Visualization and optimization

- Matplotlib
- Jupyter
- scipy.optimize.minimize



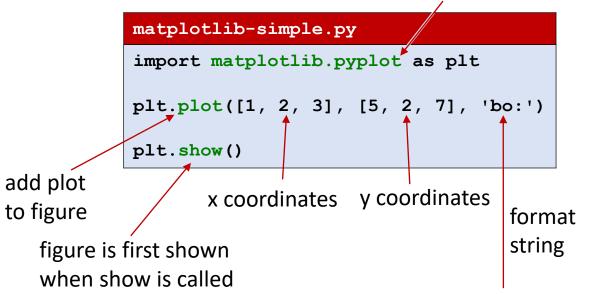
Matplotlib is a Python 2D plotting library which produces publication quality figures in a variety of hardcopy formats and interactive environments across platforms. Matplotlib can be used in Python scripts, the Python and IPython shells, the Jupyter notebook, web application servers, and four graphical user interface toolkits.

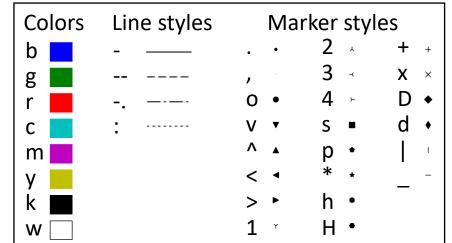
Matplotlib tries to make easy things easy and hard things possible. You can generate plots, histograms, power spectra, bar charts, errorcharts, scatterplots, etc., with just a few lines of code. For simple plotting the pyplot module provides a MATLAB-like interface, particularly when combined with IPython. For the power user, you have full control of line styles, font properties, axes properties, etc, via an object oriented interface or via a set of functions familiar to MATLAB users.

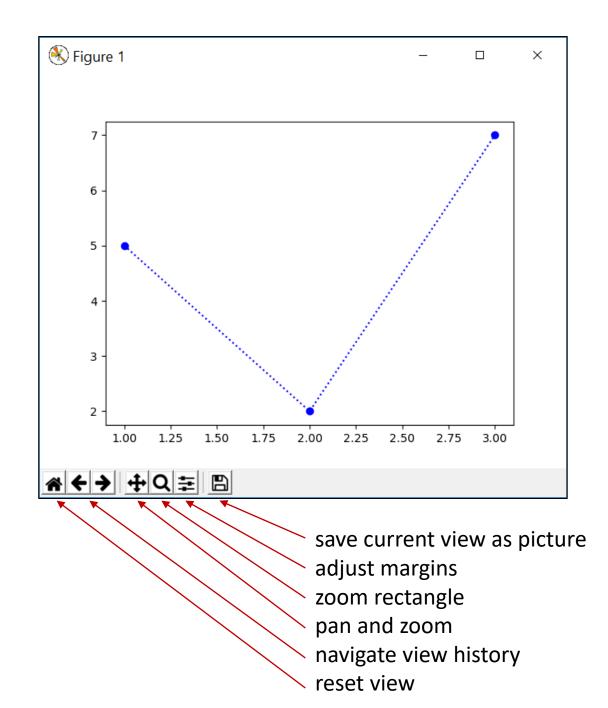
pip install matplotlib <u>matplotlib.org</u>

Plot

pyplot module ≈ MATLAB-like plotting framework

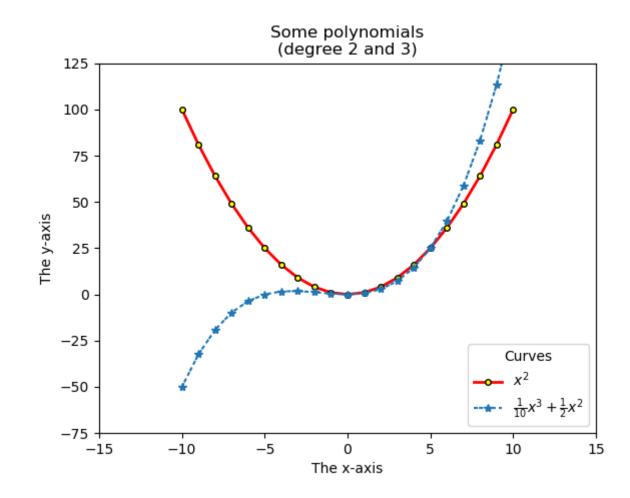






Plot – some keyword arguments

```
matplotlib-plot.py
import matplotlib.pyplot as plt
X = range(-10, 11)
Y1 = [x ** 2 for x in X]
Y2 = [x ** 3 / 10 + x ** 2 / 2 \text{ for } x \text{ in } X]
plt.plot(X, Y1, color='red', label='$x^2$',
    linestyle='-', linewidth=2,
    marker='o', markersize=4,
    markeredgewidth=1,
    markeredgecolor='black',
    markerfacecolor='vellow')
plt.plot(X, Y2, '*', dashes=(2, 0.5, 2, 1.5),
    label=r'$\frac{1}{10}x^3+\frac{1}{2}x^2$')
plt.xlim(-15, 15)
plt.ylim(-75, 125)
plt.title('Some polynomials\n(degree 2 and 3)')
plt.xlabel('The x-axis')
plt.ylabel('The y-axis')
plt.legend(title='Curves')
plt.show() # finally show figure
```



matplotlib.org/api/ as gen/matplotlib.pyplot.plot.html Colors: matplotlib.org/gallery/color/named colors.html

Scatter (points with individual size and color)

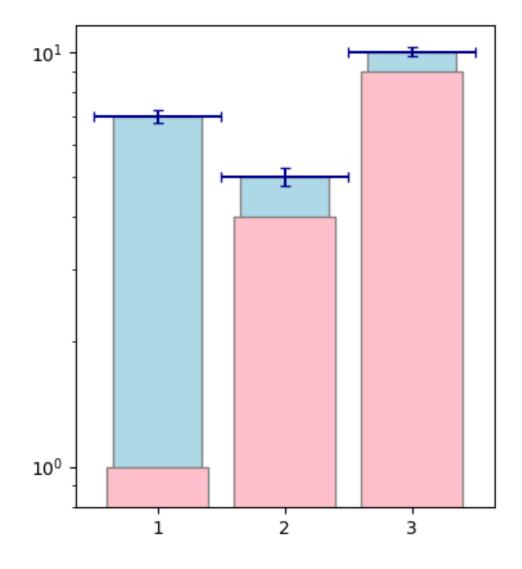
```
transparency
matplotlib-scatter.py
                                                                                     A scatter plot
import matplotlib.pyplot as plt
                                                                                                   s = x^2. c = x
n = 13
                                                                                                                    - 10
X = range(n)
S = [x ** 2 for x in X]
E = [2 ** x for x in X]
plt.scatter(X, [4] * n, s=E, label='s = $2^x$', alpha=.2)
plt.scatter(X, [3] * n, s=S, label='s = x^2;
plt.scatter(X, [2] * n, s=X, label='s = x')
plt.scatter(X, [1] * n, s=S, c=X, cmap='plasma',
    label='s = x^2, c = x,
                                           colormap (predefined)
    edgecolors='gray', linewidth=0.5)
                                           color of each point
plt.colorbar()
                                           size ≈ area of each point
                                                                                                          12
plt.ylim(0.5, 5.5)
                                           point boundary width
plt.xlim(0.5, 13.5)
                                           point boundary color
                                                                                                         colorbar
plt.title('A scatter plot')
plt.legend(loc='upper center', frameon=False, ncol=4,
                                                                                                        (of most recently
           handletextpad=0)
                                                                                                        used colormap)
plt.show()
```

manual placement of legend box (default automatic); remove frame; place legends in 4 columns (default 1); reduce space between marks and label

matplotlib.org/api/ as gen/matplotlib.pyplot.scatter.html matplotlib.org/tutorials/colors/colormaps.html

Bars

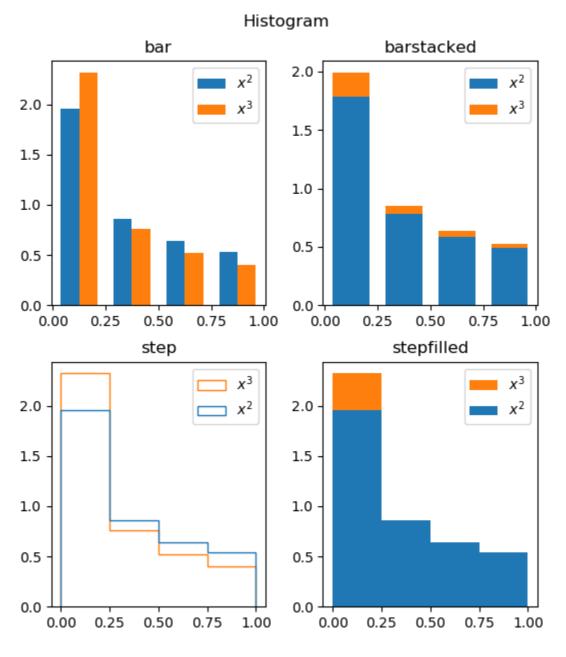
```
matplotlib-bars.py
import matplotlib.pyplot as plt
x = [1, 2, 3]
y = [7, 5, 10]
plt.bar(x, y,
      color='lightblue', # bar background color
      linewidth=1,  # bar boundary width
      edgecolor='gray', # bar boundary color
      tick_label=x,  # ticks on x-axis
      width=0.7,  # width, default 0.8
      xerr=0.5, # x length
      capsize=3, # capsize in points
      ecolor='darkblue', # error bar color
      log=True) # y-axis log scale
plt.bar(x, [v**2 for v in x],
      color='pink',
      linewidth=1,
      edgecolor='gray')
plt.show()
```



matplotlib.org/api/ as gen/matplotlib.pyplot.bar.html

Histogram

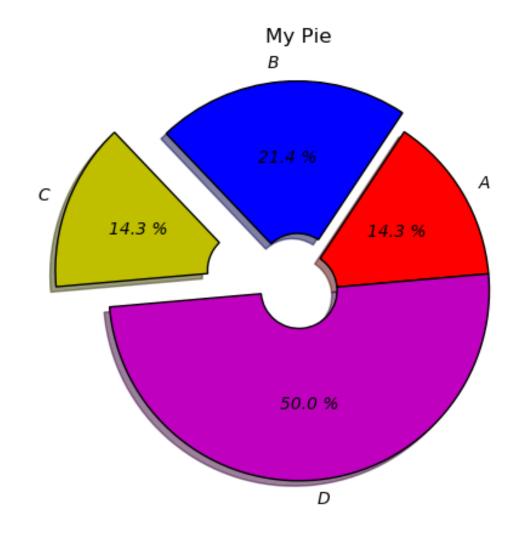
```
matplotlib-histogram.py
import matplotlib.pyplot as plt
from random import random
values1 = [random()**2 for in range(1000)]
values2 = [random()**3 for in range(100)]
bins = [0.0, 0.25, 0.5, 0.75, 1.0]
for i, ht in enumerate(
        ['bar', 'barstacked', 'step', 'stepfilled'],
        start=1):
   plt.subplot(2, 2, i) # start new plot
   plt.hist([values1, values2], # data sets
            bins,
                         # bucket boundaries
           → histtype=ht, # default ht='bar'
            rwidth=0.7, # fraction of bucket width
            label=['$x^2$', '$x^3$'], # labels
            density=True) # norm. prob. density
                         # plot title
   plt.title(ht)
   plt.xticks(bins) # ticks on x-axis
   plt.legend()
plt.suptitle('Histogram') # figure title
plt.show()
```



matplotlib.org/api/ as gen/matplotlib.pyplot.hist.html

Pie

```
matplotlib-pie.py
import matplotlib.pyplot as plt
plt.title('My Pie')
plt.pie([2, 3, 2, 7],  # relative wedge sizes
      labels=['A','B','C','D'],
      colors=['r', 'b', 'y', 'm'],
      explode=(0, 0.1, 0.3, 0), # radius fraction
      startangle=5,  # angle above horizontal
      counterclock=True, # default True
      rotatelabels=False, # default False
                  # default False
      shadow=True,
      color='black', # text color
          style='italic'), # text style
      width=0.8, # width (missing center)
          linewidth=1,  # wedge boundary width
          edgecolor='black'), # boundary color
      autopct='%1.1f %%') # percent formatting
plt.show()
```

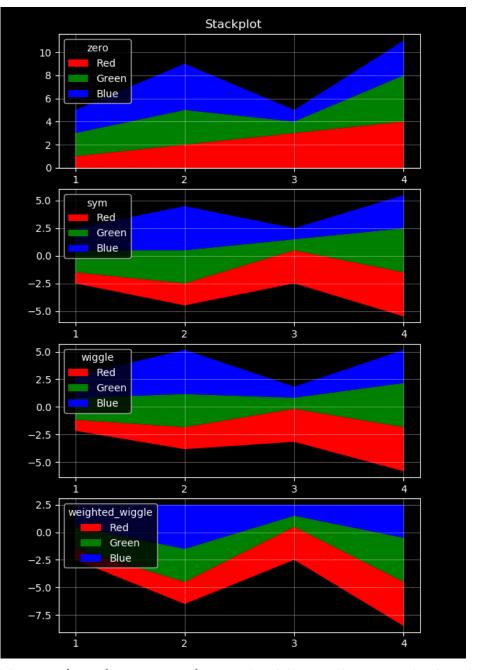


Stackplot

```
matplotlib-stackplot.py
import matplotlib.pyplot as plt
x = [1, 2, 3, 4]
y1 = [1, 2, 3, 4]
                              Stacked Graphs – Geometry & Aesthetics
y2 = [2, 3, 1, 4]
                               Lee Byron & Martin Wattenberg, 2008
y3 = [2, 4, 1, 3]
plt.style.use('dark background')
for i, base in enumerate(
        ['zero', 'sym', 'wiggle', 'weighted wiggle'],
        start=1):
    plt.subplot(4, 1, i)
    plt.stackplot(x, y1, y2, y3,
                   colors=['r', 'q', 'b'],
                   labels=['Red', 'Green', 'Blue'],
                 → baseline=base)
    plt.grid(axis='both', # 'x', 'y', or 'both'
        linewidth=0.5, linestyle='-', alpha=0.5)
    plt.legend(title=base, loc='upper left')
    plt.xticks(x) # a tick for each value in x
plt.suptitle('Stackplot')
plt.show()
```

To list all available styles:

```
print(plt.style.available)
```



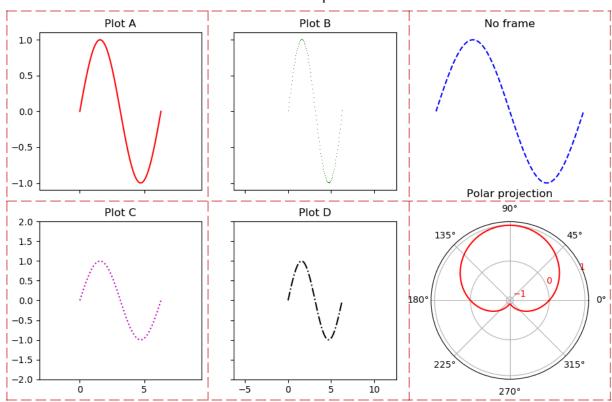
matplotlib.org/api/_as_gen/matplotlib.pyplot.stackplot.html

```
matplotlib-subplot.py
import matplotlib.pyplot as plt
from math import pi, sin
x \min, x \max, n = 0, 2 * pi, 100
x = [x min + (x max - x min) * i / n for i in range(n + 1)]
y = [\sin(v) \text{ for } v \text{ in } x]
ax1 = plt.subplot(2, 3, 1) # 2 rows, 3 columns
ax1.label outer()
                            # removes x-axis labels
plt.xlim(-pi, 3 * pi)
                            # increase x-axis range
plt.plot(x, y, 'r-')
plt.title('Plot A')
ax2 = plt.subplot(2, 3, 2)
ax2.label outer()
                            # removes x- and y-axis labels
plt.xlim(-2 * pi, 4 * pi)
                           # increase x-axis range
plt.plot(x, y, 'g,')
plt.title('Plot B')
ax3 = plt.subplot(2, 3, 3, frameon=False) # remove frame
                            # remove x-axis ticks & labels
ax3.set xticks([])
ax3.set yticks([])
                            # remove x-axis ticks & labels
plt.plot(x, y, 'b--')
plt.title('No frame')
ax4 = plt.subplot(2, 3, 4, sharex=ax1) # share x-axis range
plt.ylim(-2, 2)
                           # increase v-axis range
plt.plot(x, y, 'm:')
plt.title('Plot C')
ax5 = plt.subplot(2, 3, 5, sharex=ax2, sharey=ax4) # share ranges
ax5.set xticks(range(-5, 15, 5)) # specific x-ticks & x-labels
ax5.label outer()
                            # removes y-axis labels
plt.plot(x, y, 'k-.')
plt.title('Plot D')
ax6 = plt.subplot(2, 3, 6, projection='polar') # polar projection
ax6.set yticks([-1, 0, 1])
                                              # v-labels
ax6.tick params(axis='y', labelcolor='red') # color of y-labels
plt.plot(x, y, 'r')
plt.title('Polar projection\n') # \n to avoid overlap with 90°
plt.suptitle('2 x 3 subplots', fontsize=16)
plt.show()
```

Subplot

(2 rows, 3 columns)

2 x 3 subplots



- Subplots are numbered 1..6 row-by-row, starting top-left
- subplot returns an axes to access the plot in the figure

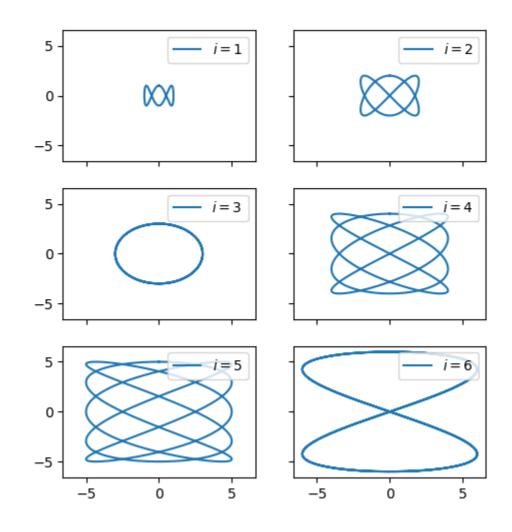
matplotlib.org/api/ as gen/matplotlib.pyplot.subplot.html

Subplots

```
matplotlib-subplots.py
import matplotlib.pyplot as plt
from math import pi, sin, cos
times = [2 * pi * t / 1000 for t in range(1001)]
fig, ((ax1, ax2), (ax3, ax4), (ax5, ax6)) = \
   plt.subplots(3, 2, sharex=True, sharey=True)
for i, ax in enumerate([ax1, ax2, ax3, ax4, ax5, ax6],
                       start=1):
    x = [i * sin(i * t) for t in times]
    y = [i * cos(3 * t) for t in times]
    ax.plot(x, y, label=f'$i = {i}$') # plot to axes
    ax.legend(loc='upper right') # axes legend
fig.suptitle('subplots', fontsize=16) # figure title
plt.show()
```

create 6 axes in 3 rows with 2 colums share the x- and y-axis ranges (automatically applies label_outer to created axes) returns a pair (figure, axes)

subplots

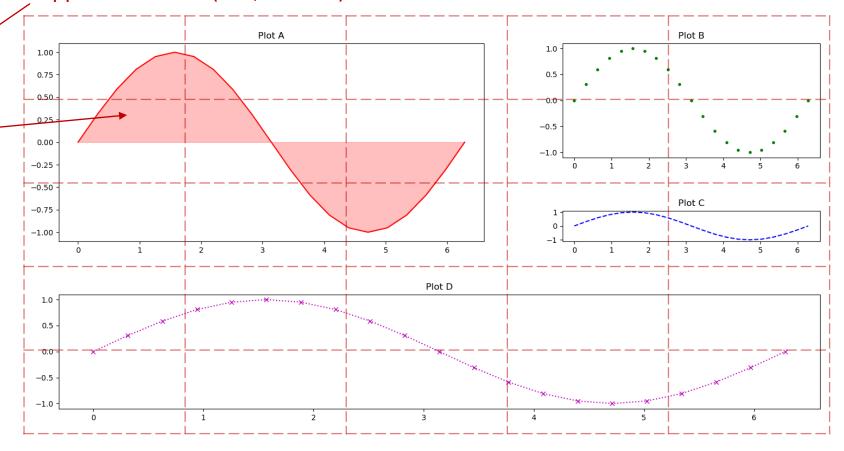


matplotlib.org/api/ as gen/matplotlib.pyplot.subplots.html

```
matplotlib-subplot2grid.py
import matplotlib.pyplot as plt
import math
x \min, x \max, n = 0, 2 * math.pi, 20
x = [x min + (x max - x min) * i / n]
     for i in range(n + 1)]
y = [math.sin(v) for v in x]
plt.subplot2grid((5, 5), (0,0),
                 rowspan=3, colspan=3)
plt.fill between(x, 0.0, y, -
                 alpha=0.25, color='r')
plt.plot(x, y, 'r-')
plt.title('Plot A')
plt.subplot2grid((5, 5), (0,3),
                 rowspan=2, colspan=2)
plt.plot(x, y, 'q.')
plt.title('Plot B')
plt.subplot2grid((5, 5), (2,3),
                 rowspan=1, colspan=2)
plt.plot(x, y, 'b--')
plt.title('Plot C')
plt.subplot2grid((5, 5), (3,0),
                 rowspan=2, colspan=5)
plt.plot(x, y, 'mx:')
plt.title('Plot D')
plt.tight layout() # adjust padding
plt.show()
```

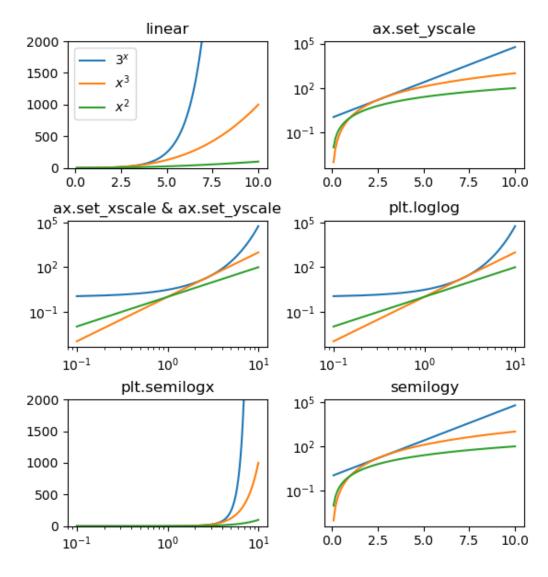
subplot2grid (5 x 5)

upper left corner (row, column)



matplotlib-log.py import matplotlib.pyplot as plt x = [i / 10 for i in range(1, 101)]v1 = [i ** 2 for i in x]y2 = [i ** 3 for i in x]y3 = [3 ** i for i in x]for i in range (1, 7): ax = plt.subplot(3, 2, i)plt.plot(x, y3, label='\$3^x\$') $plt.plot(x, y2, label='$x^3$')$ $plt.plot(x, y1, label='$x^2$')$ if i == 1: plt.ylim(0, 2000) plt.xscale('linear') # default plt.yscale('linear') # default plt.legend() plt.title('linear') if i == 2: plt.yscale('log') plt.title('ax.yscale') if i == 3: ax.set xscale('log') ax.set yscale('log') plt.title('ax.set xscale & ax.set yscale') if i == 4: plt.loglog() plt.title('plt.loglog') if i == 5: plt.ylim(0, 2000) plt.semilogx() plt.title('plt.semilogx') if i == 6: plt.semilogy() plt.title('semilogy') plt.show()

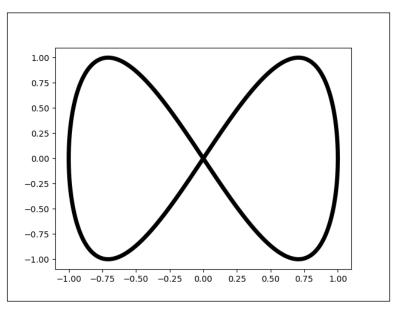
log scales



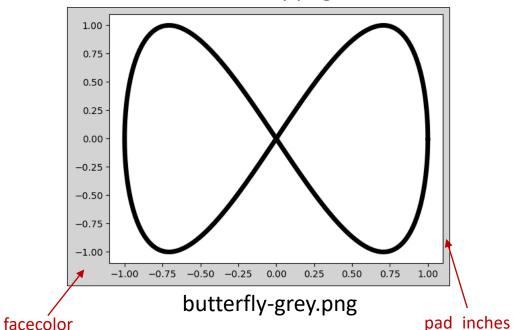
■ There are many ways to make the x- and/or y-axis logarithmic with pyplot

Saving figures

```
matplotlib-savefig.py
import matplotlib.pyplot as plt
from math import pi, sin, cos
n = 1000
points = [(\cos(2 * pi * i / n),
           sin(4 * pi * i / n)) for i in range(n)]
x, y = zip(*points)
plt.plot(x, y, 'k-', linewidth=5)
plt.savefig('butterfly.png') # save plot as PNG
plt.savefig('butterfly-grey.png',
    dpi=100,
                              # dots per inch
    bbox inches='tight',  # crop to bounding box
    pad inches=0.1,
                             # space around figure
    facecolor='lightgrey',
                             # background color
    format='png')
                             # optional if file extension
plt.savefig('butterfly.pdf') # save plot as PDF
                              # interactive viewer
plt.show()
```

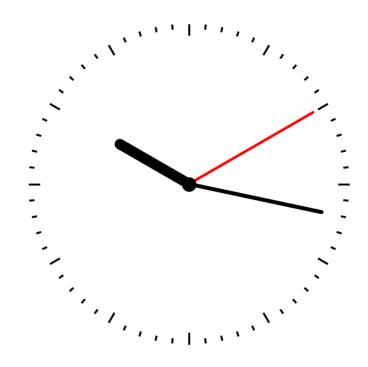


butterfly.png



A crude animation

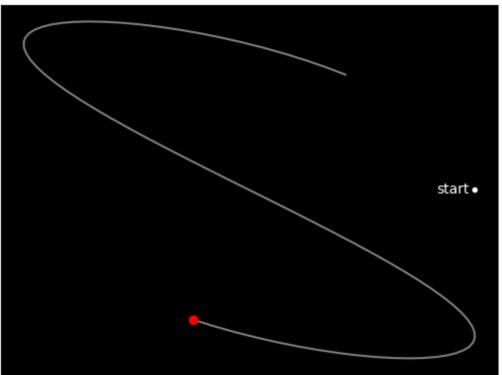
```
clock.py
import matplotlib.pyplot as plt
from math import pi, sin, cos
import datetime
def plot clock(hour, minute, second):
   plt.axis('off')
                                # hide x and y axes
   plt.gca().set aspect('equal') # don't squeeze circle
   for i in range(60): # show second marks
       angle = 2 * pi * i / 60
       x, y = cos(angle), sin(angle)
       start = 0.98 if i % 5 else .94 # every 5'th mark should be longer
       plt.plot([start * x, x], [start * y, y], c='black') # mark
   for angle, length, style in [
        (second / 60, .90, dict(c='red', lw=2, solid capstyle='round')),
        (minute / 60, .85, dict(c='black', lw=3, solid capstyle='round')),
        (hour / 12, .50, dict(c='black', lw=8, solid capstyle='round'))
   1:
       angle = 2 * pi * (0.25 - angle)
       x, y = length * cos(angle), length * sin(angle)
       plt.plot([0, x], [0, y], **style) # clock arm
   plt.plot(0, 0, 'o', ms=10, c='black') # center dot
while True:
   now = datetime.datetime.now() # UTZ
   plot clock(now.hour, now.minute, now.second)
   plt.pause(1) # show figure and pause 1 second
   plt.clf() # clear figure
```



```
matplotlib-animation.py
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from math import pi, cos, sin
n, tail length = 200, 75
points = []
                           # tail length recent poins
def point(i):
    t = 2 * pi * i / n
    return (\cos(3 * t), \sin(2 * t))
fig = plt.figure()
                           # new figure
ax = plt.gca()
                           # get current axes
ax.set facecolor('black') # set background color
plt.xlim(-1.1, 1.1)
                           # set x-axis range
plt.ylim(-1.1, 1.1)
                           # set y-axis range
plt.xticks([])
                           # remove x-ticks & labels
plt.yticks([])
                           # remove y-ticks & labels
plt.title('Moving point')
                           # plot title
x, y = point(0)
plt.plot(x, y, 'w.')
                          # start point
plt.text(x - 0.025, y, 'start', color='w', # text label
   ha='right', va='center')
                                           # alignment
tail, = plt.plot([], [], 'w-', alpha=0.5) # init. tail
head, = plt.plot([], [], 'ro') # init. current point
                      # frame = value from frames
def move(frame):
    points.append(point(frame))
    del points[:-tail length]
                                  # limit tail
    tail.set data(*zip(*points)) # update tail points
   head.set data(*points[-1])
                                  # update head point
animation = FuncAnimation(fig,
                                # figure to animate -
                      # function called for each frame
    func=move,
    frames=range(n), # array like to iterate over
                      # milliseconds between frames
   interval=25,
   repeat=True,
                      # repeat frames when done
    repeat delay=0)
                      # wait milliseconds before repeat
plt.show()
```

matplotlib.animation.FuncAnimation

Moving point



plot returns "Line2D" objects representing the plotted data "Line2D" objects can be updated using set_data

To make an animation you need to repeatedly update the "line2D" objects **FuncAnimation** repeatedly calls **func** in regular intervals **interval**, each time with the next value from **frames** (if frames is None, then the frame values provided to func will be the infinite sequence 0,1,2,3,...)

matplotlib.org/api/ as gen/matplotlib.animation.FuncAnimation.html

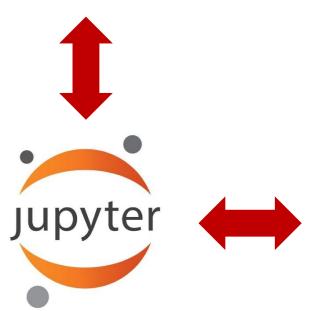


The Jupyter Notebook

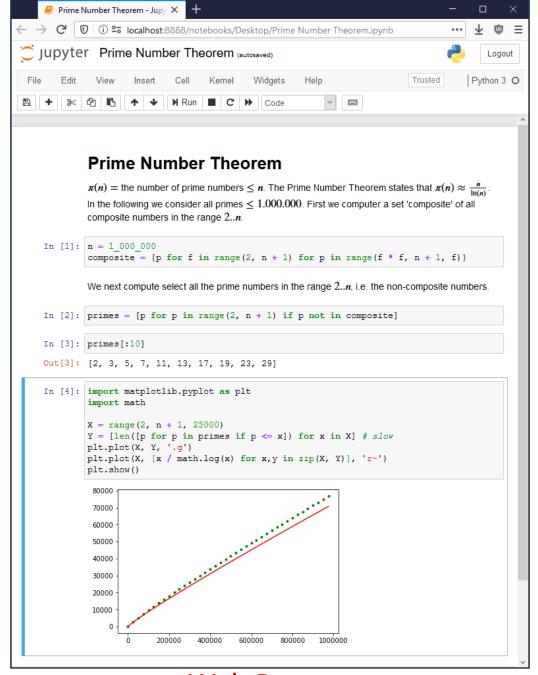
The Jupyter Notebook is an open-source web application that allows you to create and share documents that contain live code, equations, visualizations and narrative text. Uses include: data cleaning and transformation, numerical simulation, statistical modeling, data visualization, machine learning, and much more.

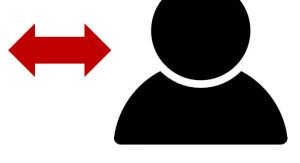






Jupyter Server (e.g. running on local machine)



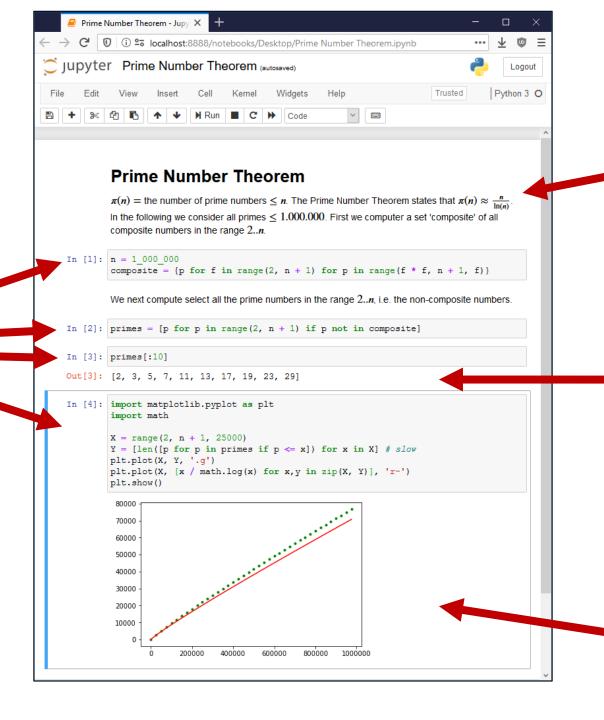


User

Web Browser

cells

python code



formatted text:
Markdown /
LaTeX / HTML /

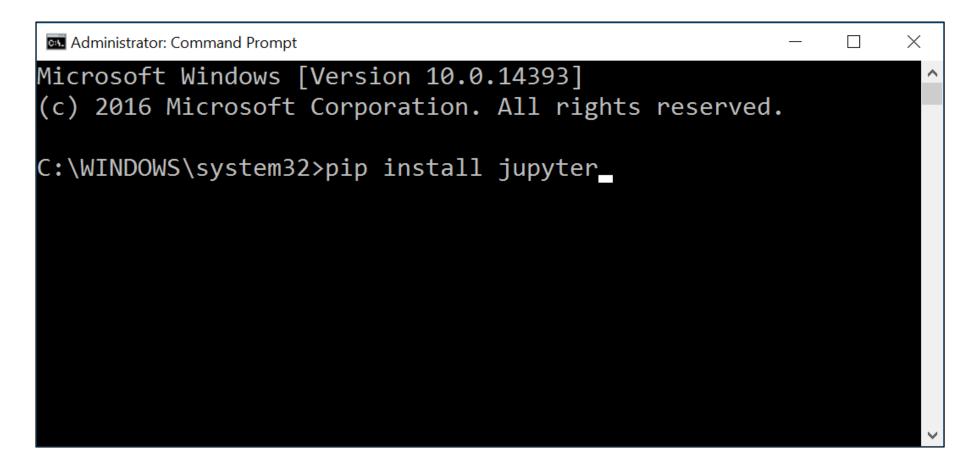
• • •

python shell output

matplotlib / numpy / ... output

Jupyter - installing

Open a windows shell and run: pip install jupyter

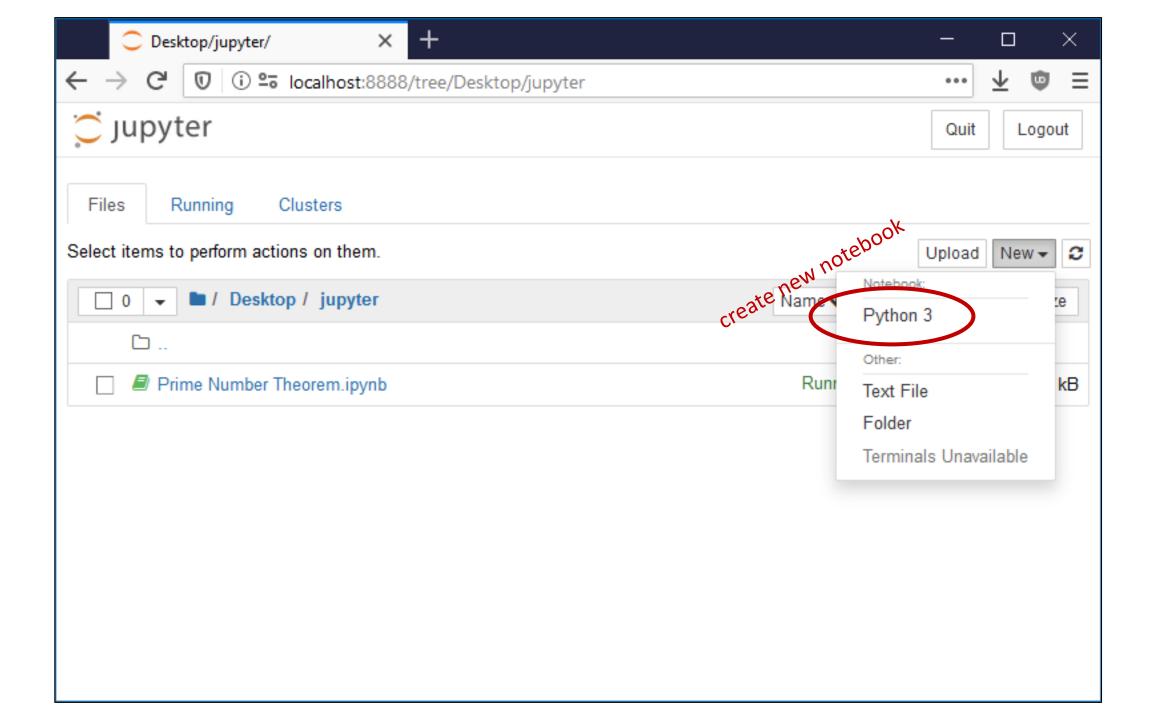


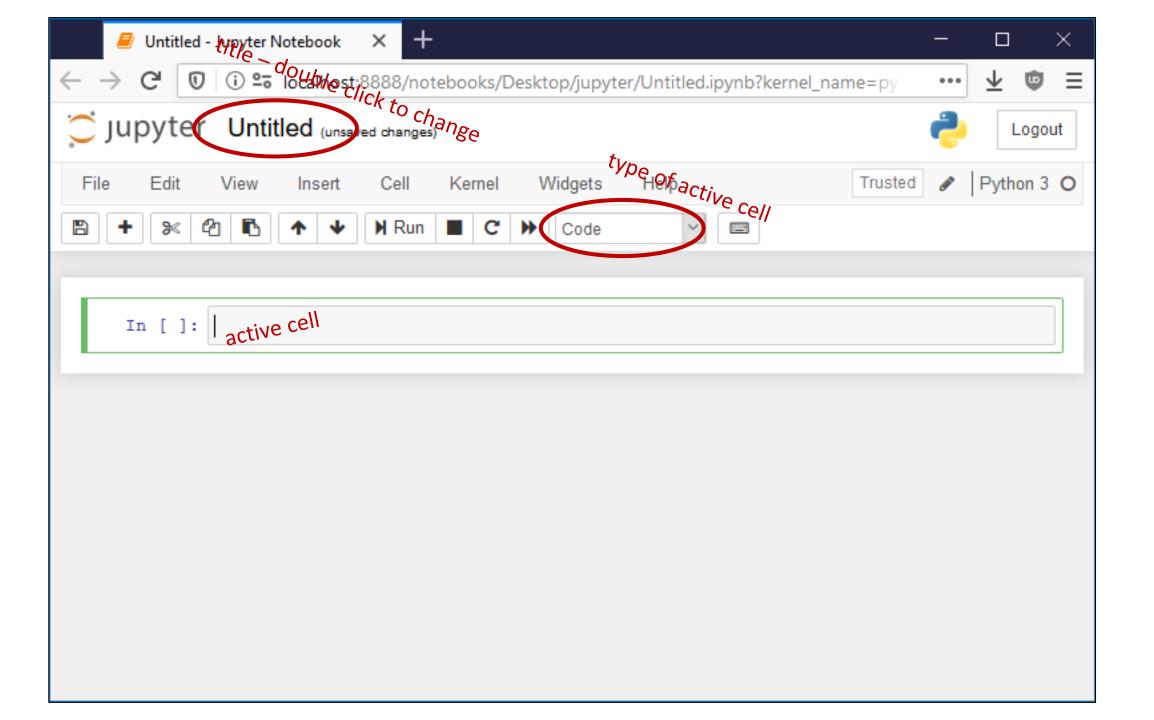
Jupyter – launching the jupyter server

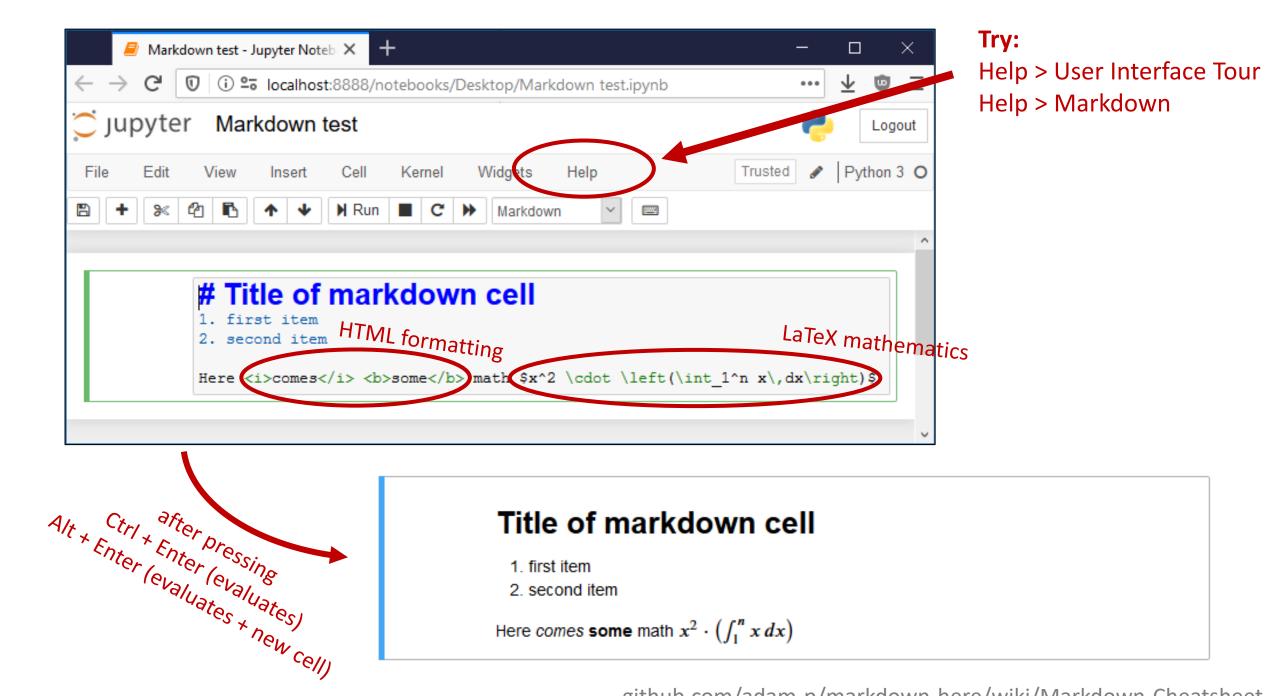
Open a windows shell and run: jupyter notebook

```
Command Prompt - jupyter notebook
                                                                                         Microsoft Windows [Version 10.0.14393]
(c) 2016 Microsoft Corporation. All rights reserved.
C:\Users\au121>jupyter notebook
[I 17:36:08.584 NotebookApp] Serving notebooks from local directory: C:\Users\au121
[I 17:36:08.584 NotebookApp] 0 active kernels
[I 17:36:08.584 NotebookApp] The Jupyter Notebook is running at:
[I 17:36:08.584 NotebookApp] http://localhost:8888/?token=18644cfeedab986bbdcc068f61c21b06b62
548a090169463
[I 17:36:08.584 NotebookApp] Use Control-C to stop this server and shut down all kernels (twi
ce to skip confirmation).
[C 17:36:08.584 NotebookApp]
    Copy/paste this URL into your browser when you connect for the first time,
    to login with a token:
                                                                                                      url
        http://localhost:8888/?token=18644cfeedab986bbdcc068f61c21b06b62548a090169463
[I 17:36:09.154 NotebookApp] Accepting one-time-token-authenticated connection from ::1
```

If this does not work, then try python -m notebook





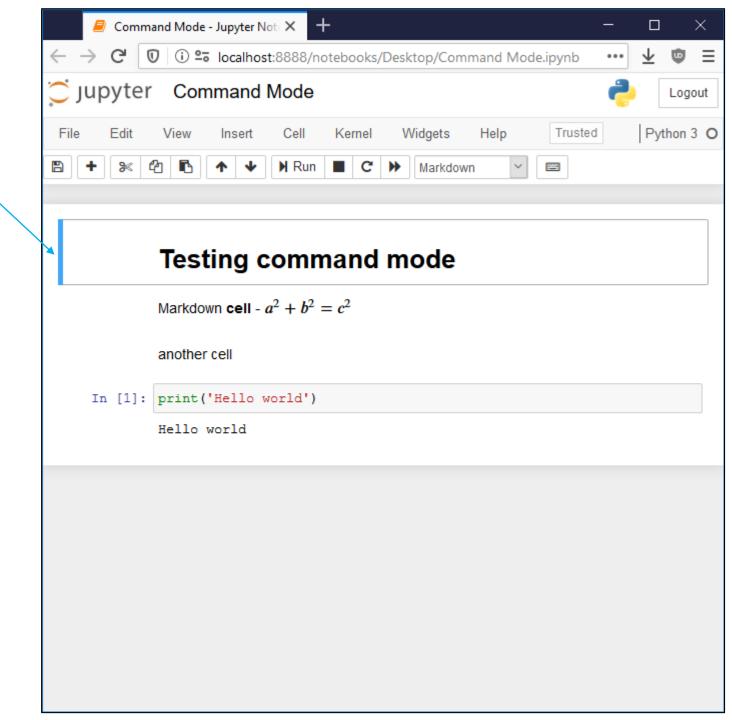


Here comes some math $x^2 \cdot \left(\int_1^n x \, dx \right)$

Command Mode

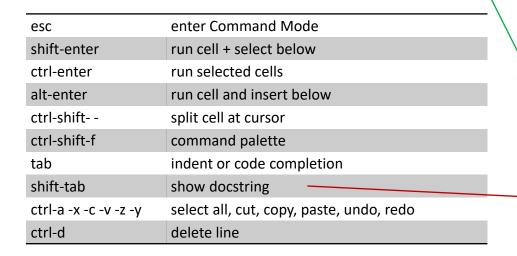
- Used to naviagte between cells
- Current cell is marked with blue bar.
- Keyboard shortcuts

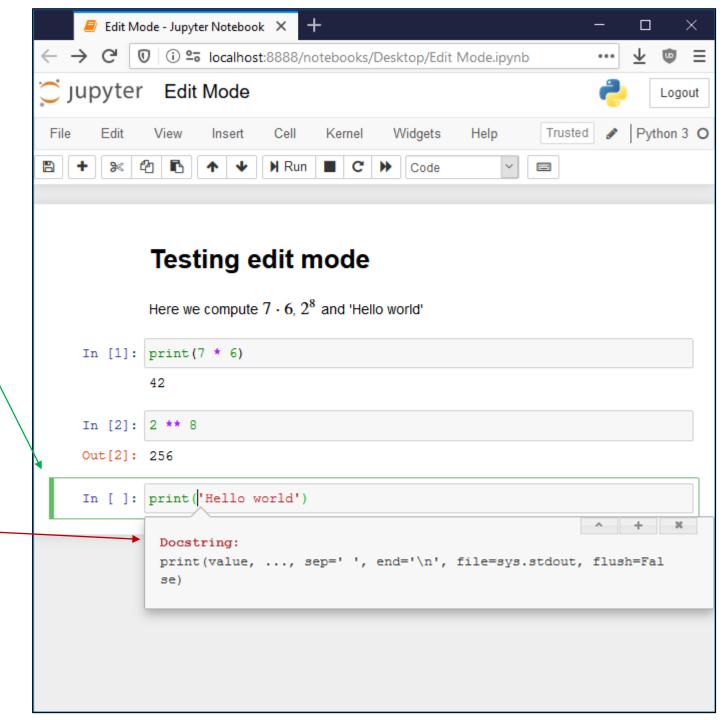
| h | show keyboard shortcuts |
|---------------------|--|
| enter | enter Edit Mode on current cell |
| shift-enter | run cell + select below |
| ctrl-enter | run selected cells |
| alt-enter | run cell and insert below |
| YMR | change cell type (code, markdown, raw text) |
| 123456 | change heading level |
| ctrl-A | select all cells |
| down up | move to next/previous cell |
| space shift-space | scroll down/up |
| shift-up shift-down | extend selected cells |
| A B | insert cell above/below |
| X C V shift-V Z DD | cut, copy, paste below/above, undo, delete cells |
| shift-L | toggle line numbers in cells |
| shift-M | merge selected cells (or with cell below) |
| 0 | toggle output of selected cells |
| shift-O | toggle scrollbar on selected cells (long output) |



Edit Mode

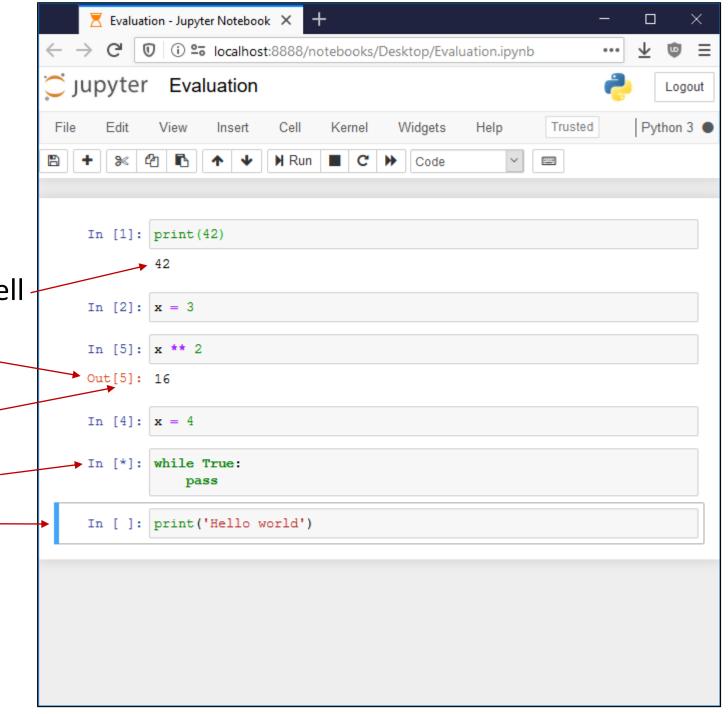
- Used to edit current cell
- Current cell is marked with green bar
- Keyboard shortcuts





Evaluating cells

- To evaluate cell ctrl-enter, alt-enter, shift-enter
- Output from program shown below cell
- Result of last evaluated line —
- Order of code cells evaluated
 Note "x ** 2" computed after "x = 4"
- [*] are cells being evaluated / waiting.
- [] not yet evaluated
- Recompute all cells top-down
 - → or Kernel > Restart & Run all



Magic lines

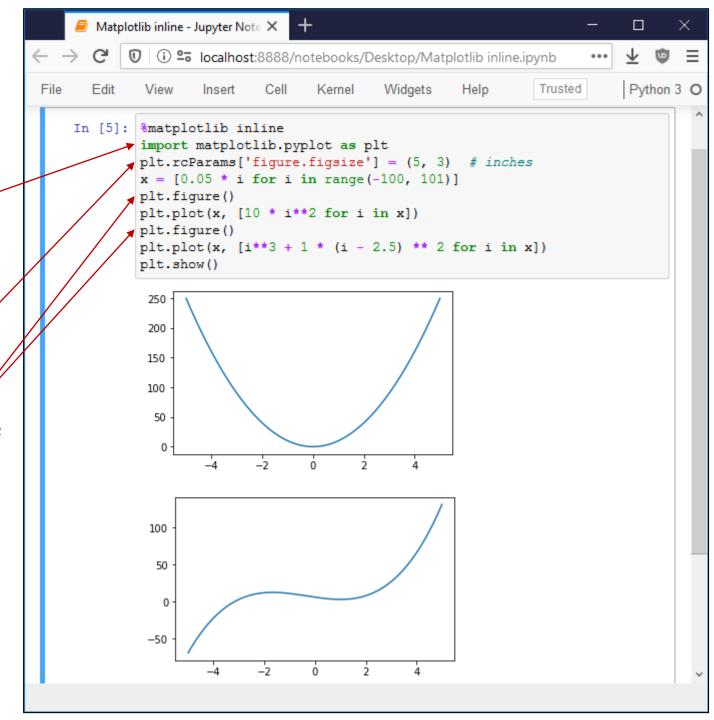
- Jupyter code cells support magic commands (actually it is IPython)
- % is a line magic
- % is a cell magic

| %lsmagic | list magic commands |
|-------------------------|--|
| %quickref | quick reference sheet to IPython |
| %pwd | print working directory (current folder) |
| %cd directory | change directory (absolut or relative) |
| %ls | list content of current directory |
| %pip or %conda | run pip or conda from jupyter |
| %load script | insert external script into cell |
| %run <i>program</i> | run external program and show output |
| %automagic | toggle if %-prefix is required |
| %matplotlib inline | no zoom & resize, allows multiple plots |
| %matplotlib notebook | a single plot can be zoomed & resized |
| %%writefile <i>file</i> | write content of cell to a file |
| %%time | measure time for cell execution |
| %timeit expression | time for simple expression |

```
Magic lines - Jupyter Notebook X
          (i) % localhost:8888/notebooks/Desktop/Magic lines.ipynb
                               Kernel
                                        Widgets
                                                  Help
                                                            Trusted
                                                                        Python 3 O
         View
                Insert
In [1]: %pwd
       'C:\\Users\\au121\\Desktop'
   [2]: *cd my folder
        C:\Users\au121\Desktop\my folder
In [3]: %ls
         Volume in drive C is OSDisk
         Volume Serial Number is 3CDB-90D8
         Directory of C:\Users\au121\Desktop\my folder
        26-03-2020 14:11
                              <DIR>
        26-03-2020 14:11
                              <DIR>
        25-03-2020 14:57
                                           24 my document.txt
                        1 File(s)
                                               24 bytes
                        2 Dir(s) 382.033.829.888 bytes free
In [4]: open('my document.txt').readlines()
Out[4]: ['Document INSIDE folder\n']
In [5] %%time
        for x in range (1000000):
             s += x ** 2
        Wall time: 492 ms
```

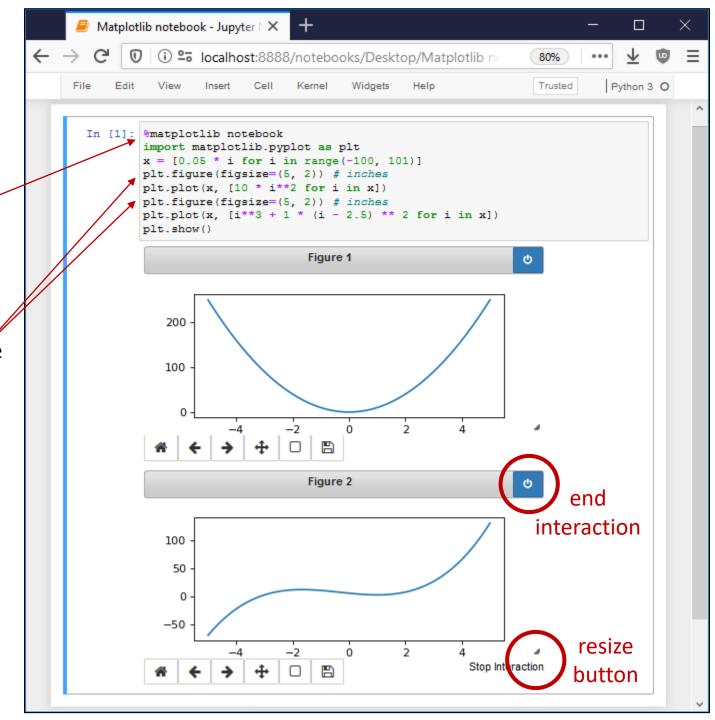
Jupyter and matplotlib

- %matplotlib inline pyplot figures are shown without interactive zoom and pan (default)
- Consider changing default figure size plt.rcParams['figure.figsize']
- Start each figure with plt.figure
- Final call to show can be omitted



Jupyter and matplotlib

- %matplotlib notebook pyplot figures are shown with interactive zoom and pan
- Start each figure with plt.figure
 (also allows setting figure size)
- Final call to show can be omitted

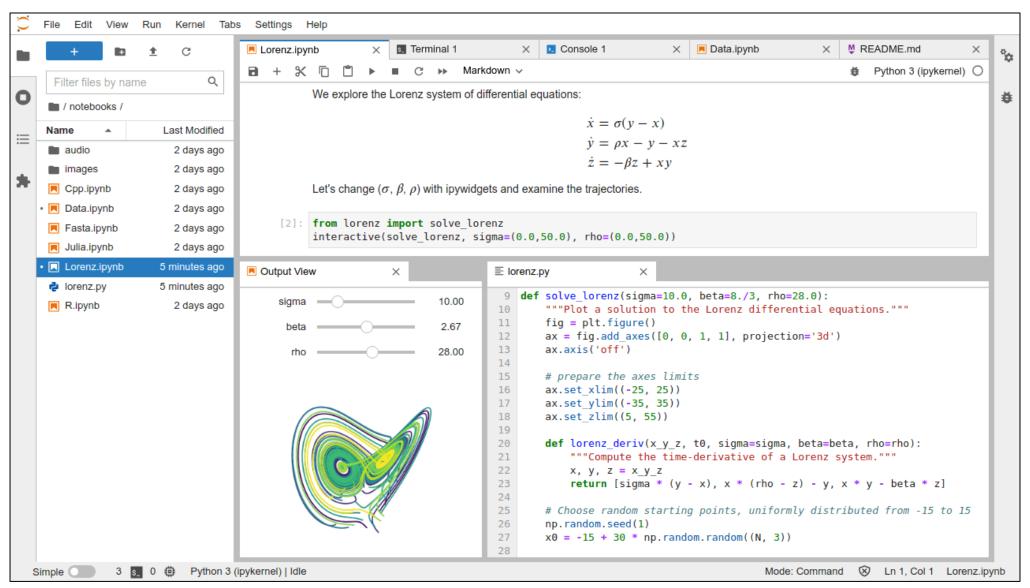




- Widespread tool used for data science applications
- Documentation, code for data analysis, and resulting visualizations are stored in one common format
- Easy to update visualizations
- Works with about 100 different programming languages (not only Python 3), many special features,
- Easy to share data analysis

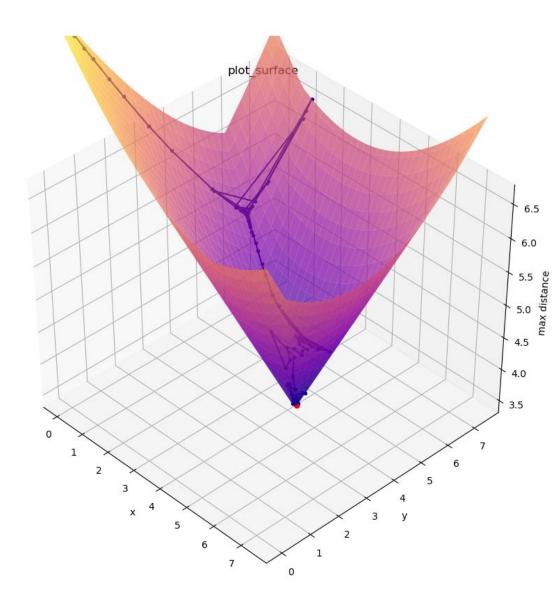
Many online tutorials and examples are available

JupyterLab: A Next-Generation Notebook Interface

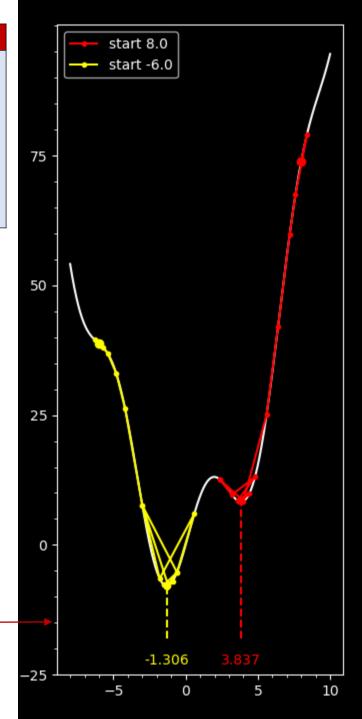


scipy.optimize.minimize

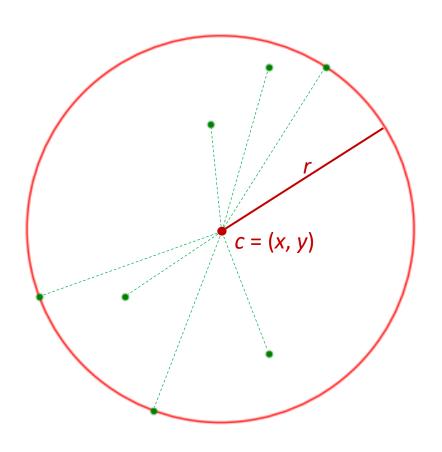
- Find point p minimizing function f
- Supports 13 algorithms but no guarantee that result is correct
- Knowledge about optimization will help you know what optimization algorithm to select and what parameters to provide for better results
- WARNING
 Many solvers return the wrong value when used as a black box



```
Python shell
minimize.py
                                          print(solution)
                                          final simplex: (array([[-1.3064209],
from math import sin
                                            [-1.30649414]]), array([-7.94582337, -7.94582336]))
import matplotlib.pyplot as plt
                                                   fun: -7.94582337348758
from scipy.optimize import minimize
                                               message: 'Optimization terminated successfully.'
                                                  nfev: 38
trace = [] # remember calls to f
                                                   nit: 19
                                                status: 0
def f(x):
                                               success: True
    value = x ** 2 + 10 * sin(x)
                                                    x: array([-1.3064209])
    trace.append((x, value))
    return value
X = [-8 + 18 * i / 9999  for i in range (1000)]
Y = [f(x) \text{ for } x \text{ in } X]
plt.style.use('dark background')
plt.plot(X, Y, 'w-')
for start, color in \[(8, 'red'), (-6, 'yellow')]:
    trace = []
    solution = minimize(f, [start], method='nelder-mead')
    x, y = solution.x[0], solution.fun
    plt.plot(*zip(*trace), '.-', c=color, label=f'start {start:.1f}') # trace
    plt.plot(*trace[0], 'o', c=color)
                                                            # first trace point
    plt.text(x, -23, f'{x:.3f}', c=color, ha='center') # show minimum x
    plt.plot([x, x], [-18, y], '--', c=color)
                                                            # dash to minimum
plt.xticks(range(-5, 15, 5))
plt.yticks(range(-25, 100, 25))
plt.minorticks on()
plt.legend()
                           minimize tries to find a local minimum for f
plt.show()
                           by repeatedly evaluating f for different x values
```



Example: Minimum enclosing circle



- Find c such that $r = \max_{p} |p c|$ is minimized
- A solution is characterized by either
 - 1) three points on circle, where the triangle contains the circle center
 - 2) two opposite points on diagonal
- Try a standard numeric minimization solver
- Computation involves max and \sqrt{x} , which can be hard for numeric optimization solvers

Python/scipy vs MATLAB

Some basic differences

- "end" closes a MATLAB block
- ";" at end of command avoids command output
- a(i) instead a[i]
- 1st element of a list a(1)
- a(i:j) includes both a(i) and a(j)

like R, Mathematica, Julia, AWK, Smalltalk, ...





Quickstart About Python

Why Python Python 3 Python vs Matlab Speed For whom

About Pyzo Guide Learn

Python vs Matlab

We regularly hear of people (and whole research groups) that transition from Matlab to Python. The scientific Python ecosystem is maturing fast and Python is an appealing alternative, because it's free, open source, and becoming ever more powerful. This page tries to explain the differences between these two tools.

Matlab and Python and their ecosystems

Python, by definition, is a programming language. The most common implementation is that in C (also known as <u>CPython</u>) and is what is mostly refered to as "Python". Apart from the programming language and interpreter, Python also consists of an extensive standard library. This library is aimed at programming in general and contains modules for os specific stuff, threading, networking, databases, etc.

Matlab is a commercial numerical computing environment and programming language. The concept of Matlab refers to the whole package, including the IDE. The standard library does not contain as much generic programming functionality, but does include matrix algebra and an extensive library for data processing and plotting. For extra functionality the Mathworks provides toolkits (but these cose you extra).

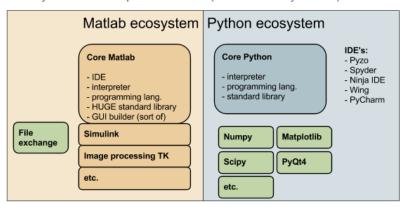
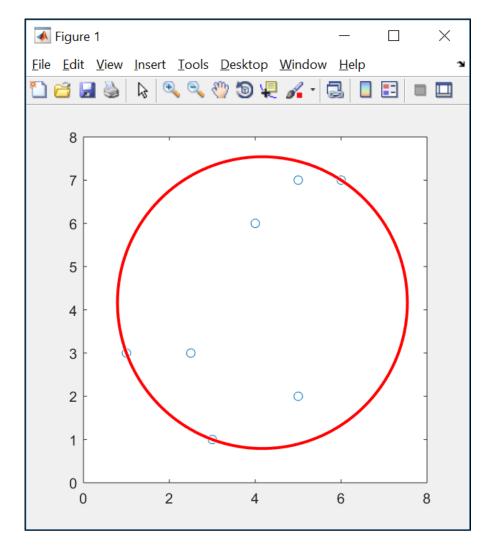


Diagram illustrating the differences between Python and Matlab in terms of their ecosystem.

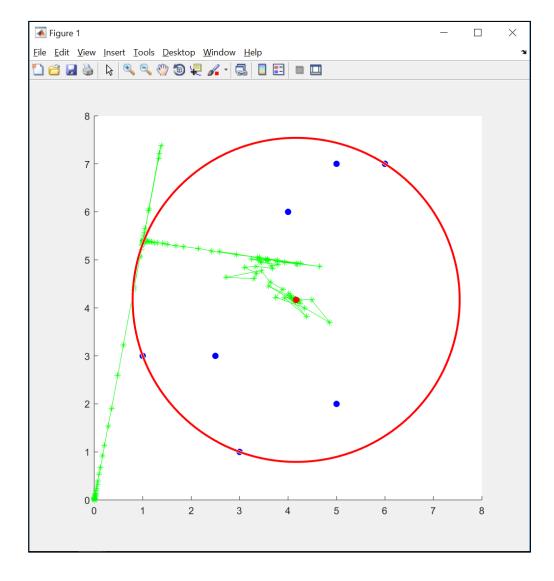
Minimum enclosing circle in MATLAB

```
enclosing circle.m
% Minimum enclosing circle of a point set
% fminsearch uses the Nelder-Mead algorithm
global x y
x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0];
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0];
c = fminsearch(@(x) max distance(x), [0,0]);
plot(x, y, "o");
viscircles(c, max distance(c));
function dist = max distance(p)
    global x y
    dist = 0.0;
    for i=1:length(x)
        dist = max(dist, pdist([p; x(i), y(i)],
                                'euclidean'));
    end
end
```



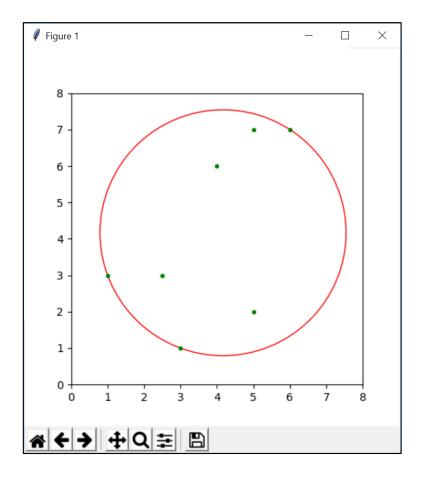
Minimum enclosing circle in MATLAB (trace)

```
enclosing circle trace.m
global x y trace x trace y
x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0];
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0];
trace x = [];
trace y = [];
c = fminsearch(@(x) max distance(x), [0,0]);
hold on
plot(x, y, "o", 'color', 'b', 'MarkerFaceColor', 'b');
plot(trace x, trace y, "*-", "color", "g");
plot(c(1), c(2), "o", 'color', 'r', 'MarkerFaceColor', 'r');
viscircles(c, max distance(c), "color", "red");
function dist = max distance(p)
    global x y trace x trace y
    trace x = [trace x, p(1)];
    trace y = [trace y, p(2)];
    dist = 0.0:
    for i=1:length(x)
        dist = max(dist, pdist([p; x(i), y(i)], 'euclidean' ));
    end
end
```



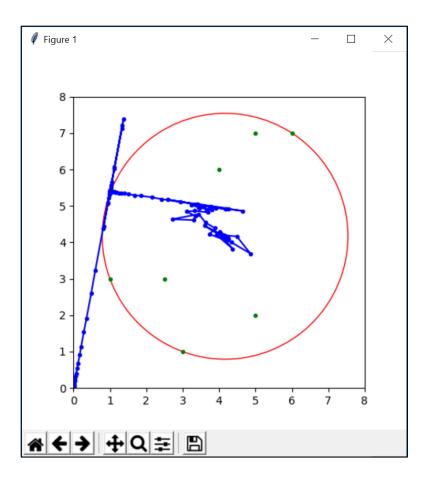
Minimum enclosing circle in Python

```
enclosing circle.py
from scipy.optimize import minimize
                                        import modules
import matplotlib.pyplot as plt
x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0]
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0]
def dist(p, q):
    return ((p[0] - q[0]) ** 2 + (p[1] - q[1]) ** 2)) ** 0.5
def max distance(c):
    return max(dist(p, c) for p in zip(x, y))
c = minimize(max distance, [0.0, 0.0], method="nelder-mead").x
ax = plt.qca()
                                             optimization method
ax.set xlim((0, 8))
ax.set ylim((0, 8))
                        manually set axis (force circle inside plot)
ax.set aspect("equal")
plt.plot(x, y, "g.")
ax.add artist(plt.Circle(c, max distance(c),
                          color="r", fill=False))
plt.show()
```



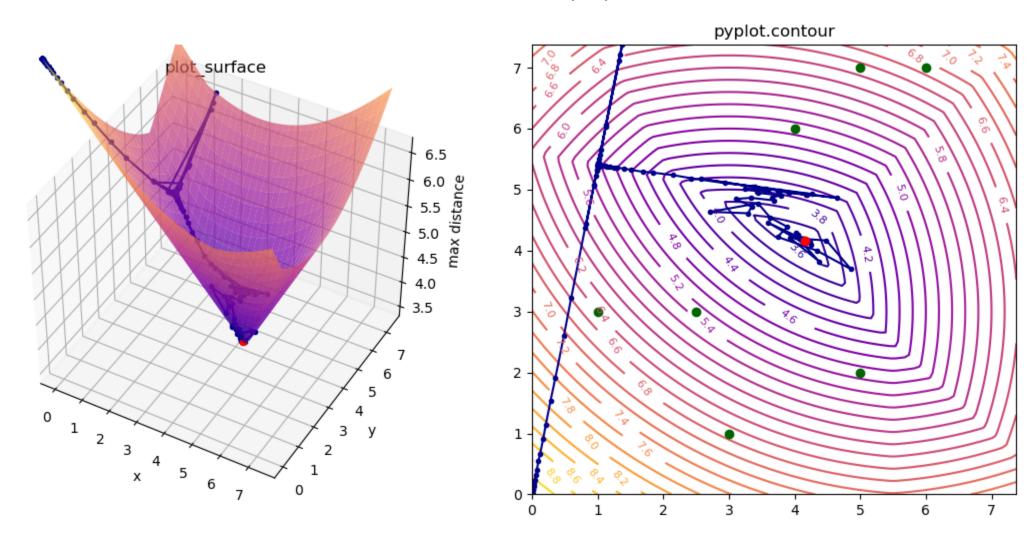
Minimum enclosing circle in Python (trace)

```
enclosing circle trace.py
from scipy.optimize import minimize
import matplotlib.pyplot as plt
x = [1.0, 3.0, 2.5, 4.0, 5.0, 6.0, 5.0]
y = [3.0, 1.0, 3.0, 6.0, 7.0, 7.0, 2.0]
trace = []
def dist(p, q):
    return ((p[0] - q[0]) ** 2 + (p[1] - q[1]) ** 2) ** 0.5
def max distance(c):
    trace.append(c)
    return max([dist(p, c) for p in zip(x, y)])
c = minimize(max distance, [0.0, 0.0],
             method="nelder-mead").x
ax = plt.qca()
ax.set xlim((0, 8))
ax.set ylim((0, 8))
ax.set aspect("equal")
plt.plot(x, y, "g.")
plt.plot(*zip(*trace), "b.-")
ax.add artist(plt.Circle(c, max distance(c),
                         color="r", fill=False))
plt.show()
```



Minimum enclosing circle – search space

Maximum distance to an input point



enclosing_circle_search_space.py (previous slide)

```
from scipy.optimize import minimize
import matplotlib.pyplot as plt
import numpy as np
from mpl toolkits.mplot3d import Axes3D
points = [(1.0, 3.0), (3.0, 1.0), (2.5, 3.0),
  (4.0, 6.0), (5.0, 7.0), (6.0, 7.0), (5.0, 2.0)
# Minimum enclosing circle solver
trace = []
def distance(p, q):
    return ((p[0]-q[0])**2 + (p[1]-q[1])**2)**0.5
def distance max(q):
    dist = max(distance(p, q) for p in points)
    trace.append((*q, dist))
    return dist
solution = minimize(distance max, [0.0, 0.0],
                    method='nelder-mead')
center = solution.x
radius = solution.fun
# unzip point coordinates
points x, points y = zip(*points)
trace x, trace y, trace z = zip(*trace)
# Bounding box [x min, x max] x [y min, y max]
xs, ys = points x + trace x, points y + trace y
x \min, x \max = \min(xs), \max(xs)
y \min, y \max = \min(ys), \max(ys)
# enforce apsect ratio
x max = max(x max, x min + y max - y min)
y max = max(y max, y min + x max - x min)
```

```
# Minimum enclosing circle - 3D surface plot
 (plot surface requires X, Y, Z are 2D numpy.arrays)
X, Y = np.meshgrid(np.linspace / x min, x max, 100),
                   np.linspace(y min, y max, 100))
Z = np.zeros(X.shape)
for px, py in points:
    Z = np.maximum(Z, (X -/px)**2 + (Y - py)**2)
Z = np.sqrt(Z)
ax = plt.subplot(1, 2, 1/2, projection='3d')
ax.plot surface(X, Y, Z, cmap='plasma', alpha=0.7)
ax.plot(trace x, trace y, trace z, '.-', c='darkblue')
ax.scatter(*center, radius, 'o', c='red')
ax.set xlabel('x')
ax.set ylabel('y')
ax.set zlabel('max distance')
ax.set title('plot surface')
# Minimum enclosing circle - contour plot
plt.subplot(1, 2, 2)
plt.title('pyplot.contour')
plt.plot(trace x, trace y, '.-', color='darkblue')
plt.plot(points x, points y, 'o', color='darkgreen')
plt.plot(*center, 'o', c='red')
qcs = plt.contour(X, Y, Z, levels=30, cmap='plasma')
plt.clabel(gcs, inline=1, fontsize=8, fmt='%.1f')
plt.suptitle('Maximum distance to an input point')
plt.tight layout()
plt.show()
```

numpy arrays

scipy.minimize $f(c) = \max_{n} |p - c|$

