

Automatically creates <, <=, >, >= if at least one of the functions is implemented and == is implemented

**Python shell**

```
> donald < grandma
| True
> grandma >= gladstone
| False
> grandma <= gladstone
| True
> donald > gladstone
| False
```

**class_decorator.py**

```python
def add_lessequal(cls):
    '''Class decorator to add __le__ given __eq__ and __lt__.'''
    cls.__le__ = lambda self, other : self == other or self < other
    return cls  # the original class cls with attribute __le__ added

def add_lessequal(cls):  # alternative
    class sub_cls(cls):
        def __le__(self, other):
            return self == other or self < other

    return sub_cls  # new subclass of class cls

@add_lessequal  # Vector = add_lessequal(Vector)
class Vector:
    def __init__(self, x, y):
        self.x = x
        self.y = y

    def _length_squared(self):
        return self.x ** 2 + self.y ** 2

    def __eq__(self, other):
        # Required, otherwise Vector(1, 2) == Vector(1, 2) is False
        return self._length_squared() == other._length_squared()

    def __lt__(self, other):
        return self._length_squared() < other._length_squared()

#    def __le__(self, other):
#        return self._length_squared() <= other._length_squared()

#    def __le__(self, other):
#       return self == other or self < other
```

**Python shell**

```
> u = Vector(3, 4)
> v = Vector(2, 5)
> u.__eq__(v)
| False
> u.__ne__(v)
| True  # not u.__eq__(v)
> u.__lt__(v)
| True
> u.__gt__(v)
| NotImplemented  # special value
> u.__le__(v)
| True  # added by @add_lessequal
> u.__ge__(v)
| NotImplemented  # special value
> u == v
| False
> u != v
| True
> u < v
| True
> u > v  # v < u
| False
> u <= v
| True
> u >= v  # v <= u
| False
```

https://docs.python.org/3/reference/datamodel.html#basic-customization

# Summary

- *@decorator_name*
- Pyton decorators are just syntatic sugar
- Adds functionality to a function without having to augment each call to the function or each return statement in the function
- There are decorators for functions, class methods, and classes
- There are many decorators in the Python Standard Library
- Decorators are easy to use
- ...and (slightly) harder to write

# Dynamic programming

- memoization
- decorator memoized
- systematic subproblem computation

# Binomial coefficient

```
--(7, 5)
  |--(6, 5)
  |   |--(5, 5)
  |    --(5, 4)
  |       |--(4, 4)
  |        --(4, 3)
  |           |--(3, 3)
  |            --(3, 2)
  |               |--(2, 2)
  |                --(2, 1)
  |                   |--(1, 1)
  |                    --(1, 0)
   --(6, 4)
      |--(5, 4)
      |   |--(4, 4)
      |    --(4, 3)
      |       |--(3, 3)
      |        --(3, 2)
      |           |--(2, 2)
      |            --(2, 1)
      |               |--(1, 1)
      |                --(1, 0)
       --(5, 3)
          |--(4, 3)
          |   |--(3, 3)
          |    --(3, 2)
          |       |--(2, 2)
          |        --(2, 1)
          |           |--(1, 1)
          |            --(1, 0)
           --(4, 2)
              |--(3, 2)
              |   |--(2, 2)
              |    --(2, 1)
              |       |--(1, 1)
              |        --(1, 0)
               --(3, 1)
                  |--(2, 1)
                  |   |--(1, 1)
                  |    --(1, 0)
                   --(2, 0)
```

identical computations

recursion tree for `binomial(7,5)`

$$\binom{n}{k} = \begin{cases} 1 & \text{if } k = 0 \text{ or } k = n \\ \binom{n-1}{k} + \binom{n-1}{k-1} & \text{otherwise} \end{cases}$$

**binomial_recursive.py**

```python
def binomial(n, k):
    if k == 0 or k == n:
        return 1
    return binomial(n - 1, k) + binomial(n - 1, k - 1)
```

# Dynamic Programming
# ≡
# Remember solutions already found (memoization)

- Technique sometimes applicable when running time otherwise becomes exponential
- Only applicable if stuff to be remembered is manageable

# Binomial Coefficient
# Dynamic programming using a dictionary

recursion tree for
`binomial(7,5)`

```
--(7, 5)
  |--(6, 5)
  |  |--(5, 5)
  |   --(5, 4)
  |     |--(4, 4)
  |      --(4, 3)
  |        |--(3, 3)
  |         --(3, 2)
  |           |--(2, 2)
  |            --(2, 1)
  |              |--(1, 1)
  |               --(1, 0)
   --(6, 4)
     |--(5, 4)
      --(5, 3)
        |--(4, 3)
         --(4, 2)
           |--(3, 2)
            --(3, 1)
              |--(2, 1)
               --(2, 0)
```

**binomial_dictionary.py**

```python
answers = {}  # answers[(n, k)] = binomial(n, k)

def binomial(n, k):
    if (n, k) not in answers:
        if k == 0 or k == n:
            answer = 1
        else:
            answer = binomial(n - 1, k) + binomial(n - 1, k - 1)
        answers[(n, k)] = answer
    return answers[(n, k)]
```

**Python shell**

```
> binomial(6, 3)
| 20
> answers
| {(3, 3): 1, (2, 2): 1, (1, 1): 1, (1, 0): 1, (2, 1): 2, (3, 2):
  3, (4, 3): 4, (2, 0): 1, (3, 1): 3, (4, 2): 6, (5, 3): 10, (3,
  0): 1, (4, 1): 4, (5, 2): 10, (6, 3): 20}
```

- Use a dictionary `answers` to store already computed values

reuse value stored in dictionary `answers`

# Question – What is the order of the size of the dictionary **`answers`** after calling **`binomial(n, k)`** ?

**binomial_dictionary.py**

```
answers = {}  # answers[(n, k)] = binomial(n, k)

def binomial(n, k):
    if (n, k) not in answers:
        if k == 0 or k == n:
            answer = 1
        else:
            answer = binomial(n - 1, k) + binomial(n - 1, k - 1)
        answers[(n, k)] = answer
    return answers[(n, k)]
```

a) max(n, k)

b) n + k

c) n * k

d) $n^k$

e) $k^n$

f) Don't know

# Binomial Coefficient
# Dynamic programming using decorator

- Use a decorator (@memoize) that implements the functionality of remembering the results of previous function calls

binomial_decorator.py

```
def memoize(f):
    # answers[args] = f(*args)
    answers = {}

    def wrapper(*args):
        if args not in answers:
            answers[args] = f(*args)
        return answers[args]

    return wrapper
```

```
@memoize
def binomial(n, k):
    if k == 0 or k == n:
        return 1
    else:
        return binomial(n - 1, k) + binomial(n - 1, k - 1)
```

www.python-course.eu/python3_memoization.php

**binomial_decorator_trace.py**

```python
def trace(f):  # decorator to trace recursive calls
    indent = 0

    def wrapper(*args):
        nonlocal indent

        spaces = '|  ' * indent
        arg_str = ', '.join(map(repr, args))
        print(spaces + f'{f.__name__}({arg_str})')
        indent += 1
        result = f(*args)
        indent -= 1
        print(spaces + f'> {result}')
        return result

    return wrapper

def memoize(f):
    answers = {}

    def wrapper(*args):
        if args not in answers:
            answers[args] = f(*args)
        return answers[args]

    wrapper.__name__ = f.__name__ + '_memoize'

    return wrapper

@trace
@memoize
def binomial(n, k):
    if k == 0 or k == n:
        return 1
    return binomial(n - 1, k) + binomial(n-1, k-1)

print(binomial(5, 2))
```

**Python shell (without @memoize)**

```
binomial(5, 2)
|  binomial(4, 2)
|  |  binomial(3, 2)
|  |  |  binomial(2, 2)
|  |  |  > 1
|  |  |  binomial(2, 1)
|  |  |  |  binomial(1, 1)
|  |  |  |  > 1
|  |  |  |  binomial(1, 0)
|  |  |  |  > 1
|  |  |  > 2
|  |  > 3
|  |  binomial(3, 1)
|  |  |  binomial(2, 1)
|  |  |  |  binomial(1, 1)
|  |  |  |  > 1
|  |  |  |  binomial(1, 0)
|  |  |  |  > 1
|  |  |  > 2
|  |  |  binomial(2, 0)
|  |  |  > 1
|  |  > 3
|  > 6
|  binomial(4, 1)
|  |  binomial(3, 1)
|  |  |  binomial(2, 1)
|  |  |  |  binomial(1, 1)
|  |  |  |  > 1
|  |  |  |  binomial(1, 0)
|  |  |  |  > 1
|  |  |  > 2
|  |  |  binomial(2, 0)
|  |  |  > 1
|  |  > 3
|  |  binomial(3, 0)
|  |  > 1
|  > 4
> 10
10
```

saved recursive calls when using memoization

**Python shell (with @memoize)**

```
binomial_memoize(5, 2)
|  binomial_memoize(4, 2)
|  |  binomial_memoize(3, 2)
|  |  |  binomial_memoize(2, 2)
|  |  |  > 1
|  |  |  binomial_memoize(2, 1)
|  |  |  |  binomial_memoize(1, 1)
|  |  |  |  > 1
|  |  |  |  binomial_memoize(1, 0)
|  |  |  |  > 1
|  |  |  > 2
|  |  > 3
|  |  binomial_memoize(3, 1)
|  |  |  binomial_memoize(2, 1)
|  |  |  > 2
|  |  |  binomial_memoize(2, 0)
|  |  |  > 1
|  |  > 3
|  > 6
|  binomial_memoize(4, 1)
|  |  binomial_memoize(3, 1)
|  |  > 3
|  |  binomial_memoize(3, 0)
|  |  > 1
|  > 4
> 10
10
```

without assigning `wrapper.__name__` the name shown would be `wrapper`

# Dynamic programming using cache decorator

**bionomial_cache.py**

```
from functools import cache

@cache
def binomial(n, k):
    if k == 0 or k == n:
        return 1
    else:
        return binomial(n - 1, k) + binomial(n - 1, k - 1)
```

- The decorators `@cache` (since Python 3.9) and `@lru_cache(maxsize=None)` in the standard library `functools` supports the same as the decorator `@memoize`
- By default `@lru_cache` at most remembers (caches) 128 previous function calls, always evicting **L**east **R**ecently **U**sed entries from its dictionary

docs.python.org/3/library/functools.html#functools.lru_cache

# Subset sum using dynamic programming

- In the subset sum problem (Exercise 13.4) we are given a number `x` and a list of numbers `L`, and want to determine if a subset of `L` has sum `x`

```
L = [3, 7, 2, 11, 13, 4, 8]    x = 22 = 7 + 11 + 4
```

- Let S(v, k) denote if it is possible to achieve value v with a subset of `L`[:k], i.e. S(v, k) = True if and only if a subset of the first k values in `L` has sum v
- S(v, k) can be computed from the following recurrence:

$$S(v, k) = \begin{cases} \text{True} & \text{if } k = 0 \text{ and } v = 0 \\ \text{False} & \text{if } k = 0 \text{ and } v \neq 0 \\ S(v, k-1) \text{ or } S(v - L[k-1], k-1) & \text{otherwise} \end{cases}$$

# Subset sum using dynamic programming

**subset_sum_dp.py**

```
def subset_sum(x, L):
    @memoize
    def solve(value, k):
        if k == 0:
            return value == 0
        return solve(value, k - 1) or solve(value - L[k - 1], k - 1)
    return solve(x, len(L))
```

**Python shell**

```
> subset_sum(11, [2, 3, 8, 11, -1])
| True
> subset_sum(6, [2, 3, 8, 11, -1])
| False
```

# Question – What is a bound on the size order of the memoization table if all values are possitive integers?

**subset_sum_dp.py**

```
def subset_sum(x, L):
    @memoize
    def solve(value, k):
        if k == 0:
            return value == 0
        return solve(value, k-1) or solve(value - L[k-1], k-1)
    return solve(x, len(L))
```

**Python shell**

```
> subset_sum(11, [2, 3, 8, 11, -1])
| True
> subset_sum(6, [2, 3, 8, 11, -1])
| False
```

a) `len(L)`
b) `sum(L)`
c) `x`
d) $2^{\texttt{len(L)}}$
e) `len(L)`
f) `len(L)*sum(L)`
g) Don't know

# Subset sum using dynamic programming

subset_sum_dp.py

```
def subset_sum_solution(x, L):
    @memoize
    def solve(value, k):
        if k == 0:
            if value == 0:
                return []
            else:
                return None
        solution = solve(value, k - 1)
        if solution != None:
            return solution
        solution = solve(value - L[k - 1], k - 1)
        if solution != None:
            return solution + [L[k - 1]]
        return None

    return solve(x, len(L))
```

Python shell

```
> subset_sum_solution(11, [2, 3, 8, 11, -1])
| [3, 8]
> subset_sum_solution(6, [2, 3, 8, 11, -1])
| None
```

# Knapsack problem

| | Volume | Value |
|---|---|---|
| 0 | 3 | 6 |
| 1 | 3 | 7 |
| 2 | 2 | 8 |
| 3 | 5 | 9 |

- Given a **knapsack** with volume **capacity C**, and set of **objects** with different **volumes and value**.
- **Objective**: Find a subset of the objects that fits in the knapsack (sum of volume ≤ capacity) and has maximal value.
- **Example**: If C = 5 and the volume and weights are given by the table, then the maximal value 15 can be achieved by the 2nd and 3rd object.
- Let V(c, k) denote the maximum value achievable by a subset of the first k objects within capacity c.

$$V(c, k) = \begin{cases} 0 & \text{if } k = 0 \\ V(c, k-1) & \text{volume}[k-1] > c \\ \max\{V(c, k-1), \text{value}[k-1] + V(c - \text{volume}[k-1], k-1)\} & \text{otherwise} \end{cases}$$

# Knapsack – maximum value

knapsack.py

```
def knapsack_value(volume, value, capacity):
    @memoize
    def solve(c, k):  # solve with capacity c and objects 0..k-1
        if k == 0:  # no objects to put in knapsack
            return 0
        v = solve(c, k - 1)  # try without object k-1
        if volume[k - 1] <= c:  # try also with object k-1 if space
            v = max(v, value[k - 1] + solve(c - volume[k - 1], k - 1))
        return v

    return solve(capacity, len(volume))
```

Python shell

```
> volumes = [3, 3, 2, 5]
> values = [6, 7, 8, 9]
> knapsack_value(volumes, values, 5)
| 15
```

# Knapsack – maximum value and objects

knapsack.py

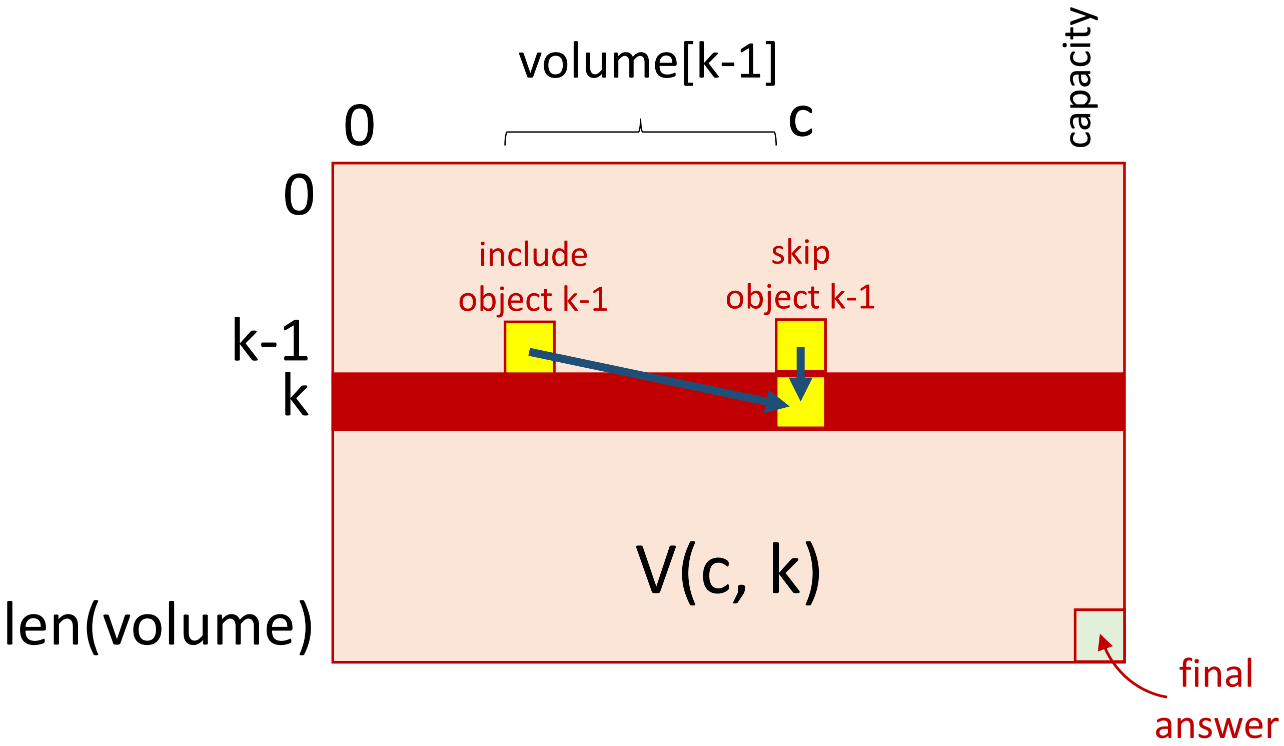
```
def knapsack(volume, value, capacity):
    @memoize
    def solve(c, k):  # solve with capacity c and objects 0..k-1
        if k == 0:  # no objects to put in knapsack
            return 0, []
        v, solution = solve(c, k - 1)  # try without object k-1
        if volume[k - 1] <= c:  # try also with object k-1 if space
            v2, sol2 = solve(c - volume[k - 1], k - 1)
            v2 = v2 + value[k - 1]
            if v2 > v:
                v = v2
                solution = sol2 + [k - 1]
        return v, solution

    return solve(capacity, len(volume))
```

Python shell

```
> volumes = [3, 3, 2, 5]
> values = [6, 7, 8, 9]
> knapsack(volumes, values, 5)
| (15, [1, 2])
```

# Knapsack - Table

$$V(c, k) = \begin{cases} 0 & \text{if } k = 0 \\ V(c, k-1) & \text{value}[k-1] > c \\ \max\{V(c, k-1), \text{value}[k-1] + V(c - \text{volume}[k-1], k-1)\} & \text{otherwise} \end{cases}$$




- systematic fill out table
- only need to remember two rows

# Knapsack – Systematic table fill out

**knapsack_systematic.py**

```
def knapsack(volume, value, capacity):
  ① solutions = [(0, [])] * (capacity + 1)
  ② for obj in range(len(volume)): ⑤
          for c in reversed(range(volume[obj], capacity + 1)): ④
              prev_v, prev_solution = solutions[c - volume[obj]]
              v = value[obj] + prev_v
              if solutions[c][0] < v:
                ③ solutions[c] = v, prev_solution + [obj]

     return solutions[capacity]
```

**Python shell**

```
> volumes = [3, 3, 2, 5]
> values = [6, 7, 8, 9]
> knapsack(volumes, values, 5)
| (15, [1, 2])
```

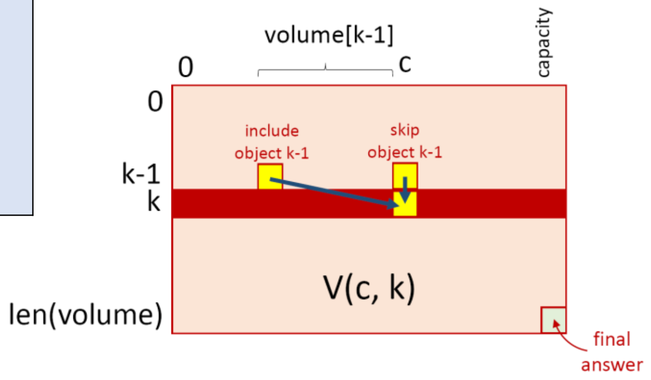
① base case k = 0

② consider each object

③ `solutions[c:]` current row
`solutions[:c]` previous row

④ compute next row right-to-left

⑤ `solutions[:volume[obj]]`
unchanged from previous row



# Summary

- Dynamic programming is a general approach for recursive problems where one tries to avoid recomputing the same expresions repeatedly

- **Solution 1: Memoization**
  - add dictionary to function to remember previous results
  - decorate with a @memoize decorator
- **Solution 2: Systematic table fill out**
  - can need to compute more values than when using memoization
  - can discard results not needed any longer (reduced memory usage)

# Coding competitions and online judges

If you like to practice your coding skills, there are many online "judges" with numerous exercises and where you can upload and test your solutions.

- Project Euler
- Kattis
- Google Code Jam
- CodeForces
- Topcoder

See cs.au.dk/~gerth/code/

# Google Code Jam

codingcompetitions.withgoogle.com/codejam

code jam

print "hello, world!"

- **Coding competition**
- **Qualification round 2022** (April 2, 01:00 – April 3, 04:00)

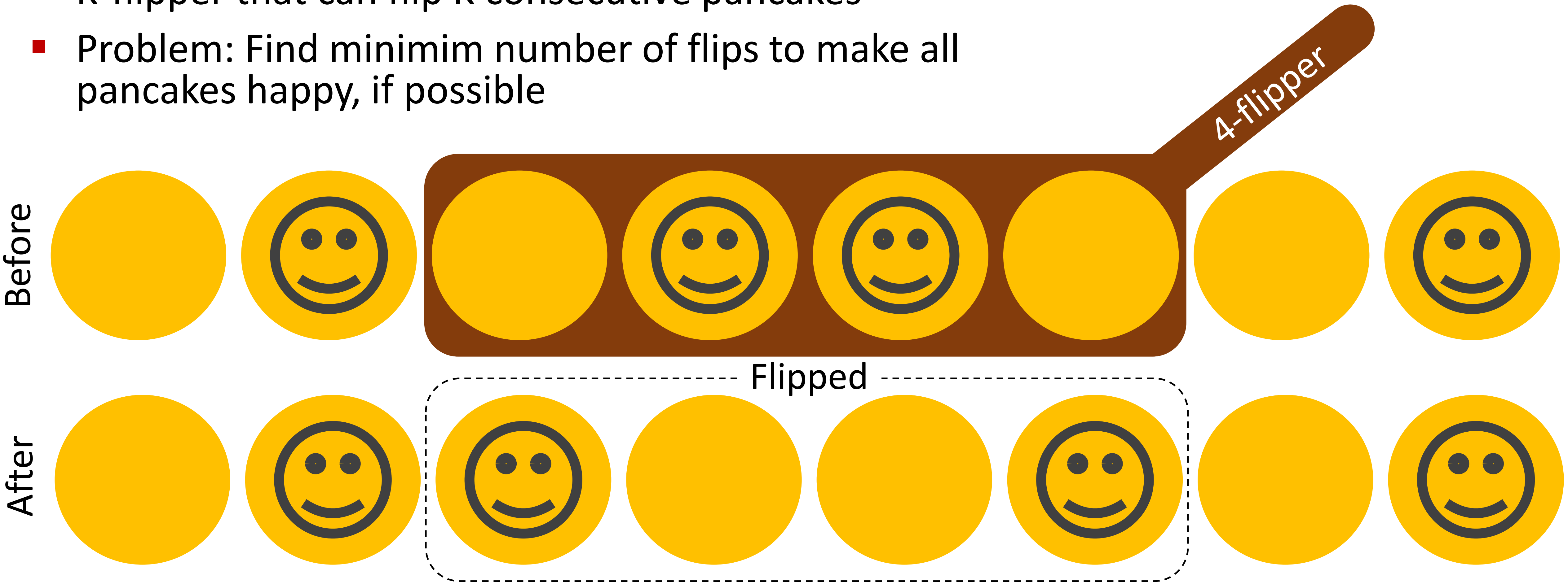
- In 2021 there was 37.000 participants for the qualification round

Scoreboard 2017

| Rank | Contestant | Score | Penalty | A. Oversized Pancake Flipper 5pt | A. Oversized Pancake Flipper 10pt | B. Tidy Numbers 5pt | B. Tidy Numbers 15pt | C. Bathroom Stalls 5pt | C. Bathroom Stalls 10pt | C. Bathroom Stalls 15pt | D. Fashion Show 10pt | D. Fashion Show 25pt |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | FatalEagle | 100 | 55:12 | 4:02 | 4:29 | 9:02 | 9:25 | 22:30 | 22:50 | 23:13 | 54:48 | 55:12 |
| 2 | ACMonster | 100 | 57:32 | 3:41 | 4:02 | 11:37 | 11:58 | 25:03 | 25:27 | 26:01 | 57:08 | 57:32 |
| 3 | y0105w49 | 100 | 58:00 | 8:28 | 8:58 | 16:20 | 16:47 | 31:34<br>1 wrong try | 32:03 | 32:36 | 53:27 | 54:00 |
| 621 | MathCrusader | 75 | 11:52:45 | 1:52:49 | 1:53:20 | 2:11:37<br>1 wrong try | 2:12:05 | 2:18:48 | 2:19:18 | 2:19:48 | 11:24:45<br>6 wrong tries | --<br>1 wrong try |
| 622 | Siddhant22 | 75 | 12:07:37 | 2:15:58<br>1 wrong try | 2:18:01 | 3:05:10 | 3:06:39 | 4:42:18 | 4:43:32 | 4:46:14 | 11:59:37<br>1 wrong try | Time expired |
| **623** | **daimi89**<br>Me | **75** | **12:18:31** | 6:21:53 | 6:22:58 | 7:15:07 | 7:15:54 | 7:52:33 | 7:53:13 | 7:54:04 | 12:14:31<br>1 wrong try | -- |
| 624 | plamenko | 75 | 12:19:28 | 11:20 | 11:56 | 27:38 | 28:23 | 42:34 | 43:17 | 43:55 | 12:11:28<br>2 wrong tries | --<br>1 wrong try |

# Google Code Jam - Qualification Round 2017
# Problem A: Oversized Pancake Flipper (description)

- N pancakes each with exactly one happy chocolate side
- K-flipper that can flip K consecutive pancakes
- Problem: Find minimim number of flips to make all pancakes happy, if possible



# Visualization and optimization

- Matplotlib
- Jupyter
- scipy.optimize.minimize

# matplotlib

*Matplotlib is a Python 2D plotting library which produces publication quality figures in a variety of hardcopy formats and interactive environments across platforms. Matplotlib can be used in Python scripts, the Python and IPython shells, the Jupyter notebook, web application servers, and four graphical user interface toolkits.*

*Matplotlib tries to make easy things easy and hard things possible. You can generate plots, histograms, power spectra, bar charts, errorcharts, scatterplots, etc., with just a few lines of code. For simple plotting the pyplot module provides a MATLAB-like interface, particularly when combined with IPython. For the power user, you have full control of line styles, font properties, axes properties, etc, via an object oriented interface or via a set of functions familiar to MATLAB users.*

pip install matplotlib

matplotlib.org

# Plot

pyplot module ≈ MATLAB-like plotting framework

**matplotlib-simple.py**

```
import matplotlib.pyplot as plt

plt.plot([1, 2, 3], [5, 2, 7], 'bo:')

plt.show()
```

add plot to figure

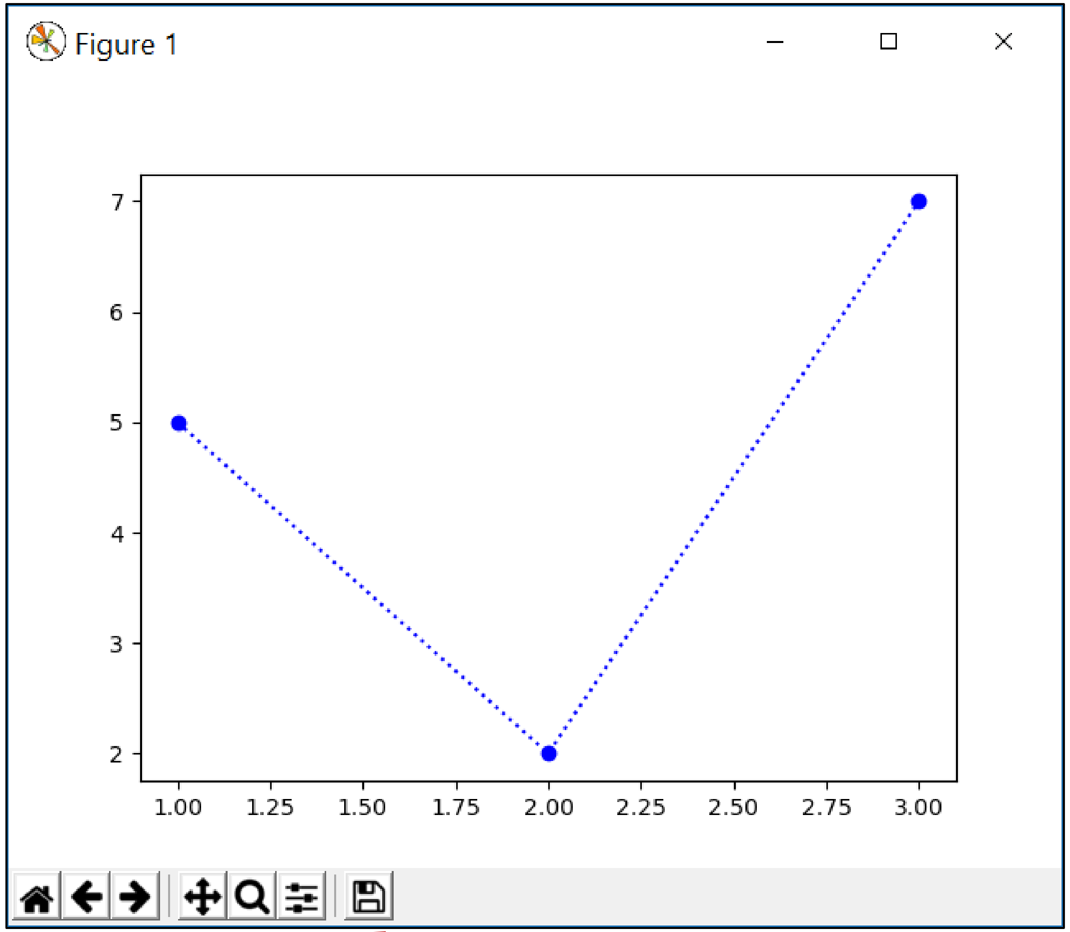
x coordinates

y coordinates

format string

figure is first shown when show is called

| Colors | Line styles | Marker styles | | |
|---|---|---|---|---|
| b | - | . | 2 | + |
| g | -- | , | 3 | x |
| r | -. | o | 4 | D |
| c | : | v | s | d |
| m | | ^ | p | \| |
| y | | < | * | _ |
| k | | > | h | |
| w | | 1 | H | |




save current view as picture

adjust margins

zoom rectangle

pan and zoom

navigate view history

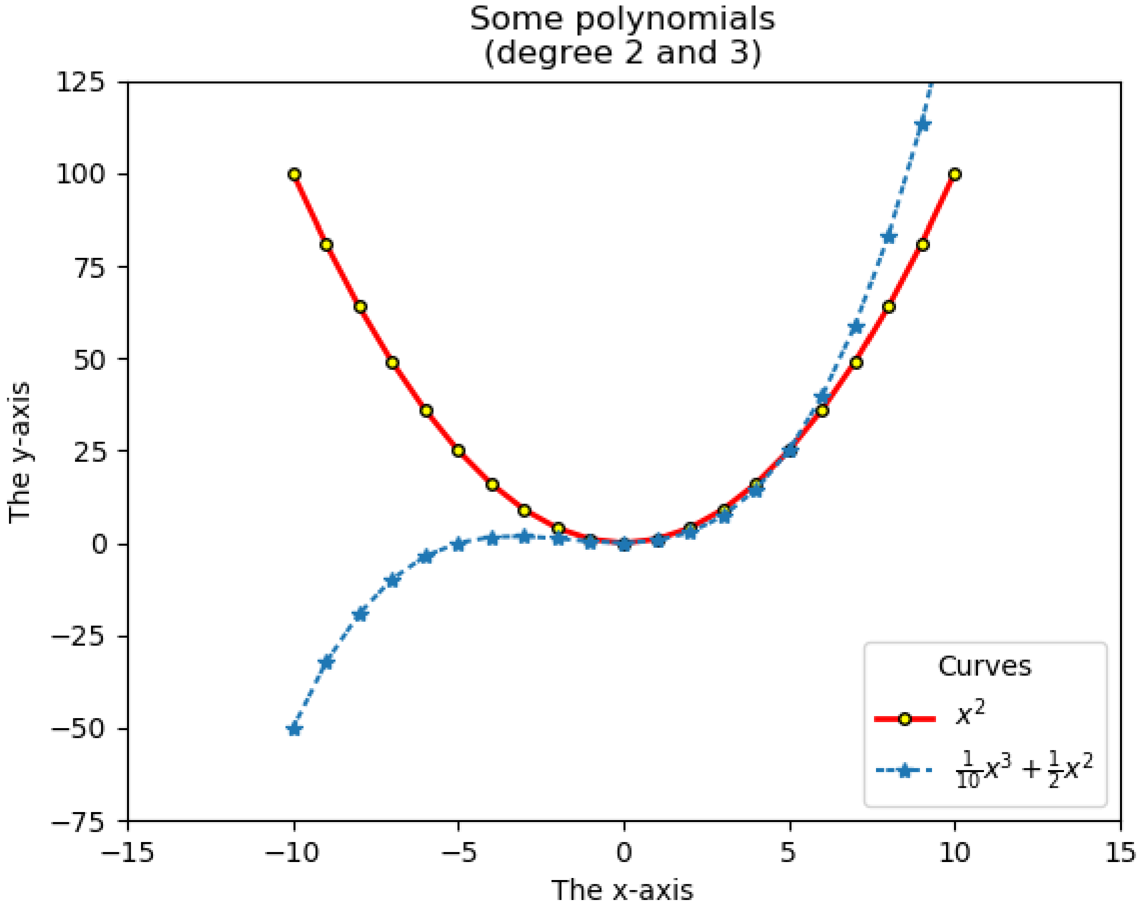
reset view

# Plot – some keyword arguments

**matplotlib-plot.py**

```
import matplotlib.pyplot as plt

X = range(-10, 11)
Y1 = [x ** 2 for x in X]
Y2 = [x ** 3 / 10 + x ** 2 / 2 for x in X]

plt.plot(X, Y1, color='red', label='$x^2$',
    linestyle='-', linewidth=2,
    marker='o', markersize=4,
    markeredgewidth=1,
    markeredgecolor='black',
    markerfacecolor='yellow')

plt.plot(X, Y2, '*', dashes=(2, 0.5, 2, 1.5),
    label=r'$\frac{1}{10}x^3+\frac{1}{2}x^2$')

plt.xlim(-15, 15)
plt.ylim(-75, 125)
plt.title('Some polynomials\n(degree 2 and 3)')

plt.xlabel('The x-axis')
plt.ylabel('The y-axis')
plt.legend(title='Curves')

plt.show()  # finally show figure
```

LaTeX




matplotlib.org/api/_as_gen/matplotlib.pyplot.plot.html

Colors: matplotlib.org/gallery/color/named_colors.html

# Scatter (points with individual size and color)

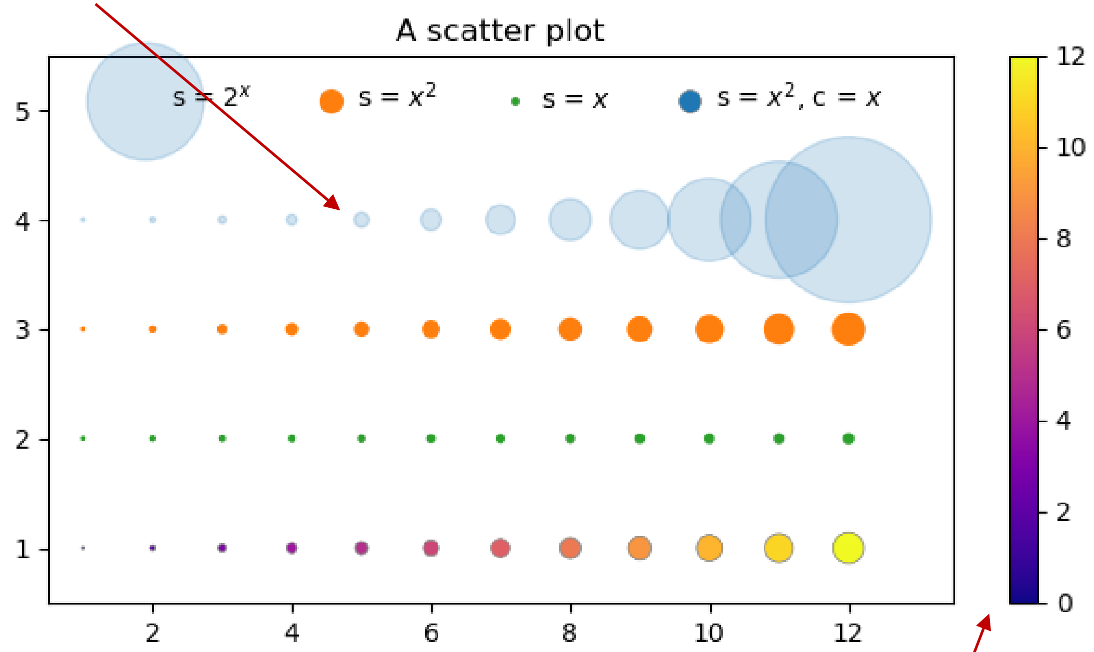
**matplotlib-scatter.py**

```
import matplotlib.pyplot as plt

n = 13
X = range(n)
S = [x ** 2 for x in X]
E = [2 ** x for x in X]

plt.scatter(X, [4] * n, s=E, label='s = $2^x$', alpha=.2)
plt.scatter(X, [3] * n, s=S, label='s = $x^2$')
plt.scatter(X, [2] * n, s=X, label='s = $x$')
plt.scatter(X, [1] * n, s=S, c=X, cmap='plasma',
    label='s = $x^2$, c = $x$',
    edgecolors='gray', linewidth=0.5)
plt.colorbar()

plt.ylim(0.5, 5.5)
plt.xlim(0.5, 13.5)
plt.title('A scatter plot')
plt.legend(loc='upper center', frameon=False, ncol=4,
           handletextpad=0)
plt.show()
```

transparency

colormap (predefined)
color of each point
size ≈ area of each point
point boundary width
point boundary color

manual placement of legend box (default automatic); remove frame; place legends in 4 columns (default 1); reduce space between marks and label



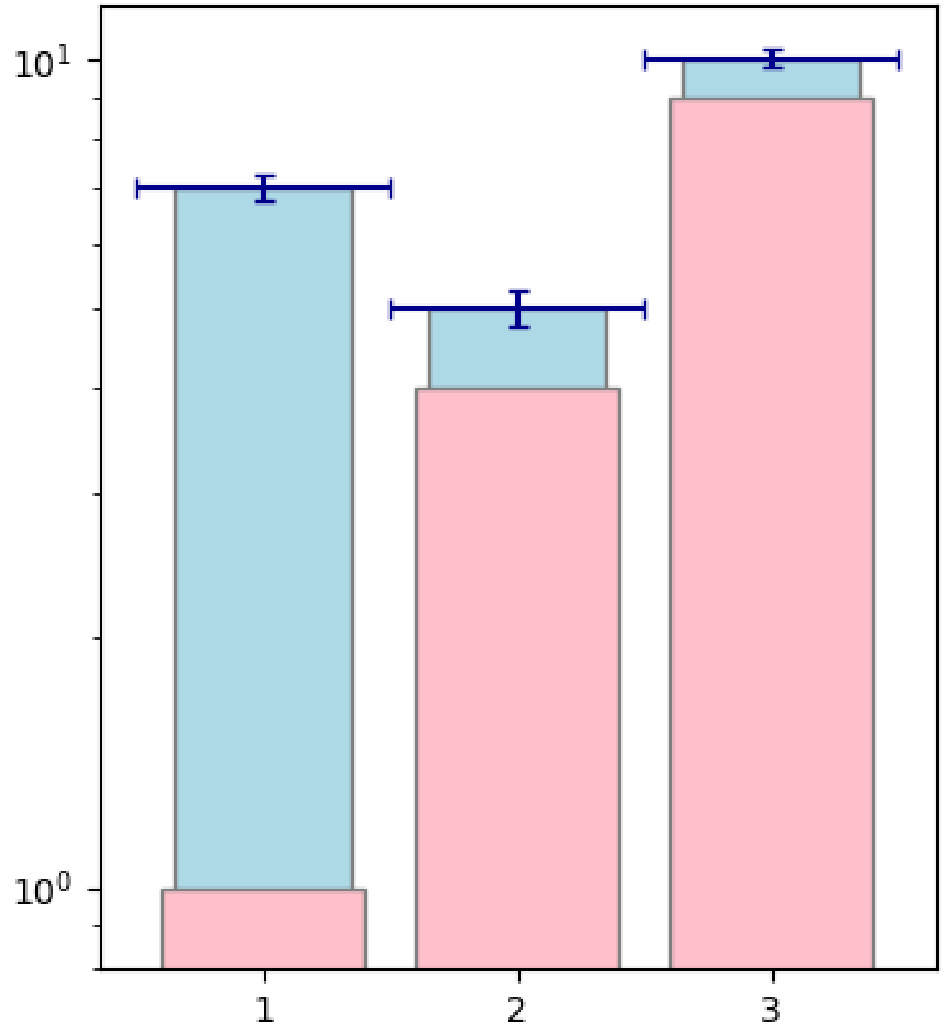

**colorbar**
(of most recently used colormap)

matplotlib.org/api/_as_gen/matplotlib.pyplot.scatter.html
matplotlib.org/tutorials/colors/colormaps.html

# Bars

**matplotlib-bars.py**

```
import matplotlib.pyplot as plt
x = [1, 2, 3]
y = [7, 5, 10]
plt.bar(x, y,
        color='lightblue',   # bar background color
        linewidth=1,         # bar boundary width
        edgecolor='gray',    # bar boundary color
        tick_label=x,        # ticks on x-axis
        width=0.7,           # width, default 0.8
        yerr=0.25,           # Error bar: y length
        xerr=0.5,            #   x length
        capsize=3,           #   capsize in points
        ecolor='darkblue',   #   error bar color
        log=True)            # y-axis log scale
plt.bar(x, [v**2 for v in x],
        color='pink',
        linewidth=1,
        edgecolor='gray')
plt.show()
```

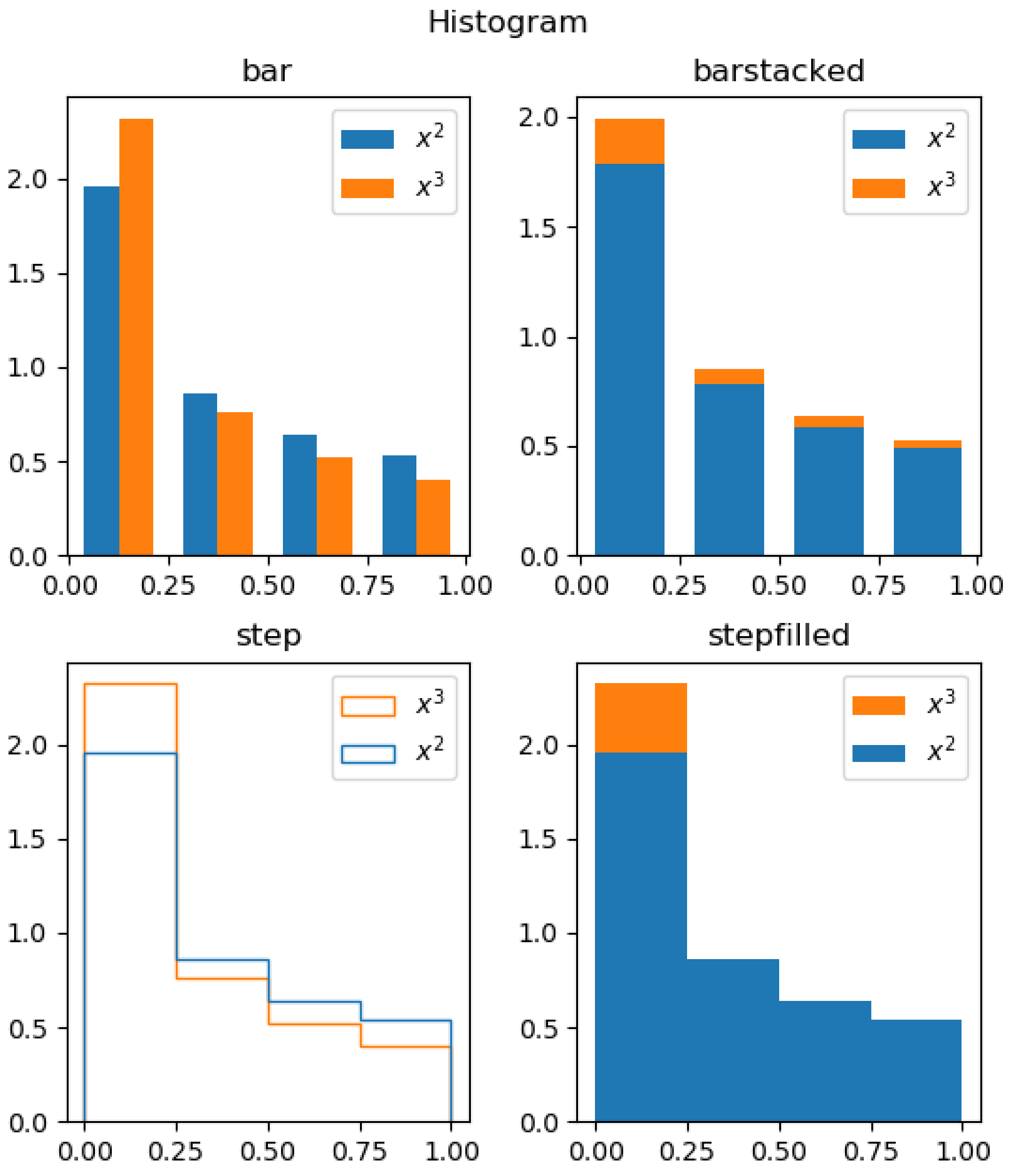



matplotlib.org/api/_as_gen/matplotlib.pyplot.bar.html

# Histogram

**matplotlib-histogram.py**

```
import matplotlib.pyplot as plt
from random import random

values1 = [random()**2 for _ in range(1000)]
values2 = [random()**3 for _ in range(100)]

bins = [0.0, 0.25, 0.5, 0.75, 1.0]

for i, ht in enumerate(
                ['bar', 'barstacked', 'step', 'stepfilled'],
                start=1):
    plt.subplot(2, 2, i)   # start new plot
    plt.hist([values1, values2],  # data sets
             bins,         # bucket boundaries
             histtype=ht, # default ht='bar'
             rwidth=0.7,  # fraction of bucket width
             label=['$x^2$', '$x^3$'],  # labels
             density=True)  # norm. prob. density
    plt.title(ht)          # plot title
    plt.xticks(bins)       # ticks on x-axis
    plt.legend()

plt.suptitle('Histogram') # figure title
plt.show()
```

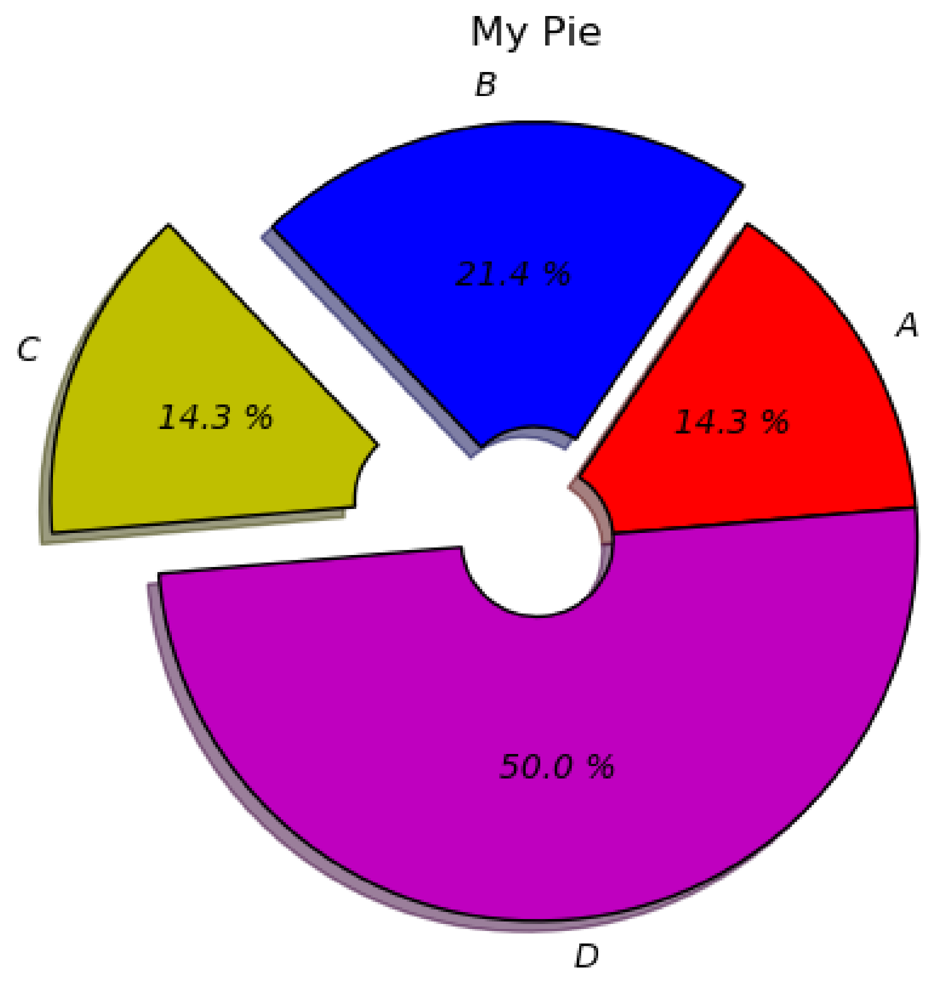



matplotlib.org/api/_as_gen/matplotlib.pyplot.hist.html

# Pie

**matplotlib-pie.py**

```
import matplotlib.pyplot as plt

plt.title('My Pie')
plt.pie([2, 3, 2, 7],            # relative wedge sizes
        labels=['A','B','C','D'],
        colors=['r', 'b', 'y', 'm'],
        explode=(0, 0.1, 0.3, 0),  # radius fraction
        startangle=5,            # angle above horizontal
        counterclock=True,       # default True
        rotatelabels=False,      # default False
        shadow=True,             # default False
        textprops=dict(          # text properties, dict
            color='black',       # text color
            style='italic'),     # text style
        wedgeprops=dict(         # wedge properties, dict
            width=0.8,           # width (missing center)
            linewidth=1,         # wedge boundary width
            edgecolor='black'),   # boundary color
        autopct='%1.1f %%')      # percent formatting
plt.show()
```




matplotlib.org/api/_as_gen/matplotlib.pyplot.pie.html

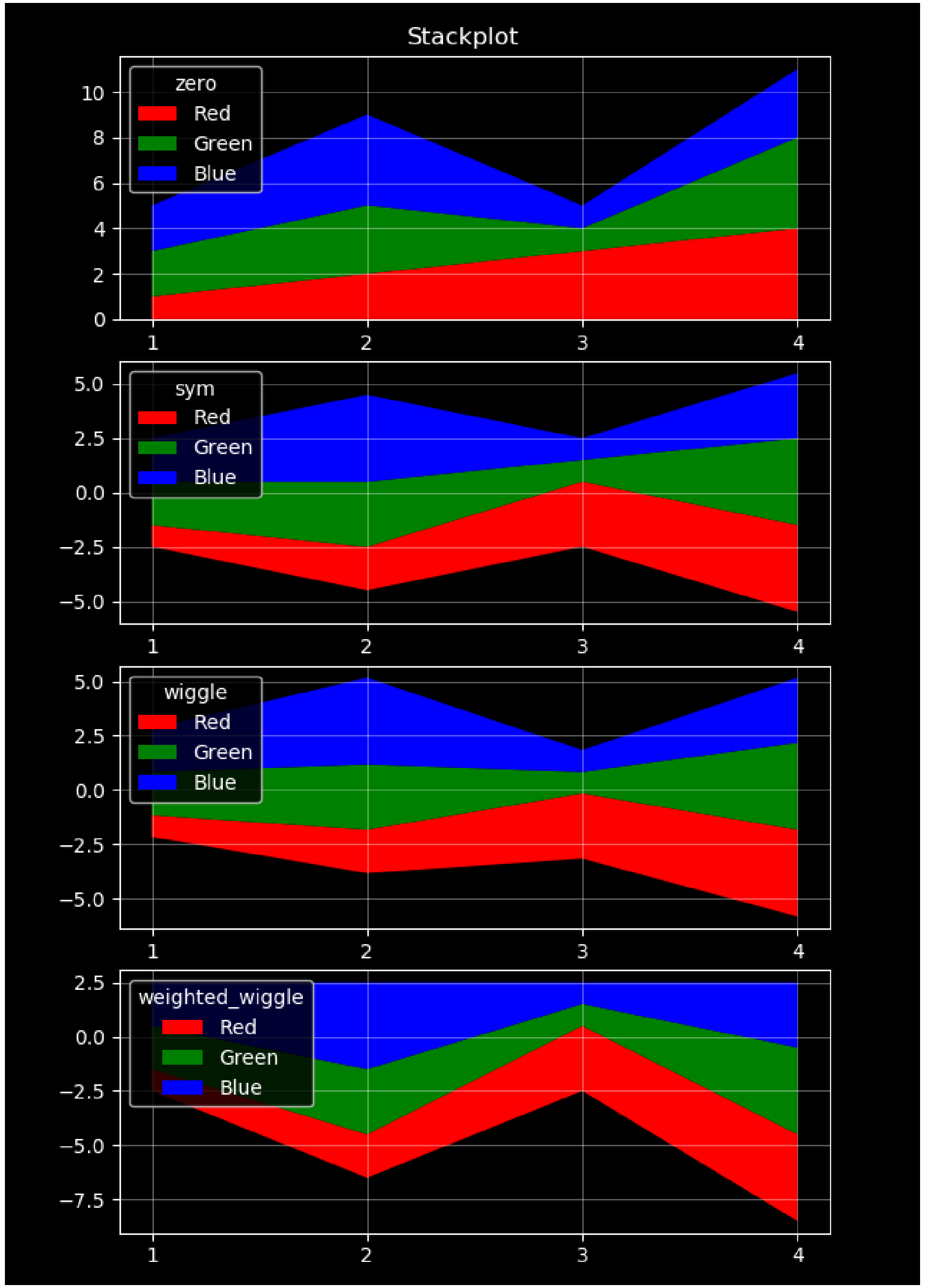
# Stackplot

**matplotlib-stackplot.py**

```python
import matplotlib.pyplot as plt
x = [1, 2, 3, 4]
y1 = [1, 2, 3, 4]
y2 = [2, 3, 1, 4]
y3 = [2, 4, 1, 3]
plt.style.use('dark_background')
for i, base in enumerate(
        ['zero', 'sym', 'wiggle','weighted_wiggle'],
        start=1):
    plt.subplot(4, 1, i)
    plt.stackplot(x, y1, y2, y3,
                  colors=['r', 'g', 'b'],
                  labels=['Red', 'Green', 'Blue'],
                  baseline=base)
    plt.grid(axis='both',  # 'x', 'y', or 'both'
        linewidth=0.5, linestyle='-', alpha=0.5)
    plt.legend(title=base, loc='upper left')
    plt.xticks(x)  # a tick for each value in x
plt.suptitle('Stackplot')
plt.show()
```

Stacked Graphs – Geometry & Aesthetics
*Lee Byron & Martin Wattenberg*, 2008

To list all available styles:
`print(plt.style.available)`



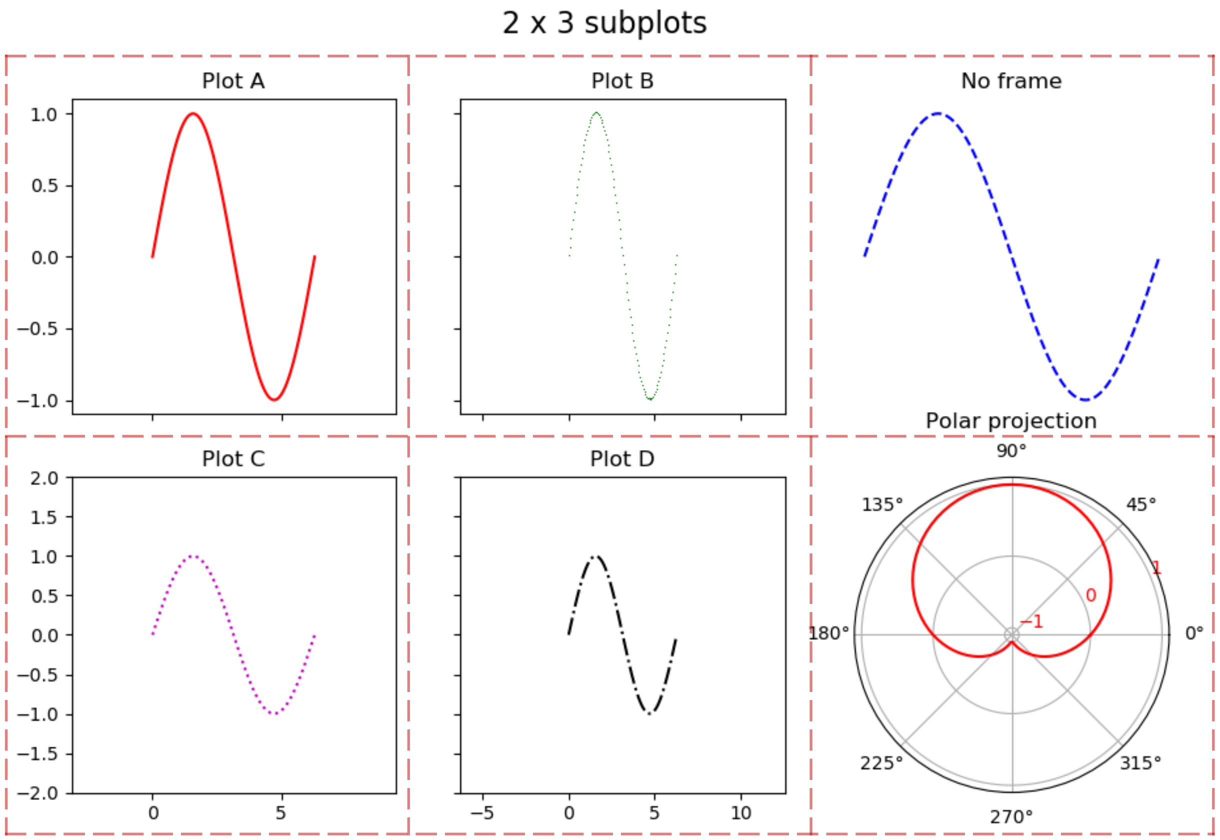

matplotlib.org/api/_as_gen/matplotlib.pyplot.stackplot.html

**matplotlib-subplot.py**

```
import matplotlib.pyplot as plt
from math import pi, sin

x_min, x_max, n = 0, 2 * pi, 100
x = [x_min + (x_max - x_min) * i / n for i in range(n + 1)]
y = [sin(v) for v in x]

ax1 = plt.subplot(2, 3, 1)   # 2 rows, 3 columns
ax1.label_outer()            # removes x-axis labels
plt.xlim(-pi, 3 * pi)        # increase x-axis range
plt.plot(x, y, 'r-')
plt.title('Plot A')

ax2 = plt.subplot(2, 3, 2)
ax2.label_outer()            # removes x- and y-axis labels
plt.xlim(-2 * pi, 4 * pi)    # increase x-axis range
plt.plot(x, y, 'g,')
plt.title('Plot B')

ax3 = plt.subplot(2, 3, 3, frameon=False)  # remove frame
ax3.set_xticks([])           # remove x-axis ticks & labels
ax3.set_yticks([])           # remove x-axis ticks & labels
plt.plot(x, y, 'b--')
plt.title('No frame')

ax4 = plt.subplot(2, 3, 4, sharex=ax1)  # share x-axis range
plt.ylim(-2, 2)              # increase y-axis range
plt.plot(x, y, 'm:')
plt.title('Plot C')

ax5 = plt.subplot(2, 3, 5, sharex=ax2, sharey=ax4) # share ranges
ax5.set_xticks(range(-5, 15, 5))  # specific x-ticks & x-labels
ax5.label_outer()            # removes y-axis labels
plt.plot(x, y, 'k-.')
plt.title('Plot D')

ax6 = plt.subplot(2, 3, 6, projection='polar')  # polar projection
ax6.set_yticks([-1, 0, 1])                       # y-labels
ax6.tick_params(axis='y', labelcolor='red')     # color of y-labels
plt.plot(x, y, 'r')
plt.title('Polar projection\n')  # \n to avoid overlap with 90°

plt.suptitle('2 x 3 subplots', fontsize=16)

plt.show()
```

# Subplot

## (2 rows, 3 columns)




- Subplots are numbered 1..6 row-by-row, starting top-left
- subplot returns an axes to access the plot in the figure

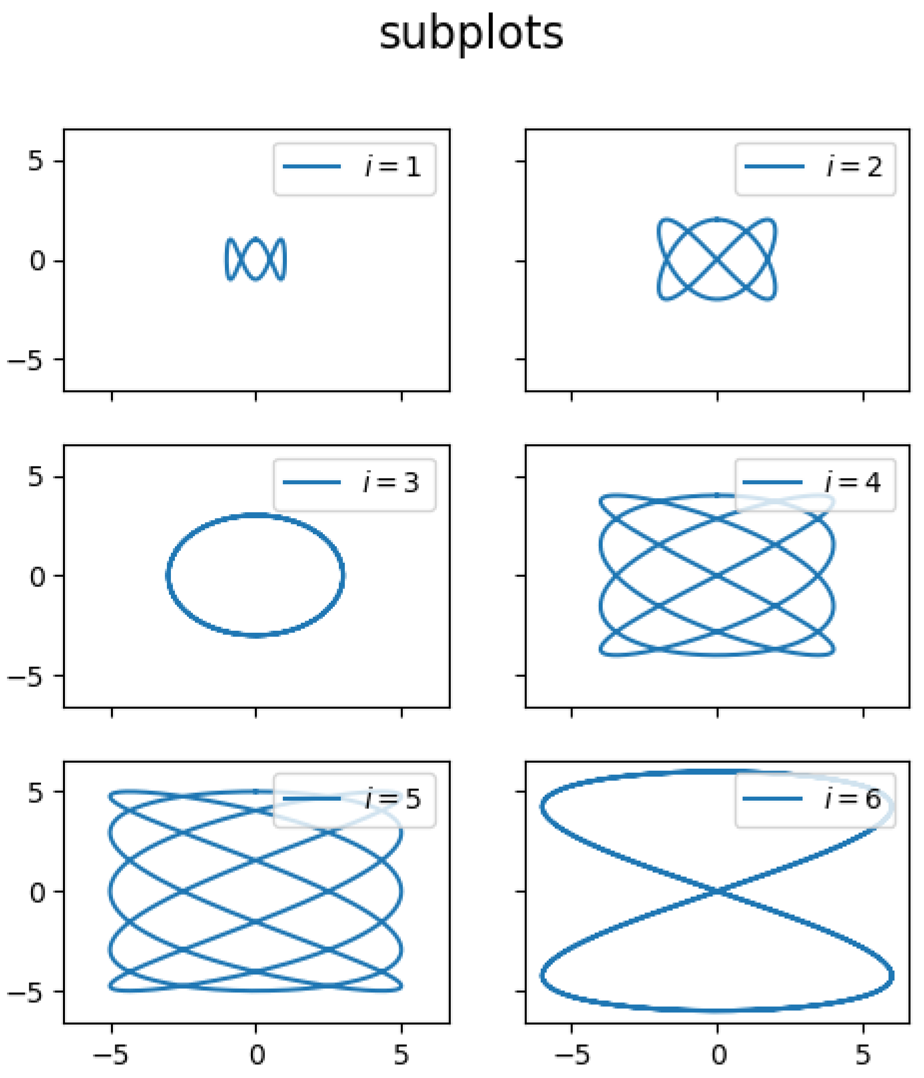
matplotlib.org/api/_as_gen/matplotlib.pyplot.subplot.html

# Subplots

**matplotlib-subplots.py**

```
import matplotlib.pyplot as plt
from math import pi, sin, cos
times = [2 * pi * t / 1000 for t in range(1001)]

fig, ((ax1, ax2), (ax3, ax4), (ax5, ax6)) = \
    plt.subplots(3, 2, sharex=True, sharey=True)

for i, ax in enumerate([ax1, ax2, ax3, ax4, ax5, ax6],
                       start=1):
    x = [i * sin(i * t) for t in times]
    y = [i * cos(3 * t) for t in times]
    ax.plot(x, y, label=f'$i = {i}$')  # plot to axes
    ax.legend(loc='upper right')       # axes legend
fig.suptitle('subplots', fontsize=16)  # figure title
plt.show()
```

create 6 axes in 3 rows with 2 colums
share the x- and y-axis ranges (automatically applies label_outer to created axes)
returns a pair (figure, axes)



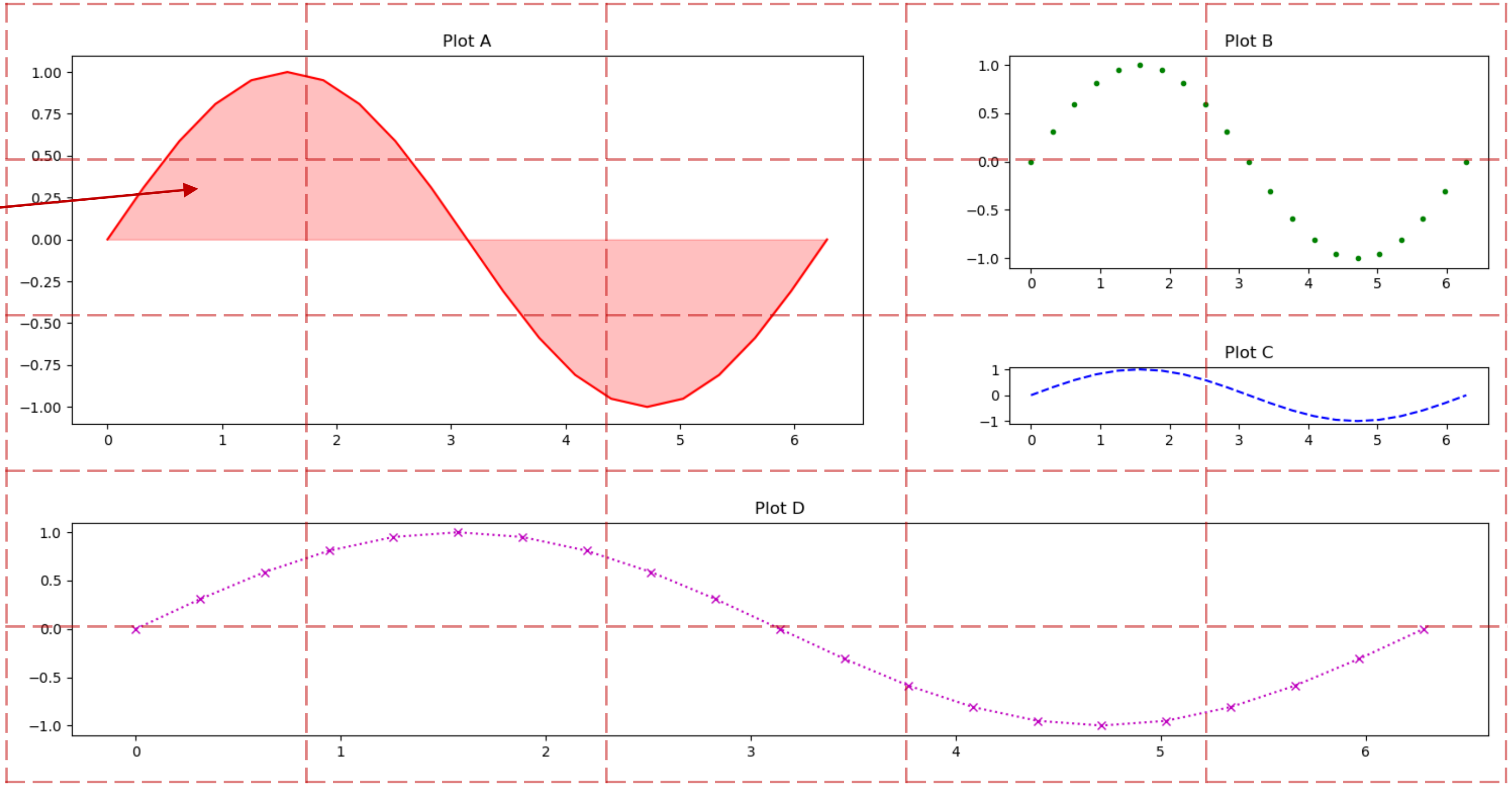

matplotlib.org/api/_as_gen/matplotlib.pyplot.subplots.html

# subplot2grid (5 x 5)

matplotlib-subplot2grid.py

```
import matplotlib.pyplot as plt
import math

x_min, x_max, n = 0, 2 * math.pi, 20

x = [x_min + (x_max - x_min) * i / n
      for i in range(n + 1)]
y = [math.sin(v) for v in x]

plt.subplot2grid((5, 5), (0,0),
                 rowspan=3, colspan=3)
plt.fill_between(x, 0.0, y,
                 alpha=0.25, color='r')
plt.plot(x, y, 'r-')
plt.title('Plot A')
plt.subplot2grid((5, 5), (0,3),
                 rowspan=2, colspan=2)
plt.plot(x, y, 'g.')
plt.title('Plot B')
plt.subplot2grid((5, 5), (2,3),
                 rowspan=1, colspan=2)
plt.plot(x, y, 'b--')
plt.title('Plot C')
plt.subplot2grid((5, 5), (3,0),
                 rowspan=2, colspan=5)
plt.plot(x, y, 'mx:')
plt.title('Plot D')
plt.tight_layout()  # adjust padding
plt.show()
```

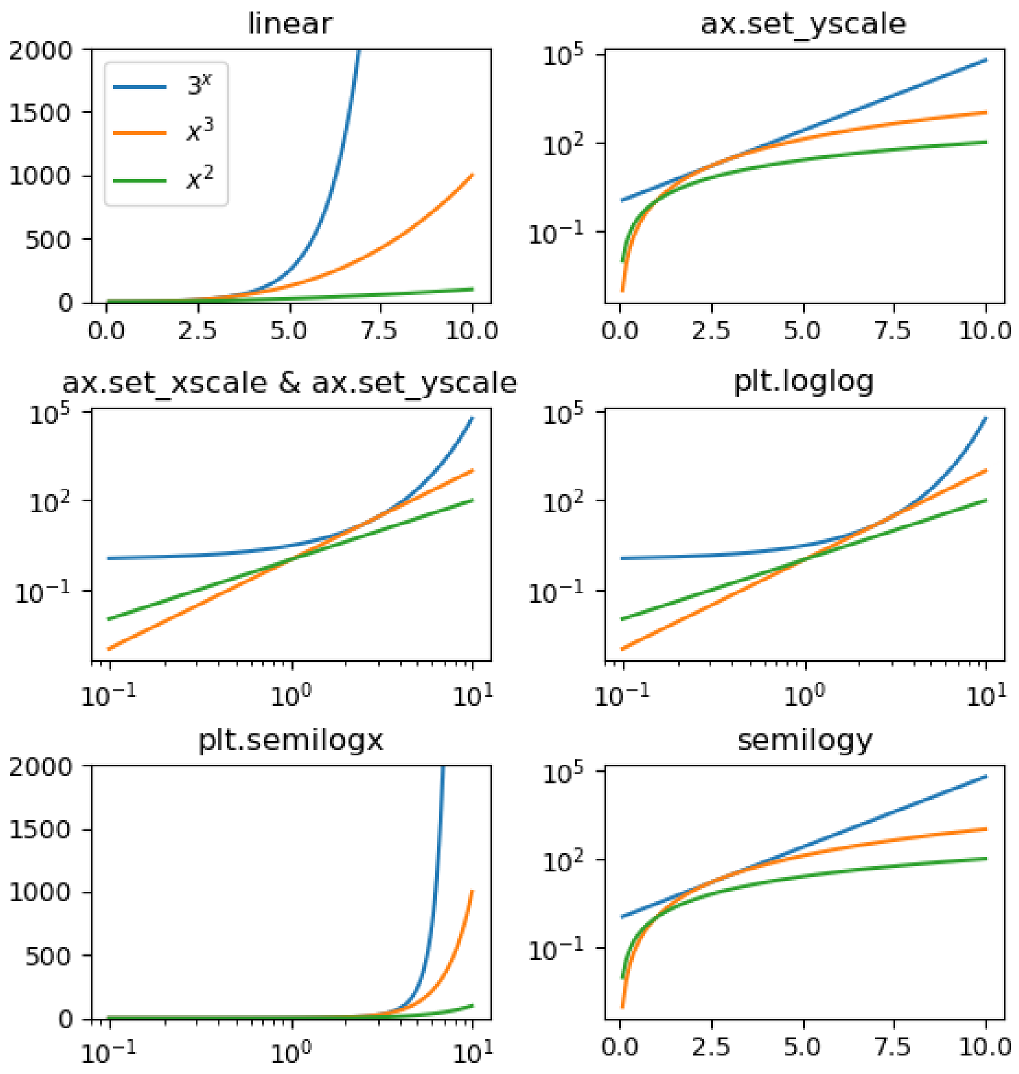
upper left corner (row, column)




matplotlib.org/api/_as_gen/matplotlib.pyplot.subplot2grid.html matplotlib-log.py

```
import matplotlib.pyplot as plt
x = [i / 10 for i in range(1, 101)]
y1 = [i ** 2 for i in x]
y2 = [i ** 3 for i in x]
y3 = [3 ** i for i in x]
for i in range(1, 7):
    ax = plt.subplot(3, 2, i)
    plt.plot(x, y3, label='$3^x$')
    plt.plot(x, y2, label='$x^3$')
    plt.plot(x, y1, label='$x^2$')
    if i == 1:
        plt.ylim(0, 2000)
        plt.xscale('linear')  # default
        plt.yscale('linear')  # default
        plt.legend()
        plt.title('linear')
    if i == 2:
        plt.yscale('log')
        plt.title('ax.yscale')
    if i == 3:
        ax.set_xscale('log')
        ax.set_yscale('log')
        plt.title('ax.set_xscale & ax.set_yscale')
    if i == 4:
        plt.loglog()
        plt.title('plt.loglog')
    if i == 5:
        plt.ylim(0, 2000)
        plt.semilogx()
        plt.title('plt.semilogx')
    if i == 6:
        plt.semilogy()
        plt.title('semilogy')
plt.show()
```

# log scales

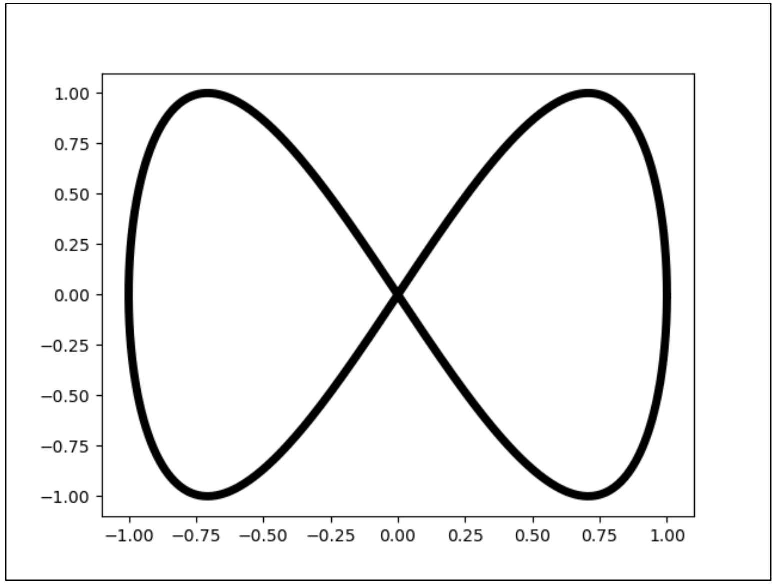



- There are many ways to make the x- and/or y-axis logarithmic with pyplot

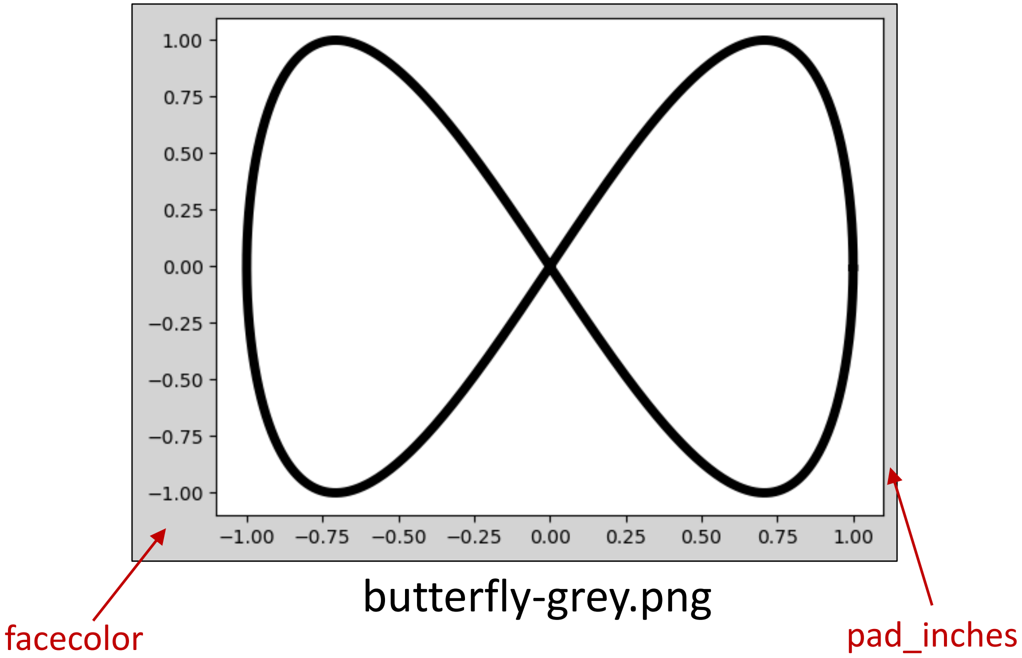
# Saving figures

matplotlib-savefig.py

```
import matplotlib.pyplot as plt
from math import pi, sin, cos

n = 1000
points = [(cos(2 * pi * i / n),
           sin(4 * pi * i / n)) for i in range(n)]
x, y = zip(*points)
plt.plot(x, y, 'k-', linewidth=5)

plt.savefig('butterfly.png')  # save plot as PNG

plt.savefig('butterfly-grey.png',
    dpi=100,                  # dots per inch
    bbox_inches='tight',      # crop to bounding box
    pad_inches=0.1,           # space around figure
    facecolor='lightgrey',    # background color
    format='png')             # optional if file extension

plt.savefig('butterfly.pdf')  # save plot as PDF

plt.show()                    # interactive viewer
```




butterfly.png




butterfly-grey.png

matplotlib.org/api/_as_gen/matplotlib.pyplot.savefig.html

# A crude animation

**clock.py**

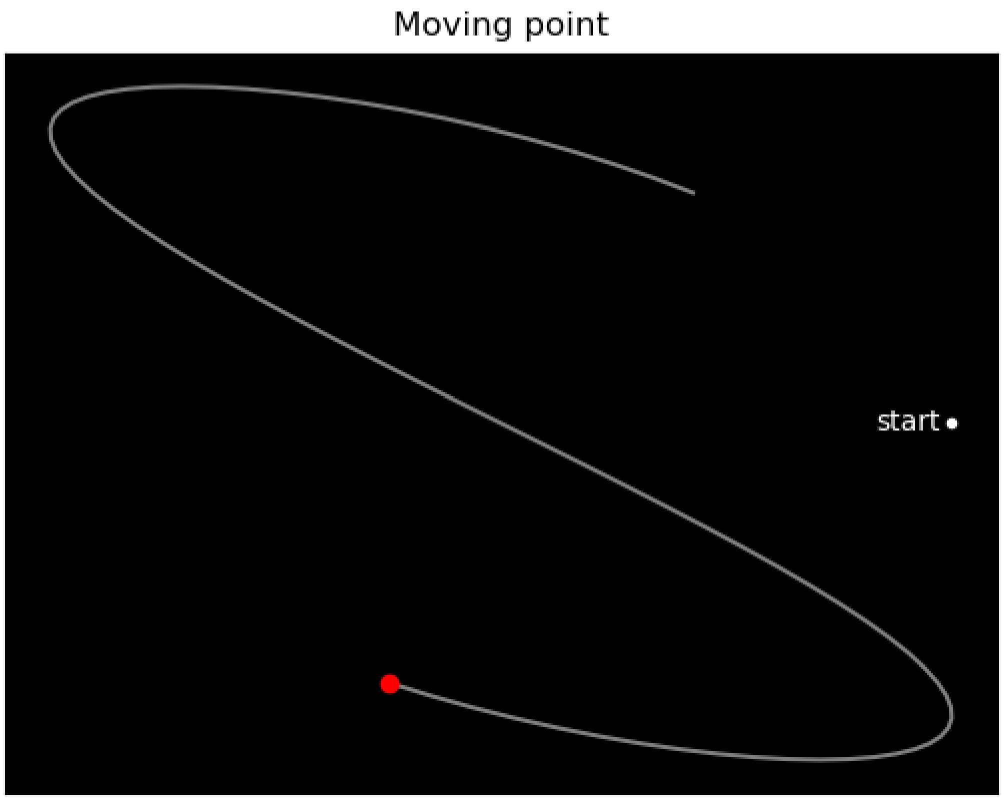
```
import matplotlib.pyplot as plt
from math import pi, sin, cos
import datetime

def plot_clock(hour, minute, second):
    plt.axis('off')                    # hide x and y axes
    plt.gca().set_aspect('equal')  # don't squeeze circle
    for i in range(60):                # show second marks
        angle = 2 * pi * i / 60
        x, y = cos(angle), sin(angle)
        start = 0.98 if i % 5 else .94  # every 5'th mark should be longer
        plt.plot([start * x, x], [start * y, y], c='black')   # mark
    for angle, length, style in [
        (second / 60, .90, dict(c='red',   lw=2, solid_capstyle='round')),
        (minute / 60, .85, dict(c='black', lw=3, solid_capstyle='round')),
        (hour   / 12, .50, dict(c='black', lw=8, solid_capstyle='round'))
    ]:
        angle = 2 * pi * (0.25 - angle)
        x, y = length * cos(angle), length * sin(angle)
        plt.plot([0, x], [0, y], **style)  # clock arm
    plt.plot(0, 0, 'o', ms=10, c='black')  # center dot

while True:
    now = datetime.datetime.now()  # UTZ
    plot_clock(now.hour, now.minute, now.second)
    plt.pause(1)  # show figure and pause 1 second
    plt.clf()     # clear figure
```

matplotlib.org/stable/api/_as_gen/matplotlib.pyplot.pause.html

**matplotlib-animation.py**

```
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from math import pi, cos, sin

n, tail_length = 200, 75
points = []                   # tail_length recent poins

def point(i):
    t = 2 * pi * i / n
    return (cos(3 * t), sin(2 * t))

fig = plt.figure()            # new figure
ax = plt.gca()                # get current axes
ax.set_facecolor('black')     # set background color
plt.xlim(-1.1, 1.1)           # set x-axis range
plt.ylim(-1.1, 1.1)           # set y-axis range
plt.xticks([])                # remove x-ticks & labels
plt.yticks([])                # remove y-ticks & labels
plt.title('Moving point')     # plot title

x, y = point(0)
plt.plot(x, y, 'w.')          # start point
plt.text(x - 0.025, y, 'start', color='w', # text label
    ha='right', va='center')               # alignment
tail, = plt.plot([], [], 'w-', alpha=0.5)  # init. tail
head, = plt.plot([], [], 'ro')     # init. current point

def move(frame):        # frame = value from frames
    points.append(point(frame))
    del points[:-tail_length]      # limit tail
    tail.set_data(*zip(*points))   # update tail points
    head.set_data(*points[-1])     # update head point

animation = FuncAnimation(fig,     # figure to animate
    func=move,          # function called for each frame
    frames=range(n),    # array like to iterate over
    interval=25,        # milliseconds between frames
    repeat=True,        # repeat frames when done
    repeat_delay=0)     # wait milliseconds before repeat

plt.show()
```

# matplotlib.animation.FuncAnimation



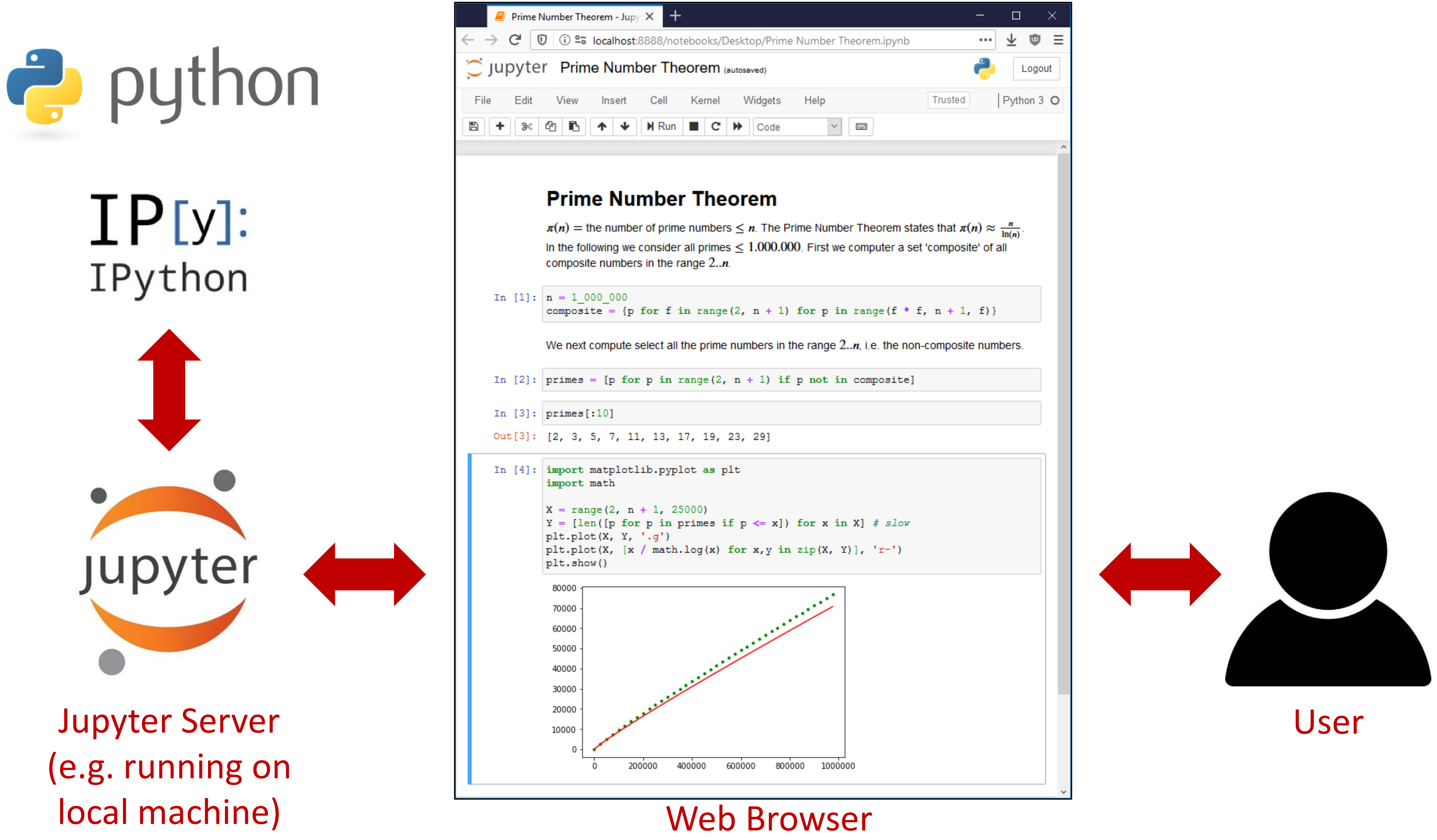

- `plot` returns "Line2D" objects representing the plotted data
- "Line2D" objects can be updated using `set_data`
- To make an animation you need to repeatedly update the "line2D" objects
- `FuncAnimation` repeatedly calls func in regular intervals interval, each time with the next value from frames (if frames is None, then the frame values provided to func will be the infinite sequence 0,1,2,3,...)

matplotlib.org/api/_as_gen/matplotlib.animation.FuncAnimation.html

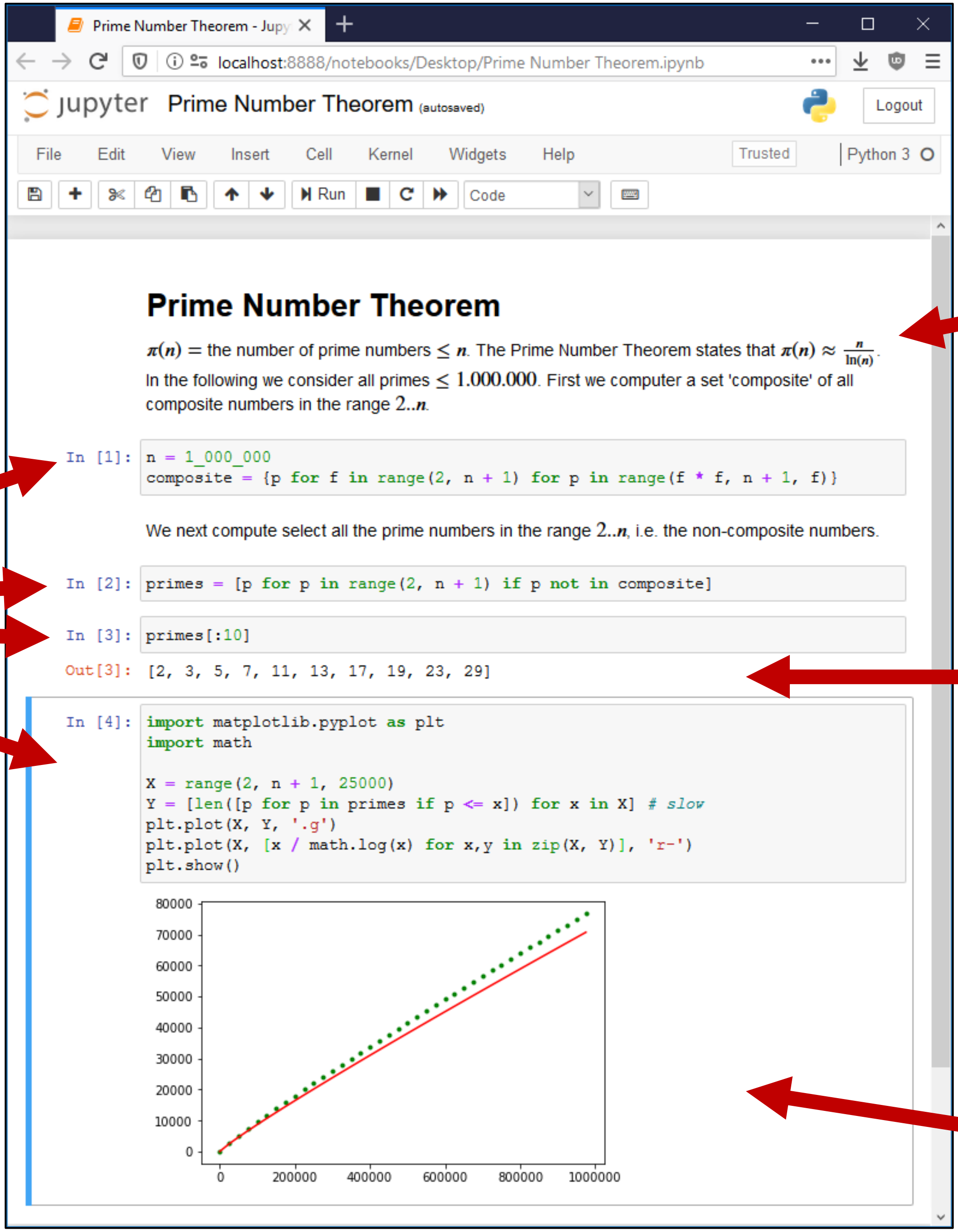
***The Jupyter Notebook***

*The Jupyter Notebook is an open-source web application that allows you to create and share documents that contain live code, equations, visualizations and narrative text. Uses include: data cleaning and transformation, numerical simulation, statistical modeling, data visualization, machine learning, and much more.*

jupyter.org


python
IP[y]:
IPython
jupyter
Jupyter Server
(e.g. running on
local machine)
Prime Number Theorem - Jupy
localhost:8888/notebooks/Desktop/Prime Number Theorem.ipynb
jupyter Prime Number Theorem (autosaved)
Logout
File Edit View Insert Cell Kernel Widgets Help
Trusted
Python 3
Run
Code
Prime Number Theorem
π(n) = the number of prime numbers ≤ n. The Prime Number Theorem states that π(n) ≈ n/ln(n).
In the following we consider all primes ≤ 1.000.000. First we computer a set 'composite' of all composite numbers in the range 2..n.
In [1]: n = 1_000_000
composite = {p for f in range(2, n + 1) for p in range(f * f, n + 1, f)}
We next compute select all the prime numbers in the range 2..n, i.e. the non-composite numbers.
In [2]: primes = [p for p in range(2, n + 1) if p not in composite]
In [3]: primes[:10]
Out[3]: [2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
In [4]: import matplotlib.pyplot as plt
import math
X = range(2, n + 1, 25000)
Y = [len([p for p in primes if p <= x]) for x in X] # slow
plt.plot(X, Y, '.g')
plt.plot(X, [x / math.log(x) for x,y in zip(X, Y)], 'r-')
plt.show()
80000
70000
60000
50000
40000
30000
20000
10000
0
0
200000
400000
600000
800000
1000000
Web Browser
User

cells




formatted text: Markdown / LaTeX / HTML / ...

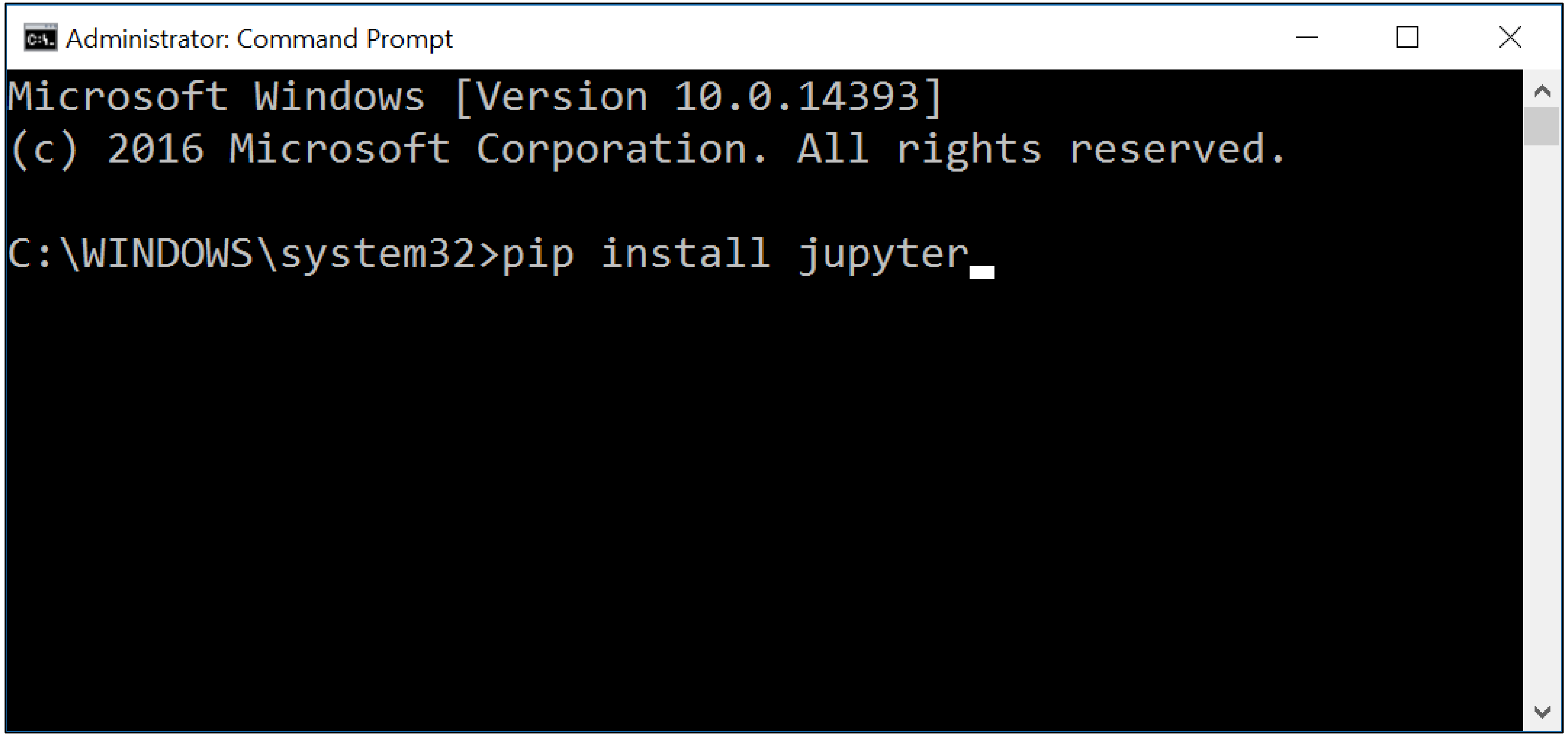
python code

python shell output

matplotlib / numpy / ... output

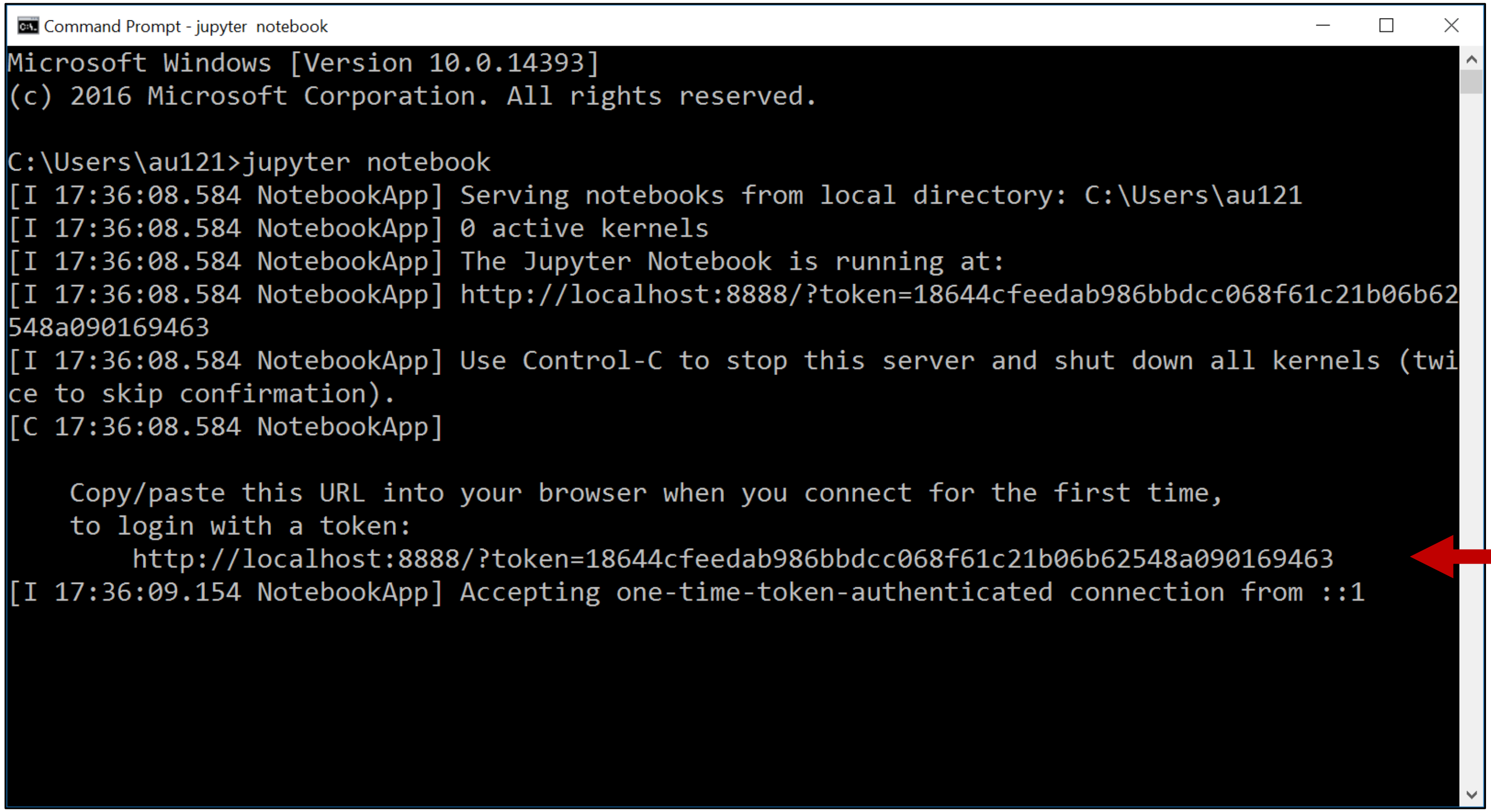
# Jupyter - installing

- Open a windows shell and run: `pip install jupyter`



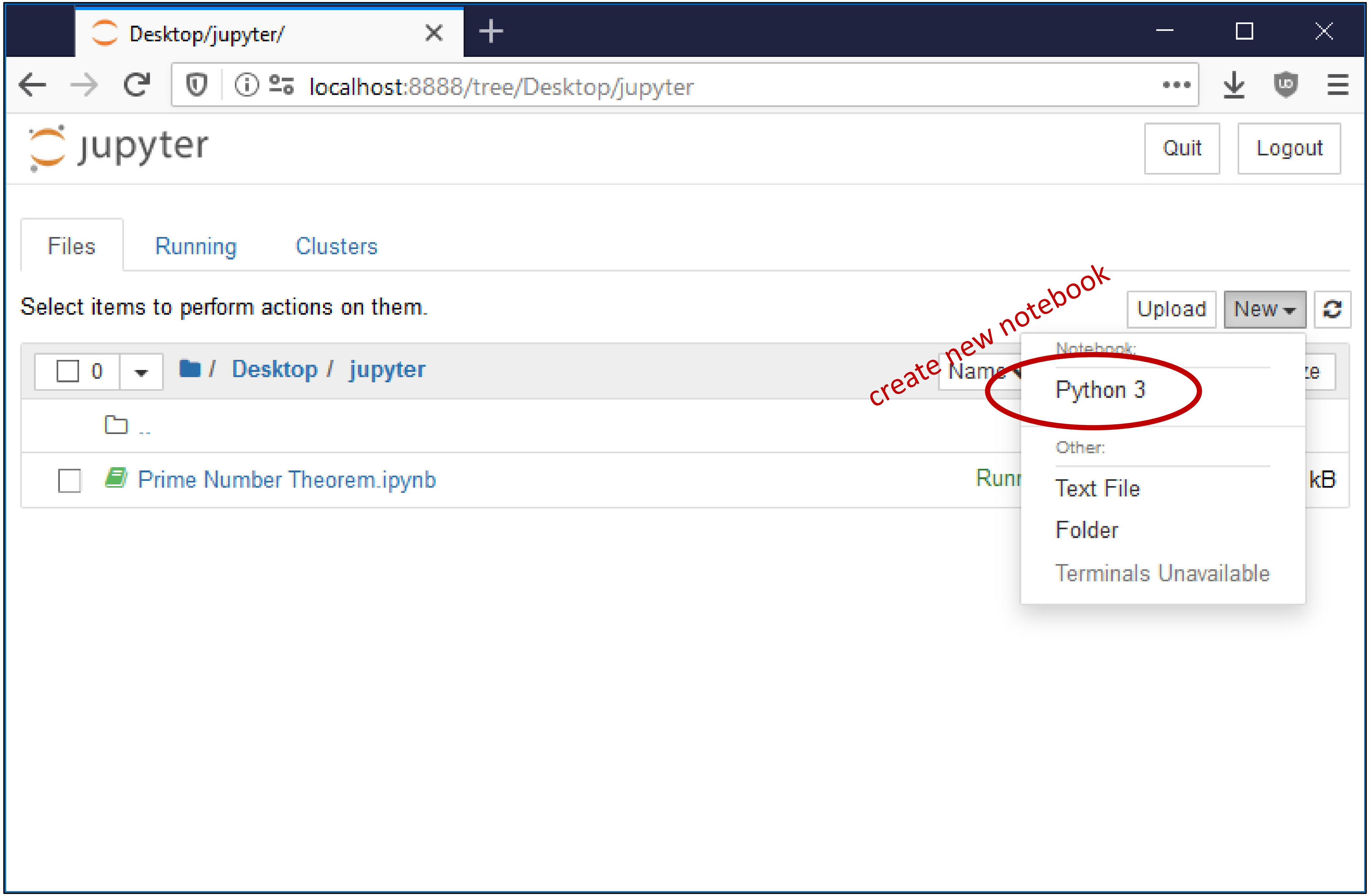
# Jupyter – launching the jupyter server

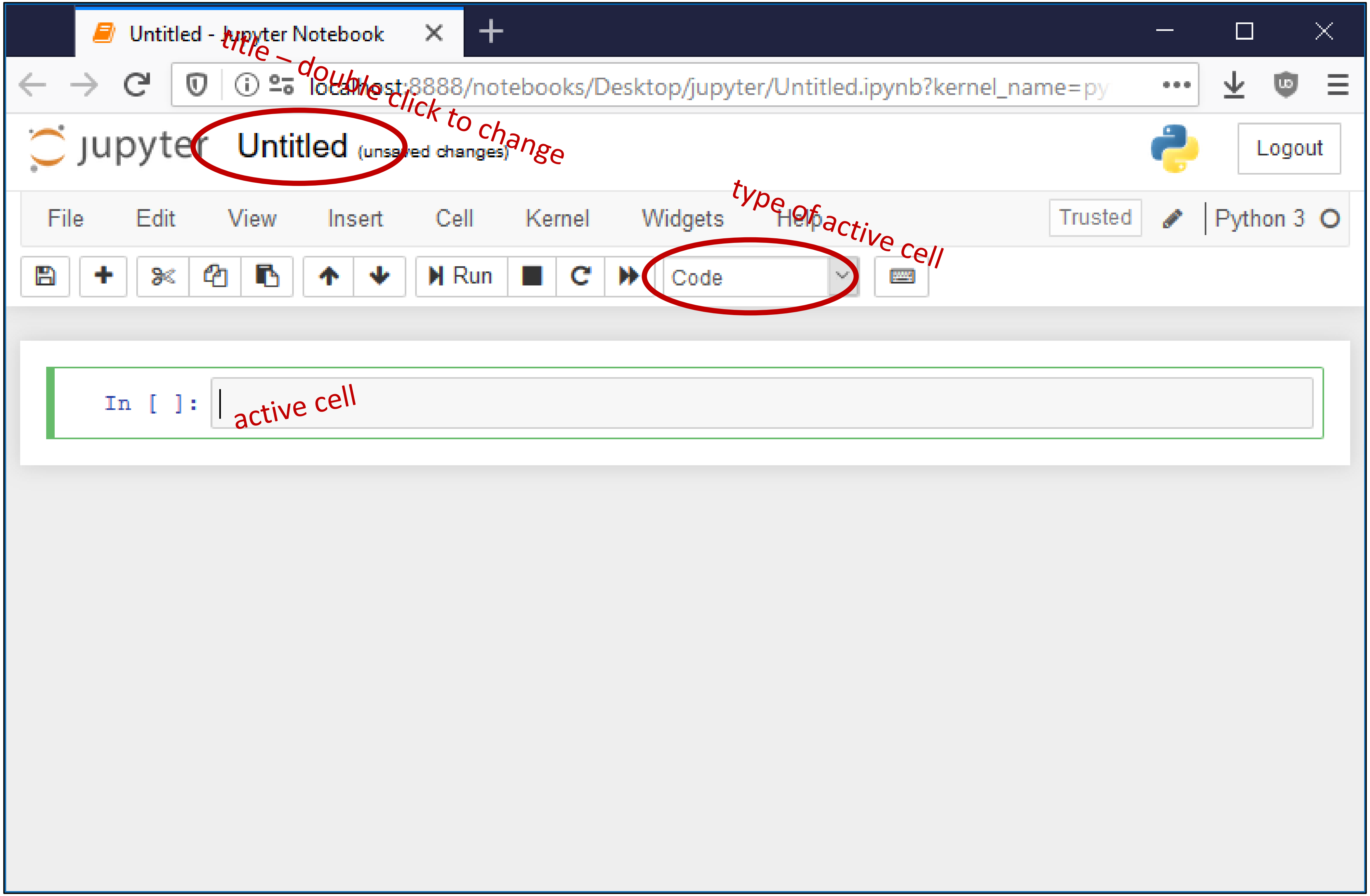
- Open a windows shell and run: `jupyter notebook`




url

- If this does not work, then try `python -m notebook`

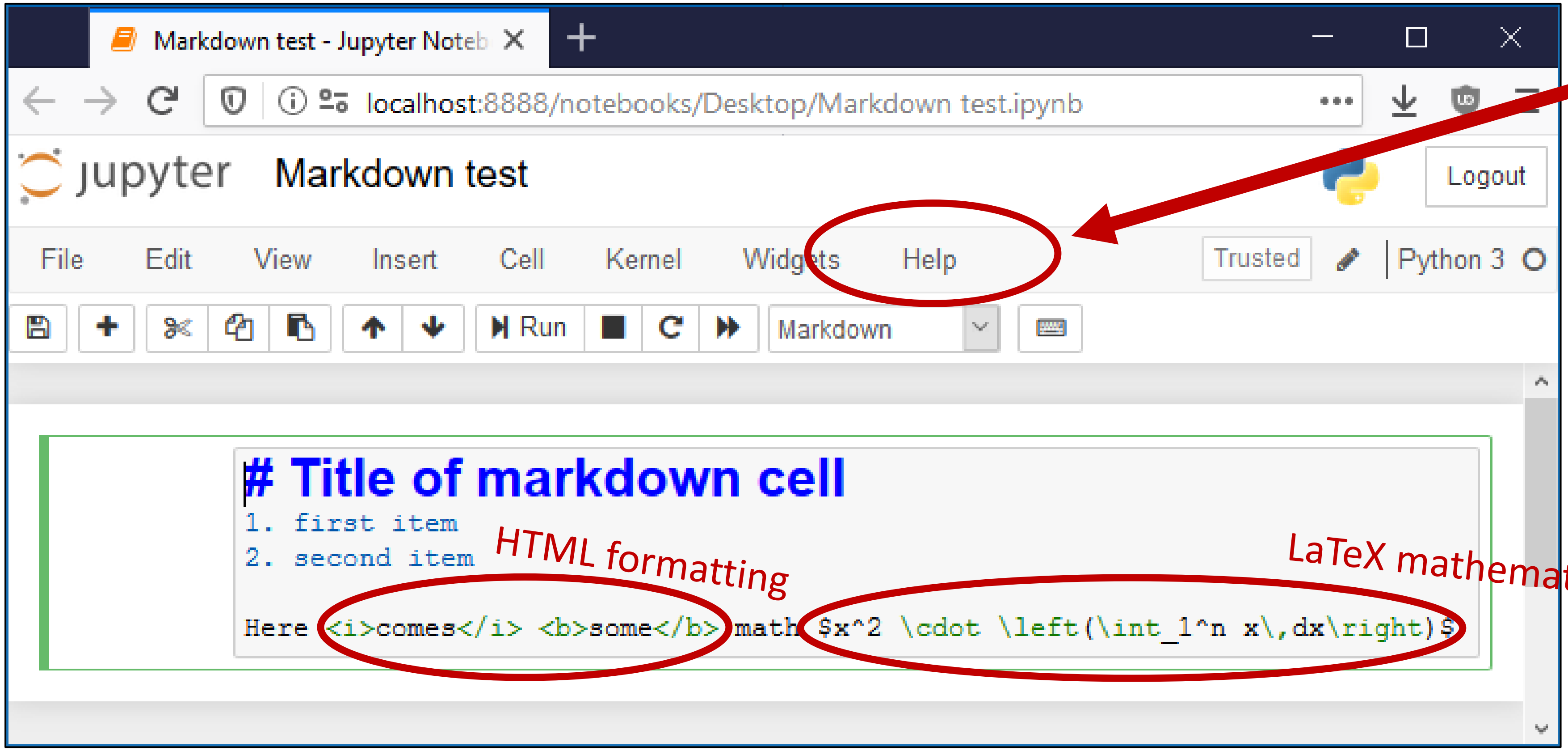

Desktop/jupyter/
localhost:8888/tree/Desktop/jupyter
jupyter
Quit
Logout
Files
Running
Clusters
Select items to perform actions on them.
Upload
New
create new notebook
0
/ Desktop / jupyter
Name
Notebook:
Python 3
Other:
Text File
Folder
Terminals Unavailable
..
Prime Number Theorem.ipynb
kB


Untitled - Jupyter Notebook
localhost:8888/notebooks/Desktop/jupyter/Untitled.ipynb?kernel_name=py
jupyter
Untitled
(unsaved changes)
Logout
File
Edit
View
Insert
Cell
Kernel
Widgets
Help
Trusted
Python 3
Run
Code
In [ ]:
title – double click to change
type of active cell
active cell

**Try:**

Help > User Interface Tour

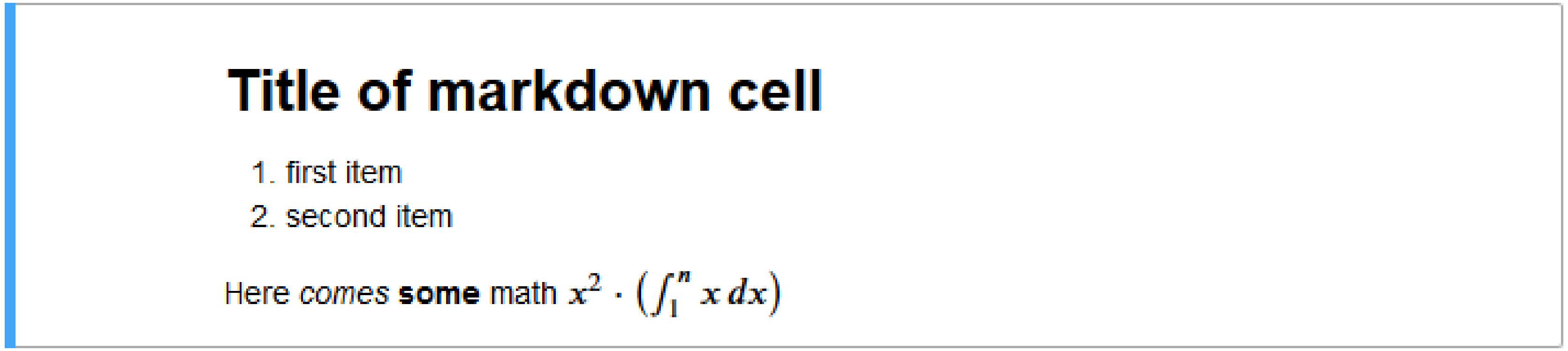
Help > Markdown




after pressing

Ctrl + Enter (evaluates)

Alt + Enter (evaluates + new cell)

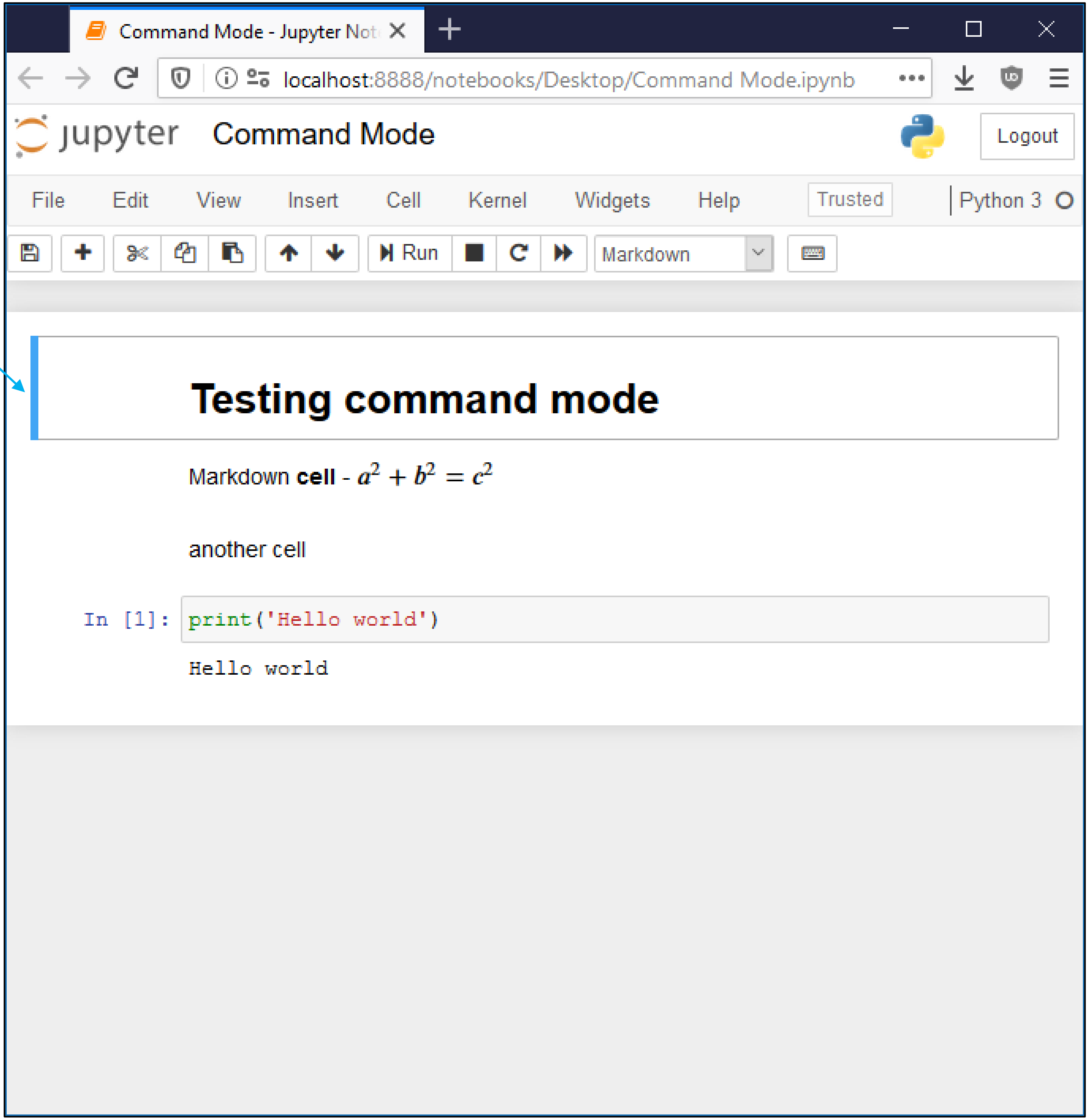



github.com/adam-p/markdown-here/wiki/Markdown-Cheatsheet

# Command Mode

- Used to naviagte between cells
- Current cell is marked with blue bar
- Keyboard shortcuts

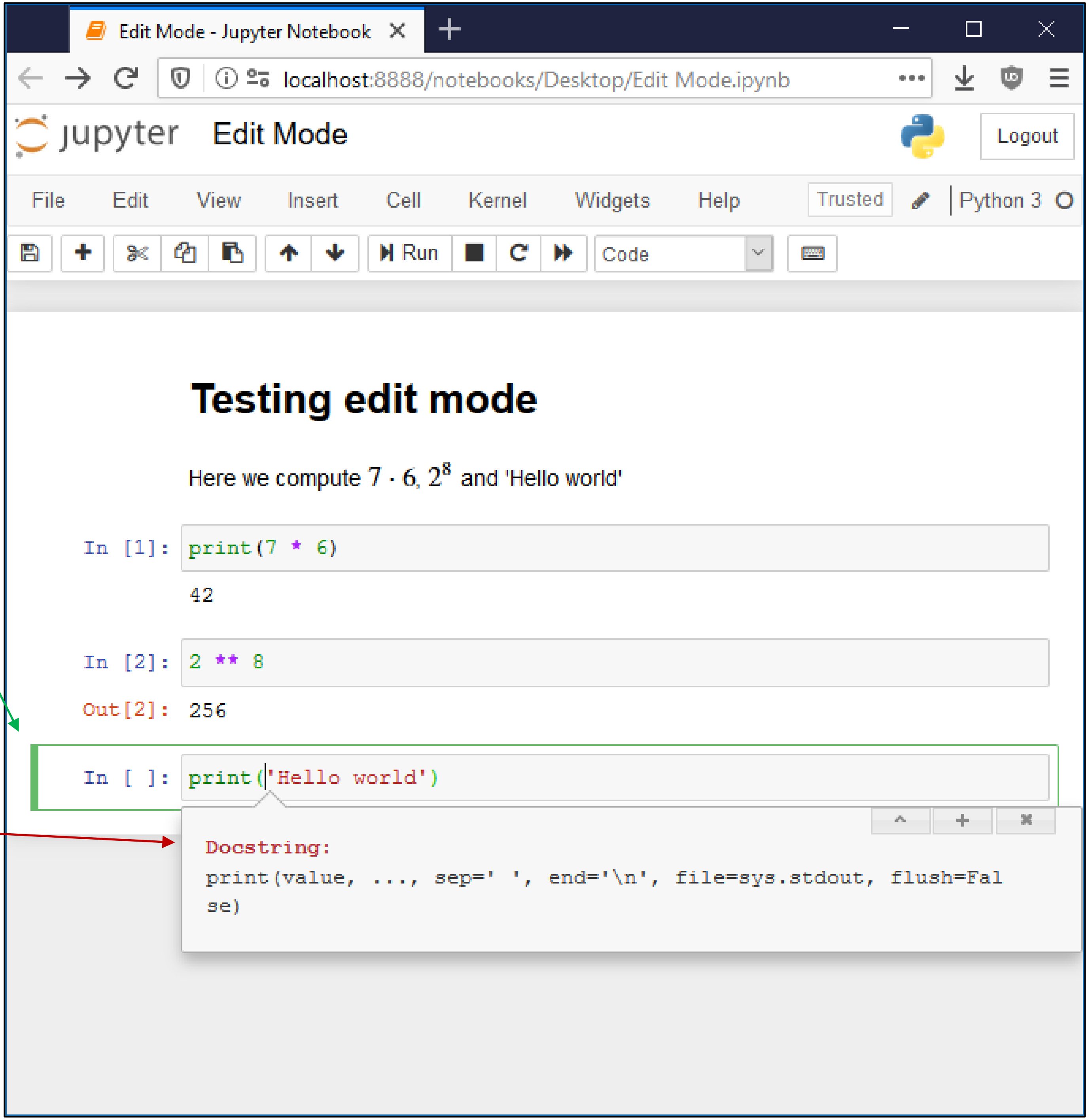
| | |
|---|---|
| h | show keyboard shortcuts |
| enter | enter Edit Mode on current cell |
| shift-enter | run cell + select below |
| ctrl-enter | run selected cells |
| alt-enter | run cell and insert below |
| Y M R | change cell type (code, markdown, raw text) |
| 1 2 3 4 5 6 | change heading level |
| ctrl-A | select all cells |
| down up | move to next/previous cell |
| space shift-space | scroll down/up |
| shift-up shift-down | extend selected cells |
| A B | insert cell above/below |
| X C V shift-V Z DD | cut, copy, paste below/above, undo, delete cells |
| shift-L | toggle line numbers in cells |
| shift-M | merge selected cells (or with cell below) |
| O | toggle output of selected cells |
| shift-O | toggle scrollbar on selected cells (long output) |



# Edit Mode

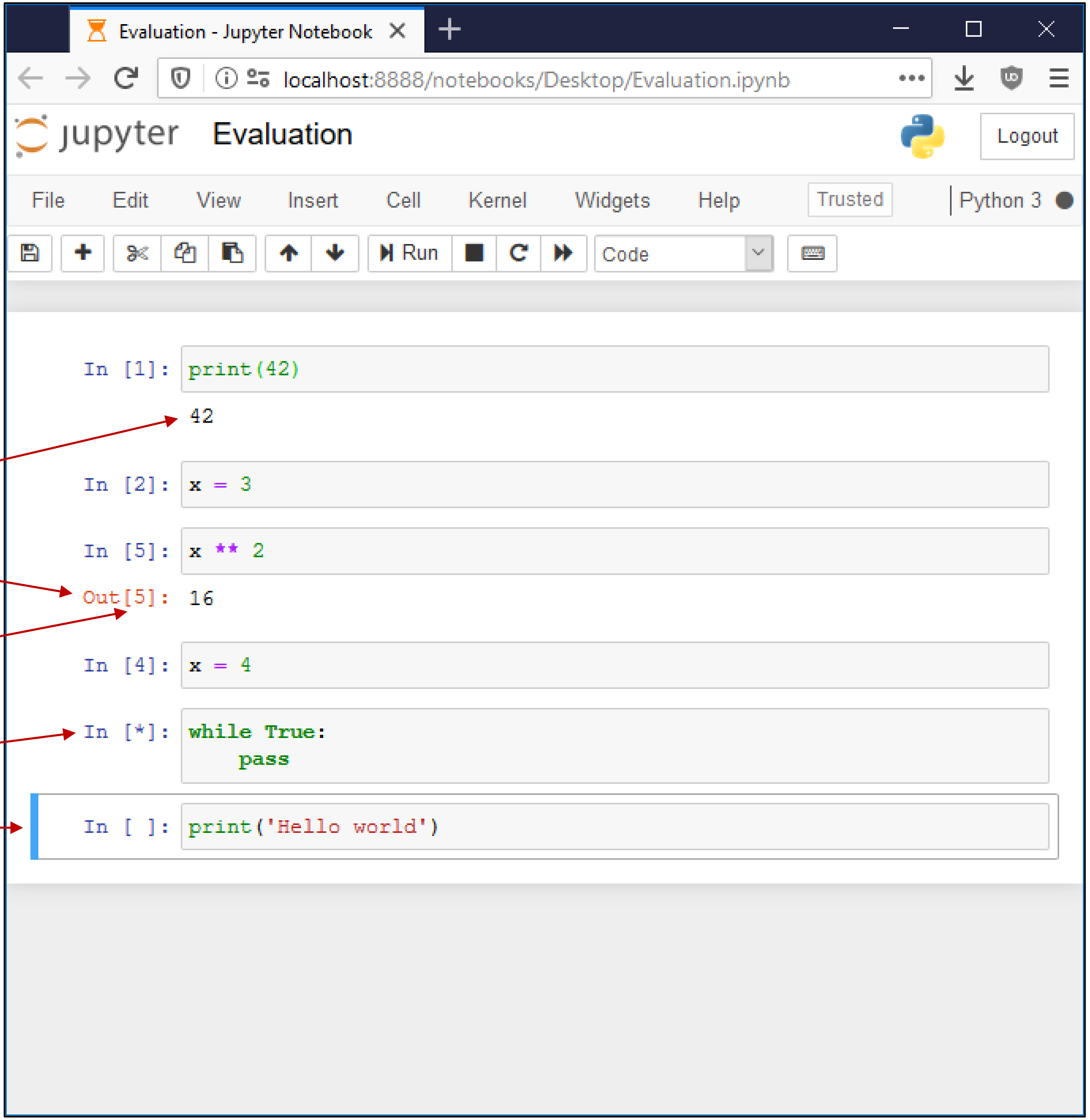
- Used to edit current cell
- Current cell is marked with green bar
- Keyboard shortcuts

| | |
|---|---|
| esc | enter Command Mode |
| shift-enter | run cell + select below |
| ctrl-enter | run selected cells |
| alt-enter | run cell and insert below |
| ctrl-shift- - | split cell at cursor |
| ctrl-shift-f | command palette |
| tab | indent or code completion |
| shift-tab | show docstring |
| ctrl-a -x -c -v -z -y | select all, cut, copy, paste, undo, redo |
| ctrl-d | delete line |



# Evaluating cells

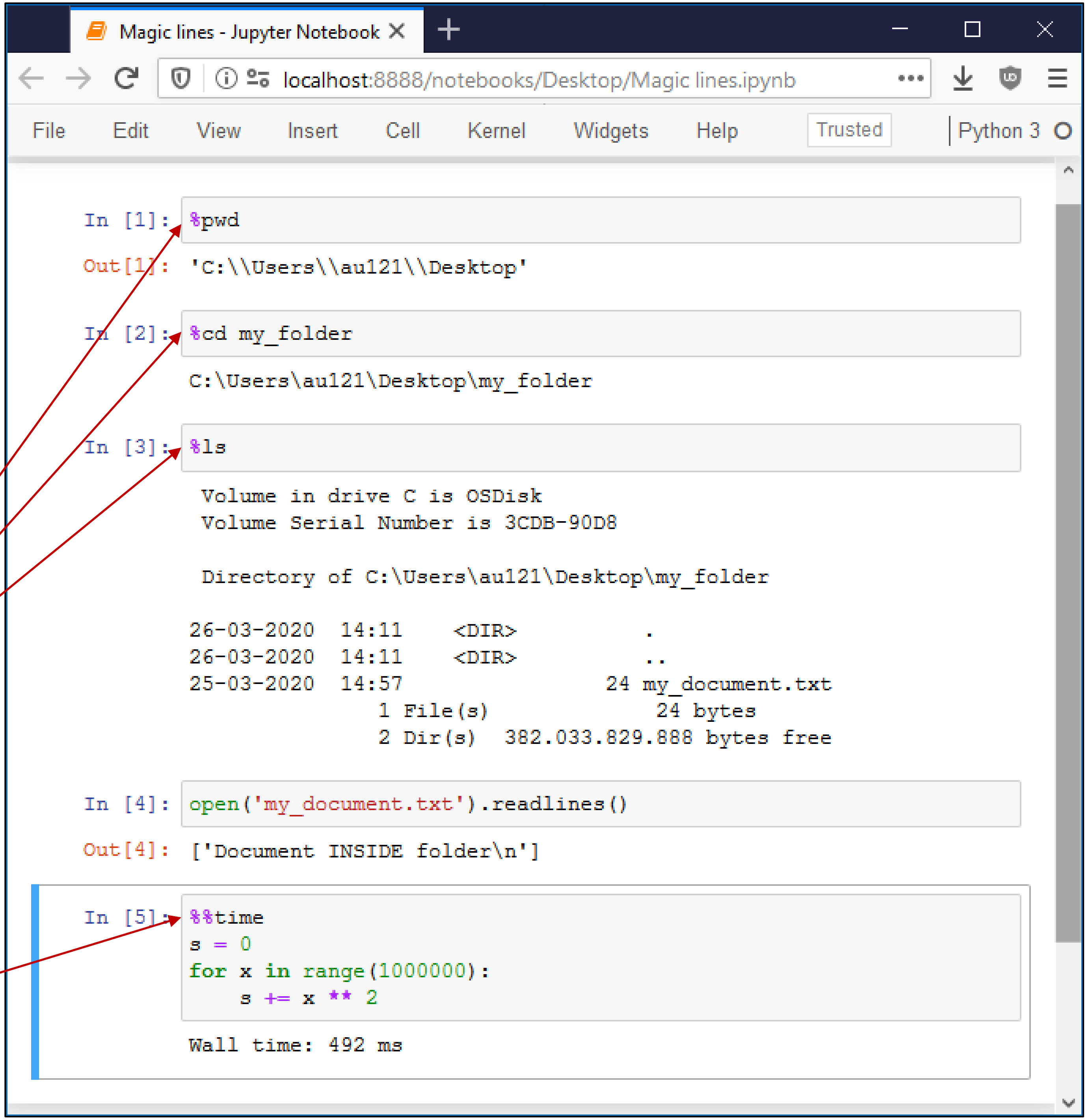
- To evaluate cell
  ctrl-enter, alt-enter, shift-enter
- Output from program shown below cell
- Result of last evaluated line
- Order of code cells evaluated
  Note ”x ** 2” computed after ”x = 4”
- [*] are cells being evaluated / waiting
- [ ] not yet evaluated
- Recompute all cells top-down
   or Kernel > Restart & Run all



# Magic lines

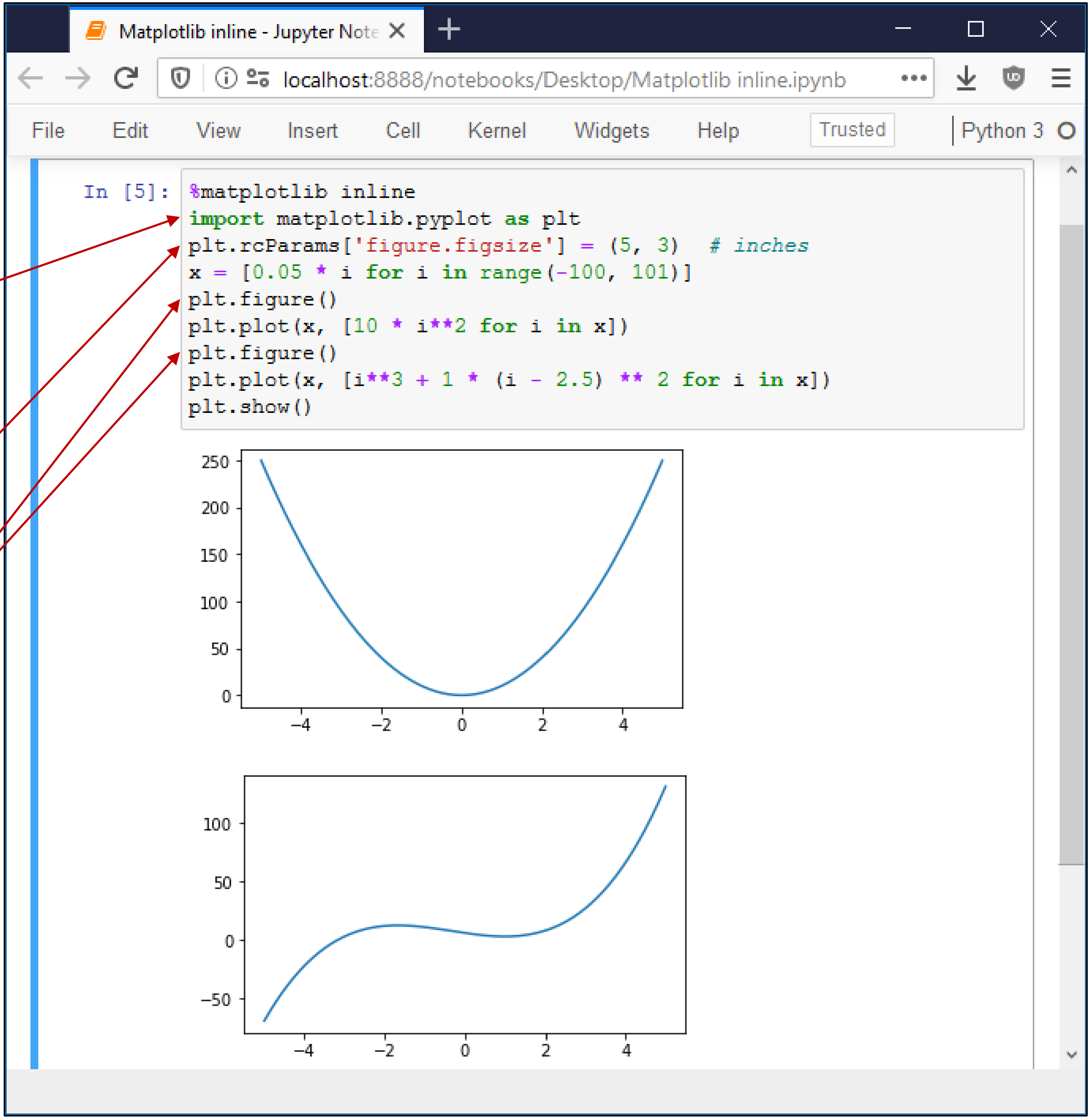
- Jupyter code cells support *magic commands* (actually it is IPython)
- % is a *line magic*
- %% is a *cell magic*

| | |
|---|---|
| %lsmagic | list magic commands |
| %quickref | quick reference sheet to IPython |
| %pwd | print working directory (current folder) |
| %cd *directory* | change directory (absolut or relative) |
| %ls | list content of current directory |
| %pip *or* %conda | run pip or conda from jupyter |
| %load *script* | insert external script into cell |
| %run *program* | run external program and show output |
| %automagic | toggle if %-prefix is required |
| %matplotlib inline | no zoom & resize, allows multiple plots |
| %matplotlib notebook | a single plot can be zoomed & resized |
| %%writefile *file* | write content of cell to a file |
| %%time | measure time for cell execution |
| %timeit *expression* | time for simple expression |



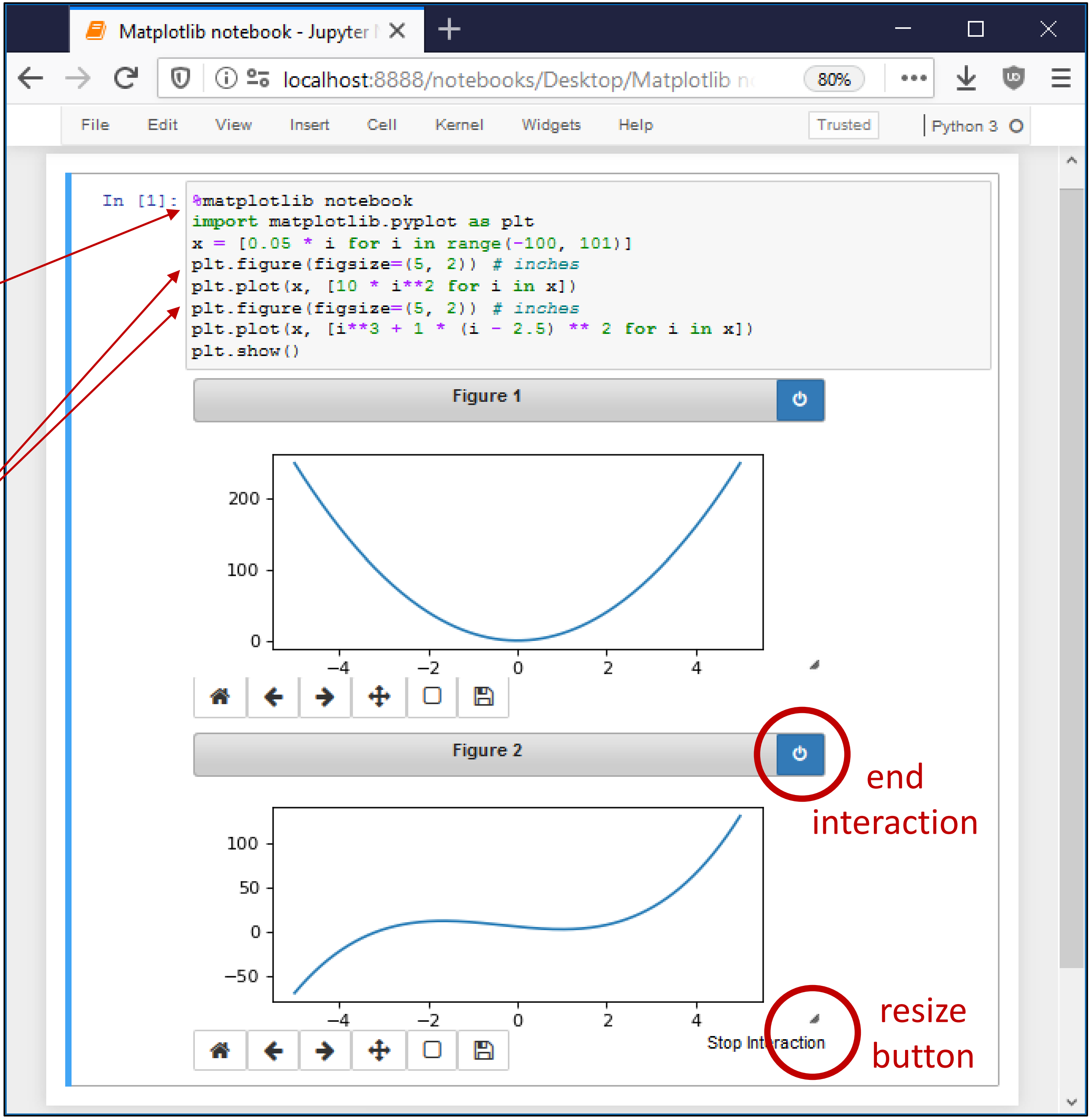
# Jupyter
# and matplotlib

- `%matplotlib inline`
  pyplot figures are shown *without* interactive zoom and pan (default)
- Consider changing default figure size
  `plt.rcParams['figure.figsize']`
- Start each figure with `plt.figure`
- Final call to `show` can be omitted



# Jupyter and matplotlib

- `%matplotlib notebook`
  pyplot figures are shown *with* interactive zoom and pan
- Start each figure with `plt.figure` (also allows setting figure size)
- Final call to `show` can be omitted



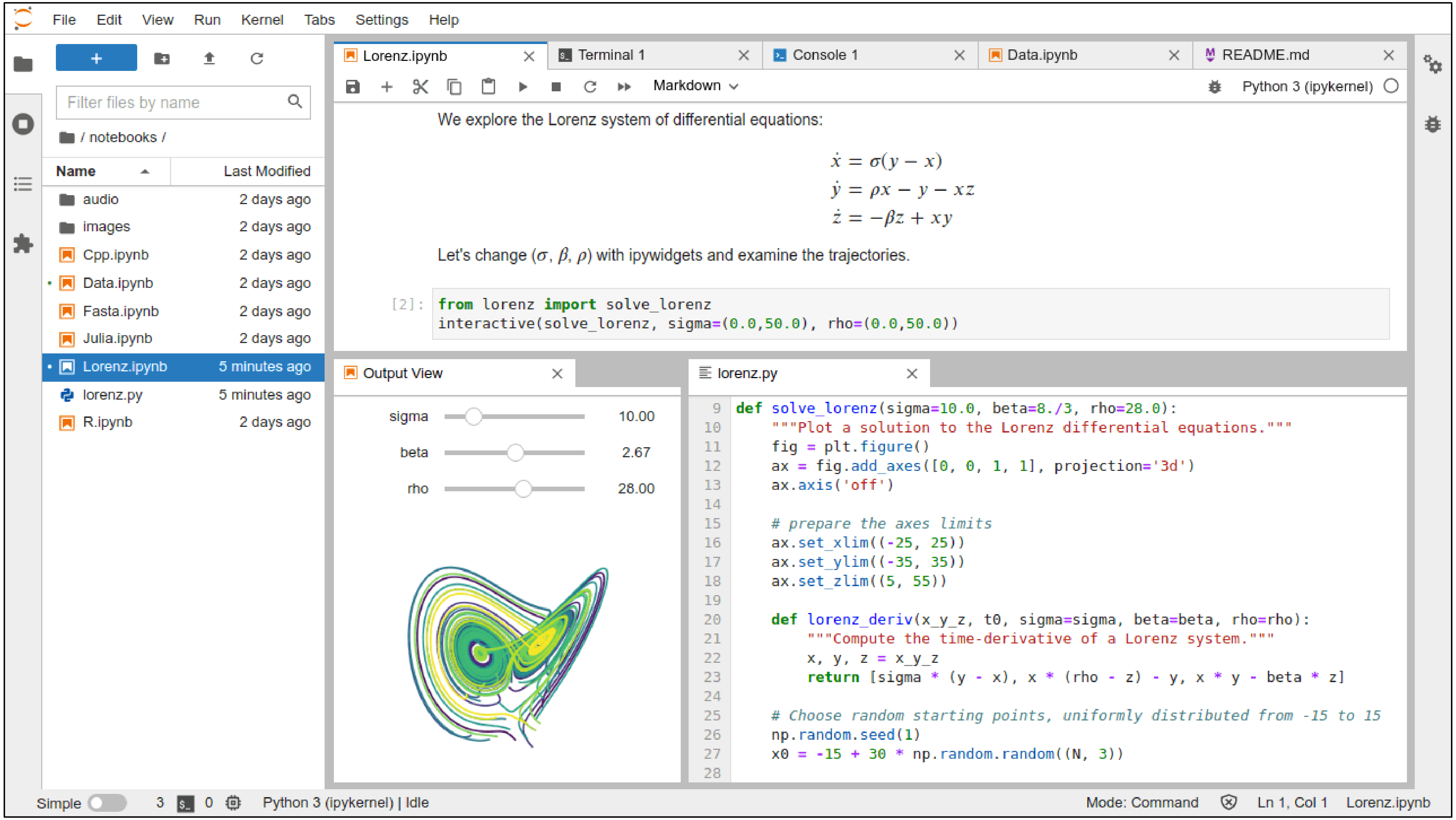
jupyter

- Widespread tool used for data science applications
- Documentation, code for data analysis, and resulting visualizations are stored in one common format
- Easy to update visualizations
- Works with about 100 different programming languages (not only Python 3), many special features, ....
- Easy to share data analysis

- *Many online tutorials and examples are available*

https://www.youtube.com/results?search_query=jupyter+python

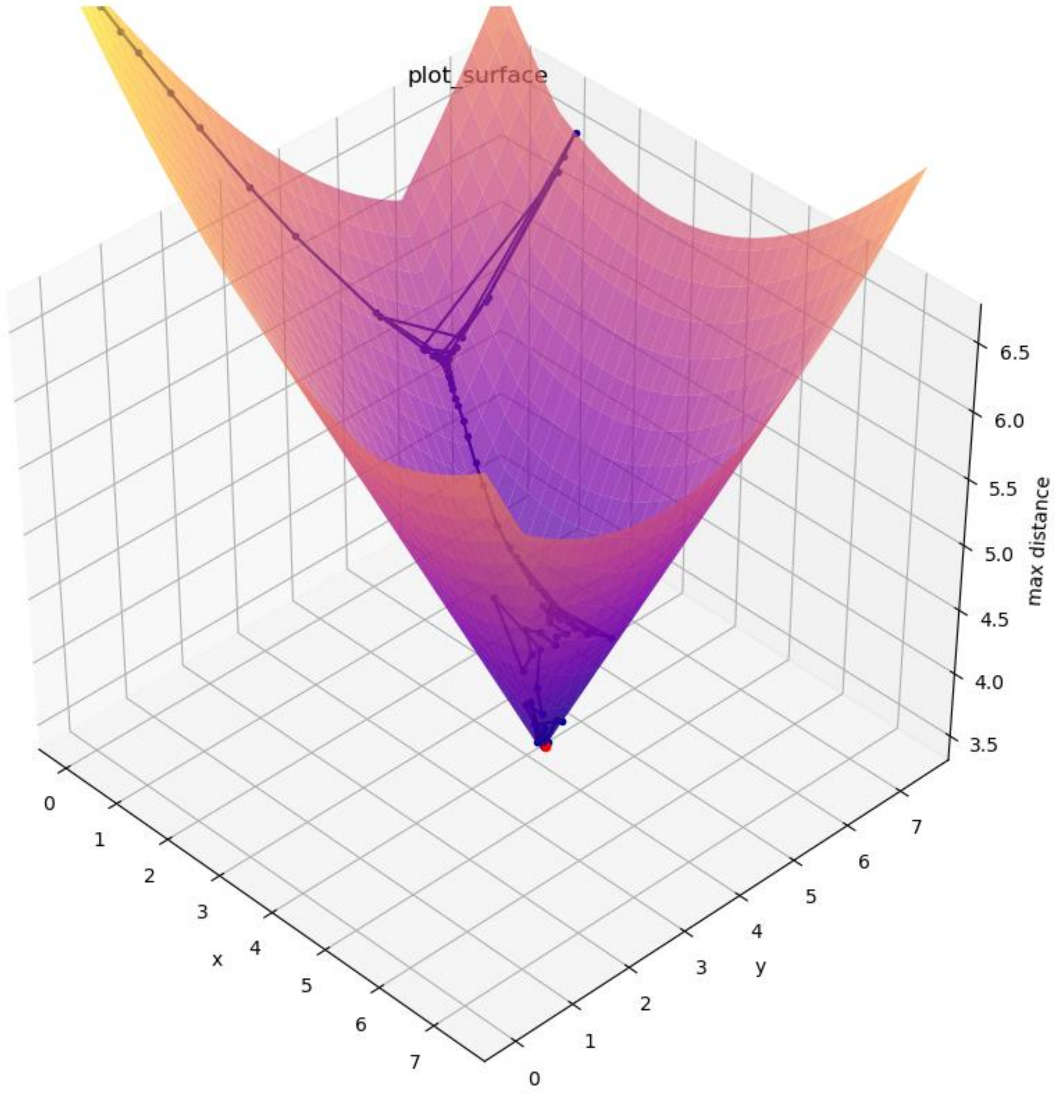
# JupyterLab: A Next-Generation Notebook Interface




jupyter.org    jupyterlab.readthedocs.io

# scipy.optimize.minimize

- Find point p minimizing function f
- Supports 13 algorithms – but no guarantee that result is correct
- Knowledge about optimization will help you know what optimization algorithm to select and what parameters to provide for better results
- ⚠ WARNING ⚠
  Many solvers return the wrong value when used as a black box




pip install scipy

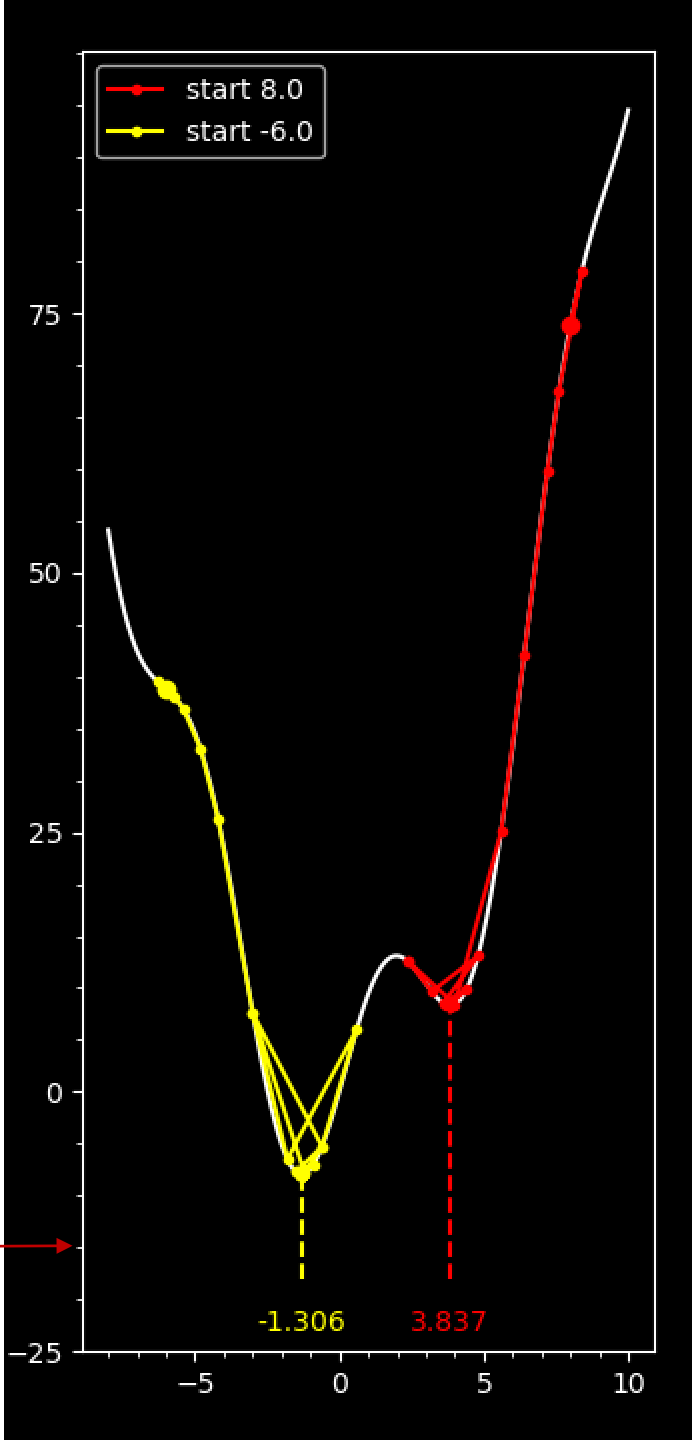
docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.minimize.html

**minimize.py**

```
from math import sin
import matplotlib.pyplot as plt
from scipy.optimize import minimize

trace = []  # remember calls to f

def f(x):
    value = x ** 2 + 10 * sin(x)
    trace.append((x, value))
    return value

X = [-8 + 18 * i / 9999 for i in range(1000)]
Y = [f(x) for x in X]

plt.style.use('dark_background')
plt.plot(X, Y, 'w-')
for start, color in [(8, 'red'), (-6, 'yellow')]:
    trace = []
    solution = minimize(f, [start], method='nelder-mead')

    x, y = solution.x[0], solution.fun
    plt.plot(*zip(*trace), '.-', c=color, label=f'start {start:.1f}') # trace
    plt.plot(*trace[0], 'o', c=color)                               # first trace point
    plt.text(x, -23, f'{x:.3f}', c=color, ha='center')              # show minimum x
    plt.plot([x, x], [-18, y], '--', c=color)                       # dash to minimum
plt.xticks(range(-5, 15, 5))
plt.yticks(range(-25, 100, 25))
plt.minorticks_on()
plt.legend()
plt.show()
```

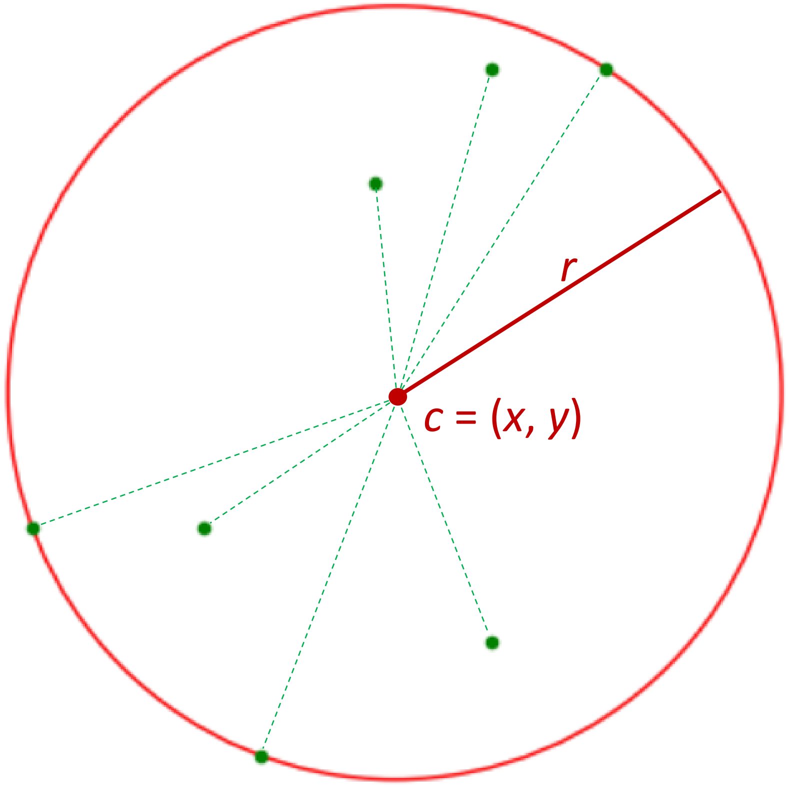
**Python shell**

```
> print(solution)
| final_simplex: (array([[-1.3064209 ],
|   [-1.30649414]]), array([-7.94582337, -7.94582336]))
|          fun: -7.94582337348758
|      message: 'Optimization terminated successfully.'
|         nfev: 38
|          nit: 19
|       status: 0
|      success: True
|            x: array([-1.3064209])
```




**minimize** tries to find a local minimum for **f** by repeatedly evaluating **f** for different **x** values

# Example: Minimum enclosing circle



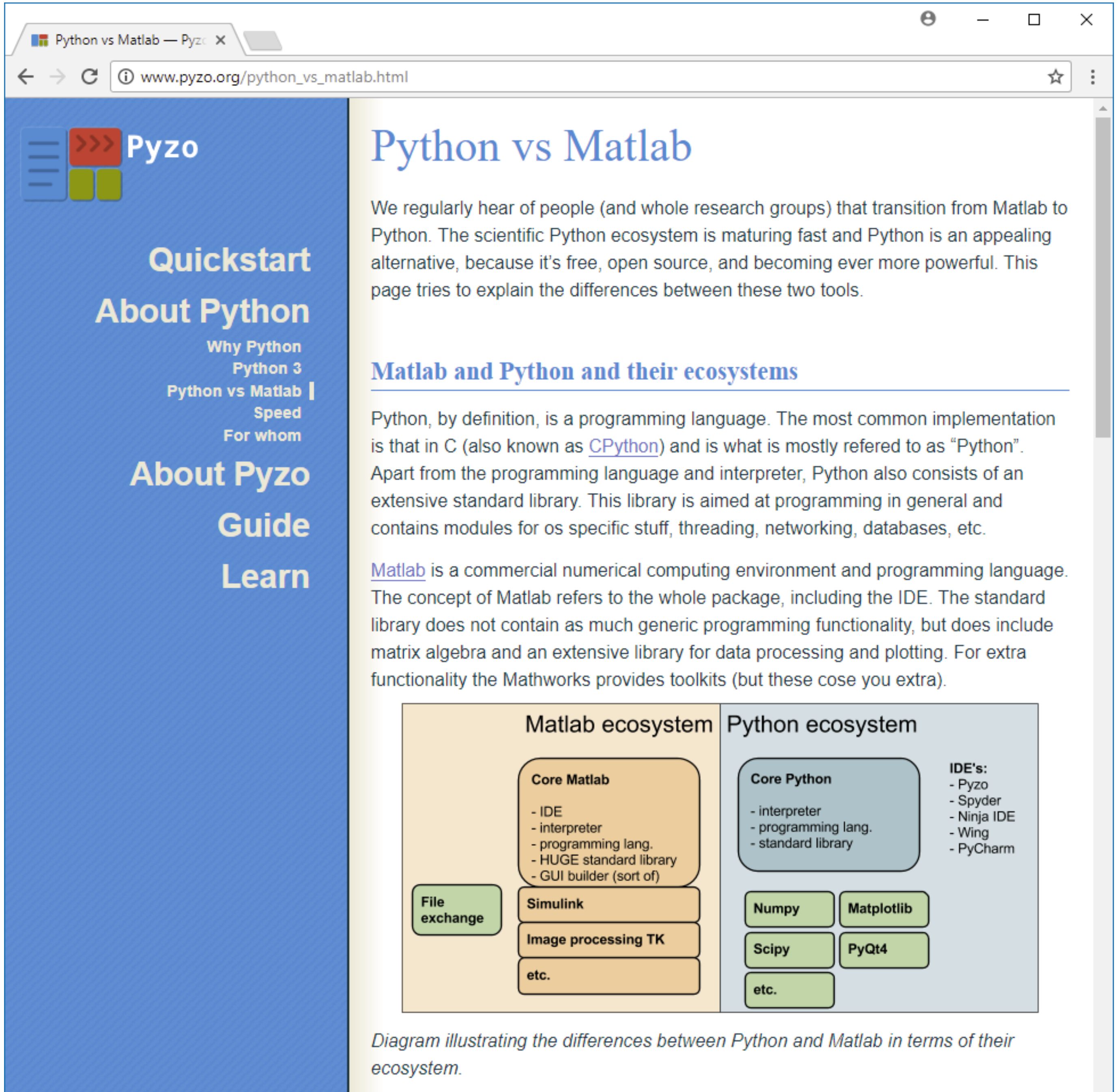

- Find $c$ such that $r = \max_p |p - c|$ is minimized
- A solution is characterized by either
  1) three points on circle, where the triangle contains the circle center
  2) two opposite points on diagonal
- Try a standard numeric minimization solver
- ⚠ Computation involves max and $\sqrt{x}$, which can be hard for numeric optimization solvers

# Python/scipy vs MATLAB

Some basic differences

- "**end**" closes a MATLAB block
- ";" at end of command avoids command output
- a(i) instead a[i]
- 1st element of a list a(1)
- a(i:j) includes both a(i) and a(j)

like R, Mathematica, Julia, AWK, Smalltalk, ...


Python vs Matlab — Pyzo

www.pyzo.org/python_vs_matlab.html

Pyzo

Quickstart

About Python

Why Python

Python 3

Python vs Matlab

Speed

For whom

About Pyzo

Guide

Learn

# Python vs Matlab

We regularly hear of people (and whole research groups) that transition from Matlab to Python. The scientific Python ecosystem is maturing fast and Python is an appealing alternative, because it's free, open source, and becoming ever more powerful. This page tries to explain the differences between these two tools.

## Matlab and Python and their ecosystems

Python, by definition, is a programming language. The most common implementation is that in C (also known as CPython) and is what is mostly refered to as "Python". Apart from the programming language and interpreter, Python also consists of an extensive standard library. This library is aimed at programming in general and contains modules for os specific stuff, threading, networking, databases, etc.

Matlab is a commercial numerical computing environment and programming language. The concept of Matlab refers to the whole package, including the IDE. The standard library does not contain as much generic programming functionality, but does include matrix algebra and an extensive library for data processing and plotting. For extra functionality the Mathworks provides toolkits (but these cose you extra).

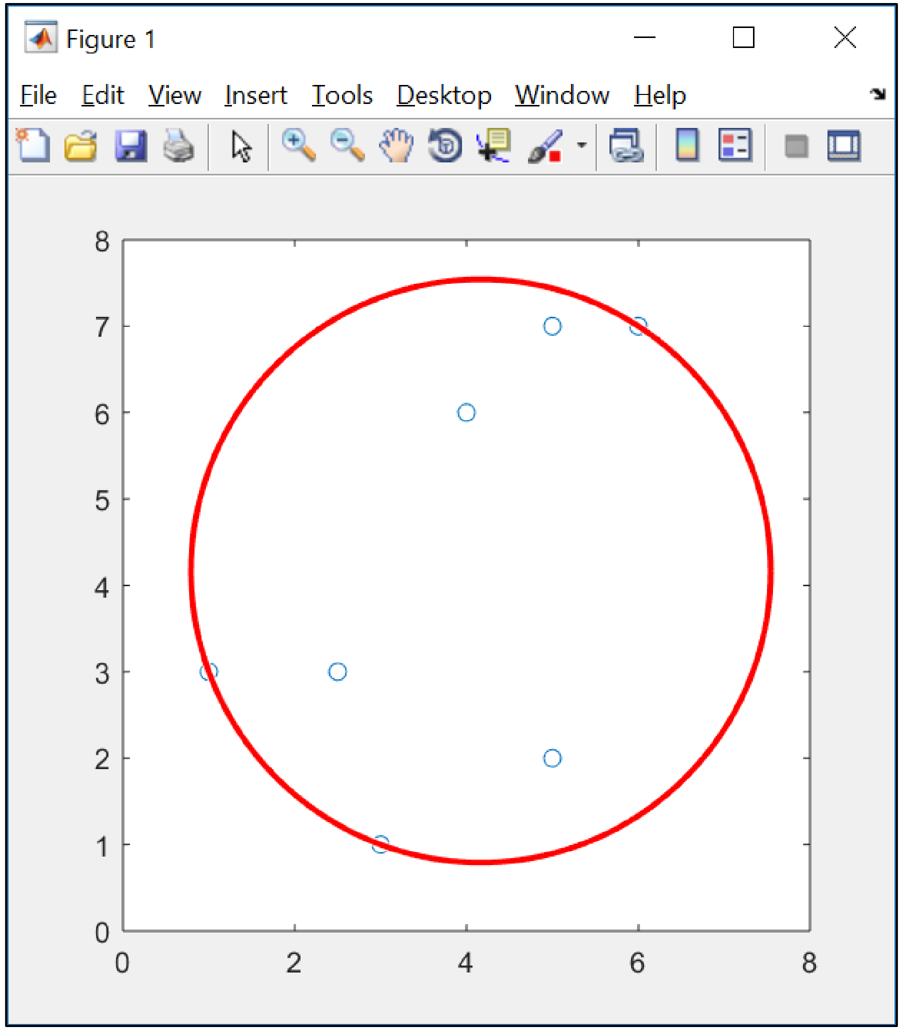


*Diagram illustrating the differences between Python and Matlab in terms of their ecosystem.*

# Minimum enclosing circle in MATLAB

**enclosing_circle.m**

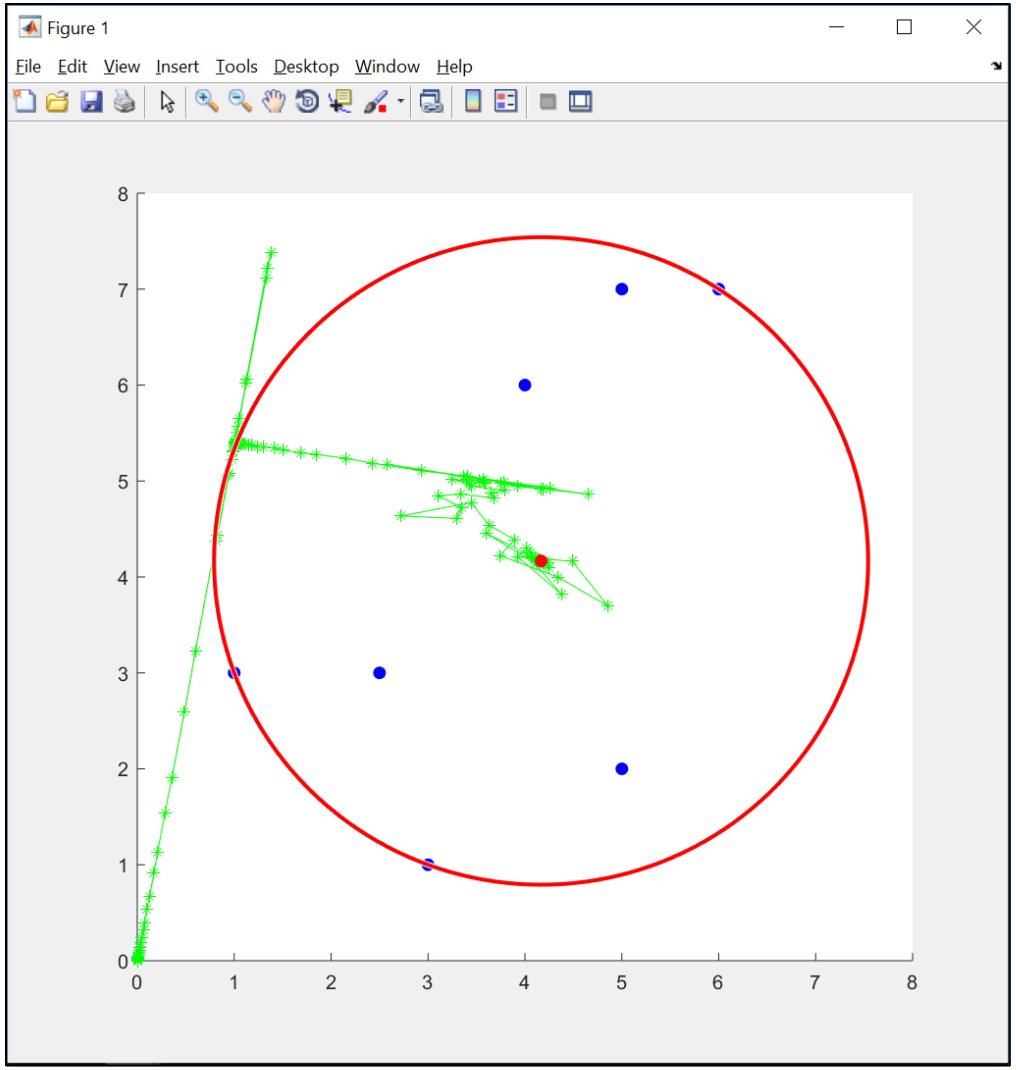
```
% Minimum enclosing circle of a point set
% fminsearch uses the Nelder-Mead algorithm

global x y
x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0];
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0];
c = fminsearch(@(x) max_distance(x), [0,0]);
plot(x, y, "o");
viscircles(c, max_distance(c));

function dist = max_distance(p)
    global x y
    dist = 0.0;
    for i=1:length(x)
        dist = max(dist, pdist([p; x(i), y(i)],
                                'euclidean'));
    end
end
```



# Minimum enclosing circle in MATLAB (trace)

enclosing_circle_trace.m

```
global x y trace_x trace_y

x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0];
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0];
trace_x = [];
trace_y = [];

c = fminsearch(@(x) max_distance(x), [0,0]);

hold on
plot(x, y, "o", 'color', 'b', 'MarkerFaceColor', 'b');
plot(trace_x, trace_y, "*-", "color", "g");
plot(c(1), c(2), "o", 'color', 'r', 'MarkerFaceColor', 'r');
viscircles(c, max_distance(c), "color", "red");

function dist = max_distance(p)
    global x y trace_x trace_y
    trace_x = [trace_x, p(1)];
    trace_y = [trace_y, p(2)];

    dist = 0.0;
    for i=1:length(x)
        dist = max(dist, pdist([p; x(i), y(i)], 'euclidean' ));
    end
end
```



# Minimum enclosing circle in Python

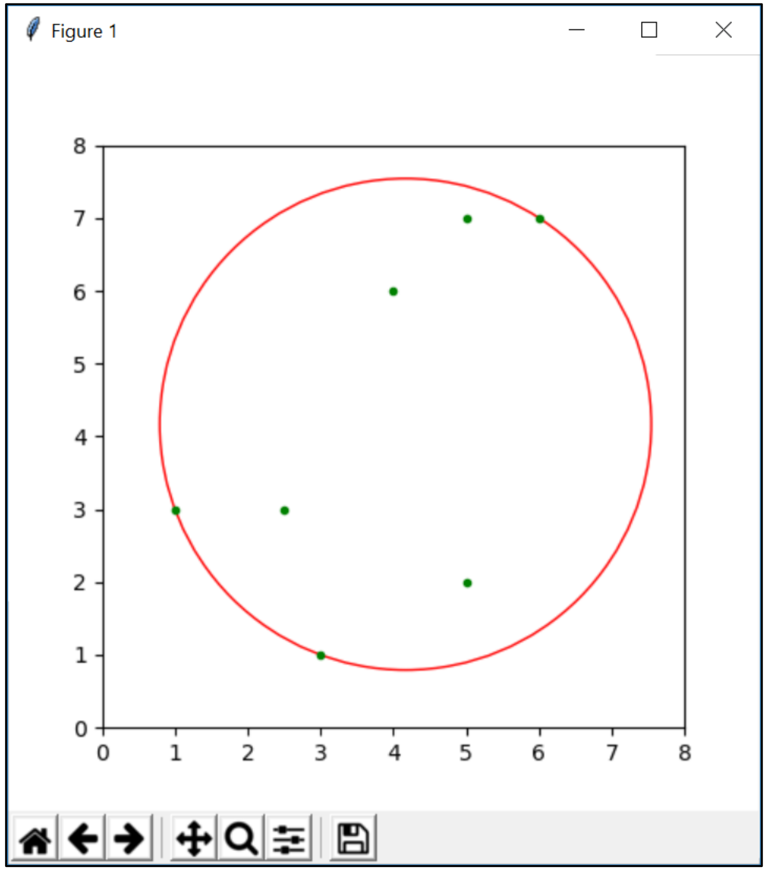
**enclosing_circle.py**

```
from scipy.optimize import minimize
import matplotlib.pyplot as plt

x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0]
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0]

def dist(p, q):
    return ((p[0] - q[0]) ** 2 + (p[1] - q[1]) ** 2)) ** 0.5
def max_distance(c):
    return max(dist(p, c) for p in zip(x, y))

c = minimize(max_distance, [0.0, 0.0], method="nelder-mead").x
ax = plt.gca()
ax.set_xlim((0, 8))
ax.set_ylim((0, 8))
ax.set_aspect("equal")
plt.plot(x, y, "g.")
ax.add_artist(plt.Circle(c, max_distance(c),
                         color="r", fill=False))
plt.show()
```

import modules

optimization method

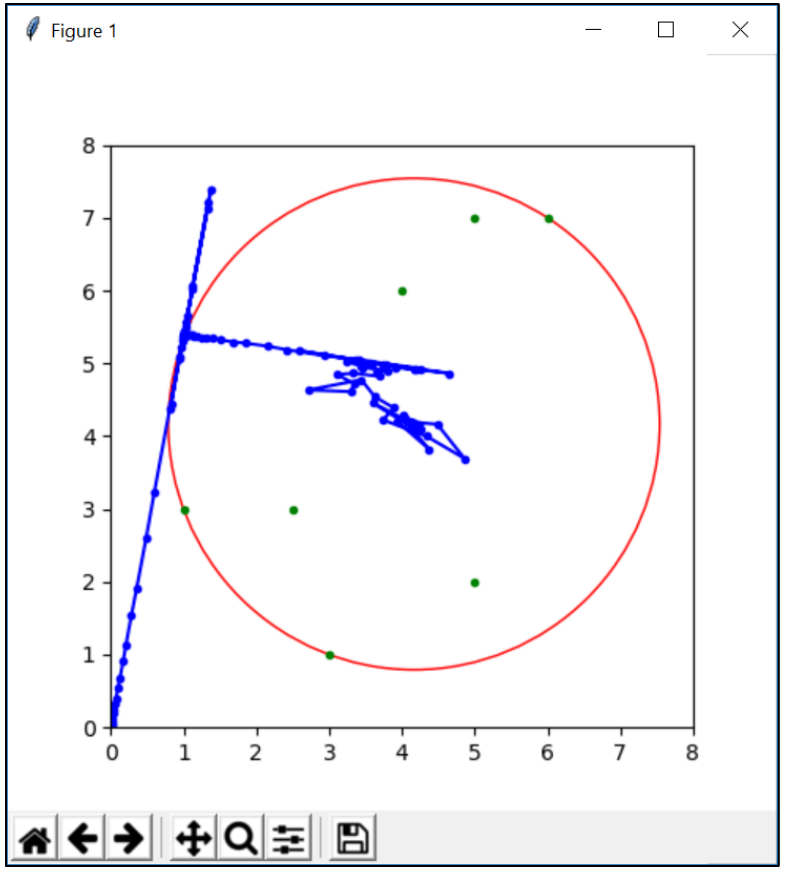
manually set axis (force circle inside plot)



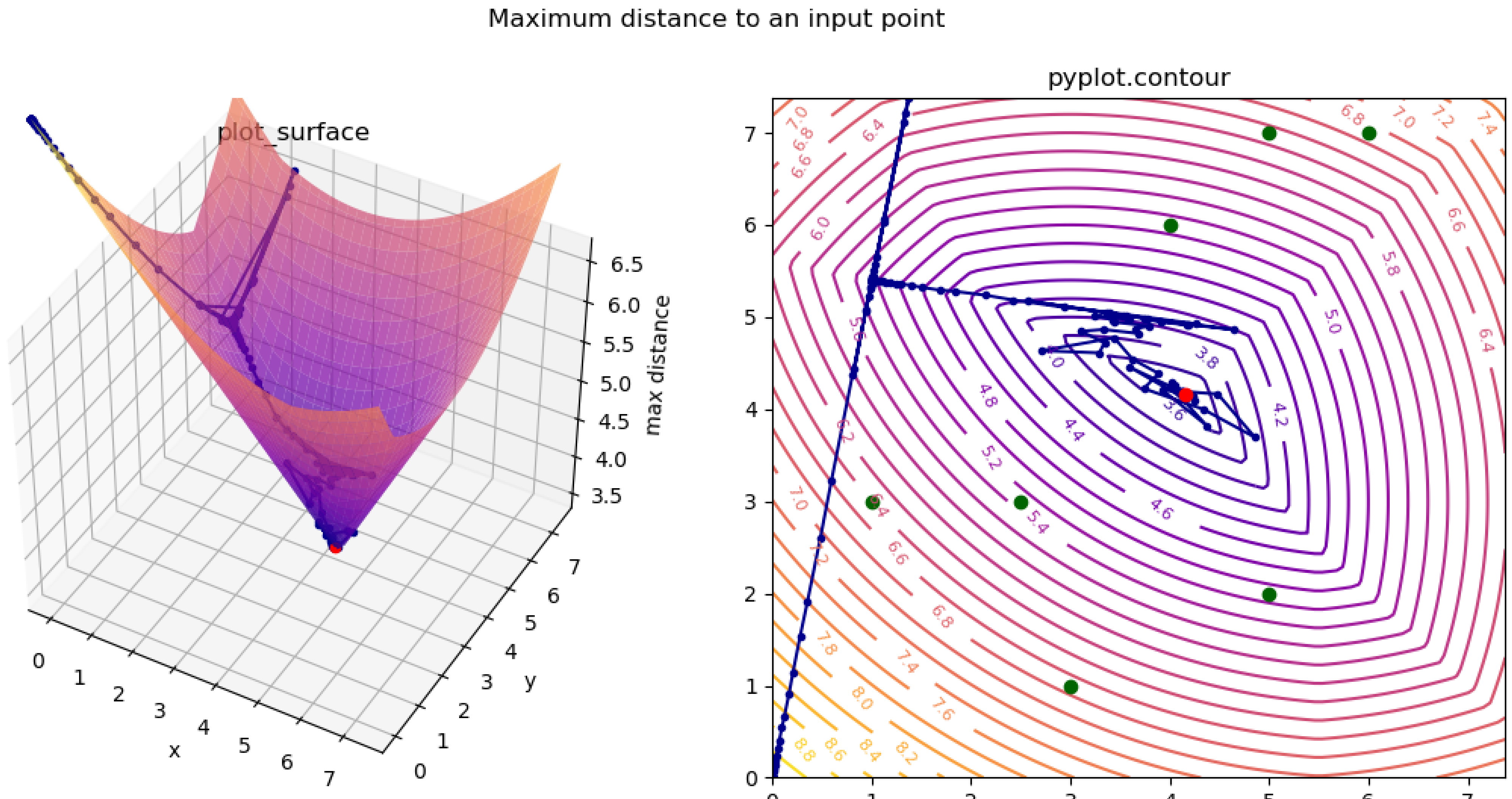
# Minimum enclosing circle in Python (trace)

enclosing_circle_trace.py

```
from scipy.optimize import minimize
import matplotlib.pyplot as plt

x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0]
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0]
trace = []

def dist(p, q):
    return ((p[0] - q[0]) ** 2 + (p[1] - q[1]) ** 2) ** 0.5
def max_distance(c):
    trace.append(c)
    return max([dist(p, c) for p in zip(x, y)])

c = minimize(max_distance, [0.0, 0.0],
             method="nelder-mead").x
ax = plt.gca()
ax.set_xlim((0, 8))
ax.set_ylim((0, 8))
ax.set_aspect("equal")
plt.plot(x, y, "g.")
plt.plot(*zip(*trace), "b.-")
ax.add_artist(plt.Circle(c, max_distance(c),
                         color="r", fill=False))
plt.show()
```



# Minimum enclosing circle – search space



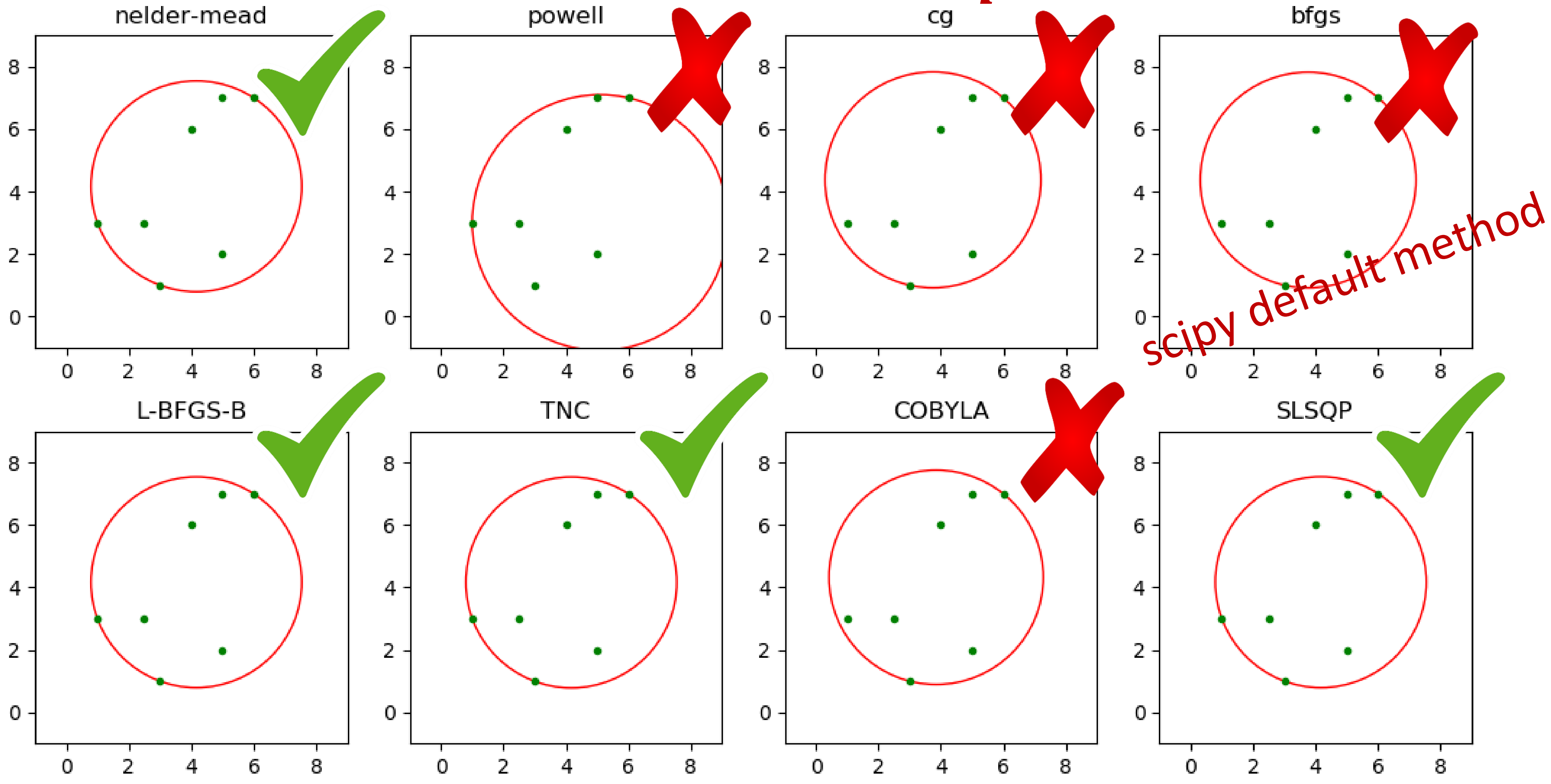
## enclosing_circle_search_space.py (previous slide)

```
from scipy.optimize import minimize
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D

points = [(1.0, 3.0), (3.0, 1.0), (2.5, 3.0),
  (4.0, 6.0), (5.0, 7.0), (6.0, 7.0), (5.0, 2.0)]

#  Minimum enclosing circle solver

trace = []

def distance(p, q):
    return ((p[0]-q[0])**2 + (p[1]-q[1])**2)**0.5

def distance_max(q):
    dist = max(distance(p, q) for p in points)
    trace.append((*q, dist))
    return dist

solution = minimize(distance_max, [0.0, 0.0],
                    method='nelder-mead')
center = solution.x
radius = solution.fun

#  unzip point coordinates

points_x, points_y = zip(*points)
trace_x, trace_y, trace_z = zip(*trace)

#  Bounding box [x_min, x_max] x [y_min, y_max]

xs, ys = points_x + trace_x, points_y + trace_y
x_min, x_max = min(xs), max(xs)
y_min, y_max = min(ys), max(ys)
# enforce apsect ratio
x_max = max(x_max, x_min + y_max - y_min)
y_max = max(y_max, y_min + x_max - x_min)
```

```
#  Minimum enclosing circle - 3D surface plot
#  (plot_surface requires X, Y, Z are 2D numpy.arrays)

X, Y = np.meshgrid(np.linspace(x_min, x_max, 100),
                   np.linspace(y_min, y_max, 100))
Z = np.zeros(X.shape)
for px, py in points:
    Z = np.maximum(Z, (X - px)**2 + (Y - py)**2)
Z = np.sqrt(Z)

ax = plt.subplot(1, 2, 1, projection='3d')
ax.plot_surface(X, Y, Z, cmap='plasma', alpha=0.7)
ax.plot(trace_x, trace_y, trace_z, '.-', c='darkblue')
ax.scatter(*center, radius, 'o', c='red')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('max distance')
ax.set_title('plot_surface')

#  Minimum enclosing circle - contour plot

plt.subplot(1, 2, 2)
plt.title('pyplot.contour')
plt.plot(trace_x, trace_y, '.-', color='darkblue')
plt.plot(points_x, points_y, 'o', color='darkgreen')
plt.plot(*center, 'o', c='red')
qcs = plt.contour(X, Y, Z, levels=30, cmap='plasma')
plt.clabel(qcs, inline=1, fontsize=8, fmt='%.1f')

plt.suptitle('Maximum distance to an input point')
plt.tight_layout()
plt.show()
```

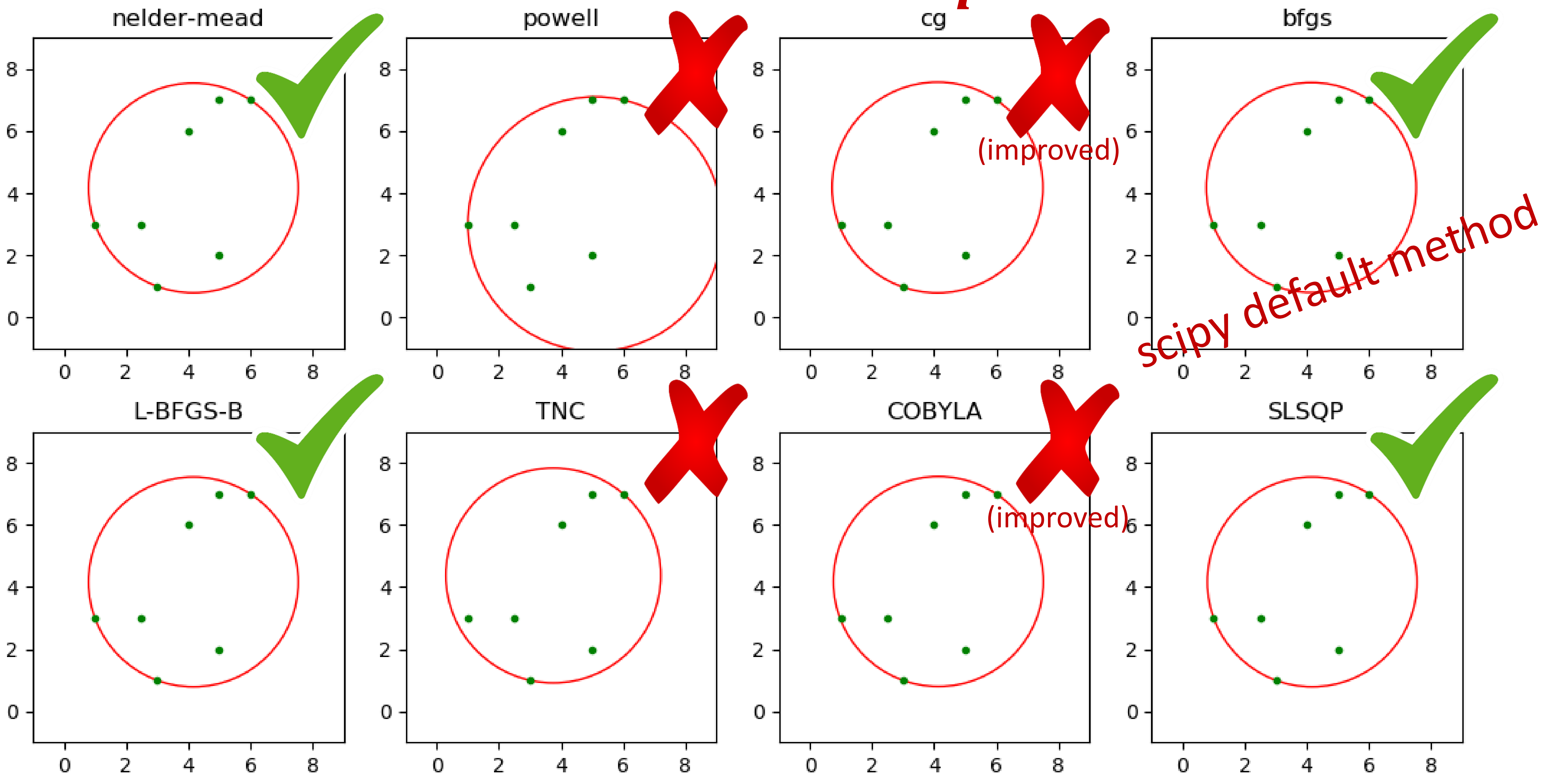
numpy arrays

# scipy.minimize $f(c) = \max_{p} |p - c|$




docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.minimize.html scipy.minimize $f(c) = \max_p |p - c|^2$

avoids $\sqrt{\ }$



# Multi-dimensional data

- NumPy
- matrix multiplication, @
- numpy.linalg.solve, numpy.polyfit

*Array programming with NumPy*
Harris et al.
Nature, volume 585, pages 357–362, 2020
DOI 10.1038/s41586-020-2649-2

- NumPy is a Python package for dealing with multi-dimensional data

www.numpy.org

docs.scipy.org/doc/numpy/user/quickstart.html

# pylab ?

- Guttag [2nd edition] uses pylab in the examples, but...

> *"pylab is a convenience module that bulk imports matplotlib.pyplot (for plotting) and numpy (for mathematics and working with arrays) in a single name space. Although many examples use pylab, it is no longer recommended."*

matplotlib.org/faq/usage_faq.html

# NumPy arrays (example)

**Python shell**

```
> range(0, 1, .3)
| TypeError: 'float' object cannot be
  interpreted as an integer
> [1 + i / 4 for i in range(5)]
| [1.0, 1.25, 1.5, 1.75, 2.0]
```

python only supports ranges of int

generate 5 uniform values in range [1,2]

**Python shell**

```
> import numpy as np
> np.arange(0, 1, 0.3)
| array([0. , 0.3, 0.6, 0.9])
> type(np.arange(0, 1, 0.3))
| <class 'numpy.ndarray'>
> help(numpy.ndarray)
| +2000 lines of text
> np.linspace(1, 2, 5)
| array([1.  , 1.25, 1.5 , 1.75, 2.  ])
```

numpy can generate ranges with float

returns a ”NumPy array” (not a list)

”arange” ≡ ”array range” and generates the array explicitly

generate n uniformly spaced values

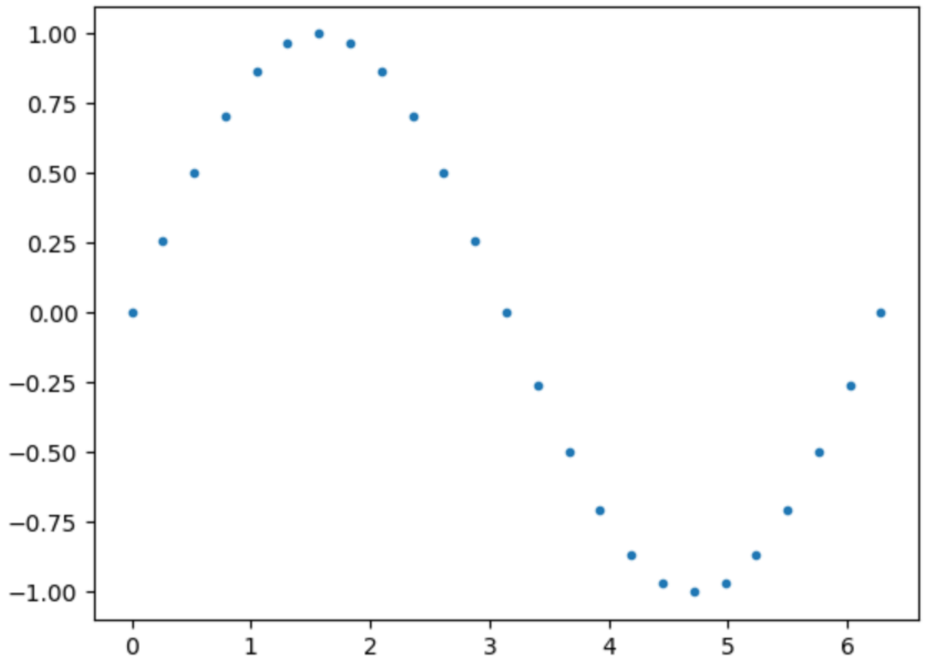
# Plotting a function (example)

**sin.py**

```
import matplotlib.pyplot as plt
import math
n = 25
x = [2 * math.pi * i / (n - 1) for i in range(n)]
y = [math.sin(v) for v in x]
plt.plot(x, y, '.')
plt.show()
```

**sin_numpy.py**

```
import matplotlib.pyplot as plt
import numpy as np
x = np.linspace(0, 2 * np.pi, 25)
y = np.sin(x)
plt.plot(x, y, '.')
plt.show()
```

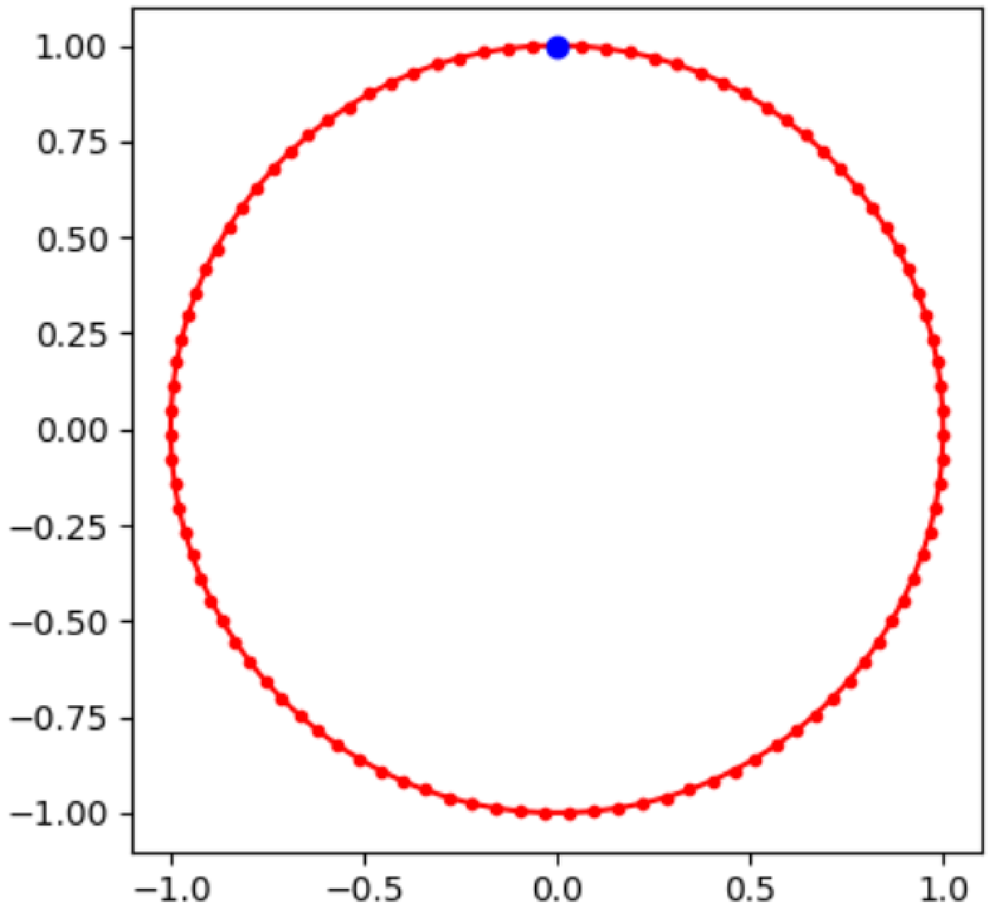



- `np.sin` applies the `sin` function *to each* element of `x`
- `pyplot` accepts NumPy arrays
- `math.pi == np.pi` $\approx \frac{22}{7}$

# A circle

circle.py

```
import matplotlib.pyplot as plt
import numpy as np

a = np.linspace(0, 2 * np.pi, 100)
x = np.sin(a)
y = np.cos(a)

plt.plot(x, y, 'r.-')
plt.plot(x[0], y[0], 'bo')
plt.show()
```

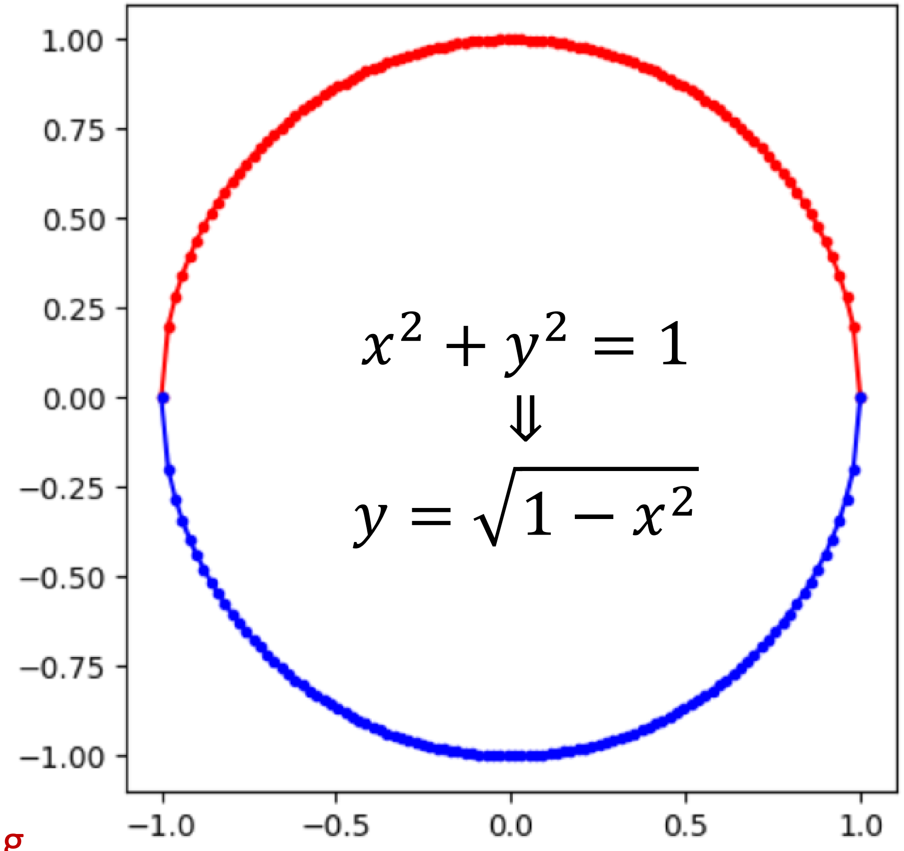


# Two half circles

**half_circles.py**

```python
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(-1, 1, 100)
plt.plot(x, np.sqrt(1 - x ** 2), 'r.-')
plt.plot(x, -np.sqrt(1 - x ** 2), 'b.-')
plt.show()
```

compact expression computing
something quite comlicated




- `x` is a NumPy array
- `**` NumPy method `__pow__` squaring each element in `x`
- binary – NumPy method `__rsub__` that for each element e in `x` computes 1 - e
- `np.sqrt` NumPy method computing the square root of each element in `x`
- unary – NumPy method `__neg__` that negates each element in `x`

# Creating one-dimensional NumPy arrays

**Python shell**

```
> np.array([1, 2, 3])
| array([1, 2, 3])
> np.array((1, 2, 3))
| array([1, 2, 3])
> np.array(range(1, 4))
| array([1, 2, 3])
> np.arange(1., 4., 1.)
| array([1., 2., 3.])
> np.linspace(1, 3, 3)
| array([1., 2., 3.])
> np.zeros(3)
| array([0., 0., 0.])
> np.ones(3)
| array([1., 1., 1.])
> np.full(3, 7)
| array([7, 7, 7])
> np.random.random(3)
| array([0.73761651,
  0.60607355, 0.3614118 ])
```

```
> np.array([1, 2, 3]).dtype   # type of all values
| dtype('int32')
> np.arange(3, dtype='float')
| array([0., 1., 2.])
> np.arange(3, dtype='int16') # 16 bit integers
| array([0, 1, 2], dtype=int16)
> np.arange(3, dtype='int32') # 32 bit integers
| array([0, 1, 2])
> 1000 ** np.arange(5)
| array([1, 1000, 1000000, 1000000000,
  -727379968], dtype=int32)   # OOPS.. overflow
> 1000 ** np.arange(5, dtype='O')
| array([1, 1000, 1000000, 1000000000,
  1000000000000], dtype=object) # Python integer
> np.arange(3, dtype='complex')
| array([0.+0.j, 1.+0.j, 2.+0.j])
```

Elements of a NumPy array are not arbitrary precision integers by default – you can select between +25 number representations

docs.scipy.org/doc/numpy-1.13.0/user/basics.types.html

# Mantissa size in various numpy floats

**Python shell**

```
> for data_type in ['half', 'float', 'single', 'double', 'longdouble', 'float32', 'float64']:
    x = np.array([1], dtype=data_type)
    for i in range(100):
      if x == x + (x / 2) ** i:
        break
    print(data_type, i - 1, 'bits mantissa')
| half 10 bits mantissa
| float 52 bits mantissa
| single 23 bits mantissa
| double 52 bits mantissa
| longdouble 52 bits mantissa
| float32 23 bits mantissa                # platform independent
| float64 52 bits mantissa                # platform independent
```

$$1.\overbrace{\underbrace{00001}_{i}00000}^{\text{mantissa}}{}_2$$

[docs.scipy.org/doc/numpy-1.13.0/user/basics.types.html](docs.scipy.org/doc/numpy-1.13.0/user/basics.types.html)

# Creating multi-dimensional NumPy arrays

**Python shell**

```
> np.array([[1, 2, 3], [4, 5, 6]])
| array([[1, 2, 3],
|        [4, 5, 6]])
> np.arange(1, 7).reshape(2, 3)
| array([[1, 2, 3],
|        [4, 5, 6]])
> x = np.arange(12).reshape(2, 2, 3)
> x
| array([[[ 0,  1,  2],
|         [ 3,  4,  5]],
|
|        [[ 6,  7,  8],
|         [ 9, 10, 11]]])
> numpy.zeros((2, 5), dtype='int32')
| array([[0, 0, 0, 0, 0],
|        [0, 0, 0, 0, 0]])
```

```
> x.size
| 12
> x.ndim
| 3
> x.shape
| (2, 2, 3)
> x.dtype
| dtype('int32')
> x.flatten()
| array([0,1,2,3,4,5,6,7,8,9,10,11])
> list(x.flat)   # flat is an iterator
| [0,1,2,3,4,5,6,7,8,9,10,11]
> np.eye(3)      # diagonal 2D array
| array([[1., 0., 0.],
|        [0., 1., 0.],
|        [0., 0., 1.]])
```

# View vs Copy

- Numpy is optimized to handle big multi-dimensional arrays
- To *avoid copying* data, **views** allows one to look at the underlying data in different ways (data can be shared by multiple views)
- `reshape`, `ravel` and slicing are examples creating views
- `flatten` and `ravel` both turn multiple dimensional arrays into one dimensional arrays but `flatten` creates an explicit copy whereas `ravel` creates a space efficient view

**Python shell**

```
> x = np.arange(6)
> y = x.reshape(2, 3)   # view
> y[0][0] = 42          # updates x
> x
| array([42, 1, 2, 3, 4, 5])
> y
| array([[42, 1, 2],
|        [ 3, 4, 5]])
> z = y.flatten()       # copy
> z[5] = 0
> z
| array([42, 1, 2, 3, 4, 0])
> x
| array([42, 1, 2, 3, 4, 5])
> w = y.ravel()        # view
> w[5] = -1
> w
| array([42, 1, 2, 3, 4, -1])
> x
| array([42, 1, 2, 3, 4, -1])
```

# NumPy operations

**Python shell**

```
> x = numpy.arange(3)
> x
| array([0, 1, 2])
> x + x  # elementwise addition
| array([0, 2, 4])
> 1 + x  # add integer to each element
| array([1, 2, 3])
> x * x  # elementwise multiplication
| array([0, 1, 4])
> np.dot(x, x)  # dot product
| 5
> np.cross([1, 2, 3], [3, 2, 1]) # cross product
| array([-4,  8, -4])
```

```
> a = np.arange(6).reshape(2,3)
> a
| array([[0, 1, 2],
|        [3, 4, 5]])
> a.T  # matrix transposition, view
| array([[0, 3],
|        [1, 4],
|        [2, 5]])
> a @ a.T  # matrix multiplication
| array([[ 5, 14],
|        [14, 50]])
> a += 1
> a
| array([[1, 2, 3],
|        [4, 5, 6]])
```

PEP 465 - A dedicated infix operator for matrix multiplication

# Universal functions (apply to each entry)

**Python shell**

```
> x = np.array([[1, 2], [3, 4]])
> np.sin(x)   # also: cos, exp, sqrt, log, ceil, floor, abs
| array([[ 0.84147098,  0.90929743],
|        [ 0.14112001, -0.7568025 ]])
> np.sign(np.sin(x))
| array([[ 1.,  1.],
|        [ 1., -1.]])
> np.mod(np.arange(10), 3)   # same as: np.arange(10) % 3
| array([0, 1, 2, 0, 1, 2, 0, 1, 2, 0], dtype=int32)
```

# Axis

**Python shell**

```
> x = np.arange(1, 7).reshape(2, 3)
> x
| array([[1, 2, 3],
|        [4, 5, 6]])
> x.sum()   # = x.sum(axis=(0, 1))
| 21
> x.sum(axis=0)
| array([5, 7, 9])
> x.sum(axis=1)
| array([6, 15])
> x.min()   # = x.min(axis=(0, 1))
| 1
```

**Python shell**

```
> x.min(axis=0)
| array([1, 2, 3])
> x.min(axis=1)
| array([1, 4])
> x.cumsum()
| array([ 1,  3,  6, 10, 15, 21], dtype=int32)
> x.cumsum(axis=0)
| array([[1, 2, 3],
|        [5, 7, 9]], dtype=int32)
> x.cumsum(axis=1)
| array([[ 1,  3,  6],
|        [ 4,  9, 15]], dtype=int32)
```

# Slicing

**Python shell**

```
> x = numpy.arange(20).reshape(4,5)
> x
| array([[ 0,  1,  2,  3,  4],
|        [ 5,  6,  7,  8,  9],
|        [10, 11, 12, 13, 14],
|        [15, 16, 17, 18, 19]])
> x[2, 3]  # =  x[(2, 3)]
| 13
> x[1:4:2, 2:4:1]  # rows 1 and 3, and columns 2 and 3, view
| array([[ 7,  8],
|        [17, 18]])
> x[:, 3]
| array([ 3,  8, 13, 18])
> x[..., 3]  # ... is placeholder for ':' for all missing dimensions
| array([ 3,  8, 13, 18])
> type(...)
| <class 'ellipsis'>
```

# Broadcasting (stretching arrays to get same size)

- Numpy tries to apply *broadcasting*, if array shapes do not match, i.e. adds missing leading dimensions and repeats a dimension with only one element:

```
  [[1],[2]] + [10,20]          column + row vector
≡ [[1],[2]] + [[10,20]]        both ndim = 2
≡ [[1,1],[2,2]] + [[10,20]]
≡ [[1,1],[2,2]] + [[10,20],[10,20]]
≡ [[11,21],[12,22]]
```

- To prevent unexpected broadcasting, add an assertion to your program:

```
assert x.shape == y.shape
```

docs.scipy.org/doc/numpy/user/basics.broadcasting.html

**Python shell**

```
> x = np.array([[1, 2, 3], [4, 5, 6]])
> y = np.array([1, 2, 3])   # one row
> z = np.array([[1], [2]]) # one column
> x + 3  # add 3 to each entry
| array([[4, 5, 6],
|        [7, 8, 9]])
> x + y  # add y to each row
| array([[2, 4, 6],
|        [5, 7, 9]])
> x + z  # add z to each column
| array([[2, 3, 4],
|        [6, 7, 8]])
> y + z  # 2 rows with y + 3 columns with z
| array([[2, 3, 4],
|        [3, 4, 5]])
> z == z.T  # [[1,1],[2,2]] == [[1,2],[1,2]]
| array([[ True, False],
|        [False,  True]])
```

# Masking

**Python shell**

```
> x = np.arange(1, 11).reshape(2, 5)
> x
| array([[ 1,  2,  3,  4,  5],
|        [ 6,  7,  8,  9, 10]])
> x % 3
| array([[1, 2, 0, 1, 2],
|        [0, 1, 2, 0, 1]], dtype=int32)
> x % 3 == 0
| array([[False, False,  True, False, False],
|        [ True, False, False,  True, False]])
> x[x % 3 == 0]  # use Boolean matrix to select entries
| array([3, 6, 9])
> x[:, x.sum(axis=0) % 3 == 0]  # columns with sum divisible by 3
| array([[ 2,  5],
|        [ 7, 10]])
```

# Numpy is fast... but be aware of dtype ⚠

**Python shell**

```
> sum([x**2 for x in range(1000000)])
| 333332833333500000
> (np.arange(1000000)**2).sum()
| 584144992    # wrong since overflow when default dtype='int32'
> (np.arange(1000000, dtype="int64")**2).sum()
| 333332833333500000  # 64 bit integers do not overflow
> import timeit from timeit
> timeit('sum([x**2 for x in range(1000000)])', number=1)
| 0.5614346340007614
> timeit('(np.arange(1000000)**2).sum()', setup='import numpy as np', number=1)
| 0.014362967000124627  # ridiculous fast but also wrong result...
> timeit('(np.arange(1000000, dtype="int64")**2).sum()',
  setup='import numpy as np', number=1)
| 0.048017077999247704  # fast and correct
> np.iinfo(np.int32).min
| -2147483648
> np.iinfo(np.int32).max
| 2147483647
```

# numpy.int32 – 32 bit signed two's-complement integers

32 bits $b_{31}b_{30}b_{29}b_{28}\cdots b_2b_1b_0$
represent the value

$$-b_{31}\cdot 2^{31}+\sum_{i=0}^{30} b_i\cdot 2^i$$

| | |
|---|---|
| 10000000000000000000000000000000 | -2147483648 |
| 10000000000000000000000000000001 | -2147483647 |
| ⋮ | |
| 11111111111111111111111111111110 | -2 |
| 11111111111111111111111111111111 | -1 |
| 00000000000000000000000000000000 | 0 |
| 00000000000000000000000000000001 | 1 |
| ⋮ | |
| 01111111111111111111111111111110 | 2147483646 |
| 01111111111111111111111111111111 | 2147483647 |

Note: There is one more negative number than positive

**Python shell**

```
> np.int32(- 2 ** 31)
| -2147483648
> np.int32(- 2 ** 31) + 1
| -2147483647
> np.int32(- 2 ** 31) - 1
| 2147483647
> np.int32(2 ** 31)
| OverflowError: Python int too
  large to convert to C long
> np.int32(2 ** 31 - 1)
| 2147483647
> np.int32(2 ** 31 - 1) + 1
| -2147483648
> abs(np.int32(-2147483647))
| 2147483647
> abs(np.int32(-2147483648))
| -2147483648
```

numpy.org/doc/stable/user/basics.types.html
en.wikipedia.org/wiki/Two's_complement

docs.scipy.org/doc/numpy/reference/routines.linalg.html

# Linear algebra

**Python shell**

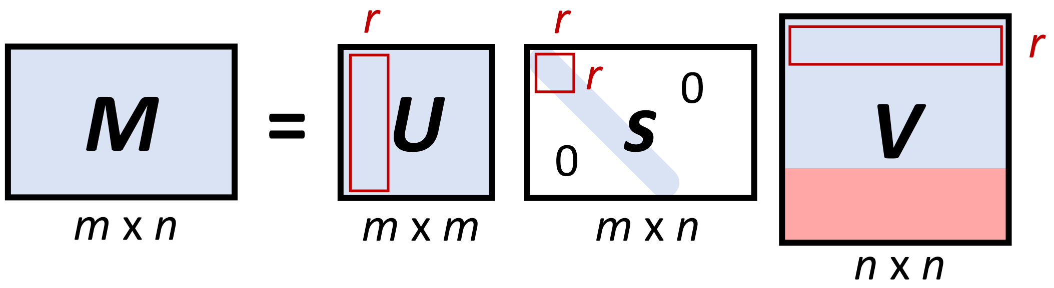
```
> x = np.arange(1, 5, dtype=float).reshape(2, 2)
> x
| array([[1., 2.],
|        [3., 4.]])
> x.T  # matrix transpose
| array([[1., 3.],
|        [2., 4.]])
> np.linalg.det(x)  # matrix determinant
| -2.0000000000000004
> np.linalg.inv(x)  # matrix inverse
| array([[-2. ,  1. ],
|        [ 1.5, -0.5]])
> np.linalg.eig(x)  # eigenvalues and eigenvectors
| (array([-0.37228132,  5.37228132]),
   array([[-0.82456484, -0.41597356], [0.56576746, -0.90937671]]))
> y = np.array([[5.], [7.]])
> np.linalg.solve(x, y)  # solve linear matrix equations
| array([[-3.],   # z1
|        [ 4.]])  # z2
```

**numpy.matrix**

*"It is no longer recommended to use this class, even for linear algebra. Instead use regular arrays. The class may be removed in the future."*

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

# Singular value decomposition, `np.linalg.svd`

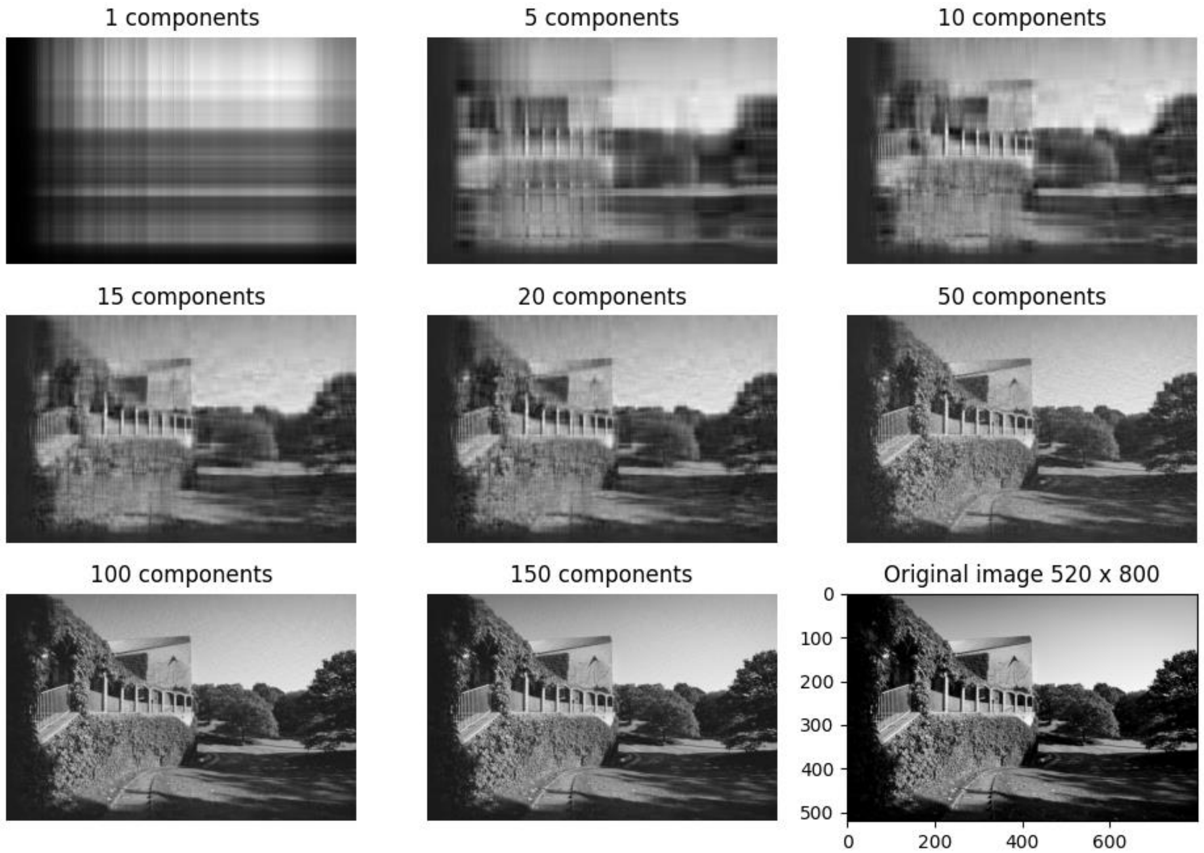



- *U* and *V* unitary matrix ($UU^T = I$)
- *S* diagonal matrix, decreasing singular values

np.ndarray shape=(520, 800) dtype=uint8 min=0 max=252

**image-reconstruction.py**

```
import numpy as np
import cv2   # Computer Vision, opencv.org
import matplotlib.pyplot as plt

color = cv2.imread('university.jpg')                # color image
gray = cv2.cvtColor(color, cv2.COLOR_BGR2GRAY)   # convert to gray

u, s, v = np.linalg.svd(gray)   # Calculating the SVD

for i, r in enumerate([1, 5, 10, 15, 20, 50, 100, 150], start=1):
    rank_r = u[:, :r] @ np.diag(s[:r]) @ v[:r, :]
    plt.subplot(3, 3, i)
    plt.imshow(rank_r, cmap='gray', vmin=0, vmax=255)
    plt.title(f'{r} components')
    plt.axis('off')
plt.subplot(3, 3, 9)
plt.imshow(gray, cmap='gray')
plt.title(f'Original image {gray.shape[0]} x {gray.shape[1]}')
plt.show()
```



# ... and in color

**image-reconstruction-color.py**

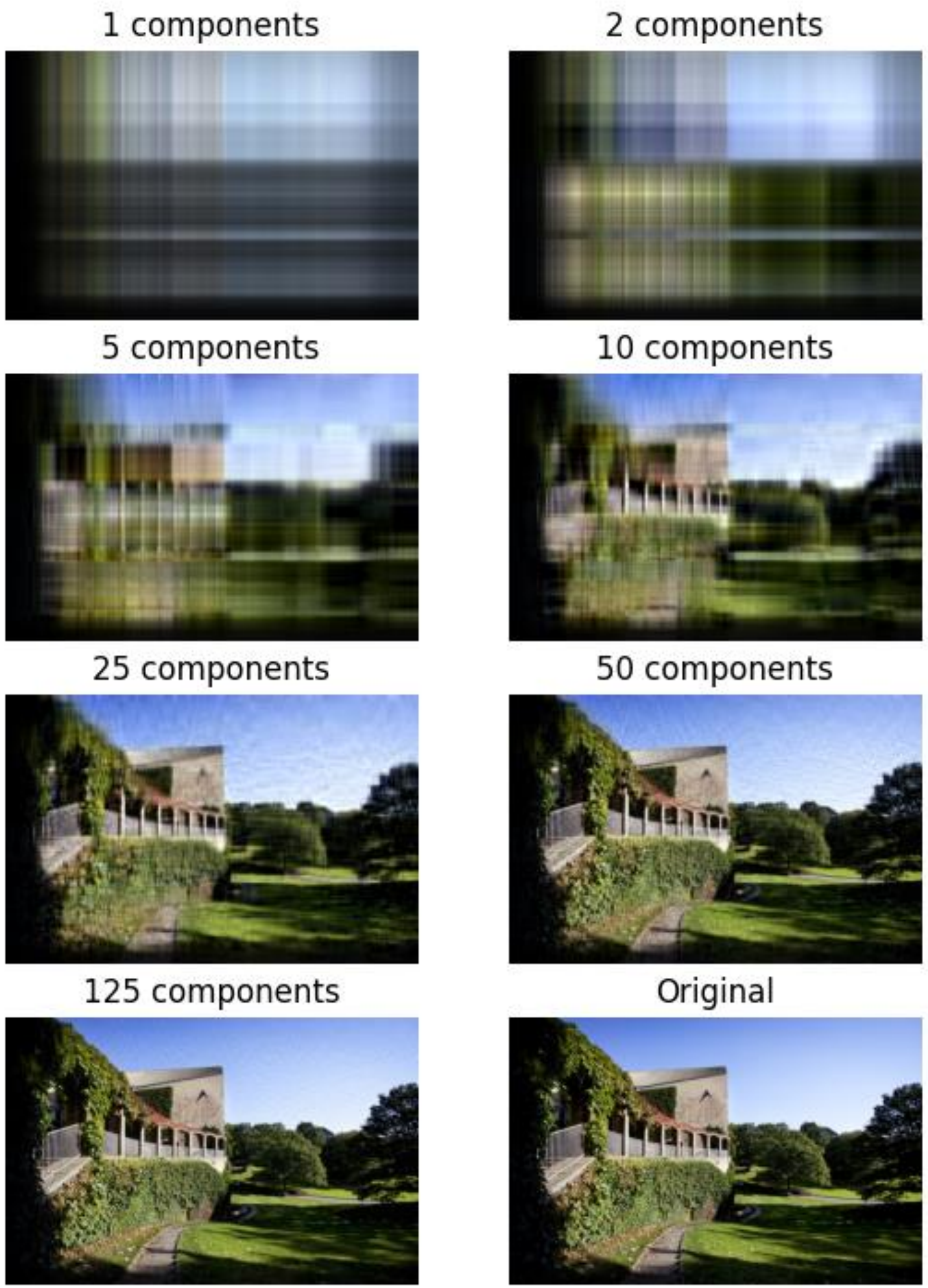
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.image import imread

color = imread('university.jpg')
color = color / 255   # convert integers 0..255 to floats 0..1

plt.subplot(4, 2, 8)
plt.imshow(color)
plt.axis('off')
plt.title(f'Original')

height, width, colors = color.shape

u, s, v = np.linalg.svd(color.reshape((height, width * colors)),
                        full_matrices=False)

for i, r in enumerate([1, 2, 5, 10, 25, 50, 125], start=1):
    rank_r = u[:, :r] @ np.diag(s[:r]) @ v[:r, :]
    plt.subplot(4, 2, i)
    plt.imshow(rank_r.reshape((height, width, colors)))
    plt.title(f'{r} components')
    plt.axis('off')
plt.show()
```

shape=(520, 800, 3)
3 = (red, blue, green)

shape=(520, 2400)

shape=(520, 800, 3)

v shape=(520, 2400)
instead of (2400, 2400)



# ... and in color (stacked)

image-reconstruction-color-stacked.py

```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.image import imread

color = imread('university.jpg')
color = color / 255  # convert integers 0..255 to floats 0..1

plt.subplot(4, 2, 8)
plt.imshow(color)
plt.axis('off')
plt.title(f'Original')

u, s, v = np.linalg.svd(color.transpose(2, 0, 1), full_matrices=False)

print(f'{u.shape=} {s.shape=} {v.shape=}')

for i, r in enumerate([1, 2, 5, 10, 25, 50, 125], start=1):
    rank_r = (u[:, :, :r] * s[:, None, :r]) @ v[:, :r, :]
    plt.subplot(4, 2, i)
    plt.imshow(rank_r.transpose(1, 2, 0))
    plt.title(f'{r} components')
    plt.axis('off')
plt.show()
```

shape = (520, 800, 3)

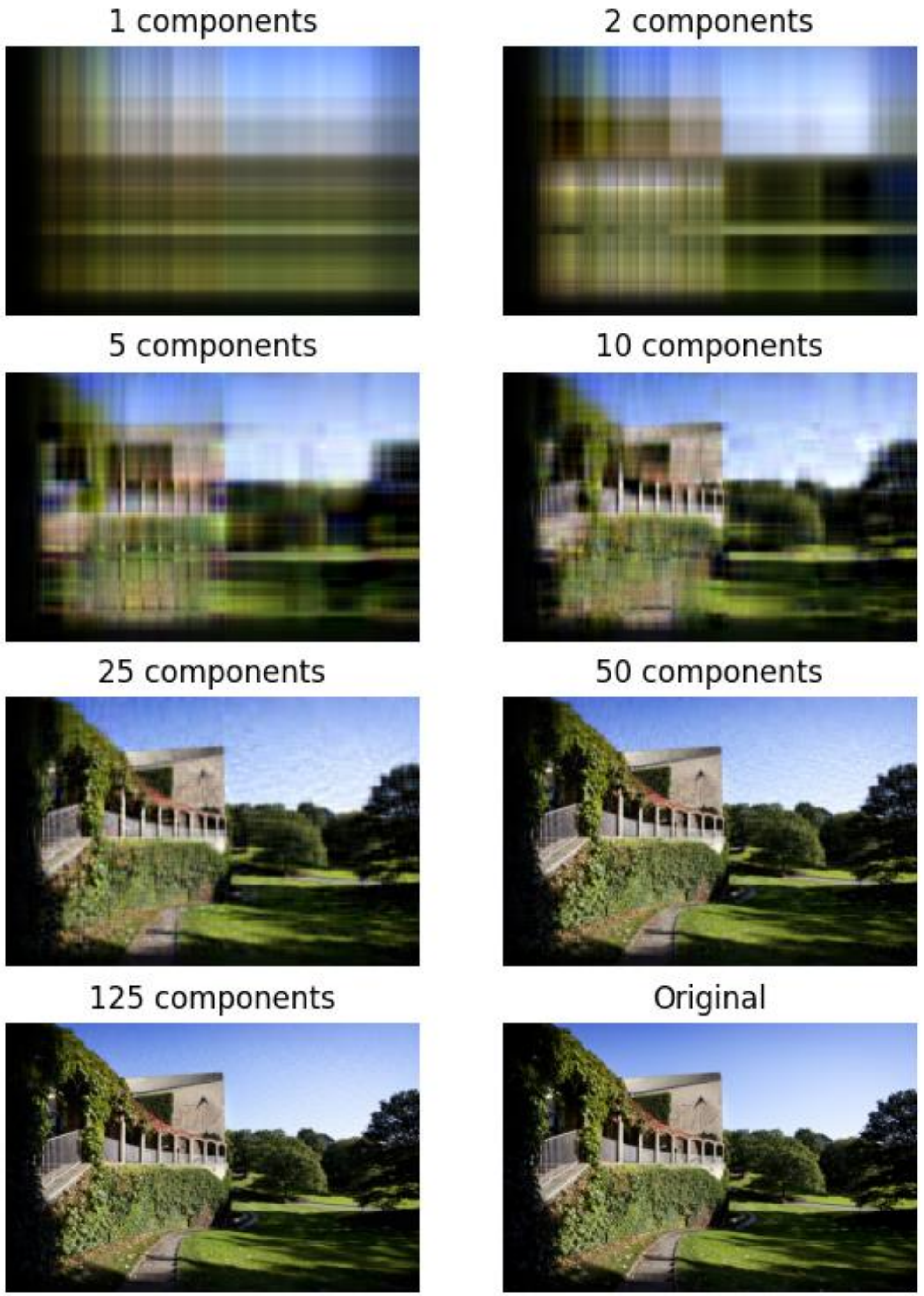
shape = (3, 520, 800)

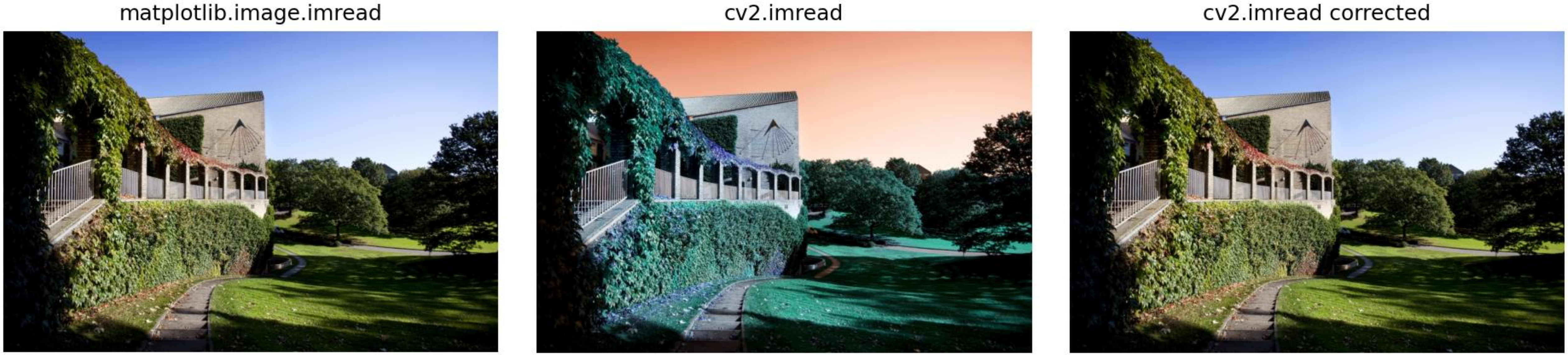
shape = (520, 800, 3)

element-wise multiplication (*), broadcasting (None), stacked for each color (:)

Python shell

```
| u.shape=(3, 520, 520) s.shape=(3, 520) v.shape=(3, 520, 800)
```

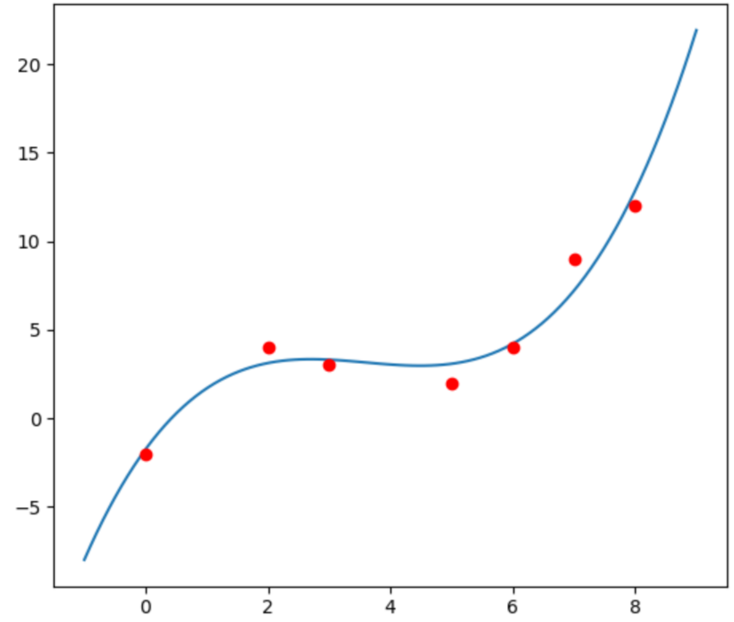





**color-image.py**

```
import matplotlib.pyplot as plt
import matplotlib.image
import cv2

img1 = matplotlib.image.imread('university.jpg')
img2 = cv2.imread('university.jpg')  # cv2 uses BGR instead of RGB
img3 = img2[:, :, ::-1]              # change color order BGR to RGB

Images = [(img1, 'matplotlib.image.imread'), (img2, 'cv2.imread'), (img3, 'cv2.imread corrected')]

for i, (img, title) in enumerate(images, start=1):
    plt.subplot(1, 3, i)
    plt.imshow(img)
    plt.axis('off')
    plt.title(title)

plt.show()
```

# numpy.polyfit




- Given n points with $(x_0, y_0), \ldots, (x_{n-1}, y_{n-1})$
- Find polynomial p of degree d that minimizes

$$\sum_{i=0}^{n-1} (y_i - p(x_i))^2$$

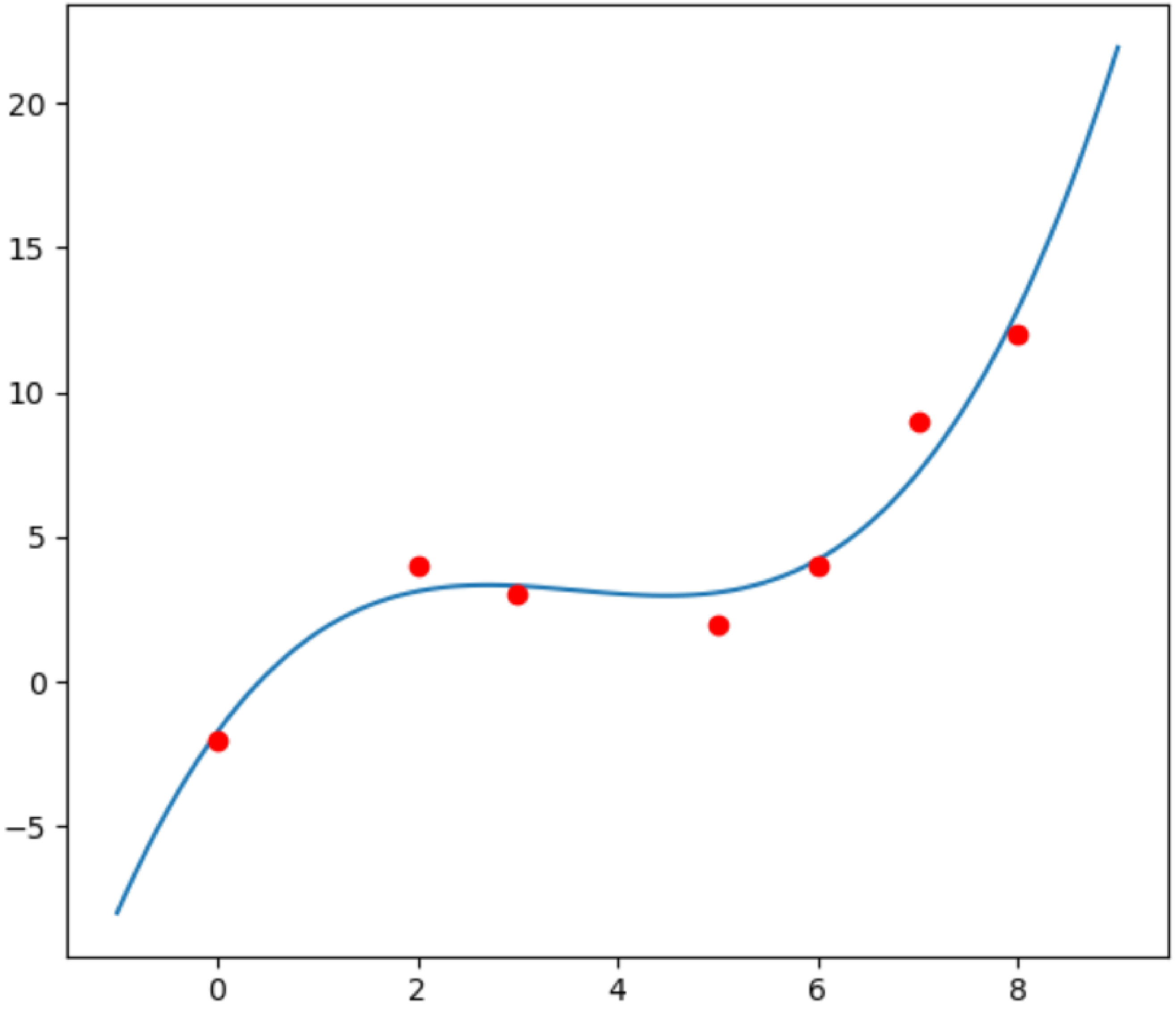
- know as least squares fit / linear regression / polynomial regression

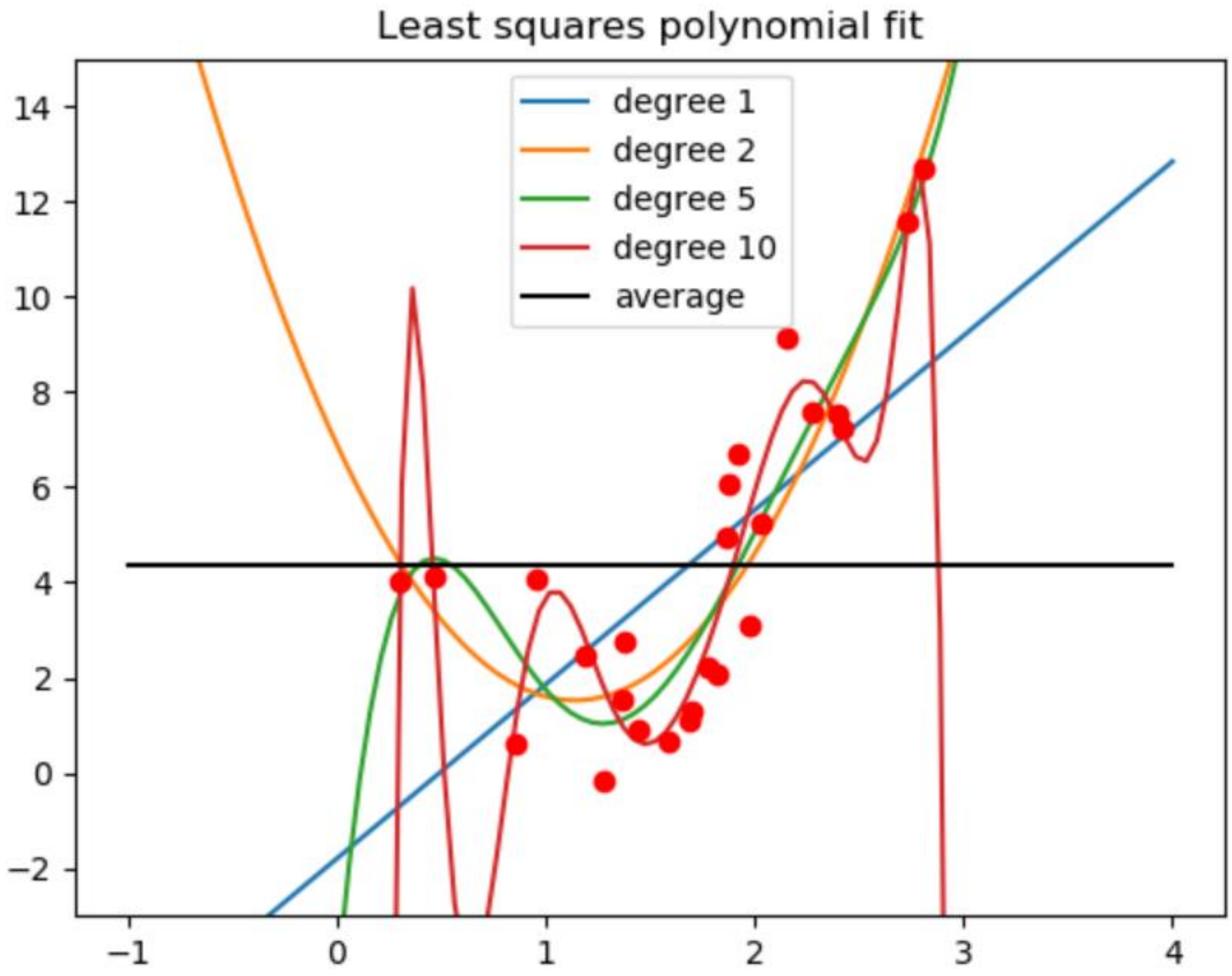
docs.scipy.org/doc/numpy/reference/generated/numpy.polyfit.html

**fit.py**

```
import matplotlib.pyplot as plt
import numpy as np

x = [0, 2, 3, 5, 6, 7, 8]
y = [-2, 4, 3, 2, 4, 9, 12]

coefficients = np.polyfit(x, y, 3)

fx = np.linspace(-1, 9, 100)
fy = np.polyval(coefficients, fx)

plt.plot(fx, fy, '-')
plt.plot(x, y, 'ro')

plt.show()
```

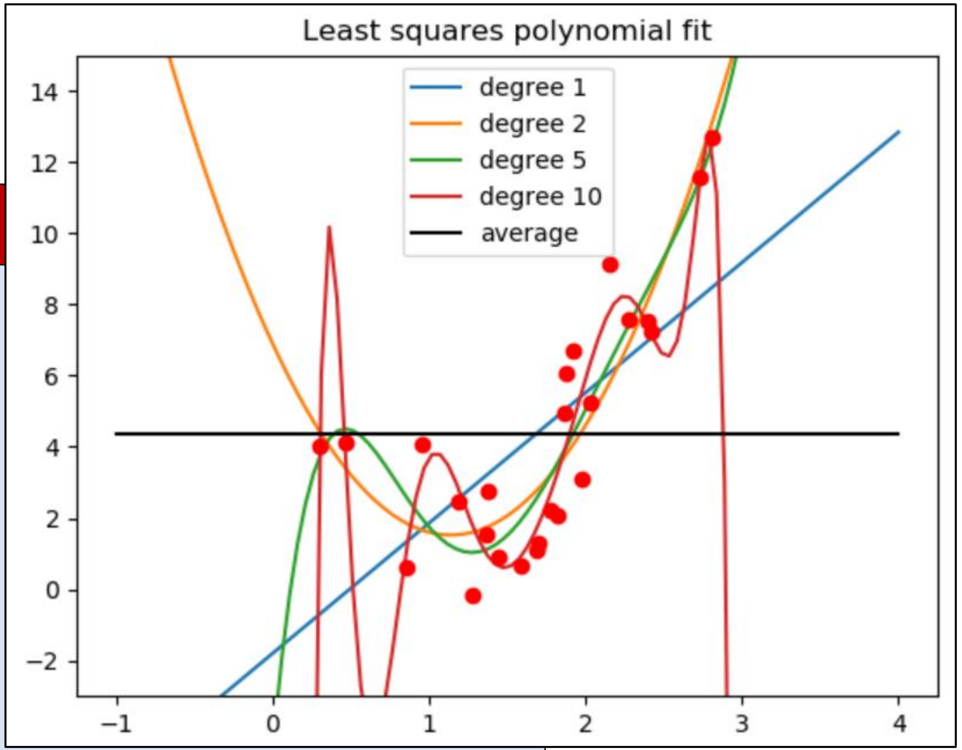
degree




Least squares polynomial fit
degree 1
degree 2
degree 5
degree 10
average
14
12
10
8
6
4
2
0
−2
−1
0
1
2
3
4

**polyfit.py**

```
import matplotlib.pyplot as plt
import numpy as np

x = 3 * np.random.random(25)
noise = np.random.random(x.size) ** 2
y = 5 * x ** 2 - 12 * x + 7 + 5 * noise

for degree in [1, 2, 5, 10]:
    coefficients = np.polyfit(x, y, degree)
    fx = np.linspace(-1, 4, 100)
    fy = np.polyval(coefficients, fx)
    plt.plot(fx, fy, '-', label="degree %s" % degree)

avg = np.average(y)
plt.plot(x, y, 'ro')
plt.plot([-1, 4], [avg, avg], 'k-', label="average")
plt.ylim(-3, 15)
plt.title('Least squares polynomial fit')
plt.legend()
plt.show()
```

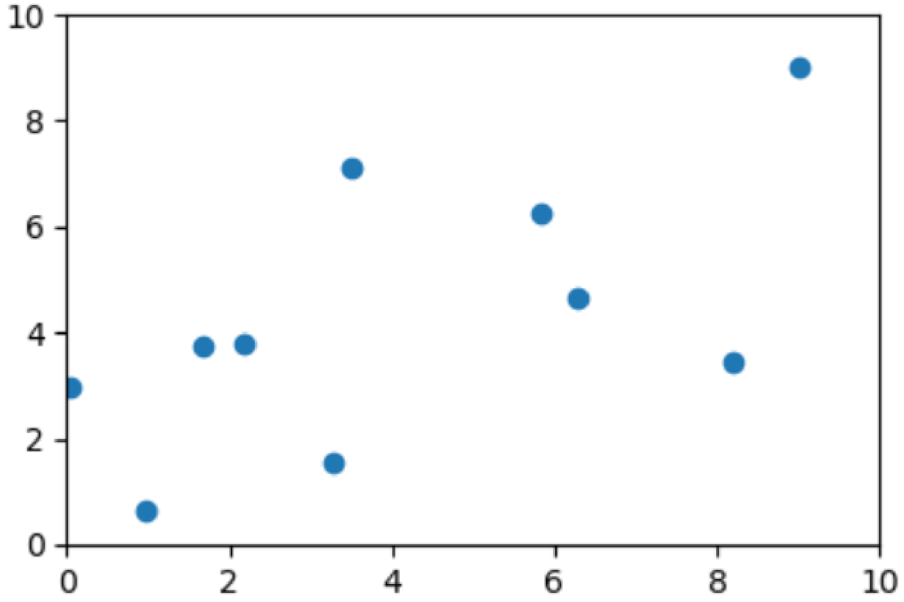


# Animating bouncing balls

- matplotlib figures can be animated using

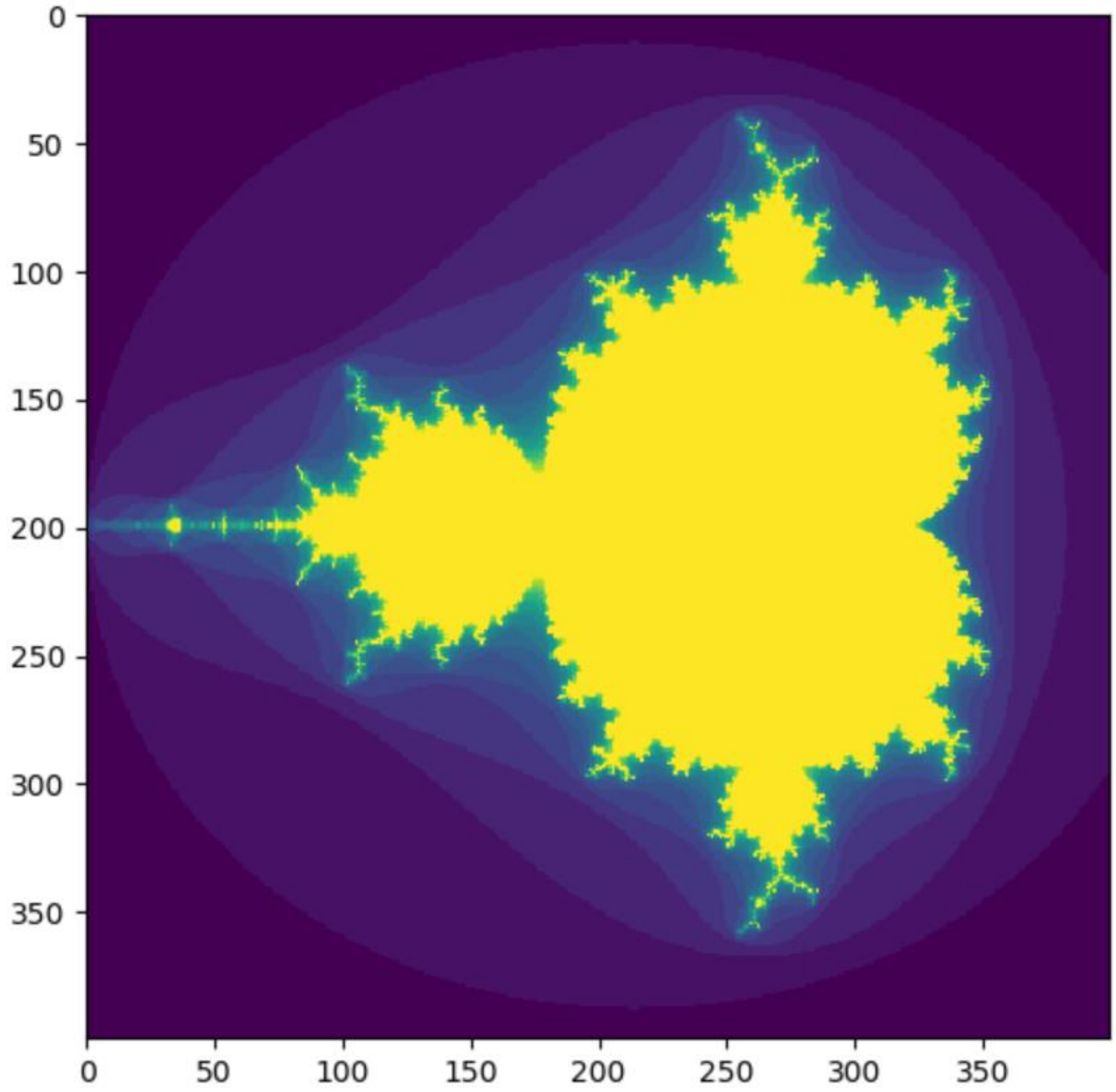
  `matplotlib.animation.FuncAnimation`

  that as arguments takes the figure to be updated/ redrawn, a function to call for each update, and an interval in milliseconds between updates



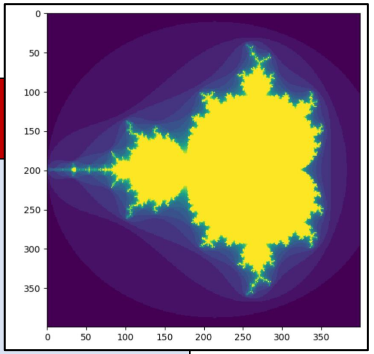

balls.py

```
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from numpy import zeros, maximum, minimum
from numpy.random import random

g = 0.01
N = 10
x, y = 10.0 * random(N), 1.0 + 9.0 * random(N)
dx, dy = random(N) / 5, zeros(N)

fig = plt.figure()
plt.xlim(0, 10)
plt.ylim(0, 10)
balls, = plt.plot(x, y, 'o') # returns Line2D obj

def move(frame):
    global x, y, dx, dy

    x += dx
    bounce = (x > 10.0) | (x < 0.0) # numpy mask
    dx[bounce] = -dx[bounce]
    x = minimum(10.0, maximum(0.0, x))

    y += dy
    bounce = y < 0.0  # numpy mask
    y[bounce] -= dy[bounce]
    dy[bounce] = -dy[bounce]
    dy -= g

    balls.set_data(x, y)  # update positions

# removing 'ani =' causes program to fail...
ani = FuncAnimation(fig, move, interval=25)
plt.show()
```



0
50
100
150
200
250
300
350
0
50
100
150
200
250
300
350

**mandelbrot.py**




```python
import numpy as np
import matplotlib.pyplot as plt
def mandelbrot(h, w, maxit=20):
    '''Returns an image of the Mandelbrot fractal of size (h, w).'''
    x = np.linspace(-2.0, 0.8, w).reshape(1, w)   # row vector
    y = np.linspace(-1.4, 1.4, h).reshape(h, 1)   # column vector
    c = x + y * 1j                                # broadcast & complex
    z = c
    divtime = np.full(z.shape, maxit, dtype=int)  # all values = maxit
    for i in range(maxit):
        z = z * z + c                             # elementwise
        diverge = z * np.conj(z) > 4              # who is diverging
        div_now = diverge & (divtime == maxit)    # who is diverging now
        divtime[div_now] = i                      # note when
        z[diverge] = 0                            # limit divergence
    return divtime                                #   (avoids overflows)
plt.imshow(mandelbrot(400, 400))
plt.show()
```

code from docs.scipy.org/doc/numpy/user/quickstart.html

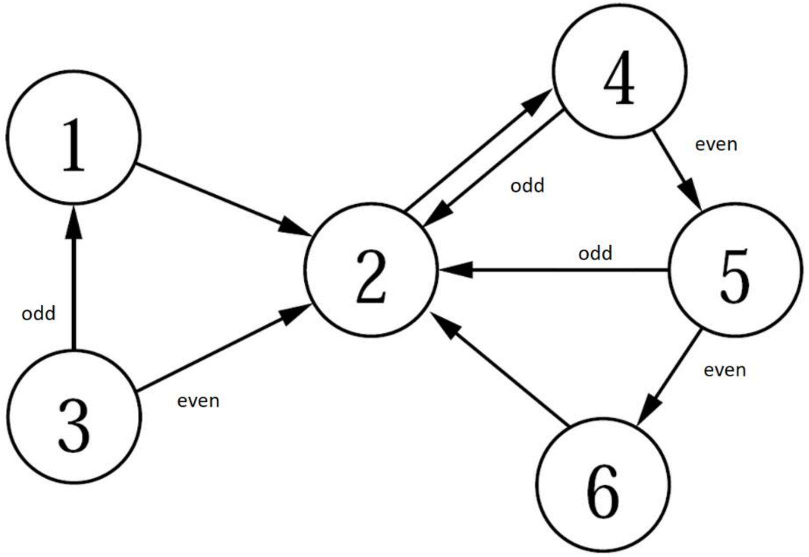
# Linear programming

- Example Numpy: PageRank
- scipy.optimize.linprog
- Example linear programming: Maximum flow

# PageRank

# PageRank - A NumPy / Jupyter / matplotlib example

- Google's original search engine ranked webpages using PageRank
- View the internet as a graph where nodes correspond to webpages and directed edges to links from one webpage to another webpage
- Google's PageRank algorithm was described in (ilpubs.stanford.edu:8090/361/, 1998)



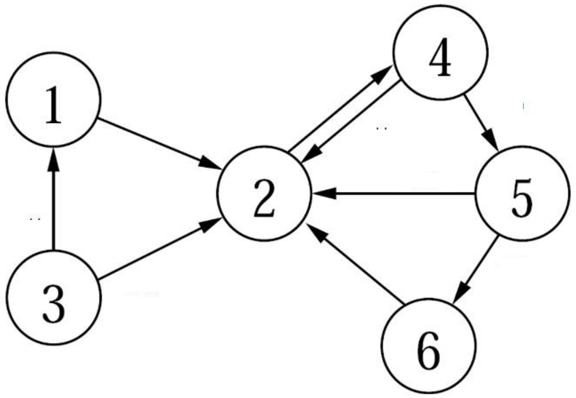

**The Anatomy of a Large-Scale Hypertextual Web Search Engine**


Sergey Brin and Lawrence Page

*Computer Science Department,*
*Stanford University, Stanford, CA 94305, USA*
sergey@cs.stanford.edu and page@cs.stanford.edu

# Five different ways to compute PageRank probabilities

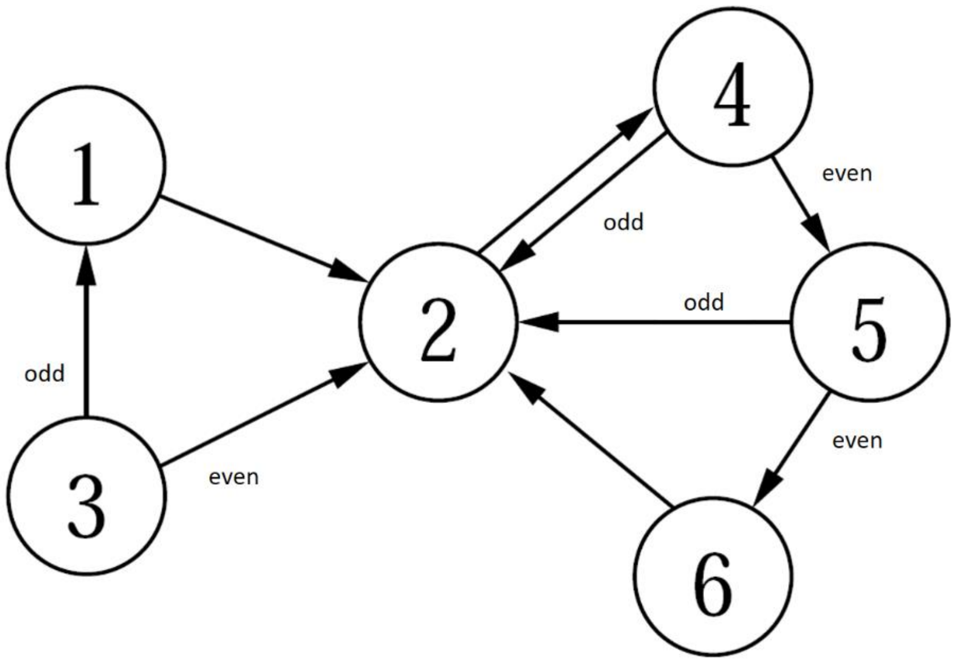
1) Simulate random process manually by rolling dices
2) Simulate random process in Python
3) Computing probabilities using matrix multiplication
4) Repeated matrix squaring
5) Eigenvector for $\lambda = 1$



# Random surfer model (simplified)

The PageRank of a node (web page) is the fraction of the time one visits a node by performing an *infinite random traversal* of the graph starting at node 1, and in each step

- with probability 1/6 jumps to a random page (probability 1/6 for each node)
- with probability 5/6 follows an outgoing edge to an adjacent node (selected uniformly)

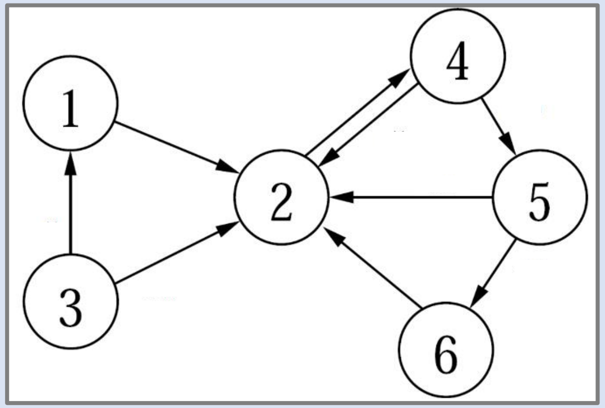



The above can be simulated by using a dice: Roll a *dice*. If it shows 6, jump to a random page by rolling the dice again to figure out which node to jump to. If the dice shows 1-5, follow an outgoing edge - if two outgoing edges roll the dice again and go to the lower number neighbor if it is odd.

# Adjacency matrix and degree vector

**pagerank.ipynb**

```
import numpy as np

# Adjacency matrix of the directed graph in the figure
# (note that the rows/colums are 0-indexed, whereas in the figure the nodes are 1-indexed)

G = np.array([[0, 1, 0, 0, 0, 0],
              [0, 0, 0, 1, 0, 0],
              [1, 1, 0, 0, 0, 0],
              [0, 1, 0, 0, 1, 0],
              [0, 1, 0, 0, 0, 1],
              [0, 1, 0, 0, 0, 0]])

n = G.shape[0]  # number of rows in G
degree = np.sum(G, axis=1, keepdims=True)  # column vector with row sums = out-degrees

# The below code handles sinks, i.e. nodes with outdegree zero (no effect on the graph above)

G = G + (degree == 0)  # add edges from sinks to all nodes
degree = np.sum(G, axis=1, keepdims=True)
```



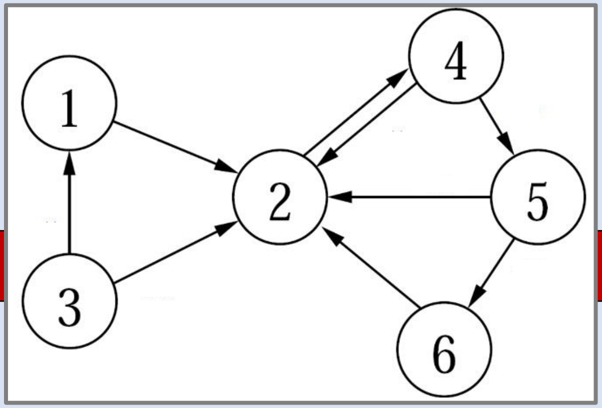
# Simulate random walk (random surfer model)

pagerank.ipynb

```python
from random import randint
STEPS = 1000000
# adjacency_list[i] is a list of all j where (i, j) is an edge of the graph.
adjacency_list = [[j for j, e in enumerate(row) if e] for row in G]
count = np.zeros(n)            # histogram over number of node visits
state = 0                      # start at node with index 0
for _ in range(STEPS):
    count[state] += 1          # increment count for state
    if randint(1, 6) == 6:     # original paper uses 15% instead of 1/6
        state = randint(0, 5)
    else:
        state = adjacency_list[state][randint(0, degree[state] - 1)]
print(adjacency_list, count / STEPS, sep='\n')
```

Python shell

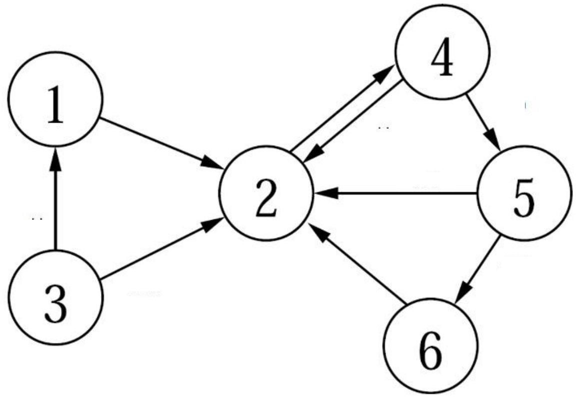
```
| [[1], [3], [0, 1], [1, 4], [1, 5], [1]]
  [0.039365 0.353211 0.02751  0.322593 0.1623   0.095021]
```

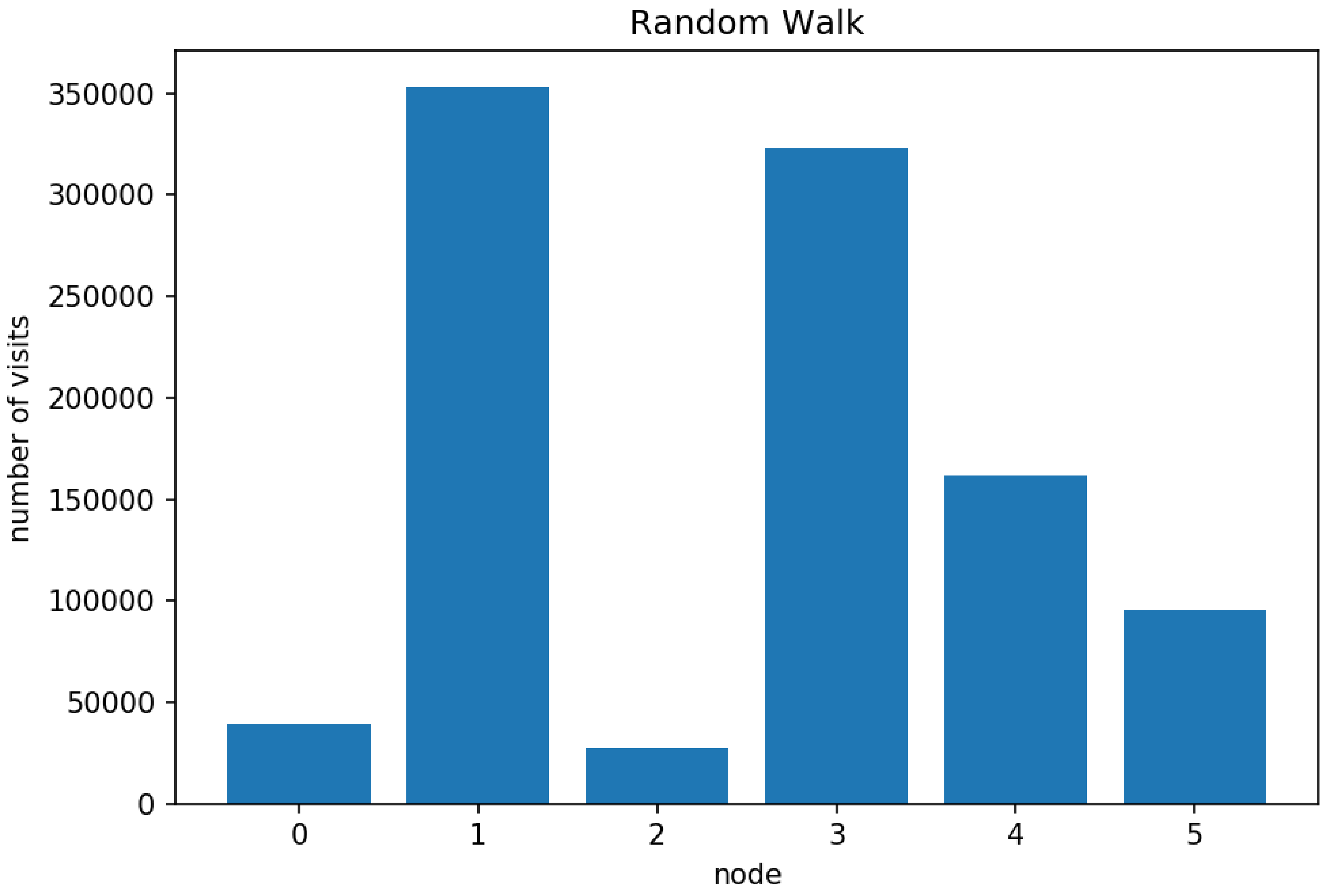


# Simulate random walk (random surfer model)

**pagerank.ipynb**

```
import matplotlib.pyplot as plt
plt.bar(range(6), count)
plt.title("Random Walk")
plt.xlabel("node")
plt.ylabel("number of visits")
plt.show()
```



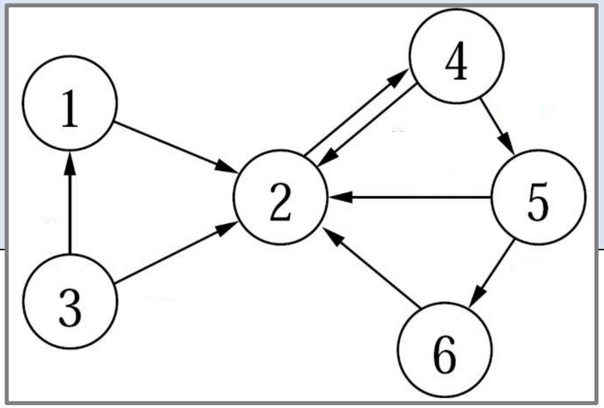
# Transition matrix *A*

**pagerank.ipynb**

```
A = G / degree   # Normalize row sums to one. Note that 'degree'
                 # is an n x 1 matrix, whereas G is an n x n matrix.
                 # The elementwise division is repeated for each column of G
print(A)
```

**Python shell**

```
[[0.  1.  0.  0.  0.  0. ]
 [0.  0.  0.  1.  0.  0. ]
 [0.5 0.5 0.  0.  0.  0. ]
 [0.  0.5 0.  0.  0.5 0. ]
 [0.  0.5 0.  0.  0.  0.5]
 [0.  1.  0.  0.  0.  0. ]]
```



# Repeated matrix multiplication

We now want to compute the probability $p^{(i)}{}_j$ to be in vertex $j$ after $i$ steps. Let $p^{(i)} = (p^{(i)}{}_0, \ldots, p^{(i)}{}_{n-1})$.

Initially we have $p^{(0)} = (1, 0, \ldots, 0)$.

We compute a matrix $M$, such that $p^{(i)} = M^i \cdot p^{(0)}$ (assuming $p^{(0)}$ is a column vector).

If we let $\mathbf{1}_n$ denote the $n \times n$ matrix with 1 in each entry, then $M$ can be computed as:

$$p_j^{(i+1)} = \frac{1}{6} \cdot \frac{1}{n} + \frac{5}{6} \sum_k p_k^{(i)} \cdot A_{k,j}$$

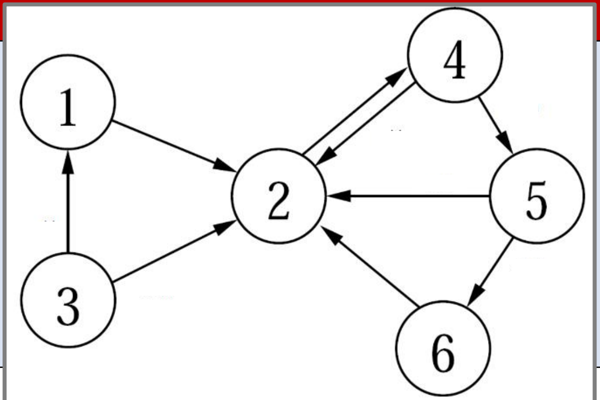
$$p^{(i+1)} = \underbrace{\left( \frac{1}{6} \cdot \frac{1}{n} \mathbf{1}_n + \frac{5}{6} A^{\mathrm{T}} \right)}_{M} \cdot p^{(i)}$$

**pagerank.ipynb**

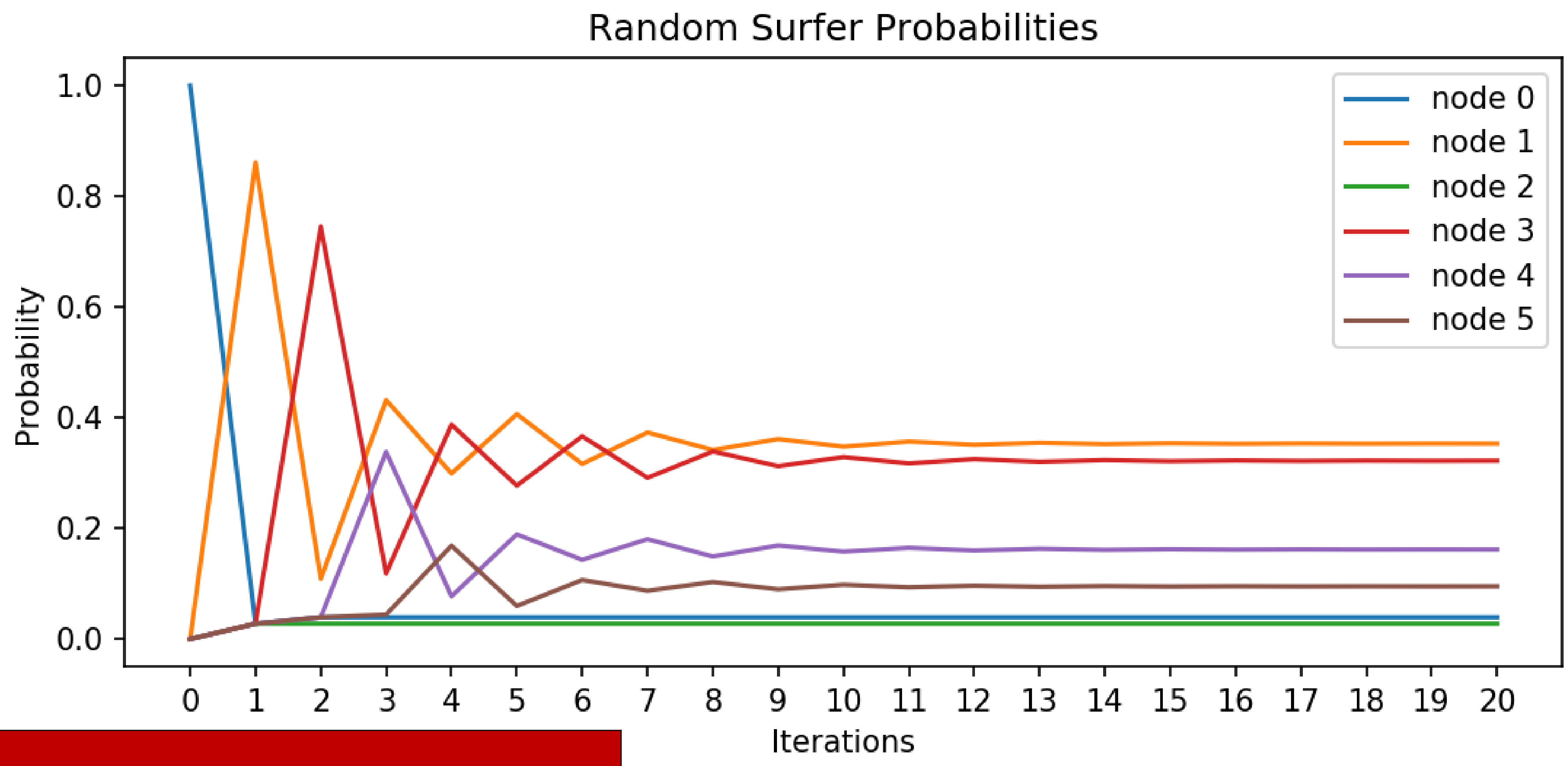
```
ITERATIONS = 20
p_0 = np.zeros((n, 1))
p_0[0, 0] = 1.0
M = 1 / (6 * n) + 5 / 6 * A.T
p = p_0
prob = p  # 'prob' will contain each
          # computed 'p' as a new column
for _ in range(ITERATIONS):
    p = M @ p
    prob = np.append(prob, p, axis=1)
print(p)
```

**Python shell**

```
| [[0.03935185]
   [0.35326184]
   [0.02777778]
   [0.32230071]
   [0.16198059]
   [0.09532722]]
```

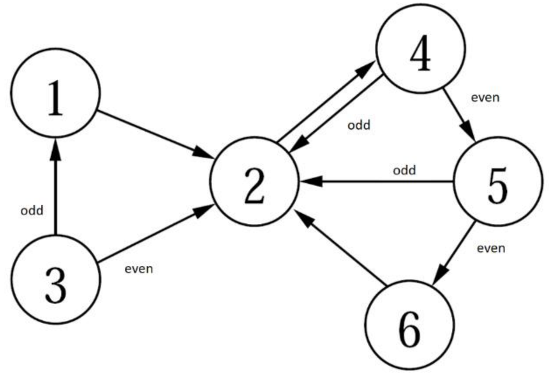


# Rate of convergence




**pagerank.ipynb**

```
x = range(ITERATIONS + 1)
for node in range(n):
    plt.plot(x, prob[node], label="node %s" % node)
plt.xticks(x)
plt.title("Random Surfer Probabilities")
plt.xlabel("Iterations")
plt.ylabel("Probability")
plt.legend()
plt.show()
```



# Repeated squaring

$$\underbrace{M\cdot(\cdots(M\cdot(M}_{k\text{ multiplications, }k\text{ power of 2}}\cdot p^{(0)}))\cdots) = M^k\cdot p^{(0)} = M^{2^{\log k}}\cdot p^{(0)} = (\cdots((M^{\overbrace{2})^2)^2\cdots)^2}^{\log k}\cdot p^{(0)}$$

**pagerank.ipynb**

```python
from math import log
MP = M
for _ in range(1 + int(log(ITERATIONS, 2))):
    MP = MP @ MP
p = MP @ p_0
print(p)
```

**Python shell**

```
[[0.03935185]
 [0.35332637]
 [0.02777778]
 [0.32221711]
 [0.16203446]
 [0.09529243]]
```

# PageRank : Computing eigenvector for λ = 1

- We want to find a vector $p$, with $|p| = 1$, where $Mp = p$, i.e. an *eigenvector* $p$ for the eigenvalue $\lambda = 1$

**pagerank.ipynb**

```
eigenvalues, eigenvectors = np.linalg.eig(M)
idx = eigenvalues.argmax()              # find the largest eigenvalue (= 1)
p =  np.real(eigenvectors[:, idx])      # .real returns the real part of complex numbers
p /= p.sum()                            # normalize p to have sum 1
print(p)
```

**Python shell**

```
| [0.03935185 0.3533267  0.02777778 0.32221669 0.16203473 0.09529225]
```

# PageRank : Note on practicality

- In practice an explicit matrix for billions of nodes is infeasible, since the number of entries would be order of $10^{18}$
- Instead use sparse matrices (in Python modul `scipy.sparse`) and stay with repeated multiplication

# Linear programming

# scipy.optimize.linprog

- scipy.optimize.linprog can solve *linear programs* of the following form, where one wants to find an $n$ x 1 vector $x$ satisfying:

**Minimize:** $c^T \cdot x$

**Subject to:** $A_{ub} \cdot x \leq b_{ub}$

$A_{eq} \cdot x = b_{eq}$

*dimension*

$c : n \times 1$

$A_{ub} : m \times n$ $\quad b_{ub} : m \times 1$

$A_{eq} : k \times n$ $\quad b_{eq} : k \times 1$

Some other open-source optimization libraries PuLP and Pyomo

For industrial strength linear solvers, use solvers like Cplex or Gurobi

# Linear programming example

**Maximize**

$3 \cdot x_1 + 2 \cdot x_2$

**Subject to**

$2 \cdot x_1 + 1 \cdot x_2 \leq 10$

$5 \cdot x_1 + 6 \cdot x_2 \geq 4$

$-3 \cdot x_1 + 7 \cdot x_2 = 8$

$\Updownarrow$

**Minimize**

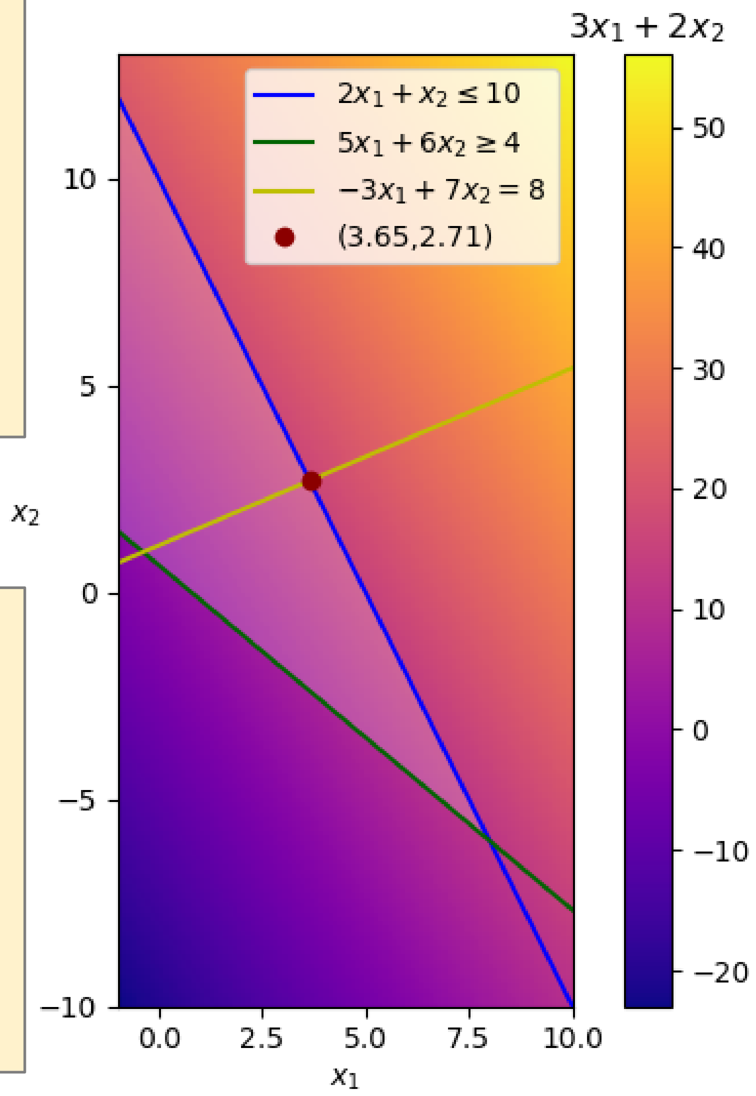
$-(3 \cdot x_1 + 2 \cdot x_2)$

**Subject to**

$2 \cdot x_1 + 1 \cdot x_2 \leq 10$

$-5 \cdot x_1 + -6 \cdot x_2 \leq -4$

$-3 \cdot x_1 + 7 \cdot x_2 = 8$




**linear_programming.py**

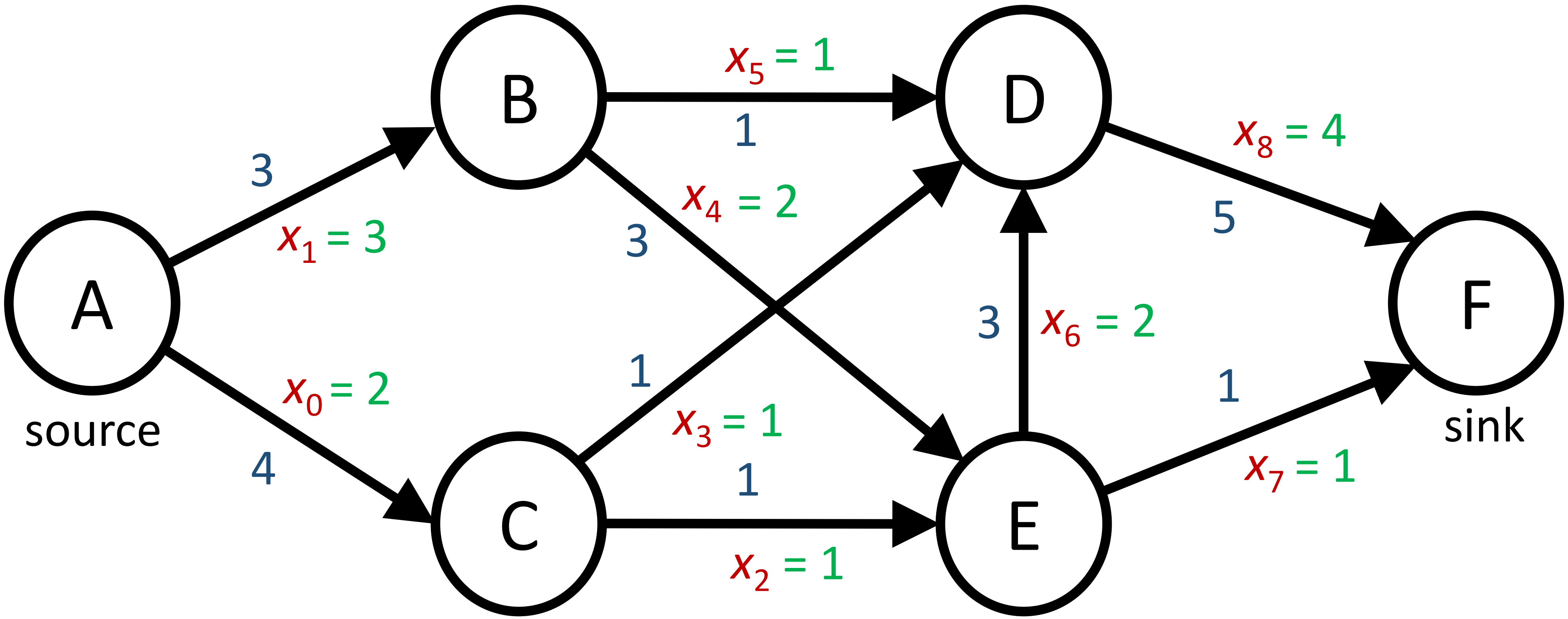
```
import numpy as np
from scipy.optimize import linprog
c = np.array([3, 2])
A_ub = np.array([[ 2,  1],
                 [-5, -6]]) # multiplied by -1
b_ub = np.array([10, -4])
A_eq = np.array([[-3, 7]])
b_eq = np.array([8])
res = linprog(-c,  # maximize = minimize the negated
              A_ub=A_ub,
              b_ub=b_ub,
              A_eq=A_eq,
              b_eq=b_eq)
print(res)  # res.x is the optimal vector
```

**Python shell**

```
|   fun: -16.35294117647059
  message: 'Optimization terminated successfully.'
      nit: 3
    slack: array([ 0.        , 30.47058824])
   status: 0
  success: True
        x: array([3.64705882, 2.70588235])
```

# Maxmium flow

# Solving maximum flow using linear programming




We will use the scipy.optimize.linprog function to solve the *maximum flow* problem on the above directed graph. We want to send as much *flow* from node A to node F. Edges are numbered 0..8 and each edge has a maximum *capacity*.

**Maximize**

$x_7 + x_8$ flow value

**Subject to**

capacity constraints:

$x_0 \leq 4$

$x_1 \leq 3$

$x_2 \leq 1$

$x_3 \leq 1$

$x_4 \leq 3$

$x_5 \leq 1$

$x_6 \leq 3$

$x_7 \leq 1$

$x_8 \leq 5$

flow conservation:

$x_1 = x_4 + x_5$

$x_0 = x_2 + x_3$

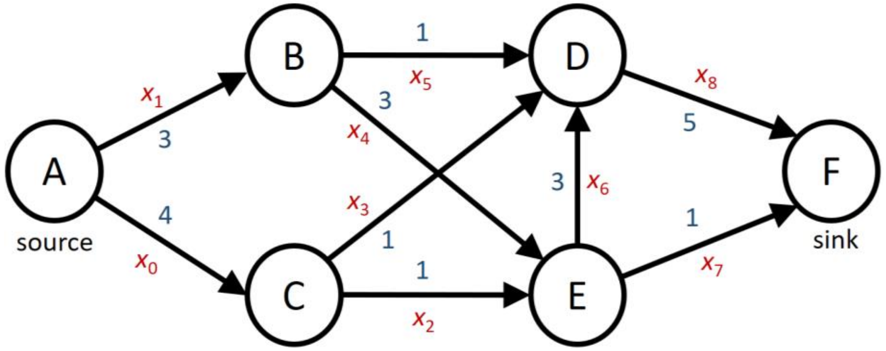
$x_3 + x_5 + x_6 = x_8$

$x_2 + x_4 = x_6 + x_7$

Note: solution not unique

# Solving maximum flow using linear programming

- $x$ is a vector describing the flow along each edge
- $c$ is a vector that to add the flow along the edges (7 and 8) to the sink (F), i.e. a function computing *the flow value*
- $A_{ub}$ and $b_{ub}$ is a set of *capacity constraints*, for each edge flow ≤ capacity
- $A_{eq}$ and $b_{eq}$ is a set of *flow conservation* constraints, for each non-source and non-sink node (B, C, D, E), requiring that the flow into equals the flow out of a node




**Minimize**

$\boxed{-x_7 - x_8}$ $c^T \cdot x$ flow value

**Subject to**

$x_0 \leq 4$
$x_1 \leq 3$
$x_2 \leq 1$
$x_3 \leq 1$
$x_4 \leq 3$
$x_5 \leq 1$
$x_6 \leq 3$
$x_7 \leq 1$
$x_8 \leq 5$

$A_{ub} \cdot x \leq b_{ub}$ $\Updownarrow$ $I \cdot x \leq \text{capacity}$

capacity constraints

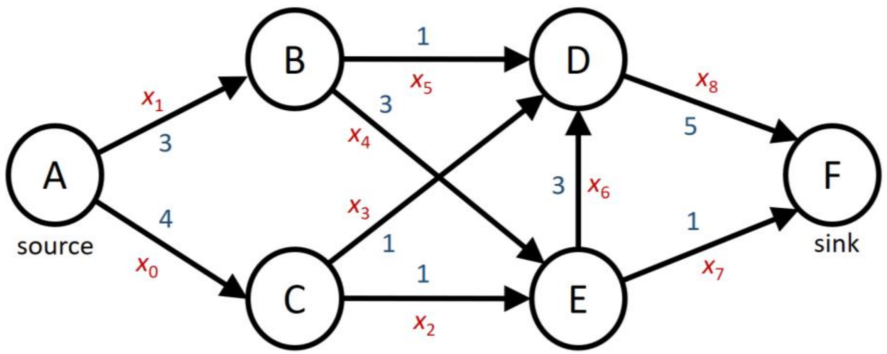
$A_{eq} \cdot x = b_{eq} = 0$

$0 = -x_1 + x_4 + x_5$
$0 = -x_0 + x_2 + x_3$
$0 = -x_3 - x_5 - x_6 + x_8$
$0 = -x_2 - x_4 + x_6 + x_7$

flow conservation

maximum-flow.py

```
import numpy as np
from scipy.optimize import linprog

#                             0  1  2  3  4  5  6  7  8
conservation = np.array([[ 0,-1, 0, 0, 1, 1, 0, 0, 0],   # B
                          [-1, 0, 1, 1, 0, 0, 0, 0, 0],   # C
                          [ 0, 0, 0,-1, 0,-1,-1, 0, 1],   # D
                          [ 0, 0,-1, 0,-1, 0, 1, 1, 0]]) # E

#                  0  1  2  3  4  5  6  7  8
sinks = np.array([0, 0, 0, 0, 0, 0, 0, 1, 1])

#                     0  1  2  3  4  5  6  7  8
capacity = np.array([4, 3, 1, 1, 3, 1, 3, 1, 5])

res = linprog(-sinks,
              A_eq=conservation,
              b_eq=np.zeros(conservation.shape[0]),
              A_ub=np.eye(capacity.size),
              b_ub=capacity)

print(res)
```




Python shell

```
     fun: -5.0
 message: 'Optimization terminated successfully.'
     nit: 9
   slack: array([2., 0., 0., 0., 1., 0., 1., 0., 1.])
  status: 0
 success: True
       x: array([2., 3., 1., 1., 2., 1., 2., 1., 4.])
```

the solution found varies
with the scipy version

# Generators, iterators

- __iter__, __next__
- yield
- generator expression
- measuring memory usage

# Iterable & Iterator

**Python shell**

```
> L = ['a', 'b', 'c']
> type(L)
| <class 'list'>
> it = L.__iter__()
> type(it)
| <class 'list_iterator'>
> it.__next__()
| 'a'
> it.__next__()
| 'b'
> it.__next__()
| 'c'
> it.__next__()
| StopIteration   # Exception
```

**Python shell**

```
> L = ['a', 'b', 'c']
> it = iter(L)   # calls L.__iter__()
> next(it)       # calls it.__next__()
| 'a'
> next(it)
| 'b'
> next(it)
| 'c'
> next(it)
| StopIteration
```

iterator ≈ pointer into list

['a', 'b', 'c']

- Lists are iterable (must support `__iter__`)
- `iter` returns an iterator (must support `__next__`)

Some iterables in Python: string, list, set, tuple, dict, range, enumerate, zip, map, reversed

# Iterator

- next(*iterator_object*) returns the next element from the iterator, by calling the *iterator_object*.`__next__()`. If no more elements to report, raises exception `StopIteration`
- next(*iterator_object*, *default*) returns *default* when no more elements are available (no exception is raised)
- for-loops and list comprehensions require iterable objects
  `for x in range(5):` and `[2**x for x in range(5)]`
- The iterator concept is also central to Java and C++

# `for` loop

**Python shell**

```
> for x in ['a', 'b', 'c']:
      print(x)
| a
| b
| c
```

result of `next` on iterator

iterable object (can call `iter` on it to generate an iterator)

≡

**Python shell**

```
> L = ['a', 'b', 'c']
> it = iter(L)
> while True:
      try:
          x = next(it)
      except StopIteration:
          break
      print(x)
| a
| b
| c
```

# 8.3. The `for` statement

The `for` statement is used to iterate over the elements of a sequence (such as a string, tuple or list) or other iterable object:

```
for_stmt ::=  "for" target_list "in" expression_list ":" suite
              ["else" ":" suite]
```

The expression list is evaluated once; it should yield an iterable object. An iterator is created for the result of the `expression_list`. The suite is then executed once for each item provided by the iterator, in the order returned by the iterator. Each item in turn is assigned to the target list using the standard rules for assignments (see Assignment statements), and then the suite is executed. When the items are exhausted (which is immediately when the sequence is empty or an iterator raises a `StopIteration` exception), the suite in the `else` clause, if present, is executed, and the loop terminates.

docs.python.org/3/reference/compound_stmts.html#the-for-statement

# `for` loop over changing iterable

Changing (extending) the list while scanning
The iterator over a list is just an index into the list

**Python shell**

```
> L = [1, 2]
> for x in L:
      print(x, L)
      L.append(x + 2)
| 1 [1, 2]
| 2 [1, 2, 3]
| 3 [1, 2, 3, 4]
| 4 [1, 2, 3, 4, 5]
| 5 [1, 2, 3, 4, 5, 6]
...
```

**Python shell**

```
> L = [1, 2]
> for x in L:
      print(x, L)
      L[:0] = [L[0] - 2, L[0] - 1]
| 1 [1,2]
| 0 [-1,0,1,2]
| -1 [-3,-2,-1,0,1,2]
| -2 [-5,-4,-3,-2,-1,0,1,2]
| -3 [-7,-6,-5,-4,-3,-2,-1,0,1,2]
...
```

# range

**Python shell**

```
> r = range(1, 6)   # 1,2,3,4,5
> type(r)
| <class 'range'>
> it = iter(r)
> type(it)
| <class 'range_iterator'>
> next(it)
| 1
> next(it)
| 2
> for x in it:
      print(x)
| 3
| 4
| 5
```

iterable expected but got iterator ?

**Python shell**

```
> it
| <range_iterator object at 0x03E7FFC8>
> iter(it)
| <range_iterator object at 0x03E7FFC8>
> it is iter(it)
| True
```

Calling `iter` on a `range_iterator` just returns the iterator itself, i.e. can use the iterator wherever an iterable is expected

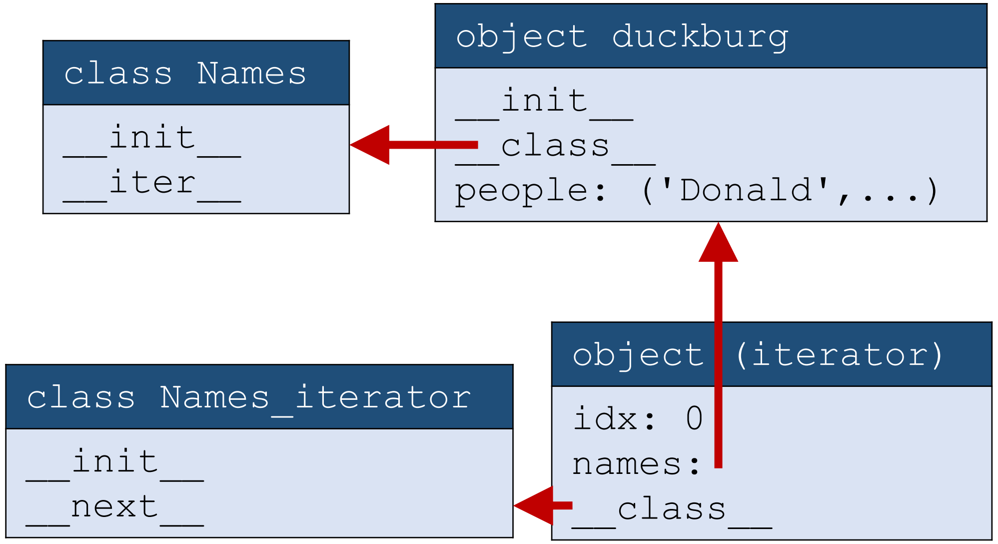
# Creating an interable class

**names.py**

```python
class Names:
    def __init__(self, *arg):
        self.people = arg
    def __iter__(self):
        return Names_iterator(self)
class Names_iterator:
    def __init__(self, names):
        self.idx = 0
        self.names = names
    def __next__(self):
        if self.idx >= len(self.names.people):
            raise StopIteration
        self.idx += 1
        return self.names.people[self.idx - 1]
duckburg = Names('Donald', 'Goofy', 'Mickey', 'Minnie')
for name in duckburg:
    print(name)
```

**Python shell**

```
| Donald
| Goofy
| Mickey
| Minnie
```



# An infinite iterable

**infinite_range.py**

```
class infinite_range:
    def __init__(self, start=0, step=1):
        self.start = start
        self.step = step
    def __iter__(self):
        return infinite_range_iterator(self)
class infinite_range_iterator:
    def __init__(self, inf_range):
        self.range = inf_range
        self.current = self.range.start
    def __next__(self):
        value = self.current
        self.current += self.range.step
        return value
    def __iter__(self):  # make iterator iterable
        return self
```

**Python shell**

```
> r = infinite_range(42, -3)
> it = iter(r)
> for idx, value in zip(range(5), it):
      print(idx, value)
| 0 42
| 1 39
| 2 36
| 3 33
| 4 30
> for idx, value in zip(range(5), it):
      print(idx, value)
| 0 27
| 1 24
| 2 21
| 3 18
| 4 15
> print(sum(r))   # don't do this
| (runs forever)
```

`sum` and `zip` take iterables
(`zip` stops when shortest iterable is exhausted)

# Creating an iterable class (iterable = iterator)

**my_range.py**

```python
class my_range:
    def __init__(self, start, end, step):
        self.start = start
        self.end = end
        self.step = step
        self.x = start

    def __iter__(self):
        return self  # self also iterator

    def __next__(self):
        if self.x >= self.end:
            raise StopIteration
        answer = self.x
        self.x += self.step
        return answer

r = my_range(1.5, 2.0, 0.1)
```

**Python shell**

```
> list(r)
| [1.5, 1.6,
  1.7000000000000002,
  1.8000000000000003,
  1.9000000000000004]
> list(r)
| []
```

- Note that objects act both as an iterable and an iterator
- This e.g. also applies to `zip` objects
- Can only iterate over a `my_range` once

# itertools

| **Function** | **Description** |
|---|---|
| `count(start, step)` | Inifinite sequence: `start`, `stat + step`, … |
| `cycle(seq)` | Infinite repeats of the elements from `seq` |
| `repeat(value[, times])` | Infinite repeats of `value` or `times` repeats |
| `chain(seq0,...,seqk)` | Concatenate sequences |
| `starmap(func, seq)` | `func(*seq[0]), func(*seq[1]),` … |
| `permutations(seq)` | Genereate all possible permutations of `seq` |
| `islice(seq, start, stop, step)` | Create a slice of `seq` |
| … | … |

https://docs.python.org/3/library/itertools.html

# Example : Java iterators

**vector-iterator.java**

```
import java.util.Vector;
import java.util.Iterator;

class IteratorTest {
   public static void main(String[] args) {
       Vector<Integer> a = new Vector<Integer>();
       a.add(7);
       a.add(42);
       // "C" for-loop & get method
       for (int i=0; i<a.size(); i++)
          System.out.println(a.get(i));
       // iterator
       for (Iterator it = a.iterator(); it.hasNext(); )
          System.out.println(it.next());
       // for-each loop - syntax sugar since Java 5
       for (Integer e : a)
          System.out.println(e);
   }
}
```

In Java iteration does not stop using exceptions, but instead the iterator can be tested if it is at the end of the iterable

# Example : C++ iterators

**vector-iterator.cpp**

```
#include <iostream>
#include <vector>
int main() {
  // Vector is part of STL (Standard Template Library)
  std::vector<int> A = {20, 23, 26};
  // "C" indexing - since C++98
  for (int i = 0; i < A.size(); i++)
    std::cout << A[i] << std::endl;
  // iterator - since C++98
  for (std::vector<int>::iterator it = A.begin(); it != A.end(); ++it)
    std::cout << *it << std:: endl;
  // "auto" iterator - since C++11
  for (auto it = A.begin(); it != A.end(); ++it)
    std::cout << *it << std:: endl;
  // Range-based for-loop - since C++11
  for (auto e : A)
    std::cout << e << std:: endl;
}
```

In C++ iterators can be tested if they reach the end of the iterable

move iterator to next element

# Generators

# Generator expressions

**Python shell**

```
> [x ** 2 for x in range(5)]   # list comprehension
| [0, 1, 4, 9, 16]  # list
> (x ** 2 for x in range(3))   # generator expression
| <generator object <genexpr> at 0x03D9F8A0>
> o = (x ** 2 for x in range(3))
> next(o)
| 0
> next(o)
| 1
> next(o)
| 4
> next(o)
| StopIteration
```

- A generator expression
  `(... for x in ...)`
  looks like a list comprehension, except square brackets are replaced by parenthesis
- Is an iterable and iterator, that uses less memory than a list comprehension
- computation is done *lazily*, i.e. first when needed

https://docs.python.org/3/reference/expressions.html#generator-expressions

# Nested generator expressions

**Python shell**

```
> squares = (x ** 2 for x in range(1, 6))   # generator expression
> ratios = (1 / y for y in squares)   # generator expression
> ratios
| <generator object <genexpr> at 0x031FC230>
> next(ratios)
| 1.0
> next(ratios)
| 0.25
> print(list(ratios))
| [0.1111111111111111, 0.0625, 0.04]   # remaining 3
```

- Each fraction is first computed when requested by `next(ratios)` (implicitly called repeatedly in `list(ratios)`)
- The next value of `squares` is first computed when needed by `ratios`

# Generator expressions as function arguments

**Python shell**

```
> doubles = (x * 2 for x in range(1, 6))
> sum(doubles)   # sum takes an iterable
| 30
> sum((x * 2 for x in range(1, 6)))
| 30
> sum(x * 2 for x in range(1, 6))   # one pair of parenthesis omitted
| 30
```

- Python allows to omit a pair of parenthesis when a generator expression is the only argument to a function

  `f(... for x in ...)` ≡ `f((... for x in ...))`

[PEP 289 – Generator Expressions]()

# Generator functions

**two.py**

```
def two():
    yield 1
    yield 2
```

**Python shell**

```
> two()
| <generator object two at 0x03629510>
> t = two()
> next(t)
| 1
> next(t)
| 2
> next(t)
| StopIteration
```

- A *generator function* contains one or more `yield` statements
- Python automatically makes a call to a generator function into an iterable and iterator (provides `__iter__` and `__next__`)
- Calling a generator function returns a *generator object*
- Whenever `next` is called on a generator object, the excuting of the function continues until the next `yield` *exp* and the value of *exp* is returned as a result of `next`
- Reaching the end of the function or a return statement, will raise `StopIteration`
- Once consumed, can't be reused

https://docs.python.org/3/reference/expressions.html#yield-expressions

# Generator functions (II)

**my_generator.py**

```python
def my_generator(n):
    yield 'Start'
    for i in range(n):
        yield chr(ord('A') + i)
    yield 'Done'
```

**Python shell**

```
> g = my_generator(3)
> print(g)
| <generator object my_generator at 0x03E2F6F0>
> print(list(g))
| ['Start', 'A', 'B', 'C', 'Done']
> print(list(g))  # generator object g exhausted
| []
> print(*my_generator(5))  # * takes an iterable (PEP 448)
| Start A B C D E Done
```

# Generator functions (III)

**my_range_generator.py**

```
def my_range(start, end, step):
    x = start
    while x < end:
        yield x
        x += step
```

**Python shell**

```
> list(my_range(1.5, 2.0, 0.1))
| [1.5, 1.6, 1.7000000000000002, 1.8000000000000003, 1.9000000000000004]
```

# Pipelining generators

**Python shell**

```
> def squares(seq):      # seq should be an iterable object
      for x in seq:      # use iterator to run through seq
          yield x ** 2   # generator
> list(squares(range(5)))
| [0, 1, 4, 9, 16]
> list(squares(squares(range(5))))   # pipelining generators
| [0, 1, 16, 81, 256]
> sum(squares(squares(range(100000000))))   # pipelining generators
| 1999999950000000333333333333333330000000
> sum((x ** 2) ** 2 for x in range(100000000))   # generator expression
| 1999999950000000333333333333333330000000
> sum([(x ** 2) ** 2 for x in range(100000000)])   # list comprehension
| MemoryError  # when using a 32-bit version of Python, limited to 2 GB
```

# yield vs yield from

**Python shell**

```
> def g():
      yield 1
      yield [2, 3, 4]
      yield 5
> list(g())
| [1, [2, 3, 4], 5]
```

**Python shell**

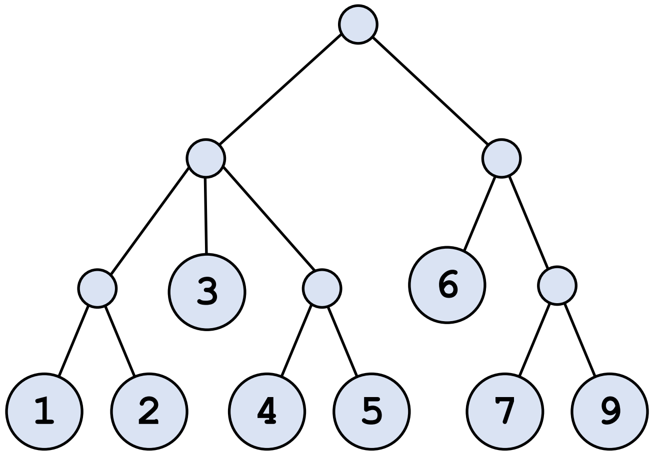
```
> def g():
      yield 1
      yield from [2, 3, 4]
      yield 5
> list(g())
| [1, 2, 3, 4, 5]
```

- `yield from` available since Python 3.3
- `yield from` *`exp`* ≈ `for x in` *`exp`*`: yield x`

# Recursive `yield from`

**Python shell**

```
> def traverse(T):  # recursive generator
      if isinstance(T, tuple):
          for child in T:
              yield from traverse(child)
      else:
          yield T
> T = (((1, 2), 3, (4, 5)), (6, (7, 9)))
> traverse(T)
| <generator object traverse at 0x03279F30>
> list(traverse(T))
| [1, 2, 3, 4, 5, 6, 7, 9]
```



# Making objects iterable using yield

vector2D.py

```
class vector2D:
    def __init__(self, x_value, y_value):
        self.x = x_value
        self.y = y_value
    def __iter__(self):  # generator
        yield self.x
        yield self.y
    def __iter__(self):  # alternative generator
        yield from (self.x, self.y)
v = vector2D(5, 7)
print(list(v))
print(tuple(v))
print(set(v))
```

Python shell

```
| [5, 7]
| (5, 7)
| {5, 7}
```

# Generators vs iterators

- Iterators can often be reused (can copy the current state)
- Generators cannot be reused (only if a new generator is created, starting over again)

- David Beazley's tutorial on
"*Generators: The Final Frontier*", PyCon 2014 (3:50:54)
Throughout advanced discussion of generators, e.g. how to use `.send` method to implement coroutines
https://www.youtube.com/watch?v=D1twn9kLmYg

# Measuring memory usage

# Measuring memory usage (memory profiling)

- Macro level:

  Task Manager (Windows)
  Activity Monitor (Mac)
  top (Linux)

- Variable level:

  `getsizeof` from `sys` module

- Detailed overview:

  Module `memory_profiler`

  Allows detailed space usage of the code line-by-line (using @profile function decorator) or a plot of total space usage over time

  pip install memory-profiler

**Python shell**

```
> import sys
> sys.getsizeof(42)
| 14  # size of the integer 42 is 14 bytes
> sys.getsizeof(42 ** 42)
| 44  # the size increases with value
> sys.getsizeof('42')
| 27  # size of a string
> import numpy as np
> sys.getsizeof(np.array(range(100), dtype='int32'))
| 448  # also works on Numpy arrays
> squares = [x ** 2 for x in range(1000000)]
> sys.getsizeof(squares)
| 4348736
> g = (x ** 2 for x in range(1000000))
> sys.getsizeof(g)
| 64
```

# Module memory-profiler

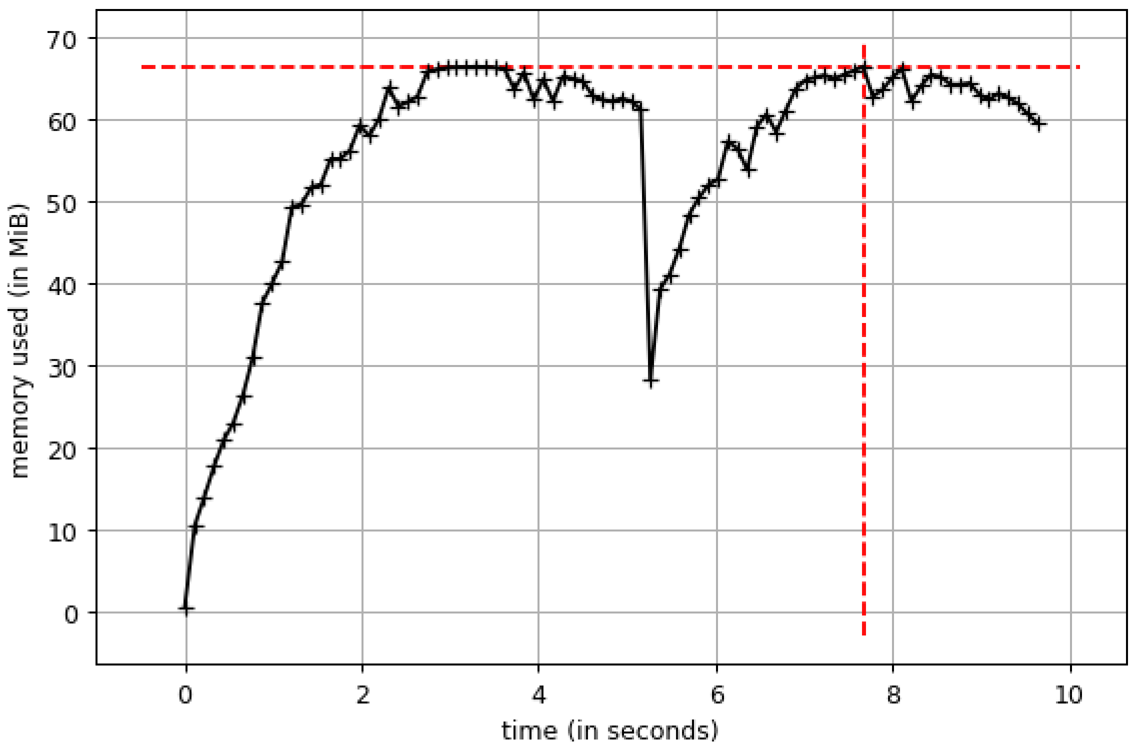
pypi.org/project/memory-profiler/

**memory_usage.py**

```python
from memory_profiler import profile

@profile  # prints new statistics for each call
def use_memory():
    s = 0
    x = list(range(20_000_000))
    s += sum(x)
    y = list(range(10_000_000))
    s += sum(x)

use_memory()
```

**Python Shell**

```
| Filename: C:/.../memory_usage.py
|
| Line #  Mem usage  Increment   Line Contents
| ================================================
|      3   32.0 MiB   32.0 MiB  @profile
|      4                        def use_memory():
|      5   32.0 MiB    0.0 MiB      s = 0
|      6  415.9 MiB  383.9 MiB      x = list(range(20_000_000))
|      7  415.9 MiB    0.0 MiB      s += sum(x)
|      8  607.8 MiB  191.9 MiB      y = list(range(10_000_000))
|      9  607.8 MiB    0.0 MiB      s += sum(x)
```

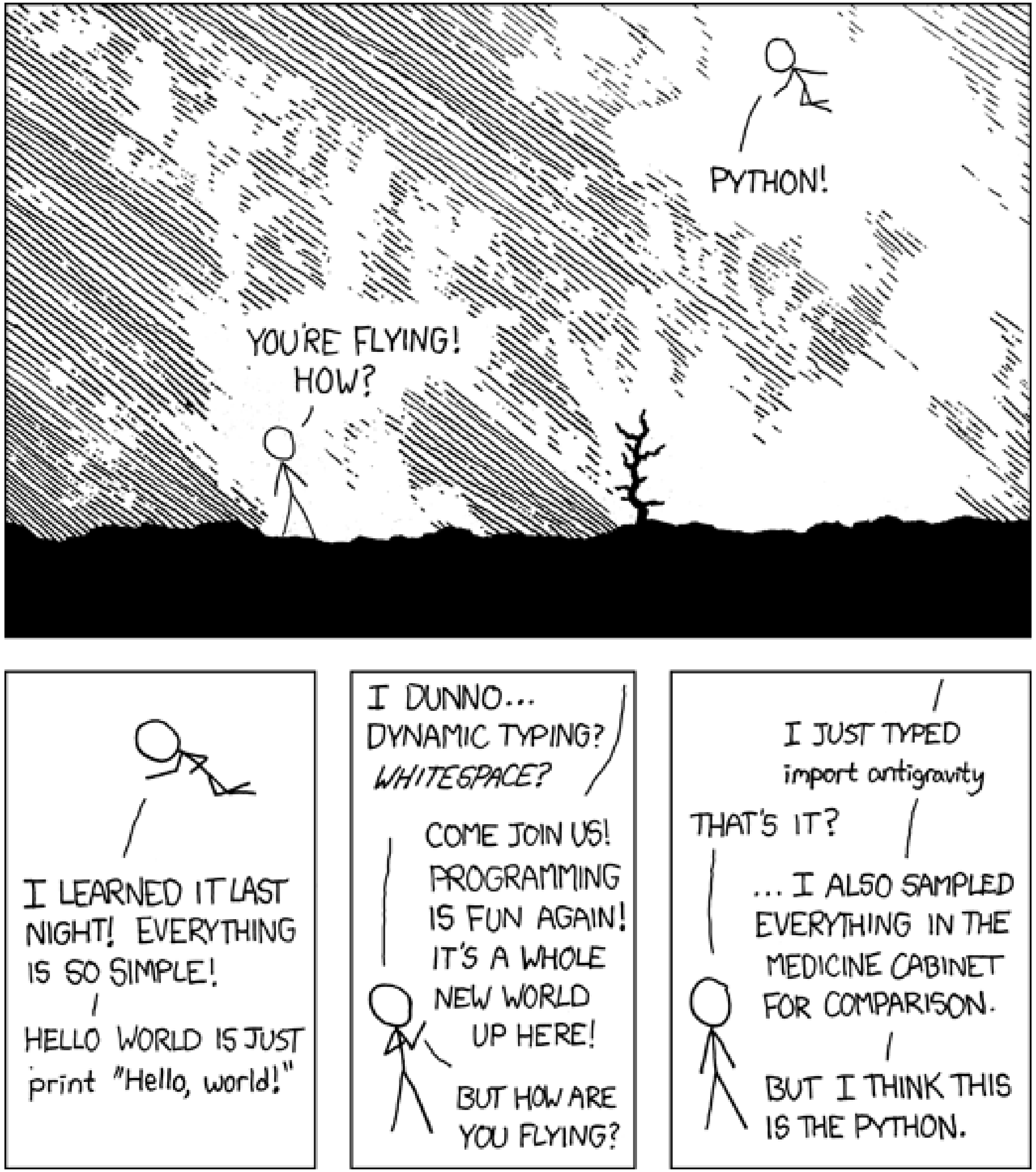



**memory_sin_usage.py**

```python
from math import sin, pi

for a in range(1000):
    x = list(range(int(1000000 * sin(pi * a / 250))))
```

**Windows Shell**

```
> pip install memory-profiler
> mprof run memory_sin_usage.py
| mprof: Sampling memory every 0.1s
| running as a Python program...
> mprof plot
```




xkcd.com/353

# Modules and packages

- import – from – as
- __name__, "__main__"

docs.python.org/3/tutorial/modules.html

# Python modules and packages

- A Python module is a *module_name*.py file containing Python code
- A Python package is a collection of modules

**Why do you need modules ?**

- A way to structure code into smaller logical units
- Encapsulation of functionality
- Reuse of code in different programs

- Your can write your own modules and packages or use any of the +350.000 existing packages from pypi.org
- The Python Standard Library consists of the modules listed on docs.python.org/3/library

# Defining and importing a module

**mymodule.py**

```python
""" This is a 'print something' module """

from random import randint

print("Running my module")

def print_something(n):
    W = ['Eat', 'Sleep', 'Rave', 'Repeat']
    words = (W[randint(0, len(W) - 1)] for _ in range(n))
    print(' '.join(words))

def the_name():
    print('__name__ = "' + __name__ +'"')
```

**using_mymodule.py**

```python
import mymodule
mymodule.the_name()
mymodule.print_something(5)
from mymodule import print_something
print_something(5)
```

**Python shell**

```
| Running my module
| __name__ = "mymodule"
| Eat Sleep Sleep Sleep Rave
| Eat Sleep Rave Repeat Sleep
```

- A module is only run once when imported several times

# Some modules mentioned in the course

| Module (example functions) | Description |
|---|---|
| math (pi sqrt ceil log sin) | *basic math* |
| random (random randint) | *random number generator* |
| numpy (array shape) | *multi-dimensional data* |
| pandas | *data tables* |
| SQLlite | *SQL database* |
| scipy<br>scipy.optimize (minimize linprog)<br>scipy.spatial (ConvexHull) | *mathematical optimization* |
| matplotlib<br>matplotlib.pyplot (plot show style)<br>matplotlib.backends.backend_pdf (PdfPages)<br>mpl_toolkits.mplot3d (Axes3D) | <br>*plotting data*<br>*print plots to PDF*<br>*3D plot tools* |
| doctest (testmod)<br>unittest (assertEqual assertTrue) | *testing using doc strings*<br>*unit testing* |
| time (time)<br>datetime (date.today) | *current time, coversion of time values* |
| timeit (timeit) | *time execution of simple code* |
| heapq | *use a list as a heap* |

| Module (example functions) | Description |
|---|---|
| functools (cache lru_cache total_ordering) | *higher order functions and decorators* |
| itertools (islice permutations) | *Iterator tools* |
| collections (Counter deque) | *datat structures for collections* |
| builtins | *module containing the Python builtins* |
| os (path) | *operating system interface* |
| sys (argv path) | *system specific functions* |
| Tkinter<br>PyQt | *graphic user interface* |
| xml | *xml files (eXtensible Markup Language)* |
| json | *JSON (JavaScript Object Notation) files* |
| csv | *comma separated files* |
| openpyxl | *EXCEL files* |
| re | *regular expression, string searching* |
| string (split join lower ascii_letters digits) | *string functions* |

# Ways of importing modules

**import.py**

```
# Import a module name in the current namespace
# All definitions in the module are available as <module>.<name>

import math
print(math.sqrt(2))

# Import only one or more specific definitions into current namespace

from math import sqrt, log, ceil
print(ceil(log(sqrt(100), 2)))

# Import specific modules/definitions from a module into current namespace under new names

from math import sqrt as kvadratrod, \  # long import line broken onto multiple lines
                 log as logaritme
import matplotlib.pyplot as plt
print(logaritme(kvadratrod(100)))

# Import all definitions form a module in current namespace
# Deprecated, since unclear what happens to the namespace

from math import *
print(pi)  # where did 'pi' come from?
```

**Python shell**

```
| 1.4142135623730951
| 4
| 2.302585092994046
| 3.141592653589793
```

# __all__ vs import *

- A module can control what is imported by `import *` by defining `__all__`

**Python shell**

```
> min
| <built-in function min>
> sum
| <built-in function sum>
> import numpy
> numpy.min
| <function amin at 0x0000024768E69F30>  # numpy.min == numpy.amin
> numpy.sum
| <function sum at 0x0000024768E69510>
> from numpy import *
> sum
| <function sum at 0x0000024768E69510>  # numpy.sum
> min
| <built-in function min>  # builtin min
> numpy.__all__
| [..., 'sum', ...]   # 'min' is not in list
```

**all.py**

```
__all__ = ['f']

def f():
    print('this is f')

def g():
    print('this is g')
```

**Python shell**

```
> import all
> all.f()
| this is f
> all.g()
| this is g
> from all import *
> f()
| this is f
> g()
| NameError: name 'g' is not defined
```

# Performance of different ways of importing

**`from math import sqrt`**

appears to be faster than

**`math.sqrt`**

**sqrt_performance.py**

```
from time import time

import math
start = time()
x = sum(math.sqrt(x) for x in range(10000000))
end = time()
print("math.sqrt", end - start)

from math import sqrt
start = time()
x = sum(sqrt(x) for x in range(10000000))
end = time()
print("from math import sqrt", end - start)

def test(sqrt=math.sqrt):  # abuse of keyword argument
    start = time()
    x = sum(sqrt(x) for x in range(10000000))
    end = time()
    print("bind sqrt to keyword argument", end - start)

test()
```

**Python shell**

```
| math.sqrt 4.05187726020813
| from math import sqrt 3.5011463165283203
| bind sqrt to keyword argument 3.261594772338867
```

# Listing definitions in a module: dir(*module*)

**Python shell**

```
> import math
> import matplotlib.pyplot as plt
> dir(math)
| ['__doc__', '__loader__', '__name__', '__package__', '__spec__', 'acos',
  'acosh', 'asin', 'asinh', 'atan', 'atan2', 'atanh', 'ceil', 'copysign',
  'cos', 'cosh', 'degrees', 'e', 'erf', 'erfc', 'exp', 'expm1', 'fabs',
  'factorial', 'floor', 'fmod', 'frexp', 'fsum', 'gamma', 'gcd', 'hypot',
  'inf', 'isclose', 'isfinite', 'isinf', 'isnan', 'ldexp', 'lgamma', 'log',
  'log10', 'log1p', 'log2', 'modf', 'nan', 'pi', 'pow', 'radians', 'sin',
  'sinh', 'sqrt', 'tan', 'tanh', 'tau', 'trunc']
> help(math)
| Help on built-in module math:
| NAME
|     math
| DESCRIPTION
| ...
```

https://docs.python.org/3/library/functions.html#dir

# __name__

double.py

```python
""" Module double """
def f(x):
    """
    Some doc test code:
    >>> f(21)
    42
    >>> f(7)
    14
    """
    return 2 * x
print('__name__ =', __name__)
if __name__ == "__main__":
    import doctest
    doctest.testmod(verbose=True)
```

Python shell

```
| __name__ = __main__
  ...
  2 passed and 0 failed.
  Test passed.
```

using_double.py

```python
import double
print(__name__)
print(double.f(5))
```

Python shell

```
| __name__ = double
  __main__
  10
```

- The variable `__name__` contains the name of the module, or `'__main__'` if the file is run as the main file by the interpreter
- Can e.g. be used to test a module if the module is run independently

# module `importlib`

- Implements the `import` statement (Python internal implementation details)
- `importlib.reload(`*module*`)`
  - Reloads a previously imported *module*. Relevant if you have edited the code for the module and want to load the new version in the Python interpreter, without restarting the full program from scratch.

**a_constant.py**

```
the_constant = 7
```

**Python shell**

```
> import a_constant   # import module
> a_constant.the_constant
| 7
> from a_constant import the_constant
> the_constant
| 7
# Update 7 to 42 in a_constant.py
> a_constant.the_constant   # new value not reflected
| 7
> import a_constant   # void, module already loaded
> a_constant.the_constant
| 7  # unchanged
> import importlib
> importlib.reload(a_constant)
| <module 'a_constant' from 'C:\\...\\a_constant.py'>
> a_constant.the_constant
| 42
> the_constant
| 7  # imported attributes are not updated by reload
> from a_constant import the_constant  # force update
> the_constant
| 42  # the new value
```

# Packages

- A package is a collection of modules (and subpackages) in a folder = package name
- Only folders having an `__init__.py` file are considered packages
- The `__init__.py` can be empty, or contain code that will be loaded when the package is imported, e.g. importing specific modules

**mypackage/__init__.py**

```
```

**mypackage/a.py**

```python
print("Loading mypackage.a")
def f():
    print("mypackage.a.f")
```

**using_mypackage.py**

```python
import mypackage.a
mypackage.a.f()
```

**Python shell**

```
| Loading mypackage.a
| mypackage.a.f
```

# A package with a subpackage

**mypackage/__init__.py**

```python
print('loading mypackage')
```

**mypackage/a.py**

```python
print('Loading mypackage.a')
def f():
    print('mypackage.a.f')
```

**mypackage/mysubpackage/__init__.py**

```python
print('loading mypackage.mysubpackage')
import mypackage.mysubpackage.b
```

**mypackage/mysubpackage/b.py**

```python
print('Loading mypackage.mysubpackage.b')
def g():
    print('mypackage.mysubpackage.b.g')
```

**using_mysubpackage.py**

```python
import mypackage.a
mypackage.a.f()
import mypackage.mysubpackage
mypackage.mysubpackage.b.g()
from mypackage.mysubpackage.b import g
g()
```

**Python shell**

```
| loading mypackage
| Loading mypackage.a
| mypackage.a.f
| loading mypackage.mysubpackage
| Loading mypackage.mysubpackage.b
| mypackage.mysubpackage.b.g
| mypackage.mysubpackage.b.g
```

# `__pycache__` folder

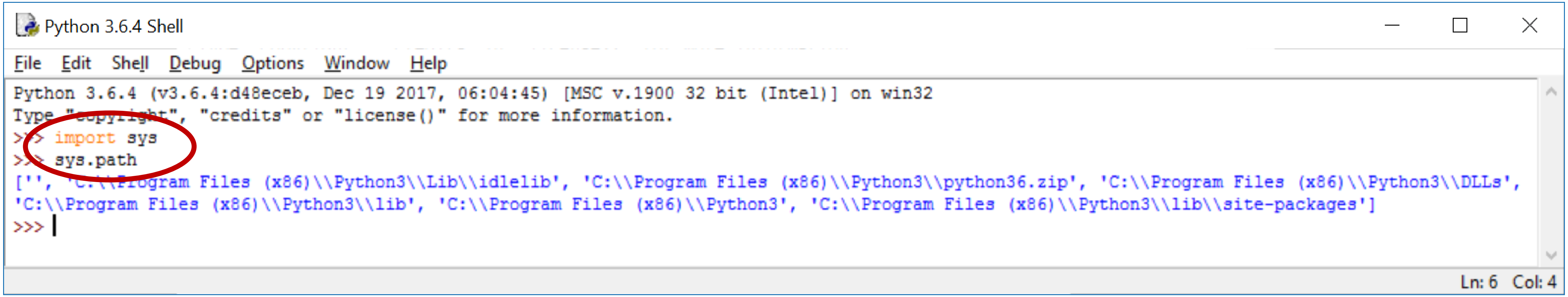
- When Python loads a module the first time it is *compiled* to some intermediate code, and stored as a .pyc file in the `__pycache__` folder.
- If a .pyc file exists for a module, and the .pyc file is newer than the .py file, then `import` loads .pyc – saving time to load the module (but does not make the program itself faster).
- It is safe to delete the `__pycache__` folder – but it will be created again next time a module is loaded.

# Path to modules

Python searches the following folders for a module in the following order:

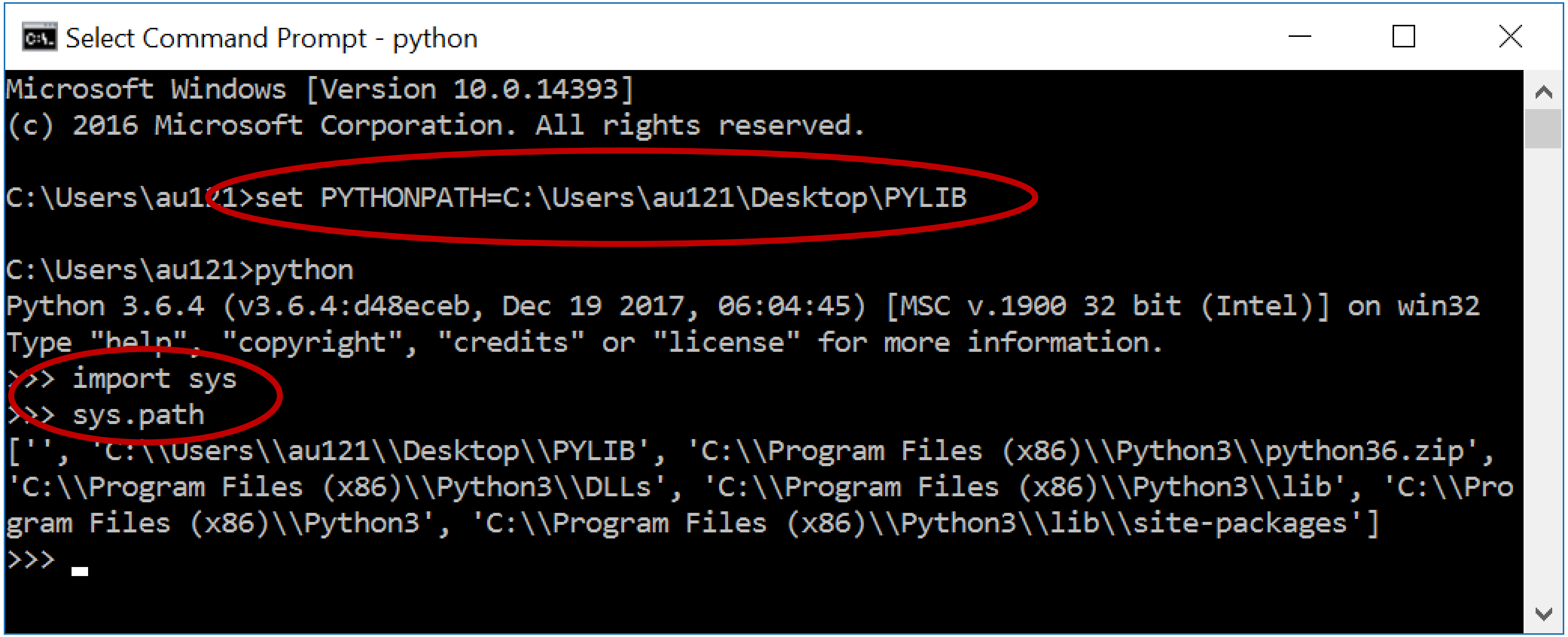
1) The directory containing the input script / current directory
2) *Environment* variable `PYTHONPATH`
3) Installation defaults

The function `path` in the modul `sys` returns a list of the paths



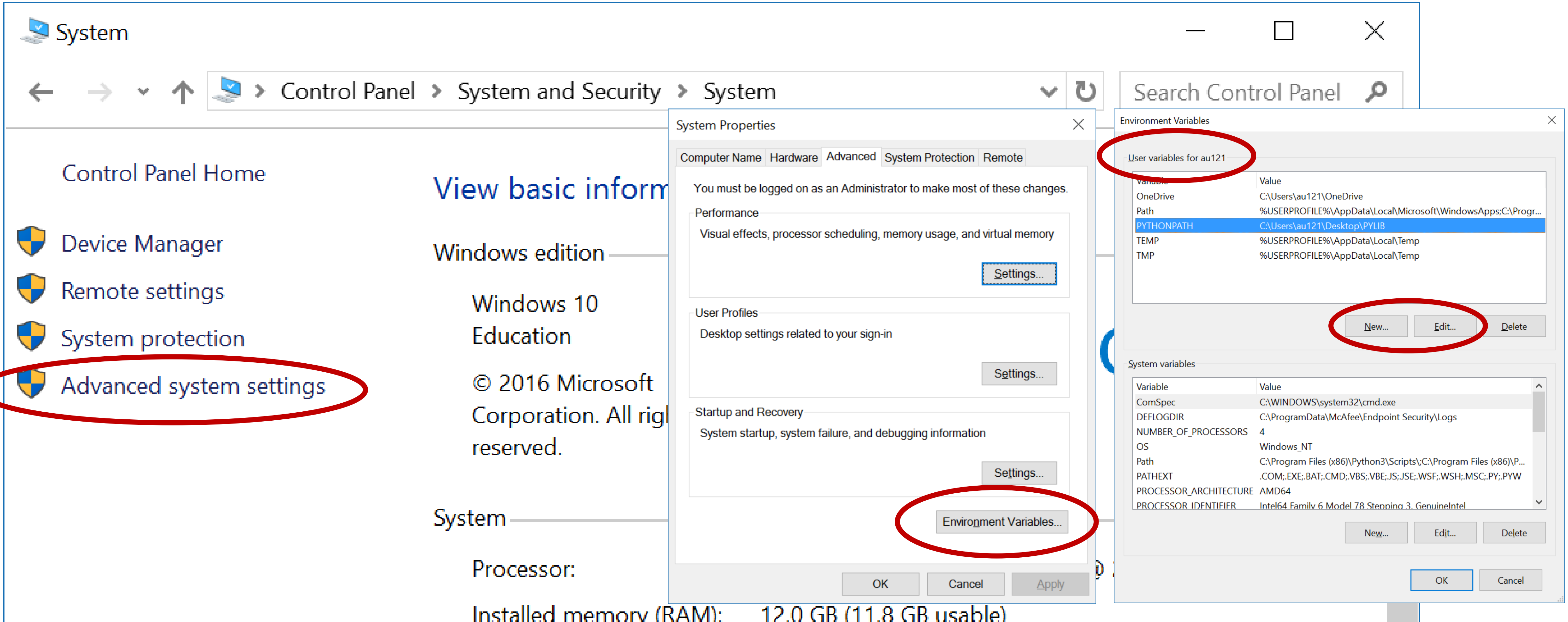
# Setting PYTHONPATH from windows shell

- set PYTHONPATH=*paths separated by semicolon*
  (only valid until shell is closed)



# Setting PYTHONPATH from control panel

- Control panel > System > Advanced system settings > Environment Variables > User variables > Edit or New PYTHONPATH



**Python shell** www.python.org/dev/peps/pep-0020/

```
> import this
| The Zen of Python, by Tim Peters
|
| Beautiful is better than ugly.
| Explicit is better than implicit.
| Simple is better than complex.
| Complex is better than complicated.
| Flat is better than nested.
| Sparse is better than dense.
| Readability counts.
| Special cases aren't special enough to break the rules.
| Although practicality beats purity.
| Errors should never pass silently.
| Unless explicitly silenced.
| In the face of ambiguity, refuse the temptation to guess.
| There should be one-- and preferably only one --obvious way to do it.
| Although that way may not be obvious at first unless you're Dutch.
| Now is better than never.
| Although never is often better than *right* now.
| If the implementation is hard to explain, it's a bad idea.
| If the implementation is easy to explain, it may be a good idea.
| Namespaces are one honking great idea -- let's do more of those!
```

# module **heapq** (Priority Queue)

- Implements a binary **heap** (Williams 1964).
- Stores a set of elements in a standard list, where arbitrary elements can be inserted efficiently and the smallest element can be extracted efficiently

**`heapq.heappush`**
**`heapq.heappop`**

docs.python.org/3/library/heapq.html

J. W. J. Williams. *Algorithm 232: Heapsort*. Communications of the ACM (1964)

**heap.py**

```
import heapq
from random import random

H = []  # a heap is just a list

for _ in range(10):
    heapq.heappush(H, random())

while True:
    x = heapq.heappop(H)
    print(x)
    heapq.heappush(H, x + random())
```

**Python shell**

```
| 0.20569933892764458
  0.27057819339616174
  0.31115615362876237
  0.484106227215225
  0.5054280956005357
  0.509387117524076
  0.598647195480462
  0.7035150735555027
  0.707392968582622
  0.7091224012815325
  0.714213496127318
  0.727868481291271
  0.8051275413759873
  0.827952376728290
  0.862602236320289
```

# Valid heap

- A *valid heap* satisfies for all i:

  $L[i] \leq L[2 \cdot i + 1]$ and $L[i] \leq L[2 \cdot i + 2]$

- **heapify(L)** rearranges the elements in a list to make the list a valid heap

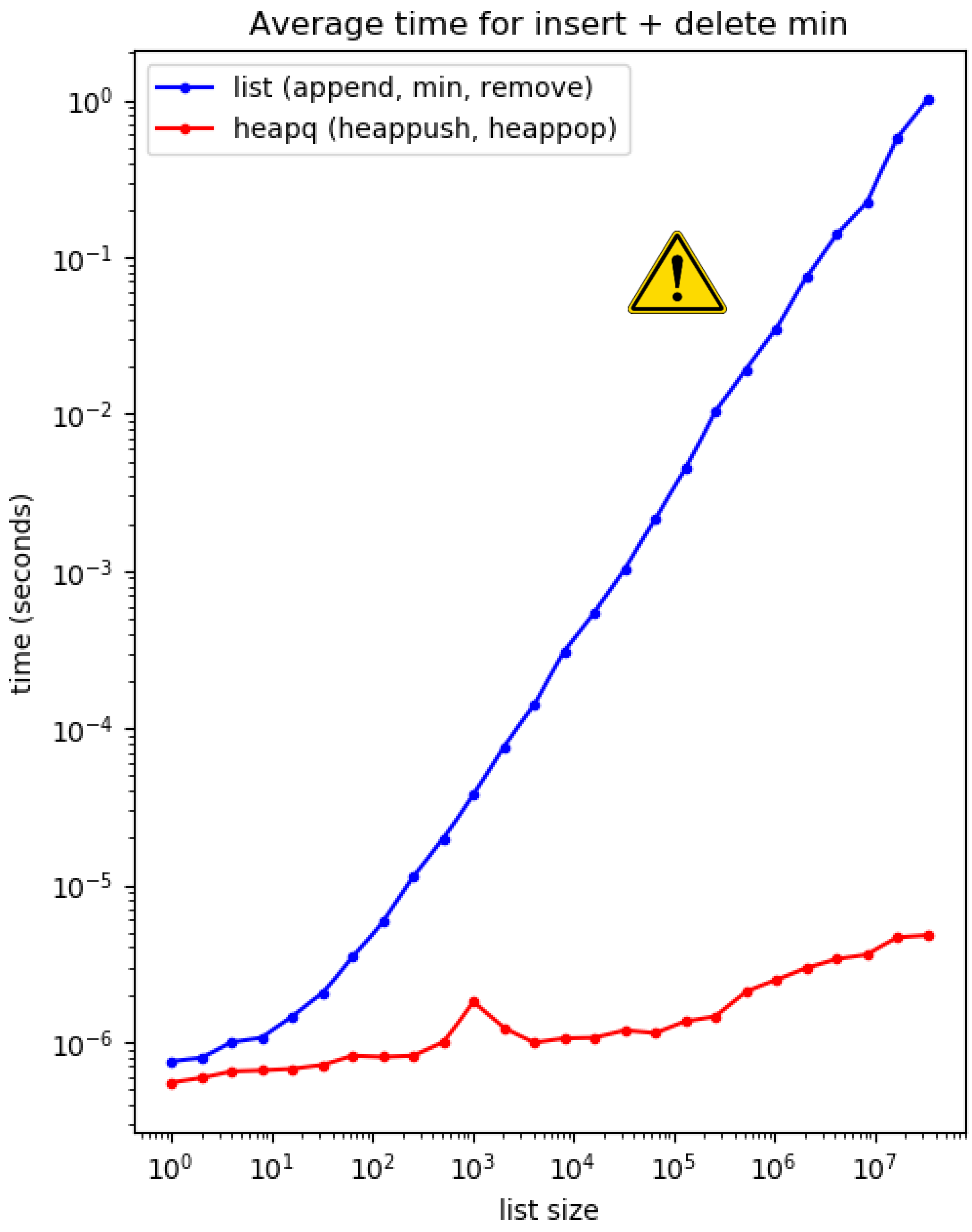
**Python shell**

```
> from random import randint
> L = [randint(1, 20) for _ in range(10)]
> L  # just random numbers
| [18, 1, 15, 17, 4, 14, 11, 3, 4, 9]
> import heapq
> heapq.heapify(L)  # make L a valid heap
> L
| [1, 3, 11, 4, 4, 14, 15, 17, 18, 9]
> print(heapq.heappop(L))
| 1
> L
| [3, 4, 11, 4, 9, 14, 15, 17, 18]
> heapq.heappush(L, 7)
> L
| [3, 4, 11, 4, 7, 14, 15, 17, 18, 9]
```

J. W. J. Williams. *Algorithm 232: Heapsort*. Communications of the ACM (1964)

# Why **`heapq`** ?

- `min` and `remove` on a list take *linear time* (runs through the whole list)
- heapq supports `heappush` and `heappop` in *logarithmic time*
- For lists of length 30.000.000 the performance gain is a factor 200.000



**heap_performance.py (generating plot on previous slide)**

```
import heapq
from random import random
import matplotlib.pyplot as plt
from time import time
import gc  # garbage collection

size = []
time_heap = []
time_list = []

for i in range(26):
    n = 2 ** i
    size.append(n)

    L = [random() for _ in range(n)]
    R = max(1, 2 ** 23 // n)
B   gc.collect()
    start = time()
    for _ in range(R):
        L.append(random())
        x = min(L)
        L.remove(x)
    end = time()
    time_list.append((end - start) / R)
A   L = None  # avoid MemoryError
    L = [random() for _ in range(n)]
    heapq.heapify(L)  # make L a legal heap
B   gc.collect()
    start = time()
    for _ in range(100000):
        heapq.heappush(L, random())
        x = heapq.heappop(L)
    end = time()
    time_heap.append((end - start) / 100000)

plt.title("Average time for insert + delete min")
plt.xlabel("list size")
plt.ylabel("time (seconds)")
plt.plot(size, time_list, 'b.-',
         label='list (append, min, remove)')
plt.plot(size, time_heap, 'r.-',
         label='heapq (heappush, heappop)')
plt.xscale('log')
plt.yscale('log')
plt.legend()
plt.show()
```

A Avoid out of memory error for largest experiment, by allowing old **L** to be garbage collected

B Reduce noise in experiments by forcing Python garbage collection before measurement

# Working with text

- file formats
- CSV, JSON, XML, Excel
- regular expressions
- module re, finditer

# Some file formats

| File extension | Content |
|---|---|
| .html | HyperText Markup Language |
| .mp3 | Audio File |
| .png .jpeg .jpg | Image files |
| .svg | Scalable Vector Graphics file |
| .json | JavaScript Object Notation |
| .csv | Comma separated values |
| .xml | eXtensible Markup Language |
| .xlmx | Micosoft Excel 2010/2007 Workbook |

| File extension | Description |
|---|---|
| .exe | Windows executable file |
| .app | Max OS X Application |
| .py | Python program |
| .pyc | Python compiled file |
| .java | Java program |
| .cpp | C++ program |
| .c | C program |
| .txt | Raw text file |

# PIL – the Python Imaging Library

- pip install Pillow

**rotate_image.py**

```
from PIL import Image
img = Image.open("Python-Logo.png")
img_out = img.rotate(45, expand=True)
img_out.save("Python-rotated.png")
```

- For many file types there exist Python packages handling such files, e.g. for images Pillow supports 40+ different file formats
- For more advanced computer vision tasks you should consider OpenCV


Python-Logo.png


Python-rotated.png

python-pillow.org

# CSV files - Comma Separated Values

- Simple 2D tables are stored as rows in a file, with values separated by comma
- Strings stored are quoted if necessary
- Values read are strings
- The deliminator (default comma) can be changed by keyword argument `delimiter`.
  Other typical deliminators are tabs '`\t`', and semicolon '`;`'

docs.python.org/3/library/csv.html

**csv-example.py**

```
import csv
FILE = 'csv-data.csv'
data = [[1, 2, 3],
        ['a', '"b"'],
        [1.0, ['x',"y"], 'd']]
with open(FILE, 'w', newline="") as outfile:
    csv_out = csv.writer(outfile)
    for row in data:
        csv_out.writerow(row)
with open(FILE, 'r', newline="") as infile:
    for row in csv.reader(infile):
        print(row)
```

**Python shell**

```
| ['1', '2', '3']
  ['a', '"b"']
  ['1.0', "['x', 'y']", 'd']
```

**csv-data.csv**

```
1,2,3
a,"""b"""
1.0,"['x', 'y']",d
```

# CSV files - Tab Separated Values

csv-tab-separated.py

```python
import csv

FILE = 'tab-separated.csv'

with open(FILE) as infile:
    for row in csv.reader(infile, delimiter='\t'):
        print(row)
```

Python shell

```
['1', '2', '3']
['4', '5', '6']
['7', '8', '9']
```

tab-separated.csv

```
1	2	3
4	5	6
7	8	9
```

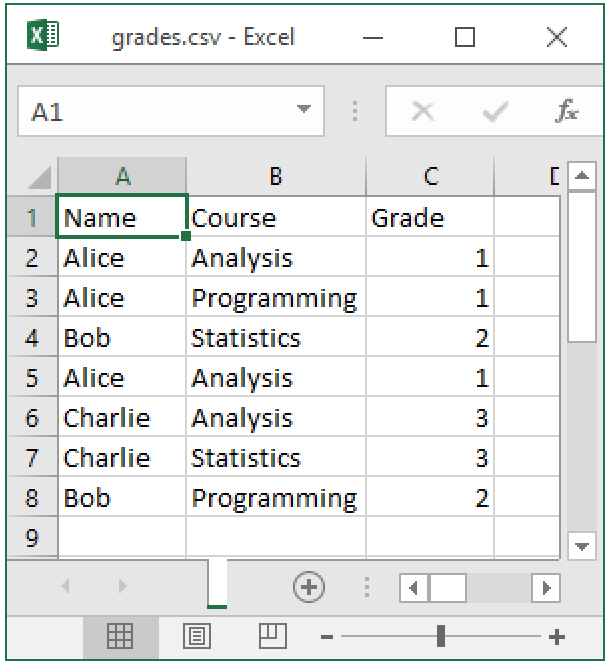
# Reading an Excel generated CSV file

**average.py**

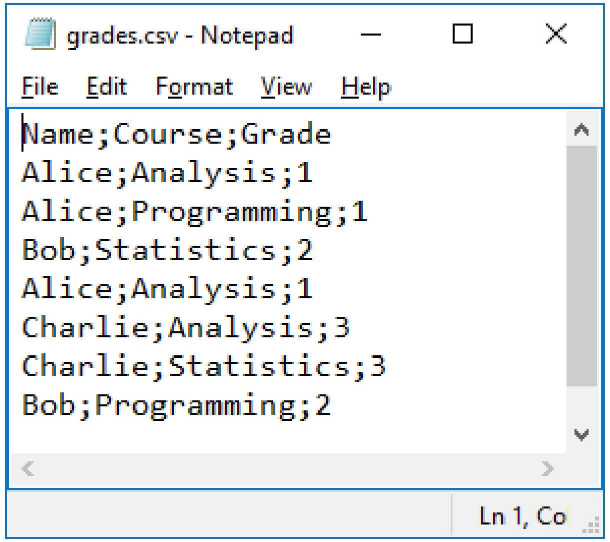
```python
import csv

with open('grades.csv') as file:
    data = csv.reader(file, delimiter=';')  # data = iterator over the rows
    header = next(data)                     # ['Name', 'Course', 'Grade']
    count = {}
    total = {}
    for row in data:                        # iterate over data rows
        course = row[header.index('Course')]
        grade  = int(row[header.index('Grade')])
        count[course] = count.get(course, 0) + 1
        total[course] = total.get(course, 0) + grade
print('Average grades:')
width = max(map(len, count))  # maximum course name length
for course in count:
    print(f'{course:>{width}s} : {total[course] / count[course]:.2f}')
```

**Python shell**

```
| Average grades:
|    Analysis : 1.67
| Programming : 1.50
|  Statistics : 2.50
```




Saving a file in Excel as
**CSV (Comma delimited) (*.csv)**
apparently uses ';' as the separator...



# CSV files
# - Quoting

- The amount of quoting is controlled with keyword argument `quoting`
- `csv.QUOTE_MINIMAL` etc. can be used to select the quoting level
- Depending on choice of quoting, numeric values and strings cannot be distinguished in CSV file (`csv.reader` will read all as strings anyway)

**csv-quoting.py**

```python
import csv
import sys
data = [[1, 1.0, '1.0'], ['abc', '"', '\t"', ',']]
quoting_options = [(csv.QUOTE_MINIMAL,    "QUOTE_MINIMAL"),
                   (csv.QUOTE_ALL,        "QUOTE_ALL"),
                   (csv.QUOTE_NONNUMERIC, "QUOTE_NONNUMERIC"),
                   (csv.QUOTE_NONE,       "QUOTE_NONE")]
for quoting, name in quoting_options:
    print(name)
    csv_out = csv.writer(sys.stdout, quoting=quoting, escapechar='\\')
    for row in data:
        csv_out.writerow(row)
```

**Python shell**

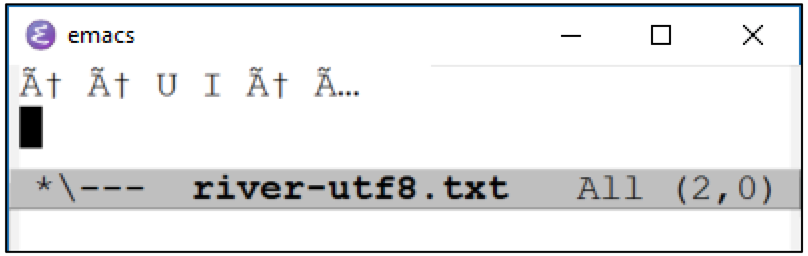
```
| QUOTE_MINIMAL  # cannot distinguish 1.0 and "1.0"
| 1,1.0,1.0
| abc,"""","     """,","
| QUOTE_ALL  # cannot distinguish 1.0 and "1.0"
| "1","1.0","1.0"
| "abc","""","   """,","
| QUOTE_NONNUMERIC
| 1,1.0,"1.0"
| "abc","""","   """,","
| QUOTE_NONE  # cannot distinguish 1.0 and "1.0"
| 1,1.0,1.0
| abc,\",        \",\,
```

# File encodings...

```
river-utf8.py   (size 17 bytes, encoding UTF-8)
Æ Æ U I Æ Å
```

```
river-windows1252.py   (size 13 bytes, encoding Windows-1252)
Æ Æ U I Æ Å
```

- Text files can be *encoded* using many different encodings (UTF-8, UTF-16, UTF-32, Windows-1252, ANSI, ASCII, ISO-8859-1, ...)
- Different encodings can result in different file sizes, in particular when containing non-ASCII symbols
- Programs often try to predict the encoding of text files (often with success, but not always)
- Opening files assuming wrong encoding can give strange results....




Opening UTF-8 encoded file but trying to decode using Windows-1252


Opening Windows-1252 encoded file but trying to decode using UTF-8

en.wikipedia.org/wiki/Character_encoding

**encoding.py**

```
for filename in ['river-utf8.txt', 'river-windows1252.txt']:
    print(filename)
    f = open(filename, 'rb')  # open input in binary mode, default = text mode = 't'
    line = f.readline()       # type(line) = bytes = immutable list of integers in 0..255
    print(line)                                   # byte literals look like strings, prefixed 'b'
    print(list(line))                             # print bytes as list of integers
    f = open(filename, 'r', encoding='utf-8')     # try to open file as UTF-8
    line = f.readline()                           # fails if input line is not utf-8
    print(line)
```

**river-utf8.py**

```
Æ Æ U I Æ Å
```

**Python shell**

```
| river-utf8.txt
| b'\xc3\x86 \xc3\x86 U I \xc3\x86 \xc3\x85\r\n'  # \x = hexadecimal value follows
| [195, 134, 32, 195, 134, 32, 85, 32, 73, 32, 195, 134, 32, 195, 133, 13, 10]
| Æ Æ U I Æ Å
| river-windows1252.txt
| b'\xc6 \xc6 U I \xc6 \xc5\r\n'
| [198, 32, 198, 32, 85, 32, 73, 32, 198, 32, 197, 13, 10]
| UnicodeDecodeError: 'utf-8' codec can't decode byte 0xc6 in position 0: invalid continuation
  byte
> 'Æ Æ U I Æ Å'.encode('utf8')  # convert string to (an immutable array of) bytes
| b'\xc3\x86 \xc3\x86 U I \xc3\x86 \xc3\x85'
> 'Æ Æ U I Æ Å'.encode('utf8').decode('Windows-1252')  # decode bytes to string
| 'Ã† Ã† U I Ã† Ã…'
```

# Reading CSV files with specific encoding

**read_shopping.py**

```
import csv
with open("shopping.csv", encoding="Windows-1252") as file:
    for article, amount in csv.reader(file):
        print("Buy", amount, article)
```

**Python shell**

```
| Buy 2 æbler
| Buy 4 pærer
| Buy 3 jordbær
| Buy 10 gulerøder
```

**shopping.csv**

```
æbler,2
pærer,4
jordbær,3
gulerøder,10
```

CSV file saved with Windows-1252 encoding

# JSON

*"**JSON** (**J**ava**S**cript **O**bject **N**otation) is a lightweight data-interchange format. It is easy for humans to read and write. It is easy for machines to parse and generate. It is based on a subset of the JavaScript Programming Language, Standard ECMA-262 3rd Edition - December 1999. JSON is an ideal data-interchange language."*

www.json.org

- Human readable file format
- Easy way to save a Python expression to a file
- Does *not* support all Python types, e.g. sets are not supported, and tuples are saved (and later loaded) as lists

# JSON example

json-example.py

```python
import json
FILE = 'json-data.json'
data = ((None, True), (42.7, (42,)), [3,2,4], (5,6,7),
        {'b':'banana', 'a':'apple', 'c': 'coconut'})

with open(FILE, 'w') as outfile:
    json.dump(data, outfile, indent=2, sort_keys=True)

with open(FILE) as infile:
    indata = json.load(infile)

print(indata)
```

Python shell

```
| [[None, True], [42.7, [42]], [3, 2, 4], [5, 6, 7], {'a':
  'apple', 'b': 'banana', 'c': 'coconut'}]
```

json-data.json

```json
[
  [
    null,
    true
  ],
  [
    42.7,
    [
      42
    ]
  ],
  [
    3,
    2,
    4
  ],
  [
    5,
    6,
    7
  ],
  {
    "a": "apple",
    "b": "banana",
    "c": "coconut"
  }
]
```

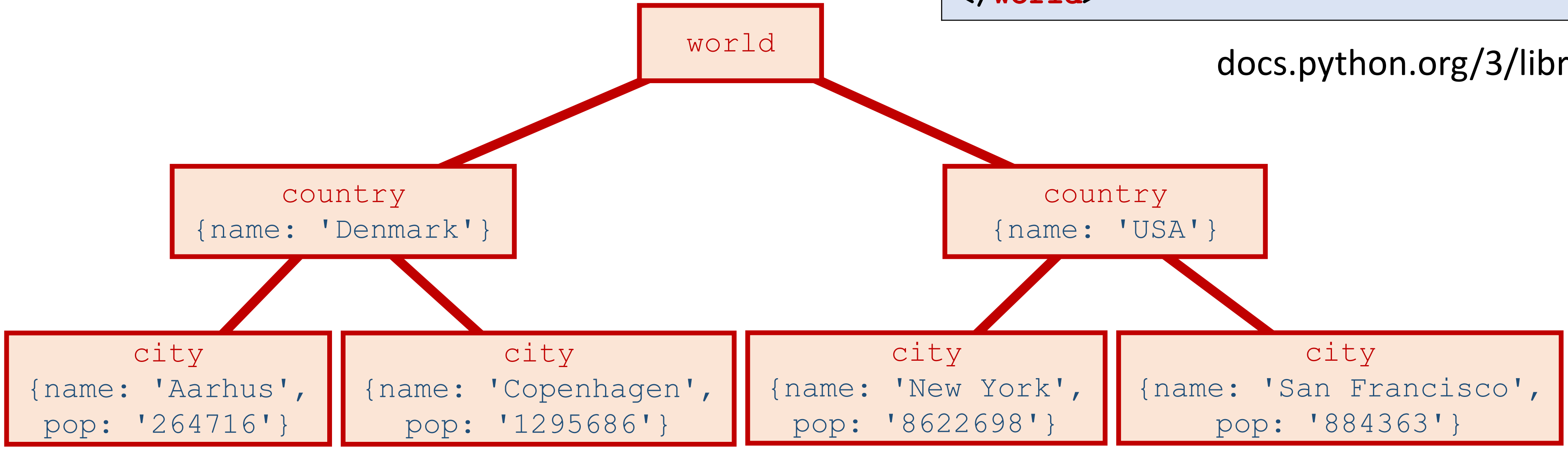
# XML - eXtensible Markup Language

- XML is a widespread used data format to store hierarchical data with tags and attributes

cities.xml

```xml
<?xml version="1.0"?>
<world>
  <country name="Denmark">
    <city name="Aarhus" pop="264716"/>
    <city name="Copenhagen" pop="1295686"/>
  </country>
  <country name="USA">
    <city name="New York" pop="8622698"/>
    <city name="San Francisco" pop="884363"/>
  </country>
</world>
```

docs.python.org/3/library/xml.html



xml-example.py

```python
import xml.etree.ElementTree as ET
FILE = 'cities.xml'
tree = ET.parse(FILE)  # parse XML file to internal representation
root = tree.getroot()  # get root element
for country in root:
    for city in country:
        print(city.attrib['name'],  # get value of attribute for an element
              'in',
              country.attrib['name'],
              'has a population of',
              city.attrib['pop'])
print(root.tag, root[0][1].attrib)  # the tag & indexing the children of an element
print([city.attrib['name'] for city in root.iter('city')])  # .iter finds elements
```

Python shell

```
| Aarhus in Denmark has a population of 264716
  Copenhagen in Denmark has a population of 1295686
  New York in USA has a population of 8622698
  San Francisco in USA has a population of 884363
  world {'name': 'Copenhagen', 'pop': '1295686'}
  ['Aarhus', 'Copenhagen', 'New York', 'San Francisco']
```

cities.xml

```xml
<?xml version="1.0"?>
<world>
  <country name="Denmark">
    <city name="Aarhus" pop="264716"/>
    <city name="Copenhagen" pop="1295686"/>
  </country>
  <country name="USA">
    <city name="New York" pop="8622698"/>
    <city name="San Francisco" pop="884363"/>
  </country>
</world>
```

# XML tags with text

**city-descriptions.xml**

```xml
<?xml version="1.0"?>
<world>
  <country name="Denmark">
    <city name="Aarhus" pop="264716">The capital of Jutland</city>
    <city name="Copenhagen" pop="1295686">The capital of Denmark</city>
  </country>
  <country name="USA">
    <city name="New York" pop="8622698">Known as Big Apple</city>
    <city name="San Francisco" pop="884363">Home of the Golden Gate Bridge</city>
  </country>
</world>
```

**xml-descriptions.py**

```python
import xml.etree.ElementTree as ET
FILE = 'city-descriptions.xml'
tree = ET.parse(FILE)
root = tree.getroot()

for city in root.iter('city'):
    print(city.get('name'), "-", city.text)
```

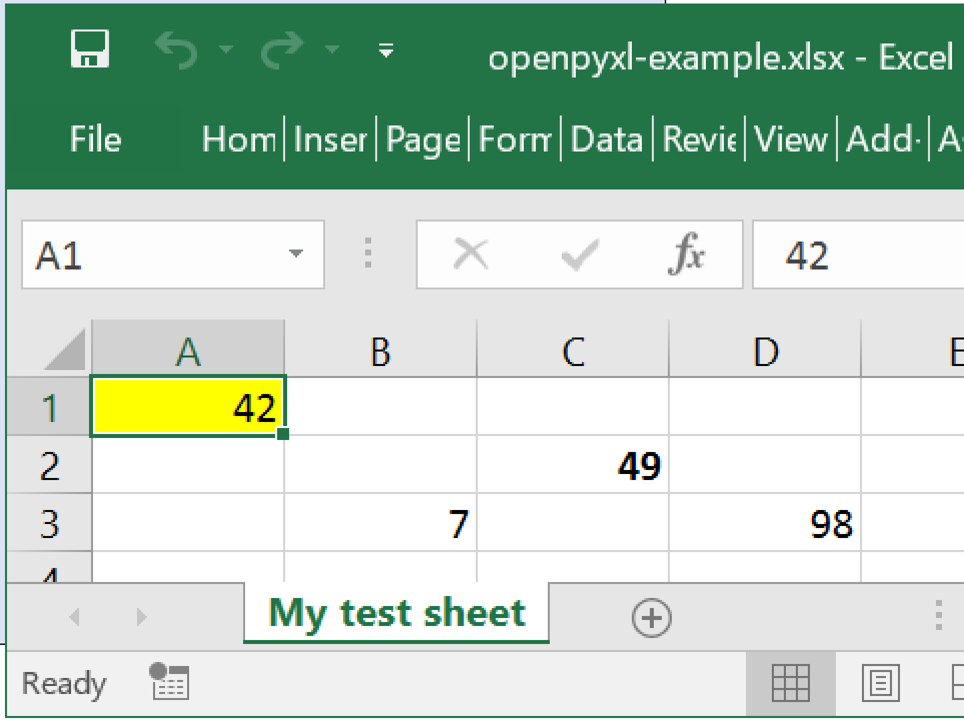
**Python shell**

```
Aarhus - The capital of Jutland
Copenhagen - The capital of Denmark
New York - Known as Big Apple
San Francisco - Home of the Golden Gate Bridge
```

# Openpyxl - Microsoft Excel 2010 manipulation

openpyxl-example.py

```
from openpyxl import Workbook
from openpyxl.styles import Font, PatternFill

wb = Workbook()   # create workbook
ws = wb.active    # active worksheet

ws['A1'] = 42
ws['B3'] = 7
ws['C2'] = ws['A1'].value + ws['B3'].value
ws['D3'] = '=A1+B3+C2'

ws.title = 'My test sheet'

ws['A1'].fill = PatternFill('solid', fgColor='ffff00')
ws['C2'].font = Font(bold=True)

wb.save("openpyxl-example.xlsx")
```




openpyxl.readthedocs.io/en/stable/tutorial.html

# String searching using `find`

- Search for first occurence of *`substring`* in *`str[start, end]`*

  `str.find(substring[, start[, end]])`

- Returns -1 if no occurence found.
- `.index` similar as `.find`, except raises ValueError exception if substring not found

**string-search.py**

```python
text = 'this is a string - a list of characters'
pattern = 'is'
idx = text.find(pattern)
while idx >= 0:
    print(idx, end=" ")
    idx = text.find(pattern, idx + 1)
```

**Python shell**

```
| 2 5 22
```

docs.python.org/3/library/stdtypes.html#textseq

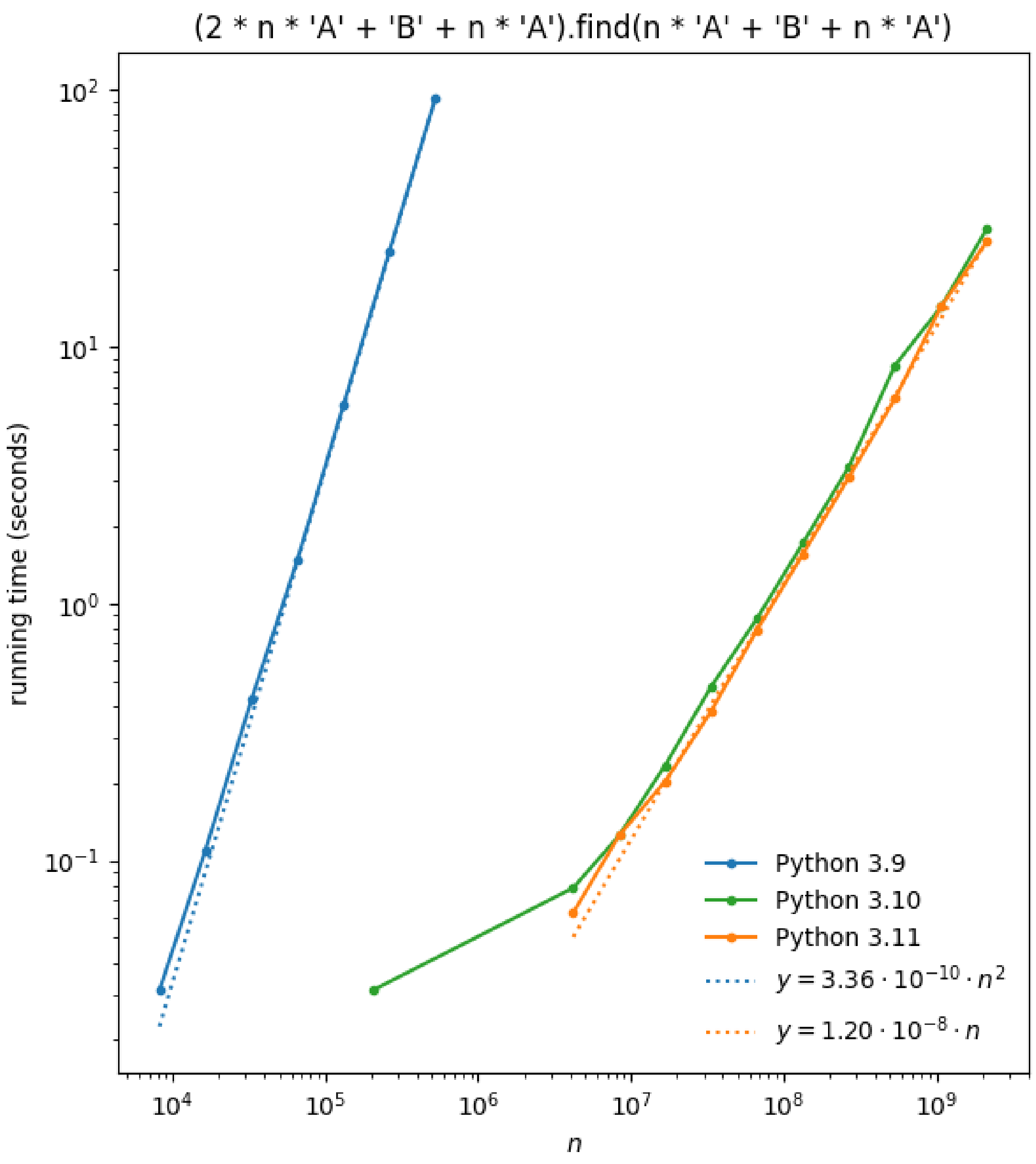
# Is *str*.find fast?

- Typically linear
- Until Python 3.9 in some cases quadractic

  `"A`$^{2n}$`BA`$^{n}$`".find("A`$^{n}$`BA`$^{n}$`")`

- docs.python.org/3/whatsnew/3.10.html

  *"Substring search functions such as str1 in str2 and str2.find(str1) now sometimes use Crochemore & Perrin's "Two-Way" string searching algorithm to avoid quadratic behavior on long strings."*



# Regular expression
## – A powerful language to describe sets of strings

- **Examples**
  - `abc` denotes a string of letters
  - `ab*c` any string starting with `a`, followed by an arbitrary number of `b`s and terminated by `c`, i.e. {`ac`, `abc`, `abbc`, `abbbc`, `abbbbc`, ...}
  - `ab+c` equivalent to `abb*c`, i.e. there must be at least one `b`
  - `a\wc` any three letter string, starting with `a` and ending with `c`, where second character is any character in `[a-zA-Z0-9_]`
  - `a[xyz]c` any three letter string, starting with `a` and ending with `c`, where second character is either `x`, `y` or `z`
  - `a[^xyz]c` any three letter string, starting with `a` and ending with `c`, where second character is *none* of `x`, `y` or `z`
  - `^xyz` match at start of string (prefix)
  - `xyz$` match at end of string (suffix)
  - ...
- See docs.python.org/3/library/re.html for more

# String searching using regular expressions

- `re.search(`*`pattern`*`, `*`text`*`)`
  - find the first occurence of *`pattern`* in *`text`* – returns `None` or a *match object*
- `re.findall(`*`pattern`*`, `*`text`*`)`
  - returns a list of non-overlapping occurence of *`pattern`* in *`text`* – returns a list of substrings
- `re.finditer(`*`pattern`*`, `*`text`*`)`
  - iterator returning a match object for each non-overlapping occurence of *`pattern`* in *`text`*

**Python shell**

```
> import re
> text = 'this is a string - a list of characters'
> re.findall(r'i\w*', text)  # prefix with 'r' for raw string literal
| ['is', 'is', 'ing', 'ist']
> for m in re.finditer(r'a[^at]*t', text):
      print('text[%s, %s] = %s' % (m.start(), m.end(), m.group()))
| text[8, 12] = a st
  text[19, 25] = a list
  text[33, 36] = act
```

docs.python.org/3/library/re.html

# Substitution and splitting using regular expressions

- `re.sub(`*`pattern, replacement, text`*`)`
  - replace any occurence of the *`pattern`* in *`text`* by *`replacement`*
- `re.split(`*`pattern, text`*`)`
  - split *`text`* at all occurences of *`patern`*

**Python shell**

```
> text = 'this is a string - a list of characters'
> re.sub(r'\w*i\w*', 'X', text)   # all words containing i
| 'X X a X - a X of characters'
> re.split(r'[^\w]+a[^\w]+', text)   # split around word 'a'
| ['this is', 'string', 'list of characters']
```

docs.python.org/3/library/re.html

# Regular expression substitution: \b \w \1 \2 ...

- Assume we want to replace ”a” with ”an” in front of words starting with the vowels a, e, i, o and u.

**Python shell**

```
> txt = 'A elephant, a zebra and a ape'   # two places to correct
> re.sub('a', 'an', txt)
| 'A elephannt, an zebran annd an anpe'   # replaces all letters 'a' with 'an'
> re.sub(r'\ba\b', 'an', txt)             # raw string + \b boundary of word
| 'A elephant, an zebra and an ape'       # all lower 'a' replaced
> re.sub(r'\b[aA]\b', 'an', txt)
| 'an elephant, an zebra and an ape'      # both 'a' and 'A' replaced by 'an'
> re.sub(r'\b([aA])\b', r'\1n', txt)      # use () and \1 to reinsert match
| 'An elephant, an zebra and an ape'      # kept 'a' and 'A'
> re.sub(r'\b([aA])\s+[aeiou]', r'\1n', txt)      # \s+ = one or more whitespace
| 'Anlephant, a zebra and anpe'           # missing original whitespace + vowel
> re.sub(r'\b([aA])(\s+[aeiou])', r'\1n\2', txt)  # reinsert both () using \1 \2
| 'An elephant, a zebra and an ape'
```

# Fun with strings: Lindenmayer systems (L-systems)

| | |
|---|---|
| **Axiom** | A |
| (1) | AB |
| (2) | ABA |
| (3) | ABAAB |
| (4) | ABAABABA |

**Rules**

A → AB

B → A

First four iterations of parallel rewriting

**L_system.py**

```python
S = 'A'   # axiom
rules = {'A': 'AB', 'B': 'A'}
for i in range(8):
    S = ''.join(rules.get(c, c) for c in S)
    print(S)
```

**Python shell**

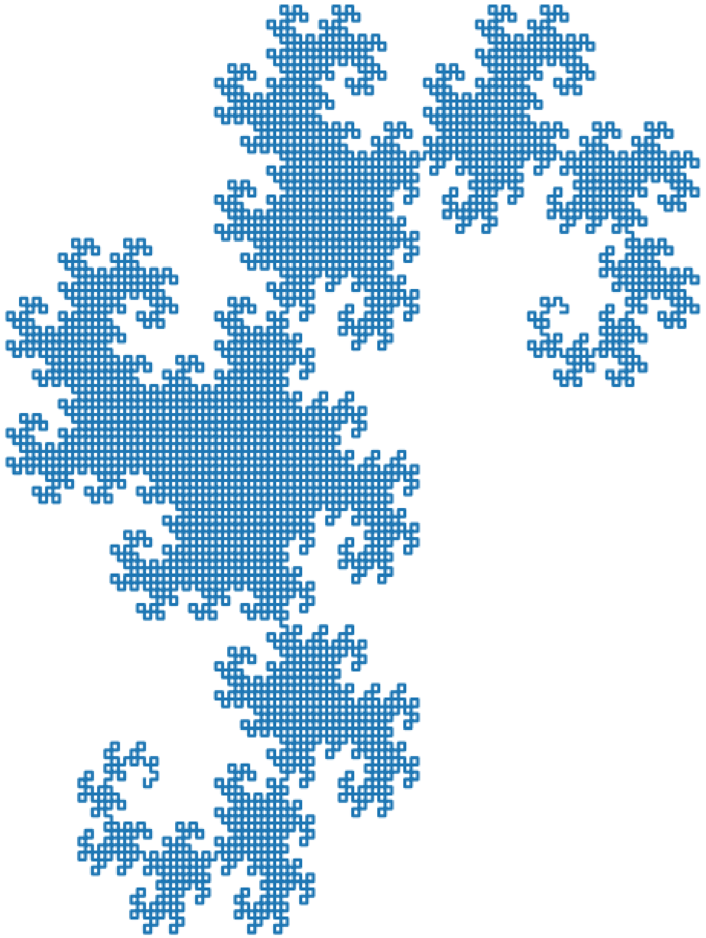
```
| AB
| ABA
| ABAAB
| ABAABABA
| ABAABABAABAAB
| ABAABABAABAABABAABABA
| ABAABABAABAABABAABABAABAABABAABAAB
```

*"L-systems were introduced and developed in 1968 by Aristid Lindenmayer, a Hungarian theoretical biologist and botanist at the University of Utrecht. Lindenmayer used L-systems to describe the behaviour of plant cells and to model the growth processes of plant development."*

\- Wikipedia

en.wikipedia.org/wiki/L-system

# Heighway Dragon



**dragon.py**

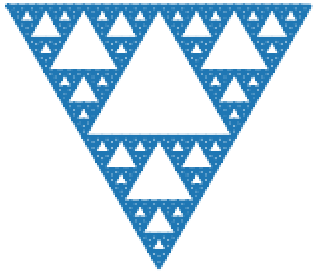
```
import matplotlib.pyplot as plt
from math import sin, cos, radians

axiom = 'FX'
rules = {'X': 'X+YF+', 'Y': '-FX-Y'}

def apply_rules(axiom, rules, repeat):
    for _ in range(repeat):
        axiom = ''.join(rules.get(symbol, symbol) for symbol in axiom)
    return axiom

def walk(commands, position=(0, 0), angle=0, turn=90):
    path = [position]
    for move in commands:
        if move == 'F':
            position = (position[0] + cos(radians(angle)),
                        position[1] + sin(radians(angle)))
            path.append(position)
        elif move == '-': angle -= turn
        elif move == '+': angle += turn
    return path

path = walk(apply_rules(axiom, rules, 13))
plt.plot(*zip(*path), '-')
plt.title('Heighway dragon')
plt.show()
```

Interprete the symbols of the resulting string as a walk where 'F' = draw line forward, and '+' and '-' are turn left and right 90° (X and Y are skipped)

FX+YF++-FX-YF++-FX+YF+--FX-YF++-FX+YF++-FX-YF+--FX+YF+--FX-YF++-FX+YF++-FX-YF++-FX+YF+--FX-YF+--FX+YF++-FX-YF+--FX+YF+--FX-YF++-FX+YF++-FX-YF++-FX+YF+--FX-YF++-FX+YF++-FX-YF+--FX+Y

# More space filling curves…

Sierpinski triangle



Axiom F-G-G
F → F-G+F+G-F
G→ GG

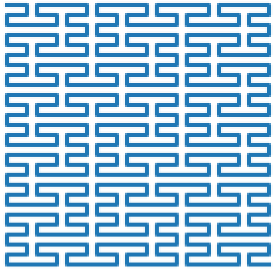
Forward F and G
Turns 120°

Sierpinski arrowhead curve

Axiom A
A → B-A-B
B → A+B+A

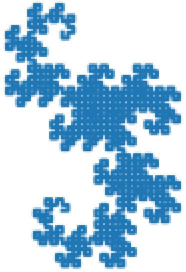
Forward A and B
Turns 60°

Peano curve



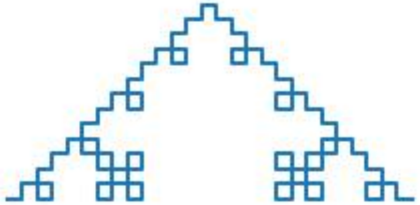
Axiom L
L → LFRFL-F-RFLFR+F+LFRFL
R→ RFLFR+F+LFRFL-F-RFLFR

Heighway dragon



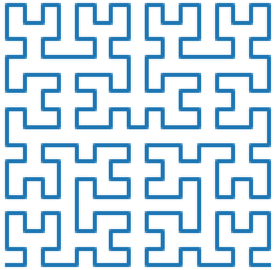
Axiom FX
X → X+YF+
Y → -FX-Y

Koch curve



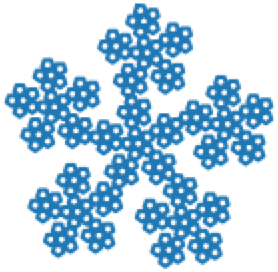
Axiom F
F → F+F-F-F+F

Hilbert curve



Axiom L
L → +RF-LFL-FR+
R → -LF+RFR+FL-

McWorter Pentigree curve



Axiom F-F-F-F-F
F → F-F-F++F+F-F

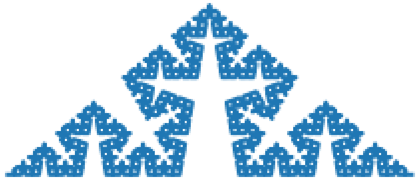
Turns 72°

Tree

Axiom F
F → F[+FF][-FF]F[-F][+F]F

Turns 36°
[ and ] return to start point when done

Cesero fractal



Axiom F
F → F+F--F+F

Turns 80°

# More space filling curves... (source code)

space-filling-L_systems.py

```python
import matplotlib.pyplot as plt
from math import sin, cos, radians

def walk(commands,
         pos=(0, 0),
         forward=frozenset('F'),
         angle=0,
         turn=90):
  paths = [[pos]]
  stack = []
  for move in commands:
    if move in forward:
      pos = (pos[0]+cos(radians(angle)),
             pos[1]+sin(radians(angle)))
      paths[-1].append(pos)
    elif move == '-': angle -= turn
    elif move == '+': angle += turn
    elif move == '[':
      stack.append((pos, angle))
    elif move == ']':
      pos, angle = stack.pop()
      paths.append([pos])
  return paths
```

```python
def apply_rules(axiom, rules, repeat=1):
    for _ in range(repeat):
        axiom = ''.join(rules.get(symbol, symbol) for symbol in axiom)
    return axiom

curves = [  # Lindenmayer systems (L-systems)
  ('Sierpinski triangle', 'F-G-G', {'F': 'F-G+F+G-F', 'G': 'GG'}, 5, {'turn': 120, 'forward': {'F','G'}}),
  ('Sierpinski arrowhead curve', 'A', {'A': 'B-A-B', 'B': 'A+B+A'}, 5, {'turn': 60, 'forward': {'A','B'}}),
  ('Peano curve', 'L', {'L': 'LFRFL-F-RFLFR+F+LFRFL', 'R': 'RFLFR+F+LFRFL-F-RFLFR'}, 3, {}),
  ('Heighway dragon','FX', {'X': 'X+YF+', 'Y': '-FX-Y'}, 10, {}),
  ('Koch curve', 'F', {'F': 'F+F-F-F+F'}, 3, {}),
  ('Hilbert curve', 'L', {'L': '+RF-LFL-FR+', 'R': '-LF+RFR+FL-'}, 4, {}),
  ('McWorter Pentigree curve', 'F-F-F-F-F', {'F': 'F-F-F++F+F-F'}, 3, {'turn': 72}),
  ('Tree', 'F', {'F': 'F[+FF][-FF]F[-F][+F]F'}, 3, {'turn': 36}),
  ('Cesero fractal', 'F', {'F': 'F+F--F+F'}, 5, {'turn': 80})
]

for idx, (title, axiom, rules, repeat, walk_arg) in enumerate(curves, start=1):
    paths = walk(apply_rules(axiom, rules, repeat), **walk_arg)
    ax = plt.subplot(3, 3, idx, aspect='equal')
    ax.set_title(title)
    for path in paths:
        plt.plot(*zip(*path), '-')
plt.axis('off')
plt.show()
```

# Relational data

- SQLite
- pandas

# Two tables

| Table: country | | | |
|---|---|---|---|
| name | population | area | capital |
| 'Denmark' | 5748769 | 42931 | 'Copenhagen' |
| 'Germany' | 82800000 | 357168 | 'Berlin' |
| 'USA' | 325719178 | 9833520 | 'Washington, D.C.' |
| 'Iceland' | 334252 | 102775 | 'Reykjavik' |

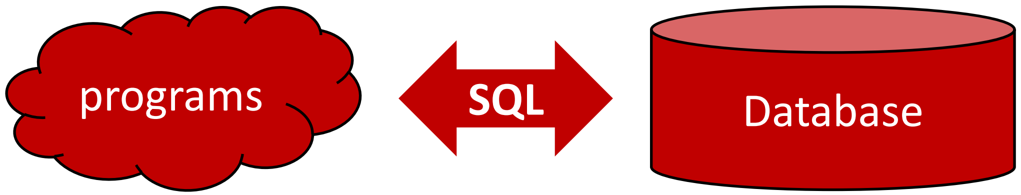
| Table: city | | | |
|---|---|---|---|
| name | country | population | established |
| 'Copenhagen' | 'Denmark' | 775033 | 800 |
| 'Aarhus' | 'Denmark' | 273077 | 750 |
| 'Berlin' | 'Germany' | 3711930 | 1237 |
| 'Munich' | 'Germany' | 1464301 | 1158 |
| 'Reykjavik' | 'Iceland' | 126100 | 874 |
| 'Washington D.C.' | 'USA' | 693972 | 1790 |
| 'New Orleans' | 'USA' | 343829 | 1718 |
| 'San Francisco' | 'USA' | 884363 | 1776 |

| Table: country | | | |
|---|---|---|---|
| name | population | area | capital |
| 'Denmark' | 5748769 | 42931 | 'Copenhagen' |
| 'Germany' | 82800000 | 357168 | 'Berlin' |
| 'USA' | 325719178 | 9833520 | 'Washington, D.C.' |
| 'Iceland' | 334252 | 102775 | 'Reykjavik' |

# SQL

## pronounced ˌɛsˌkjuːˈɛl or ˈsiːkwəl

- SQL = Structured Query **Language**
- **Database = collection of tables** stored persistently on disk
- ANSI and ISO standards since 1986 and 1987, respectively; origin early 70s
- Widespread used SQL databases (can handle many tables/rows/users): Oracle, MySQL, Microsoft SQL Server, PostgreSQL and IBM DB2
- **SQLite** is a very lightweight version storing a database in a single file, without a separate database server
- SQLite is included in both iOS and Android mobil phones




The Course "Database Systems" gives a more in-depth introduction to SQL (MySQL)

# SQL examples

| Table: country | | | |
|---|---|---|---|
| name | population | area | capital |
| 'Denmark' | 5748769 | 42931 | 'Copenhagen' |
| 'Germany' | 82800000 | 357168 | 'Berlin' |
| 'USA' | 325719178 | 9833520 | 'Washington, D.C.' |
| 'Iceland' | 334252 | 102775 | 'Reykjavik' |

- CREATE TABLE country (name, population, area, capital)
- INSERT INTO country VALUES ('Denmark', 5748769, 42931, 'Copenhagen')
- UPDATE country SET population=5748770 WHERE name='Denmark'
- SELECT name, capital FROM country WHERE population >= 1000000

  > [('Denmark', 'Copenhagen'), ('Germany', 'Berlin'), ('USA', 'Washington, D.C.')]
- SELECT * FROM country WHERE capital = 'Berlin'

  > [('Germany', 82800000, 357168, 'Berlin')]
- SELECT country.name, city.name, city.established FROM city, country WHERE city.name=country.capital AND city.population < 500000

  > [('Iceland', 'Reykjavik', 874), ('USA', 'Washington, D.C.', 1790)]
- DELETE FROM country WHERE name = 'Germany'
- DROP TABLE country

www.w3schools.com/sql/

## SQLite

sqlite-example.py

```python
import sqlite3
connection = sqlite3.connect('example.sqlite')  # creates file if necessary
c = connection.cursor()
c.executescript('''DROP TABLE IF EXISTS country;  -- multiple SQL statements
                   DROP TABLE IF EXISTS city''')
countries = [('Denmark', 5748769, 42931, 'Copenhagen'),
             ('Germany', 82800000, 357168, 'Berlin'),
             ('USA', 325719178, 9833520, 'Washington, D.C.'),
             ('Iceland', 334252, 102775, 'Reykjavik')]
cities = [('Copenhagen', 'Denmark', 775033, 800),
          ('Aarhus', 'Denmark', 273077, 750),
          ('Berlin', 'Germany', 3711930, 1237),
          ('Munich', 'Germany', 1464301, 1158),
          ('Reykjavik', 'Iceland', 126100, 874),
          ('Washington, D.C.', 'USA', 693972, 1790),
          ('New Orleans', 'USA', 343829, 1718),
          ('San Francisco', 'USA', 884363, 1776)]
c.execute('CREATE TABLE country (name, population, area, capital)')
c.execute('CREATE TABLE city (name, country, population, established)')
c.executemany('INSERT INTO country VALUES (?,?,?,?)', countries)
c.executemany('INSERT INTO city VALUES (?,?,?,?)', cities)
connection.commit()  # save data to database before closing
connection.close()
```

docs.python.org
/3/library/sqlite3.html

# SQLite query examples

try to avoid using the asterisk (*) as a good habit
www.sqlitetutorial.net/sqlite-select

**sqlite-example.py**

```
for row in c.execute('SELECT * FROM country'):   # * = all columns, execute returns iterator
    print(row)                                   # row is by default a Python tuple

for row in c.execute('''SELECT * FROM city, country     -- all pairs of rows from city × country
                        WHERE city.name = country.capital AND city.population < 700000'''):
    print(row)

print(*c.execute('''SELECT country.name,
                           COUNT(city.name) AS cities,
                           100 * SUM(city.population) / country.population
                    FROM city JOIN country ON city.country = country.name  -- SQL join 2 tables
                    WHERE city.population > 500000                       -- only consider big cities
                    GROUP BY city.country                          -- output has one row per group of rows
                    ORDER BY cities DESC, SUM(city.population) DESC'''))   # ordering of output
```

**Python shell**

```
| ('Denmark', 5748769, 42931, 'Copenhagen')
| ('Germany', 82800000, 357168, 'Berlin')
| ('USA', 325719178, 9833520, 'Washington, D.C.')
| ('Iceland', 334252, 102775, 'Reykjavik')

| ('Reykjavik', 'Iceland', 126100, 874, 'Iceland', 334252, 102775, 'Reykjavik')
| ('Washington, D.C.', 'USA', 693972, 1790, 'USA', 325719178, 9833520, 'Washington, D.C.')

| ('Germany', 2, 6) ('USA', 2, 0) ('Denmark', 1, 13)
```

# SQL injection

Right way

```
c.execute('INSERT INTO users VALUES (?)', (user,))
```

**unsafe-example.py**

```
import sqlite3
connection = sqlite3.connect('users.sqlite')
c = connection.cursor()
c.execute('CREATE TABLE users (name)')
while True:
    user = input('New user: ')
    c.executescript('INSERT INTO users VALUES ("%s")' % user)
    connection.commit()
    print(list(c.execute('SELECT * FROM users')))
```

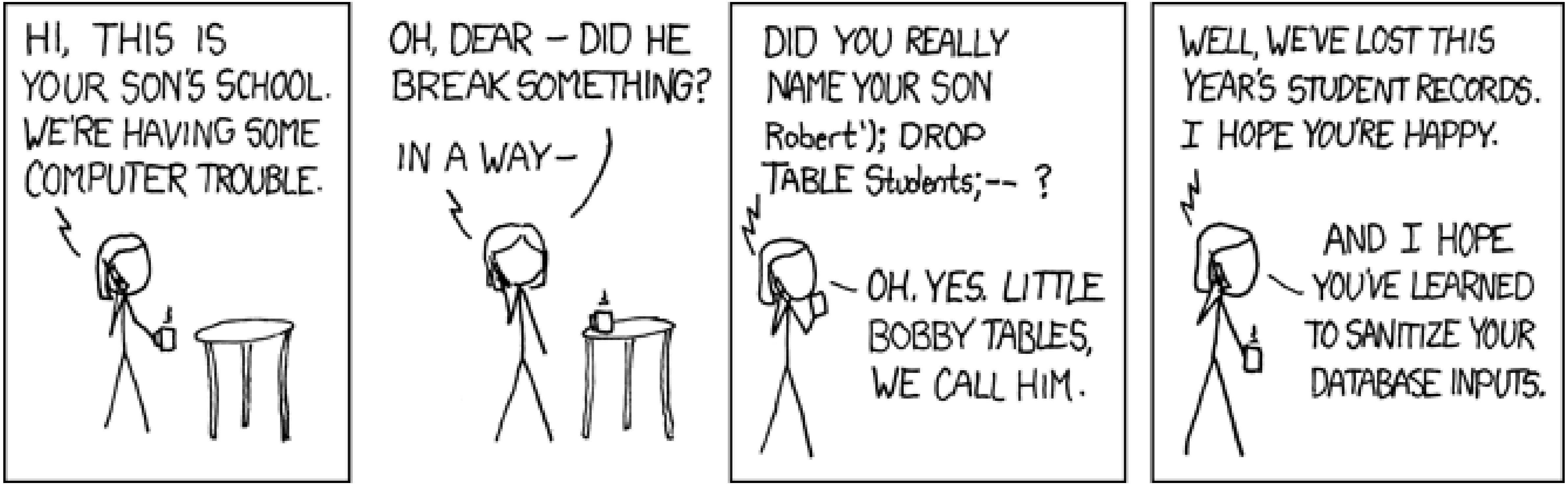
can execute a string containing several SQL statements

Insecure: NEVER use % on user input

**Python shell**

```
> New user: gerth
| [('gerth',)]
> New user: guido
| [('gerth',), ('guido',)]
> New user: evil"); DROP TABLE users; --
| sqlite3.OperationalError: no such table: users
```

```
INSERT INTO users VALUES ("evil"); DROP TABLE users; --")
```


HI, THIS IS YOUR SON'S SCHOOL. WE'RE HAVING SOME COMPUTER TROUBLE.
OH, DEAR – DID HE BREAK SOMETHING?
IN A WAY–
DID YOU REALLY NAME YOUR SON Robert'); DROP TABLE Students;-- ?
OH. YES. LITTLE BOBBY TABLES, WE CALL HIM.
WELL, WE'VE LOST THIS YEAR'S STUDENT RECORDS. I HOPE YOU'RE HAPPY.
AND I HOPE YOU'VE LEARNED TO SANITIZE YOUR DATABASE INPUTS.

xkcd.com/327/

# Pandas

- Comprehensive Python library for data manipulation and analysis, in particular tables and time series
- Pandas **data frames** = tables
- Supports interaction with SQL, CSV, JSON, ...
- Integrates with Jupyter, numpy, matplotlib, ...

pandas.pydata.org

# Pandas integration with Jupyter

- Tables (Pandas data frames) are rendered nicely in Jupyter

In [1]:
```python
import pandas as pd
students = pd.read_csv('students.csv')
students
```

Out[1]:

| | Name | City |
|---|---|---|
| 0 | Donald Duck | Copenhagen |
| 1 | Goofy | Aarhus |
| 2 | Mickey Mouse | Aarhus |

**students.csv**

```
Name,City
"Donald Duck","Copenhagen"
"Goofy","Aarhus"
"Mickey Mouse","Aarhus”
```

# Reading tables (data frames)

- Pandas provide functions for reading different data formats, e.g. SQLite and .csv files, into pandas.DataFrames

pandas-example.py

```
import pandas as pd

import sqlite3
connection = sqlite3.connect('example.sqlite')
countries = pd.read_sql_query('SELECT * FROM country', connection)
cities = pd.read_sql_query('SELECT * FROM city', connection)

students.to_sql('students', connection, if_exists='replace')
print(students)
```

Python shell

```
|              Name        City
| 0   Donald Duck  Copenhagen
| 1         Goofy      Aarhus
| 2  Mickey Mouse      Aarhus
```

# Selecting columns and rows

| Table: country | | | |
|---|---|---|---|
| name | population | area | capital |
| 'Denmark' | 5748769 | 42931 | 'Copenhagen' |
| 'Germany' | 82800000 | 357168 | 'Berlin' |
| 'USA' | 325719178 | 9833520 | 'Washington, D.C.' |
| 'Iceland' | 334252 | 102775 | 'Reykjavik' |

**Python shell**

```
> countries['name']                # select column
> countries.name                   # same as above
> countries[['name', 'capital']]   # select multiple columns, note double-[]
> countries.head(2)                # first 2 rows
> countries[1:3]                   # slicing rows, rows 1 and 2
> countries[::2]                   # slicing rows, rows 0 and 2
> countries.at[1, 'area']          # indexing cell by (row label, column name)
> cities[(cities['name'] == 'Berlin') | (cities['name'] == 'Munich')]  # select rows
|      name  country  population  established
| 2  Berlin  Germany     3711930         1237  # note original row labels
| 3  Munich  Germany     1464301         1158
> pd.DataFrame([[1,2], [3, 4], [5,6]], columns=['x', 'y'])   # create DF from list
> pd.DataFrame(np.random.random((3,2)), columns=['x', 'y'])  # from numpy
```

pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html

# Row labels

**Python shell**

```
> df = pd.DataFrame(np.arange(1, 13).reshape(3, 4),
                    index=['q', 'w', 'e'],           # row labels
                    columns=['c', 'a', 'd', 'e'])   # column names
> df
|    c   a   d   e
| q  1   2   3   4   # row labels can be strings
| w  5   6   7   8
| e  9  10  11  12
> df.loc['w':'e', ['e', 'a']]   # slice of labeled rows
|     e   a
| w   8   6
| e  12  10
> df.loc['w']   # single row
| c     5
| a     6
| d     7
| e     8
| Name: w, dtype: int32
> df.iloc[:2,:2]   # use iloc to work with integer indexes
|    c  a
| q  1  2
| w  5  6
```

# Merging tables and creating a new column

**pandas-example.py**

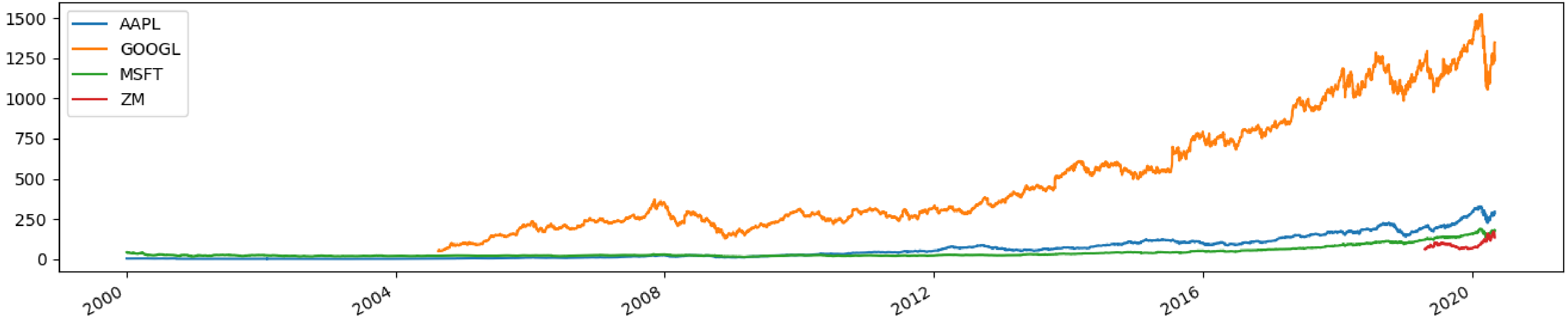
```
M = pd.merge(countries, cities, left_on='capital', right_on='name')
# both data frames had a 'name' and 'population' column
M1 = M.rename(columns={
    'population_x': 'country_population',
    'population_y': 'capital_population'
})
M2 = M1.drop(columns=['name_x', 'name_y'])
M2['%pop in capital'] = M2.capital_population / M2.country_population
M2.sort_values('%pop in capital', ascending=False, inplace=True)
print(M2[['country', '%pop in capital']])
```

**Python shell**

```
|      country  %pop in capital
| 3  Iceland          0.377260  # note row labels arepermuted
| 0  Denmark          0.134817
| 1  Germany          0.044830
| 2      USA          0.002131
```

# Pandas datareader and Matplotlib

- pandas_datareader provides access to many data sources
- dataframes have a .plot method (using matplotlib.pyplot)




**pandas-datareader.py**

```
import matplotlib.pyplot as plt
import pandas_datareader
#df = pandas_datareader.data.DataReader(['AAPL', 'GOOGL', 'MSFT', 'ZM'], 'stooq')  # ignores start=...
df = pandas_datareader.stooq.StooqDailyReader(['AAPL', 'GOOGL', 'MSFT', 'ZM'], start='2000-01-01').read()
df['Close'].plot()
plt.legend()
plt.show()
```

pandas-datareader.readthedocs.io
pandas-datareader.readthedocs.io/en/latest/readers/stooq.html

# Hierarchical / Multi-level indexing (MultiIndex)

**Python shell**

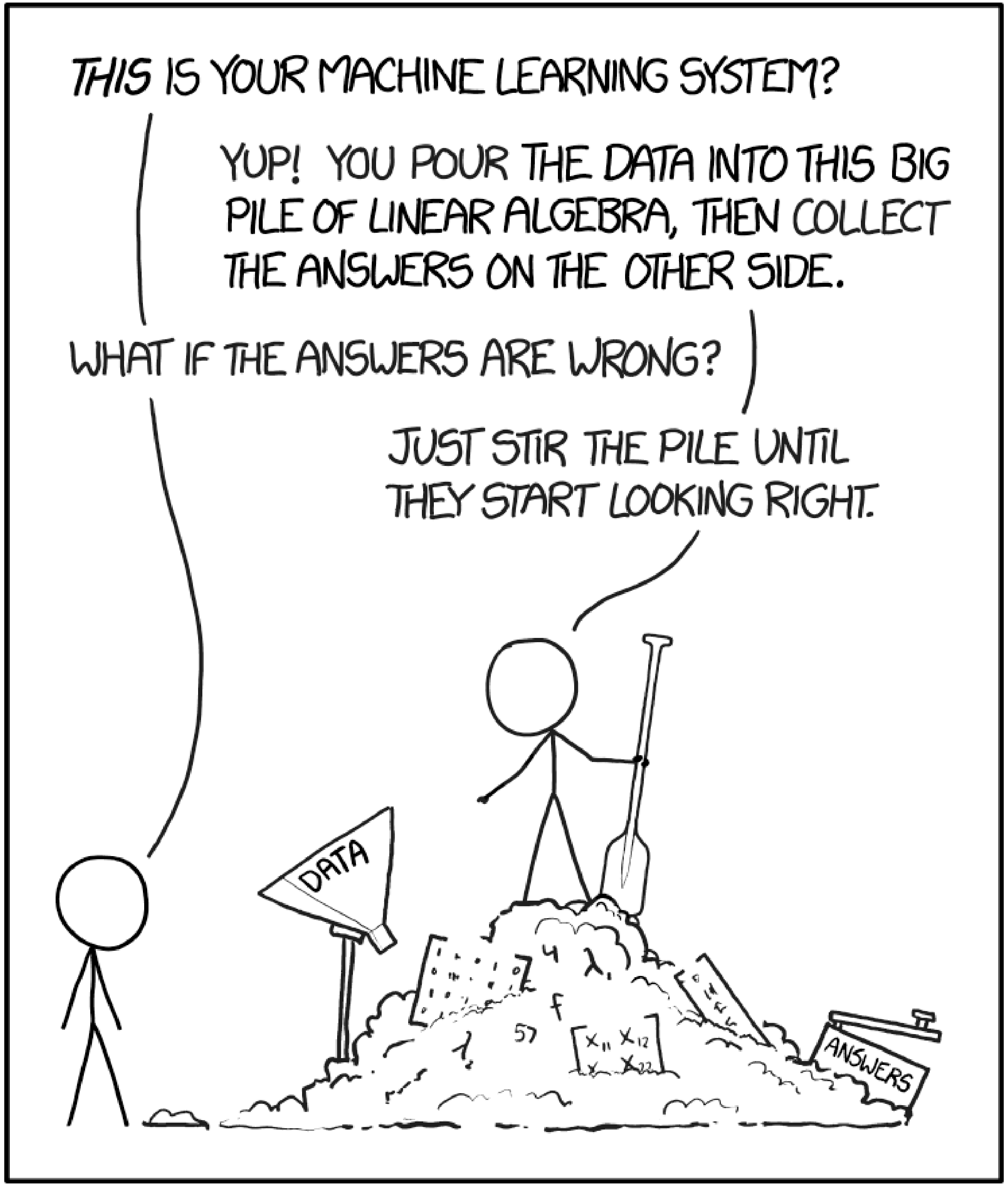
```
> df.tail(2)
| Attributes    Close                          ...      Volume
| Symbols        AAPL     GOOGL     MSFT       ...       GOOGL        MSFT          ZM
| Date                                         ...
| 2020-04-29   287.73   1342.18   177.43       ...   5417888.0  51286559.0  22033320.0
| 2020-04-30   293.80   1346.70   179.21       ...   2788644.0  53627543.0  16648922.0
> df['Close'].tail(2)
| Symbols        AAPL     GOOGL     MSFT       ZM
| Date
| 2020-04-29   287.73   1342.18   177.43   146.48
| 2020-04-30   293.80   1346.70   179.21   135.17
> df['Close']['GOOGL'].tail(2)
| Date
| 2020-04-29    1342.18
| 2020-04-30    1346.70
| Name: GOOGL, dtype: float64
> df.loc[:, pd.IndexSlice[:,'GOOGL']].tail(2)
| Attributes      Close      High       Low      Open      Volume
| Symbols         GOOGL     GOOGL     GOOGL     GOOGL       GOOGL
| Date
| 2020-04-29   1342.18   1360.15   1326.73   1345.00   5417888.0
| 2020-04-30   1346.70   1350.00   1321.50   1331.36   2788644.0
```

Both rows and columns can have multi-level indexing

**Python shell**

```
> df.columns
| MultiIndex([( 'Close',  'AAPL'),
|             ( 'Close', 'GOOGL'),
|             ( 'Close',  'MSFT'),
|             ( 'Close',    'ZM'),
|             (  'High',  'AAPL'),
|             (  'High', 'GOOGL'),
|             (  'High',  'MSFT'),
|             (  'High',    'ZM'),
|             (   'Low',  'AAPL'),
|             (   'Low', 'GOOGL'),
|             (   'Low',  'MSFT'),
|             (   'Low',    'ZM'),
|             (  'Open',  'AAPL'),
|             (  'Open', 'GOOGL'),
|             (  'Open',  'MSFT'),
|             (  'Open',    'ZM'),
|             ('Volume',  'AAPL'),
|             ('Volume', 'GOOGL'),
|             ('Volume',  'MSFT'),
|             ('Volume',    'ZM')],
|  names=['Attributes', 'Symbols'])
```

pandas.pydata.org/pandas-docs/stable/user_guide/advanced.html

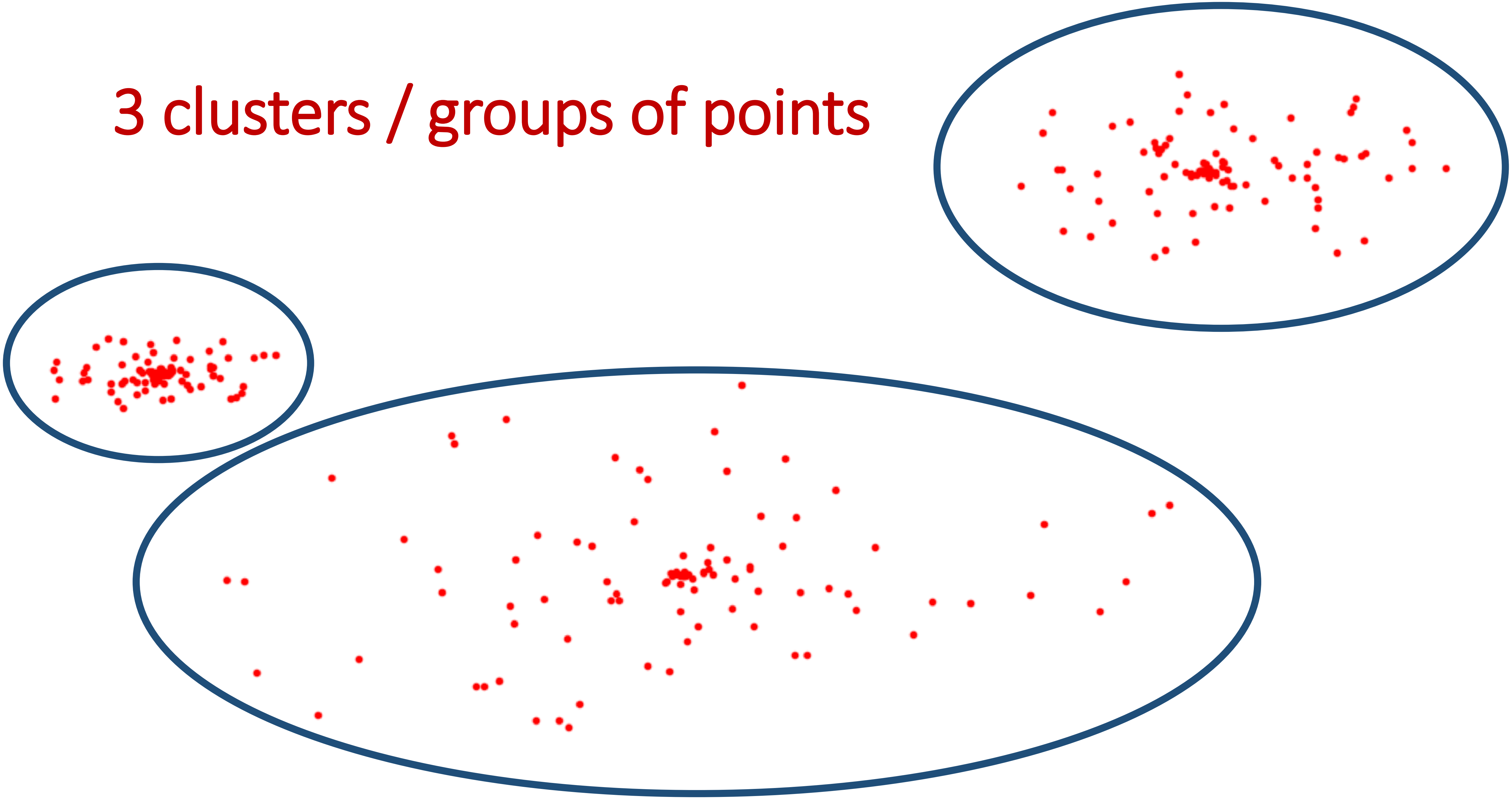



xkcd.com/1838/

# Clustering

- k-means
- scipy.cluster.vq.kmeans
- DBSCAN*
- neural networks

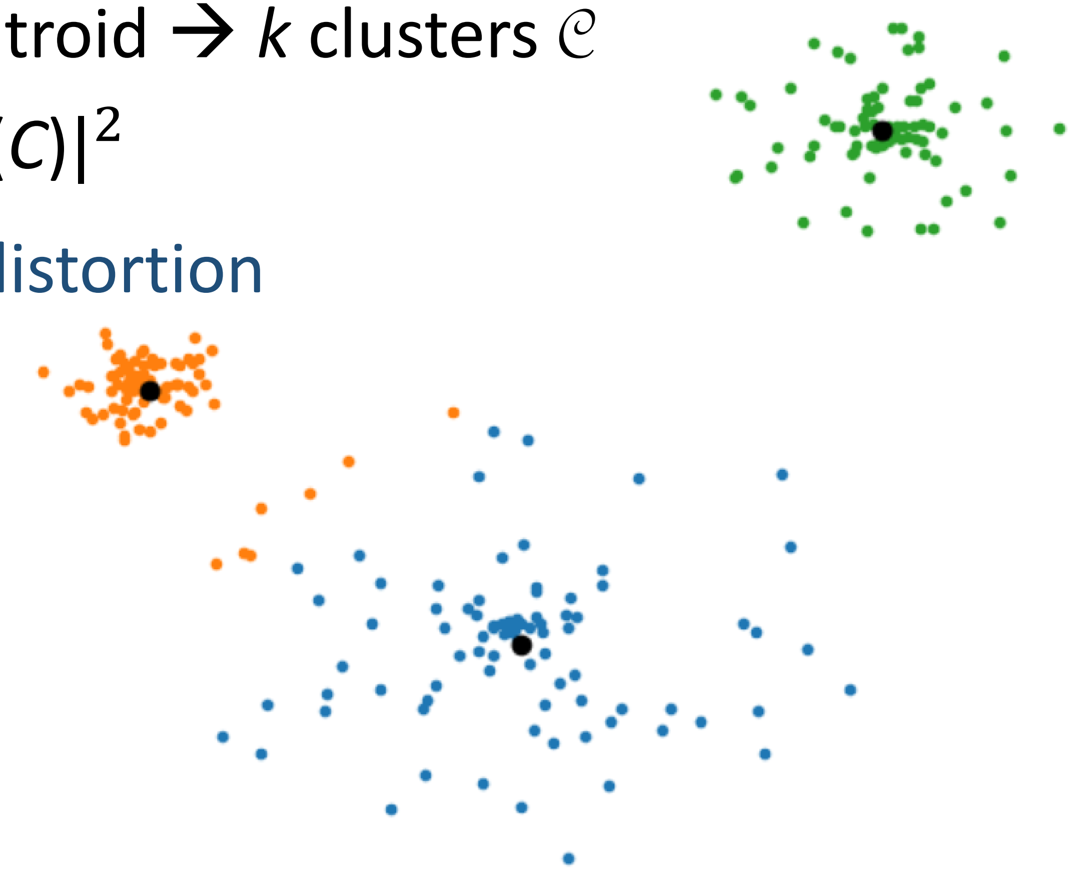
# 3 clusters / groups of points



# Clustering = Optimization problem

Example: k-means

- Find $k$ points *centroids*
- Assign each input point to nearest centroid → $k$ clusters $\mathcal{C}$
- distortion = $\sum_{C\in\mathcal{C}} \sum_{p\in C} |p - \text{centroid}(C)|^2$
- Goal : Find $k$ centroids that minimize distortion

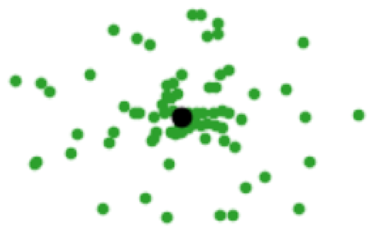


# k-means for $k = 1$

- Let the centroid point $c$ for a point set $C$ be the point minimizing the

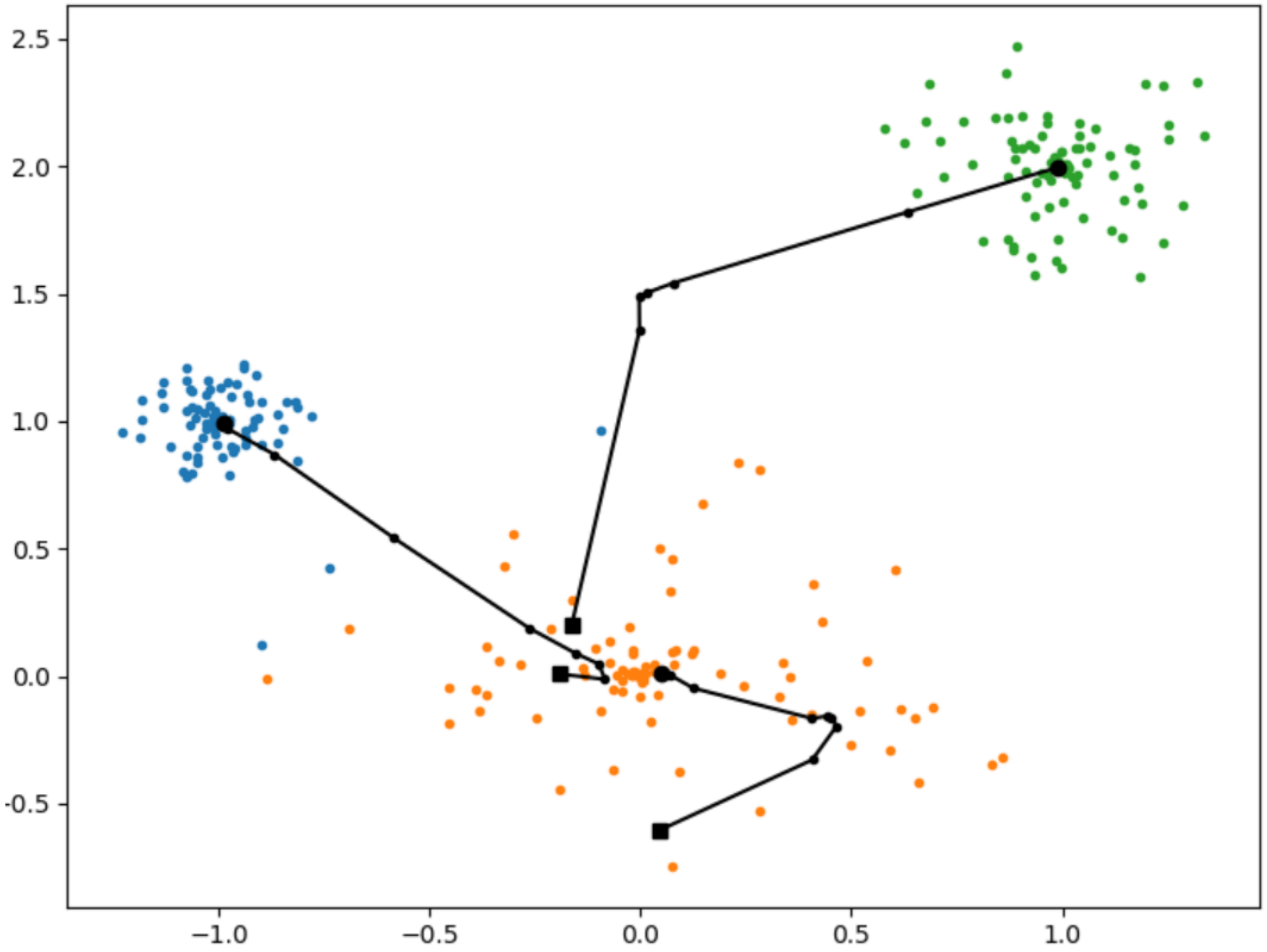
$$\text{distortion} = \sum_{p \in C} |p - c|^2$$

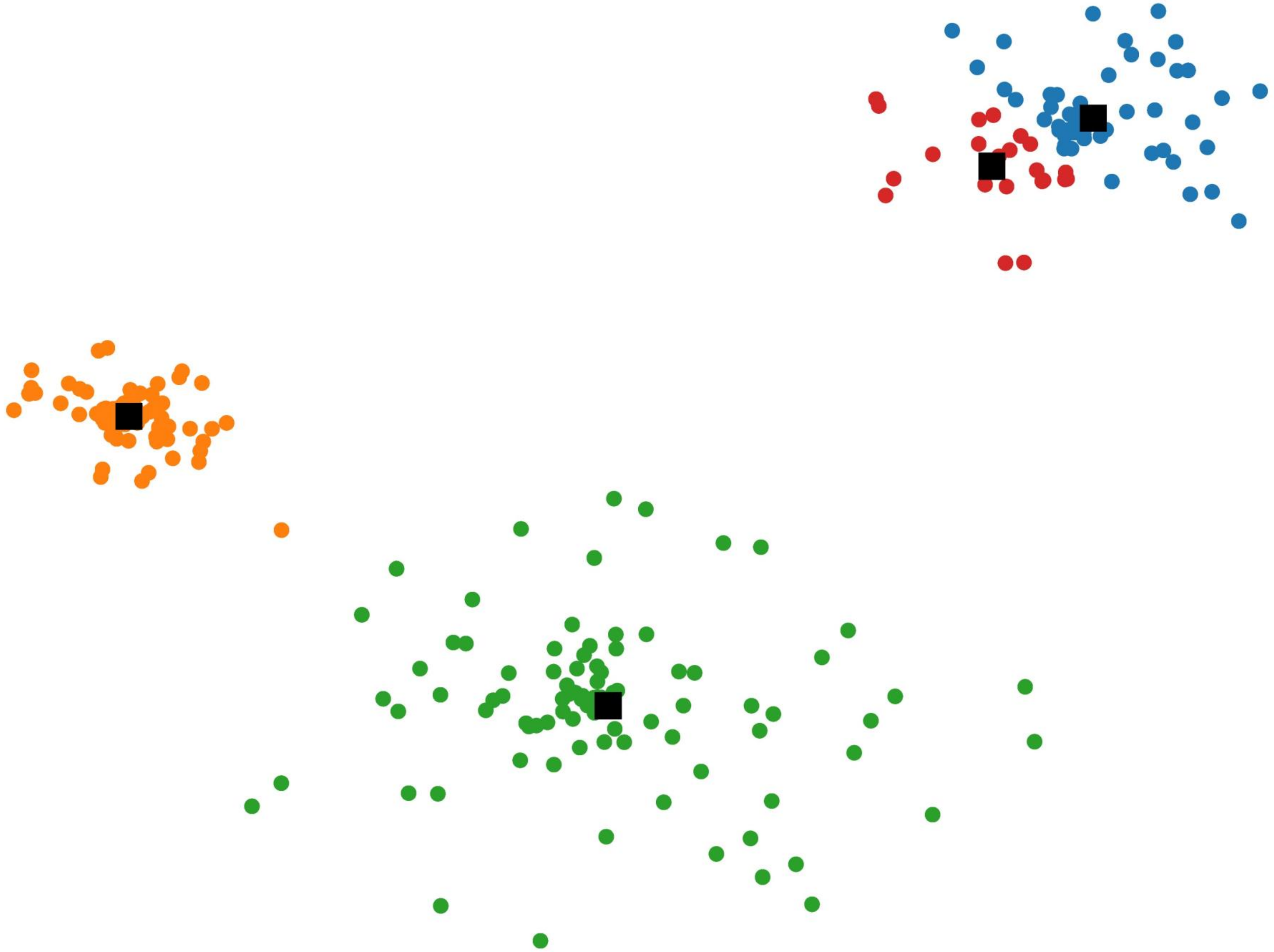
- Theorem $c = \text{average}(C)$

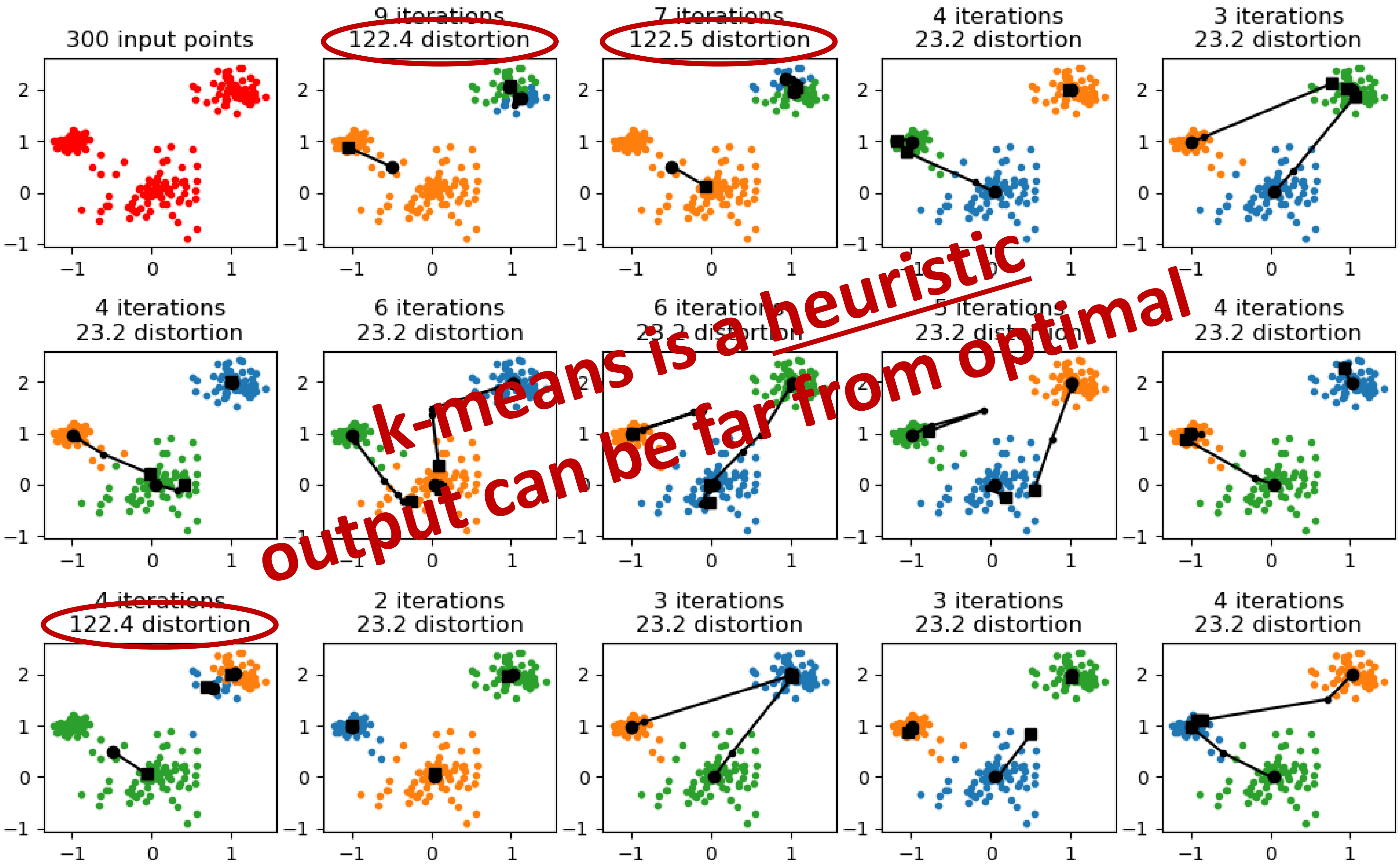


# k-means - Lloyd's method (pseudo code)

```
centroids = k distinct random input points
while centroids change:
    create clusters C by assigning points to the nearest centroid
    centroids = average of each cluster
```



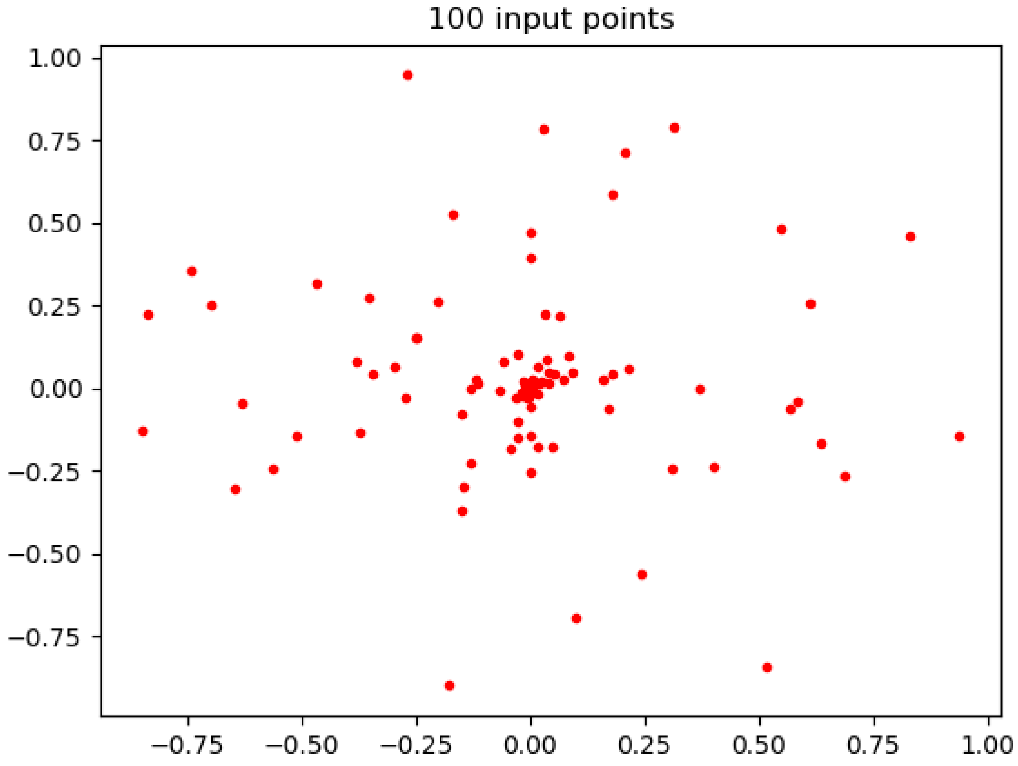



300 input points
9 iterations
122.4 distortion
7 iterations
122.5 distortion
4 iterations
23.2 distortion
3 iterations
23.2 distortion
4 iterations
23.2 distortion
6 iterations
23.2 distortion
6 iterations
23.2 distortion
5 iterations
23.2 distortion
4 iterations
23.2 distortion
4 iterations
122.4 distortion
2 iterations
23.2 distortion
3 iterations
23.2 distortion
3 iterations
23.2 distortion
4 iterations
23.2 distortion
k-means is a heuristic
output can be far from optimal

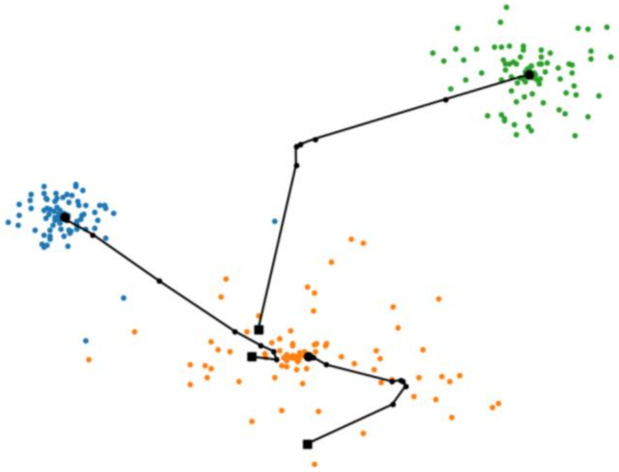
# Generating random points (just one random approach)

**k_means.py**

```
from random import random
from math import pi, cos, sin

def random_point(x, y, radius):
    angle = 2 * pi * random()
    r = radius * random() ** 2
    return x + r * cos(angle), y + r * sin(angle)

def random_points(n, x, y, radius):
    for _ in range(n):
        yield random_point(x, y, radius)
```



# k-means



k_means.py

```
from random import sample
from numpy import argmin, mean

def k_means(points, k):
    centroid = sample(points, k)
    centroids = [ centroid ]  # history for visualization
    while True:
        clusters = [[] for _ in centroid]
        for p in points:
            i = argmin([dist(p, c) for c in centroid])
            clusters[i].append(p)
        centroid = [tuple(map(mean, zip(*c))) for c in clusters]
        if centroid == centroids[-1]:
            break
        centroids.append(centroid)
        if min(len(c) for c in clusters) == 0:
            print("Not good - empty cluster")
            break
    return clusters
```

# k-mean limitations

- Can easily converge to a solution far from a global minimum
  - Solution – try several times and take the best
    (possibly since we can measure the quality (= distortion) of a solution)
- Clusters can become empty
  - Solution – discard and restart / take a random point out as a new centroid / take point furthest away from existing centroids / ....
- Sensitive to the scales of the different dimensions
  - Solution – apply some kind of initial normalization of coordinates

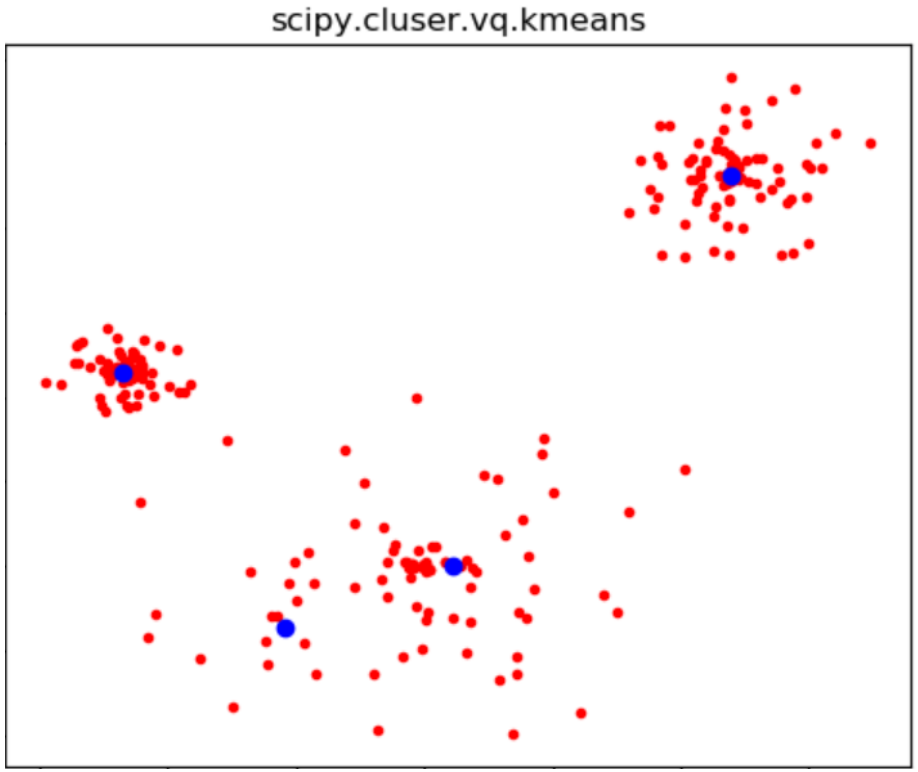
# k-means - better bounds

- The k-means++ algorithm achieves an expected guarantee to be at most a factor $8(2 + \ln k)$ from the optimal [Vassilvitskii & Arthur]
- There exist polynomial time approximation schemes that find a solution that is guaranteed $1 + \varepsilon$ of the optimal (but running time exponential in $k$ and dimension of points) [Har Peled et al.]
- **In practice: A heuristic is most often the algorithm of choice**

# scipy.cluster.vq.kmeans

k_means.py

```
from scipy.cluster.vq import kmeans, whiten
import matplotlib.pyplot as plt
points = whiten(points)   # normalize variance of points
centroids, distortion = kmeans(points, K)
plt.plot(*zip(*points), 'r.')
plt.plot(*zip(*centroids), 'bo')
plt.title('scipy.cluster.vq.kmeans')
plt.show()
```




**Note**: According to the documentation "`whiten` must be called prior to passing an observation matrix to `kmeans`"

docs.scipy.org/doc/scipy/reference/generated/scipy.cluster.vq.kmeans.html

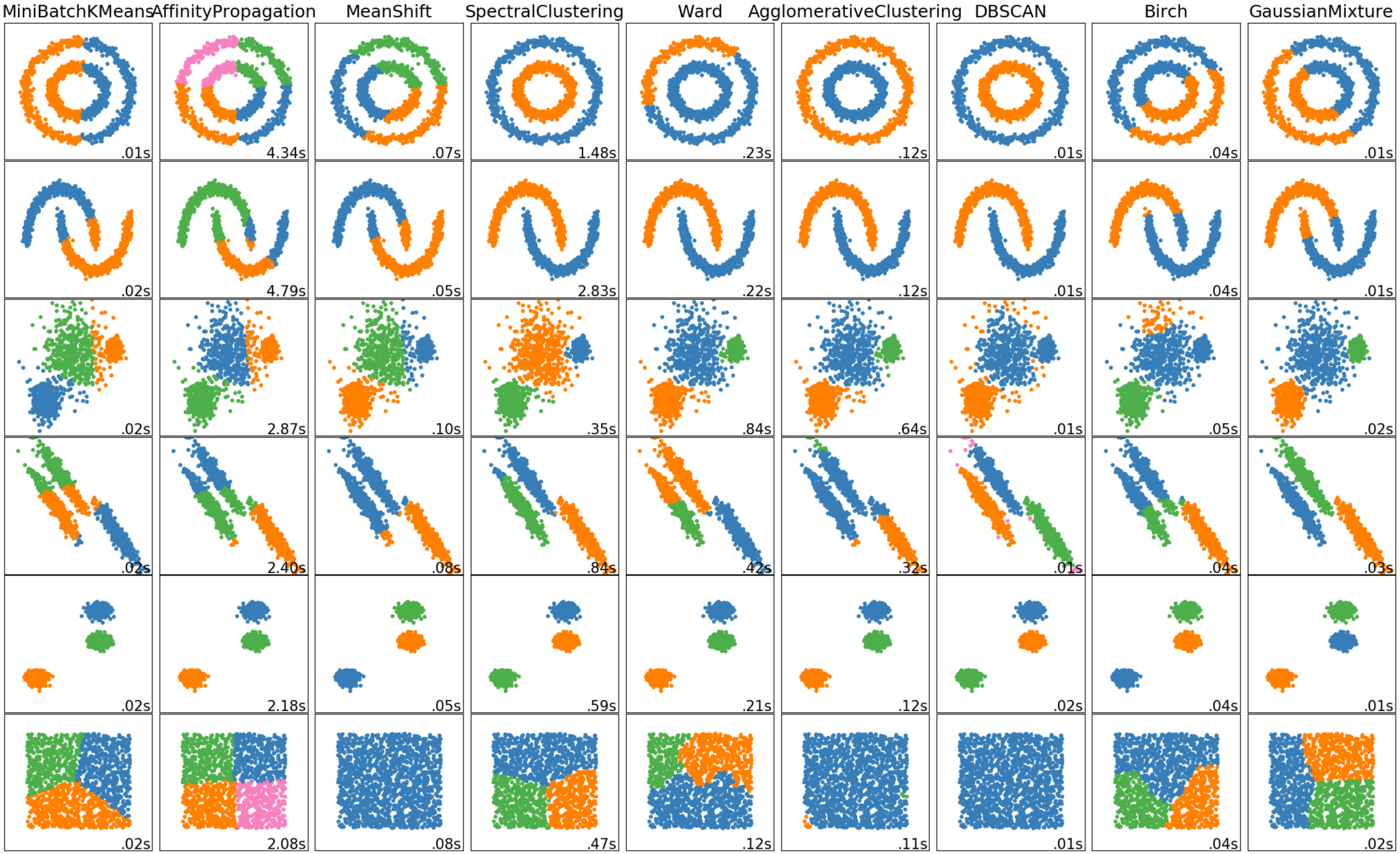
# scipy.cluster.vq.whiten

- Normalizes / scales each dimension to have unit variance 1.0

$$\mathrm{Var}(X) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 \qquad \mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

https://docs.scipy.org/doc/scipy/reference/generated/scipy.cluster.vq.kmeans.html

# Other Python clustering methods - `sklearn.cluster`



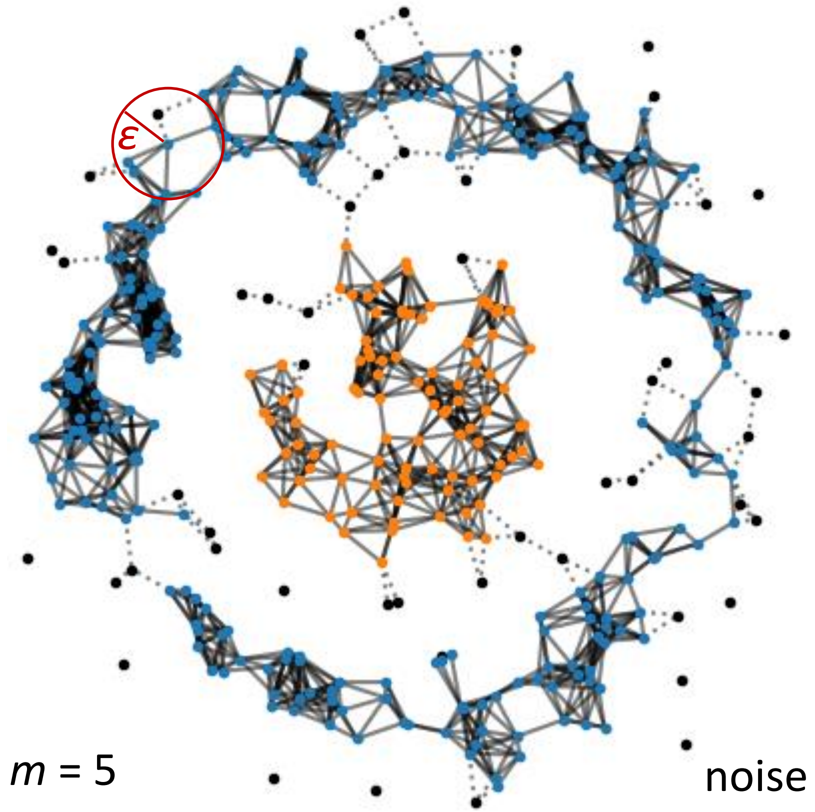

http://scikit-learn.org/stable/modules/clustering.html

# DBSCAN*

**dbscan.py**

```
def dbscan(points, epsilon, m):
    def dist(p, q):
        return sum((pi - qi) ** 2 for pi, qi in zip(p, q))

    def close(p, q):
        return dist(p, q) <= epsilon ** 2

    core, noise, clusters = [], [], []
    for p in points:
        if sum(close(p, q) for q in points) >= m:
            core.append(p)
        else:
            noise.append(p)

    while core:
        cluster = [core.pop()]
        for p in cluster:
            for q in list(core):
                if close(p, q):
                    cluster.append(q)
                    core.remove(q)
        clusters.append(cluster)
    return clusters, noise
```

- Parameters $\varepsilon$ and $m$
- $p$ is a core point when
  $$|\{q \mid |p - q| \leq \varepsilon\}| \geq m$$
- Remaining points are noise
- Core points $p$ and $q$ are in the same cluster if $|p - q| \leq \varepsilon$



# Data Mining Algorithms

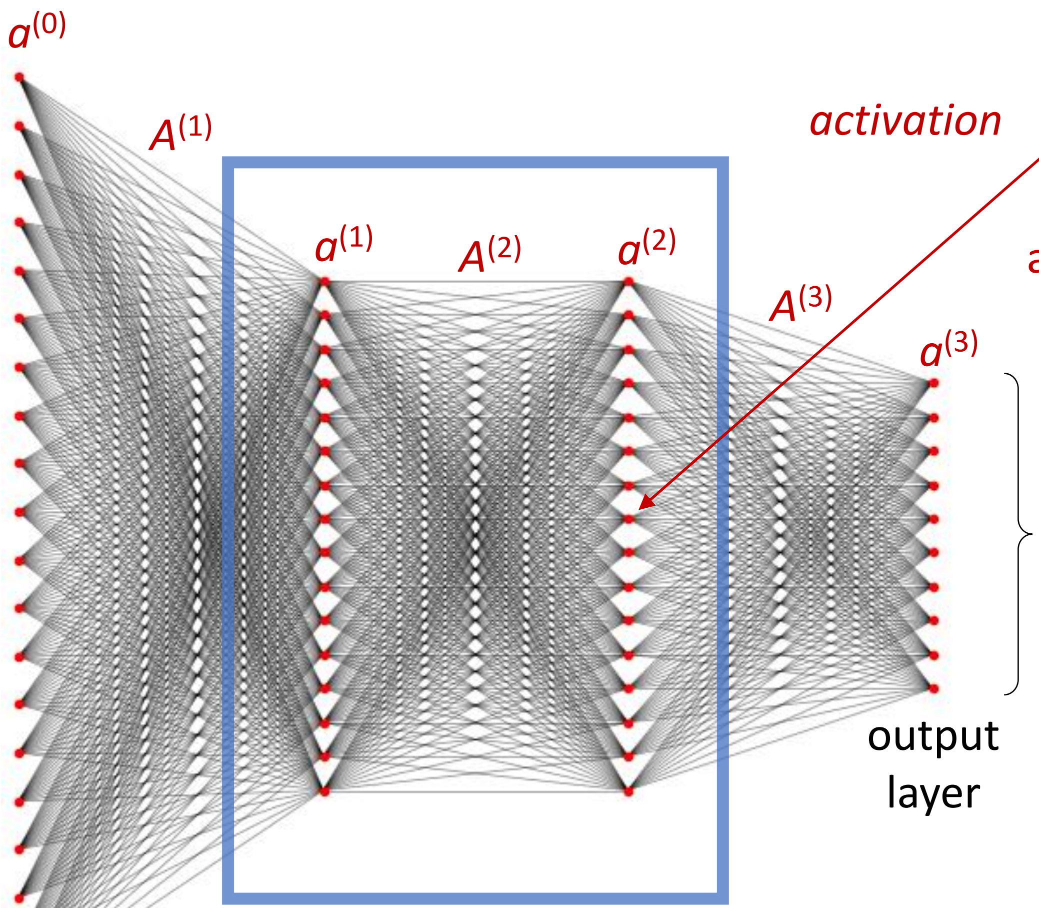
- k-means, and more generally clustering, is just one field in the area of *Data Mining*

- For more information see the webpage
  [Top 10 Data Mining Algorithms, Explained]
  a follow up to the below paper

- X. Wu et al., *Top 10 algorithms in data mining*,
  Knowledge and Information Systems, 14(1):1–37, 2008.
  DOI 10.1007/s10115-007-0114-2

www.kdnuggets.com/2015/05/top-10-data-mining-algorithms-explained.html

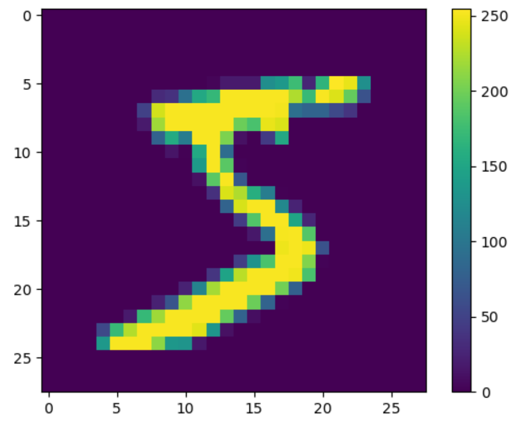
# Neural networks (one slide introduction)




*activation*

$$a_i^{(l)} = f^{(l)}\left( \sum_j a_j^{(l-1)} \cdot A_{ji}^{(l)} + b_i^{(l)} \right)$$

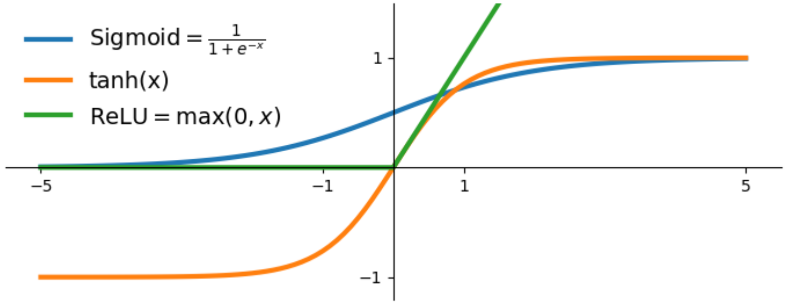
activation function    weight    bias




MNIST : 28 x 28 pixel values from [0, 255]

Classification, like MNIST, prediction = index of node with maximum output

Common activation functions




zero or more hidden layers (zero = linear classifier)

input layer

MNIST, 784 input pixels scaled to [0.0, 1.0]

e.g. mean squared error

$$\frac{1}{n}\sum_{(x,y)} |\text{out}(x) - y|^2$$

**Learning**
Find *A*s and *b*s performing well (minimize a cost function) on a set of *n* training inputs *x* with known output *y* using *backpropagation* / *stochastic gradient descend*

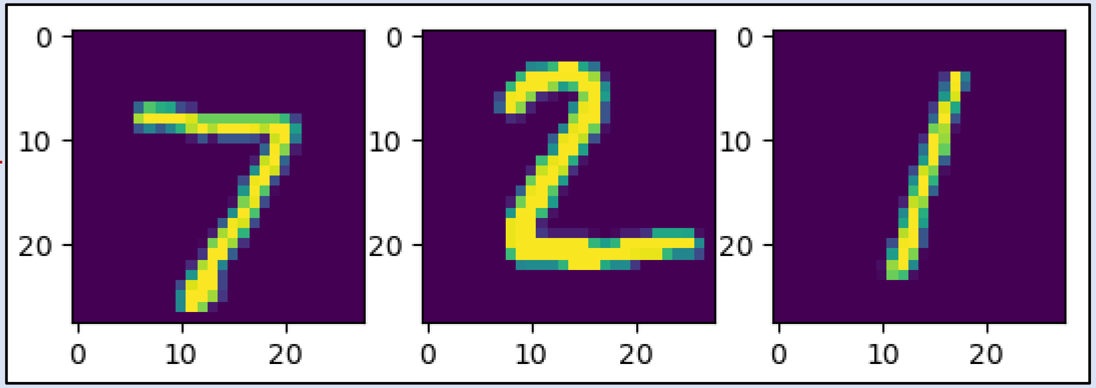
# Applying a linear classifier using Numpy: $x \cdot A + b$

**Python shell**

```
| import matplotlib.pyplot as plt
| import numpy as np
| from tensorflow import keras
| (train_images, train_labels), (test_images, test_labels) = keras.datasets.mnist.load_data()
| type(test_images)
> <class 'numpy.ndarray'>
| test_images.shape
> (10000, 28, 28)   # 10_000 images 28 x 28
| test_labels.shape
> (10000,)          # 10_000 labels
| test_labels[:3]
> array([7, 2, 1], dtype=uint8)
| for i, image in zip(range(3), test_images):
|       plt.subplot(1, 3, i + 1)
|       plt.imshow(image)
| plt.show()

| A, b = map(np.array, eval(open('mnist_linear.weights').read()))  # read A and b from file
| print(A.shape, A.dtype, b.shape, b.dtype)
> (784, 10) float64 (10,) float64
| print([np.argmax(image.reshape(28 * 28) @ A + b) for image in test_images[:3]])
> [7, 2, 1]  # correct on 9_142 of the 10_000 images for the above file, ie accuracy 91%
```

manually generated labels

network prediction



# Graphical user interfaces (GUI)

- Tkinter

**primitive_calculator.py**

```python
accumulator = 0

while True:
    print("Accumulator:", accumulator)
    print("Select:")
    print("   1: clear")
    print("   2: add")
    print("   3: subtract")
    print("   4: multiply")
    print("   5: quit")

    choice = int(input("Choice: "))

    match choice:
        case 1: accumulator = 0
        case 2: accumulator += int(input("add: "))
        case 3: accumulator -= int(input("subtract: "))
        case 4: accumulator *= int(input("multiply by: "))
        case 5: break
```

**Python shell**

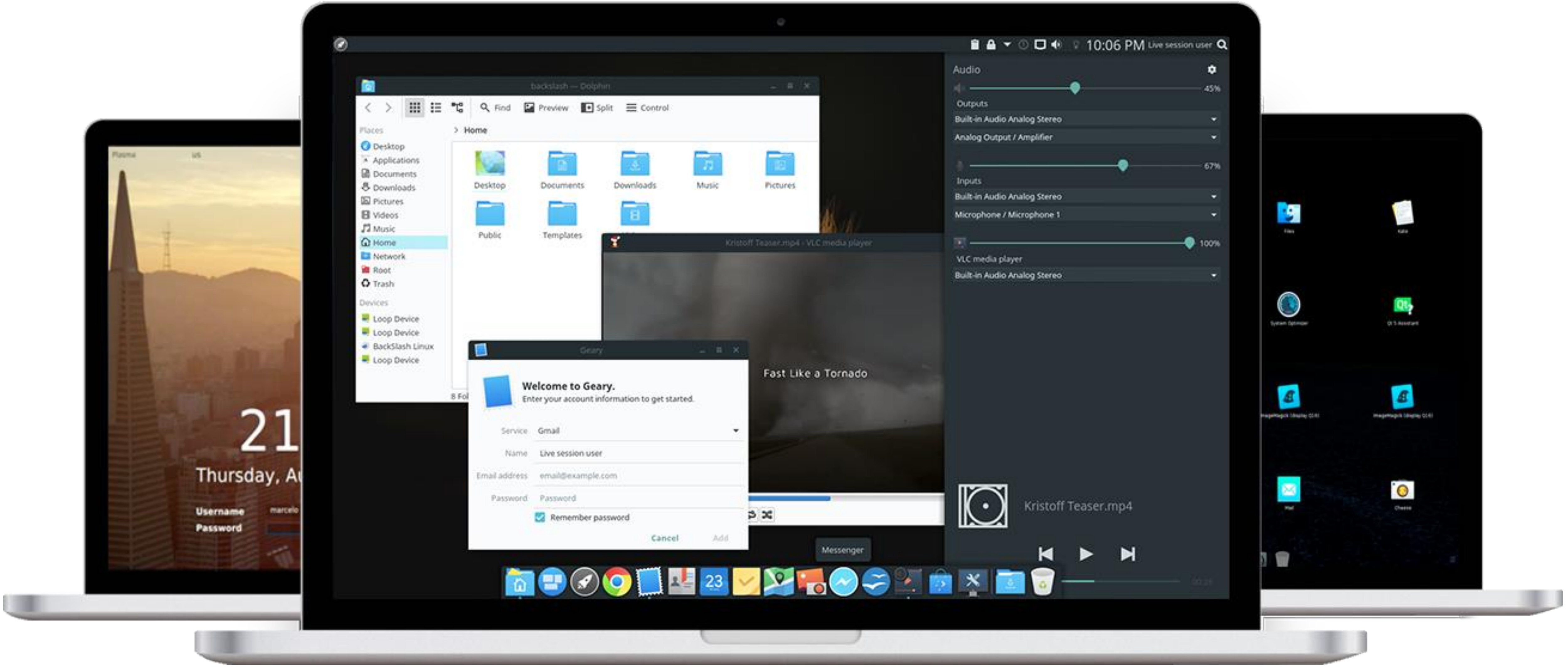
```
Accumulator: 0
Select:
   1: clear
   2: add
   3: subtract
   4: multiply
   5: quit
Choice: 2
add: 10
Accumulator: 10
Select:
   1: clear
   2: add
   3: subtract
   4: multiply
   5: quit
Choice: 2
add: 15
Accumulator: 25
Select:
...
```

# Python GUI’s (Graphical Users Interfaces)

- There is a long list of GUI frameworks and toolkits, designer tools
  - we will only briefly look at Tkinter
- GUI are, opposed to a text terminal, easier to use, more intuitive and flexible
- Windows, icons, menus, buttons, scrollbars mouse / touch / keyboard interaction etc.
- Operating system (e.g. Windows, maxOS, iOS, Linux, Android) provides basic functionality in particular a windows manager
- Writing GUI applications from scratch can be painful – frameworks try to provide all standard functionality

en.wikipedia.org/wiki/Colossal_Cave_Adventure

wiki.python.org/moin/GuiProgramming




**BackSlash Linux GUI**
**www.backslashlinux.com**

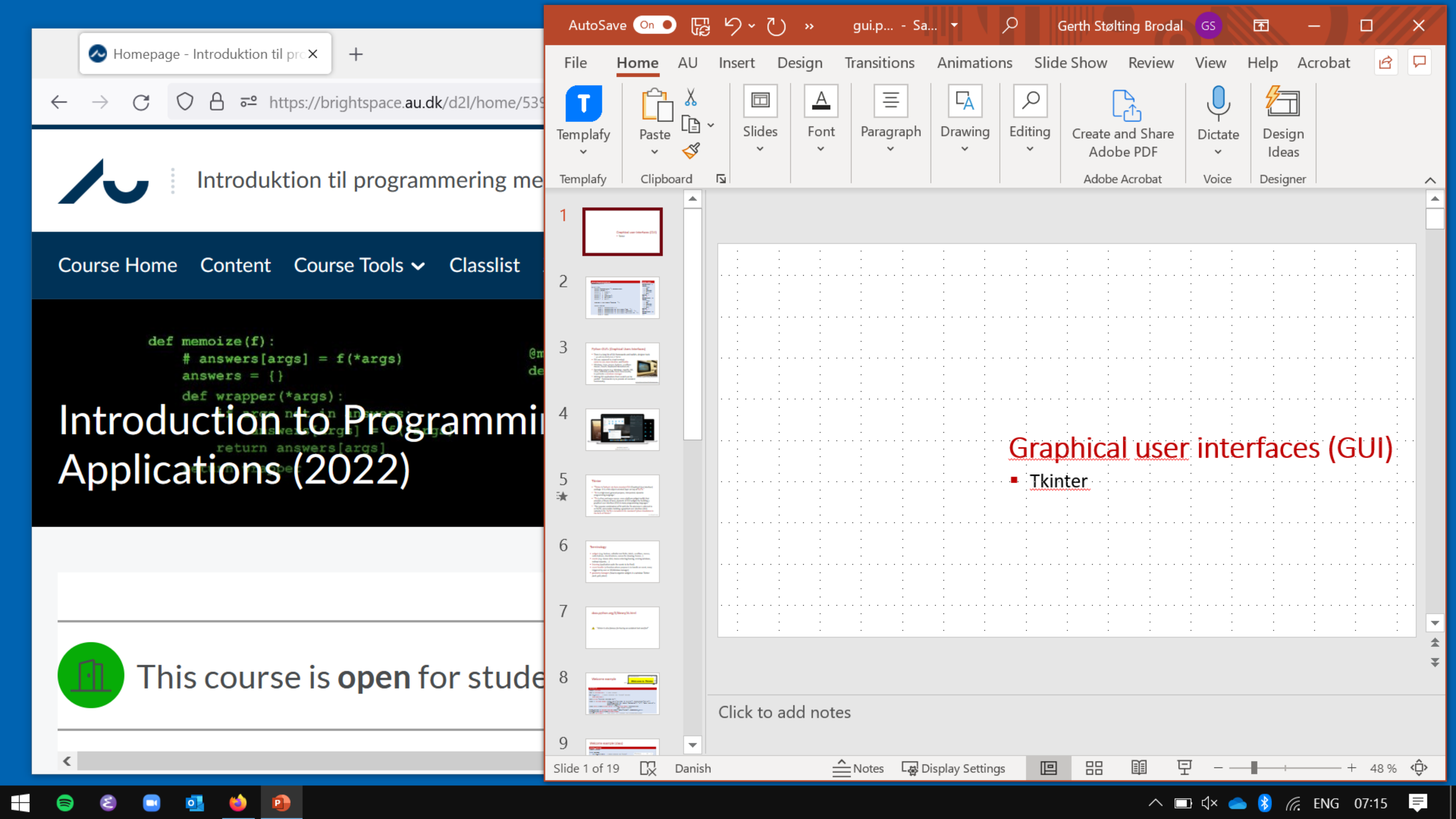
# Tkinter

- “Tkinter is Python's de-facto standard GUI (Graphical User Interface) package. It is a thin object-oriented layer on top of Tcl/Tk.”
- “Tcl is a high-level, general-purpose, interpreted, dynamic programming language.”
- “Tk is a free and open-source, cross-platform widget toolkit that provides a library of basic elements of GUI widgets for building a graphical user interface (GUI) in many programming languages.”
- “The popular combination of Tcl with the Tk extension is referred to as Tcl/Tk, and enables building a graphical user interface (GUI) natively in Tcl. Tcl/Tk is included in the standard Python installation in the form of Tkinter.”

en.wikipedia.org

# Terminology

- widgets (e.g. buttons, editable text fields, labels, scrollbars, menus, radio buttons, check buttons, canvas for drawing, frames...)
- events (e.g. key press, mouse click, mouse entering/leaving, resizing windows, redraw requests, ...)
- listening (application waits for events to be fired)
- event handler (a function whose purpose is to handle an event, many triggered by user or OS/Window manager)
- geometry managers (how to organize widgets in a window: Tkinter *pack*, *grid*, *place*)


Homepage - Introduktion til pro
https://brightspace.au.dk/d2l/home/539
Introduktion til programmering me
Course Home
Content
Course Tools
Classlist
def memoize(f):
# answers[args] = f(*args)
answers = {}
def wrapper(*args):
return answers[args]
Introduction to Programmi
Applications (2022)
This course is open for stude
AutoSave On
gui.p... - Sa...
Gerth Stølting Brodal
File
Home
AU
Insert
Design
Transitions
Animations
Slide Show
Review
View
Help
Acrobat
Templafy
Paste
Slides
Font
Paragraph
Drawing
Editing
Create and Share Adobe PDF
Dictate
Design Ideas
Templafy
Clipboard
Adobe Acrobat
Voice
Designer
Graphical user interfaces (GUI)
Tkinter
Click to add notes
Slide 1 of 19
Danish
Notes
Display Settings
48 %
ENG
07:15

# docs.python.org/3/library/tk.html

*"tkinter is also famous for having an outdated look and feel"*

- Comes with Python
- Alternative PyQt

# Welcome example

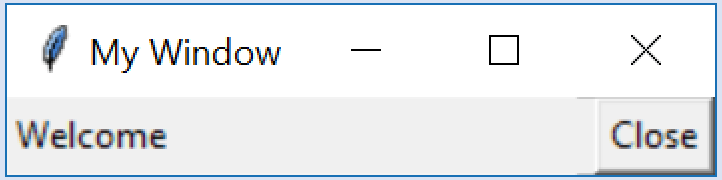

**welcome.py**

```
import tkinter
root = tkinter.Tk()   # root window
def do_quit():   # event handler for "Close" button
    root.destroy()
root.title("Tkinter Welcome GUI")
label = tkinter.Label(root, text="Welcome to Tkinter", background="yellow",
                      anchor=tkinter.SE, font=("Helvetica", "24", "bold italic"),
                      padx=10, pady=10)
label.pack(side=tkinter.LEFT, fill=tkinter.BOTH, expand=True)
                                         # parent window
close_button = tkinter.Button(root, text="Close", command=do_quit)
close_button.pack(side=tkinter.RIGHT)
tkinter.mainloop()   # loop until all windows are closed/destroyed
```

# Welcome example (class)

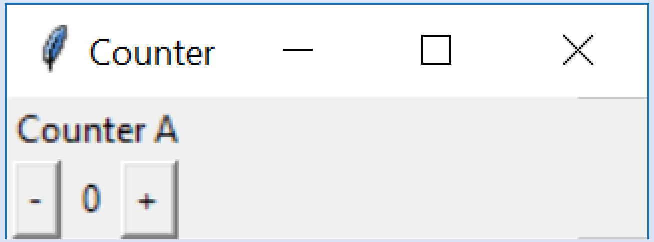
welcome_class.py

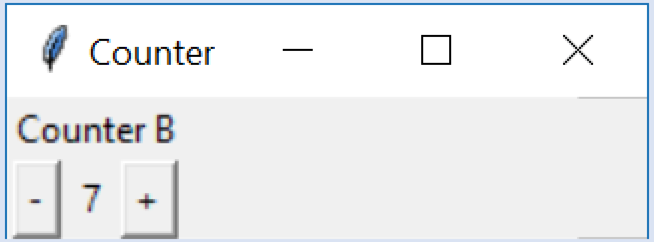
```
import tkinter

class Welcome:
    def do_quit(self):   # event handler for "Close"
        self.root.destroy()

    def __init__(self, window_title):
        self.root = tkinter.Tk()
        self.root.title(window_title)

        self.label = tkinter.Label(self.root, text="Welcome")
        self.label.pack(side=tkinter.LEFT)

        self.close_button = tkinter.Button(self.root, text="Close", command=self.do_quit)
        self.close_button.pack(side=tkinter.RIGHT)

Welcome("My Window")

tkinter.mainloop()
```



**increment.py (part I)**

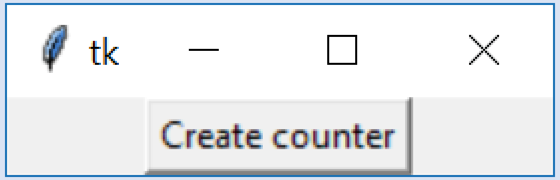
```
import tkinter

class Counter:
    def do_quit(self):
        self.root.destroy()
    def add(self, x):
        self.counter += x
        self.count.set(self.counter)
    def __init__(self, message):
        self.counter = 0
        self.root = tkinter.Toplevel()  # new window
        self.root.title("Counter")
        self.label = tkinter.Label(self.root, text=message)
        self.label.grid(row=0, columnspan=3)
        self.minus_button = tkinter.Button(self.root, text="-", command=lambda: self.add(-1))
        self.minus_button.grid(row=1, column=0)
        self.count = tkinter.IntVar()
        self.count_label = tkinter.Label(self.root, textvariable=self.count)
        self.count_label.grid(row=1, column=1)
        self.plus_button = tkinter.Button(self.root, text="+", command=lambda: self.add(+1))
        self.plus_button.grid(row=1, column=2)
```



**increment.py (part II)**

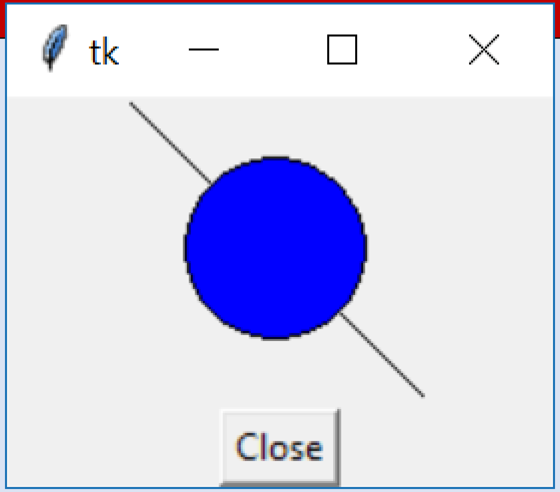
```
class Counter_app:
    def __init__(self):
        self.counters = 0

        self.root = tkinter.Tk()

        self.create = tkinter.Button(self.root, text="Create counter", command=self.new_counter)
        self.create.pack()

    def new_counter(self):
        Counter("Counter " + chr(ord('A') + self.counters))
        self.counters += 1

Counter_app()

tkinter.mainloop()
```

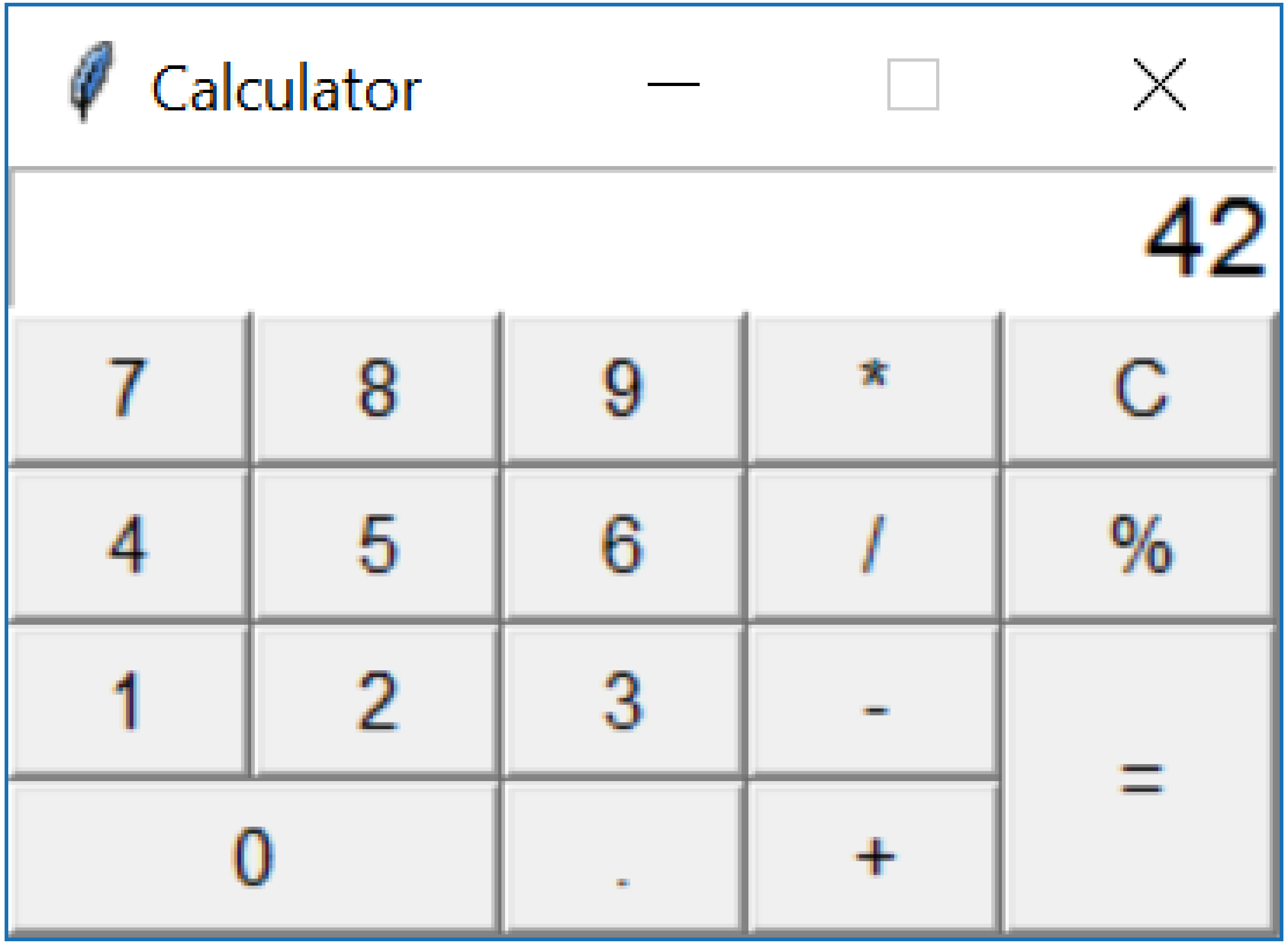


# Canvas

canvas.py

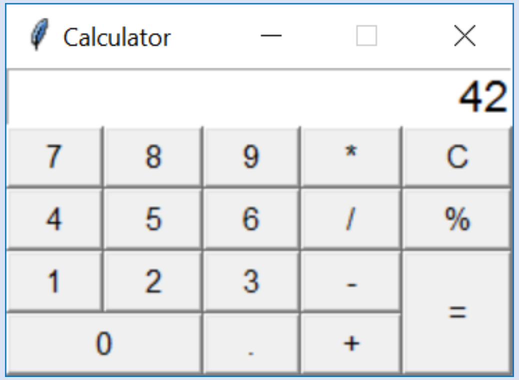
```
import tkinter

root = tkinter.Tk()

canvas = tkinter.Canvas(root, width=100, height=100)
canvas.pack()
canvas.create_line(0, 0, 100, 100)
canvas.create_oval(20, 20, 80, 80, fill="blue")

close = tkinter.Button(root, text="Close", command=root.destroy)
close.pack()

tkinter.mainloop()
```




Calculator
42
7
8
9
*
C
4
5
6
/
%
1
2
3
-
=
0
.
+

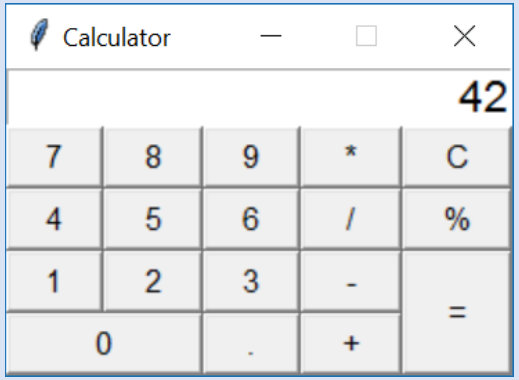
**calculator.py (Part I)**

```
import tkinter
from tkinter import messagebox

class Calculator:
    def __init__(self, root):
        self.root = root

        self.display = tkinter.Entry(self.root, font=("Helvetica", 16), justify=tkinter.RIGHT)
        self.display.insert(0, "0")
        self.display.grid(row=0, column=0, columnspan=5)  # grid = geometry manager

        self.button(1, 0, '7')
        self.button(1, 1, '8')
        self.button(1, 2, '9')
        self.button(1, 3, '*')
        self.button(1, 4, 'C', command=self.clearText)  # 'C' button
        self.button(2, 0, '4')
        self.button(2, 1, '5')
        self.button(2, 2, '6')
        self.button(2, 3, '/')
        self.button(2, 4, '%')
        self.button(3, 0, '1')
        self.button(3, 1, '2')
        self.button(3, 2, '3')
        self.button(3, 3, '-')
        self.button(3, 4, '=', rowspan=2, command=self.calculateExpression)  # '=' button
        self.button(4, 0, '0', columnspan=2)
        self.button(4, 2, '.')
        self.button(4, 3, '+')
```



## calculator.py (Part II)

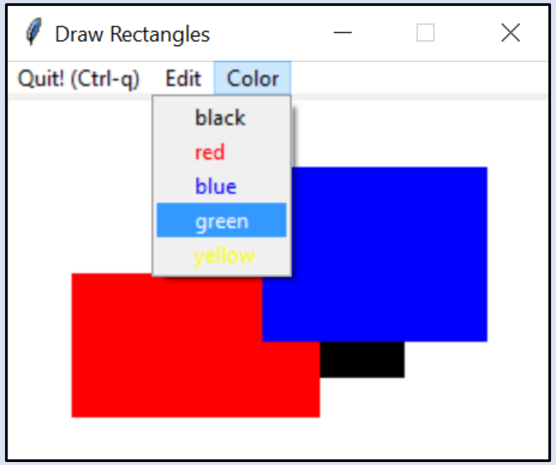
```python
    def button(self, row, column, text, command=None, columnspan=1, rowspan=1):
        if command == None:
            command = lambda: self.appendToDisplay(text)
        B = tkinter.Button(self.root, font=("Helvetica", 11), text=text, command=command)
        B.grid(row=row, column=column, rowspan=rowspan, columnspan=columnspan, sticky="NWNESWSE")

    def clearText(self):
        self.replaceText("0")

    def replaceText(self, text):
        self.display.delete(0, tkinter.END)
        self.display.insert(0, text)

    def appendToDisplay(self, text):
        if self.display.get() == "0":
            self.replaceText(text)
        else:
            self.display.insert(tkinter.END, text)

    def calculateExpression(self):
        expression = self.display.get().replace("%", "/ 100")
        try:
            result = eval(expression)  # DON'T DO THIS !!!
            self.replaceText(result)
        except:
            messagebox.showwarning("Message", "Invalid expression")

root = tkinter.Tk()
root.title("Calculator")
root.resizable(0, 0)

Calculator(root)

tkinter.mainloop()
```



# Creating a menu

rectangles.py

```
class Rectangles:
    Colors = ['black', 'red', 'blue', 'green', 'yellow']

    def create_menu(self):
        menubar = tkinter.Menu(self.root)
        menubar.add_command(label="Quit! (Ctrl-q)", command=self.do_quit)

        editmenu = tkinter.Menu(menubar, tearoff=0)
        editmenu.add_command(label="Clear", command=self.clear_all)
        editmenu.add_command(label="Delete last (Ctrl-z)", command=self.delete_last_rectangle)

        colormenu = tkinter.Menu(menubar, tearoff=0)
        for color in self.Colors:  # list of color names
            colormenu.add_command(label=color,
                                  foreground=color,
                                  command=self.get_color_handler(color))

        menubar.add_cascade(label="Edit", menu=editmenu)
        menubar.add_cascade(label="Color", menu=colormenu)
        self.root.config(menu=menubar)  # Show menubar

    def get_color_handler(self, color):
        return lambda : self.set_color(color)

    def set_color(self, color):
        self.current_color = color
...
```

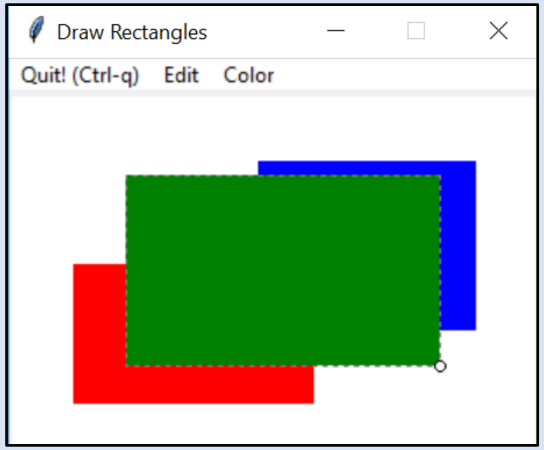


# Binding key and mouse events

- Whenever a key is pressed, mouse button is pressed/released, mouse is moved, mouse enters/leaves objects etc. events are triggered that can be bound to call a user defined event handler

**rectangles.py (continued)**

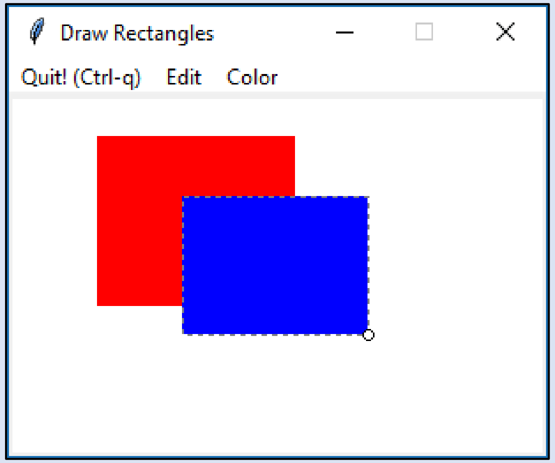
```
...
  self.root = tkinter.Tk()
  self.root.bind('<Control-q>', self.do_quit)
  self.root.bind('<Control-z>', self.delete_last_rectangle)
...
  self.canvas = tkinter.Canvas(self.root, width=300, height=200,
                               background='white')
  self.canvas.bind('<Button-1>', self.create_rectangle_start)
  self.canvas.bind('<B1-Motion>', self.create_rectangle_mouse_move)
  self.canvas.bind('<ButtonRelease-1>', self.create_rectangle_end)
...
```



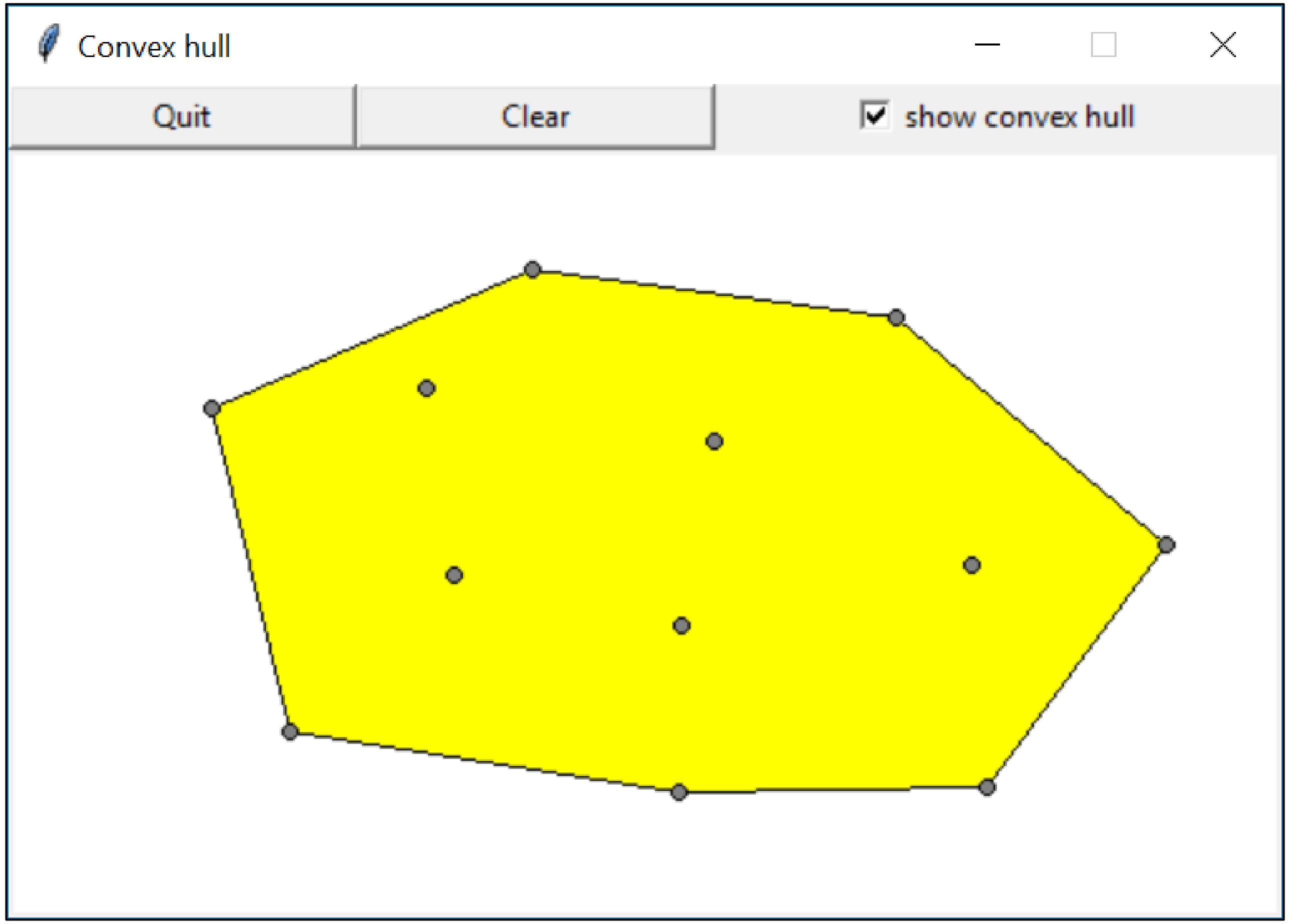
# Handling mouse events

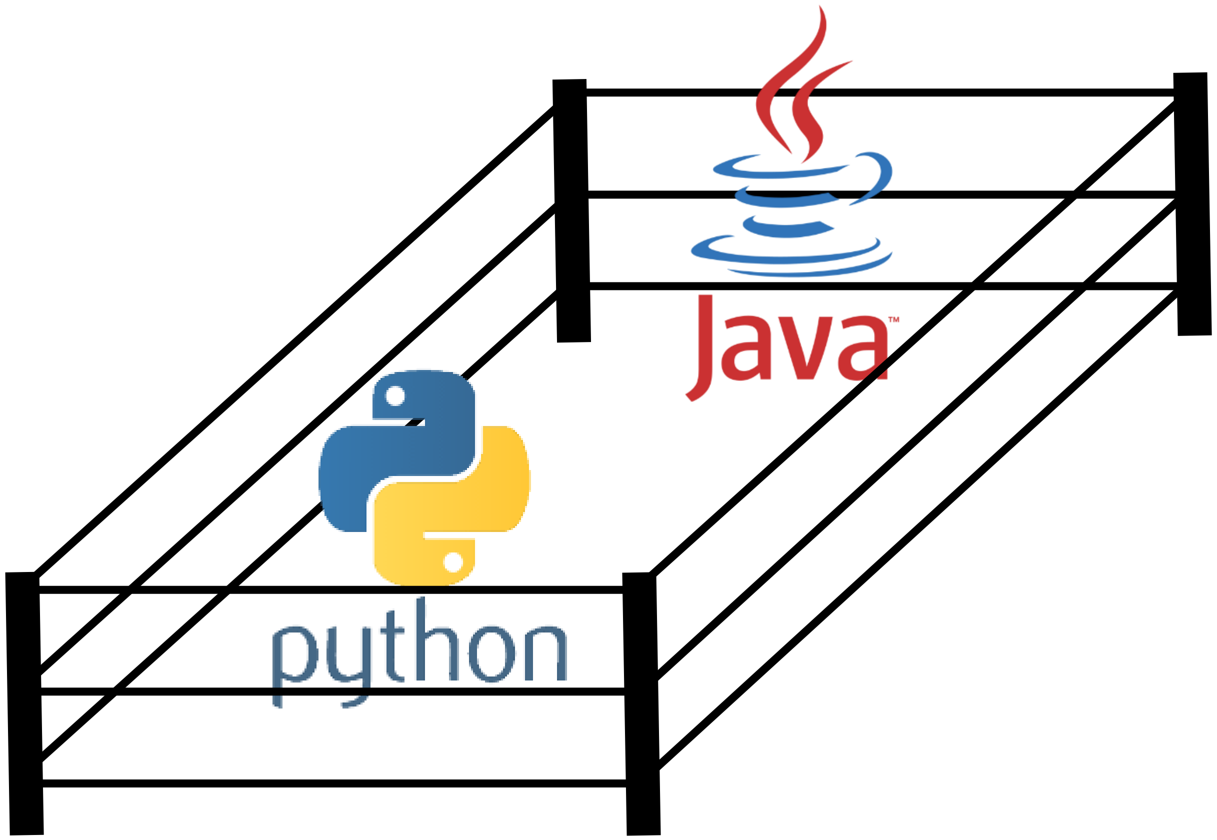
rectangles.py (continued)

```
def create_rectangle_start(self, event):
    radius = 3
    x, y = event.x, event.y
    self.top_pos = (x, y)
    self.bottom_pos = (x, y)
    self.rectangle = self.canvas.create_rectangle(x, y, x, y,  # top-left = bottom-right
                                     fill=self.current_color, width=1, outline='grey', dash=(3, 5))
    self.corner = self.canvas.create_oval(x - radius, y - radius, x + radius, y + radius, fill='white')
def create_rectangle_mouse_move(self, event):
    if self.corner:
        x, y = event.x, event.y
        x_, y_ = self.bottom_pos
        self.bottom_pos = (x, y)
        self.canvas.coords(self.rectangle, *self.top_pos, *self.bottom_pos)
        self.canvas.move(self.corner, x - x_, y - y_)
def create_rectangle_end(self, event):
    if self.corner:
        self.canvas.delete(self.corner)
        self.corner = None
        if self.bottom_pos != self.top_pos:
            self.rectangles.append(self.rectangle)
            self.canvas.itemconfig(self.rectangle, width=0)
        else:  # empty rectangle, skip
            self.canvas.delete(self.rectangle)
        self.rectangle = None
```



# Exercise 25.1 (convex hull GUI)



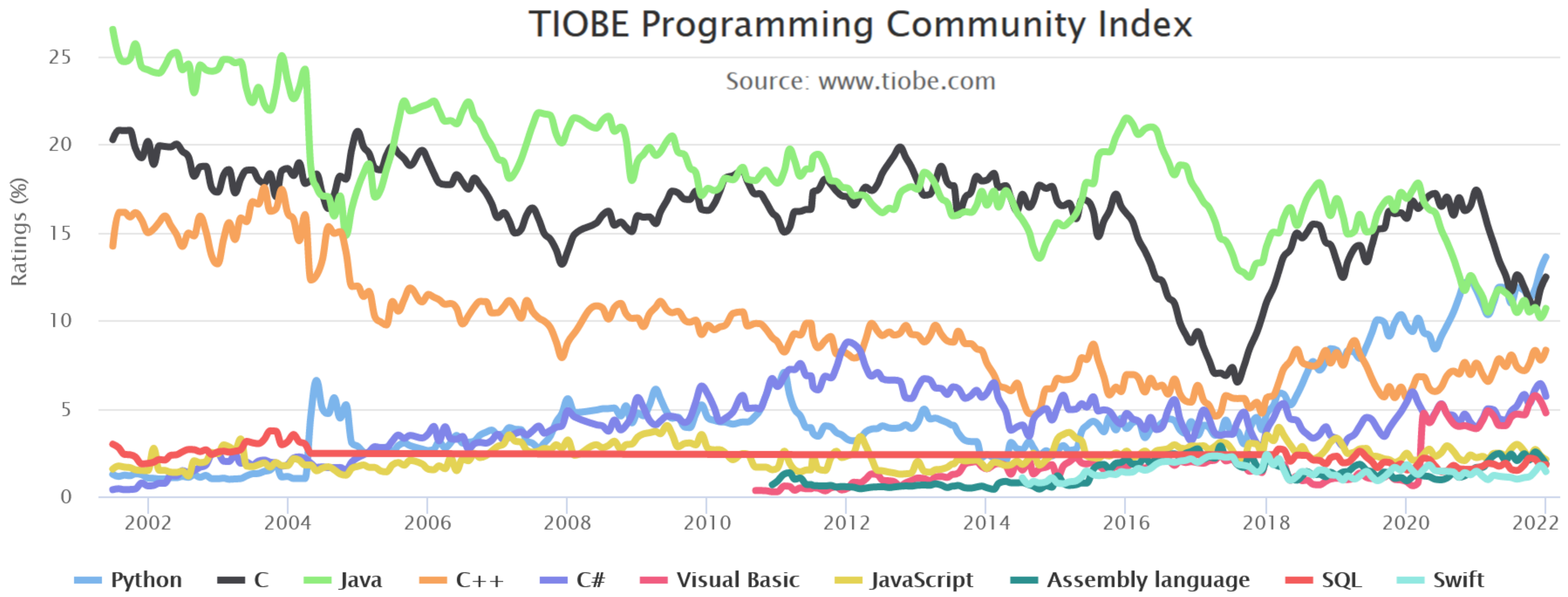


# Java vs Python

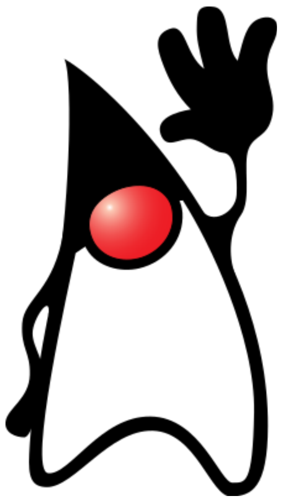
runestone.academy/ns/books/published/java4python/

# Why should you know something about Java?

- Java is an example of a statically typed object oriented language (like C and C++) opposed to Python's being dynamically typed
- One of the most widespread used programming languages
- Used in other courses at the Department of Computer Science



# Java history



**PyPy** is adopting the same ideas to Python (Just-in-Time compilation)

- Java 1.0 released 1995 by Sun Microsystems (acquired by Oracle 2010)
- ”Write Once, Run Anywhere”
- 1999 improved performance by the **Java HotSpot Performance Engine**
- Current version Java 18 (released March 2022)
- Java compiler generates **Java bytecode** that is executed on a **Java virtual machine (JVM)**

# Installing Java

- To compile Java programs into **bytecode** you need a compiler, e.g. from Java SE Development Kit (**JDK**):

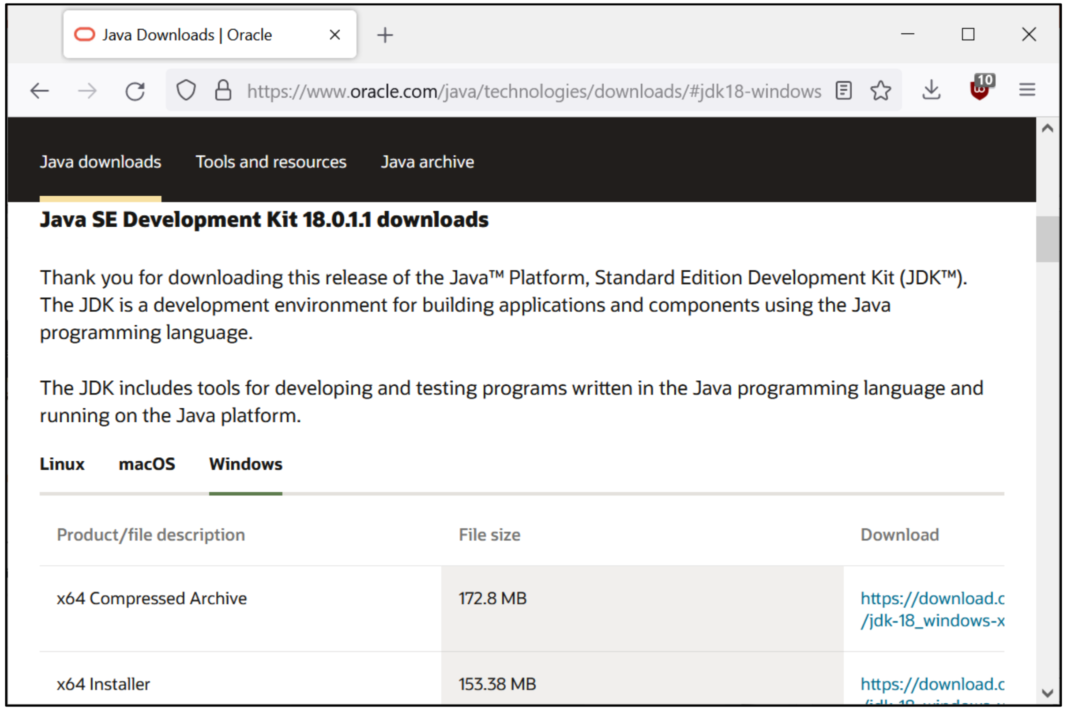
  www.oracle.com/java/technologies/downloads/

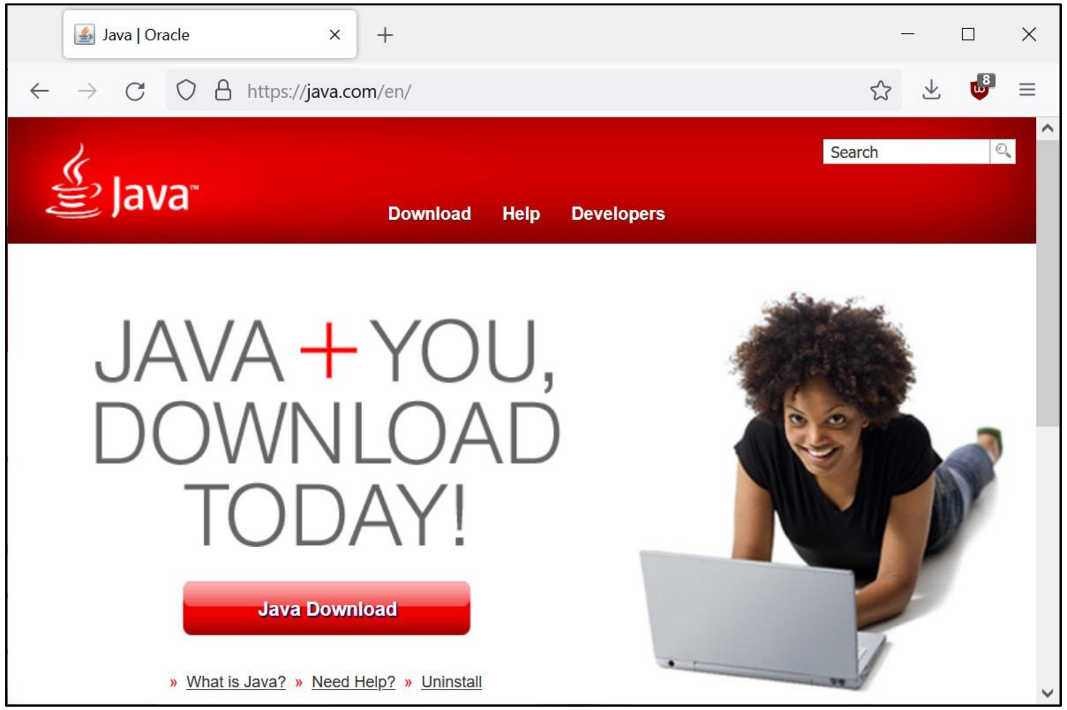
  (you might need to add the JDK directory to your PATH, e.g. C:\Program Files\Java\jdk-18.0.1.1\bin)

- To only run compiled Java programs:

  java.com/download

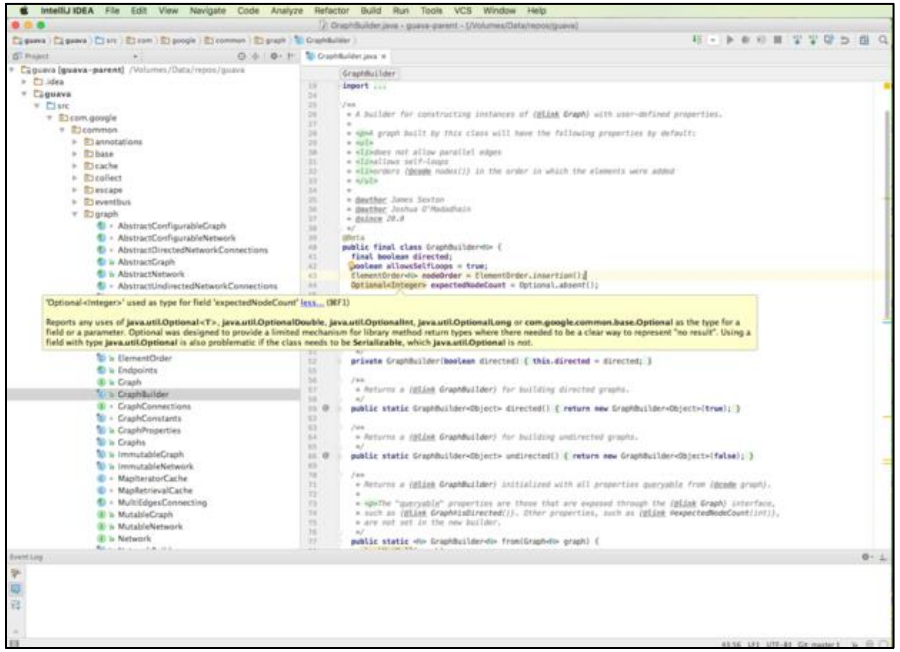
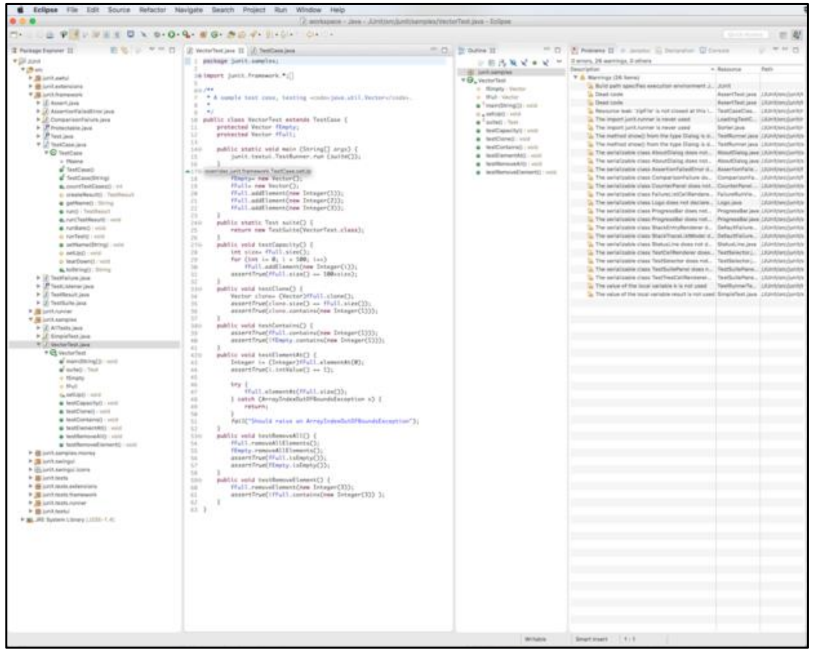
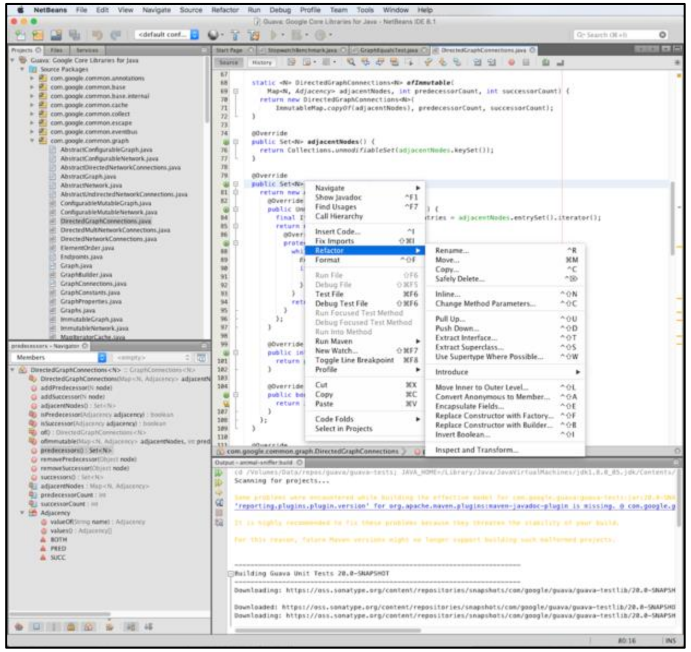
  (If you use JDK, you should not download this)



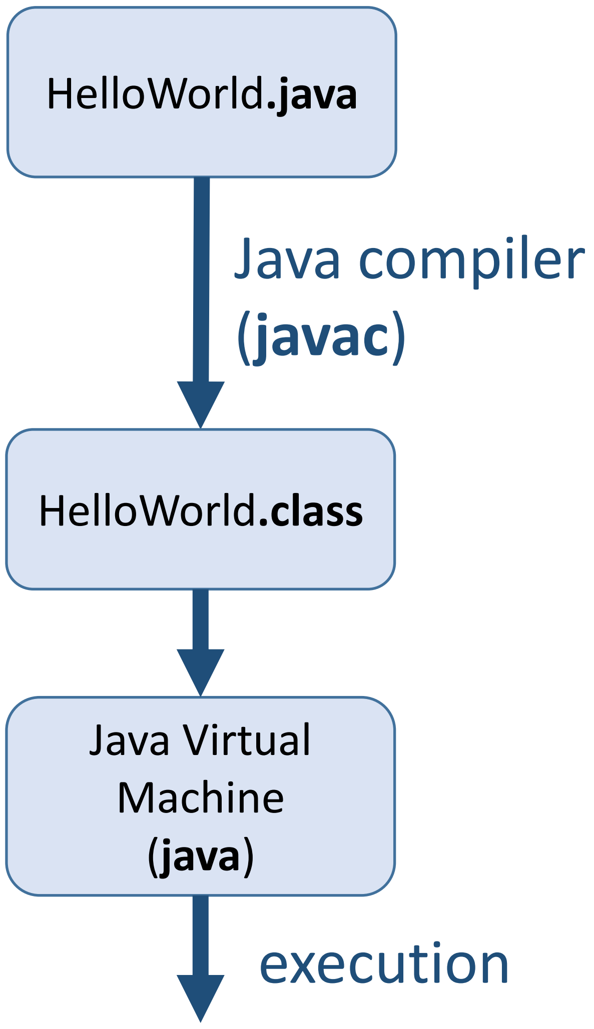



# Java IDE

- Many available, some popular:
  Visual Studie Code, IntelliJ IDEA, Eclipse, and NetBeans
- An IDE for beginners: BlueJ

# Compiling and running a Java program




**HelloWorld.java**

```java
public class HelloWorld {
    public static void main( String[] args ) {
        System.out.println( "Hello World!" );
    }
}
```

Command Prompt

```
C:\Users\au121\Desktop>javac HelloWorld.java

C:\Users\au121\Desktop>dir HelloWorld*
 Volume in drive C is OSDisk
 Volume Serial Number is 3CDB-90D8

 Directory of C:\Users\au121\Desktop

04-05-2020  17:40               426 HelloWorld.class
09-05-2019  21:18               132 HelloWorld.java
               2 File(s)            558 bytes
               0 Dir(s)  372.191.944.704 bytes free

C:\Users\au121\Desktop>java HelloWorld
Hello World!

C:\Users\au121\Desktop>
```

# Java : `main`

- *name*`.java` must be equal to the `public class` *name*
- A class can only be excuted using `java` *name* (without `.class`) if the class has a class method `main` with signature

  `public static void main(String[] args)`
- (`main` is inherited from C and C++ sharing a lot of syntax with Java)
- Java convention is that class names should use CamelCase

**PrintArguments.java**

```java
public class PrintArguments {
    public static void main( String[] args ) {
        for (int i=0; i<args.length; i++)
            System.out.println( args[i] );
    }
}
```

**shell**

```
> java PrintArguments x y z
| x
| y
| z
```

a `static` method in
a class is a class method
(exists without creating objects)

method name

there can be several classes
in a file – but only one class
should be `public` and have
same name as file

type of return value
(`void` = no return value)

class name containing `main`
must have same name as file

type of argument,
array of `String` values

the main method must
be `public` to be visible
outside class

**PrintArguments.java**

```java
public class PrintArguments {
    public static void main( String[] args ) {
        for (int i=0; i<args.length; i++)
            System.out.println( args[i] );
    }
}
```

name of argument

For-loop equivalent to

```
int i=0
while (i<args.length) {
    code
    i++;
}
```

i += 1

declare new `int` variable
locally inside for-loop

the print statement is found
in the `System` class

java arrays are indexed from
`0` to `args.length - 1`
and the length of an array
object is fixed once created

# Argument list also exists in Python...

**PrintArguments.py**

```
import sys
print(sys.argv)
```

**shell**

```
> python PrintArguments.py a b 42
| ['PrintArguments.py', 'a', 'b', '42']
```

**Primitive.java**

```java
/**
 * A Java docstring to be processed using 'javadoc'
 */
// comment until end-of-line
public class Primitive {
    public static void main( String[] args ) {
        int x;  // type of variable must be declared before used
        x = 1;  // remember ';' after each statement
           int y=2;  // indentation does not matter
    int a=3, b=4;  // multiple declarations and initialization
        System.out.println(x + y + a + b);
        int[] v={1, 2, 42, 3};  // array of four int
        System.out.println(v[2]);  // prints 42, arrays 0-indexed
/* multi-line comment
       that continues until here */
        v = new int[3];  // new array of size three, containing zeros
        System.out.println(v[2]);  // prints 0
        if (x == y) {  // if-syntax '(' and ')' mandatory
             a = 1;
             b = 2;
        } else {  // use '{' and '}' to create block of statements
             a = 4; b = 3;  // two statements on one line
        }
}}
```

# Why state types – Python works without...

- Just enforcing a different programming style (also C and C++)
- Helps users to avoid mixing up values of different types
- (Some) type errors can be caught at compile time
- More efficient code execution

**type_error.py**

```
x = 3
y = "abc"
print("program running...")
print(x / y)
```

**Python shell**

```
| program running...
...
| ----> 4 print(x / y)
| TypeError: unsupported operand type(s) for
  /: 'int' and 'str'
```

**TypeError.java**

```
public class TypeError {
    public static void main( String[] args ) {
        int x = 3;
        String y = "abc";
        System.out.println(x / y);
    }
}
```

**shell**

```
> javac TypeError.java
| javac TypeError.java
| TypeError.java:5: error: bad operand types for
  binary operator '/'
|           System.out.println(x / y);
|                                ^
|   first type:  int
|   second type: String
| 1 error
```

# Basic Java types

| Type | Values |
|---|---|
| boolean | true or false |
| byte | 8 bit integer |
| char | character (16-bit UTF) |
| short | 16 bit integer |
| int | 32 bit integer |
| long | 64 bit integer |
| float | 32 bit floating bout |
| double | 64 bit floating point |
| class BigInteger | arbitrary precision integers |
| class String | strings |

**BigIntegerTest.java**

```
import java.math.*;   // import everything
// import java.math.BigInteger;  // alternativ

public class BigIntegerTest {
    public static void main( String[] args ) {
        BigInteger x = new BigInteger("2");
        while (true) {
            // BigIntegers are immutable
            x = x.multiply(x);
            // java.math.BigInteger.toString()
            System.out.println(x);
        }
    }
}
```

**shell**

```
| 4
| 16
| 256
| 65536
| 4294967296
| 18446744073709551616
| 340282366920938463463374607431768211456
| ...
```

# Java arrays

- The size of a builtin Java array can not be modified when first created. If you need a bigger array you have to instantiate a new array.
- Or better use a standard collection class like **ArrayList**
- **ArrayList** is a generic class (type of content is given by <*element type*>; generics available since Java 5, 2004)
- The for-each loop was introduced in Java 5

**ConcatenateArrayLists.java**

```
import java.util.*;  // java.util contains ArrayList

public class ConcatenateArrayList {
    public static void main( String[] args ) {
        // ArrayList is a generic container
        ArrayList<String> a = new ArrayList<String>();
        ArrayList<String> b = new ArrayList<String>();
        ArrayList<String> c = new ArrayList<String>();
        a.add("A1");  // in Python .append
        a.add("A2");
        b.add("B1");
        c.addAll(a);  // in Python .extend
        c.addAll(b);
        for (String e : c) {  // foreach over iterator
            System.out.println(e);
        }
    }
}
```

**shell**

```
A1
A2
B1
```

# Tired of writing all these types...

- In Java 7 (2011) the "diamond operator" `<>` was introduced for type inference for generic instance creation to reduce verbosity
- In Java 10 (2018) the `var` keyword was introduced to type infer variables

**ArrayListTest.java**

```java
import java.util.*;  // java.util contains ArrayList

public class ArrayListTest {
    public static void main( String[] args ) {
       // ArrayList is a generic container
        ArrayList<String> a = new ArrayList<String>(); // Full types
        List<String> b = new ArrayList<String>();      // ArrayList is subclass of class List
        ArrayList<String> c = new ArrayList<>();       // <> uses type inference
        List<String> d = new ArrayList<>();            // <> and ArrayList subclass of List
        var e = new ArrayList<String>();               // use var to infer type of variable
        var v = Math.floor(1.5);                       // not obvious what type v is (double)
}}
```

# Function arguments

- Must declare the number of arguments and their types, and the return type
- The argument types are part of the *signature* of the function
- Several functions can have the same name, but different type signatures
- Python keyword arguments, * and ** do not exist in Java ☹

**Functions.java**

```java
public class Functions {
    private static int f(int x) {
        return x * x;
    }
    private static int f(int x, int y) {
        return x * y;
    }
    private static String f(String a, String b) {
        return a + b; // string concatenation
    }

    public static void main( String[] args ) {
        System.out.println(f(7));
        System.out.println(f(3, 4));
        System.out.println(f("abc", "def"));
    }
}
```

**shell**

```
49
12
abcdef
```

**functions.py**

```python
def f(x, y=None):
    if y == None:
        y = x
    if type(x) is int:
        return x * y
    else:
        return x + y
print(f(7), f(3, 4), f('abc', 'def'))
```

# Class

- Constructor = method with name equal to class name (no return type)
- `this` = referes to current object (Python “self”)
- Use `private` / `public` on attributes / methods to give access outside class
- Use `new` *name*(arguments) to create new objects
- There can be multiple constructors, but with distinct type signatures

AClass.java

```
class Rectangle {
    private int width, height;  // declare attributes
    // constructor, class name, no return type
    public Rectangle(int width, int height) {
        this.width = width;
        this.height = height;
    }
    public Rectangle(int side) {
        width = side;  // same as this.width = side
        height = side;
    }
    public int area() {
        return width * height;
    }
}
public class AClass {
    public static void main( String[] args ) {
        Rectangle r = new Rectangle(6, 7);
        System.out.println(r.area());
    }
}
```

shell

```
| 42
```

# Inheritance

- Java supports single inheritance using `extends`
- Attributes and methods that should be accessible in a subclass must be declared protected (or public)
- Constructors are not inherited but can be called using `super`

**Inheritance.java**

```java
class BasicRectangle {
    // protected allows subclass to access attributes
    protected int width, height;
    public BasicRectangle(int width, int height) {
        this.width = width; this.height = height;
    }
}

class Rectangle extends BasicRectangle {
    public Rectangle(int width, int height) {
        // call constructor of super class
        super(width, height);
    }
    public int area() {
        return width * height;
    }
}

public class Inheritance {
    public static void main( String[] args ) {
        Rectangle r = new Rectangle(6, 7);
        System.out.println(r.area());
    }
}
```

**shell**

```
| 42
```

# Generic class

- Class that is parameterized by one or more types (comma separated)
- Primitive types cannot be type parameters
- Instead use *wrappers*, like Integer for int

GenericPair.java

```
class Pair<element> {
    private element x, y;
    public Pair(element x, element y) {
        this.x = x; this.y = y;
    }
    element first() { return x; }
    element second() { return y; }
}
public class GenericPair {
    public static void main( String[] args ) {
        var p = new Pair<Integer>(6, 7);
        System.out.println(p.first() * p.second());
    }
}
```

shell

```
| 42
```

# Interface

- Java does not support multiple inheritance like Python
- But a class can implement an arbitrary number of interfaces
- An interface specifices a set of attributes and methods a class must have
- The type of a variable can be an interface, and the variable can hold any object where the class is stated to implement the interface

**RectangleInterface.java**

```java
interface Shape {
    public int area(); // method declaration
}

class Rectangle implements Shape {
    private int width, height;

    // constructor, class name, no return type
    public Rectangle(int width, int height) {
        this.width = width; this.height = height;
    }

    public int area() {
        return width * height;
    }
}

public class RectangleInterface {
    public static void main( String[] args ) {
        Shape r = new Rectangle(6, 7);
        System.out.println(r.area());
    }
}
```

# Abstract classes

- Abstract class = class that cannot be instantiated, labeled `abstract`
- Abstract method = method declared without definition, labeled `abstract`, must be in abstract class
- An abstract class can be extended to a non-abstract class by providing the missing method definitions

AbstractRectangle.java

```
abstract class Shape {
    abstract public int circumference();
    abstract public int area();
    public double fatness() {
        // convert int from area() to double before /
        return (double)area() / circumference();
    }
}

class Rectangle extends Shape {
    private int width, height;
    // constructor, class name, no return type
    public Rectangle(int width, int height) {
        this.width = width; this.height = height;
    }
    public int area() {
        return width * height;
    }
    public int circumference() {
        return 2 * (width + height);
    }
}

public class AbstractRectangle {
    public static void main( String[] args ) {
        Shape r = new Rectangle(6, 7);
        System.out.println(r.fatness());
    }
}
```

# Default methods in interfaces

- Before Java 8 all methods in an interface were abstract (no definition)
- Since Java 8 interfaces can have **`default`** methods with definition
- The distinction between abstract classes and interfaces gets blurred
  - a class can only extend one abstract class
  - a class can implement more interfaces

  ⇒ multiple “inheritance” is possible in Java

**DefaultInterface.java**

```
interface Shape {
    public int circumference();
    public int area();
    default public double fatness() {
        // convert int from area() to double before /
        return (double)area() / circumference();
    }
}

class Rectangle implements Shape {
    private int width, height;
    // constructor, class name, no return type
    public Rectangle(int width, int height) {
        this.width = width; this.height = height;
    }
    public int area() {
        return width * height;
    }
    public int circumference() {
        return 2 * (width + height);
    }
}

public class DefaultRectangle {
    public static void main( String[] args ) {
        Shape r = new Rectangle(6, 7);
        System.out.println(r.fatness());
    }
}
```

# Multiple Inheritance

- Class `C` implements both interfaces `A` and `B`
- Inherits default methods `sayA` and `sayB`
- Cannot inherit `sayHi`, since in both `A` and `B`. Must be overriden in `C`.
- Can use `@override` to enforce compiler to check if method exists in super class
- `new A(){}` creates an instance of an *anonymous class* (extending or implementing A)

MultipleInheritance.java

```java
interface A {
    default public void sayA() { System.out.println("say A"); }
    default public void sayHi() { System.out.println("A say's Hi"); }
}
interface B {
    default public void sayB() { System.out.println("say B"); }
    default public void sayHi() { System.out.println("B say's Hi"); }
}
class C implements A, B {
    @Override // (optional) requests compiler to
              // check if sayHi exists in supertype
    public void sayHi() {
        System.out.println("C say's Hi");
        (new A(){}).sayHi();  // instantiate an anonymous class
    }
    public void test() {
        sayA();
        sayB();
        sayHi();
    }
}
public class MultipleInheritance {
    public static void main( String[] args ) {
        new C().test();
    }
}
```

Shell output

```
say A
say B
C say's Hi
A say's Hi
```

# Lambda expression

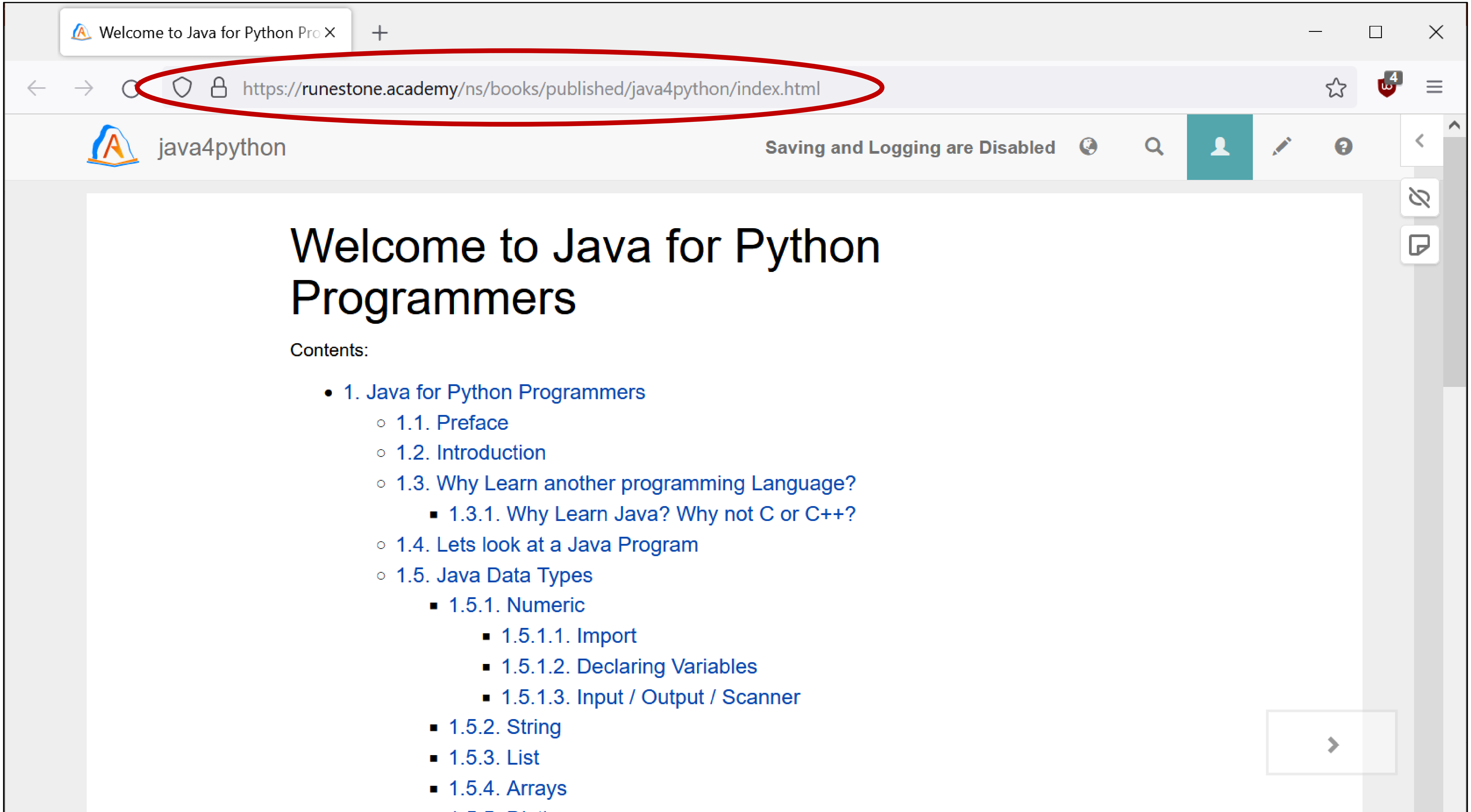
- Lambda expressions are possible since Java 8
- Syntax : *argument -> expression*

LambdaPrinting.java

```
import java.util.*;  // ArrayList

public class LambdaPrinting {
    public static void main(String[] args) {
        var elements = new ArrayList<Integer>();

        for (int i = 1; i <= 3; i++)
            elements.add(i);

        elements.forEach(e -> System.out.println(e));
    };
}
```

LambdaPrinting.java

```
1
2
3
```


Welcome to Java for Python Pro
https://runestone.academy/ns/books/published/java4python/index.html
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