Part 1 (DLT)

$$x = P X$$

x → pixel coordinate

X → world coordinate

We already know some world coordinates and their corresponding pixel coordinates. Now, the task is to calculate intrinsic & extrinsic parameters.

$$P = K R [I_3 | -X_0] X$$

K → calibration matrix

R → Rotation matrix

X_0 → shift vector

Total unknowns = 3 translations
+ 3 rotations
+ 5 calibration

= 11

Since each world → image mapping gives us 2 equations (one for x & one for y), we need atleast $\frac{11}{2} = 5.5$, ie, 6 mappings

Now,

$$x_i = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} X_i \qquad \begin{array}{l} \to A^T \\ \to B^T \\ \to C^T \end{array}$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} A^T X_i \\ B^T X_i \\ C^T X_i \end{bmatrix}$$

$$x_i = \frac{u_i}{w_i} = \frac{A^T X_i}{C^T X_i}$$

$$y_i = \frac{v_i}{w_i} = \frac{B^T X_i}{C^T X_i}$$

OR

$$-X_i^T A \qquad\qquad + x_i X_i^T C = 0$$

$$- X_i^T B \quad + y_i X_i^T C = 0$$

Now, let $p = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$

$$\Rightarrow a_{x_i}^T p = 0$$

$$\& \quad a_{y_i}^T p = 0$$

where,

$$a_{x_i}^T = \left(-X_i^T , 0^T , x_i X_i^T \right)$$

$$a_{y_i}^T = \left(0^T , -X_i^T , y_i X_i^T \right)$$

So we need to solve:

$$a_{x_i}^T p = 0$$

$$a_{y_i}^T p = 0$$

where $i = 1, 2, \dots, 6$ (or more)

$$\Rightarrow \underbrace{\begin{bmatrix} a_{x_1}^T \\ a_{y_1}^T \\ a_{x_2}^T \\ a_{y_2}^T \\ \vdots \\ \vdots \end{bmatrix}}_{M} p = 0$$

Now, solve this using SVD.

In practice, the RHS comes close to 0, so we minime the squared error to find perfect p.

Part 2 (when DLT fail)

Obviously atleast 6 points are needed.

Other than this, if all the points lie on a single plane, rank deficiency will occur on M so we will not get any solution.

Also, it will fail if all those points and X_0 (projection center) lie on a twisted cubic curve.